Diffusive dynamics: from EFT to observables

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华中核理论中心



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Content



EFT approach from action principle



Application in diffusive dynamics near CEP

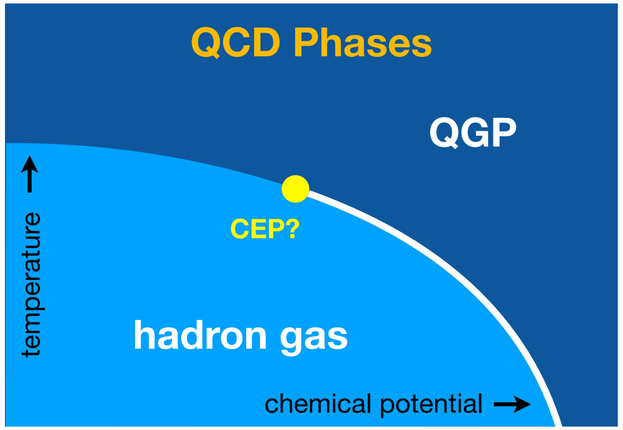
Phases of QCD

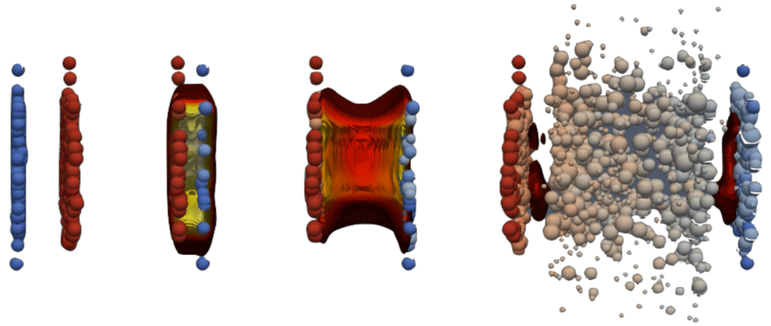
A tough and general question

What are the phases of QCD?

A simpler and more specific question

Where is the critical end point?





CGC Glasma QGP hadrons

diagram of QCD, and could result in a discovery of what is perhaps the most fundamental feature of the landscape. Once the critical point E is discovered, it will be prominent on the map of the phase diagram which will appear in any future textbook on QCD.

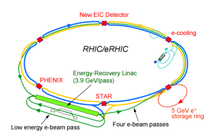
Sincerely,

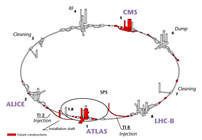
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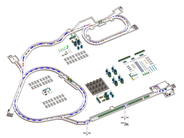
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University of Illinois at Chicago
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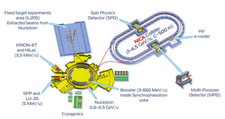
Prof. Frank Wilczek
Institute for Advanced Study, Princeton
wilczek@ias.edu

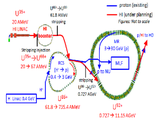








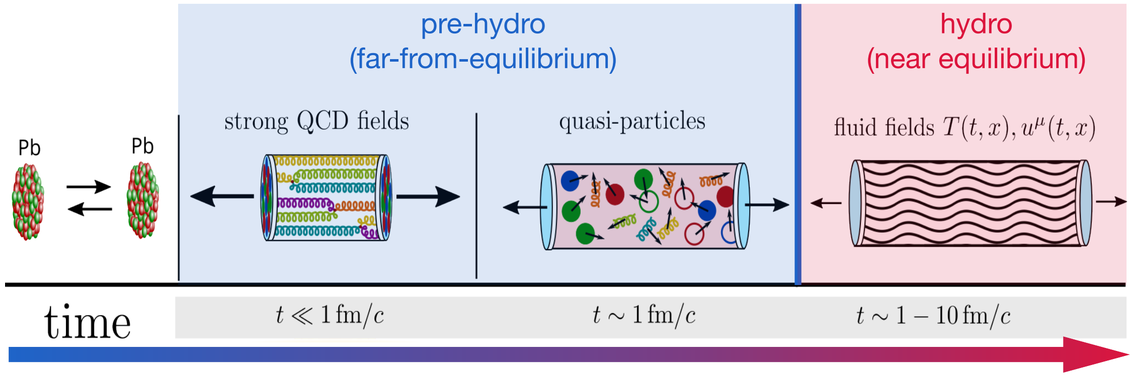




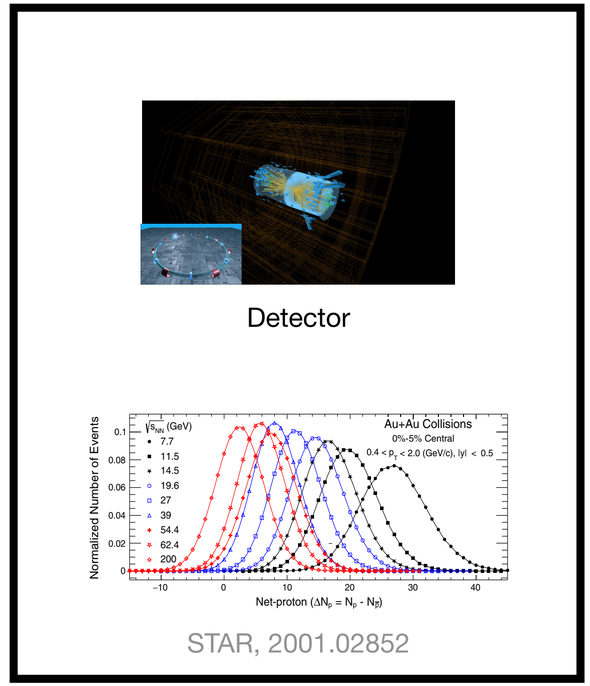
RHIC (US) LHC (EU) FAIR (DE) HIAF (CN) NICA (RU) J-PARC (JP)

2000 2010 2020 here we are 2030

HIC experiments constrain QCD properties

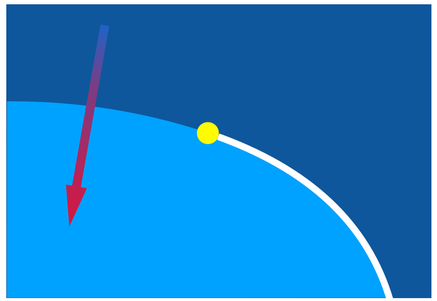


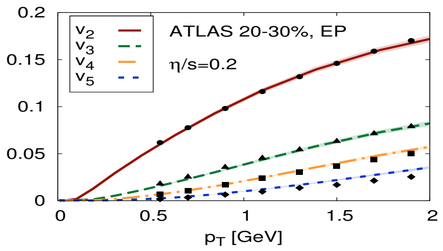
measure



constrain

input





Gale et al, 1301.5893

Far-from-equilibrium process

Non-hydro DOFs is an important measure in systems that are small or at early time

Stochastic process

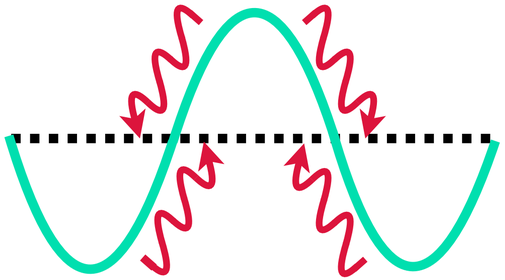
Fluctuations is an important measure in systems that are small or with critical point

Why diffusion?

- The slowest hydrodynamic mode near CEP is diffusive charges.

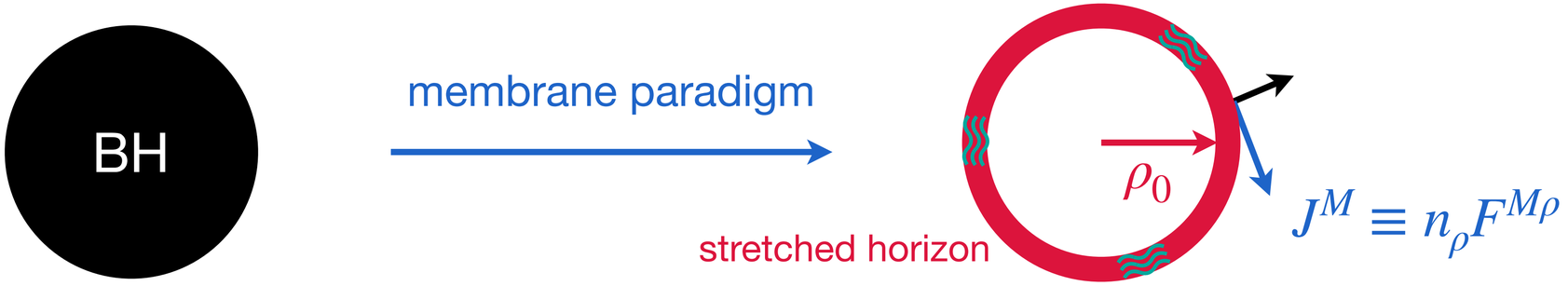
Fick's law (1855)

$$\partial_t n = D\partial^2 n$$



Bottom up EOM, applied near equilibrium

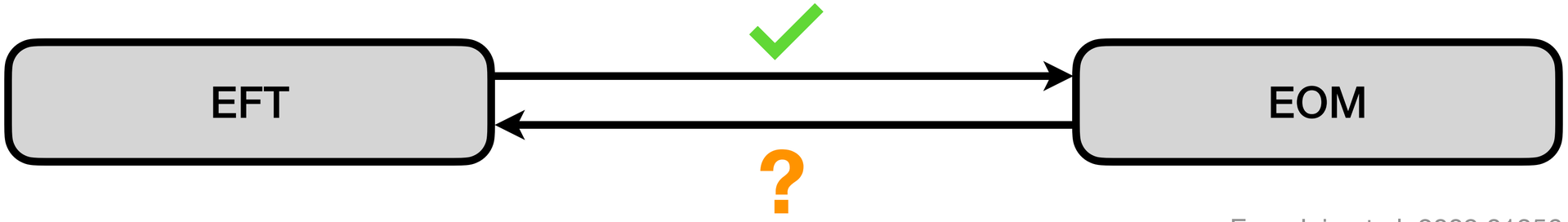
- Diffusive dynamics is ubiquitous.



Why EFT?

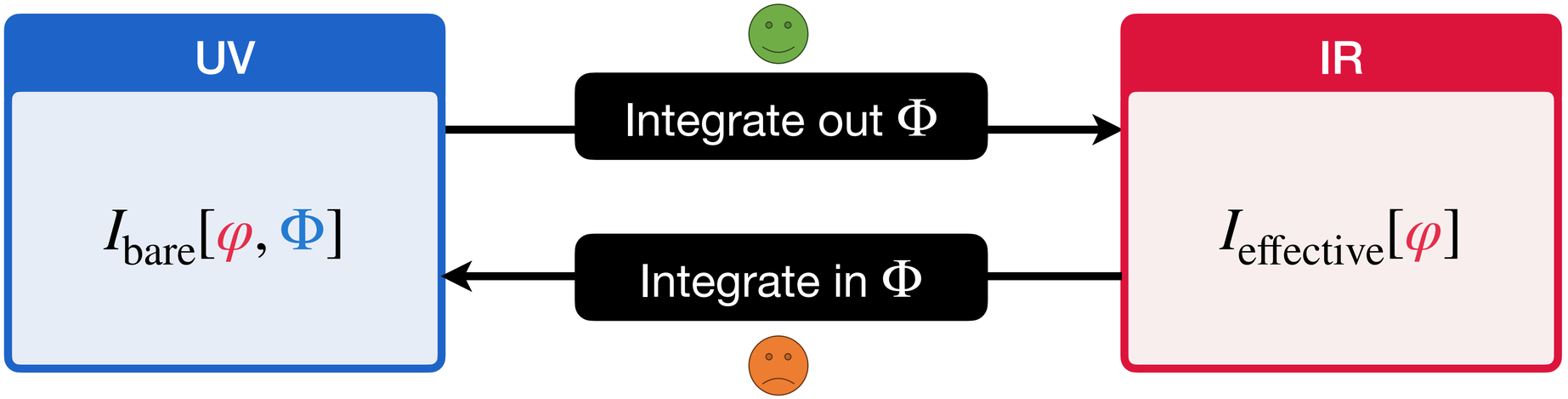
- EFT is convenient for nonlinear loop-calculations in QFT.





E.g., Jain et al, 2009.01356

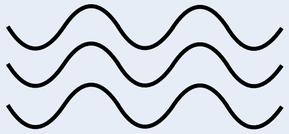
- Unlike universal near-equilibrium phenomena governed by conservation, far-from-equilibrium phenomena exhibit a rich landscape without obvious organizing principles.



Strongly and weakly coupled regimes far from equilibrium

Strongly coupled regime

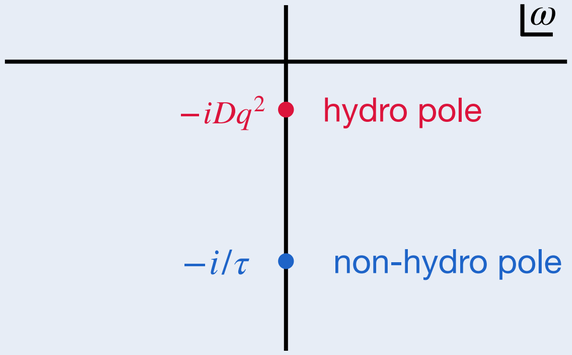
Collective excitations



Fluid-like

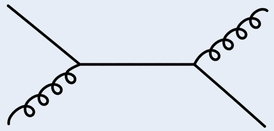
$$(1+\tau\partial_t)\partial_t n = D\partial^2 n$$
$$\tau \sim \ell_{\text{mic}} \sim 1/T$$

Mode structure: poles



Weakly coupled regime

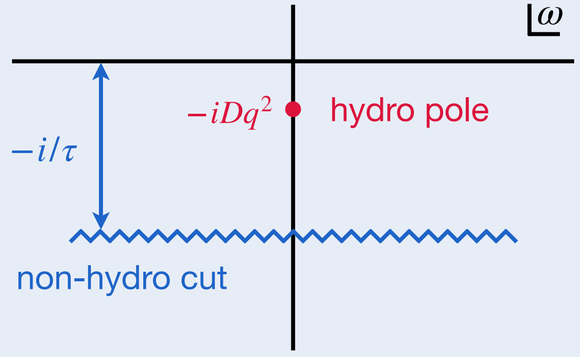
Quasiparticles



Particle-like

$$(\partial_t + v\cdot\partial) f(x,p) \approx -(f-f_{\text{eq}})/\tau$$
$$\tau \sim \ell_{\text{mfp}} \sim 1/(\alpha_s^2 T)$$

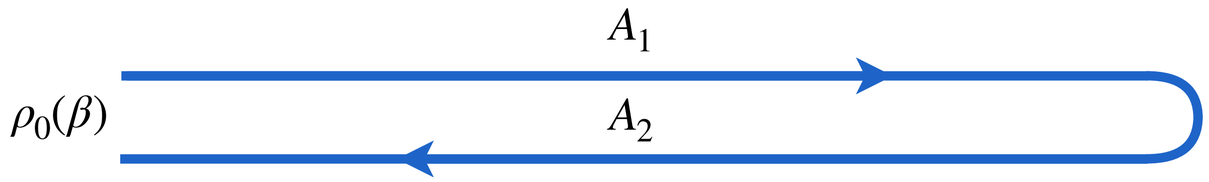
Mode structure: cuts



Schwinger-Keldysh formalism

$$Z[A_1, A_2] = \mathrm{Tr}\, U[A_1]\rho_0 U[A_2]^\dagger$$

$\mathrm{Tr}\,\rho = 1$



$$A \equiv A^r = \frac{1}{2}(A_1 + A_2) \qquad A^a = A_1 - A_2$$

Schwinger, 1961;
Keldysh, 1965
Martin et al, 1973
Glorioso et al, 1805.09331
…

Unitarity constraints

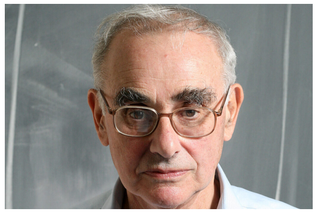
Reflectivity	$I^*[A\,, A_a] = -\,I[A\,, -A_a]$
Normalizability	$I[A\,, A_a = 0] = 0$
Positivity	$\mathrm{Im}\, I[A\,, A_a] \geq 0$

Kubo-Martin-Schwinger (KMS) invariance

$$I[A, A_a] = I[\tilde{A}, \tilde{A}_a] \quad \Longrightarrow \quad \tilde{A}(-t) = A(t), \quad \tilde{A}_{1,2}(-t) = A_{1,2}(t \pm i\beta/2)$$

Explicit and emergent symmetries

The action is required to be invariant under

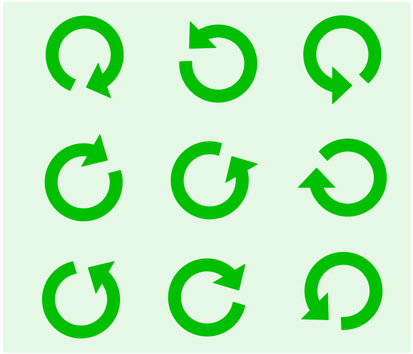


Jeffrey Goldstone
1933.09.03-2026.10.12

- *Explicit* **U(1) gauge transformation**

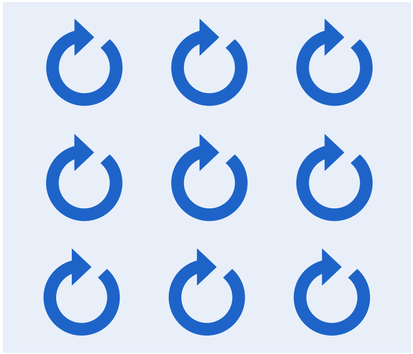
$$\phi \to \phi + \theta(x), \quad A_\mu \to A_\mu - \partial_\mu \theta(x), \quad \Longrightarrow \quad \mathbb{A}_\mu = A_\mu + \partial_\mu \phi \to \mathbb{A}_\mu$$

- *Emergent* **shift** due to SSB



Unbroken (disordered) phase: local/independent shift

$$\phi \to \phi + \lambda(\mathbf{x}), \quad \mathbb{A}_\mu \to \mathbb{A}_\mu + \partial_\mu \lambda(\mathbf{x})$$



Broken (ordered) phase: global/common shift

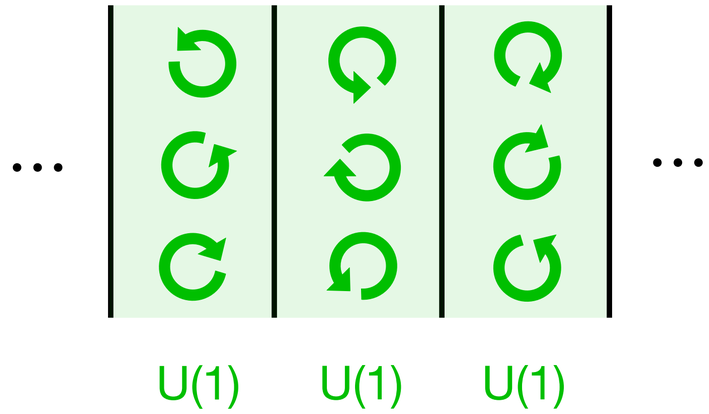
$$\phi \to \phi + \lambda, \quad \mathbb{A}_\mu \to \mathbb{A}_\mu$$

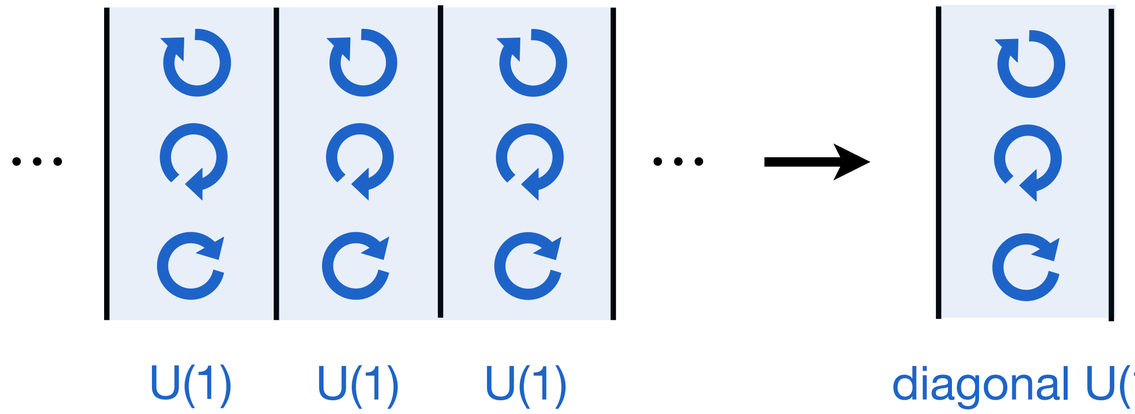
Scenarios for multiple U(1)s

Generalization to a collection of systems (layers) with multiple U(1)s. XA, Brants, Heller and Yin, 2511.11555, PRL

I All U(1)s are **unbroken**

II All U(1)s are **broken** into a diagonal U(1)



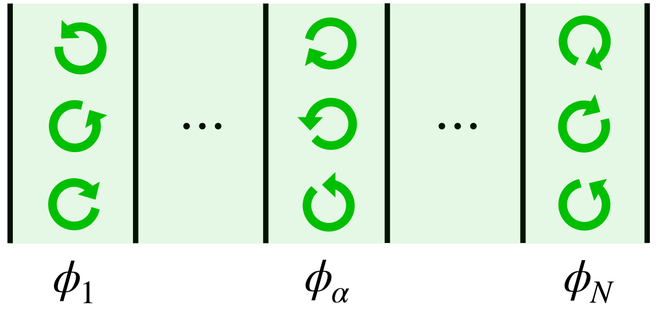


III **Hybridization** of the two

Scenario I: all U(1)s are unbroken

Layer-dependent shift

$$I_{\mathrm{U}}[\{\phi_\alpha, \phi^a_\alpha\}] = I_{\mathrm{U}}[\{\phi_\alpha + \lambda_\alpha(\mathbf{x}), \phi^a_\alpha\}]$$

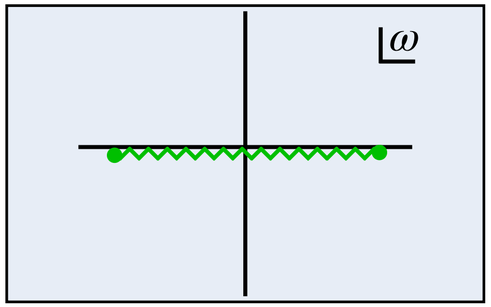


Action for unbroken scenario

$$I_{\mathrm{U}}[\{\phi_\alpha\}] = \sum_{\alpha=1}^{N} \int_x \chi_0 \Big\{ (\partial_t \phi)\partial_t \phi^a - \cancel{(\partial_i \phi)\partial_i \phi^a} + \frac{1}{2}\left[(\mathbf{v}\cdot\partial)\phi\, \partial_t \phi^a + (r \leftrightarrow a)\right] \Big\}_\alpha$$

EOM: kinetic-like theory

Boltzmann equation for $\delta f = \chi_0(\partial_t \phi)$

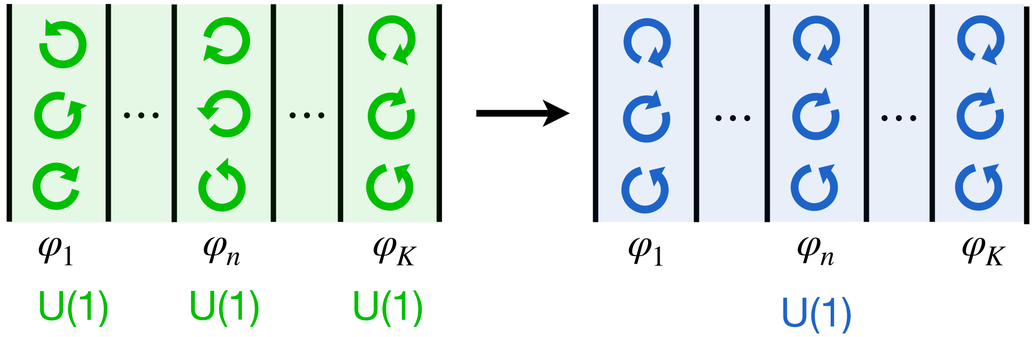


Delacretaz et al, 2203.05004
York et al, 0811.0729
Mo et al, 2512.23960

Scenario II: all U(1)s are broken into a diagonal U(1)

Layer-independent (diagonal) shift

$$I_{\mathrm{B}}[\{\mathbb{A}_n, \mathbb{A}_n^a\}] = I_{\mathrm{B}}\{\mathbb{A}_n + \partial\lambda(x), \mathbb{A}_n^a\}]$$



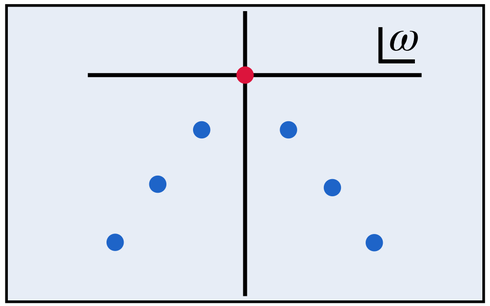
Action for broken scenario

$$I_{\mathrm{B}}[\{\mathbb{A}_n\}] = \sum_{n=1}^{K} \int_x \left\{ f^2 (\Delta\mathbb{A})_t (\Delta\mathbb{A})_t^a - g^2 (\Delta\mathbb{A})_i (\Delta\mathbb{A})_i^a + \epsilon\, \mathbf{E}\cdot\mathbf{E}^a - \kappa\, \mathbf{B}\cdot\mathbf{B}^a + \sigma\, \mathbf{E}\cdot\mathbb{A}^a \right\}_n$$

$$\Delta\mathbb{A}_n = \mathbb{A}_{n+1} - \mathbb{A}_n$$

EOM: holographic-like theory

Maxwell equations for hidden photon $\mathbb{A}_\mu$



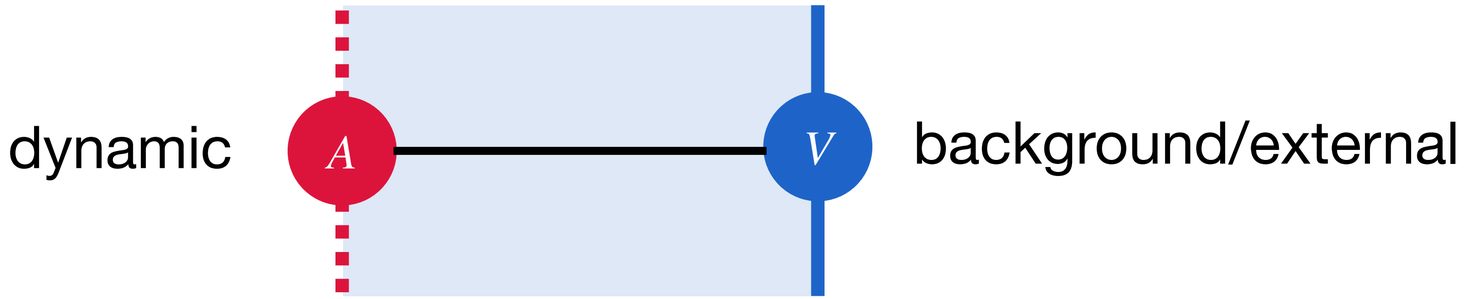
Nickel et al, 1009.3094

Scenario II: all U(1)s are broken into a diagonal U(1)

$K = 1$ (minimum configuration) reproduces Maxwell-Cattaneo (MC) diffusion.

Maxwell 1867; Cattaneo, 1948; Nickel et al, 1009.3094

$$U_A(1) \times U_V(1) \to U(1)$$



Morse Diagram

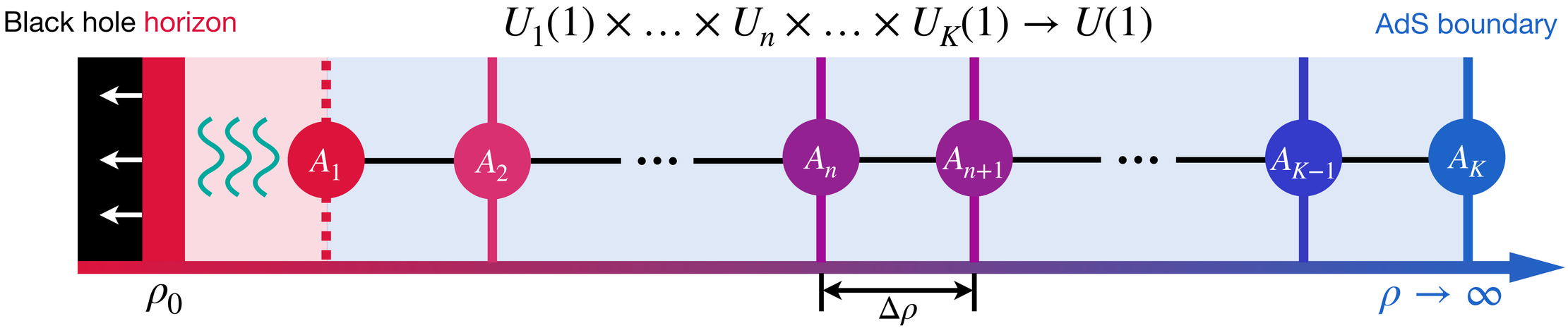
Action for MC

$$I_{\mathrm{B}} \xrightarrow{K=1} \int_x \left(f^2 (\mathbb{V}-\mathbb{A})_t (\mathbb{V}-\mathbb{A})^a_t - g^2 (\mathbb{V}-\mathbb{A})_i (\mathbb{V}-\mathbb{A})^a_i + \epsilon\, \mathbf{E}\cdot\mathbf{E}^a - \kappa\, \mathbf{B}\cdot\mathbf{B}^a + \sigma\, \mathbf{E}\cdot\mathbb{A}^a \right)$$

Scenario II: all U(1)s are broken into a diagonal U(1)

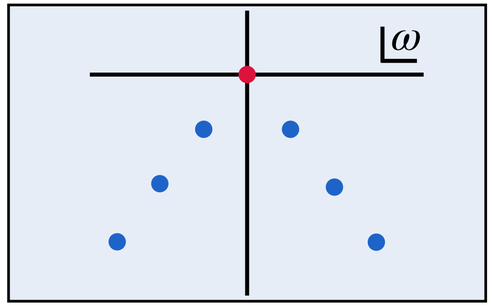
$K \to \infty$ (continuous limit) reproduces holography via dimensional deconstruction.

Arkani-Hamed et al, 0104005

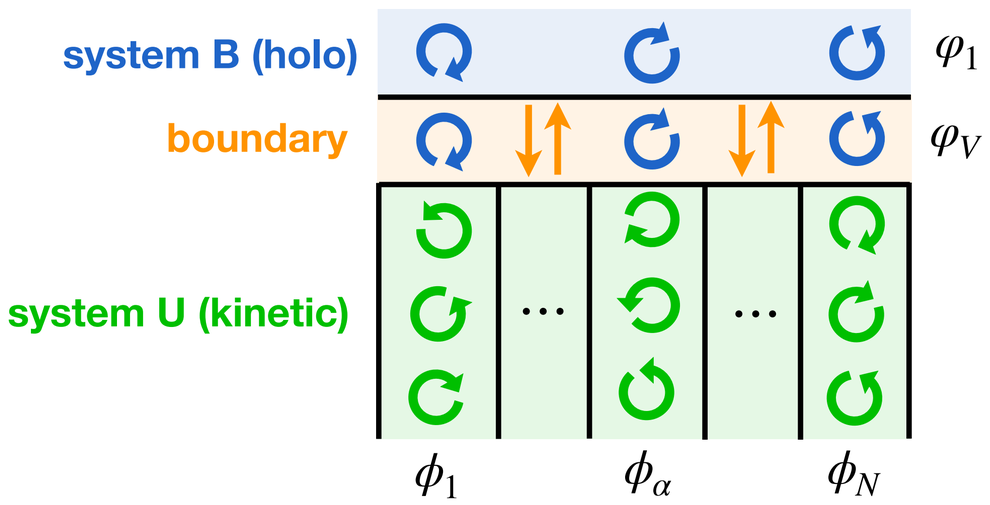


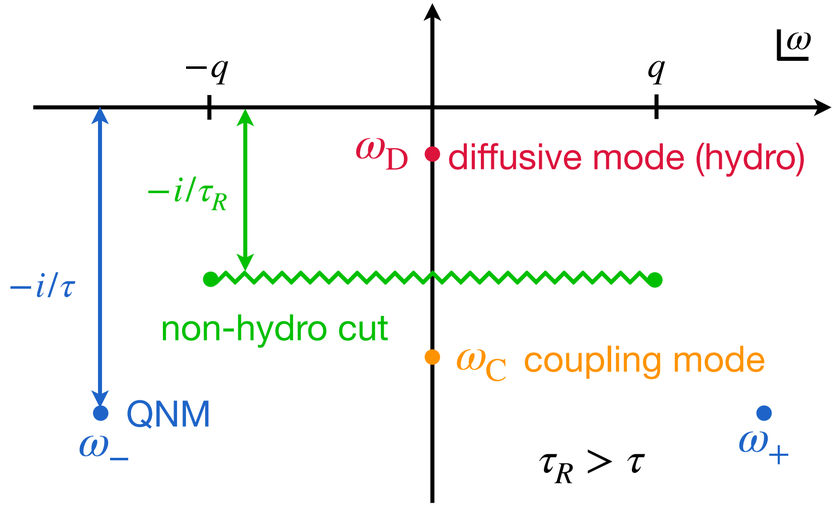
Maxwell action for 5D holography

$$I_{\mathrm{B}} \xrightarrow{K\to\infty} -\frac{1}{4}\int_{x,\rho}\sqrt{-g}\, g^{MM'} g^{NN'} F_{MN} F^{a}_{M'N'}$$



Scenario III: Hybridization of the two





Action for hybrid scenario

$$I_H = I_U[\{\phi_\alpha\}] + I_B[\{\varphi_1, \varphi_V\}] + \Delta I[\phi_\alpha, \varphi_V]$$

$$\Delta I = -\frac{\chi_U}{\tau_R} \sum_\alpha \int_x (\partial_t \phi_\alpha - \partial_t \varphi_V)(\phi_\alpha^a - \varphi_V^a)$$

- Mutual equilibration of two open systems with chemical potential $\mu_U \sim \partial_t \phi_\alpha$, $\mu_B = \partial_t \varphi_V$.
- Transition between strongly and weakly coupled regimes.

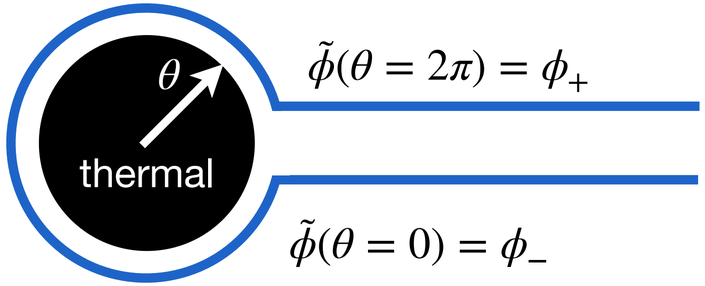
Fluctuations and shift symmetry

XA, Brants, Heller and Yin, 2511.11555, PRL

"Integrated in" action **invariant** under **shift**

Shift on auxiliary field

$$\tilde{I}[\tilde{\phi}(\theta,x)] = \tilde{I}[\tilde{\phi}(\theta,x) + \lambda(\mathbf{x})]$$



Local action for $\tilde{\phi}$

$$\tilde{I}[\tilde{\phi}] = \int_x \int_0^{2\pi} d\theta \, \frac{\sigma}{2} \left[i(\partial_\theta \tilde{\phi})^2 - \frac{1}{2\pi T} (\partial_\theta \tilde{\phi}) \, \partial_t \tilde{\phi} \right]$$

integrate out $\tilde{\phi}$

Nonlocal action for ϕ

$$I_{\text{diss}} + I_{\text{fluct}} = \int_{\mathbf{x},\omega} \phi^a(-\omega) \left[E\phi(\omega) + iQ\phi^a(\omega) \right]$$

KMS/FDR

$$Q = \coth\left(\frac{\omega}{2T}\right) \text{Im}E$$

Content



EFT approach from action principle

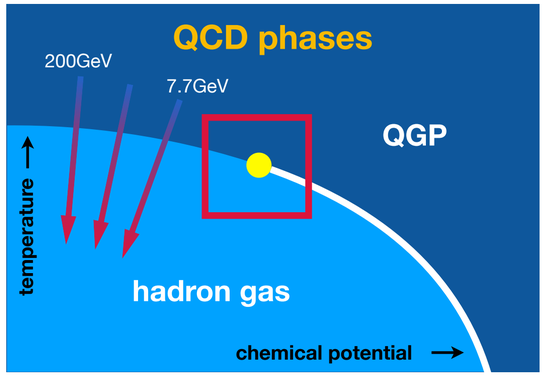


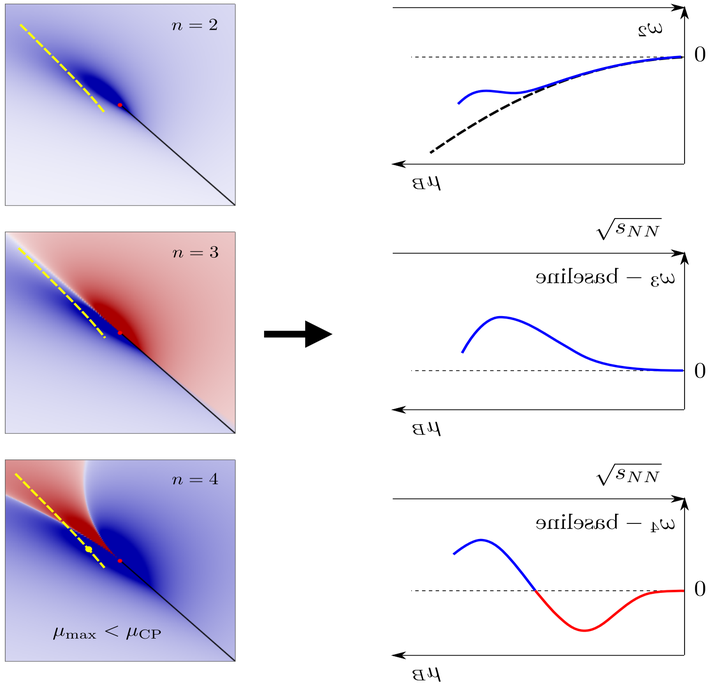
Application in diffusive dynamics near CEP

Cumulant as a probe of CEP

Theory

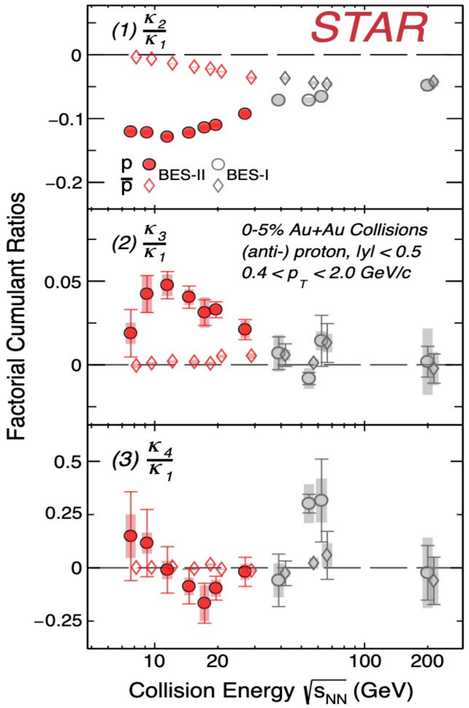
$$C_n \sim \frac{\partial^n P(\mu, T)}{\partial \mu^n}$$





Experiment

$$C_n = \langle (N - \langle N \rangle)^n \rangle_{\rm c}$$



STAR, 2504.00817; Stephanov, 2410.02861

“Dynamical calculations are called for”

PHYSICAL REVIEW LETTERS **135,** 142301 (2025)

Editors' Suggestion | Featured in Physics

Precision Measurement of Net-Proton-Number Fluctuations in Au + Au Collisions at RHIC

(STAR Collaboration)

(Received 26 March 2025; accepted 18 July 2025; published 29 September 2025)

We report precision measurements on cumulants (C_n) and factorial cumulants (κ_n) of (net) proton number distributions up to fourth order in Au + Au collisions over center-of-mass energies $\sqrt{s_{NN}} =$ 7.7–27 GeV from phase II of the Beam Energy Scan program at RHIC. (Anti)protons are selected at midrapidity ($|y| < 0.5$) within a transverse momentum range of $0.4 < p_T < 2.0$ GeV/c. Relative to various noncritical-point model calculations and peripheral collision 70%–80% data, the net proton C_4/C_2 measurement in 0%–5% collisions shows a minimum around 19.6 GeV for significance of deviation at ~2–5σ. A minimum in C_4/C_2 with respect to a noncritical baseline is expected to be a characteristic feature of the signature associated with a critical point in the QCD phase diagram. In addition, deviations from noncritical baselines around the same collision energy region are also seen in proton factorial cumulant ratios, especially in κ_2/κ_1 and κ_3/κ_1. Dynamical model calculations including a critical point are called for in order to understand these precision measurements.

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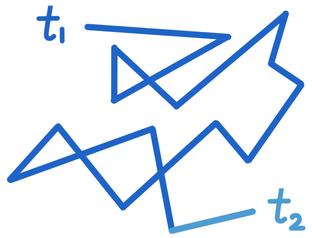
Dynamic approaches

- Different approaches are complementary and related

1) Stochastic sampling

$$\dot{\varphi}_i = F_i + \xi_i, \quad \langle \xi_i(x)\xi_j(x')\rangle = 2Q_{ij}\delta_{xx'}$$

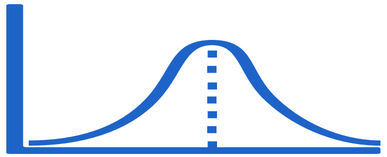
Straightforward but consuming; noise problem



2) Deterministic equations

$$\dot{\varphi}_i = Q_{ij}(S_{,j} + S_{,jk\ell}G_{k\ell}) + \ldots, \quad S = \log P_{\rm eq}$$

analytic control; need truncation



XA, Basar and Stephanov, 2604.14110, PRC

3) Schwinger-Keldysh (SK) formalism

$$\mathbb{L}(\varphi_r, \psi_a) = (\dot{\varphi}_r - F_i(\varphi_r))\varphi_i^a + i\varphi_i^a Q_{ij}\varphi_j^a$$

Top-down; less straightforward



SK Lagrangian for MC

MC Lagrangian ($K = 1$ case of holography-like theory in the classical limit $\omega/T \ll 1$)

$$\mathscr{L} = \underbrace{f^2\,(\mathbb{V}-\mathbb{A})_t}_{n}(\mathbb{V}-\mathbb{A})^a_t \;\underbrace{-g^2(\mathbb{V}-\mathbb{A})_i}_{J_i}(\mathbb{V}-\mathbb{A})^a_i + \underbrace{i\lambda\mathbb{A}_{ai}\tilde{\mathbb{A}}^i_a}_{\lambda\,(\beta E^i-\partial^i\alpha)\mathbb{A}^a_i+i\lambda\mathbb{A}_{ai}\mathbb{A}^i_a} + \text{maxwell}$$

$$\tilde{\mathbb{A}}^i_a = \mathbb{A}^i_a + i\beta\partial_t\mathbb{A}^i_r$$

zero source $\mathbb{V}^\mu_a = \partial^\mu\varphi_a$

integrate out J^i at $g \to \infty$

MC Lagrangian (bottom-up MSR)

$$\mathscr{L} = \underbrace{-(\partial_t n + \partial_i J^i)\varphi_a}_{\text{conservation}} \qquad \tau \sim \lambda/g^2$$

$$\underbrace{-(J^i + \tau\partial_t J^i + \lambda\partial^i\alpha)\mathbb{A}_{ai}}_{\text{MC equation}} + \underbrace{i\lambda\mathbb{A}^i_a\mathbb{A}_{ai}}_{\text{noise}}$$

Fick Lagrangian

$$\mathscr{L} = n\mathbb{V}^a_t + i\lambda\mathbb{V}_{ai}(\mathbb{V}^i_a + i\partial^i\alpha)$$

$$= \underbrace{-(\partial_t n + \partial_i\lambda\partial^i\alpha)\varphi_a}_{\text{Fick equation}} \underbrace{- i\varphi_a\partial_i\lambda\partial^i\varphi_a}_{\text{noise}}$$

EFT constraint on transport coefficients

SK Lagrangian

$$\mathscr{L} = n\mathbb{V}_t^a + J^i(\mathbb{V} - \mathbb{A})_i^a + i\lambda\mathbb{A}_{ai}\tilde{\mathbb{A}}_a^i$$

$$\tilde{\mathbb{A}}_a^\mu = \mathbb{A}_a^\mu + i\beta\partial_t\mathbb{A}_r^\mu$$
$$\tilde{\mathbb{V}}_a^\mu = \mathbb{V}_a^\mu + i\beta\partial_t\mathbb{V}_r^\mu$$

KMS invariance requires $\tilde{\mathscr{L}} - \mathscr{L} = i\beta\left[n\partial_t\mu - g^{-2}J_i\partial_t J^i\right] = i\beta\partial_t p$

$$\Longrightarrow \quad dp = nd\mu - \frac{1}{2g^2}d\mathbf{J}^2 \quad \Longrightarrow \quad \left(\frac{\partial g^{-2}}{\partial\mu}\right)_{\mathbf{J}} = 2\left(\frac{\partial n}{\partial \mathbf{J^2}}\right)_\mu$$

The independence of n and $\mathbf{J}$ requires $\left(\frac{\partial g^{-2}}{\partial\mu}\right)_{\mathbf{J}} \Longrightarrow g^2 = \frac{\tau}{\sigma} = \text{constant}$

Jain et al, 2309.00511
See also Michal's talk on NTFP

This implies that τ and σ must have the same scaling behavior near CEP.

Diffusion dynamics: from Fick to MC

Charge conservation

$$\partial_t n + \vec{\partial} \cdot \vec{J} = 0$$

Fick

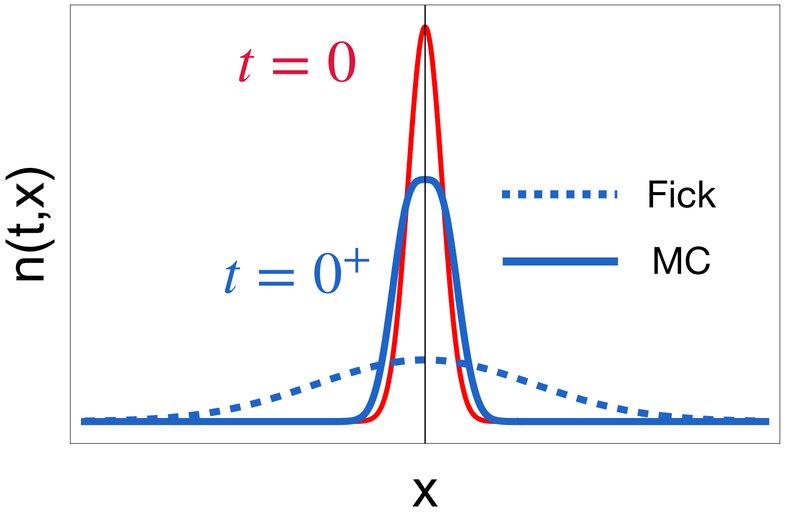
$$\vec{J} = -D\vec{\partial} n$$

or

Maxwell-Cattaneo (MC)

$$(\tau\partial_t + 1)\,\vec{J} = -D\vec{\partial} n$$

To close the system we use EOS st $D = \lambda\alpha'$



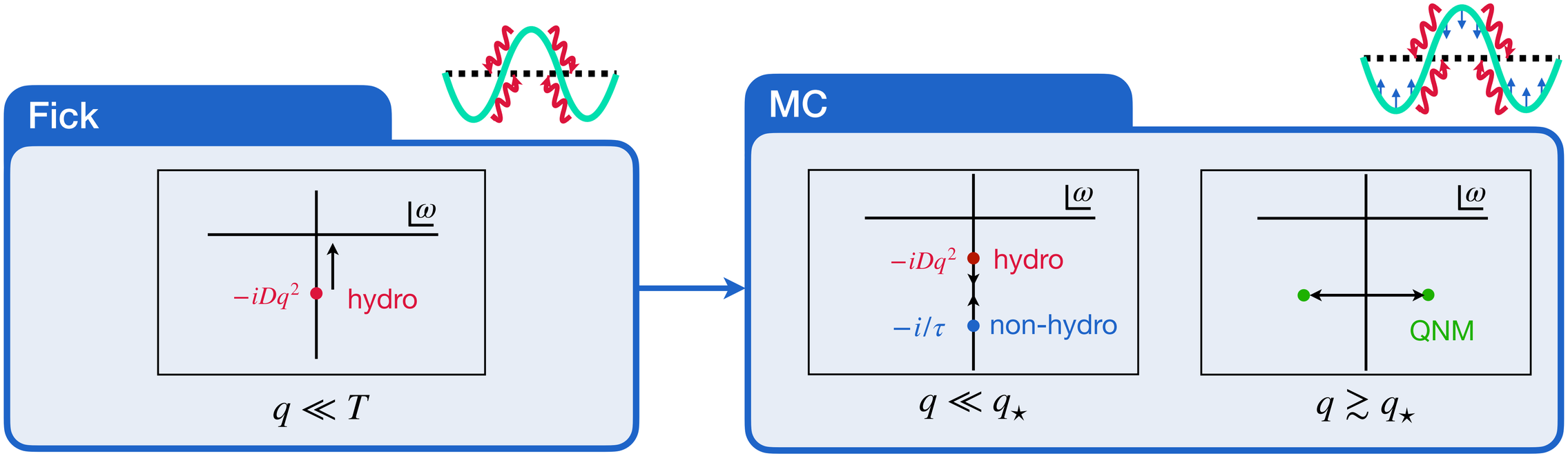
$\tau \to 0$ limit: Fick

Mode analysis of MC diffusion

- The modes

$$G_R(\omega, \mathbf{q}) \sim \frac{1}{-i\omega(1 - i\omega\tau) + Dq^2} \quad \Longrightarrow \quad \omega_\pm = \frac{-i \pm i\sqrt{1 - 4\tau D q^2}}{2\tau}, \quad q_\star = \frac{1}{2\sqrt{\tau D}}$$

- MC equation retains one non-hydro mode and thus applies beyond hydro:

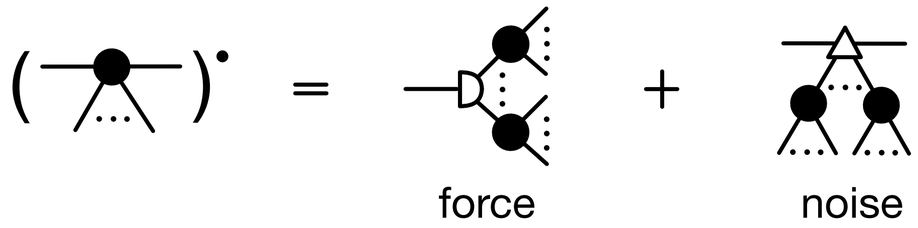


Schwinger–Dyson equations for multi-point functions

- Equations for multi-point functions are given by variation method.

$$\int \mathscr{D}\chi \mathscr{D}\chi_a \frac{\delta}{\delta \chi_a}\left(e^{iI[\chi,\chi_a]}\mathcal{O}[\chi]\right) = \Big\langle \underbrace{\frac{\delta I[\chi,\chi_a]}{\delta\chi_a}}_{\text{EOM for } \chi}\mathcal{O}[\chi]\Big\rangle = 0$$

$$\sum_{i=1}^{N}\left\langle (\text{EOM for } \delta\chi_i)\prod_{j\neq i}\delta\chi_j\right\rangle_c \quad\Longrightarrow\quad \partial_t\left\langle\prod_{i=1}^{N}\delta\chi_i\right\rangle_c = \sum_{i=1}^{N}\left\langle \delta F[\chi_i]\prod_{j\neq i}\delta\chi_j\right\rangle_c$$



Sogabe et al, 2111.14667
Abbasi et al, 2212.11499

Equation for MC correlators

- Equation for 2-pt correlators in the longitudinal channel for n and $g \equiv \dot{n} = = -\partial \cdot J$:

$$W_{ij\ldots}(\mathbf{q}_i, \mathbf{q}_j) = \text{Wigner Transform } [\langle \delta\chi_i(\mathbf{x}_1)\delta\chi_j(\mathbf{x}_2)\ldots\rangle], \quad \chi = n, g$$

EOM for 2-pt functions (MC) Abbasi, XA and Wu, 2606.02446, PRD

$$\partial_t W_{nn} = 2W_{ng},$$
$$\partial_t W_{ng} = W_{gg} - \tau^{-1} W_{ng} - u^2 q^2 W_{nn},$$
$$\partial_t W_{gg} = -2\tau^{-1} W_{gg} - 2u^2 q^2 W_{ng} + 2\lambda\tau^{-2} q^2 .$$

$$u^2 = \frac{D}{\tau} = \frac{g^2}{f^2}$$

$\tau \to 0$

EOM for 2-pt functions (Fick) XA, Basar, Stephanov and Yee, 2009.10742, PRL

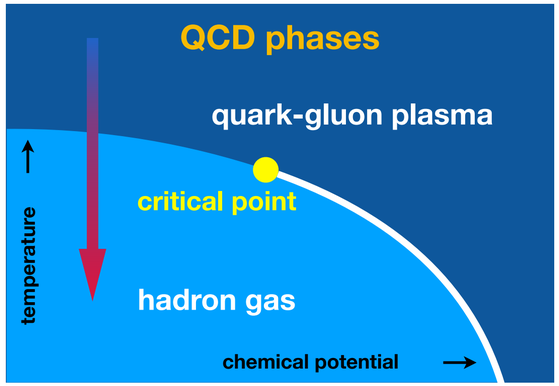
$$\partial_t W_{nn} = -2Dq^2 W_{nn} + 2\lambda q^2 .$$

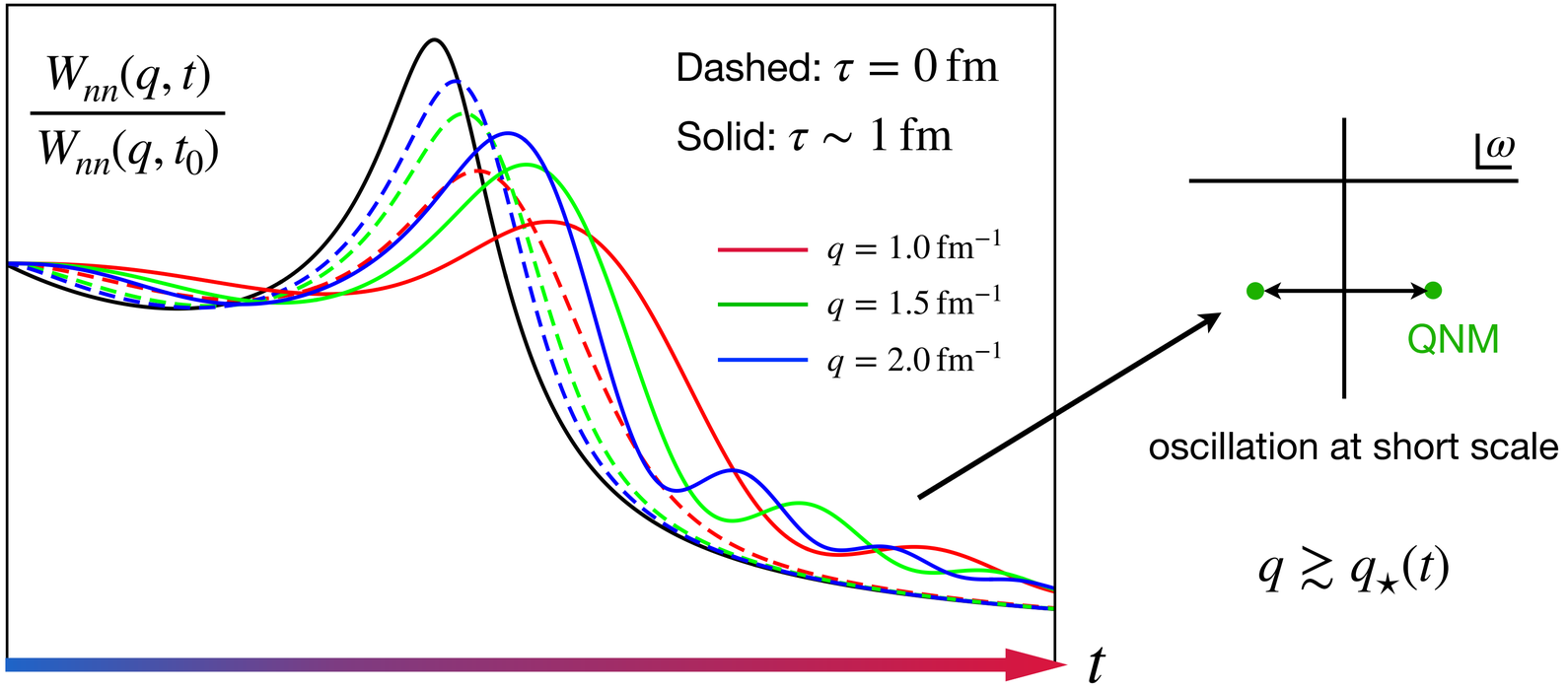
The results for higher-point functions are straightforward.

2-pt function dynamics

- Evolution at a particular wavenumber q:

Abbasi, XA and Wu, 2606.02446, PRD



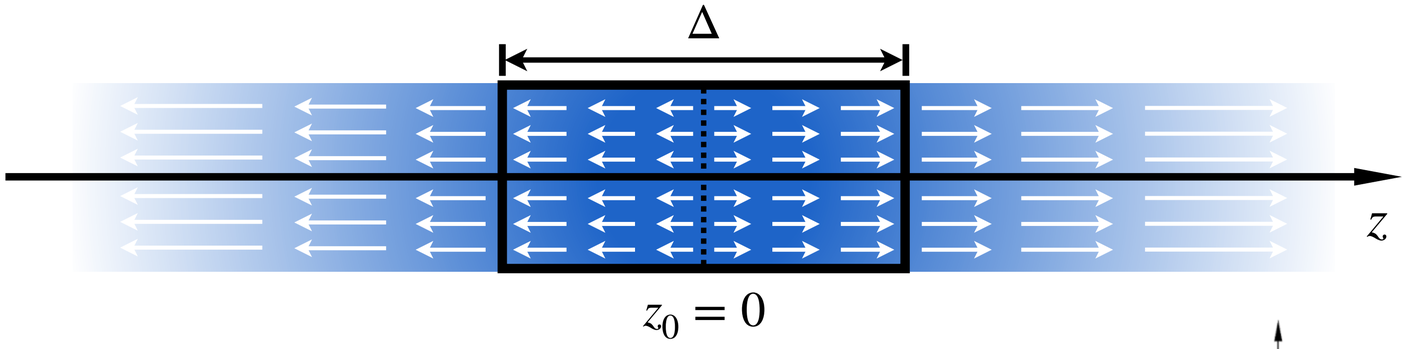


The behavior of higher-point functions is similar.

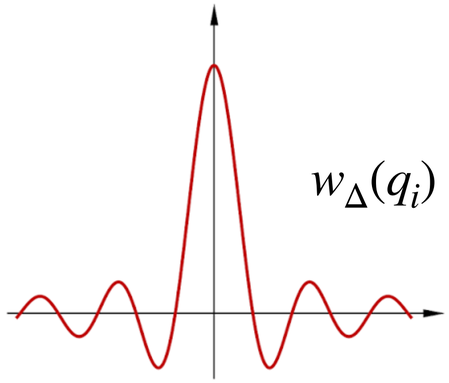
Freezeout in a boost-invariant fluid

- Experiments collect data at acceptance window

Abbasi, XA and Wu, 2606.02446, PRD



$$Q_\Delta = \int_z w_\Delta(z)\, n(t_f, z), \quad w_\Delta(z) = \Theta(\Delta - |z|)$$

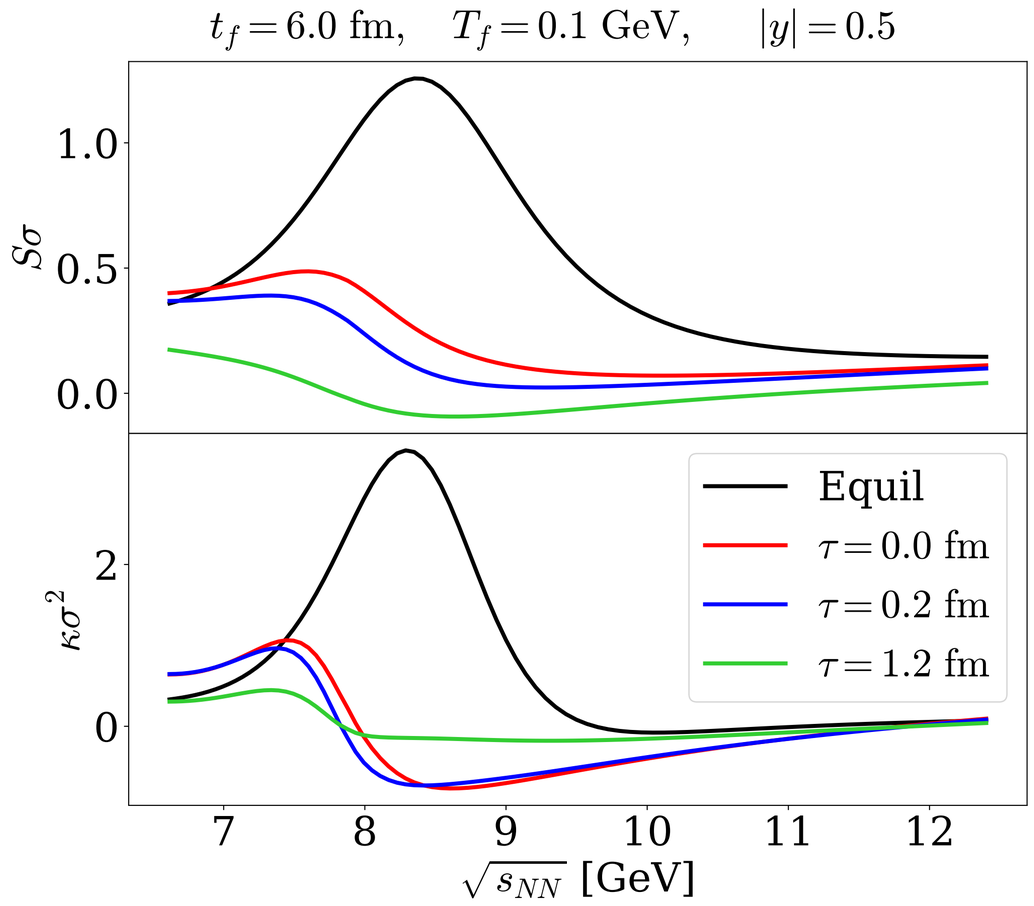


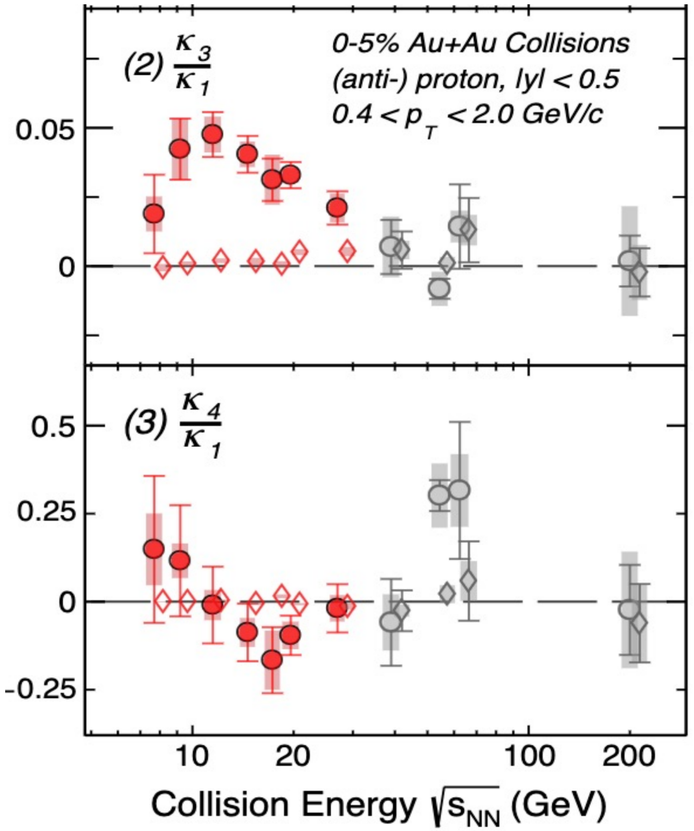
$$w_\Delta(q_i) = \frac{2\sin(q_i\Delta)}{q_i}$$

$$C_n[w] = \int \left[\prod_i \frac{d^3q_i}{(2\pi)^3}\right] (2\pi)^3\delta^{(3)}\left(\sum_i q_i\right) W^c_{n\ldots n}(q_1, \ldots, q_N; t_f) \prod_i w_\Delta(q_i)$$

Cumulants and energy dependence

Abbasi, XA and Wu, 2606.02446, PRD





The toy model features the qualitative similarity.

Recap



Far-from-equilibrium phenomena are organized by one principle.

EFT is powerful in constraining parameters and calculating nonlinear effects.



A framework for fluctuation dynamics incorporating nonhydro behavior is plausible.

The far-from-equilibrium phenomena may affect the CEP experiment results.

Thank You