C3NT Workshop on QCD Critical End Point and Effective Field Theory Methods

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AI-Assisted Quasi-Parton Model and the 4D QCD EoS

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With Fu-Peng Li, Hong-Liang Lü and Guang-You Qin

Phys. Lett. B **844**, 138088 (2023) • *Phys. Lett. B* **868**, 139692 (2025) • arXiv:2604.22352

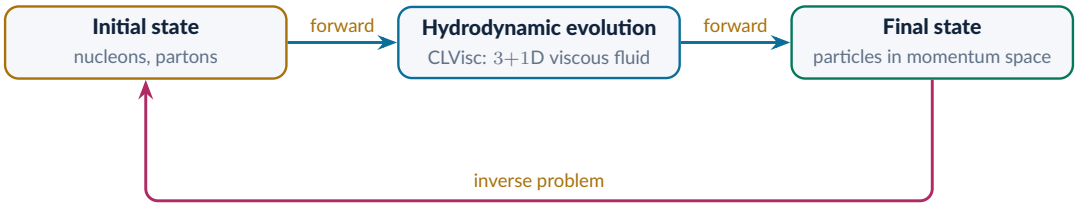
PART 01

Why a four-dimensional EoS

Heavy-ion collisions as an inverse problem • from finite μ_B to finite μ_B, μ_Q, μ_S • what lattice QCD can and cannot give

The equation of state is an input we must provide

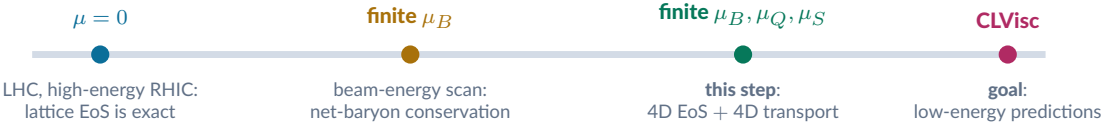
Fluid dynamics needs two inputs: the **equation of state** $P(\varepsilon)$ and the **transport coefficients** η/s, ζ/s.



CLVisc: L.-G. Pang, Q. Wang, X.-N. Wang, PRC **86**, 024911 (2012); L.-G. Pang, H. Petersen, X.-N. Wang, PRC **97**, 064918 (2018).

The frontier has moved from one dimension to four

Net baryon is conserved in CLVisc; **net charge and strangeness are next**.



Already done

- Net baryon is conserved in CLVisc;
- EoS at finite μ_B is learned.

Still missing

- EoS, shear and bulk viscosity in 4D;
- Conserving n_Q, n_S needs the **diffusion matrix** $k_{qq'}$.

Finite-μ_B CLVisc: X.-Y. Wu, G.-Y. Qin, L.-G. Pang, X.-N. Wang, PRC **105**, 034909 (2022).

Lattice QCD cannot reach the low-energy region

The **sign problem** blocks Monte Carlo at finite μ; expansions lose convergence beyond $\mu_B/T \gtrsim 3$.

LATTICE QCD

- At $\mu = 0$: **exact first-principles** EoS and generalized susceptibilities;

$$\chi^{B,Q,S}_{i,j,k} = \frac{\partial^{\,i+j+k}\,(p/T^4)}{\partial\hat{\mu}_B^{\,i}\,\partial\hat{\mu}_Q^{\,j}\,\partial\hat{\mu}_S^{\,k}}\Big|_{\hat{\mu}_B=\hat{\mu}_Q=\hat{\mu}_S=0}, \quad \hat{\mu}_X = \mu_X/T$$

definition as used here: Eq. (15) of arXiv:2604.22352

- At finite μ Monte Carlo is blocked: use **Taylor** ($\mu_B/T \lesssim 2.5$) or T' (~ 3.5) expansions;
- They give thermodynamics, but **no effective degrees of freedom**.

Lattice QCD supplies the thermodynamics, not the effective masses behind it.

PART 02

Quasi-particle models and PINNs

The idea: dress interactions into a mass • three pain points of conventional fits • physics as a loss term

Dress the interactions into a mass

All interactions enter **one medium-dependent mass** $m(T)$; thermodynamics then follows ideal-gas statistics.

$$\ln Z_g = -\frac{d_g V}{2\pi^2}\int_0^\infty p^2 dp \,\ln\left[1 - e^{-E_g/T}\right]$$

$$\ln Z_{q_i} = +\frac{d_{q_i} V}{2\pi^2}\int_0^\infty p^2 dp \,\ln\left[1 + e^{-(E_{q_i}-\mu_{q_i})/T}\right]$$

$$\ln Z_{\bar{q}_i} = +\frac{d_{q_i} V}{2\pi^2}\int_0^\infty p^2 dp \,\ln\left[1 + e^{-(E_{q_i}+\mu_{q_i})/T}\right]$$

$E = \sqrt{p^2 + m^2(T)}$; gluons $d = 16$, quarks $d = 6$.

ONE POTENTIAL

All come from the **same** $\ln Z$.

THE CATCH

A running mass needs a bag term.

THE FIX

Every relation holds by design.

Conventional fits have three pain points

Assuming a functional form puts the bias **into the assumption itself**.

1 Parametrization bias

Assume $m(T) = a + bT^2 + c/T + \cdots$ and fit. If the true shape lies outside that family, the mass is wrong.

2 Underdetermined

Three unknown functions $m_{u/d}, m_s, m_g$, but at $\mu_B = 0$ the lattice gives only $s(T)$ and $\Delta(T)$.

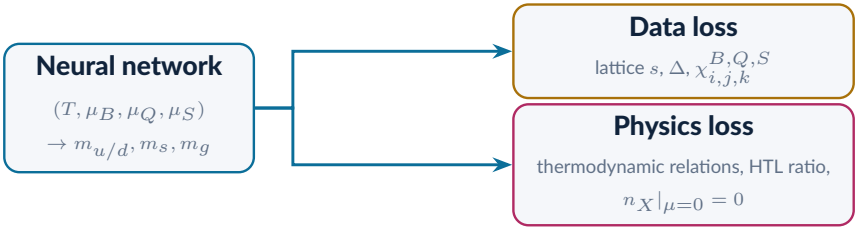
3 No anchor in 4D

Four inputs $\times$ three functions: no reference point, and expansions break down at $\mu_B/T \gtrsim 3$.

The way out: a **network** as a non-parametric representation + **automatic differentiation** turn thermodynamic relations into training constraints.

Put the physics in the loss function

$\mathcal{L}_{\text{PINN}} = \mathcal{L}_{\text{data}} + \lambda\,\mathcal{L}_{\text{physics}}$ — the known physics becomes a **training constraint**, not a modelling assumption.



REPRESENT
No prescribed form.

DIFFERENTIATE
Exact derivatives.

CONSTRAIN
Physics as constraints.

Thermodynamic self-consistency comes for free

The PINN quasi-particle method is **thermodynamically self-consistent**.

$$p = T\left(\frac{\partial \ln Z}{\partial V}\right)_{T,\mu}$$

$$s = \left(\frac{\partial p}{\partial T}\right)_{\mu}, \qquad n_X = \left(\frac{\partial p}{\partial \mu_X}\right)_{T}$$

$$\varepsilon = Ts - p + \sum_X \mu_X n_X$$

MAXWELL RELATIONS

Mixed derivatives commute:
$\partial s/\partial \mu_X|_T = \partial n_X/\partial T|_\mu$.

EULER RELATION

$\varepsilon = Ts - p + \sum_X \mu_X n_X$ is the integral form of $p = T \ln Z/V$ — an identity.

Thermodynamic self-consistency = every observable is a derivative of one $p(T, \mu_B, \mu_Q, \mu_S)$; no bag constant $B(T)$ is needed.

Three steps, one unchanged pipeline

The network backbone never changes: only the **input dimension** and the **constraint terms** are added.

STEP 1 • 2023

$m_i(T)$

input T (1D)

constraints s, Δ& HTL ratio

test $P, \varepsilon, \eta/s$

PLB **844**, 138088

STEP 2 • 2025

$m_i(T, \mu_B)$

input T, μ_B (2D)

constraints $+\chi_2^B, \chi_4^B$

test $c_s^2, \eta/s(T, \mu_B)$

PLB **868**, 139692

STEP 3 • 2026

$m_i(T, \mu_B, \mu_Q, \mu_S)$

input T, μ_B, μ_Q, μ_S (4D)

constraints $\chi_{i,j,k}^{B,Q,S}$, $n_X|_{\mu=0}$

test T' expansion, C_{BS}

arXiv:2604.22352

ResNet backbone: 8 hidden layers $\times$ 32 neurons, Gauss quadrature layer (25 pt 2023; 50 pt 2025/26).

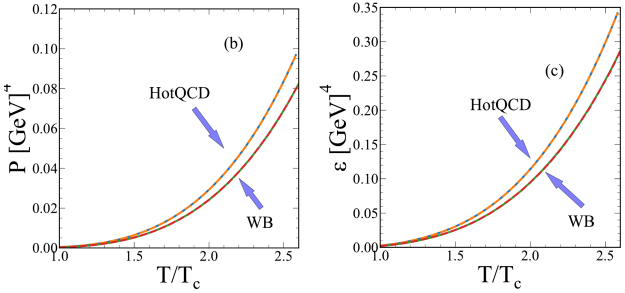
PART 03

Results along the three-step program

$m(T)$: trained on two quantities, predicted many • $m(T, \mu_B)$: susceptibilities as constraints • 4D DLQPM against the lattice

Work I: two inputs in, many observables out

Quantities that **never entered the loss** come out correct — that is the self-consistency test.



Left: pressure P; right: energy density ε (both in GeV^4); solid = DNN, dashed = lattice. Blind test $\leq 0.5\%$ (HotQCD), $\leq 1.7\%$ (ε, WB); P: $\leq 6.3\%$ at T_c ($P \to 0$).

IN THE LOSS

s and Δ — the only training inputs.

BLIND TEST

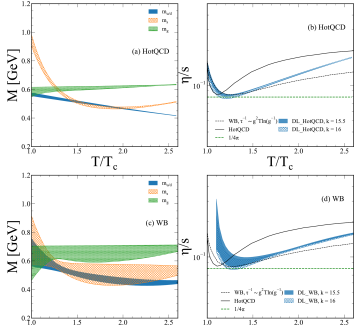
P and ε — never trained on.

PHYSICS OUT

η/s minimum at $1.25\,T_c$ (JETSCAPE).

Work I: the network learns $m(T)$ from two observables

Trained on $s(T)$ and $\Delta(T)$ only — the masses come out, and so does the η/s they imply.



Learned masses (left) and the η/s they imply (right); HotQCD (upper), WB (lower); bands $k = 15.5$ and 16, green dashed $1/4\pi$.

MODEL

Three ResNets, sigmoid-bounded output.

INTEGRATION LAYER

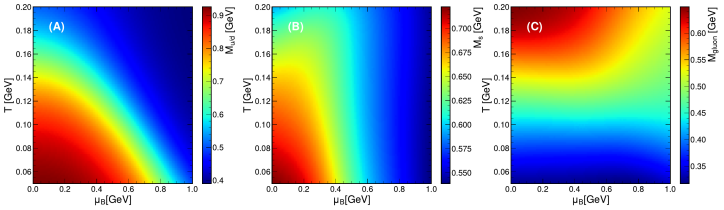
25-point Gauss quadrature layer.

CONSTRAINT

$$R_{g/q} = \sqrt{\frac{3}{2}\left(\frac{N_C}{3} + \frac{N_f}{6}\right)} = \frac{3}{2}.$$

Work II: the mass becomes a surface

The μ_B dependence is **produced by the network itself**, not assumed.



Quasi-particle masses in the (T, μ_B) plane: (A) u/d quark, (B) s quark, (C) gluon.

INPUT

(T, μ_B); architecture identical to Step 1.

NEW CONSTRAINTS

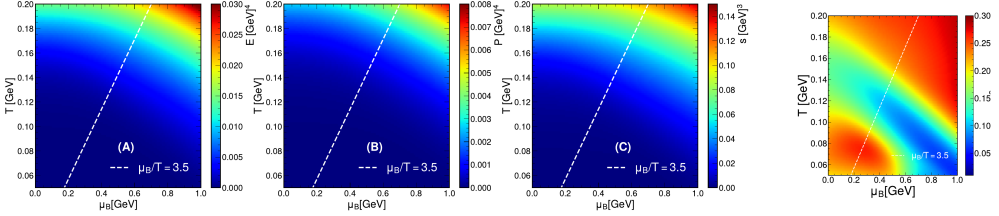
χ_2^B, χ_4^B and $n_B|_{\mu_B=0} = 0$.

TREND

Quarks fall with T, μ_B; the gluon rises with T.

Work II: the EoS and the speed of sound

P, ε, s and c_s^2 were not trained on — they are a **blind extrapolation** to finite μ_B.



Left: ε, P, s; right: c_s^2 (colour 0.05–0.30), both in the (T, μ_B) plane; along T, c_s^2 dips then rises.

AGREEMENT

Matches the lattice over $0 \leq \mu_B/T \leq 3.5$.

CROSSOVER

c_s^2 dips then rises: hadron gas $\rightarrow$ QGP.

Work II: where the QGP is least viscous

The η/s minimum sits **above the chemical freeze-out line** — the hottest, densest matter needs a higher temperature.

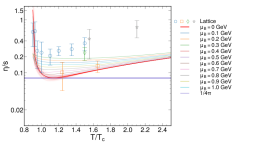


FIG. 7. (Color online) The temperature dependence of the shear viscosity to entropy density ratio η/s. The red curve represents our theoretical predictions obtained using the DLQP mass framework. The discrete data points correspond to lattice QCD calculations reported in [55–57]. Additionally, we also present the μ_B-dependent behavior of η/s as a function of temperature.

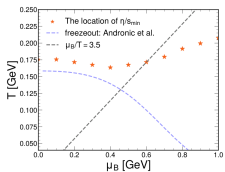


FIG. 8. (Color online) The location of the minimum value of η/s in the $T-\mu_B$ plane. The orange points represent the values computed using the masses learned by the DNN. The blue dashed line corresponds to the freeze-out result.[58].

We gratefully acknowledge the extensive computing resources provided by the Nuclear Computing Center of Central China Normal University.

[1] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C **58**, 1804 (1998), arXiv:nucl-th/9804027.
[2] J. M. Lattimer and M. Prakash, Astrophys. J. **550**, 426
[16] A. Bazavov *et al.* (HotQCD), Phys. Lett. B **795**, 15 (2019), arXiv:1812.08235 [hep-lat].
[17] A. Bazavov *et al.*, Phys. Rev. D **95**, 054504 (2017),

MINIMUM AT $\mu_B = 0$

175 MeV

position of the η/s minimum.

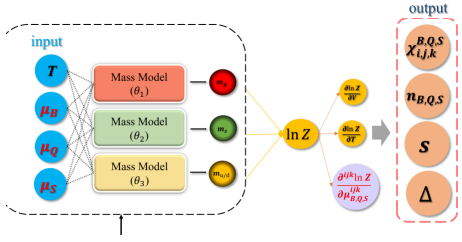
Consistent with lattice estimates and with Step 1.

MOVES WITH μ_B

The minimum shifts down for $\mu_B/T \leq 3.5$, then turns upward.

Work III: four chemical potentials, one $\ln Z$

The input becomes (T, μ_B, μ_Q, μ_S) and the loss gains a **five-component** structure.



The same statistical-mechanics and autodiff pipeline, now in four dimensions.

QUARK CHEMICAL POTENTIALS

$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q, \;\; \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q,$
$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$

FIVE-PART LOSS

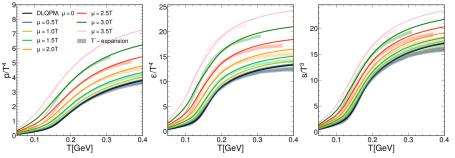
s and Δ/T; $\chi^{B,Q,S}_{i,j,k}$; $n_B = n_Q = n_S = 0$ at $\mu_X = 0$; HTL ratio.

TRAINING DATA

Lattice s, Δ, χ at $\mu_{B,Q,S} = 0$; 70 temperatures in 0.05–0.4 GeV.

Work III: the 4D EoS against the lattice

Agreement in **value and temperature behaviour** over the full range $\mu/T = 0 \to 3.5$.



p/T^4, ε/T^4, s/T^3; colours from $\mu/T = 0$ (black) to 3.5 (pink); grey band = lattice T' expansion.

VALID RANGE

$T \gtrsim 0.1$ GeV: agreement everywhere.

CAVEAT

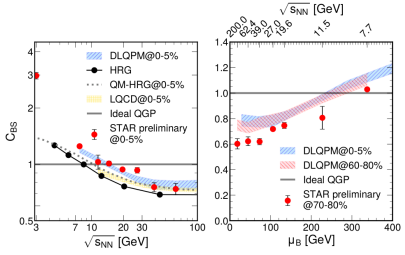
Below 0.1 GeV the model sits slightly high.

ADVANTAGE

One continuous, differentiable 4D function.

Work III: susceptibilities and the C_{BS} correlation

Adding μ_Q and μ_S is not optional: the higher-order susceptibilities improve **markedly**.



OBSERVABLE

$$C_{BS} = -3\chi_{11}^{BS}/\chi_2^{SS}$$

AGAINST DATA

Consistent with preliminary STAR data for 0–5% centrality, $\sqrt{s_{NN}} = 7.7$–100 GeV.

CAVEAT

Peripheral (60–80%): about 10% above data.

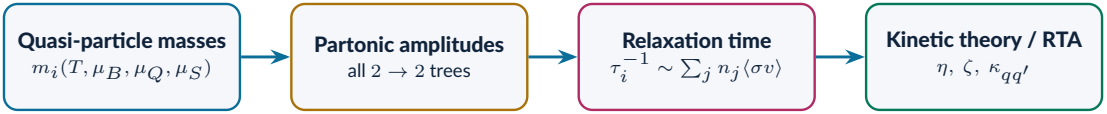
PART 04

Towards dissipative hydrodynamics

From the learned masses to the 4D transport coefficients • the multi-charge diffusion matrix $k_{qq'}$ • embedding into CLVisc

From the learned masses to 4D transport coefficients

One chain: the **same** quasi-particle masses feed the cross sections and the transport coefficients.



TARGET PRODUCTS

$\eta/s(T, \mu_B, \mu_Q, \mu_S)$, ζ/s, diffusion.

INGREDIENT

All partonic $2 \to 2$ tree diagrams.

CONSISTENCY

The **same** masses as in Step 3.

Three conserved currents, one coupled diffusion matrix

Three conserved currents: convection is diagonal, diffusion mixes through k_{ij}.

$$\partial_\mu N_B^\mu = \partial_\mu N_Q^\mu = \partial_\mu N_S^\mu = 0, \qquad N_i^\mu = n_i u^\mu + V_i^\mu, \quad u_\mu V_i^\mu = 0,$$

$$V_i^\mu = -\sum\nolimits_{j=B,Q,S} k_{ij}\, \nabla^\mu \left(\frac{\mu_j}{T}\right), \qquad i = B, Q, S,$$

$$Dn_i + n_i\,\theta + \nabla_\mu V_i^\mu = 0, \qquad D \equiv u^\mu \partial_\mu, \;\; \theta \equiv \nabla_\mu u^\mu.$$

$\nabla^\mu \equiv (g^{\mu\nu} - u^\mu u^\nu)\partial_\nu$ projects transverse to u^μ (metric $+,-,-,-$).

CONVECTION

Each charge rides with the fluid; $n_i\theta$ dilutes it.

DIFFUSION

All three gradients drive V_i^μ; k_{ij} couples them.

CONSTRAINTS

$n_Q = \frac{Z}{A} n_B$, $n_S = 0$;
$k_{ij} = k_{ji} \succeq 0$.

Multicomponent Israel–Stewart: J. A. Fotakis *et al.*, PRD **106**, 036009 (2022), arXiv:2203.11549; sign convention $k_{ij} = -\kappa_{ij} \succeq 0$.

Conserving three charges needs a matrix, not a number

With B, Q and S all conserved, dissipation is described by $k_{qq'}$, a 3×3 matrix.

	B	Q	S
B	k_{00}	k_{01}	k_{02}
Q	k_{10}	k_{11}	k_{12}
S	k_{20}	k_{21}	k_{22}

TWO ROUTES

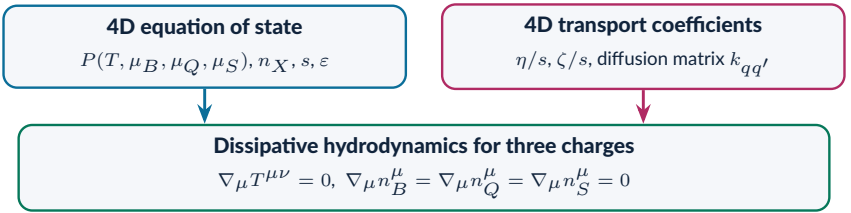
Kubo relations (correlators) and kinetic theory (linearized Boltzmann).

SAME INPUT

Both routes use the PINN masses and cross sections.

Embedding it all into CLVisc

CLVisc already has net-baryon conservation; the 4D EoS unlocks **net charge and net strangeness**.



NOW

Net baryons conserved.

NEXT

Bring n_Q, n_S into the evolution.

GOAL

Low-energy predictions.

Summary in four lines

Physics-driven inference = data + equations + constraints: the network represents, the physics constrains, autodiff differentiates.

1 Why

Fluid dynamics needs a 4D EoS and 4D transport coefficients.

2 How

Network + autodiff + physics loss removes the parametrization bias.

3 Results

$m(T) \to m(T, \mu_B) \to$ 4D; agrees with the lattice for $\mu/T \leq 3.5$.

4 Next

Cross sections $\to$ relaxation times $\to$ 4D $\eta/s, \zeta/s, k_{qq'}$.

Thermodynamic self-consistency is structural: everything descends from one ln $Z[m(T, \mu)]$.

Outlook

In one sentence: turn every non-perturbative QGP input into a **differentiable, constrainable, self-consistent** network module.

PHYSICS

- Complete 4D $\eta/s, \zeta/s$ from all tree diagrams;
- $k_{qq'}$ together with the 4D EoS;
- n_Q, n_S inside CLVisc;
- behaviour near the QCD critical point.

METHODS

- Uncertainty quantification (Bayesian NNs);
- attention, graph networks, neural operators;
- PINN/generative models accelerating relativistic hydrodynamics simulations;
- differentiable EoS/transport plugins.

References

1. F.-P. Li, H.-L. Lü, L.-G. Pang, G.-Y. Qin, *Deep-learning quasi-particle masses from QCD equation of state*, **PLB 844**, 138088 (2023), arXiv:2211.07994.
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3. F.-P. Li, L.-G. Pang, G.-Y. Qin, *Four-dimensional QCD equation of state from a quasi-parton model with PINNs*, arXiv:2604.22352 (2026).
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The complete list (quasi-particle history, transport coefficients, reviews) is in the BACKUP section.

Thank you

Questions and comments welcome

In collaboration with Fu-Peng Li, Hong-Liang Lü and Guang-You Qin

Phys. Lett. B **844**, 138088 (2023) • *Phys. Lett. B* **868**, 139692 (2025) • arXiv:2604.22352

BACKUP

Details behind the claims

Training setup and loss functions • a short history of quasi-particle models • the wider PINN programme • complete reference list

Backup: network, integration layer and loss

Reproducibility layer: the parts that make the method work.

ARCHITECTURE

- Three independent ResNets, one per mass function;
- 8 hidden layers $\times$ 32 neurons, swish activation;
- sigmoid output bound keeps masses positive;
- Gauss quadrature as a differentiable integration layer (25 pt 2023; 50 pt 2025/26).

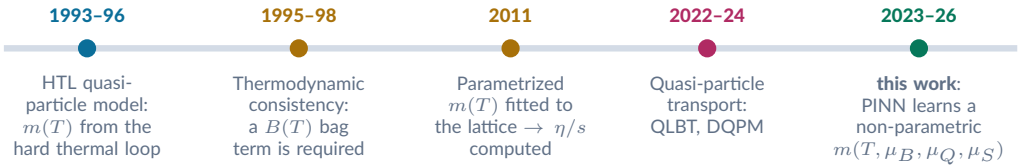
LOSS TERMS

- Work I: $\mathcal{L} = \|s_{\mathrm{NN}} - s_{\mathrm{LQCD}}\|^2 + \|\Delta_{\mathrm{NN}} - \Delta_{\mathrm{LQCD}}\|^2 + \mathcal{L}_{\mathrm{MC}}$;
- Work II: $+\,\chi_2^B, \chi_4^B, n_B|_{\mu_B=0} = 0$;
- Work III: five components – $s, \Delta/T$; $\chi_{i,j,k}^{B,Q,S}$; $n_X|_{\mu=0} = 0$; HTL.

Automatic differentiation in reverse mode: one forward and one backward pass give derivatives with respect to all inputs.

Backup: a short history of quasi-particle models

Thirty years of dressing: from HTL masses to learned mass functions.



1990s

Goloviznin–Satz, Peshier *et al.*, Gorenstein–Yang, Lévai–Heinz.

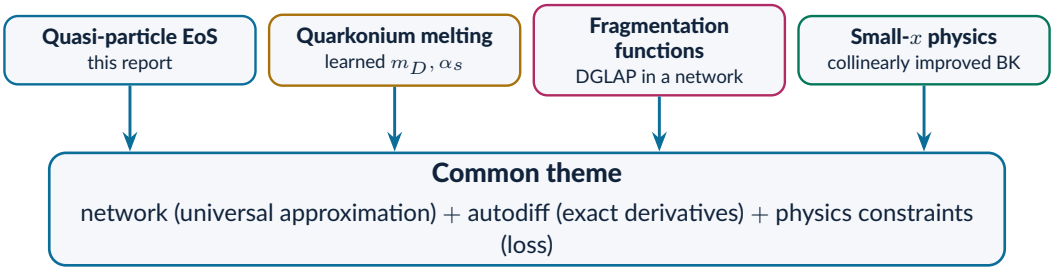
2011 onwards

Plumari *et al.*; Liu *et al.* (QLBT); Soloveva *et al.* (DQPM).

The recurring theme: a parametrized mass must be fitted, and the bag term must be handled.

Backup: the wider PINN programme in high-energy nuclear physics

The quasi-particle EoS is one instance of "network as representation + equations as constraints".



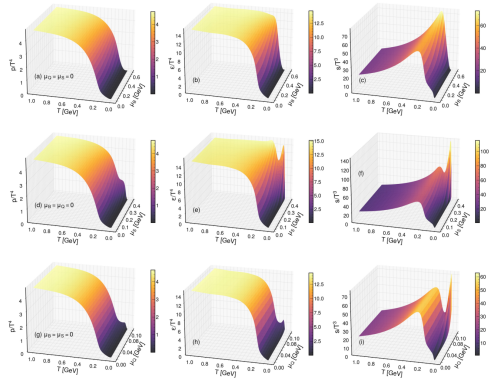
Reviews: L.-G. Pang, Int. J. Mod. Phys. E **33**, 2430009 (2024); K. Zhou *et al.*, Prog. Part. Nucl. Phys. **135**, 104084 (2024).

Backup: complete reference list

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Backup: EoS surfaces and the diffusion matrix

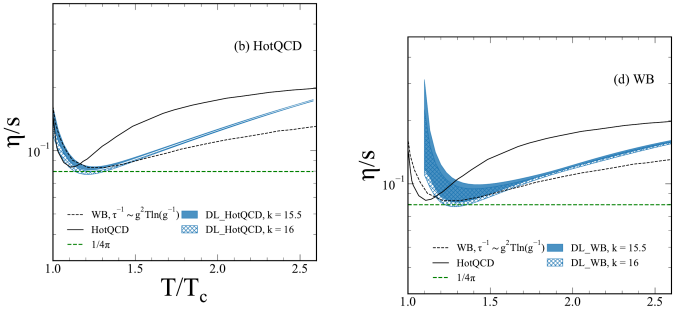
The DLQPM returns continuous surfaces in every (T, μ) plane, from one and the same ln Z.



- p/T^4, ε/T^4, s/T^3 in the (T, μ_B), (T, μ_S) and (T, μ_Q) planes;
- the growth along the chemical-potential directions is steeper at low T;
- these are exactly the interpolable, differentiable EoS tables fluid codes need.

Backup: the η/s that follows from the learned masses

Work I in full: the shear viscosity over entropy density from the learned mass functions, both lattice sets.



Enlarged η/s panels — same curves as the mass figure on p.14; $k = 15.5, 16$; minimum at $1.25\ T_C$ (JETSCAPE 60% CL).