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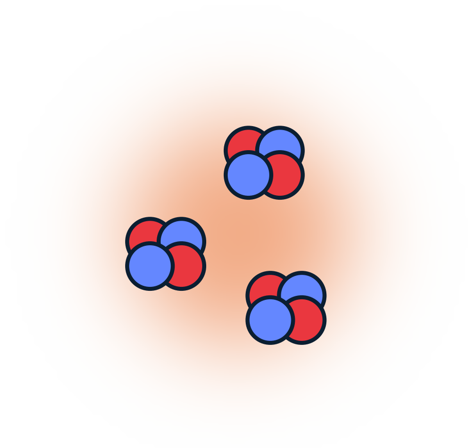
USTC Jianwei Seminar

Exotic Cluster Structures in Light Nuclei



Bo Zhou (周波)

Fudan University

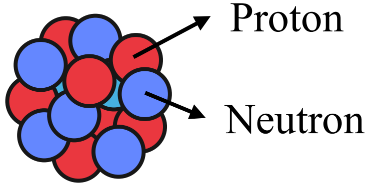
Jan. 16, 2026@USTC

Outline

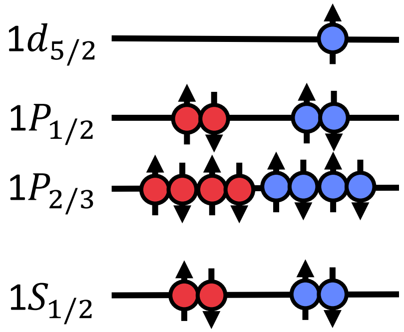
- **Introduction:** Cluster and Model
- **Nα nuclei**: From the Holy state to Condensate State
- **Nα+X nuclei**: Novel clustering structure in nuclei
- **Summary and Prospect**

Nuclear Cluster Physics

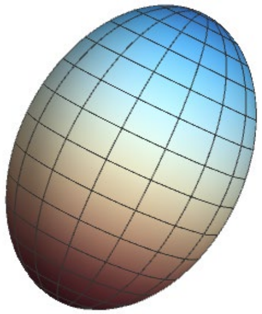
Nuclear many-body problem



$$H\,|\Psi\rangle = E\,|\Psi\rangle$$

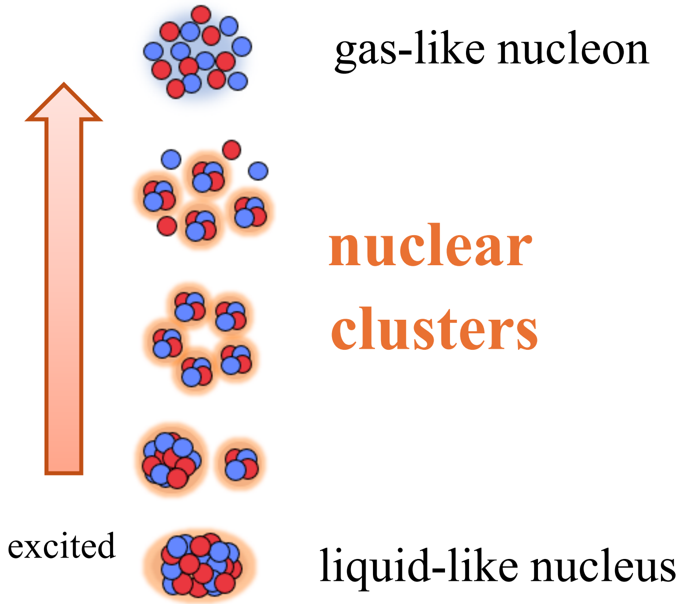


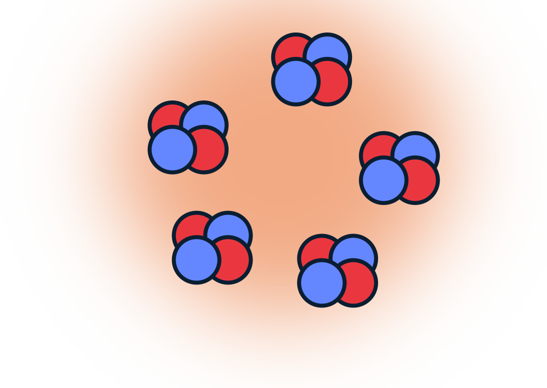
single-particle shell model



collective model

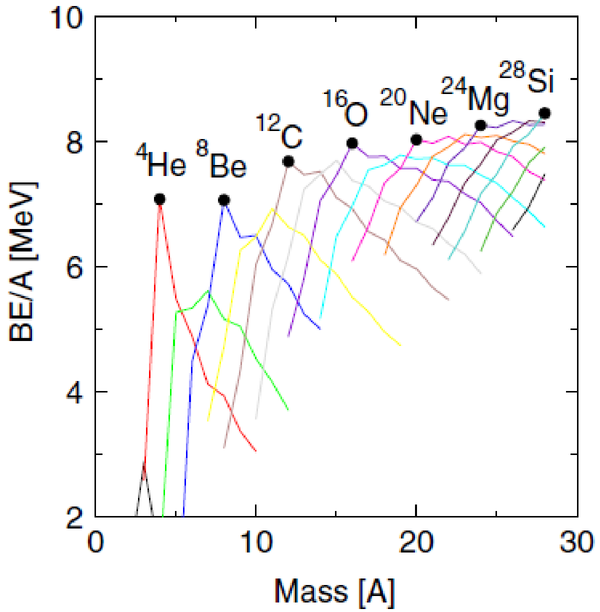
A "phase transition" can occur ,

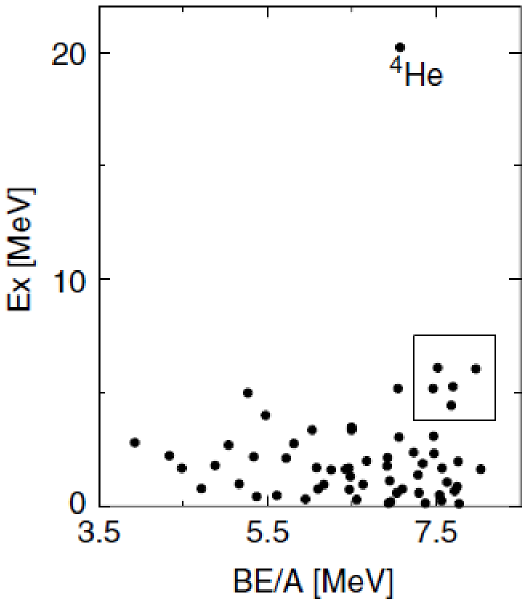


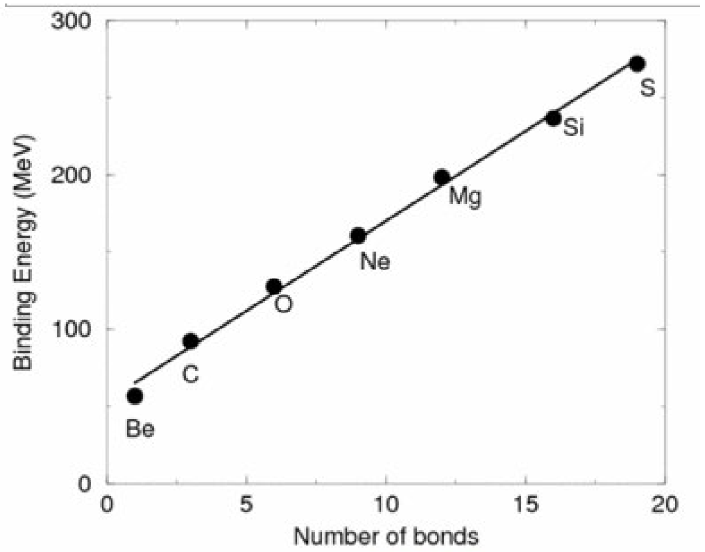


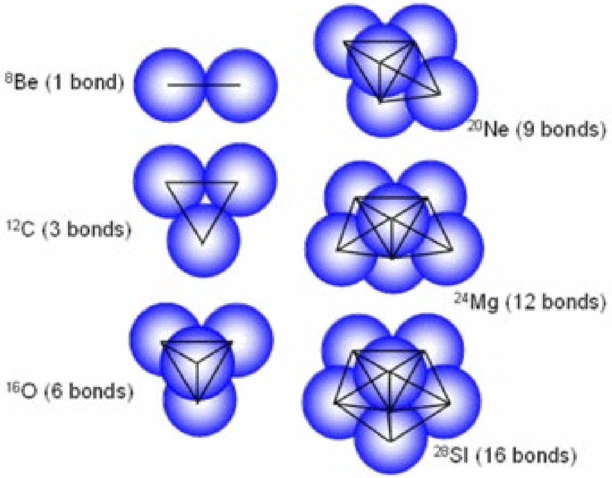
nuclear **cluster** structure

Early developments for clustering in nuclei









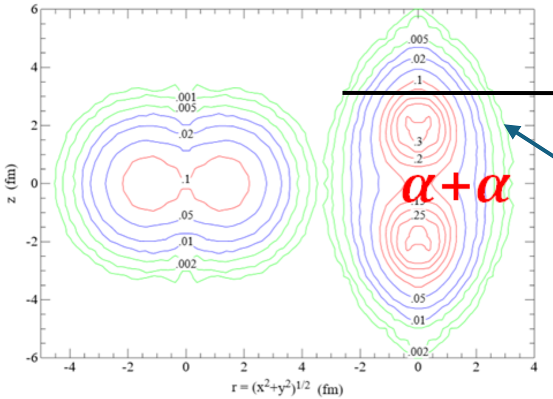
The lines show common elements. The maxima for a given isotope chain coincides with nuclei with even (and equal) numbers of protons and neutrons.

Excitation energy of first excited states plotted versus binding energy per nucleon for nuclei up to A = 20. Good clusters should have both high binding energies and first excited states.

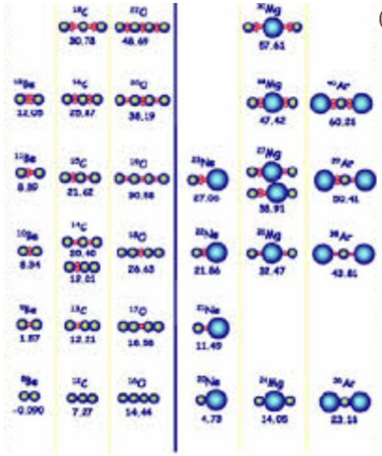
L. R. Hafstad and E. Teller, Phys. Rev. 1938

M. Freer (2010), Scholarpedia, 5(6):9652.

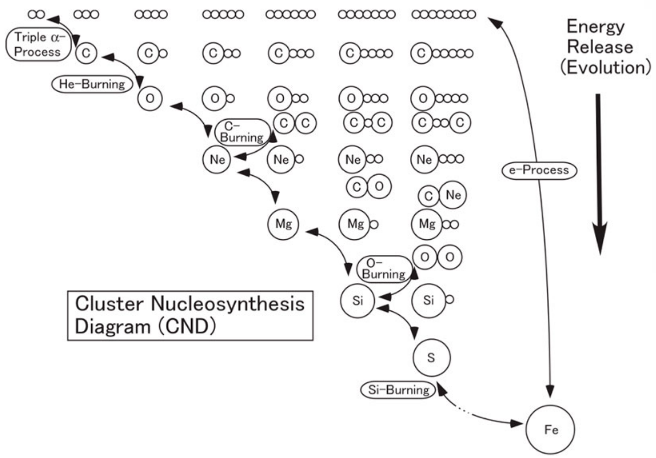
Ikeda diagram for light nuclei



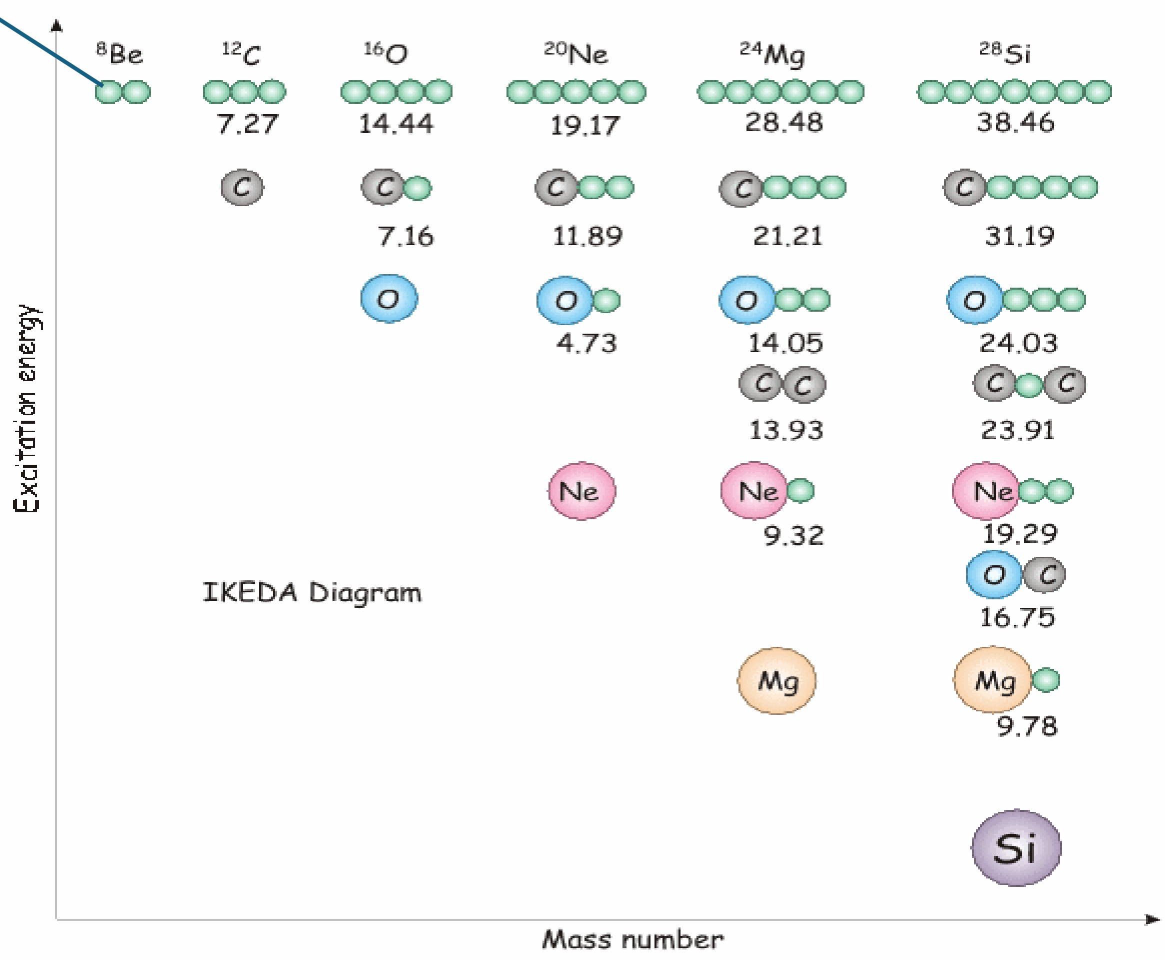
Monte Carlo for the ground state density of ^{8}Be



014001(2000)



Cluster Nucleosynthesis Diagram (CND)



Threshold Rule: clusters appear near decay thresholds.

Supplement of the Progress of Theoretical Physics, Extra Number, 1968

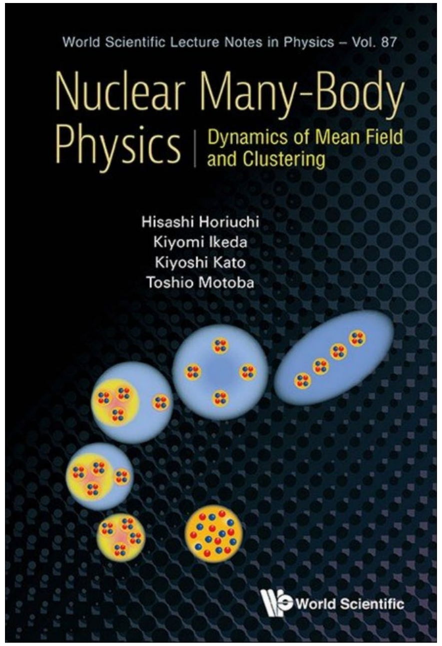
The Systematic Structure-Change into the Molecule-like Structures in the Self-Conjugate 4n Nuclei

Kiyomi IKEDA,*) Noboru TAKIGAWA and Hisashi HORIUCHI

Department of Physics, University of Tokyo, Tokyo

(Received November 6, 1968)

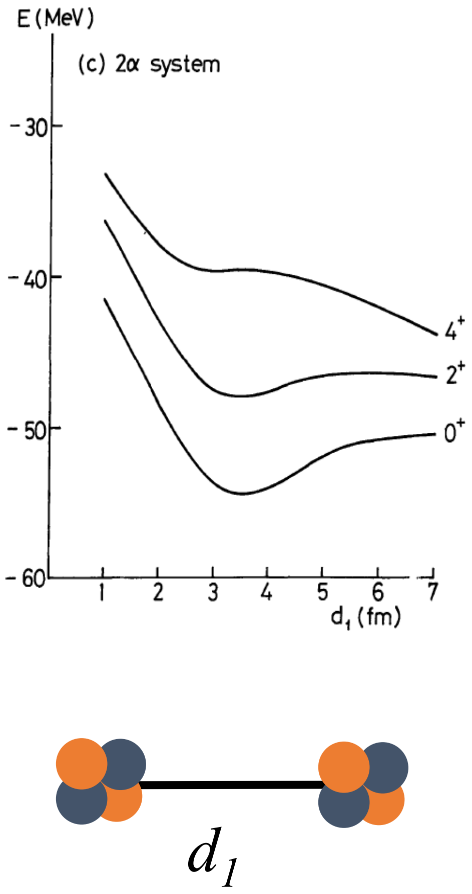
The rotational bands with the diatomic-molecule-like structure in the self-conjugate $4n$ light nuclei, such as, α-α, α-^{12}C and α-^{16}O, appear systematically at near the threshold energy for the decay into the relevant subunit nuclei. The relations between the structure change into the molecule-like structure and the threshold energy for the decay into the subunit nuclei are discussed. According to this discussions, the diagram for the systematic structure changes into the molecule-like structures through the alpha particle release is presented as the function of the mass number and the energy. Upon this diagram the rotational bands with $K=0^{\pm}$ in light $4n$ nuclei can be summarized. The order of the degree of the polarization toward the separation of the subunit nuclei for the diatomic molecule-like structure case is discussed qualitatively.

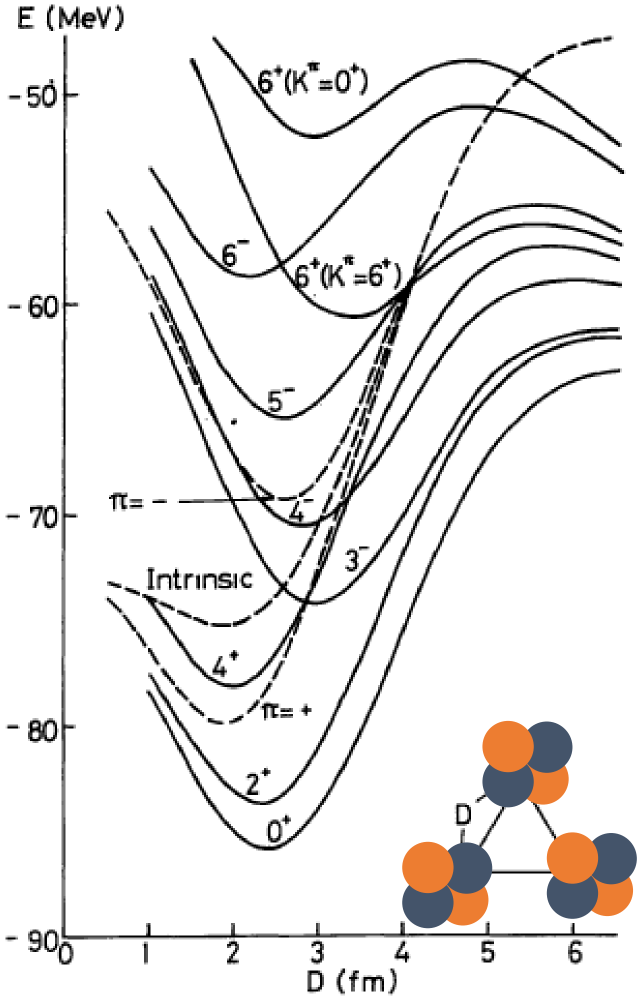


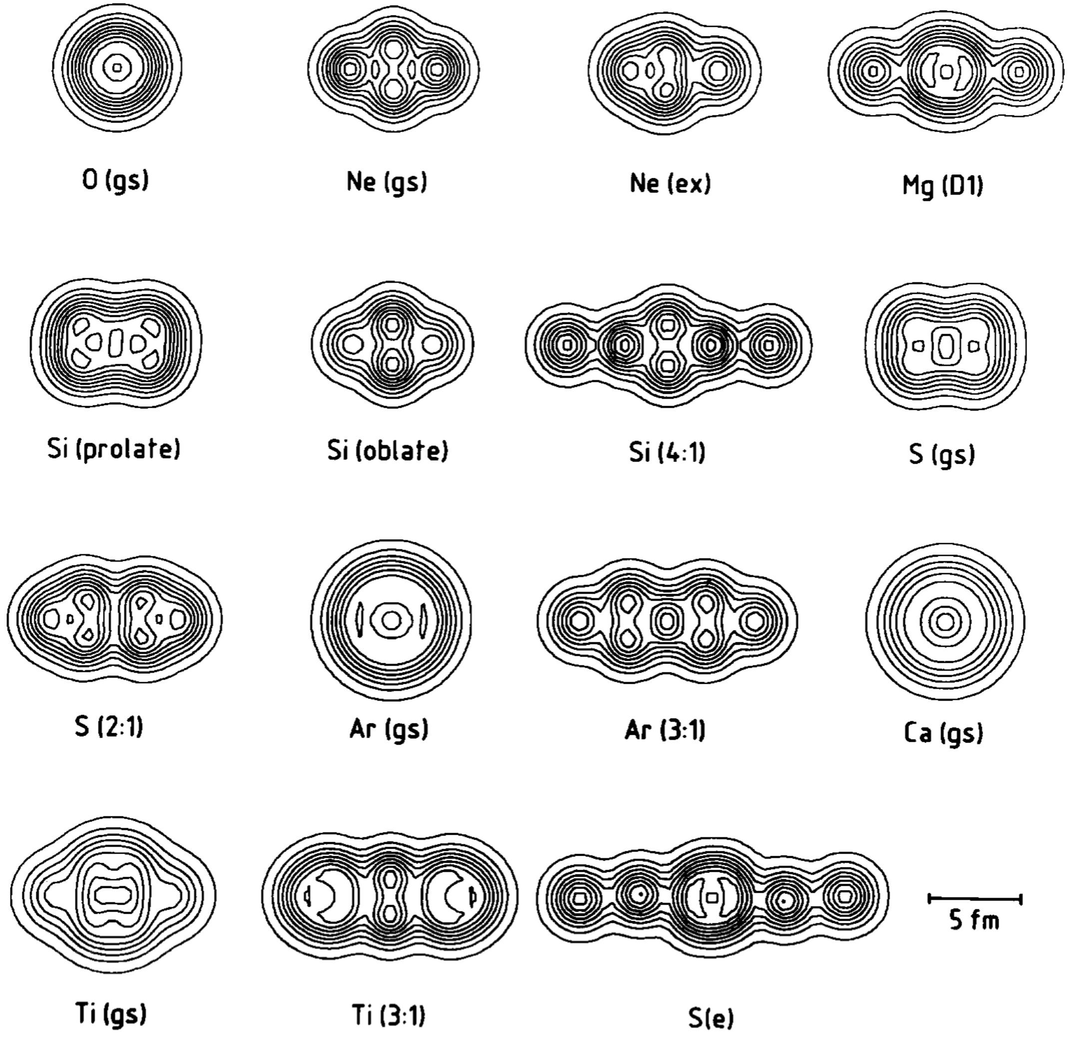
https://doi.org/10.1142/13718

Geometry structure of Nα nuclei

In nuclear microscopic cluster model,



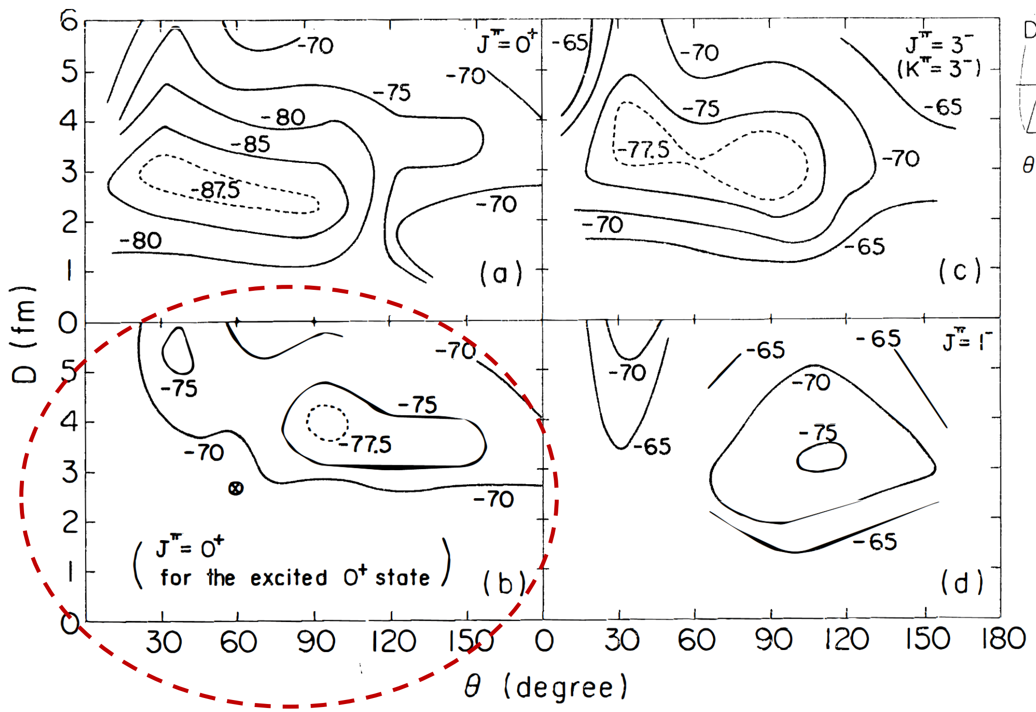


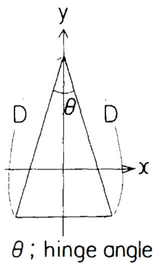


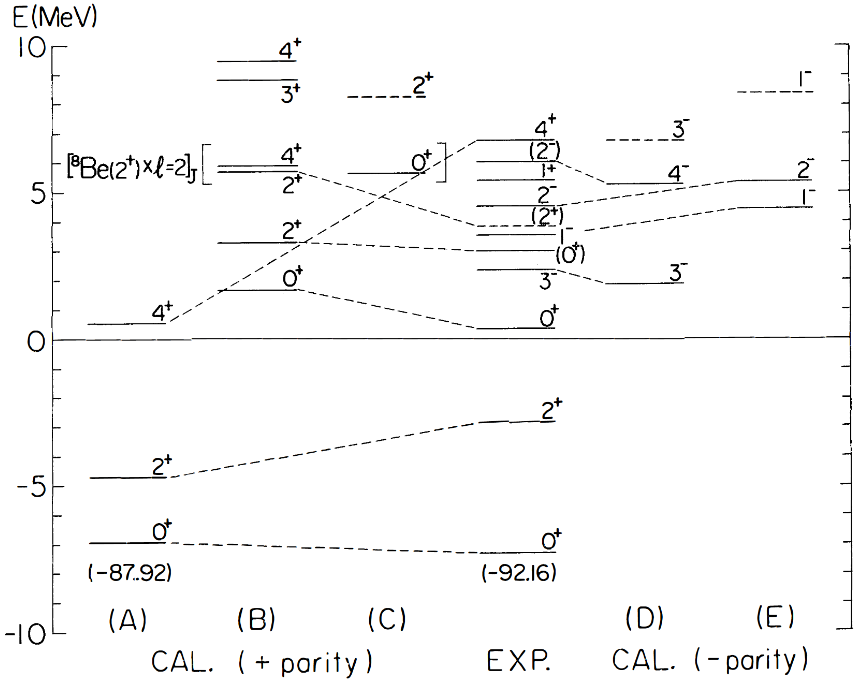
Prog. Theor. Phys. Suppl, No.68,1980

J. Zhang et al.,NPA 575,61(1994)

Structure of the Excited States in ^{12}C







Microscopic cluster calculations for ^{12}C,

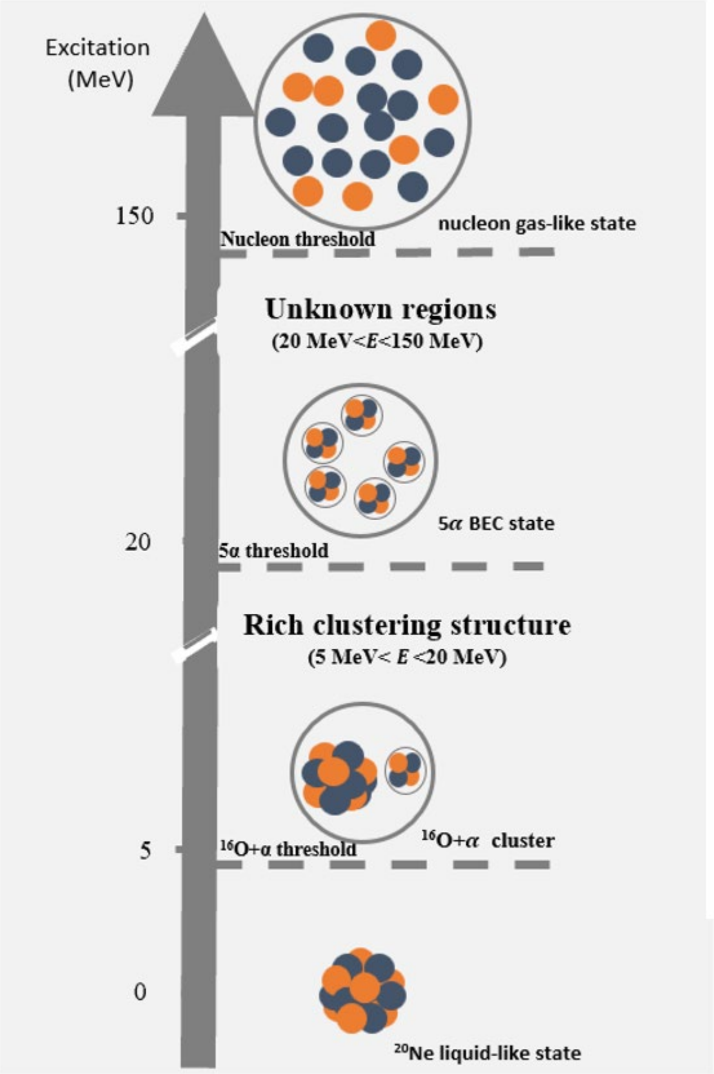
[1] H.Horiuchi, Prog.Theor.Phys. 51,1266 (1974); 53,447(1975)**(OCM)**

[2] Y.Fukushima and M.Kamimura in Proceedings of the International Conference on Nuclear Structure (1977).

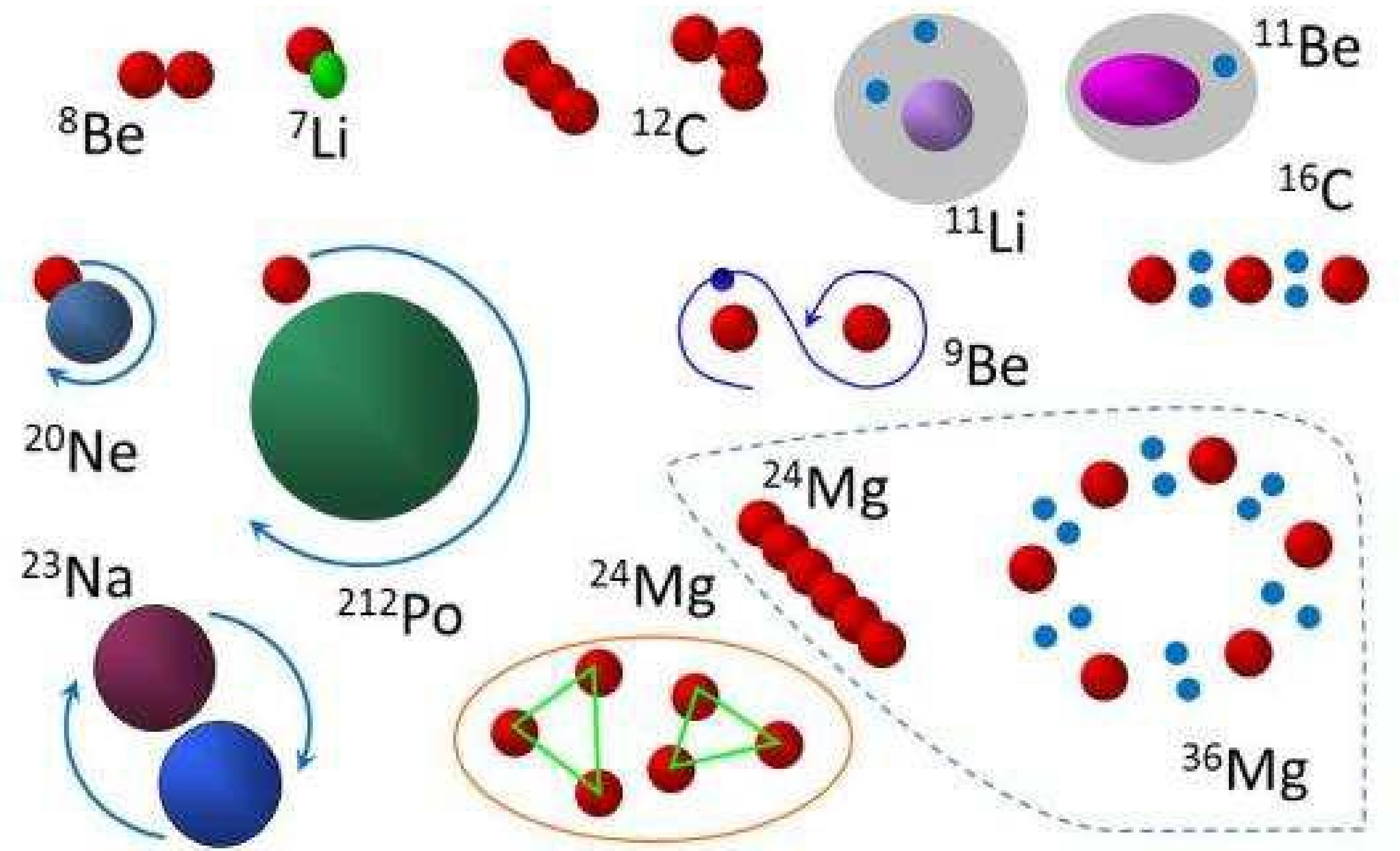
M.Kamimura, Nucl.Phys.A 351,456(1981)**(RGM)**

[3] E.Uegaki, S.Okabe, Y.Abe and H.Tanaka, Prog.Theor.Phys. 57,1262(1977); 59,1031(1978); 62,1621(1979) **(GCM)**

Rich clustering structure in nuclear systems.



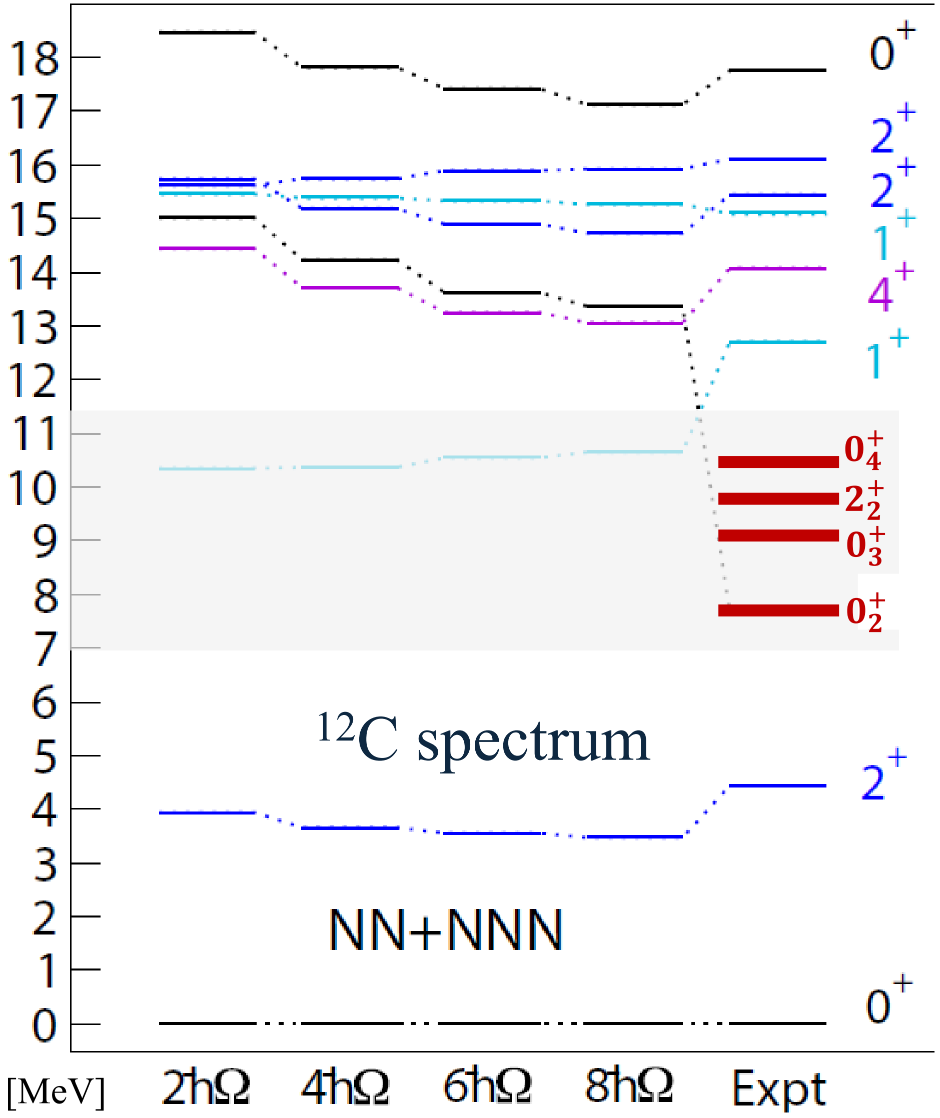
Evolution of structure of ^{20}Ne



DOI:10.1088/1742-6596/863/1/012002

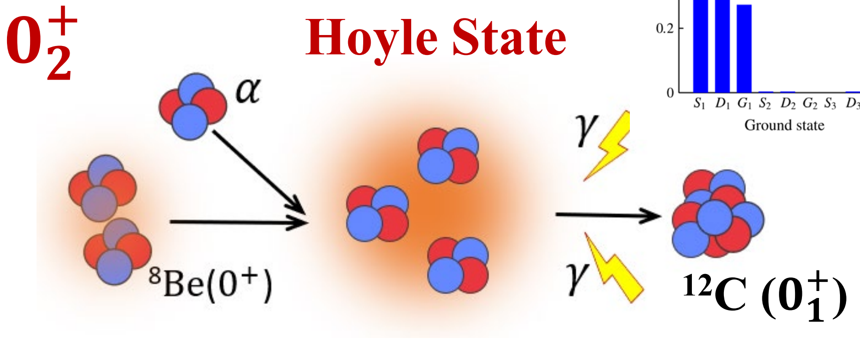
$$\Psi = \mathcal{A}\{\chi(\xi_1, \dots, \xi_{n-1})\phi_1 \dots \phi_n\}$$

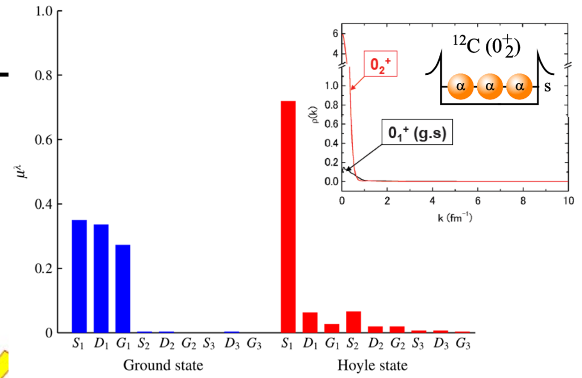
Cluster states of ^{12}C



Recent No-Core-Shell-Model calculations

V.Somà,P.Navrátil, *et al.* PRC,101,014318 (2020)





Hoyle state & Bose-Einstein Condensate.

Rev. Mod. Phys. **89**, 011002 (2017)

$0_{3,4}^+$

Two broad resonance states with large decay width

Phys. Rev. C **84**, 054308 (2011)

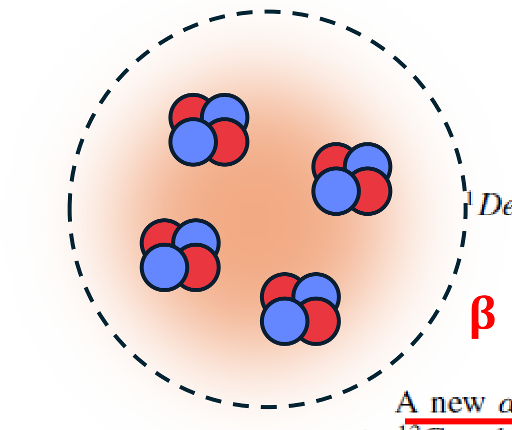
2_2^+

Long puzzle and it now has been confirmed for its existence.

Phys. Rev. Lett. **110**, 152502 (2013)

well-developed clustering states

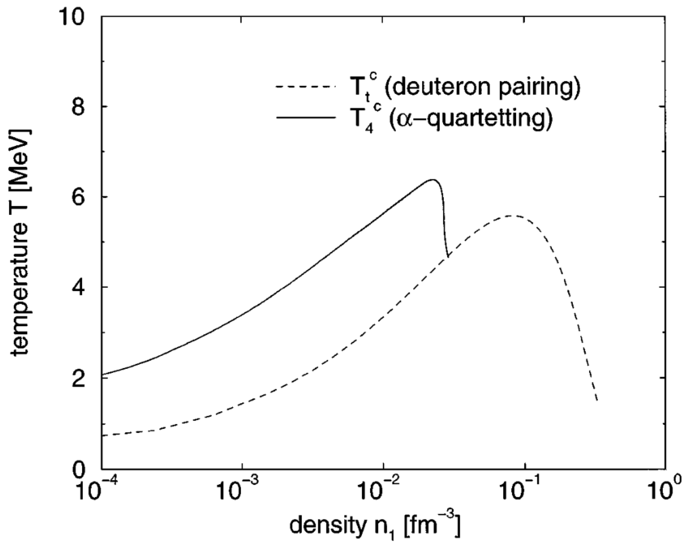
VOLUME 87, NUMBER 19 PHYSICAL REVIEW LETTERS 5 NOVEMBER 2001



Alpha Cluster Condensation in ^{12}C and ^{16}O

A. Tohsaki,[1] H. Horiuchi,[2] P. Schuck,[3] and G. Röpke[4]
[1]Department of Fine Materials Engineering, Shinshu University, Ueda 386-8567, Japan
[2]Department of Physics, Kyoto University, Kyoto 606-8502, Japan
[3]Institut de Physique Nucléaire, F-91406 Orsay Cedex, France
[4]FB Physik, Universität Rostock, D-18051 Rostock, Germany
(Received 29 June 2001; published 17 October 2001)

A new α-cluster wave function is proposed which is of the α-particle condensate type. Applications to ^{12}C and ^{16}O show that states of low density close to the 3 and 4 α-particle thresholds in both nuclei are possibly of this kind. It is conjectured that all self-conjugate $4n$ nuclei may show similar features.



THSR wave function

$$\Phi^{THSR}(\beta)=\int d^3R_1\ldots d^3R_n \mathrm{Exp}[-\frac{R_1^2+\ldots+R_n^2}{\beta^2}]\Phi^{Brink}(R_1,\ldots,R_n)$$

$$\propto \phi_G \mathcal{A}\{\prod_{i=1}^{n}[\mathrm{Exp}(-\frac{2(X_i - X_G)^2}{B^2})\phi(\alpha_i)]\}$$

$$B^2 = b^2+2\beta^2$$

$$\phi(\alpha) \propto \exp[-\sum_{1\le i<j\le 4}(r_i-r_j)^2/(8b^2)]$$

β can be considered as the size parameter of the nucleus

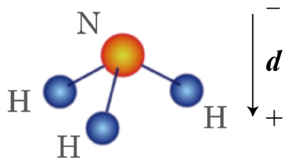
), NUMBER 15 PHYSICAL REVIEW LETTERS 13

Four-Particle Condensate in Strongly Coupled Fermion Systems

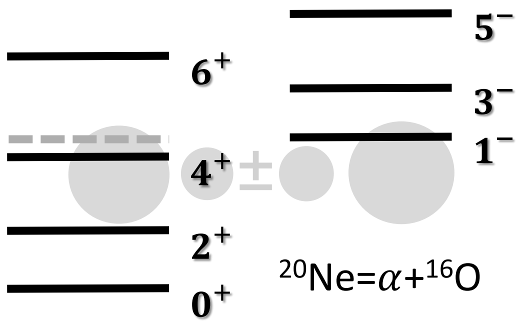
G. Röpke, A. Schnell, and P. Schuck*
University of Rostock, FB Physik, Universitätsplatz 1, 18051 Rostock, Germany

P. Nozières
Institut Laue-Langevin, B.P. 156, 38042 Grenoble Cedex 9, France
(Received 25 September 1997)

Four-particle correlations in fermion systems at finite temperatures are investigated with special attention to the formation of a condensate. Instead of the instability of the normal state with respect to the onset of pairing described by the Gorkov equation, a new equation is obtained which describes the onset of "quartetting." Applying to symmetric nuclear matter, below a critical density, the four-particle condensation (α-like quartetting) is found to be favored over deuteron condensation (triplet pairing). This pairing-quartetting competition is expected to be a general feature of interacting fermion systems, such as the exciton-biexciton system in excited semiconductors. Possible experimental consequences are pointed out. [S0031-9007(98)05829-3]



Nonlocalized clustering

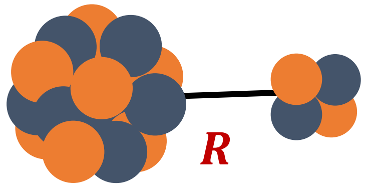


Inversion doublet rotational bands in ^{20}Ne

H.Horiuchi and K.Ikeda,PTP**40**,277(1968)

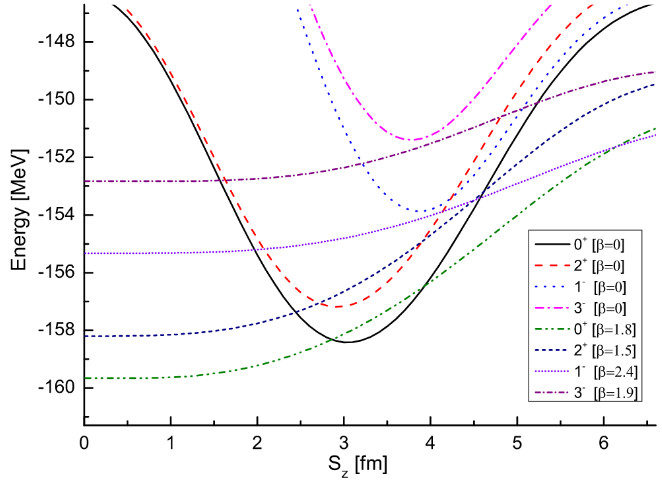
Clusters make the localized motion confined by the inter-cluster distance parameter *R*.

$$\mathcal{A}\{\exp[-\frac{8\,(\boldsymbol{r}-\boldsymbol{R})^2}{5\boldsymbol{b}^2}]\phi(\alpha)\phi(^{16}\mathrm{O})\}$$



Brink cluster model

Single THSR wave function
≈ Superposed Brink wave functions

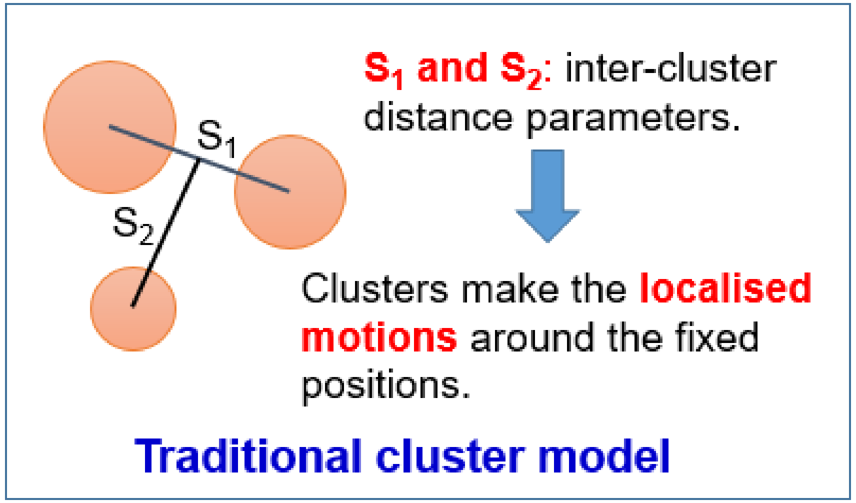


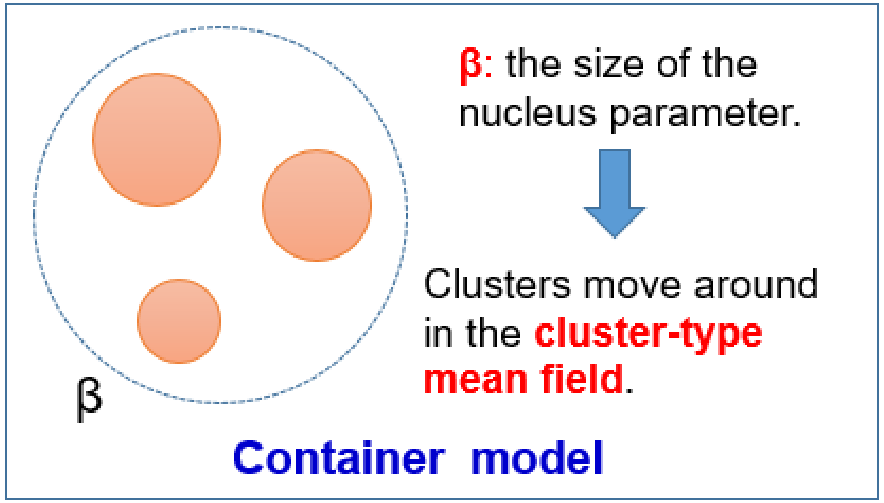
$$\mathcal{A}\{\exp[-\frac{8\,r^2}{5\boldsymbol{B}^2}]\phi(\alpha)\phi(^{16}\mathrm{O})\}$$

Clusters make the nonlocalized motion in a container whose size is described by parameter $\boldsymbol{\beta}$ ($\boldsymbol{B}^2=\boldsymbol{b}^2+2\boldsymbol{\beta}^2$)

Container picture

Container picture for various cluster systems





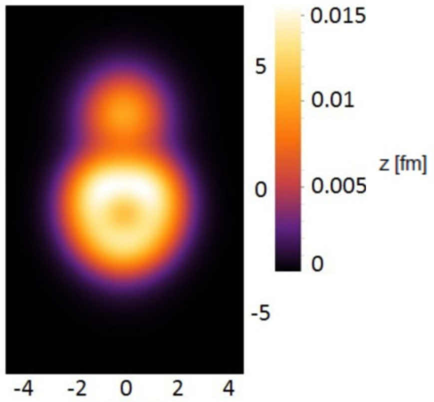
B. Zhou, Y. Funaki, H. Horiuchi, Z. Ren,et al.,PRL. **110**, 262501 (2013) PRC89, 034319 (2014).

Y. Funaki et al. / Progress in Particle and Nuclear Physics 82 (2015) 78–132 113

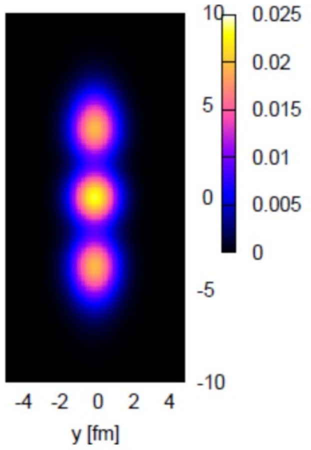
Table 5
The maximum squared overlaps between the single THSR wave functions and RGM/GCM wave functions. The corresponding $\boldsymbol{\beta}$ values, where $(\beta_x = \beta_y, \beta_z) = (\beta_\perp, \beta_z)$, are shown in parentheses in a unit of fm.

	Max.$(\beta_\perp, \beta_z)$					
	^{8}Be	^{12}C	^{20}Ne	3αLCS	4αLCS	$^{9}_{\Lambda}$Be
0^+	1.000(1.8, 7.8)	0_1^+:0.93(1.5, 1.5) $(0_1^+$:0.978)[a] 0_2^+:0.993(5.3, 1.5)	0.993(0.9, 2.5)	0.987(0.1, 5.1)	0.944(0.1, 8.2)	0.995(1.6, 3.0)
2^+			0.988(0.0, 2.2)	0.989(0.1, 5.4)	0.942(0.1, 8.4)	0.994(0.1, 3.0)
4^+			0.978(0.0, 1.8)	0.981(0.1, 6.6)	0.931(0.1, 9.0)	0.977(0.1, 2.1)
			1.000(3.7, 1.4)			
3^-			0.999(3.7, 0.0)			

[a] The value by the use of the extended version of the THSR wave function, with the parameter values $(\beta_{1\perp}, \beta_{1z}, \beta_{2\perp}, \beta_{2z}) = (0.1, 2.3, 2.8, 0.1)$, which we will discuss in Section 3.9.3.



two-body



linear-chain

Future work: New trial wave function for the nuclear clusterstructure of nuclei

An antisymmetric A-nucleon Slater Determinant wave function,

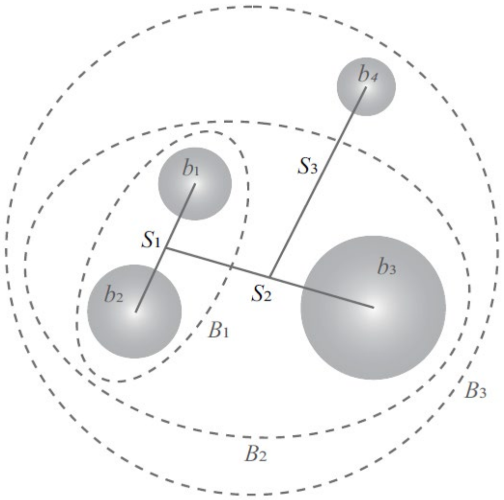
$$\Phi_0(\boldsymbol{r}) = \frac{1}{\sqrt{A!}}\mathcal{A}[\phi_1(\boldsymbol{r}_1)\cdots\phi_A(\boldsymbol{r}_A)]$$

The single-nucleon wave function is,

$$\phi_i(\boldsymbol{r}_i) = (\frac{1}{\pi b_i^2})^{3/4} e^{-\frac{r_i^2}{2b_i^2}} \otimes \chi_{\tau_i} \otimes \chi_{\sigma_i}.$$

A shift operator is used,

$$\widehat{\boldsymbol{D}}(\boldsymbol{R})\Phi_0(\boldsymbol{r}) = \Phi_0(\boldsymbol{r} - \boldsymbol{R})$$

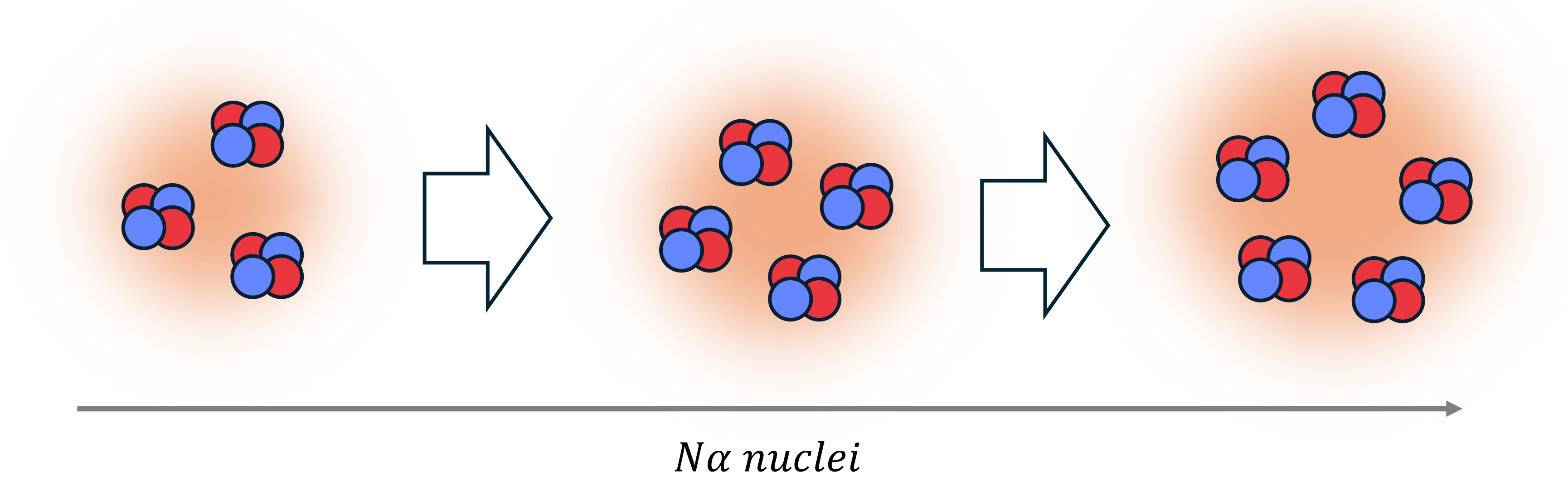


Integrated Gaussian Method
(AMD, FMD, THSR, Brink)

$$\begin{aligned}
\Psi_{\text{new}} &= \widehat{\boldsymbol{L}}_{n-1}(\beta)\widehat{\boldsymbol{G}}_n(\beta_0)\widehat{\boldsymbol{D}}(\boldsymbol{Z})\Phi_0(\boldsymbol{r}) \\
&= \int d^3\widetilde{T}_1\cdots d^3\widetilde{T}_{n-1}\exp[-\sum_{i=1}^{n-1}\frac{\widetilde{T}_i^2}{\beta_i^2}]\int d^3R_1\cdots d^3R_n\exp[-\sum_{i=1}^{n}(\frac{A_i}{\beta_0^2-2b_i^2})(\boldsymbol{R}_i-\boldsymbol{Z}_i-\boldsymbol{T}_i)^2]\Phi_0(\boldsymbol{r}-\boldsymbol{R}) \\
&= n_0\exp[-\frac{A}{\beta_0^2}X_g^2]\ \mathcal{A}\{\prod_{i=1}^{n-1}\exp[-\frac{1}{2B_i^2}(\boldsymbol{\xi}_i-\boldsymbol{S}_i)^2]\prod_{i=1}^{n}\phi_i^{\text{int}}(b_i)\}.
\end{aligned}$$

B. Zhou, Prog. Theor. Exp. Phys. 2018, 041D01 (2018)

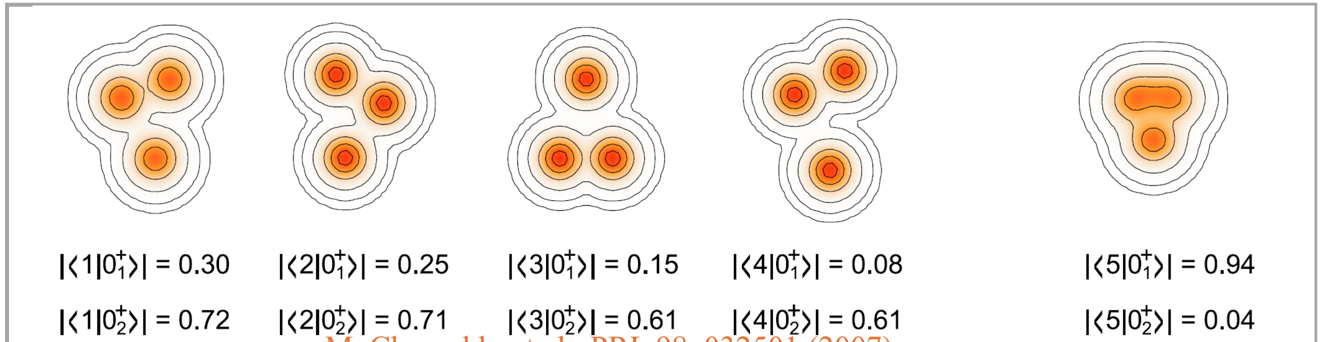
OO and NeNe collisions
ATLAS arXiv:2509.05171
ALICE arXiv:2509.06428
CMS PAS: https://cds.cern.ch/record/2942004



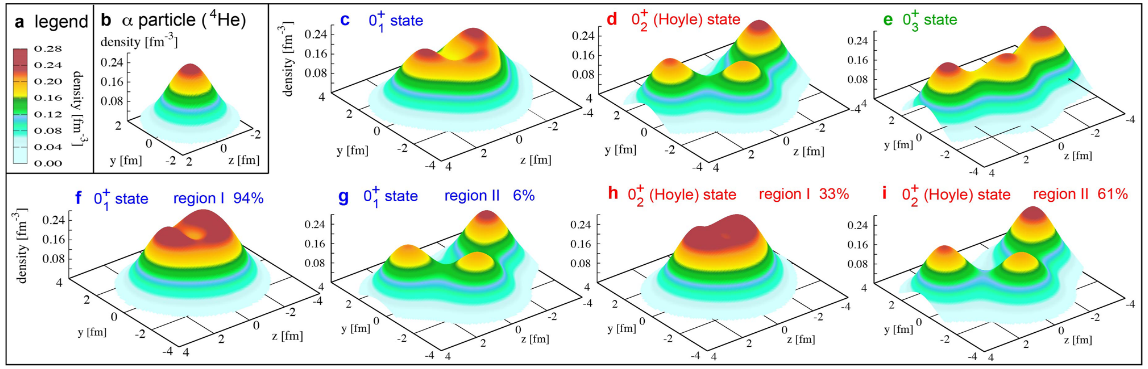
Search for the Nα novel cluster states

gas-like cluster state
no-geometry shape
excited states

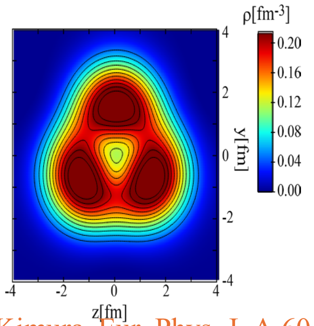
Shape/Structure of the ^{12}C



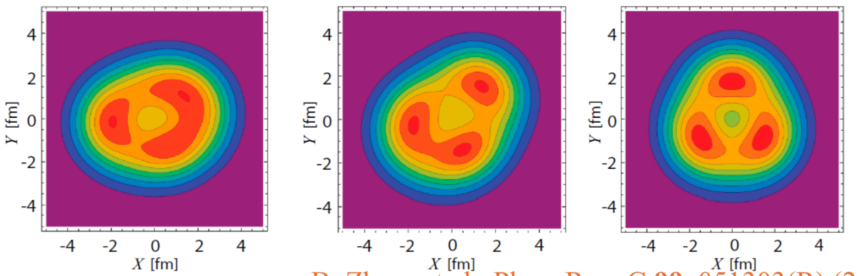
M. Chernykh, et al., PRL 98, 032501 (2007)



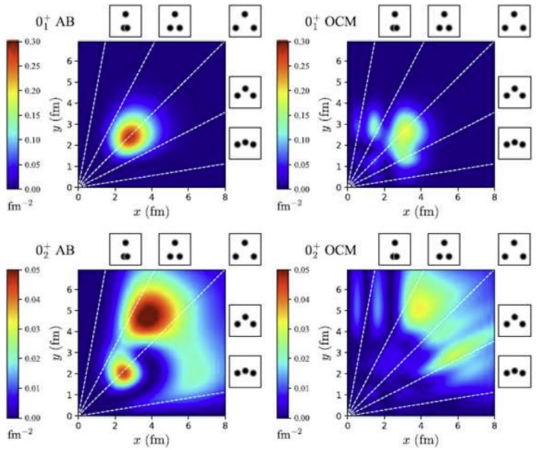
Otsuka, et al., Nat Commun 13, 2234 (2022)



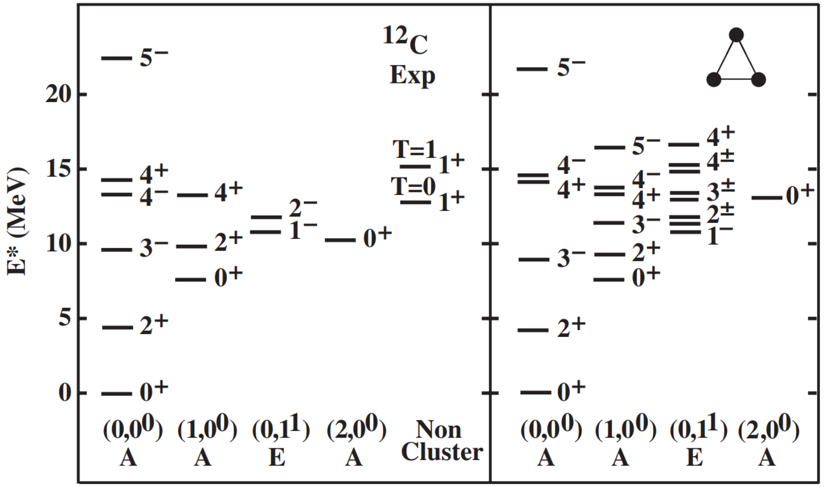
M.Kimura, Eur. Phys. J. A 60, 77 (2024)



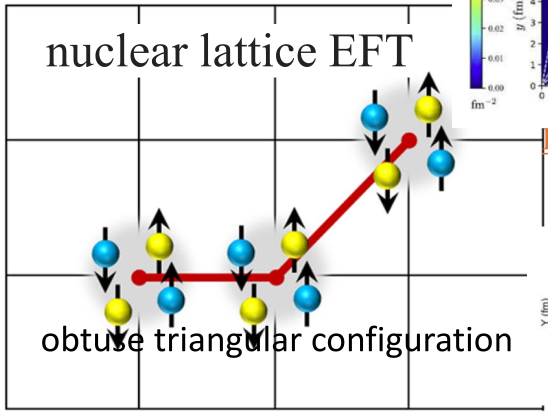
B. Zhou, et al., Phys. Rev. C **99**, 051303(R) (2019).



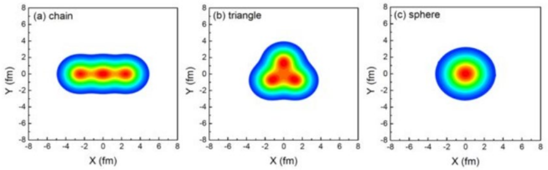
L.Fortunato, *Few-Body Syst* **65**, 1 (2024).



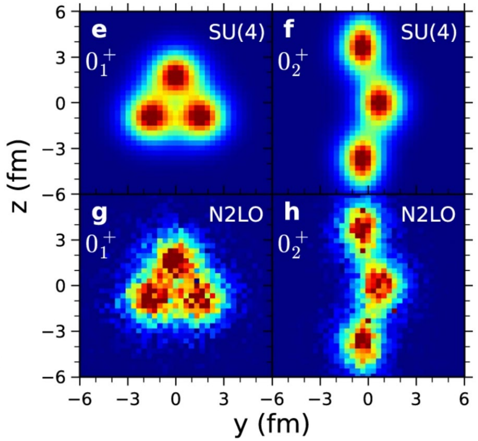
D J Marín-Lámbarri,et al., PRL 113, 012502 (2014)



E.Epelbaum,et al,,PRL 109,252501 (2012)

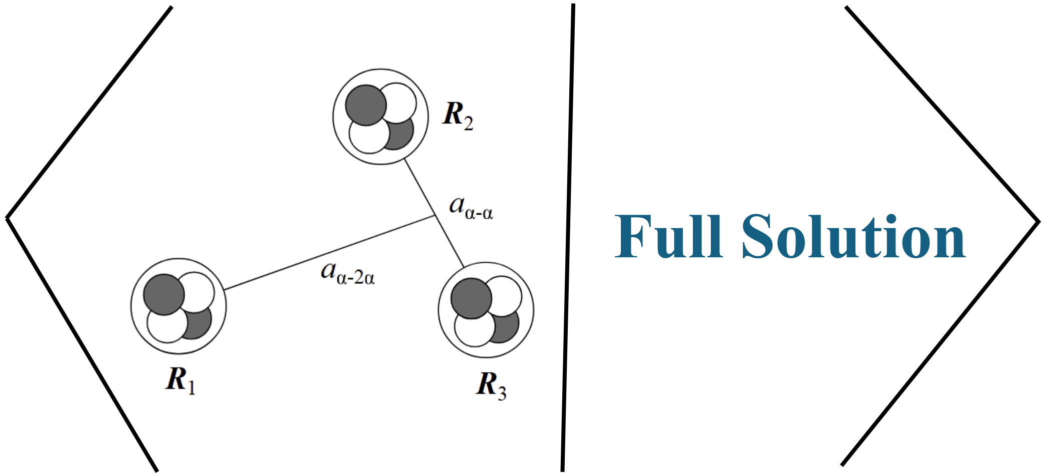


B. S. Huang, et al., PRC95,034606 (2017)



S.Shen, et al., *Nat Commun* **14**, 2777 (2023)

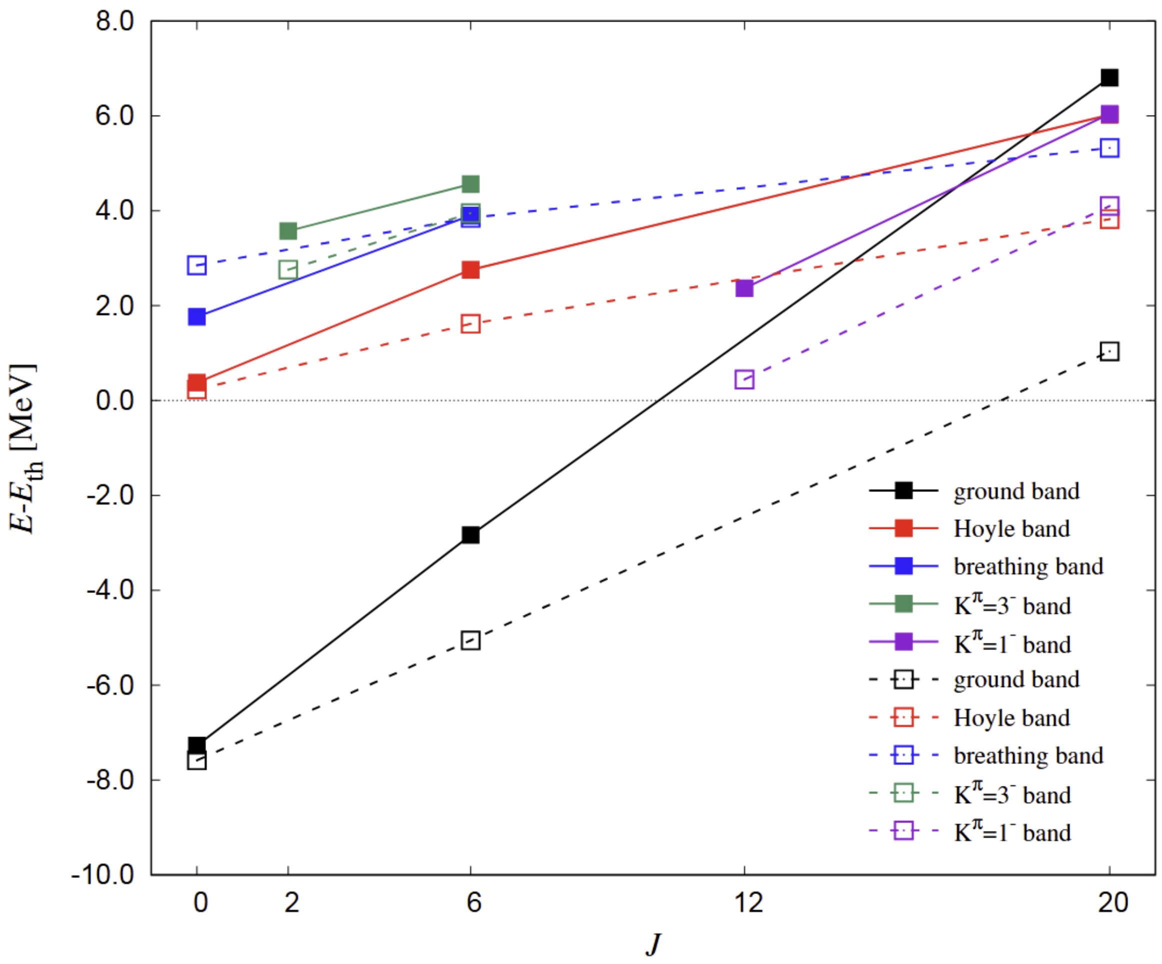
Two-body overlap function (Two-body RWA)



$$\mathcal{Y}^{J\pi}_{L_{\alpha\text{-}2\alpha}L_{\alpha\text{-}\alpha}L}(a_{\alpha\text{-}2\alpha}, a_{\alpha\text{-}\alpha}) = \sqrt{\frac{12!}{4!4!4!}}$$

$$\times \left\langle \frac{\delta(r_{\alpha\text{-}2\alpha} - a_{\alpha\text{-}2\alpha})\delta(r_{\alpha\text{-}\alpha} - a_{\alpha\text{-}\alpha})}{r^2_{\alpha\text{-}2\alpha} r^2_{\alpha\text{-}\alpha}} [[Y_{L_{\alpha\text{-}2\alpha}}(\hat{r}_{\alpha\text{-}2\alpha}) \otimes Y_{L_{\alpha\text{-}\alpha}}(\hat{r}_{\alpha\text{-}\alpha})]_L \otimes [\Phi_\alpha \otimes \Phi_\alpha \otimes \Phi_\alpha]_0]_{JM} \middle| \Psi^{J\pi}_M \right\rangle$$

$$S^2_{3\alpha} = \int_0^\infty \int_0^\infty \left| a_{\alpha\text{-}2\alpha} a_{\alpha\text{-}\alpha} \mathcal{Y}^{J\pi}_c(a_{\alpha\text{-}2\alpha}, a_{\alpha\text{-}\alpha}) \right|^2 da_{\alpha\text{-}2\alpha} da_{\alpha\text{-}\alpha}.$$



How to analyze the clustering structure in light nuclei ?

De-Ye Tao, et al., arXiv:2412.20928,NST (2025)

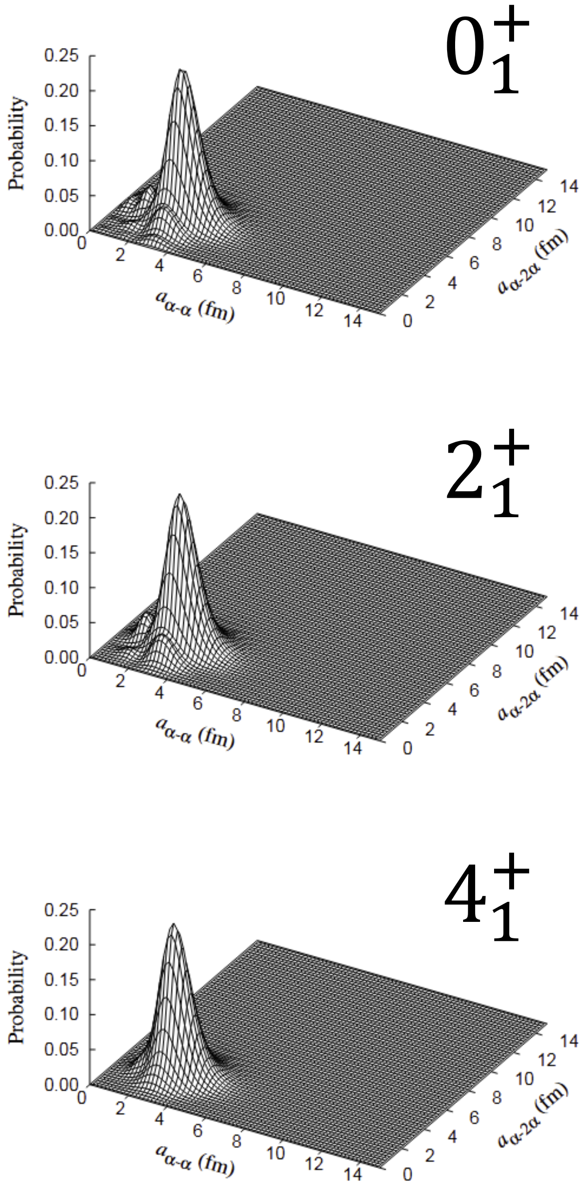


FIG. 1. The two-cluster probabilities for the 0_1^+, 2_1^+, and 4_1^+ states.

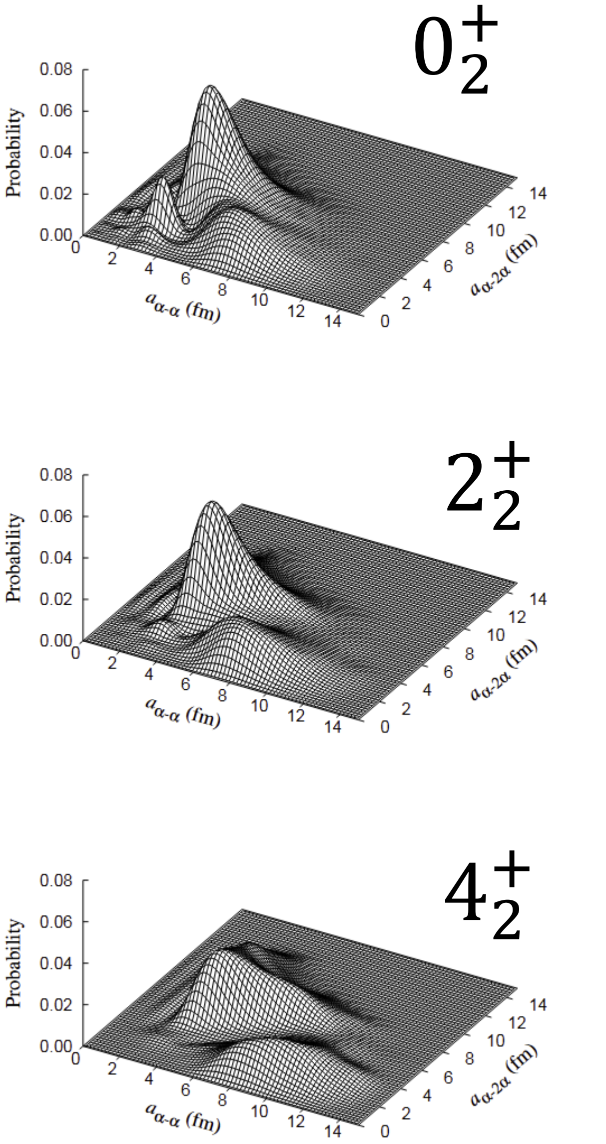


FIG. 2. The two-cluster probabilities for the 0_2^+, 2_2^+, and 4_2^+ states.

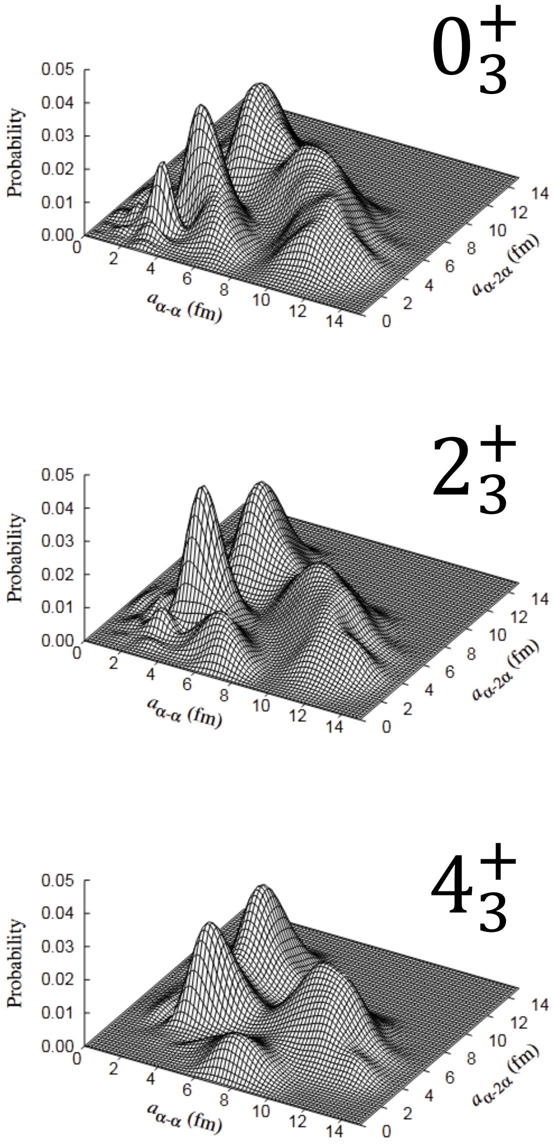
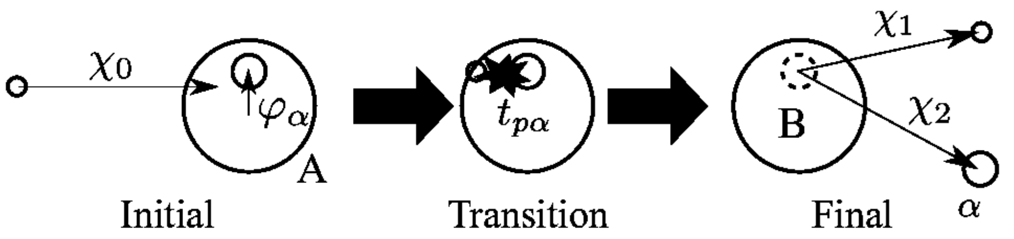


FIG. 3. The two-cluster probabilities for the 0_3^+, 2_3^+, and 4_3^+ states.

De-Ye Tao, et al., PRC112 (2025), Visualization of rotational bands of ^{12}C (under review)

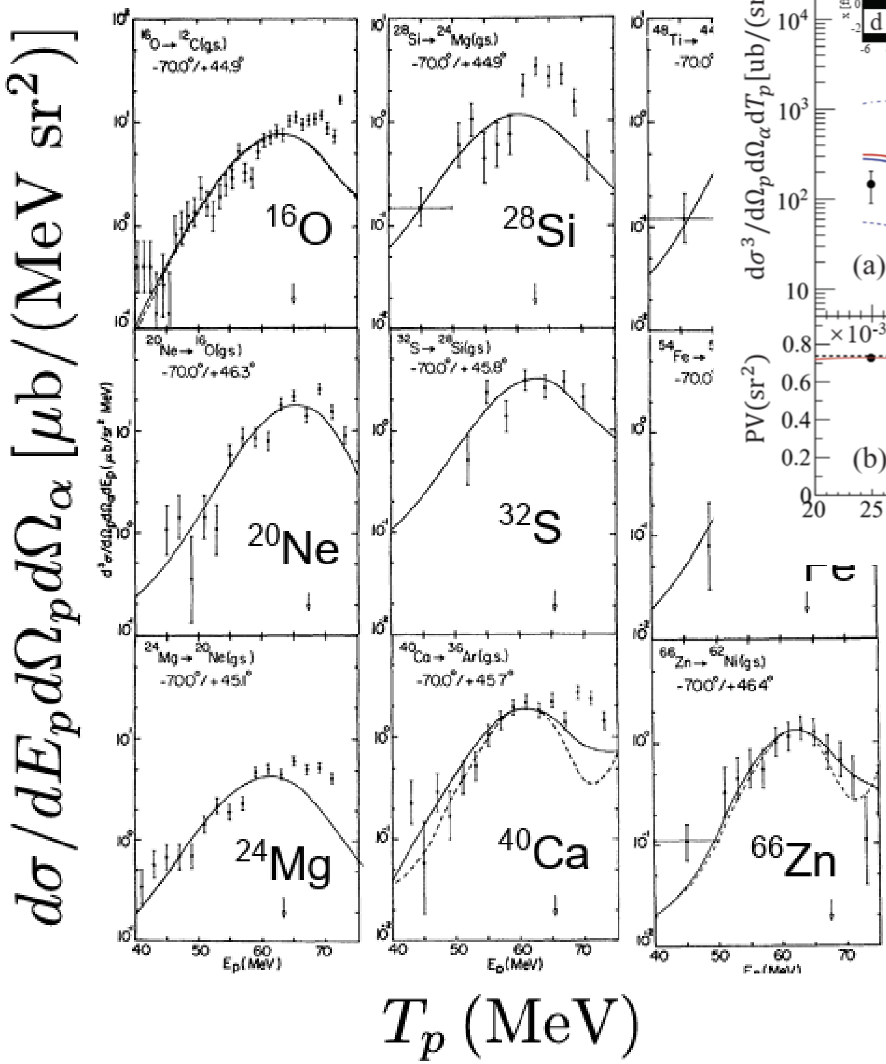
The clustering structure from experiments



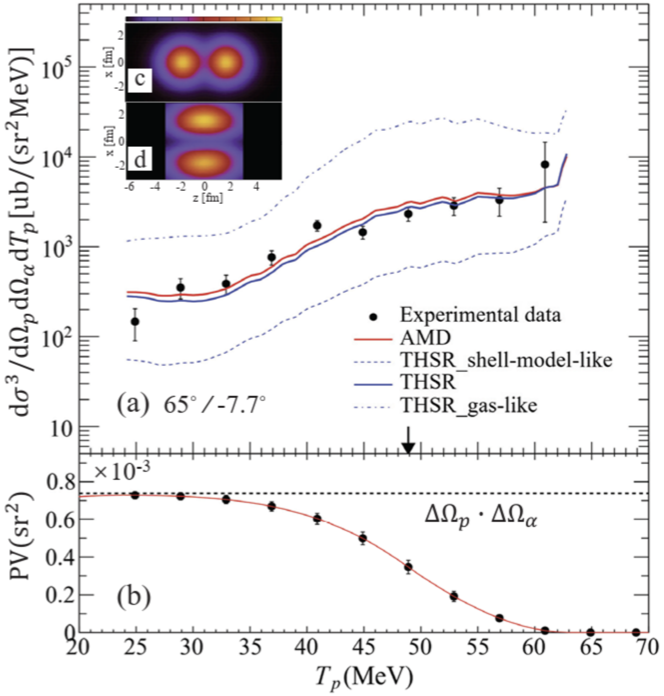
$$T = \langle \chi_1 \chi_2 \Phi_\alpha \Phi_{\rm B} \,|\, t_{p\alpha} \,|\, \chi_0 \Phi_{\rm A} \rangle = \langle \chi_1 \chi_2 \,|\, t_{p\alpha} \,|\, \chi_0 \varphi_\alpha \rangle$$

φ_α: Cluster wave function $\langle [\Phi_\alpha \otimes \Phi_{\rm B}] \,|\, \Phi_{\rm A} \rangle$

$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} \propto |T|^2$$



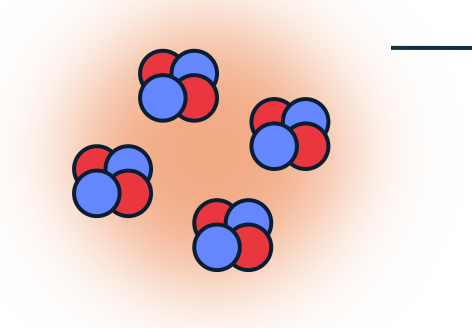
T. A. Carey et al., Phys. Rev. C 29, 1273 (1984).



Pengjie Li,et al.,PRL131 (2023)

K.Yoshida et al., PRC100(2019): Quantitative description of the ^{20}Ne(p, pα)^{16}O reaction as a means of probing the surface α amplitude

The clustering structure of ^{16}O



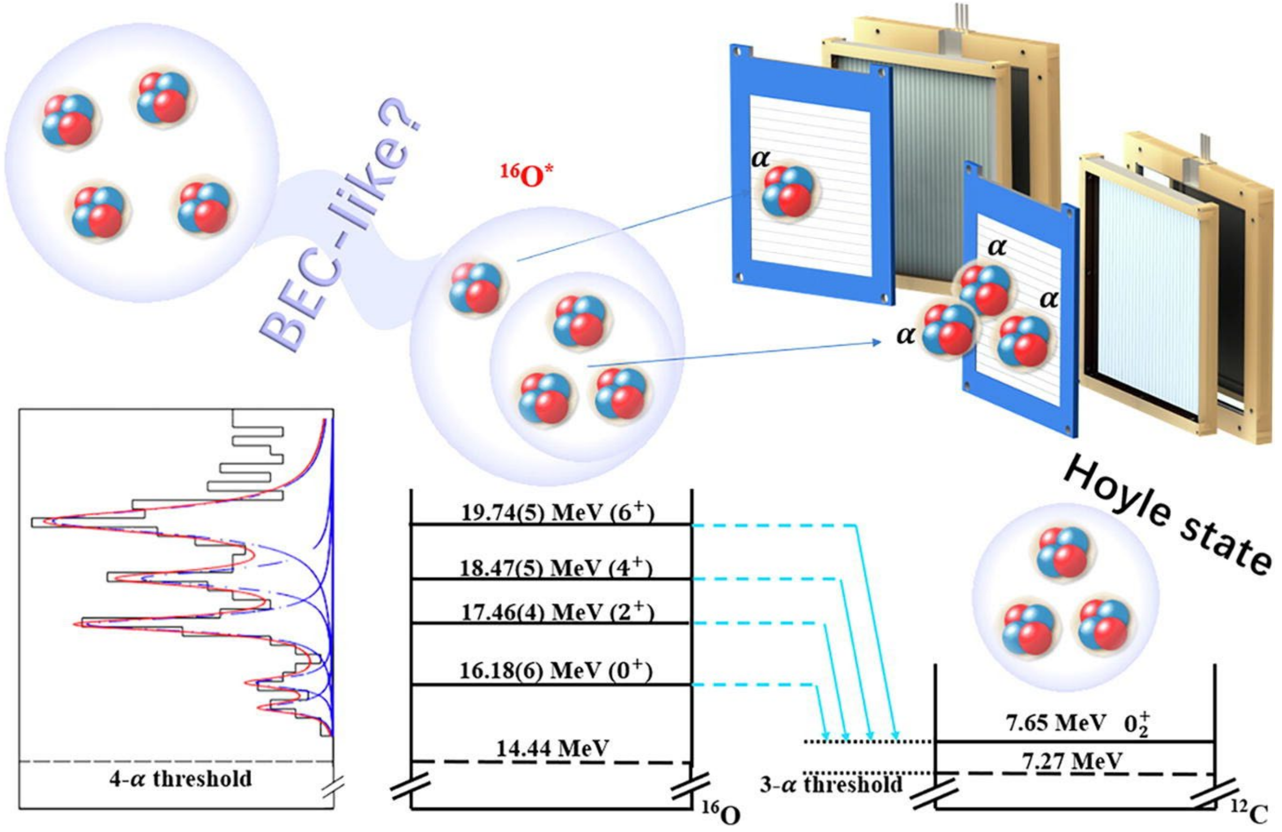
$$\Phi^{\mathrm{B}}(\boldsymbol{R}_1,\boldsymbol{R}_2,\boldsymbol{R}_3,\boldsymbol{R}_4)=\mathcal{A}[\phi_1(\boldsymbol{R}_1)\cdots\phi_4(\boldsymbol{R}_1)\phi_5(\boldsymbol{R}_2)\cdots\phi_{16}(\boldsymbol{R}_4)],$$

$$\phi_k(\boldsymbol{R}_j)=\frac{1}{(\pi b^2)^{3/4}}\exp\left[-\frac{1}{2b^2}(\boldsymbol{r}_k-\boldsymbol{R}_j)^2\right]\chi_k\tau_k$$

$$\boxed{\Psi^{J\pi}_{M}=\sum_{i,K}c_{i,K}\hat{P}^{J}_{MK}\hat{P}^{\pi}\Phi^{\mathrm{B}}(\{\boldsymbol{R}\}_i),}$$

$$\hat{H}=-\frac{\hbar^2}{2m}\sum_i\nabla_i^2-T_{\mathrm{c.m.}}+\hat{V}_{NN}+\hat{V}_{\mathrm{C}}.$$

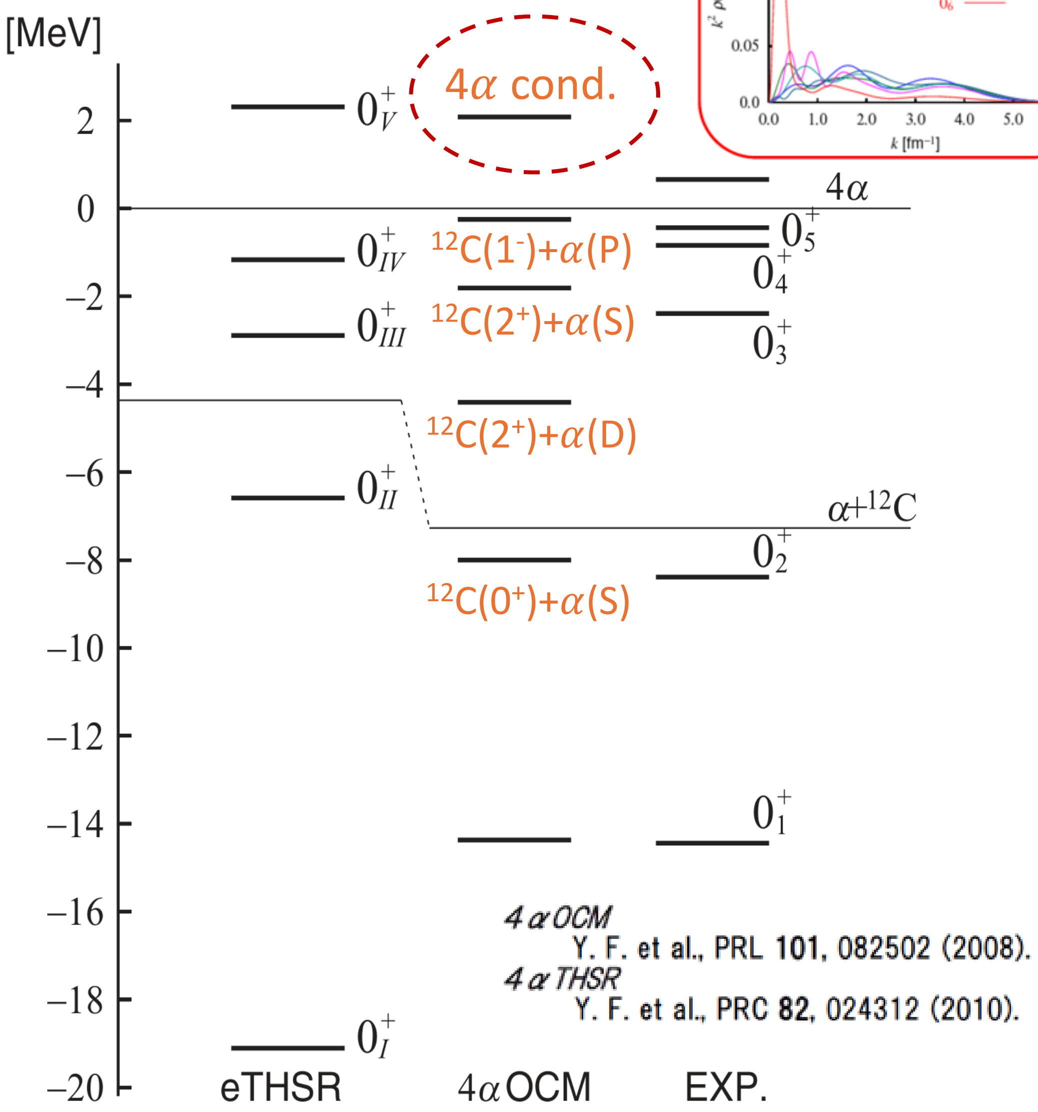
$$\hat{V}_{NN}=\frac{1}{2}\sum_{i\neq j}V^{(2)}_{ij}+\frac{1}{6}\sum_{i\neq j,j\neq k,i\neq k}V^{(3)}_{ijk},$$

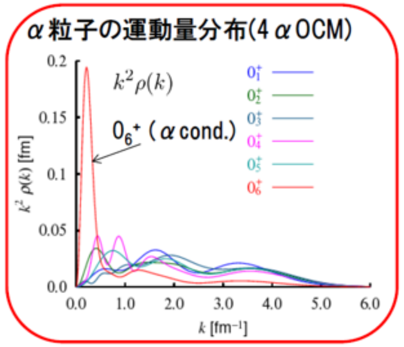


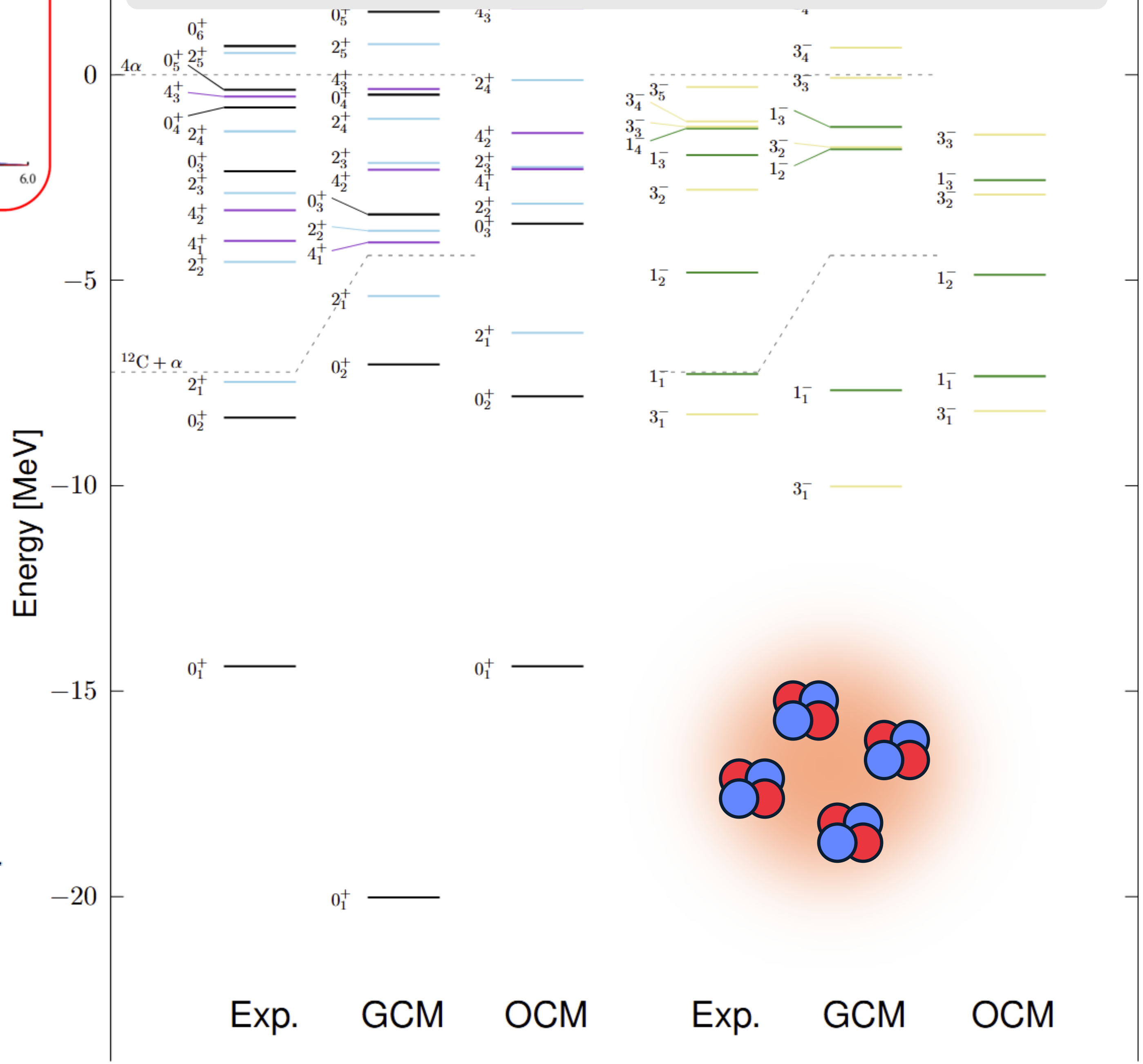
Prof. Yanlin Ye's group

Science Bulletin, 68(11), 1119-1126(2023)

still very difficult in this region





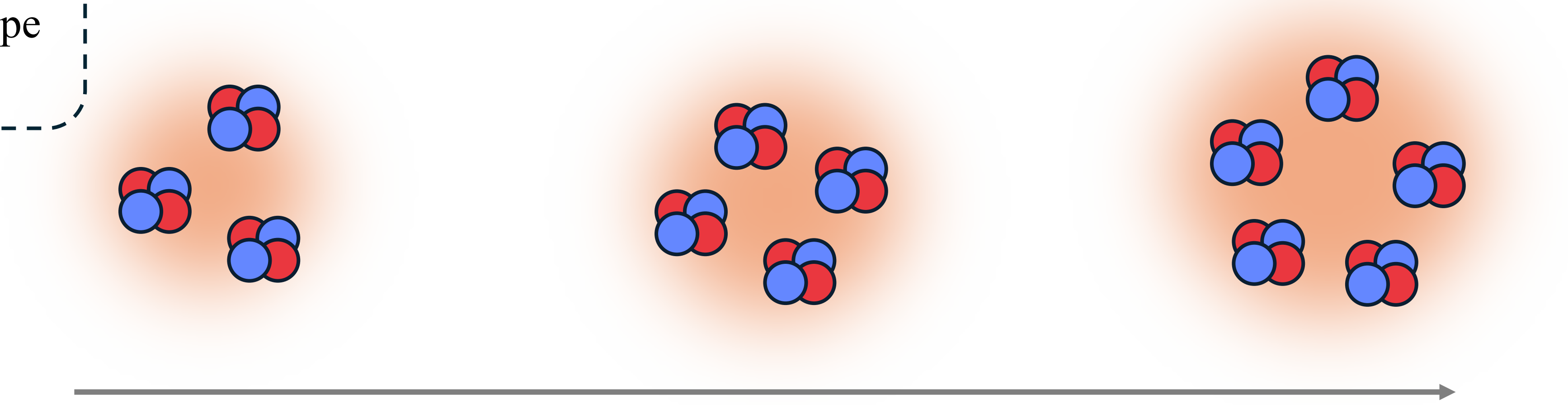


S.Y Zhou et al. (under review)

gas-like cluster state

no-geometry shape

excited states



Nα nuclei

Search for the 5α condensate state

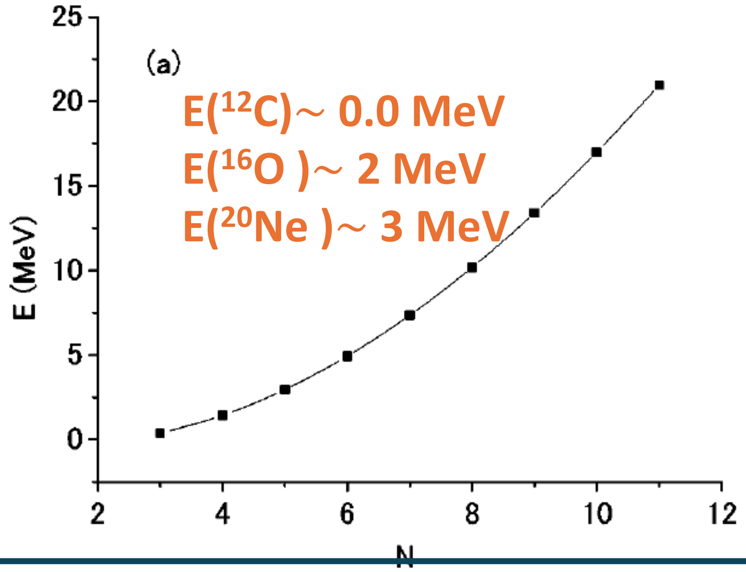
3α condensate	4α condensate	5α condensate
(Hoyle state)	(0_6^+ state)	(?)
2001 (THSR)	2008~ (OCM,THSR)	2019~

study of alpha condensate in finite nuclei

Multi-alpha condensation

Dilute multi-α cluster condensed states with spherical and axially deformed shapes are studied with the Gross-Pitaevskii equation and Hill-Wheeler equation where the α cluster is treated as a structureless boson,
it is predicted to exist in heavier self-conjugate 4*N* nuclei up to *N*=10.

T. Yamada and P. Schuck, Phys. Rev. C 69, 024309 (2004).



Some candidates for α condensate were found from experiments for ^{12}C and ^{16}O.

Rev. Mod. Phys. **89**, 011002 (2017).

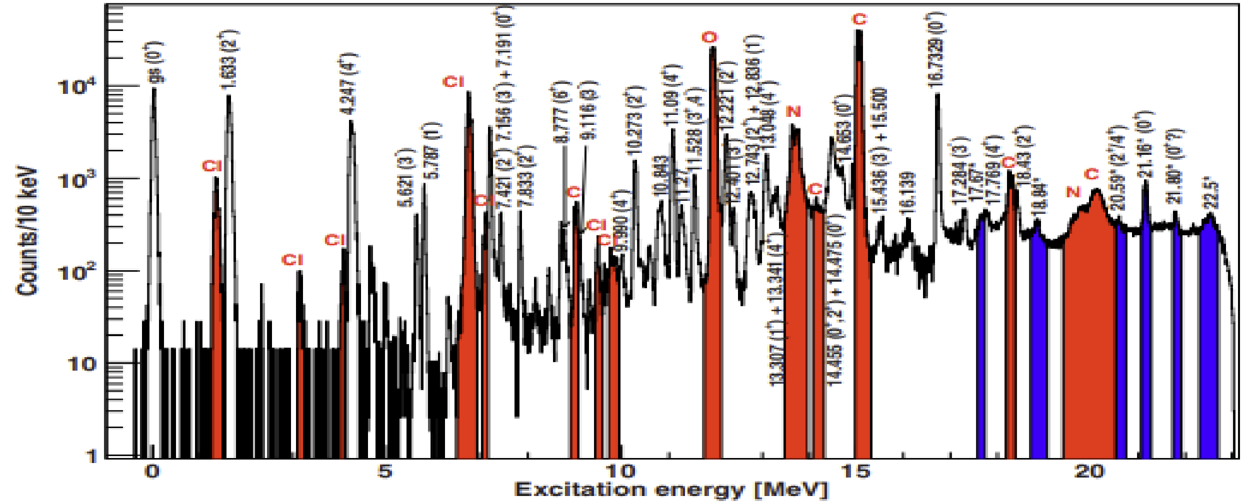
No experimental signatures for α condensation were observed

Phys. Rev. C **100**, 034320 (2019)

An experimental way of testing Bose-Einstein condensation of clusters in the atomic nucleus is reported. The enhancement of cluster emission and the multiplicity partition of possible emitted clusters could be direct signatures for the condensed states.

PRL **96,** 192502 (2006)

Recent experiment for 5α condensation

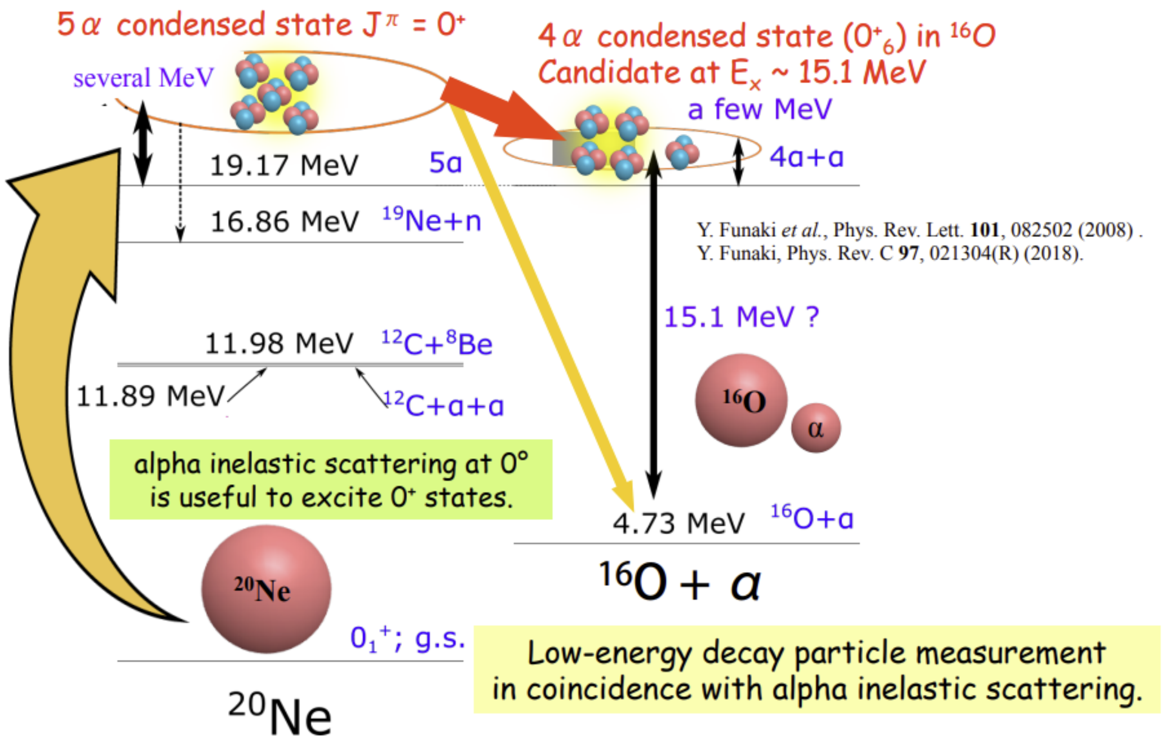


E_x calculated MeV	Model	E_x measured [MeV]
6.05	$2\hbar\omega$	6.725
6.70	USDB	7.191
12.58	$2\hbar\omega$	13.642
14.43	USDB	14.455/14.475
14.67	USDB	14.653
16.76	USDB	16.73
18.06	USDB	17.90
19.02	USDB	18.840(56)
20.47	$2\hbar\omega$	21.160(53)
22.14	5-α [31]	22.500(52)

PHYSICAL REVIEW C 91, 034317 (2015)

Spectroscopy of narrow, high-lying, low-spin states in ^{20}Ne

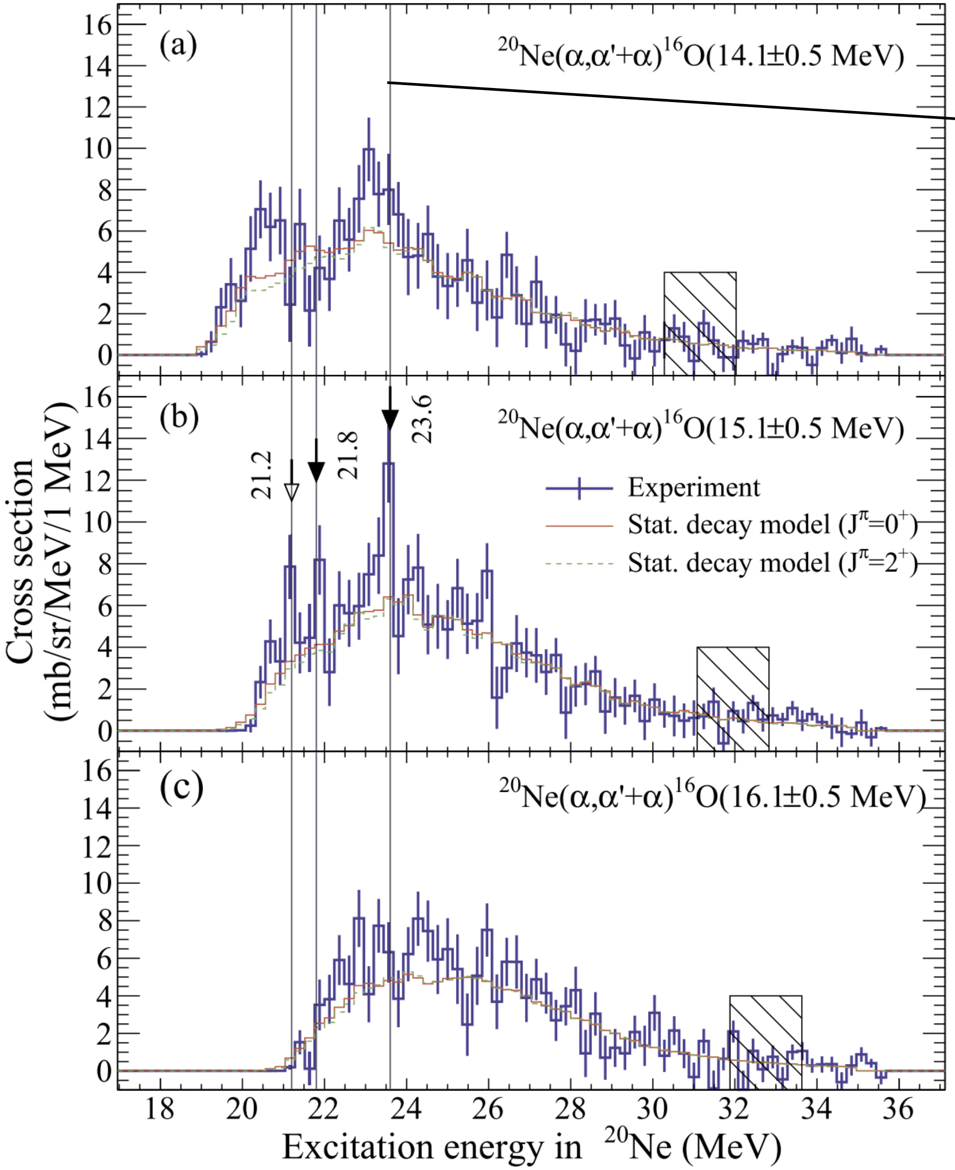
J. A. Swartz,[1,2,*] B. A. Brown,[3,4] P. Papka,[1,2] F. D. Smit,[2] R. Neveling,[2] E. Z. Buthelezi,[2] S. V. Förtsch,[2] M. Freer,[5] Tz. Kokalova,[5] J. P. Mira,[1,2] F. Nemulodi,[1,2] J. N. Orce,[6] W. A. Richter,[2,6] and G. F. Steyn[2]



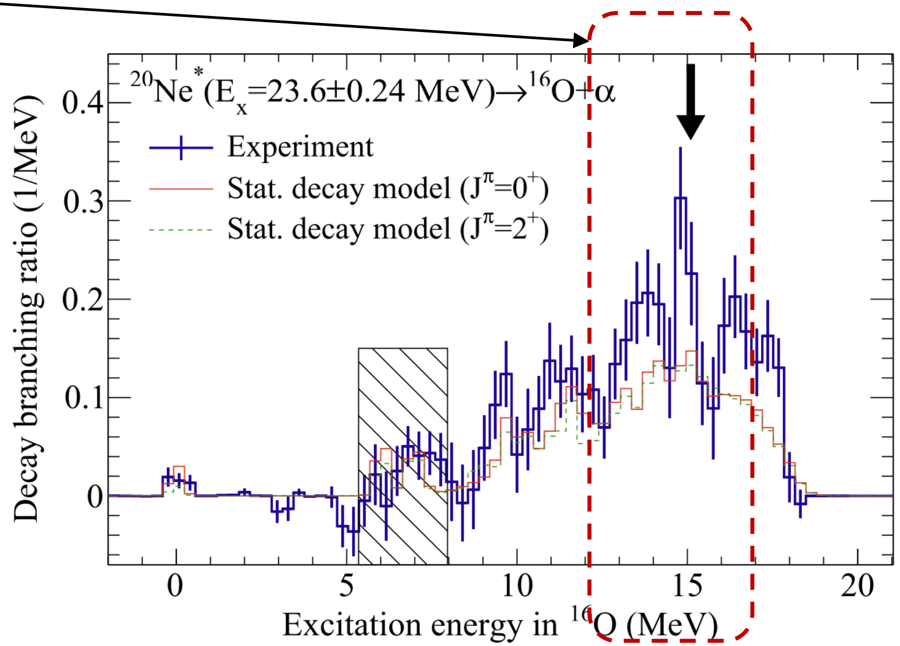
courtesy from Kawabata

The state at ***Ex=22.5 MeV***, which could not be interpreted by the shell-model calculations, is a tentative candidate for the 5α cluster state.

Recent experiment for 5α condensation



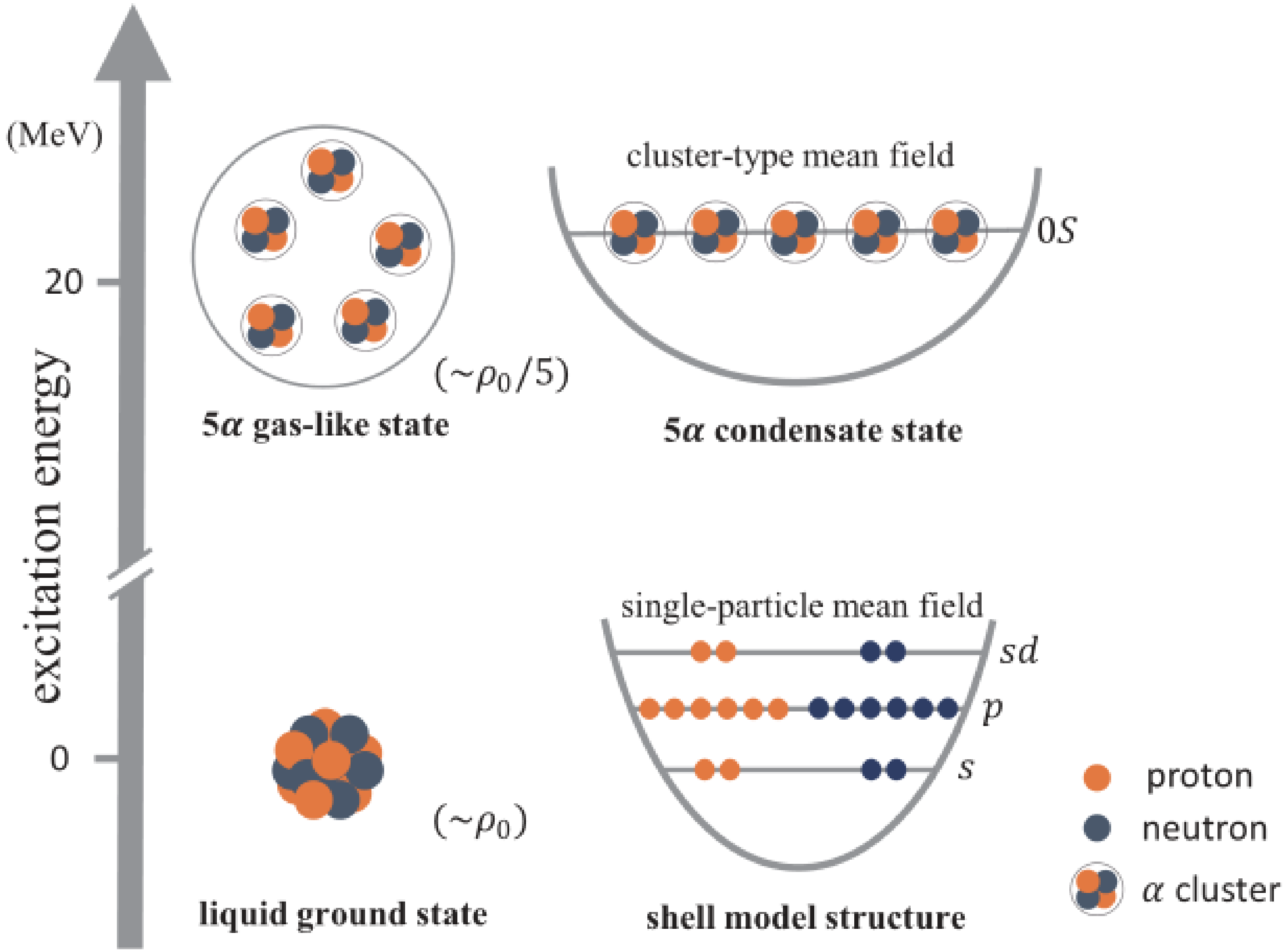
23.6-MeV state enhances in the decay to the ^{16}O (0_6^+) $+\alpha$ channel

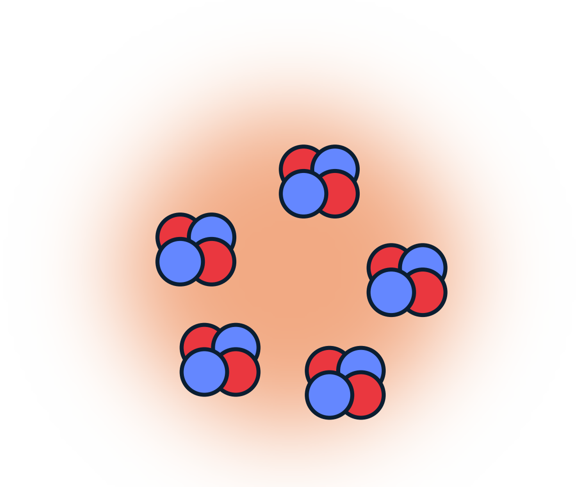


- ❑ 3.3 MeV above the 5α threshold
- ❑ strongly coupled to ^{16}O(0_6^+) state

S. Adachi et al., *Candidates for the 5α condensed state in ^{20}Ne*, PLB **819**,136411 (2021).

Search for the 5alpha condensate state



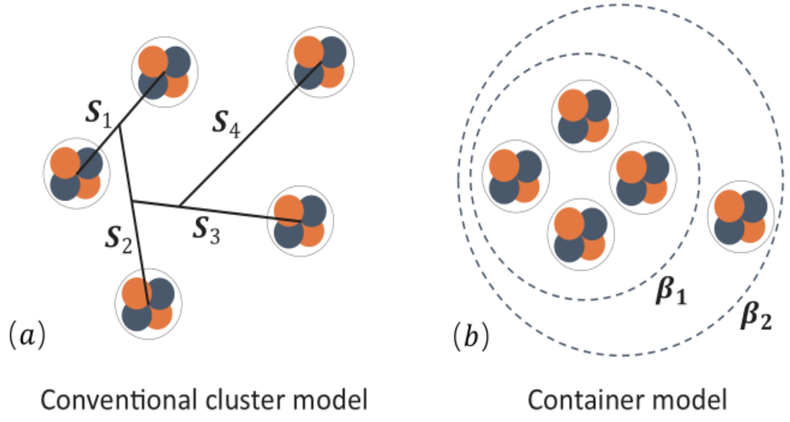


5α BEC state ?

COREnet: Study of α condensation in ^{20}Ne within the THSR wave function Approved.

The 5alpha microscopic wave function

To solve the configurations problem:



| **Schematic illustrations of two distinct microscopic cluster models. a**, The conventional cluster model of Φ^{B}, in which the inter-cluster variables $\{S_i\}$ are the Jacobi coordinates of $\{R_i\}$. **b**, Container picture for $4\alpha + \alpha$ cluster structure of ^{20}Ne. The β_1 is the size variable for the description of 4α and β_2 for the description of the relative motion between 4α and α clusters.

$$\Psi(\beta_1,\beta_2) = \int d^3R_1 d^3R_2 d^3R_3 d^3R_4 d^3R_5 \,\mathrm{Exp}\left[-\frac{1/2S_1^2 + 2/3S_2^2 + 3/4S_3^2}{\beta_1^2} - \frac{4/5S_4^2}{\beta_2^2}\right]\Phi^{\mathrm{B}}(R_1,R_2,R_3,R_4,R_5) \tag{1}$$

$$= n_0\,\mathscr{A}\left\{\mathrm{Exp}\left[-\frac{2\xi_1^2 + 8/3\xi_2^2 + 3\xi_3^2}{2(b^2+2\beta_1^2)}\right]\mathrm{Exp}\left[-\frac{16/5\xi_4^2}{2(b^2+2\beta_2^2)}\right]\prod_{i=1}^{5}\varphi_i^{\mathrm{int}}(\alpha)\right\},$$

where the conventional Brink cluster wave function Φ^{B},

$$\Phi^{\mathrm{B}}(R_1,R_2,R_3,R_4,R_5) = \frac{1}{\sqrt{20!}}\mathscr{A}[\phi_1(R_1)\ldots\phi_5(R_2)\ldots\phi_{20}(R_5)],$$

$$\propto \phi_g\,\mathscr{A}\left\{\mathrm{Exp}\left[-\frac{2(\xi_1-S_1)^2 + 8/3(\xi_2-S_2)^2 + 3(\xi_3-S_3)^2}{2b^2}\right]\mathrm{Exp}\left[-\frac{16/5(\xi_4-S_4)^2}{2b^2}\right]\prod_{i=1}^{5}\varphi_i^{\mathrm{int}}(\alpha)\right\}, \tag{4}$$

with the single-nucleon wave function,

$$\phi_i(R_k) = \left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{1}{2b^2}(r_i-R_k)^2}\chi_i\tau_i.$$

Three-body effective interaction

To solve the interaction problem:

The Hamiltonian for ^{20}Ne in this work can be written as:

$$\mathcal{H} = -\frac{\hbar^2}{2M}\sum_i \nabla_i^2 - T_G + \sum_{i<j} V_{ij}^C + \sum_{i<j} V_{ij}^{(2)} + \sum_{i<j<k} V_{ijk}^{(3)},$$

The effective nucleon-nucleon potential part is taken a Gaussian forr.., which is expressed as:

$$V_{ij}^{(2)} = \sum_n v_n^{(2)} \exp\left\{-\left(\frac{r_{ij}}{r_n^{(2)}}\right)^2\right\}(W_n^{(2)} + M_n^{(2)} P_{ij})$$

and

$$V_{ijk}^{(3)} = \sum_n v_n^{(3)} \exp\left\{-\left(\frac{r_{ij}}{r_n^{(3)}}\right)^2 - \left(\frac{r_{jk}}{r_n^{(3)}}\right)^2\right\} \times(W_n^{(3)} + M_n^{(3)} P_{ij})(W_n^{(3)} + M_n^{(3)} P_{jk}),$$

Tohsaki F1 three-body interaction was used.

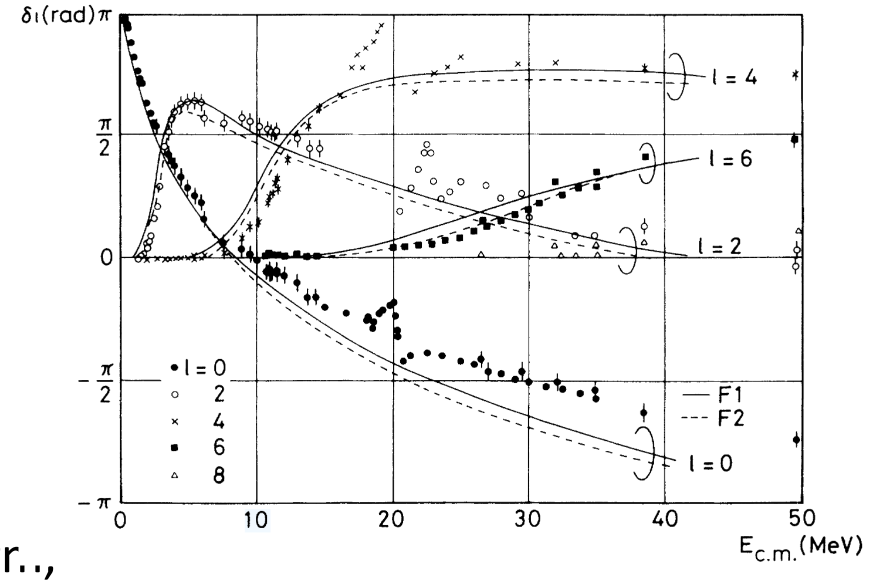


TABLE III. Physical quantities for double closed shell nuclei together with nuclear matter.

		F1	F2	BB	SII	Expt.
α	$E_{\rm bin}$ (MeV)	27.5	27.0	27.4	26.0	28.3
	a (fm^{-2})	0.50	0.50	0.50	0.41	0.51
^{16}O	$E_{\rm bin}$ (MeV)	123.0	124.0	93.0	121.4	127.6
	a (fm^{-2})	0.35	0.35	0.31	0.32	0.34
	$E_{\rm mp}$ (MeV)	23.3	27.7	23.2	32.7	26.4
^{40}Ca	$E_{\rm bin}$ (MeV)	334.0	340.2	250.8	325.5	342.1
	a (fm^{-2})	0.26	0.27	0.25	0.26	0.27
	$E_{\rm mp}$ (MeV)	20.8	24.1	19.0	27.0	23.5
NM	E/A (MeV)	17.0	16.7	15.7	16.0	16.0
	k_f (fm^{-1})	1.27	1.29	1.45	1.30	1.30
	K (MeV)	309	332	193	342	300

A. Tohsaki, Phys. Rev. C **49**, 1814 (1994).

Radius-Constraint Method + Stabilization Method

To solve the resonance problem:

Radius-Constraint Method,

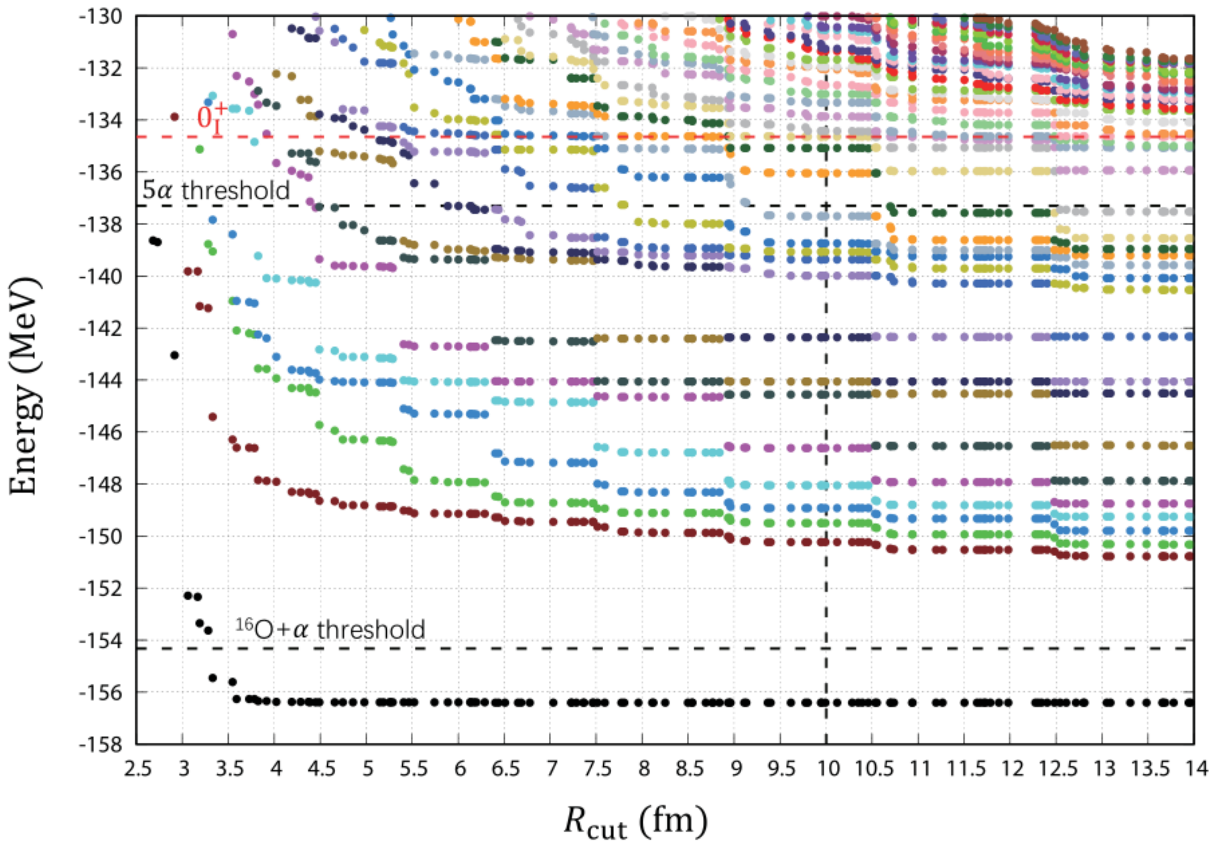
$$\sum_{\beta_1',\beta_2'} \left\langle \widehat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1,\beta_2) \middle| \sum_{i=1} \frac{1}{20}(r_i - X_G)^2 \middle| \widehat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1',\beta_2') \right\rangle \times g^{(\gamma)}(\beta_1',\beta_2')$$

$$= \{R^{(\gamma)}\} g^{(\gamma)}(\beta_1,\beta_2) \left\langle \widehat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1,\beta_2) \middle| \widehat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1',\beta_2') \right\rangle$$

$$\Psi^{0^+}_{GCM} = \sum_{\beta_1,\beta_2} g^{(\gamma)}(\beta_1,\beta_2) \widehat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1',\beta_2')$$

Here ,$R^{(\gamma)} \le R_{cut}$ and R_{cut} is the radius cut-off parameter.

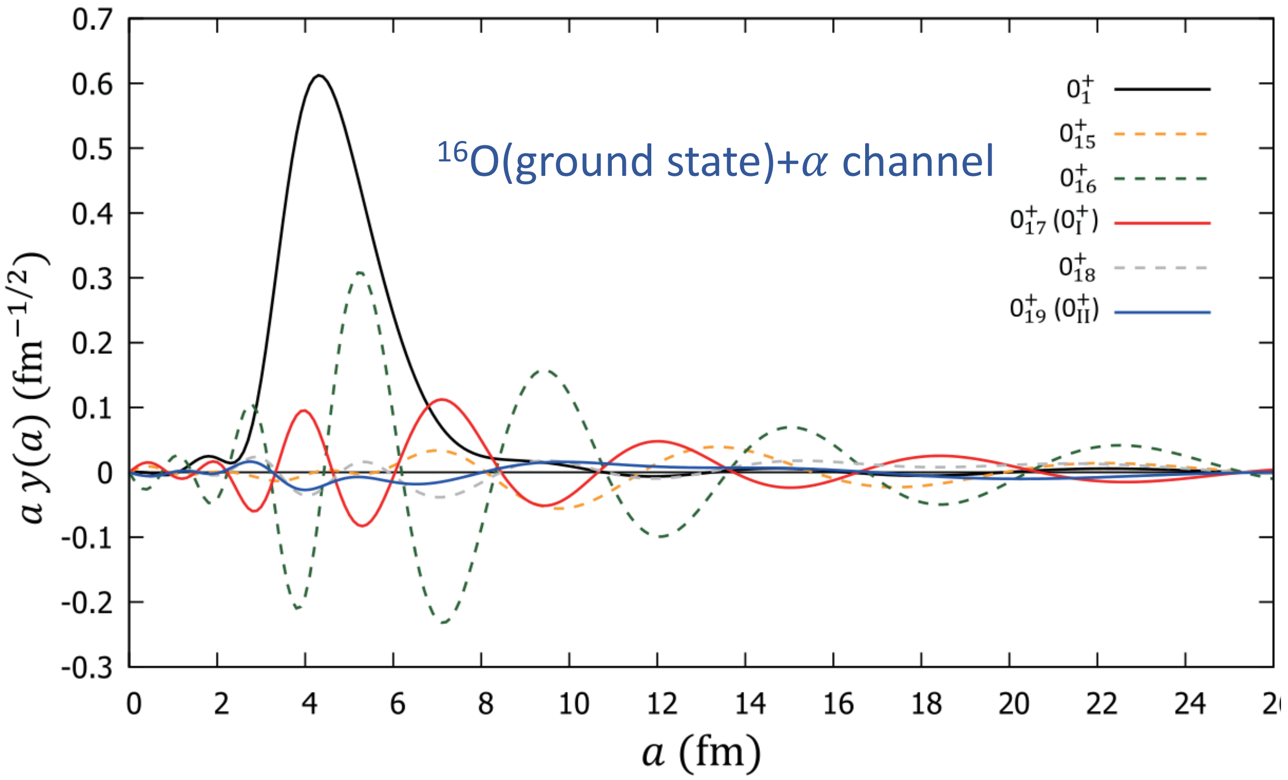
Stabilization Method

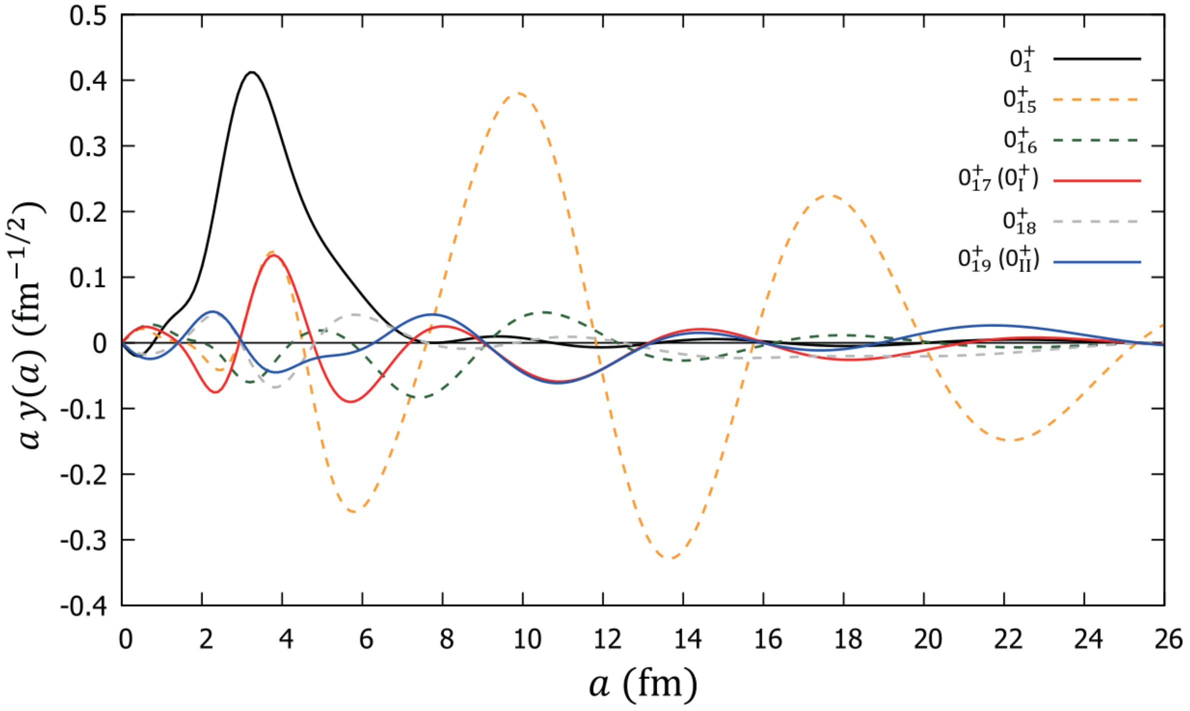


Above the 5alpha threshold: $\mathbf{0^+_{15}} \sim \mathbf{0^+_{19}}$

Five 0^+ states above the threshold

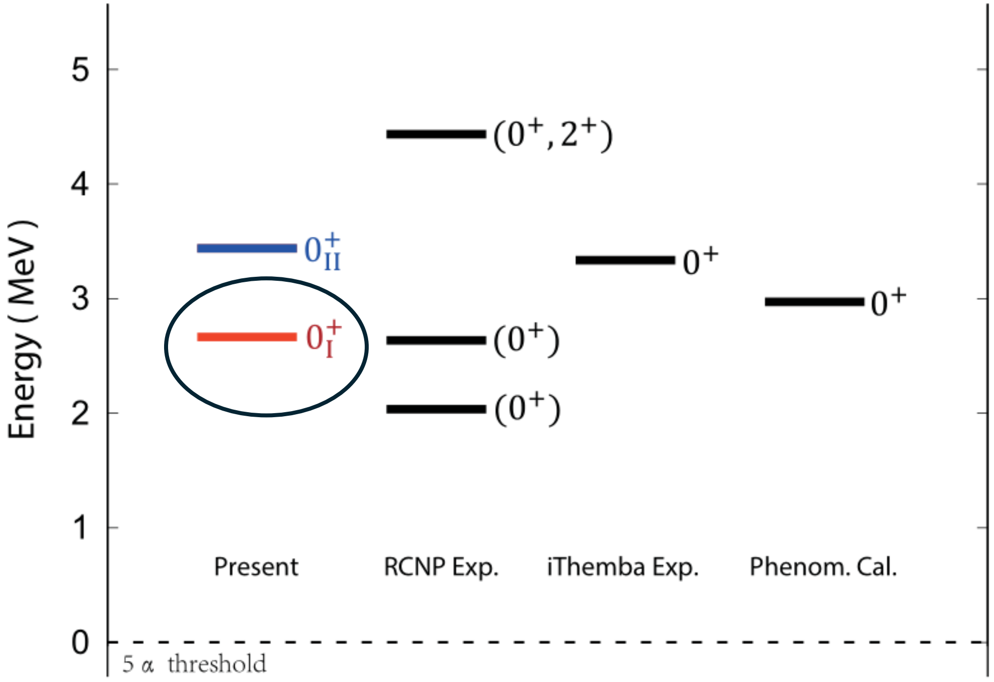
To solve the resonance problem



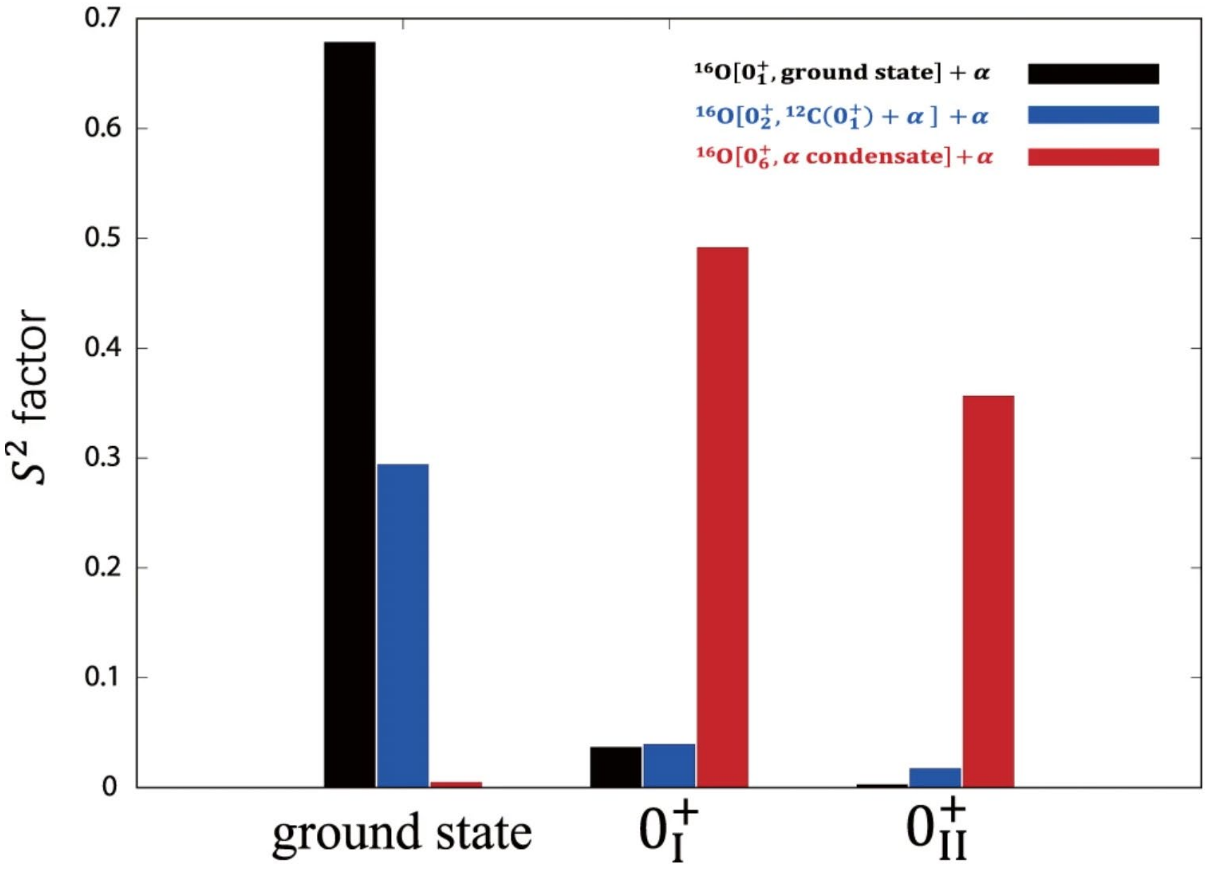


$$y(a) = \sqrt{\frac{20!}{4!16!}} \left\langle \left[\left[\Psi_{\mathrm{gcm}}^{0_s^+}(^{16}\mathrm{O}) \varphi_5(\alpha) \right]_{0^+} Y_{00}(\hat{\xi}_4) \right]_{0^+} \frac{\delta(\xi_4 - a)}{\xi_4^2} \middle| \Psi_{\mathrm{gcm}}^{0_\lambda^+}(^{20}\mathrm{Ne}) \right\rangle$$

Two of Five 0^+ states above the threshold



Two 0^+ states around 3 MeV are found.

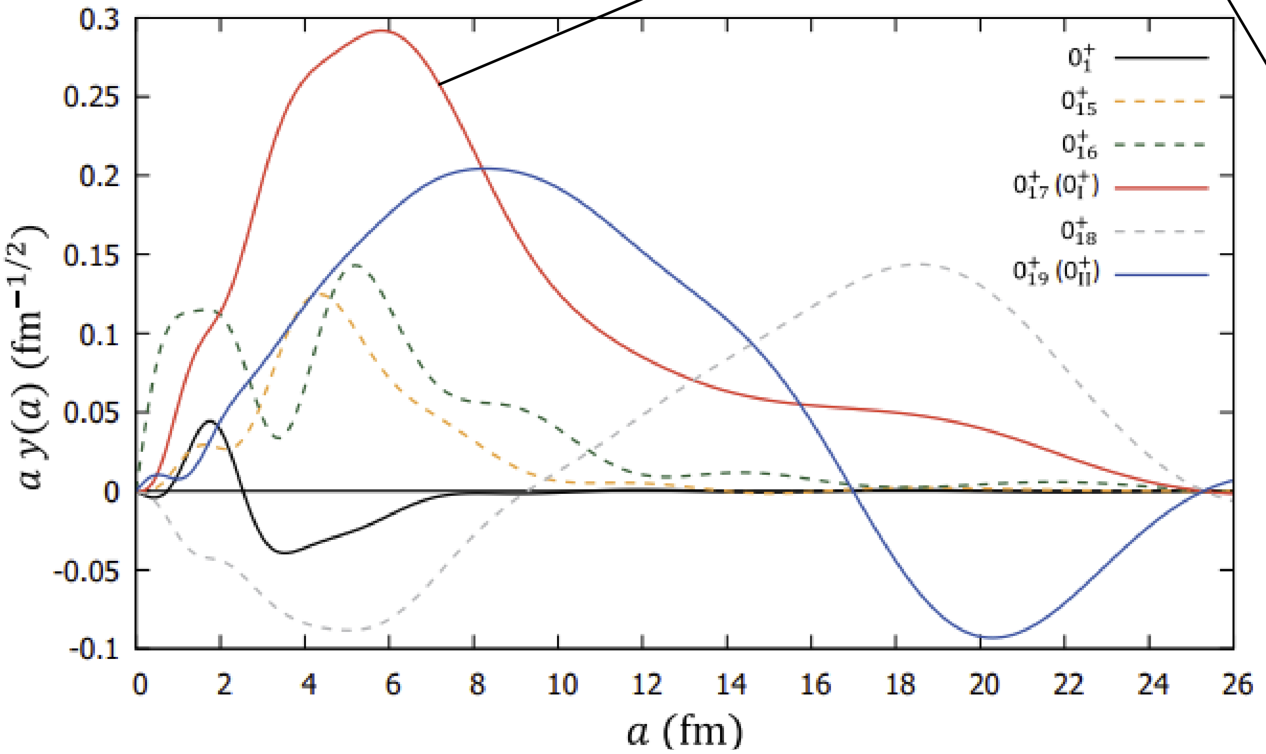


The 5alpha condensate state

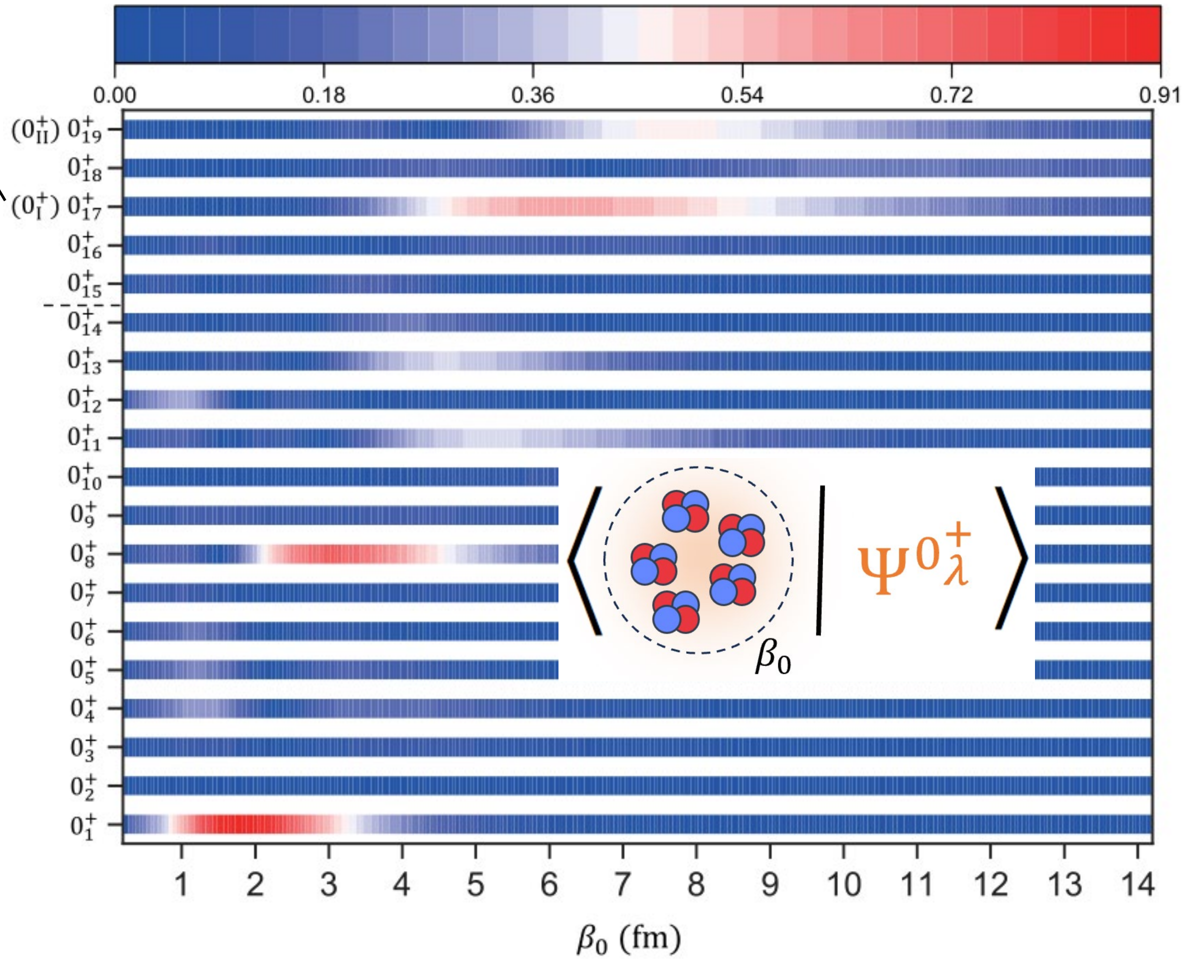
Reduced width amplitude

larger amplitude

Simple way to confirm condensate state



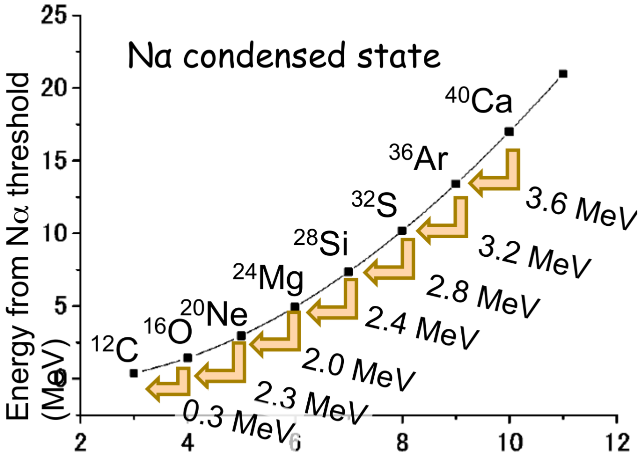
$$y(a) = \sqrt{\frac{20!}{4!16!}} \left\langle \left[\left[\Psi^{0_s^+}_{\rm gcm}(^{16}\mathrm{O})\varphi_5(\alpha)\right]_{0^+} Y_{00}(\hat{\xi}_4)\right]_{0^+} \frac{\delta(\xi_4 - a)}{\xi_4^2} \middle| \Psi^{0_\lambda^+}_{\rm gcm}(^{20}\mathrm{Ne}) \right\rangle$$



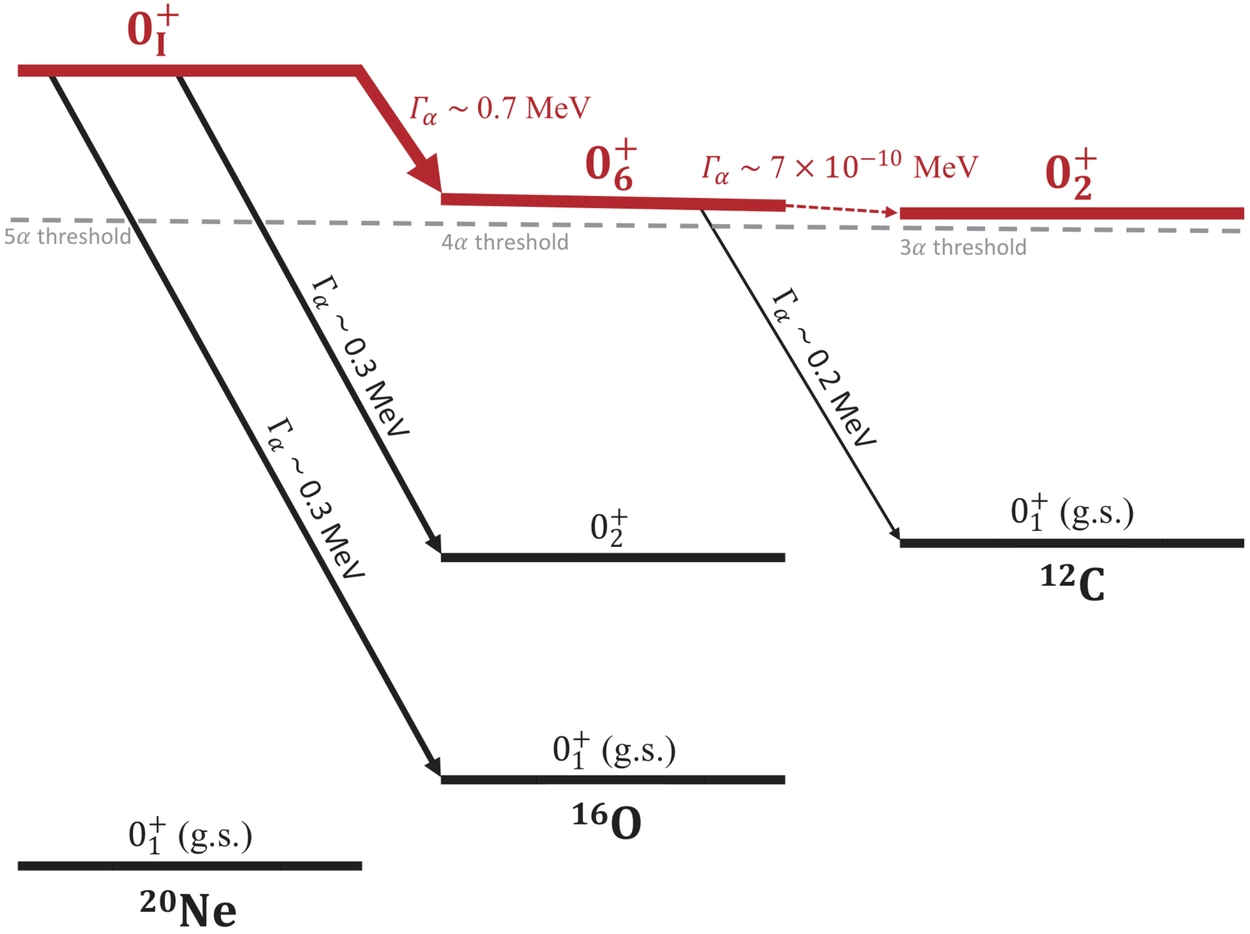
B. Zhou, et al., *Nat. Commun.* **14**, 8206 (2023).

The decay scheme and connections

Exotic clustering structure ?

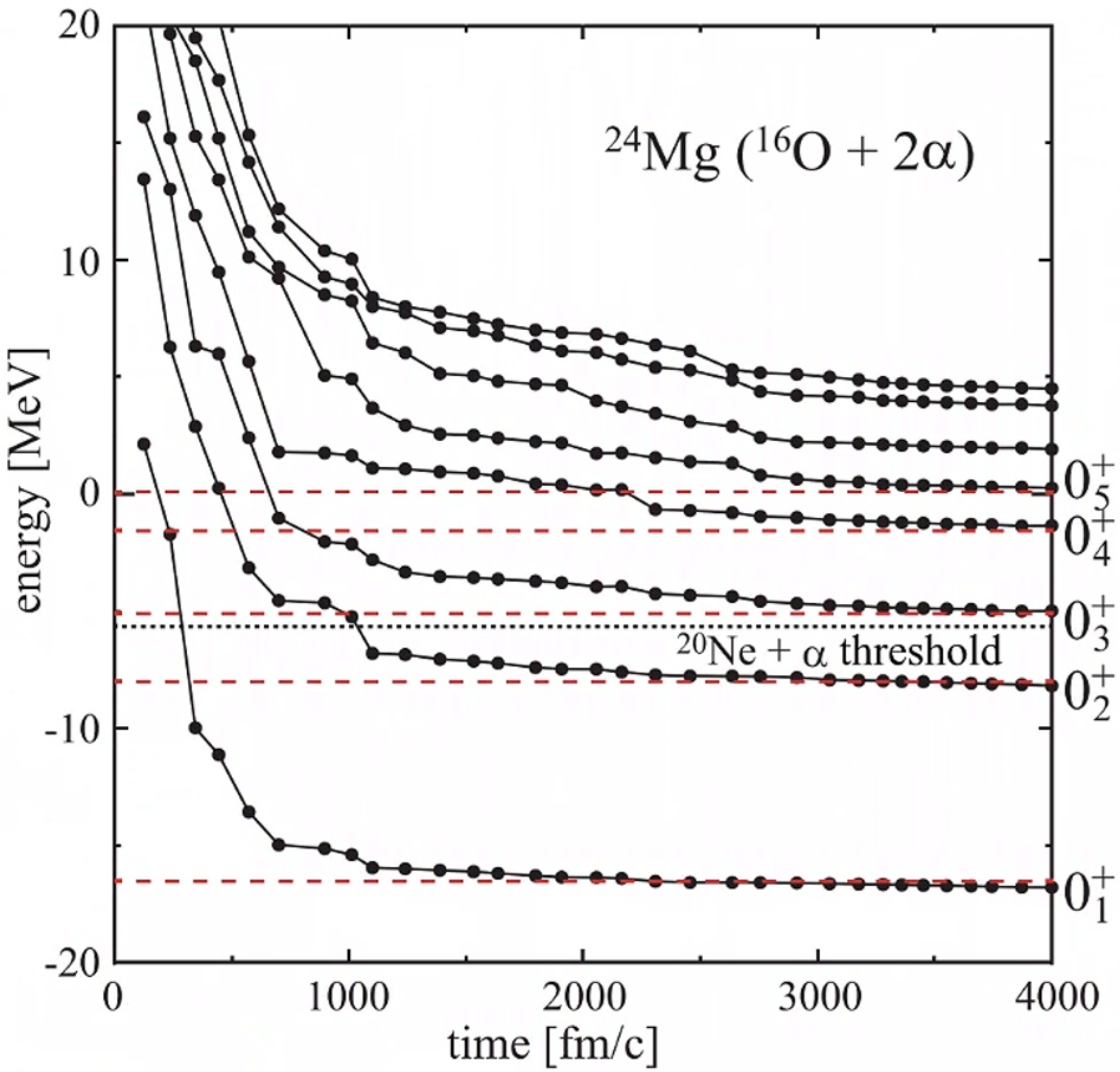


T. Yamada and P. Schuck, PRC 69, 024309 (2004).

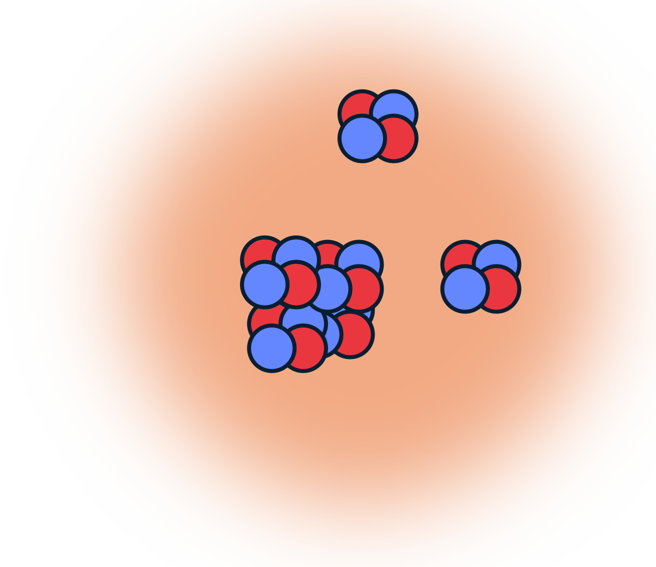


B. Zhou, et al., *Nat. Commun.* **14**, 8206 (2023).

Candidates of a 2α condensate surrounding the ^{16}O nucleus



State	E	Γ	B(IS0)	$\sqrt{\langle r^2\rangle}$	$\sqrt{\langle r^2\rangle_{\text{val}}}$
0_1^+	−16.78			2.67	3.47
0_2^+	−8.16		28.4	2.70	3.53
0_3^+	−4.95	$\leqslant$0.01	145.0	2.97	4.13
0_4^+	−1.26	0.17	160.7	2.94	4.08
0_5^+	0.37		4.28	3.03	4.28

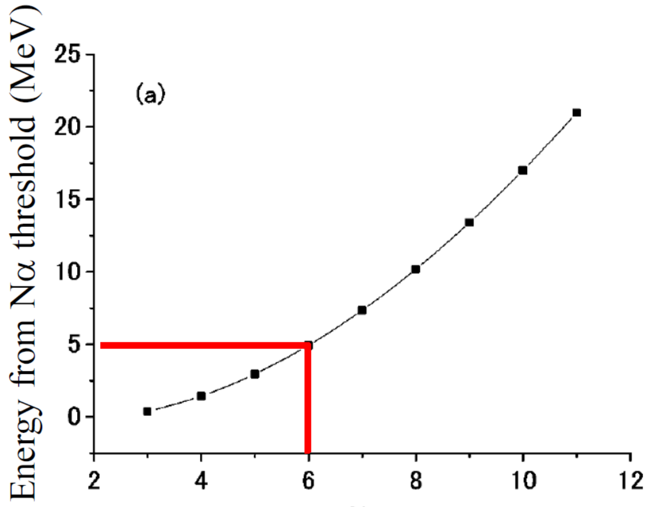


2α condensation around the core nucleus ^{16}O

Htet, et al., PRC112, 014322 (2025)

The 6α clustering structure probed by Inelastic Scattering

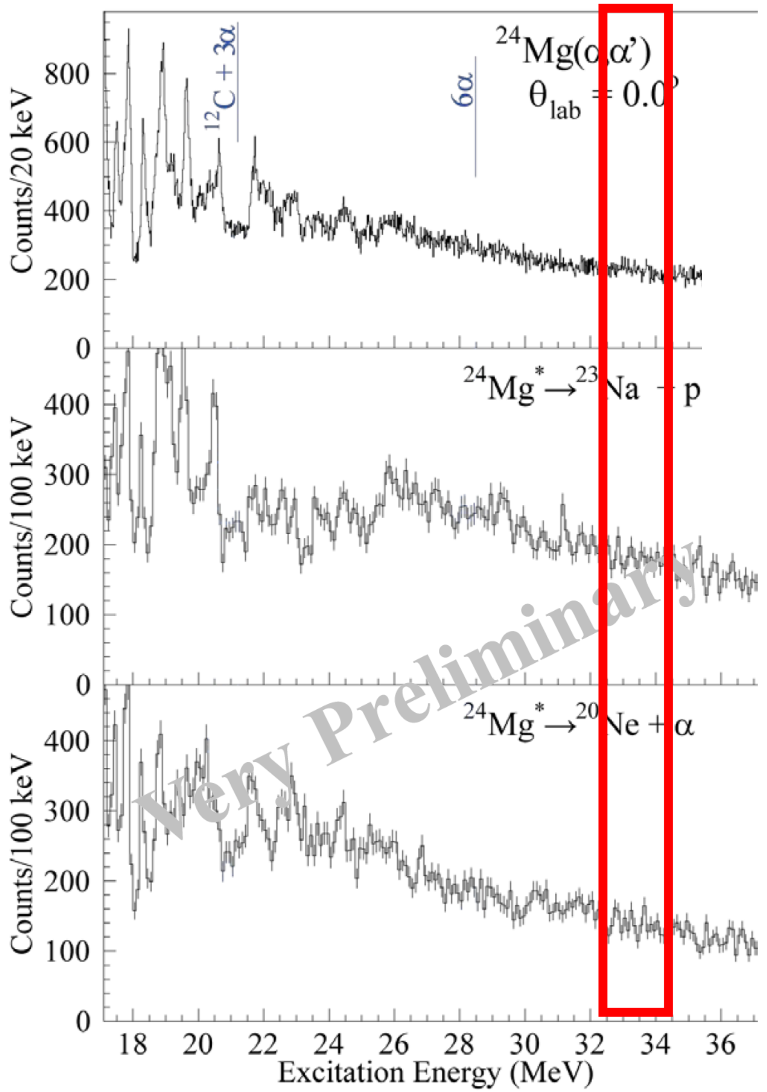
6α condensed state was searched for in the highly excited region.



T. Yamada and P. Schuck, Phys. Rev. C **69**, 024309 (2004).

- 6α condensed state is expected at 5 MeV above the 6α threshold.
  - $E_x \sim 28.5 + 5 = 33.5$ MeV
- No significant structure suggesting the 6α condensed state.
  - Several small structures indistinguishable from the statistical fluctuation. → Need more statistics.

by measuring the 12C+12C scattering



PLB,848 (2024)

A. Tohsaki et al. / Nuclear Physics A738 (2004) 259–263 261

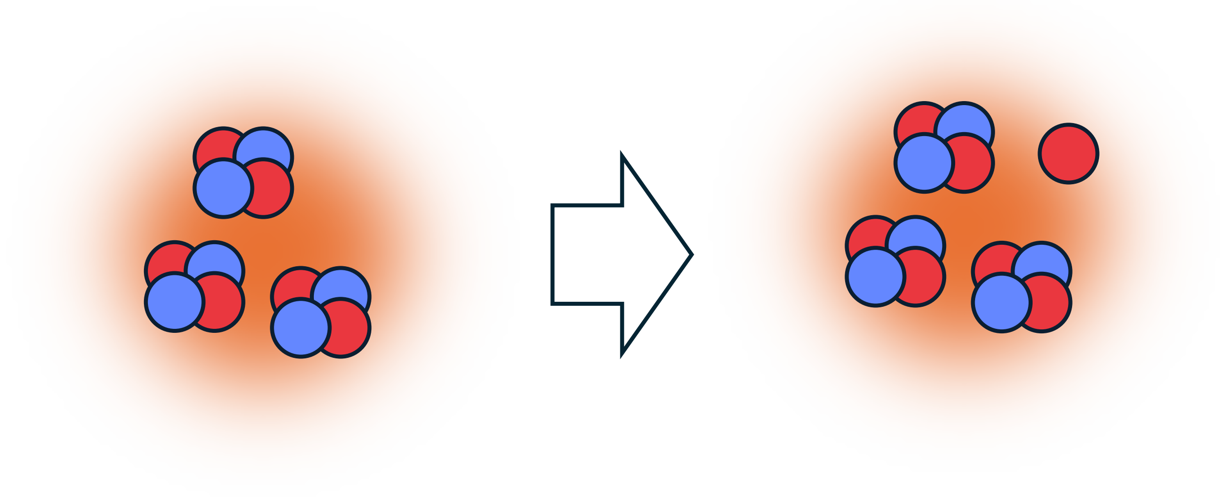
Table 1
The independent number of permutations for each kernel. Here, the case of the norm kernel for ^{24}Mg is added. The final row shows a full number of permutations without any reduction for the norm kernel.

	^{8}Be(2α)	^{12}C(3α)	^{16}O(4α)	^{20}Ne(5α)	^{24}Mg(6α)
norm	3	9	35	185	1614
kinetic	7	34	242	2546	
two-body	9	58	669	10912	
three-body	40	366	6773	156617	
$(n!)^4$	16	1296	3.32×10^5	2.07×10^8	2.79×10^{11}

$$\langle \Psi^{\mathrm{THSR}}_{n\alpha}(\beta)\,|\mathcal{O}|\,\Psi^{\mathrm{THSR}}_{n\alpha}(\beta')\rangle = \sum_{p=0}^{m_p^{(1)}-1} W_p^{(1)} I_p^{(1)}$$

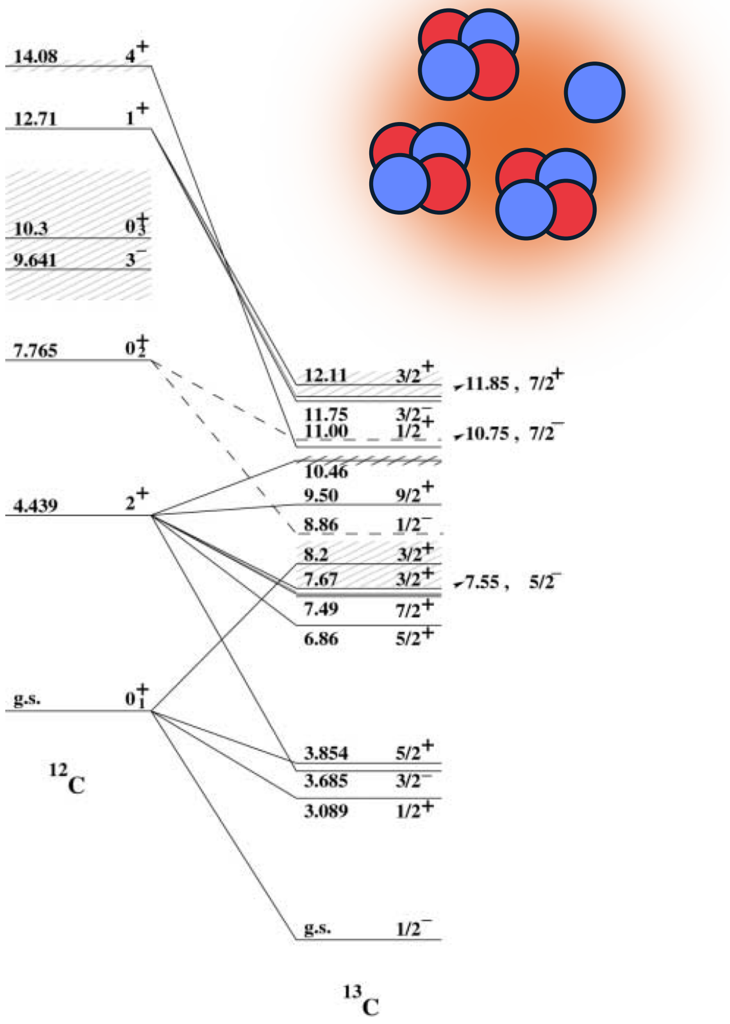
Still tough in theory—
but AI makes a big difference.

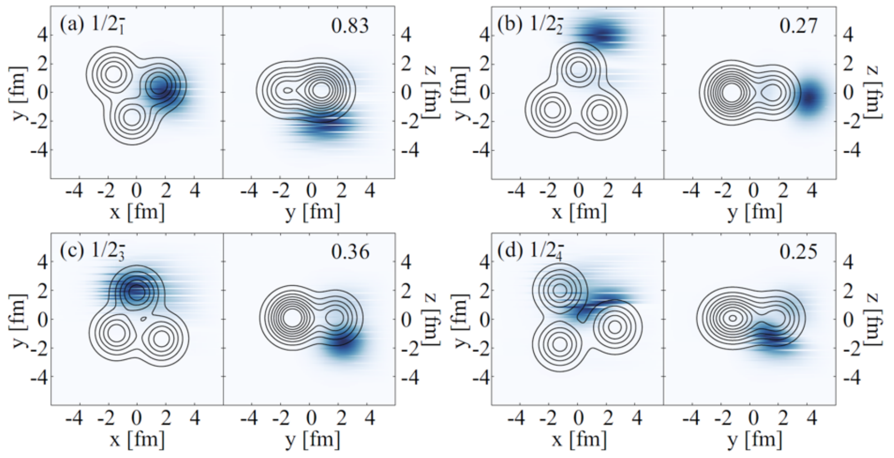
KAWABATA Takahiro

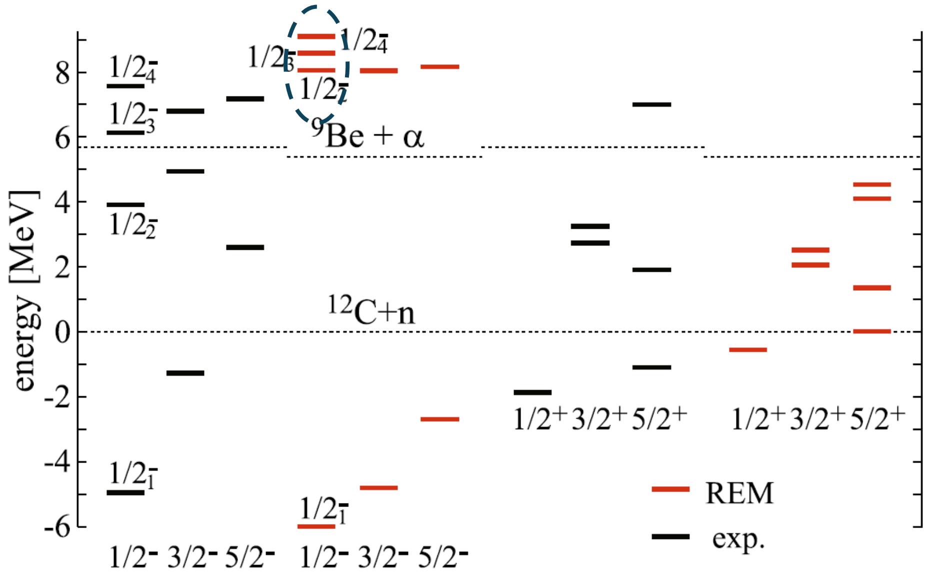


Clustering structure of $3\alpha+p$ in ^{13}N

Search for the Hoyle-analogy state in ^{13}C





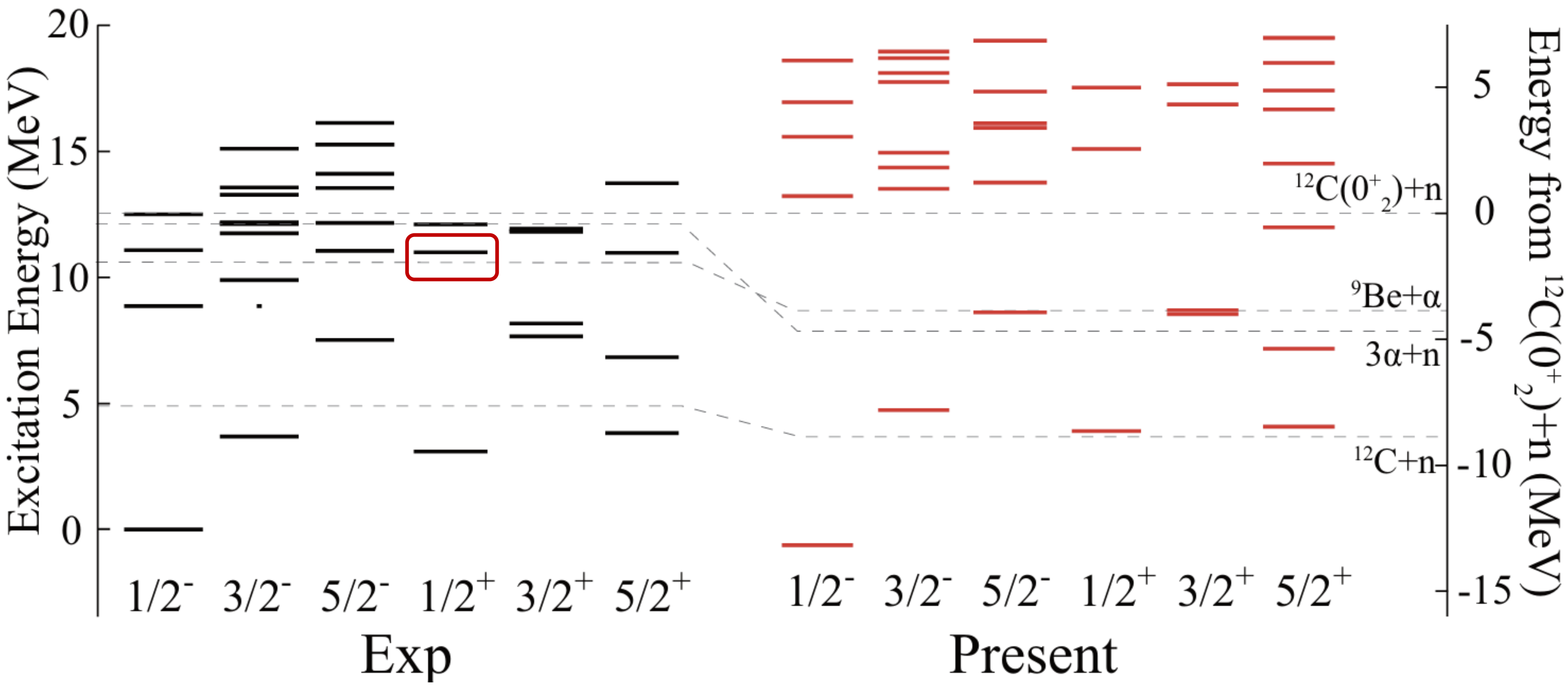


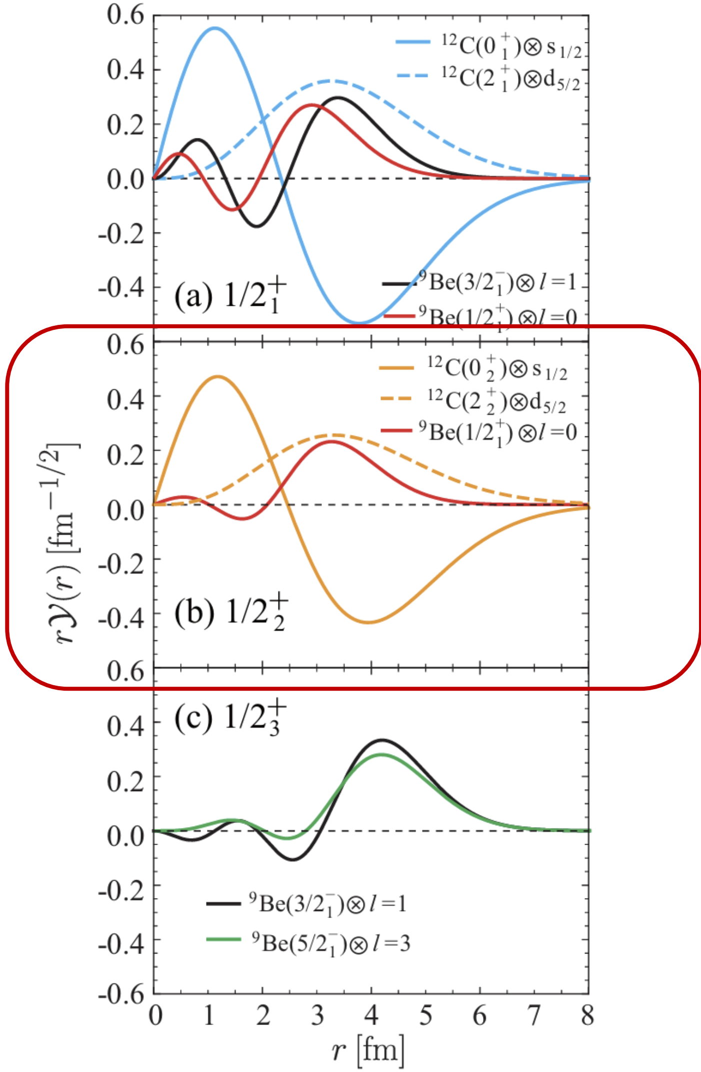
^{12}C(states) + single particle states ⇨ ^{13}C (states)

M. Milin and W. von Oertzen, Eur Phys J A **14**, 295 (2002).

S. Shin, et al., Phys. Rev. C **103**, 054313 (2021).

Search for the Hoyle-analogy state in ^{13}C

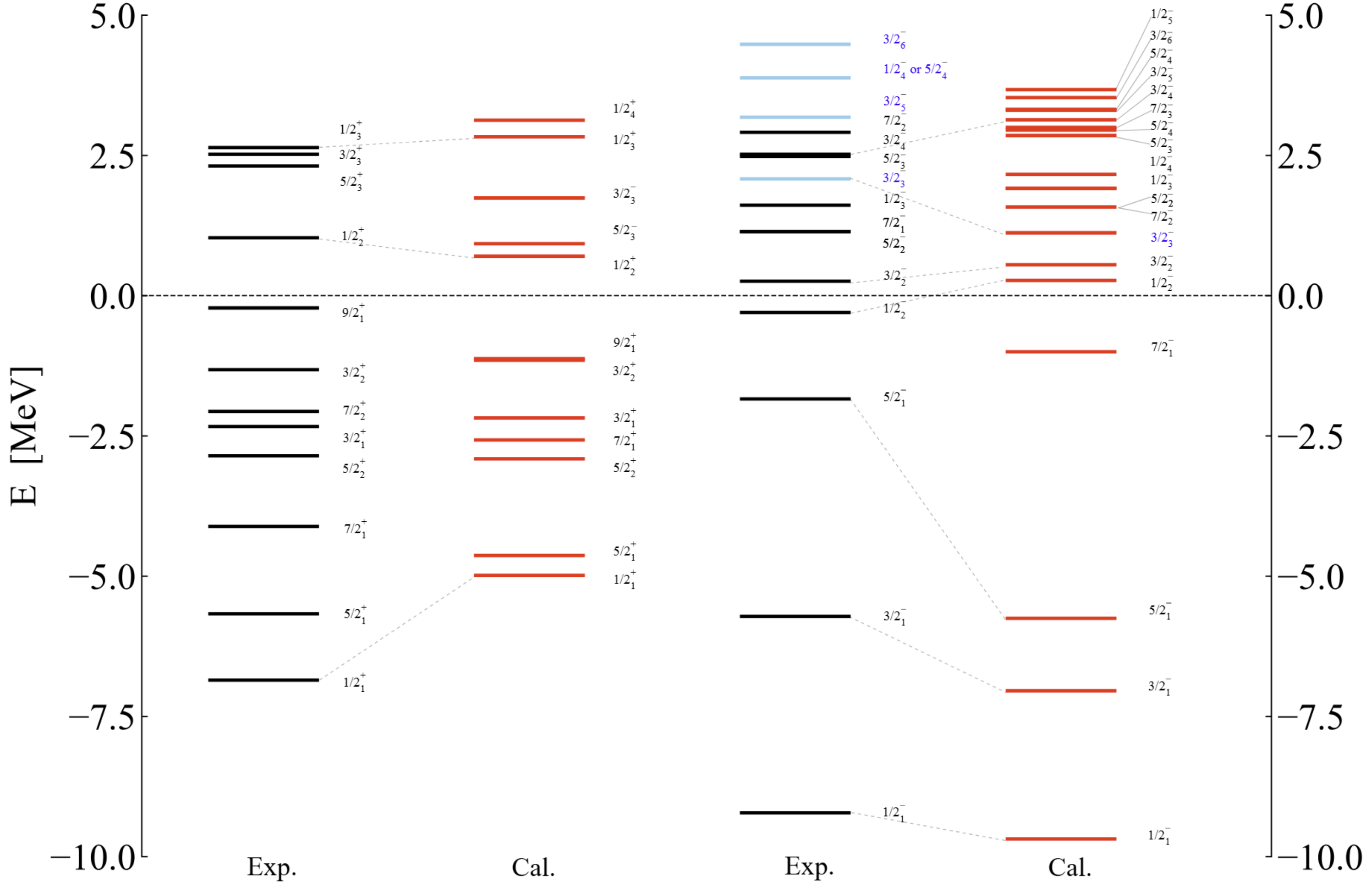


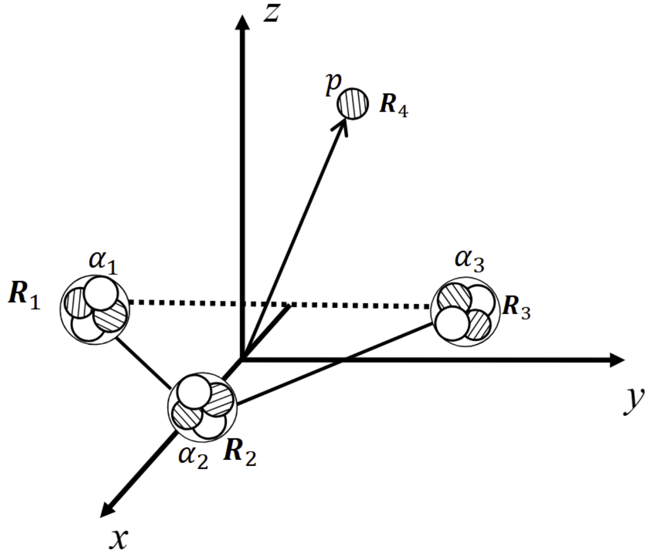


It is found that the $1/2^+_2$ state is dominantly composed of the Hoyle state configuration and can be regarded as a Hoyle analogue state.

Chiba et al.,Phys. Rev. C 101, 024317 (2020)

Hoyle-analog state in ^{13}N





Transition	Present	Experiment
$B(E1, 3/2_1^- \rightarrow 1/2_1^+)$	0.016	0.036
$B(E1, 1/2_1^+ \rightarrow 1/2_1^-)$	0.0007	0.036 ± 0.004
$B(E2, 3/2_1^- \rightarrow 1/2_1^-)$	4.87	
$B(E2, 5/2_1^- \rightarrow 1/2_1^-)$	3.71	
$B(E2, 1/2_1^- \rightarrow 3/2_1^+)$	21.58	

Y.Y Cao et al. Phys. Rev. C 112, 034313 (2025)

Hoyle-analog state in ^{13}N

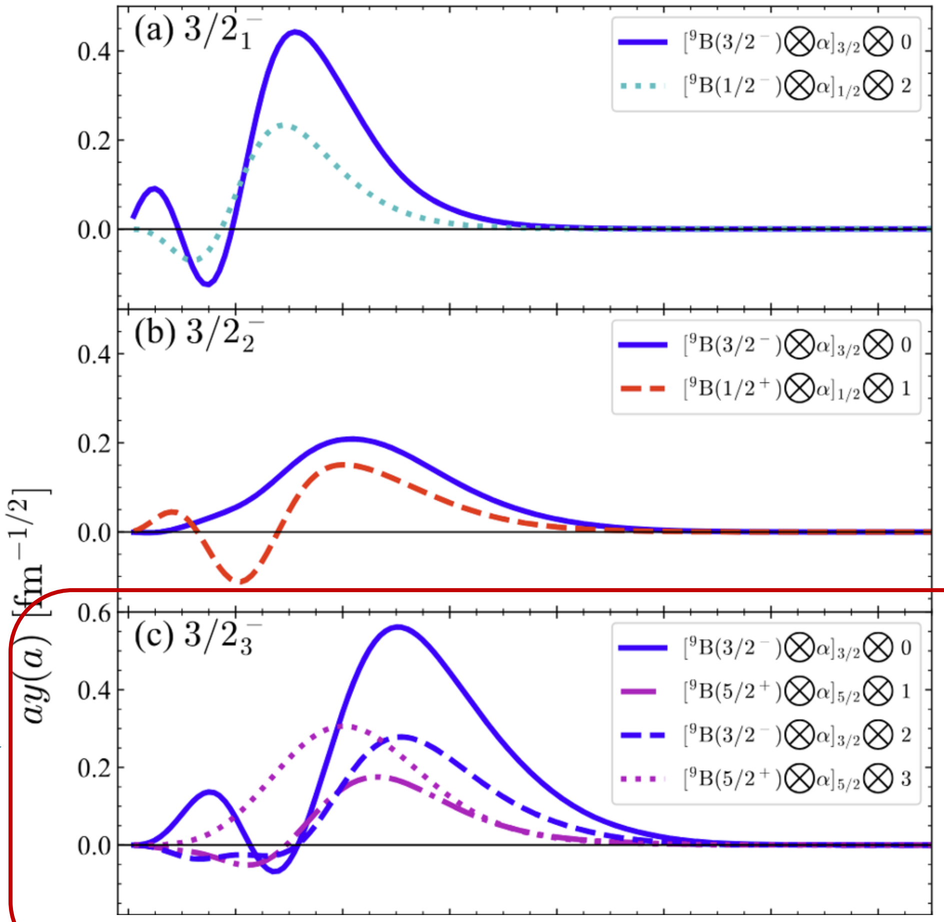
PHYSICAL REVIEW C **109**, 054308 (2024)

Cluster structure of $3\alpha + p$ states in ^{13}N

J. Bishop,[1,2] G. V. Rogachev,[1,3,4] S. Ahn,[5] M. Barbui,[1] S. M. Cha,[5] E. Harris,[1,3] C. Hunt,[1,3] C. H. Kim,[6] D. Kim,[5] S. H. Kim,[6] E. Koshchiy,[1] Z. Luo,[1,3] C. Park,[5] C. E. Parker,[1] E. C. Pollacco,[7] B. T. Roeder,[1] M. Roosa,[1,3] A. Saastamoinen,[1] and D. P. Scriven[1,3]

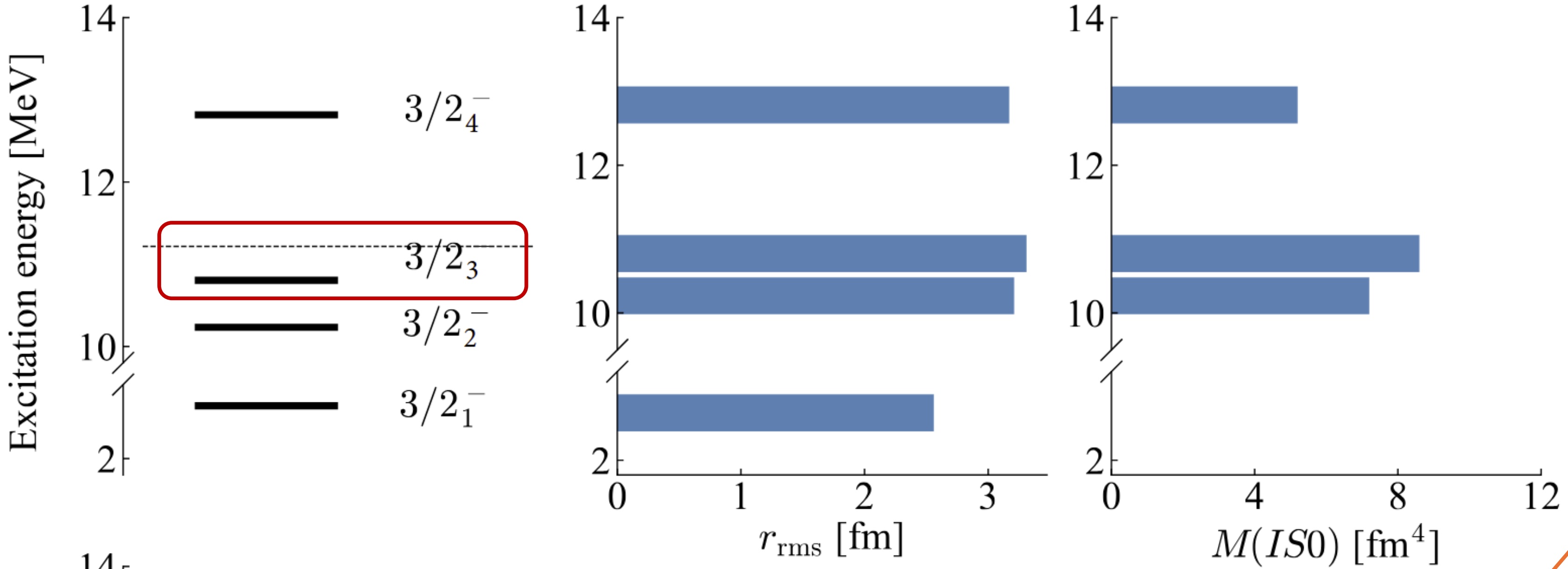
[1]*Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA*
[2]*School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom*
[3]*Department of Physics & Astronomy, Texas A&M University, College Station, Texas 77843, USA*
[4]*Nuclear Solutions Institute, Texas A&M University, College Station, Texas 77843, USA*
[5]*Center for Exotic Nuclear Studies, Institute for Basic Science, 34126 Daejeon, Republic of Korea*
[6]*Department of Physics, Sungkyunkwan University, Suwon 16419, Republic of Korea*
[7]*IRFU, CEA, Université Paris-Saclay, F-91191 Gif-Sur-Yvette, France*

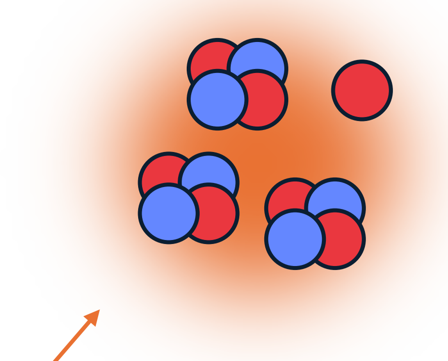
Background: Cluster states in ^{13}N are extremely difficult to measure due to the unavailability of ^{9}B $+\alpha$ elastic-scattering data.
Purpose: Using β-delayed charged-particle spectroscopy of ^{13}O, clustered states in ^{13}N can be populated and measured in the $3\alpha + p$ decay channel.
Methods: One-at-a-time implantation and decay of ^{13}O was performed with the Texas Active Target Time Projection Chamber. 149 $\beta 3\alpha p$ decay events were observed and the excitation function in ^{13}N reconstructed.
Results: Four previously unknown α-decaying excited states were observed in ^{13}N at an excitation energy of 11.3, 12.4, 13.1, and 13.7 MeV decaying via the $3\alpha + p$ channel.
Conclusions: These states are seen to have a $[^9\mathrm{B(g.s)} \otimes \alpha / p + {}^{12}\mathrm{C}(0_2^+)]$, $[^9\mathrm{B}(\frac{1}{2}^+) \otimes \alpha]$, $[^9\mathrm{B}(\frac{5}{2}^+) \otimes \alpha]$, and $[^9\mathrm{B}(\frac{5}{2}^+) \otimes \alpha]$ structure, respectively. A previously seen state at 11.8 MeV was also determined to have a $[p + {}^{12}\mathrm{C(g.s.)} / p + {}^{12}\mathrm{C}(0_2^+)]$ structure. The overall magnitude of the clustering is not able to be extracted, however, due to the lack of a total width measurement. Clustered states in ^{13}N (with unknown magnitude) seem to persist from the addition of a proton to the highly α-clustered ^{12}C. Evidence of the $\frac{1}{2}^+$ state in ^{9}B was also seen to be populated by decays from ^{13}N*.

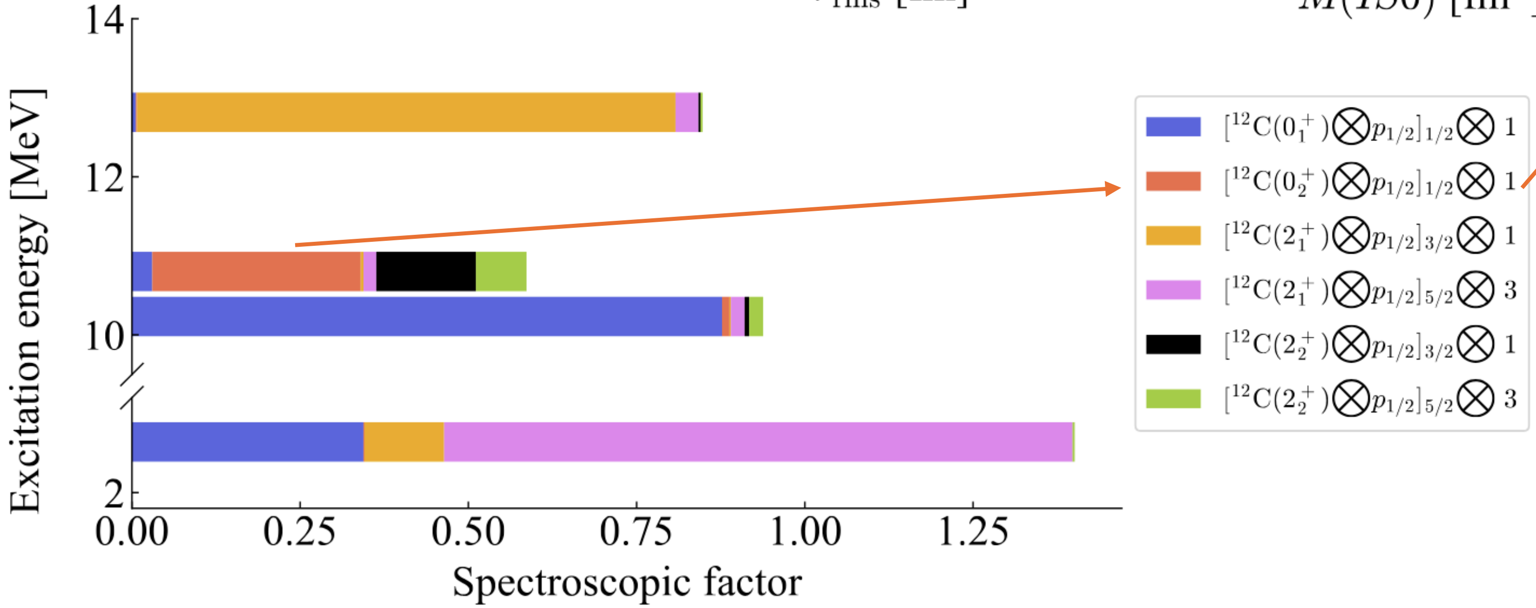


This obtained state corresponds to the state observed at 11.3 MeV

Hoyle-analog state in ^{13}N





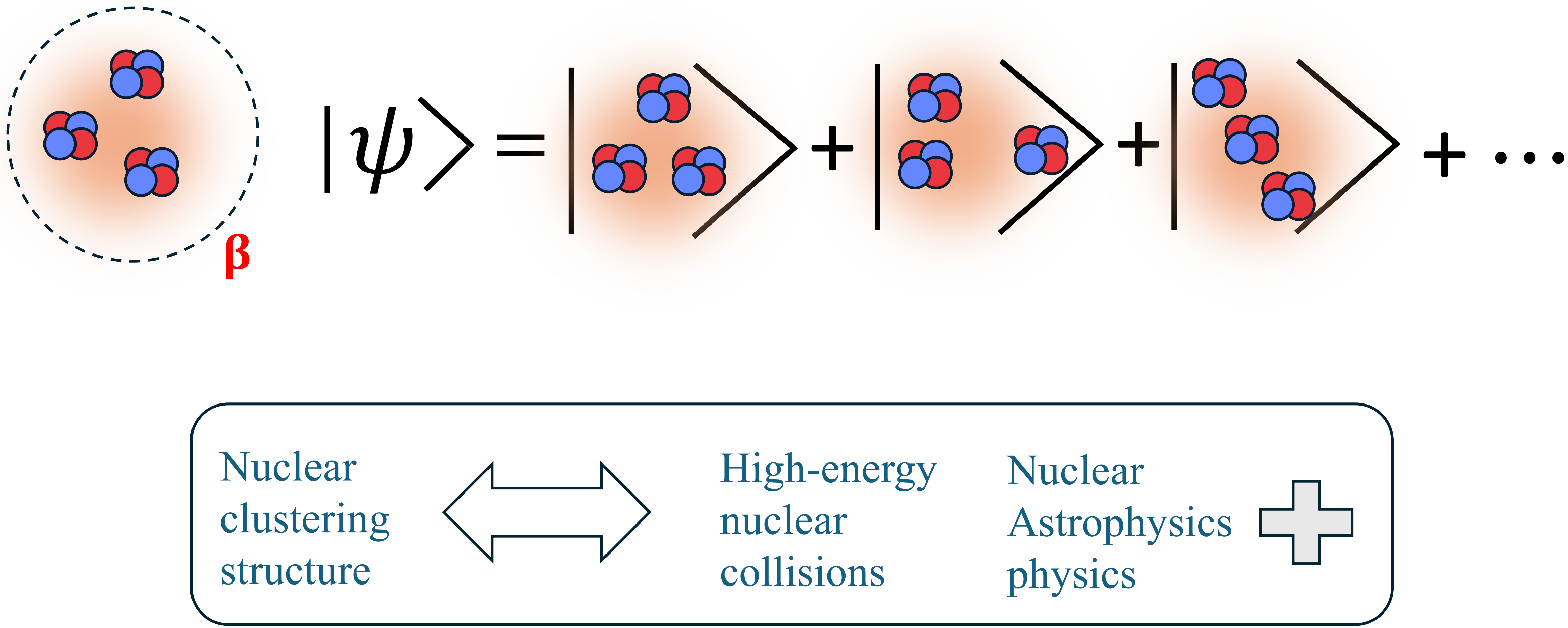


$\langle \text{Hoyle} + p | ^{13}\text{N state} \rangle$

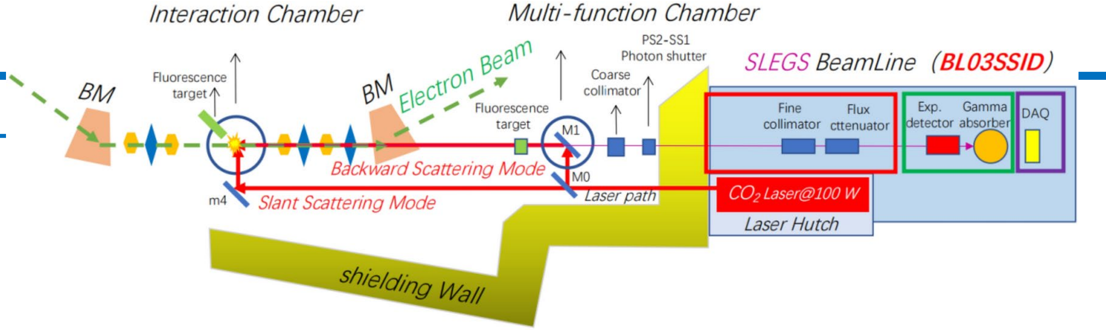
$$y^{J\pi}_{j_1\pi_1 j_2\pi_2 j_{12} l}(a) = \sqrt{\frac{A!}{(1+\delta_{C_1C_2})\,C_1!C_2!}} \times \left\langle \frac{\delta(r-a)}{r^2}\left[Y_l(\hat{r})\left[\Phi^{j_1\pi_1}_{C_1}\Phi^{j_2\pi_2}_{C_2}\right]_{j_{12}}\right]_{JM}\middle|\, \Psi^{J\pi}_M \right\rangle$$

Y. Y Cao et al. Phys. Rev. C 112, 034313 (2025)

light nuclei, ground and excited states, superposed many clustering configurations



Lorentz integral transformation (LIT)



The response function of photonuclear reaction

$$R(\omega, \mathbf{q}) = \sum_f |\langle \Psi_0 | O(\mathbf{q}) | \Psi_f \rangle|^2 \, \delta(E_f - E_0 - \omega),$$

in which ω is energy of incident photon, $O(\mathbf{q})$ is the respondence of nuclear system against photon injection. For evaluating the response function, define the Lorentz integral transformation

$$\mathcal{L}(\sigma, \mathbf{q}) = \int_{0^-}^{\infty} d\omega \frac{R(\omega, \mathbf{q})}{(\omega - \sigma_R)^2 + \sigma_I^2},$$

After integrating over ω, and letting $\sigma = -\sigma_R + i\sigma_I$, we obtain

$$\mathcal{L}(\sigma, \mathbf{q}) = \sum_f \langle \Psi_0 | O^\dagger(\mathbf{q}) \frac{1}{(E_f - E_0 + \sigma^*)} |\Psi_f\rangle\langle\Psi_f| \frac{1}{(E_f - E_0 + \sigma)} O(\mathbf{q}) |\Psi_0\rangle$$

$$= \sum_f \langle \Psi_0 | O^\dagger(\mathbf{q}) \frac{1}{(H - E_0 + \sigma^*)} |\Psi_f\rangle\langle\Psi_f| \frac{1}{(H - E_0 + \sigma)} O(\mathbf{q}) |\Psi_0\rangle$$

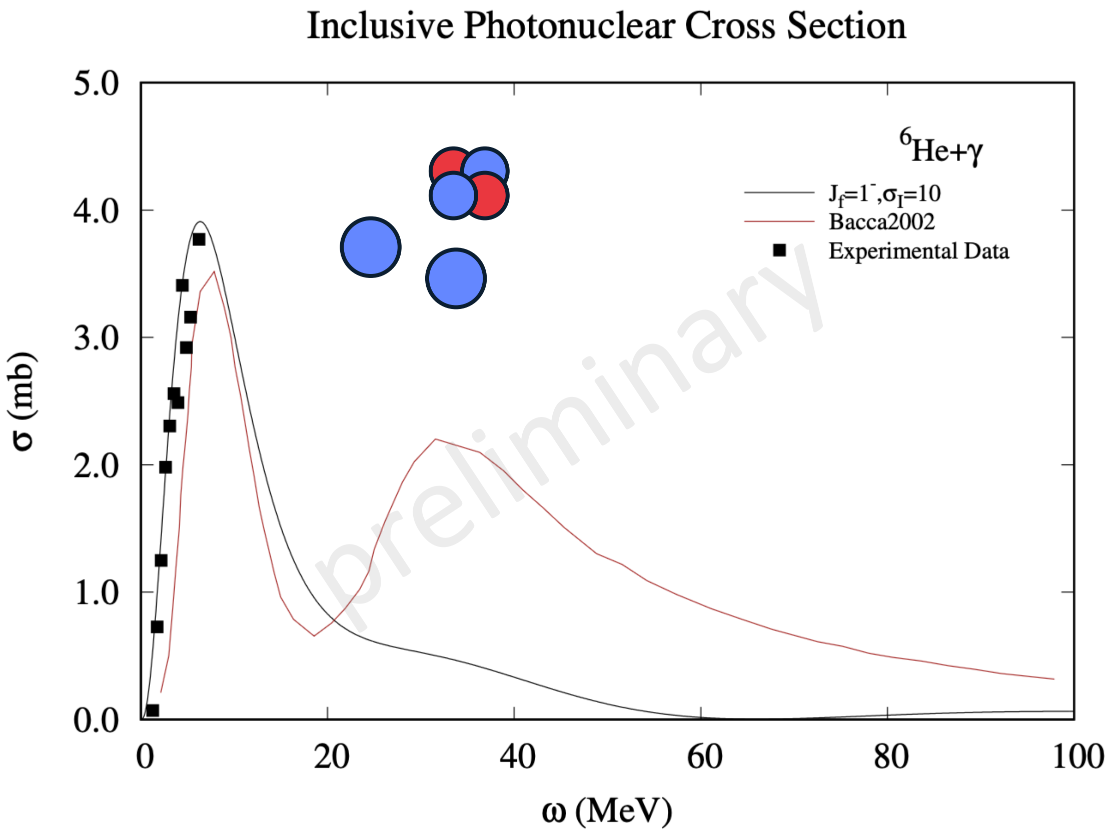
$$= \langle \Psi_0 | O^\dagger(\mathbf{q}) \frac{1}{(H - E_0 + \sigma^*)} \frac{1}{(H - E_0 + \sigma)} O(\mathbf{q}) |\Psi_0\rangle$$

$$\left(\hat{H} - E_0 - \sigma_R + i\sigma_I\right) |\tilde{\Psi}\rangle = \hat{D}_z |\Psi_0\rangle$$

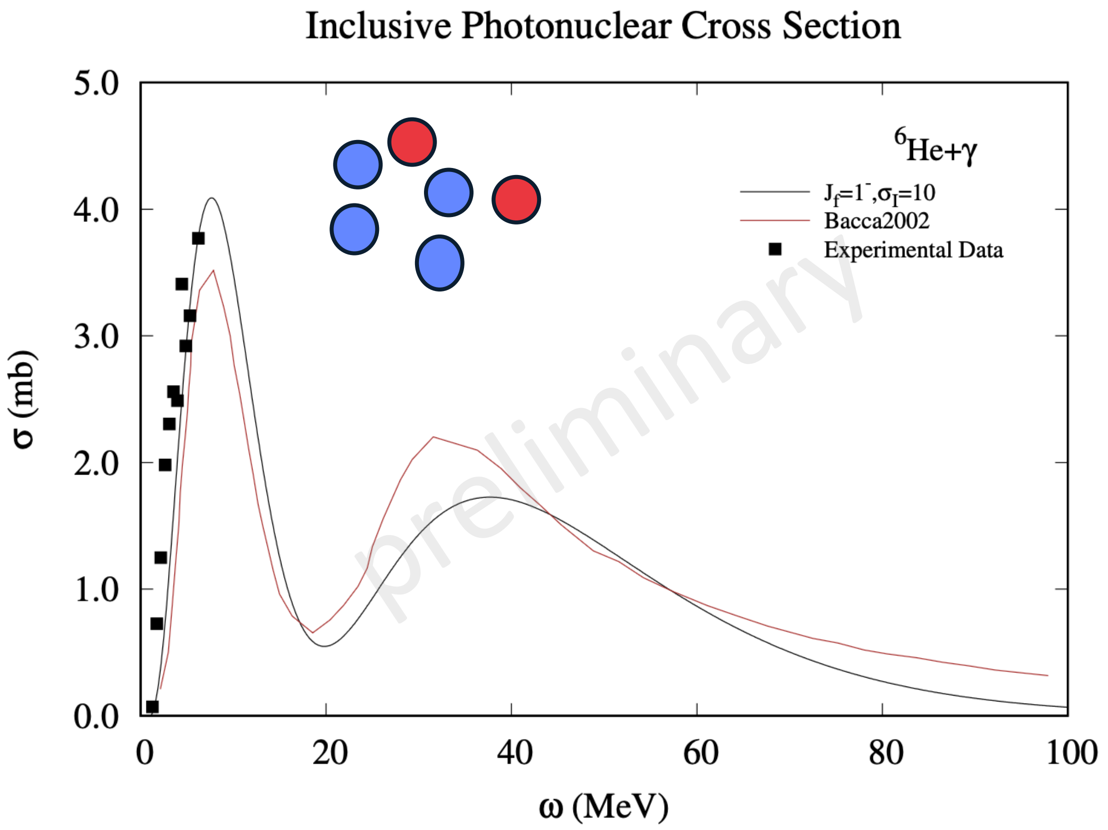
$$\mathcal{L}(\sigma, \mathbf{q}) = \left\langle \tilde{\Psi} \,\middle|\, \tilde{\Psi} \right\rangle$$

V. D. Efros, et al., J. Phys. G. 34, R459 (2007).

Photonuclear reaction cross sections of ^{6}He and ^{6}Li



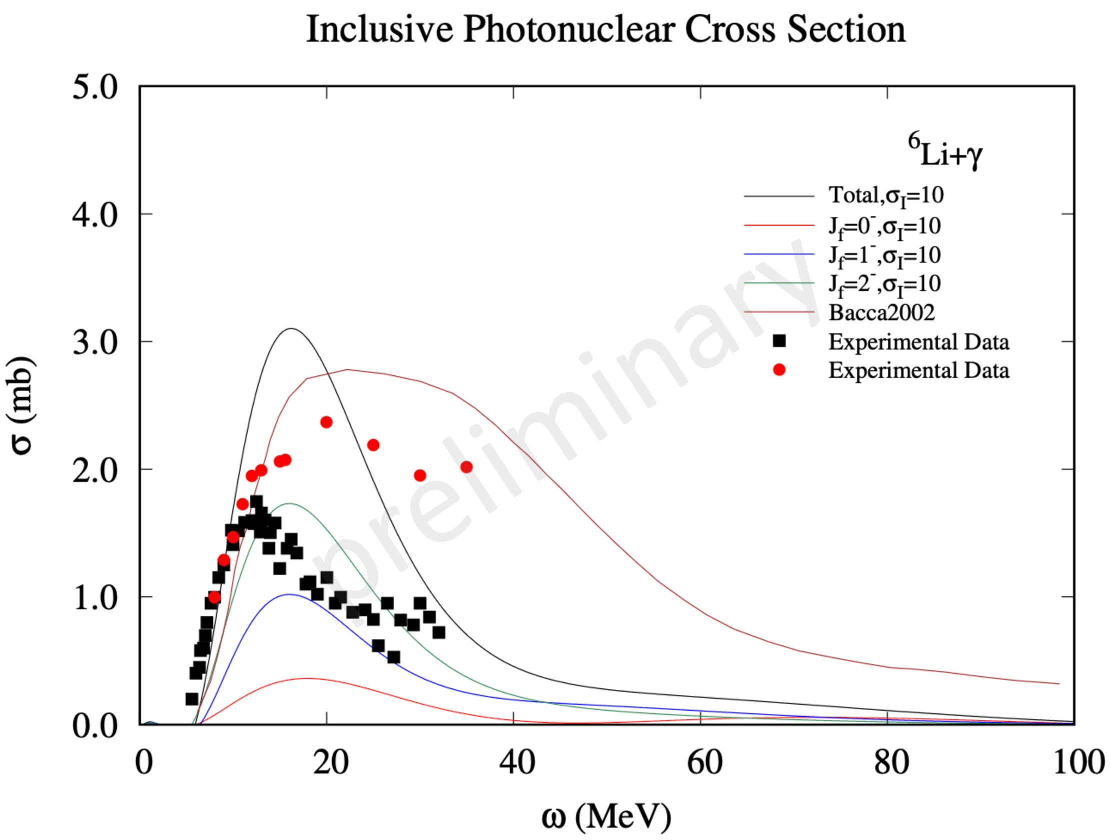
^{6}He+γ cross sections calculated by LIT with a+n+n cluster model space



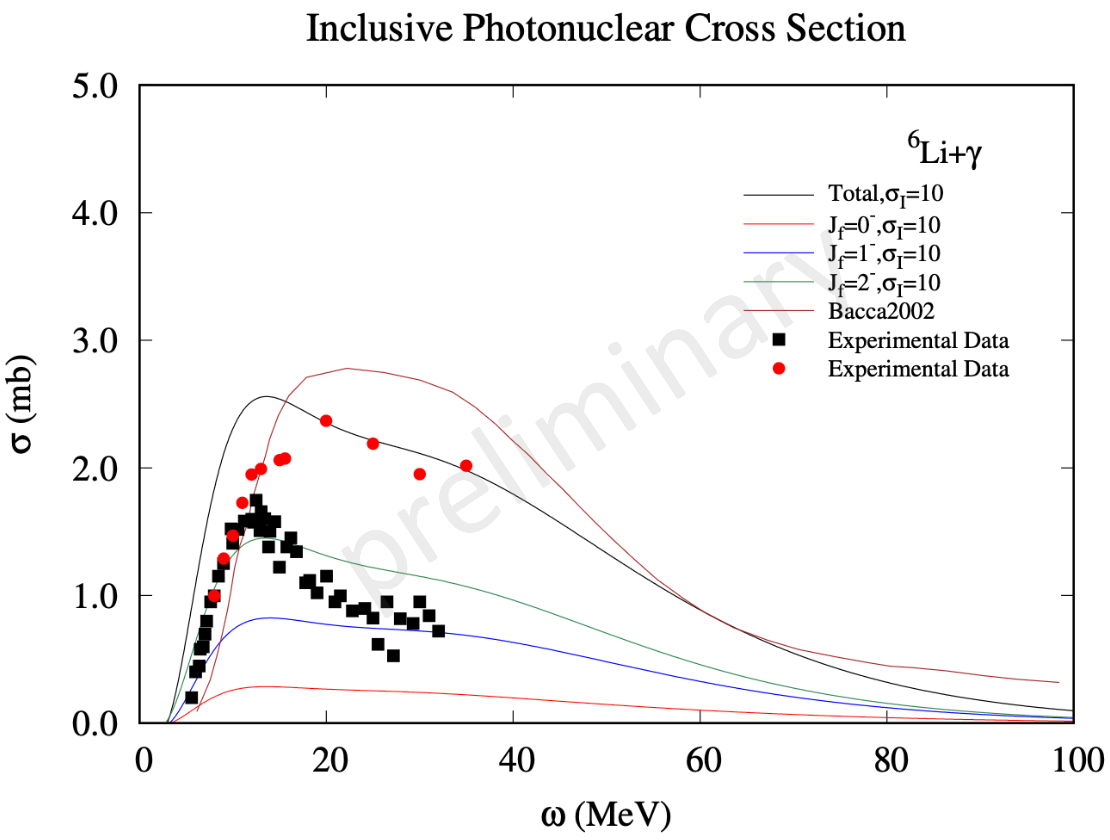
^{6}He+γ cross sections calculated by LIT with six-body model space

S. Bacca, et al., Phys.Rev.Lett.89, 052502 (2002).

Photonuclear reaction cross sections of ^{6}He and ^{6}Li



^{6}Li+γ cross sections calculated by LIT with a+n+p cluster model space



^{6}Li+γ cross sections calculated by LIT with six-body model space

S. Bacca, et al., Phys.Rev.Lett.89, 052502 (2002).

PHYSICAL REVIEW LETTERS **131**, 242501 (2023)

Editors' Suggestion Featured in Physics

Observation of the Exotic 0_2^+ Cluster State in ^{8}He

Z. H. Yang,[1,2,*,†] Y. L. Ye,[1,*,‡] B. Zhou,[3,4,5] H. Baba,[2] R. J. Chen,[6] Y. C. Ge,[1] B. S. Hu,[1] H. Hua,[1] D. X. Jiang,[1] M. Kimura,[2,5,7] C. Li,[2] K. A. Li,[6] J. G. Li,[1] Q. T. Li,[1] X. Q. Li,[1] Z. H. Li,[1] J. L. Lou,[1] M. Nishimura,[2] H. Otsu,[2] D. Y. Pang,[8] W. L. Pu,[1] R. Qiao,[1] S. Sakaguchi,[2,9] H. Sakurai,[2] Y. Satou,[10] Y. Togano,[2] K. Tshoo,[10] H. Wang,[2,11] S. Wang,[2] K. Wei,[1] J. Xiao,[1] F. R. Xu,[1] X. F. Yang,[1] K. Yoneda,[2] H. B. You,[1] and T. Zheng[1]

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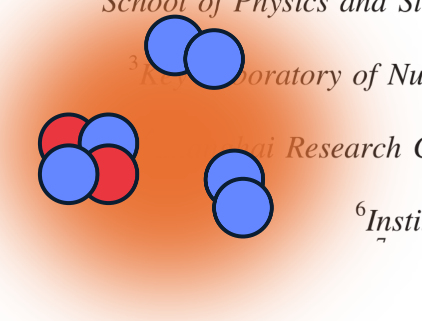
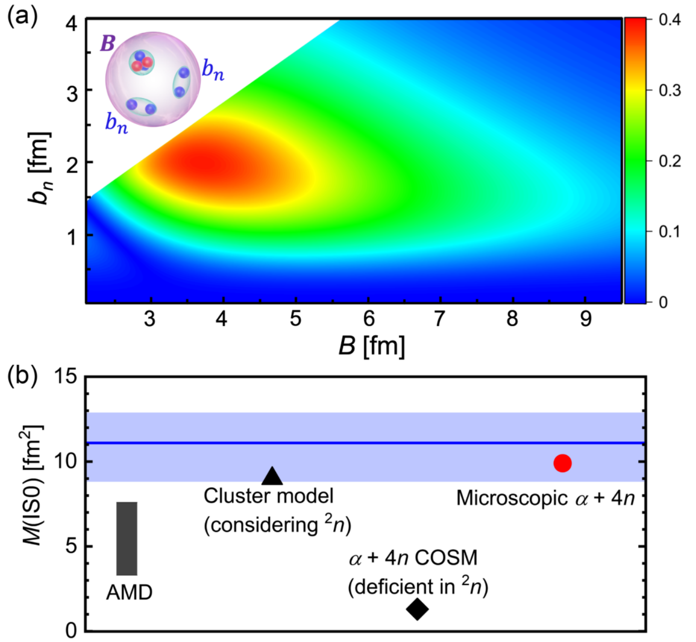
...ai Research Center for ... a

[5]*Departm...*

[6]*Institute of Mo...*



$$\Phi(\boldsymbol{B}, b_n) \propto \mathcal{A}\left\{\exp\left[-\frac{4\xi_1^2}{3B^2} - \frac{3\xi_2^2}{2B^2}\right] \times \phi_\alpha(b_\alpha)\phi_{^2_1n}(b_n)\phi_{^2_2n}(b_n)\right\},$$



PTEP

Prog. Theor. Exp. Phys. **2018**, 041D01 (10 pages)
DOI: 10.1093/ptep/pty034

Letter

New trial wave function for the nuclear cluster structure of nuclei

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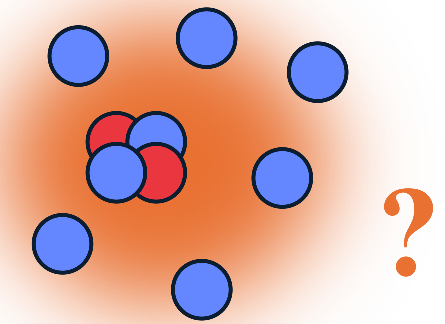
Received December 5, 2017; Revised February 21, 2018; Accepted March 2, 2018; Published April 16, 2018

A new trial wave function is proposed for nuclear cluster physics, in which an exact solution to the long-standing center-of-mass problem is given. In the new approach, the widths of the

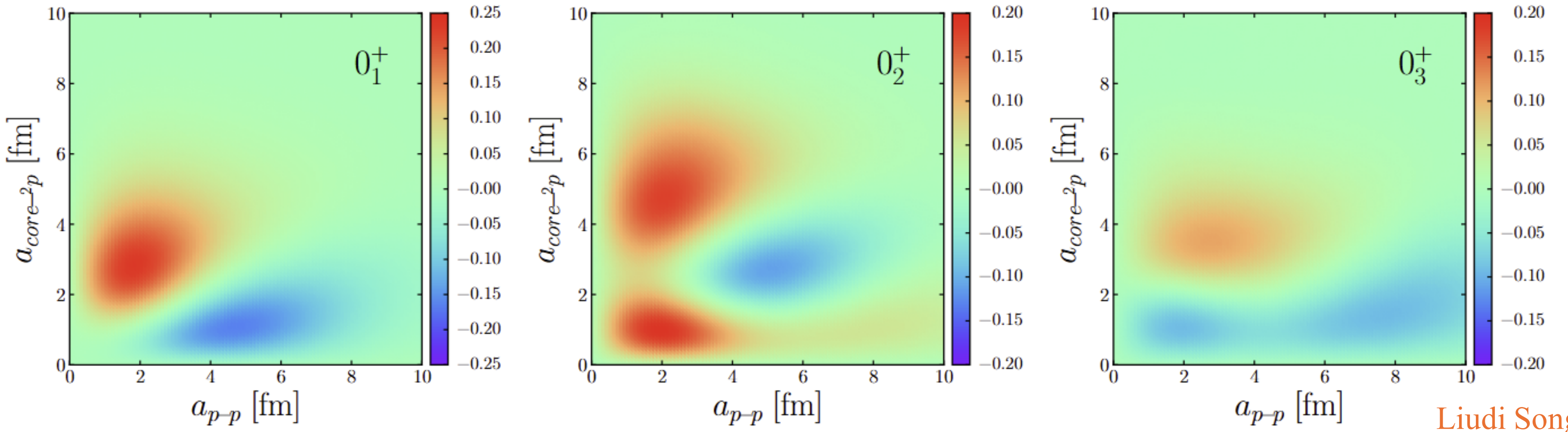
$$\Psi(\mathbf{r}) = \Phi_{\mathrm{g}}(\mathbf{r}_g)\Phi_{\mathrm{int}}(\mathbf{r}_i - \mathbf{r}_j)$$

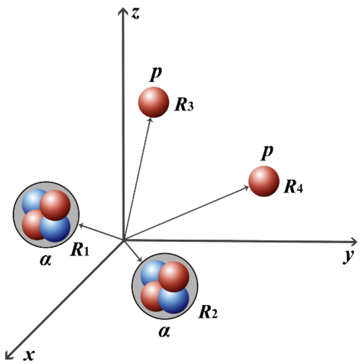
$$\begin{aligned}\Psi_{\mathrm{new}} &= \widehat{L}_{n-1}(\beta)\widehat{G}_n(\beta_0)\widehat{D}(\boldsymbol{Z})\Phi_0(\boldsymbol{r})\\ &= \int d^3\widetilde{T}_1\cdots d^3\widetilde{T}_{n-1}\exp[-\sum_{i=1}^{n-1}\frac{\widetilde{T}_i^2}{\beta_i^2}]\int d^3R_1\cdots d^3R_n\exp[-\sum_{i=1}^{n}(\frac{A_i}{\beta_0^2-2b_i^2})(\boldsymbol{R}_i-\boldsymbol{Z}_i-\boldsymbol{T}_i)^2]\Phi_0(\boldsymbol{r}-\boldsymbol{R})\\ &= n_0\exp[-\frac{A}{\beta_0^2}X_g^2]\,\mathcal{A}\{\prod_{i=1}^{n-1}\exp[-\frac{1}{2B_i^2}(\boldsymbol{\xi}_i-\boldsymbol{S}_i)^2]\prod_{i=1}^{n}\phi_i^{\mathrm{int}}(b_i)\}.\end{aligned}$$

a tool for studying the cluster correlations



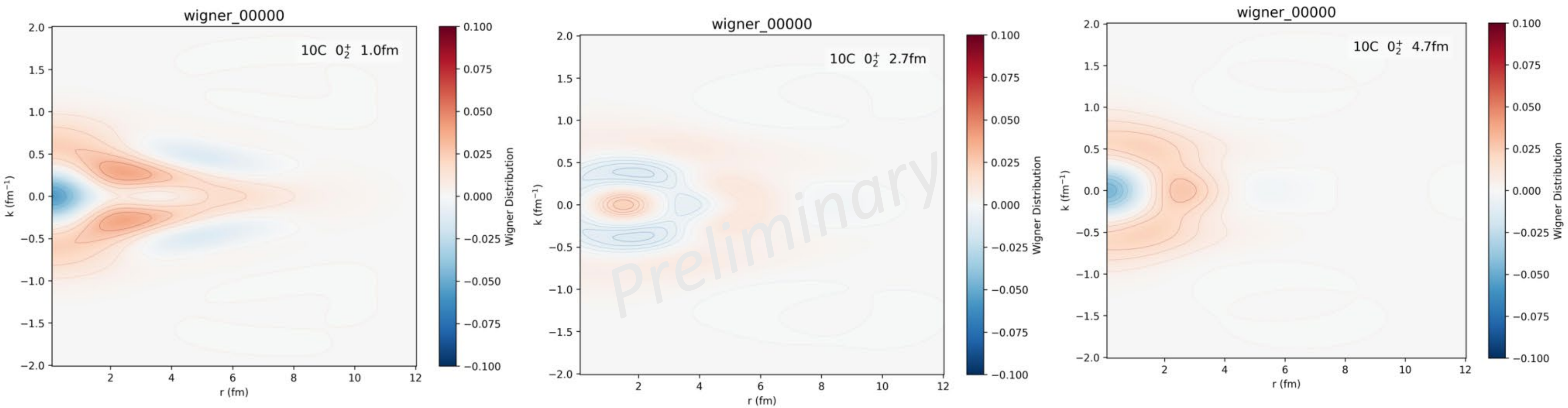
2α + 2p Clustering Structure and Diproton Correlations



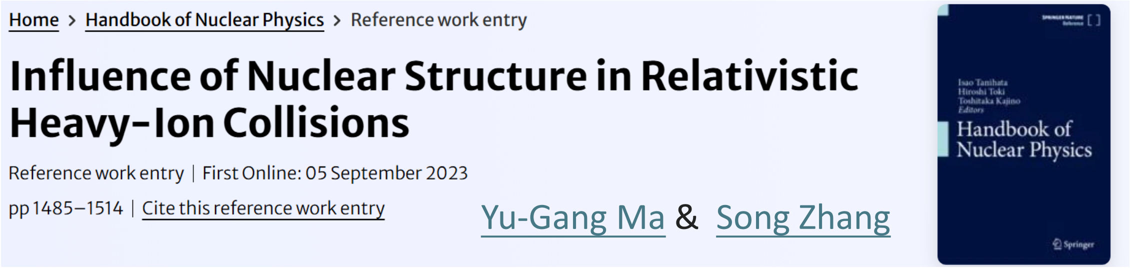


Liudi Song, et al., PRC (accepted)

FIG. 5: The calculated TCOA of $[^8\text{Be}(0^+) \otimes [p \otimes p]_0]_0 \otimes [0 \otimes 0]_0$ channel in the 0^+ states of ^{10}C.



Intersection of nuclear structure and high energy nuclear collisions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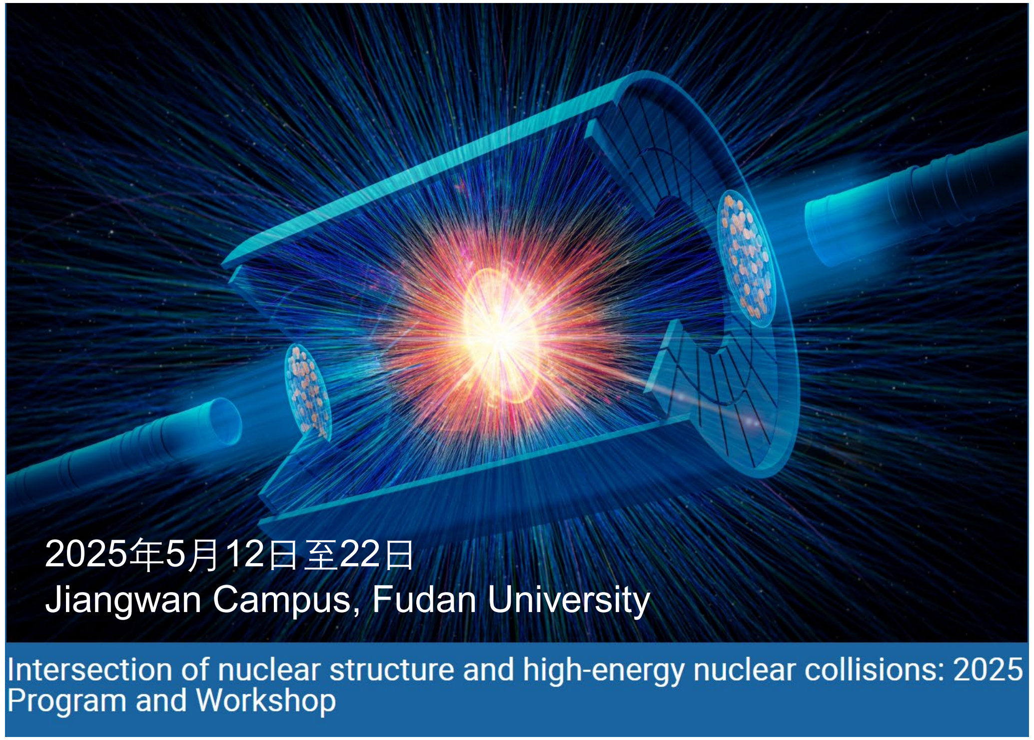
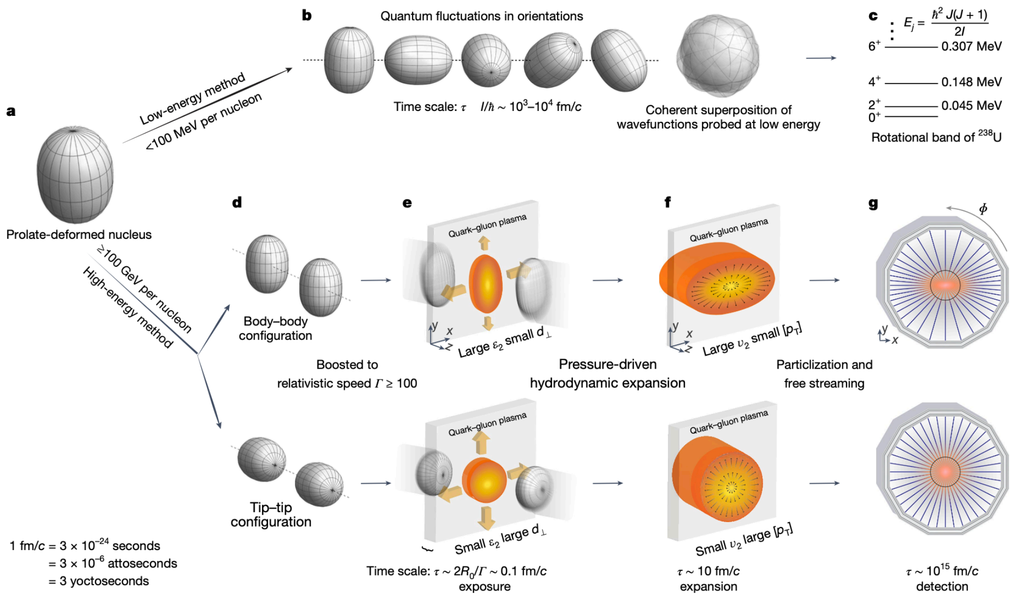
Influence of Nuclear Structure in Relativistic Heavy-Ion Collisions

Reference work entry | First Online: 05 September 2023

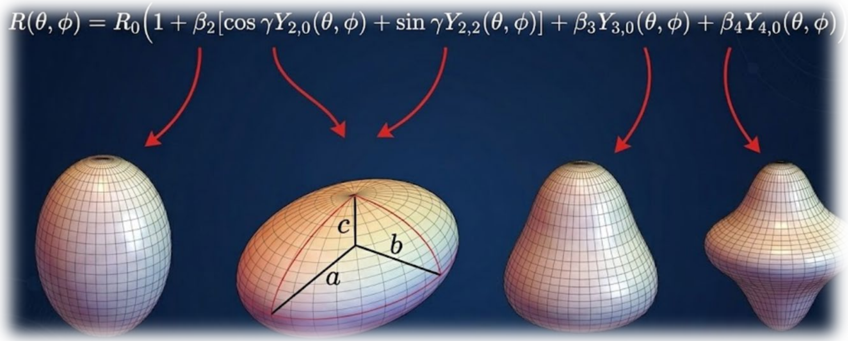
pp 1485–1514 | Cite this reference work entry

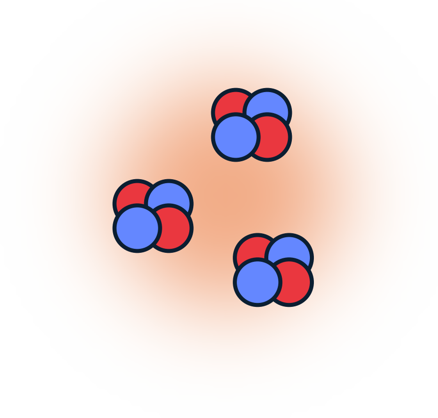
Yu-Gang Ma & Song Zhang





Chunjian Zhang (for the STAR Collaboration)





from collective to cluster motion

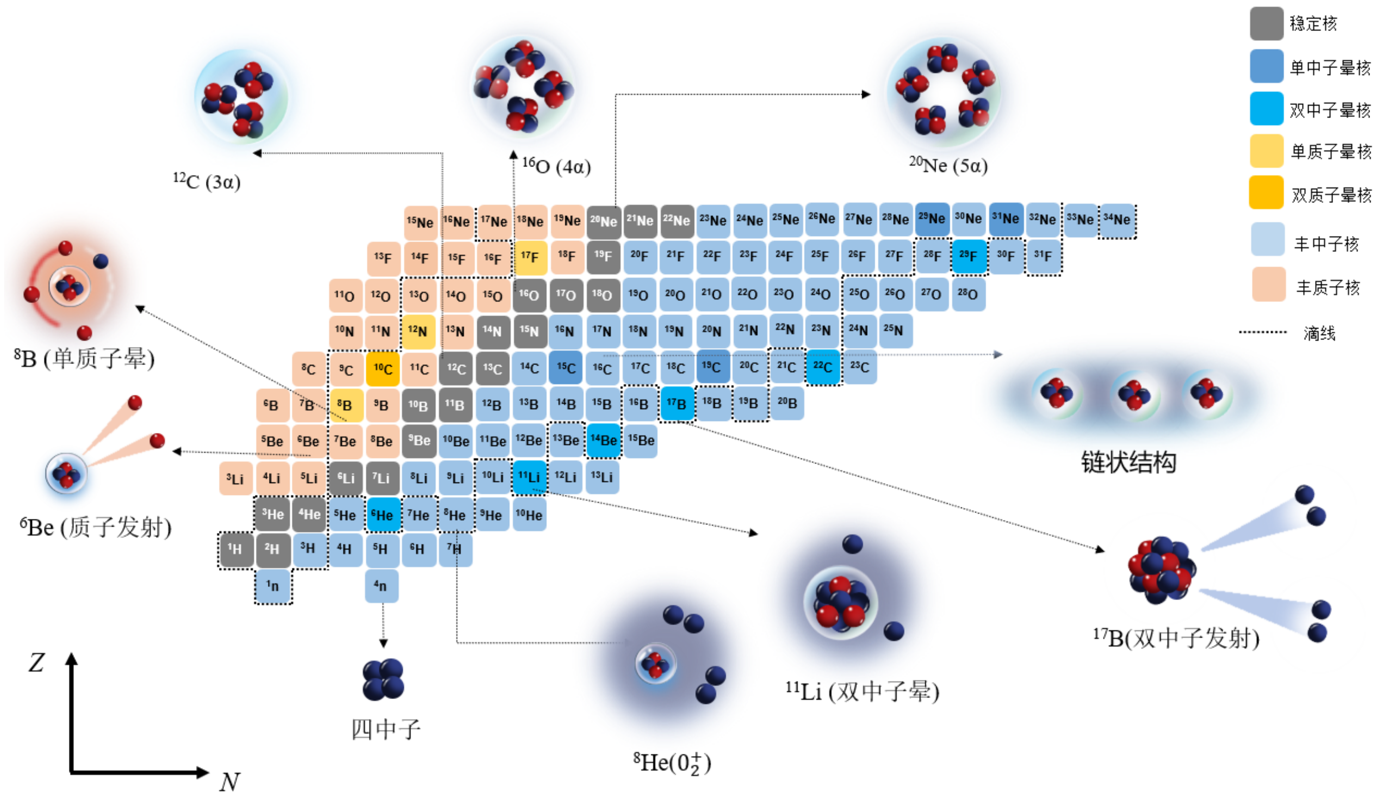
Summary and Prospect

- **Light nuclei exhibit a rich variety of cluster structures** and, near the cluster threshold, can display novel clustering configurations.

Future (2026~)

- **Integrated Gaussian Method** enhanced with:
  New interaction paradigms,
  AI-driven many-body approaches

- **In-depth interdisciplinary research** in areas such as nuclear reactions, nuclear astrophysics, and high-energy heavy-ion collisions, among others.



Thanks for my collaborators and your attentions.