Learning quark-gluon plasma properties with neural ODEs & QCD Phase diagram

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w/. Rong-Gen Cai, Li Li, Yuan-Xu Wang, Phys.Rev.D 106 (2022) 12, L121902,
w/. Rong-Gen Cai, Li Li, Hongan Zeng, Sci. China Phys. Mech. Astron. 69 (2026) 4, 24 0414, 2406.12772[hep-th].
w/. Chengwei Tong, Jie Zhou, Yuanxu Wang, Rong-Gen Cai, Li Li, 2605.23834[hep-th]
w/. Chris Lau, Chengwei Tong, …Working In Progress.

Outline

I. Motivations

II. Equation of States (EoS) data driven

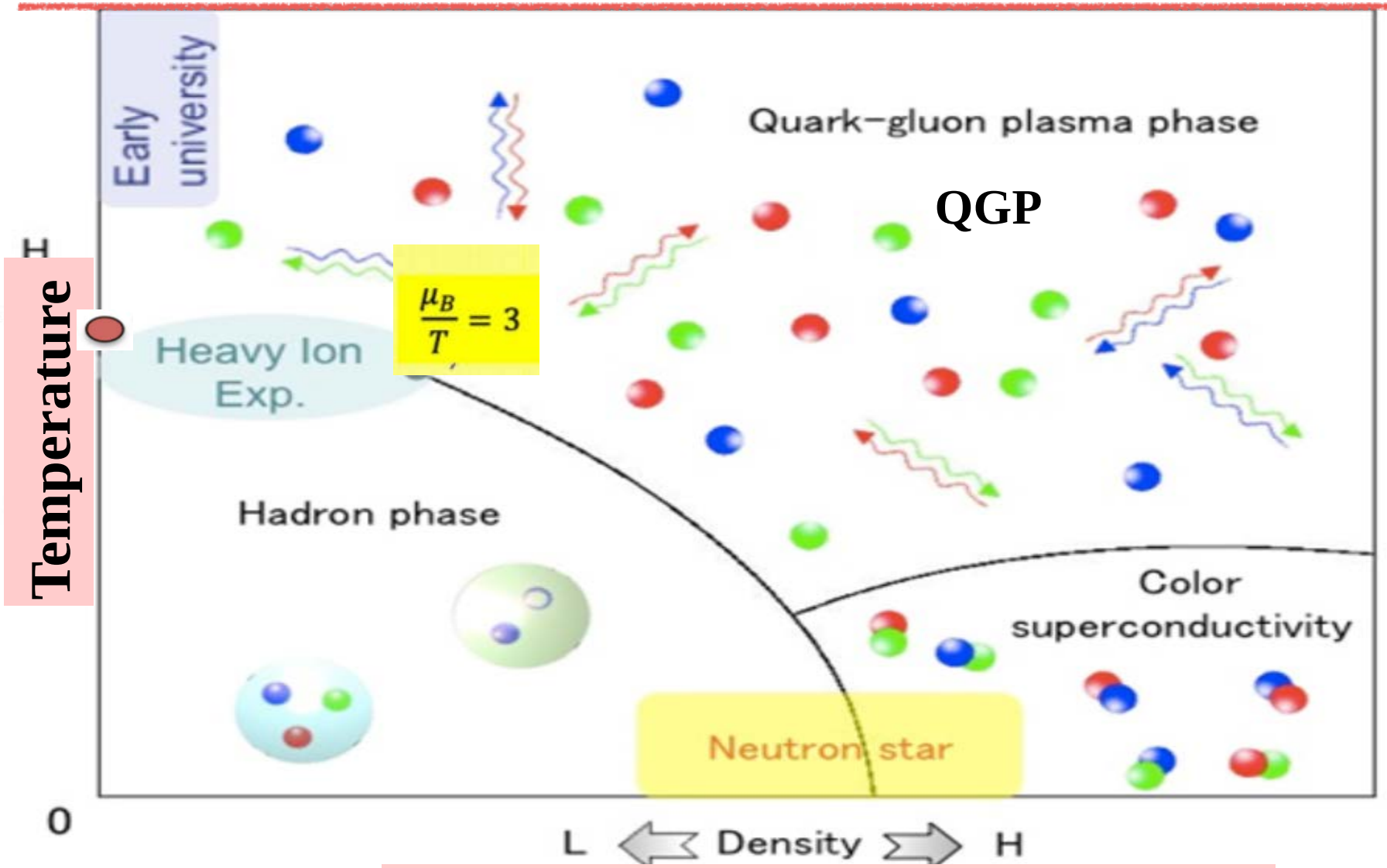
III. Magnetic EoS data driven

IV. EoS and Hydro data driven

V. Summary

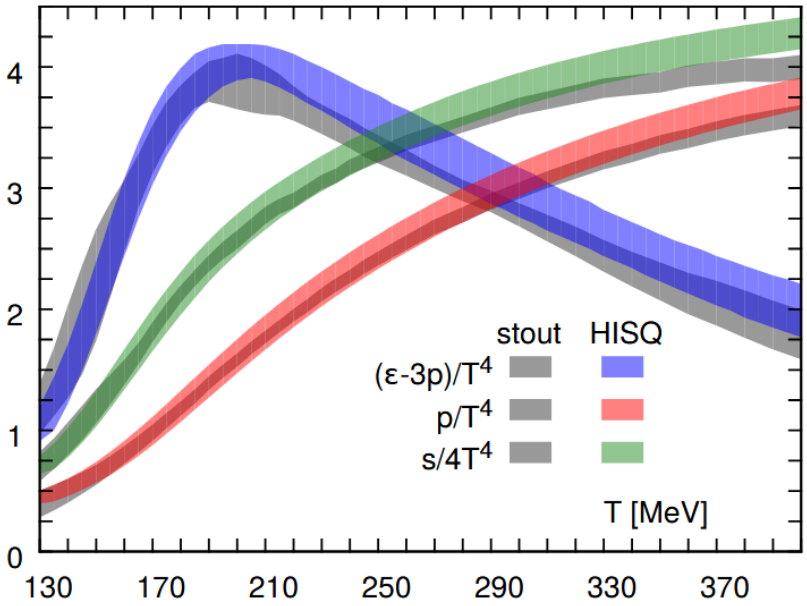
Motivations

A schematic view of QCD Phase diagram



Baryon Number chemical potential

2+1 flavor QCD Data Set of QGP: EoS & Hydro



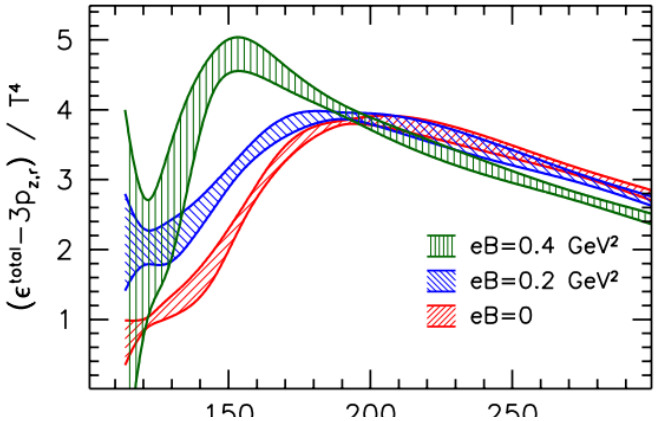
Equation of state in (2+1)-flavor QCD #2

HotQCD Collaboration • A. Bazavov (Iowa U.) et al. (Jul 23, 2014)

Published in: *Phys.Rev.D* 90 (2014) 094503 • e-Print: 1407.6387 [hep-lat]

pdf DOI cite claim reference search 1,769 citations

Magnetic EoS



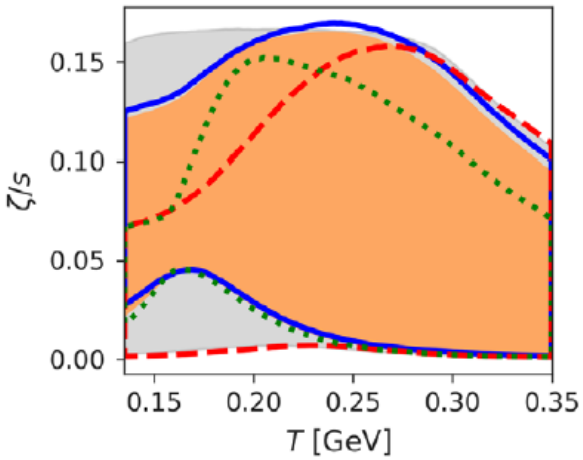
The QCD equation of state in background magnetic fields #2

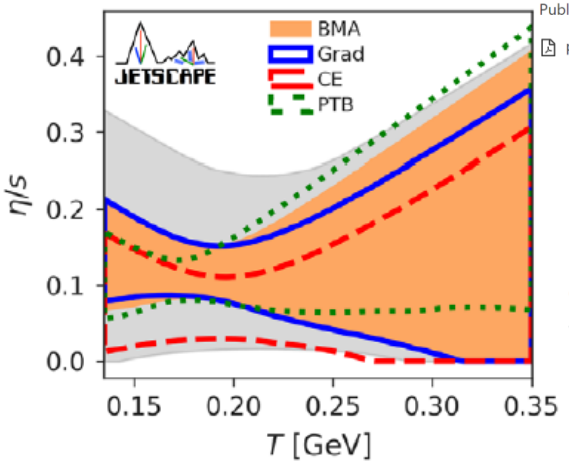
G.S. Bali (Regensburg U. and Tata Inst.), F. Bruckmann (Regensburg U.), G. Endrődi (Regensburg U.), S.D. Katz (Eotvos U.), A. Schäfer (Regensburg U.) (Jun 2, 2014)

Published in: *JHEP* 08 (2014) 177 • e-Print: 1406.0269 [hep-lat]

pdf DOI cite claim reference search 305 citations

Hydro





Phenomenological constraints on the transport properties of QCD matter with data-driven model averaging #1

JETSCAPE Collaboration • D. Everett (Ohio State U.) et al. (Oct 8, 2020)

Published in: *Phys.Rev.Lett.* 126 (2021) 24, 242301 • e-Print: 2010.03928 [hep-ph]

pdf DOI cite claim reference search 256 citations

Motivations

Low energy QCD is strong coupled system

P. Braun-Munzinger and J. Wambach, Rev. Mod. Phys. 81 (2009), 1031-1050 [arXiv:0801.4256 [hep-ph]].
S. Gupta, X. Luo, B. Mohanty, H. G. Ritter and N. Xu, Science 332 (2011), 1525-1528 [arXiv:1105.3934[hep-ph]].

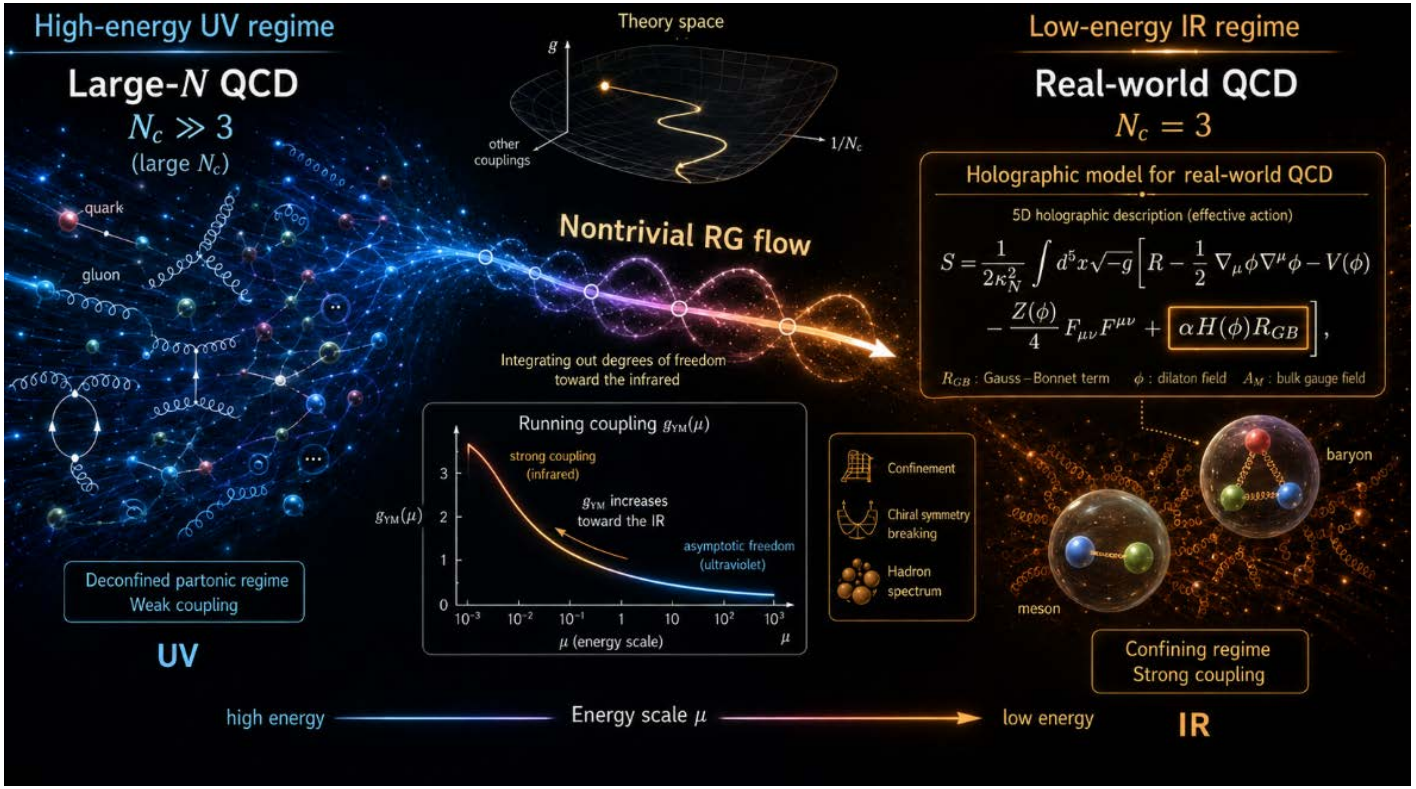
Finite density QCD v.s. sign Problem

O. Philipsen, Prog. Part. Nucl. Phys. 70 (2013), 55-107 [arXiv:1207.5999 [hep-lat]].

Non-perbative Theoretical Approaches

Schwinger–Dyson equation (DSE)
Nambu–Jona-Lasinio models (NJL, PNJL)
Functional renormalization group (FRG)
…

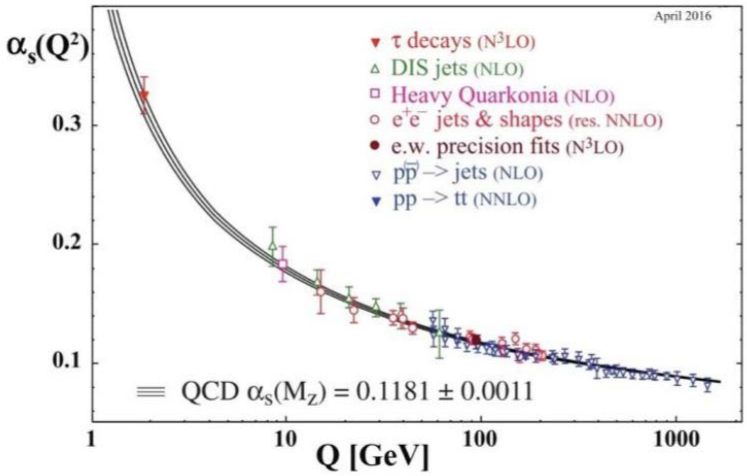
From AdS/CFT to AdS/QCD



We choose all the non-trival RG flow and non pertubative effectively captured by Model parameters for simplicity.

Typically, investigate QCD phase diagram via holography (Many original hQCD references from top-down and bottom up.):
O. DeWolfe, S. S. Gubser and C. Rosen, Phys. Rev. D 84 (2011), 126014 [arXiv:1108.2029 [hep-th]].
R. G. Cai, S. He and D. Li, JHEP 03 (2012), 033 [arXiv:1201.0820 [hep-th]].
U. Gursoy, M. Jarvinen and G. Nijs, Phys. Rev. Lett. 120 (2018) no.24,242002 [arXiv:1707.00872 [hep-th]].
J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo,C. Ratti and R. Rougemont,Phys. Rev. D 104 (2021) no.3, 034002 [arXiv:2102.12042[nucl-th]]. ……

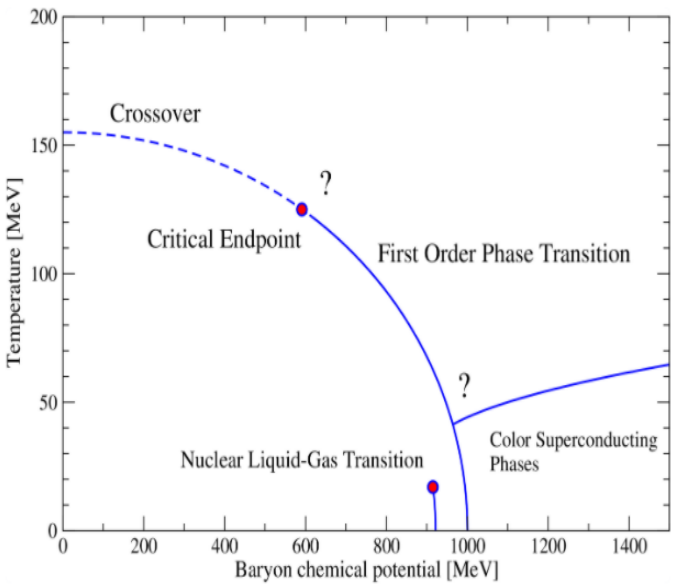
2+1 flavor EoS data



EoS data-driven hQCD model

To capture the characteristic properties of Thermal (BH) and dense (U(1) gauge field) QCD matter.

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\Big],$$



ϕ Break conformal symmetry

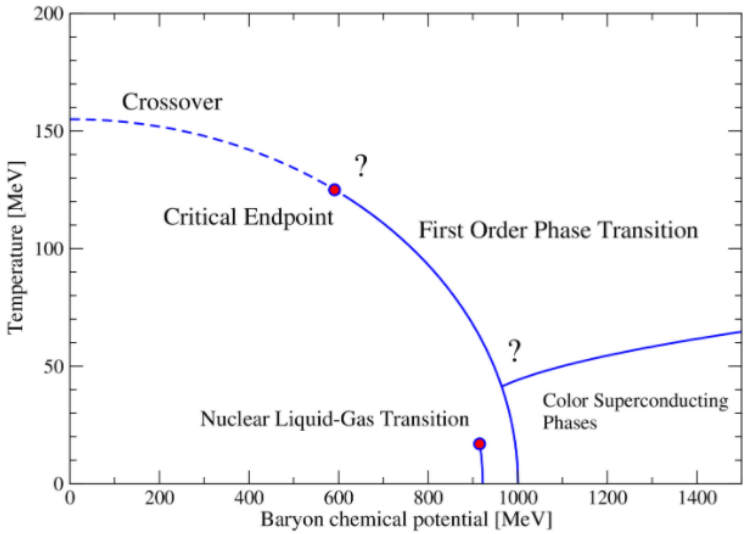
$F^{\mu\nu}$ Introduce baryon chemical potential

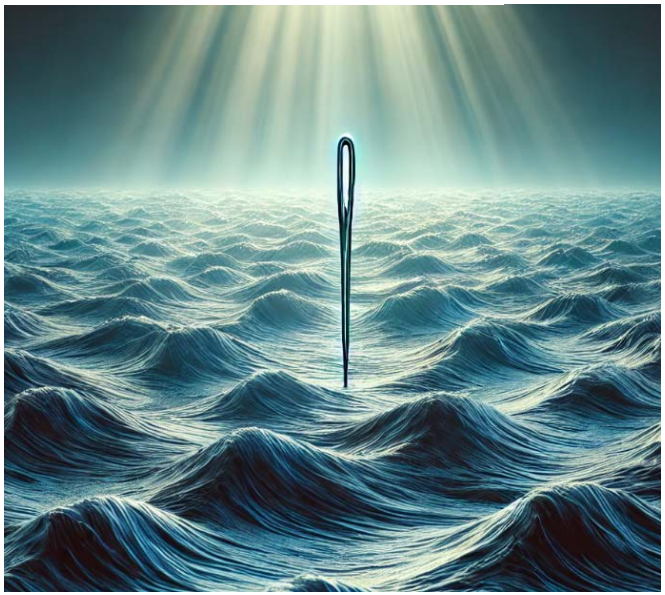
EoS data-driven hQCD model

Gravity Action

Finding a needle in the ocean
From 2011 to 2022

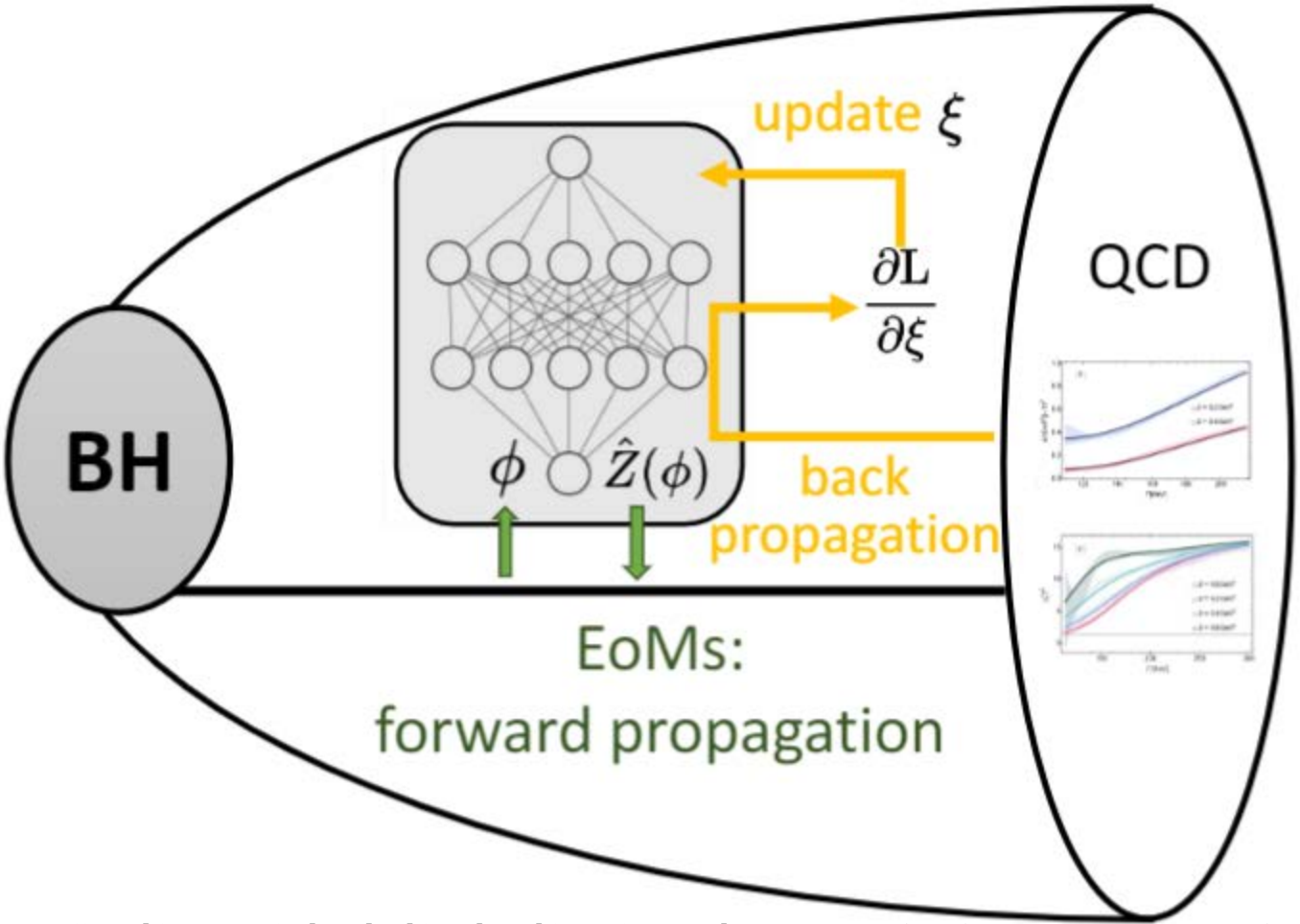
$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\Big],$$





AI: Transformer helps us!

Rong-Gen Cai, SH, Li Li, Hongan Zeng, 2406.12772



In hQCD: Deriving action: Koji Hashimoto, Keisuke Ohashi, Takayuki Sumimoto, Phys.Rev.D 105 (2022) 10, 106008; PTEP 2023(2023)3,033B01 ,….. Learning metric: Tetsuya Akutagawa, Koji Hashimoto, Takayuki Sumimoto, Phys.Rev.D 102 (2020) 2, 026020; Koji Hashimoto, Sotaro Sugishita, Akinori Tanaka, Akio Tomiya, Phys.Rev.D 98 (2018) 10, 106014, Hong-Ye Hu, Shuo-Hui Li, Lei Wang, and Yi-Zhuang You, Physical Review Research, 2(2):023369, 2020.[12] Yu-Kun Yan, Shao-Feng Wu, Xian-Hui Ge, and Yu Tian. Physical Review D, 102(10):101902, 2020; Mugeon Song, Maverick SH Oh, Yongjun Ahn, and Keun-Young Kim ,Chinese Physics C, 45(7):073111, 2021.Zhuo-Fan Gu, Yu-Kun Yan, and Shao-Feng Wu, 2401.09946, 2024; Xun Chen and Mei Huang, Physical Review D, 109(5):L051902, 2024. Byoungjoon Ahn, Hyun-Sik Jeong, Keun-Young Kim, and Kwan Yun, 2406.07395, 2024; Xun Chen, Mei Huang, Kai Zhou, …….

2+1-flavor QCD data-driven HQCD model

$$S=\frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\Big[\mathcal{R}-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu}-V(\phi)\Big],$$

Model Parameters

$$V(\phi)=-12\cosh[c_1\phi]+(6c_1^2-\frac{3}{2})\phi^2+c_2\phi^6,$$

$$Z(\phi)=\frac{1}{1+c_3}\mathrm{sech}[c_4\phi^3]+\frac{c_3}{1+c_3}e^{-c_5\phi},$$

Effective Newton constant κ_N^2 Scalar source+Renormalization **b**

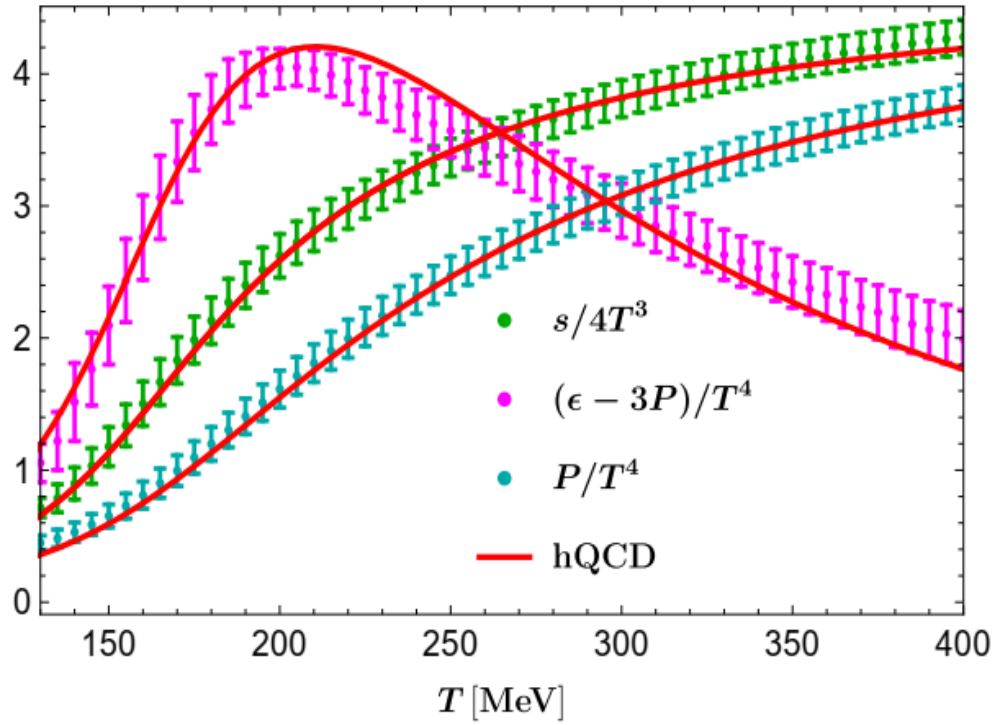
model	c_1	c_2	c_3	c_4	c_5	κ_N^2	ϕ_s(GeV)	b
pure $SU(3)$	0.735	0				$2\pi(4.88)$	1.523	-0.36458
2 flavor	0.710	0.0002	0.530	0.085	30	$2\pi(3.72)$	1.227	-0.25707
2+1 flavor	0.710	0.0037	1.935	0.085	30	$2\pi(1.68)$	1.085	-0.27341

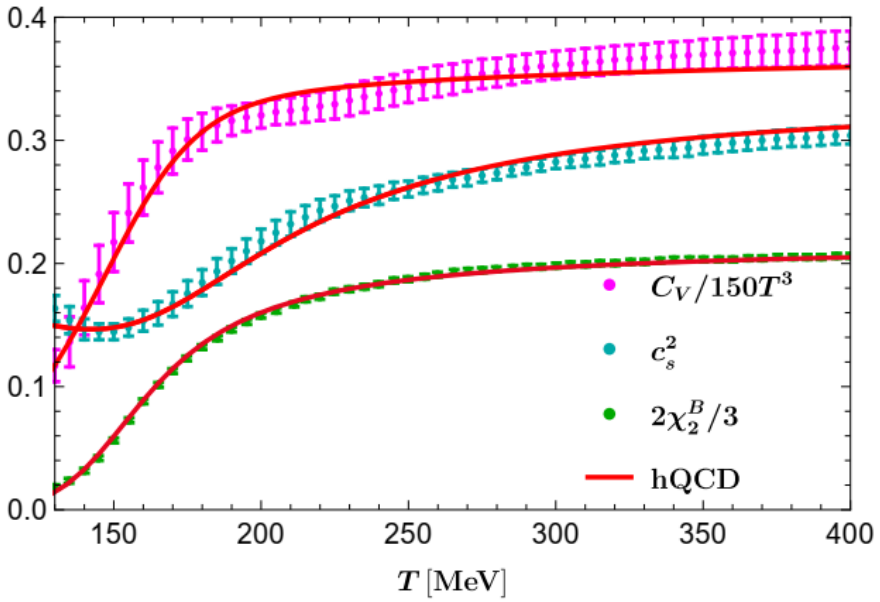
Rong-Gen Cai, SH, Li Li, Yuan-Xu Wang, 2201.02004
Yanqin Zhao, SH, Defu Hou, Li Li, Zhibin Li, 2310.13432

Fix model parameters by Equation of States By hand tuning

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, 2201.02004

c_1, c_2, c_3, c_4, c_5 Effective Newton constant κ_N^2 Scalar source ϕ_s+Renormalization **b**



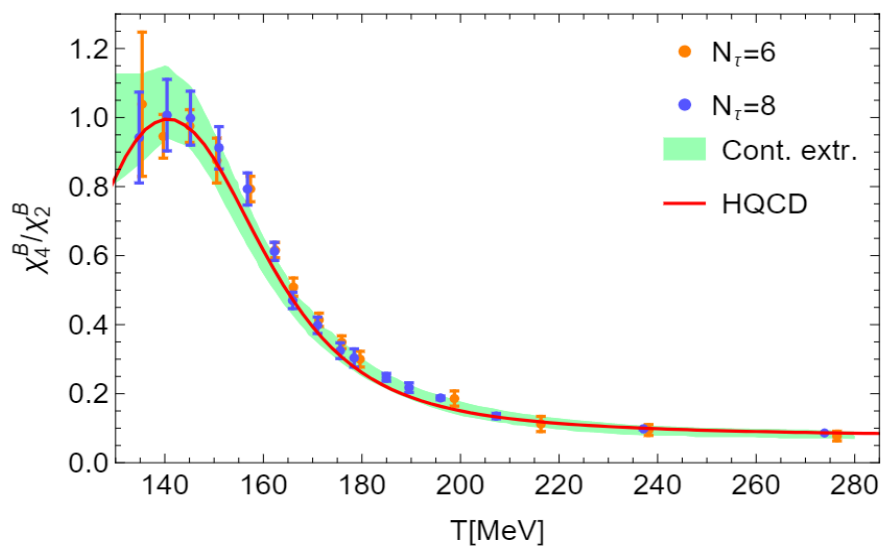


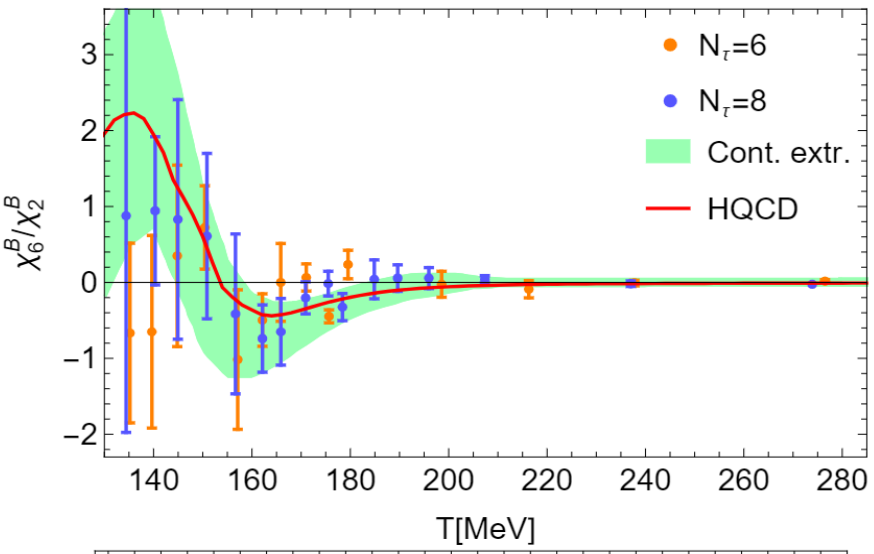
A. Bazavov *et al.* [HotQCD], Phys. Rev. D 90 (2014), 094503[arXiv:1407.6387 [hep-lat]].

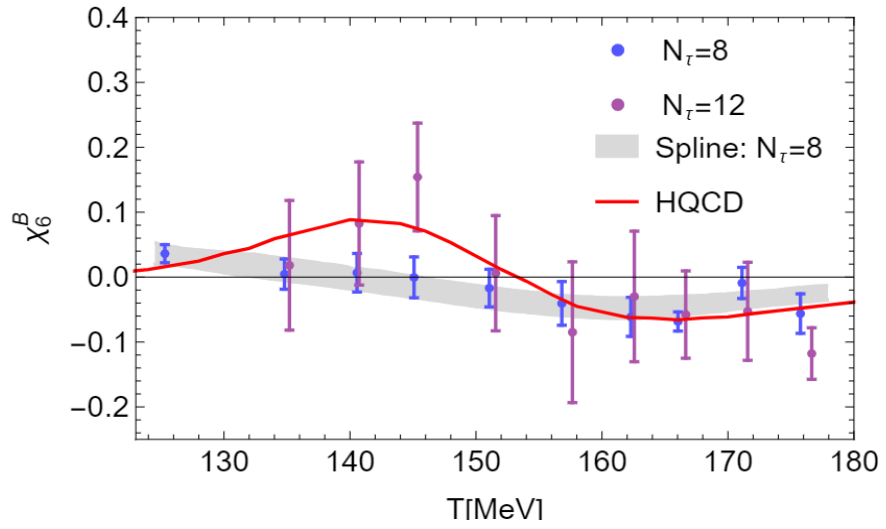
Higher precision checking EOS, Vanishing Mu_B

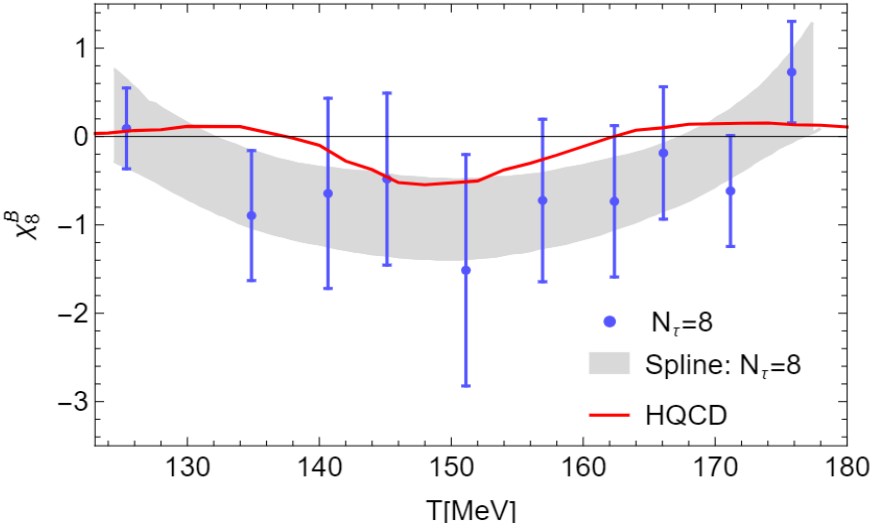
Generalized higher-order baryon susceptibility.

$$\chi_n^B = \frac{\partial^n P/T^4}{\partial \hat{\mu}_B^n}.$$







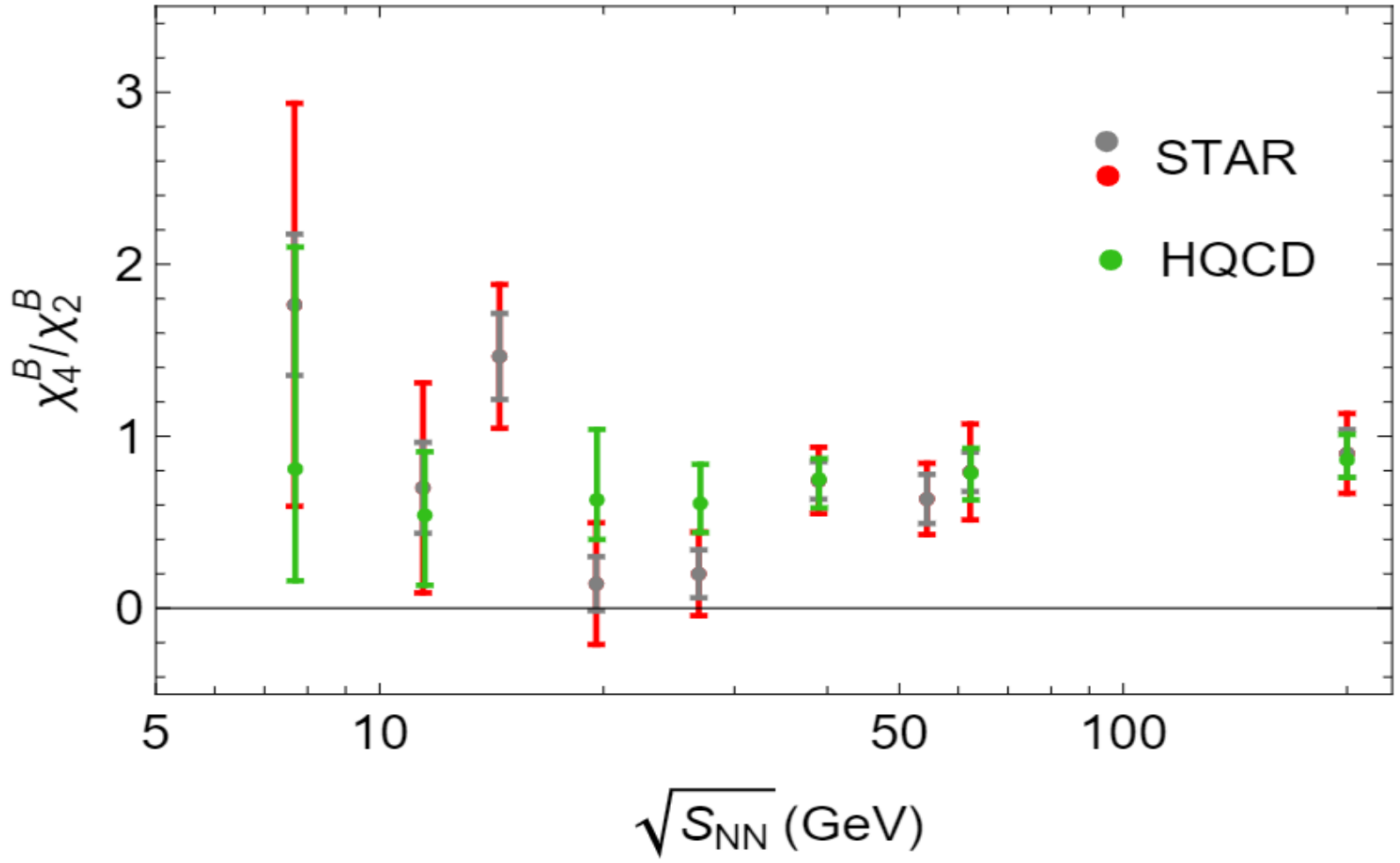


A. Bazavov *et al.* [HotQCD], Phys. Rev. D 95 (2017) 5, 05404 [arXiv:1701.04325[hep-lat]].

Higher precision checking EOS, finite Mu_B

Higher-order baryon susceptibility.

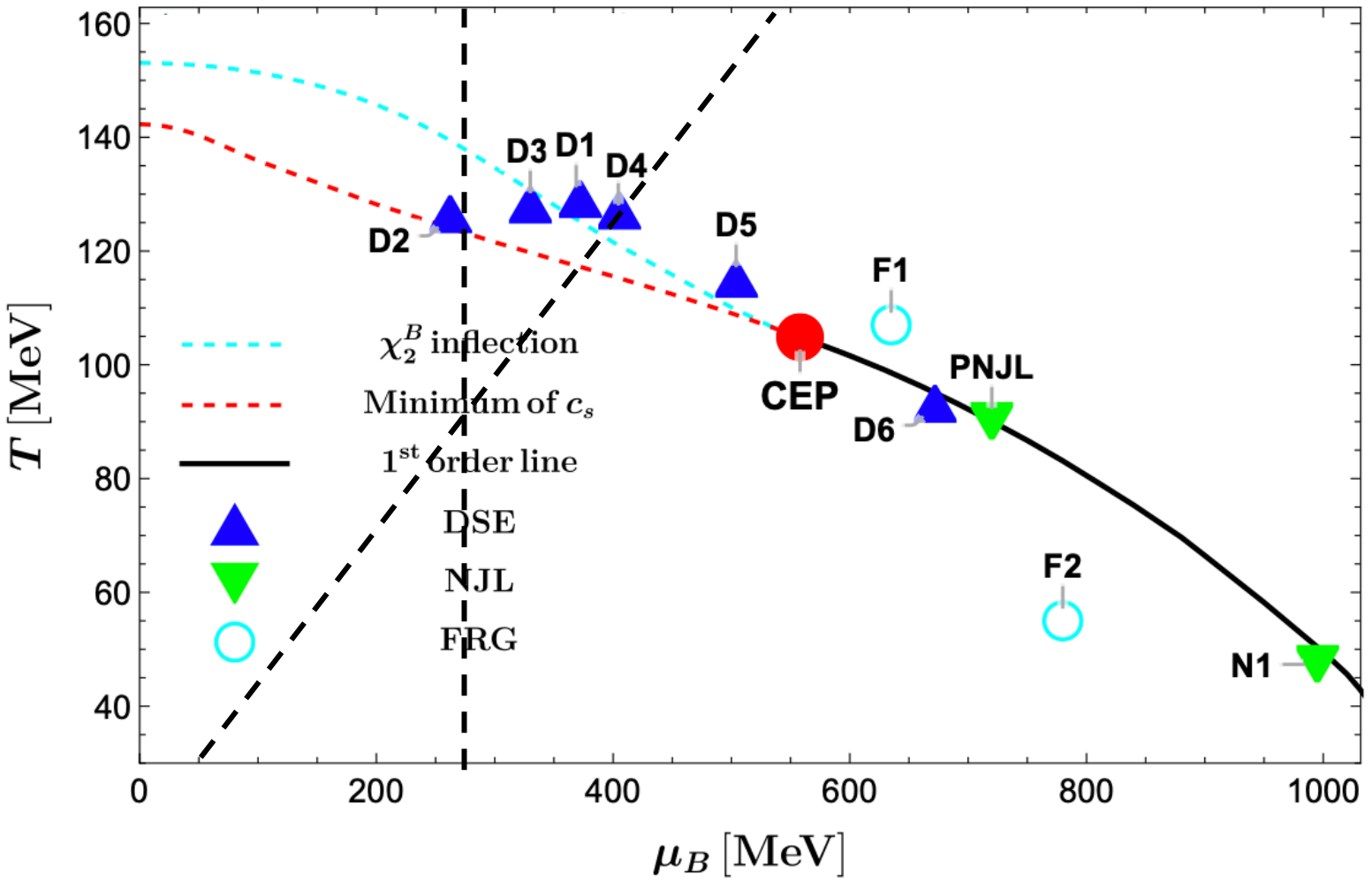
$$\chi_n^B = \frac{\partial^n P/T^4}{\partial \hat{\mu}_B^n}.$$



w/. Li Li, Zhibin Li, Jing-Min Liang, 2305.13874

STAR Collaboration, M. S. Abdallah et al., Phys. Rev. Lett. 128 no. 20, (2022) 202303, arXiv:2112.00240 [nucl-ex].

QCD phase diagram from our holographic model



$\mu_B/T \leq 3, \mu_B < 300\text{MeV}$ was excluded by lattice simulation

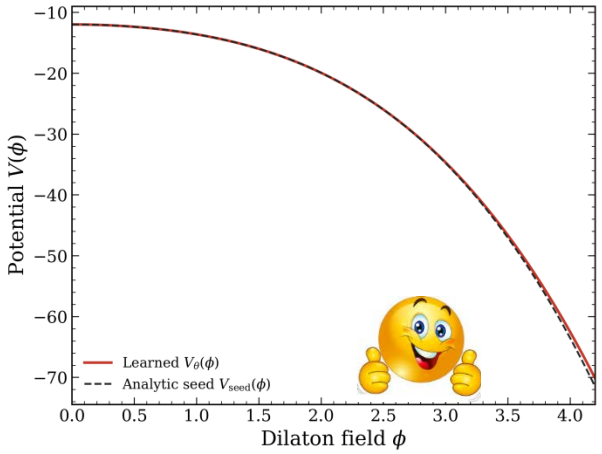
A. Bazavov, etc. Phys. Rev. D 95 (2017) no.5, 054504

Human Learning versus Machine Learning

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, 2201.02004

$$V(\phi) = -12\cosh[c_1\phi] + (6c_1^2 - \frac{3}{2})\phi^2 + c_2\phi^6$$

versus

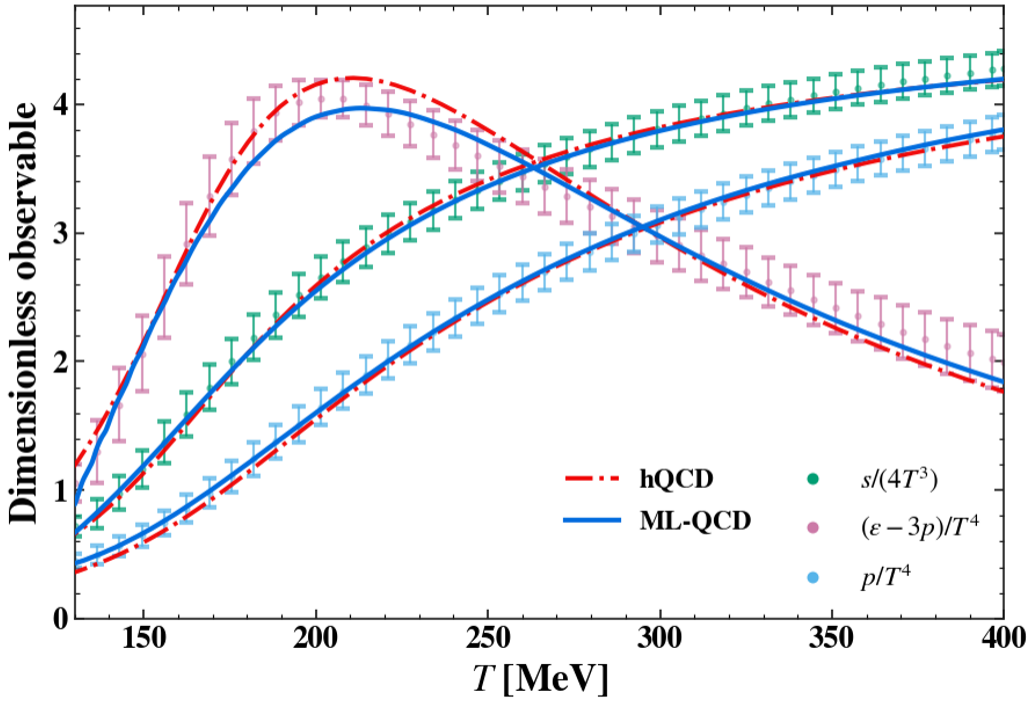


Human Learning

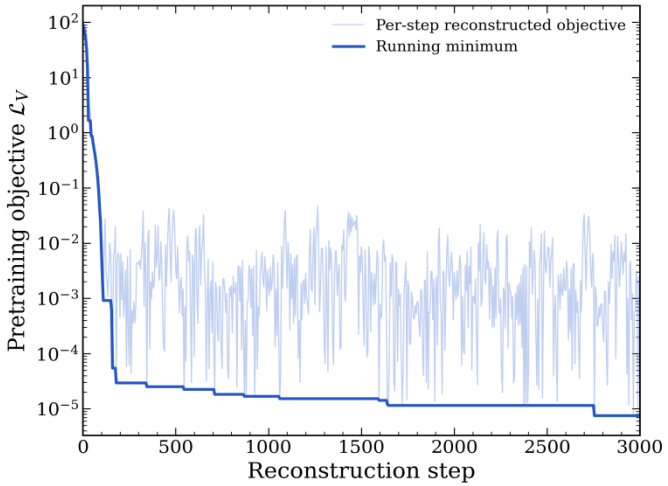
Fit to HotQCD fixed lattice EOS

$$\chi_a^2 = \left\langle \left(\frac{y_a - m_a}{\sigma_a} \right)^2 \right\rangle$$

$$\mathcal{L}_{\text{EOS}} = 10\chi_I^2 + \chi^2_{s/(4T^3)} + \chi^2_{p/T^4}$$



Collocation domain	$\phi \in [0, 4]$ with $N = 1024$ uniform points
Optimizer	Adam, learning rate 10^{-4}
Training length	3000 steps
Random seed	19012026

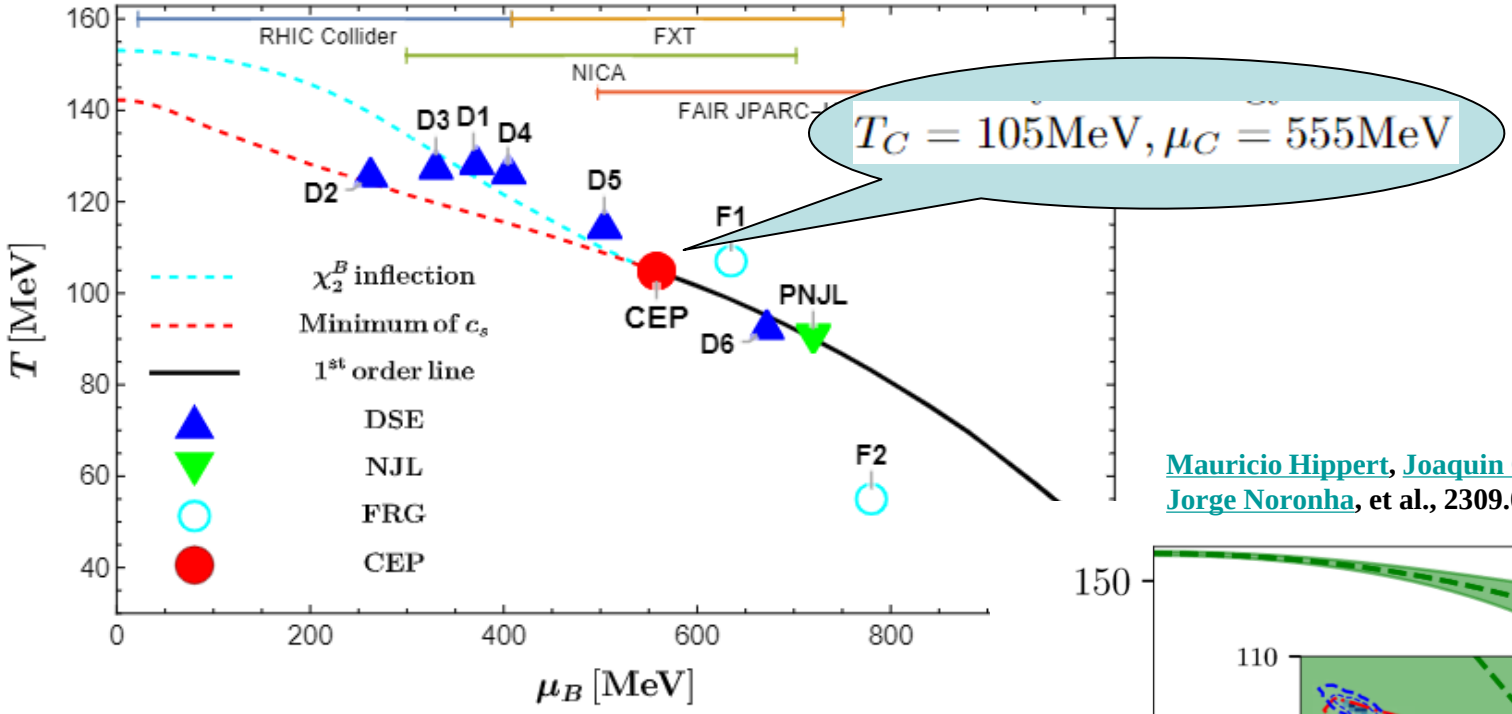


Working in Progress

A. Bazavov *et al.* [HotQCD], Phys. Rev. D 95 (2017) 5, 05404 [arXiv:1701.04325[hep-lat]].

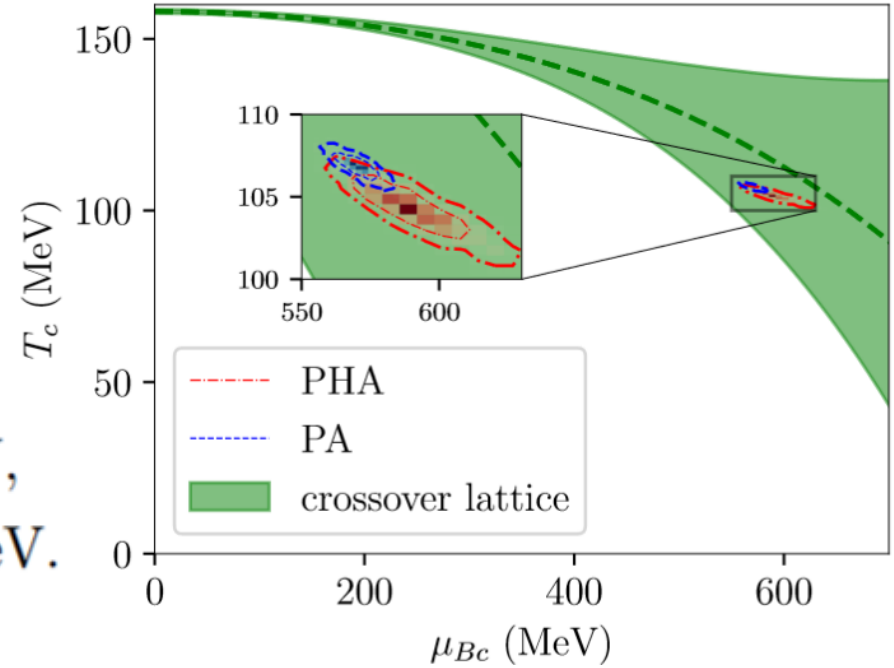
QCD phase diagram : Our Model v.s. Bayesian location

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, 2201.02004



$T_C = 105\text{MeV}, \mu_C = 555\text{MeV}$

Mauricio Hippert, Joaquin Grefa, T. Andrew Manning, Jorge Noronha, et al., 2309.00579

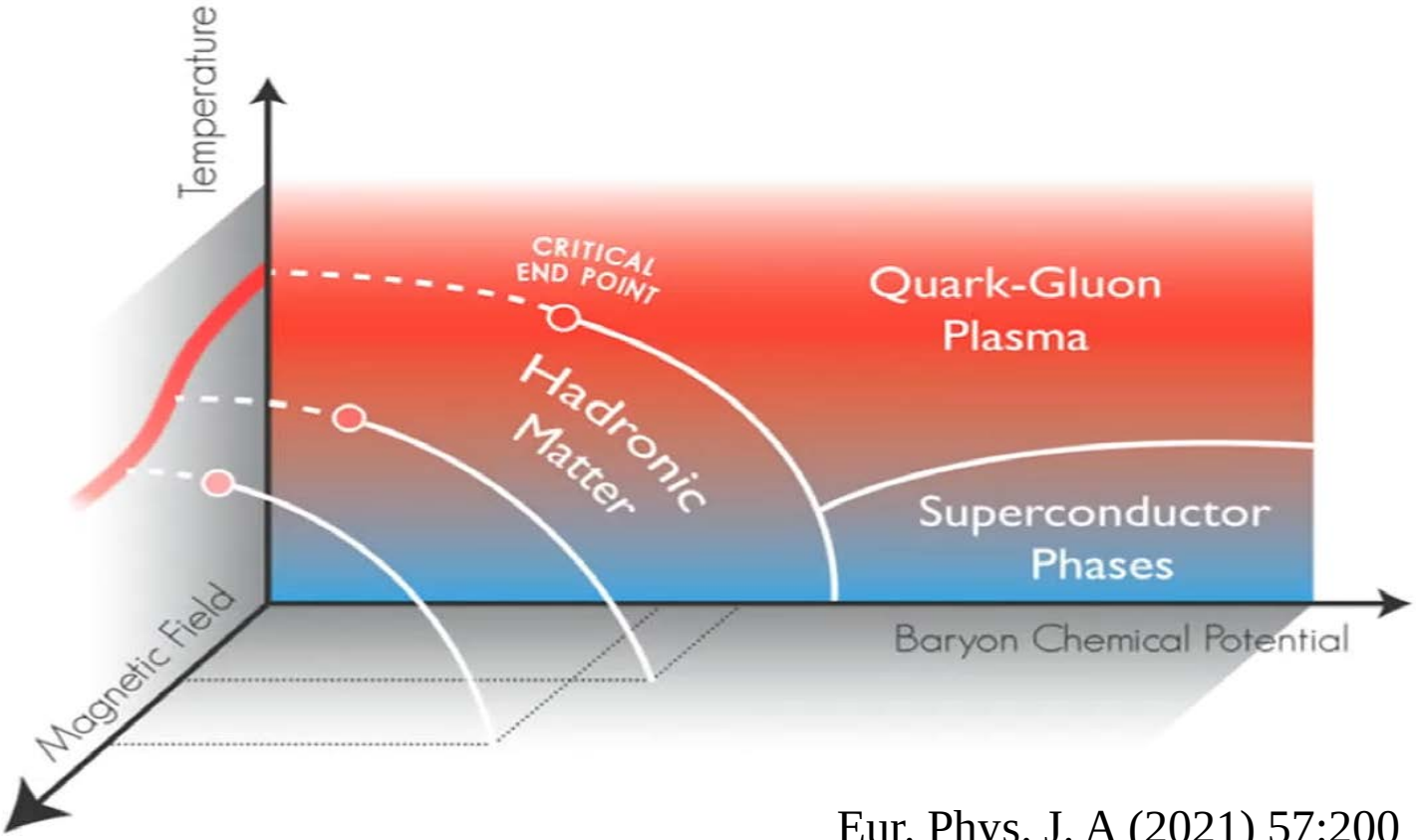


$$(T_c,\, \mu_{Bc})_{PHA} = (104 \pm 3,\, 589^{+36}_{-26})\ \text{MeV},$$
$$(T_c,\, \mu_{Bc})_{PA} = (107 \pm 1,\, 571 \pm 11)\ \text{MeV}.$$

Rong-Gen Cai, SH, Li Li, Hongan Zeng, 2406.12772

Magnetic EoS

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{Z_B(\phi)}{4}F^B_{\mu\nu}F^{B\mu\nu} - V(\phi)\right]$$



Eur. Phys. J. A (2021) 57:200

Magnetic EoS driven hQGP

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\hat{Z}(\phi)}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - V(\phi)\Big],$$

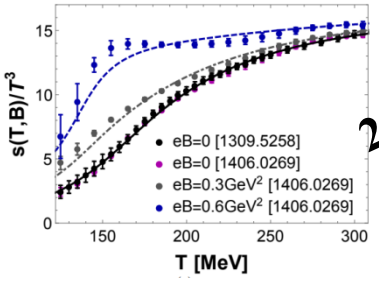
Aim: Learning

Magnetic coupling

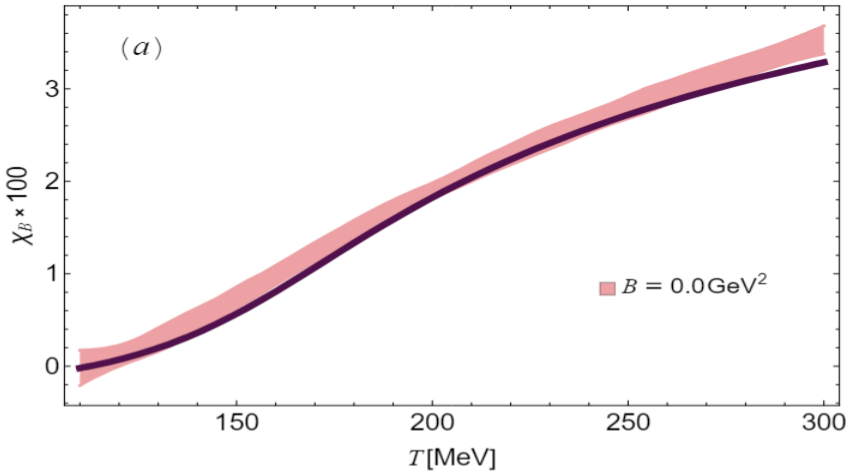
$$ds^2 = -f(r)e^{-\eta(r)}dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2 + g(r)dz^2)$$

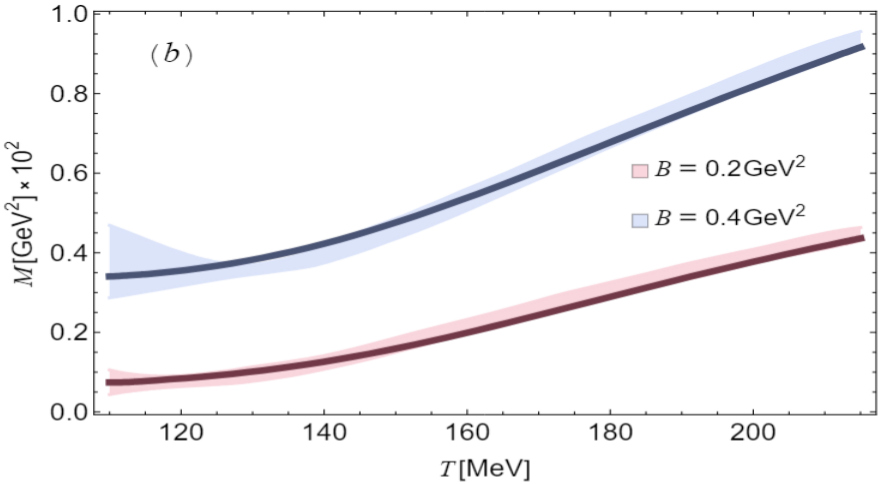
$$\phi = \phi(r), \quad A = A(r), \quad \hat{A} = \frac{B}{2}(xdy - ydx)$$

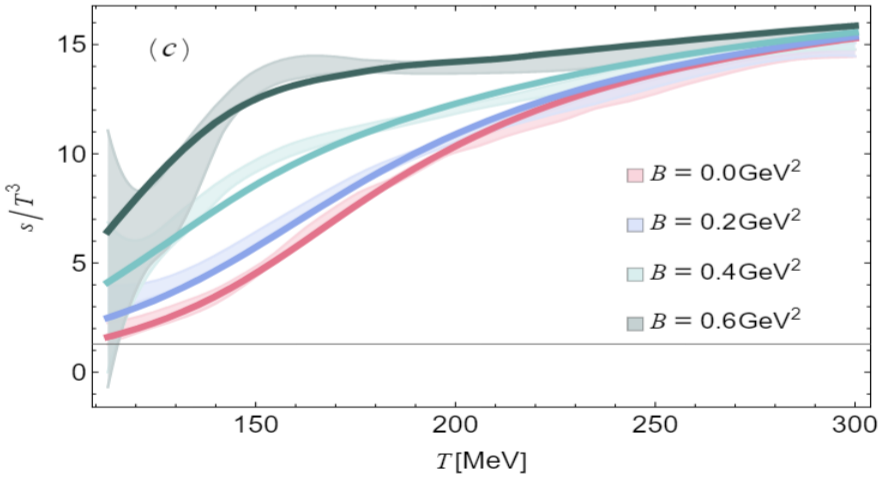
Data Fitting to Fix Model

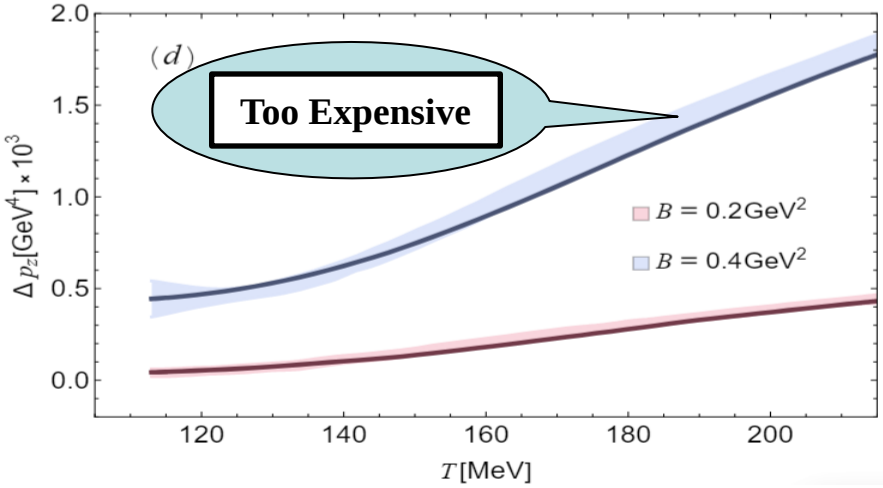


2307.03885







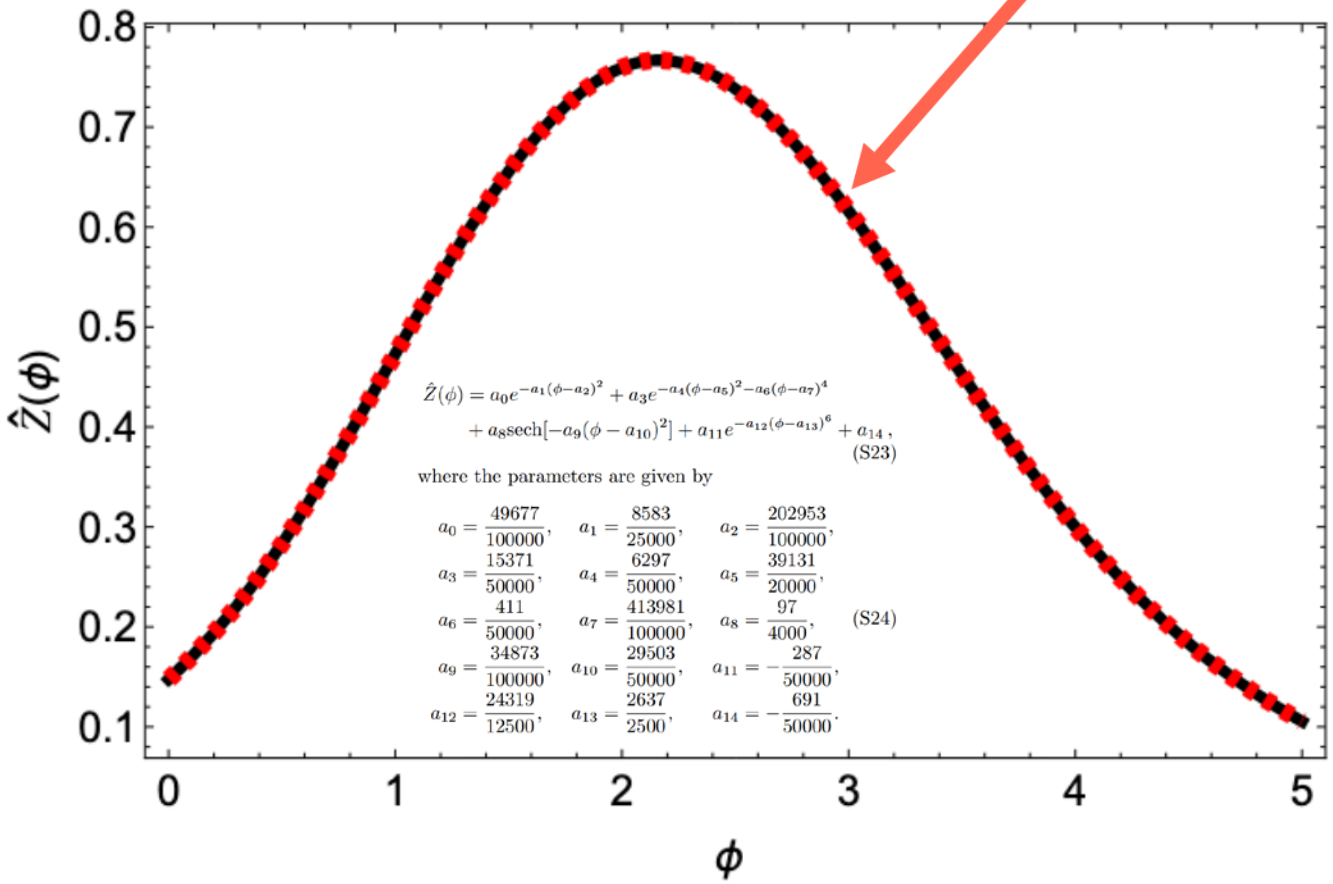


Bali G S, Bruckmann F, Endrődi G, et al. JHEP 08 (2014) 177

Rong-Gen Cai, SH, Li Li, Hongan Zeng, 2406.12772

Neural Network

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi) - \frac{Z_B(\phi)}{4}F^B_{\mu\nu}F^{B\mu\nu}\right]$$



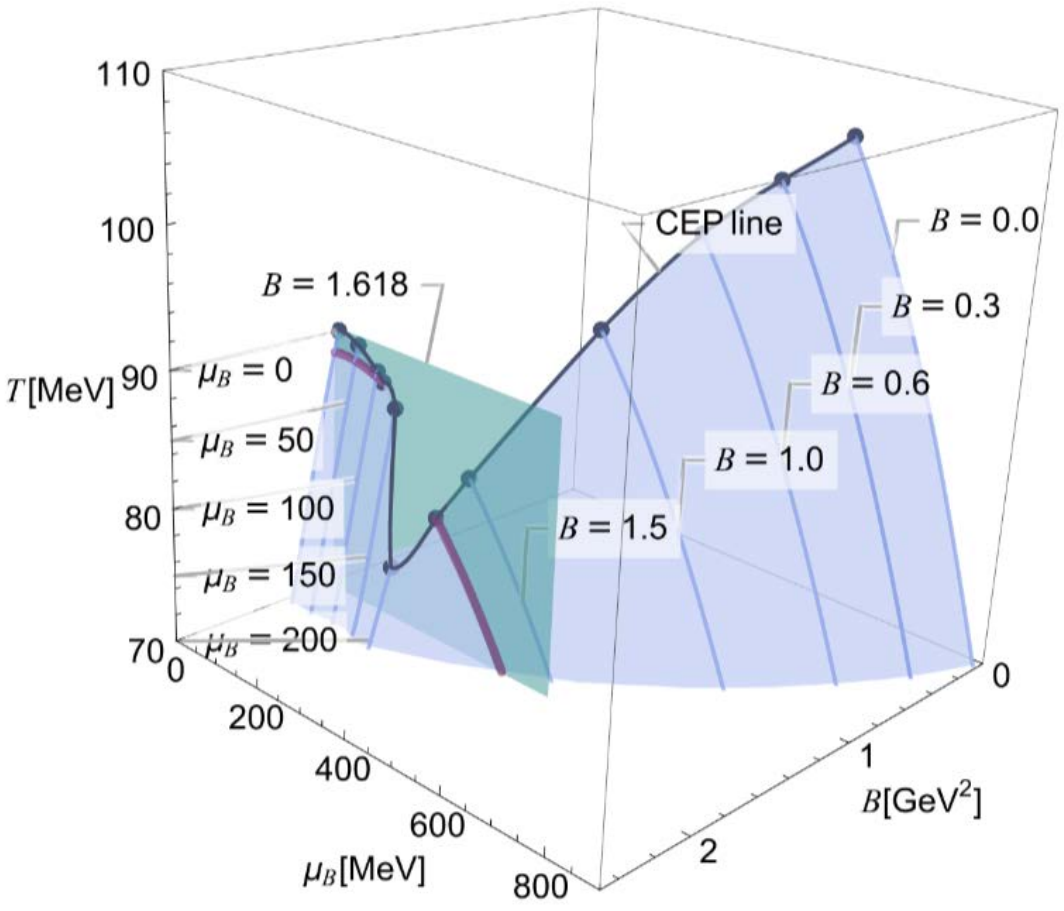
$$\hat{Z}(\phi) = a_0 e^{-a_1(\phi-a_2)^2} + a_3 e^{-a_4(\phi-a_5)^2 - a_6(\phi-a_7)^4} + a_8 \mathrm{sech}[-a_9(\phi-a_{10})^2] + a_{11} e^{-a_{12}(\phi-a_{13})^6} + a_{14}\,, \tag{S23}$$

where the parameters are given by

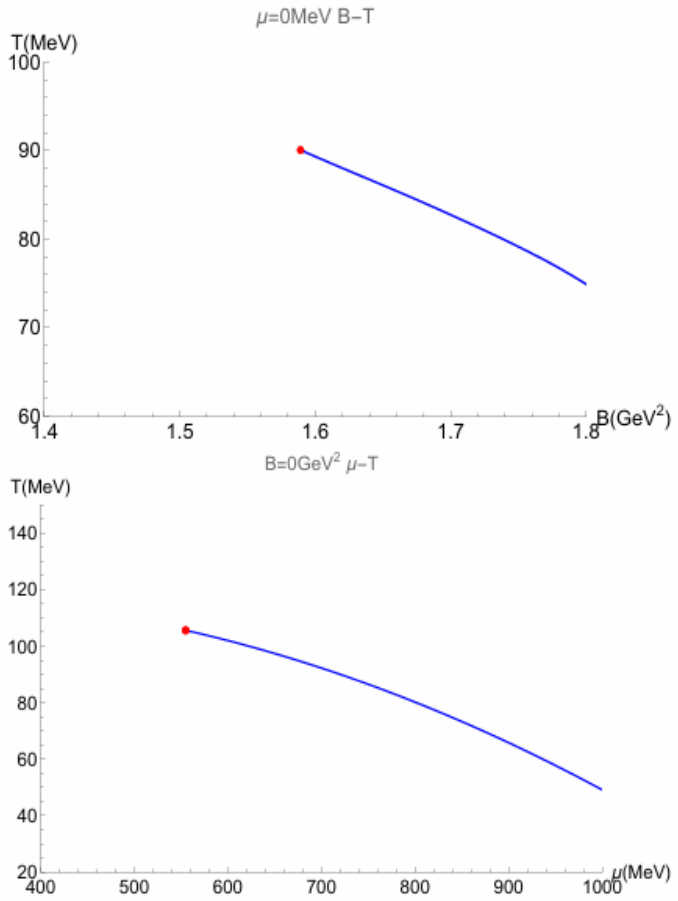
$$\begin{aligned}
a_0 &= \frac{49677}{100000}, & a_1 &= \frac{8583}{25000}, & a_2 &= \frac{202953}{100000},\\
a_3 &= \frac{15371}{50000}, & a_4 &= \frac{6297}{50000}, & a_5 &= \frac{39131}{20000},\\
a_6 &= \frac{411}{50000}, & a_7 &= \frac{413981}{100000}, & a_8 &= \frac{97}{4000},\\
a_9 &= \frac{34873}{100000}, & a_{10} &= \frac{29503}{50000}, & a_{11} &= -\frac{287}{50000},\\
a_{12} &= \frac{24319}{12500}, & a_{13} &= \frac{2637}{2500}, & a_{14} &= -\frac{691}{50000}.
\end{aligned} \tag{S24}$$

Magnetic Phase diagram

F. Cuteri, "QCD thermodynamics: an overview of recent progress," PoS **LATTICE2022** (2023), 243



$B_c, T_c = 1.58 GeV^2, 90 MeV$

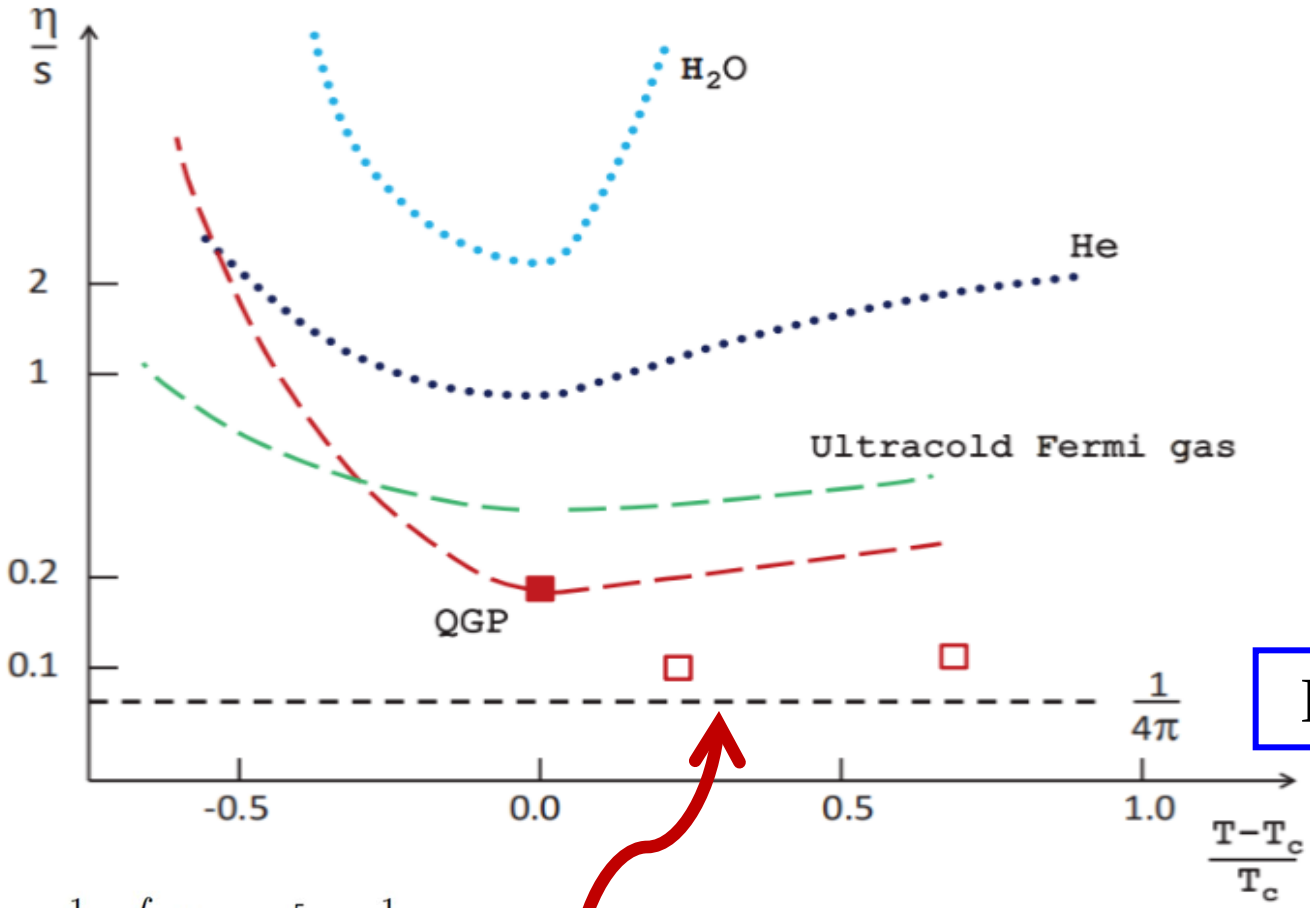


$\mu_c, T_c = 555 MeV, 105 MeV$

Rong-Gen Cai, SH, Li Li, Hongan Zeng, 2406.12772

EoS & Hydro data-driven HL & ML

Shear & Bulk Viscosity of Real QCD



KSS bound

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\Big],$$

data-driven hQCD model

To capture the characteristic properties of Thermal (BH) and dense (U(1) gauge field) QCD matter.

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\Big[R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi) - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} + \alpha H(\phi)R_{GB}\Big]$$

Gauss-Bonet terms to realize Hydro properties of QGP

$$V(\phi) = -12\cosh[c_1\phi] + (6c_1^2 - \frac{3}{2})\phi^2 + c_2\phi^6\,,$$

$$Z(\phi) = \frac{1}{1+c_3}\mathrm{sech}[c_4\phi^3] + \frac{c_3}{1+c_3}e^{-c_5\phi}\,,$$

$\alpha H(\phi)R_{GB}$

Numerical **full backreaction** gravitational background

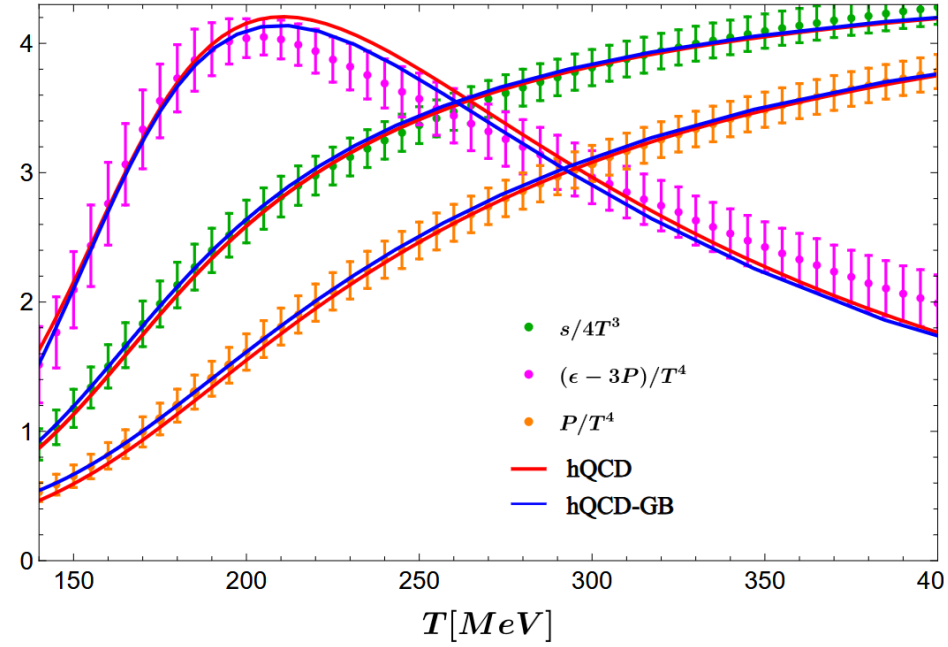
Chengwei Tong, Jie Zhou, Yuanxu Wang, Rong-Gen Cai, SH, Li Li, 2605. 23834[hep-th].

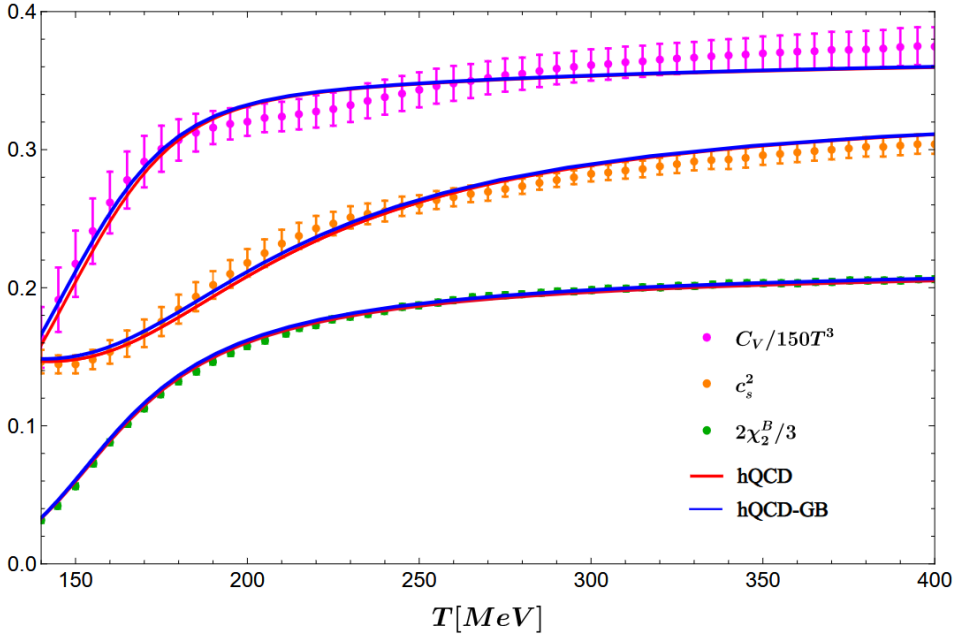
Fix model parameters by EoS

c_1	c_2	c_3	c_4	c_5	c_6	b	ϕ_s	κ_N^2
0.7074	0.0041	1.95	0.09	30	0	-0.2744	1065	$2\pi(1.62)$

$H(\phi) = 1$

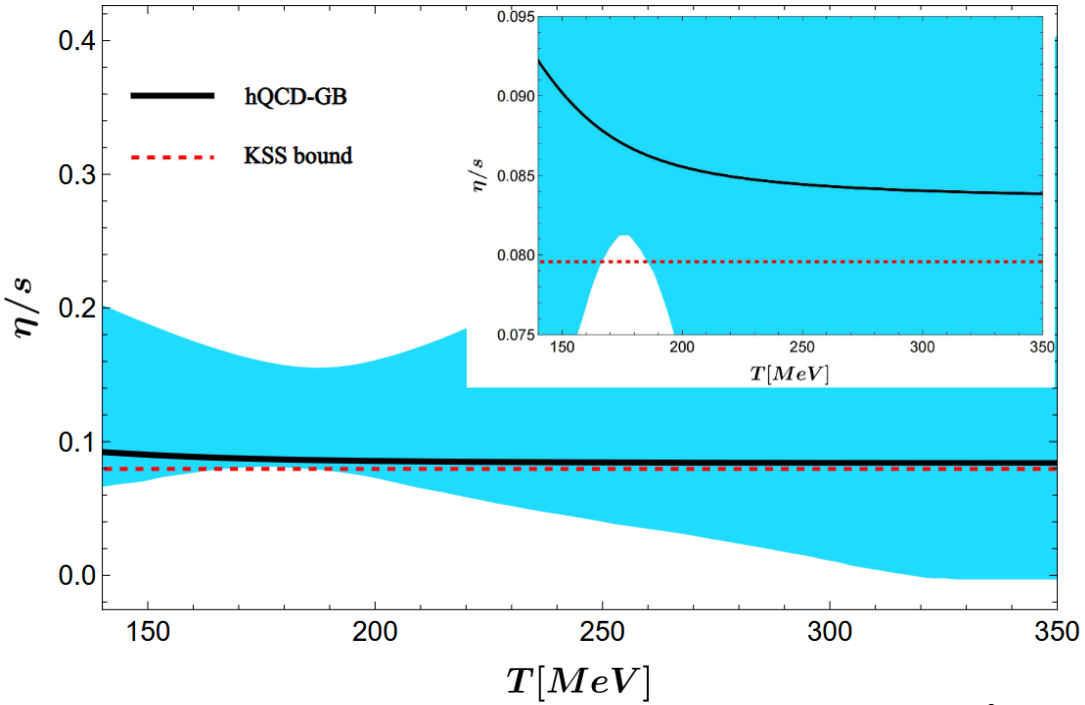
$\alpha = -0.006.$

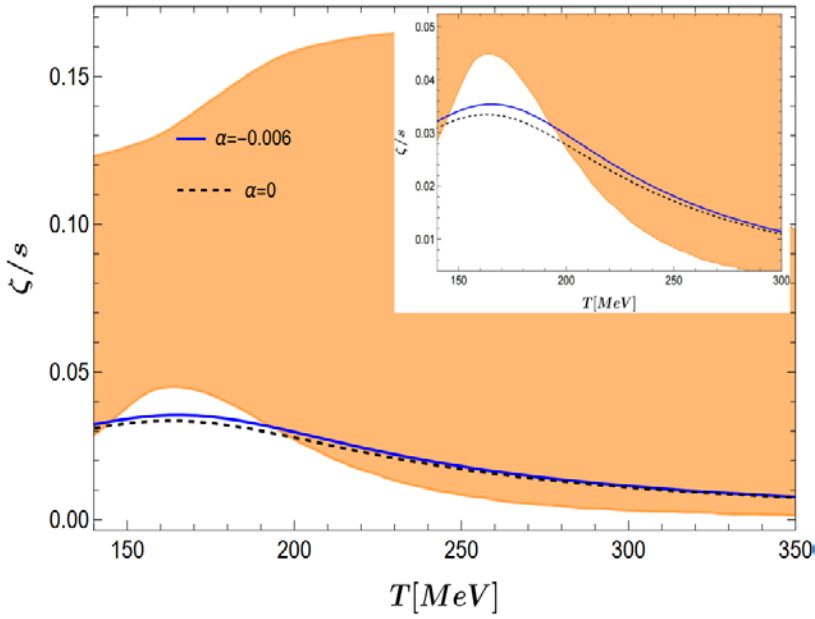




model	c_1	c_2	c_3	c_4	c_5	κ_N^2	ϕ_s(GeV)	b
pure $SU(3)$	0.735	0				$2\pi(4.88)$	1.523	-0.36458
2 flavor	0.710	0.0002	0.530	0.085	30	$2\pi(3.72)$	1.227	-0.25707
2+1 flavor	0.710	0.0037	1.935	0.085	30	$2\pi(1.68)$	1.085	-0.27341

Shear & Bulk Viscosity of hQCD + $\alpha H(\phi) R_{GB}$

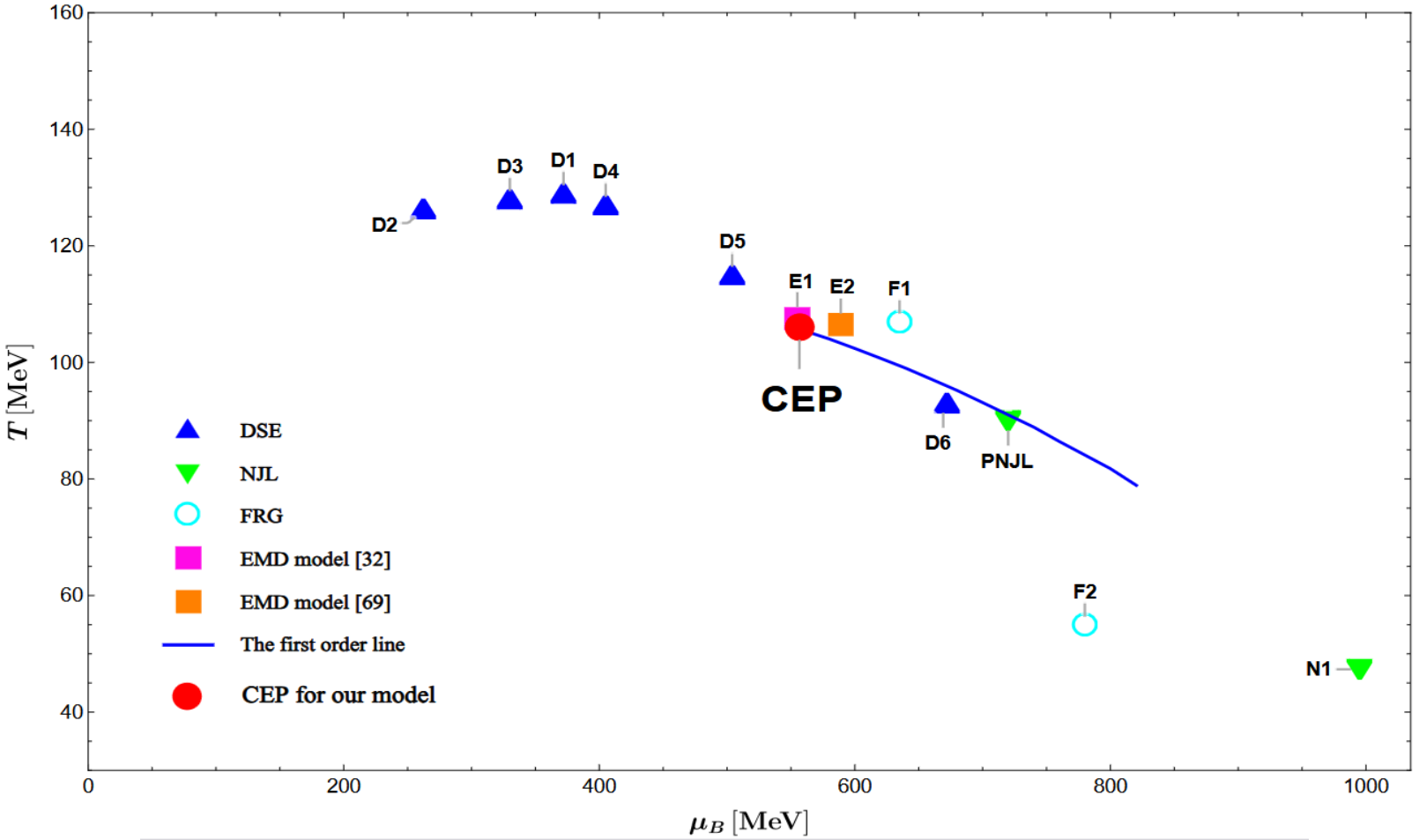




Bayesian analysis 90% credible

D. Everett, etc., Physical Review Letters 126, 10.1103 (2021).

Chengwei Tong, Jie Zhou, Yuanxu Wang, Rong-Gen Cai, SH, Li Li, 2605. 23834[hep-th].



W.-j. Fu, J.M. Pawlowski and F. Rennecke, *QCD phase structure at finite temperature and density*, *Phys. Rev. D* **101** (2020) 054032 [1909.02991].

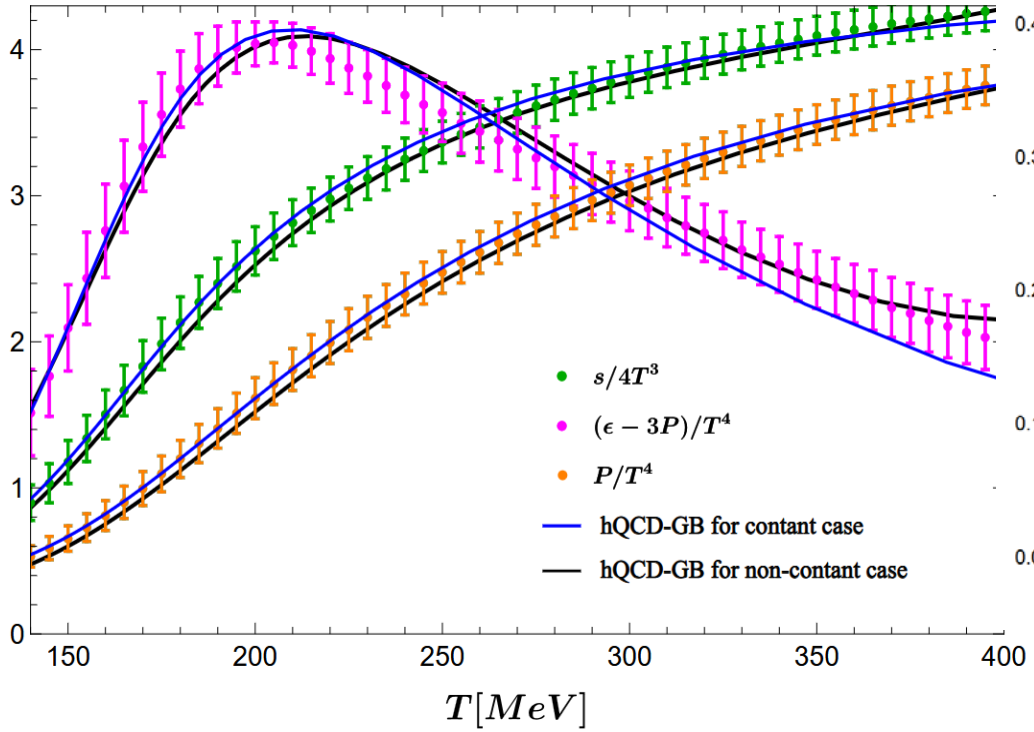
H. Zhang, D. Hou, T. Kojo and B. Qin, *Functional renormalization group study of the quark-meson model with ω meson*, *Phys. Rev. D* **96** (2017) 114029 [1709.05654].

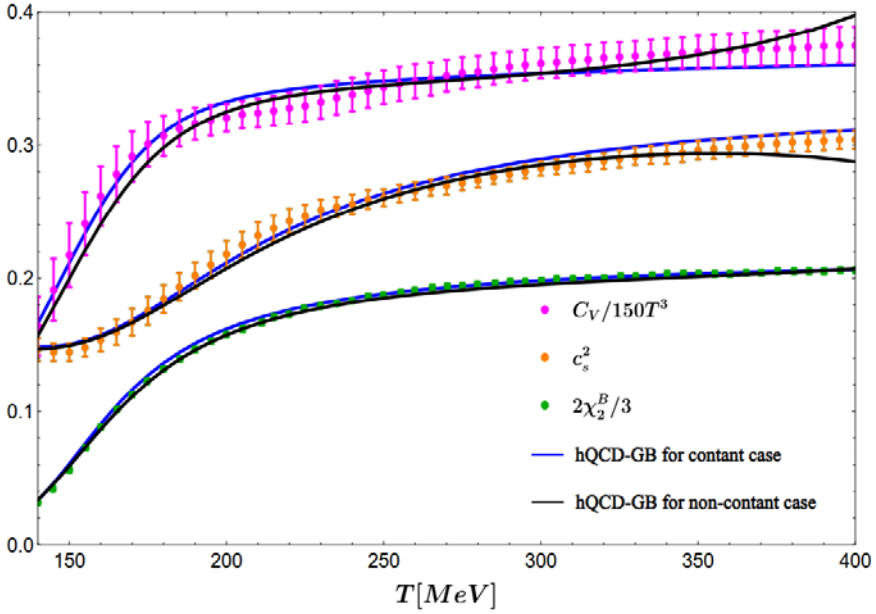
W.-j. Fu, X. Luo, J.M. Pawlowski, F. Rennecke and S. Yin, *Ripples of the QCD critical point*, *Phys. Rev. D* **111** (2025) L031502 [2308.15508].

Fix model parameters by EoS

c_1	c_2	c_3	c_4	c_5	c_6	b	ϕ_s	κ_N^2	α
0.733	0.0046	2.075	0.07	30	0.10957	-0.2744	985	$2\pi(1.49)$	-0.008

$$H(\phi) = 1 - \phi^6/(1+2\phi^6)$$

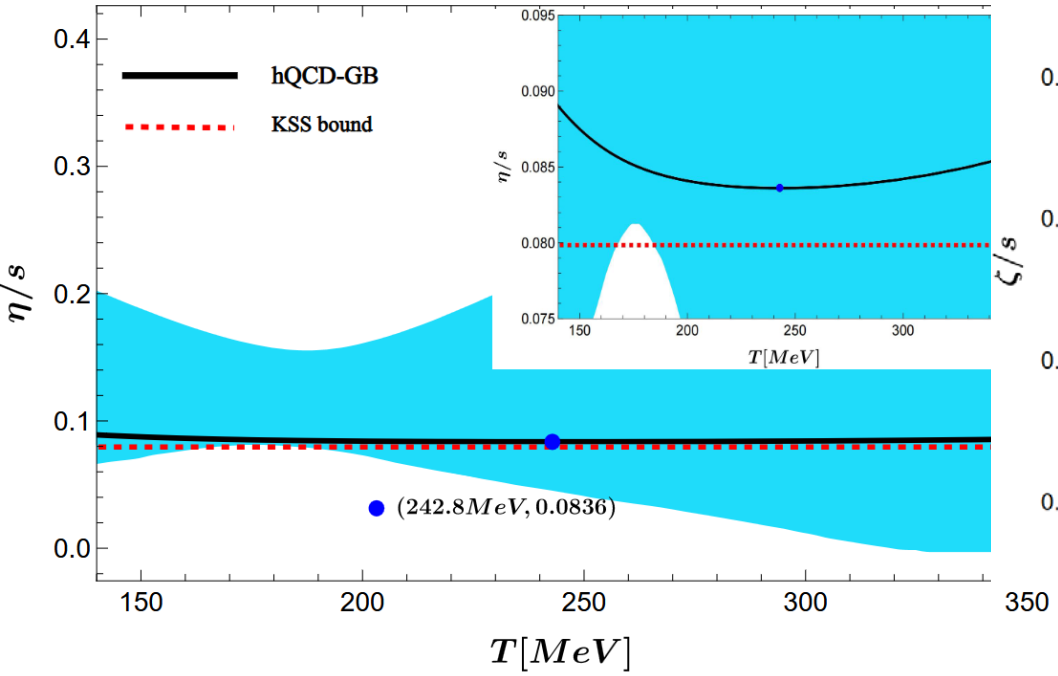


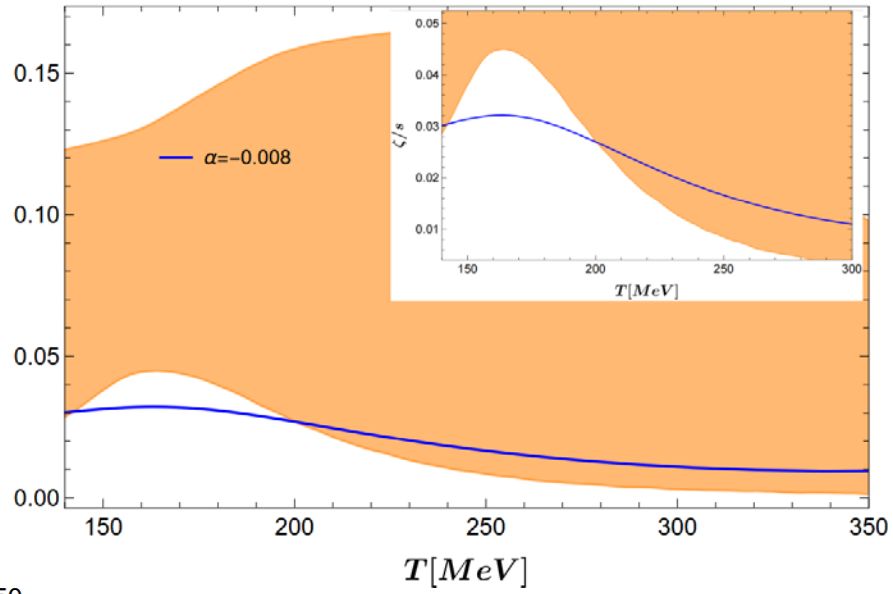


EoS have been Improved!

Chengwei Tong, Jie Zhou, Yuanxu Wang, Rong-Gen Cai, SH, Li Li, 2605. 23834[hep-th].

Shear & Bulk Viscosity of hQCD + $\alpha H(\phi) R_{GB}$





Bayesian analysis 90% credible

D. Everett, etc., Physical Review Letters 126, 10.1103 (2021).

Chengwei Tong, Jie Zhou, Yuanxu Wang, Rong-Gen Cai, SH, Li Li, 2605. 23834[hep-th].

Human Learning versus Machine Learning

Next: learn the higher-derivative coupling too

$$\alpha H(\phi) R_{GB}$$

LATTICE + JETSCAPE

EoS and transport constraints

NEURAL ODE

Learn H(φ) through the bulk evolution

PREDICTIONS

T–μB phase map and η/s, ζ/s

The magnetic study supplies the method; the transport study supplies the next function to infer.
We have to do global fitting.

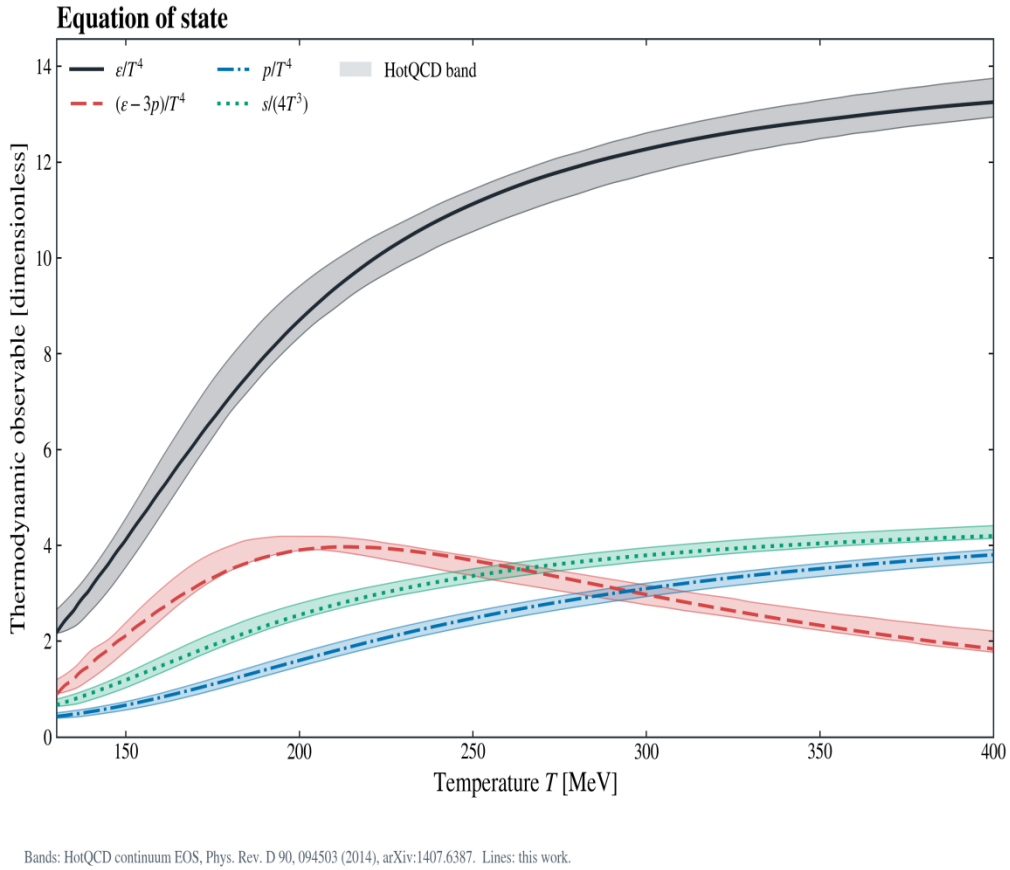
A newer candidate is calibrated against EoS and transport bands

Equation of state

97. 9 – 100%

reported band coverage

The candidate tracks the supplied HotQCD thermodynamic bands across all plotted observables.



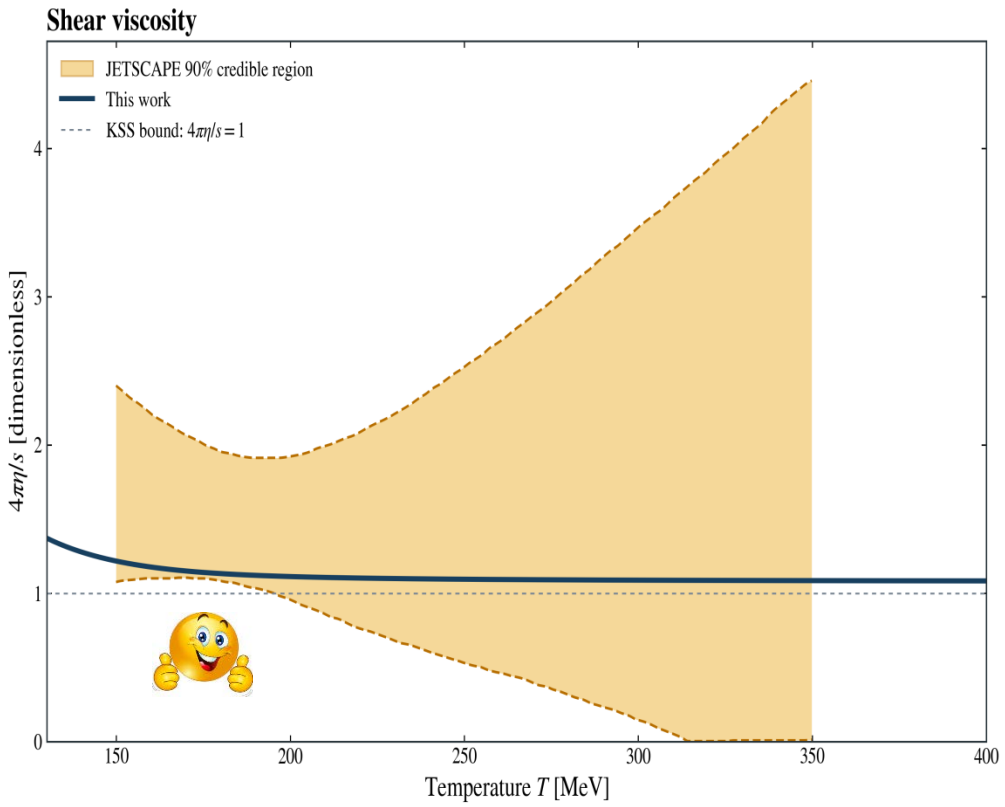
Working in Progress

The shear candidate fits the band — with an important caveat

Within the supplied JETSCAPE 90% region

Caveat

The high-T arm is near-flat.
This is not a strict U-shape.



Band: JETSCAPE Bayesian 90% credible region, Phys. Rev. Lett. 126, 242301 (2021), arXiv:2010.03928.

Working in Progress

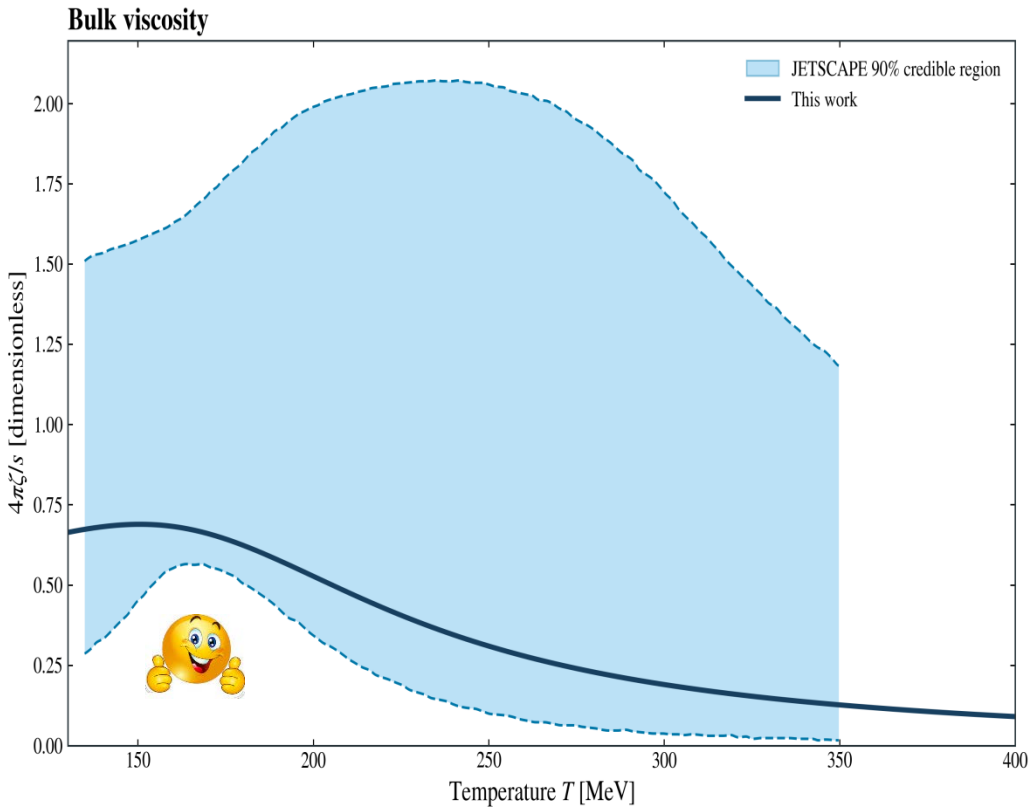
The bulk candidate retains a pronounced peak

Peak near the crossover

100%

reported band coverage

The candidate rises, peaks and decreases within the supplied credible region.



Band: JETSCAPE Bayesian 90% credible region, Phys. Rev. Lett. 126, 242301 (2021), arXiv:2010.03928.

Working in Progress

TAKE-HOME

Data learn the bulk action. Gravity connects the observables.



• Neural ODEs infer holographic coupling functions from lattice trajectories.
• The calibrated model predicts magnetic phase structure and critical behavior.
• Gauss–Bonnet corrections preserve the EoS while generating structured η/s and ζ/s.

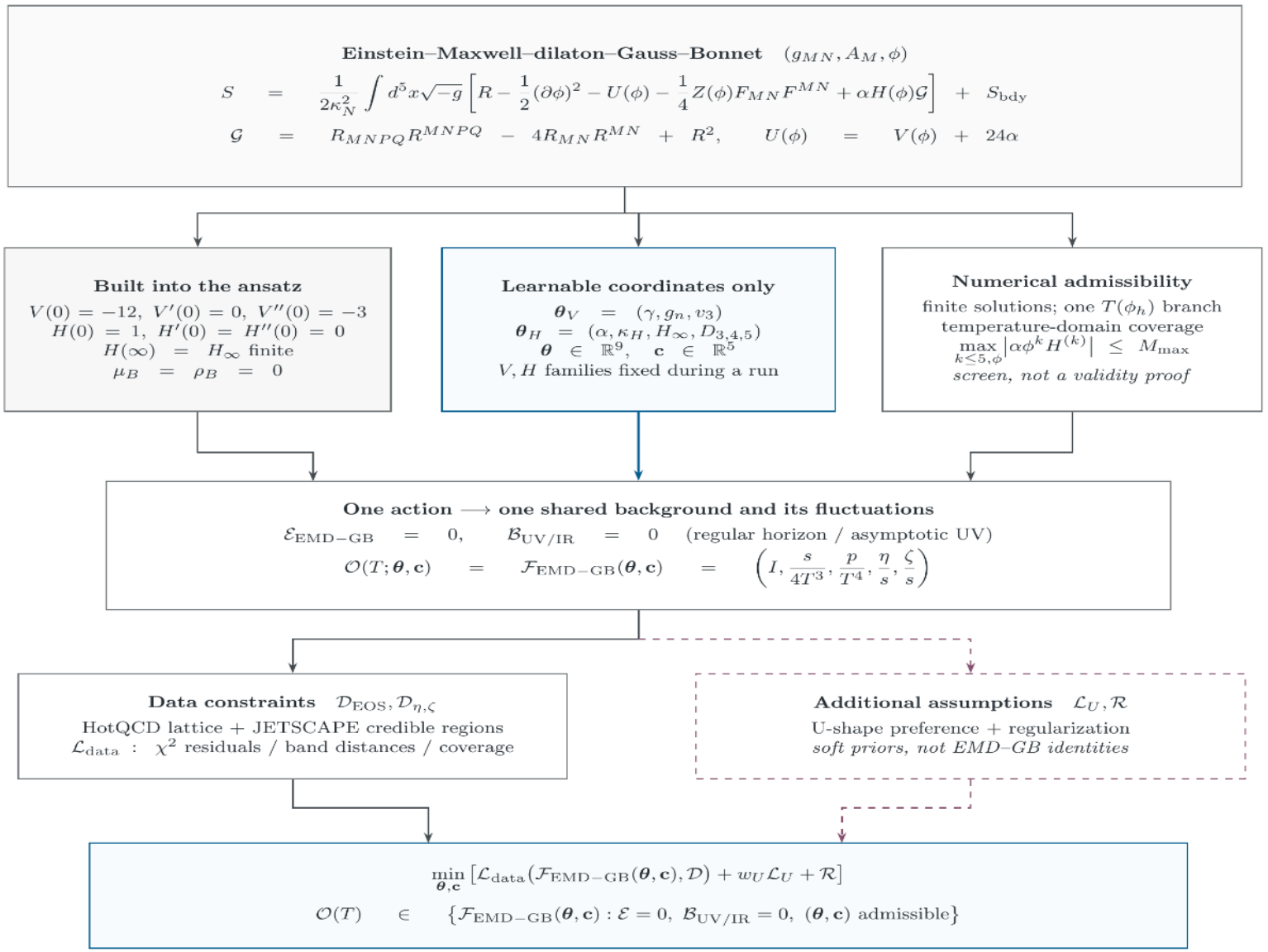
More data and More Physics constrains.

If you are interested in the associated code, please contact us by *hesong@nbu.edu.cn*.

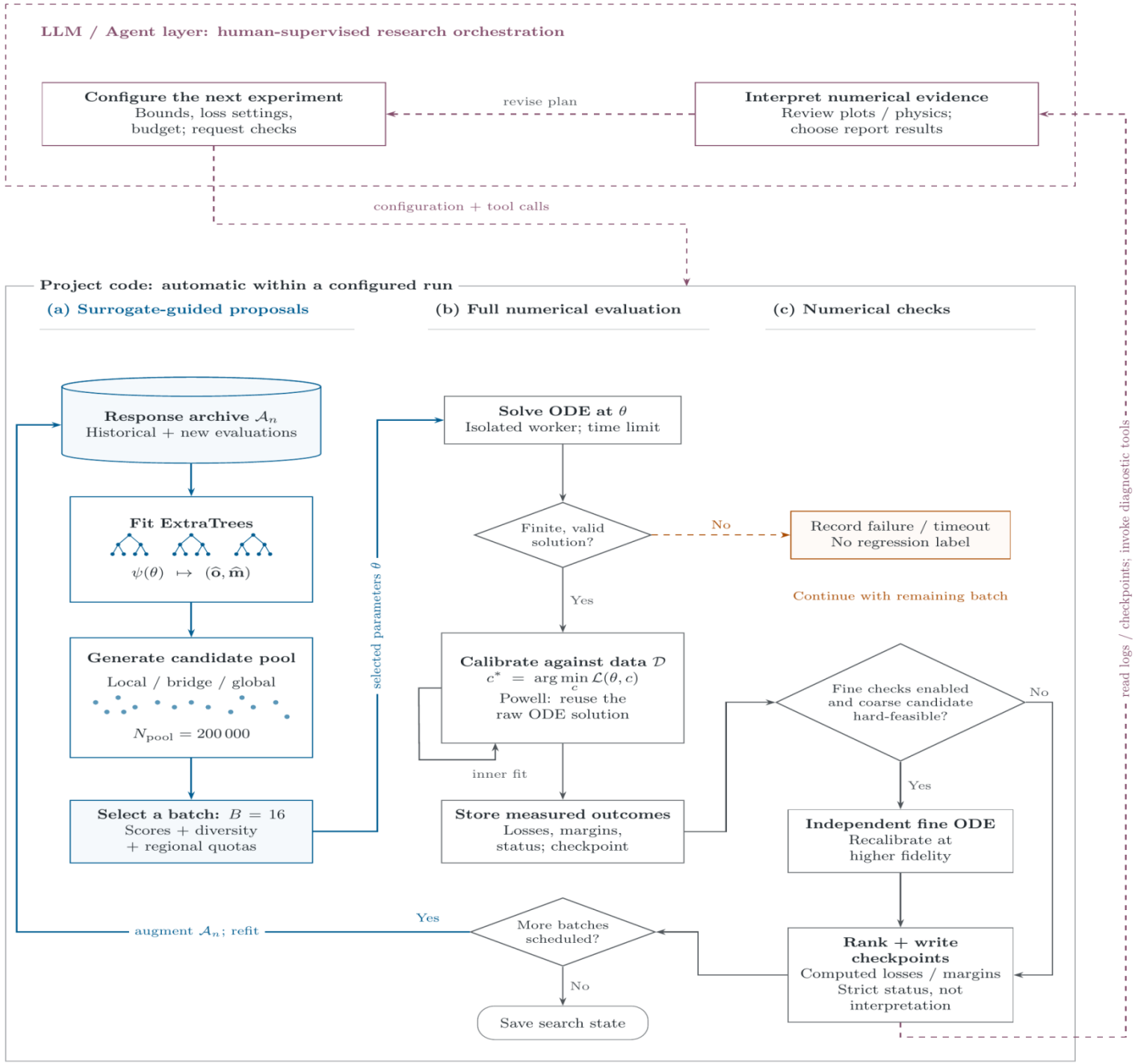
Song He
@Ningbo University

Thank You !

Physics-constrained inference in EMD–GB



Fit model parameters, not independent observable curves. Numerical EOM / boundary conditions use the retained approximation.



Solid: numerical code / data flow. Dashed purple: external configuration and evidence review.

Loss and acceptance metrics are computed by code, not judged by an LLM. No LLM call inside the search loop.