Understanding Spin polarization with (1+1)D Ideal Spin Hydrodynamics

Matteo Buzzegoli

West University of Timișoara

International Symposium on Spin Polarization in Relativistic Heavy Ion Collisions 2026

13/05/26

Based on [ArXiv:2605.08219]
In collaboration with
Aleksandar Gecic and Rajeev Singh

Spin polarization in heavy-ion collisions

Peripheral collisions → Angular momentum →

Thermal Vorticity of the fluid: ϖ

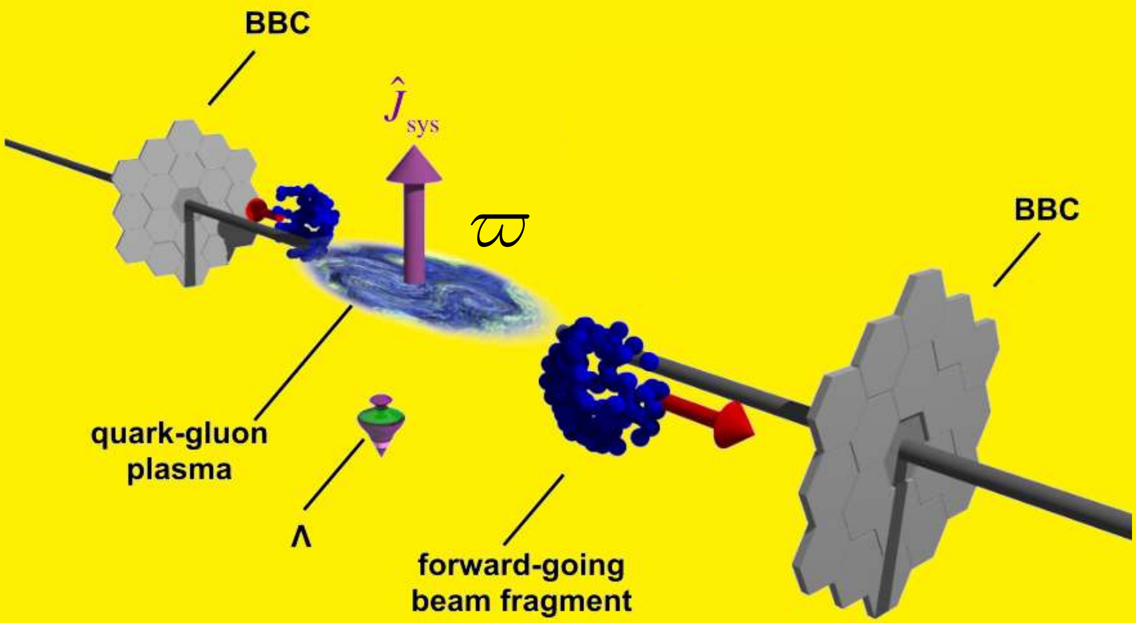
Include acceleration, rotation and gradients of temperature

$$\varpi^{\mu\nu} = -\frac{1}{2}\left(\partial^{\mu}\beta^{\nu} - \partial^{\nu}\beta^{\mu}\right)$$

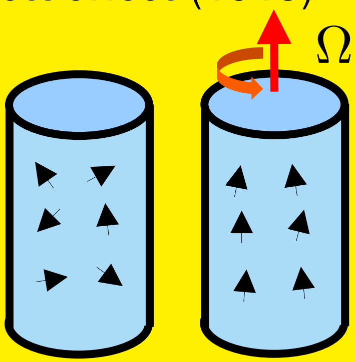
$$\beta^{\mu} = u^{\mu}/T$$

Global polarization w.r.t. reaction plane

Similar to Barnett effect (1915)



[STAR, Nature 548 (2017)]
[Z. T. Liang, X. N. Wang, PRL 94 (2005) 102301]
[F. Becattini, F. Piccinini, Ann. Phys. 323 (2008) 2452]



Ferromagnetic → Magnetization

Motivation

- Understand Spin polarization in heavy-ion collisions
- Study spin dynamics
- This talk focuses on Au+Au collisions at high energy

Agreement between hydrodynamic predictions and the data

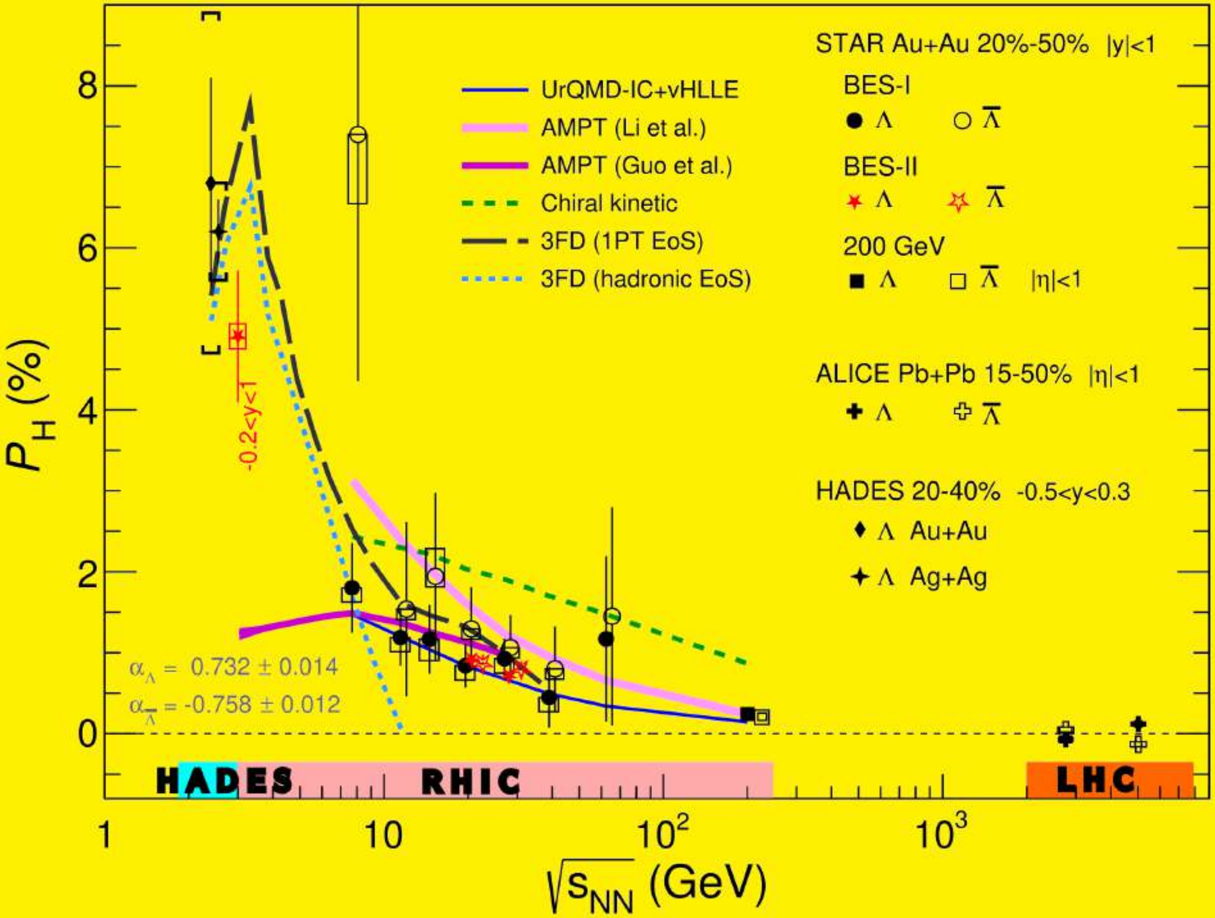
[F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338:32 (2013)]

Global Spin Polarization ← Integrate over momenta

$$P^\mu_H(p) = \frac{1}{4m_H}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

$$\varpi^{\mu\nu} = -\frac{1}{2}\left(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu\right)$$

$$n_F = \left(\mathrm{e}^{\beta\cdot p-\zeta}+1\right)^{-1}$$



Σ is a 3D hyper-surface where the hadron is formed.

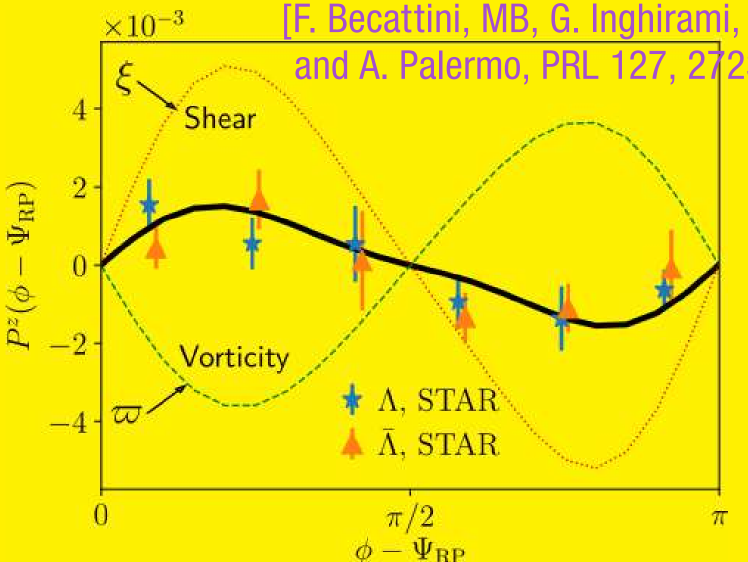
Different models of the collision, same formula for polarization

[F. Becattini, MB, T. Niida, S. Pu, A. H. Tang and Q. Wang, Int. J. Mod. Phys. E 33 (2024) no.06, 2430006]

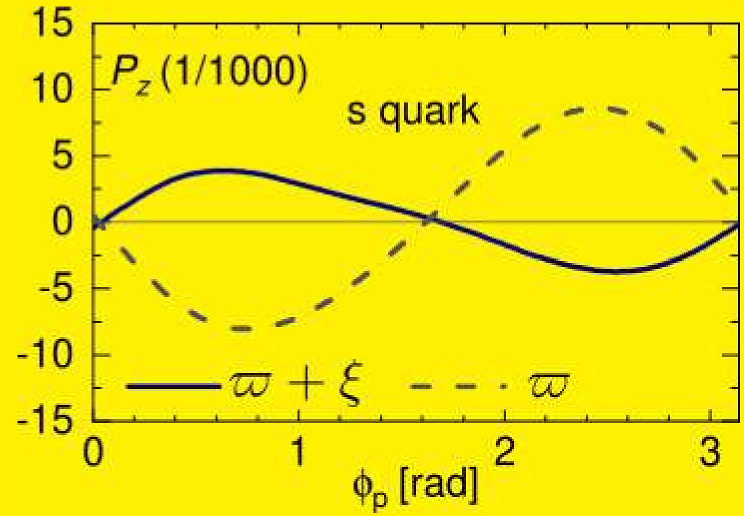
Local spin polarization

- “Local”: Momentum dependent polarization (along beam direction)
- Thermal shear-induced polarization can explain the local spin polarization data

[F. Becattini, MB, G. Inghirami, I. Karpenko, and A. Palermo, PRL 127, 272302 (2021)]



[Fu, Liu, Pang, Song, Yin, PRL 127 (2021)]



However,

- Isothermal local equilibrium vs strange quark scenario
- Sensitivity of shear flow to properties of plasma (EoS and bulk viscosity)

[Jiang, Wu, Cao, Zhang, PRC 108, 064904 (2023); Palermo, Grossi, Karpenko and Becattini, EPJC 84 (2024)]

- Pseudo-gauge dependence? [Becattini, Florkowski, Speranza, PLB 789 (2019);Speranza, Weickgenannt, EPJA 57 (2021)]
- Does spin reach equilibrium fast enough?

[M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov and H.-U. Yee, JHEP 08 (2022); D. Wagner, M. Shokri and D.H. Rischke, Phys. Rev. Res. 6 (2024)]

→ Develop Spin hydrodynamics and include a Spin potential

Spin hydrodynamics

Spin hydrodynamics is necessary when the spin relaxation time scale is much longer than the time scale of e.g. kinetic equilibration [Becattini, Florkowski, Speranza, PLB 789 (2019); M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov and H. U. Yee, JHEP 11 (2021)]

Include the spin tensor $S^{\lambda,\mu\nu}$ and the spin potential ω in the hydro equations:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j^\mu = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

$$T^{\mu\nu} = T^{\mu\nu}(\beta, \zeta, \omega), \quad j^\mu = j^\mu(\beta, \zeta, \omega), \quad S^{\lambda,\mu\nu} = S^{\lambda,\mu\nu}(\beta, \zeta, \omega)$$

Spin hydrodynamics and pseudo-gauge dependence

Spin hydrodynamics is necessary when the spin relaxation time scale is much longer than the time scale of e.g. kinetic equilibration [Becattini, Florkowski, Speranza, PLB 789 (2019); M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov and H. U. Yee, JHEP 11 (2021)]

Include the spin tensor $S^{\lambda,\mu\nu}$ and the spin potential ω in the hydro equations:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j^\mu = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

$$T^{\mu\nu} = T^{\mu\nu}(\beta, \zeta, \omega), \quad j^\mu = j^\mu(\beta, \zeta, \omega), \quad S^{\lambda,\mu\nu} = S^{\lambda,\mu\nu}(\beta, \zeta, \omega)$$

Pseudo-gauge (PG) symmetry:

$$\widehat{P}^\mu = \int \mathrm{d}\Sigma_\mu \, \widehat{T}^{\mu\nu}, \quad \widehat{J}^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda \left[x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} + \widehat{S}^{\lambda,\mu\nu} \right]$$

The charges are invariant under the redefinition:

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\nabla_\lambda \left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu} \right)$$

$$\widehat{S}'^{\lambda,\mu\nu} = \widehat{S}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu} + \nabla_\rho \widehat{Z}^{\mu\nu,\lambda\rho}$$

The spin tensor is not unique!

Local thermal equilibrium and hydrodynamics are not PG invariant!

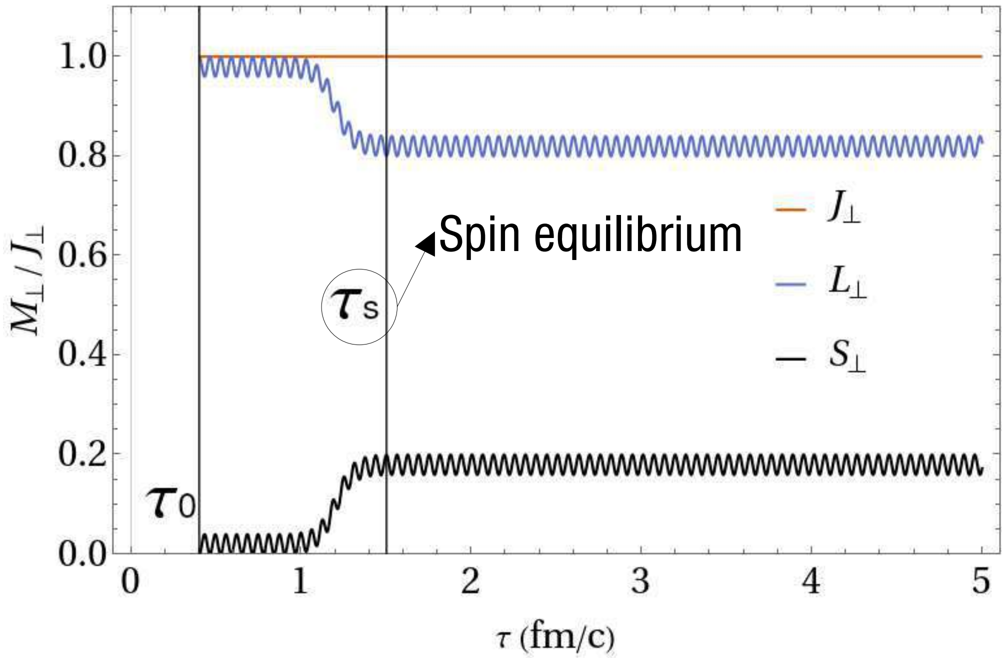
[W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019); E. Speranza and N. Weickgenannt, EPJA 57, 155 (2021); F. Becattini and L. Tinti, PRD 84 (2011) and PRD 87 (2013); K. Fukushima and S. Pu, PLB 817 (2021); A. Das, W. Florkowski, R. Ryblewski and R. Singh, PRD 103 (2021)]

Spin polarization is PG dependent!

[MB, PRC 105 (2022)]

Spin Dynamics in HIC

Realistic (illustration)



Orbital angular momentum is transferred to Spin angular momentum

[Z.-T. Liang and X.-N. Wang, PRL 94 (2005)]

Angular momentum is conserved:

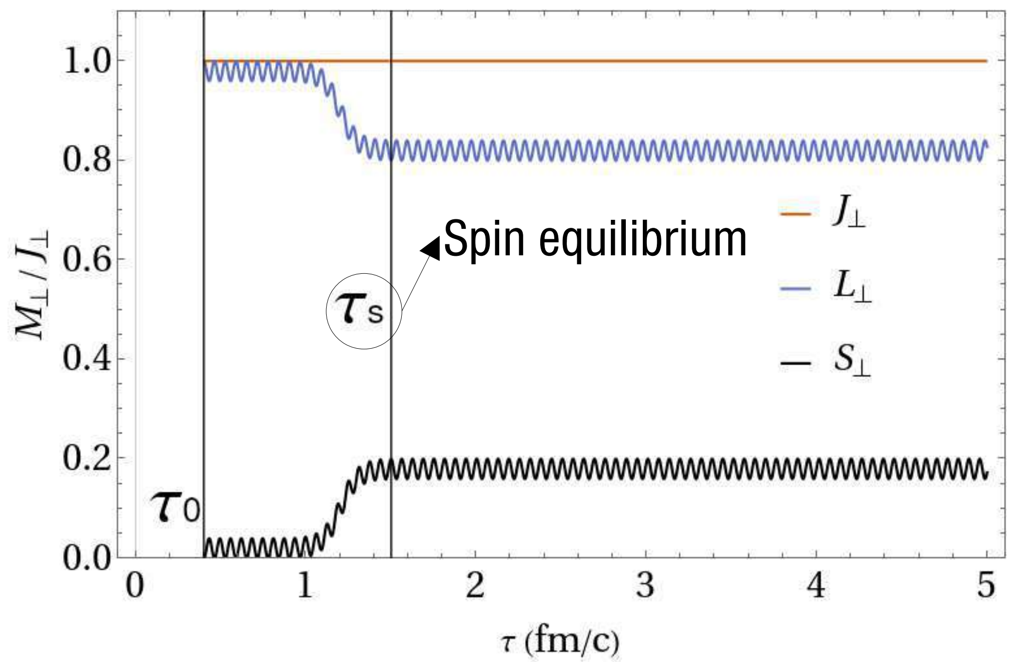
$$\boldsymbol{J}_{\text{initial}} = \boldsymbol{L}_{\text{initial}} = \boldsymbol{J}_{\text{final}} = \boldsymbol{L}_{\text{final}} + \boldsymbol{S}_{\text{final}}$$

Most successful models for Pz assume equilibration of spin (and use Belinfante pseudo-gauge)

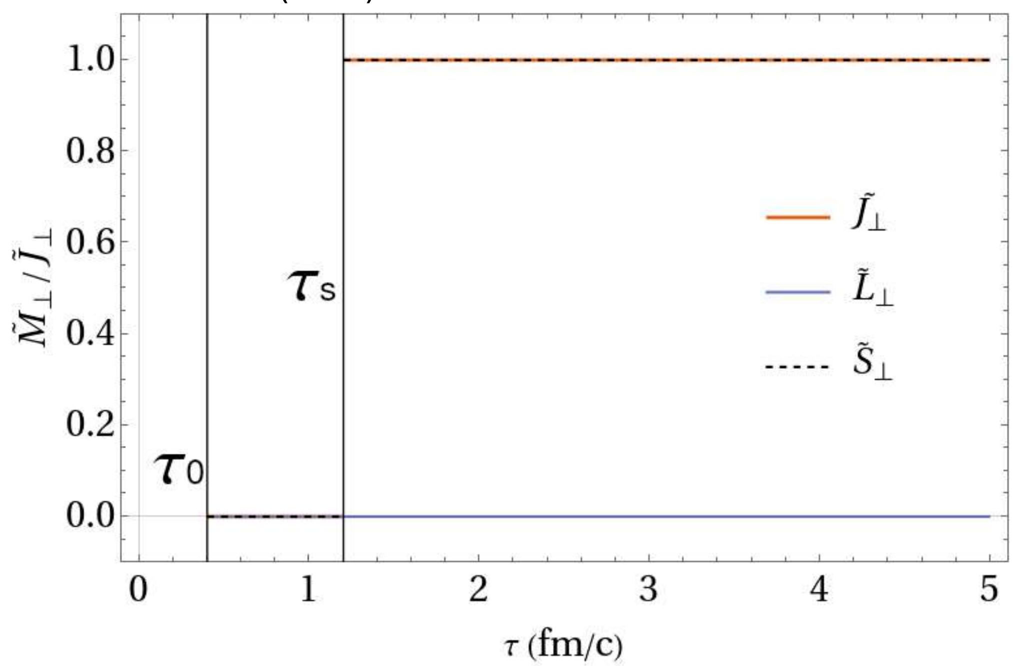
[B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, PRL 127 (2021)]
[F. Becattini, MB, G. Inghirami, I. Karpenko, and A. Palermo, PRL 127 (2021)]
[S. Alzhrani, S. Ryu and C. Shen, PRC 106 (2022)]
[Z. F. Jiang, X. Y. Wu, S. Cao and B. W. Zhang, PRC 108 (2023)]
[A. Palermo, E. Grossi, I. Karpenko, and F. Becattini, EPJ C 84 (2024)]
[A. Arslan, W.-B. Dong, C. Gale, S. Jeon, Q. Wang, and X.-Y. Wu, PRC 113 (2026)]

Spin Dynamics in HIC

Realistic (illustration)



Ideal Model (1+1)D



Angular momentum is conserved:

$$\boldsymbol{J}_{\text{initial}} = \boldsymbol{L}_{\text{initial}} = \boldsymbol{J}_{\text{final}} = \boldsymbol{L}_{\text{final}} + \boldsymbol{S}_{\text{final}}$$

Ideal Spin Hydrodynamics

[R. Singh, Int. J. Mod. Phys. A 38 (2023) no.20, 2330011]
[W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709]

- We assume that spin degrees of freedom equilibrate at time $\tau_s > \tau_i$
- The Spin tensor is thereafter conserved
- We work with the de Groot-van Leeuwen-van Weert (GLW) pseudo-gauge
- We use ideal (non-interacting, non-dissipative) hydrodynamics

Spin Hydrodynamics (GLW)

Density operator in local thermal equilibrium (LTE)

$$\widehat{\rho}_{\rm LTE} = \frac{1}{Z}\exp\left[-\int_\Sigma {\rm d}\Sigma_\mu(x)\left(\widehat{T}^{\mu\nu}(x)\beta_\nu(x) - \zeta(x)\widehat{j}^\mu(x) - \frac{1}{2}\omega_{\alpha\beta}(x)\widehat{S}^{\mu,\alpha\beta}(x)\right)\right]$$

New hydrodynamic variable: spin potential $\omega_{\alpha\beta}$

When spin is not instantaneously equilibrated, its dynamics must be included.

[R. Singh, Int. J. Mod. Phys. A 38 (2023) no.20, 2330011]

[W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709]

The GLW pseudo-gauge has a symmetric EMT and a non-vanishing spin tensor (for a free field)

$$\partial_\lambda \widehat{S}^{\lambda,\mu\nu} = \widehat{T}^{\mu\nu} - \widehat{T}^{\nu\mu} = 0$$

Spin polarization in GLW pseudo-gauge is completely dictated by spin potential

[MB, PRC 105 (2022)]

$$P_\mu(p) = \frac{\int_{\Sigma_{\rm FO}} {\rm d}^3\Sigma \cdot p\, S_\mu(x,p)}{\int_{\Sigma_{\rm FO}} {\rm d}^3\Sigma \cdot p\, n_f(x,p)} \qquad S_\mu(x,p) = -\frac{1}{4m_\Lambda}\widetilde{\omega}_{\mu\beta}p^\beta n_f(x,p)\left(1 - n_f(x,p)\right)$$

Note that contributions from thermal vorticity, gradients of temperature and thermal shear are included, but for a free field they are either vanishing or simplify in just the contribution from spin potential.

Spin polarization in GLW can be obtained in (1+1)D!

Modeling Λ polarization in Au+Au collisions at 200 GeV using relativistic spin hydrodynamics

[MB, A. Gecic, R. Singh, ArXiv:2605.08219]

Ideal Spin Hydrodynamics (1+1)D (GLW pseudo-gauge)

Small spin polarization assumption: 1st order in spin potential
(no spin potential in EMT)

Free field EMT: $\partial_\mu T^{\mu\nu} = 0\,, \quad \text{with} \quad T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - P\,\Delta^{\mu\nu}, \quad m = 300~\text{MeV},$

Spin potential: $\omega_{\mu\nu} = 2\, a_{[\mu} u_{\nu]} + \epsilon_{\mu\nu\alpha\beta}\, u^\alpha\, \omega^\beta$

Spin tensor: $S^{\lambda,\mu\nu} = u^\lambda \left(S_1 \omega^{\mu\nu} + S_2 u^{[\mu} a^{\nu]} \right) + S_3 \left(u^{[\mu} \omega^{\nu]\lambda} + \eta^{\lambda[\mu} a^{\nu]} \right)$

S_1, S_2, S_3 known functions of u, T and m.

Basis:

$$u^\alpha = (u^0, 0, 0, u^3)\,,$$
$$x^\alpha = (0, 1, 0, 0)\,,$$
$$y^\alpha = (0, 0, 1, 0)\,,$$
$$z^\alpha = (u^3, 0, 0, u^0)\,.$$

$\partial_\lambda S^{\lambda,\mu\nu} = 0$ + Initialization $\omega_{\mu\nu}|_{\tau_s}$

$$\left.\begin{aligned}
2\,\partial_\mu \left(S_3\, a_x\, u^\mu\right) - \partial_\mu \left(S_3\, \omega_y\, z^\mu\right) + S_3\, a_x \left(u^\mu z \cdot \partial z_\mu\right) + 2\, S_1\, \omega_y \left(u^\mu u \cdot \partial z_\mu\right) &= 0\,,\\
2\,\partial_\mu \left(S_3\, a_y\, u^\mu\right) + \partial_\mu \left(S_3\, \omega_x\, z^\mu\right) + S_3\, a_y \left(u^\mu z \cdot \partial z_\mu\right) - 2\, S_1\, \omega_x \left(u^\mu u \cdot \partial z_\mu\right) &= 0\,,\\
\partial_\mu \left(S_3\, a_z\, u^\mu\right) &= 0\,,\\
2\,\partial_\mu \left(S_1\, \omega_x\, u^\mu\right) - \partial_\mu \left(S_3\, a_y\, z^\mu\right) + S_3\, \omega_x \left(z^\mu z \cdot \partial u_\mu\right) + 2\, S_3\, a_y \left(z^\mu u \cdot \partial u_\mu\right) &= 0\,,\\
2\,\partial_\mu \left(S_1\, \omega_y\, u^\mu\right) + \partial_\mu \left(S_3\, a_x\, z^\mu\right) + S_3\, \omega_y \left(z^\mu z \cdot \partial u_\mu\right) - 2\, S_3\, a_x \left(z^\mu u \cdot \partial u_\mu\right) &= 0\,,\\
\partial_\mu \left(S_1\, \omega_z\, u^\mu\right) &= 0\,.
\end{aligned}\right\}$$

Solving the eqs. give the dynamics of spin potential:

$$\omega_{\mu\nu} = \omega_{\mu\nu}(\tau, \eta_s)$$

SJG (1+1)D Flow

[S. Shi, S. Jeon, and C. Gale, PRC 105 no. 2, (2022) L021902]

EoS: $\varepsilon = 3P, \quad c_s^2 = 1/3$

Analytical solutions:

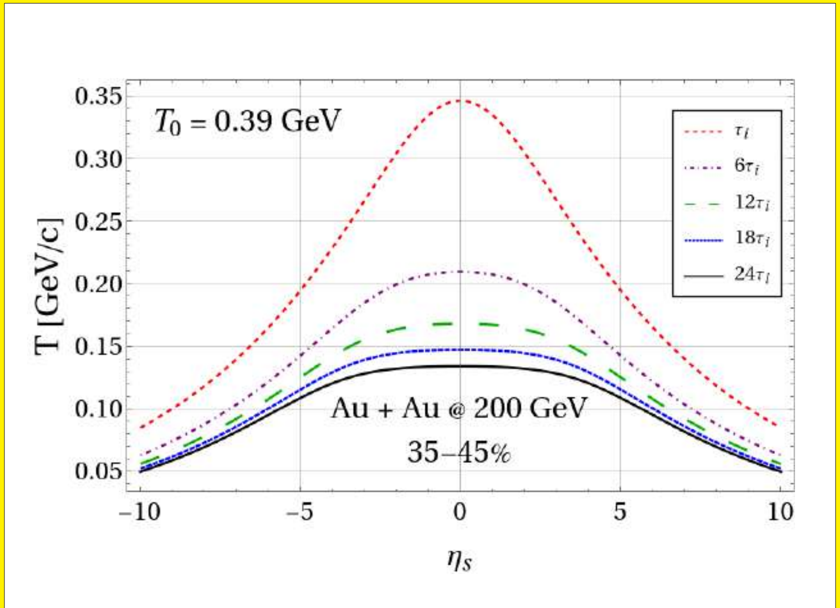
$$u^0(\tau,\eta_s) = u^\tau \cosh\eta_s + \tau u^{\eta_s} \sinh\eta_s\,,$$
$$u^3(\tau,\eta_s) = u^\tau \sinh\eta_s + \tau u^{\eta_s} \cosh\eta_s\,,$$

$$\sqrt{\frac{1-c_s^2}{1+c_s^2}} \le a \le \sqrt{\frac{1+c_s^2}{1-c_s^2}}$$

$$u^\tau = \frac{1}{2}\left[\left(\frac{t_0\,\mathrm{e}^{-\eta_s}+\tau a}{t_0\,\mathrm{e}^{\eta_s}+\frac{\tau}{a}}\right)^{\frac{1}{2}} + \left(\frac{t_0\,\mathrm{e}^{\eta_s}+\frac{\tau}{a}}{t_0\,\mathrm{e}^{-\eta_s}+\tau a}\right)^{\frac{1}{2}}\right],$$

$$\tau u^{\eta_s} = \frac{1}{2}\left[\left(\frac{t_0\,\mathrm{e}^{-\eta_s}+\tau a}{t_0\,\mathrm{e}^{\eta_s}+\frac{\tau}{a}}\right)^{\frac{1}{2}} - \left(\frac{t_0\,\mathrm{e}^{\eta_s}+\frac{\tau}{a}}{t_0\,\mathrm{e}^{-\eta_s}+\tau a}\right)^{\frac{1}{2}}\right],$$

$$T(\tau,\eta_s) = T_0\left[\frac{1}{\tau_0^2}\left[\tau^2+t_0^2+\tau t_0\left(a\mathrm{e}^{\eta_s}+\frac{\mathrm{e}^{-\eta_s}}{a}\right)\right]\right]^{\frac{1}{6a^2}-\frac{1}{3}}$$



- t_0 is a positive constant controlling the rapidity structure, specifically its width.
- a is the positive dimensionless parameter quantifying asymmetry in rapidity: e.g.
  - $a = 1$ → Au+Au
  - $a \neq 1$ → p+Pb.
- The solutions reduce to the Bjorken solutions when $t_0 = 0,\ a = 1.$
- **Non-boost invariant**

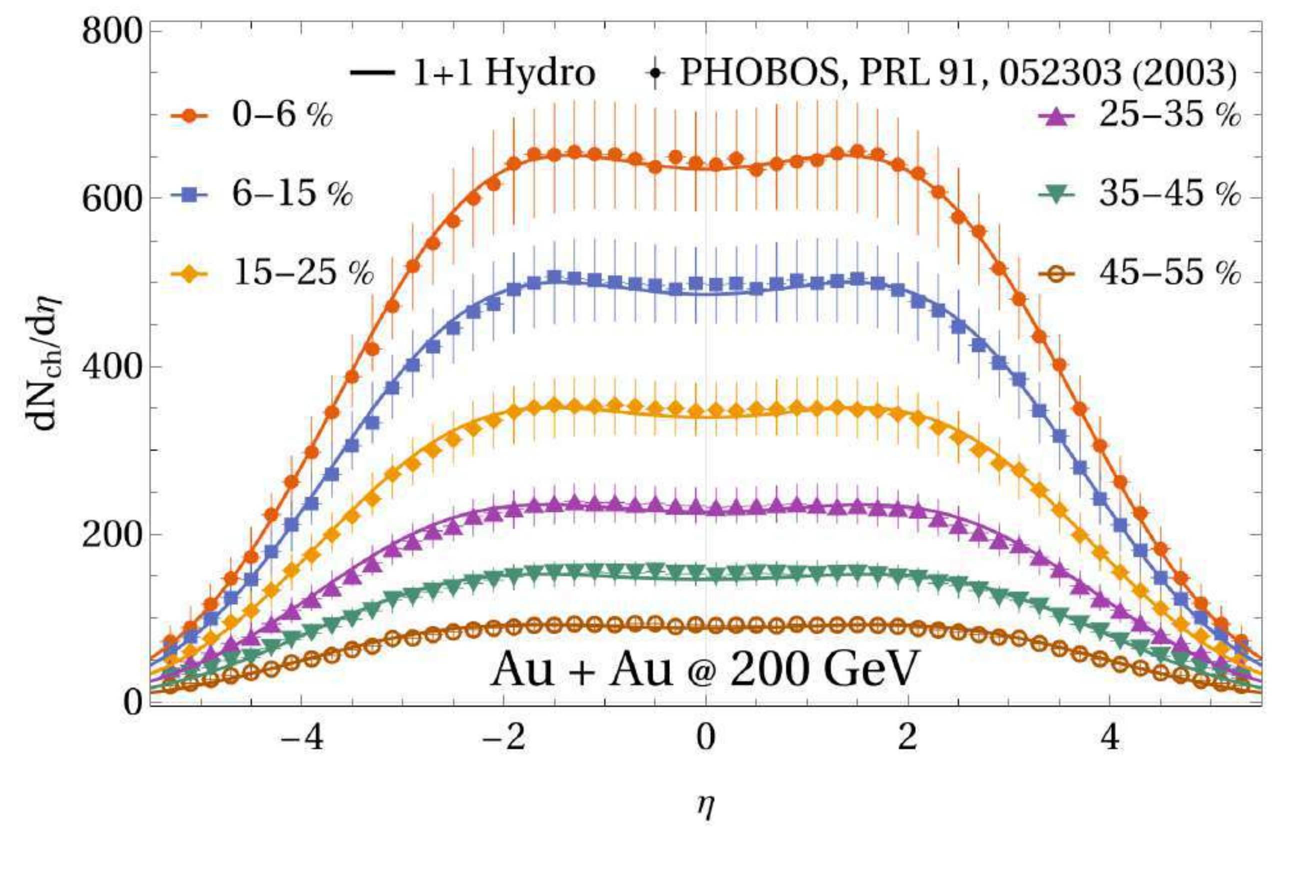
SJG (1+1)D Flow, Au-Au 200 GeV

Centrality (%)	T_0(GeV)	t_0(fm/c)
0-6	0.42	0.28
6-15	0.42	0.26
15-25	0.40	0.21
25-35	0.39	0.18
35-45	0.39	0.17
45-55	0.39	0.16
55-65	0.37	0.15
65-75	0.38	0.14

$a = 1$

$\tau_i = \tau_0 = 0.4$ fm/c

$T_{\text{FO}} = 155$ MeV



Spin potential initialization

Observation:

- Global spin polarization only along total angular momentum

We obtain:

- Only non-vanishing component along J: $S^{13} \neq 0$

[W. Florkowski, R. Ryblewski, R. Singh, and G. Sophys, PRD 105 (2022)]

Studying the explicit expressions:

- $\omega_y,\ a_y \quad [\eta_s - \text{even}]$
- $\omega_x,\ \omega_z,\ a_x,\ a_z \quad [\eta_s - \text{odd}]$

We see that

- $a_x,\ \omega_y$ only contributes to $P_x,\ P_y$,
- $a_y,\ \omega_x$ only contribute to P_z,
- No contributions from $a_z,\ \omega_z$

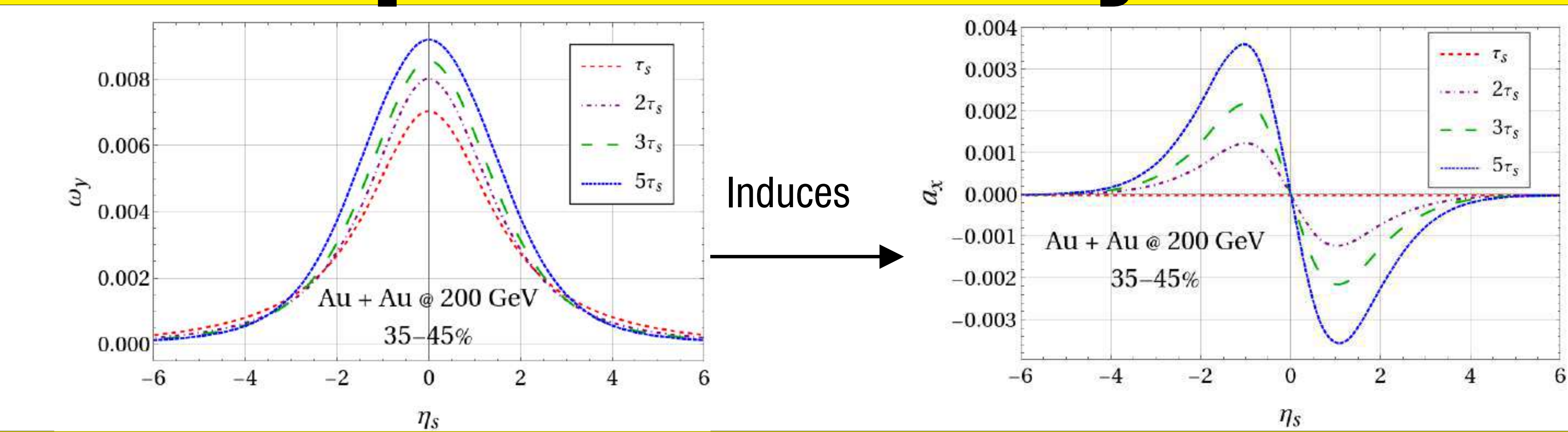
Then, we initialize the spin potential components at proper time τ_s as:

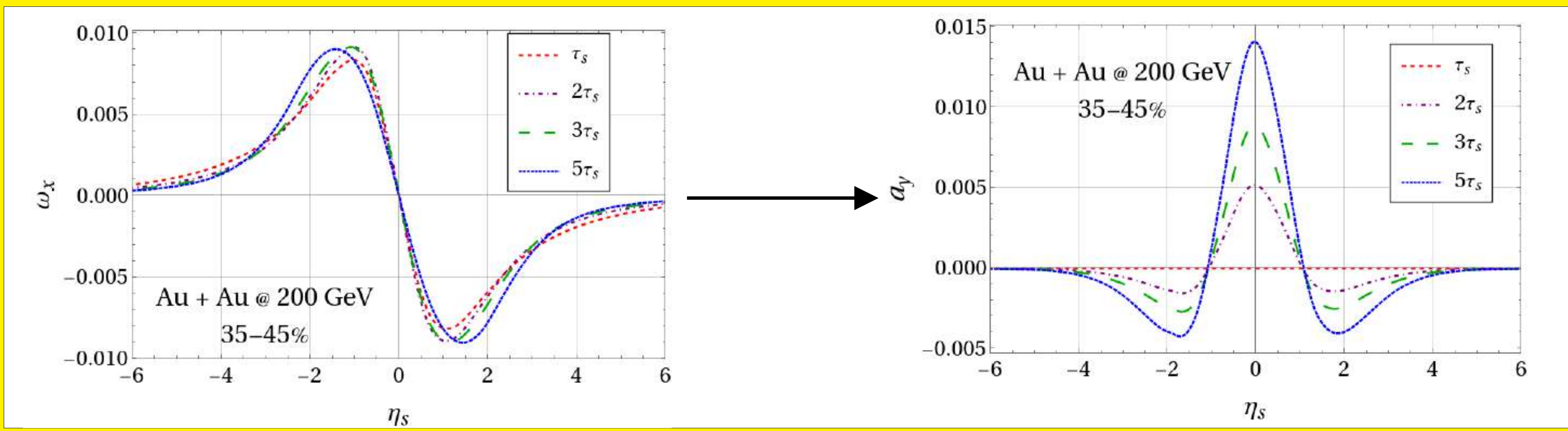
$$\omega_x(\tau_s,\ \eta_s) = \frac{b_{x0}\, u^3(\tau_s,\ \eta_s)}{u^0(\tau_s,\ \eta_s)^2}, \quad [\eta_s - \text{odd}]$$

$$\omega_y(\tau_s,\ \eta_s) = \frac{b_{y0}}{u^0(\tau_s,\ \eta_s)}, \quad [\eta_s - \text{even}]$$

We have three parameters: $b_{x0},\ b_{y0},\ \tau_s = 1\ \text{fm}/c$

Spin Potential Dynamics



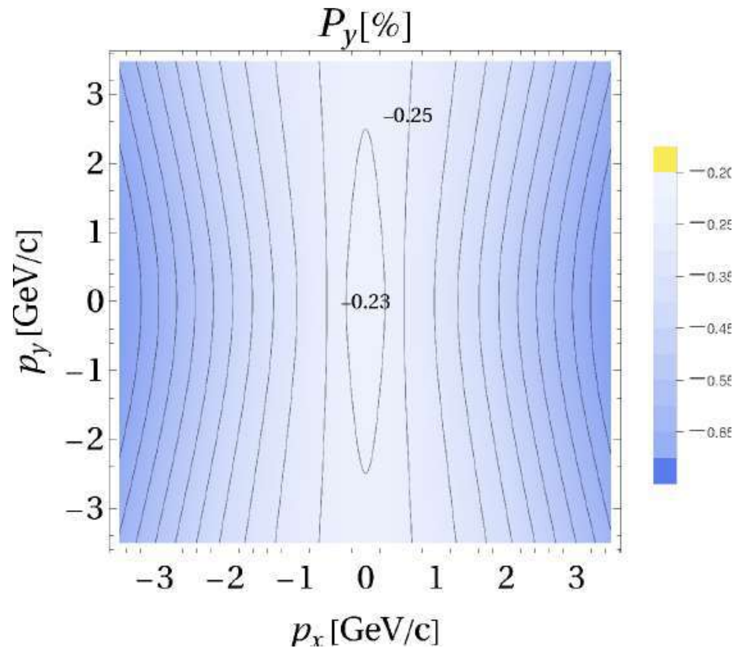


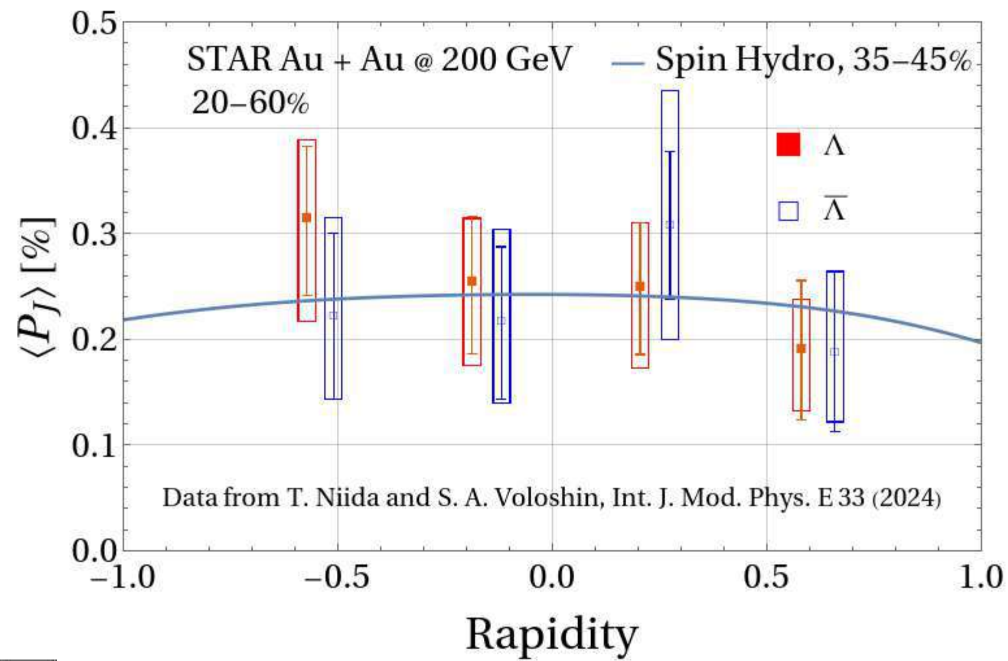
After solving the spin-hydrodynamic equation, the spin potential components are re-scaled such that they reproduce the following measurement:

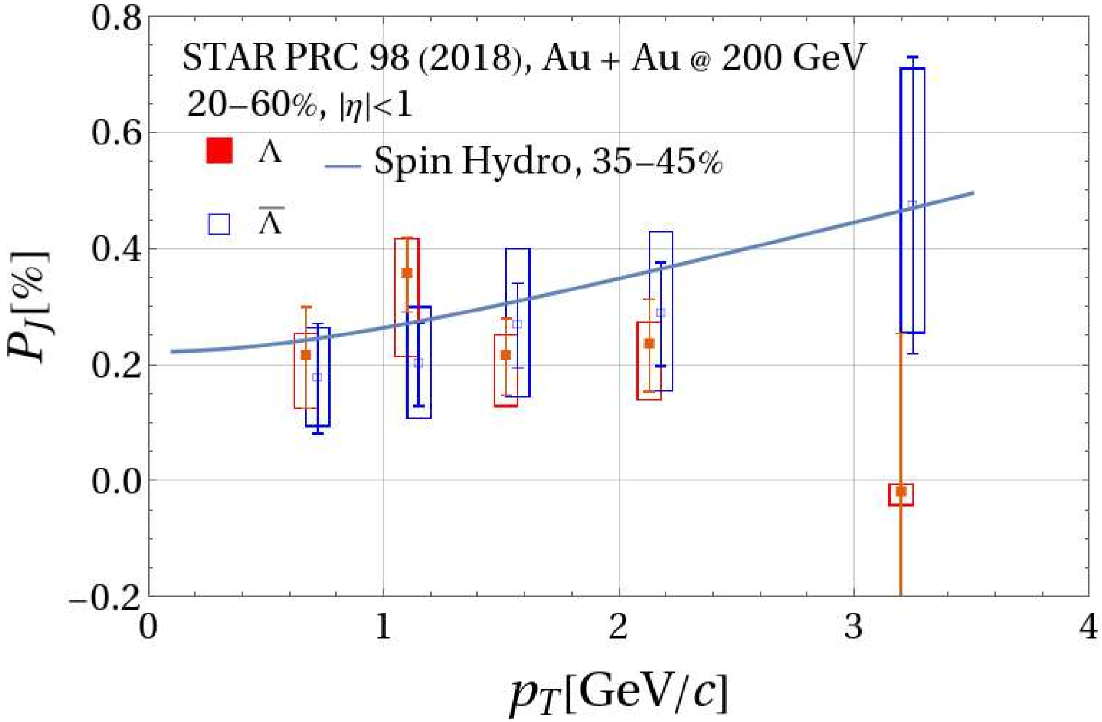
$$\langle P_J \rangle = 0.243\,\% \quad \rightarrow \; b_{y0}$$

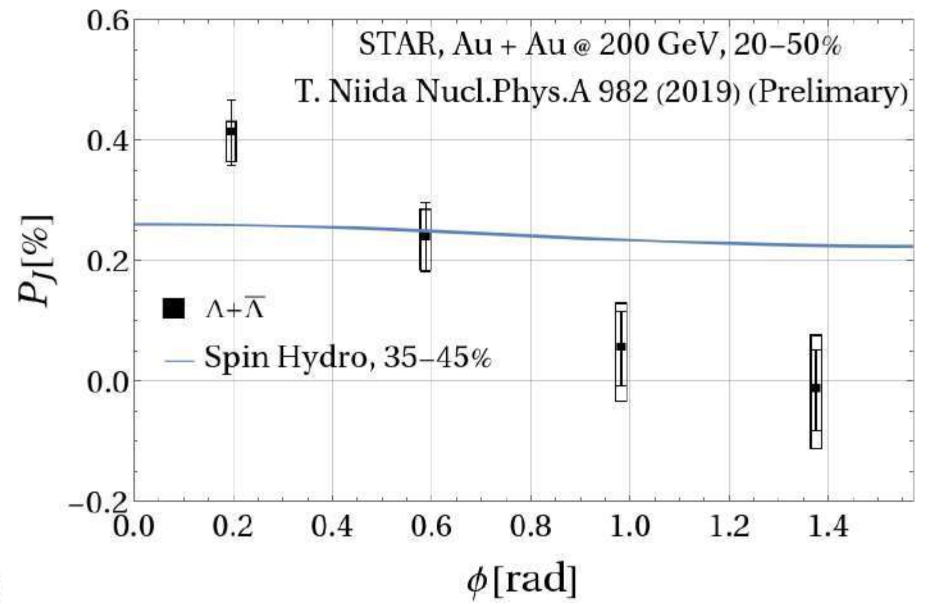
$$P_z(\phi = 0.26) = 0.156\,\% \quad \rightarrow \; b_{x0}$$

Results: transverse polarization, PJ









Results: transverse polarization, PJ

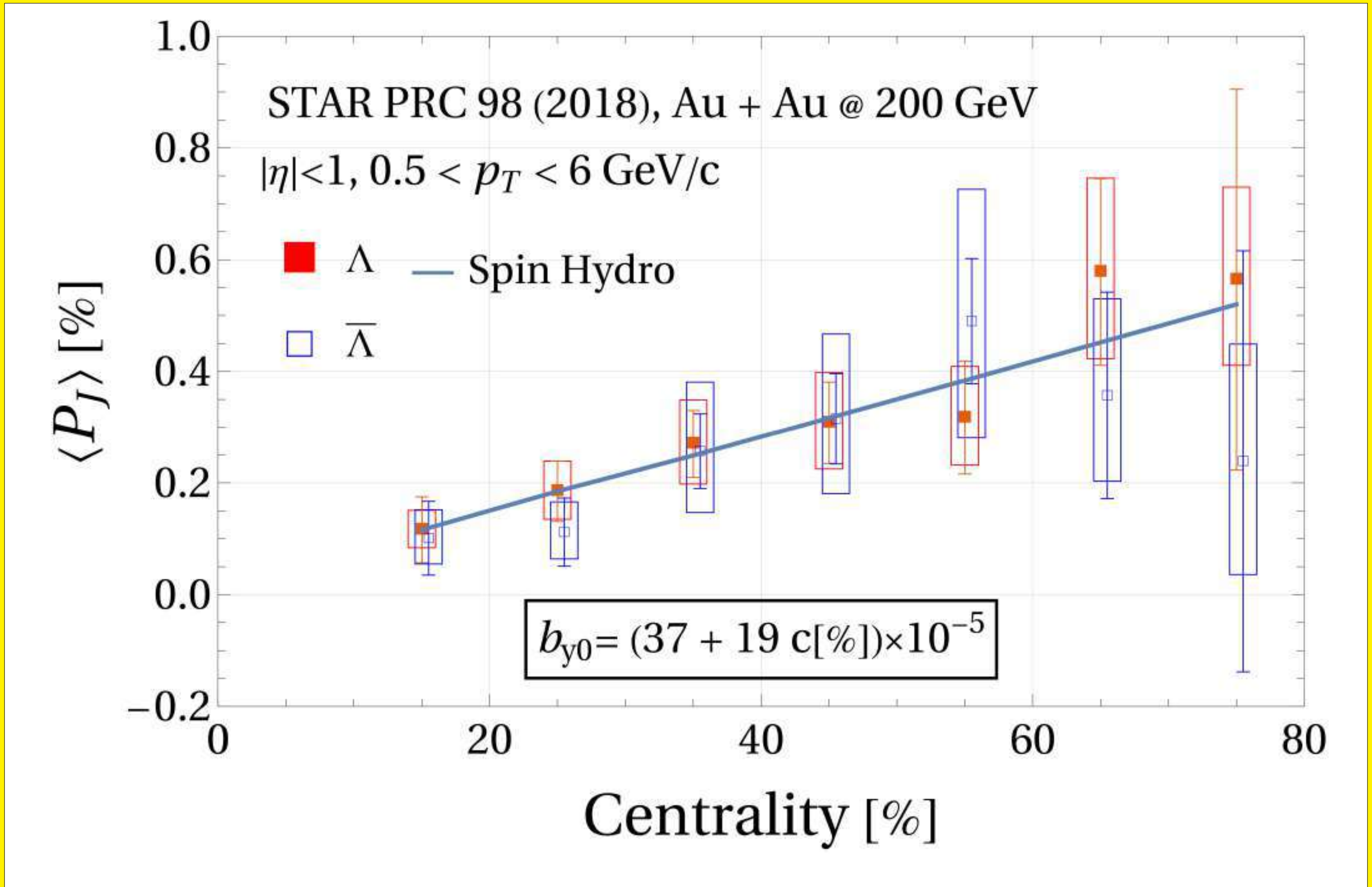
We can also obtain the centrality dependence of global spin polarization

•Use the flow parameters obtained fitting the charged particle pseudo-rapidity distribution

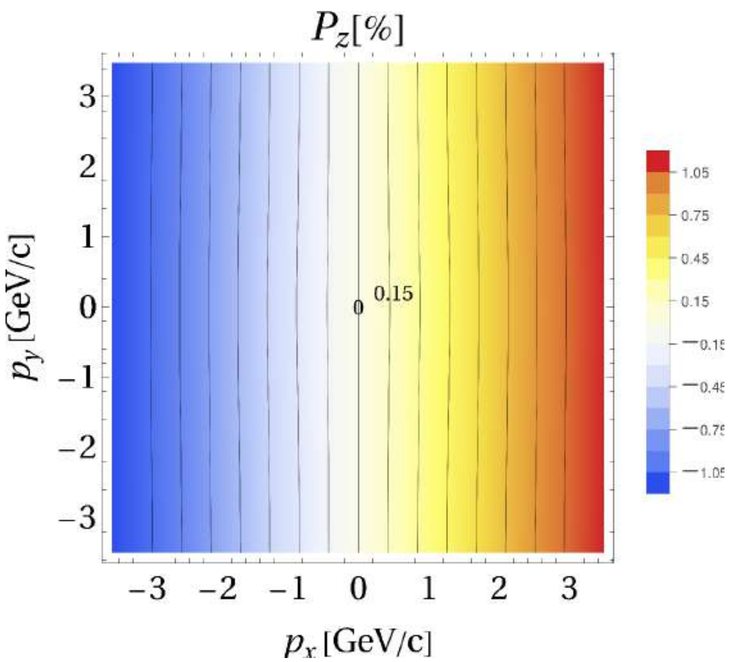
•Fit the spin potential to the first two most-central centrality classes, assume linear increase on the parameter:

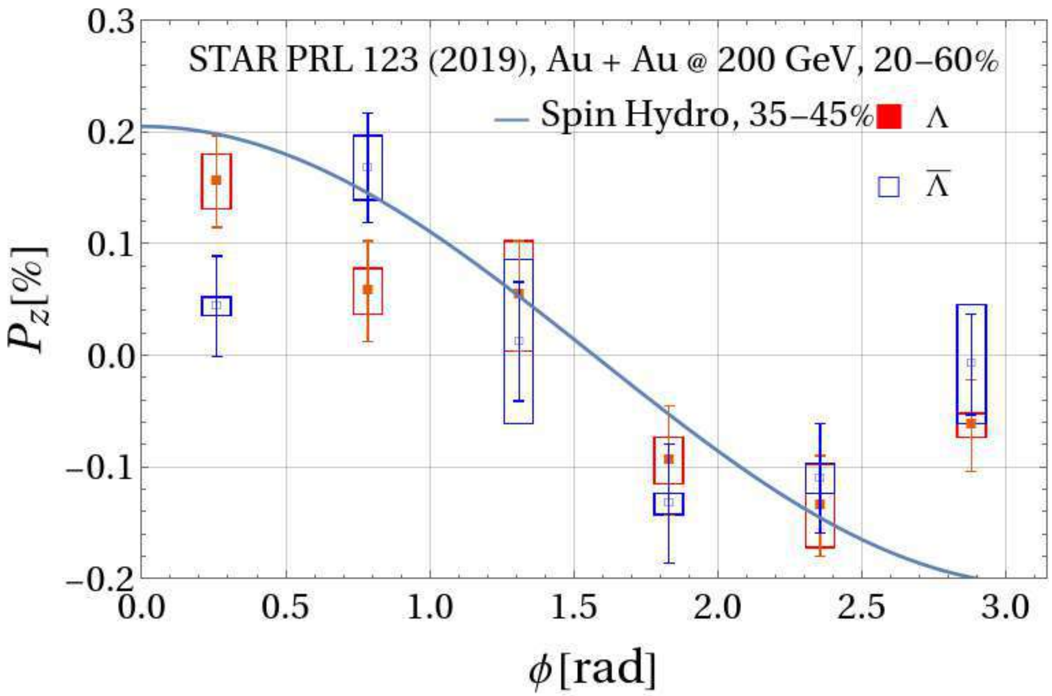
$$\omega_y \propto b_{y0}(c) = b_{y0}^{(0)} + b_{y0}^{(1)} c, \quad c = \text{centrality in } \%$$

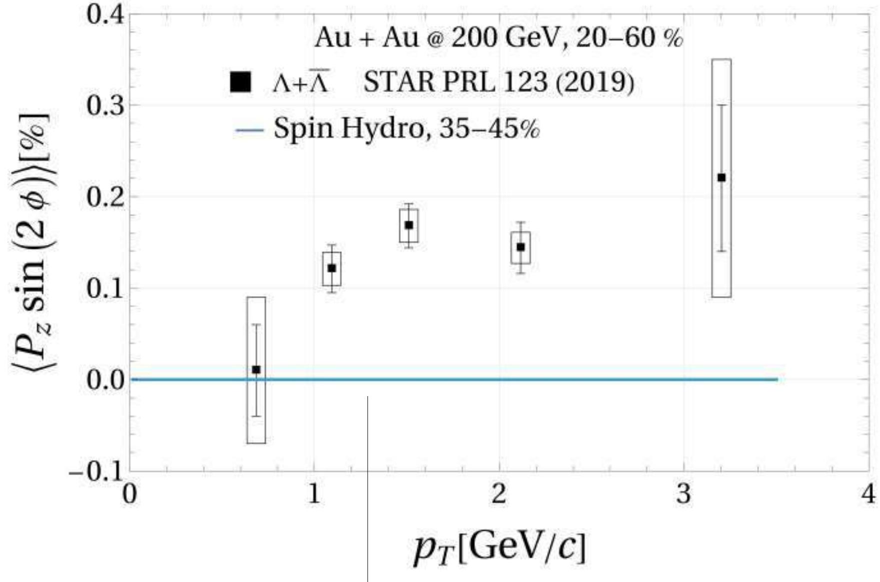
•Obtain the global spin polarization for the other centrality classes



Results: longitudinal polarization, Pz







With no transverse flow → 2nd Fourier component of longitudinal spin pol =0.

Spin hydrodynamic (1+1+2)D
1-1-2 Model

[MB, A. Gecic, R. Singh, ArXiv:2605.08219]

Transverse direction and flow

We still use the (1+1)D flow and (1+1)D spin-hydro.
At freeze-out, we use an elliptic shape in x,y and we add a transverse flow.
The spin potential components are homogeneous in the transverse direction.

$$u \to u = (\gamma_\perp u^0, \mathbf{u}_\perp, \gamma_\perp u^3), \quad \gamma_\perp = \sqrt{1 + u_\perp^2}$$

Transverse direction and flow

We still use the (1+1)D flow and (1+1)D spin-hydro.
At freeze-out, we use an elliptic shape in x,y and we add a transverse flow.

$$u \to u = (\gamma_\perp u^0, \mathbf{u}_\perp, \gamma_\perp u^3), \quad \gamma_\perp = \sqrt{1+u_\perp^2}$$

The trans. flow is parameterized by:
[Heiselberg and Levy, PRC 59 (1999)]

$$\mathbf{u}_\perp = u_\perp \frac{(R_y^2 \cos\varphi,\ R_x^2 \sin\varphi)}{\sqrt{R_y^4 \cos^2\varphi + R_x^4 \sin^2\varphi}}$$

$$\text{Deformation: } \delta = \frac{R_y^2 - R_x^2}{R_y^2 + R_x^2}, \quad R_x = \frac{R_{\rm FO}}{\sqrt{1+\delta}},\ R_y = \frac{R_{\rm FO}}{\sqrt{1-\delta}}$$

We deform the Freeze-out to an ellipse:

$$T(\tau,\eta,x,y) = T(\tau,\eta)\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Where the coordinate in the ellipse region are:

$$x = \frac{r\cos\varphi}{\sqrt{1+\delta}},\ y = \frac{r\sin\varphi}{\sqrt{1-\delta}} \qquad \varphi \in [0,\ 2\pi],\ r \in [0,\ R_{\rm FO}]$$

Transverse direction and flow

We still use the (1+1)D flow and (1+1)D spin-hydro.
At freeze-out, we use an elliptic shape in x,y and we add a transverse flow.

$$u \to u = (\gamma_\perp u^0,\, \mathbf{u}_\perp,\, \gamma_\perp u^3), \quad \gamma_\perp = \sqrt{1+u_\perp^2}$$

The trans. flow is parameterized by:
[Heiselberg and Levy, PRC 59 (1999)]

$$\mathbf{u}_\perp = u_\perp \frac{(R_y^2 \cos\varphi,\ R_x^2 \sin\varphi)}{\sqrt{R_y^4 \cos^2\varphi + R_x^4 \sin^2\varphi}}$$

$$\text{Deformation: } \delta = \frac{R_y^2 - R_x^2}{R_y^2 + R_x^2}, \quad R_x = \frac{R_{\rm FO}}{\sqrt{1+\delta}},\ R_y = \frac{R_{\rm FO}}{\sqrt{1-\delta}}$$

We deform the Freeze-out to an ellipse: $T(\tau,\eta,x,y) = T(\tau,\eta)\exp\left(-\frac{r^2}{2\sigma^2}\right)$

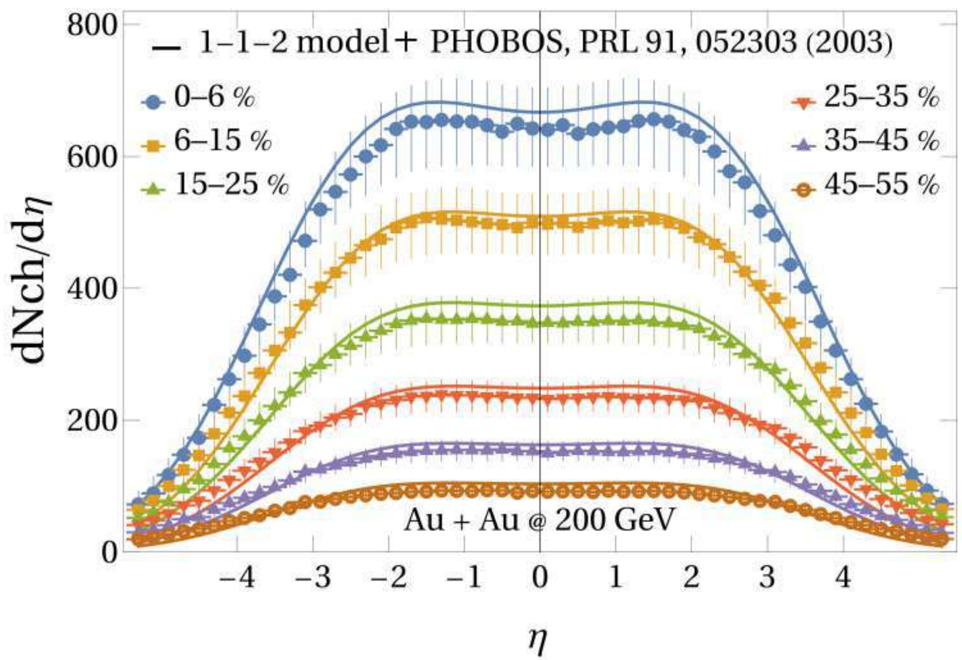
Where the coordinate in the ellipse region are: $x = \frac{r\cos\varphi}{\sqrt{1+\delta}},\ y = \frac{r\sin\varphi}{\sqrt{1-\delta}} \qquad \varphi \in [0,\ 2\pi],\ r \in [0,\ R_{\rm FO}]$

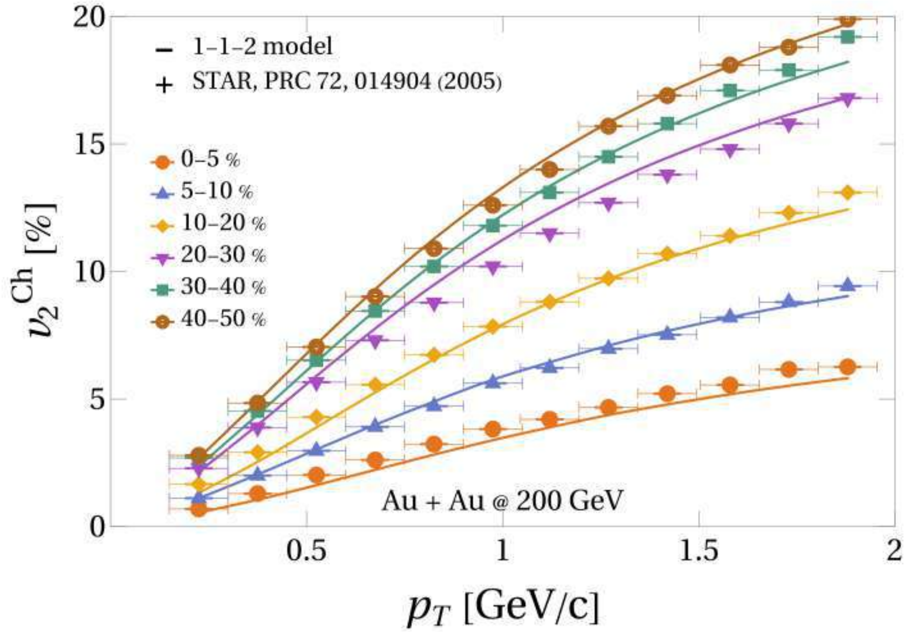
Resulting in the volume integral element:

$$\mathrm{d}\Sigma\cdot p = \left\{\tau\left[\frac{\partial\eta}{\partial\zeta}p^\tau - \frac{\partial\tau}{\partial\eta}p^\eta\right] - \tau(\eta)^2\left[\frac{p_x r\sqrt{1-\delta}\cos\varphi}{\sigma^2} + \frac{p_y r\sqrt{1-\delta}\sin\varphi}{\sigma^2}\right]\right\}\frac{r\mathrm{d}r\mathrm{d}\varphi}{\sqrt{1-\delta^2}}\mathrm{d}\zeta$$

The four new parameters $u_\perp,\ R_{\rm FO},\ \delta,\ \sigma/R_{\rm FO}$ are fitted to the hadronic observables.

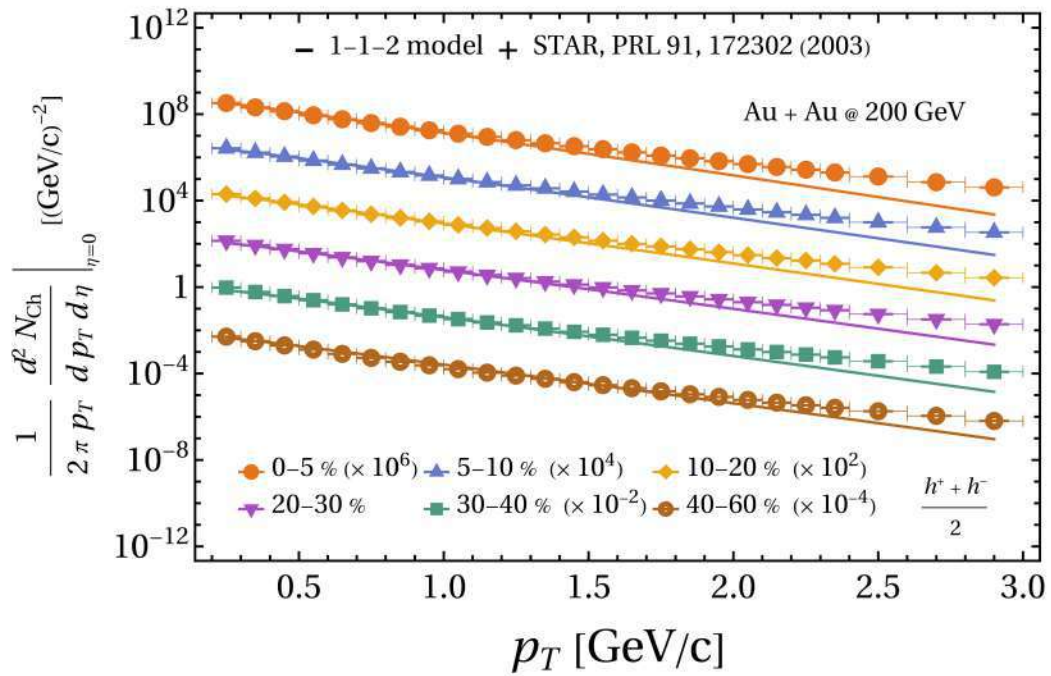
Hadron spectra





Centrality (%)	$u_\perp$	$R_{\rm FO}$(fm)	δ	$\sigma/R_{\rm FO}$
0-6	0.31	14.4	0.1	2.0
6-15	0.36	13.4	0.14	1.5
15-25	0.37	12.8	0.195	1.5
25-35	0.4	11.1	0.25	1.5
35-45	0.4	9.0	0.27	1.6
45-55	0.4	7.3	0.29	1.6
55-65	0.36	5.6	0.3	1.5
65-75	0.32	4.0	0.31	1.5

$$T_{\rm FO} = 155 \text{ MeV}$$



Spin potential 1-1-2

$$\omega_y,\ a_y \quad [\eta_s - \text{even}]$$

$$\omega_x,\ \omega_z,\ a_x,\ a_z \quad [\eta_s - \text{odd}]$$

- To match global spin polarization we still need:
- $\omega_y,\ a_x$ contribute only to Px and Py
- Now a_z can contribute to Pz and only Pz, specifically

$$P_z(p) \propto [\dots + (p_y u^1 - p_x u^2) u^3 a_z] \mathrm{d}\Sigma \cdot p \propto p_x p_y (u^1 x/\sigma_x^2 - u^2 y/\sigma_y^2) u^3 a_z + \dots$$

Quadrupole pattern

We set the following initial conditions: $\tau_s = 1\ \mathrm{fm}/c$

$$\omega_y(\tau_s, \eta_s) = \frac{b_{y0}}{u^0(\tau_s, \eta_s)}$$

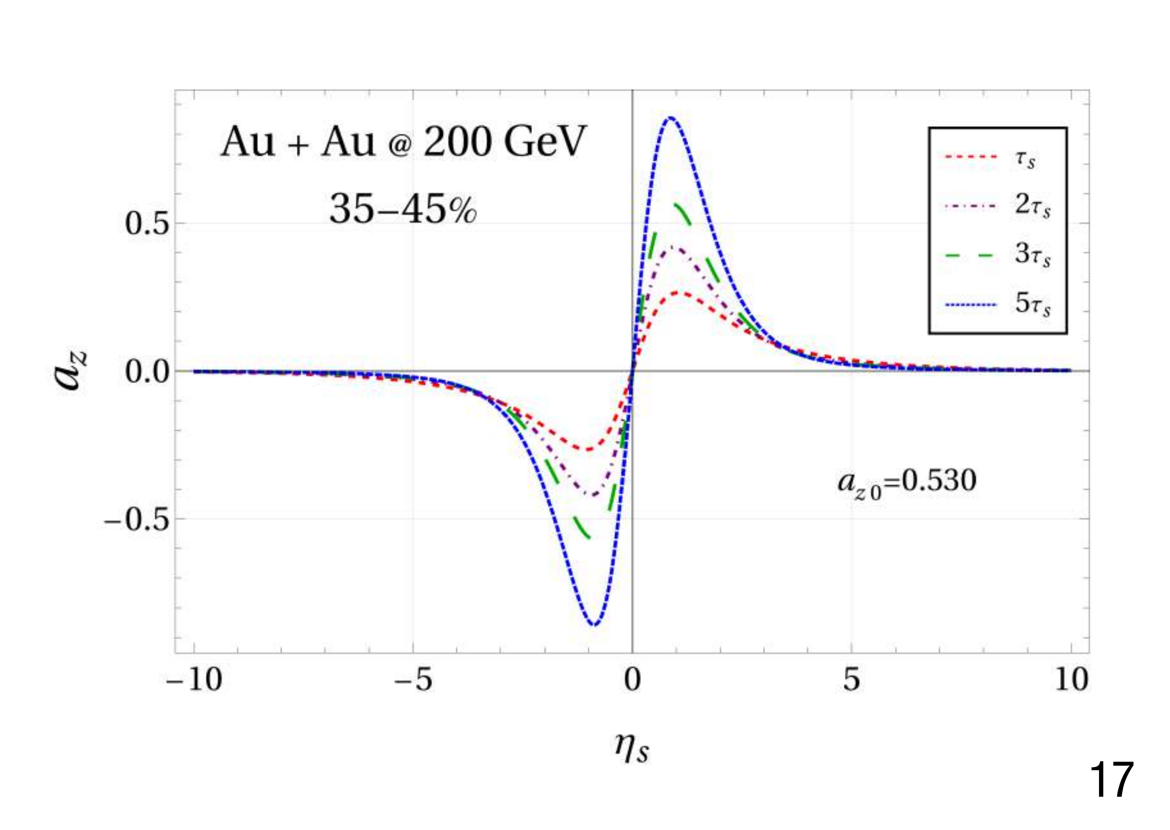
$$a_z(\tau_s, \eta_s) = \frac{a_{z0} u^3(\tau_s, \eta_s)}{u^0(\tau_s, \eta_s)^2}$$

Other components are vanishing.
We fit the parameters to the spin observable:

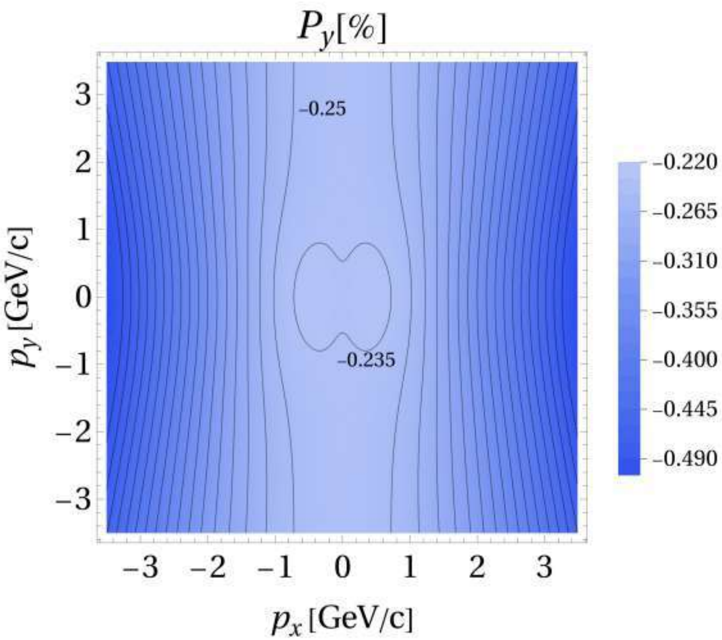
$$\langle P_J \rangle = 0.243\% \quad \rightarrow b_{y0}$$

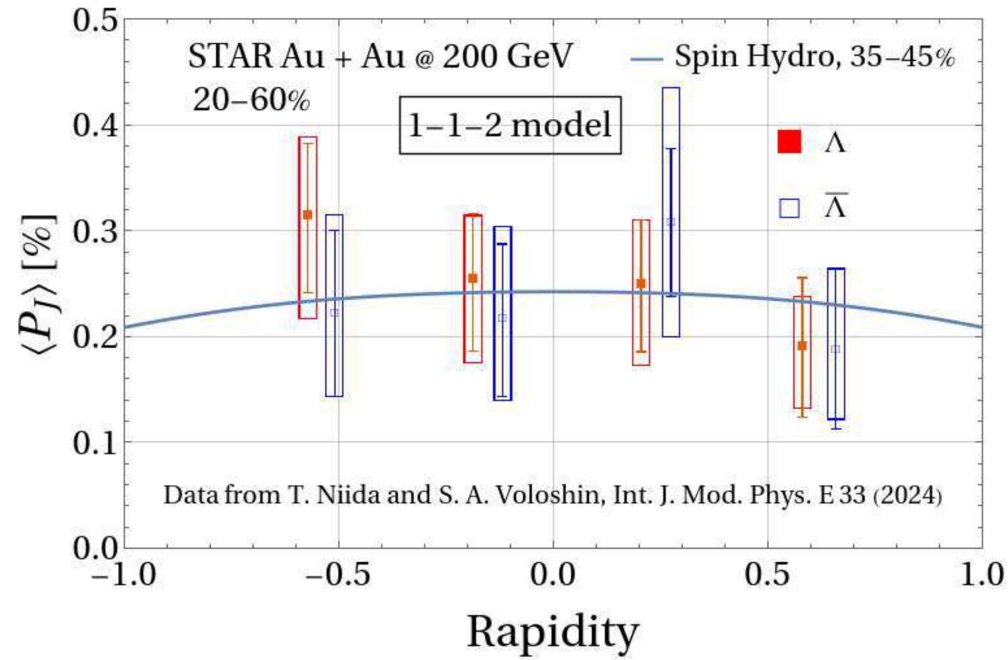
$$\langle P_z \sin(2\phi) \rangle = 0.121\% \quad \rightarrow a_{z0}$$

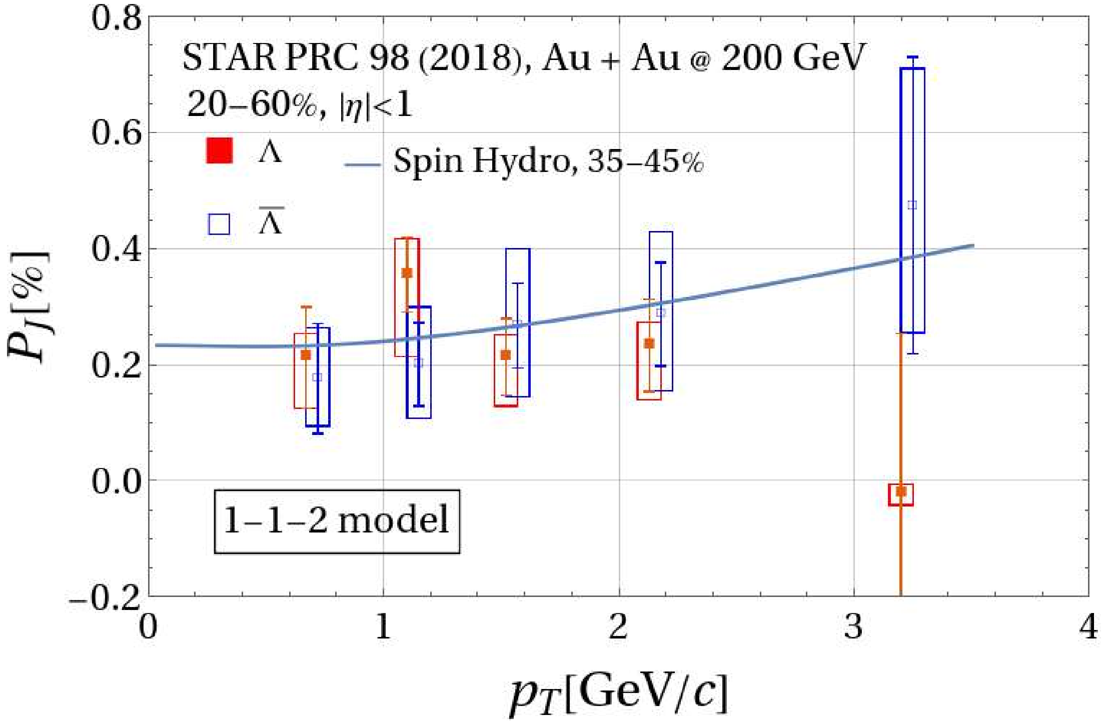
At $p_T = 1.09\ \mathrm{GeV}/c$

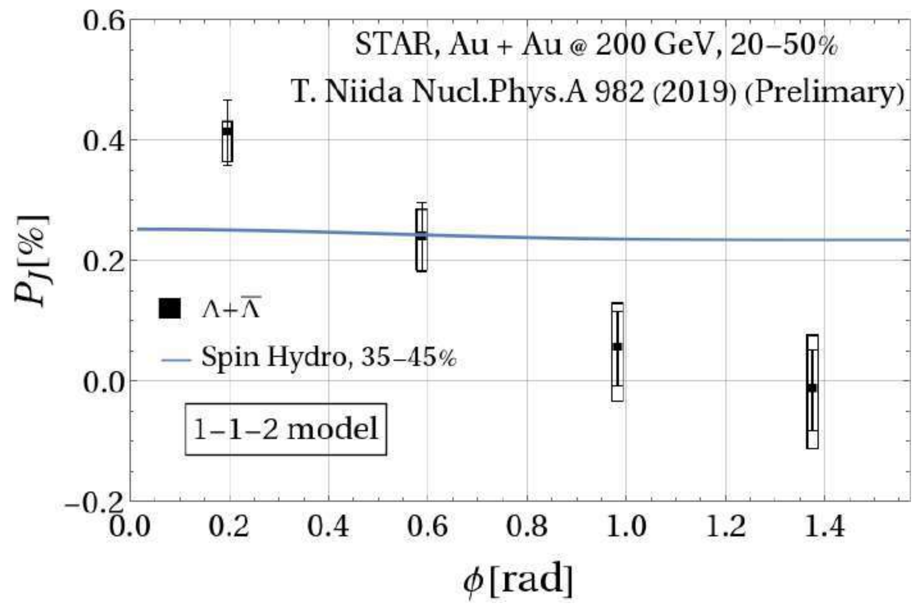


Results 1-1-2 model, PJ







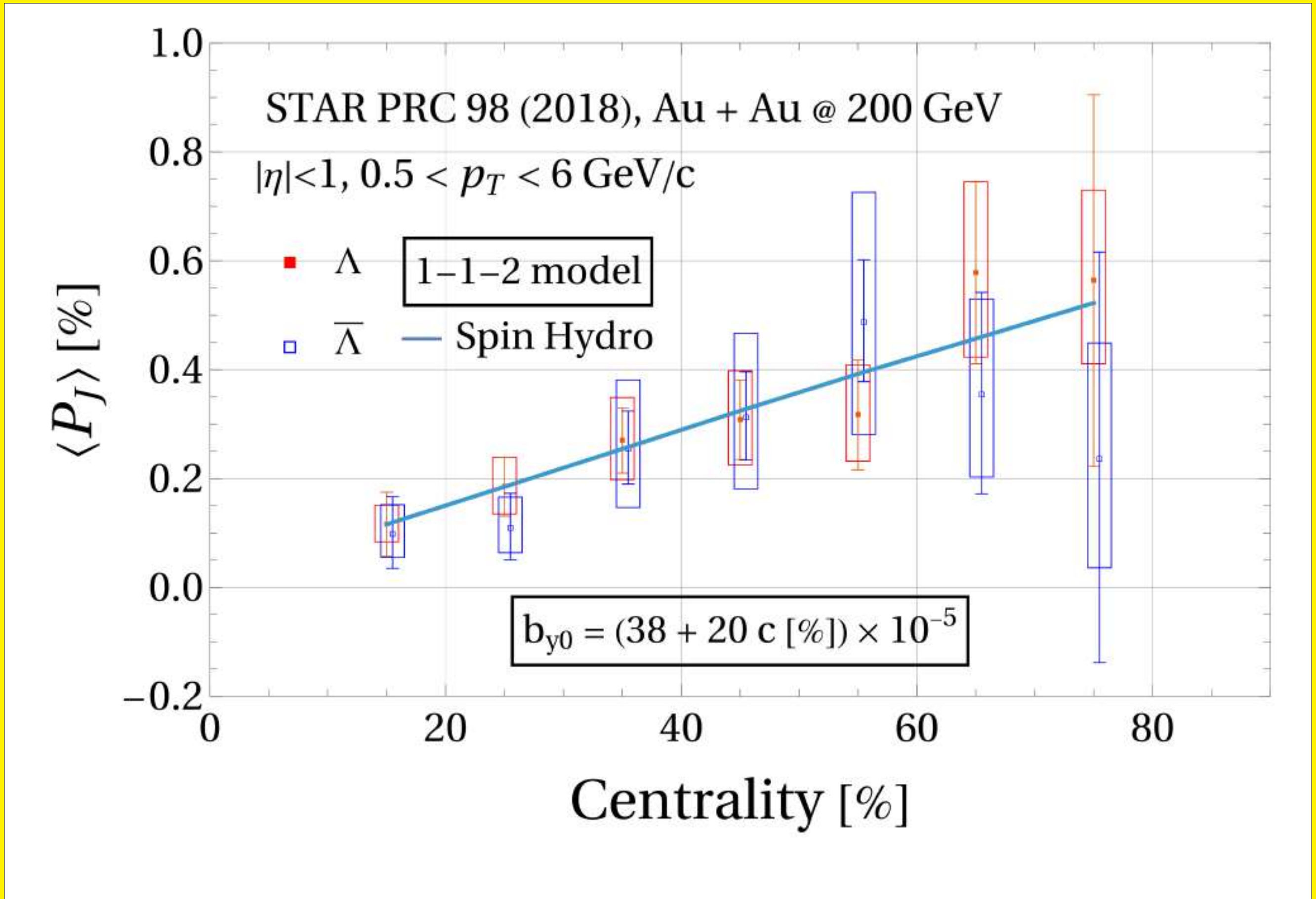


Results 1-1-2 model, PJ Centrality dependence

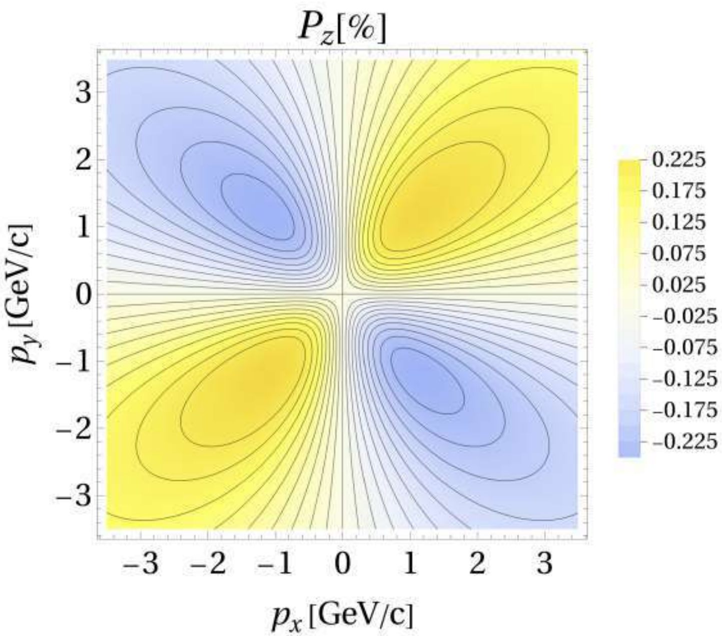
We can also obtain the centrality dependence of global spin polarization
•Use the flow parameters obtained fitting the charged particle pseudo-rapidity distribution
•Fit the spin potential to the 35% and 45% centrality classes, assume linear increase on the parameter:

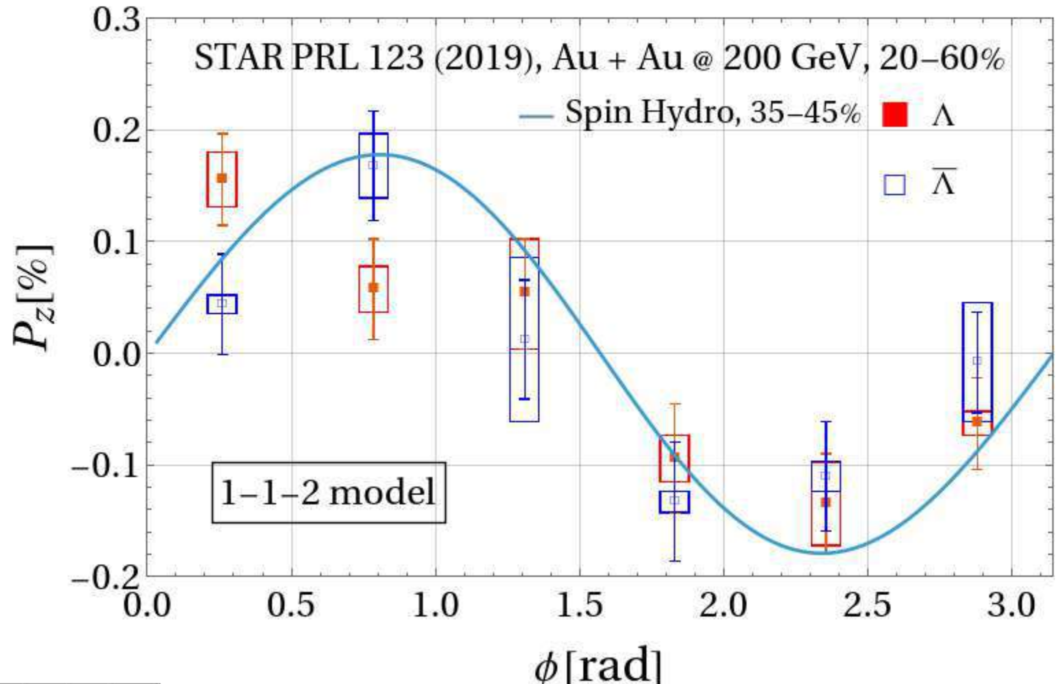
$$\omega_y \propto b_{y0}(c) = b_{y0}^{(0)} + b_{y0}^{(1)} c, \quad c = \text{centrality in } \%$$

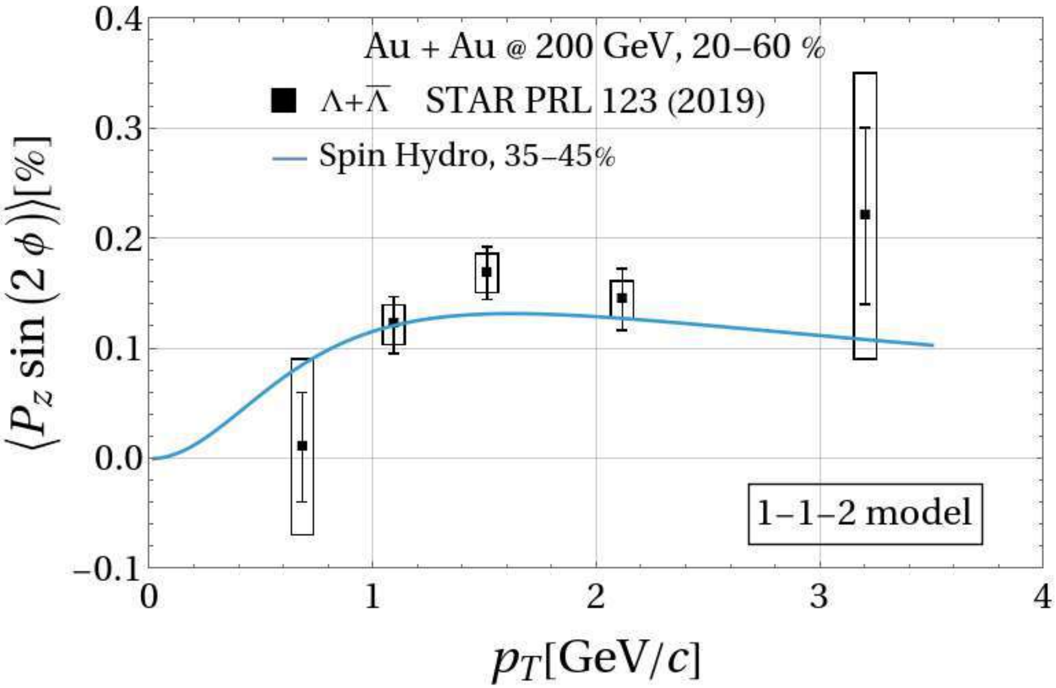
•Obtain the global spin polarization for the other centrality classes

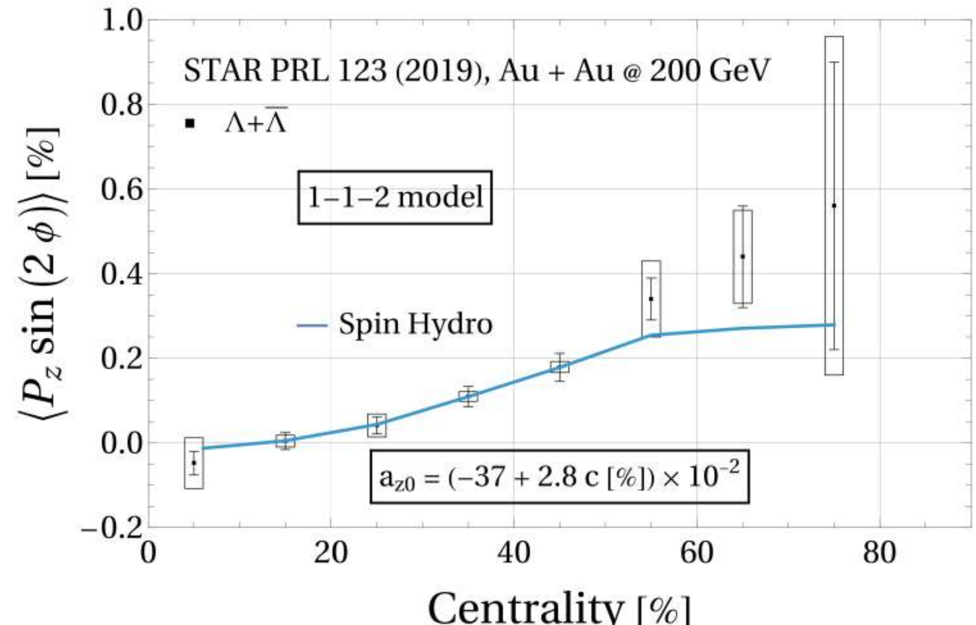


Results 1-1-2 model, Pz



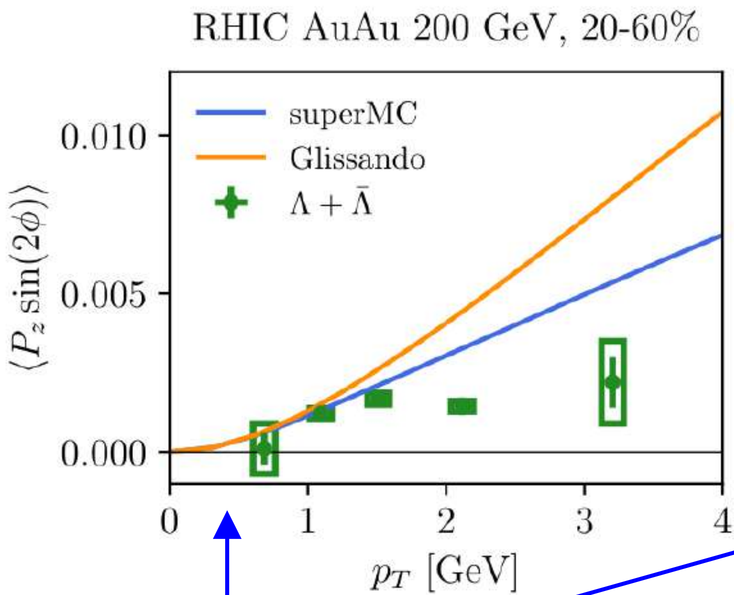




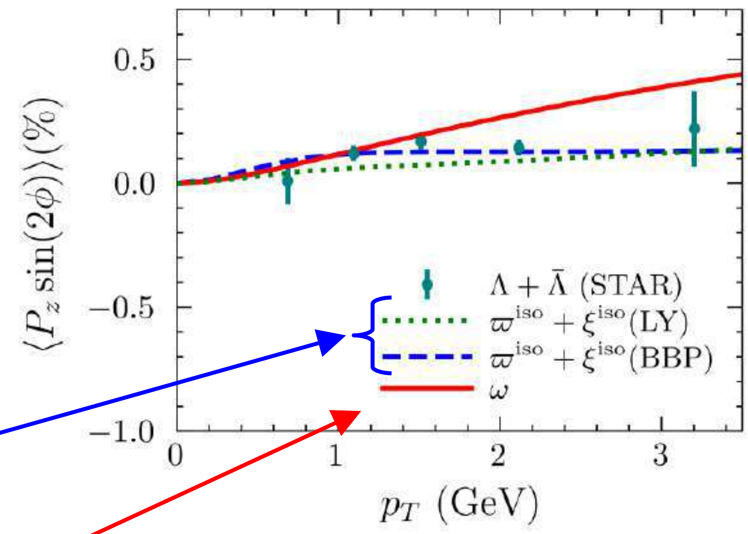


Discussion: momentum dependence

A. Palermo, E. Grossi, I. Karpenko and F. Becattini, Eur. Phys. J. C 84 (2024)

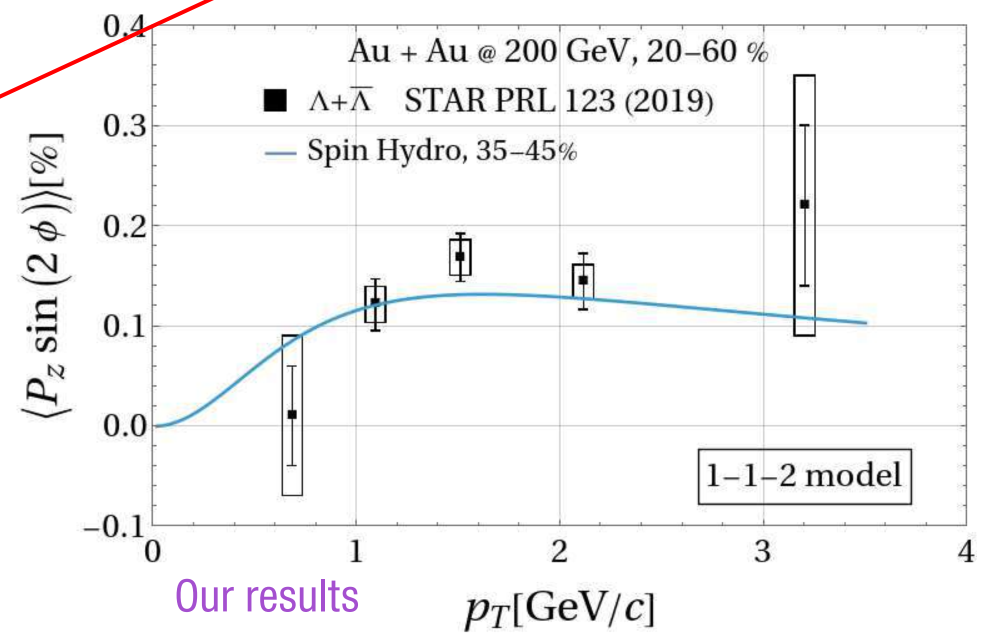


S. K. Singh, R. Ryblewski and W. Florkowski, PRC 111 (2025)



•Vorticity and Shear ϖ, ξ (Belinfante)

•Spin potential ω (GLW)



Our results

•Difference of pseudo-gauge? [MB, PRC 105 no. 4, (2022)]

•Polarization in isobar collisions show a decrease in pT. [STAR, PRL 131 (2023)]

Summary and Outlook

- We used a new family of **(1+1)D hydrodynamic** solutions and a **model of transverse directions**, and we fixed the parameter by fitting hadronic measurements.
- We solve **(1+1)D Spin hydrodynamic** equations in the GLW pseudo-gauge and we obtained the spin potential dynamics. Two parameters fitted to spin data.
- We obtain **good agreement with experimental data** for spin polarization as a function of transverse momentum, azimithal angle, centrality and rapidity.

- The method is well suited to extend the analysis to asymmteric collisions: p-Pb
- Many more applications:
  - Pb-Pb [ALICE: Spin Pol increased precision]
  - QGP in light-ions: O-O: [CMS, PRL. 136, 162301 (2026)][ALICE, upcoming spin pol data]
  - Low energies
  - Dissipative effects [J. Bhambure, R. Singh, and D. Teaney, PRC 111 (2025)]
  - Shear-induce polarization or spin potential-induced polarization?

(This is not the last slide)

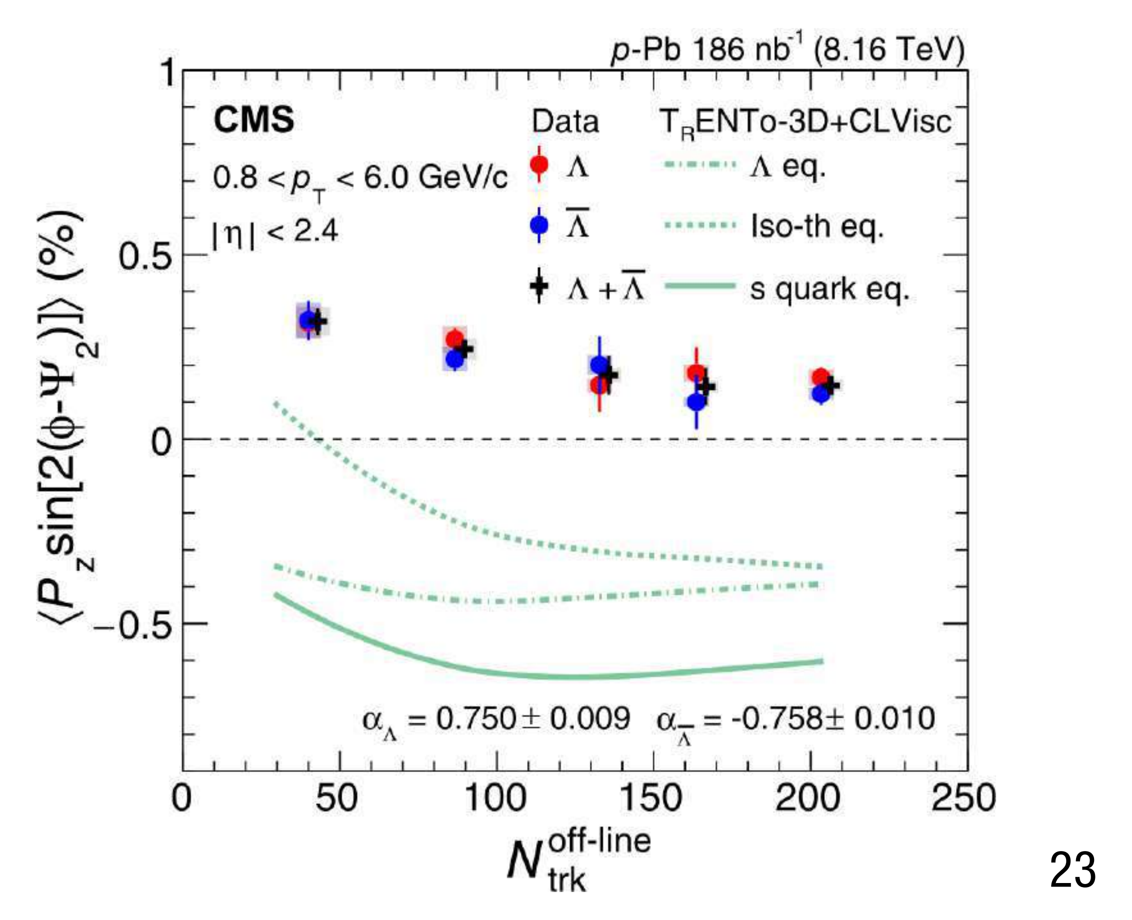
p+Pb, gradients & dissipation

- The picture of equilibrated spins might not be complete

J.I. Kapusta, E. Rrapaj and S. Rudaz, PRC 101 (2020)
A. Ayala, D. De La Cruz, S. Hernández-Ortíz, L.A. Hernández and J. Salinas, PLB, 801 (2020)
M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov and H.-U. Yee, JHEP 08 (2022) 263
D. Wagner, M. Shokri and D.H. Rischke, Phys. Rev. Res. 6 (2024)

- Spin hydrodynamics: role of gradients of spin potential and dissipative effects?
- Relevant for p+Pb collisions?

  [CMS, PRL 135, 132301 (2025)]
  Predictions:
  [C. Yi, X. Y.Wu, J. Zhu, S. Pu, and G. Y. Qin, PRC 111, 044901 (2025)]
- Hydrodynamic simulations across three scenarios fail to describe the data.
- Inclusion of fluctuations? e.g. density frame
  [J. Bhambure, R. Singh, and D. Teaney, PRC 111 (2025)]
  [J. Bhambure, A. Mazeliauskas, J. F. Paquet, R. Singh, M. Singh, D. Teaney and F. Zhou, PRC 111 (2025)]



Dissipative Spin Polarization

Spin polarization is obtained from:

$$S^\mu(k) = \frac{1}{8m} \frac{\int \mathrm{d}\Sigma \cdot k \, \mathcal{A}^\mu_+(x,k)}{\int \mathrm{d}\Sigma \cdot k \, n_F(\beta(x)\cdot k)}$$

Dissipative contributions:

[MB, JHEP 07 (2025) 255]

$$\Delta_{\mathrm{D}} \mathcal{A}^\mu_+ = \left[\bar{a}_{D\zeta_A u} u^\mu + \bar{a}_{D\zeta_A k} \frac{k^\mu_\perp}{(k\cdot u)}\right] D\zeta_A + \left[\bar{a}_{r_A u} \frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar{a}_{r_A \Delta} \Delta^{\mu\rho} + \bar{a}_{r_A k} Q^{\mu\rho}\right] \partial_{\langle\rho\rangle}\zeta_A$$

Gradients of chiral imbalance

$$+ \bar{a}_{f\epsilon}\, \epsilon^{\mu\nu\rho\sigma} \frac{k^\perp_\nu u_\sigma}{(k\cdot u)} f_\rho + \left[\bar{a}_{\Upsilon u} \frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar{a}_{\Upsilon\Delta} \Delta^{\mu\rho} + \bar{a}_{\Upsilon k} Q^{\mu\rho}\right] \Upsilon_\rho$$

Gradients of spin potential

$$+ \left[\bar{a}_{I-\Upsilon u} \frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar{a}_{I-\Upsilon\Delta} \Delta^{\mu\rho} + \bar{a}_{I-\Upsilon k} Q^{\mu\rho}\right] (I_\rho - \Upsilon_\rho) + \left[\bar{a}_{\varphi u} u^\mu + \bar{a}_{\varphi k} \frac{k^\mu_\perp}{(k\cdot u)}\right] \varphi + \bar{a}_{I_S\epsilon}\, \epsilon^{\mu\nu\alpha\rho} k^\sigma_\perp \frac{u_\nu k^\perp_\alpha}{(k\cdot u)^2} I_{S\,\rho\sigma}$$

$$+ \left[\bar{a}_{\mathrm{S}12\Delta\epsilon} \Delta^{\mu\tau} \epsilon^{\lambda\nu\rho\sigma} \frac{u_\lambda k^\perp_\nu}{(k\cdot u)} + \bar{a}_{\mathrm{S}12\epsilon} \epsilon^{\mu\nu\rho\sigma} \frac{k^\tau_\perp u_\nu}{(k\cdot u)}\right] \Phi^{S12}_{\tau,\rho\sigma} + \left[\bar{a}_{\mathrm{S}13\Delta\epsilon} \Delta^{\mu\tau} \epsilon^{\lambda\nu\rho\sigma} \frac{u_\lambda k^\perp_\nu}{(k\cdot u)} + \bar{a}_{\mathrm{S}13\epsilon} \epsilon^{\mu\nu\rho\sigma} \frac{k^\tau_\perp u_\nu}{(k\cdot u)}\right] \Phi^{S13}_{\tau,\rho\sigma}$$

Decomposition of the gradients of spin potential

$$\begin{aligned}\partial^\lambda \omega^{\mu\nu} =& u^\lambda \left(f^\mu u^\nu - f^\nu u^\mu\right) + \epsilon^{\lambda\mu\nu\rho}\Upsilon_\rho + \left(\Delta^{\lambda\mu} u^\nu - \Delta^{\lambda\nu} u^\mu\right) I + \left(I^{\lambda\mu}_S u^\nu - I^{\lambda\nu}_S u^\mu\right)\\ &+ \left(\epsilon^{\lambda\mu\alpha\beta} u^\nu - \epsilon^{\lambda\nu\alpha\beta} u^\mu\right) u_\alpha (I_\beta - \Upsilon_\beta) + \varphi\, \epsilon^{\lambda\mu\nu\rho} u_\rho + \Phi^{\lambda,\mu\nu}_{S12} + \Phi^{\lambda,\mu\nu}_{S13}\end{aligned}$$

Transport coefficients are, e.g.

$$\bar{a}_{f\epsilon} = \eta_{f\epsilon}\, \delta(k^2 - m^2)\theta(k\cdot u)\, n_F(k\cdot u)[1 - n_F(k\cdot u)]$$

A-dimensional Real Number

Pseudo-gauge (PG) ambiguity

Many open questions:

- Is it physical meaningful?
- Can different pseudo-gauges be reconciled after all?

Reminder:

- Spin polarization is PG dependent! [MB, PRC 105 (2022)]
- This occurs because the local thermal equilibrium (LTE) statistical operator $\widehat{\rho}_{\mathrm{LTE}}$ is written in terms of the EMT and spin tensor currents and it is therefore PG dependent.

[MB and A. Palermo, Phys. Rev. Lett. 133 (2024)]

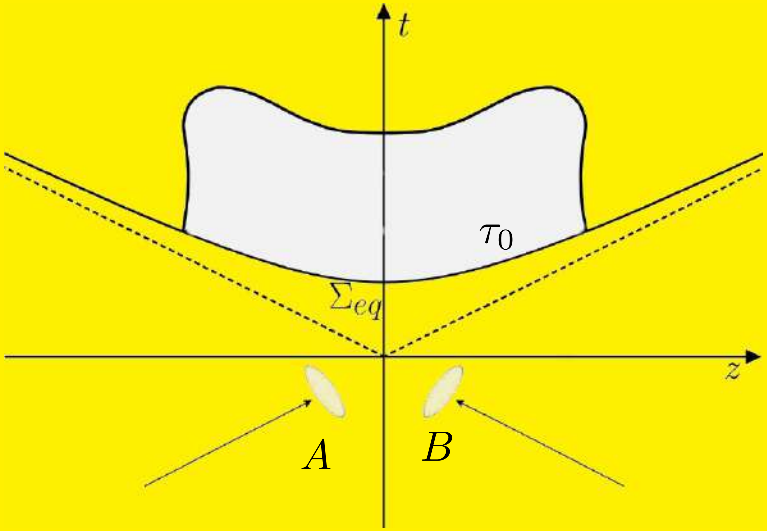
Path toward a physical mechanism to select a spin tensor.

PG invariance of Heavy-ion collisions

[F. Becattini, and C. Hoyos, 2507.09249]

The local thermal equilibrium (LTE) density operator $\widehat{\rho}_{\rm LTE}$ is pseudo-gauge dependent. It is our assumption of the initial QCD plasma state. Strictly speaking, the actual quantum state (in the Heisenberg representation) are the two colliding nuclei, which is a pseudo-gauge invariant quantum state

$$\widehat{\rho}_0 = |P_A,\, P_B\rangle\,\langle P_A,\, P_B|$$



Evolving the initial actual quantum state in Schroedinger representation

$$\lim_{t\to-\infty} \exp\left[-i\widehat{H}(\tau_0 - t)\right] |P_A,\, P_B\rangle$$

still yields a pseudo-gauge invariant state!

However, the basic (tacit) assumption in HIC is to approximate a pseudo-gauge invariant state with a non-pseudo gauge invariant state

$$\widehat{\rho}(\tau_0) = \lim_{t\to-\infty} \exp\left[-i\widehat{H}(\tau_0 - t)\right] \widehat{\rho}_0 \exp\left[i\widehat{H}(\tau_0 - t)\right]$$

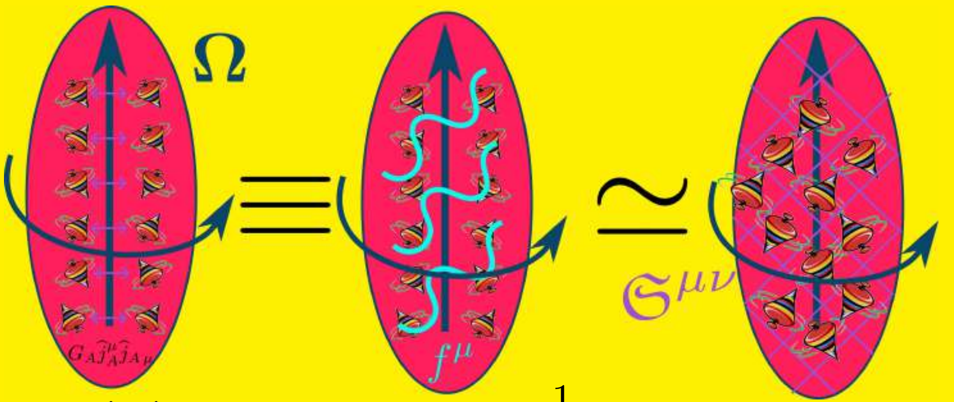
(PG invariant)

(PG dependent)

$$\widehat{\rho}_{\rm LTE} = \frac{1}{Z} \exp\left[-\int_{\Sigma_{eq}} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\mathfrak{S}_{\lambda\nu}\widehat{S}^{\mu,\lambda\nu} - \zeta\,\widehat{j}^\mu\right)\right]$$

Resolution: the spin tensor is not arbitrary but it's emerging from the interactions

Emergent spin tensor in hot QCD



$G_A \widehat{j}^{\mu}_A \widehat{j}_{A\,\mu}$

$$f^\sigma = \frac{1}{2}\epsilon^{\sigma\mu\nu\rho}(\varpi_{\mu\nu} - \mathfrak{S}_{\mu\nu})n_\rho$$

What spin tensor should be used?

Interactions can make the choice.

$$Z|_{\mathrm{NJL}+\varpi} = Z|_{\text{Spin hydro with canonical spin}}$$

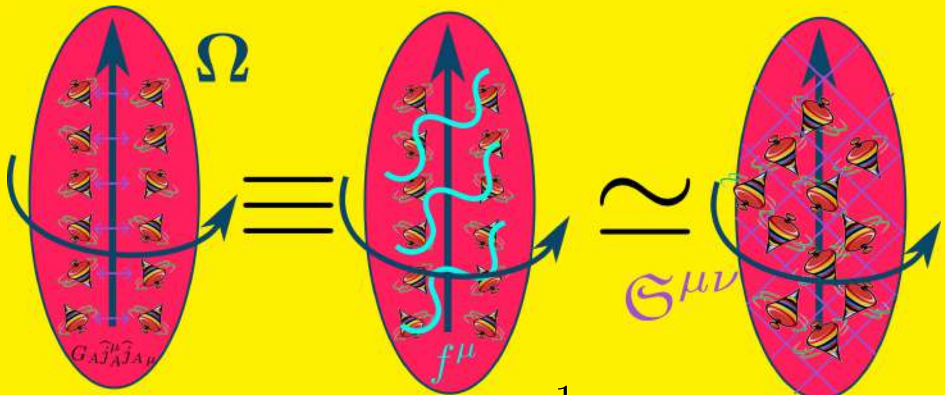
[MB and A. Palermo, Phys. Rev. Lett. 133 (2024)]

Proof of concept:

- Start with a pseudo-gauge invariant state
- Include spin-spin interactions
- → Obtain the relevant (psuedo-gauge invariant) spin tensor

	Proof of Concept	Heavy Ion Collisions
State	Global equilibrium with vorticity	$\vert P_A, P_B\rangle\langle P_A, P_B\vert$
Interactions	NJL model	QCD
Spin tensor	Canonical	?

Emergent spin tensor in hot QCD



$G_A \widehat{j}^\mu_A \widehat{j}_{A\,\mu}$

$$f^\sigma = \frac{1}{2}\epsilon^{\sigma\mu\nu\rho}(\varpi_{\mu\nu} - \mathfrak{S}_{\mu\nu})n_\rho$$

What spin tensor should be used?

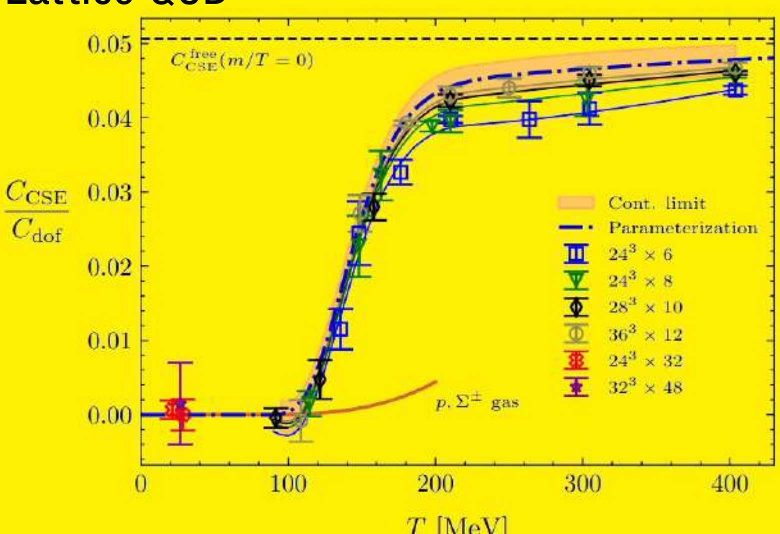
Interactions can make the choice.

$$Z|_{\mathrm{NJL}+\varpi} = Z|_{\text{Spin hydro with canonical spin}}$$

[MB and A. Palermo, Phys. Rev. Lett. 133 (2024)]

Hints that this mechanism works for heavy-ion collisions:

Lattice QCD



[B. B. Brandt, G. Endrodi, E. Garnacho-Velasco and G. Marko, JHEP 02 (2024)]

We expect the same behavior for the Axial Vortical Effect

Hot QCD + B, Ω → (AVE, CSE) → Mean canonical spin tensor → (Spin-Spin interactions) → **Spin fluid and canonical spin potential**

(This is the last slide!)

Thank you for the attention!

I acknowledge financial support by the European Union - NextGenerationEU through the grant No. 760079/23.05.2023, funded by the Romanian ministry of research, innovation and digitalization through Romania's National Recovery and Resilience Plan, call no. PNRR-III-C9-2022-I8.

Finanțat de Uniunea Europeană NextGenerationEU

Planul Național de Redresare și Reziliență

BACKUP SLIDES

GLW Spin Polarization

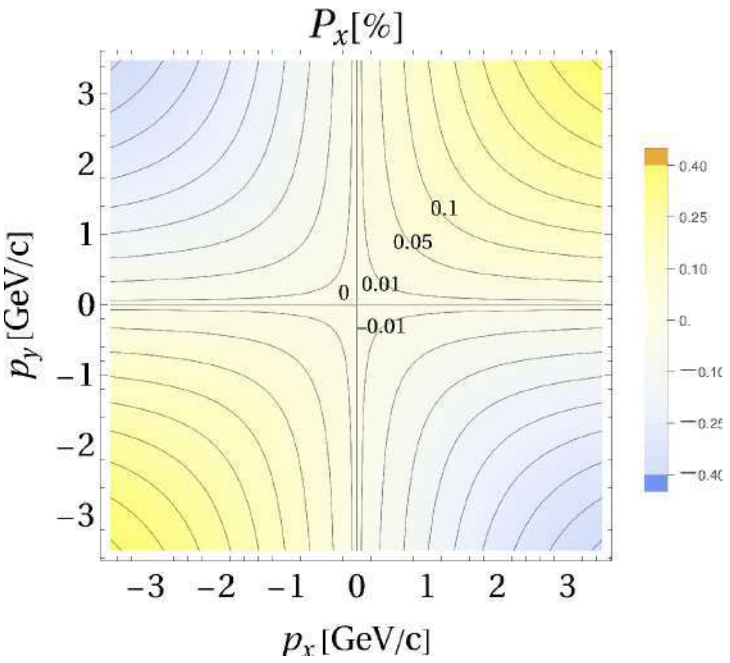
$$P_\mu(p) = \frac{\int_{\Sigma_{\rm FO}} \mathrm{d}^3\Sigma \cdot p\, S^*_\mu(x,p)}{\int_{\Sigma_{\rm FO}} \mathrm{d}^3\Sigma \cdot p\, n_f(x,p)} \qquad S_\mu(x,p) = -\frac{1}{4m_\Lambda}\widetilde{\omega}_{\mu\beta}p^\beta n_f(x,p)\,(1-n_f(x,p))$$

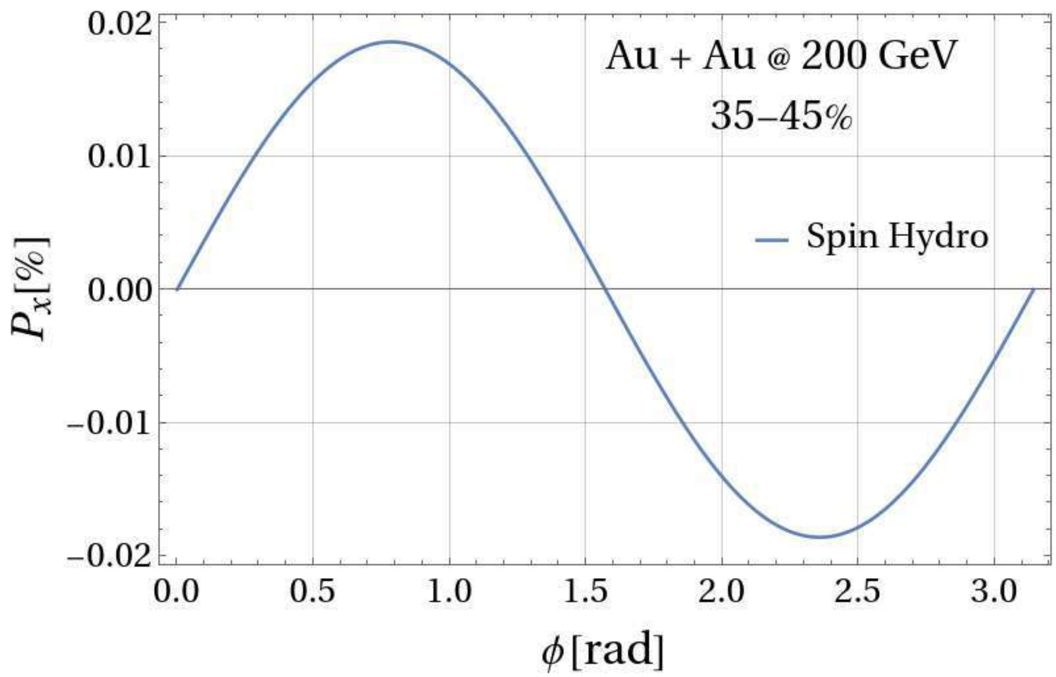
Only the rapidity-even terms survives the FO-integration:

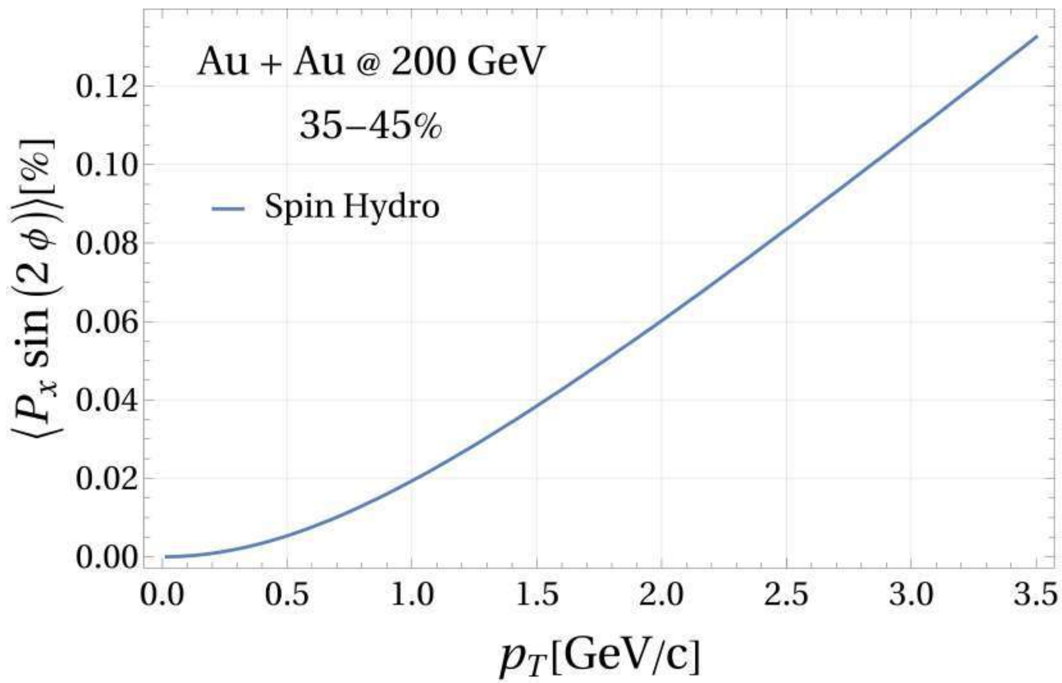
$$\begin{aligned}
(\widetilde{\omega}_{1\nu}p^\nu)^*|_{\rm even} =& \frac{p_x p_y}{m_\Lambda}(1-\alpha_p)u^3 a_x \\
&- p_y[u^1 + \frac{p_x}{m_\Lambda}u^0(\alpha_p - 1)]\omega_y \\
&- \left[\frac{p_x}{m_\Lambda}(p_x u^1 + p_y u^2)(1-\alpha_p) - m_T u^1\right]u^3\omega_z, \\
(\widetilde{\omega}_{2\nu}p^\nu)^*|_{\rm even} =& \left[\frac{p_y^2}{m_\Lambda}(1-\alpha_p) - m_T\right]u^3 a_x \\
&- \left[m_T u^0 - \frac{p_y^2}{m_\Lambda}u^0(1-\alpha_p) - p_x u^1\right]\omega_y \\
&- \left[\frac{p_y}{m_\Lambda}(p_x u^1 + p_y u^2)(1+\alpha_p) - m_T u^2\right]u^3\omega_z, \\
(\widetilde{\omega}_{3\nu}p^\nu)^*|_{\rm even} =& (p_x u^0 - m_T u^1)a_y \\
&+ (p_y u^1 - p_x u^2)u^3 a_z - p_x u^3 \omega_x.
\end{aligned}$$

$$\alpha_p = \frac{m_T}{m_\Lambda + E_p} \qquad m_T = \sqrt{m_\Lambda^2 + p_T^2}$$

Results: in-plane polarization, Px

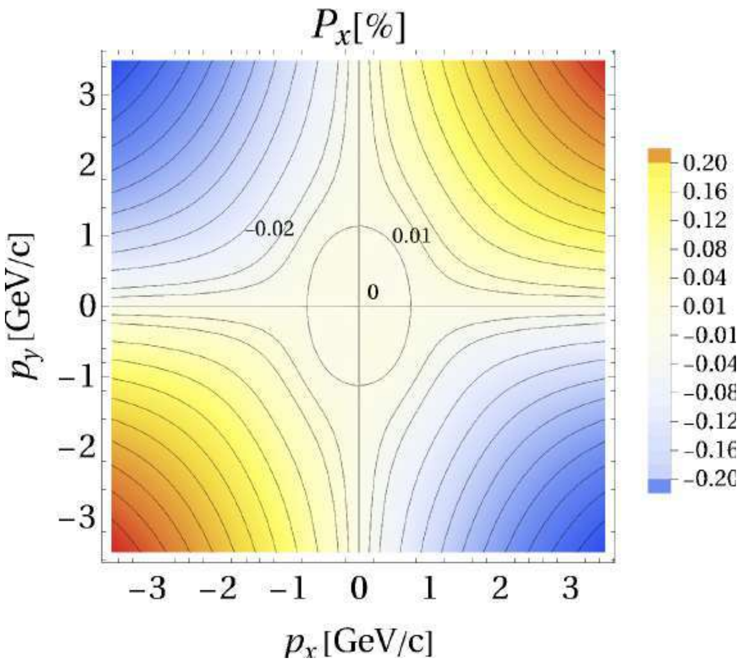


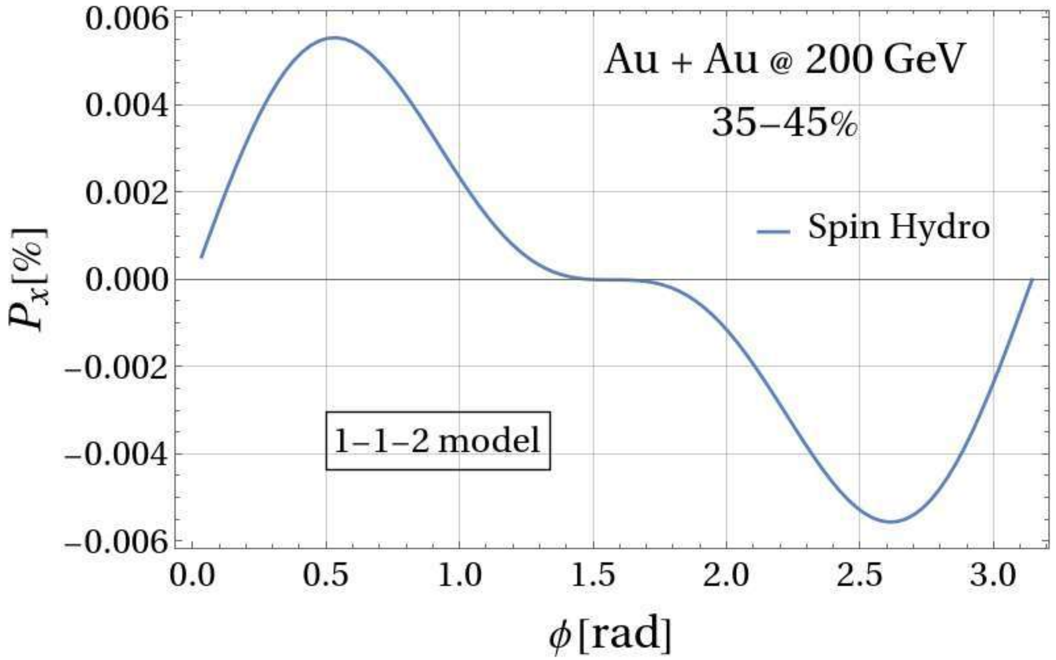


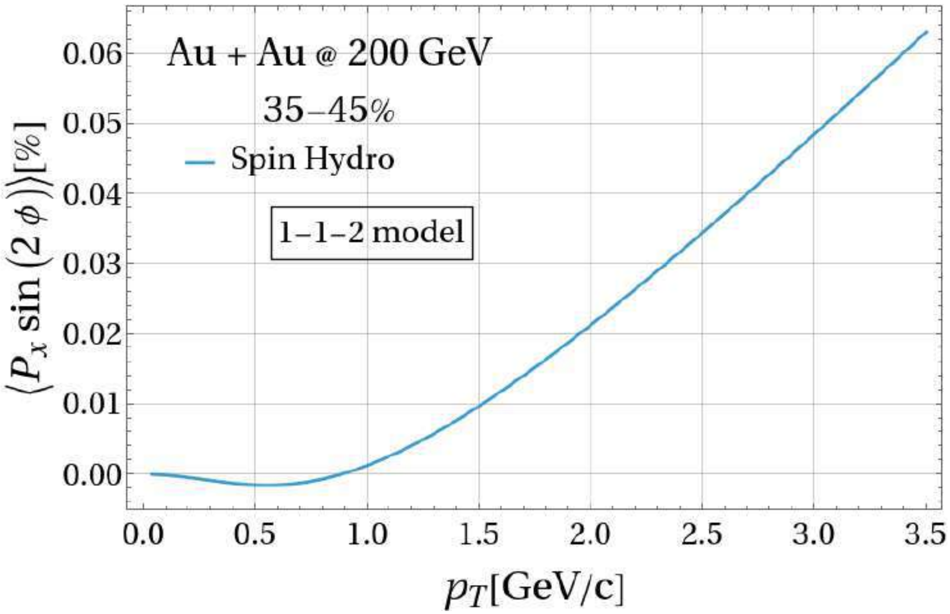


1+1D model

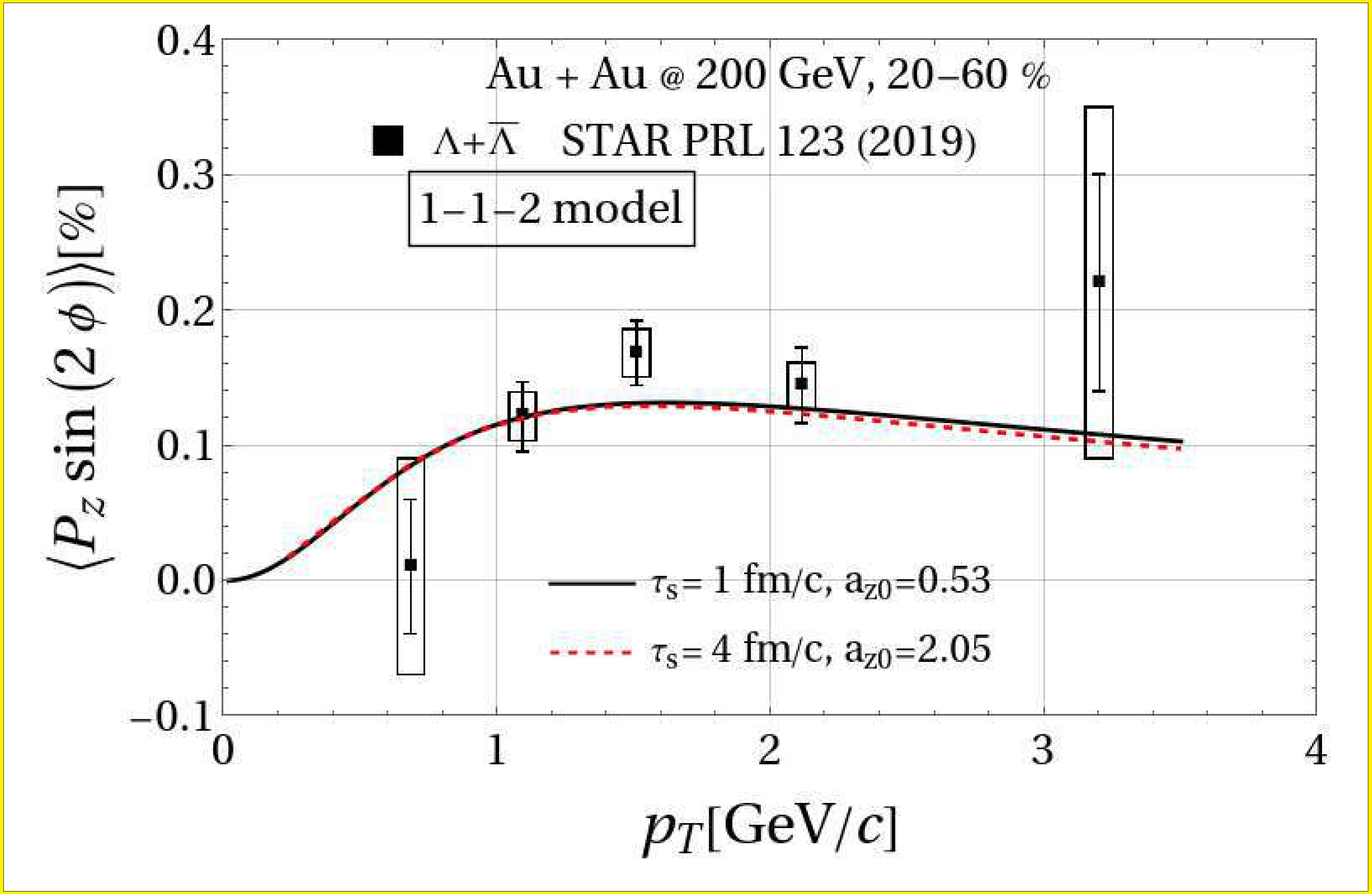
Results 1-1-2 model, Px







Dependence on spin equilibration time τ_s



We fit a_{z0} on the data for both equilibration times:

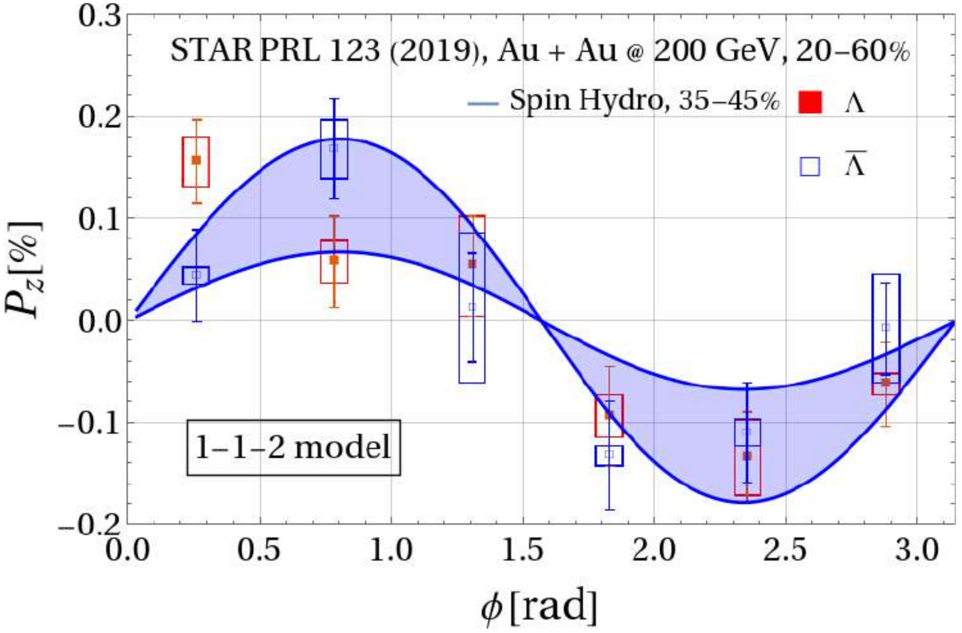
- If τ_s is large, the a_{z0} that reproduces data is too large
- a_z needs more time to develop a deeper slope in rapidity.

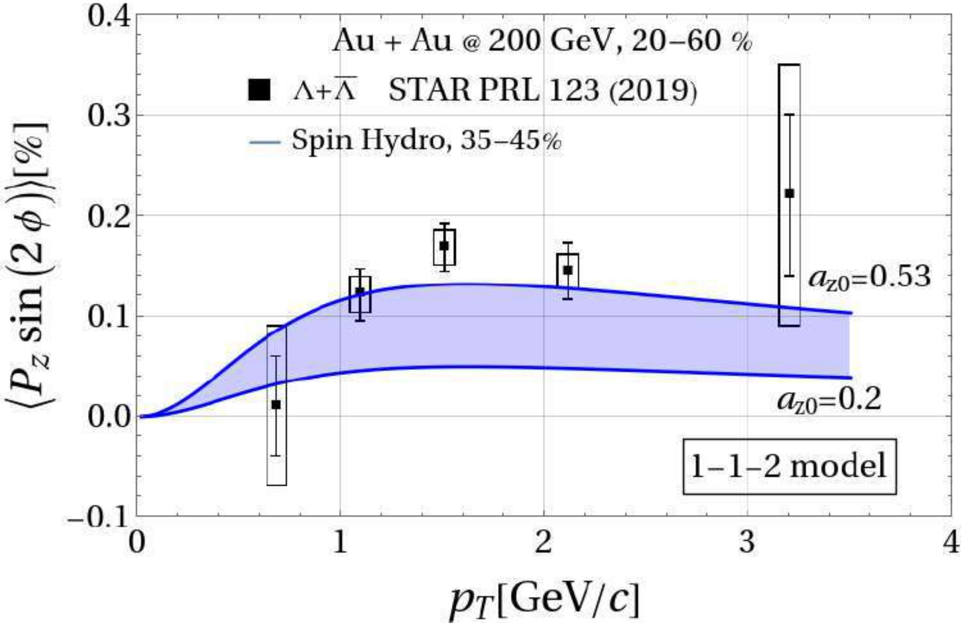
→ This model prefers early spin equilibration.

Dependence on parameter a_{z0}

The longitudinal spin polarization Pz is proportional to a_{z0}

$$a_{z0} \in [0.2,\ 0.53]$$





Dependence on transverse flow

Centrality (%)	$N_{\mathrm{ch}}^{\mathrm{exp}}$	$u_\perp$	$R_{\mathrm{FO}}(\mathrm{fm})$	δ	σ/R_{FO}	N_{ch}
35-45	1219 ± 117	0.4	9.0	0.27	1.6	1222