Critical spin fluctuation and correlation

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Content

1 Introduction

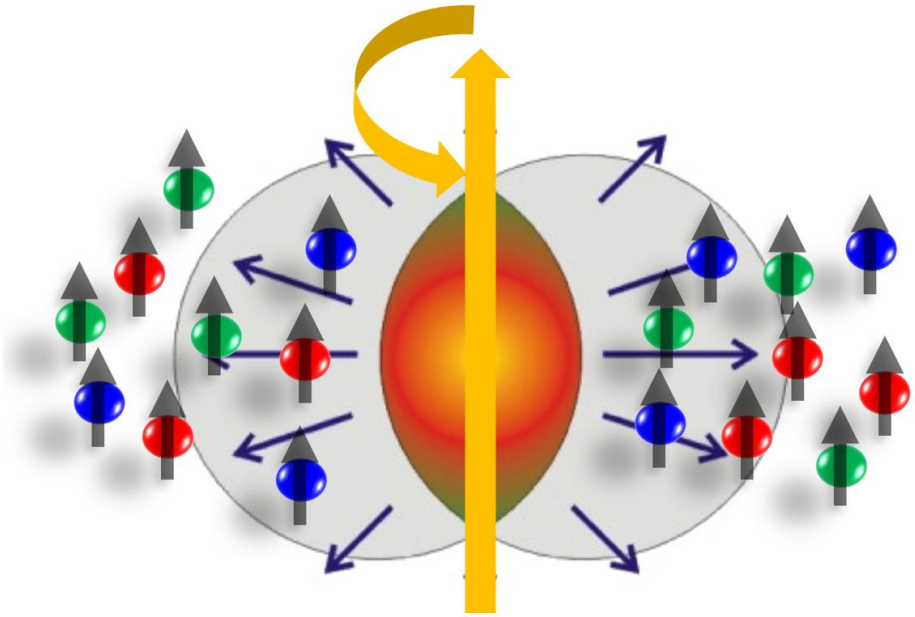
2 Static properties of spin fluctuations

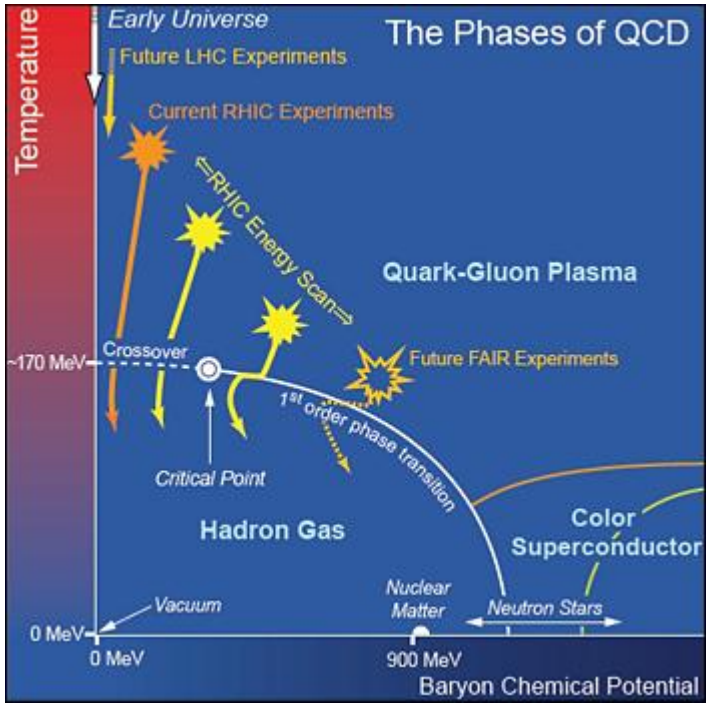
3 A NJL model study

4 Summary

1 Introduction

1 Spin polarization and critical end point (CEP)



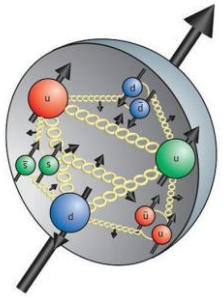


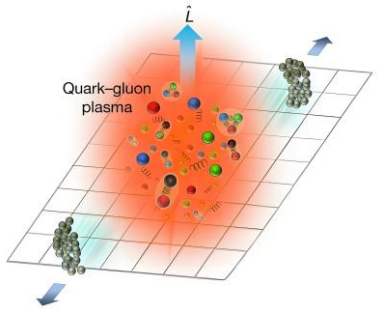
- Properties of CEP of spin polarized QCD medium?
- Location of CEP of spin polarized QCD medium?
- Would CEP affect spin polarization?
- Spin probe to the CEP?

1 Spin polarization phenomena

➢ Angular momentum (AM) distribution in strong-interaction systems

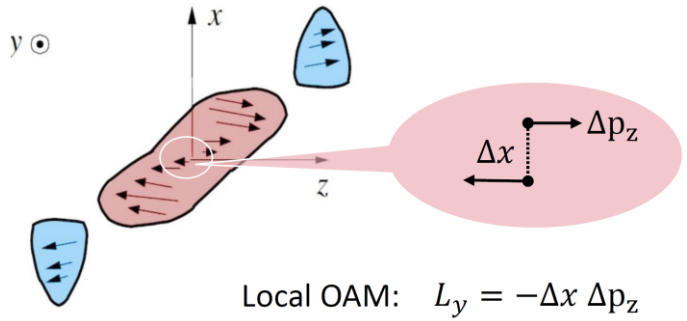
Proton
AM=1/2 fixed





Heavy ion collisions
AM varying

➢ Hyperon spin polarization --- original idea



Local OAM: $L_y = -\Delta x\, \Delta \mathrm{p_z}$

Quark polarization:

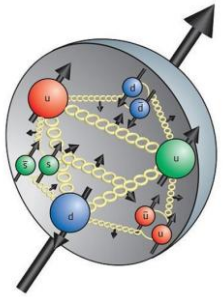
$$\mathrm{P} = -\frac{\pi \mu p}{2E(E+m)}$$

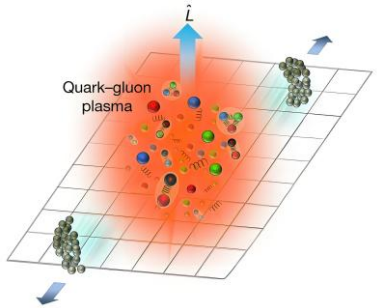
(Liang and Wang 2004)

1 Spin polarization phenomena

➢ Angular momentum (AM) distribution in strong-interaction systems

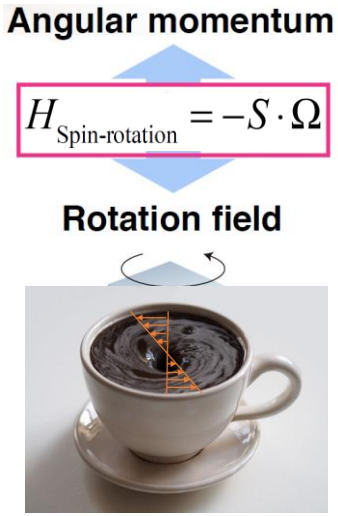
Proton
AM=1/2 fixed





Heavy ion collisions
AM varying

➢ Hyperon spin polarization --- spin-rotation (vorticity) coupling



(at thermal equilibrium)

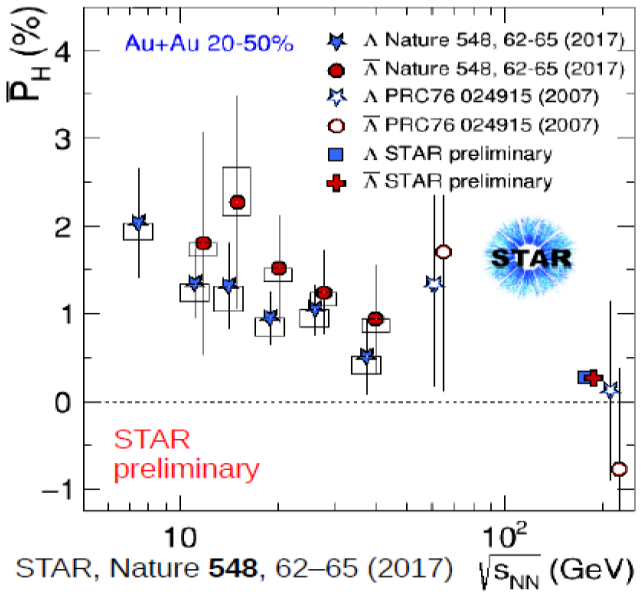
$$\frac{dN_s}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega}\cdot\boldsymbol{S})/T}$$

$$\mathrm{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$$

1 Spin polarization phenomena

➢ Global hyperon spin polarization

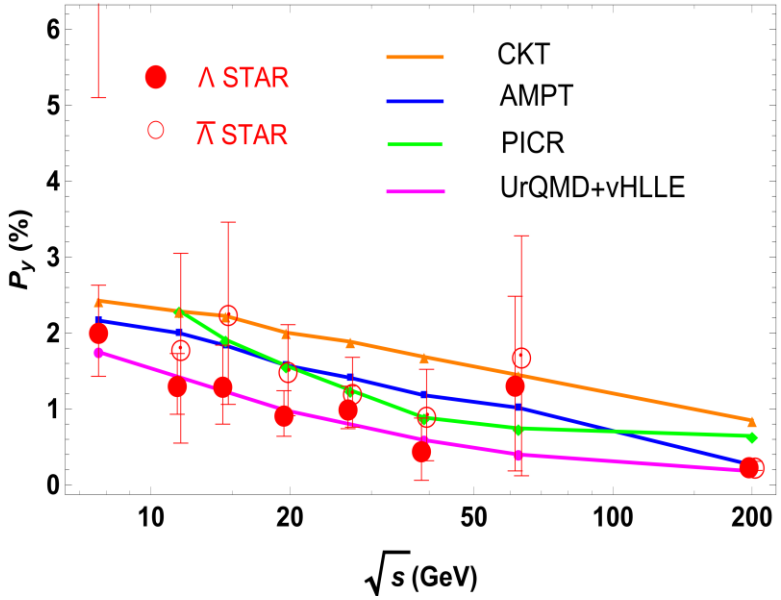
Experiment = Theory based on thermal vorticity



STAR, Nature **548**, 62–65 (2017)

$$\langle\omega\rangle = \langle T(P_\Lambda + P_{\bar{\Lambda}})\rangle_{\sqrt{s}=7-200\mathrm{GeV}}$$
$$\approx (9 \pm 1) \times 10^{21}\mathrm{s}^{-1}$$

"The most vortical fluid"



(Li-Pang-Wang-Xia 2017; Sun-Ko 2017; Wei-Deng-Huang 2019; Xie-Wang-Csernai 2017; Karpenko-Becattini 2016; and many others)

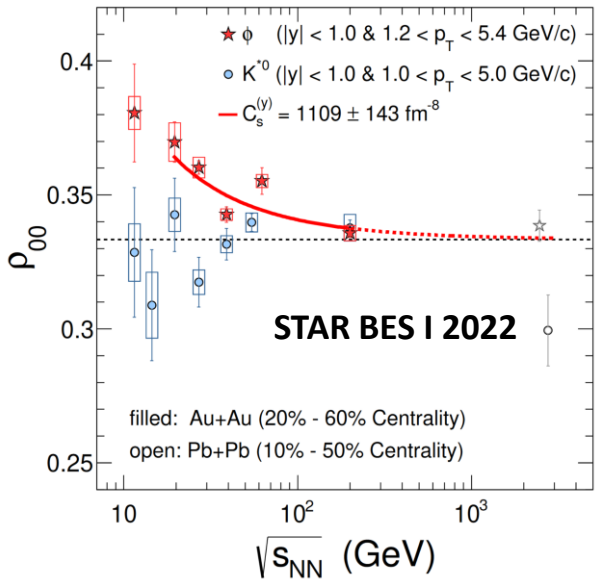
1 Spin polarization phenomena

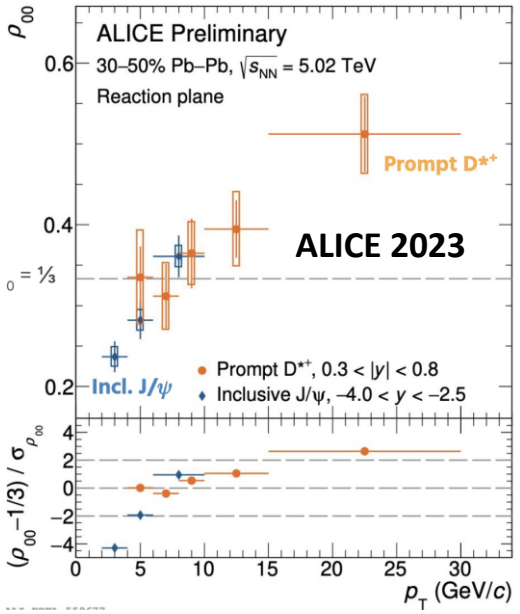
➢ Vector meson spin alignment

Spin-1 density matrix $\rho^V = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix}$

$\rho_{00} \neq \frac{1}{3}$: Spin alignment

➢ Experimental results



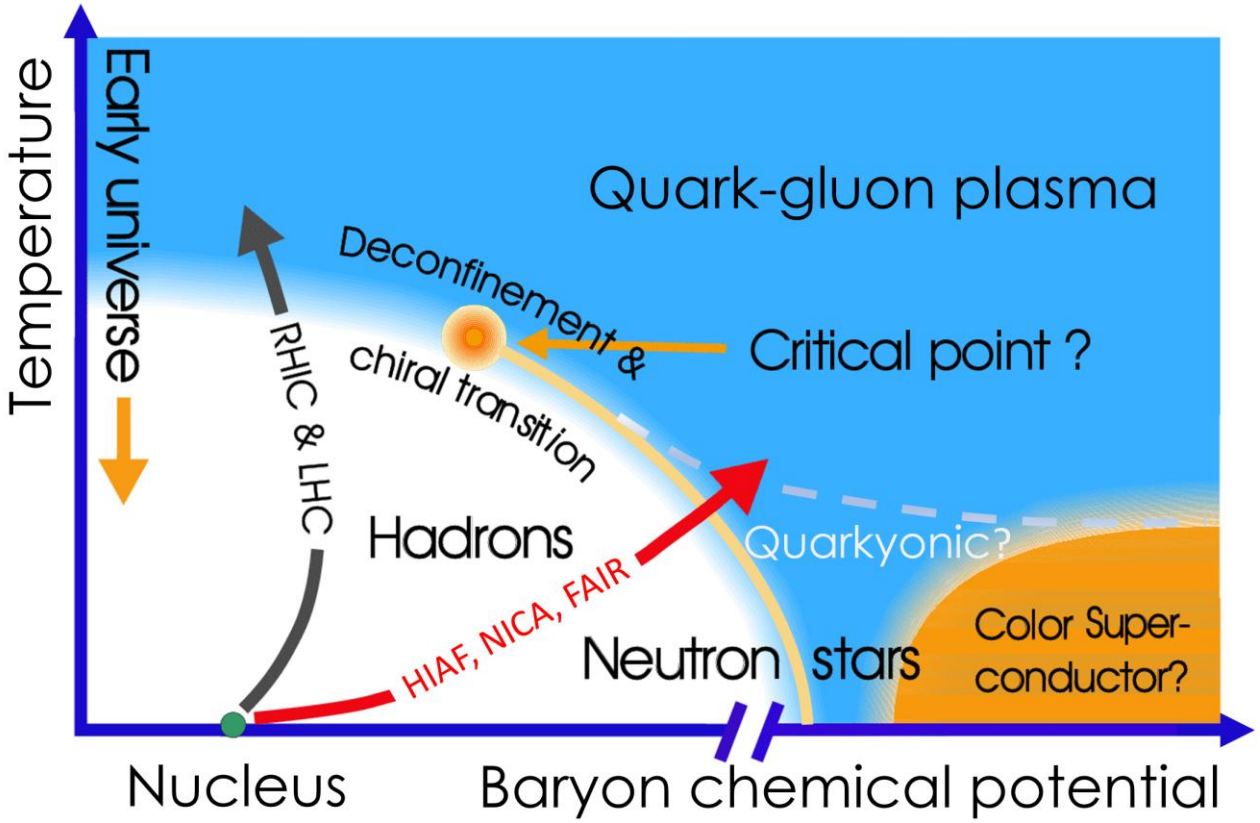


$$\rho_{00} \approx \frac{1}{3} - \frac{4}{9}\langle P_q P_{\bar{q}} \rangle$$

Strong (short-range) spin correlation

$$\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$$

1 CEP of chiral phase transition



- **CEP:** the end of first-order line
- Static universality class:
  3d Ising
- Dynamic universality class:
  Model H
- Expected to be accessible by heavy ion collisions

1 CEP of chiral phase transition

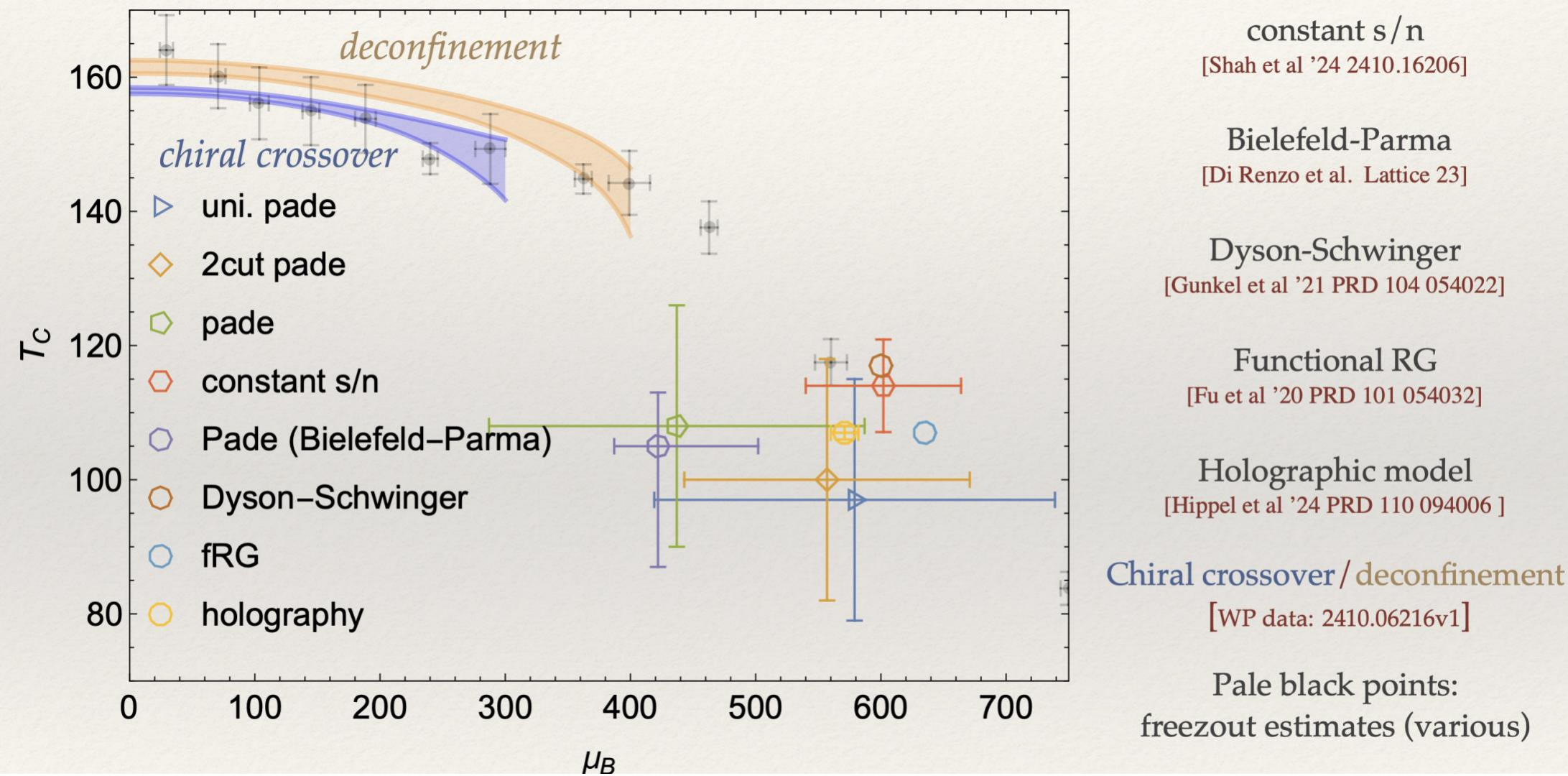


Figure courtesy : G. Basar

1 CEP of chiral phase transition

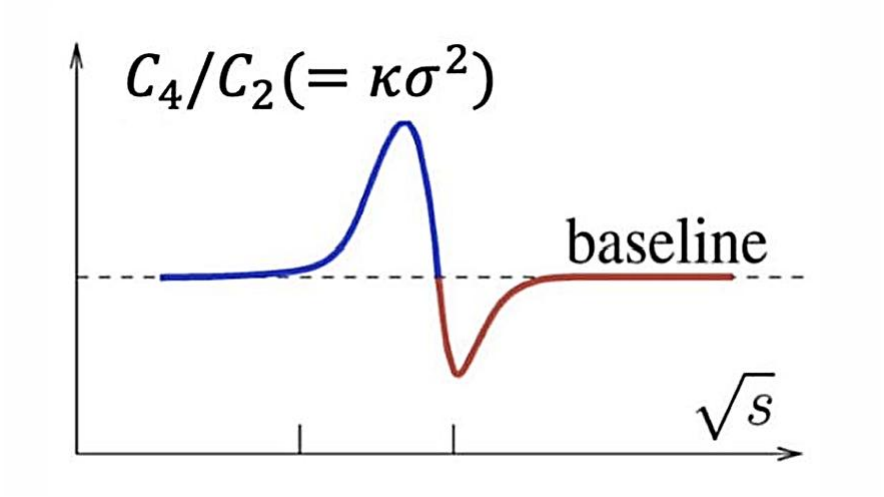
➢ Observables are usually fluctuations of conserved charges

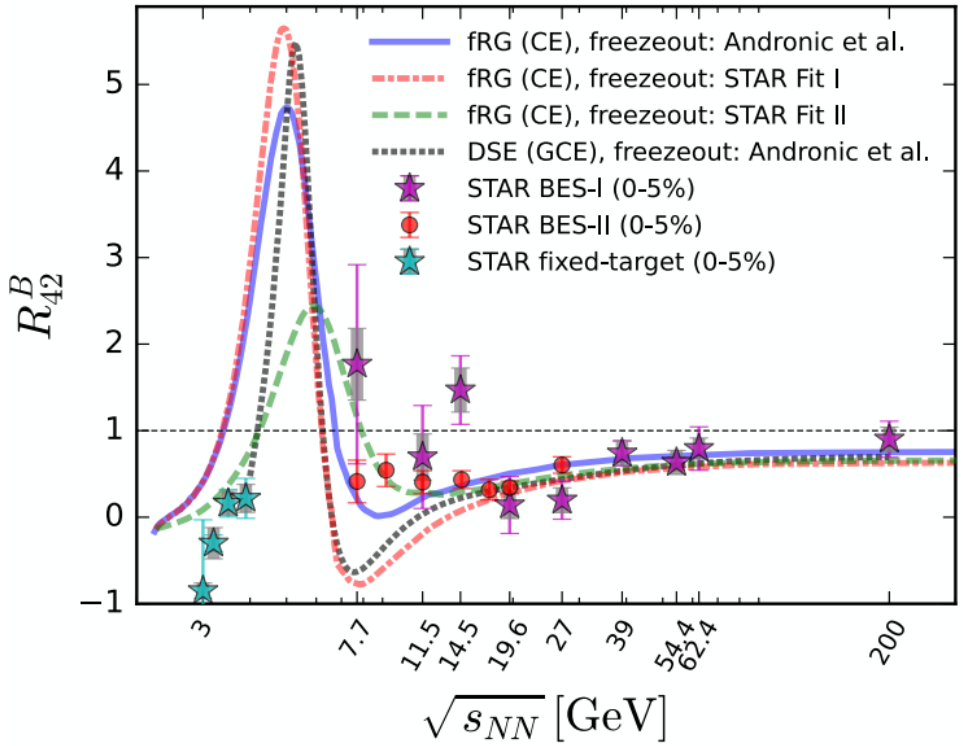
variance: $\sigma^2 = \langle(\delta N)^2\rangle = C_2$

skewness: $S = \langle(\delta N)^3\rangle/\sigma^3 = C_3/C_2^{3/2}$

kurtosis: $\kappa = \langle(\delta N)^4\rangle/\sigma^4 - 3 = C_4/C_2^2$

$$C_2 \sim \xi^2, \quad C_3 \sim \xi^{9/2}, \quad C_4 \sim \xi^7$$



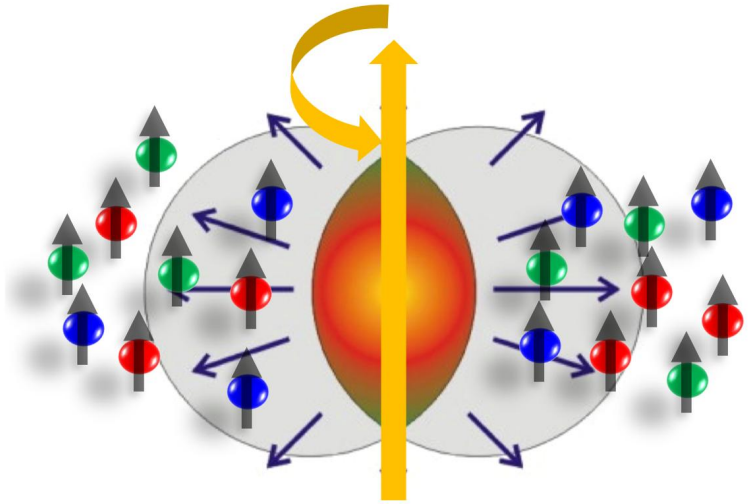


fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508

DSE: Lu, Gao, Liu, Pawlowski, arXiv: 2504.05099

Figure courtesy : W.J.Fu

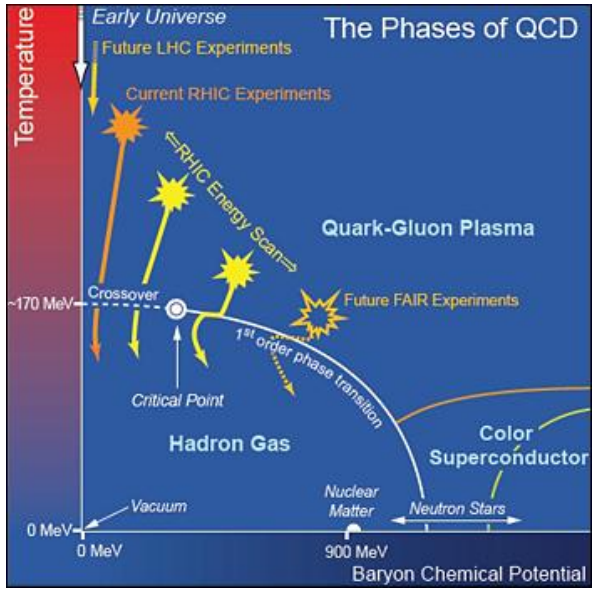
1 Bridge chiral condensate and spin density



Pseudo-vector sector

$$s^i \sim \frac{1}{2}\psi^\dagger \sigma^i \psi$$

$$\Omega \cdot s$$

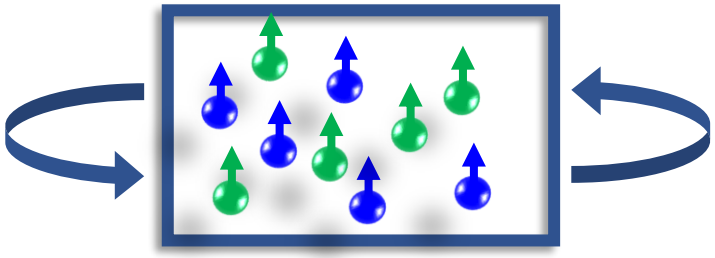


Scalar sector

$$\sigma \sim \bar{\psi}\psi$$

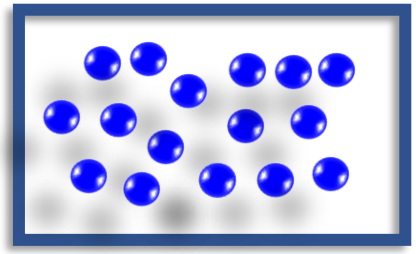
2 Static properties

2 Effect of spin polarization by rotation



Rotation

$$H = H_0 - \omega J_z$$



Chemical potential

$$H = H_0 - \mu N$$

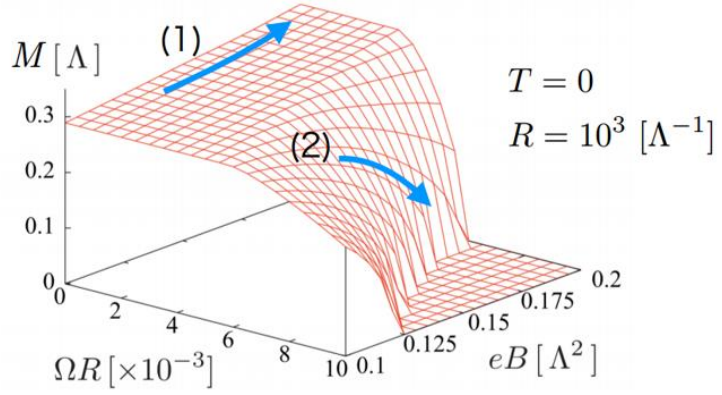
- Consider a scalar (or pseudoscalar) pair of fermions



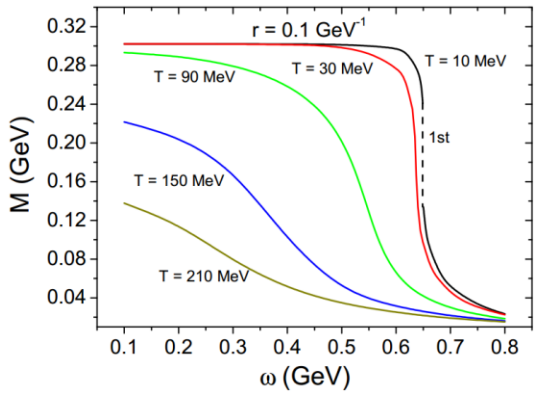
- Thus one expects that rotation tends to suppress chiral condensate σ

2 Effect of spin polarization by rotation

- Some model calculations shows that rotation suppresses chiral condensate

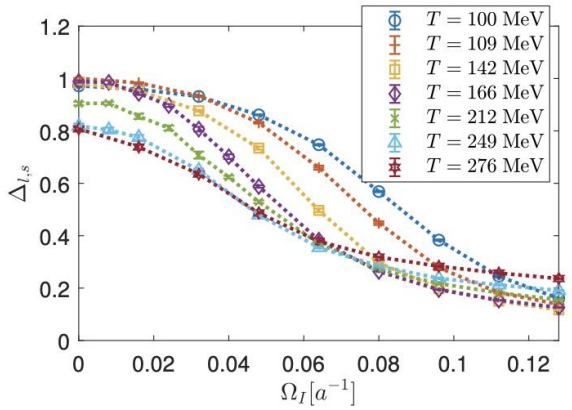


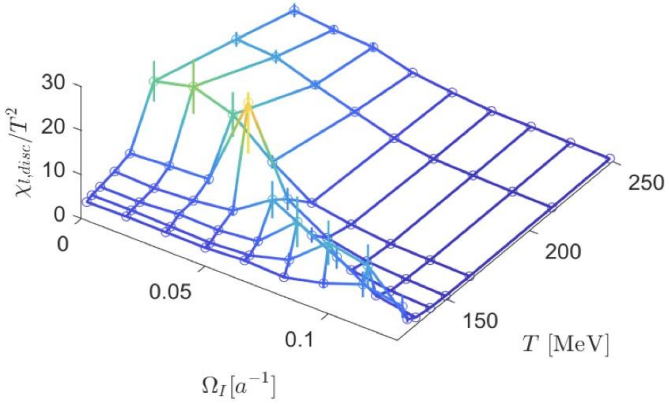
(Chen-Fukushima-Huang-Mameda 2015)



(Jiang-Liao 2015)

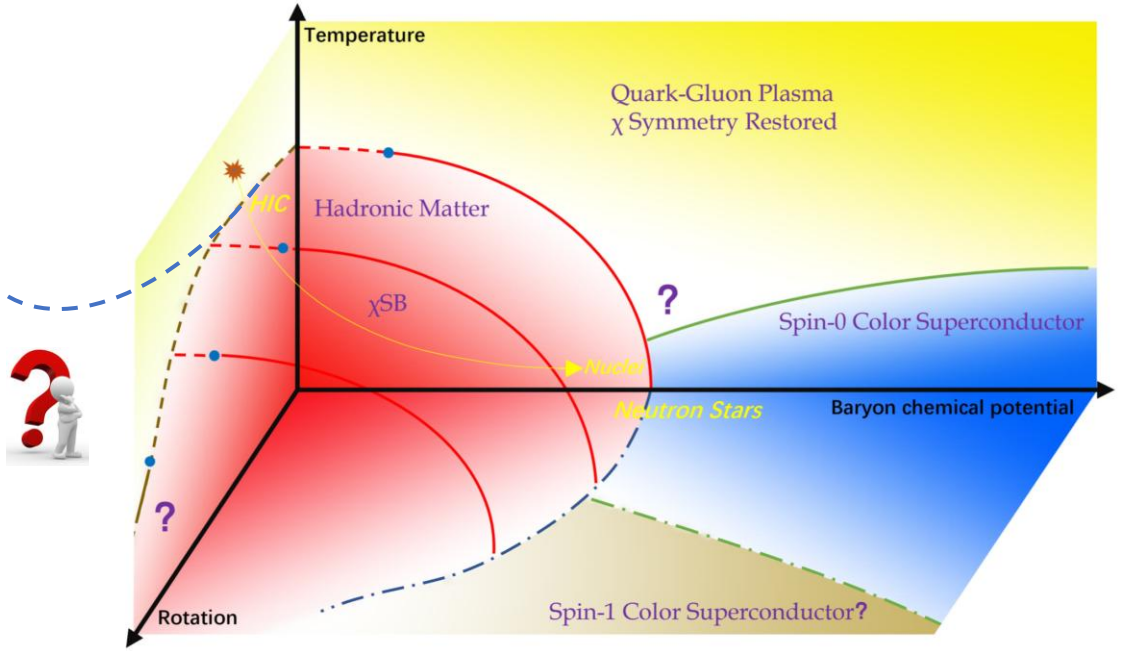
- Lattice simulations suggest that real rotation would enhance chiral condensate





(Yang-XGH 2023 for QCD; see also Braguta 2021 for Polyakov loop for pure gluon)

2 Effect of spin polarization by rotation



- Though it is still unclear whether rotation shifts CEP upward or downward, in realistic HIC environment, Ω is about a few to tens of MeV, and the location of CEP shifts by $\propto \Omega^2$, which is small

2 Static critical behavior of spin correlation

➢ The Ginzburg–Landau free energy of fluctuation fields

➢ At leading-order in Ω， only $s \equiv \hat{\boldsymbol{\Omega}} \cdot \boldsymbol{s}$ couple to chiral fields; transverse spin decouples

$$F[\sigma, n, s] = \int d^3x \left[\frac{a}{2}(\boldsymbol{\nabla}\sigma)^2 + b\,\boldsymbol{\nabla}\sigma \cdot \boldsymbol{\nabla} n + \frac{c}{2}(\boldsymbol{\nabla} n)^2 + \frac{c_s}{2}(\boldsymbol{\nabla} s)^2 + O(\partial^3) + V(\sigma, n, s) \right]$$

$$V(\sigma, n, s) = \frac{r}{2}\sigma^2 + B\,\sigma n + \frac{C}{2}n^2 + \frac{1}{2\chi_0}s^2 + g_\sigma \Omega\,\sigma s + g_n \Omega\, n s + V_{\rm int}(\sigma, n, s)$$

➢ For a moment, we turn off baryon charge

➢ Quadratic kernel $\mathcal{K}(q) = \begin{pmatrix} r + aq^2 & g_\sigma \Omega \\ g_\sigma \Omega & \chi_0^{-1} + c_s q^2 \end{pmatrix}$

➢ CEP is determined by $\det \mathcal{K}(0) = r\,\chi_0^{-1} - g_\sigma^2 \Omega^2 = 0$

➡ CEP is shifted from $r = 0$ to $r_c = \chi_0\, g_\sigma^2\, \Omega^2$

2 Static critical behavior of spin correlation

➢ Gaussian approximation

$$G_{\sigma\sigma}(q) = \frac{\chi_0^{-1}}{(r+aq^2)(\chi_0^{-1}+c_s q^2) - g_\sigma^2\Omega^2} \qquad G_{ss}(q) = \frac{r+aq^2}{(r+aq^2)(\chi_0^{-1}+c_s q^2) - g_\sigma^2\Omega^2}$$

$$G_{\sigma s}(q) = -\frac{g_\sigma\Omega}{(r+aq^2)(\chi_0^{-1}+c_s q^2) - g_\sigma^2\Omega^2}$$

➢ Near the new CEP, the singular parts

$$G_{ss}^{\rm sing}(q) = \tan^2\theta_\sigma G_\sigma^{\rm sing}(q), \quad G_{\sigma s}^{\rm sing}(q) = -\tan\theta_\sigma G_\sigma^{\rm sing}(q) \qquad \text{with} \qquad \tan\theta_\sigma \equiv \chi_0 g_\sigma \Omega$$

$$G_\sigma^{\rm sing}(q) = \frac{1}{(r-r_c)+\tilde{a}\,q^2} = \frac{\xi^2/\tilde{a}}{1+q^2\xi^2}$$

➢ The spin correlation becomes long-ranged, anisotropic

$$\langle s^i(x)\, s^j(0)\rangle^{\rm sing} = \hat{\Omega}^i\hat{\Omega}^j\; G_{ss}^{\rm sing}(x) \propto \Omega^i\Omega^j\; \frac{{\rm e}^{-|x|/\xi}}{4\pi\tilde{a}|x|}$$

Spin critical opalescence: Critical enhancement of spin correlation

2 Static critical behavior of spin correlation

➢ Including baryon charge

$$\mathcal{K}(q) = \begin{pmatrix} r + aq^2 & B + bq^2 & g_\sigma \Omega \\ B + bq^2 & C + cq^2 & g_n \Omega \\ g_\sigma \Omega & g_n \Omega & \chi_0^{-1} + c_s q^2 \end{pmatrix} \Rightarrow G_{ns}^{\rm sing}(q) = +\frac{B}{C} \tan\theta_{\rm eff}\, G_\phi^{\rm sing}(q)$$

$$\Rightarrow \boxed{\langle \delta S\, \delta N \rangle^{\rm sing} \propto \Omega\, \xi^2}$$

A stronger spin-baryon correlation than spin-spin correlation

➢ Non-Gaussian theory

$$V_{\rm int} = \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + g_3 \sigma s^2 + \cdots$$

➢ Singular spin-spin correlation inherited from chiral fluctuation

Spin cumulants $\kappa_k^s = \left. \frac{\partial^k \ln Z}{\partial \alpha_s^k} \right|_{\alpha_s = \Omega/T}$

$\alpha_s \equiv \Omega_s / T$ with Ω_s spin potential

$$\begin{cases} \kappa_2^{s,\rm sing} \propto \Omega^2\, \xi^{2-\eta} + O(\Omega^4) \\ \kappa_3^{s,\rm sing} \propto \Omega\, \xi^{2-\eta} + O(\Omega^3) \\ \kappa_4^{s,\rm sing} \propto \Omega^0\, \xi^{2-\eta} + O(\Omega^2) \end{cases}$$

2 Static critical behavior of spin correlation

➢ Including baryon charge

$$\mathcal{K}(q) = \begin{pmatrix} r + aq^2 & B + bq^2 & g_\sigma \Omega \\ B + bq^2 & C + cq^2 & g_n \Omega \\ g_\sigma \Omega & g_n \Omega & \chi_0^{-1} + c_s q^2 \end{pmatrix} \Rightarrow G_{ns}^{\rm sing}(q) = +\frac{B}{C} \tan\theta_{\rm eff}\, G_\phi^{\rm sing}(q)$$

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$$\begin{cases} \kappa_2^{s,\rm sing} \propto \Omega^2\, \xi^{2-\eta} + O(\Omega^4) \\ \kappa_3^{s,\rm sing} \propto \Omega\, \xi^{2-\eta} + O(\Omega^3) \\ \kappa_4^{s,\rm sing} \propto \Omega^0\, \xi^{2-\eta} + O(\Omega^2) \end{cases}$$

Vs

Baryon number cumulants

$$\kappa_2 = \langle \sigma_V^2 \rangle = VT\xi^2; \qquad \kappa_3 = \langle \sigma_V^3 \rangle = 2\lambda_3 VT^2 \xi^6; \qquad (3)$$
$$\kappa_4 = \langle \sigma_V^4 \rangle_c = 6VT^3[2(\lambda_3\xi)^2 - \lambda_4]\xi^8,$$

(Stephanov 2011)

3 A NJL model study

3 Rotating Nambu-Jona-Lasinio model

- NJL is a four-fermion interacting model

$$Z = \int \mathcal{D}[\bar{\psi}, \psi] \exp\left(i \int d^4x \sqrt{-g} \mathcal{L}_{\mathrm{NJL}} \right)$$

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}(i\gamma^\mu \nabla_\mu - m_0)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma^5 \boldsymbol{\tau}\psi)^2]$$

$$\nabla_\mu = \partial_\mu + i\hat{Q}A_\mu + \Gamma_\mu$$

$$g_{\mu\nu} = \begin{pmatrix} 1-\Omega^2 r^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Mean-field approximation

$$V_{\mathrm{eff}} = \frac{1}{\beta V} \int d^4x_E \left\{ \frac{\sigma^2 + \boldsymbol{\pi}^2}{2G} - \sum_{\{\xi\}} \left[\frac{\varepsilon_{\{\xi\}}}{2} + \frac{1}{\beta} \ln(1 + e^{-\beta\varepsilon_{\{\xi\}}}) \right] \Psi^\dagger_{\{\xi\}} \Psi_{\{\xi\}} \right\}$$

$\varepsilon_{\{\xi\}}$ and $\Psi_{\{\xi\}}$: Eigen-energy and eigen-wavefunction with quantum numbers $\{\xi\}$

3 Rotating Nambu-Jona-Lasinio model

- Introduce spin chemical potentials only couple to quark or antiquark spin

$$
\begin{aligned}
&V_{\text{eff}}(\Omega_q^s, \Omega_{\bar{q}}^s, \Omega, \mu_q, \mu_{\bar{q}}, \mu; r) \\
=&\frac{[m(r)-m_0]^2}{4G} - N_c N_f \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{(2\pi)^3} 2\varepsilon_p - \sum_{l=-\infty}^{\infty} N_c N_f \int_0^{\infty} \frac{\mathrm{d}^3 p}{(2\pi)^3} J_l^2(p_t r) \Big[T \ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega_q^s/2-\Omega l-\mu_q)/T}) \\
&+ T \ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega_q^s/2-\Omega l-\mu_q)/T}) + T \ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega_{\bar{q}}^s/2+\Omega l-\mu_{\bar{q}})/T}) + T \ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega_{\bar{q}}^s/2+\Omega l-\mu_{\bar{q}})/T}) \Big]
\end{aligned}
$$

- Correlation of quark/antiquark spin and particle number

$$\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle = \frac{\partial^2 V_{eff}}{\partial \Omega_q^s \partial \Omega_{\bar{q}}^s}\Big|_{\Omega_q^s=\Omega_{\bar{q}}^s=\Omega}$$

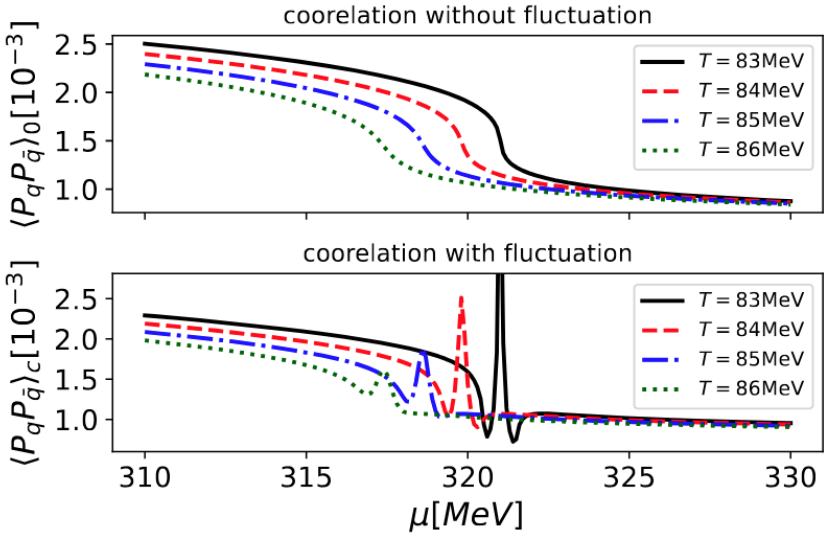
$$\langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle = \frac{\partial^2 V_{eff}}{\partial \mu_q \partial \mu_{\bar{q}}}\Big|_{\mu_q=\mu_{\bar{q}}=0}$$

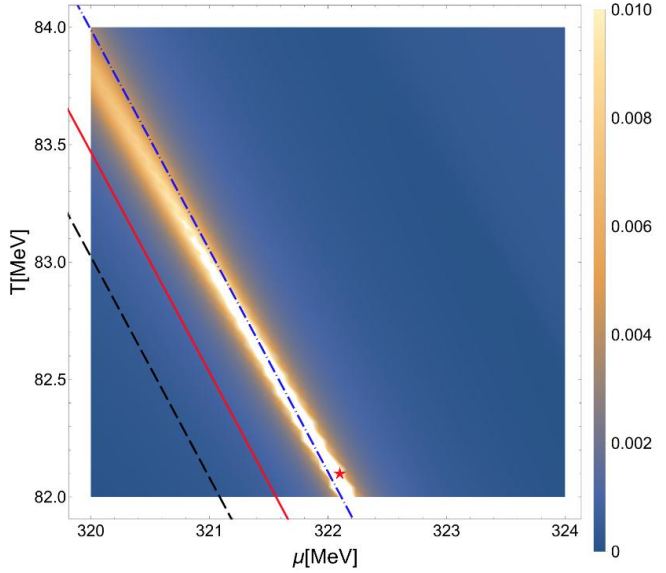
Spin polarization correlation

$$\langle P_q P_{\bar{q}} \rangle_c = \frac{4(\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle)}{\langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle}$$

3 Rotating Nambu-Jona-Lasinio model

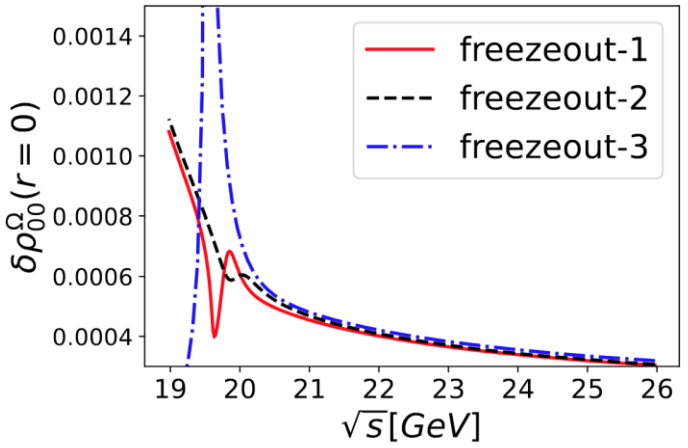
- Spin polarization correlation along constant T and along freeze out line





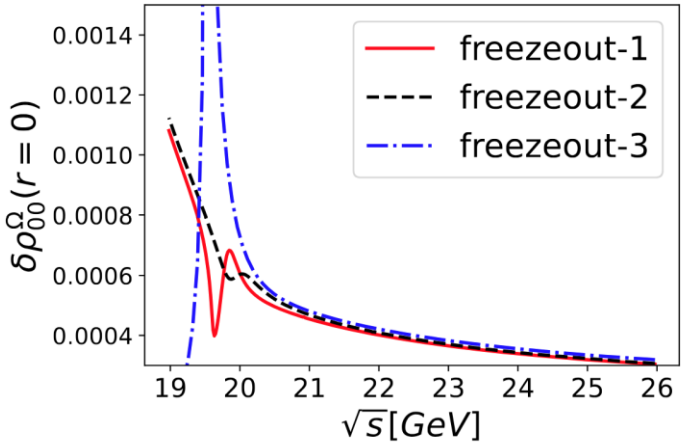
$$\rho_{00} = \frac{1 - \langle P_q P_{\bar{q}} \rangle}{3 + \langle P_q P_{\bar{q}} \rangle} \approx \bar{\rho}_{00} - \delta\rho^{\Omega}_{00}.$$

$$\delta\rho^{\Omega}_{00} = \frac{4}{9} \langle P_q P_{\bar{q}} \rangle_c$$

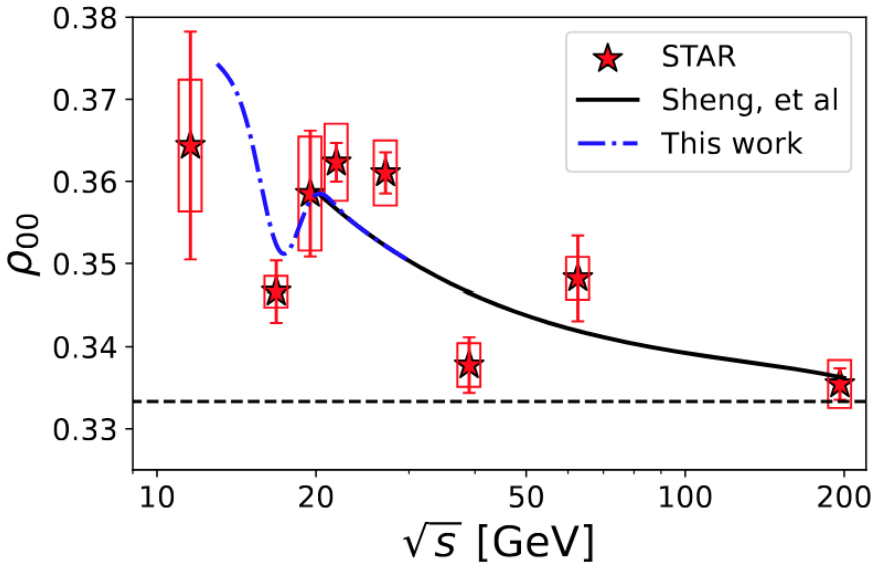


3 Rotating Nambu-Jona-Lasinio model

- Phi meson spin alignment



(Chen-Fu-Huang-Ma 2025)



$$\rho_{00} = \frac{1 - \langle P_q P_{\bar{q}} \rangle}{3 + \langle P_q P_{\bar{q}} \rangle} \approx \bar{\rho}_{00} - \delta\rho_{00}^{\Omega}.$$

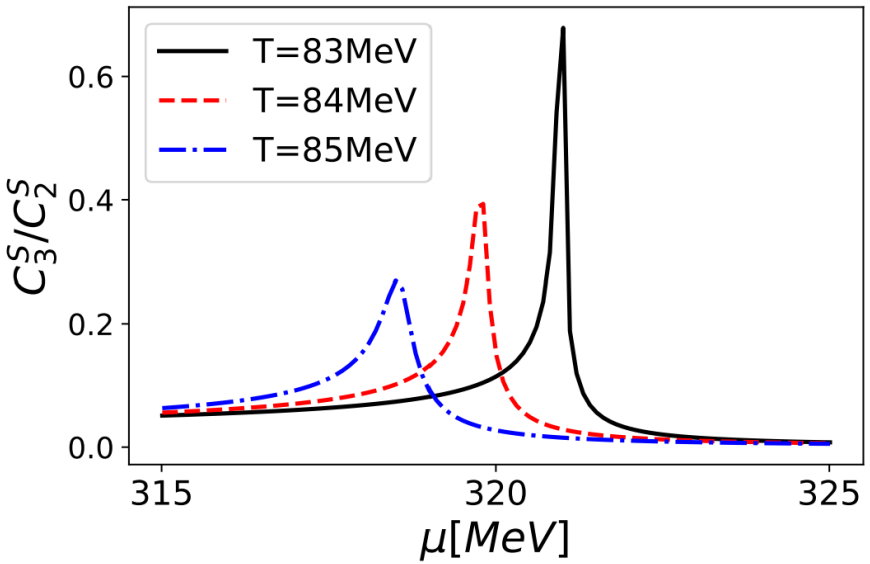
$$\delta\rho_{00}^{\Omega} = \frac{4}{9} \langle P_q P_{\bar{q}} \rangle_c$$

($\bar{\rho}_{00}$ is given in Sheng-Oliva-Liang-Wang-Wang 2023)

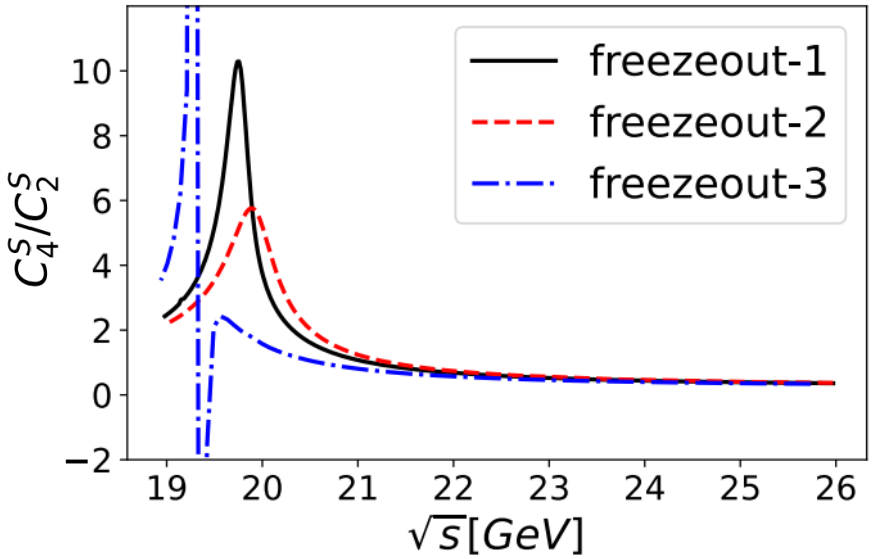
3 Rotating Nambu-Jona-Lasinio model

- Higher order cumulants

Skewness of spin fluctuation:

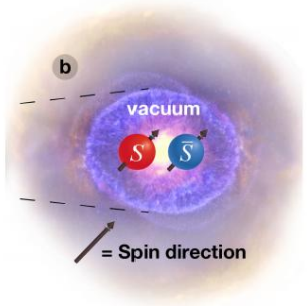


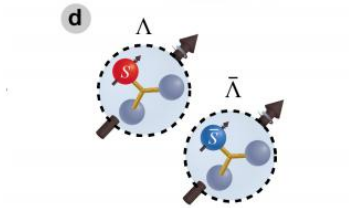
Kurtosis of spin fluctuation:



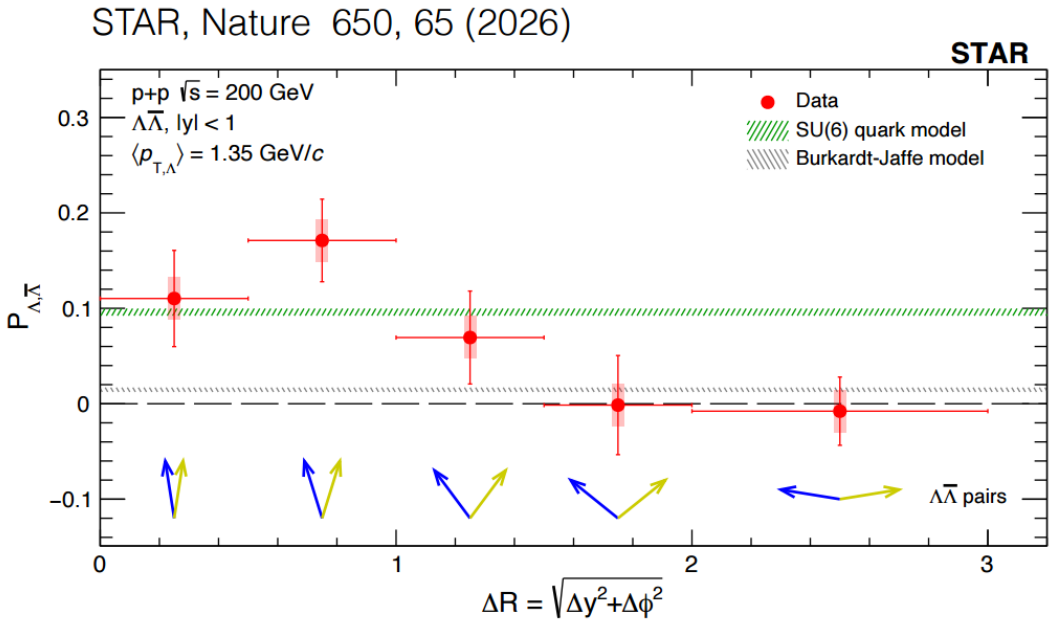
3 Whether it is relevant in experiments

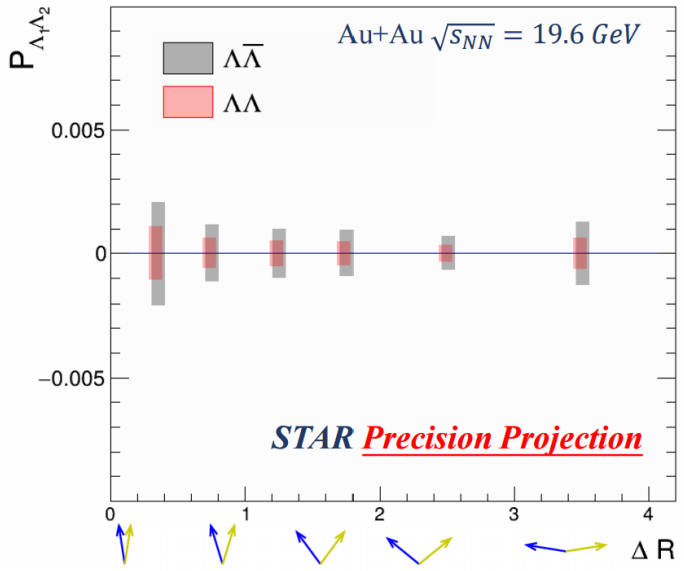
- Λ spin correlation has been measured in pp and will be in AuAu





$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}\cos\theta^{\star}} = \frac{1}{2}[1+\alpha_1\alpha_2 P_{\Lambda_1\Lambda_2}\cos\theta^{\star}]$$



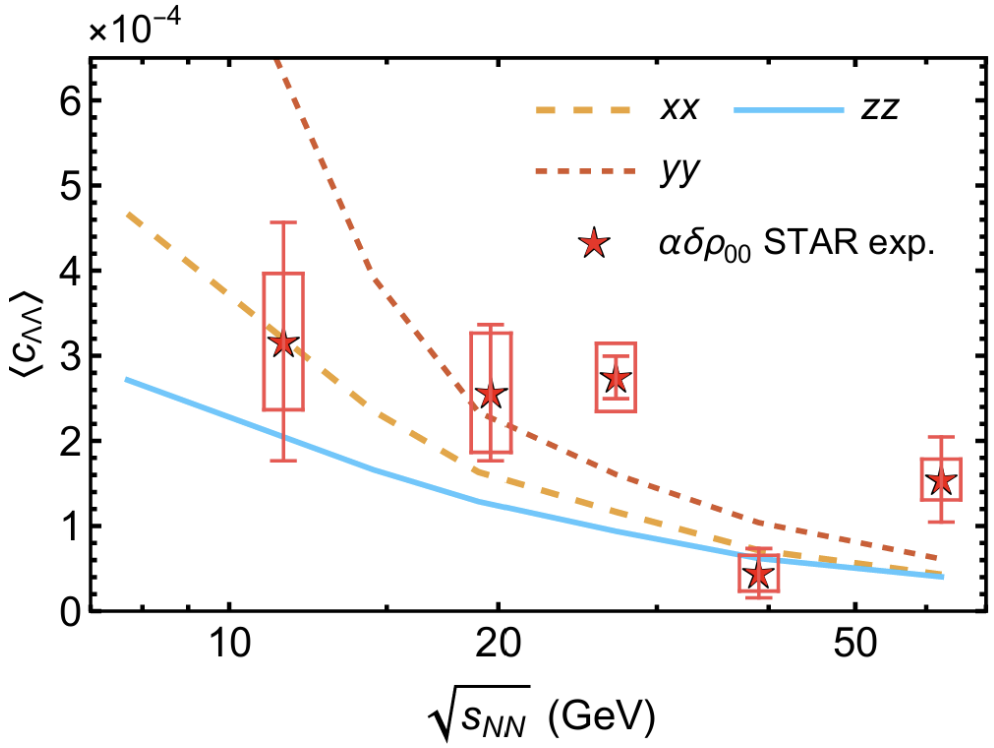


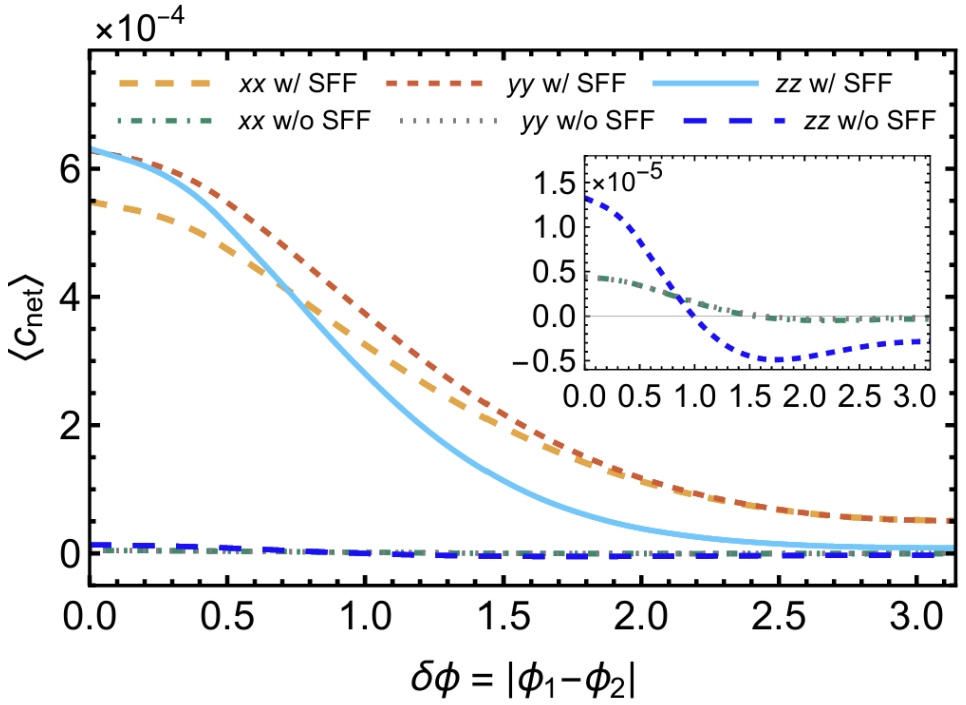
3 Comparing to another source of spin correlation

- Λ spin correlation may also been induced by strong force field

$$C_{12}^{\mu\nu}(p_1, p_2) \equiv \langle P_1^{\mu}(x_1, p_1) P_2^{\nu}(x_2, p_2) \rangle$$

(Sheng-Wu-Rischke-Wang 2025)





5 Summary

4 Summary

- Hyperon spin polarization and vector meson spin alignment probe vorticial structure, as well as the correlations in the fire ball
- In the vortical medium, critical spin fluctuation and correlation occur
- Spin-spin correlation length diverges, spin relaxation slows down
- Perhaps measurable through baryon spin correlation or vector meson spin alignment
- Spin-baryon correlation stronger than spin-spin correlation for weakly vortical medium

Thank you

5 Dynamics of critical spin correlation

4 Dynamical critical spin correlations

➢ Langevin equations (spin vorticity coupling omitted to focuse on relaxation dynamics)

$$\dot{\sigma} = -\Gamma_\sigma \frac{\delta F}{\delta \sigma} + \tilde{\lambda}\,\nabla^2 \frac{\delta F}{\delta n} + \theta_\sigma,$$
$$\dot{n} = \tilde{\lambda}\,\nabla^2 \frac{\delta F}{\delta \sigma} + \lambda\,\nabla^2 \frac{\delta F}{\delta n} + \theta_n,$$
$$\dot{s} = -\Gamma_s \frac{\delta F}{\delta s} + \theta_s,$$

$$\langle \theta_i(\boldsymbol{q},t)\theta_j(-\boldsymbol{q},t')\rangle = 2T\,\Gamma_{ij}(q)\,\delta(t-t')$$

➢ Critical slowing down of spin relaxation

$$\Gamma_s^{\rm crit}(q) = D\,q^2 \xrightarrow[q\sim\xi^{-1}]{} \xi^{-z}: \qquad \tau_s \sim \xi^z.$$

$$\langle s\,s\rangle^{\rm sing}_{\omega q} = \tan\theta^2_{\rm eff}\,\frac{2T\,G_\varphi(q)\,\Gamma(q)}{\omega^2+\Gamma(q)^2},$$
$$\langle s\,n\rangle^{\rm sing}_{\omega q} = \tan\theta_{\rm eff}\,\frac{B}{C}\,\frac{2T\,G_\varphi(q)\,\Gamma(q)}{\omega^2+\Gamma(q)^2},$$

