基于耦合道模型的奇特强子态及其相关反应研究

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中国科学技术大学　　2026.01.09

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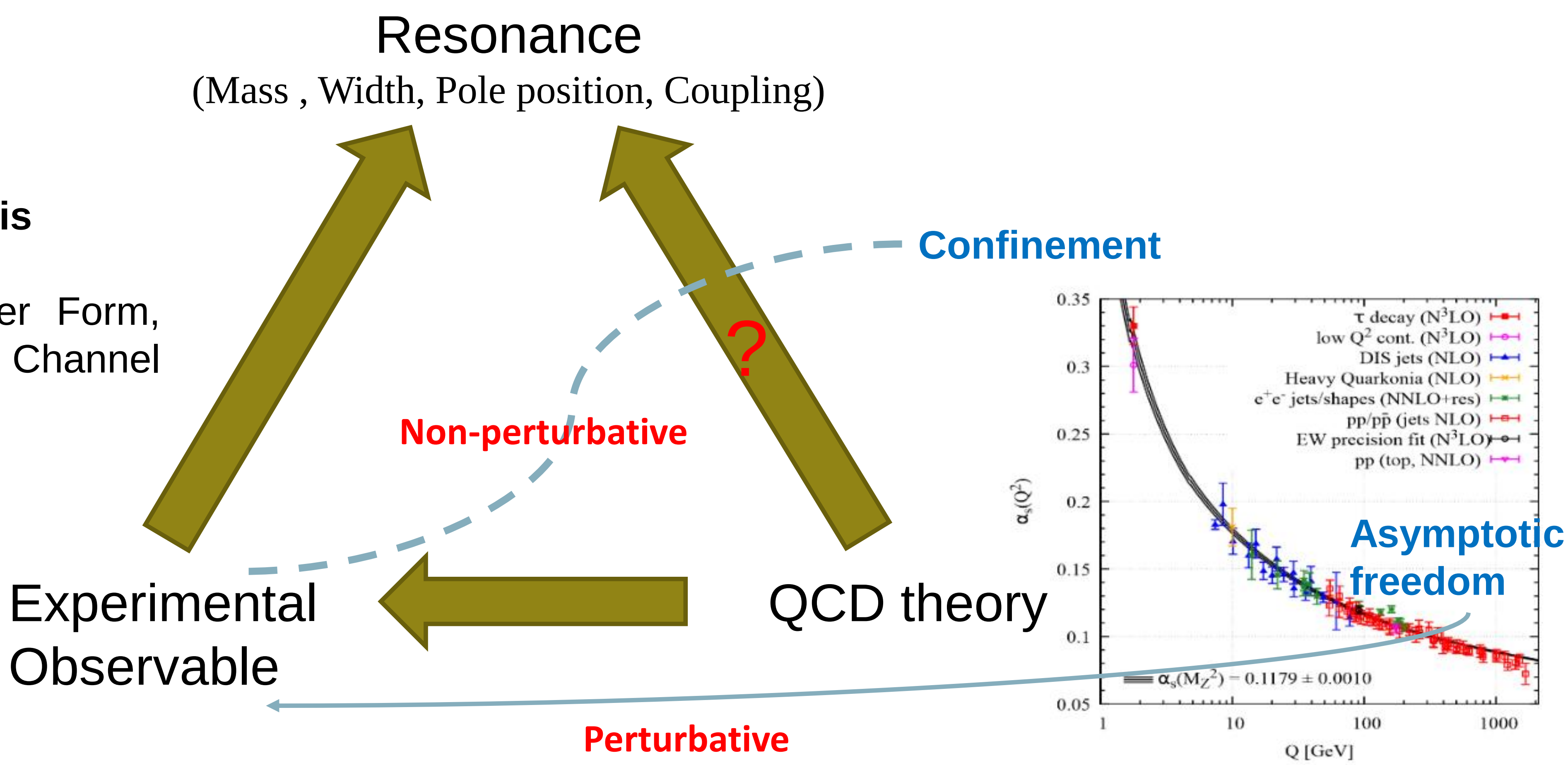
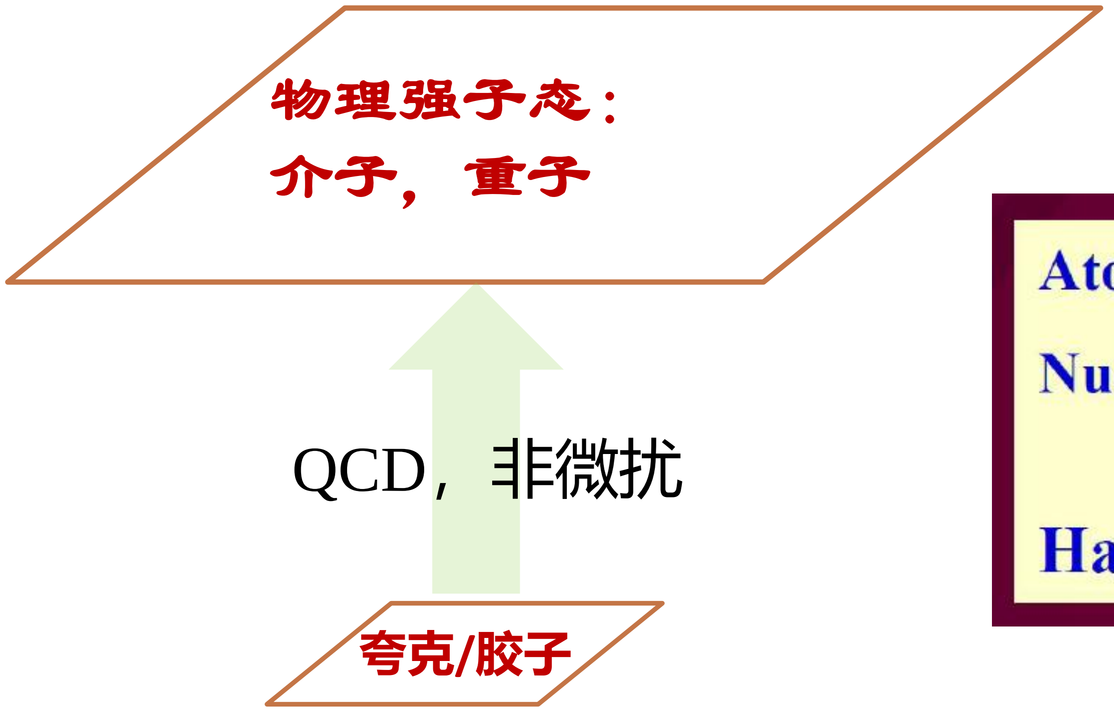
- 从T_{cc}到 $X(3872)$
- 从$D_s(2317)$到$T_{c\bar{s}}(2327)$
- 小结和展望

背景介绍：强相互作用---强子谱---强子反应

Partial Wave Analysis
With various Model:
Such as Breit-Wigner Form, Flatte Form, Coupled Channel Form, and so on



背景：强相互作用---强子谱---强子反应



强子谱？强子结构---强子内部的成分？

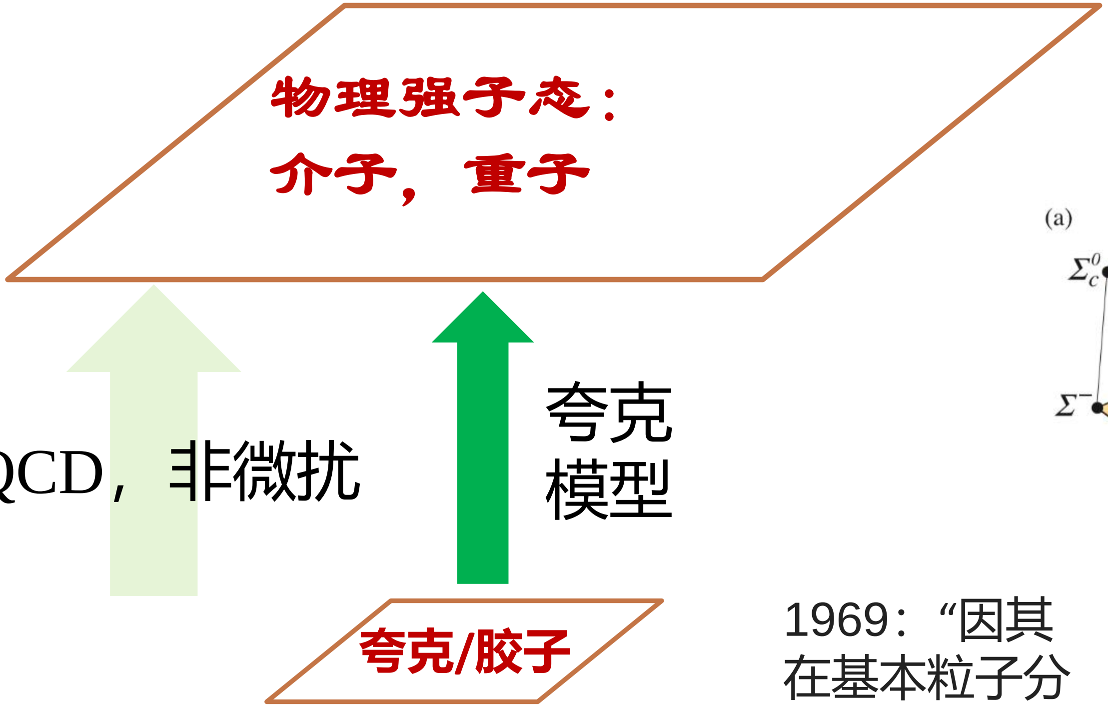
Atomic spectroscopy → Atomic Quantum Theory

Nuclear spectroscopy → Shell Model &
Collective motion Model

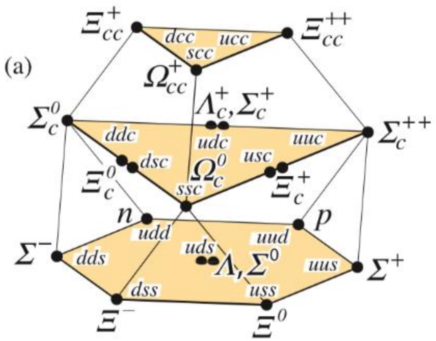
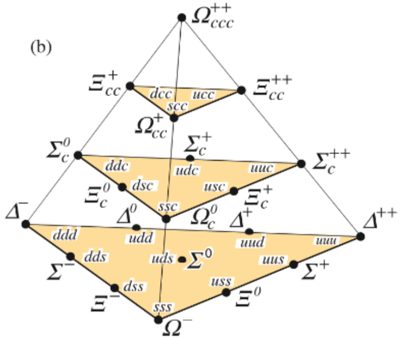
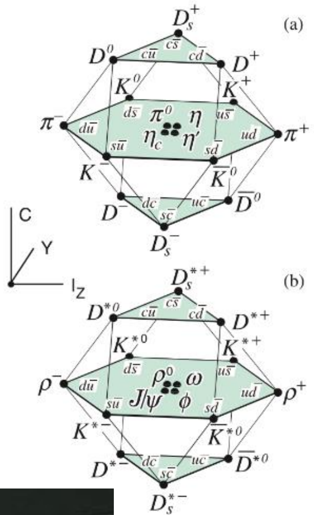
Hadron spectroscopy → ?

来自邹冰松院士的PPT截图

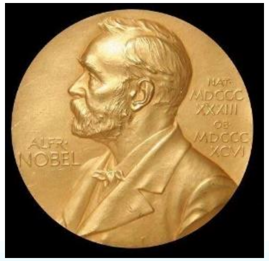
背景：强相互作用---强子谱---强子反应



强子谱？强子结构---强子内部的成分？

1969：“因其在基本粒子分类及其相互作用方面的贡献和发现。”

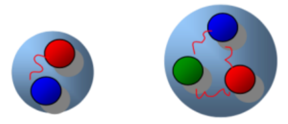


盖尔曼和茨威格分别独立提出了夸克模型

背景：强相互作用---强子谱---强子反应

物理强子态：
介子，重子//奇特态

convential hadron



(q q̄)　(qqq)

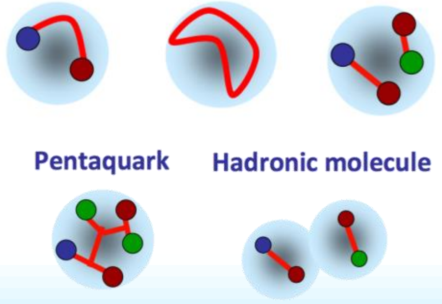
夸克模型?

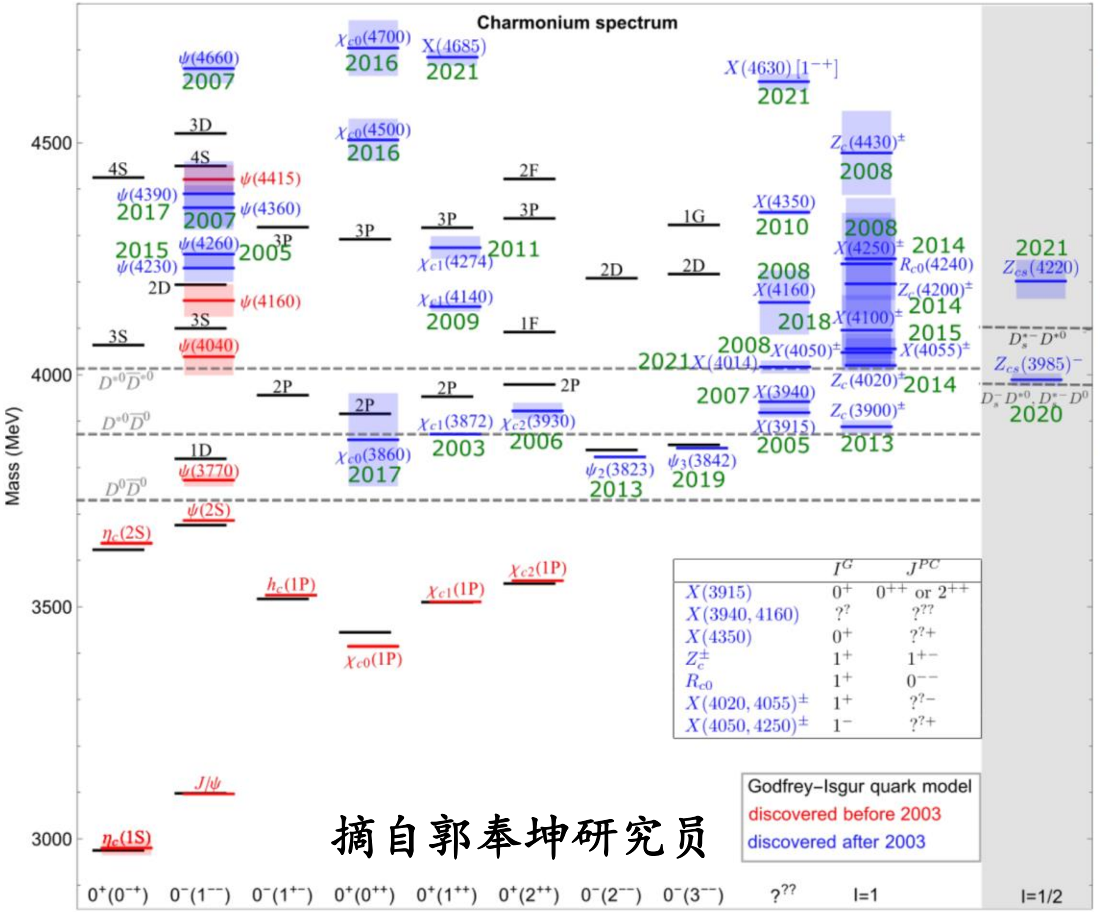
夸克/胶子

奇特态

Hybrid　Glueball　Tetraquark

Pentaquark　Hadronic molecule





	I^G	J^{PC}
$X(3915)$	0^+	0^{++} or 2^{++}
$X(3940, 4160)$	$?^?$	$?^{??}$
$X(4350)$	0^+	$?^{?+}$
$Z_c^\pm$	1^+	1^{+-}
R_{c0}	1^+	0^{--}
$X(4020, 4055)^\pm$	1^+	$?^{?-}$
$X(4050, 4250)^\pm$	1^-	$?^{?+}$

摘自郭奉坤研究员

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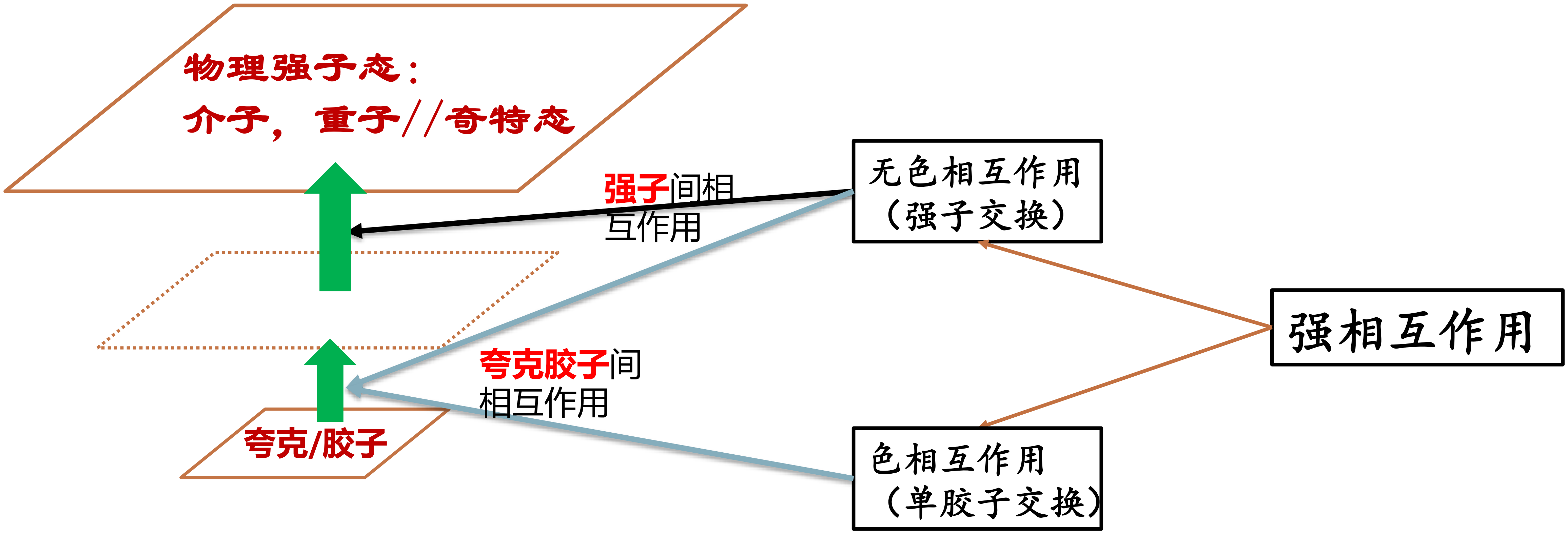
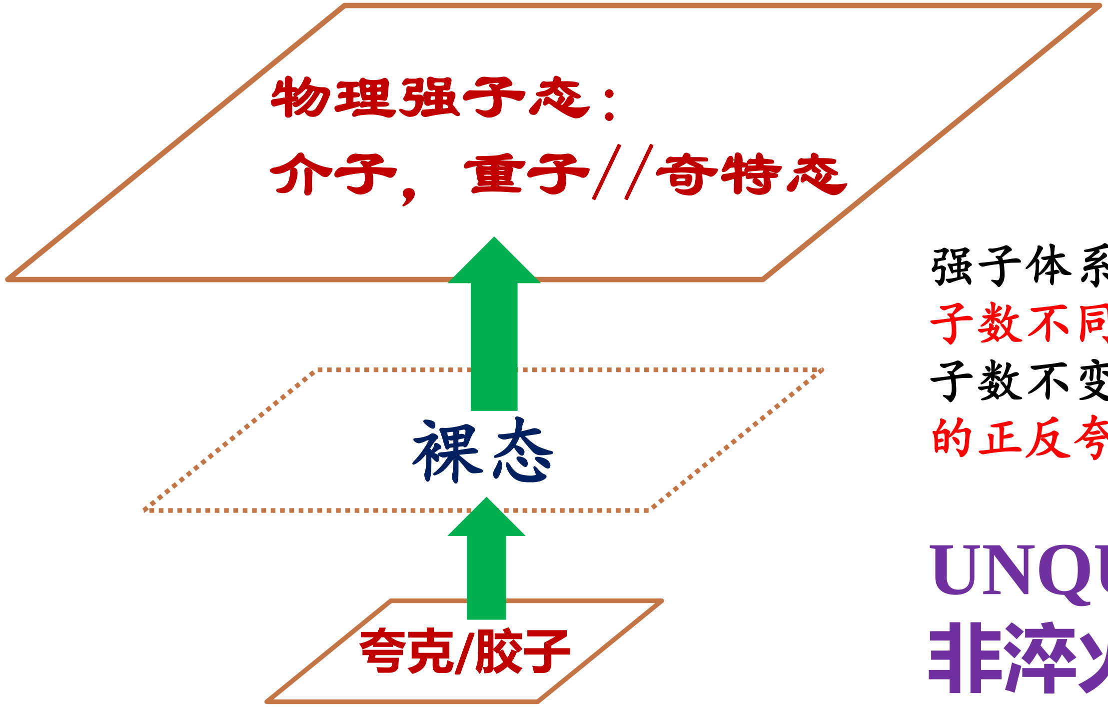
背景：强相互作用---强子谱---强子反应



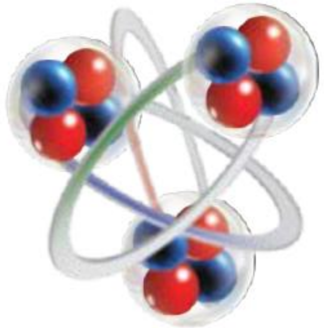
背景：强相互作用---强子谱---强子反应



核物理中： ^{12}C <= 6 p & 6 n

<= 3 α Cluster 原子团簇

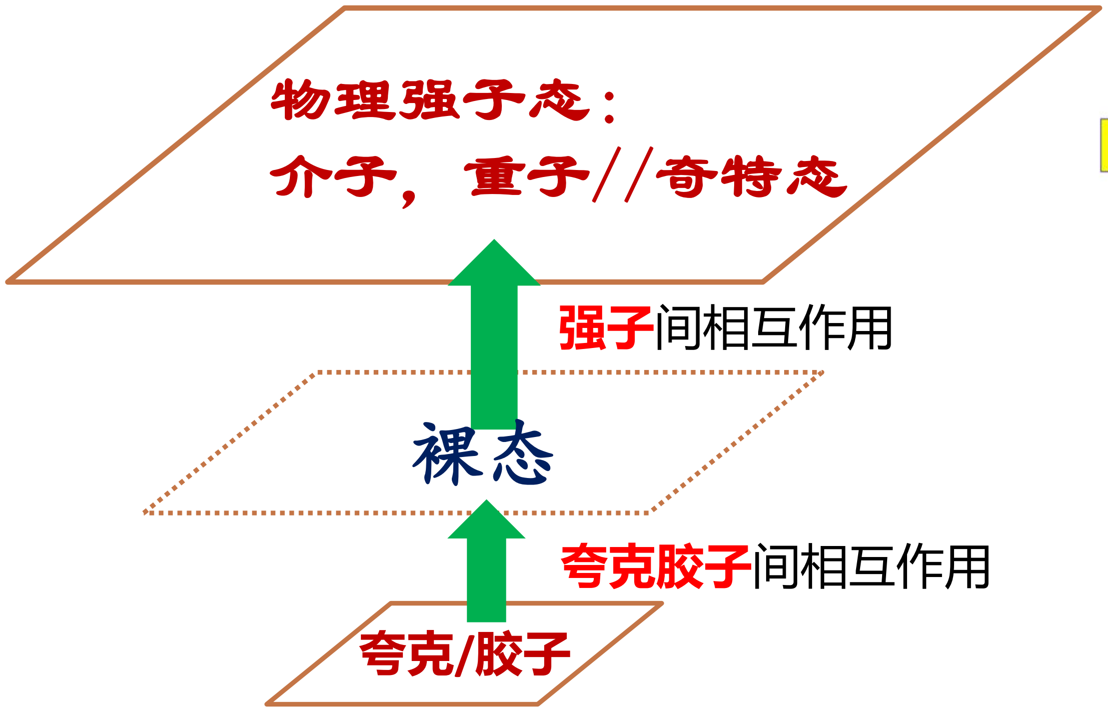
强子体系与原子核的最大差异：基础单元的粒子数不同！夸克数是可以涨落的，只是夸克量子数不变，而核子数是不变的。体现了真空中的正反夸克对的涨落，当然还可以是胶球。



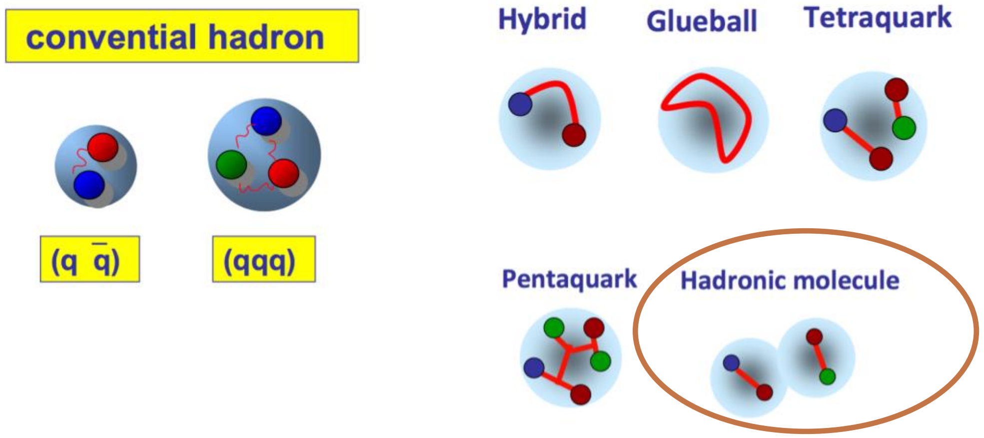
UNQUENCHED quark model

非淬火夸克模型

背景：强相互作用---强子谱---强子反应



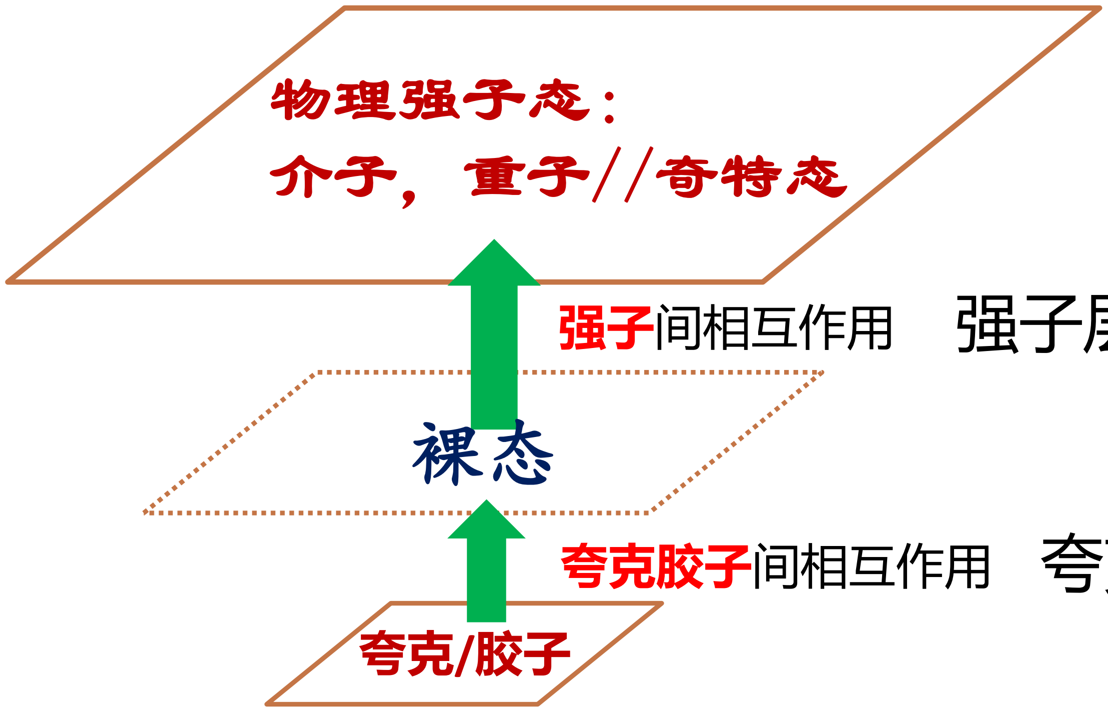
奇特态



UNQUENCHED quark model

非淬火夸克模型

背景：强相互作用---强子谱---强子反应



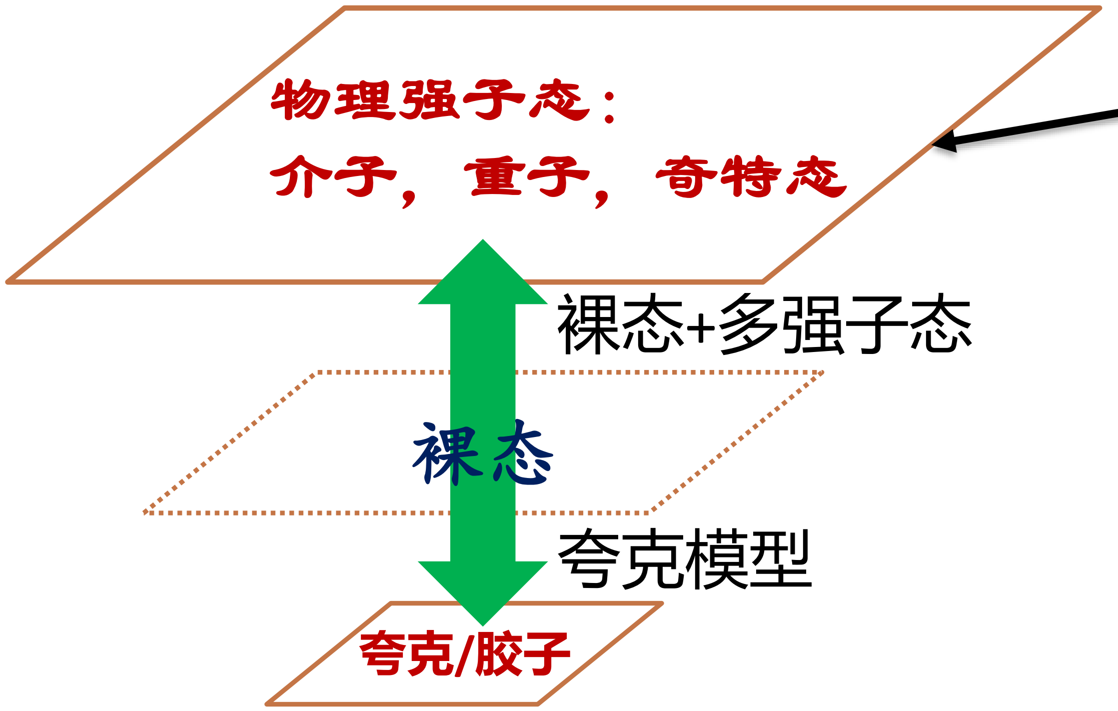
强子层次唯象模型

夸克模型

裸态(单粒子态)+
多强子态

耦合道模型

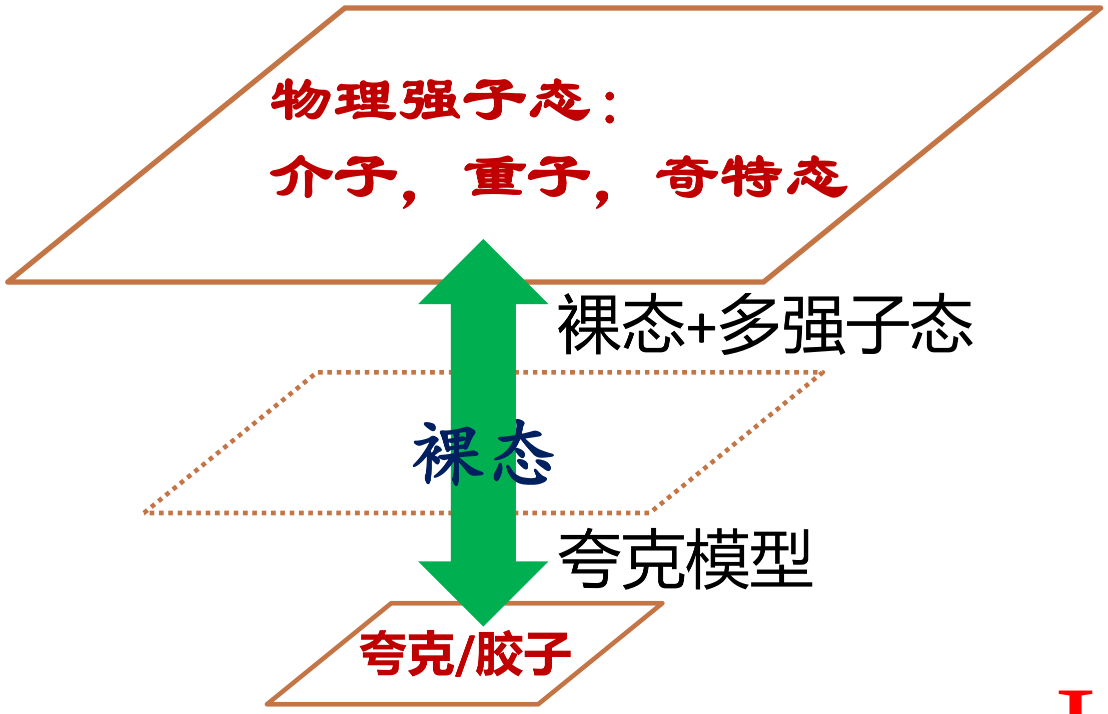
背景：强相互作用---强子谱---强子反应



实验输入！！！
衰变和散射-强子反应
格点输入！！！
有限体积能谱

HEFT方法

背景：强相互作用---强子谱---强子反应



quark level &&
hadron level.

A more comprehensive framework for systematically describing hadrons

Not only one, at least a set of hadron.

HEFT方法

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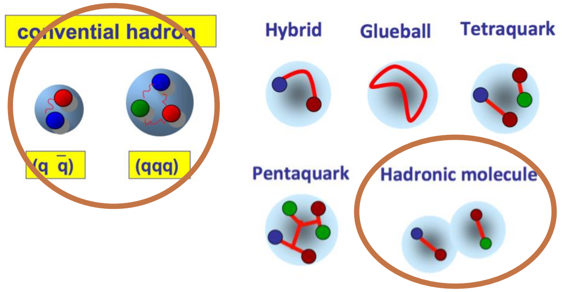
HEFT耦合道模型：裸态+多强子



$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2}\right] \langle \alpha(k_\alpha)|$$

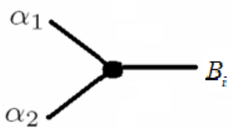
$|B_i\rangle$ bare state， bare mass m_i ⟷ Quark level

$|\alpha(k_\alpha)\rangle$ non-interaction channels ⟷ Hadron level

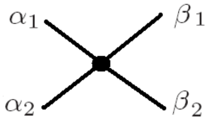
$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle \beta(k_\beta)|$$



From 3P0 model, and the wavefunction of quark model



One boson exchange (OBE)

Resonance

(Mass, width, pole position, coupling)

HEFT

T matrix (phase shift, inelasticity)

Lattice spectrum

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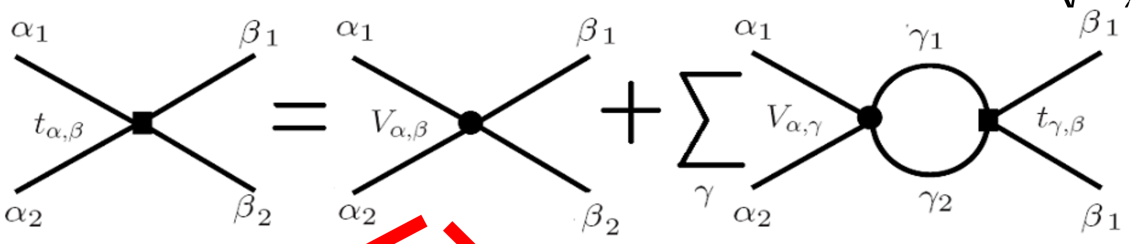
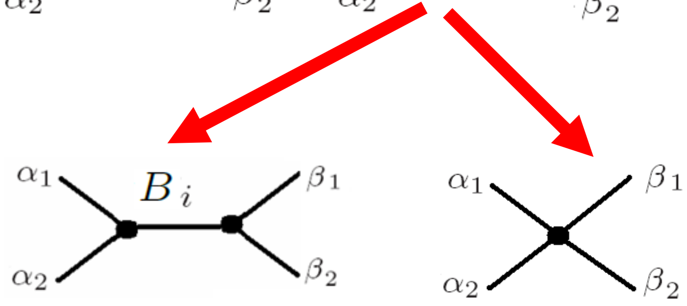
HEFT耦合道模型：裸态+多强子

Argonne-Osaka Model

- **T Matrix:**

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma^{\ 2}dk_\gamma\,\frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$



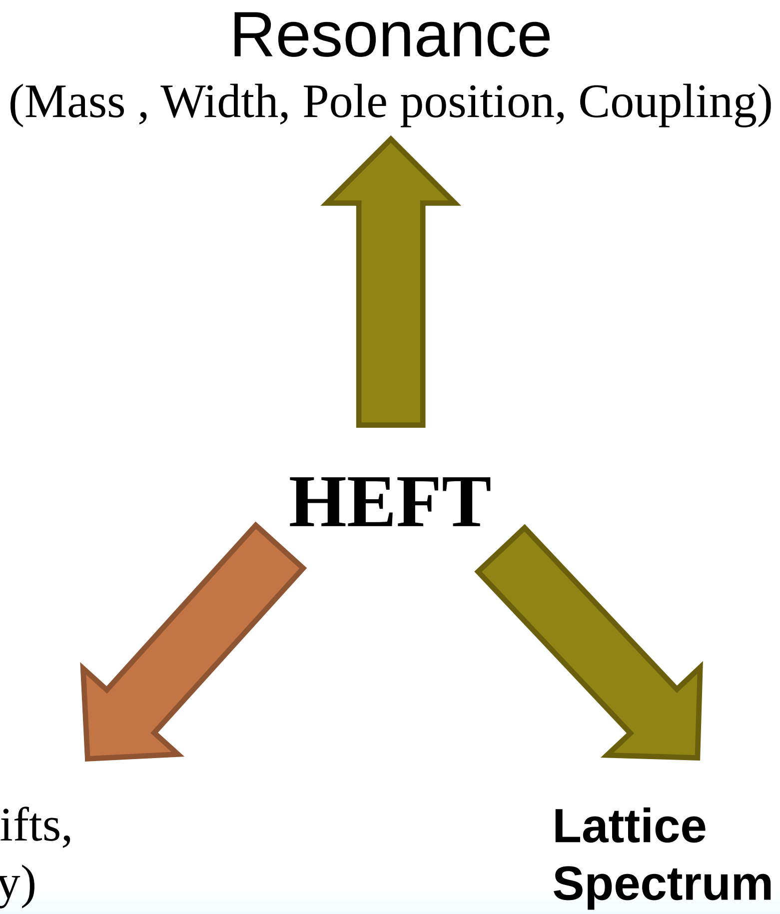


$$g_{i,\alpha}^*\frac{1}{E-m_i}g_{i,\beta} \qquad v_{\alpha,\beta}$$

$$S_{\alpha,\beta}=1-i2\sqrt{\rho_\alpha}\,t_{\alpha,\beta}(k_{0\alpha},k_{0\beta},E)\sqrt{\rho_\beta}$$

$$\rho_\alpha=\frac{\pi k_{0\alpha}\sqrt{m_{\alpha1}^2+k_{0\alpha}^2}\sqrt{m_{\alpha1}^2+k_{0\alpha}^2}}{E}$$

$$\eta e^{2i\delta_\alpha}=S_{\alpha,\alpha}$$



HEFT耦合道模型：裸态+多强子

J.-j. Wu etc. PRC90 (2014), 055206

- **Hamiltonian with discrete momentum**

Continuous ↓ Discrete

$$\int d\vec{k} \quad \text{and} \quad |\alpha(\vec{k}_\alpha)\rangle \quad \text{and} \quad \langle\beta(\vec{k}_\beta)|\alpha(\vec{k}_\alpha)\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$$

$$\downarrow$$

$$\sum_i \left(2\pi/L\right)^3 \quad \text{and} \quad \left(2\pi/L\right)^{-3/2} |\vec{k}_i, -\vec{k}_i\rangle_\alpha \quad \text{and} \quad {}_\beta\langle\vec{k}_j, -\vec{k}_j|\vec{k}_i, -\vec{k}_i\rangle_\alpha = \delta_{\alpha\beta}\delta_{ij}$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha,i} |\vec{k}_i, -\vec{k}_i\rangle_\alpha \left[\sqrt{m_{\alpha_B}^2 + k_\alpha^2} + \sqrt{m_{\alpha_M}^2 + k_\alpha^2}\right]_\alpha \langle\vec{k}_i, -\vec{k}_i|$$

$$H_I = \sum_j \left(2\pi/L\right)^{3/2} \sum_\alpha \sum_{i=1,n} \left[|\vec{k}_j, -\vec{k}_j\rangle_\alpha\, g_{i,\alpha}^+ \langle B_i| + |B_i\rangle g_{i,\alpha}\ {}_\alpha\langle\vec{k}_j, -\vec{k}_j|\right] + \sum_{i,j} \left(2\pi/L\right)^3 \sum_{\alpha,\beta} |\vec{k}_i, -\vec{k}_i\rangle_\alpha\, v_{\alpha,\beta}\ {}_\beta\langle\vec{k}_j, -\vec{k}_j|$$

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0,k_0) & v_{1,2}^V(k_0,k_0) & \cdots & v_{1,n_c}^V(k_0,k_0) & v_{1,1}^V(k_0,k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0,k_0) & v_{2,2}^V(k_0,k_0) & \cdots & v_{2,n_c}^V(k_0,k_0) & v_{2,1}^V(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0,k_0) & v_{n_c,2}^V(k_0,k_0) & \cdots & v_{n_c,n_c}^V(k_0,k_0) & v_{n_c,1}^V(k_0,k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1,k_0) & v_{1,2}^V(k_1,k_0) & \cdots & v_{1,n_c}^V(k_1,k_0) & v_{1,1}^V(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$(H_0 + H_I)\,|\Psi\rangle = E\,|\Psi\rangle$$

Eigen-Value ⟷ Lattice Spectrum

Eigen-Vector

Resonance

(Mass , Width, Pole position, Coupling)

HEFT

T matrix

(Phase Shifts, inelasticity)

Lattice Spectrum

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HEFT耦合道模型：裸态+多强子

J.-j. Wu etc. PRC90 (2014), 055206

- **Hamiltonian with discrete momentum**

Continuous	$\int d\vec{k}$ and	$\lvert\alpha(\vec{k}_\alpha)\rangle$ and	$\langle\beta(\vec{k}_\beta)\lvert\alpha(\vec{k}_\alpha)\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha-\vec{k}_\beta)$
↓	↓	↓	↓
Discrete	$\sum_i \left(\frac{2\pi}{L}\right)^3$ and	$\left(\frac{2\pi}{L}\right)^{-3/2}\lvert\vec{k}_i,-\vec{k}_i\rangle_\alpha$ and	${}_\beta\langle\vec{k}_j,-\vec{k}_j\lvert\vec{k}_i,-\vec{k}_i\rangle_\alpha = \delta_{\alpha\beta}\delta_{ij}$

$$H_0 = \sum_{i=1,n}\lvert B_i\rangle m_i\langle B_i\rvert + \sum_{\alpha,i}\lvert\vec{k}_i,-\vec{k}_i\rangle_\alpha\left[\sqrt{m_{\alpha_B}^2+k_\alpha^2}+\sqrt{m_{\alpha_M}^2+k_\alpha^2}\right]{}_\alpha\langle\vec{k}_i,-\vec{k}_i\rvert$$

$$H_I = \sum_j\left(\frac{2\pi}{L}\right)^{3/2}\sum_\alpha\sum_{i=1,n}\left[\lvert\vec{k}_j,-\vec{k}_j\rangle_\alpha g_{i,\alpha}^{+}\langle B_i\rvert + \lvert B_i\rangle g_{i,\alpha}\,{}_\alpha\langle\vec{k}_j,-\vec{k}_j\rvert\right] + \sum_{i,j}\left(\frac{2\pi}{L}\right)^3\sum_{\alpha,\beta}\lvert\vec{k}_i,-\vec{k}_i\rangle_\alpha v_{\alpha,\beta}\,{}_\beta\langle\vec{k}_j,-\vec{k}_j\rvert$$

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0,k_0) & v_{1,2}^V(k_0,k_0) & \cdots & v_{1,n_c}^V(k_0,k_0) & v_{1,1}^V(k_0,k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0,k_0) & v_{2,2}^V(k_0,k_0) & \cdots & v_{2,n_c}^V(k_0,k_0) & v_{2,1}^V(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0,k_0) & v_{n_c,2}^V(k_0,k_0) & \cdots & v_{n_c,n_c}^V(k_0,k_0) & v_{n_c,1}^V(k_0,k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1,k_0) & v_{1,2}^V(k_1,k_0) & \cdots & v_{1,n_c}^V(k_1,k_0) & v_{1,1}^V(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \ddots \end{pmatrix}$$

$$(H_0+H_I)\,|\Psi> = E\,|\Psi>$$

Eigen-Value ⟷ Lattice Spectrum

Eigen-Vector

Resonance

(Mass , Width, Pole position, Coupling)

Complex Continuum Momentum Space

HEFT

Real Continuum Momentum Space

Real Discrete Momentum Space

T matrix

(Phase Shifts, inelasticity)

Lattice Spectrum

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HEFT耦合道模型：裸态+多强子

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2. Finite-volume Hamiltonian method for coupled-channels interactions in lattice QCD PRC 90 (2014) no.5, 055206
3. Lattice QCD Evidence that the Λ(1405) Resonance is an Antikaon-Nucleon Molecule PRL 114 (2015) no.13, 132002
4. Hamiltonian effective field theory study of the N*(1535) resonance in lattice QCD PRL 116 (2016) no.8, 082004
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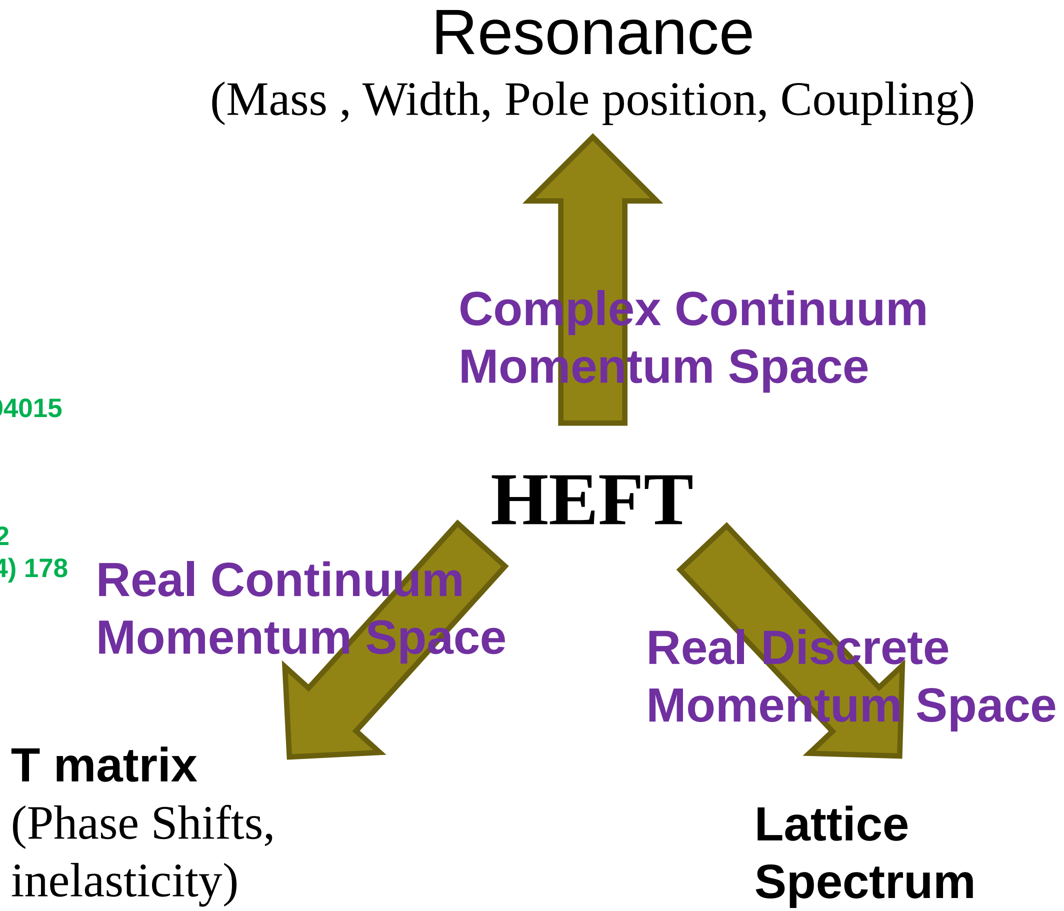
6. Structure of the Λ(1405) from Hamiltonian effective field theory PRD 95 (2017) no.1, 014506
7. Nucleon resonance structure in the finite volume of lattice QCD PRD 95 (2017) no.11, 114507
8. Structure of the Roper Resonance from Lattice QCD Constraints PRD 97(2018) no.9, 094509
9. Kaonic Hydrogen and Deuterium in Hamiltonian Effective Field Theory PLB 808(2020),135652
10. Partial Wave Mixing in Hamiltonian Effective Field Theory PRD 101(2020) no.11,114501
11. Hamiltonian effective field theory in elongated or moving finite volume PRD 103(2021) no.9, 094518
12. Regularisation in Nonperturbative Extensions of Effective Field Theory PRD 106 (2022) no.3, 034506
13. Novel Coupled Channel Framework Connecting the Quark Model and Lattice QCD for the Near-threshold Ds States PRL128(2022),112001
14. The investigations of the P-wave Bs states combining quark model and lattice QCD in the coupled channel framework JHEP01 (2023) 058
15. Effects of multiple single-particle basis states in scattering systems Annals Phys. 459(2023) 169531
16. Low-lying odd-parity nucleon resonances as quark-model like states PRD 108 (2023) 9, 094519
17. Structure of the Λ(1670) resonance PRD 109 (2024) 5, 054025
18. Investigation on the bottom analogs Tbb of Tcc+ PRD 110 (2024) 7, 074007
19. Pion photoproduction of nucleon excited states with Hamiltonian effective field theory PRD 110 (2024) 9, 094015
20. Study of the pion-mass dependence of ρ-meson properties in lattice QCD PRD 109 (2024) 3, 034505
21. Searching for the first radial excitation of the Δ(1232) in lattice QCD JPG 51 (2024) 6, 065106
22. Three-coupled-channel analysis of Zc(3900) involving $D\bar{D}^*$, πJ/ψ, and ρηc PRD 110 (2024) 11, 114029
23. New insight into the exotic states strongly coupled with the $D\bar{D}^*$ from the Tcc+ Sci.Bull. 69 (2024) 3036-3042
24. Generalized boost transformations in finite volumes and application to Hamiltonian methods JHEP 08 (2024) 178
25. Spectral parameters of the ρ resonance from lattice QCD JHEP 08 (2025) 064
26. Finite volume Hamiltonian method for two-particle systems containing long-range potential on the lattice JHEP 04 (2025) 108
27. Structure of the Ω-(2012) with Hamiltonian effective field theory PRD 112 (2025) 5, L051503
28. Nucleon resonance structure up to 2 GeV and the nature of the Roper resonance PRD 111 (2025) 11, 116002
29. Exploring the Ω− spectrum in lattice QCD JPG 52 (2025) 6, 065103
30. The odd-parity strange baryons Σ (12−)Σ(21−) below 1.8 GeV with Hamiltonian effective field theory 2509.17510



Introduction of HEFT

T matrix
(Phase Shifts, inelasticity)

Lattice Spectrum

HEFT

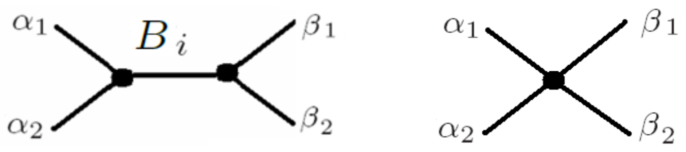
Resonance
(Mass , Width, Pole position, Coupling)

1. Build Model

2. Fix Para.

3. Extract Phys.

Unphysical π mass ??



HEFT:

1. Build a Hamiltonian model;

2. If Experimental data available, we fit Experimental data to fix the parameters in the model;
   If Lattice data available (close to physical pion mass), we fit these data;
   If both, we can use both of them constraint the model parameters.
   If we only have Lattice data with unphysical pion mass, we need another parameter for the mass dependence, such as mass slope.

3. From the fixed Hamiltonian, we can study the properties of Resonance. Especially, from the eigenvector in the finite volume, we can estimate the internal structure of the hadron.

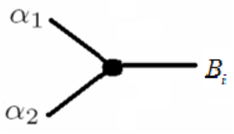
HEFT耦合道模型：裸态+强子

$$H = H_0 + H_I$$

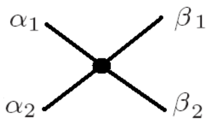
$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[\left| \alpha(k_\alpha) \right\rangle g^{+}_{i,\alpha} \left\langle B_i \right| + \left| B_i \right\rangle g_{i,\alpha} \left\langle \alpha(k_\alpha) \right| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} \left| \alpha(k_\alpha) \right\rangle v_{\alpha,\beta} \left\langle \beta(k_\beta) \right|$$



From 3P0 model, and the wavefunction of quark model



One boson exchange (OBE)

Two interactions ?

Too many solutions of a+b=5

Resonance

(Mass, width, pole position, coupling)

HEFT

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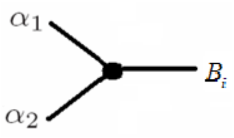

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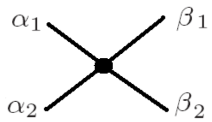

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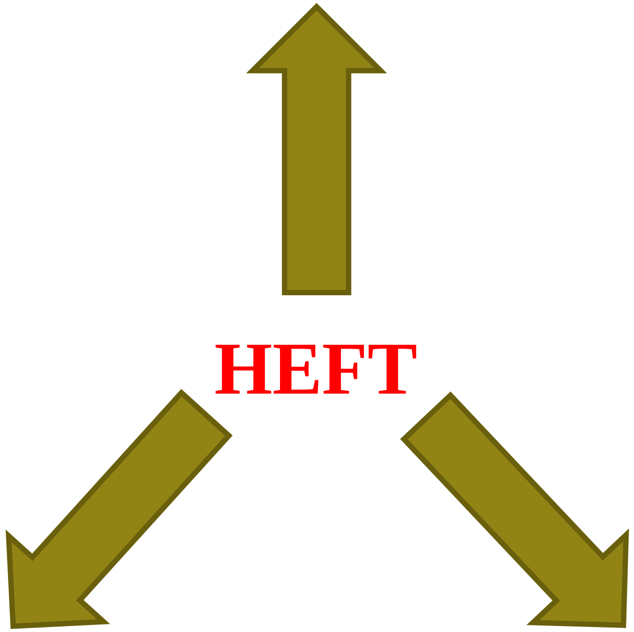

HEFT

T matrix

(phase shift, inelasticity)

Lattice Spectrum

Two interactions ?

Too many solutions of a+b=5

Model dependence!

Study X(3872) from T_{cc}

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comprehensive framework && systematically

从T_{cc}到 $X(3872)$

T_{bb}

PRD 110 (2024) 7, 074007

Yu-Shan Ren, Guang-Juan Wang, Zhi Yang, Jia-Jun Wu

$c \to b$

T_{cc} → $DD^*, D\bar{D}^*$ interaction

Scib.2024.07.012

$J/\psi\pi$, $\eta_c\rho$ → Study $Z_c(3900)$

Kang Yu, Guang-Juan Wang, Zhi Yang, Jia-Jun Wu

PRD 110 (2024) 11, 114029

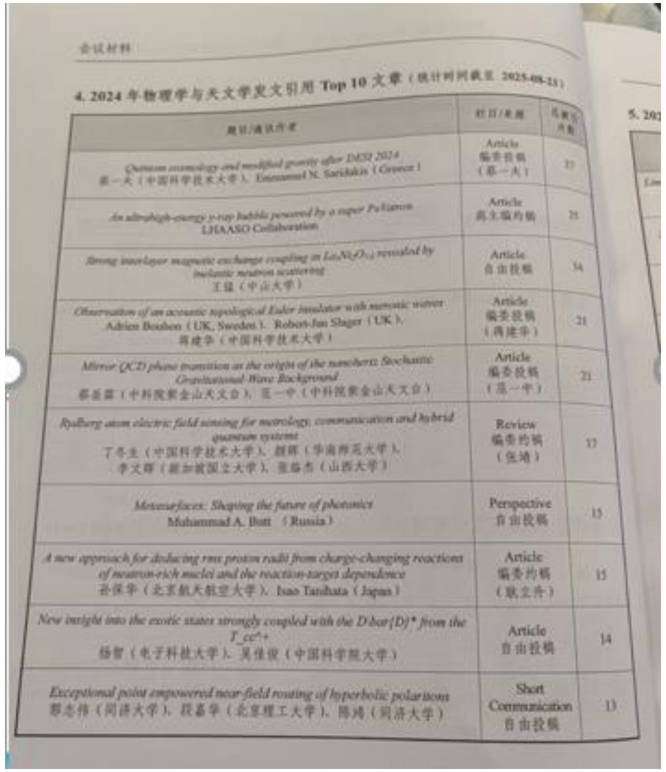
$c\bar{c}$ state

X(3872)

Study the decay of X(3872), soon…

Study the decay of X(4010)

Lattice spectrum soon…



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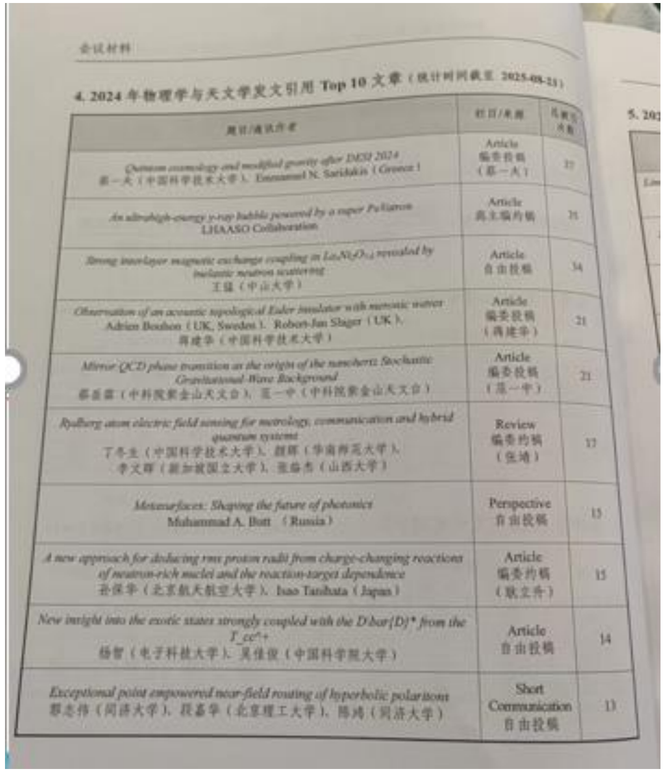
PRD 110 (2024) 11, 114029

Study $Z_c(3900)$,



Study the decay of X(3872), soon…

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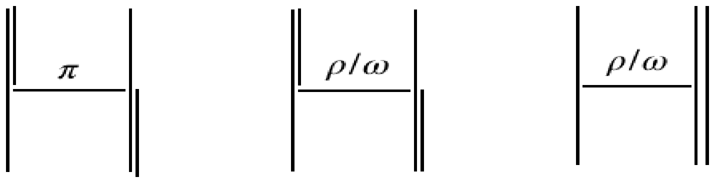
Introduction of OBE and DD^* vs $D\bar{D}^*$

The interaction between D and D[*] OBE



$$V_\pi = \frac{g^2}{f_\pi^2} \frac{(q\cdot\epsilon_\lambda)\left(q\cdot\epsilon^\dagger_{\lambda'}\right)}{q^2-m_\pi^2},$$

$$V^u_{\rho/\omega} = -2\lambda^2 g_V^2 \frac{(\epsilon^\dagger_{\lambda'}\cdot q)(\epsilon_\lambda\cdot q) - q^2\left(\epsilon_\lambda\cdot\epsilon^\dagger_{\lambda'}\right)}{q^2-m^2_{\rho/\omega}},$$

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PWA Just S-wave

$$\mathcal{V}(l,l'S,j) = \frac{1}{(2\pi)^3}\sqrt{\frac{1}{2E_D^i 2E_D^f 2E_K^i 2E_K^f}}$$

$$2\pi\int d\cos\theta V^v(\vec{p}_f,\vec{p}_i)\left(\frac{\Lambda^2}{\Lambda^2+p_f^2}\right)^2\left(\frac{\Lambda^2}{\Lambda^2+p_i^2}\right)^2$$

Heavy Quark Symmetry

$D^{(*)}D^{(*)}$

$$H_a^{(Q)} = \frac{1+\not{v}}{2}\left[P_a^{*\mu}\gamma_\mu - P_a\gamma_5\right]$$

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$$P = (D^0, D^+, D_s^+)\ \&P^* = (D^{*0}, D^{*+}, D_s^{*+})$$

$$\mathcal{L}_{MH^{(Q)}H^{(Q)}} = ig\,\mathrm{Tr}\left[H_b^{(Q)}\gamma_\mu\gamma_5 A^\mu_{ba}\bar{H}_a^{(Q)}\right]$$

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- $g = 0.57$ from the decay width of $D^* \to D\pi$, while undetermined parameters $\lambda\&\beta$.

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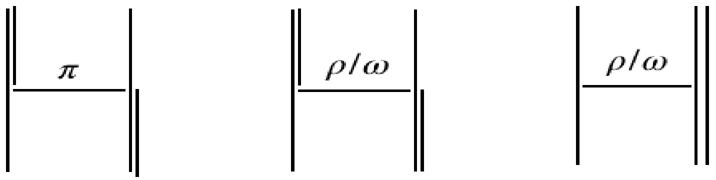

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$$H_a^{(\bar{Q})} \equiv C\left(\mathcal{C}H_a^{(Q)}\mathcal{C}^{-1}\right)^T C^{-1} = \left[P_{a\mu}^{(\bar{Q})*}\gamma^\mu - P_a^{(\bar{Q})}\gamma_5\right]\frac{1-\not{v}}{2}$$

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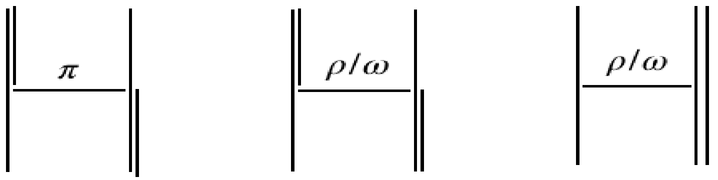

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	wave function	$I(J^{PC})$	$u-$channel : π	$u-$channel : ρ/ω	$t-$channel : ρ/ω
DD^*	$\frac{1}{\sqrt{2}}(D^+D^{*0} - D^0D^{*+})$	$0(1^+)$ $[T^+_{cc}]$	$\frac{3}{2}V_\pi$	$\frac{3}{2}V^u_\rho - \frac{1}{2}V^u_\omega$	$-\frac{3}{2}V^t_\rho + \frac{1}{2}V^t_\omega$
	$\frac{1}{\sqrt{2}}(D^+D^{*0} + D^0D^{*+})$	$1(1^+)$	$\frac{1}{2}V_\pi$	$\frac{1}{2}V^u_\rho + \frac{1}{2}V^u_\omega$	$\frac{1}{2}V^t_\rho + \frac{1}{2}V^t_\omega$
$D\bar{D}^*$	$\frac{1}{\sqrt{2}}\left([D^+D^{*-}] + [D^0\bar{D}^{*0}]\right)$	$0(1^{++})[X(3872)]$	$\frac{3}{2}V_\pi$	$-\frac{3}{2}V^u_\rho - \frac{1}{2}V^u_\omega$	$-\frac{3}{2}V^t_\rho - \frac{1}{2}V^t_\omega$
	$\frac{1}{\sqrt{2}}\left([D^+D^{*-}] - [D^0\bar{D}^{*0}]\right)$	$1(1^{++})$	$-\frac{1}{2}V_\pi$	$\frac{1}{2}V^u_\rho - \frac{1}{2}V^u_\omega$	$\frac{1}{2}V^t_\rho - \frac{1}{2}V^t_\omega$
	$\frac{1}{\sqrt{2}}\left(\{D^+D^{*-}\} + \{D^0\bar{D}^{*0}\}\right)$	$0(1^{+-})[h_c]$	$-\frac{3}{2}V_\pi$	$\frac{3}{2}V^u_\rho + \frac{1}{2}V^u_\omega$	$-\frac{3}{2}V^t_\rho - \frac{1}{2}V^t_\omega$
	$\frac{1}{\sqrt{2}}\left(\{D^+D^{*-}\} - \{D^0\bar{D}^{*0}\}\right)$	$1(1^{+-})$ $[Z_c(3900)]$	$\frac{1}{2}V_\pi$	$-\frac{1}{2}V^u_\rho + \frac{1}{2}V^u_\omega$	$\frac{1}{2}V^t_\rho - \frac{1}{2}V^t_\omega$

The mass differences leads to isospin breaking, our calculations are based on the physical states of the particles.

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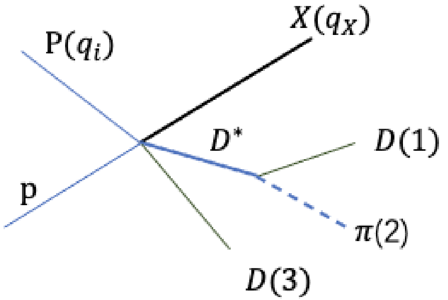
从T_{cc}到 $X(3872)$

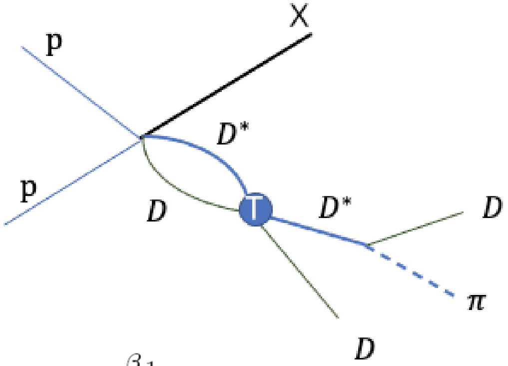
T_{cc} → $DD^*, D\bar{D}^*$ interaction

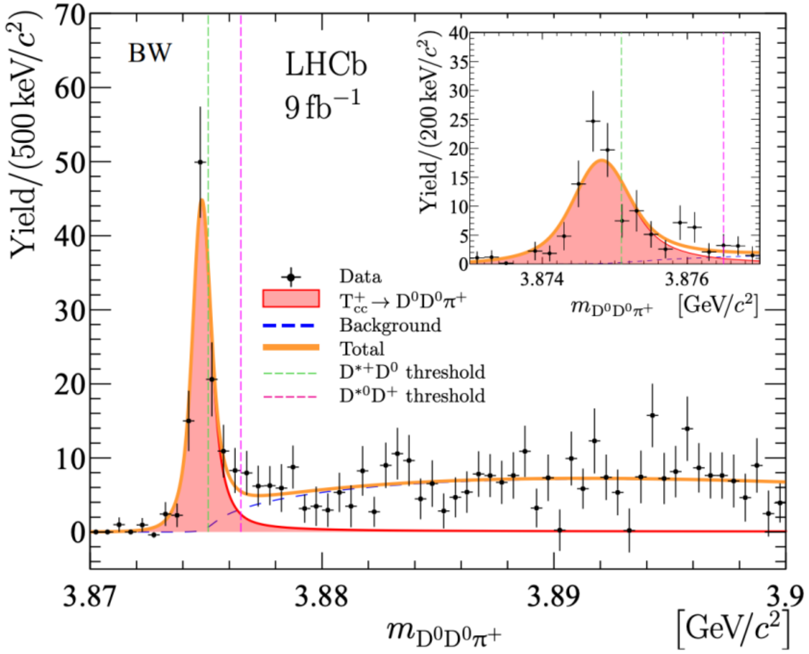
The generation of T_{cc}

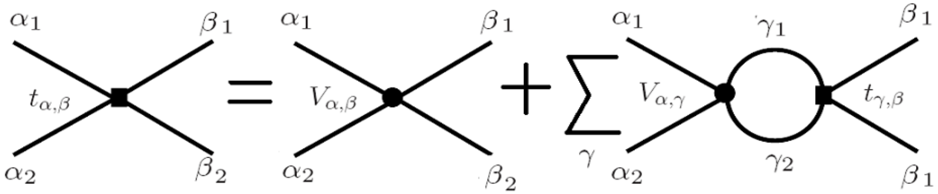
$$pp \rightarrow X\, D^0 D^0 \pi^+$$

p^t









$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_{\gamma}\int k_\gamma{}^2 dk_\gamma \frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$

$$|\mathcal{M}|^2=|a_{pp\to DD^*X}|^2\sum_{\lambda_X}\epsilon_\mu(p_X,\lambda_X)\epsilon^\dagger_{\mu'}(p_X,\lambda_X)\sum_j \mathcal{B}_{j\mu}\mathcal{B}_j^{\dagger\mu'}$$

$$\mathcal{B}_j^\mu(p_{12},p_{23})=g\left\{\frac{-i(p_\pi^\mu-\frac{p_{12}^\mu p_{12}\cdot p_\pi}{m_{D^*}^2})}{p_{12}^2-m_{D^*}^2+im_{D^*}\Gamma_{D^*}}\right\}_j+\sum_{i=1,2} ig\left\{\int dq_{D^*}q_{D^*}^2\frac{d\Omega_{q_{D^*}}}{4\pi}\frac{\sqrt{2w_{D_2}}}{\sqrt{2w_{D^*}}}\frac{\sqrt{2w_{D_{12}^*}}}{\sqrt{2w_D}}\frac{T_{ij}^{J00}(M,|q_{D^*}|,|p_{12}|)}{(M-w_{D^*}^i)-w_D^i+i\epsilon}\frac{\epsilon_a^{*\mu}(w_{D^*},q_{D^*})\epsilon_a(p_{12})\cdot p_\pi}{p_{12}^2-m_{D^*}^2+im_{D^*}\Gamma_{D^*}}\right\}_j+(p_{D_1}\to p_{D_2})$$

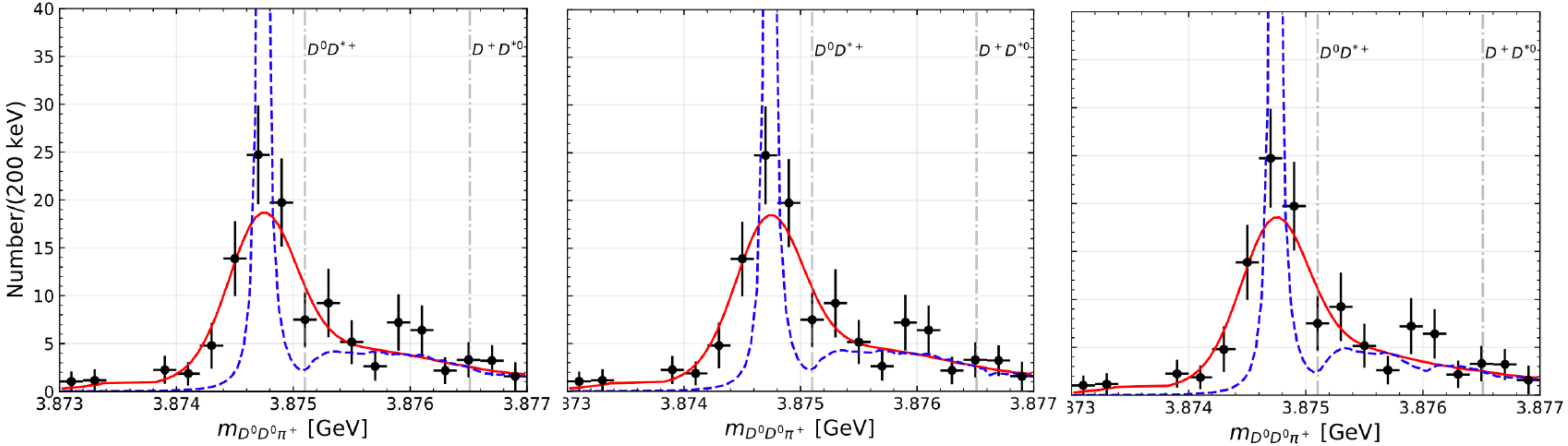
Check Robustness of Cut-off

The generation of T_{cc}

$$pp \to X\, D^0 D^0 \pi^+$$

Λ(fixed)	λ (/GeV)	β
0.8 GeV	0.890 ± 0.20	0.810 ± 0.11
1 GeV	0.683 ± 0.025	0.687 ± 0.017
1.2 GeV	0.587±0.21	0.550±0.12
1.17 GeV	0.56	0.9

Cheng, et al. PRD 106,016012



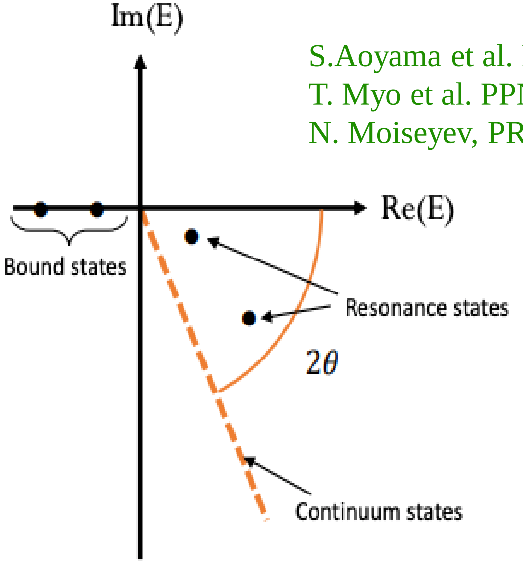
Λ (GeV)	BE (keV)	Γ (keV)	$\sqrt{\langle r^2 \rangle}$	$I=0$	$I=1$	$P(D^0D^{*+})$	$P(D^+D^{*0})$	$\frac{\text{Res}(D^0D^{*+})}{\text{Res}(D^+D^{*0})}$
0.8	-387.7	67.3	4.8 fm	95.8%	4.2%	70.0%	30.0%	$-1.063+0.001I$
1.0	-393.0	70.4	4.7 fm	95.8%	4.2%	70.0%	30.0%	$-1.055+0.001I$
1.2	-391.6	72.7	4.7 fm	95.7%	4.3%	70.3%	29.7%	$-1.052+0.001I$

The nature of T_{cc}

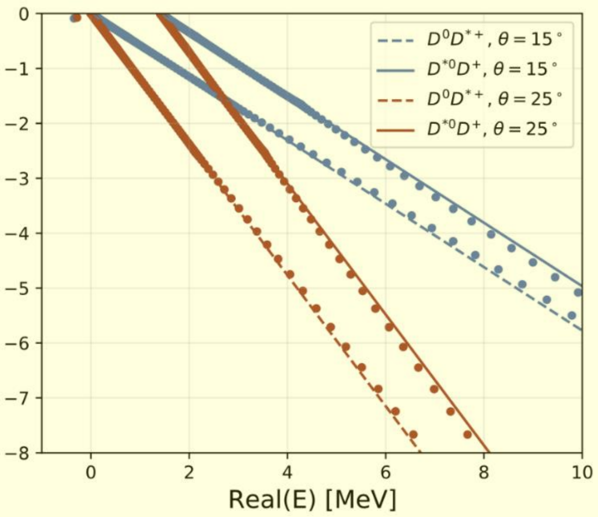
Complex scaling method

$$\boldsymbol{r} \to \boldsymbol{r}e^{i\theta}, \quad \boldsymbol{q} \to \boldsymbol{q}e^{-i\theta} \qquad H_\theta \Phi_\theta = E_\theta \Phi_\theta,$$

$$\begin{pmatrix} \overline{D^{*0}} D^0 \; / \overline{D^0} D^{*0} & \text{Coupled-channel effect} \\ \cdots & D^{*-} D^+ / D^{*+} D^- \end{pmatrix}$$



S.Aoyama et al. PTP. 116, 1 (2006).
T. Myo et al. PPNP. 79, 1 (2014)
N. Moiseyev, PR 302, 212 (1998)



- **Bound state: T_{cc}**

$m_{T_{cc}}$=3874.7 MeV,
$\Delta E = -393$ keV
$\Gamma_{T_{cc}} = 70.4$ keV

- $\sqrt{\langle r^2 \rangle} = 4.7\ fm$

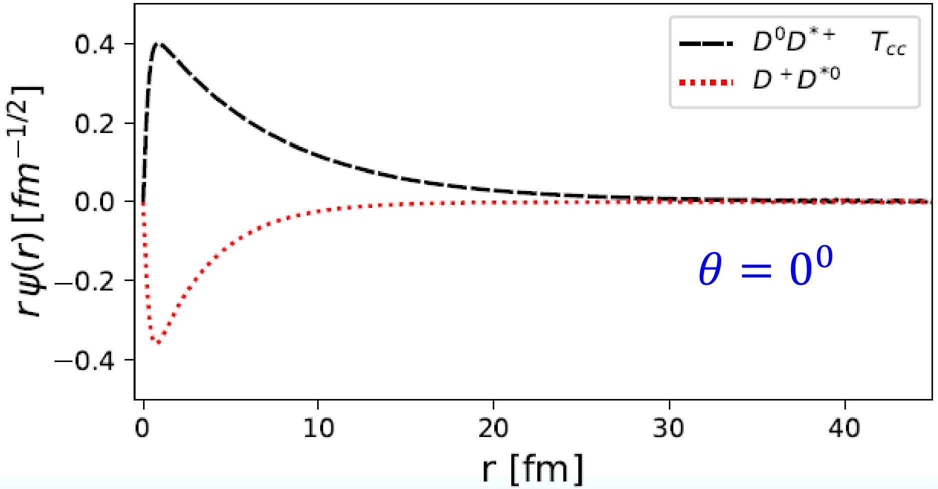
- 70.0% $D^{*+}D^0$, 30.0% D^+D^{*0}

95.8%, $DD^*(I = 0)$
4.2% $DD^*(I = 1)$

Because of mass difference

$$[I=0] = \frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+)$$

$$[I=1] = \frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+)$$



Two interactions ?
Too many solutions of a+b=5

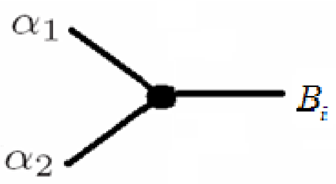
Model dependence!

Study X(3872) from T_{cc}

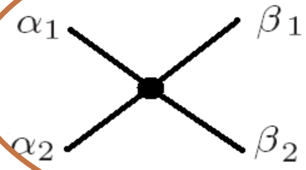
T_{cc} → $DD^*, D\bar{D}^*$ interaction

$c\bar{c}$ state ↓

X(3872)



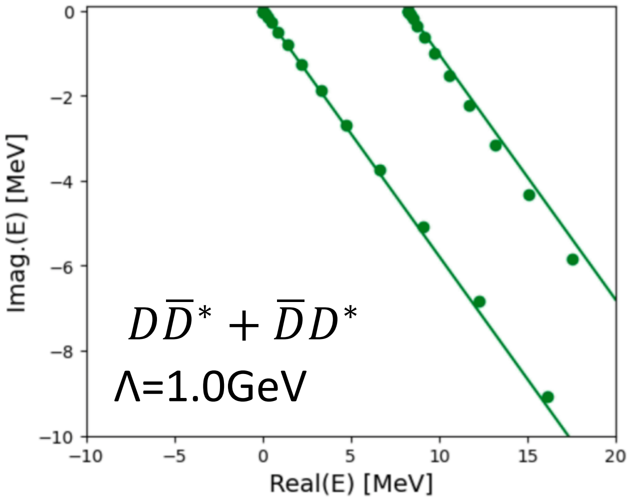
From 3P0 model, and the wavefunction of quark model

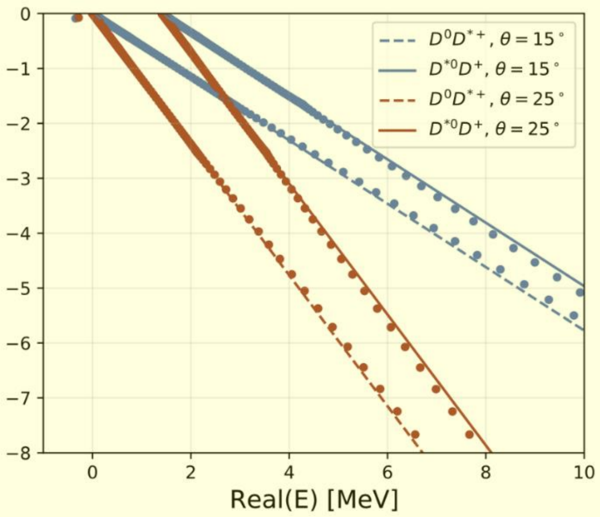


One boson exchange (OBE)

Produce X(3872) with pure $D\bar{D}^* + \bar{D}D^*$

From the interaction of DD^* to obtain the interaction of $D\bar{D}^* + \bar{D}D^*$, check X(3872) exists or not by pure $D\bar{D}^* + \bar{D}D^*$ interaction, without $c\bar{c}$ state.





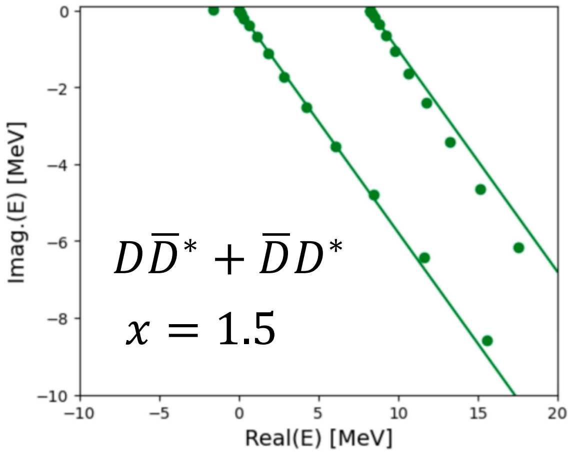
- **Bound state: T_{cc}**

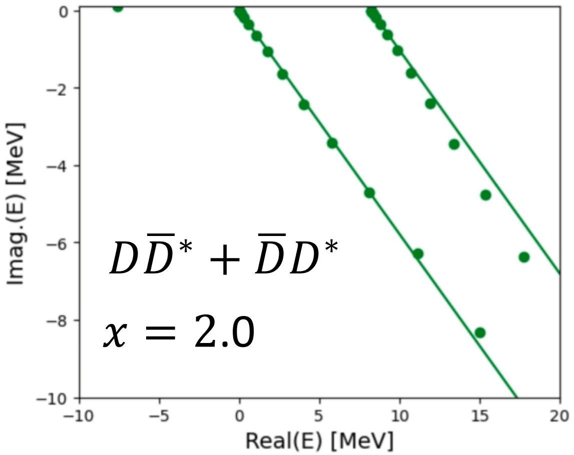
$m_{T_{cc}}$=3874.7 MeV,
$\Delta E = -393$ keV
$\Gamma_{T_{cc}} = 70.4$ keV

- $\sqrt{\langle r^2 \rangle} = 4.7\ fm$

- 70.0% $D^{*+}D^0$, 30.0% D^+D^{*0}

$$\mathbf{V'_{\overline{D^*}D} = x * V_{\overline{D^*}D}}$$



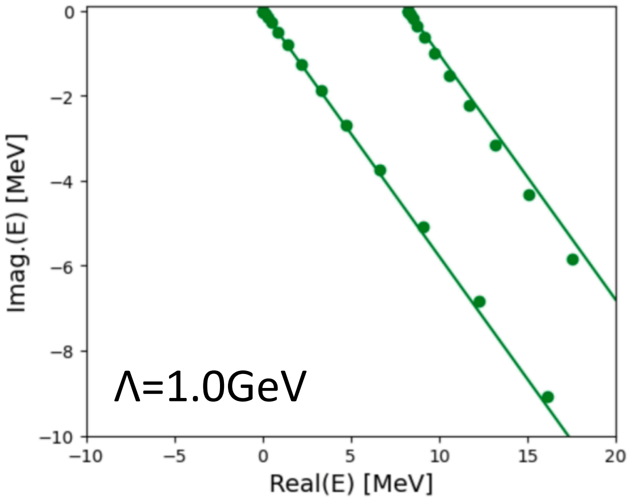


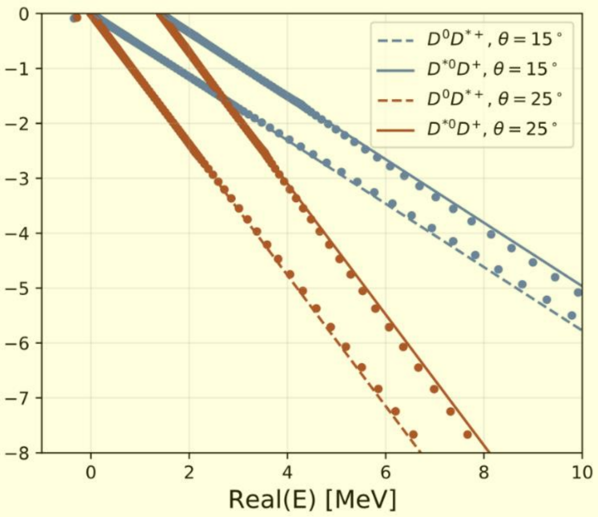
First of all, it is attractive interaction while it is not enough to form a bound state, while just a virtual state

3870.0 + 0.26 i MeV

Produce X(3872) with $D\bar{D}^* + \bar{D}D^*$ and $c\bar{c}$

From the interaction of DD^* to obtain the intraction of $D\bar{D}^* + \bar{D}D^*$, check X(3872) exists or not by pure $D\bar{D}^* + \bar{D}D^*$ interaction, without $c\bar{c}$ state.





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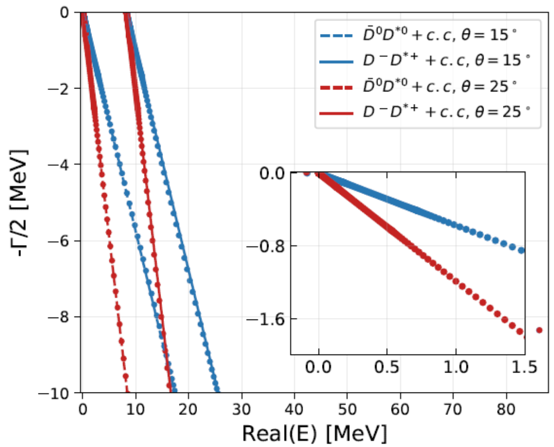
- 70.0% $D^{*+}D^0$, 30.0% D^+D^{*0}

Attractive interaction BUT not enough to form a bound state

- **A bare state shows the $c\bar{c}$ bare state component. χc1(2P, 3940) and its wave function, determined by the quark model.**
- **The interaction parameter γ = 4.69 for the 3P0 model is determined through ψ(3770) to $D\bar{D}$.**
- **Therefore the analysis of X(3872) does not introduce any additional parameters.**



(b)

- **Bound state for X(3872)**

$\Delta E = -80.4$ keV
$\Gamma_{T_{cc}} = 32.5$ keV

- $\sqrt{\langle r^2 \rangle} = 11.2\ fm$
- 94.0% $\bar{D}^{*0}D^0$, 4.8% $D^{*-}D^+$, 1.2% $c\bar{c}$

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T_{cc}, X(3872)的性质

- **T_{cc}束缚态**

$\Delta E = -393$ keV

$\Gamma_{T_{cc}} = 70.4$ keV

- $\sqrt{\langle r^2 \rangle} = 4.7\ fm$
- 70.0% $D^{*+}D^0$， 30.0% D^+D^{*0}

- **X(3872)束缚态**

$\Delta E = -80.4$ keV

$\Gamma_{T_{cc}} = 32.5$ keV

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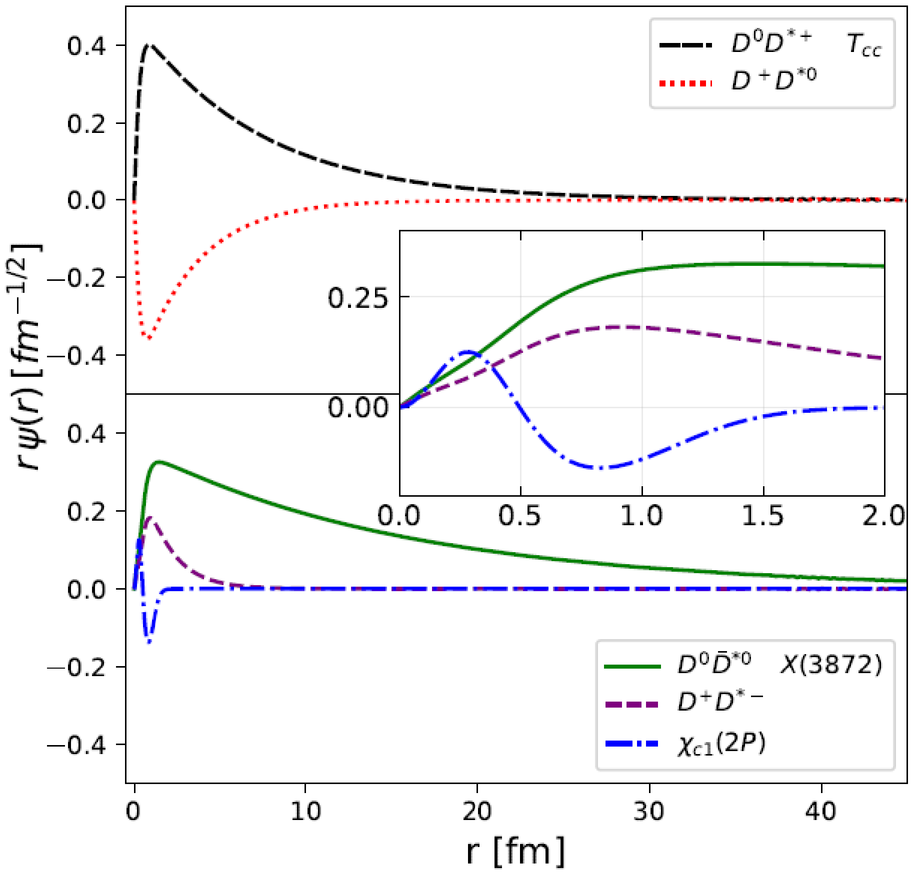

由于X(3872)的束缚能极小，所以整个$D^{*0}\bar{D}^0$波函数是一个尾巴很长很长的波函数，实际上这个很好理解，一旦这个态的质量大于$D^{*0}\bar{D}^0$阈值，就变成共振态，而共振态是不归一的，就是长程上可以变成两个强子态。而这个长尾巴实际上就提供了大部分的复合度，而在短距离的成分中，还是以$c\bar{c}$成分为主。而且在形成中，也体现出裸态的重要作用。

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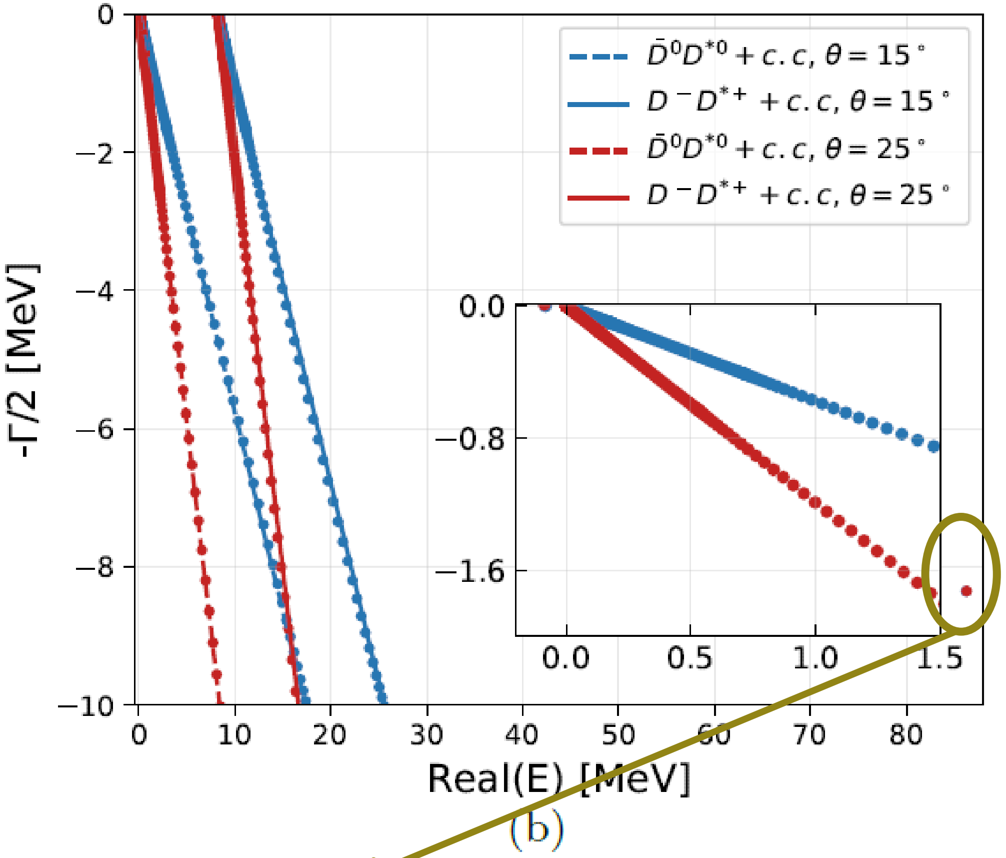
Prediction



	wave function	$I(J^{PC})$	$u-$channel : π	$u-$channel : ρ/ω	$t-$channel : ρ/ω
DD^*	$\frac{1}{\sqrt{2}}(D^+D^{*0} - D^0D^{*+})$	$0(1^+)$ $[T_{cc}^+]$	$\frac{3}{2}V_\pi$	$\frac{3}{2}V_\rho^u - \frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t + \frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}(D^+D^{*0} + D^0D^{*+})$	$1(1^+)$	$\frac{1}{2}V_\pi$	$\frac{1}{2}V_\rho^u + \frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t + \frac{1}{2}V_\omega^t$
$D\bar{D}^*$	$\frac{1}{\sqrt{2}}([D^+D^{*-}] + [D^0\bar{D}^{*0}])$	$0(1^{++})[X(3872)]$	$\frac{3}{2}V_\pi$	$-\frac{3}{2}V_\rho^u - \frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t - \frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}([D^+D^{*-}] - [D^0\bar{D}^{*0}])$	$1(1^{++})$	$-\frac{1}{2}V_\pi$	$\frac{1}{2}V_\rho^u - \frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t - \frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}(\{D^+D^{*-}\} + \{D^0\bar{D}^{*0}\})$	$0(1^{+-})[h_c]$	$-\frac{3}{2}V_\pi$	$\frac{3}{2}V_\rho^u + \frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t - \frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}(\{D^+D^{*-}\} - \{D^0\bar{D}^{*0}\})$	$1(1^{+-})$ $[Z_c(3900)]$	$\frac{1}{2}V_\pi$	$-\frac{1}{2}V_\rho^u + \frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t - \frac{1}{2}V_\omega^t$

	I(J^PC)	Our
DD^*	$0(1^+)[T_{cc}^+]$	$T_{cc}^+(3874)$
	$1(1^+)$	Unstable
$D\bar{D}^*$	$0(1^{++})[\chi_{c1}]$	$X(3872)$, $\chi_{c1}(3958)$
	$1(1^{++})[W_{c1}]$	Missing…
	$0(1^{+-})[h_c]$	$\tilde{X}(3870)$, $h_c(3961)$
	$1(1^{+-})[Z_c]$	Nopole around 3900

Black, $DD^*/D\bar{D}^*$
Red, $c\bar{c} + D\bar{D}^*$

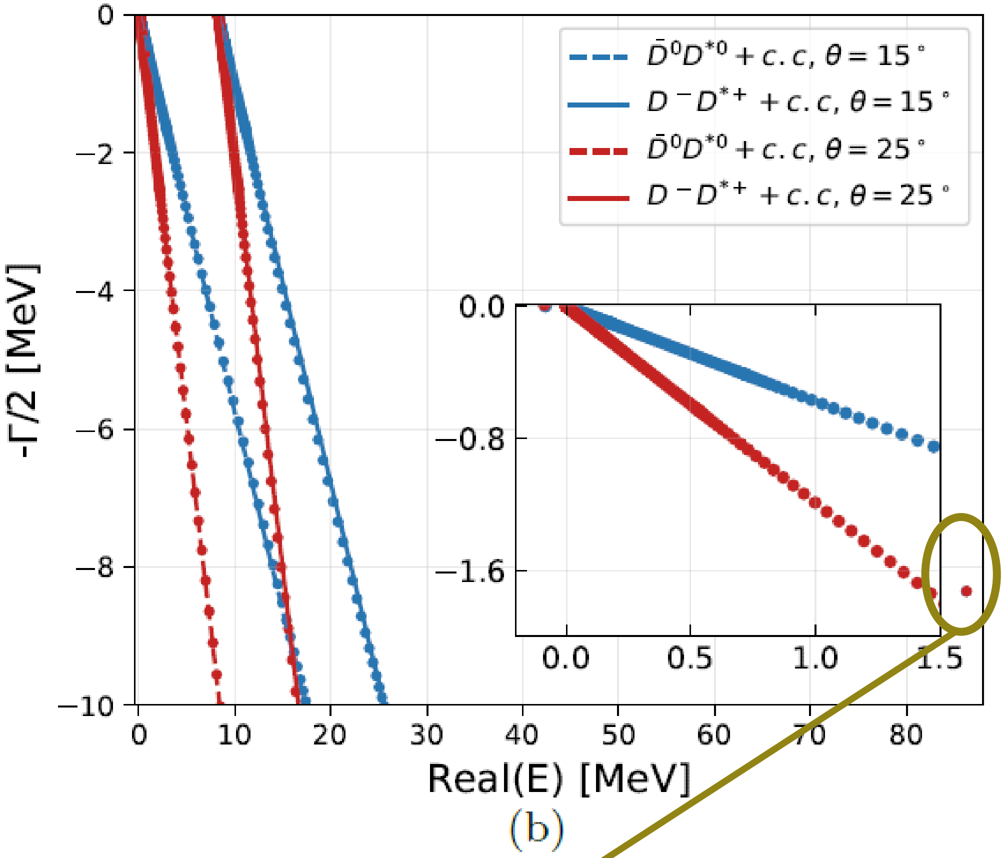
LHCb 2406.03156

Teng Ji, Dong, Guo, Hanhart, Meißner, 2502.04458

COMPASS, PLB 783,334 (2018)

- $\chi_{c1}(2p)$
  M = 3957.9 MeV $\Gamma_{\chi_{c1(2P)}} = 16.7$ MeV
- Main decay channel: $\bar{D}^* D$

Prediction



(b)

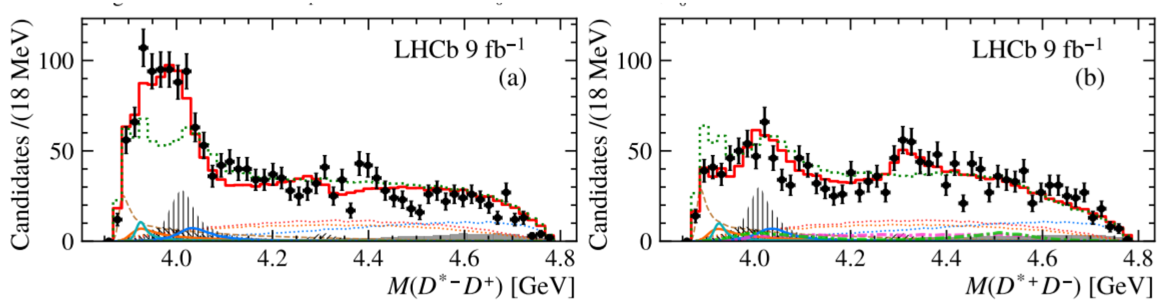
- χ_{c1}(2p)
  M = 3957.9 MeV $\Gamma_{\chi_{c1(2P)}}$ = 16.7 MeV
- Main decay channel： $\bar{D}^* D$

Observation of new charmonium(-like) states in $B^+ \to D^{*\pm} D^{\mp} K^+$ decays

LHCb Collaboration 2406.03156

$\chi_{c1}(4010)$ $J^{PC} = 1^{++}$

$m_0 = 4012.5\,^{+3.6}_{-3.9}\,^{+4.1}_{-3.7}$ $\Gamma_0 = 62.7\,^{+7.0}_{-6.4}\,^{+6.4}_{-6.6}$



X(3872) Relevant $D\bar{D}^*$ Scattering in $N_f = 2$ Lattice QCD

H. Li, C. Shi, Y. Chen, M. Gong, J. Liang et al

CLQCD 2402.14541

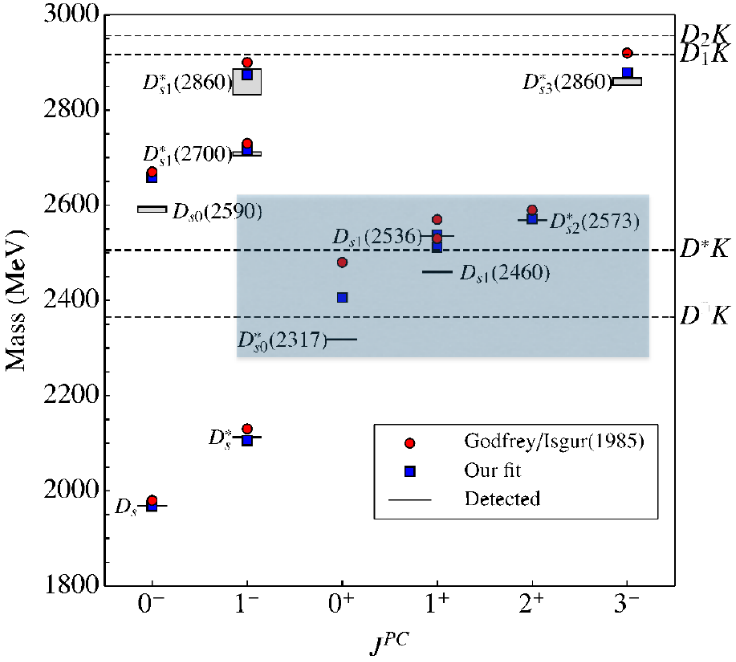
$$t(s) = \frac{K(s)}{1 - K(s) i\rho(s)}$$

$$K(s) = \frac{g}{M^2 - s} + \gamma$$

m_π(MeV)	250(3)	307(2)	362(1)	417(1)
Bound state from $E_{2,3}$				
E_B(MeV)	$-9.7^{+2.1}_{-2.2}$	$-9.7^{+1.9}_{-2.0}$	$-1.3^{+0.6}_{-0.8}$	$\mathbf{-1.3^{+0.8}_{-1.0}}$
BW fit from $E_{3,4}$				
m_R(MeV)	3924(5)	3926(6)	3969(4)	3995(4)
Γ_R(MeV)	63(23)	57(18)	37(13)	57(10)
Bound state pole and residual from $E_{2,3,4}$				
E_B(MeV)	−11(1)	−10(2)	−1.6(7)	−1.7(7)
Resonance pole and residue				
m_R(MeV)	4008(4)	4029(4)	4050(3)	4071(3)
Γ_R(MeV)	60(6)	38(9)	43(8)	50(7)
$\mathrm{Br}_{D\bar{D}^*}$(%)	∼ 100	∼ 100	∼ 100	∼ 100

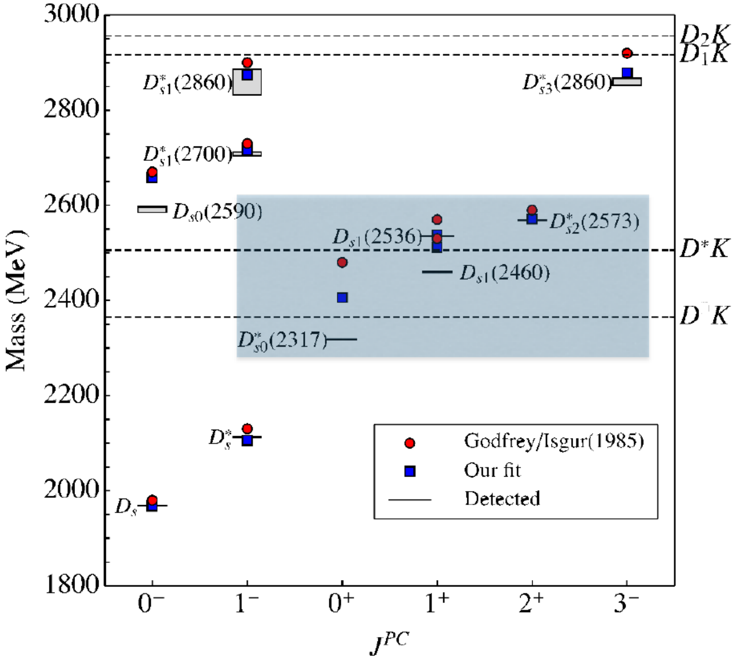
从$D_s(2317)$到$T_{c\bar{s}}(2327)$

Z. Yang, G.-J. Wang, J.-j. Wu, S.-l. Zhu, M. Oka **Phys.Rev.Lett.128(2022),112001**



$D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}(2573)$

Z. Yang, G.-J. Wang, J.-j. Wu, S.-l. Zhu, M. Oka **Phys.Rev.Lett.128(2022),112001**



	$\bar{c}s$ cores		channel	
	$B(\vert^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s0}^*(2317)$	$\vert^3P_0\rangle$	2405.9	DK	S
$D_{s1}^*(2460)$	$0.68\vert^1P_1\rangle - 0.74\vert^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D_{s1}^*(2536)$	$-0.74\vert^1P_1\rangle - 0.68\vert^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\vert^3P_2\rangle$	2571.2	DK, D^*K	D

1. Fix the bare mass and wave function from GI model;

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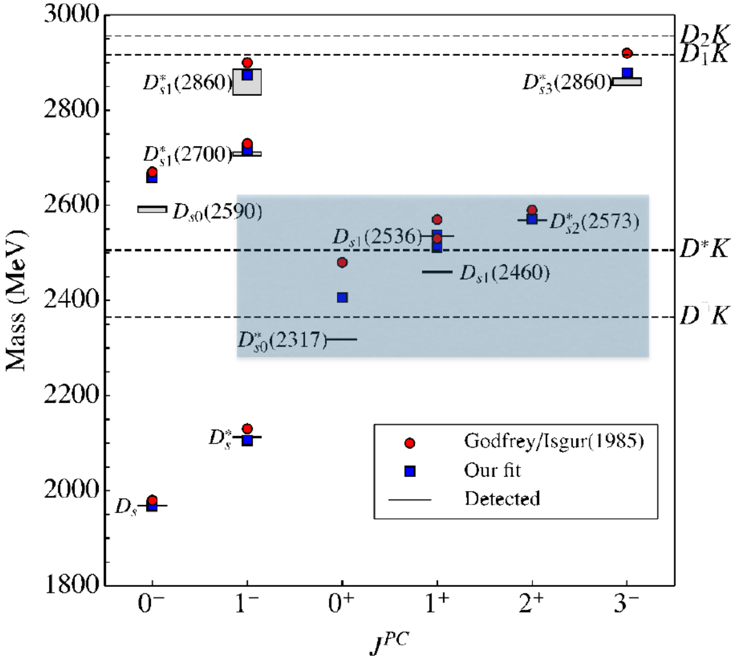
$D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}(2573)$

Z. Yang, G.-J. Wang, J.-j. Wu, S.-l. Zhu, M. Oka **Phys.Rev.Lett.128(2022),112001**



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1. Fix the bare mass and wave function from GI model;
2. The interaction of bare-channel and channel-channel;

$^3P_0$ model

at quark level

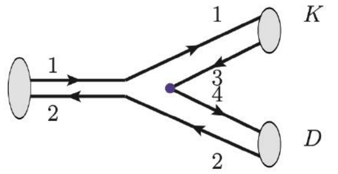
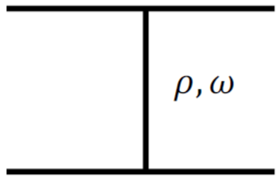
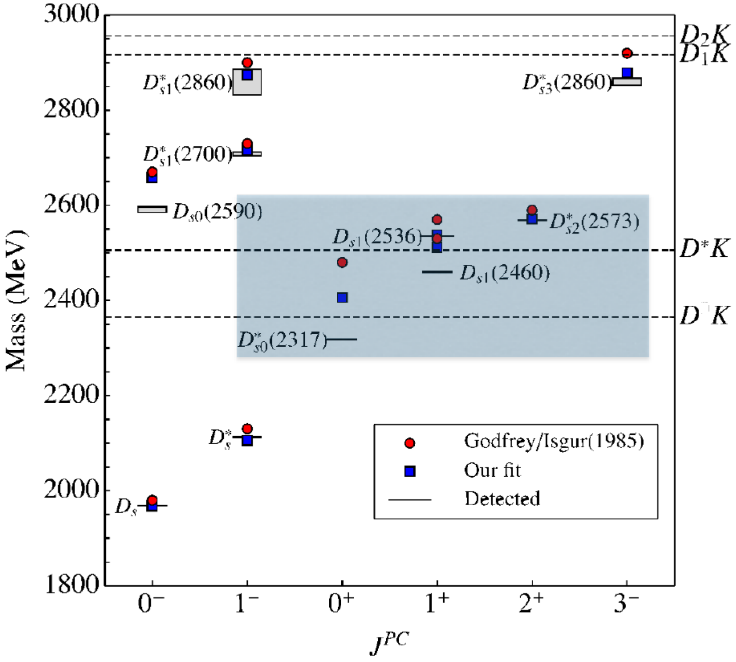


Figure 2: The diagram contribute to the process $D_s^*(2317) \to DK$.



$D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}(2573)$

Z. Yang, G.-J. Wang, J.-j. Wu, S.-l. Zhu, M. Oka **Phys.Rev.Lett.128(2022),112001**



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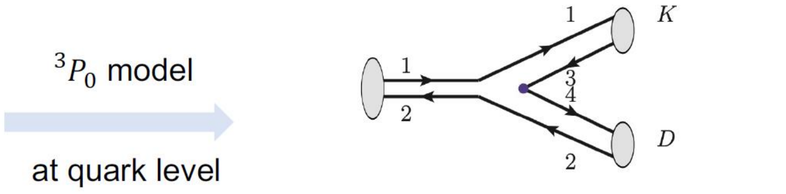

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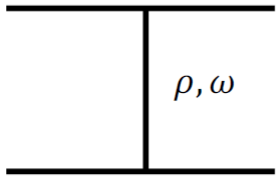
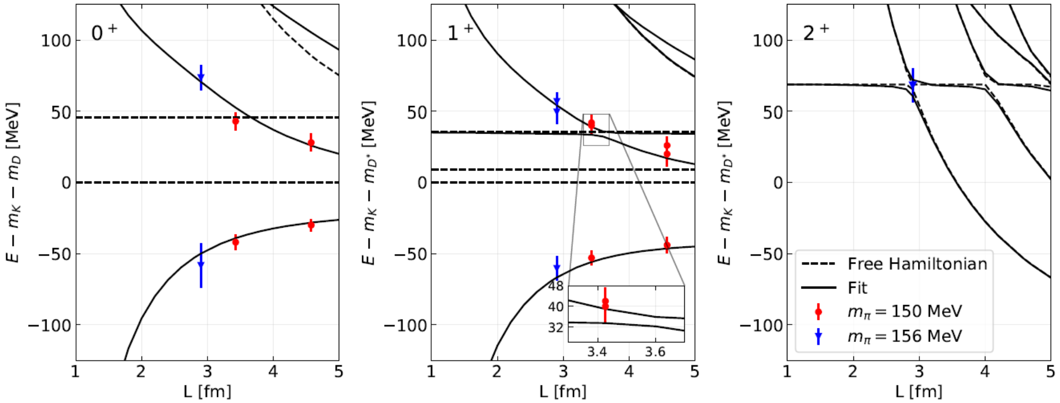


3. Fit Lattice data;



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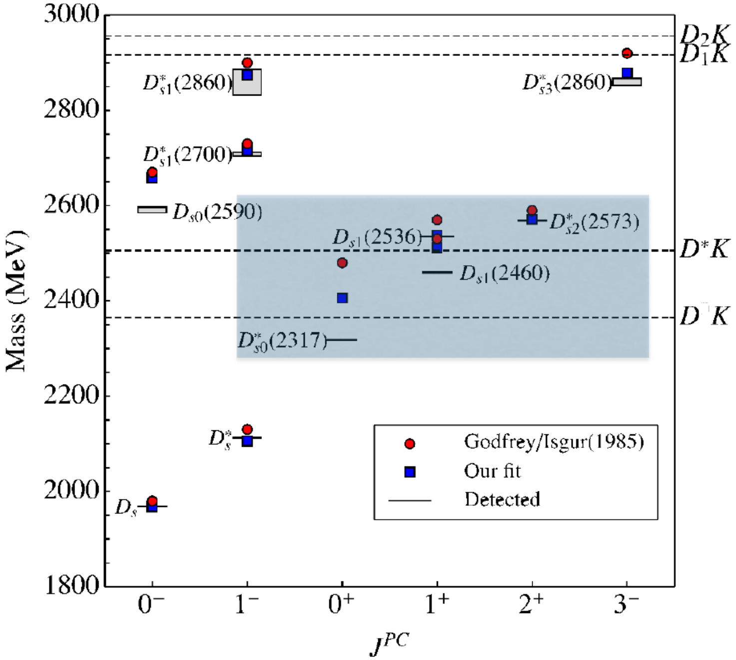
$D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}(2573)$

Z. Yang, G.-J. Wang, J.-j. Wu, S.-l. Zhu, M. Oka **Phys.Rev.Lett.128(2022),112001**



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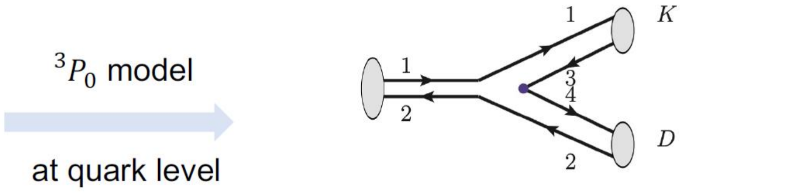
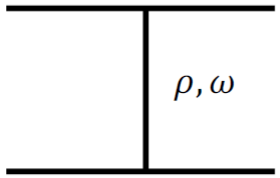
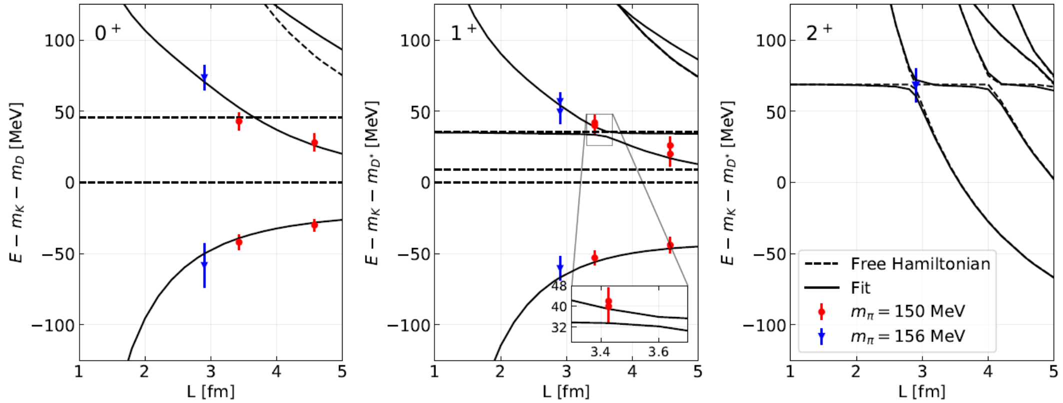


Figure 2: The diagram contribute to the process $D_s^*(2317) \to DK$.



	$P(c\bar{s})$
$D_{s0}^*(2317)$	$\approx 32.0\%$
$D_{s1}^*(2460)$	$\approx 52.4\%$
$D_{s1}^*(2536)$	$\approx 98.2\%$
$D_{s2}^*(2573)$	$\approx 95.9\%$

3. Fit Lattice data;



Conclusion:

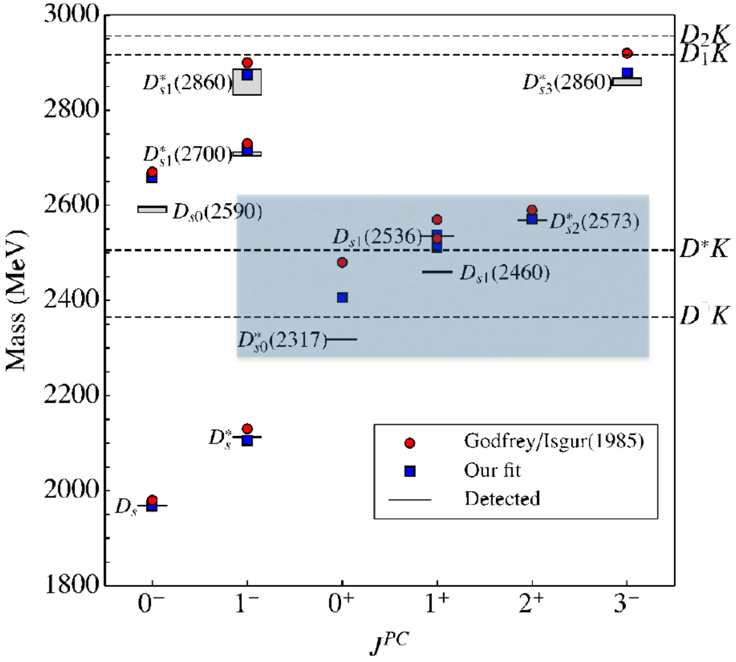
$D^*_{s0}(2317)$-DK
$D_{s1}(2460)$-D^*K **S-wave**

Mass moving vs GI Model

$D_{s1}(2536)$-D^*K
$D_{s2}(2573)$-$D^{(*)}K$ **D-wave**

Mass stable vs GI model

$D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}(2573)$



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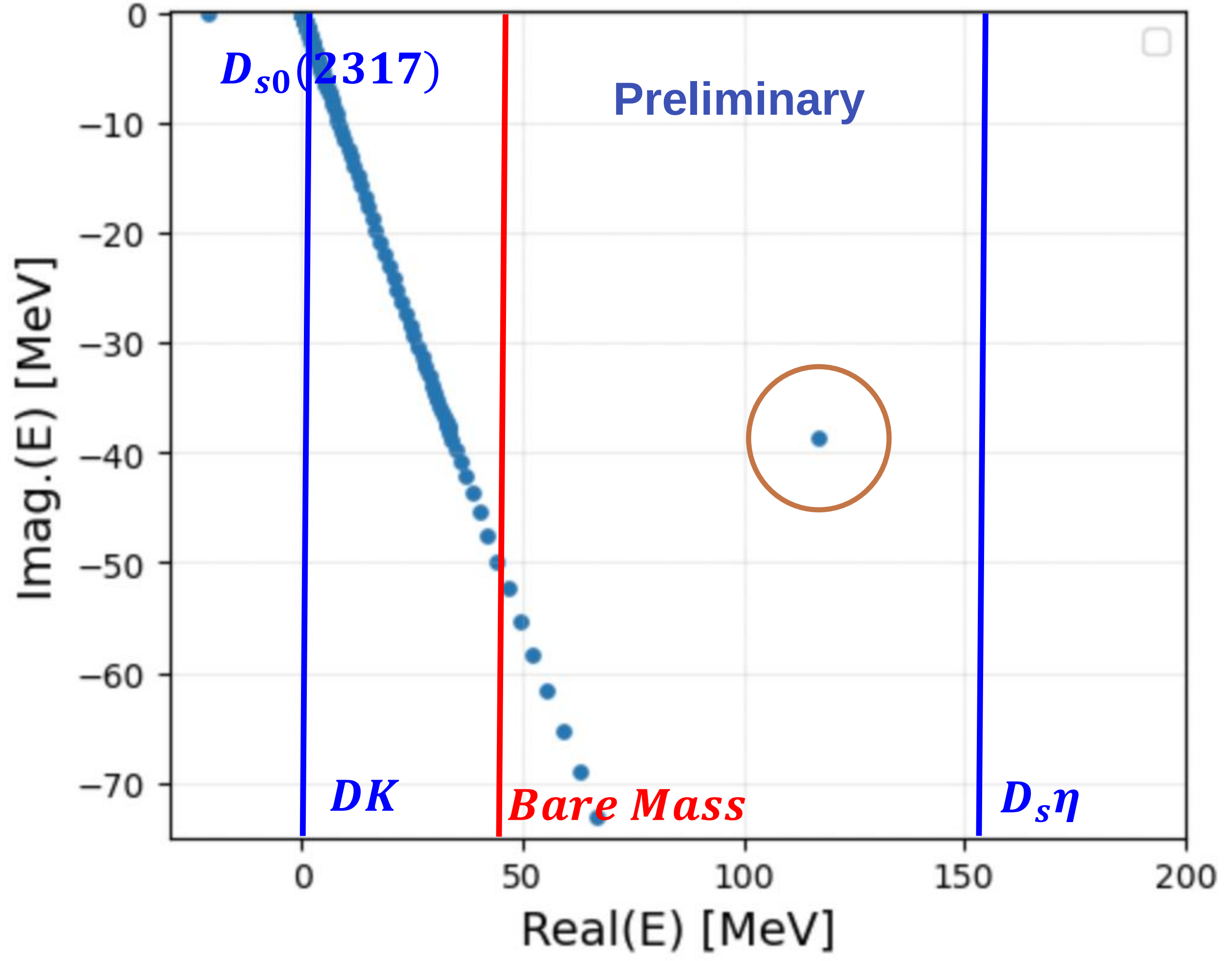
1

2

3



128(2022),112001

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	$P(c\bar{s})$
$D^*_{s0}(2317)$	$\approx 32.0\%$
$D^*_{s1}(2460)$	$\approx 52.4\%$
$D^*_{s1}(2536)$	$\approx 98.2\%$
$D^*_{s2}(2573)$	$\approx 95.9\%$

on:

)-DK

)-D*K

S-wave

ving vs GI Model

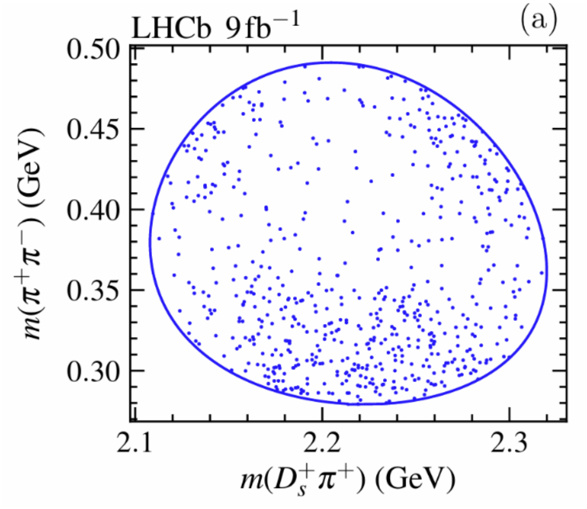
)-D*K

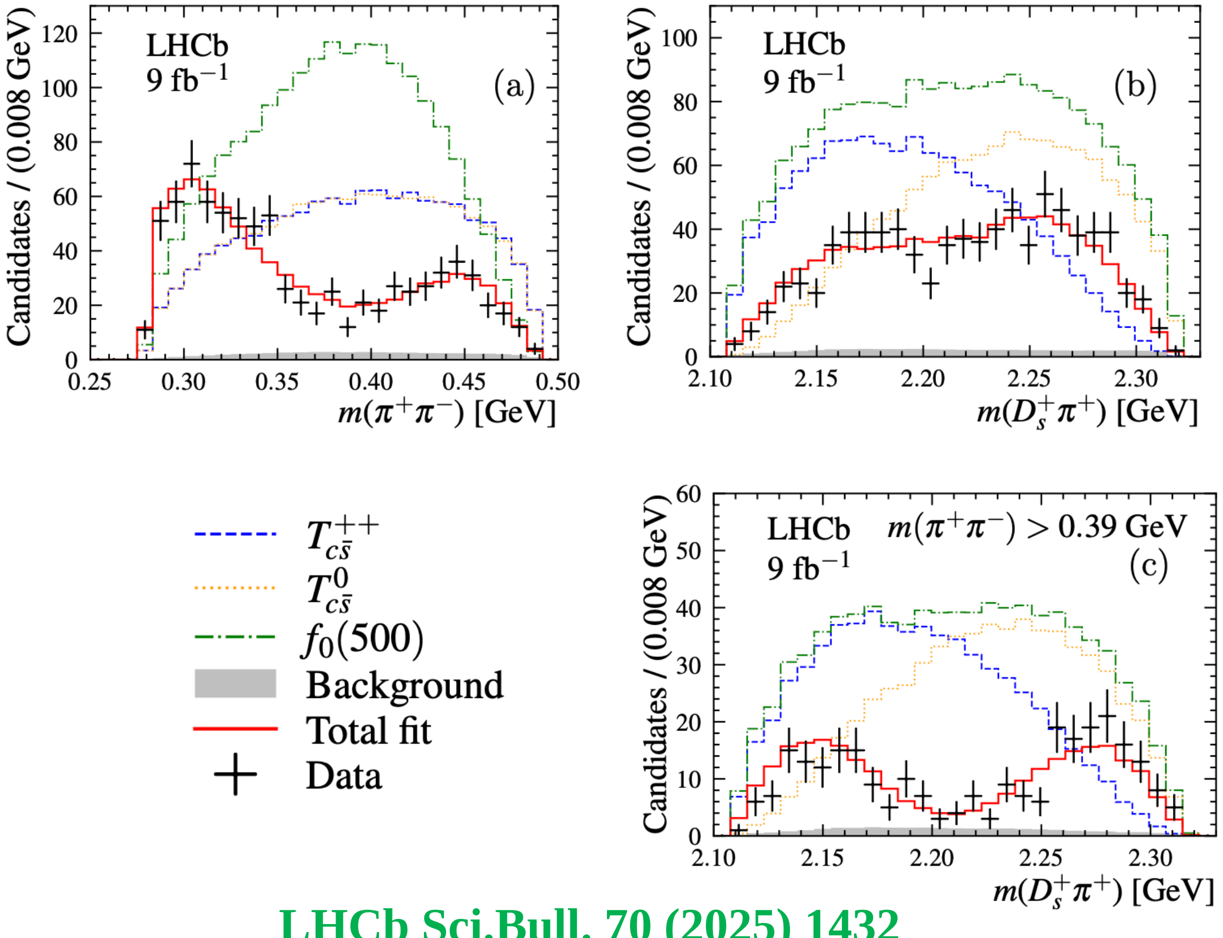
)-D(*)K

D-wave

ole vs GI model

$B \to D^{(*)} D_{s1}(2460) \to D^{(*)} D_s^+ \pi^+ \pi^-$ decays





$D_s \to D_s\pi$ Isospin breaking

$D_s^+\pi^+$, $D_s^+\pi^0$, $D_s^+\pi^-$ are isospin triplet

– Four quark state

Model	Resonance	Mass (MeV)	Width (MeV)
$f_0(500)$ + RBW $T_{c\bar{s}}(0^+)$	$f_0(500)$	$464 \pm 23 \pm 14$	$214 \pm 28 \pm 8$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^{0}$	$2312 \pm 27 \pm 11$	$264 \pm 46 \pm 21$
$f_0(500)$ + K-matrix $T_{c\bar{s}}(0^+)$	$f_0(500)$	$472 \pm 32 \pm 19$	$226 \pm 24 \pm 18$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^{0}$	$2328 \pm 12 \pm 12$	$96 \pm 16 \pm 23$

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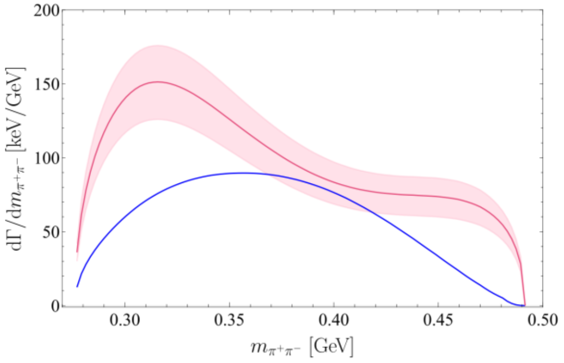
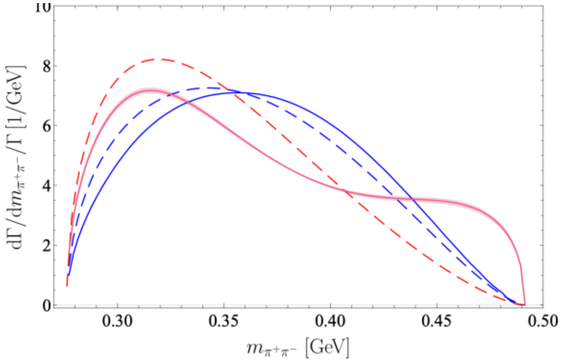
$B \to D^{(*)} D_{s1}(2460) \to D^{(*)} D_s^+ \pi^+ \pi^-$ decays





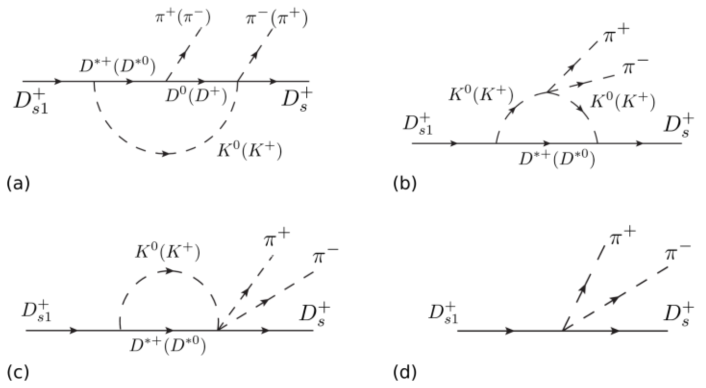
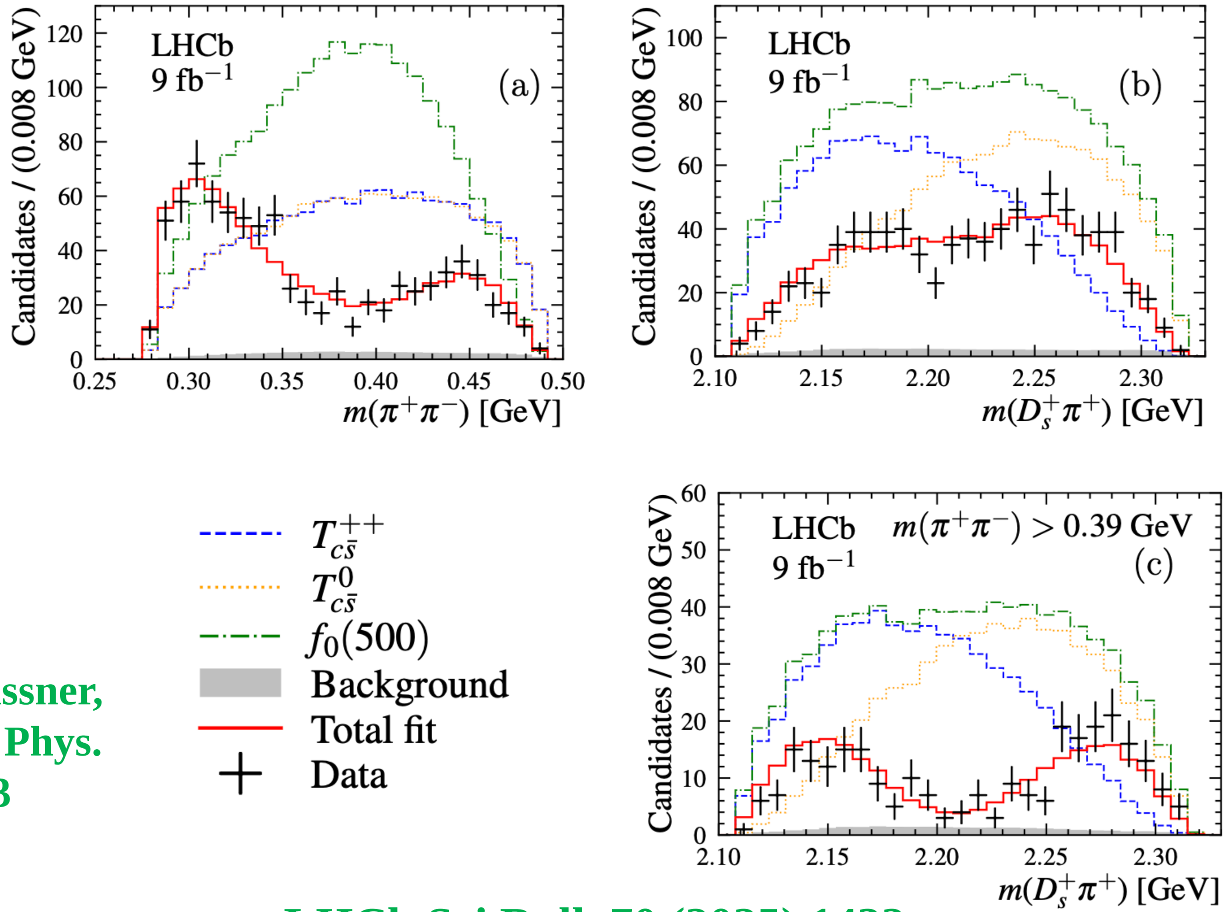


Figure 1. Diagrams for the decay $D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ with (a+b+c) and without (d) the D^*K contribution.

Tang, Lin, Guo, Hanhart and Meissner, Commun. Theor. Phys. 75 (2023)5 055203



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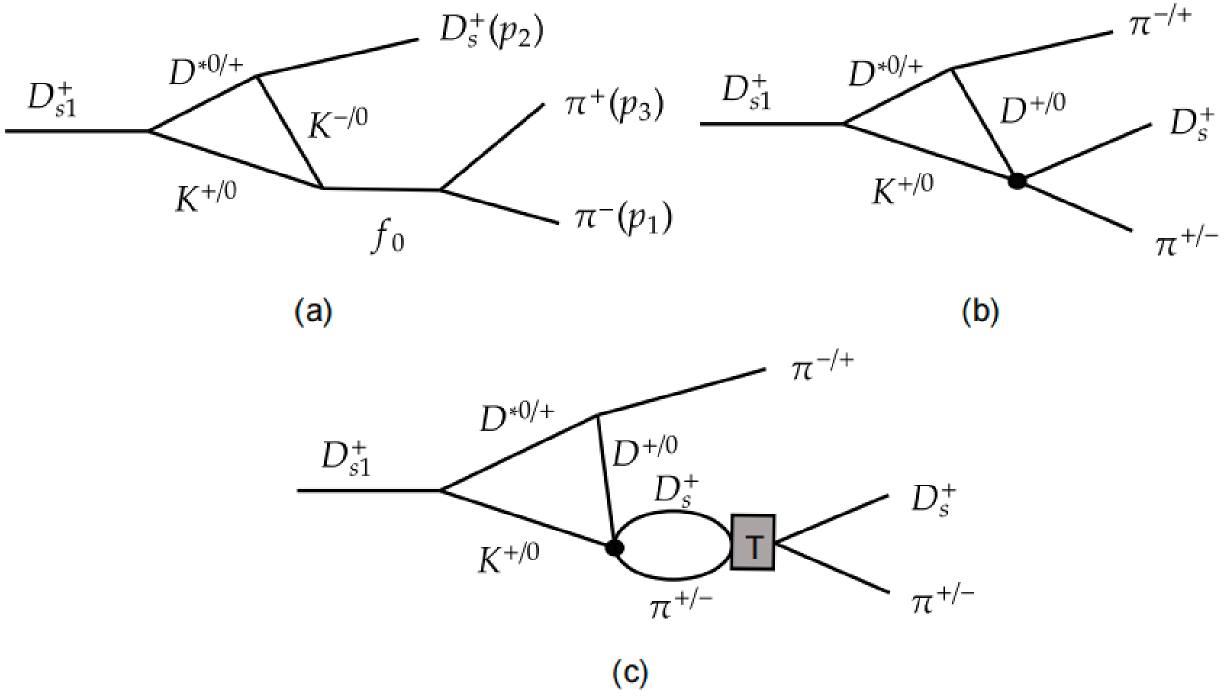
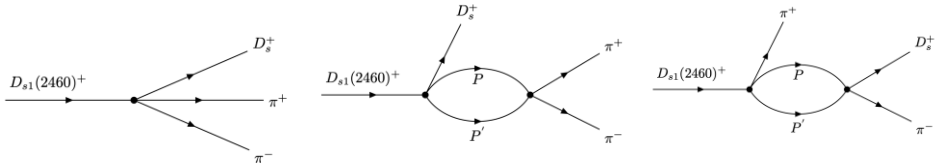
$D_{s1}(2460) \to D_s^+ \pi^+\pi^-$



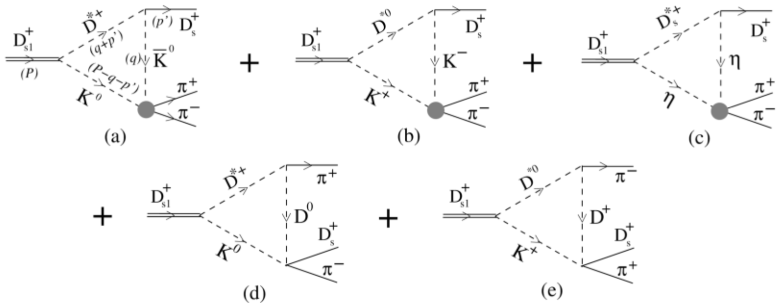
Z. Yang, G.-J. Wang, J.-J. Wu, M. Oka, 2510.01564

ZY Wang, YS Li and SQ Luo, PRD 111 (2025) 7, 076009



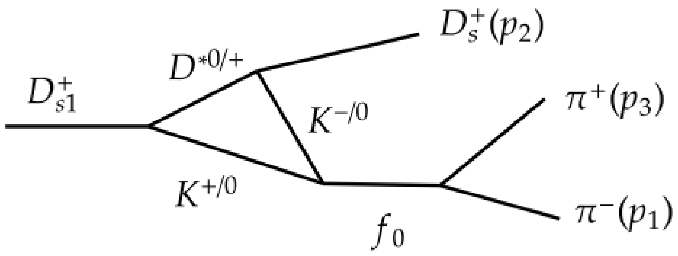
Missing the triangle loop, losing the nature of $D_{s1}(2460)$

Roca, Dias and Oset, Eur.Phys.J.C 85 (2025) 7, 808

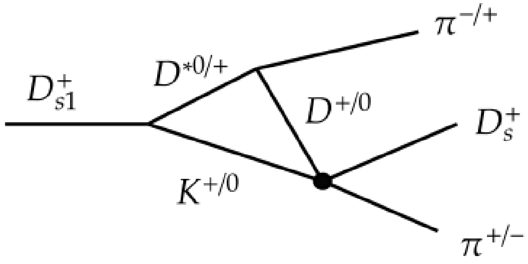


Missing the coupled channel effect of $D_s\pi - DK$

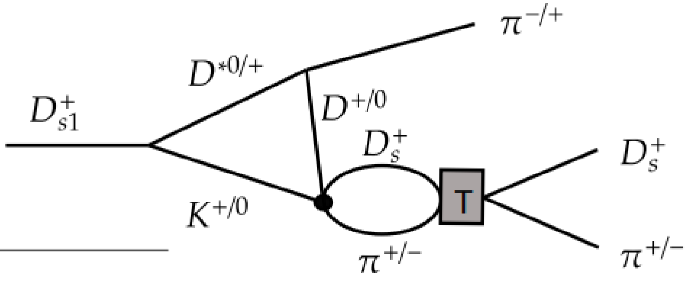
$D_{s1} \to D_s^+ \pi^+ \pi^-$



(a)



(b)



(c)

Conclusion：

$D^*_{s0}(2317)$-DK
$D_{s1}(2460)$-D^*K — S-wave

$$\mathcal{M}_S = g_S \epsilon_i^{\mu} \epsilon_{j,\mu}^{\dagger}$$

$$\mathcal{M}_D = \frac{g_D}{M^2} \epsilon_i^{\mu} \epsilon_j^{\dagger\nu} \left(q_\mu q_\nu - \frac{g_{\mu\nu} q^2}{4} \right)$$

Mass moving vs GI Model

$$D_{s1}(2460): \quad g_S \gg g_D$$

$D_{s1}(2536)$-D^*K
$D_{s2}(2573)$-$D^{(*)}$K — D-wave

$$D_{s1}(2536): \quad \frac{g_S}{g_D} = 0.08\ e^{2.67i}$$

LHCb，JHEP 10 (2023) 106 $\quad \frac{g_S}{g_D} = 0.1\ e^{-0.7i}$

Mass stable vs GI model

Parameter	
Λ_1 [GeV]	$2.18^{+0.24}_{-0.04}$
Λ_2 [GeV]	0.5 (fixed)
g_{K^*}	$55.3^{+0.8}_{-2.5}$
ϕ [Rad]	$3.78^{+0.38}_{-0.26}$
r_1	215^{+51}_{-86}
r_2	$-9.0^{+4.3}_{-0.8}$
m_{f_0} [MeV]	519^{+31}_{-89}
Γ_{f_0} [MeV]	242^{+90}_{-88}
χ^2/d.o.f.	1.43
E_p [MeV]	$2288.4^{+11.7}_{-13.4} - 89.6^{+6.5}_{-6.4} i$

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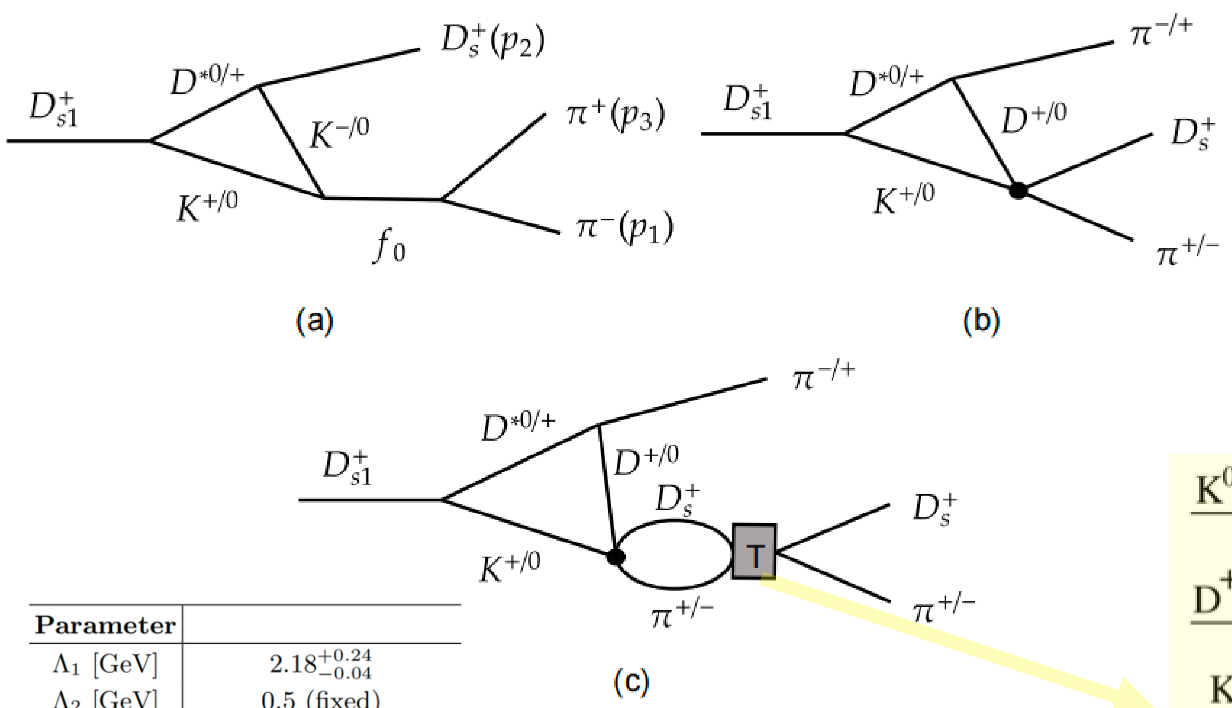
$D_{s1} \to D_s^+ \pi^+ \pi^-$



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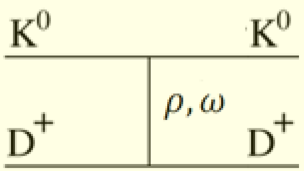
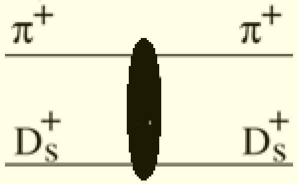
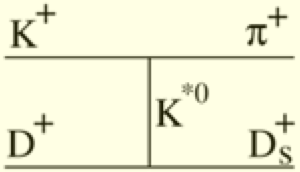
Mass stable vs GI model



$I = 0, V_\rho + V_\omega < 0$
$I = 1, V_\rho - V_\omega \sim 0$



$V \sim 0$



$V \neq 0$

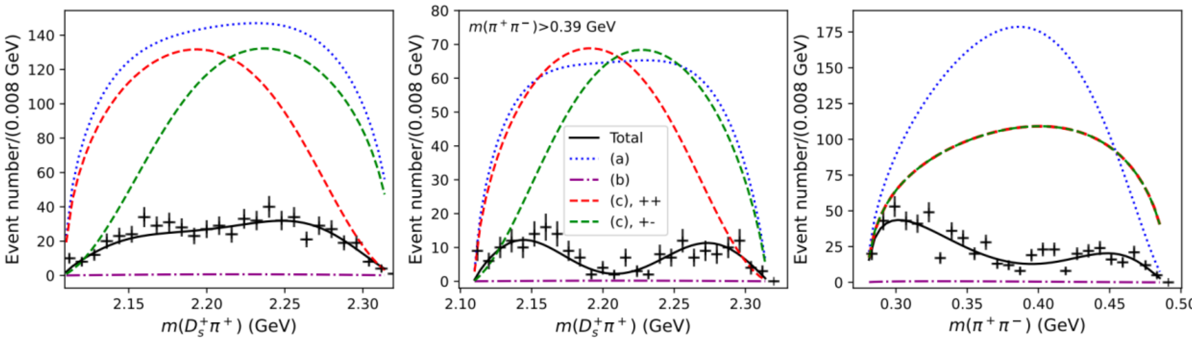
$T = (1 - VG)^{-1} V$
$\to \det T^{-1} \propto (1 - V^2 G_{DK} G_{D_s\pi})$

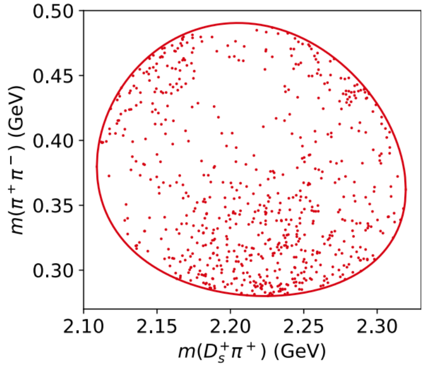
For DK, effective potential is $V^2 G_{D_s\pi}$
For $D_s\pi$, effective potential is $V^2 G_{DK}$

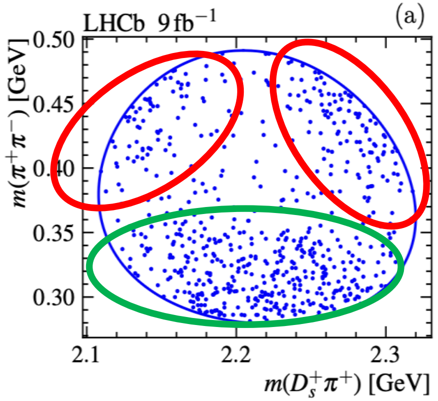
Control the value of cut off will adjust the effective potential.

$D_{s1} \to D_s^+ \pi^+ \pi^-$

$D_{s1}(2460)$

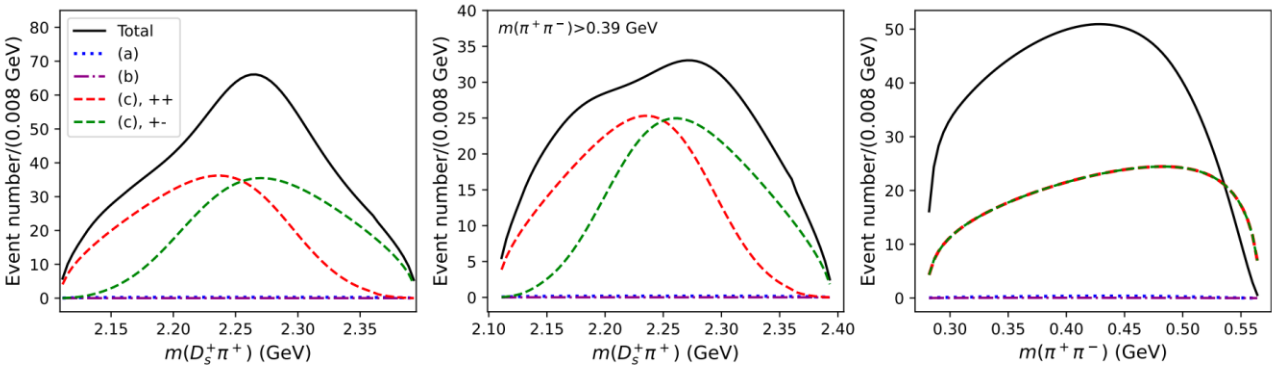


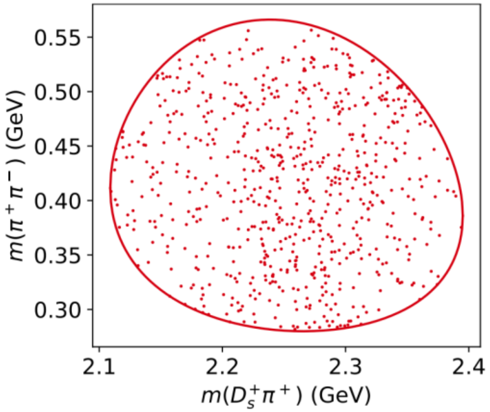




χ^2/dof
=1.43

$D_{s1}(2536)$





If a stricter cut is applied, for instance, $m(\pi^+\pi^-) > 0.45$ GeV , the two-peak structure will appear.

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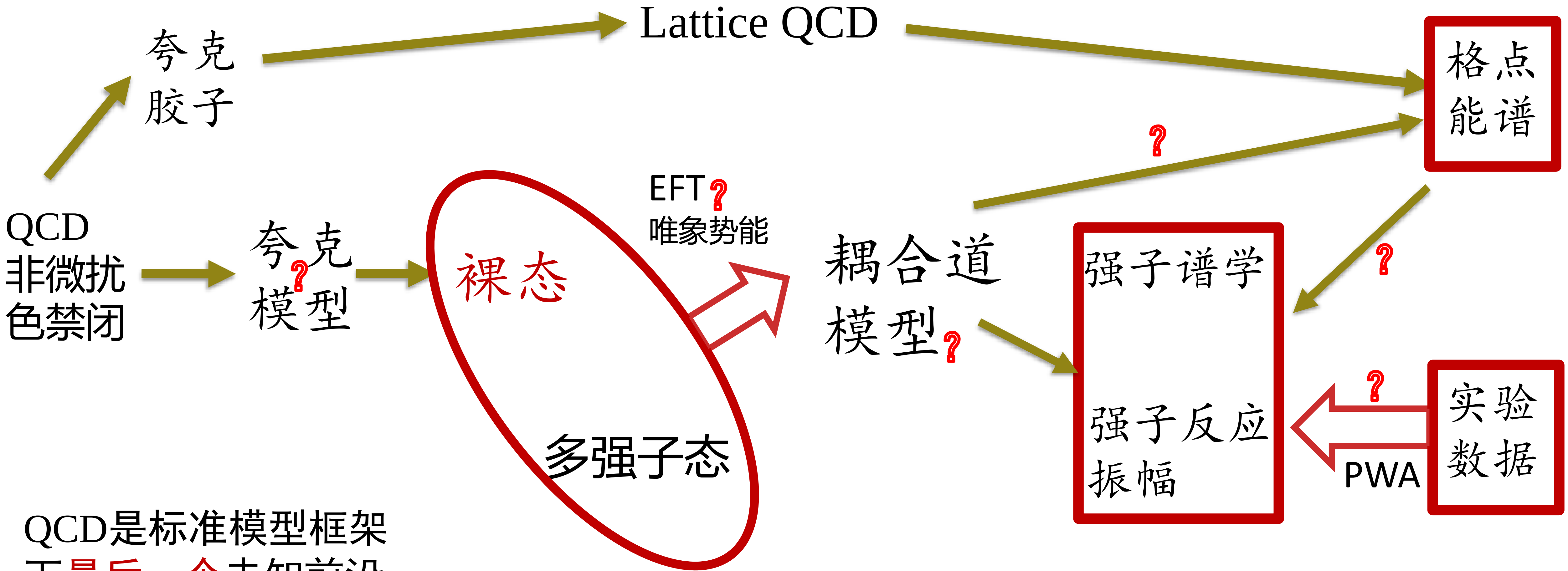
小结

- 强相互作用涉及两个层次的物理，极其复杂。
- 介绍了HEFT方法，通过裸态链接夸克胶子层次和强子层次，建立强子层次不同强子数的耦合道模型，尽可能的全面的理解强子性质。
- 利用T_{cc}获得 DD^*的相互作用，进而得到$D\bar{D}^*$的相互作用，结合$c\bar{c}$裸态细致理解X(3872), 成功预言还存在另一个态。
- 利用格点数据成功解释$D_{s0}(2317)$和$D_{s1}(2460)$的质量小于传统夸克模型预言，而$D_{s1}(2536)$和$D_{s2}(2573)$非常吻合。进而理解$D_{s1}(2460)$ 的$D_s\pi\pi$衰变，获得$T_{c\bar{s}}$态和预言$D_{s1}(2536)$的$D_s\pi\pi$衰变。

展望



QCD是标准模型框架
下最后一个未知前沿，
我相信我们终将知道。

谢谢大家观看

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