Central China
Center for Nuclear Theory
华中核理论中心

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Workshop on QCD Critical Point and Effective Field Theory methods 2026

Critical dynamics in QCD matter

Lorenz von Smekal

with: **Yong-rui Chen**, **Mattis Harhoff**, Johannes Roth,
Sören Schlichting, **Leon Sieke**, Yunxin Ye

DFG

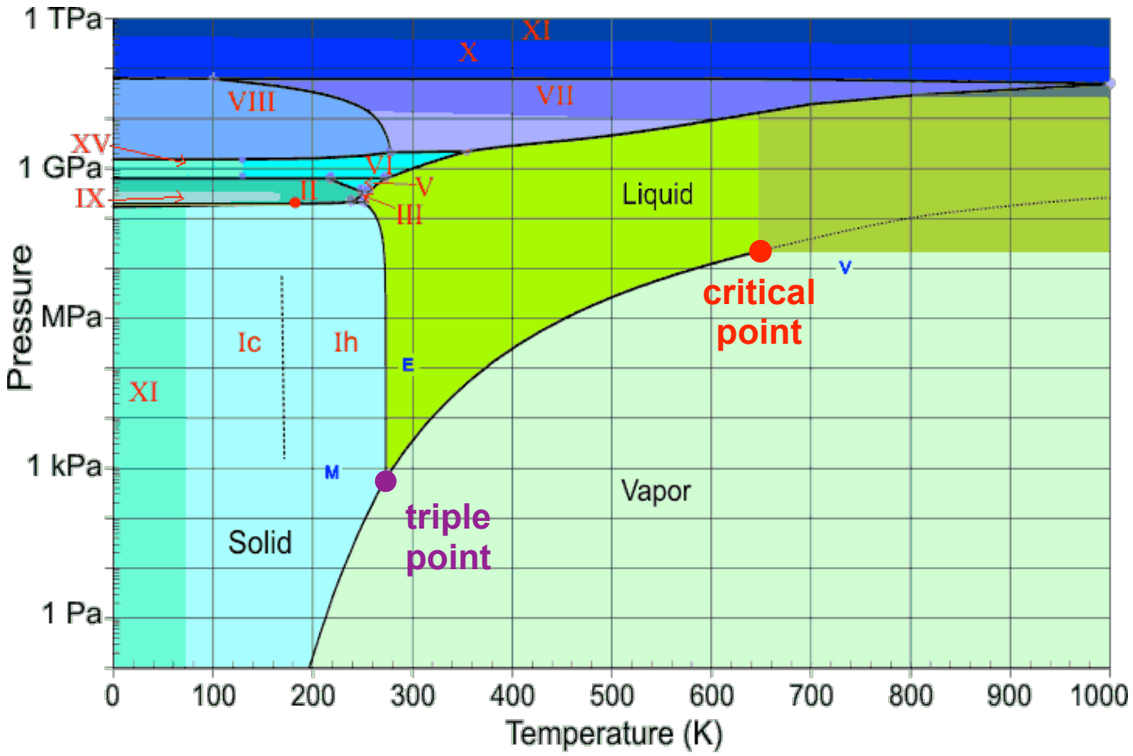
Wuhan, 19 October 2026

CRC-TR 211
Strong-interaction matter
under extreme conditions

Universality near Critical Point

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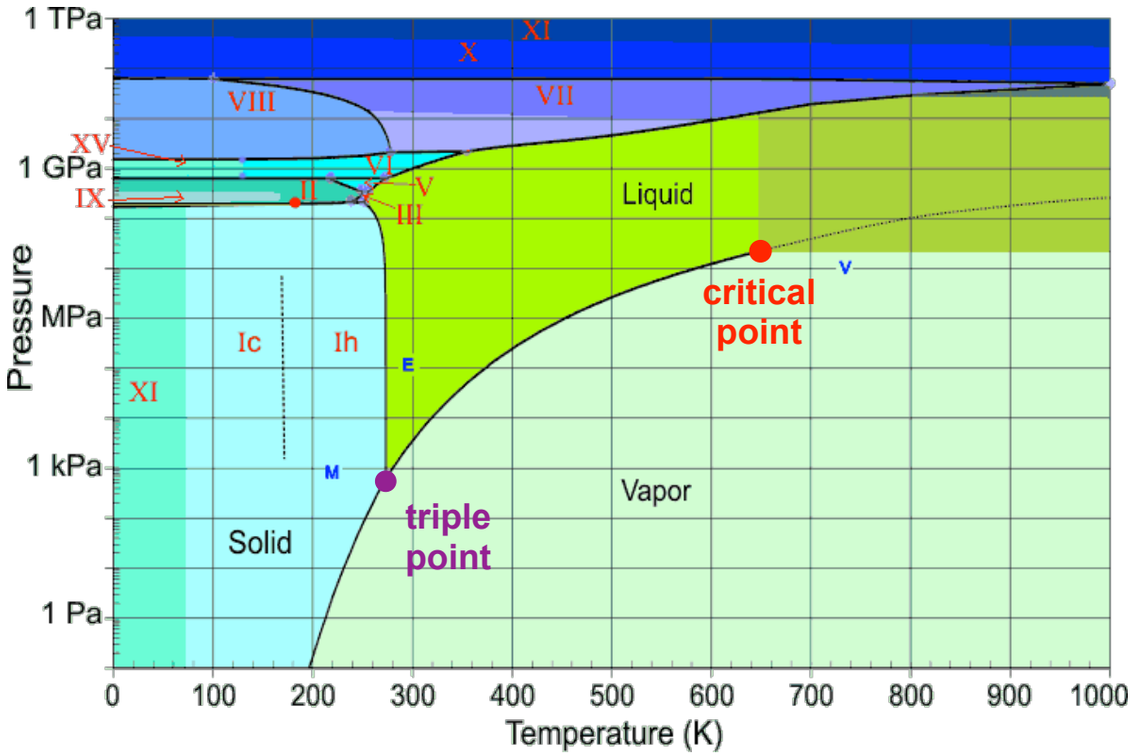
Water



http://www.lsbu.ac.uk/water/phase.html

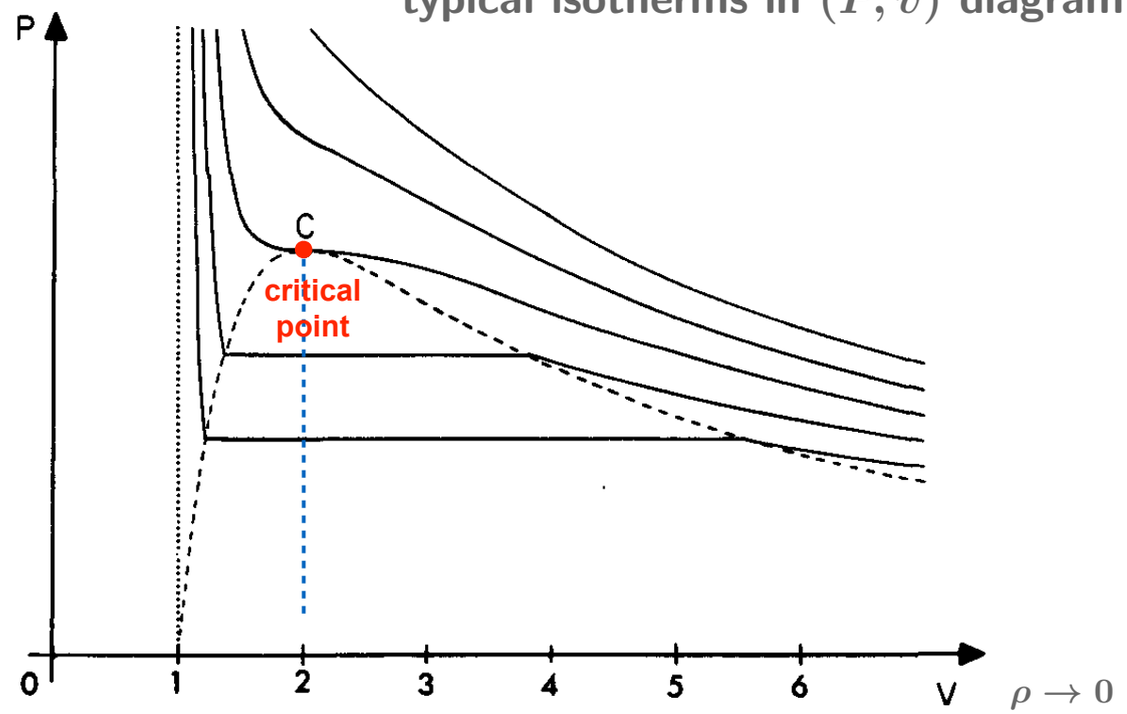
Universality near Critical Point

Water



http://www.lsbu.ac.uk/water/phase.html

typical isotherms in (P, v) diagram

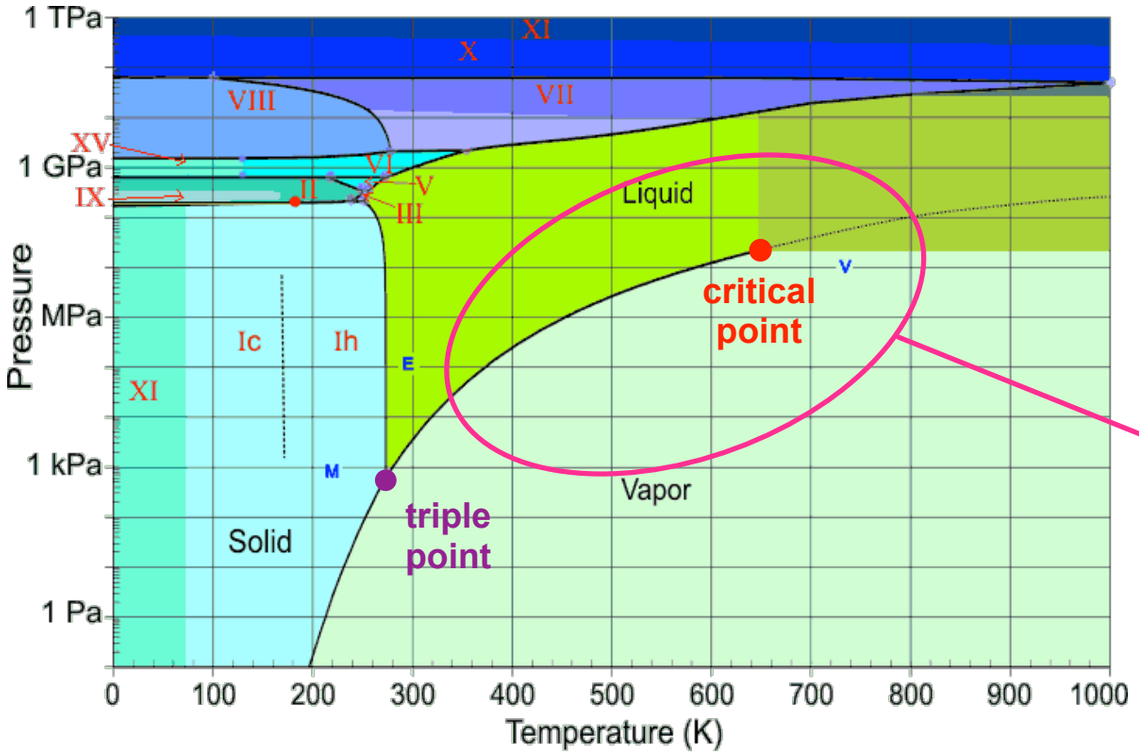


Baxter, *Exactly Solved Models in Statistical Mechanics*

Universality near Critical Point

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Water



http://www.lsbu.ac.uk/water/phase.html

critical isotherm:

$T = T_c$:

$\rho - \rho_c \sim M$,

$v_c - v \sim \pm |H|^{1/\delta}$,

$P - P_c \sim \pm |v_c - v|^{\delta}$

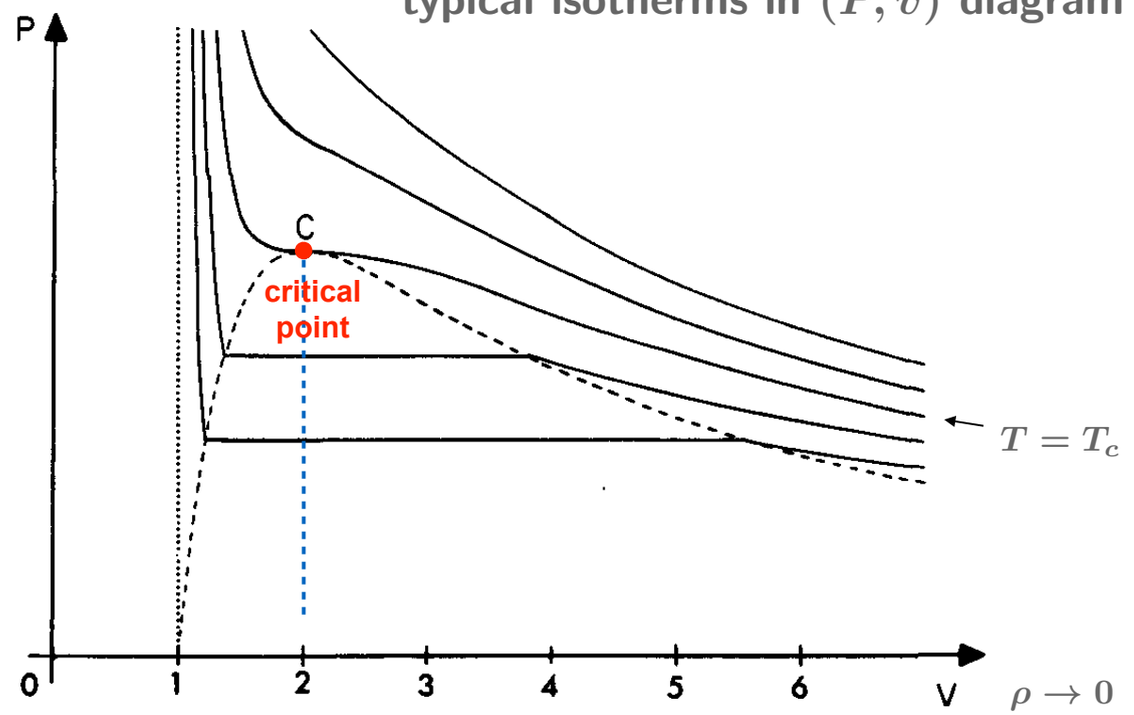
along coexistence curve:

$(H = 0, T < T_c)$

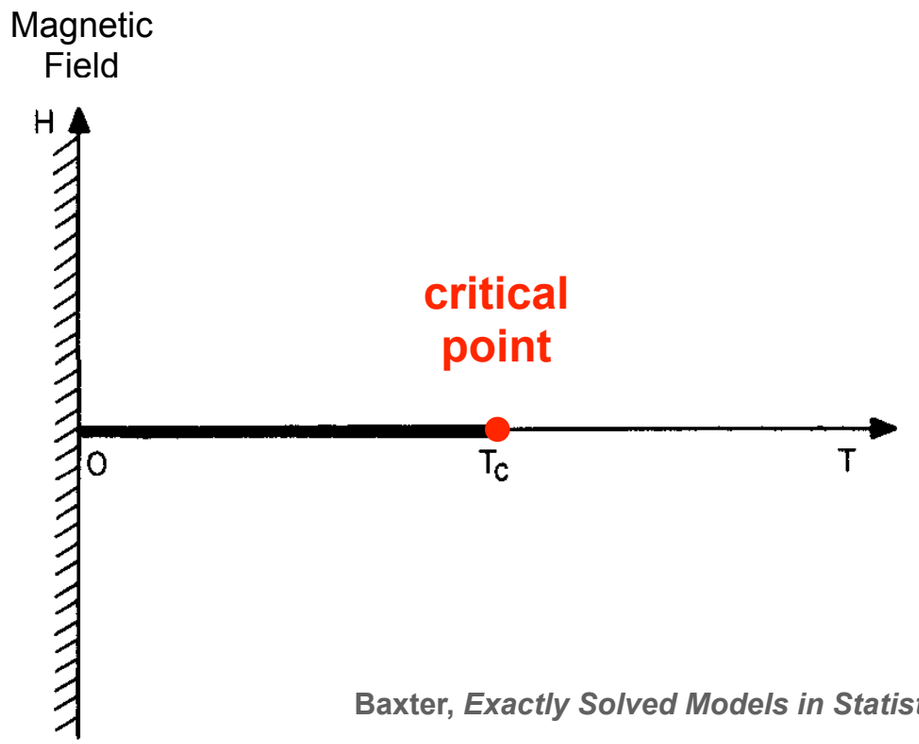
$v_c - v \sim \pm (T_c - T)^{\beta}$

$\sim \pm (P_c - P)^{\beta}$

typical isotherms in (P, v) diagram



Ising Model:



Baxter, *Exactly Solved Models in Statistical Mechanics*

Strong-Interaction Matter

QCD phase diagram

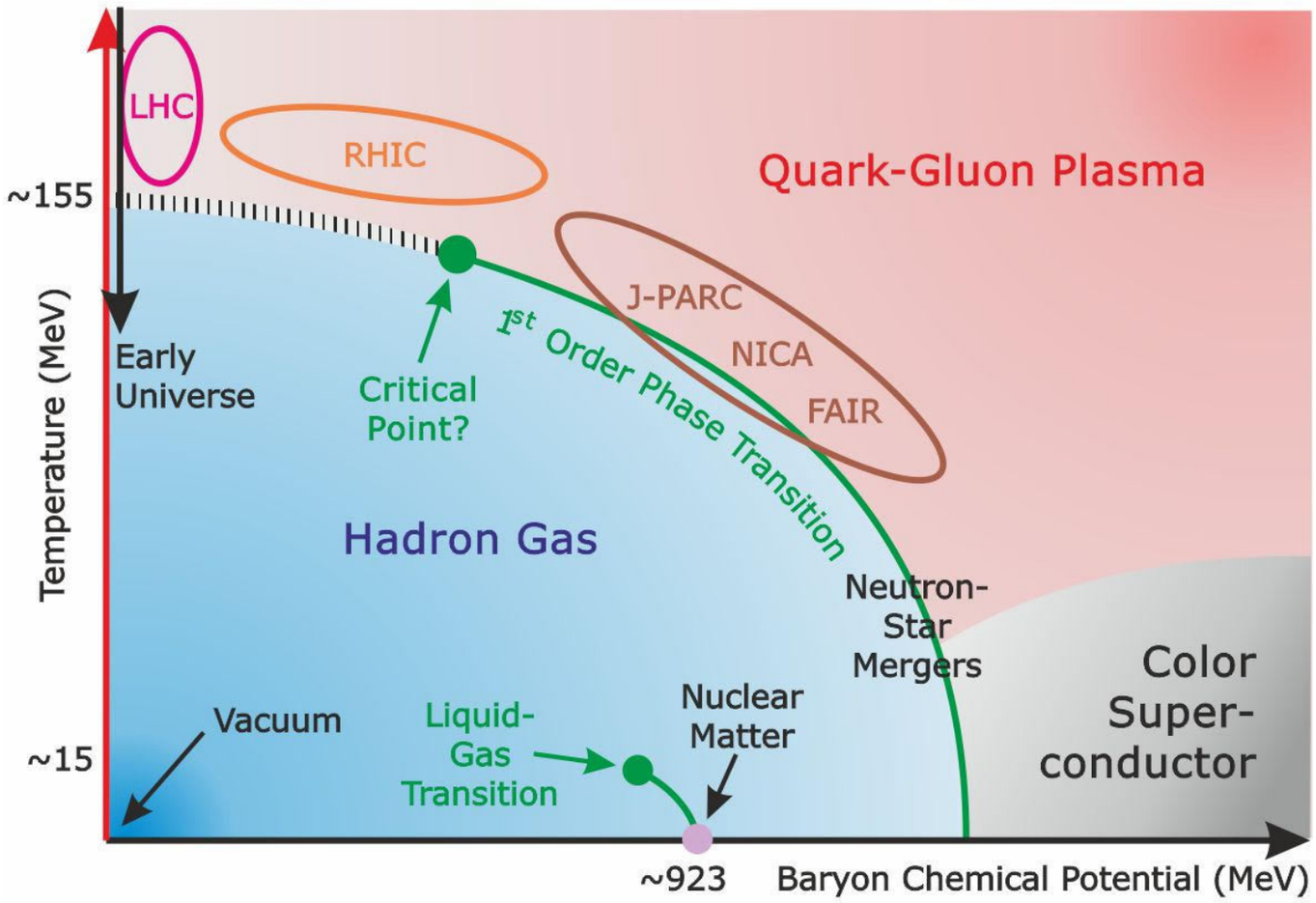


Figure: A. Steidl, CRC-TR 211

- Worldwide effort: find **critical point**

Strong-Interaction Matter

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QCD phase diagram

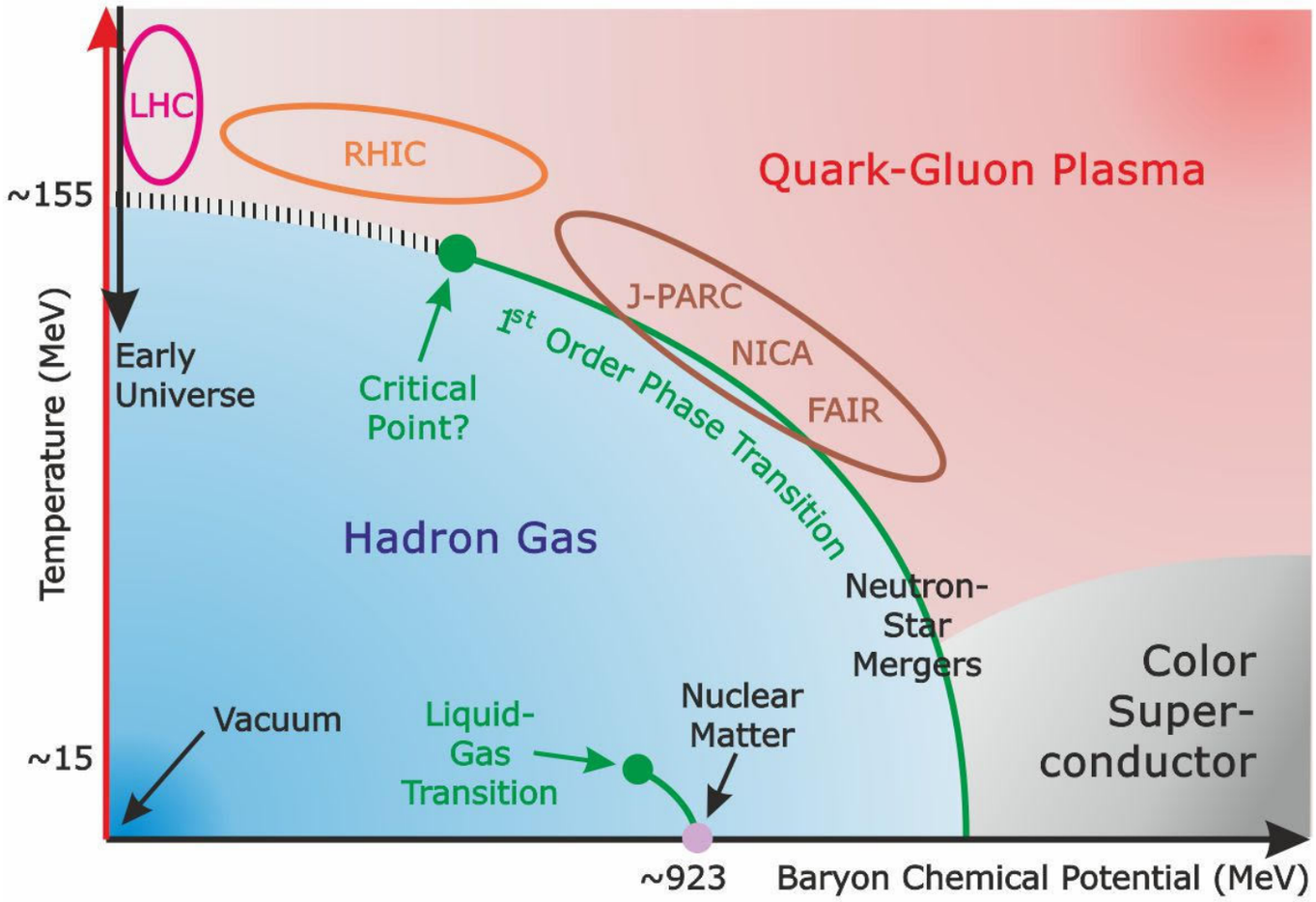
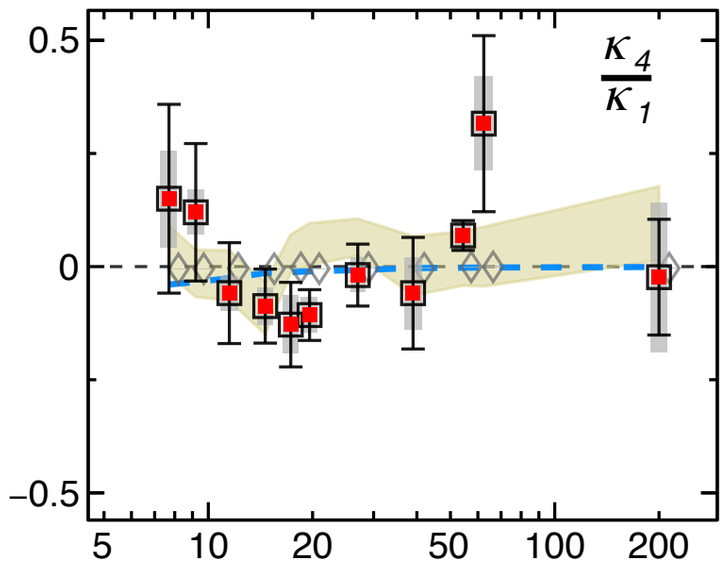


Figure: A. Steidl, CRC-TR 211

- Worldwide effort: find **critical point**
- Possible signatures in heavy-ion collisions: **critical fluctuations**

Stephanov, Rajagopal, Shuryak, PRD **60** (1999) 114028
Stephanov, PRL **107** (2011) 052301



STAR Collaboration, arXiv:2504.00817

Strong-Interaction Matter

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QCD phase diagram

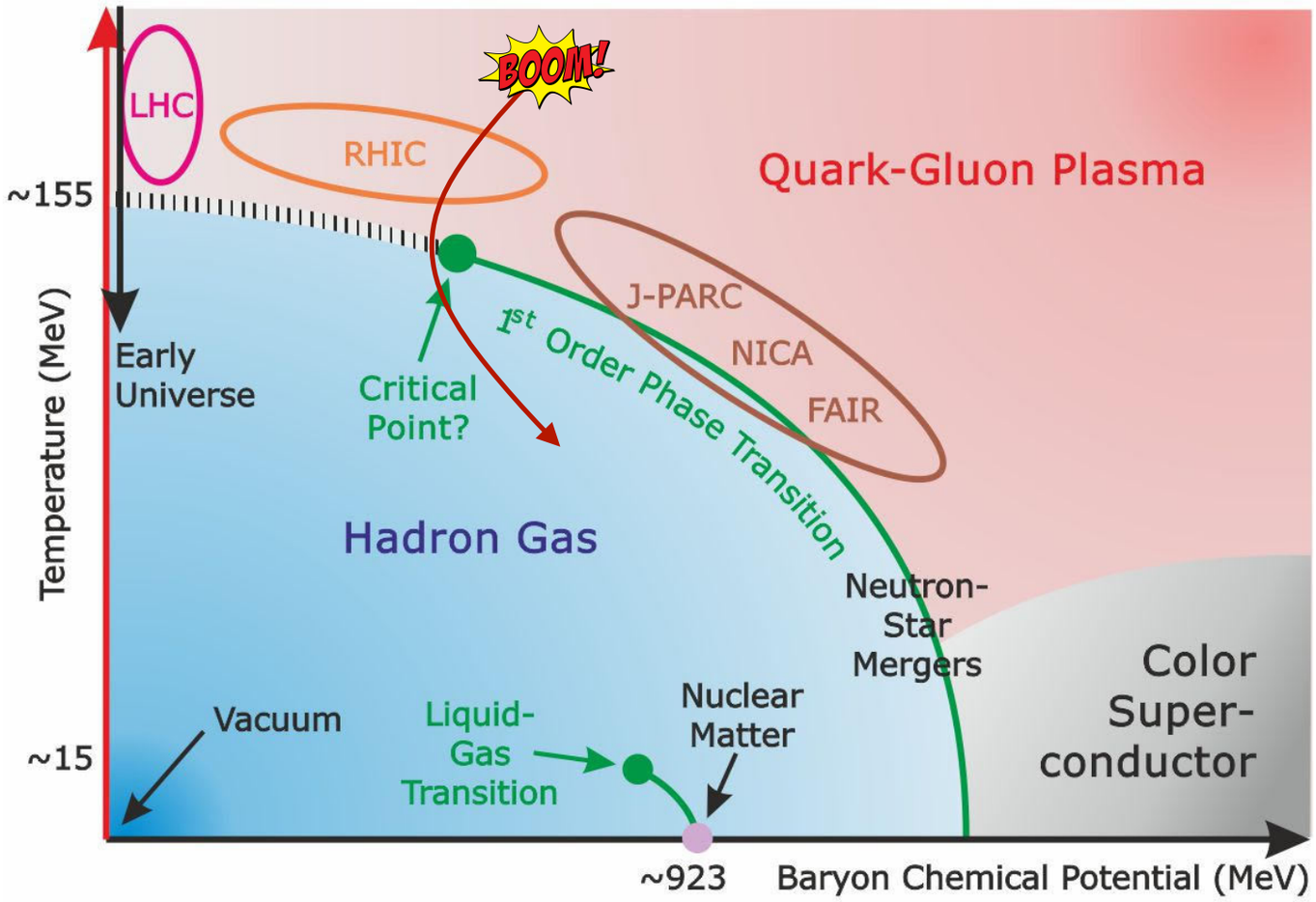
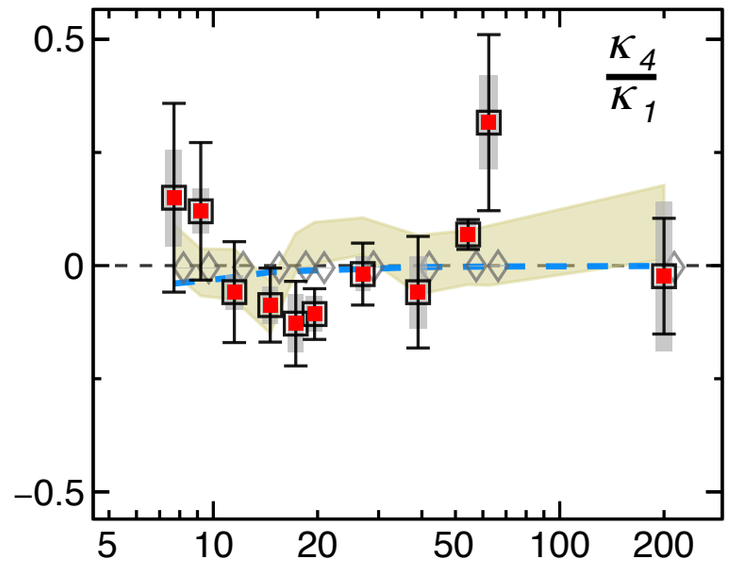


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STAR Collaboration, arXiv:2504.00817

- Expanding fireball: understand **non-equilibrium transition** and **critical dynamics**

Strong-Interaction Matter

QCD phase diagram

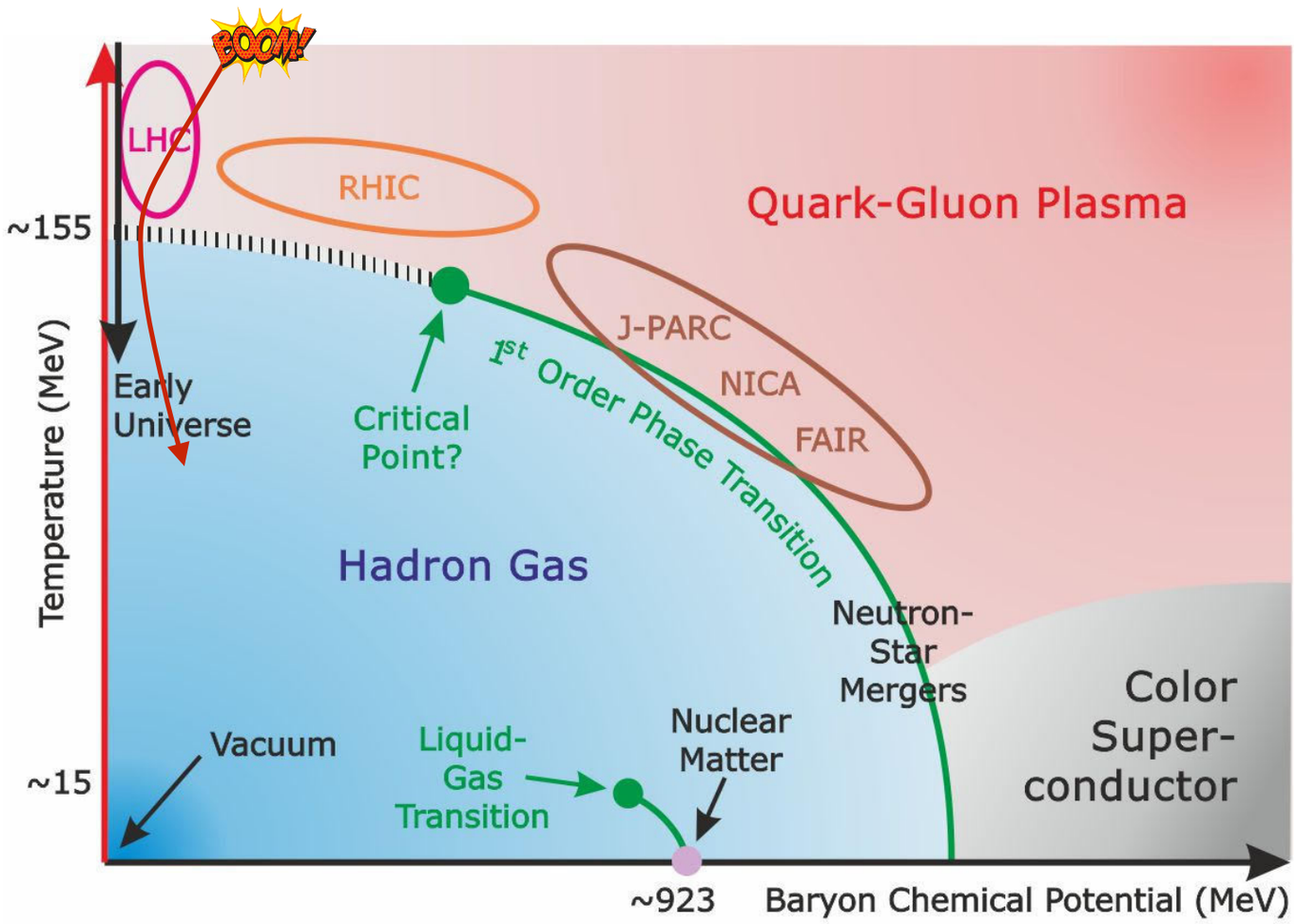


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Strong-Interaction Matter

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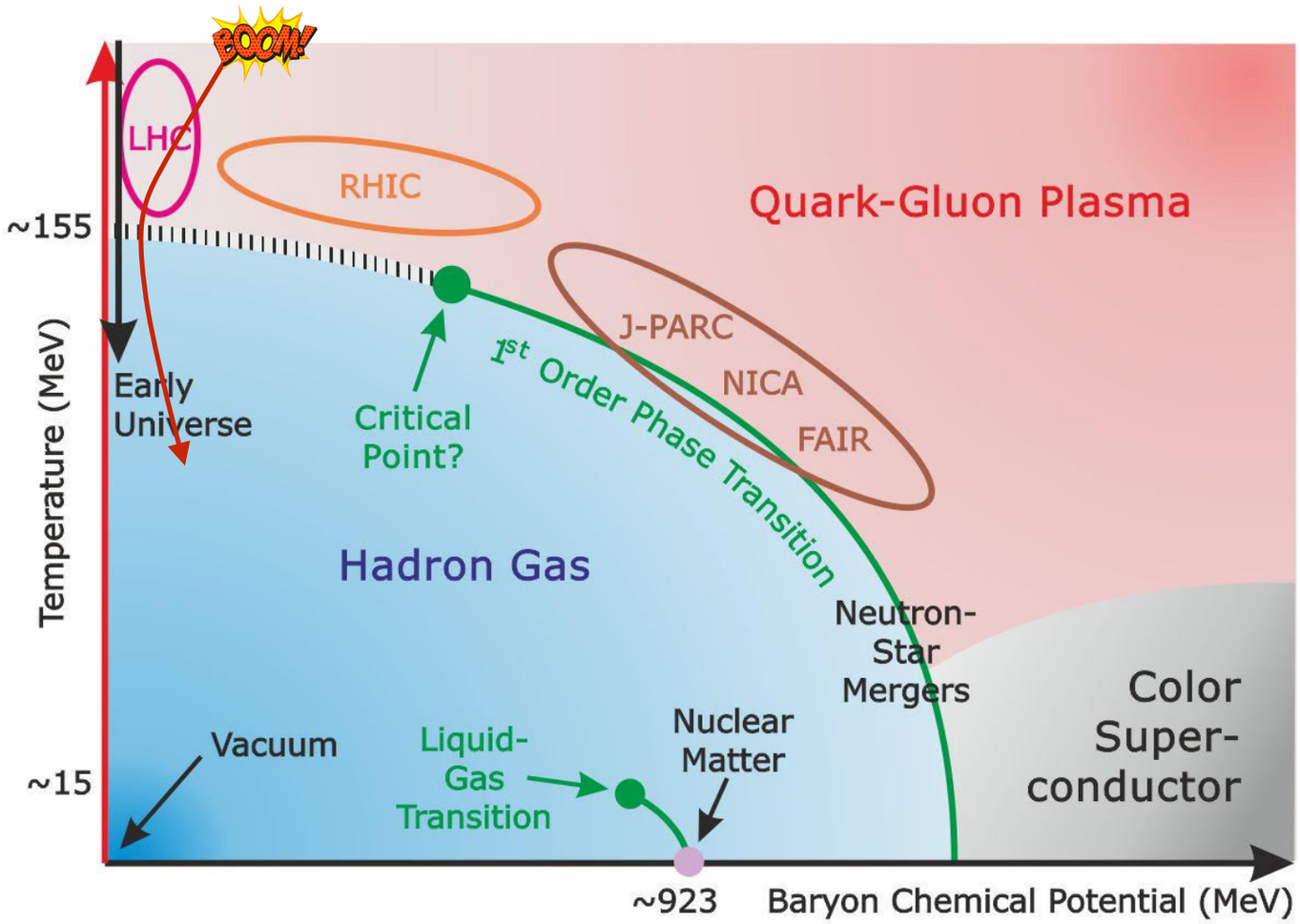
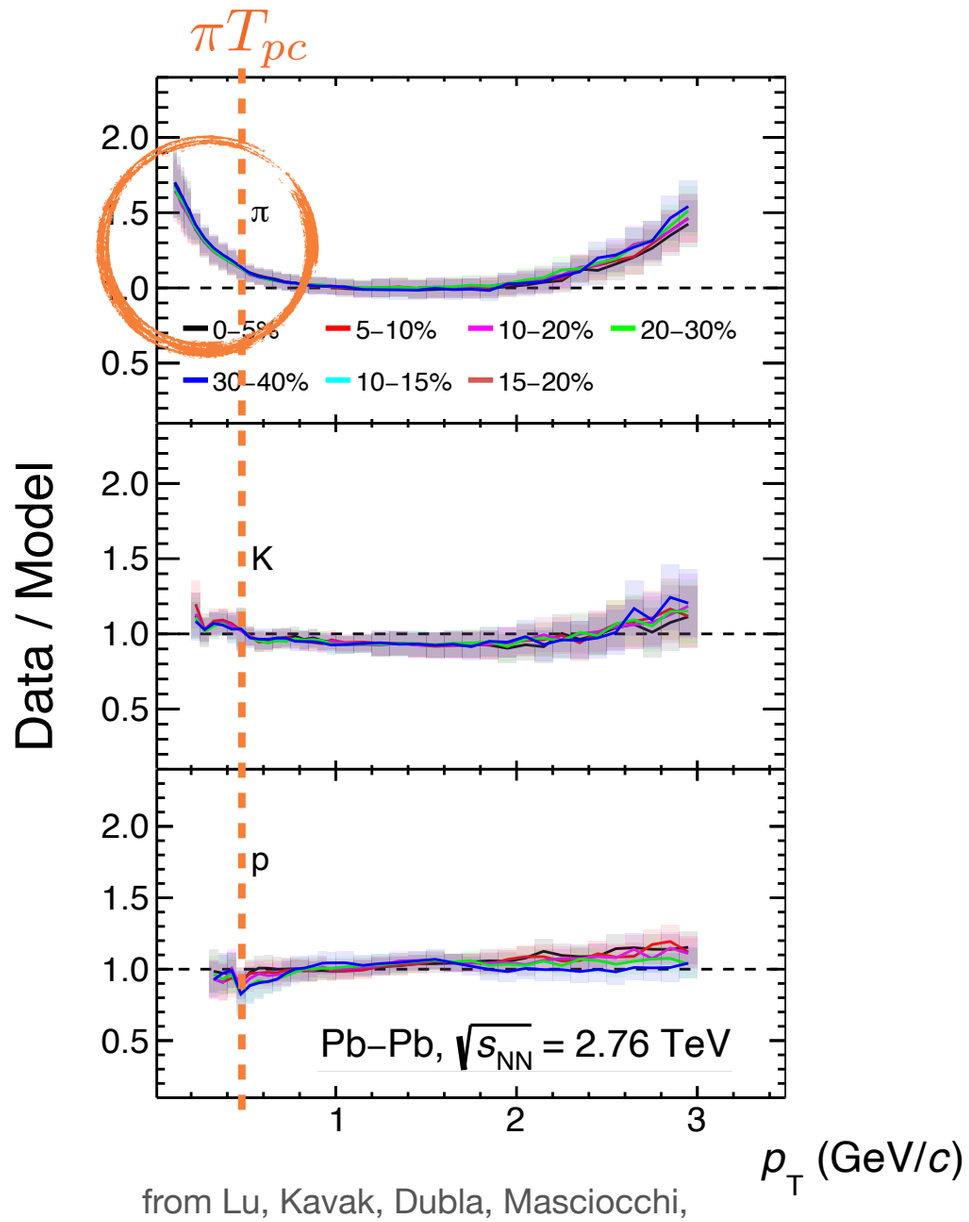


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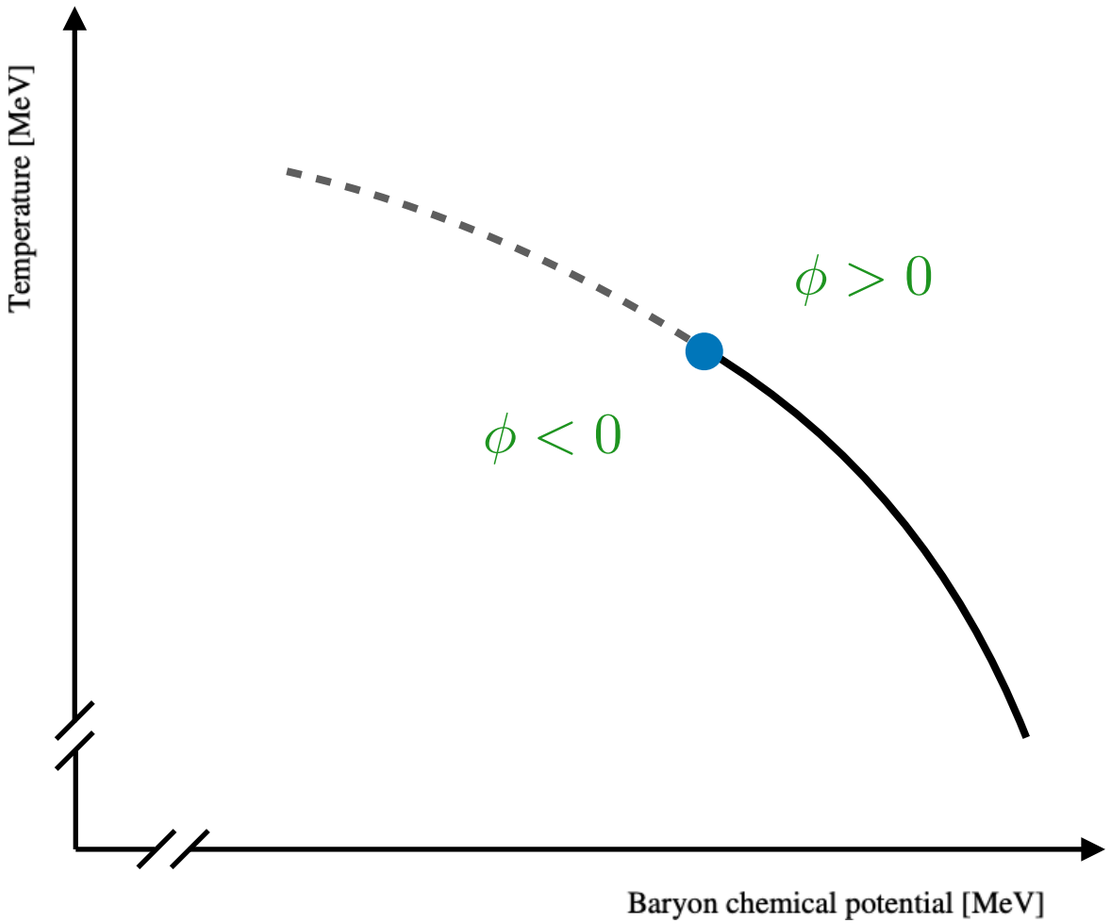
- Excess of soft pions: imprint of **chiral crossover**?

Florio, Grossi, Soloviev, Teaney, PRD **104** (2021) 034025



from Lu, Kavak, Dubla, Masciocchi, Selyuzhenkov, arXiv:2407.09207

Universality near critical point



Order parameter: ϕ

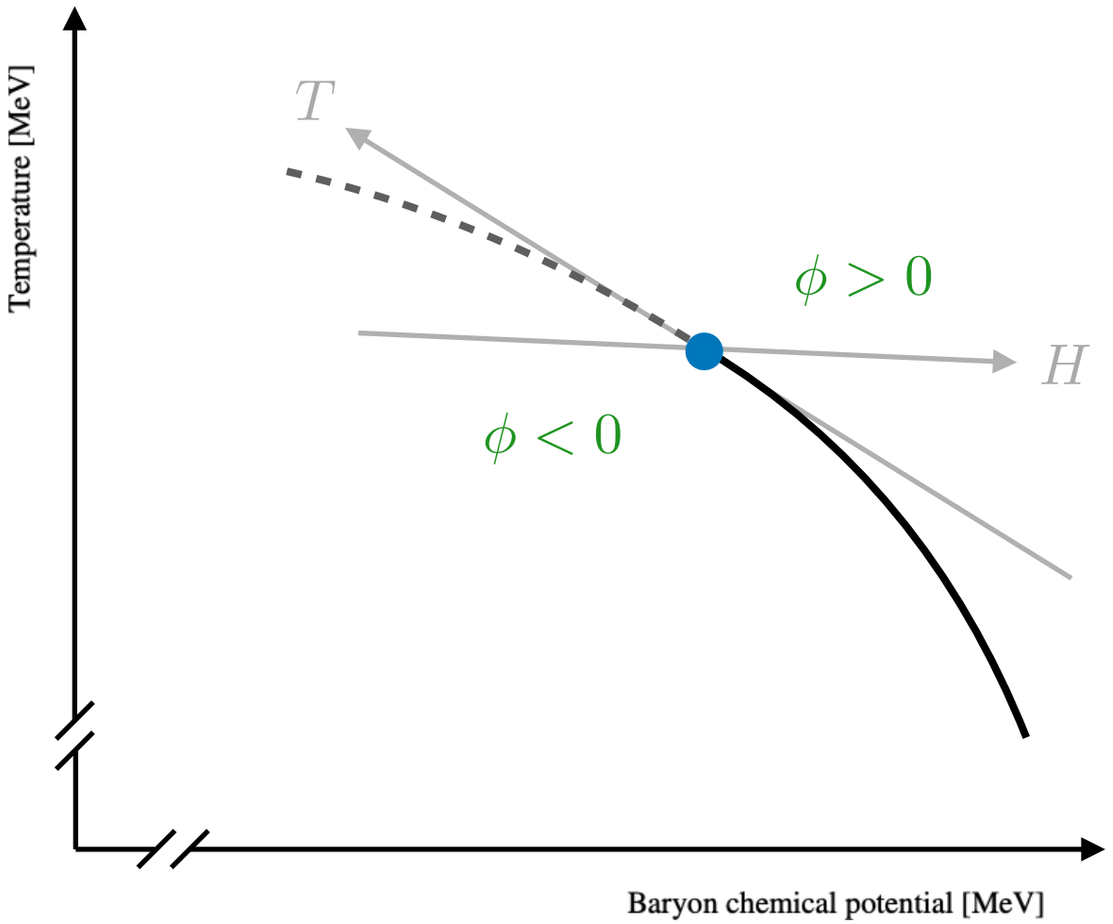
Critical point: Correlation length ξ diverges

- Microscopic details **irrelevant**
- Critical properties **only** depend on symmetries, number of dimensions, …
  ⇝ Universality (classes)

Universality near critical point

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non-universal mapping to Ising model:
Parotto et al., PRC **101** (2020) 3, 034901
Pradeep, Sogabe, Stephanov, Yee, PRC **109** (2024) 6, 064905
Pradeep, Stephanov, PRD **100** (2019) 5, 056003



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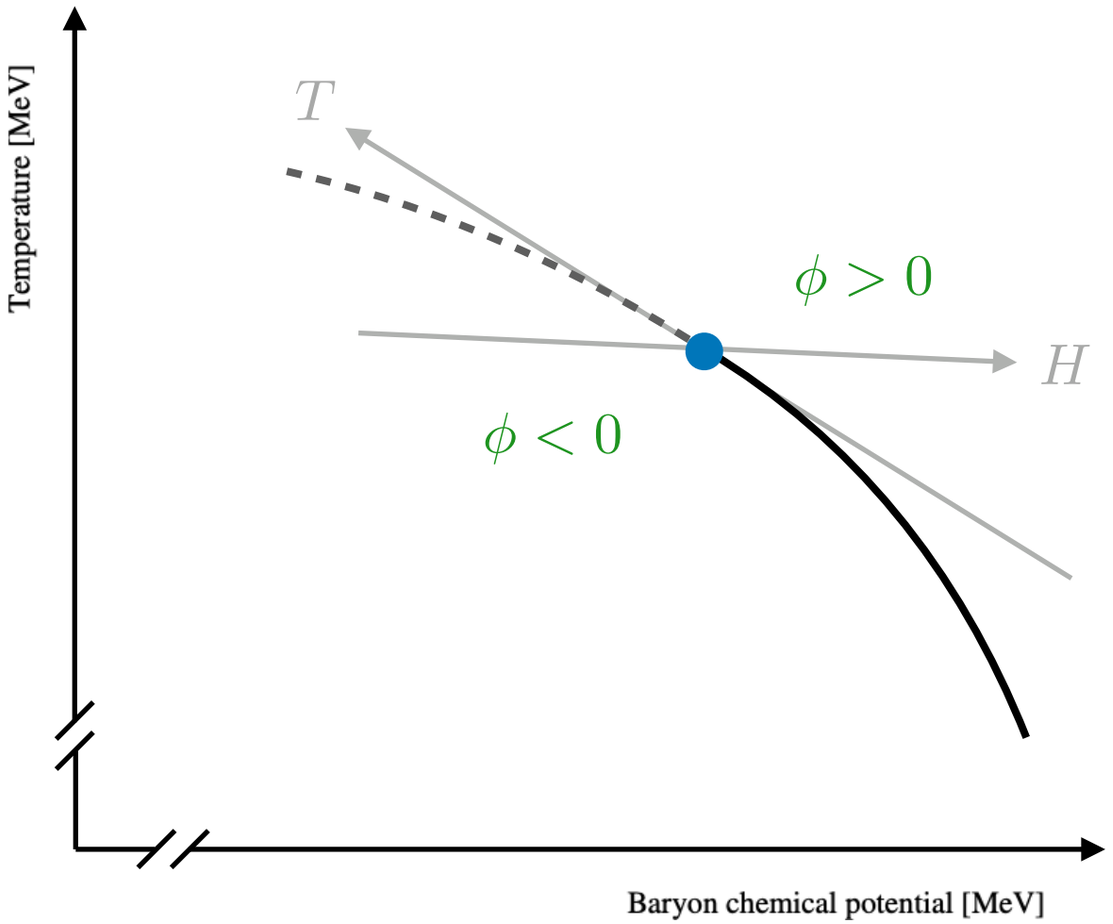
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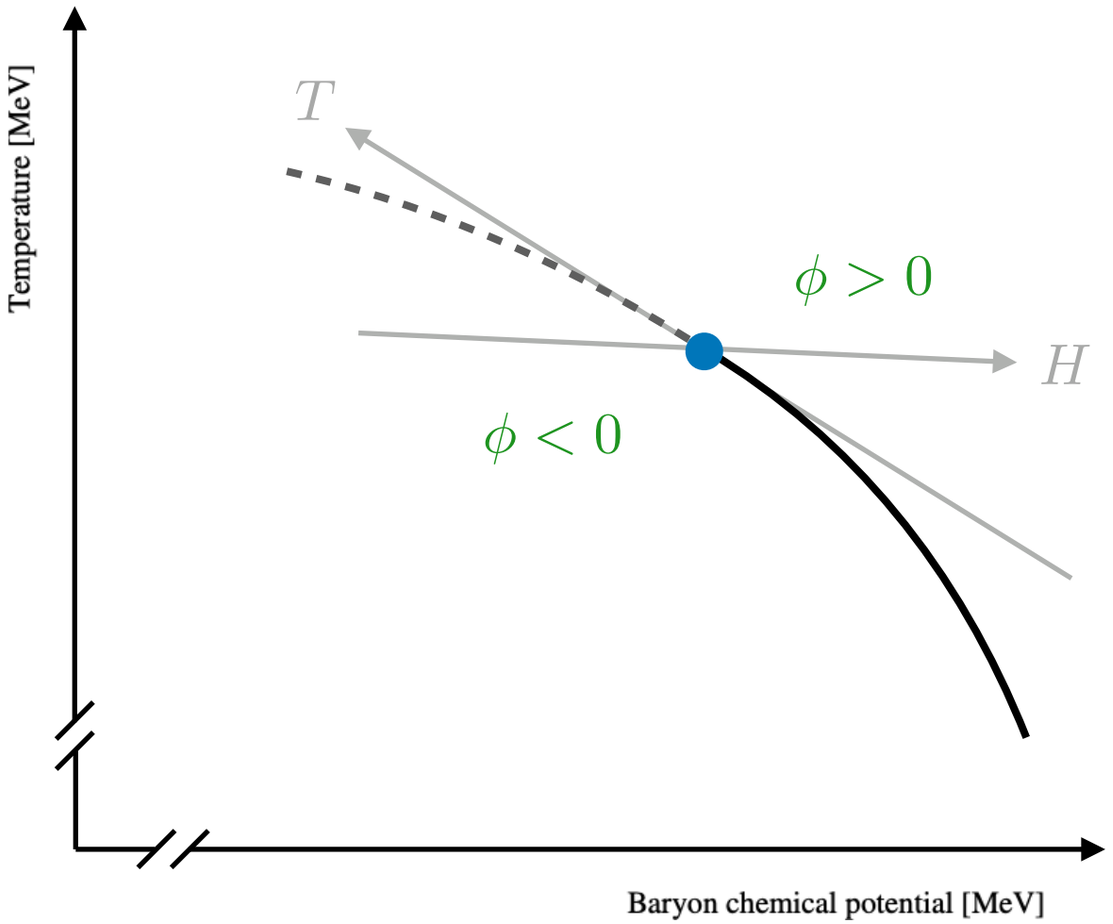
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- For example: $\xi \sim |\tau|^{-\nu}$ $\quad \nu = 0.6301(4)$
  Pelissetto, Vicari (2002)

reduced temperature $\tau = (T - T_c)/T_c$

Universality near critical point

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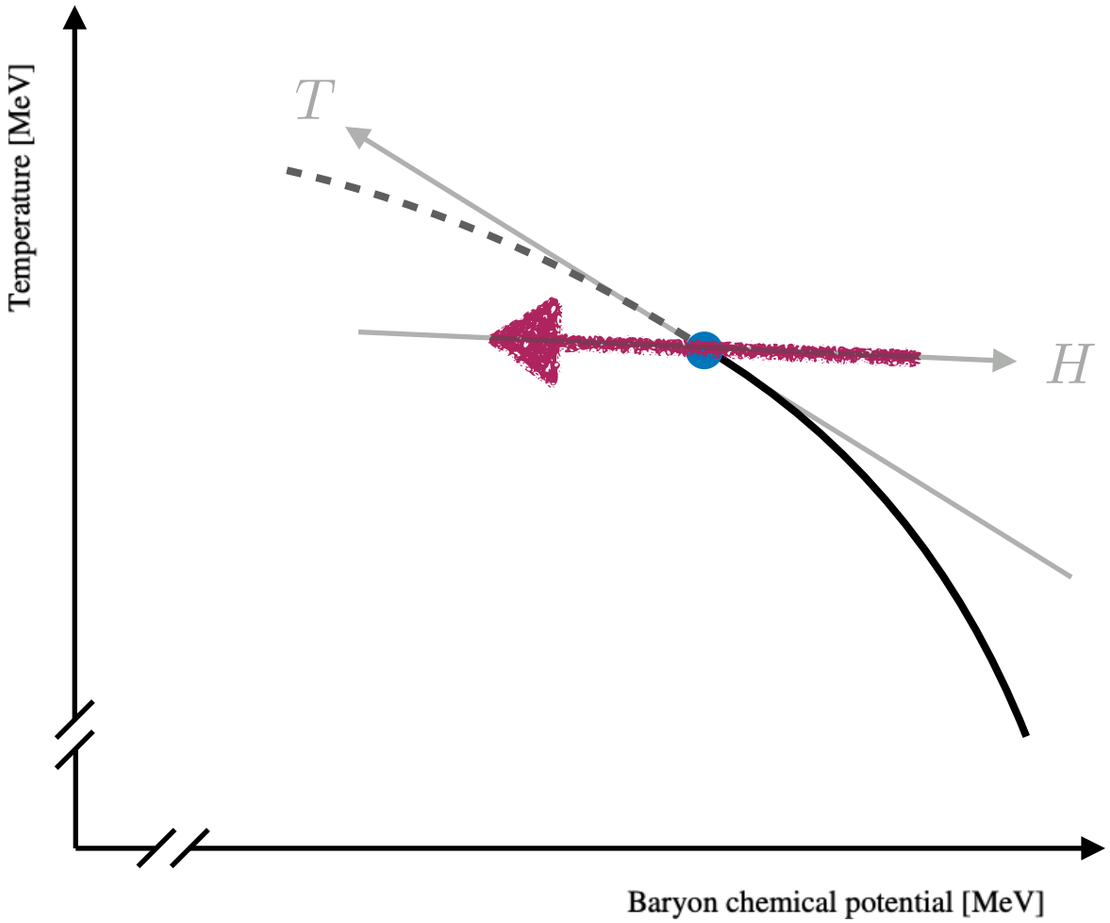
Ising model ⇒ (equilibrium) universal properties of the **QCD critical point**

Dynamics near critical point

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- study trans-critical protocol:

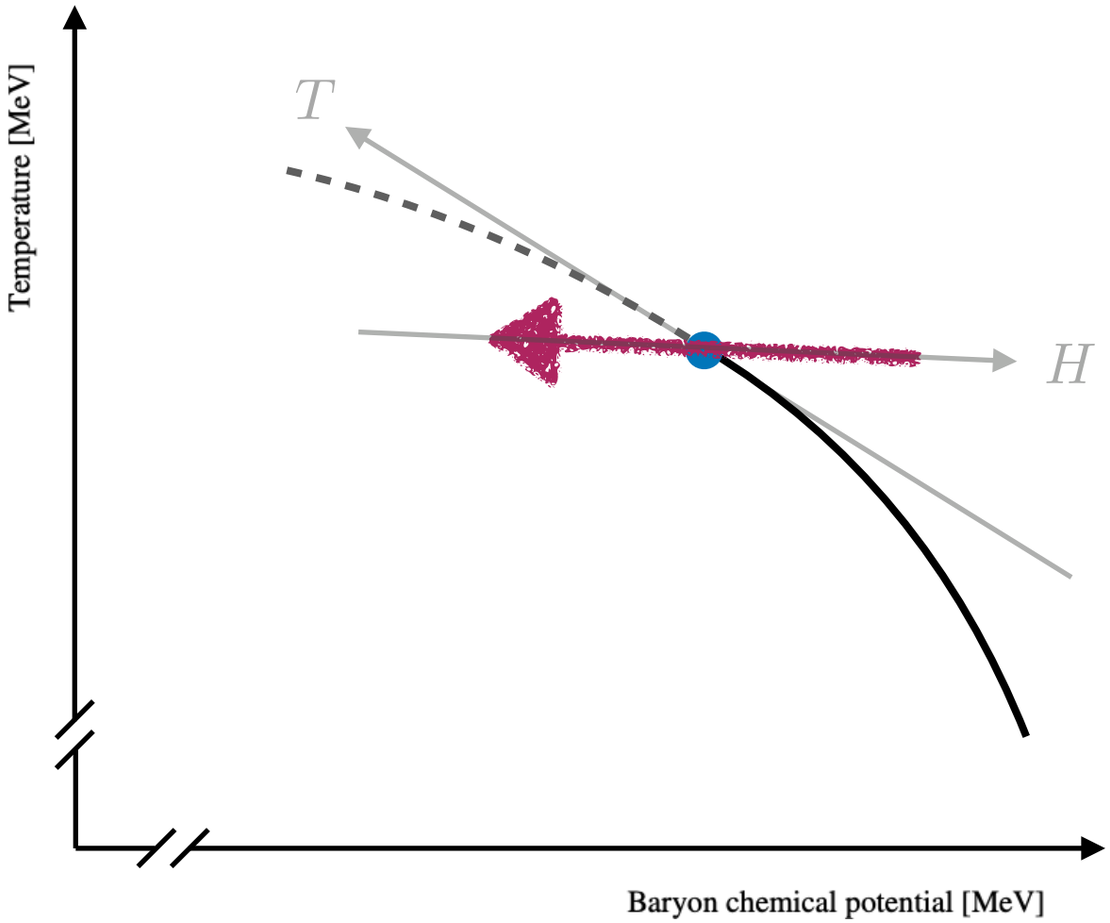
  Sieke, Harhoff, Schlichting, von Smekal, NPB **1011**, 116808 (2025)



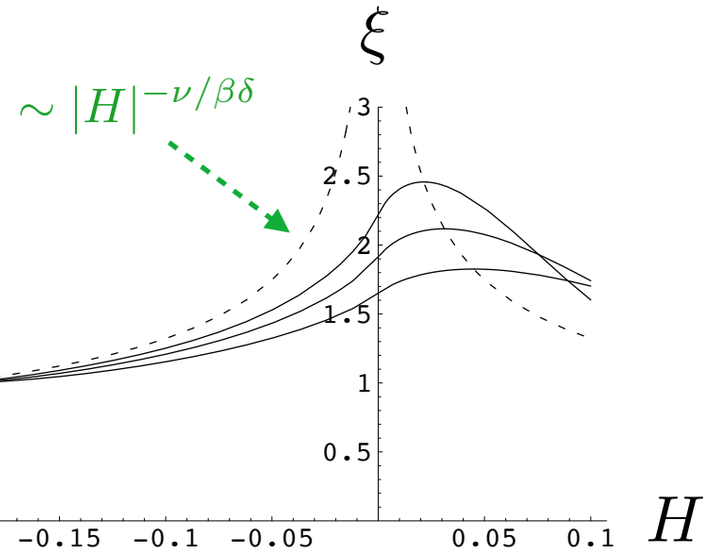
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- time evolution of correlation length ξ:

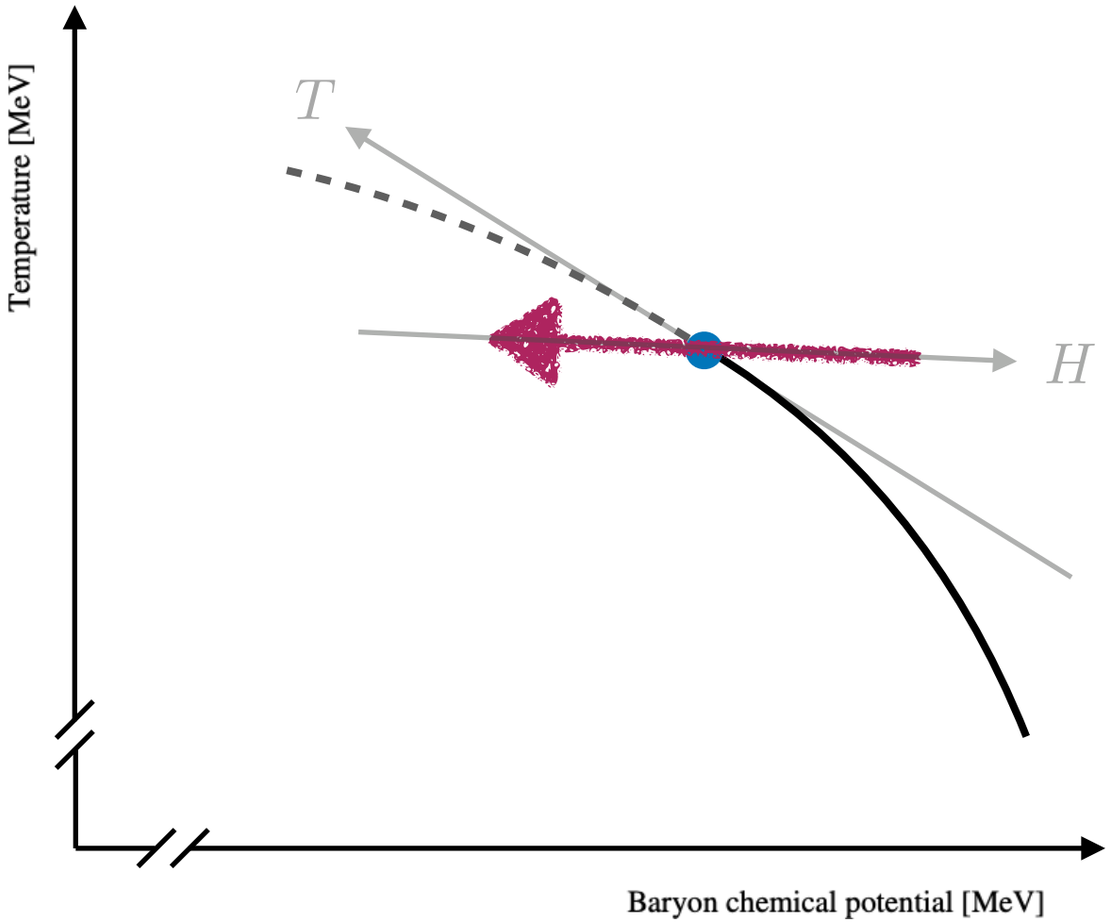


Berdnikov & Rajagopal, PRD **61**, 105017 (2000)

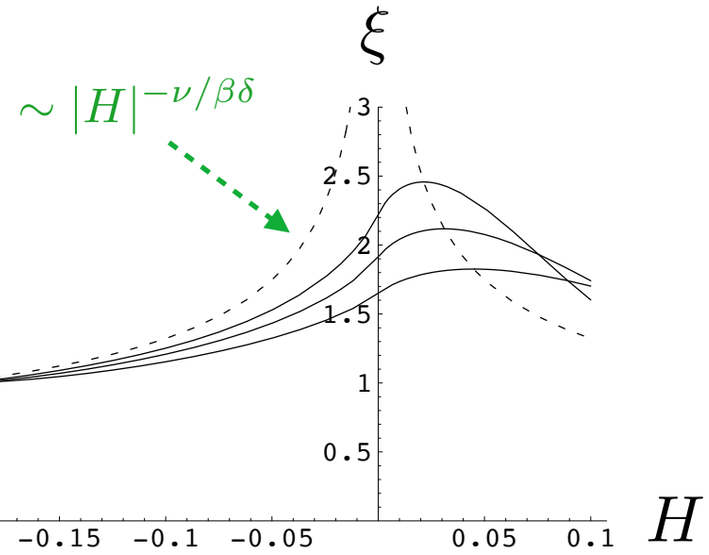
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Berdnikov & Rajagopal, PRD **61**, 105017 (2000)

↝ Critical mode falls out of (local) equilibrium!

Critical dynamics

Reason:

critical slowing down

$$\xi_t \sim \xi^z$$

correlation time — correlation length

Critical dynamics

Reason:

critical slowing down

dynamic critical exponent

$$\xi_t \sim \xi^z$$

correlation time

correlation length

Critical dynamics

Reason:

critical slowing down

dynamic critical exponent

$$\xi_t \sim \xi^z$$

correlation time — correlation length

- z determined by **dynamic universality class**
- group theories by equations of motion for critical modes
- critical mode arbitrarily slow ⇝ **hydrodynamic theory**
  (depends on: order parameter conserved or not, potential other conserved quantities, etc.)

Dynamics near Critical Point (Model H — intuitive)

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(see *Phase Transition Dynamics* by Onuki)

- conserved order parameter (specific entropy)

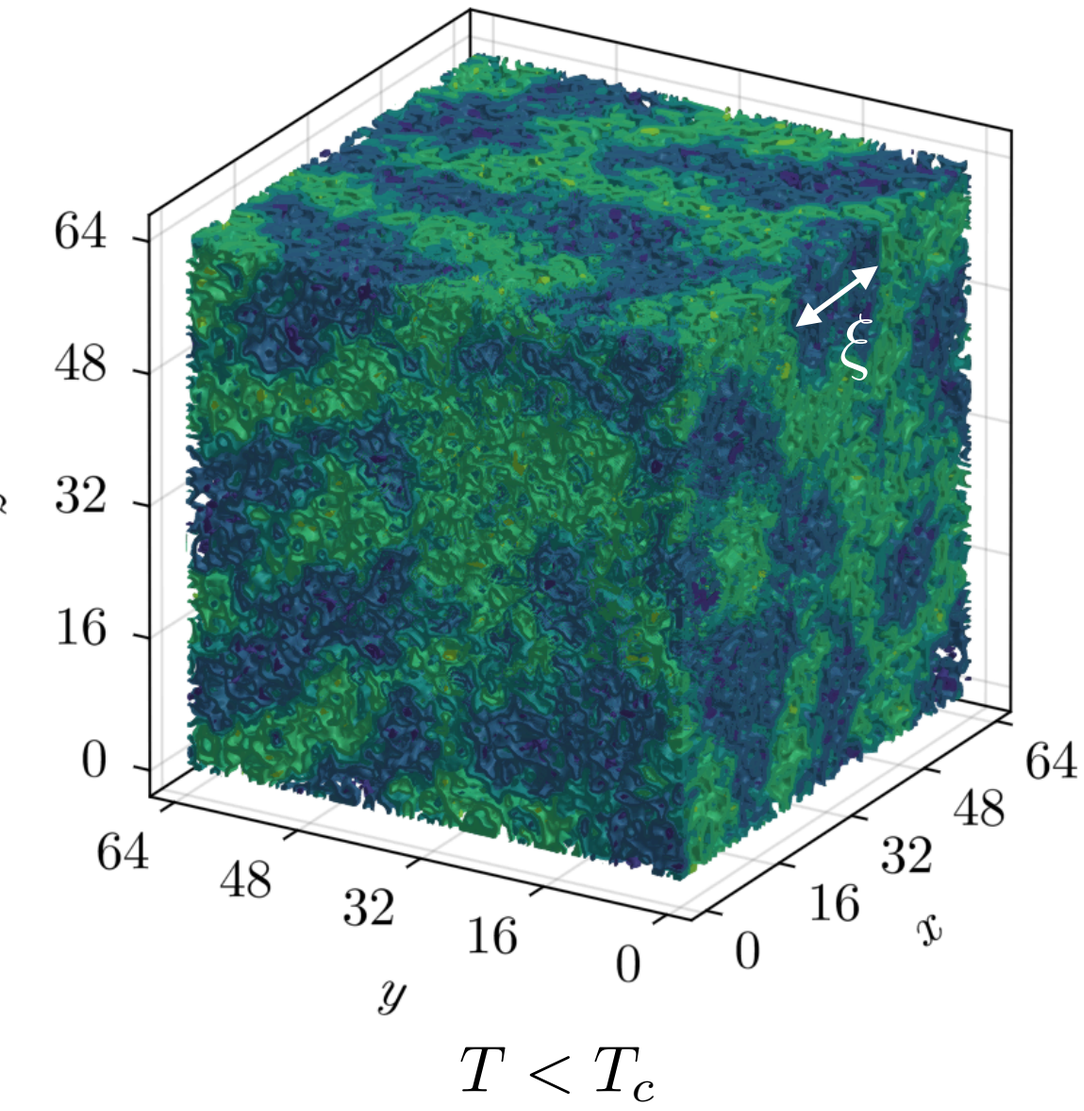


Figure from Thomas Schaefer's talk at Quark Matter 2025

Dynamics near Critical Point (Model H — intuitive)

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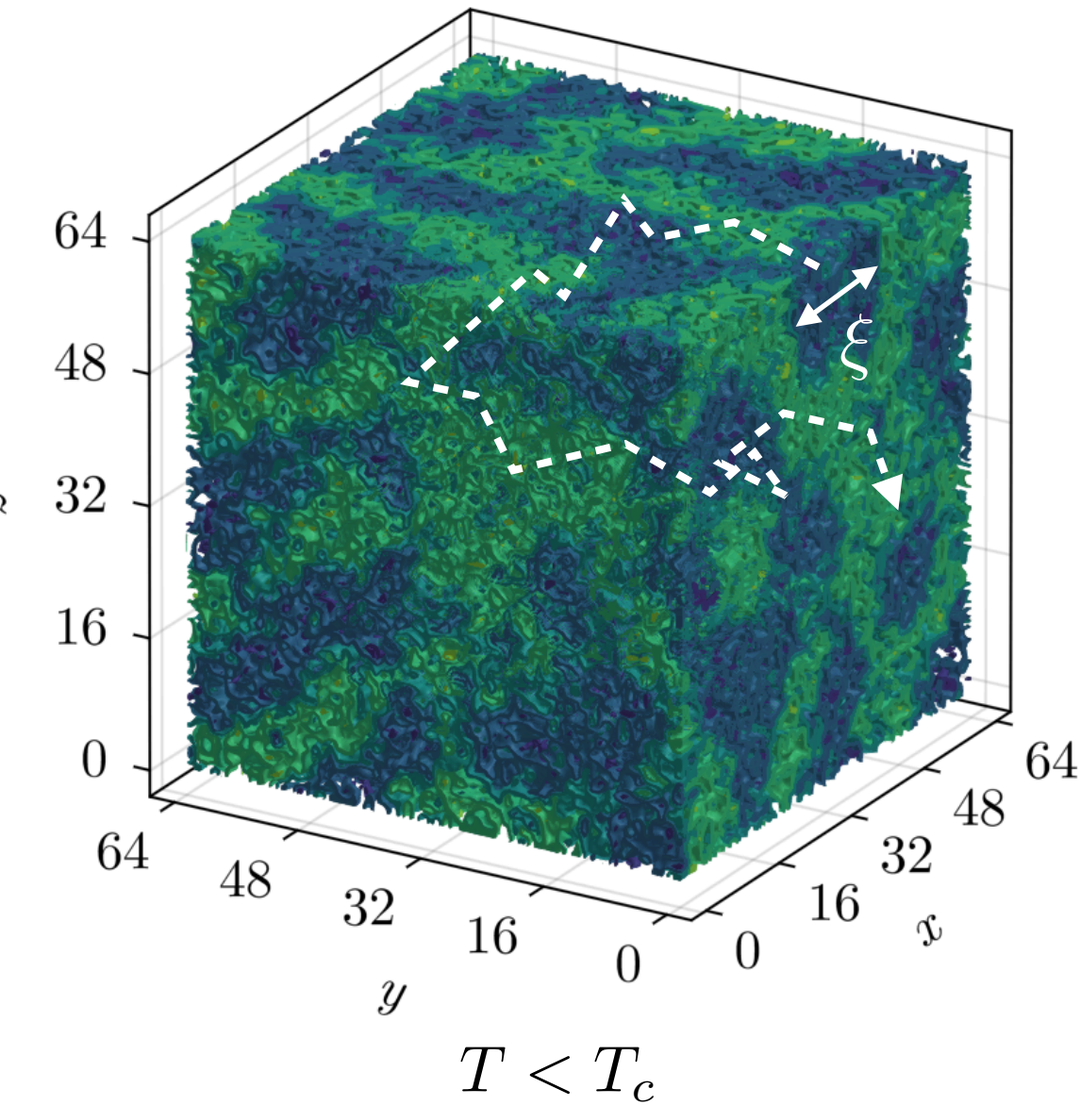


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- conserved order parameter (specific entropy)
- Clusters of linear extent $\sim \xi$ undergo Brownian motion due to advection by thermal velocity fluctuations

Dynamics near Critical Point (Model H — intuitive)

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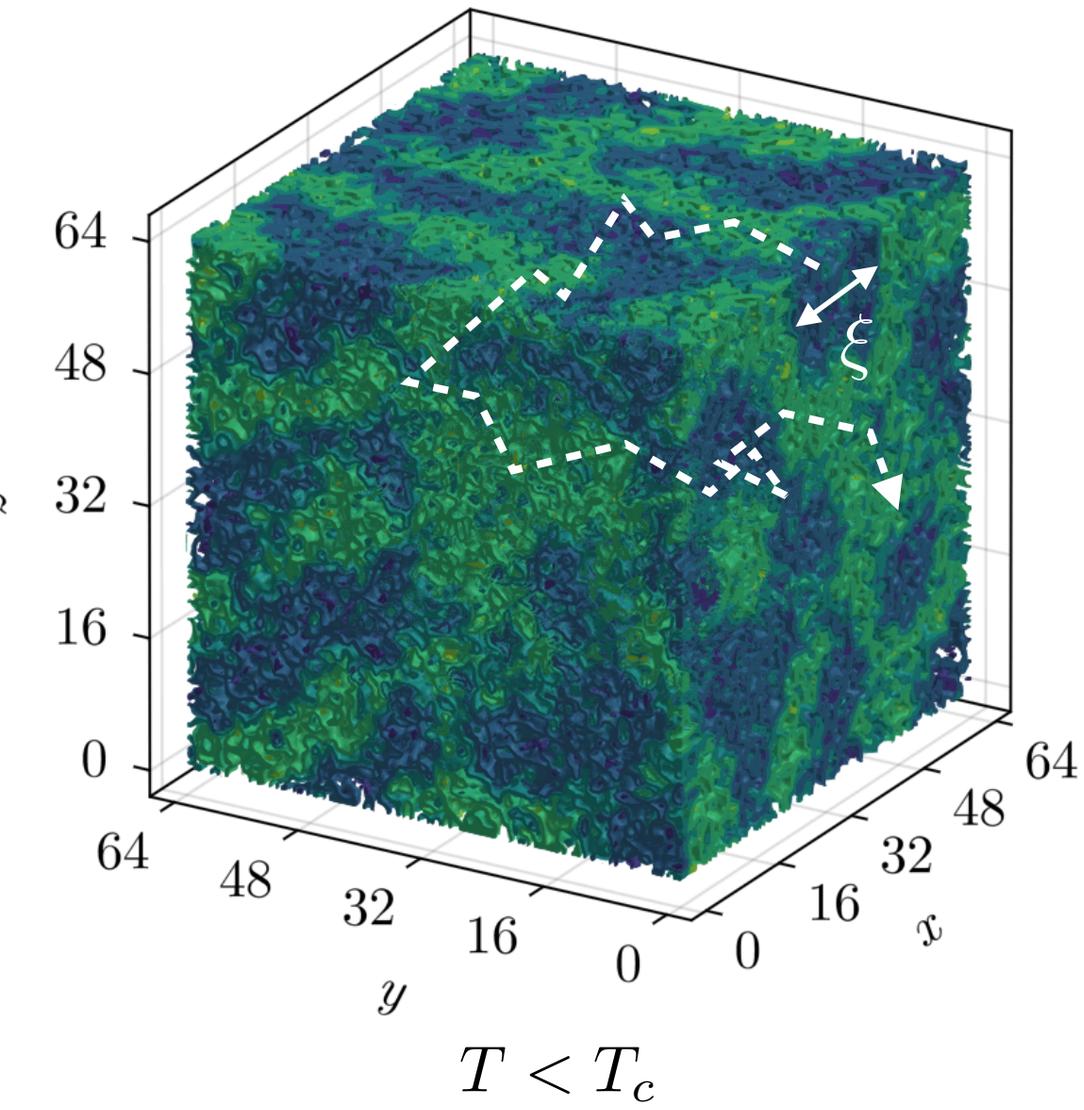


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$$D_\phi \sim \frac{g^2 T_c}{\eta \xi^{d-2}}$$

Dynamics near Critical Point (Model H — intuitive)

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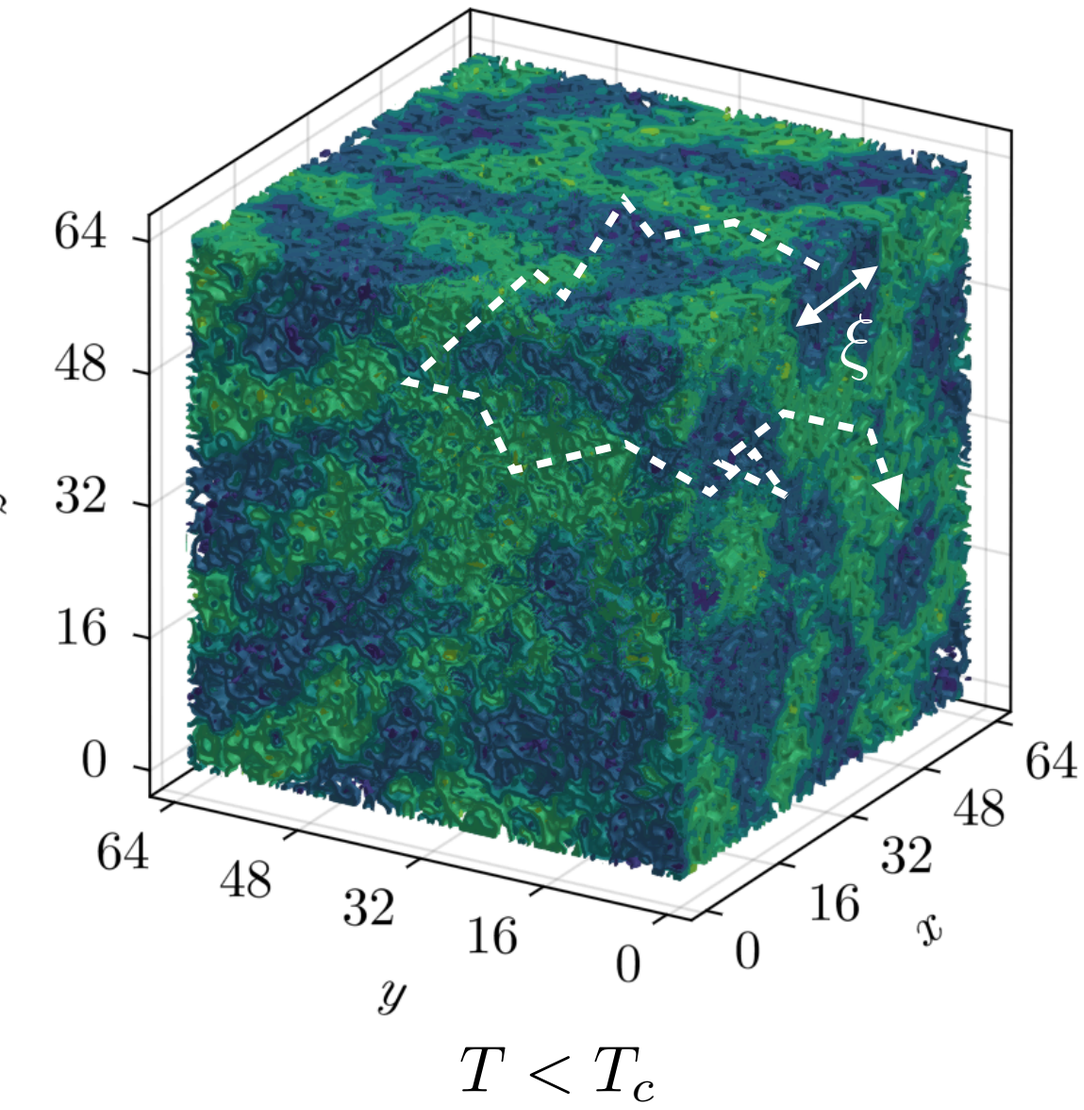


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Dynamics near Critical Point (Model H — intuitive)

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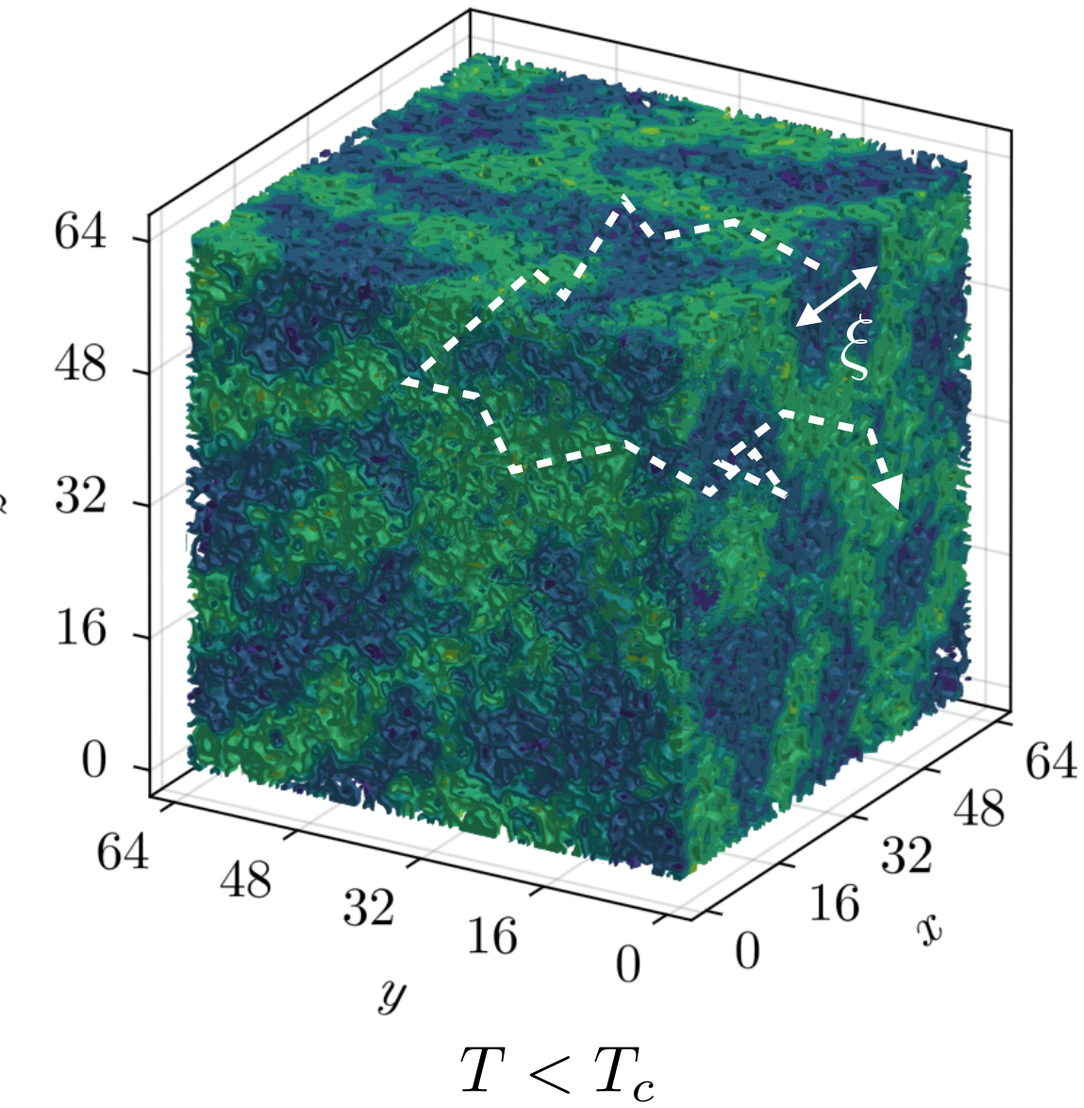


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- Match to critical scaling at $\ell \sim \xi$

$$\xi_t = D_\phi^{-1} \xi^2 \sim \frac{\eta \xi^{d-2}}{g^2 T_c} \xi^2 \sim \eta \xi^d$$

Dynamics near Critical Point (Model H — intuitive)

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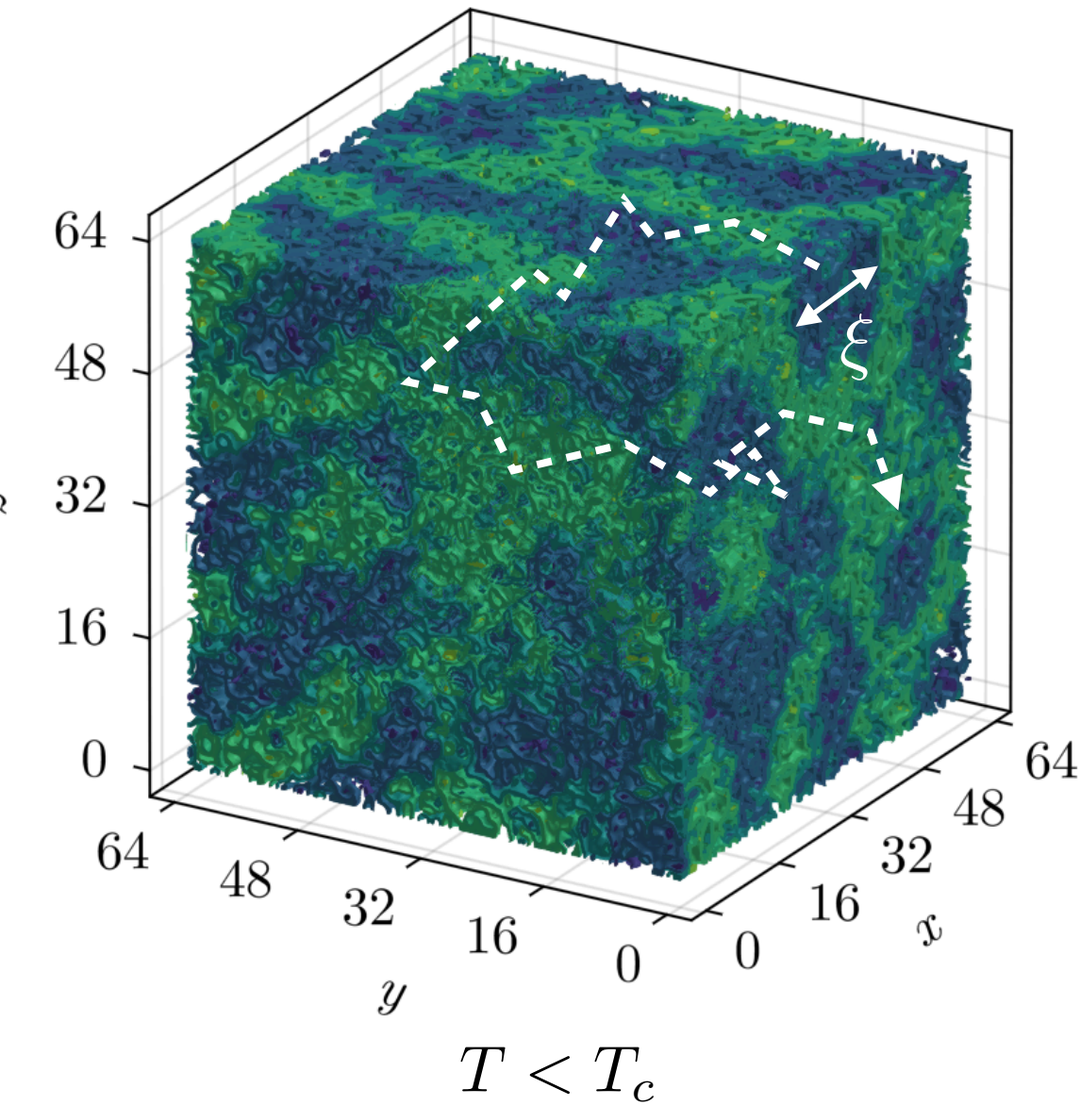


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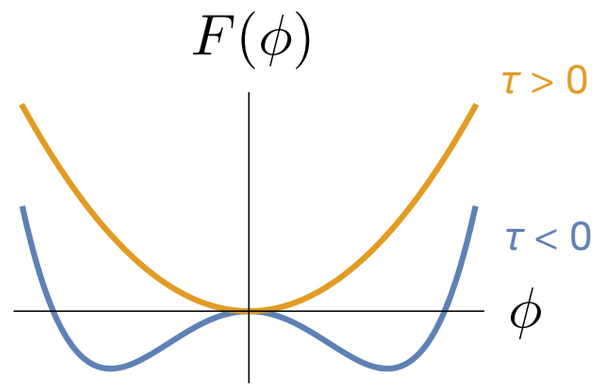
$$z = d + x_\eta$$

Dynamic universality class of QCD critical point

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- Order parameter ϕ, momentum density $\vec{j} = \rho\vec{v}$

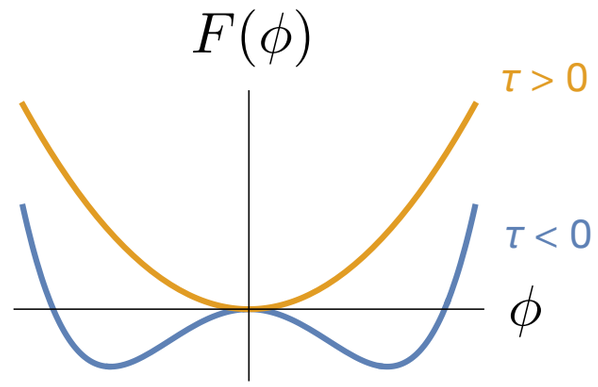
$$F = \int d^d x \left\{ \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4 + \frac{\vec{j}^2}{2\rho} \right\}$$



Dynamic universality class of QCD critical point

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- Langevin equations of motion

Hohenberg and Halperin, Rev. Mod. Phys. **49**, 435 (1977)

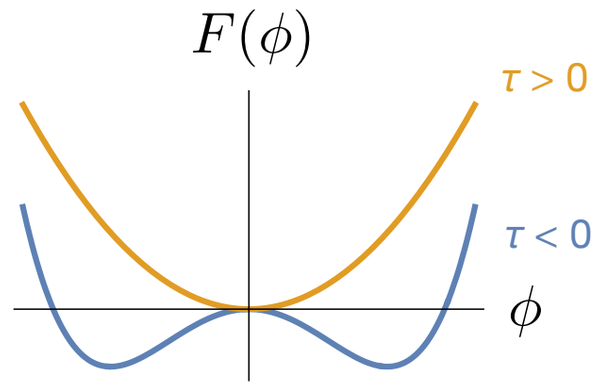
$$\frac{\partial\phi}{\partial t} = \sigma\vec{\nabla}^2\frac{\delta F}{\delta\phi} + \theta - g\,\frac{\delta F}{\delta\vec{j}}\cdot\vec{\nabla}\phi$$

$$\frac{\partial\vec{j}}{\partial t} = \mathcal{T}\left[\eta\vec{\nabla}^2\frac{\delta F}{\delta\vec{j}} + \vec{\xi} + g\,\frac{\delta F}{\delta\phi}\vec{\nabla}\phi - g\left(\frac{\delta F}{\delta\vec{j}}\cdot\vec{\nabla}\right)\vec{j}\right]$$

Dynamic universality class of QCD critical point

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diffusion noise

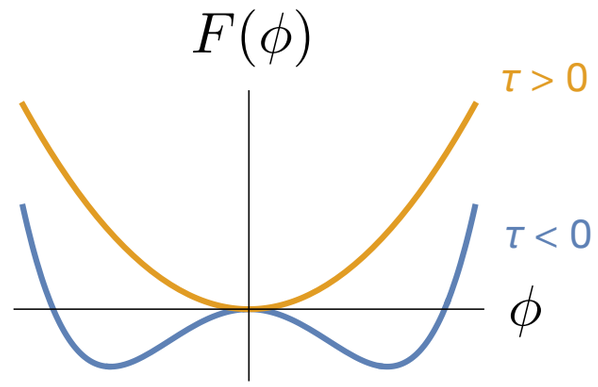
$$\frac{\partial\phi}{\partial t} = \underbrace{\sigma\vec{\nabla}^2\frac{\delta F}{\delta\phi} + \theta}_{\text{fluctuation-dissipation relation}} - g\,\frac{\delta F}{\delta\vec{j}}\cdot\vec{\nabla}\phi$$

$$\frac{\partial\vec{j}}{\partial t} = \tau\left[\eta\vec{\nabla}^2\frac{\delta F}{\delta\vec{j}} + \vec{\xi} + g\,\frac{\delta F}{\delta\phi}\vec{\nabla}\phi - g\left(\frac{\delta F}{\delta\vec{j}}\cdot\vec{\nabla}\right)\vec{j}\right]$$

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$$F = \int d^d x \left\{ \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4 + \frac{\vec{j}^2}{2\rho} \right\}$$



- Langevin equations of motion

Hohenberg and Halperin, Rev. Mod. Phys. **49**, 435 (1977)

diffusion noise

$$\frac{\partial\phi}{\partial t} = \sigma\vec{\nabla}^2\frac{\delta F}{\delta\phi} + \theta - g\,\frac{\delta F}{\delta\vec{j}}\cdot\vec{\nabla}\phi$$

fluctuation-dissipation relation

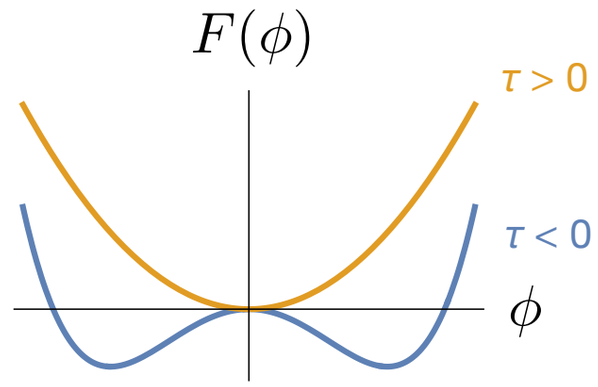
shear viscosity noise

$$\frac{\partial\vec{j}}{\partial t} = \mathcal{T}\left[\eta\vec{\nabla}^2\frac{\delta F}{\delta\vec{j}} + \vec{\xi} + g\,\frac{\delta F}{\delta\phi}\vec{\nabla}\phi - g\left(\frac{\delta F}{\delta\vec{j}}\cdot\vec{\nabla}\right)\vec{j}\right]$$

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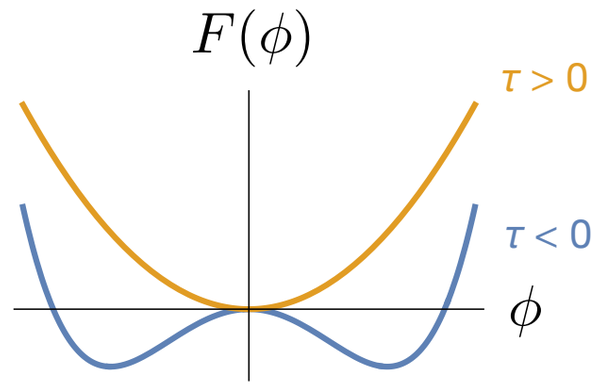
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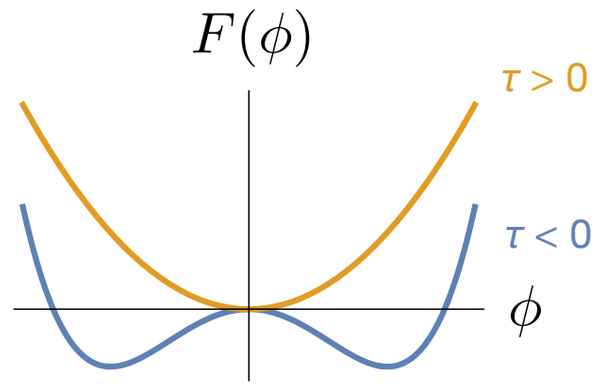
shear viscosity noise reversibility of advection

$$\frac{\partial\vec{j}}{\partial t} = \tau\left[\eta\vec{\nabla}^2\frac{\delta F}{\delta\vec{j}} + \vec{\xi} + g\,\frac{\delta F}{\delta\phi}\vec{\nabla}\phi - g\left(\frac{\delta F}{\delta\vec{j}}\cdot\vec{\nabla}\right)\vec{j}\right]$$

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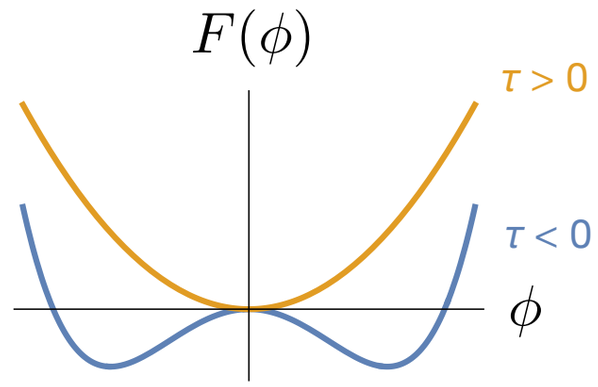
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JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

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Son and Stephanov, PRD **70**, 056001 (2004)

Model H

$z = d + x_\eta$

Liquid-gas critical point in pure fluid

Fixed points & critical exponents (Model H)

Roth, Ye, Schlichting, von Smekal,
PRD **111**, L111901 (2025)

Chen, Tan, Fu, PRD **111**, 094025 (2025)

Fixed points & critical exponents (Model H)

- dimensionless combinations of kinetic coefficients:

$$f \equiv \frac{d\,\Omega_d\,g^2 T}{(2\pi)^d}\,\frac{k^{d-4}}{\sigma_k \eta_k} \qquad w \equiv \rho\,\frac{\sigma_k k^2}{\eta_k}$$

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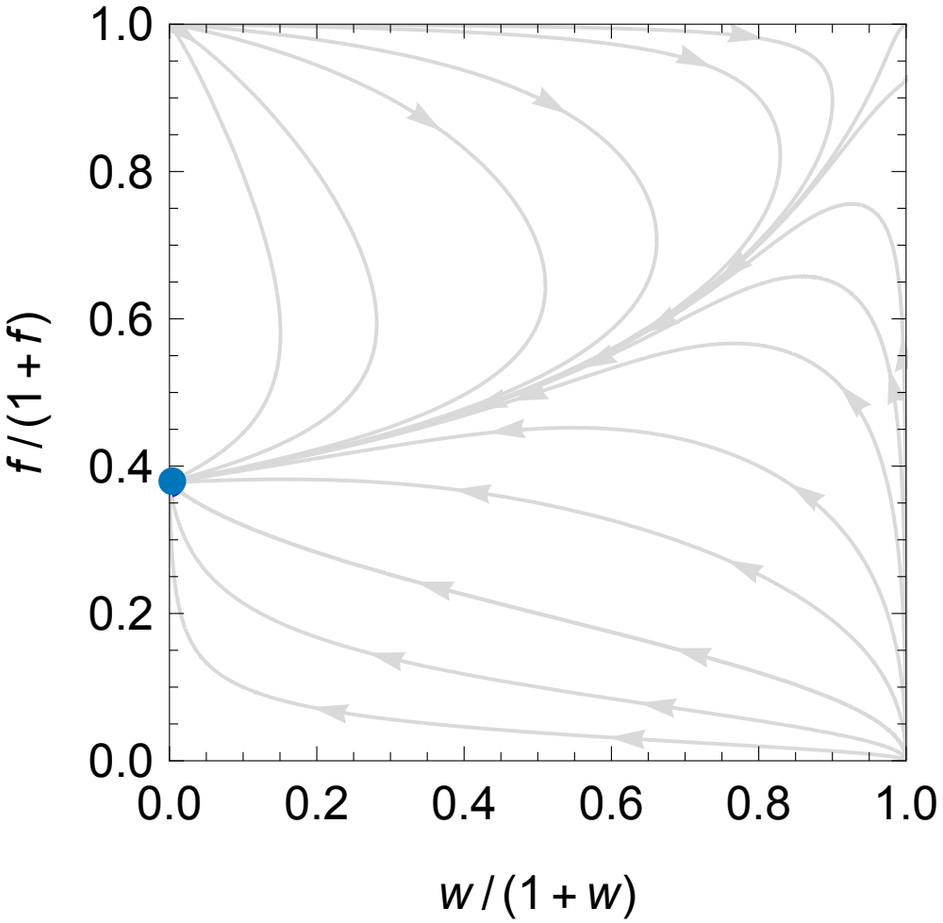
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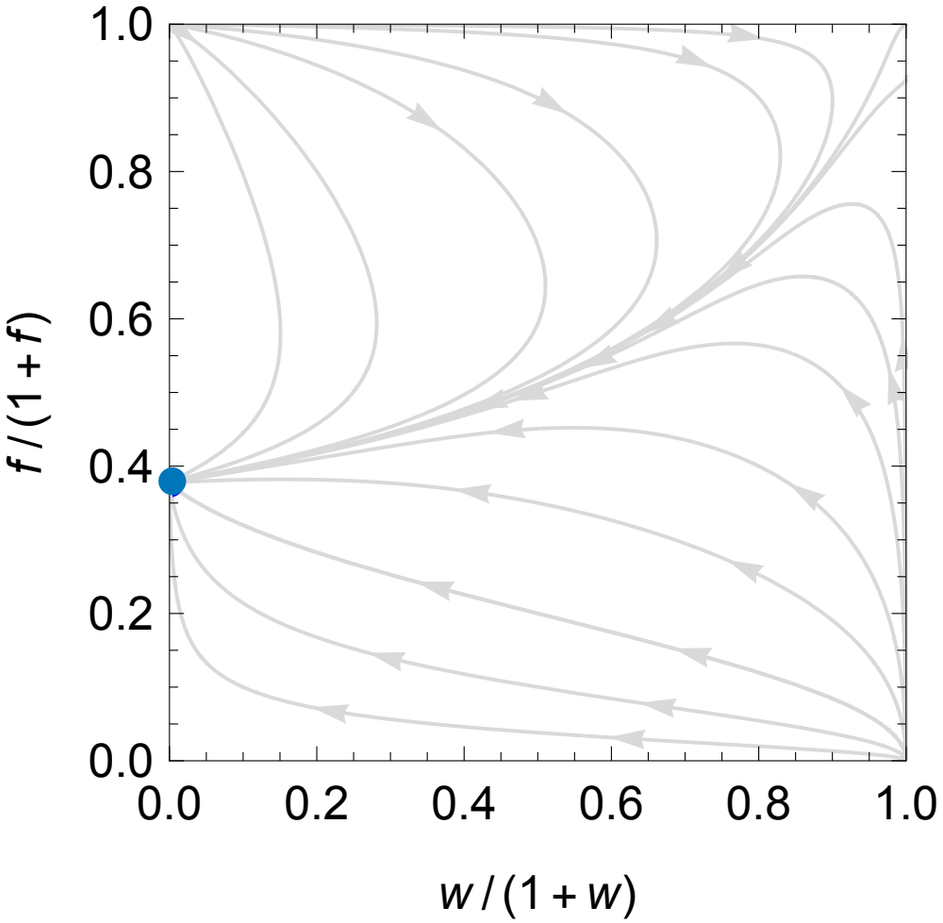
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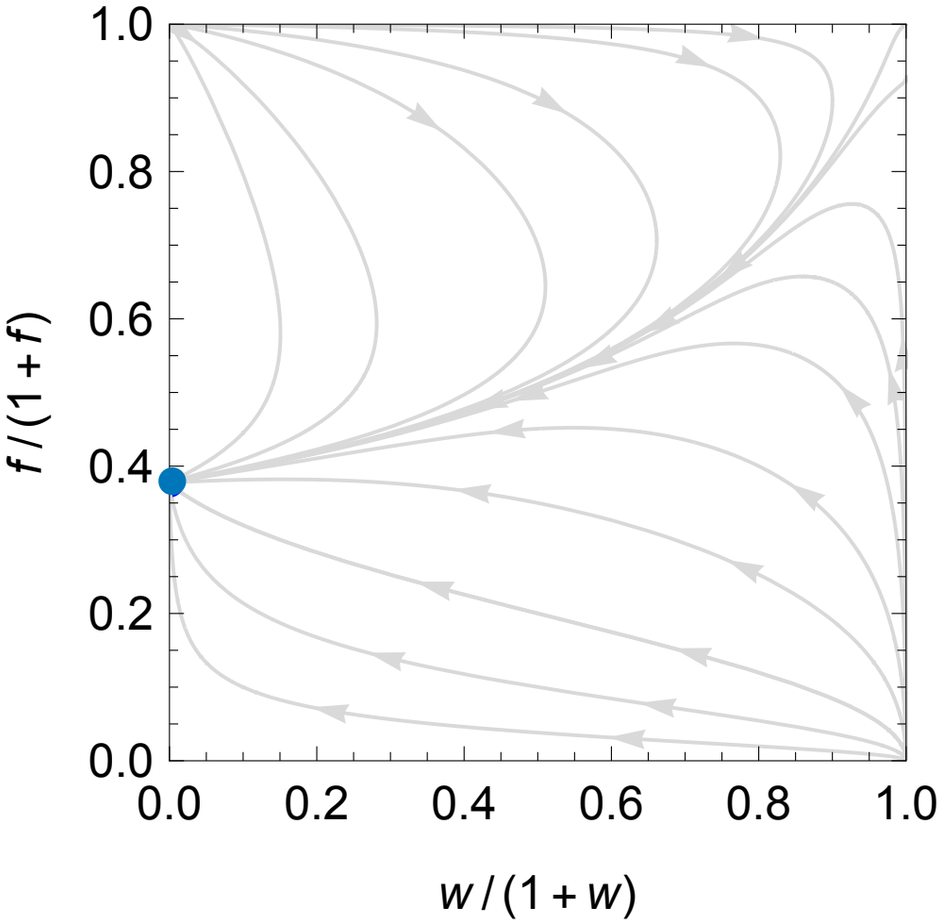
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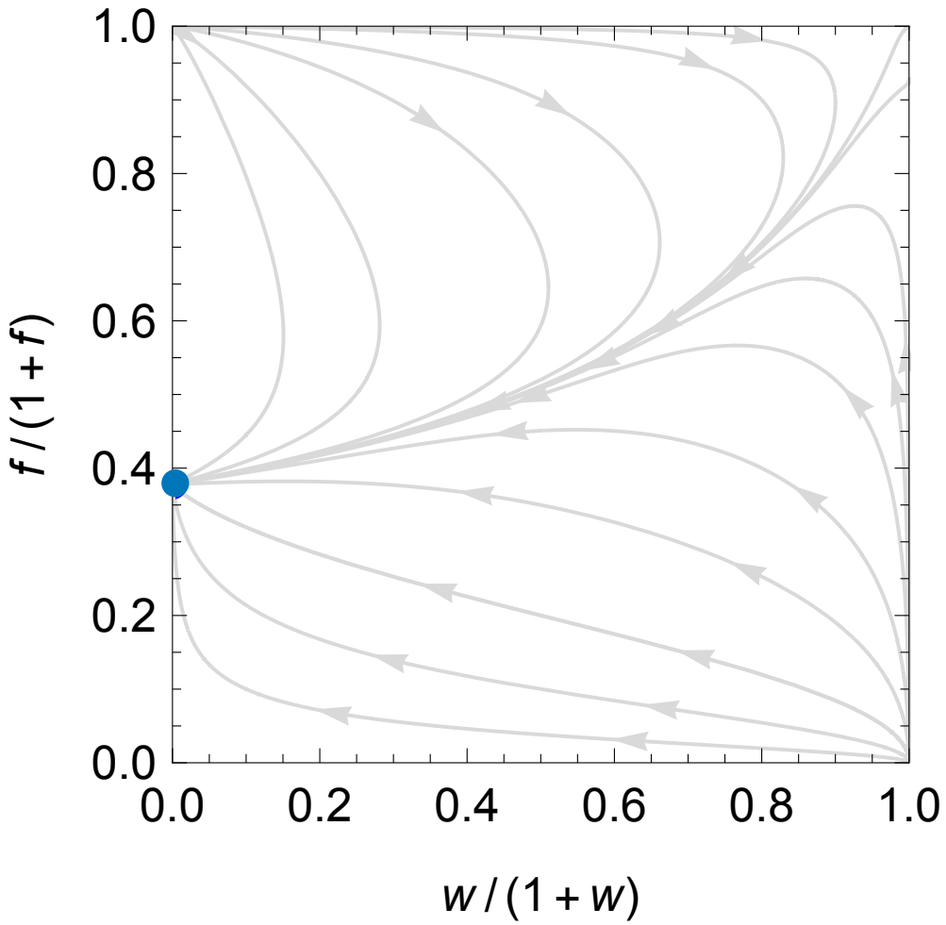
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- at fixed point (FP):

$$\sigma \sim k^{-x_\sigma}$$

∝ heat conductivity

$$\eta \sim k^{-x_\eta}$$

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JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

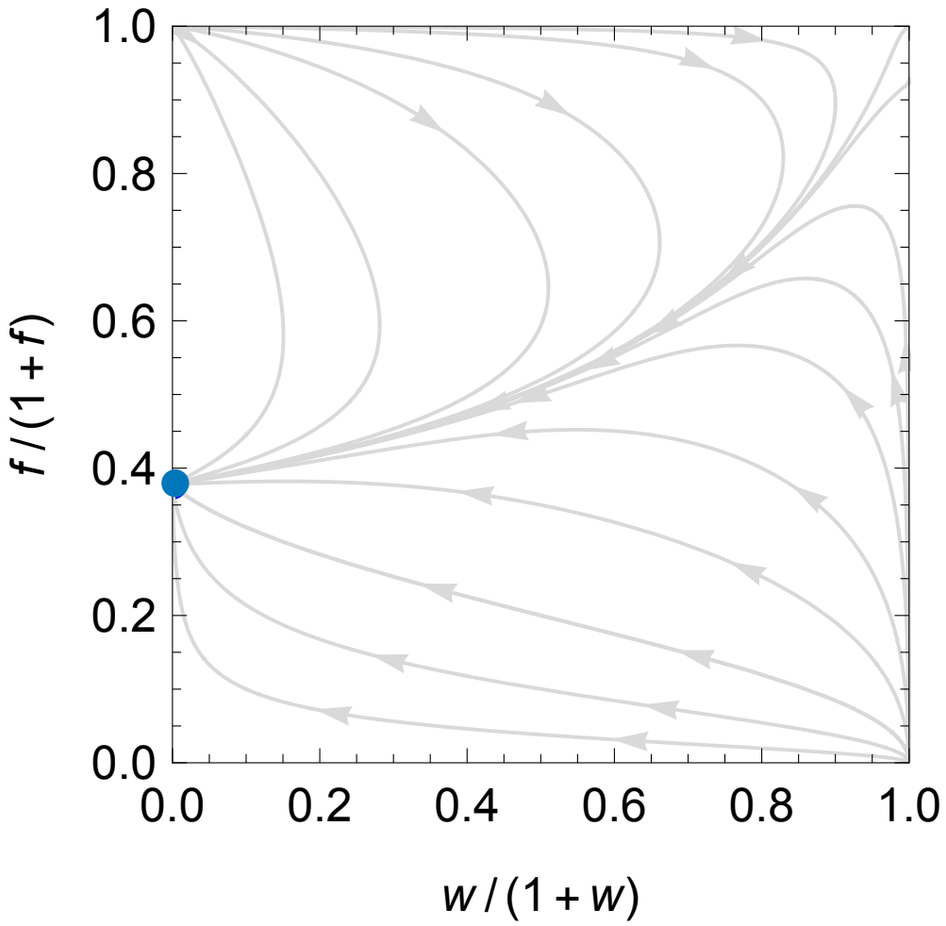
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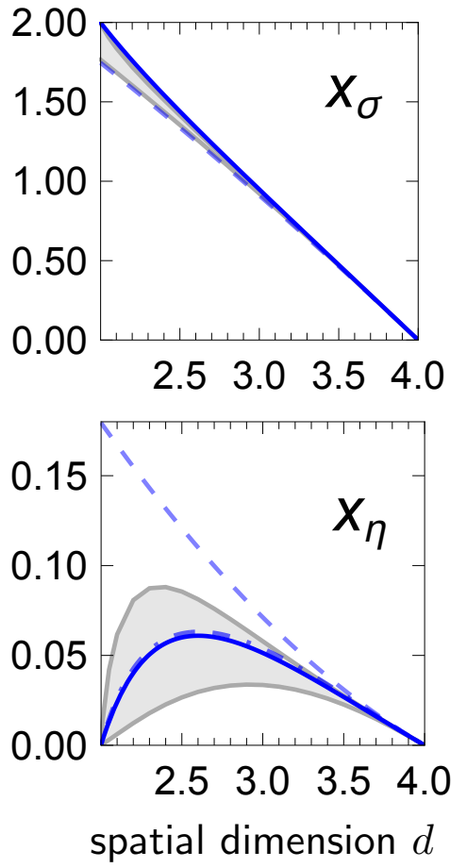
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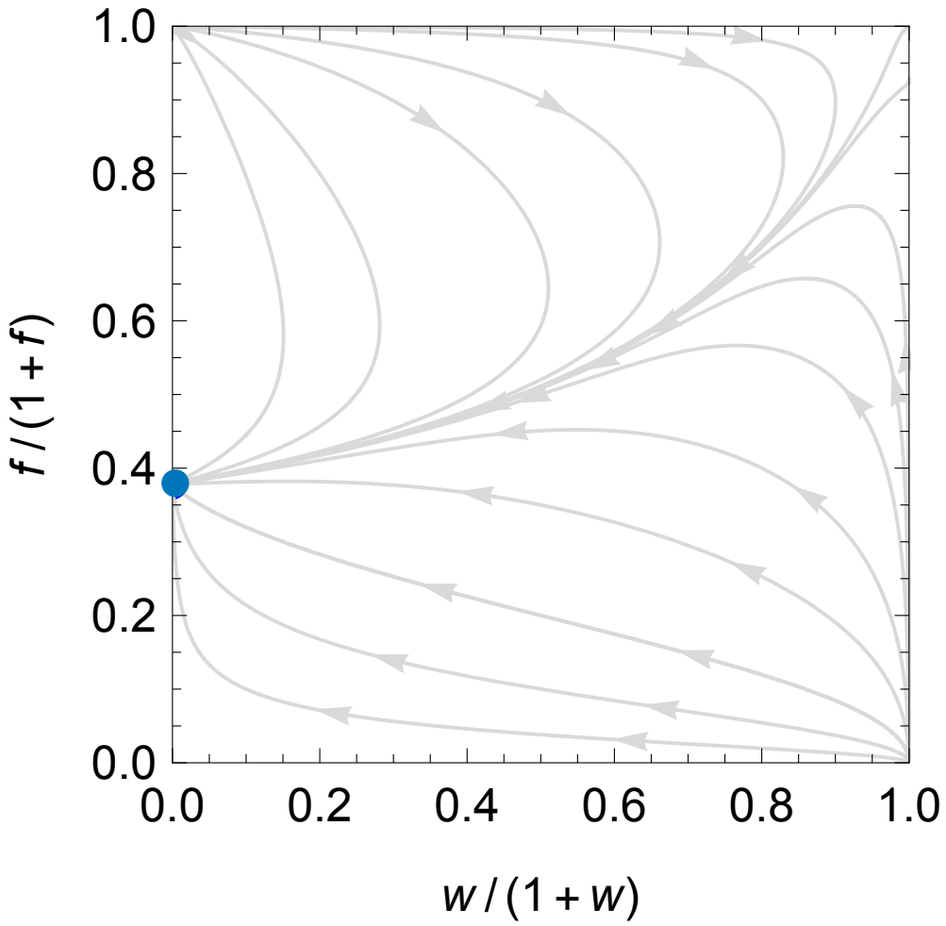
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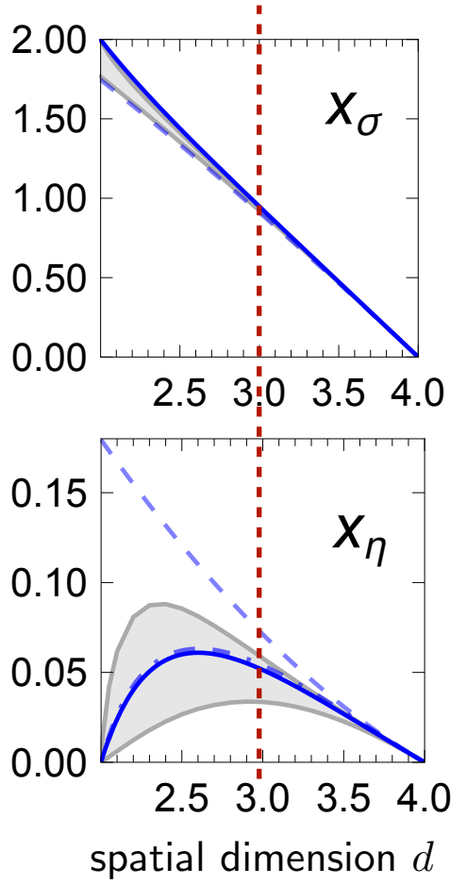
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$$\boldsymbol{d = 3}\text{:}$$

$$x_\sigma \approx 0.949$$

$$x_\eta \approx 0.051$$

$$\Downarrow$$

$$z_\phi \approx 3.051$$

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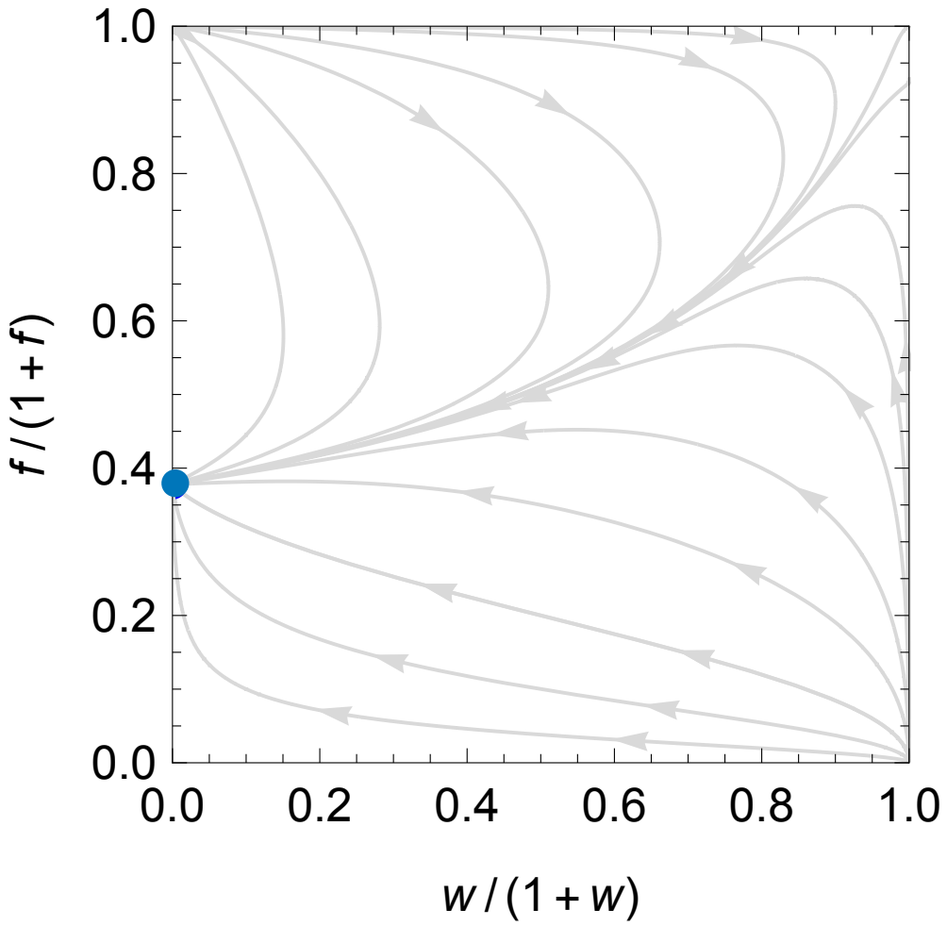
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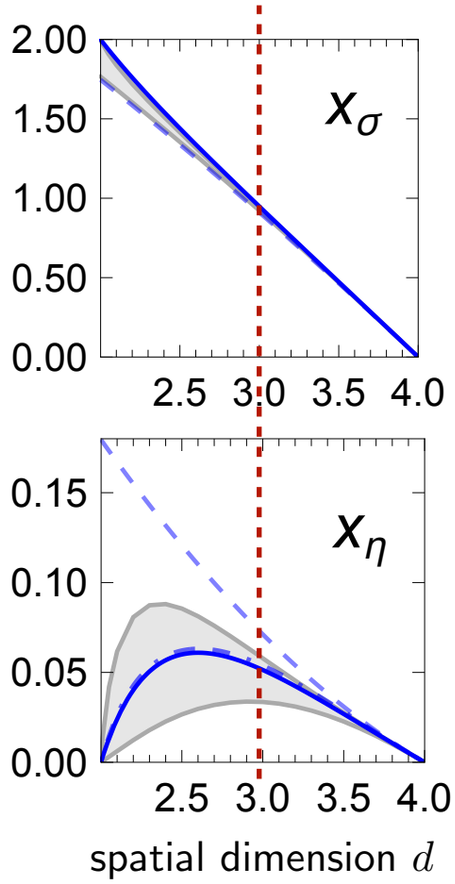
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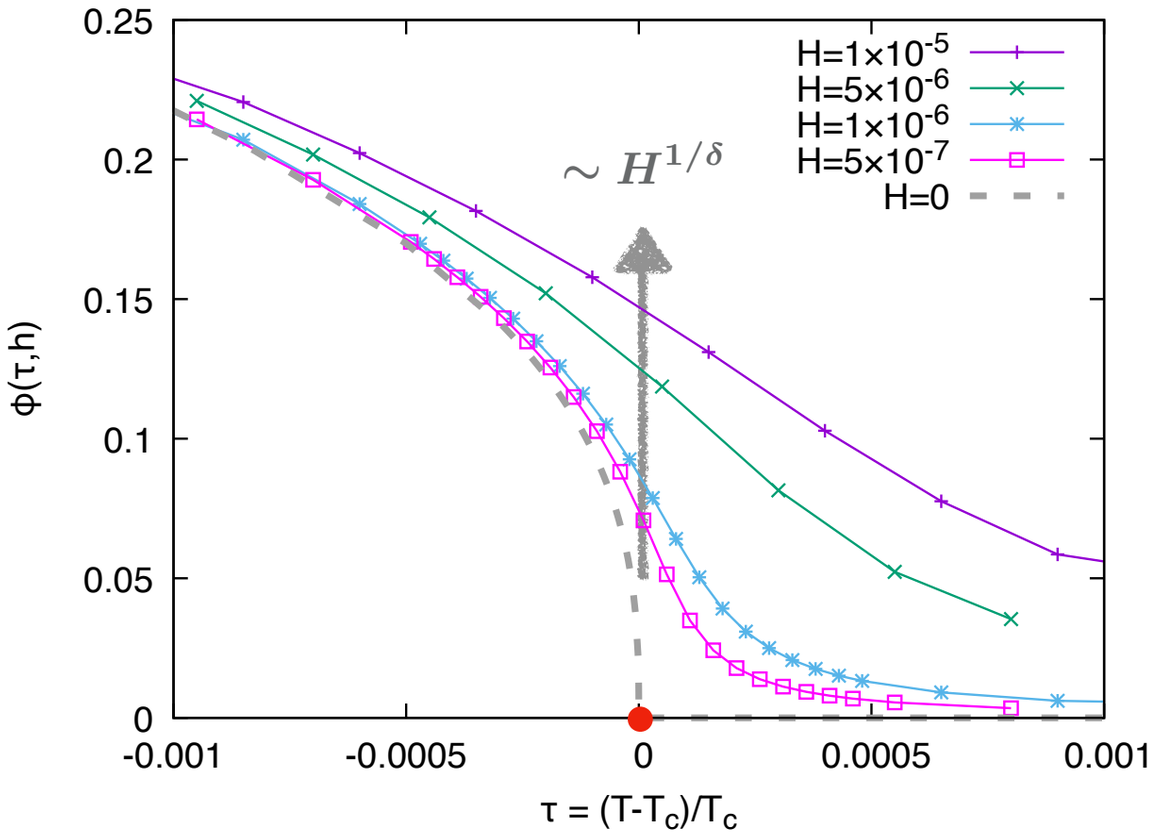
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Model H

$$z_\phi = 4 - \eta_\perp - x_\sigma$$
$$= d + x_\eta$$

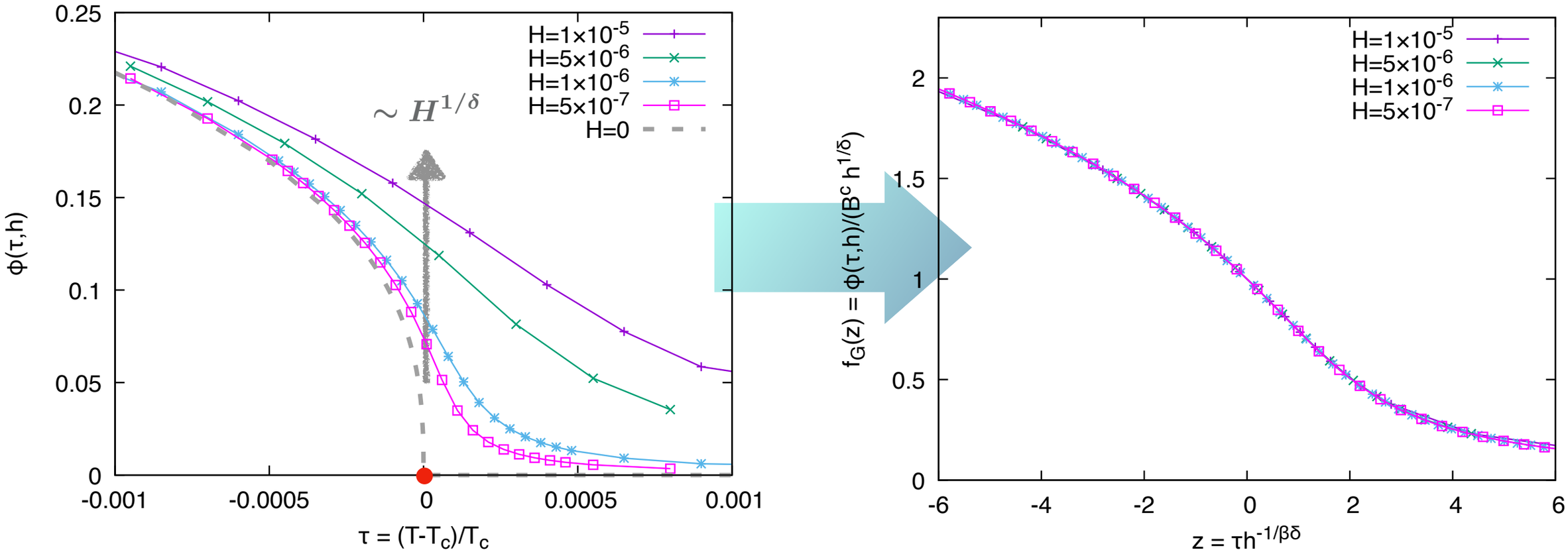
Scaling Functions

- Benchmark: Magnetic EoS (3d Ising)



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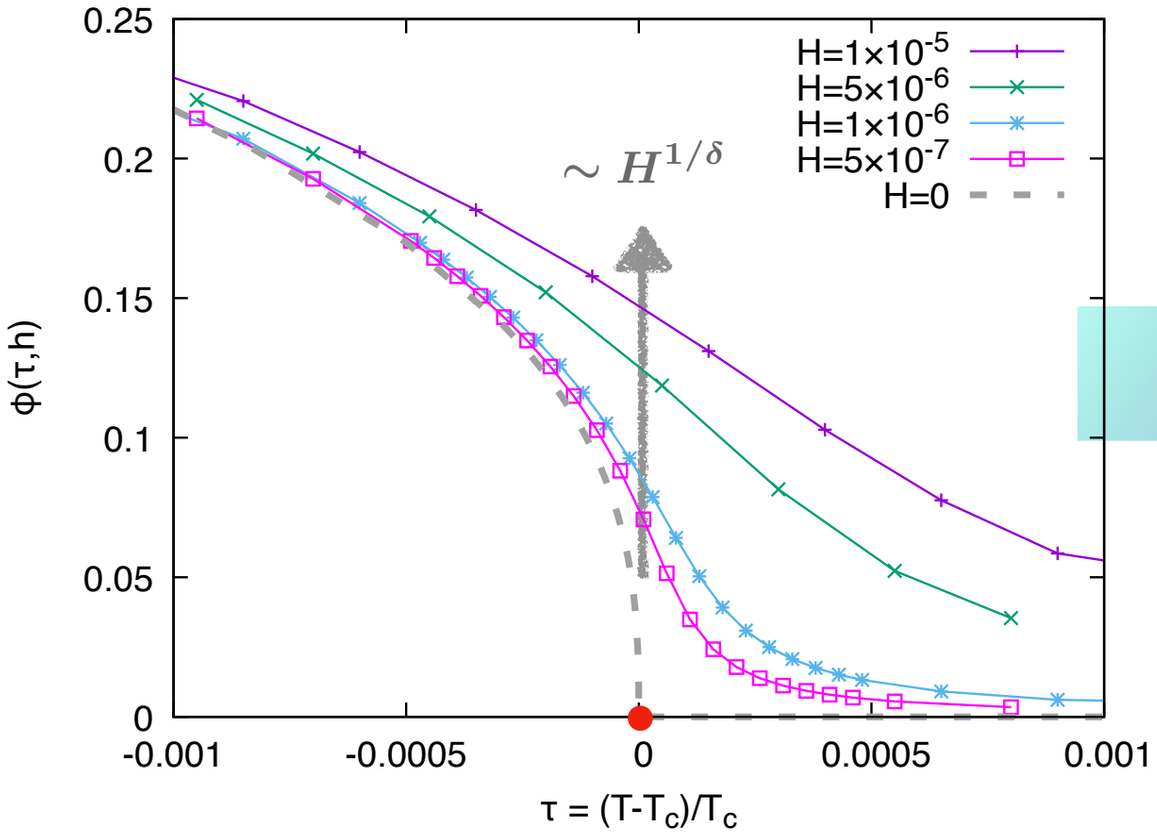
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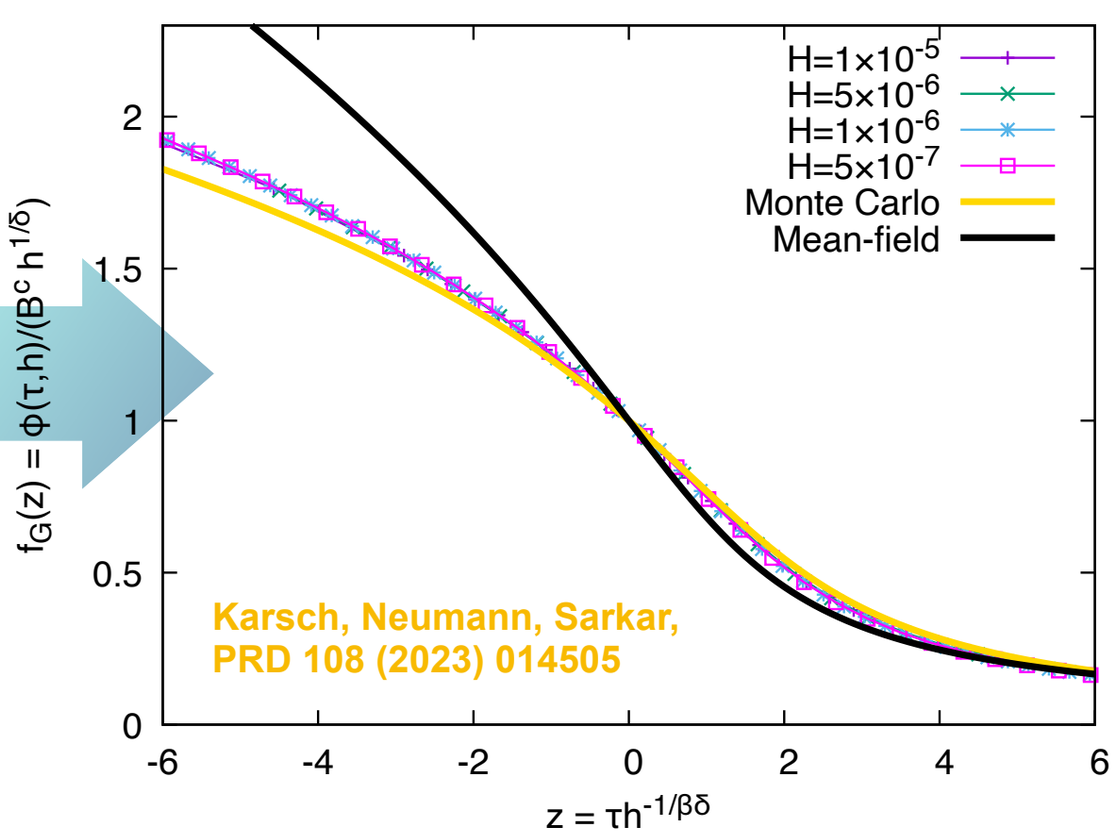
Widom-Griffiths (static) scaling function

Widom (1965), Griffiths (1967)

Scaling Functions

- Benchmark: Magnetic EoS (3d Ising)





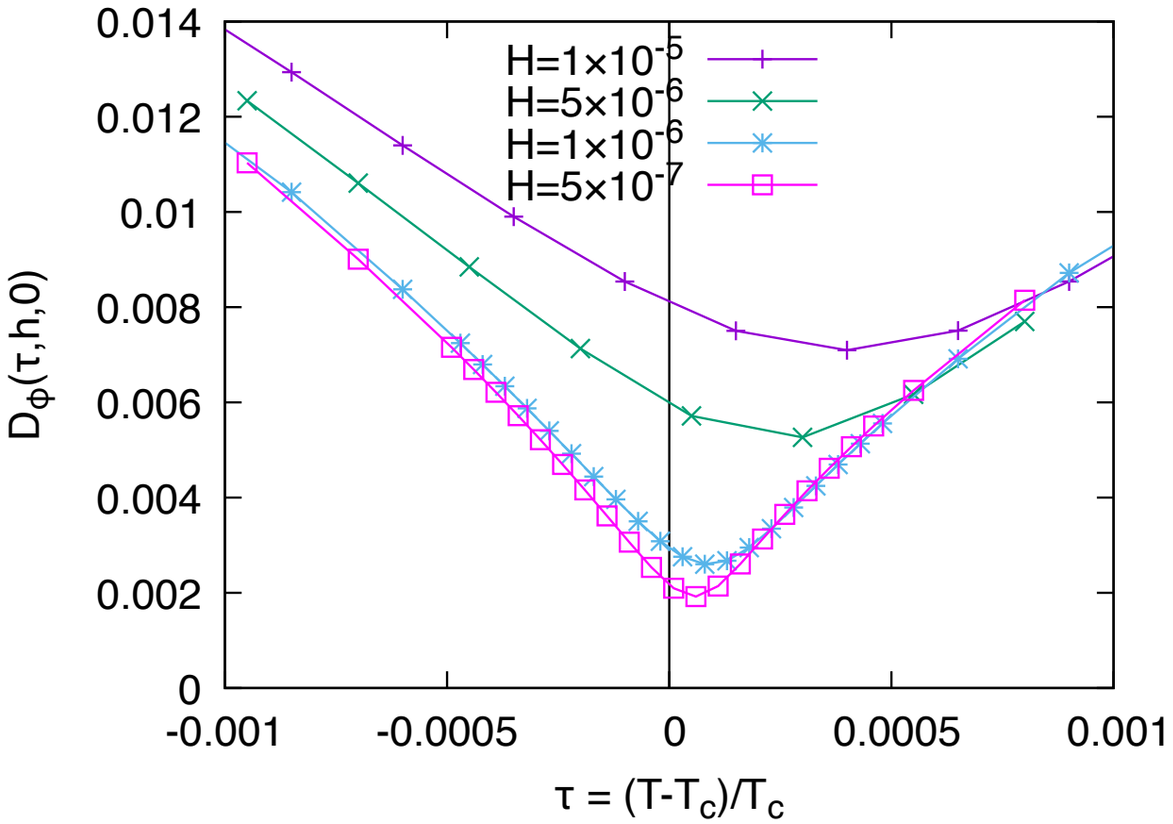
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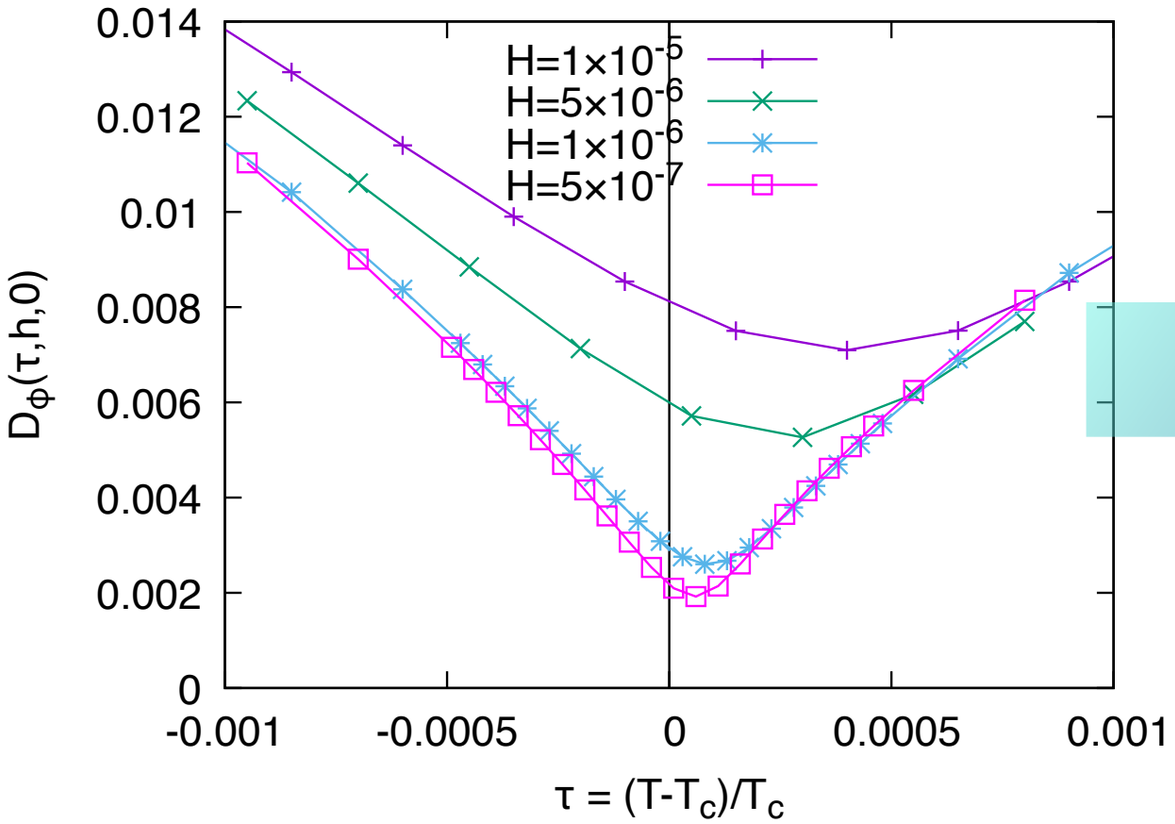
Dynamic Scaling Functions

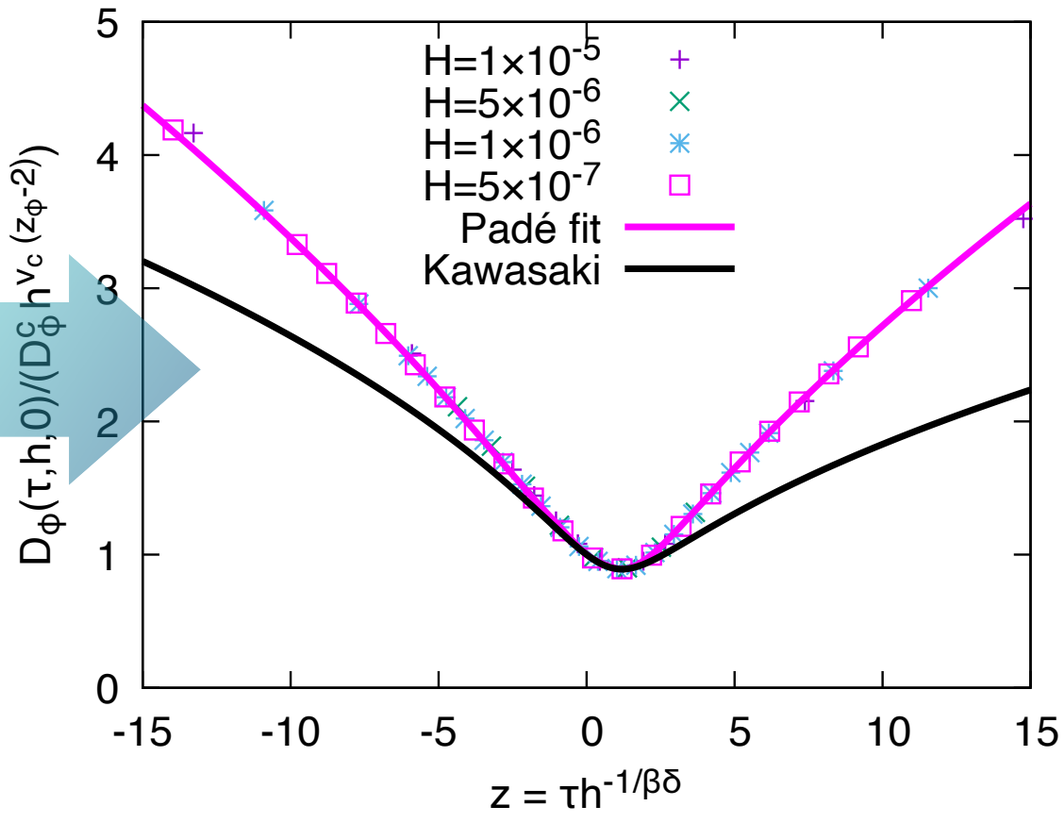
- Magnetic scaling of heat diffusion $D_\phi = \dfrac{\sigma}{\chi_\phi}$



Dynamic Scaling Functions

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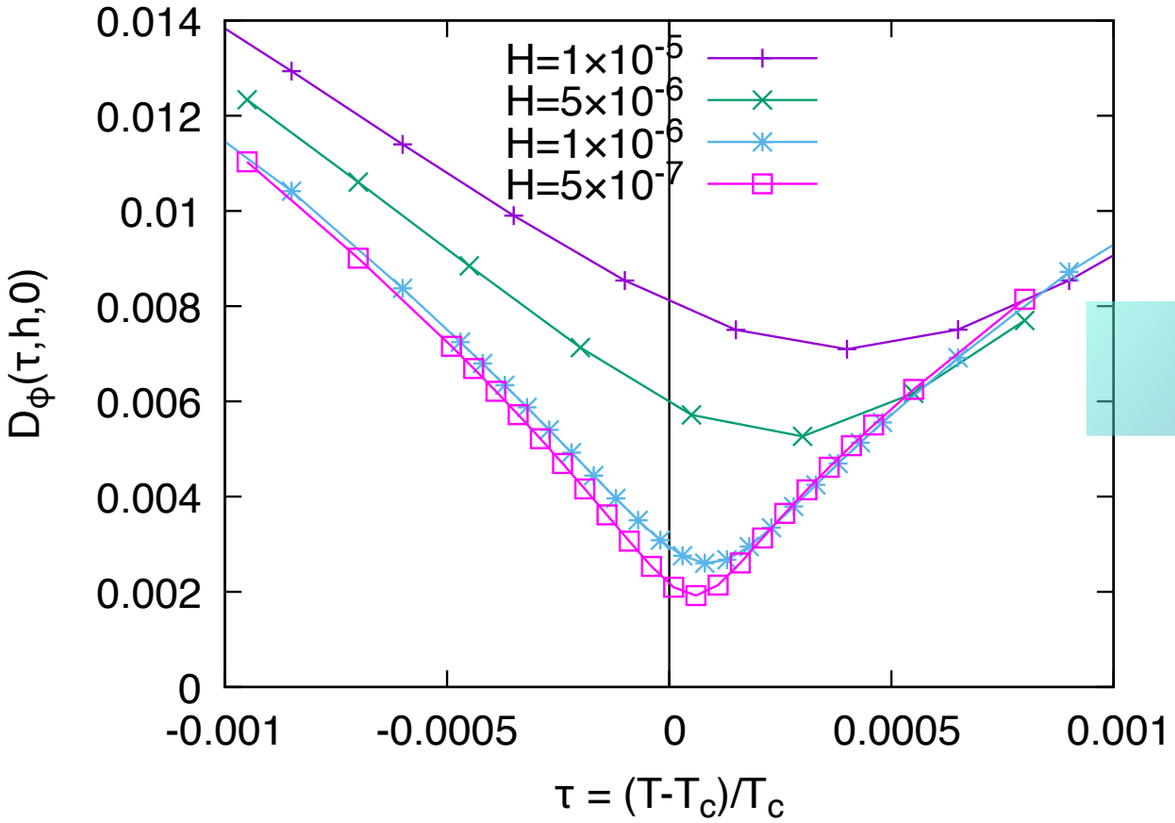
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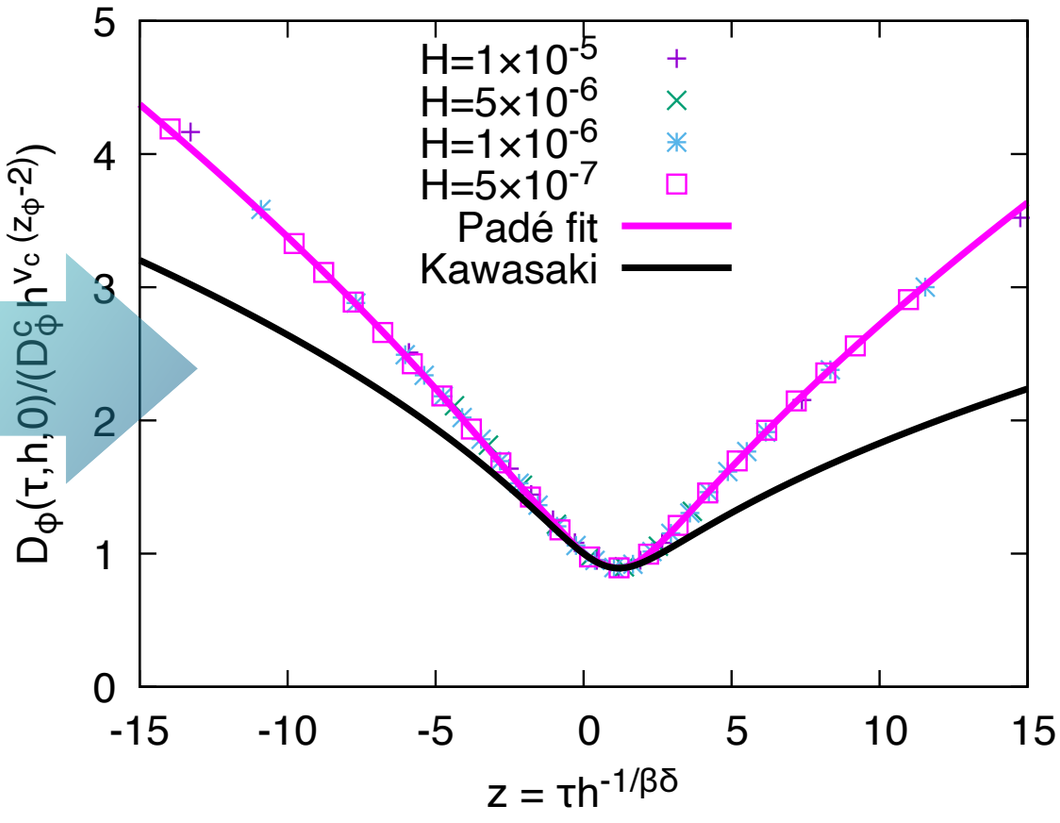
Dynamic Scaling Functions

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$$z = d + x_\eta \approx 3.06$$



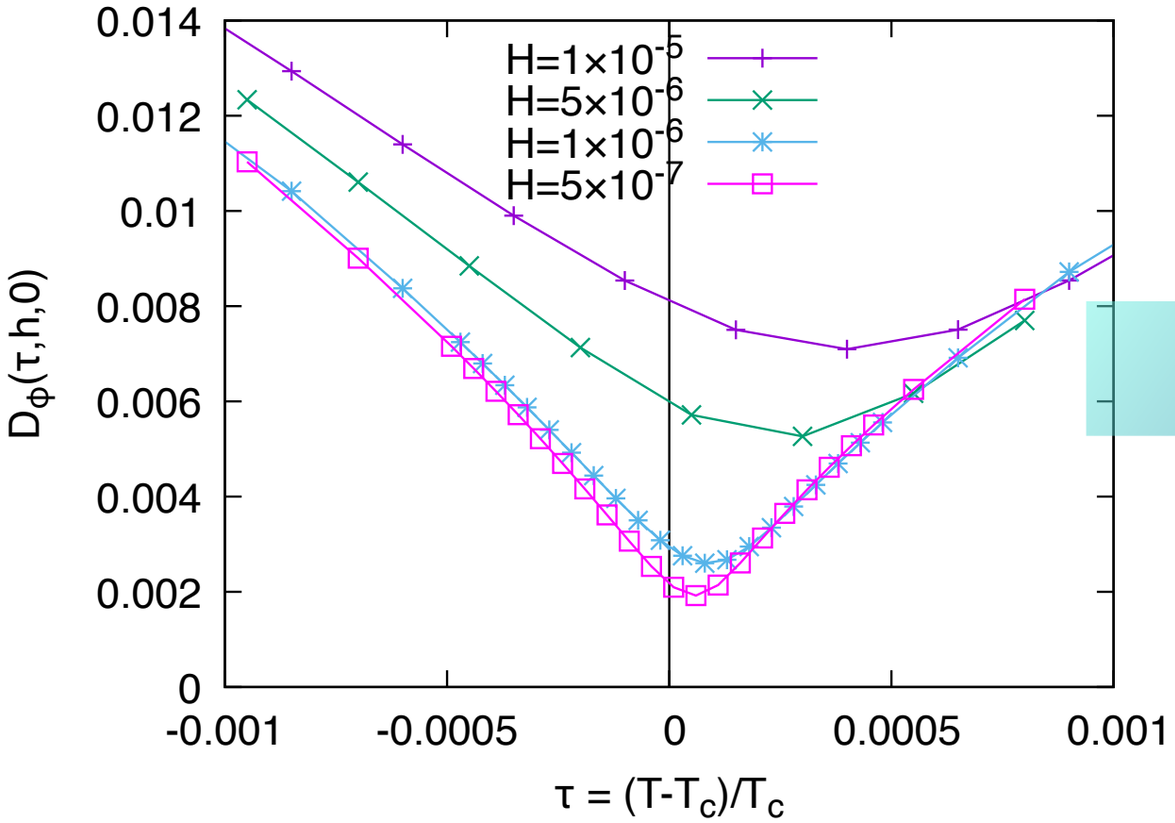


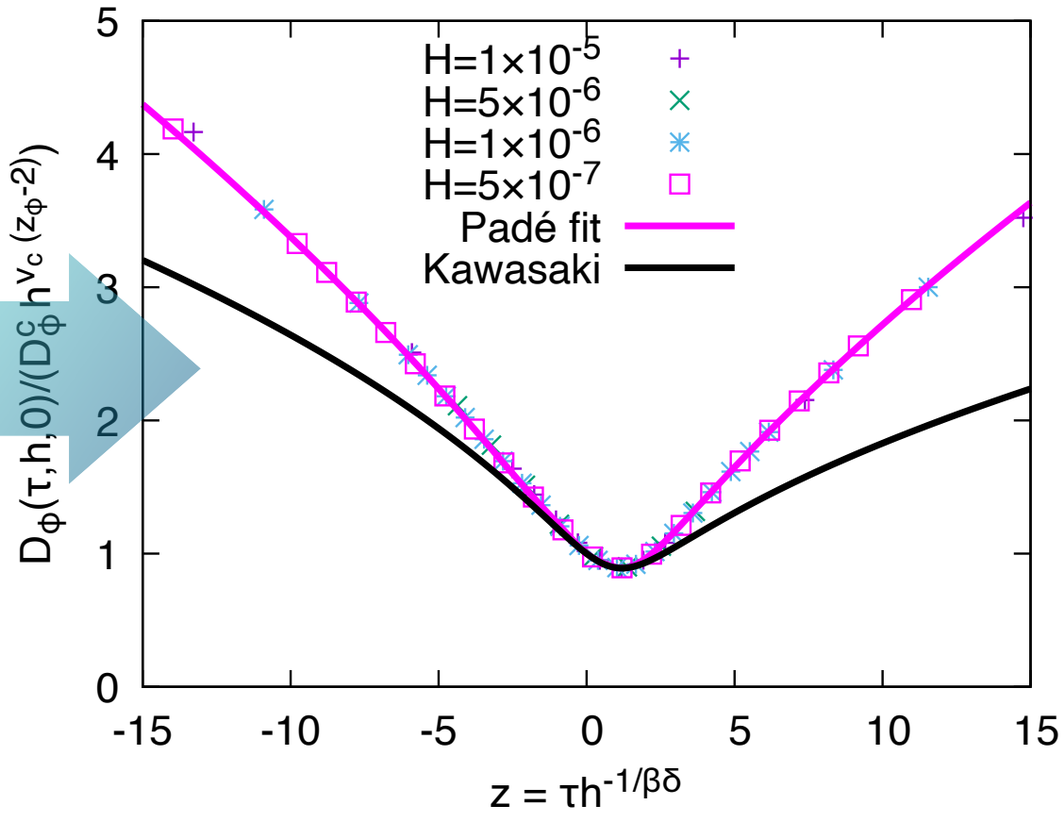
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- Critical Viscosity of Xenon (CVX) experiment: $x_\eta = 0.0690(6)$

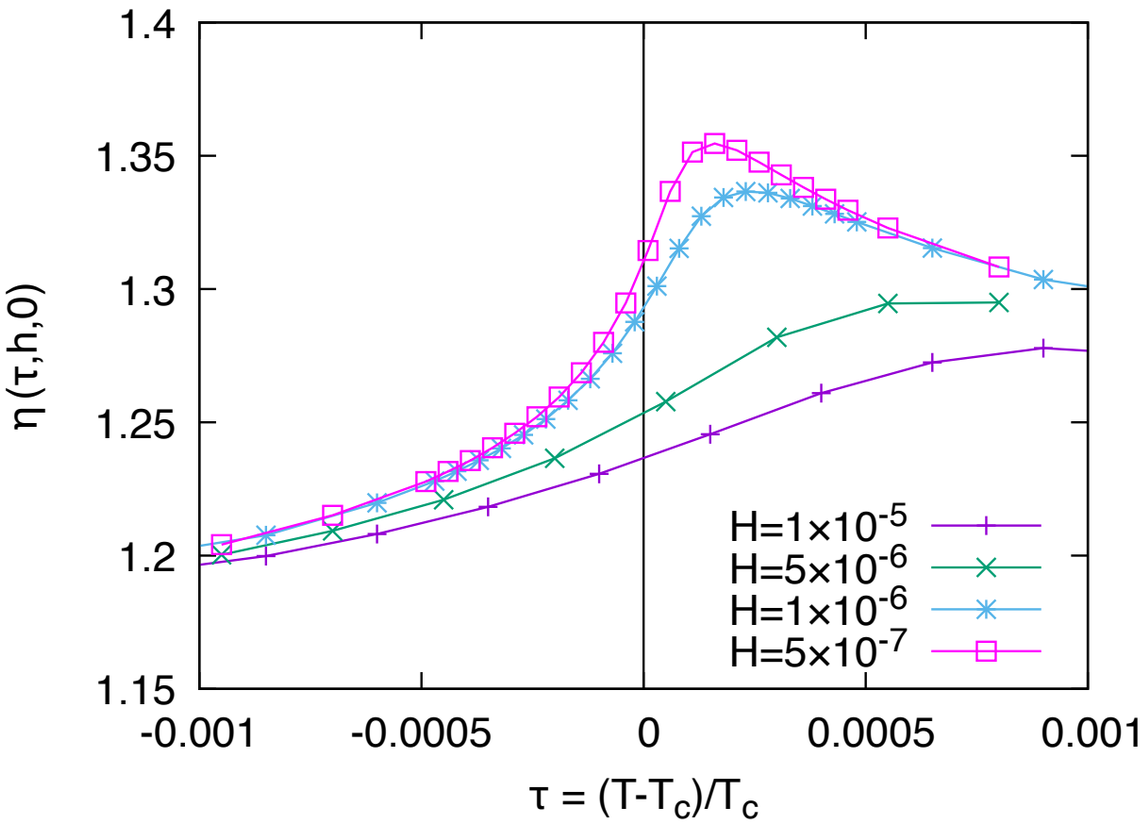
  Berg et al., Phys. Rev. E **60** (1999) 4079 — Discovery

  Berg et al., Phys. Rev. E **77** (2008) 041116 — Columbia

Dynamic Scaling Functions

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

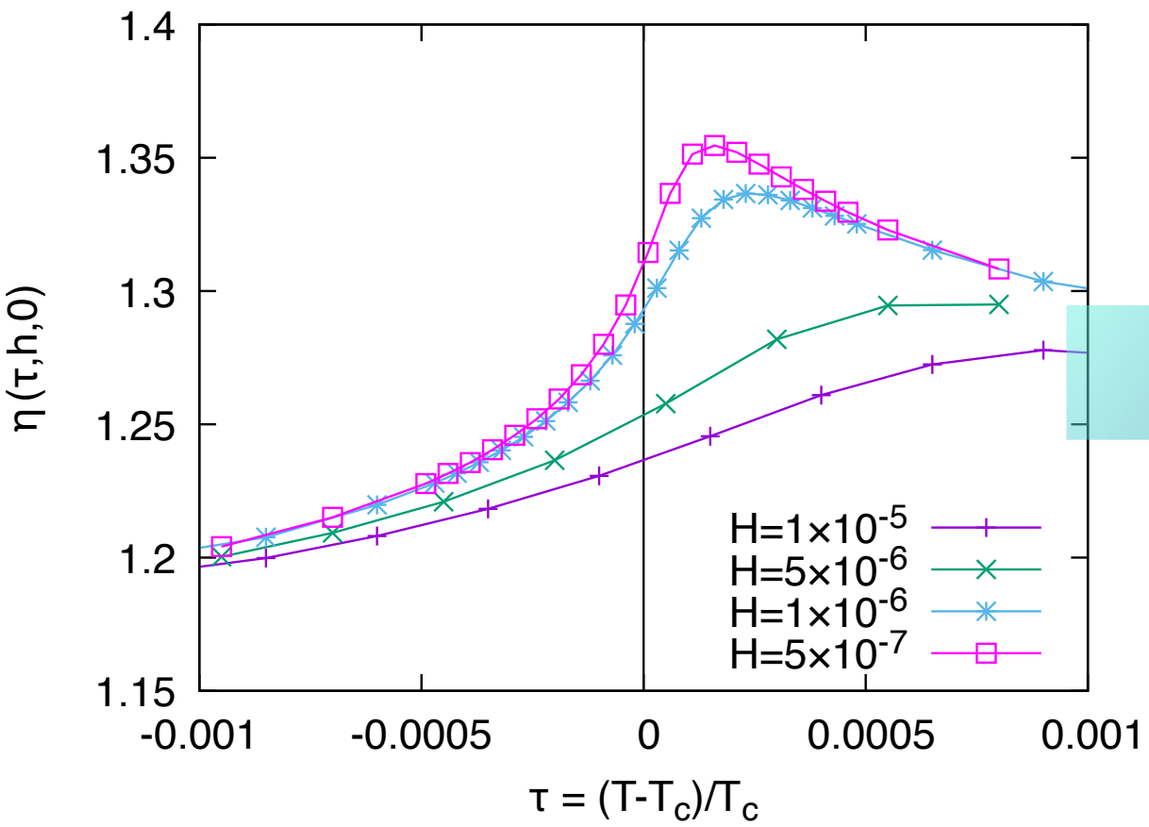
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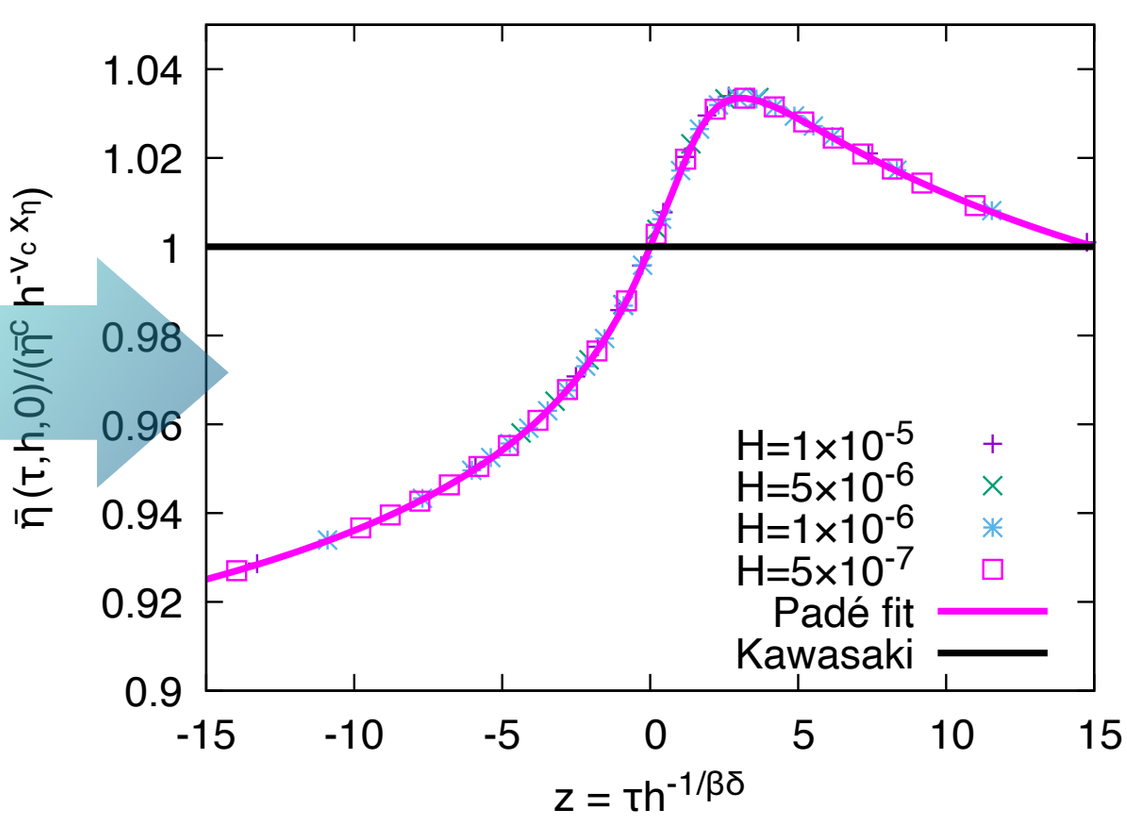


Dynamic Scaling Functions

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

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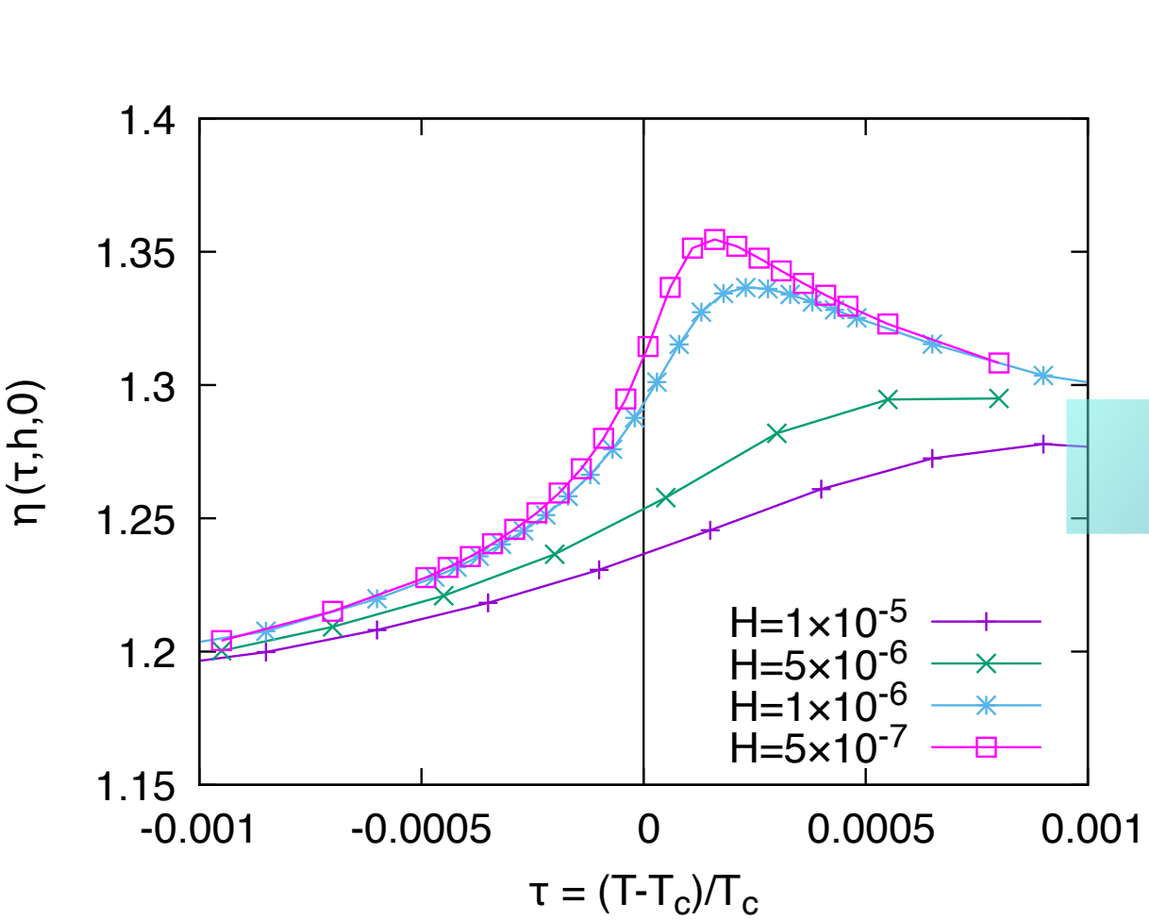


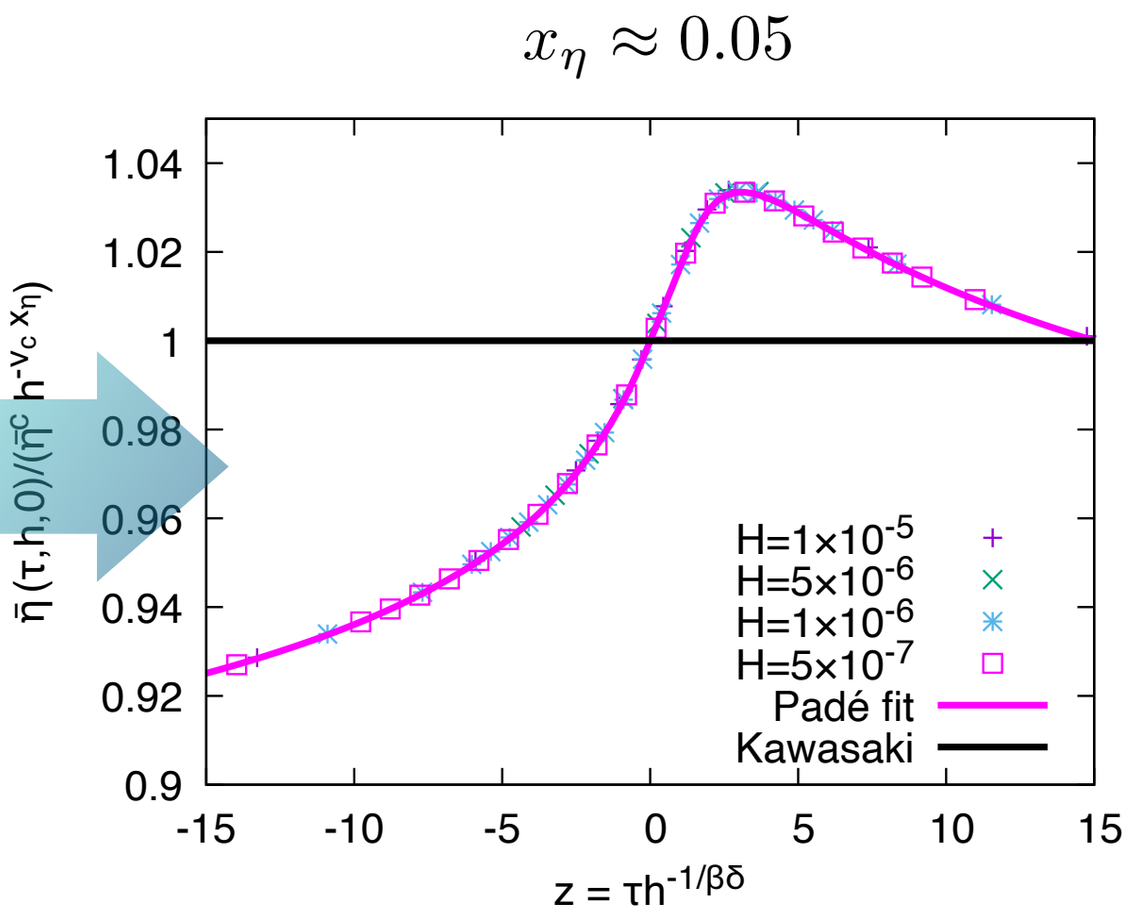
$$\eta(T,H) \propto h^{-\nu x_\eta/\beta\delta} f_\eta(\tau h^{-1/\beta\delta})$$

Dynamic Scaling Functions

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- Magnetic scaling of shear viscosity





$$\eta(T,H) \propto h^{-\nu x_\eta/\beta\delta} f_\eta(\tau h^{-1/\beta\delta})$$

Roth, Ye, Schlichting, von Smekal, arXiv:2603.17874 (2026)

Heat Diffusion: Experimental Data

- Dynamic scaling (wavenumber dependent):

$$D_\phi(\tau, p) = \frac{g^2 T_c p}{6\pi\eta(\tau)}\,\Gamma^*(p\xi(\tau))$$

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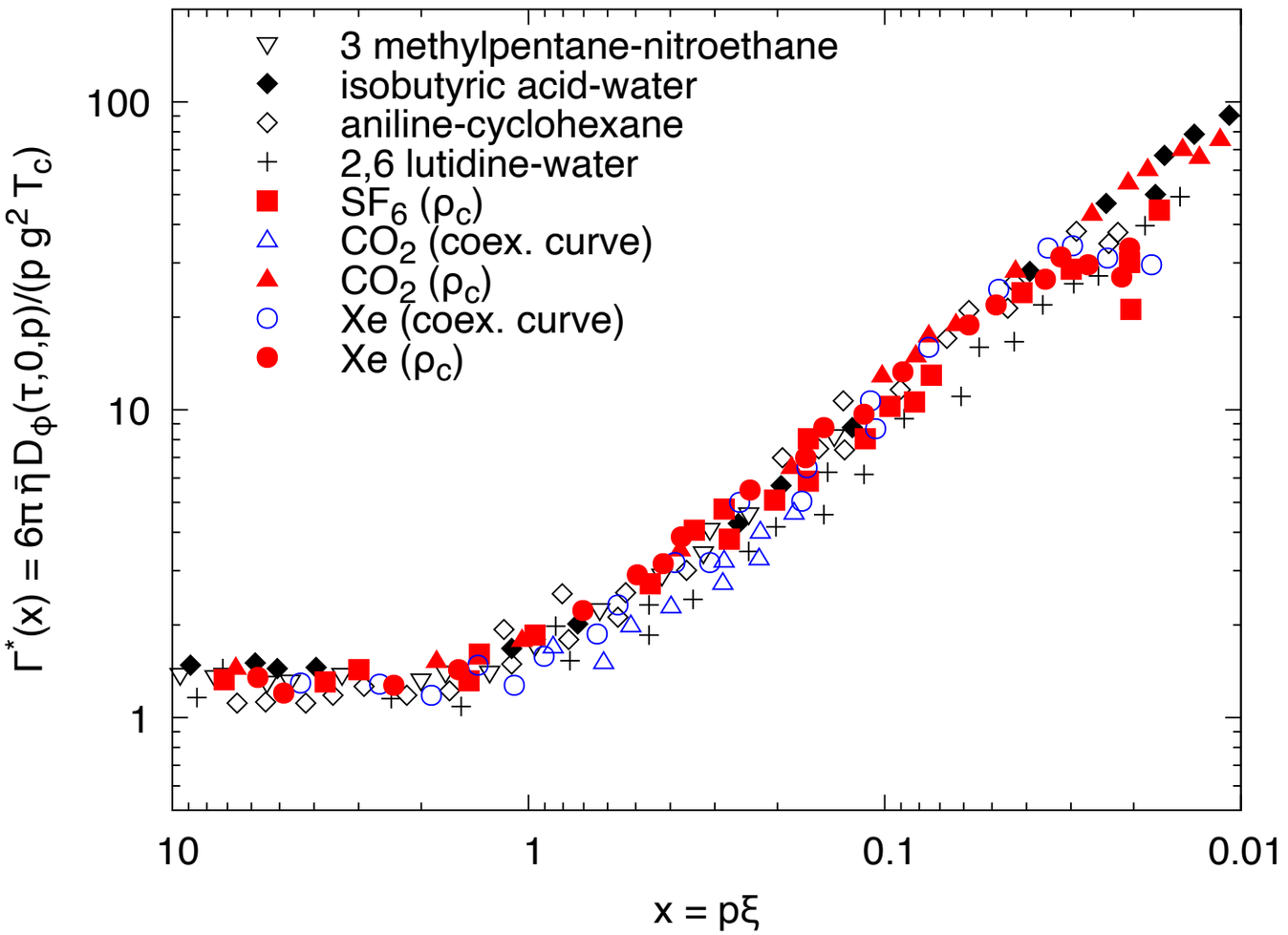
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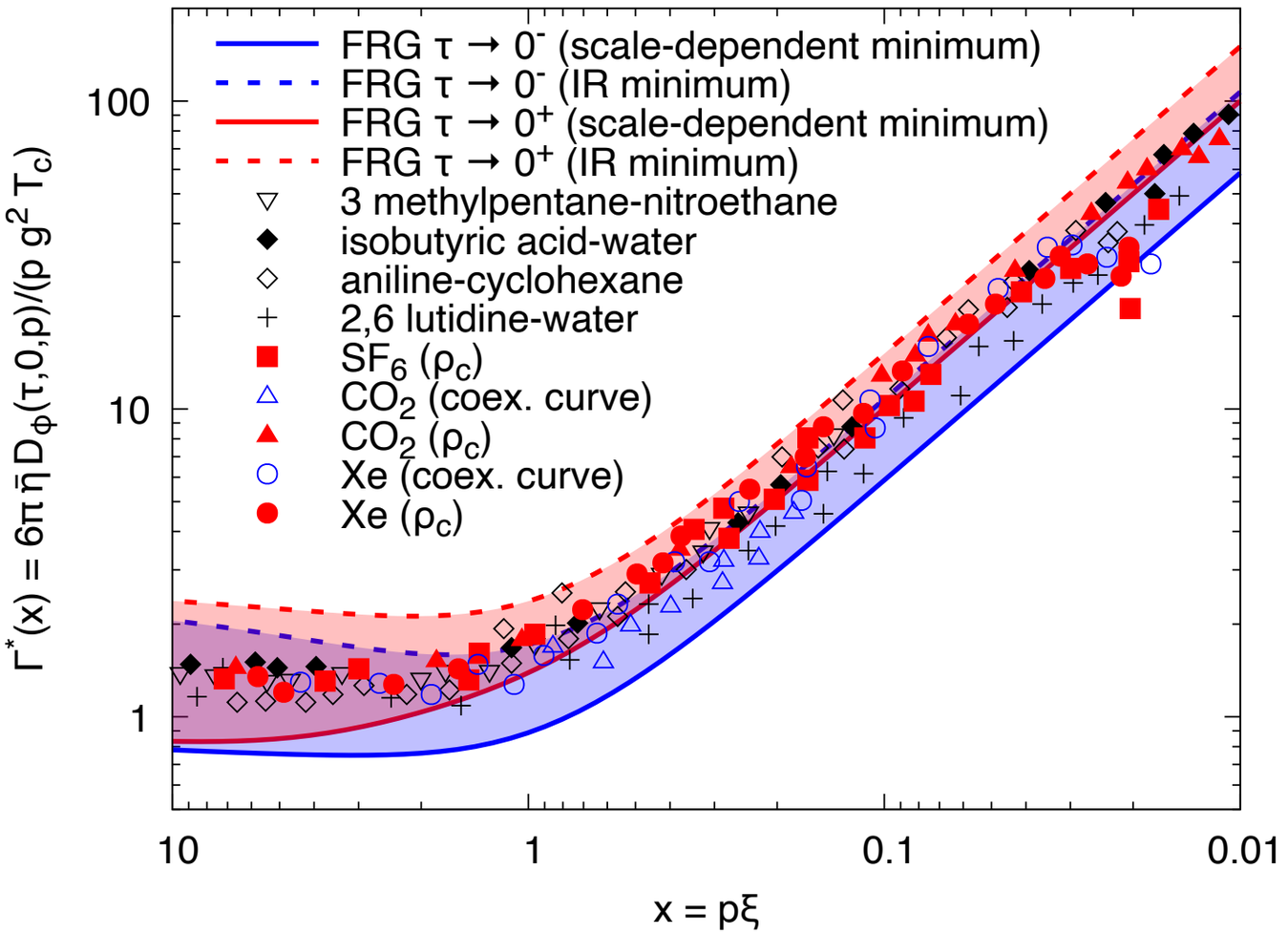
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- Experimental data for $\Gamma^*(x)$

Swinney, Henry, Phys. Rev. A **8** (1973) 2586

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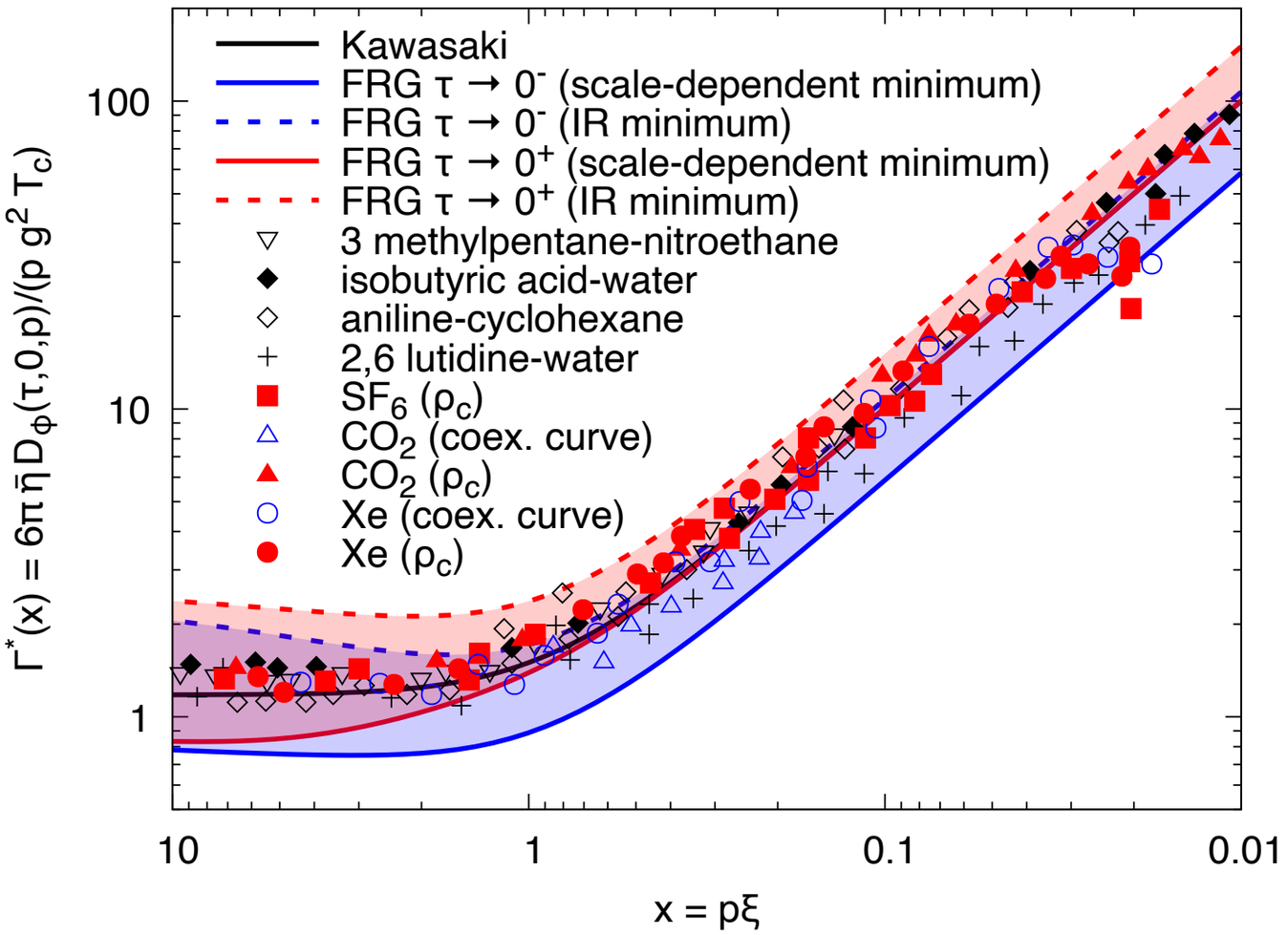
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- Kawasaki approximation:

$$\Gamma^*(x) = \frac{3}{4x^3}(1 + x^2 + (x^3 - x^{-1})\arctan x)$$

$$\rightsquigarrow\; R^\pm = \frac{1}{6\pi}$$

Kawasaki, Ann. Phys. **61** (1970) 1

Summary and More

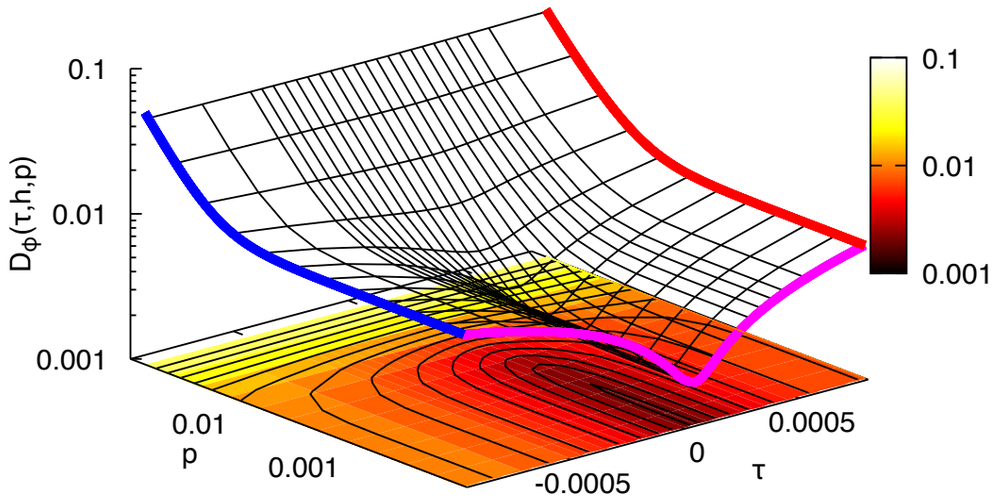
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Universal critical dynamics near:

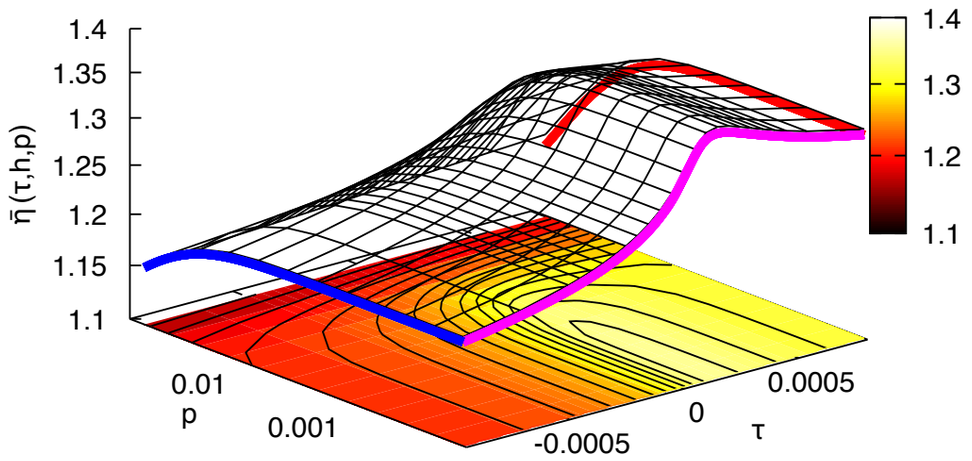
1. QCD critical point

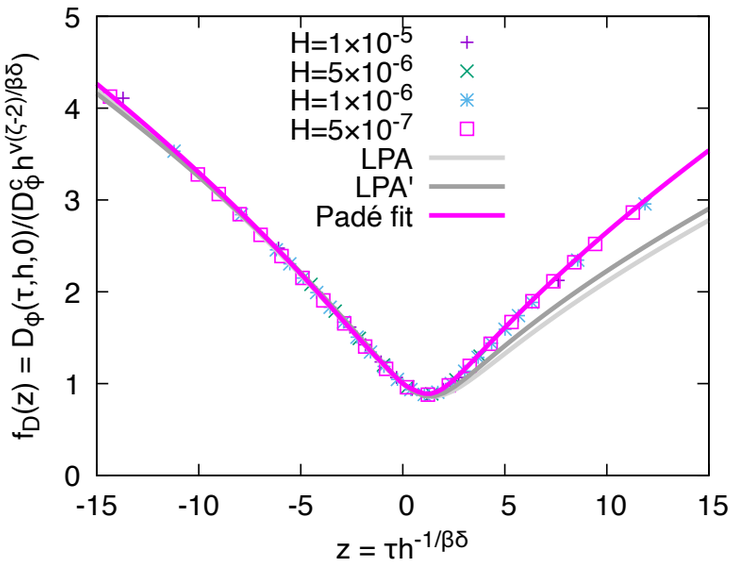
Roth, Ye, Schlichting, von Smekal, Phys. Rev. D **111** (2025) L111901; arXiv:2603.17874

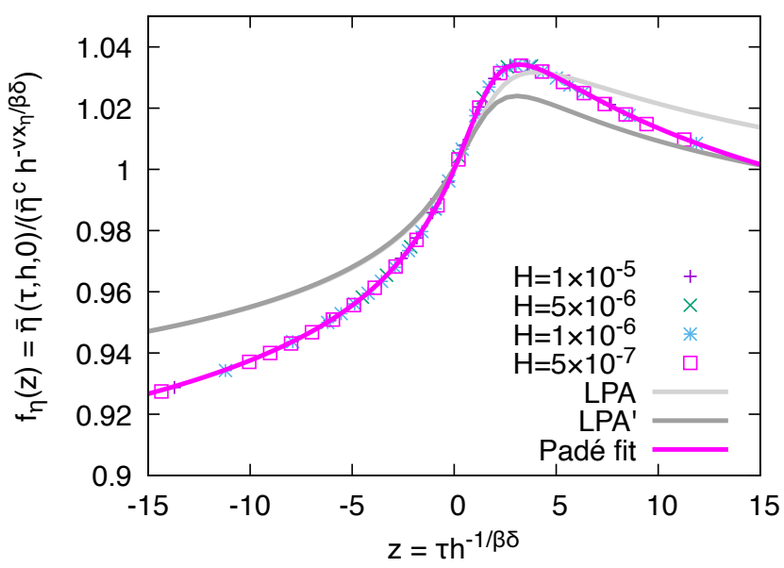
heat diffusion



shear viscosity





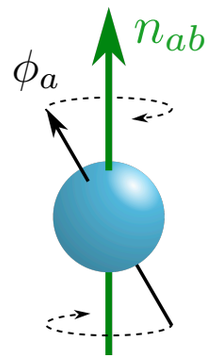


Summary and More

2. chiral phase transition

Roth, Ye, Schlichting, von Smekal, JHEP **01** (2025) 118

reversible mode couplings ↔ Larmor precession



m_π

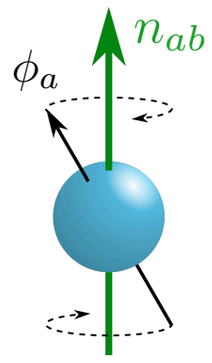
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Model G

$z = d/2$

$O(4)$ Heisenberg **antiferro**magnet

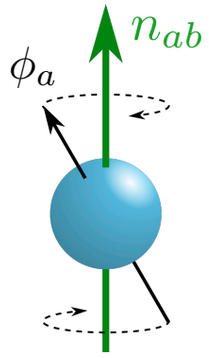
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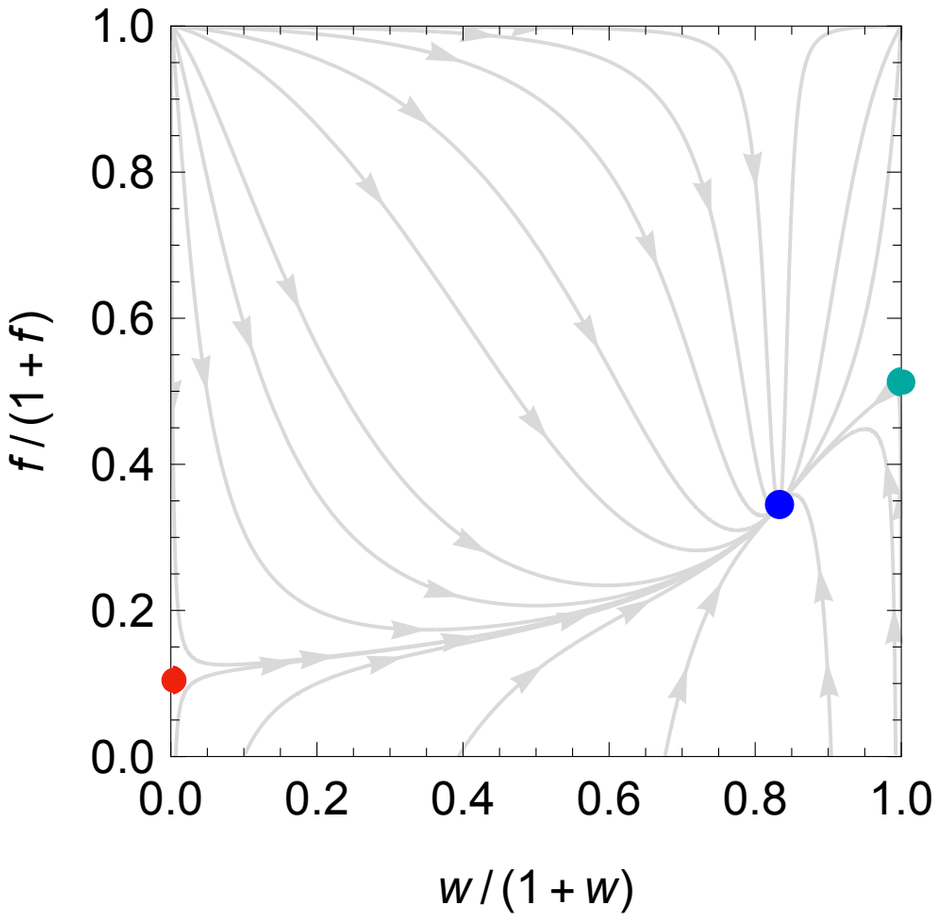
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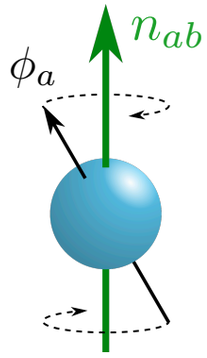
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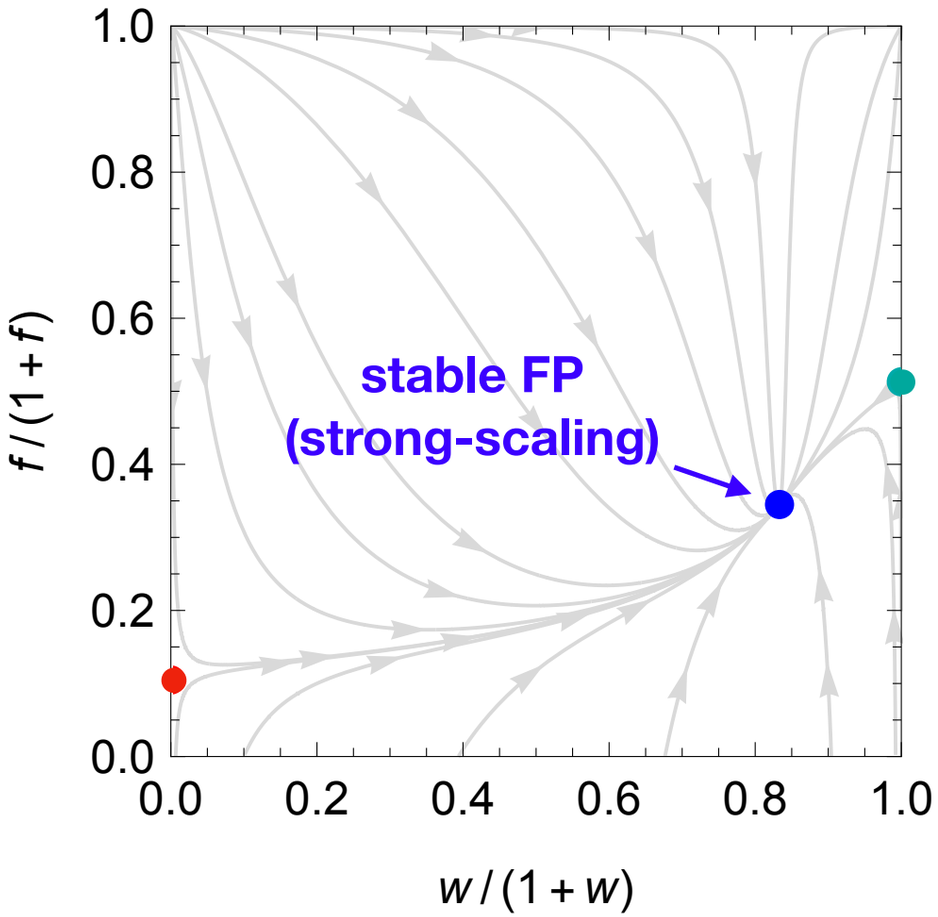
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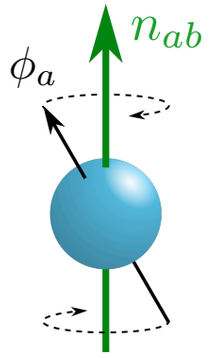
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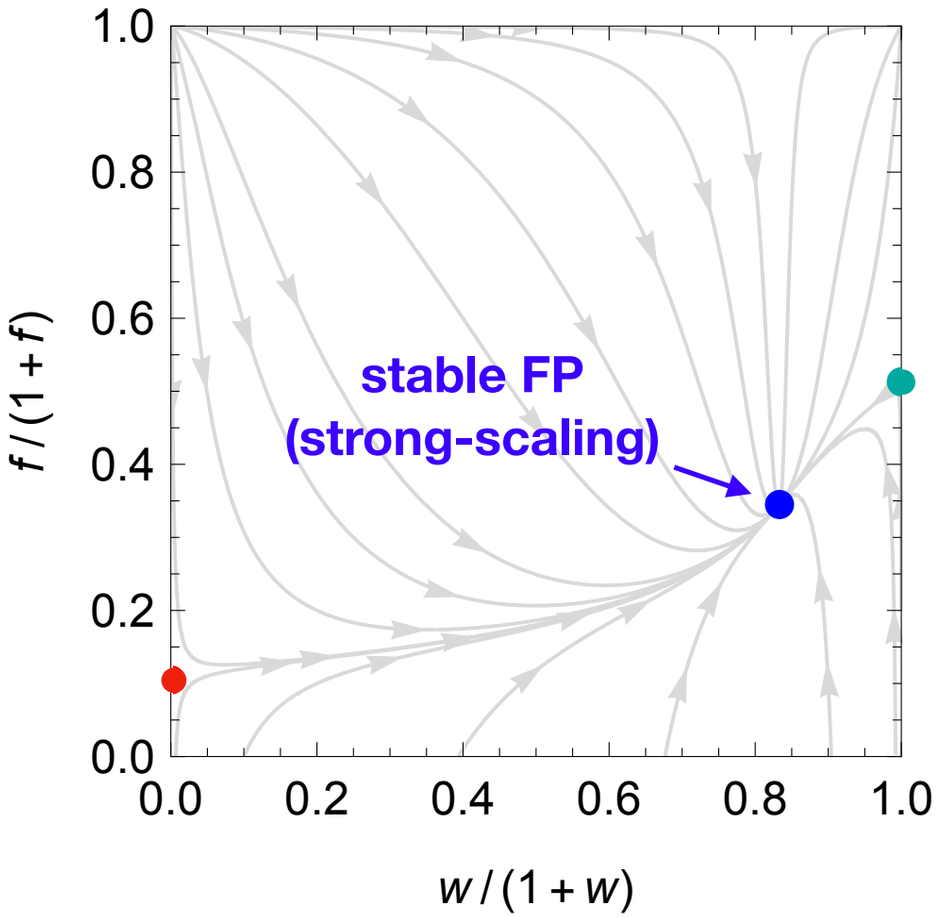
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$$\Longrightarrow \quad z_\phi = z_n = d/2 \quad \text{(exact)}$$

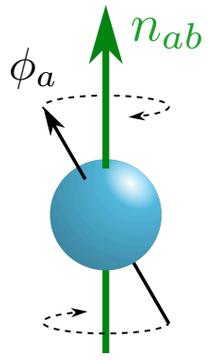
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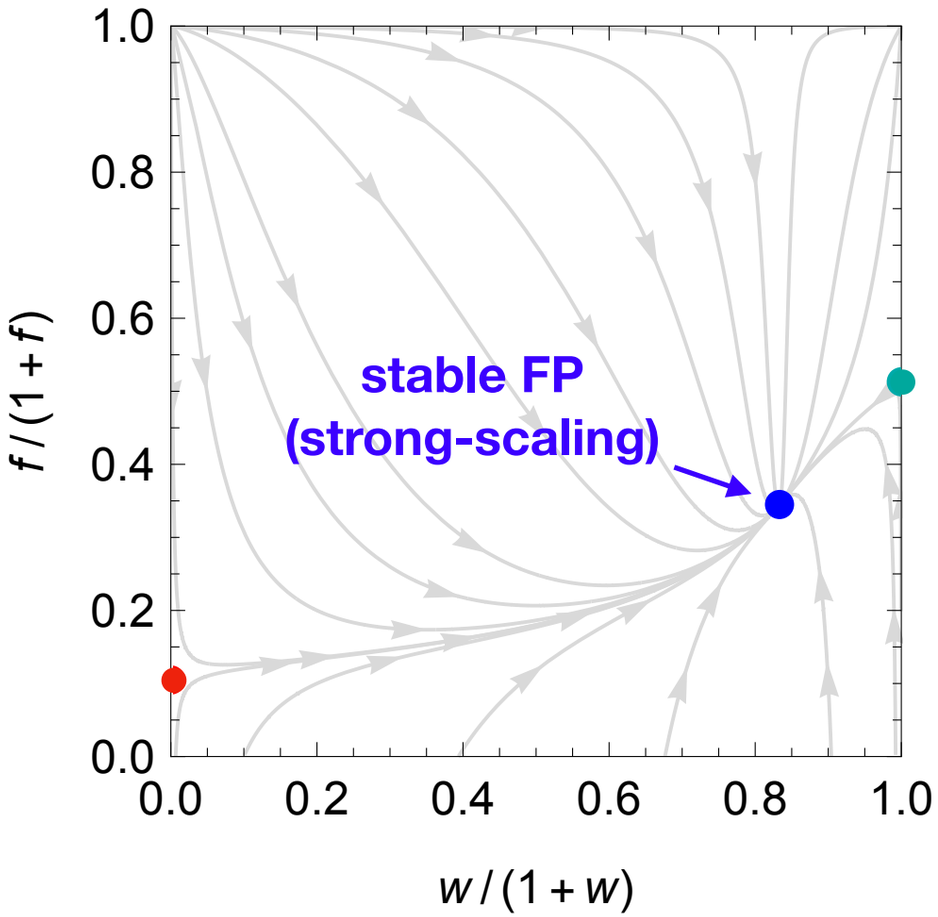
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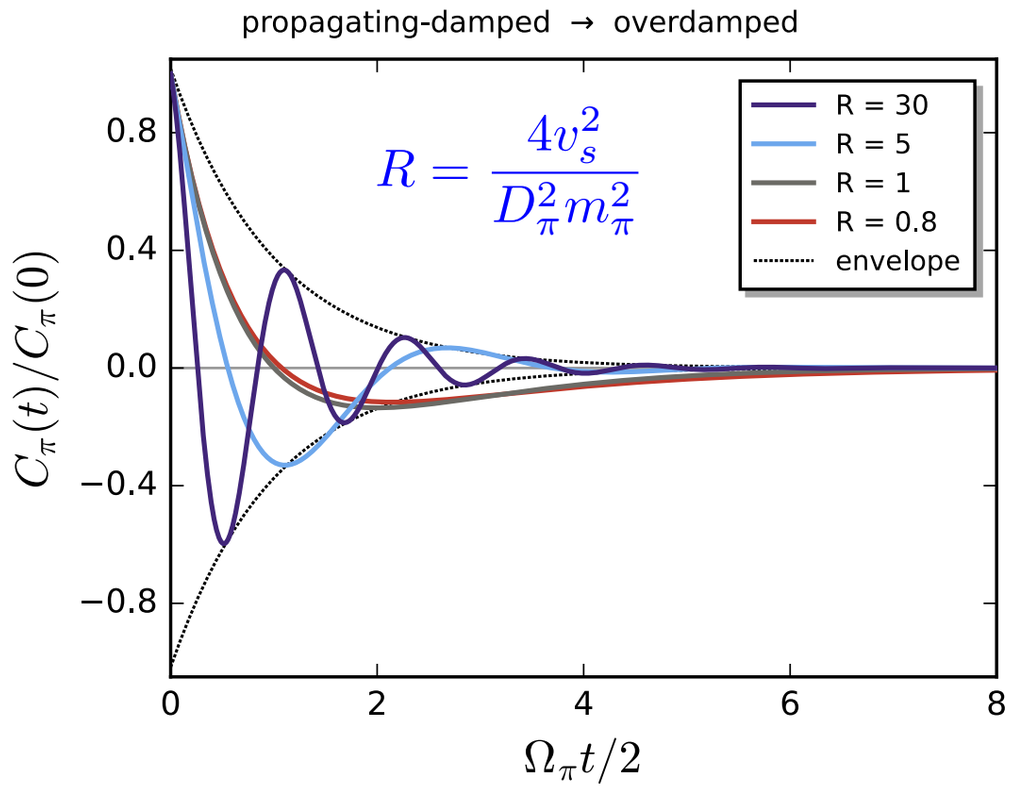
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$\Longrightarrow$ $z_\phi = z_n = d/2$ (exact)

- pions below T_c :

Yong-rui Chen, Wed. 16:45



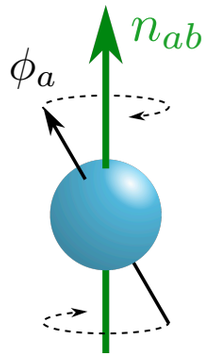
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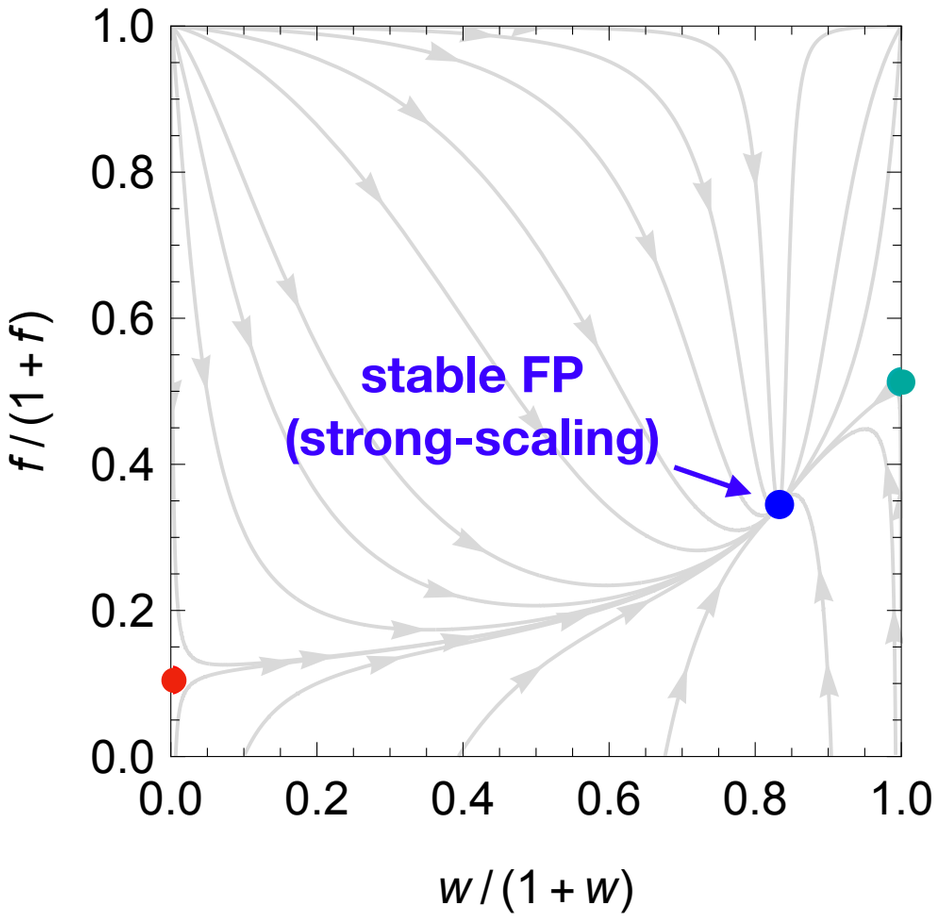
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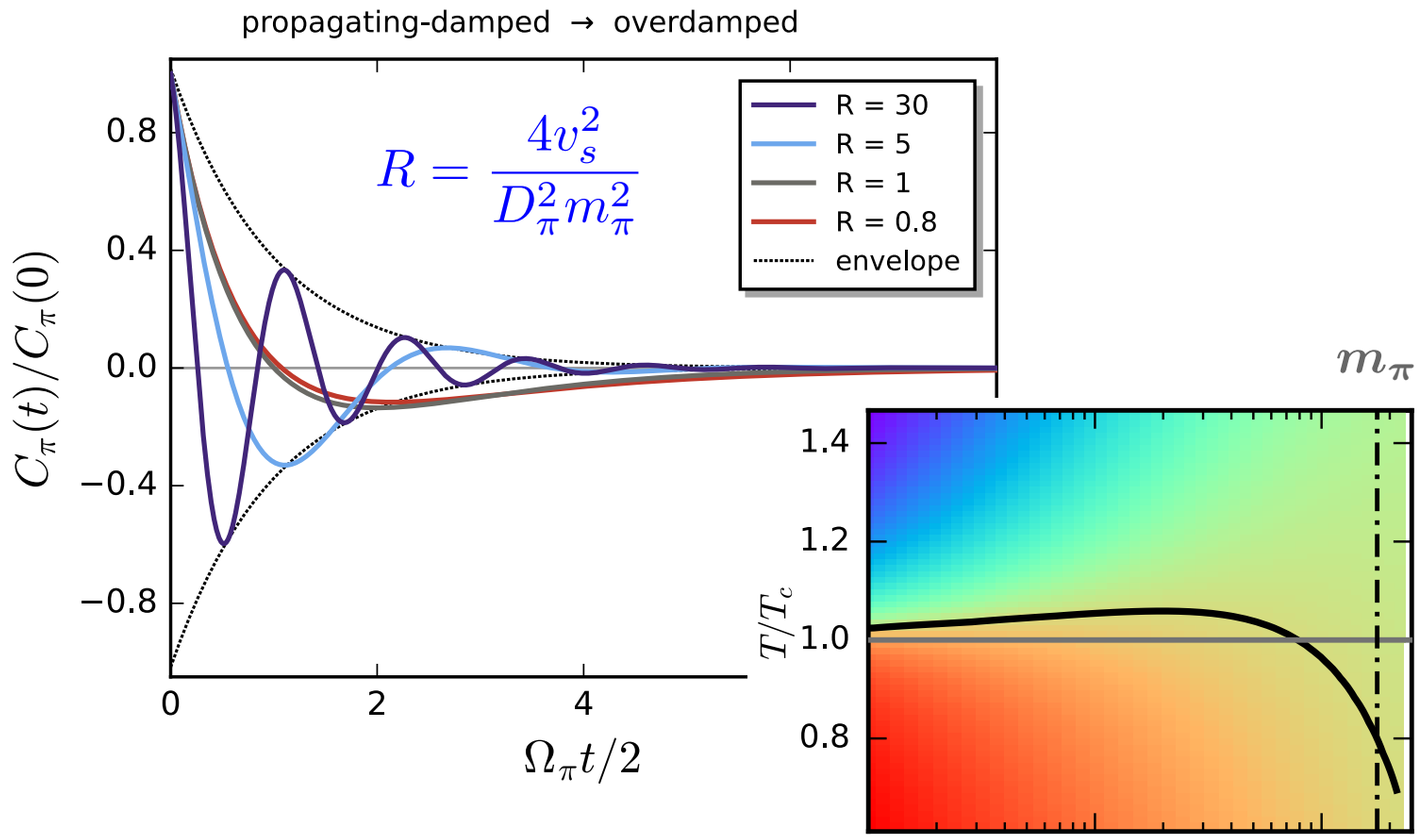
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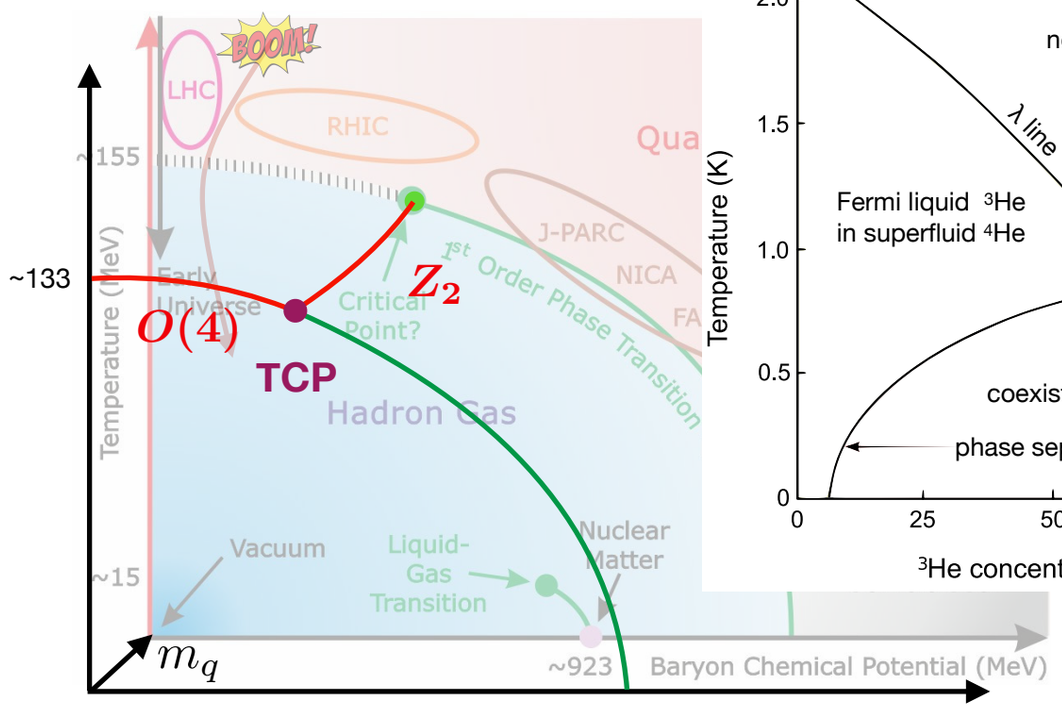
What I did not talk about:

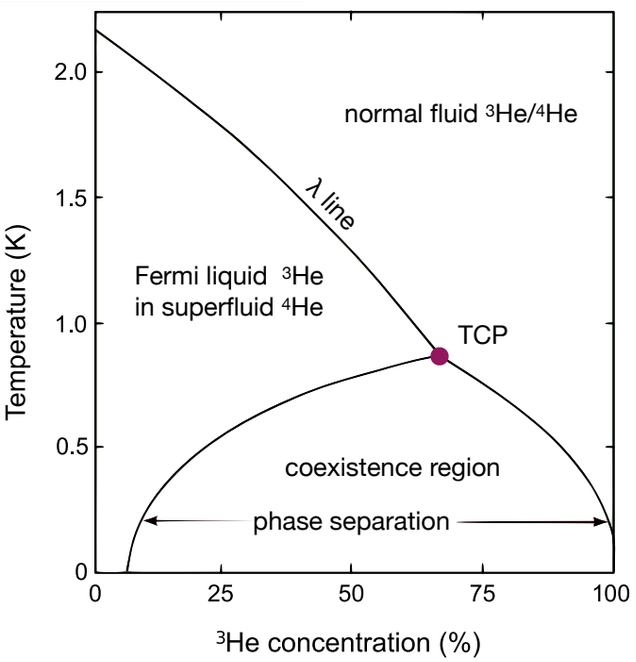
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Connect the two:

3. tricritical dynamics in QCD and ^{3}He-^{4}He mixtures

   Pradeep, Roth, von Smekal, arXiv:2608.11065





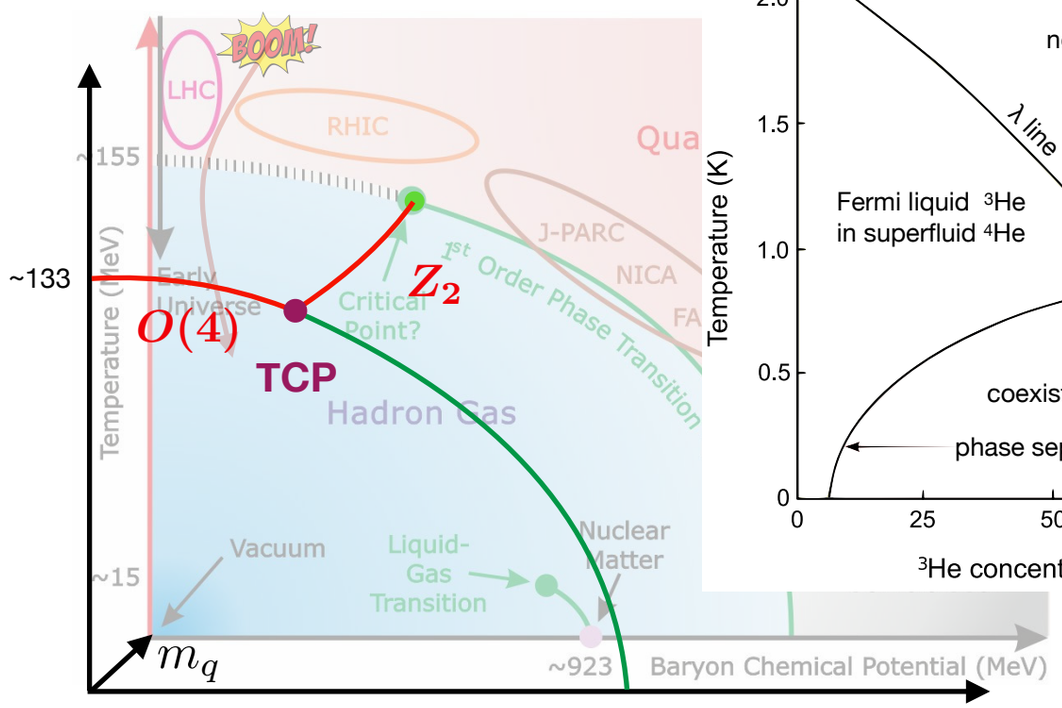
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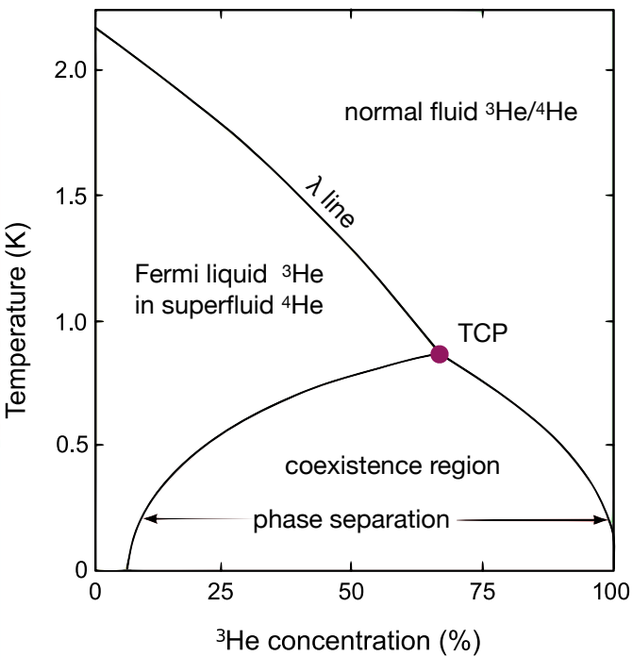
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   for dynamics near the QCD critical point

Mattis Harhoff, Tue. 16:15

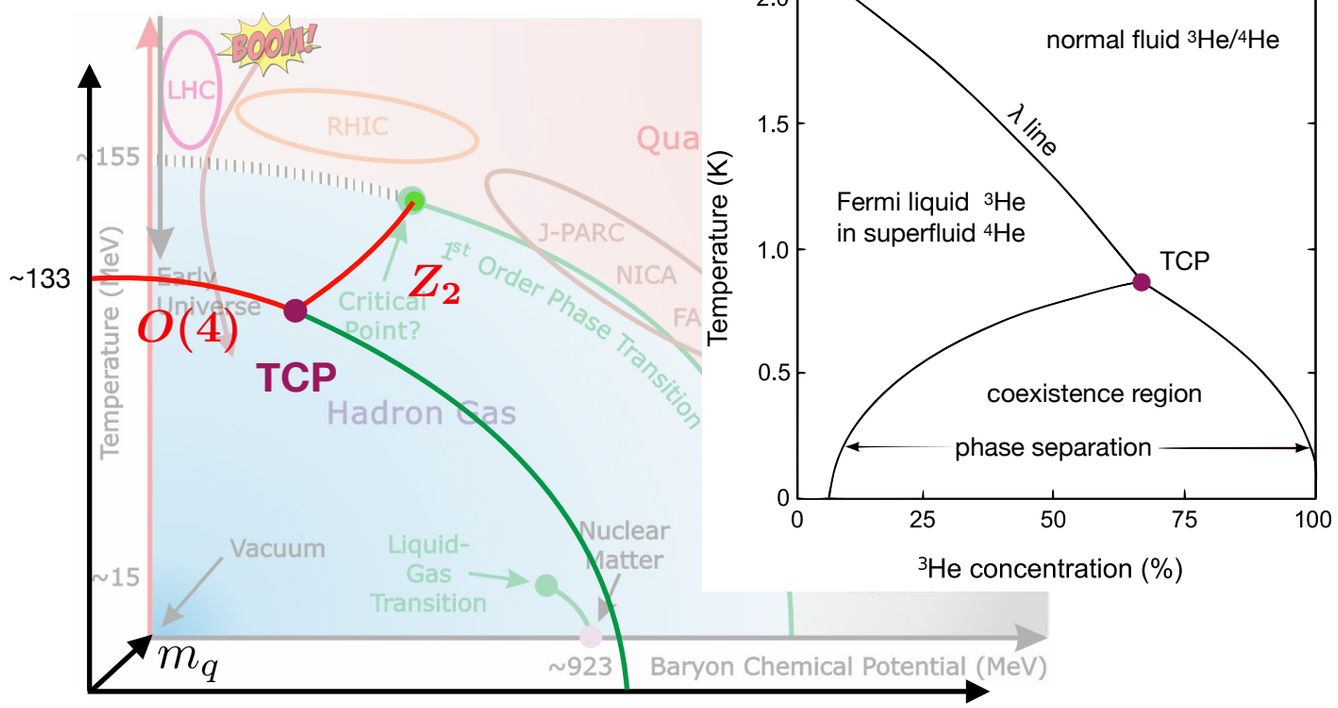
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5. Non-equilibrium phase transitions and finite-time scaling

   Kibble-Zurek in transcritical quenches:

   Sieke, Harhoff, Schlichting, von Smekal, NPB **1011** (2025) 116808

   non-equilibrium scaling across first-order transitions:

   Sieke, Fuchs, von Smekal, NPB **1029** (2026) 117585

   Leon Sieke, Tue. 16:45

The End

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Thank You!