Workshop on QCD Critical End Point
and Effective Field Theory Methods 2026
10/18-10/22, 2026



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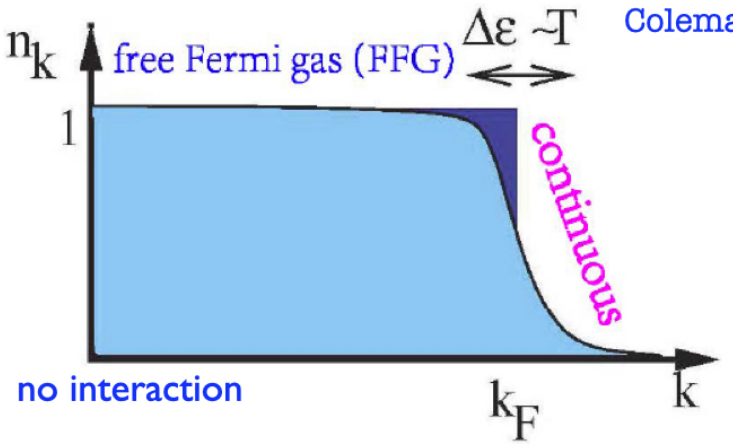
Nucleon Short-Range Correlations and Equation of State of Dense Matter

Bao-Jun Cai
Fudan University, 10/19/2026

Collaborators: Lie-Wen Chen (SJTU), Bao-An Li (East Texas A&M), Yu-Gang Ma (Fudan&ECNU)

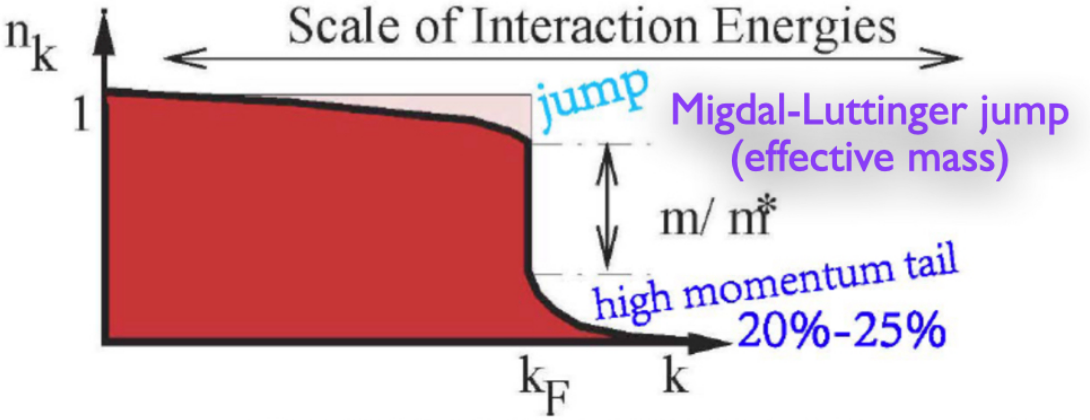
ref.: arXiv:2512.04206

The $n(k)$ at T=0



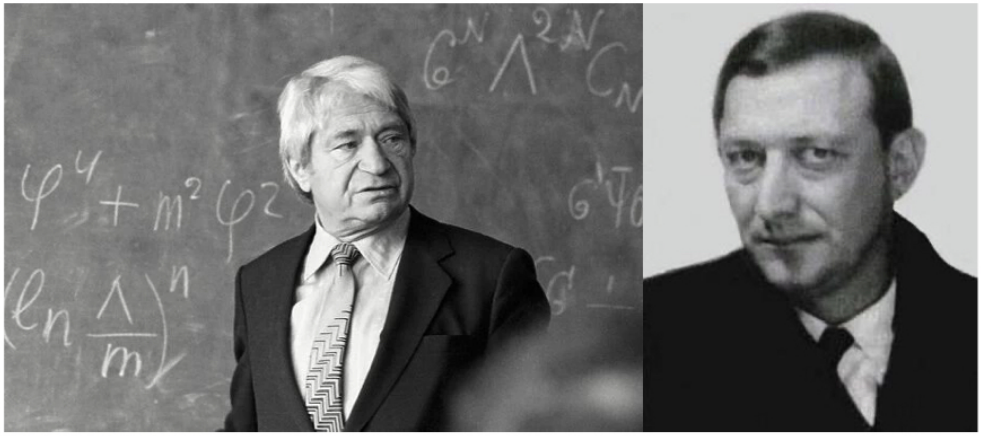
Coleman, Many-body Physics, 2015

Migdal, JETP, 1957; Luttinger, PR, 1960



$$\text{Fermi-Dirac}: \frac{1}{\exp[(\mu - E_k)/k_B T] + 1}$$

(thermal effect : $T \neq 0$)



even at T=0, there is a high momentum tail (HMT) in the single nucleon momentum distribution function $n(k)$

Short-range correlations (SRCs): ~1 fm

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two-body density : $\rho(\mathbf{x},\mathbf{x}') = [1 + \overbrace{F(\mathbf{x},\mathbf{x}')}^{\text{correlation}}]\rho(\mathbf{x})\rho(\mathbf{x}')$

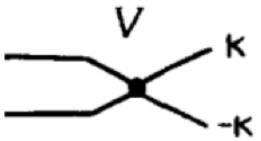
(probability of finding one nucleon at i and another at i')

↓ Fourier transform

$$n_{\mathbf{k}} = \int \rho(\mathbf{x},\mathbf{x}') e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} d\mathbf{x} d\mathbf{x}'$$

Antonov, Hodgson and Petkov, Nucleon Correlations in Nuclei, 1993

pair $\sim \psi \cdot \psi$

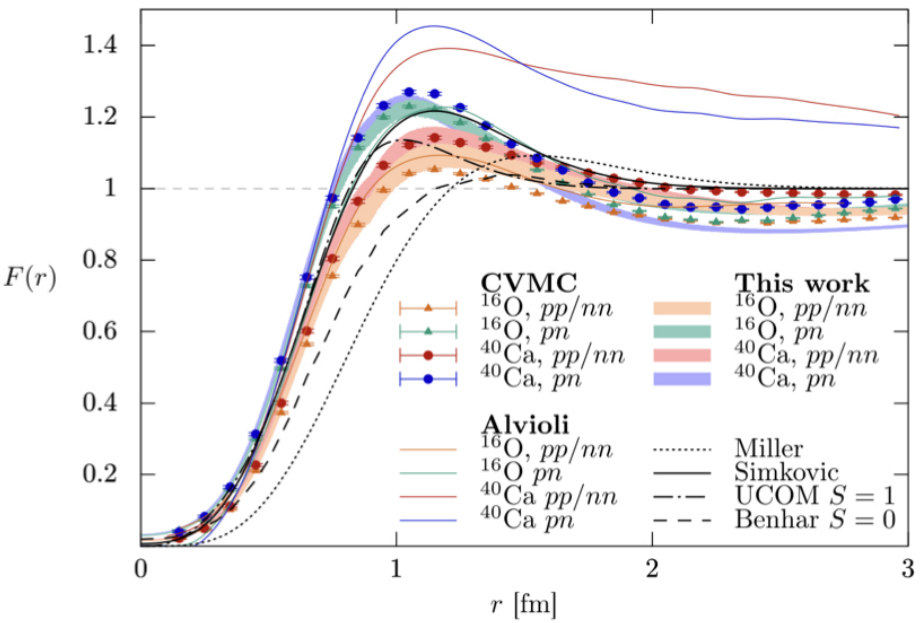


$$n_{\mathbf{k}} \sim \psi^2(k) \xrightarrow{|\mathbf{k}|\to\infty} \frac{\bar{V}^2(k)}{|\mathbf{k}|^4} \xrightarrow[V(\mathbf{x})\sim\delta(\mathbf{x})\to\bar{V}(k)=\text{const.}]{\text{contact interaction}} \sim \frac{1}{|\mathbf{k}|^4}$$

leading-order term in pion-less EFT

Amado, PRC, 1976
Hammer et al., RMP, 2020

Cruz-Torres et al., PLB, 2018



correlation occurs at ~ 1 fm
↔ short-range correlations (SRCs)

Tensor force and OPEP

about 4% high-momentum (HM) component in deuteron wave function ↔ d-wave (L=2)

(i) the presence of a large high-momentum component of the deuteron wave function:

$$\int \psi_D^2(k)\theta(k-0.3\ \mathrm{GeV}/c)\,\mathrm{d}^3k \sim (3.5\text{–}5)\% \ ;$$

(ii) dominance of the D-wave in the high-momentum component:

$$\int w^2(k)\theta(k-0.3\ \mathrm{GeV}/c)\,\mathrm{d}^3k \sim (3\text{–}4)\% \ ;$$

d-wave component denominates at large momentum k

Frankfurt and Strikman, Phys. Rep., 1988

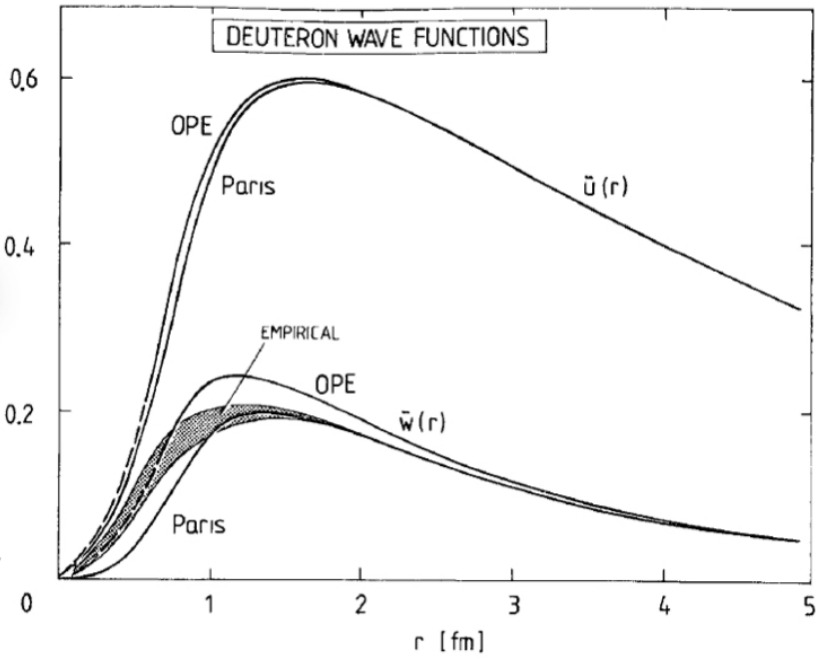
$$V_{\mathrm{OPT}}(\mathbf{x}) = \frac{1}{3}\frac{f^2}{4\pi}\left[\frac{e^{-m_\pi r}}{r} - \frac{4\pi}{m_\pi^2}\delta(\mathbf{x})\right](\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2)$$
$$+ \frac{1}{3}\frac{f^2}{4\pi}\left(1+\frac{3}{m_\pi r}+\frac{3}{m_\pi^2 r^2}\right)\frac{e^{-m_\pi r}}{r} S_{12}(\vec{\tau}_1\cdot\vec{\tau}_2)$$

tensor operator

small r:

$$V_{\mathrm{OPE}}(\mathbf{x}) \sim \frac{1}{r^3} \leftrightarrow \text{singular} \leftrightarrow \text{manifestations as SRCs}$$

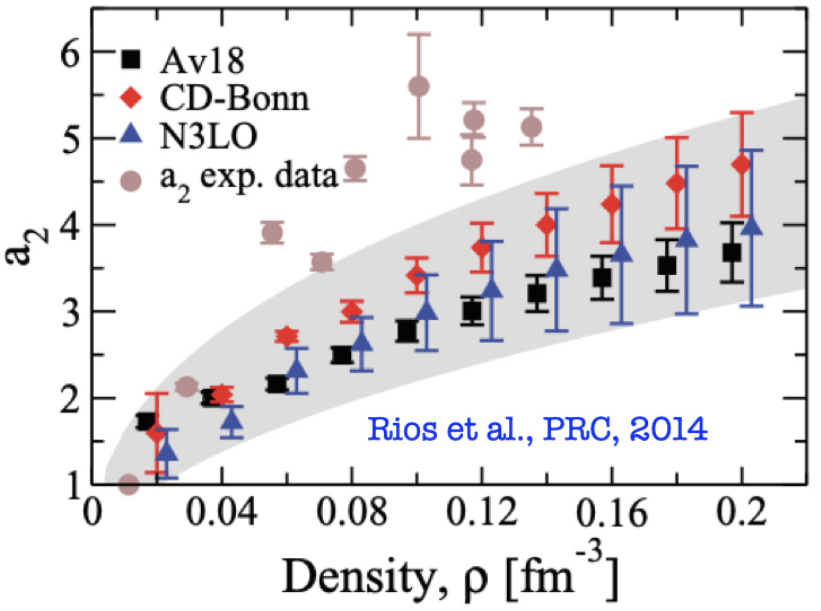
Ericson and Rosa-Clot, ARNPS, 1985
Ericson and Weise, Pion and Nuclei, 1988



u : s-wave function
w : d-wave function
↔ tensor force ↔ np≫pp

SRC pairs in nuclei and nuclear matter

$a_2(A) = n_k^A / n_k^D$: probability of finding a HM pair in nucleus A relative to deuteron



self-consistent Green's function theories

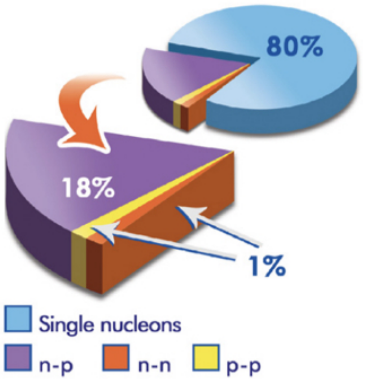
$\phi_0 \lesssim 2.5 \leftrightarrow \phi_0 k_F \lesssim 700$ MeV (exp.)

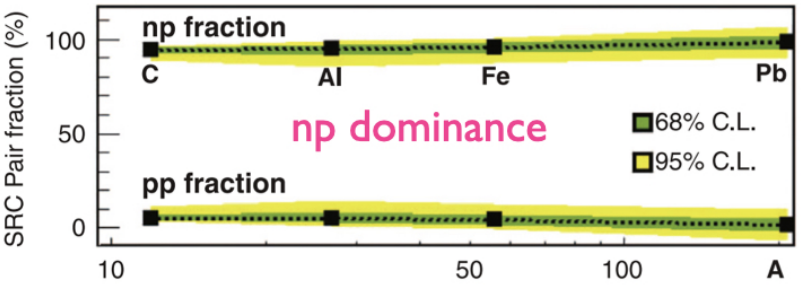
ϕ_0 : high momentum cutoff

C_0 : HMT strenghth

$$\text{SNM: } n_k^0 = C_0 \left(\frac{k_F}{|k|}\right)^4 \quad \text{for } k_F \lesssim |k| \lesssim \phi_0 k_F$$

$$x_{SNM}^{HMT} = \int_{k_F}^{\phi_0 k_F} n_k^0 dk \Big/ \int_0^{\phi_0 k_F} n_k^0 dk = 3C_0 \left(1 - \phi_0^{-1}\right)$$





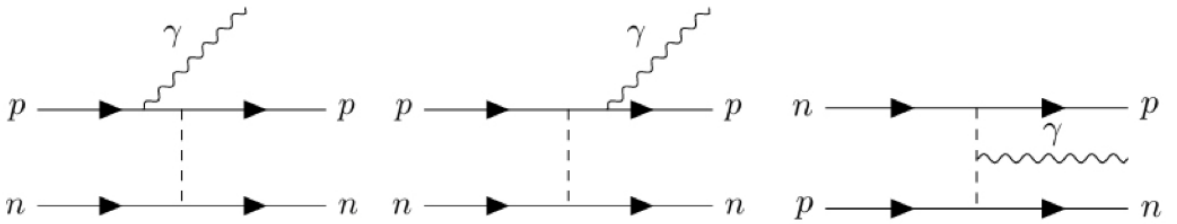
$a_2(\infty) \approx 7 \pm 1$:

SNM: $28\% \pm 4\%$; PNM: 1%-2%

Subedi et al., Science, 2008; Hen et al., Science, 2014

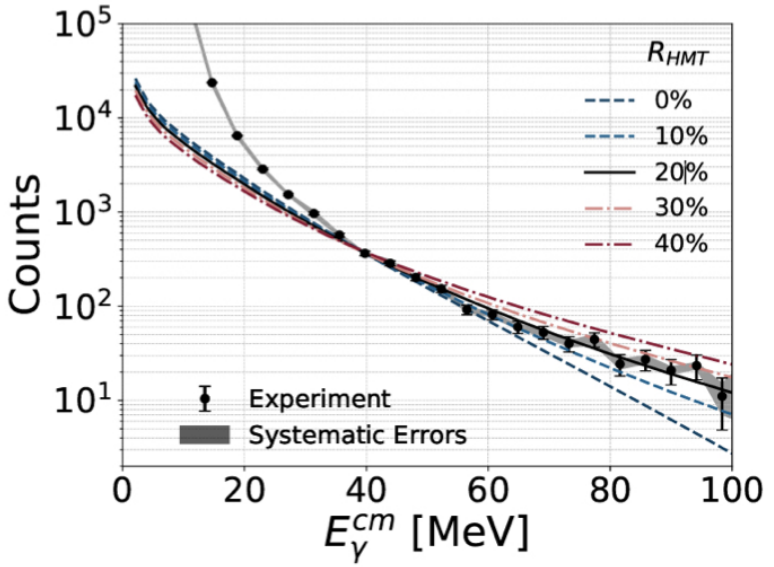
Bremsstrahlung Gamma Ray Emission

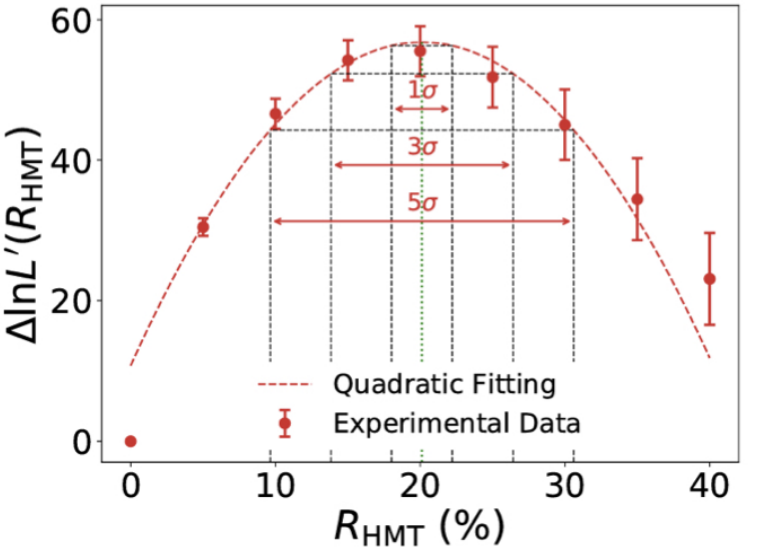
Z.G. Xiao Group, Tsinghua University



SNM: ≈ 20% in ^{124}Sn

(IBUU-MDI simulation)



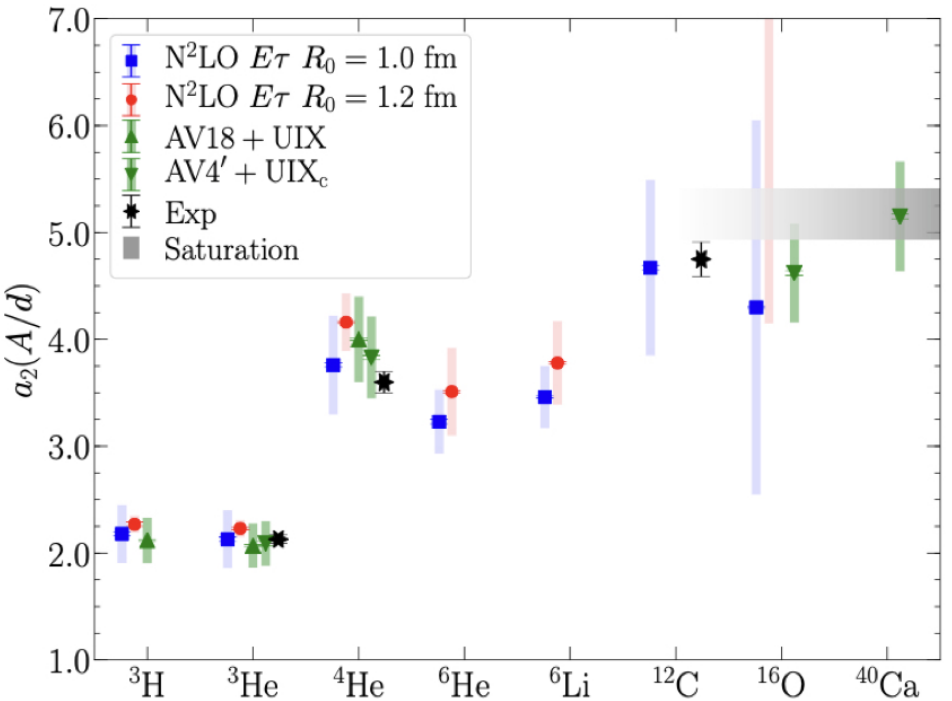


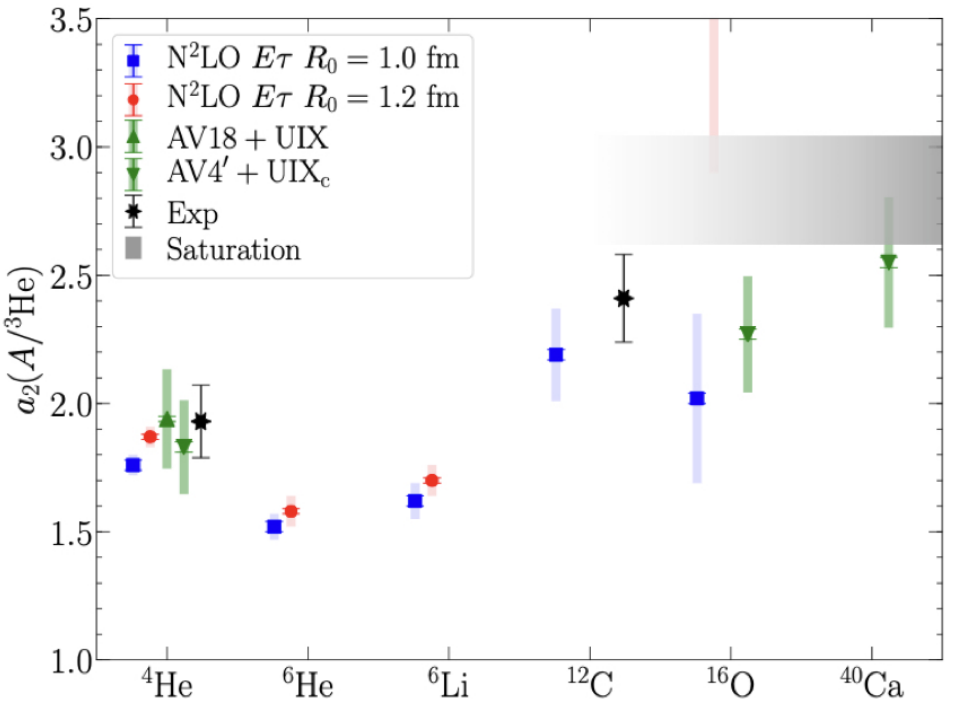
Xu et al., PRC, 2026

Scaling factor from ab initio with chiral EFT

$a_2 \approx 5.2 \pm 0.2$ consistent with Xiao's group result

$$x_{\mathrm{SNM}} \approx a_2 \cdot \underbrace{x_{\mathrm{deuteron}}}_{\approx 4\%}$$

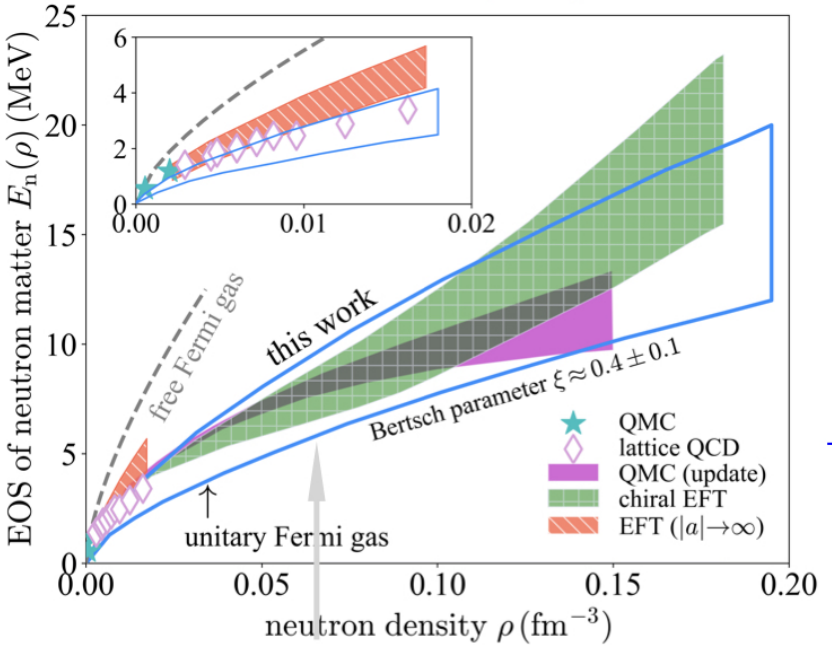




Lynn et al., JPG, 2020

Pure neutron matter: Tan's contact relations

B.J. Cai and B.A. Li, PRC, 2015



PNM: $E_n(\rho) \approx \frac{3}{5}\frac{k_F^{n,2}}{2M}\left[\xi - \zeta\left(\frac{1}{k_F^n a}\right) - \frac{5\nu}{3}\left(\frac{1}{k_F^n a}\right)^2\right]$

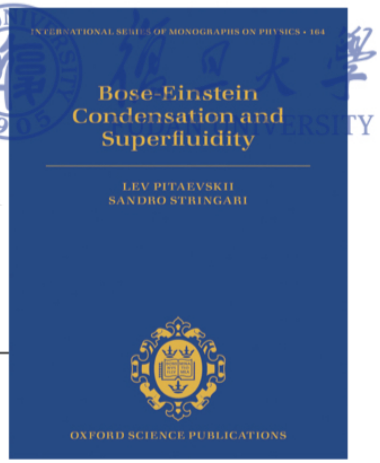
$\xi \approx 0.4$: Bertsch parameter; $\zeta \approx \nu \approx 1$: universal

Bertsch, 1999; Bulgac, PRL, 2005

18

Tan Relations and the Contact Parameter

S.N. Tan, Ann. Phys., 2008



unitarity condition: $|a| \gg d \gg r$

$a(^1S_0) \approx -18.8\,\text{fm};\ d \approx \rho_0^{-1/3} \approx 1.8\,\text{fm};\ r \approx \frac{1}{2m_\pi} \approx 0.7\,\text{fm}$

$\to C_n k_F^{n,4} = -4\pi M \frac{d(\rho E_n)}{d(a^{-1})}$ (adiabatic sweep theorem)

$C_n\left(1 - \phi_n^{-1}\right) = x_{PNM}^{HMT} \approx 1\%\text{-}2\%$

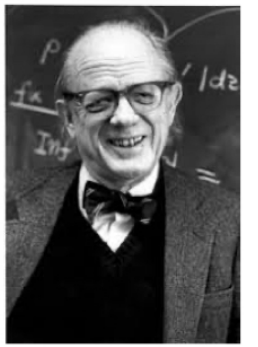
$|a| \gg d \gtrsim r$

ultracold atoms:

$a \gg d \gg r$

(Feshbach resonance)



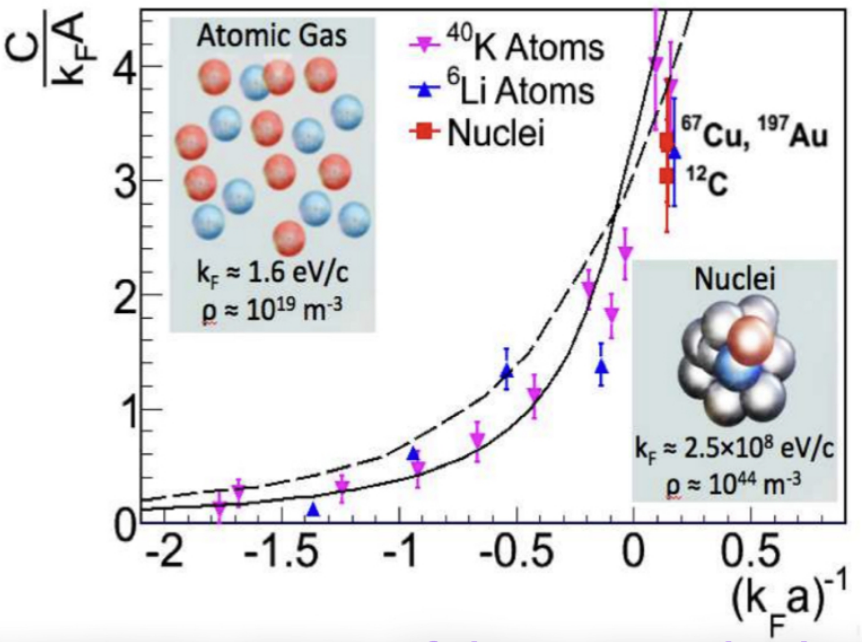


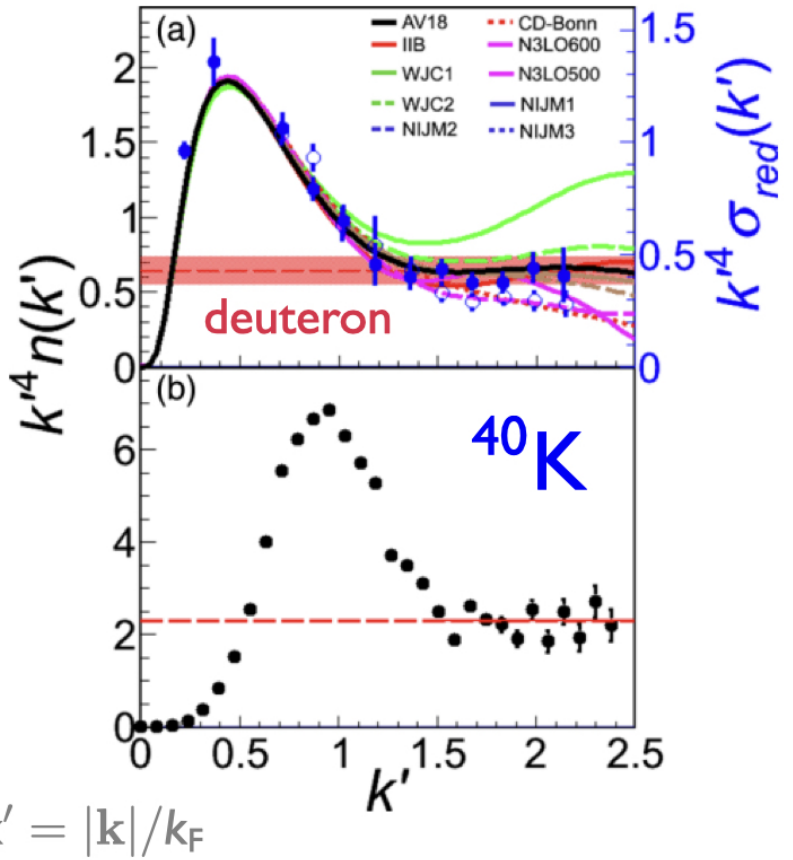
Similarity: ultracold Fermi gas & finite nuclei

ultracold Fermi gas and nuclei:

(1) differs by $\gtrsim$25 orders of magnitude in density

(2) Fermion-Fermion interactions are very different





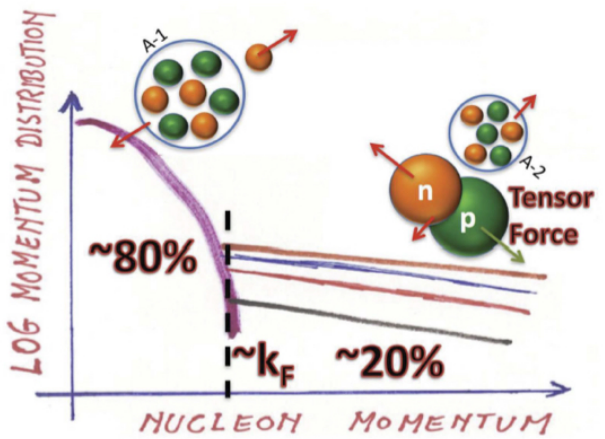
$k' = |\mathbf{k}|/k_F$

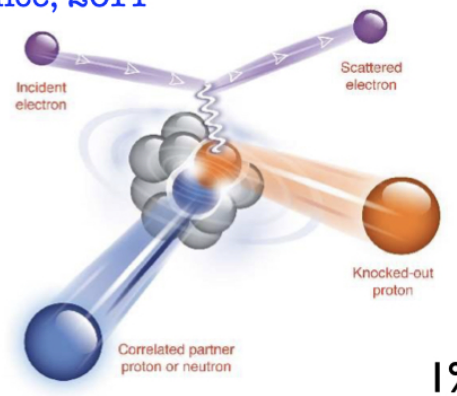
Hen et al., PRC, 2015

deep quantum nature of these many-body systems

Summary: $n(k)$ with (isospin-dependent) HMT

Hen et al., RMP, 2017; Hen et al., Science, 2014





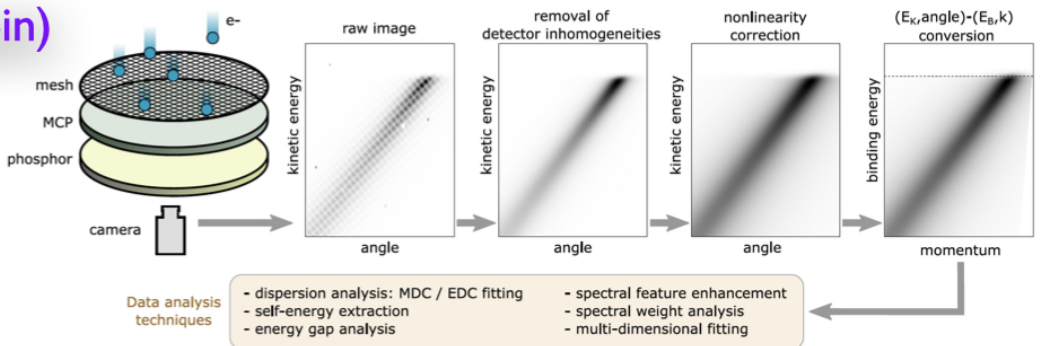
zero-momentum depletion

$$n_{\mathbf{k}}^{J}(\rho,\delta)=\begin{cases}\Delta_J+\beta_J(|\mathbf{k}|/k_F^J)^2, & 0<|\mathbf{k}|<k_F^J\\ C_J(k_F^J/|\mathbf{k}|)^4, & k_F^J<|\mathbf{k}|<\phi_J k_F^J\end{cases}$$

$$\mathcal{P}_J=\mathcal{P}_0\left(1+\tau_3^J\mathcal{P}_J\delta\right)$$

25% HMT: SNM
1%-2% HMT: PNM
} → strong isospin dependence

analogy: angle-resolved photoemission spectroscopy (ARPES)->electron wf



Sobota et al., RMP, 2021

nuclear interaction is more complicated ($\rho, \delta, \mathbf{k}$, spin)

spectral function: $\mathcal{A}(\mathbf{k},\omega)=2\mathrm{Im}\,G(\mathbf{k}-i0^+)\sim \mathrm{d}\sigma$

hole

$$\mathcal{A}(\mathbf{k},\omega)/2\pi=\overbrace{Z_{\mathbf{k}}\delta(\omega-\epsilon_{\mathbf{k}})}^{\epsilon_{\mathbf{k}}=\mathbf{k}^2/2M-\mu}+\text{continuum}$$

$$n_{\mathbf{k}}=\int_{-\infty}^{0}\frac{\mathrm{d}\omega}{2\pi}\mathcal{A}(\mathbf{k},\omega)=Z_{\mathbf{k}}\Theta(-\epsilon_{\mathbf{k}})+\frac{C}{|\mathbf{k}|^4}\Theta(\epsilon_{\mathbf{k}})$$

Early derivations on $n(k)$ with a k^{-4} HMT

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Czyz and Kottfried, Nucl. Phys., 1960

Belyakov, JETP, 1961;Sartor and Mahaux, PRC, 1980, ...

THE MOMENTUM DISTRIBUTION IN A DILUTE FERMI GAS AT ZERO TEMPERATURE

W. CZYŻ † and K. GOTTFRIED ††

Institute for Theoretical Physics, University of Copenhagen, Denmark

Received 2 September 1960

According to (11), $n(p)$ varies as p^{-4} when $(p/p_F) \to \infty$. The expressions given above are not, however, expected to be valid when $pa > 1$, since the detailed structure of the interparticle force, and not just the S-wave scattering length, comes into play for such large momenta. Therefore the p^{-4}-dependence should only be taken seriously when the inequalities $p \gg p_F$ and $pa < 1$ are satisfied simultaneously.



THE MOMENTUM DISTRIBUTION OF PARTICLES IN A DILUTE FERMI GAS

V. A. BELYAKOV

Moscow Engineering Physics Institute

Submitted to JETP editor November 26, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 1210-1212 (April, 1961)

The first two terms of the expansion in powers of the gas parameter of the momentum distribution of particles in a non-ideal Fermi gas are obtained, using a perturbation method.

$$x = |\mathbf{k}|/k_F$$

We quote the values of the function n(p) eral points:

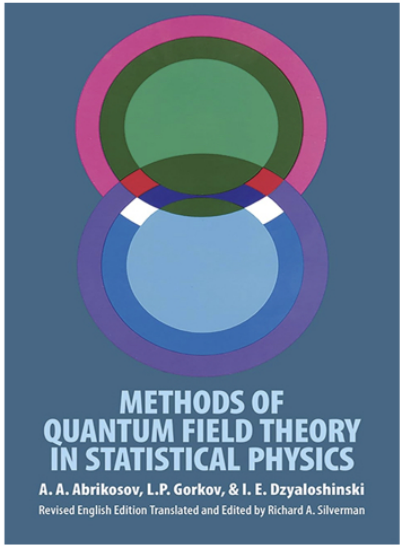
$$n(0) = 1 - 2(3na^3/\pi)^{2/3}\left(1 - \tfrac{1}{2}\ln 2\right),$$

$$n(p_0 - 0) = 1 - 2(3na^3/\pi)^{2/3}\left(\tfrac{1}{3} + \ln 2\right),$$

$$n(p_0 + 0) = 2(3na^3/\pi)^{2/3}\left(\ln 2 - \tfrac{1}{3}\right),$$

$$n(p) = (4/9x^4)(3na^3/\pi)^{2/3} \quad \text{as} \quad p \to \infty.$$

$$|k_F a| \ll 1$$



new features:

(1) sizable fraction of HMT nucleons: ~25% (SNM)

(2) isospin dependence of the HMT (SNM+PNM)

(3) without the restriction of smallness: k_F*a<<1

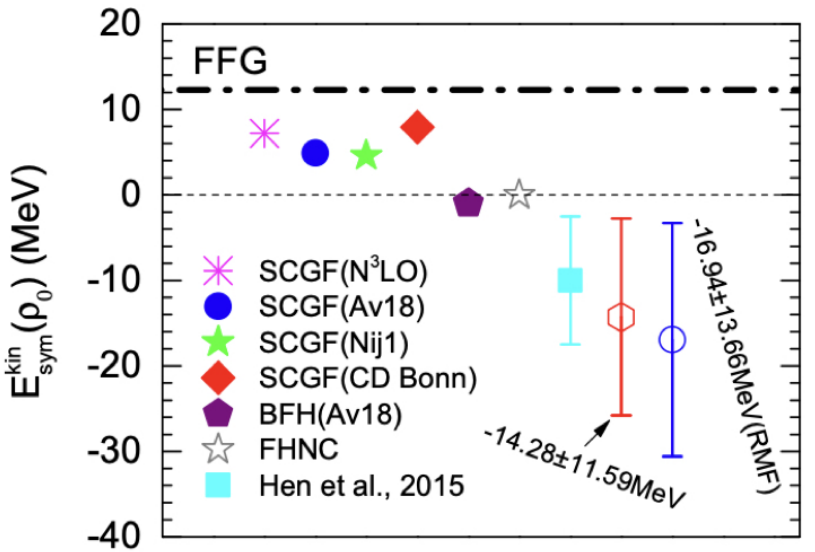
(4) experimental data available (e.g., HIAF, JLab, …)

Kinetic EOS with SRC-HMT

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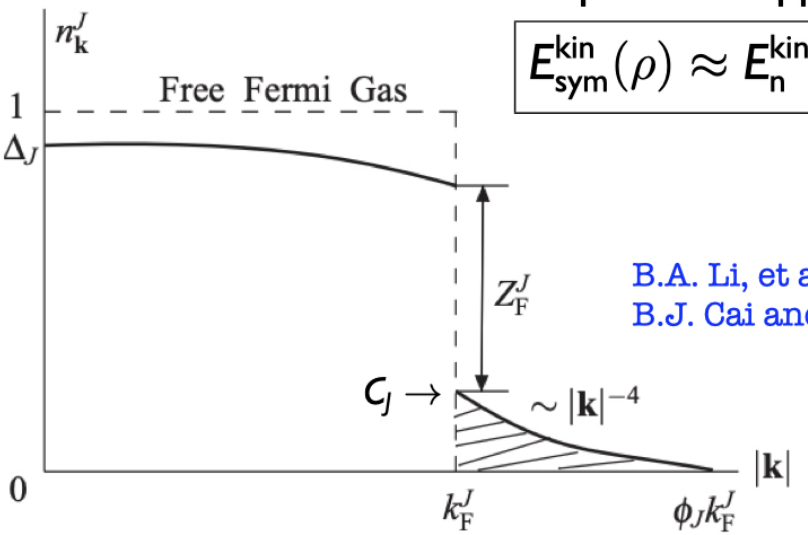
kinetic energy per nucleon (kinetic EOS):

$$E^{kin}(\rho,\delta) = \frac{1}{\rho}\underbrace{\frac{2}{(2\pi)^3}\sum_{J=n,p}\int_0^{\infty}\frac{k^2}{2M}n_k^J(\rho,\delta)dk}_{\text{kinetic energy density } \varepsilon^{kin}(\rho,\delta)}$$



parabolic approximation

$$E^{kin}_{sym}(\rho) \approx E^{kin}_{n}(\rho) - E^{kin}_{0}(\rho)$$



B.A. Li, et al., PPNP, 2018
B.J. Cai and B.A. Li, PRC, 2016

(1) about 28% np at HM state in SNM $\rightarrow E_0(\rho)$ is enhanced

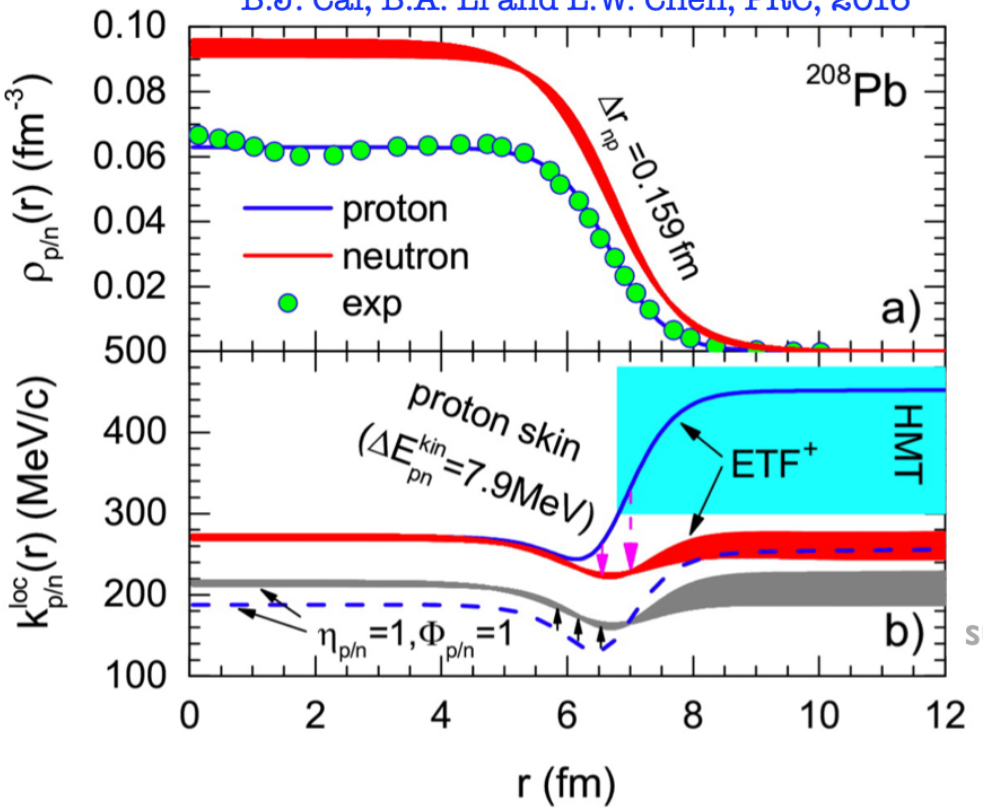
(2) 1%-2% neutrons at HM state in PNM $\rightarrow E_n(\rho) \sim$ free Fermi gas

(3) symmetry energy $\approx$ PNM-SNM (parabolic approximation)

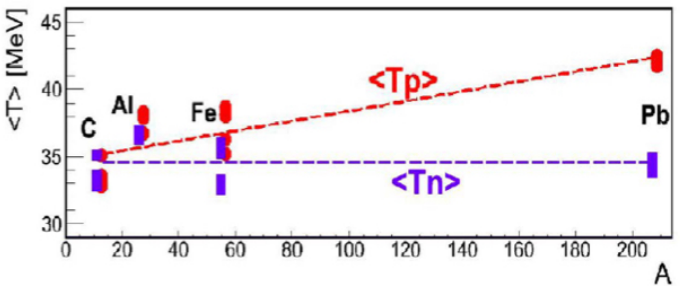
Where these high-momentum protons live?

protons move faster than neutrons in neutron skins

B.J. Cai, B.A. Li and L.W. Chen, PRC, 2016



Hen et al., Science, 2014



extended Thomas-Fermi (TF) approximation (ETF$^+$):

$$\varepsilon_J^{kin}(r) = \frac{1}{2M}\left[\tau_{TF}[\rho_J] + \tau_2[\rho_J] + \cdots\right]$$

TF term: $\tau_{TF}[\rho_J] = \frac{2}{(2\pi)^3}\int_0^{\phi_J k_F^J} k^2 n_k^J dk = \frac{3}{5}(3\pi^2)^{2/3}\rho_J^{5/3}\Phi_J$

$\Phi_J = 1 + C_J(5\phi_J + 3/\phi_J - 8) \leftrightarrow$ HMT factor

surface contribution

Weizsäcker term: $\tau_2[\rho_J] = \frac{\eta_J}{36}\frac{(\nabla\rho_J(r))^2}{\rho_J(r)} + \frac{1}{3}\Delta\rho_J(r)$

ρ_J: 2-parameter Fermi distribution

local momentum: $\varepsilon_J^{kin}(r)/\rho_J(r) = [k_J^{loc}(r)]^2/2M$

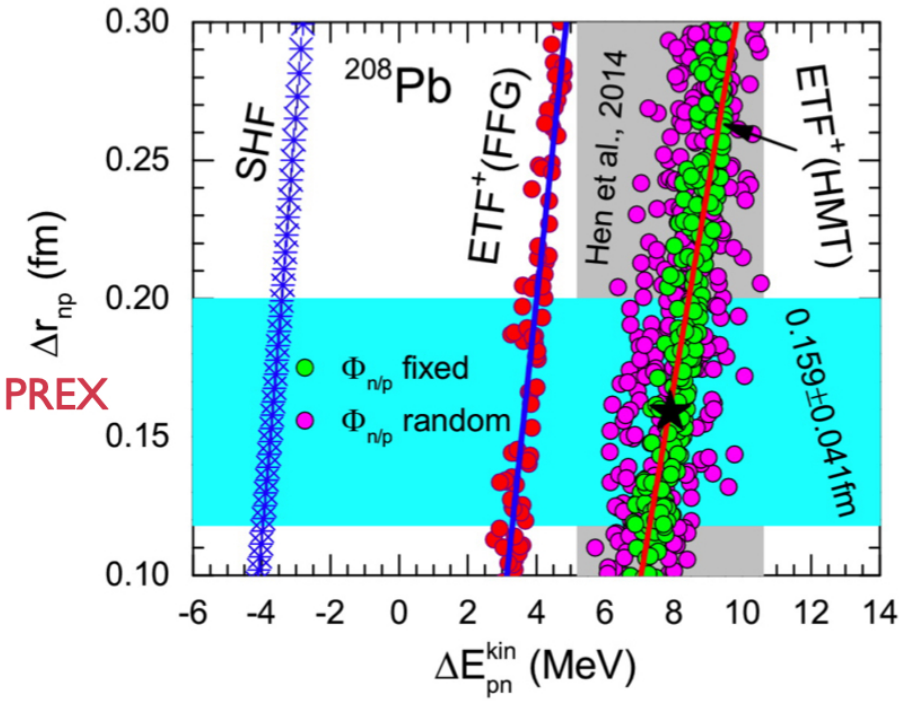
Correlation between neutron-skin and proton-skin

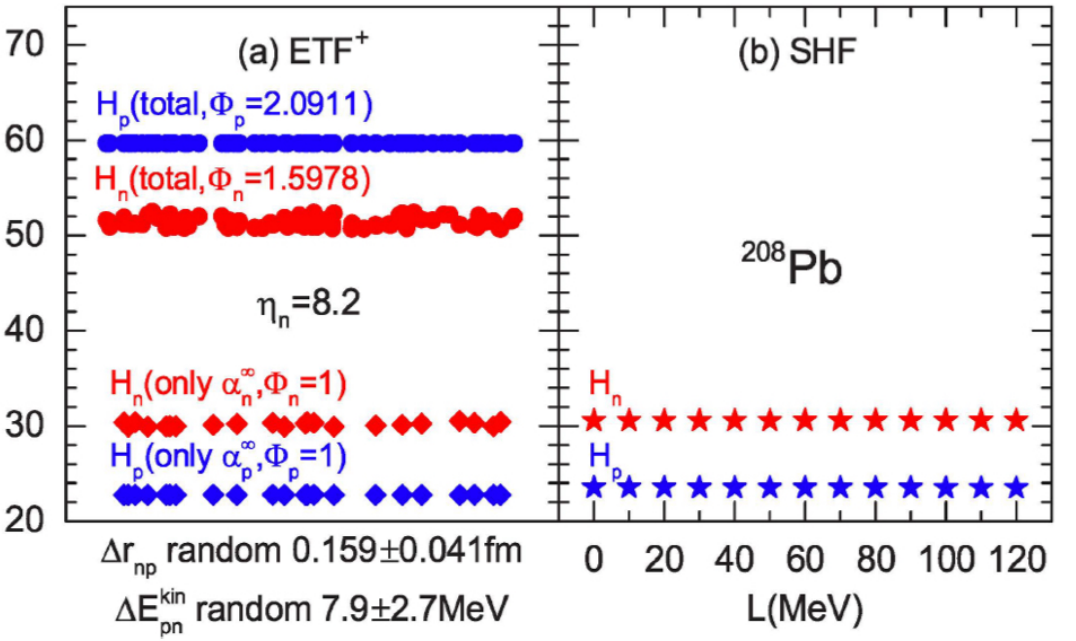
neutron skin: $\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$

proton skin: $\Delta E_{pn}^{kin} \equiv \langle T_p \rangle - \langle T_n \rangle$

$$\left(\langle T_J \rangle = \int \varepsilon_J^{kin}(r)\mathrm{d}x \Big/ \int \rho_J(r)\mathrm{d}x \right)$$

$$H_J \equiv \langle r_J^2 \rangle \langle k_J^2 \rangle = \underbrace{\langle r_J \rangle^2 \langle k_J \rangle^2}_{\approx\text{const.}\leftrightarrow \text{Liouville theorem}} \left[1 + \underbrace{\left(\frac{\delta r_J}{\langle r_J \rangle} \right)^2 + \left(\frac{\delta k_J}{\langle k_J \rangle} \right)^2}_{\text{small relative fluctuations}} \right] + \underbrace{\delta^2 r_J \delta^2 k_J}_{\approx\text{const.}\leftrightarrow \text{Heisenberg uncertainty}} \approx \text{const.}$$





B.J. Cai, B.A. Li and L.W. Chen, PRC, 2016

Need a potential —> full EOS of NM

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kinetic EOS: momentum-dependence interaction (MDI)

$$E^{kin}(\rho,\delta) = \frac{1}{\rho}\underbrace{\frac{2}{(2\pi)^3}\sum_{J=n,p}\int_0^\infty \frac{k^2}{2M} n_k^J(\rho,\delta)dk}_{\text{kinetic energy density } \varepsilon^{kin}(\rho,\delta)}$$

see, e.g, on MDI:
Gale et al., PRC, 1987
B.A. Li, PRC, 2004

potential EOS: $\Omega(k,k')$: momentum-dependence kernel

$$E^{pot}_{mom}(\rho,\delta) = \sum_{J,J'}\frac{C_{JJ'}}{\rho\rho_0}\int dkdk' f_J(x,k) f_{J'}(x,k')\Omega(k,k')$$

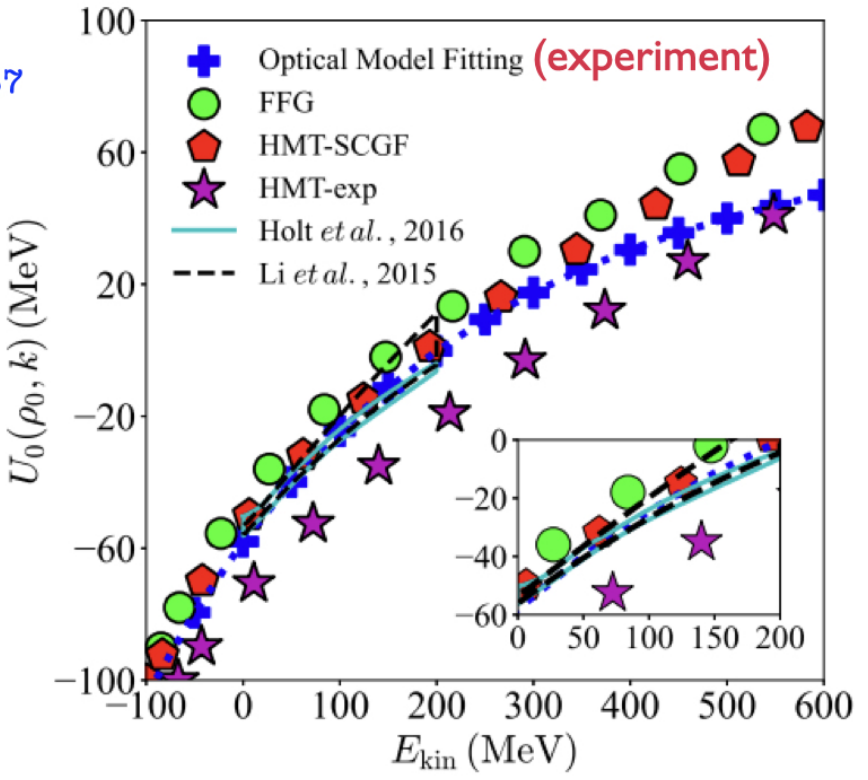
$$E^{pot}_{den}(\rho,\delta) = \frac{A_l(\rho_p^2+\rho_n^2)}{2\rho\rho_0} + \frac{A_u\rho_p\rho_n}{\rho\rho_0} + \frac{B}{\sigma+1}\left(\frac{\rho}{\rho_0}\right)^\sigma(1-x\sigma^2)$$

full EOS: x : 3-nucleon force

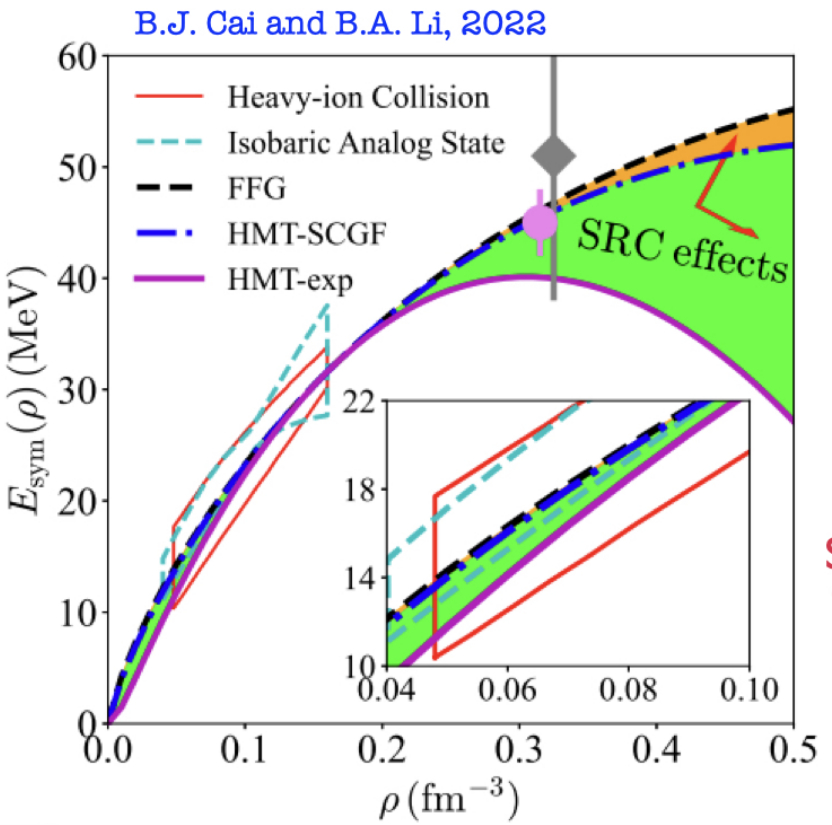
$$E(\rho,\delta) = E^{kin}(\rho,\delta) + E^{pot}_{mom}(\rho,\delta) + E^{pot}_{den}(\rho,\delta)$$

phase-space distribution: $f_J(x,k) = \frac{1}{4\pi^3}n_k^J(\rho,\delta)$

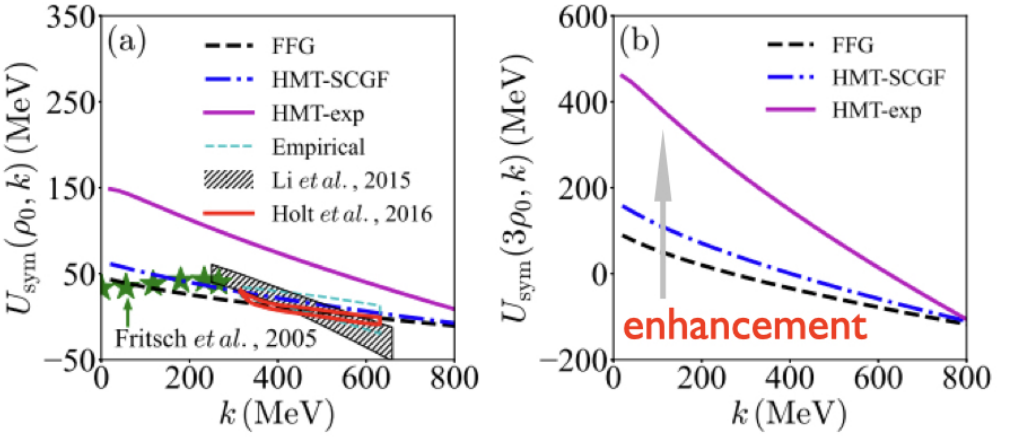
B.J. Cai and B.A. Li, 2022

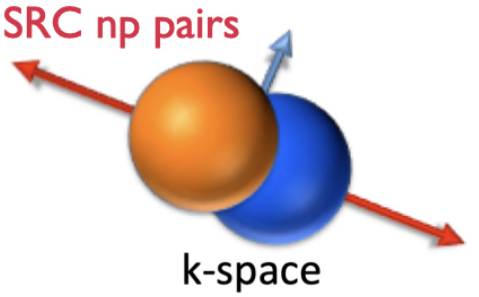


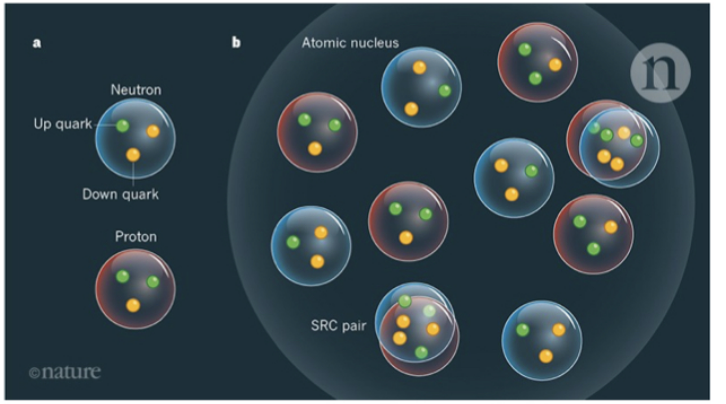
Softening/reduction of total symmetry energy



enhancement due to HMT<—> relevant for HICs







SRC nucleon pairs tend to reduce the pressure of the system, therefore softening the symmetry energy

Viewpoint from Hugenholtz-Van Hove Theorem

$$E_{sym}(\rho) = \frac{k_F^2}{6M_0^*} + \frac{1}{2}U_{sym}(\rho, k_F) + \overbrace{\Delta E_{sym}^{HMT}(\rho)}^{\text{SRC-HMT}}$$

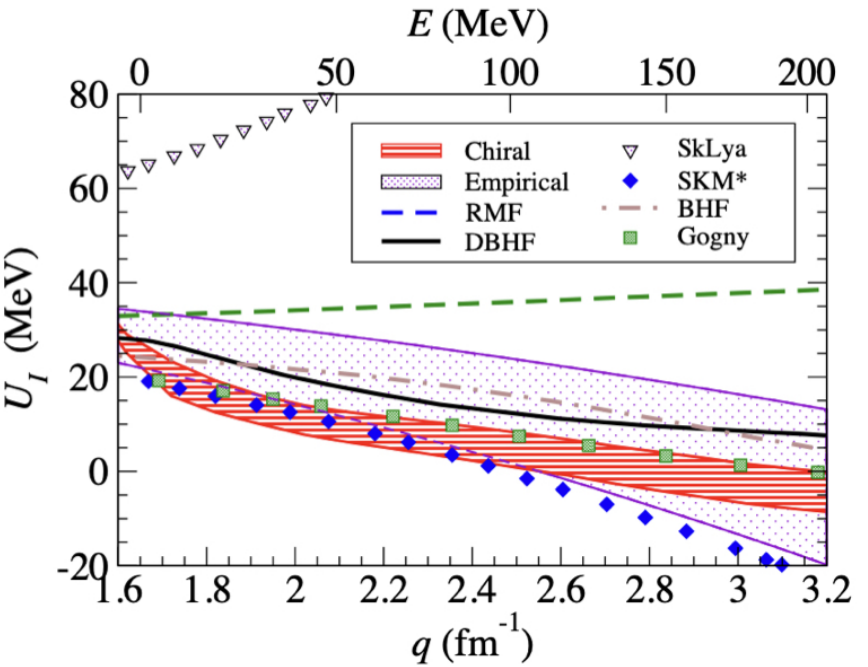
$$E_{sym}(\rho_0) \approx 30\text{-}31\ \text{MeV} \leftrightarrow \overbrace{\Delta E_{sym}^{HMT}(\rho_0)}^{\text{SRC-HMT}} \approx 0\ \text{MeV}$$

$$L(\rho) = \frac{k_F^2}{3M_0^*} + \left[\frac{k_F^2}{6}\frac{\partial^2 U_0}{\partial k^2} + \frac{k_F}{6}\frac{\partial U_0}{\partial k} + k_F\frac{\partial U_{sym}}{\partial k}\right]_{k_F} + \frac{3}{2}U_{sym}(\rho, k_F) + 3U_{sym,2}(\rho, k_F) + \underbrace{\Delta L^{HMT}(\rho)}_{\text{SRC-HMT}}$$

$$\underbrace{\Delta L^{HMT}(\rho_0)}_{\text{SRC-HMT}} \neq 0\ \text{MeV} \rightarrow L(\rho_0) \approx 27 \pm 25\ \text{MeV}$$

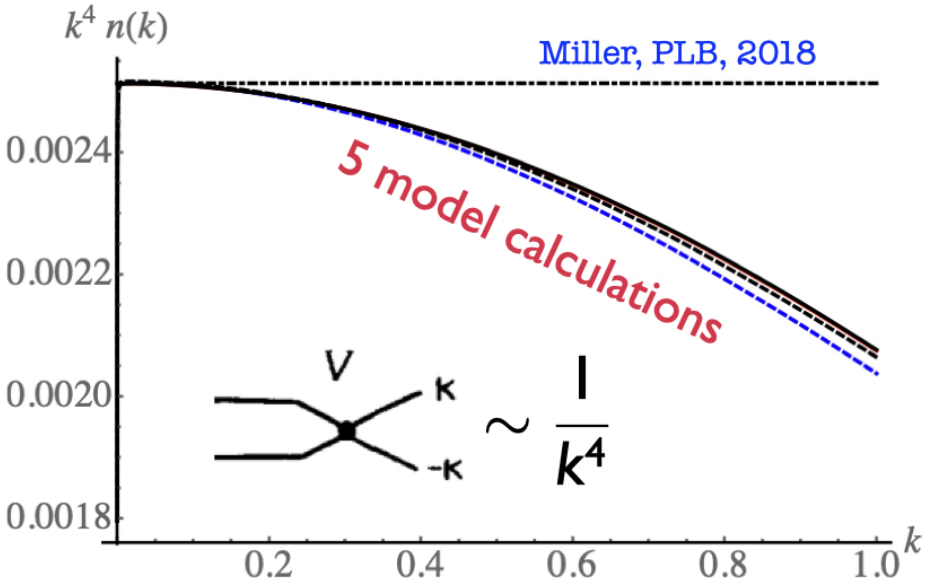
$M_0^*/M \approx 0.8 \rightarrow k_F^2/6M_0^* \approx 15\ \text{MeV}$

Optical model: $2^{-1}U_{sym}(\rho_0, k_F) \approx$ 15-16 MeV

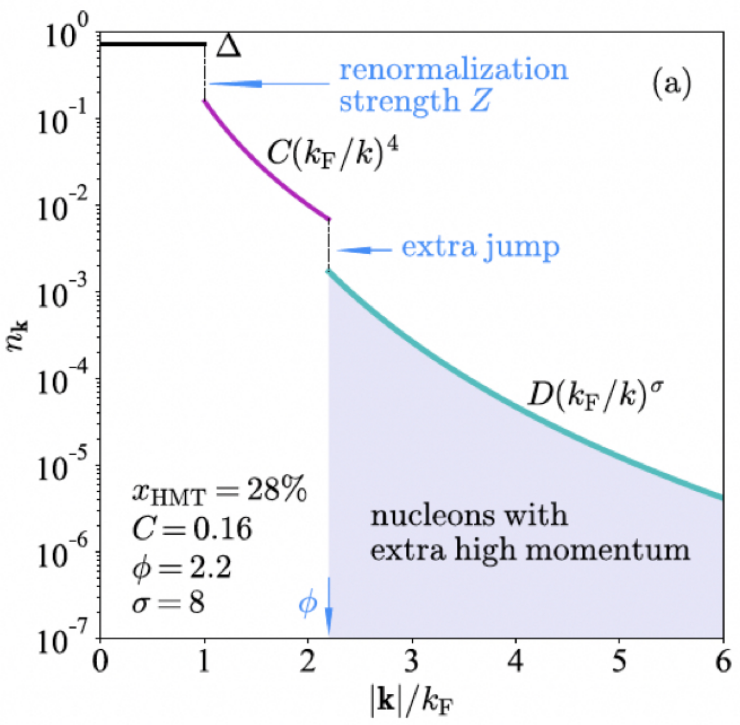


Holt, Kaiser, and Miller, PRC, 2016

HMT in the form $k^{-\sigma}$ with $\sigma \geq 4$



$$n_k = \begin{cases} \Delta, & k \leq k_F, \\ C\,(k_F/k)^4, & k_F \leq k \leq \phi k_F, \\ D\,(k_F/k)^\sigma, & k \geq \phi k_F. \end{cases}$$

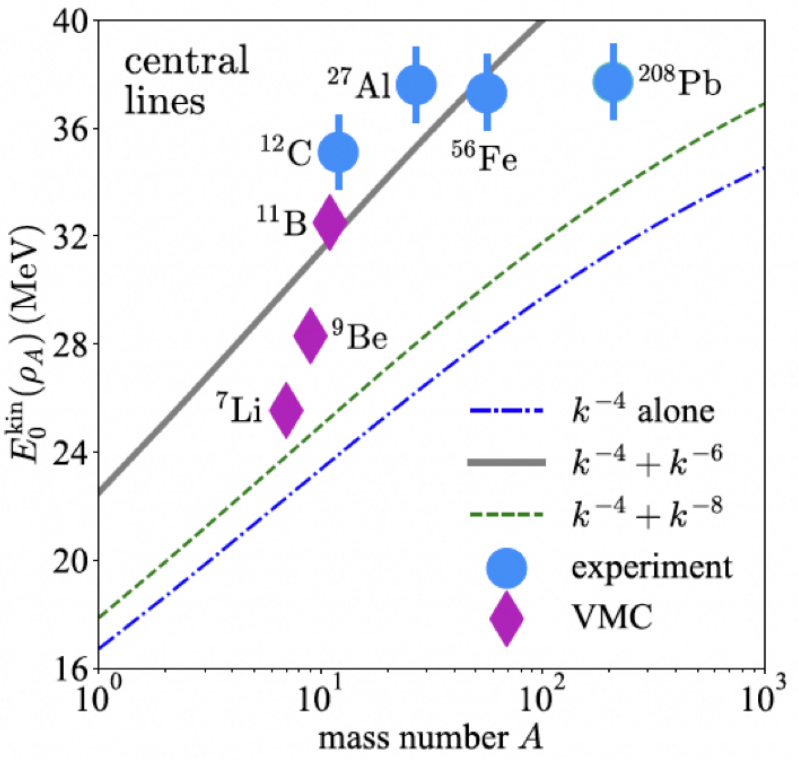


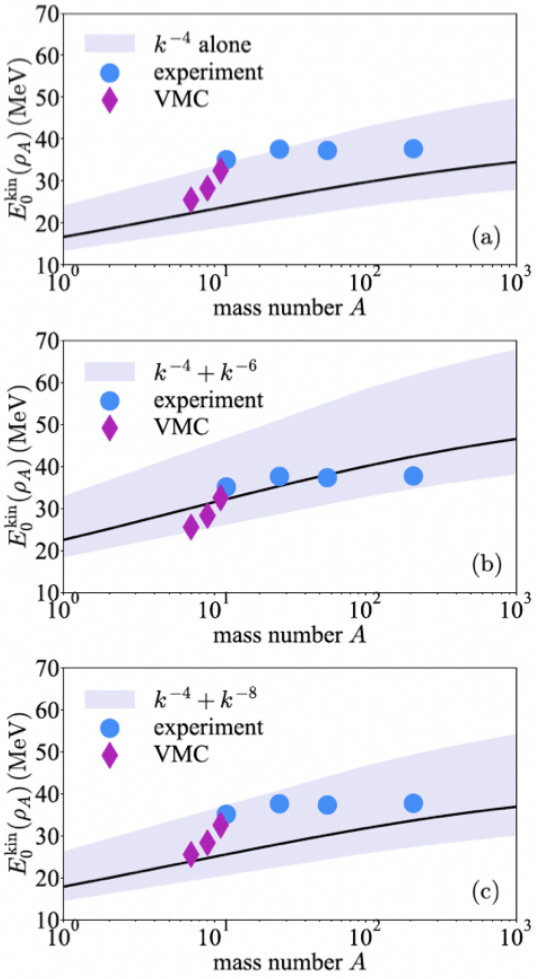
(1) HMT in the form of $k^{-\sigma}$ with $\sigma \geq 4$ exists

(2) after k^{-4} term, next contributions are $k^{-6}, k^{-8}, \cdots$

Correction to the kinetic EOS of SNM

$k^{-\sigma}$: $\delta E_0^{\mathrm{kin}}(\rho_0) \approx$ 4.1-20.2 MeV





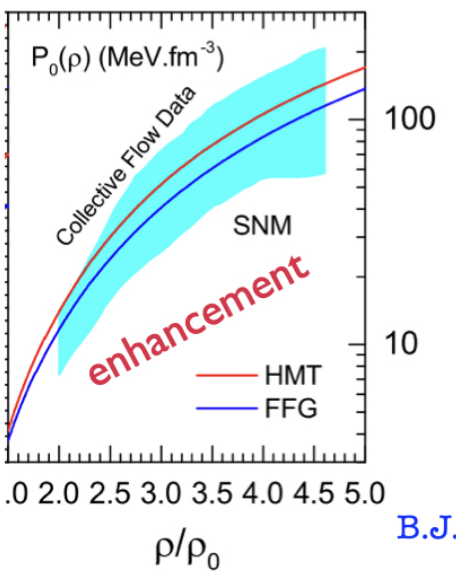
SRC-HMT on neutron star (NS) properties (RMF)

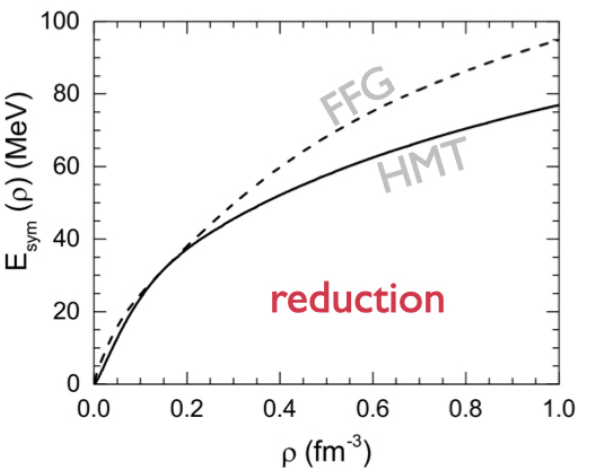
$$\int_0^{k_F^J} \overbrace{(\text{FFG step function})}^{\Theta(k_F^J-|\mathbf{k}|)} s(\mathbf{k},\rho,\delta)d\mathbf{k} \rightarrow \int_0^{\phi_J k_F^J} n_{\mathbf{k}}^J(\text{HMT}) s(\mathbf{k},\rho,\delta)d\mathbf{k}$$

SRC-HTM acts as a potential mechanism to increase NS mass to be consistent with current observations

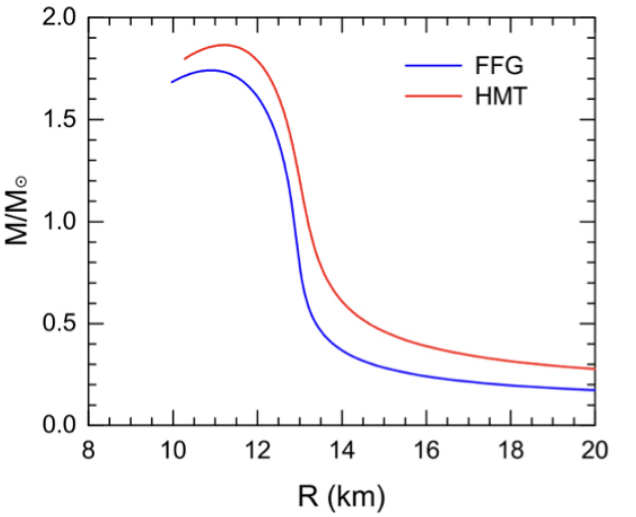
e.g., energy density $\varepsilon_J^{\text{kin}}$:

$$\frac{2}{(2\pi)^3}\int_0^{k_F^J} d\mathbf{k}\sqrt{|\mathbf{k}|^2+M_J^{*,2}} \rightarrow \frac{2}{(2\pi)^3}\left[\int_0^{k_F^J} d\mathbf{k}\Delta_J\sqrt{|\mathbf{k}|^2+M_J^{*,2}} + \underbrace{\int_{k_F^J}^{\phi_J k_F^J} d\mathbf{k} C_J \left(\frac{k_F^J}{|\mathbf{k}|}\right)^4 \sqrt{|\mathbf{k}|^2+M_J^{*,2}}}_{\text{HMT contribution}}\right]$$





Walecka model

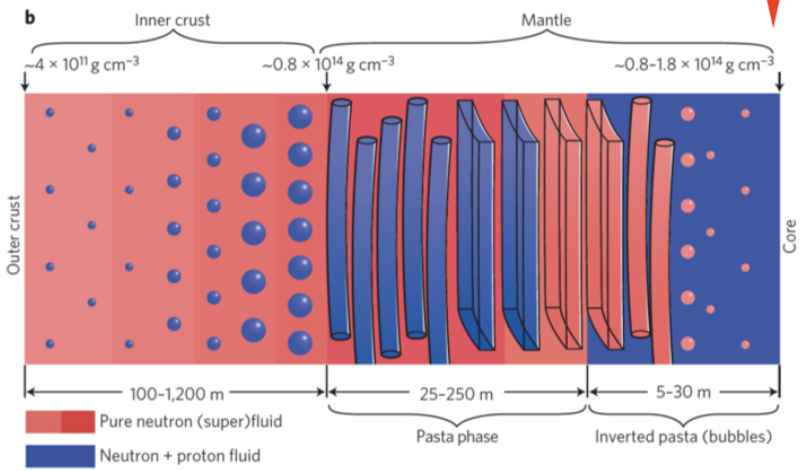


B.J. Cai and B.A. Li, PRC, 2016

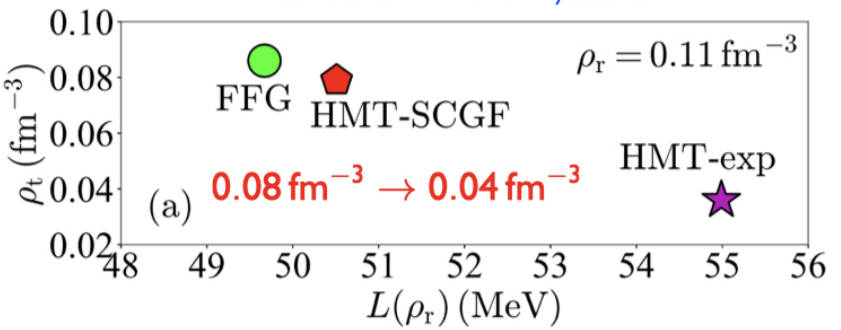
SRC-HMT on low-density properties of NSs

ρ_t : core-crust transition density in NSs

Newton, Nat. Phys., 2013



B.J. Cai and B.A. Li, 2022



Eur. Phys. J. A (2023) 59:209
https://doi.org/10.1140/epja/s10050-023-01122-4

THE EUROPEAN PHYSICAL JOURNAL A

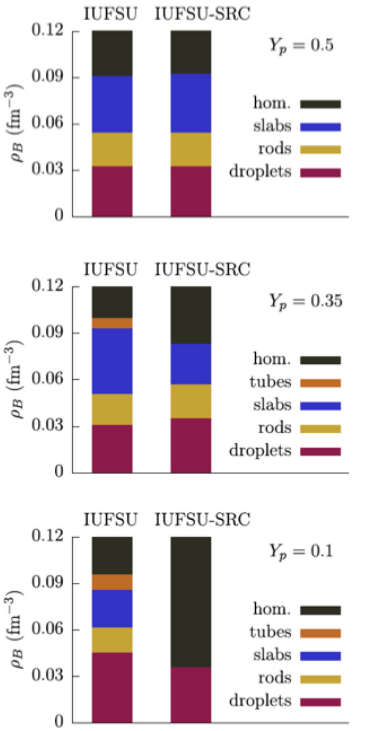
Regular Article - Theoretical Physics

Do short range correlations inhibit the appearance of the nuclear pasta?

M. R. Pelicer[1,a], D. P. Menezes[1], M. Dutra[2,3], O. Lourenço[2,3]

SRC-HTM can significantly reduce the core-crust transition density, and can even make some pasta-phases disappear in the crust of NSs

$$\mathcal{U}_{\text{ther}}(\rho, \delta) \to 0$$



Y_p: proton fraction

SRC-HMT on proton threshold of Urca process

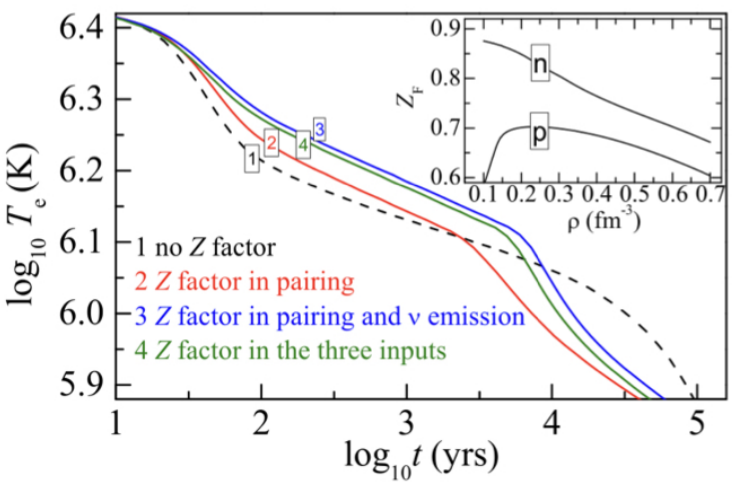
HMT cutoff: $0 \le t_J \le \phi_J$

Urca process: $n \rightarrow p + e^- + \bar{\nu}_e$

momentum consevation: $k_F^n (1 + t_n) \le k_F^p (1 + t_p) + k_F^e$

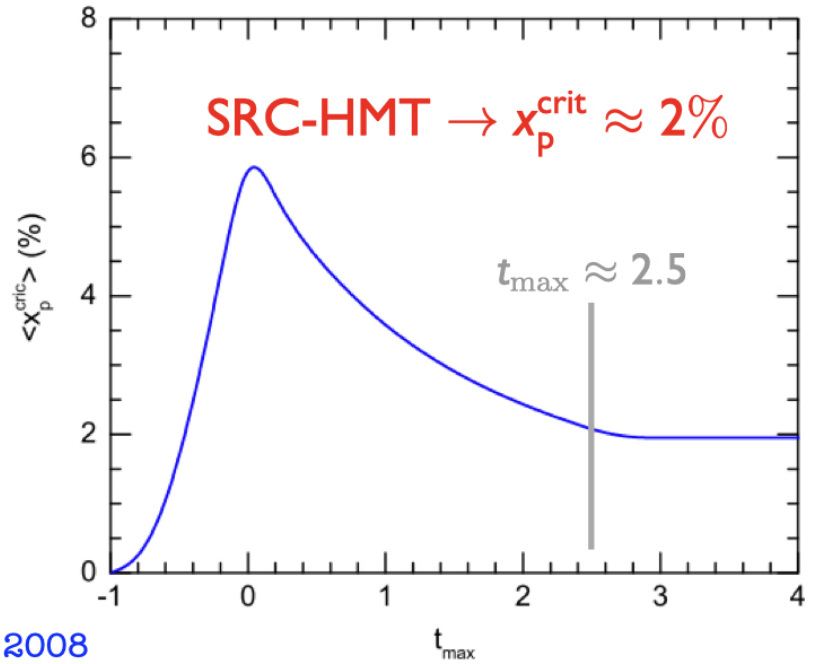
neutrality $k_F^p = k_F^e \rightarrow k_F^n (1 + t_n) \le k_F^p (2 + t_p)$

$$\rightarrow \boxed{x_p \ge x_p^{crit} = \frac{(1+t_n)^3}{(1+t_n)^3 + (2+t_p)^3} \xrightarrow[\text{free Fermi gas}]{t_n = t_p = 0} \frac{1}{9} \approx 11\%}$$



see, more refs., e.g.,
Frankfurt et al., IJMPE, 2008
J.M. Dong et al., ApJ, 2016
Souza et al., 2020
Y.P. Wang, PRC, 2024
Sedrakian, 2024

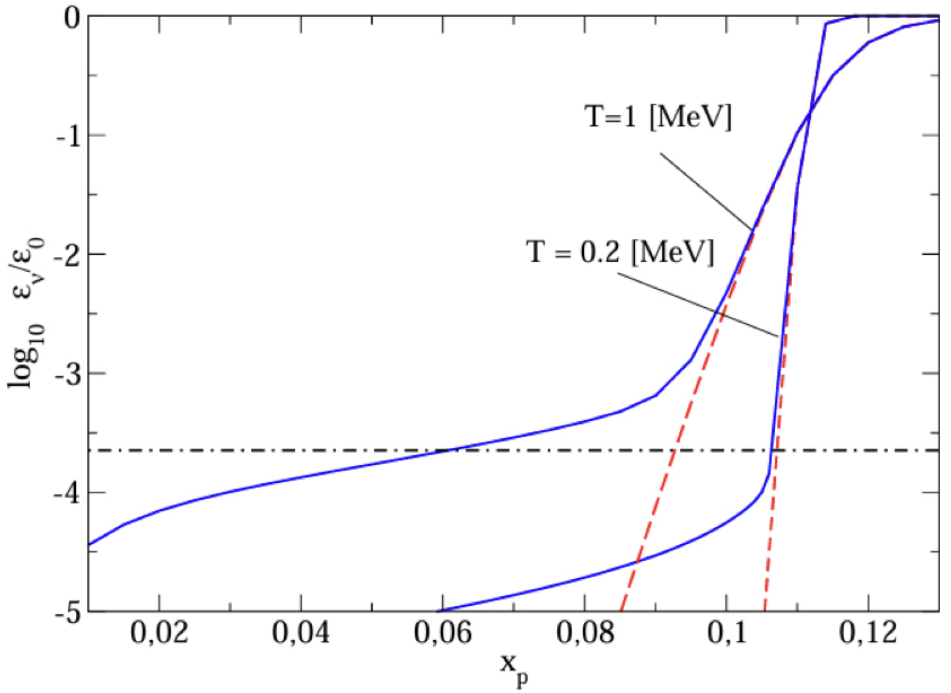
B.A. Li, et al., PPNP, 2018

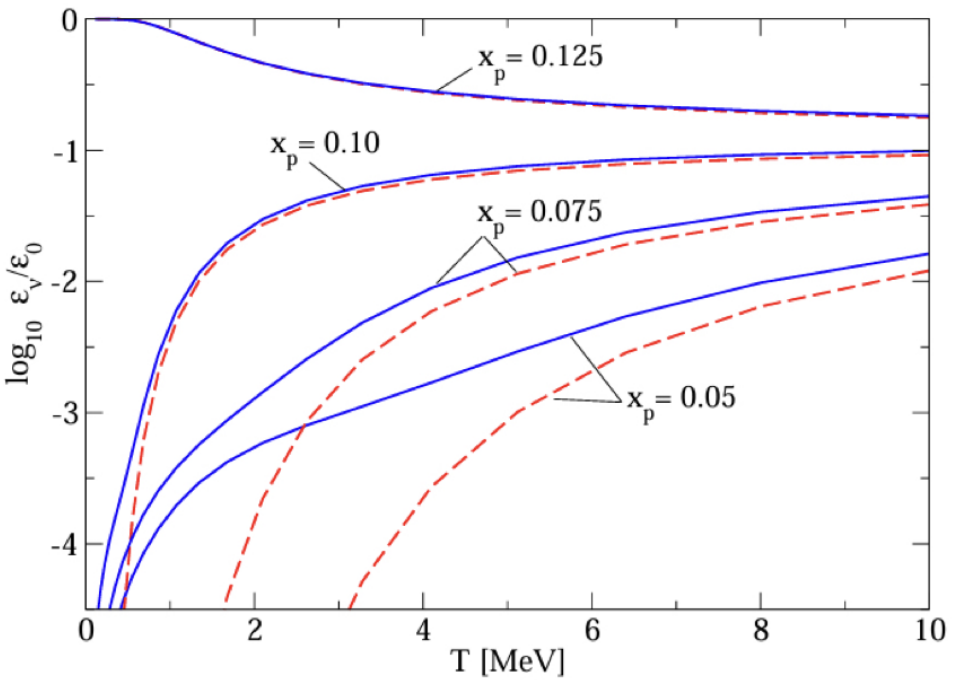


SRC-HMT may effectively influence the cooling process (neutrino emission) in NSs

Neutrino emissivity due to SRCs

SRCs replace the sharp proton fraction threshold for direct Urca with a smooth onset, enhance the neutrino emissivity at low temperatures, and potentially alter NS cooling.





Sedrakian, PRL, 2024

Summary of my talk

(1) k^{-4} HMT, $x_{\rm SNM} \approx 25\%, x_{\rm PNM} \lesssim 2\%$, isospin-dependent

(2) ps move faster than ns in neutron skins, $\Delta r_{\rm np} \leftrightarrow \Delta E^{\rm kin}_{\rm pn}$

(3) HMT makes $E^{\rm kin}_{\rm sym}(\rho) < 0$ and softens $E_{\rm sym}(\rho)$ at large ρ

(4) HMT affects NS properties, $x_{\rm p}^{\rm crit}$, $\rho_{\rm t}^{\rm cc}$, $M_{\rm NS}$, ϵ_ν, pasta

(5) $k^{-\sigma}$ with $\sigma \geq 4$ for large k: $\delta E^{\rm kin}_0(\rho_0) \approx$4.1-20.2 MeV

* size/shape/isospin dependence of HMT$\leftrightarrow$self-consistency
* dynamics (particle creation in HICs, ν-transport in NSs)
* $L \approx$ 27 MeV, optical model, PREX, $\cdots$, what is missing?

Thank the organizer for organizing this wonderful workshop!

Thank you!

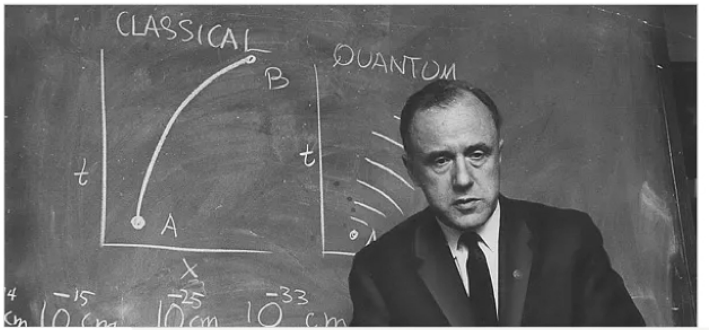
Low-dimensional nuclear/atomic systems

examples:

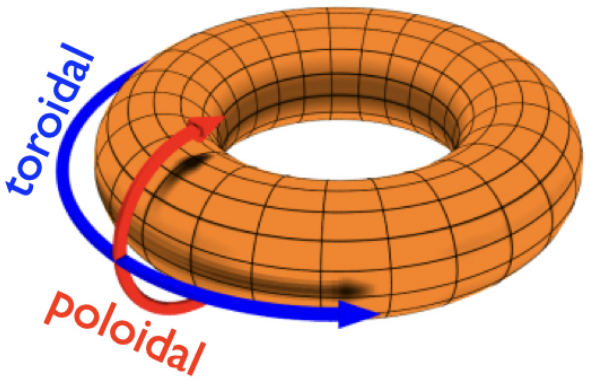
(1) toroidal/spherical bubble topologies
(Wheeler, Siemens, Wong, 1960s-1970s; Natowitz, 2010s)

(2) NS crust?

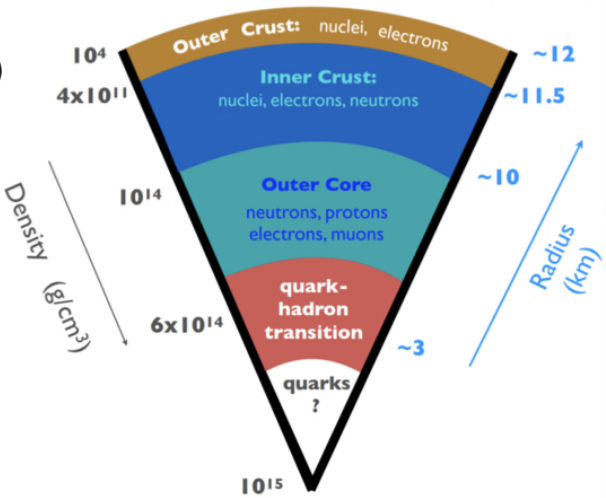
(3) ultracold 1D/2D Fermion/Boson systems
(optical lattice/box; Feshbach resonance, ⋯)



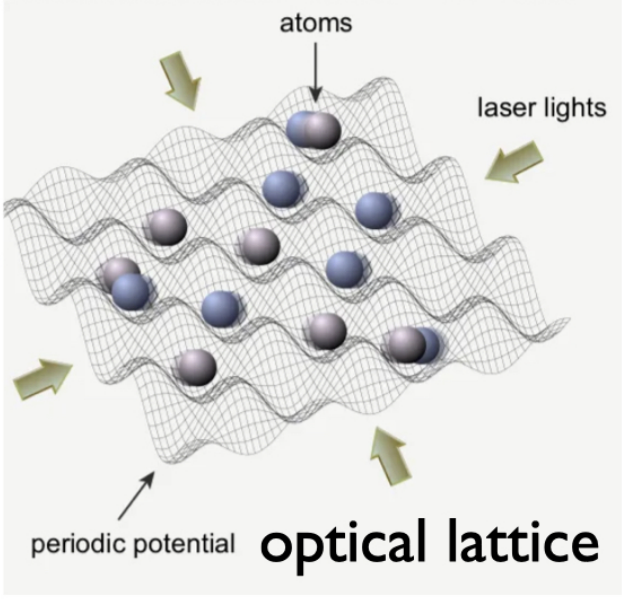
toroidal ^{12}C



Wheeler, 1963; C.Y. Wong, PLB, 1972



thickness $t \approx (1\text{-}2)$ km $\rightarrow t/4\pi R^2 \sim 10^{-3}$ km^{-1}



Example: Bertsch parameter from the epsilon expansion

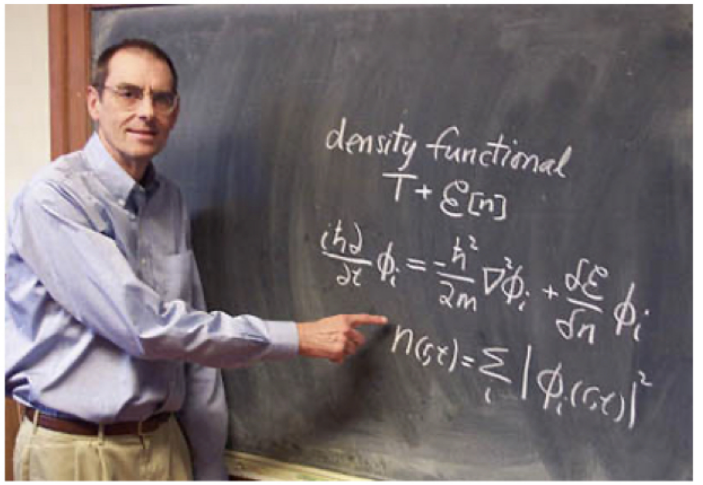
FUDAN UNIVERSITY 復旦大學

EOS $= \xi \frac{3}{5}\frac{k_F^2}{2M}$: Bertsch parameter ξ?

(unitarity condition: $|k_F a| \to \infty$)

(Lee-Yang expansion via $|k_F a| \ll 1$ breaks down)

$$\frac{E}{N} = \frac{\hbar^2 k_F^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}k_F a + \frac{4}{35\pi^2}(11 - 2\ln 2)(k_F a)^2 + 0.23(k_F a)^3 + \cdots\right]$$



D=4 is non-interacting —> extrapolation to *D*=4-1=3

PRL **97,** 050403 (2006) PHYSICAL REVIEW LETTERS week ending 4 AUGUST 2006

ϵ Expansion for a Fermi Gas at Infinite Scattering Length

Yusuke Nishida[1,2] and Dam Thanh Son[2]
[1]*Department of Physics, University of Tokyo, Tokyo 113-0033, Japan*
[2]*Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1550, USA*
(Received 2 May 2006; published 4 August 2006)

$$\epsilon = 4 - D \to \xi \approx \frac{1}{2}\epsilon^{3/2} + \frac{1}{16}\epsilon^{5/2}\ln\epsilon - 0.0246\epsilon^{5/2} \xrightarrow{\epsilon=1} 0.378$$

Arnold et al., PRA, 2007
Ku et al., Science, 2012
Forbes et al., PRL, 2011

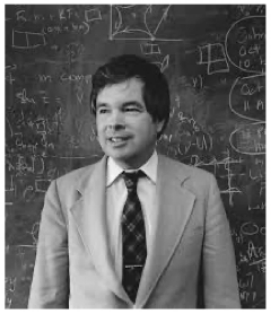
exp.: $\xi \approx 0.376$; QMC: $\xi \approx 0.383$

THE RENORMALIZATION GROUP AND THE ϵ EXPANSION*

Kenneth G. WILSON
Institute for Advanced Study, Princeton, N.J. 08540, USA
and Laboratory of Nuclear Studies, Cornell University, Ithaca, N.Y. 14850, USA†

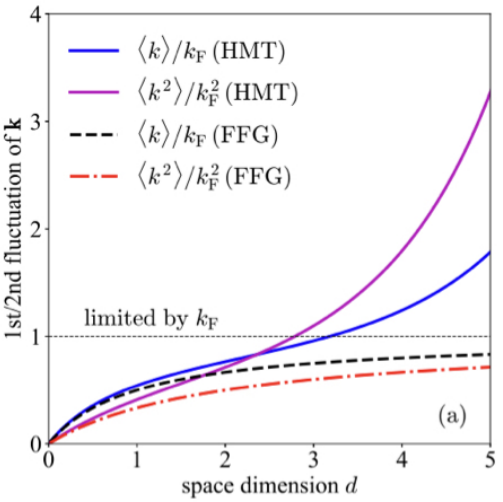
and

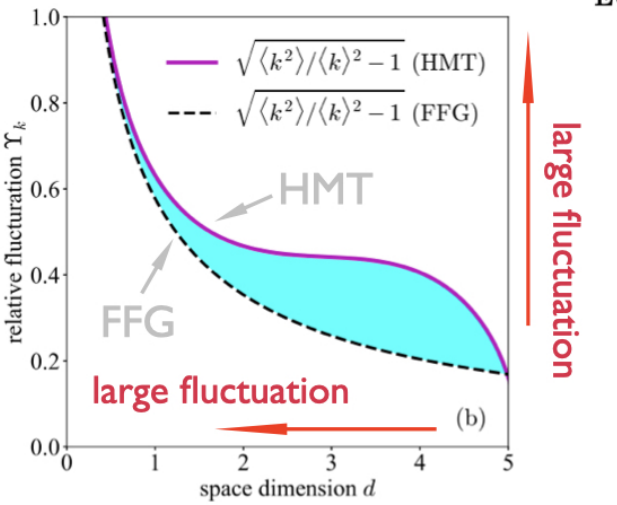
J. KOGUT‡
Institute for Advanced Study, Princeton, N.J. 08540, USA





Low-dimensional nuclear matter w/wo SRC-HMT





EQUATION OF STATE OF NEUTRON-RICH MATTER IN d-DIMENSIONS

Bao-Jun Cai[1]

Quantum Machine Learning Laboratory, Shadow Creator Inc., Shanghai 201208, China

Bao-An Li[2]

Department of Physics and Astronomy, Texas A&M University-Commerce, Commerce, TX 75429-3011, USA

We made a first systematic investigation on the nuclear matter EOS in general D dimensions (including the ϵ-expansion for the EOS).

relative fluctuation: $\Upsilon_k(d) = \sqrt{\dfrac{\langle|\mathbf{k}|^2\rangle}{\langle|\mathbf{k}|\rangle^2} - 1}$

some features:

(1) free Fermi gas: momentum scale $\leq k_F$

(2) SRC-HMT could generate large $|\mathbf{k}| > k_F$

(3) large Υ_k for low D $\leftrightarrow$ Mermin-Wagner theorem (solid state physics)

(4) low $D \rightarrow$ nearly free $\leftrightarrow$ extrapolation $\epsilon = 3 - D_{\text{free}}$

$$E_{\text{sym}}(\rho) = \frac{\bar{k}_F^2}{6M} + \frac{\bar{k}_F}{6}\frac{\partial U_0}{\partial|\mathbf{k}|}\bigg|_{|\mathbf{k}|=\bar{k}_F} + \frac{1}{2}U_{\text{sym}}(\rho,\bar{k}_F)$$
$$+\epsilon\left[\frac{\bar{k}_F^2}{6M}\left(2\sigma-\frac{1}{3}\right) + \frac{\bar{k}_F}{6}\left(\sigma\bar{k}_F\frac{\partial^2 U_0}{\partial|\mathbf{k}|^2} + \left(\sigma-\frac{1}{3}\right)\frac{\partial U_0}{\partial|\mathbf{k}|}\right) + \frac{\sigma}{2}\frac{\partial U_{\text{sym}}}{\partial|\mathbf{k}|}\cdot\bar{k}_F\right]_{|\mathbf{k}|=\bar{k}_F}$$

B.J. Cai and B.A. Li, Ann. Phys., 2022