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Chromomagnetic condensation and perturbative confinement induced by imaginary rotation in SU(2) Yang-Mills Theory

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Rotation in heavy-ion collisions

- Large vorticity / rotation in non-central heavy-ion collisions

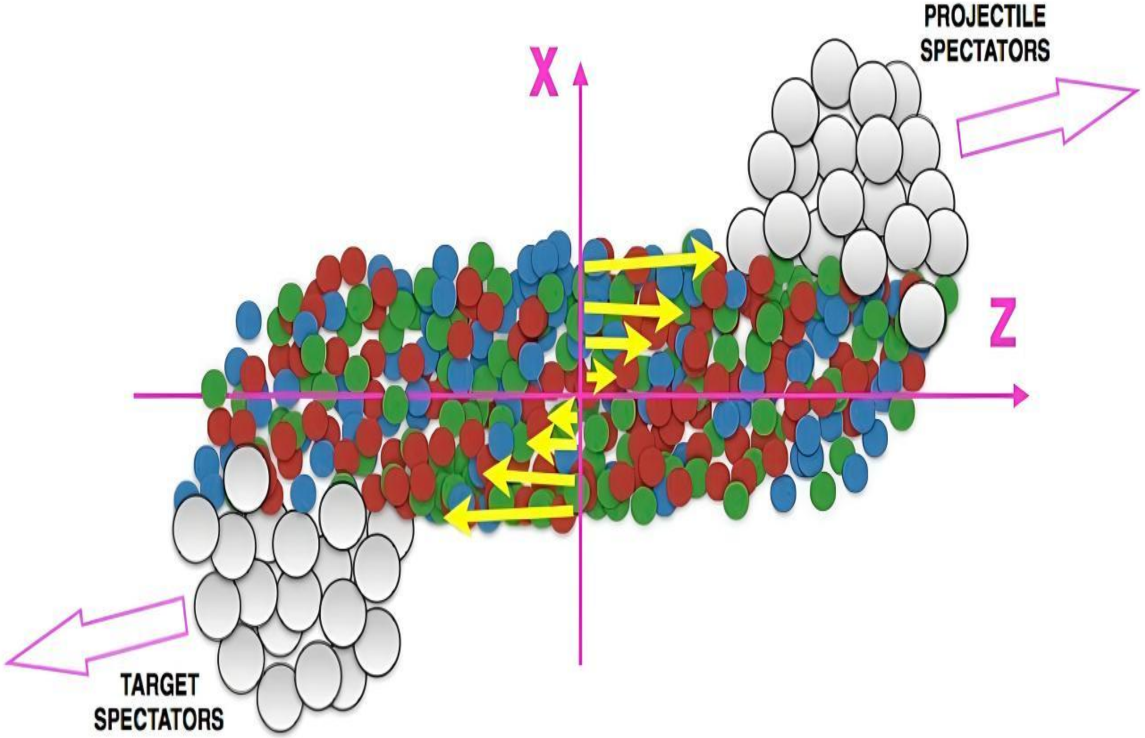
Motivation

Rotation is an important external parameter for the QCD phase structure, alongside temperature, chemical potential, and magnetic field.

Real vs. imaginary rotation

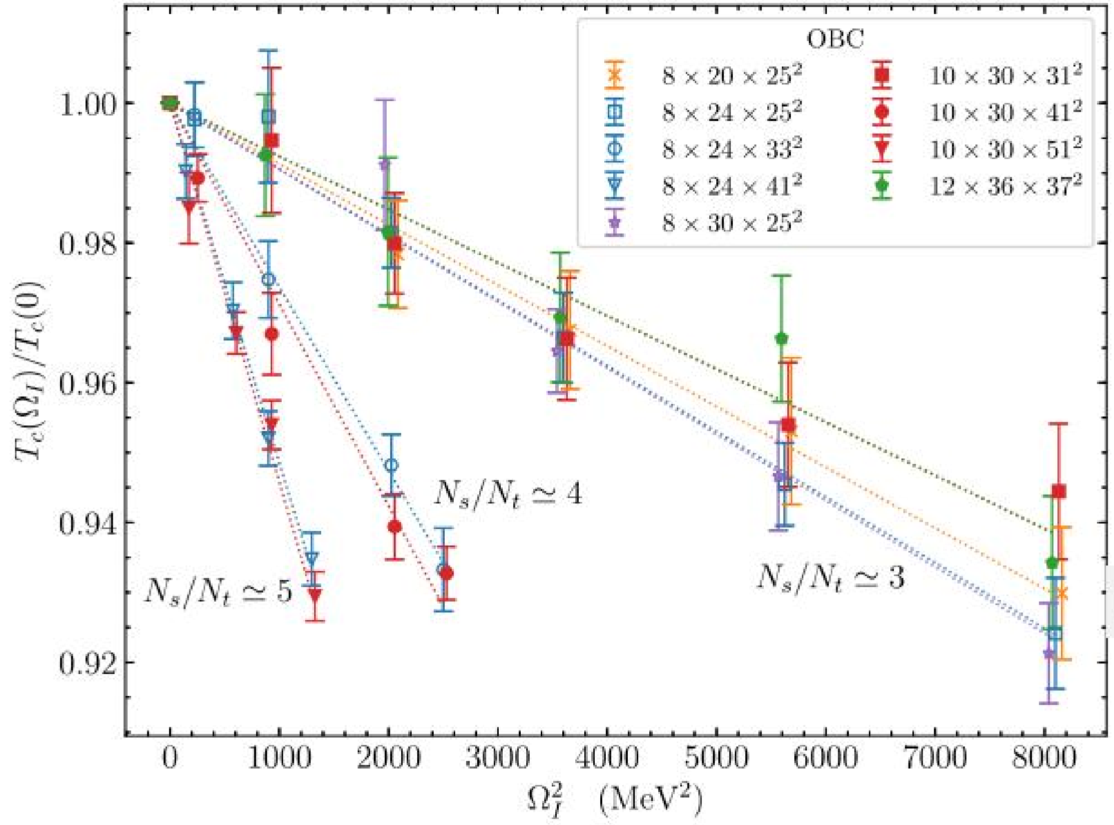
- For real rotation, lattice simulations suffer from a sign problem
- One often considers **imaginary angular velocity**

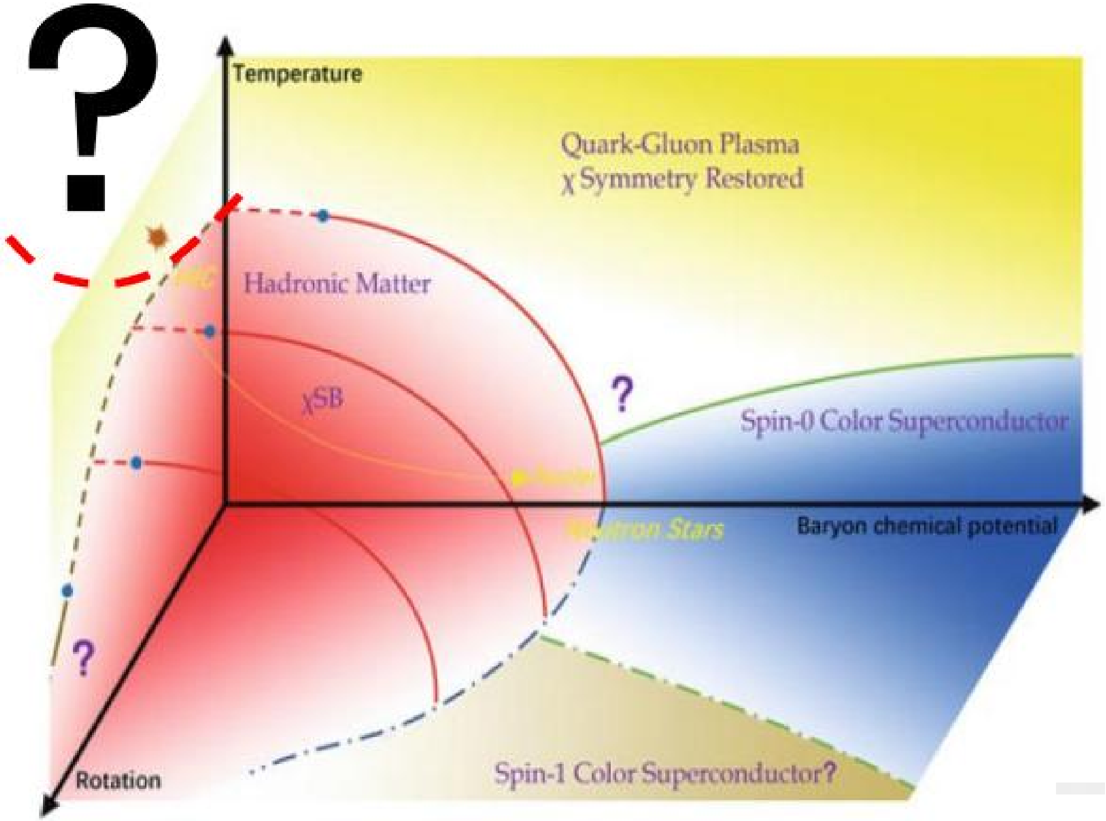
$$\Omega = -i\omega$$



Key observation

Lattice results and most model predictions for the phase transition under rotation show a clear discrepancy.





Previous result

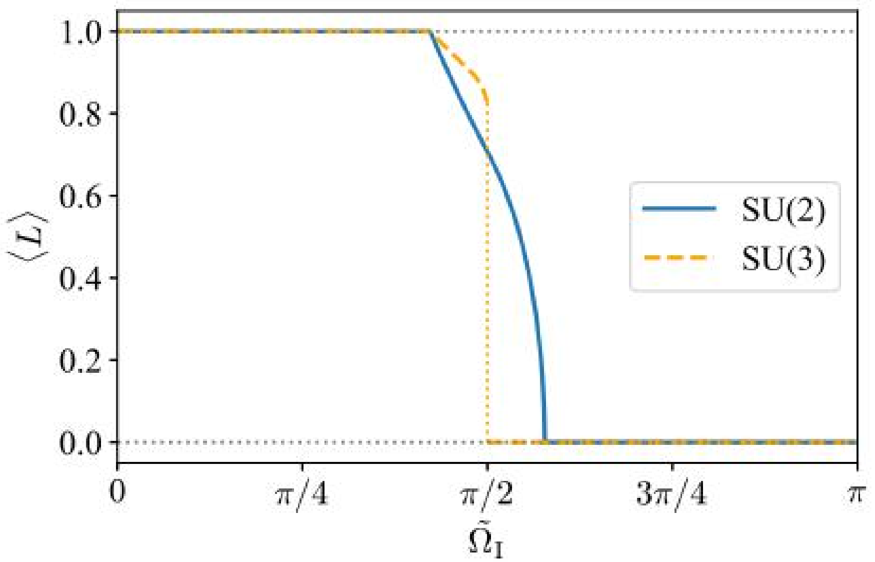
Phys. Rev. Lett. 129, 242002 (2022) found that in pure Yang-Mills theory, sufficiently large imaginary rotation can induce a **perturbative confinement phase**.

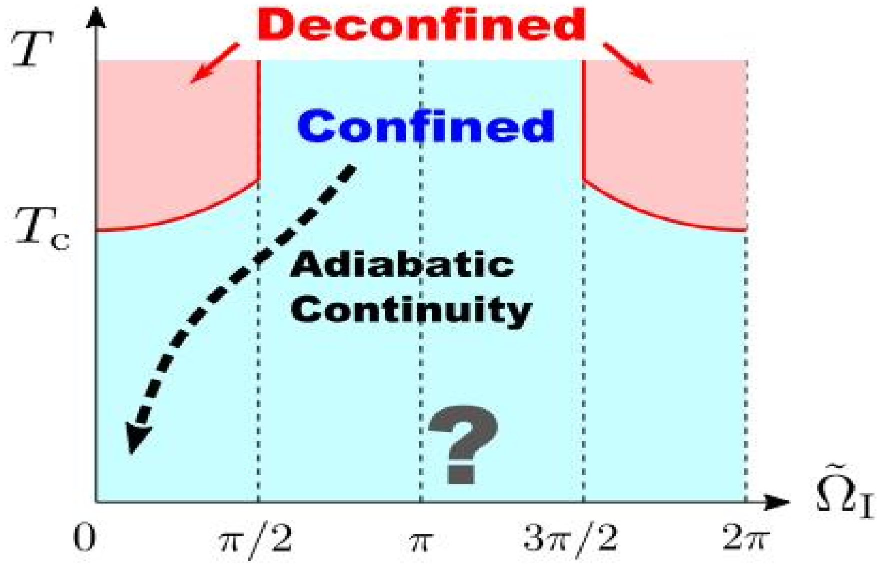
This is highly nontrivial

- Confinement → non-perturbative phenomenon
- Confinement emerges at the perturbative level

Implication

Imaginary rotation may provide a **perturbative window to explore non-perturbative physics**.





Why is the Polyakov loop an order parameter?

- The Polyakov loop is defined as

$$L = \mathrm{Tr}\,\mathcal{P}\exp\left(ig\int_0^\beta d\tau\, B_4(\vec{x},\tau)\right)$$

- It is related to the free energy of an infinitely heavy static color source:

$$|L| \propto e^{-\beta F_q}$$

- In pure gauge theories, it is an order parameter for the $\mathbb{Z}(N)$ center symmetry

For SU(2)

$$L = 2\cos(\phi/2)$$

Physical meaning

- $L = 0$: $F_q \to \infty$, confinement
- $L \neq 0$: $F_q < \infty$, deconfinement

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Savvidy vacuum [Phys. Lett.B 71, 133134 (1977)]

- Background field method $A_\mu = B_\mu + Q_\mu$
- Consider a constant chromomagnetic field background $B_\mu = \frac{H}{2}(0, y, -x, 0)$
- The one-loop effective potential may develop a nontrivial minimum at $H \neq 0$

Implications

- The Yang-Mills vacuum may favor local chromomagnetic condensation

$$\langle G^a_{\mu\nu} G^{a\,\mu\nu} \rangle \neq 0$$

[Nucl. Phys. B 469, 419–444 (1996)]

- Previous studies showed that the Nielsen-Olesen instability can be removed by higher-order fluctuations [Phys. Rev. D 75, 085007 (2007)]

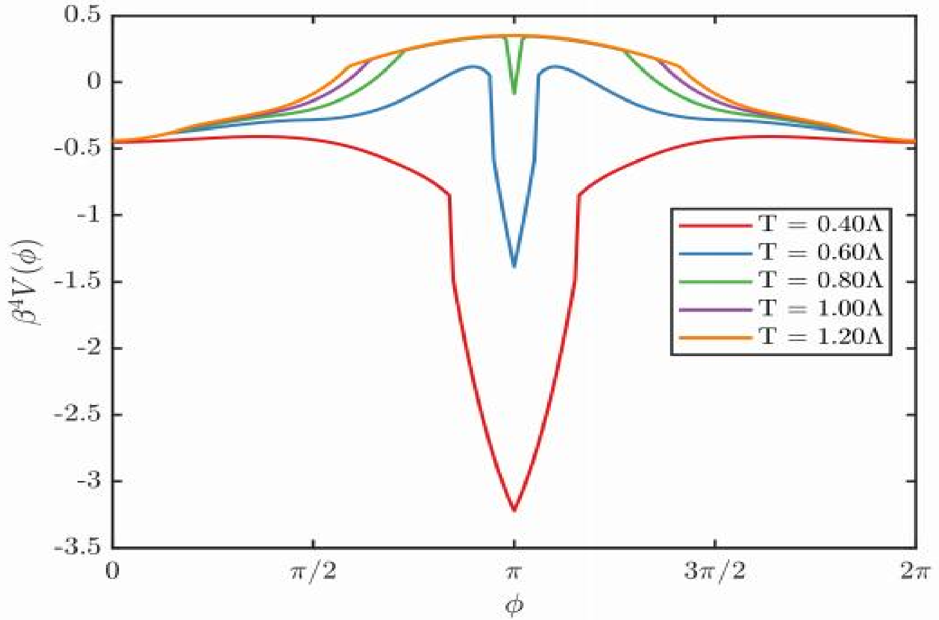
$$V_0 = \frac{11(gH)^2}{48\pi^2} \ln\left(\frac{gH}{\Lambda^2}\right) - i\frac{(gH)^2}{8\pi^2}, \quad u(-, 0, \vec{0}) = \frac{(gH)^{\frac{3}{2}}}{\pi^2 \beta} \sum_{n=1}^{\infty} \frac{1}{n} \frac{\pi}{2} \left[Y_1(n\beta\sqrt{gH}) - iJ_1(n\beta\sqrt{gH})\right] \cos n\phi$$

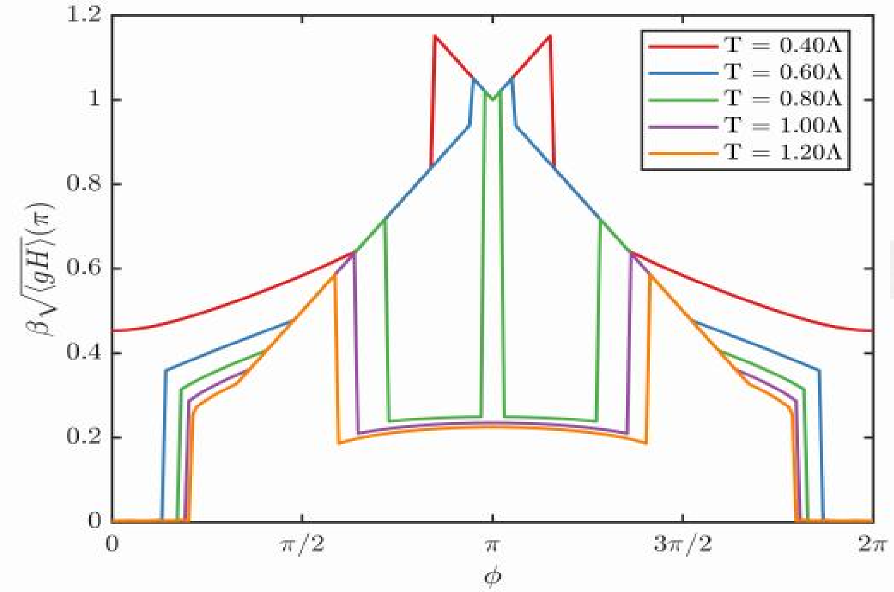
Key point

The Polyakov loop and chromomagnetic condensation are nontrivially coupled.

Under rotation

This coupling may reshape the minima of the effective potential and the nature of the phase transition.





In the low-temperature non-perturbative region, the one-loop calculation is unphysical, but it still reveals a nontrivial coupling pattern.

Core scientific questions

- If imaginary rotation can already induce a perturbative confinement phase,
    - can it also induce chromomagnetic condensation?
    - can the chromomagnetic background in turn modify the nature of the phase transition and the phase diagram?

What we do

In SU(2) Yang-Mills theory, we compute the one-loop effective potential in the presence of

- a **chromoelectric background** (Polyakov loop),
- a **chromomagnetic background**,
- and **rotation**,

and study the resulting phase structure.

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Background field and one-loop Lagrangian

$$B^a_\mu = B_\mu \delta^{a3}, \qquad B_\mu = \left(B_0, \frac{H}{2}y, -\frac{H}{2}x, 0\right)$$

$$\mathcal{L}_{\text{1-loop}} = -\frac{1}{2}\hat{Q}^+ \cdot \hat{Q}^- - igQ^+_\mu Q^-_\nu B^{\mu\nu} - \frac{1}{4}\tilde{Q}^3 \cdot \tilde{Q}^3 - \frac{1}{4}B \cdot B$$

Gauge fixing and rotation

$$\mathcal{L}_{gf} = -\mathrm{Tr}(D_\mu(B)Q^\mu)^2 = -\frac{1}{2}(\partial_\mu Q^{\mu,3})^2 - (D_\mu Q^{\mu,+})(D^*_\mu Q^{\mu,-})$$

$\partial_\mu \to \partial_\mu + \delta_{\mu 0} R_0$, [ChinPhysC 45,114102 (2021)] $\quad \partial_0 \to \partial_0 - i\omega(\hat{L}_z - s)$

Spectrum

$$[E - gB_0 + (l-s)\omega]^2 = k_z^2 + (2m+2s+1)gH$$

A common misunderstanding

$$E \to E - \omega J_z$$

This structure is generic in rotating systems, and by itself does **not** imply that the system is unphysical.

What makes the system pathological? [PhysRevLett.135.011601]

1. **Gauge invariance**: canonical angular momentum is not gauge invariant: $Z = \mathrm{Tr}\exp[\beta(H - \omega J_z)]$
2. **Thermodynamic stability**: possible unbalanced radial drift force: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = eH\omega\mathbf{r}$
3. **Thermodynamic limit**: possible bulk divergence in the infinite-area limit: $\rho_{\text{orbit}} \propto S_\perp$

Our framework

These concerns do not directly apply to our setup:

- SU(2) Yang–Mills theory
- background field method
- local rest / inertial frame

Gauge invariance: two equivalent descriptions

Curved frame

$$S[A] = -\frac{1}{2} \int d^4x \, g^{\mu\nu} g^{\alpha\beta} \, \mathrm{Tr}(F_{\mu\alpha} F_{\nu\beta})$$

$$A_\mu^U = U \left(A_\mu + \frac{i}{g} \partial_\mu \right) U^\dagger$$

$$F_{\mu\nu}^U = U F_{\mu\nu} U^\dagger$$

Local inertial frame

$$S[\bar{A}] = -\frac{1}{2} \int d^4x \, \eta^{ac} \eta^{bd} \, \mathrm{Tr}(\bar{F}_{ab} \bar{F}_{cd})$$

$$\bar{A}_a = e_a^\mu A_\mu ,$$

$$\bar{F}_{ab} = e_a^\mu e_b^\nu F_{\mu\nu} = \bar{D}_{[a} \bar{A}_{b]} - ig \bar{A}_{[a} \bar{A}_{b]} ,$$

$$\bar{D}_a \bar{A}_b = e_a^\mu D_\mu \bar{A}_b = e_a^\mu (\partial_\mu \bar{A}_b + \Gamma_{\mu bc} \bar{A}^c).$$

Equivalence

$$\eta_{ab} = e_a^\mu e_b^\nu g_{\mu\nu} \quad \Longrightarrow \quad g^{\mu\nu} g^{\alpha\beta} F^k_{\mu\alpha} F^k_{\nu\beta} = \eta^{ac} \eta^{bd} \bar{F}^k_{ab} \bar{F}^k_{cd} \quad \Longrightarrow \quad S[A] = S[\bar{A}]$$

Gauge invariance is manifest in the curved frame; therefore it must also hold in the local inertial frame.

Gauge invariance: why does $\hat{L}_z$ appear?

Rotating local frame

For rotation around the z-axis,

$$e_a^\mu = \delta_a^\mu + \omega\, \delta_a^0 \epsilon_{0\mu\nu 3} x_\nu$$

Modified gauge transformation

$$\bar{A}_a^U = e_a^\mu A_\mu^U = U \left(\bar{A}_a + \frac{i}{g}\partial_a + \boxed{\delta_a^0 \frac{\omega}{g}\hat{L}_z} \right) U^\dagger$$

Key point

- the gauge transformation law is modified in the rotating local frame
- $\hat{L}_z$ is part of the transformed structure
- it is **required**, not inserted by hand

$$S[\bar{A}^U] = S[A^U] = S[A] = S[\bar{A}]$$

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Background-field split

$$\bar{A}_a = B_a + Q_a$$

Joint transformation

$$B_\mu^U = U\left(B_\mu + \frac{i}{g}\partial_\mu + \delta_\mu^0\frac{\omega}{g}\hat{L}_z\right)U^\dagger, \qquad Q_\mu^U = UQ_\mu U^\dagger$$

Homogeneous transformation of Q

$$S[B+Q] = S[B] + S_2[B,Q] + \cdots$$
$$S_n[B,Q] = S_n[B^U, Q^U]$$

in particular,

$$S_{\text{1-loop}}[B,Q] = S_{\text{1-loop}}[B^U, Q^U]$$

Message

The one-loop Lagrangian is invariant under the joint transformation of B and Q.

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Modified background covariant derivative

$$D_\mu(B)Q^\mu = (\partial_\mu - \omega i \hat{L}_z\, \delta_{0\mu})Q^\mu - ig[B_\mu, Q^\mu]$$

$$D_\mu(B^U)Q^{U,\mu} = U\, D_\mu(B)Q^\mu\, U^\dagger$$

Hence, $$\mathcal{L}_{gf}(B,Q) = -\mathrm{Tr}(D_\mu(B)Q^\mu)^2 = \mathcal{L}_{gf}(B^U, Q^U)$$

One-loop effective action

$$e^{i\Gamma^{(1)}[B]} = \int \mathcal{D}Q\, \mathcal{D}c\, \mathcal{D}\bar{c}\; e^{i\left(S_{\text{1-loop}}[B,Q] + S_{gf}[B,Q] + S_{gh}[B,c,\bar{c}]\right)}$$

$$\mathcal{D}[Q^U]\mathcal{D}[c^U]\mathcal{D}[\bar{c}^U] = \mathcal{D}[Q]\mathcal{D}[c]\mathcal{D}[\bar{c}]$$

$$\Gamma^{(1)}[B] = \Gamma^{(1)}[B^U]$$

Conclusion

- the action is invariant under the joint transformation of (B, Q)
- invariant functional measure
- the final one-loop effective action remains gauge invariant

Why the other two concerns do not directly apply

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No unbalanced radial drift force

In the local inertial frame:

$$\mathbf{v} = 0, \qquad \mathbf{E} = 0$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0$$

In the rotating frame:

$$A_0 = -\frac{H\omega r^2}{2}$$

$$\mathbf{E} = (H\omega x, H\omega y, 0)$$

$$\mathbf{v} \times \mathbf{B} = (-H\omega x, -H\omega y, 0)$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Not a bulk thermodynamic construction

- the uniform chromomagnetic background is not interpreted as the true global vacuum
- more appropriate interpretation:
  - local $(r = 0)$
  - slowly varying
  - semiclassical background
- therefore, the bulk divergence argument for an infinite uniform system does not directly rule out our analysis

Take-home message

The cited concerns concern a different setup; they cannot be directly promoted to a no-go theorem for our formalism.

Thermodynamic potential at $r = 0$

At the rotation center, only the $l = 0$ mode survives.

$$
\begin{aligned}
V(r=0) = {} & \frac{11g^2H^2}{48\pi^2}\ln\left(\frac{gH}{\Lambda^2}\right) \\
& + \frac{gH}{2\pi\beta}\sum_{s=\pm}\int_{-\infty}^{+\infty}\frac{dk}{2\pi}\ln\left|1-2\cos(\phi+\tilde{\Omega})e^{-\beta\sqrt{k^2+sgH}}+e^{-2\beta\sqrt{k^2+sgH}}\right| \\
& - 2\frac{(gH)^{3/2}}{\pi^2\beta}\sum_{n=1}^{\infty}\frac{1}{n}\cos n\phi\cos n\tilde{\Omega}\sum_{m=0}^{\infty}\sqrt{2m+3}\,K_1\left(n\beta\sqrt{(2m+3)gH}\right)
\end{aligned}
$$

Gap equation

$$
V = V(H,\phi,\Omega) \qquad \Longrightarrow \qquad \frac{\partial V}{\partial H} = \frac{\partial V}{\partial \phi} = 0
$$

Effective-potential structure

Two competing parts

$$\beta^4 V = \beta^4 V_H + \beta^4 V_{\text{nonH}}$$

V_H

$$V_H = \frac{11 g^2 H^2}{48\pi^2} \ln\left(\frac{gH}{\Lambda^2}\right) + \frac{gH}{2\pi\beta} \sum_{s=\pm} \int \frac{dk}{2\pi} \ln\left|1 - 2\cos(\phi + \tilde{\Omega}) e^{-\beta\sqrt{k^2 + sgH}} + \cdots\right|$$

- vacuum term
- LLL / low Landau levels
- favors chromomagnetic condensation

V_{nonH}

$$V_{\text{nonH}} = -2\frac{(gH)^{3/2}}{\pi^2\beta} \sum_{n=1}^{\infty} \frac{\cos n\phi \cos n\tilde{\Omega}}{n} \sum_{m=0}^{\infty} \sqrt{2m+3}\, K_1\left(n\beta\sqrt{(2m+3)gH}\right)$$

- higher Landau levels
- suppresses chromomagnetic condensation

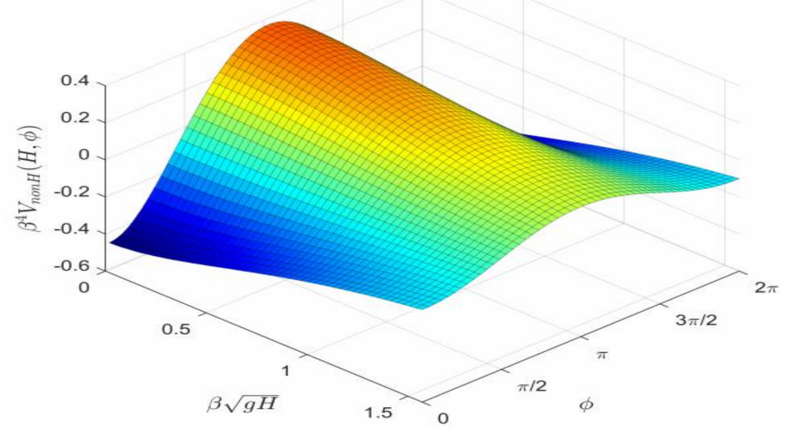
Temperature effect

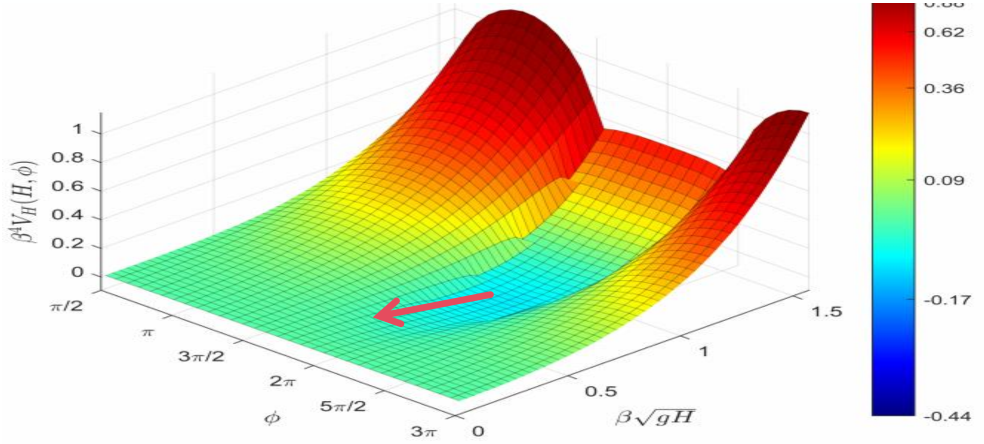
High-temperature perturbative region

- thermal effects strongly suppress chromomagnetic condensation
- the system tends to the deconfined phase
- the minimum moves toward smaller $\beta\sqrt{gH}$

At $\tilde{\Omega} = 0$

- V_{nonH} dominates the physical ground state
- H is essentially suppressed to zero
- deconfined vacuum without chromomagnetic condensation





Effect on V_H

- mainly shifts the minimum along ϕ
- the chromomagnetic-favoring tendency remains

Effect on V_{nonH}

- significantly raises its minimum
- weakens the suppression of chromomagnetic condensation

Competition reshuffling

$$\tilde{\Omega}\uparrow \implies V_{\text{nonH}} \text{ loses advantage} \implies V_H \text{ becomes competitive}$$

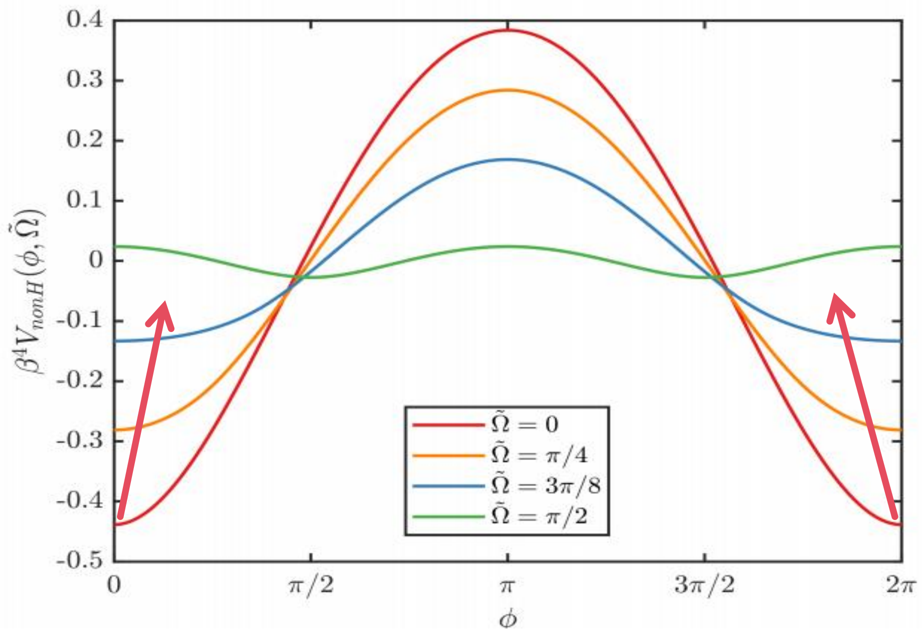
$\tilde{\Omega}$ can be confined to the interval $[0, \pi/2]$

- the overall symmetry of the potential:

$$V(H, \phi, \tilde{\Omega}) = V(H, 3\pi - \phi, \pi - \tilde{\Omega}),$$

- The physical minimum (H, ϕ) satisfies:

$$(H, \phi)\Big|_{\pi - \tilde{\Omega}} = (H, 3\pi - \phi)\Big|_{\tilde{\Omega}}.$$



Summary

- the original deconfined vacuum without chromomagnetic condensation loses advantage
- a new global minimum is expected to appear
- $H \neq 0$
- ϕ moves away from the deconfined value and toward the confining value $\phi = \pi$
- imaginary rotation may induce
  - perturbative confinement
  - chromomagnetic condensation

Conjecture

- the perturbative confinement transition may be modified by the chromomagnetic background
- the order of the transition may change
- the phase boundary may acquire nontrivial temperature dependence
- the $\tilde{\Omega}$-T phase structure may become richer

Definition

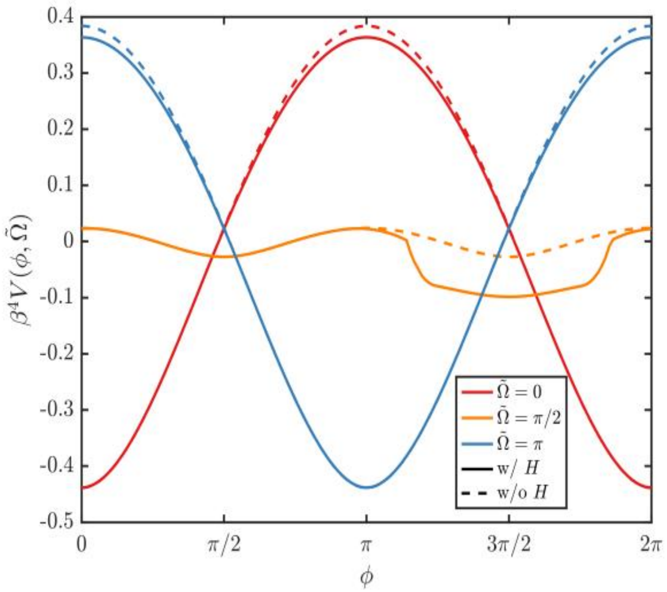
$$V(\phi) = V(\langle H \rangle, \phi), \qquad \partial_H V(H,\phi)\big|_{H=\langle H \rangle} = 0$$

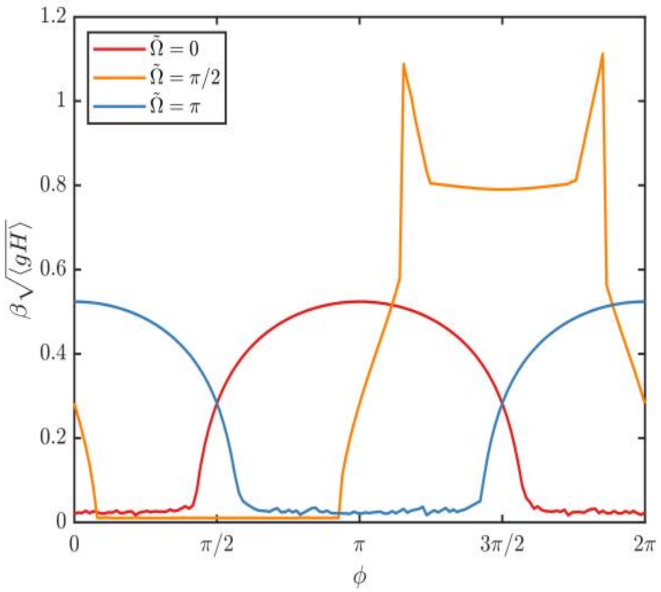
What to compare

- with chromomagnetic background
- without chromomagnetic background
- solid vs dashed curves

Key observation

- with $\beta\sqrt{gH} \neq 0$: lower potential
- nearly coincident curves: $\beta\sqrt{gH} \approx 0$
- separated curves: $\beta\sqrt{gH} \neq 0$





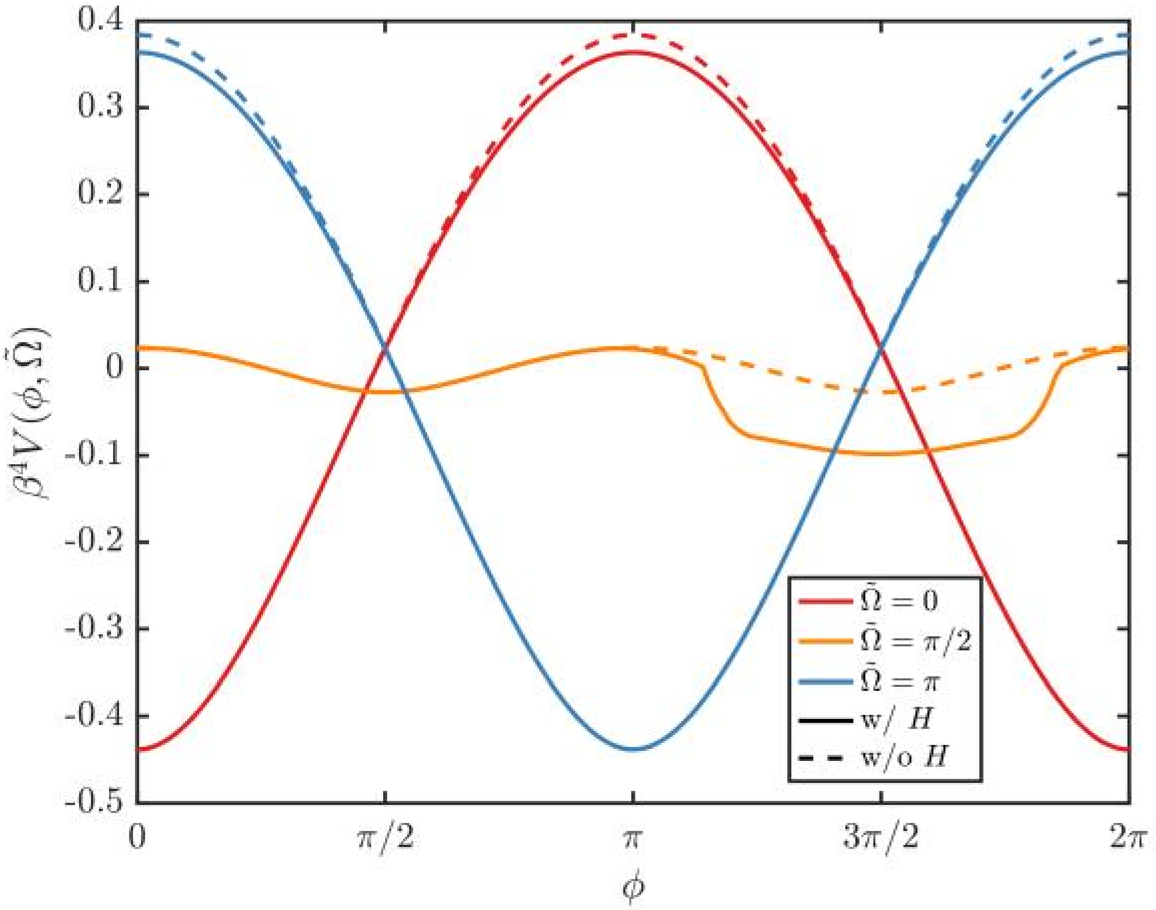
Imaginary rotation not only changes the Polyakov-loop potential, but also induces a lower-lying branch with nonzero chromomagnetic condensation.

Reading the potential

- $\tilde{\Omega} = 0$:
  - deconfined vacuum
  - $\beta\sqrt{gH} \approx 0$
- larger $\tilde{\Omega}$:
  - new minimum appears
  - accompanied by $\beta\sqrt{gH} \neq 0$
- at even larger $\tilde{\Omega}$:
  - the minimum moves toward $\phi = \pi$

Message

The chromomagnetic background is not a small correction; it changes both the depth and the location of the physical minimum.



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Direct signal from order parameters

- $|L|$ vs $\tilde{\Omega}$ and $\beta\sqrt{gH}$ vs $\tilde{\Omega}$
- $\tilde{\Omega}\uparrow \Longrightarrow V_{\text{nonH}}$ loses advantage $\Longrightarrow H \neq 0$
- simultaneous jumps
- vacuum structure changes discontinuously

Transition path

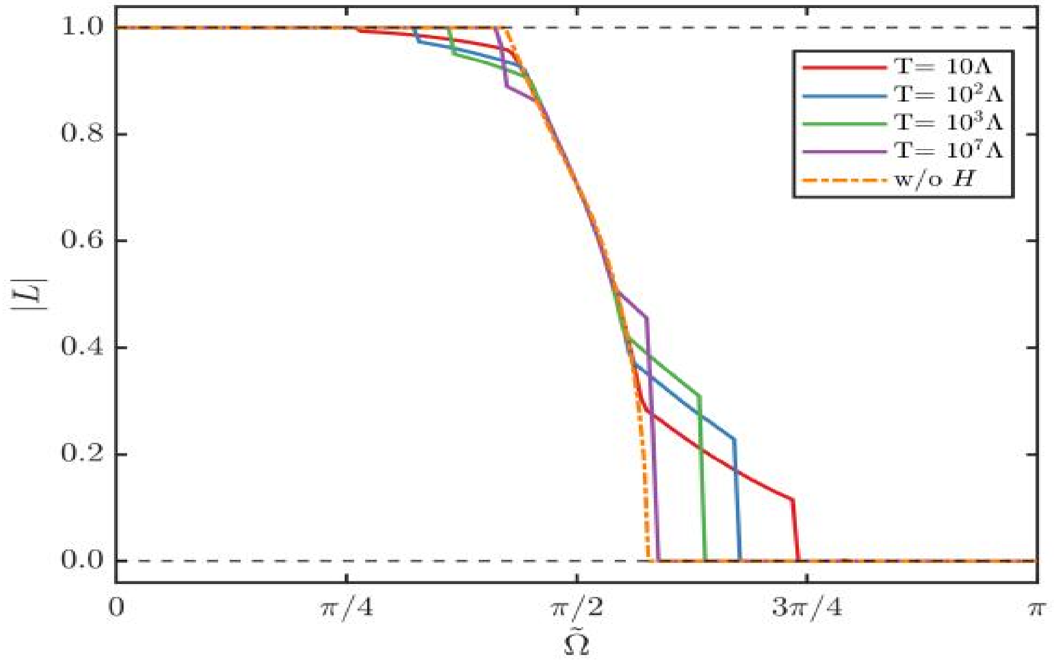
$$(H, \phi) = (0, 2\pi)$$

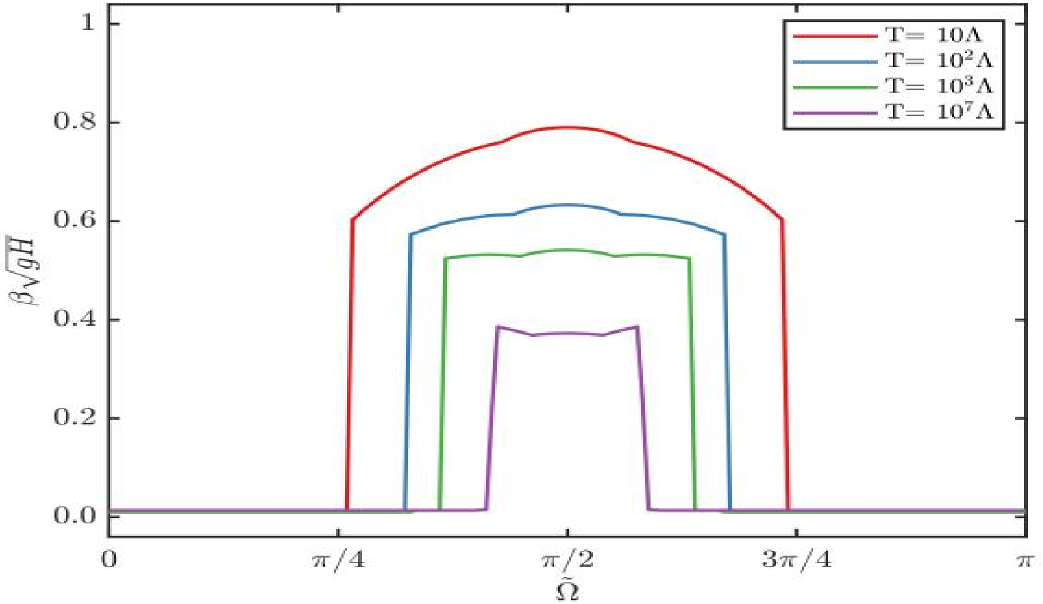
$$\Downarrow$$

$H \neq 0, \quad \phi$ moves away from the deconfined value

$$\Downarrow$$

$$(H, \phi) = (0, \pi)$$





Summary

- chromomagnetic condensation appears along the transition path
- the final perturbative confinement phase is reached at

$$H = 0, \qquad \phi = \pi$$

- the induced chromomagnetic background reshapes the route toward perturbative confinement

Conjecture

- the perturbative confinement transition may become first-order
- the phase boundary may acquire nontrivial temperature dependence
- the $\tilde{\Omega}$-T phase structure may become richer

Phase diagram in the $\tilde{\Omega}$-T plane

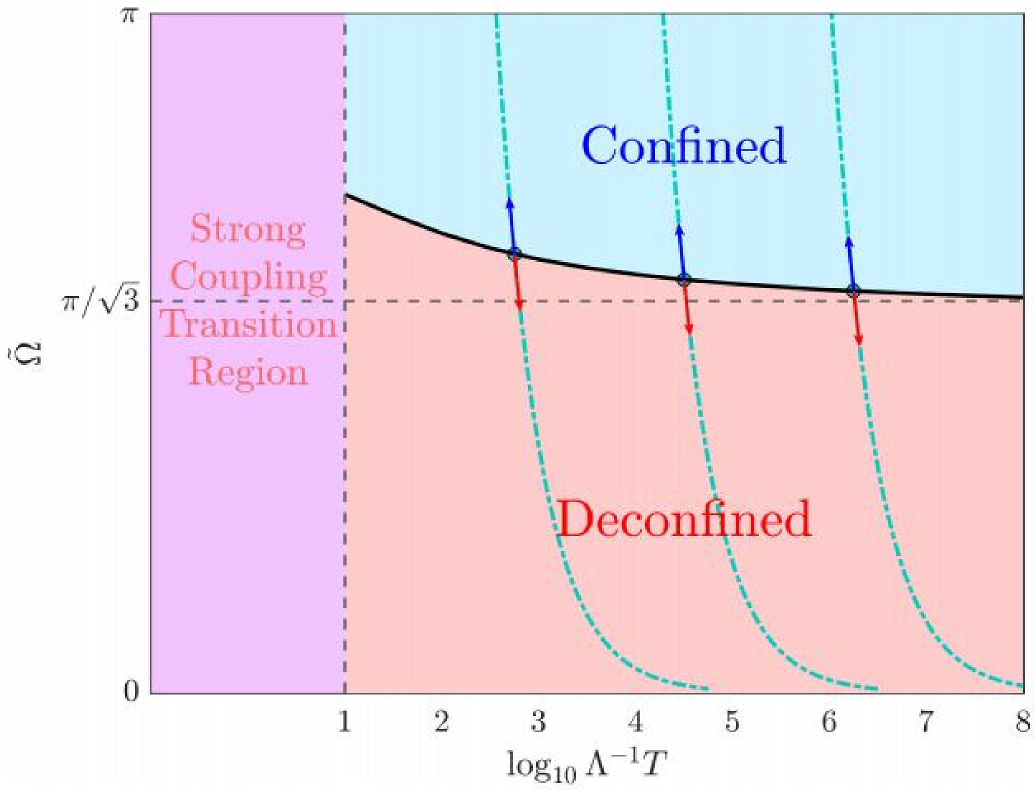
- compare with / without chromomagnetic background
- first-order phase boundary
- explicit temperature dependence

Key features

- $\tilde{\Omega}_c$ decreases with temperature
- at high temperature:

$$\tilde{\Omega}_c \to \pi/\sqrt{3}$$

- the deconfined region is enlarged
- T_c increases monotonically with Ω

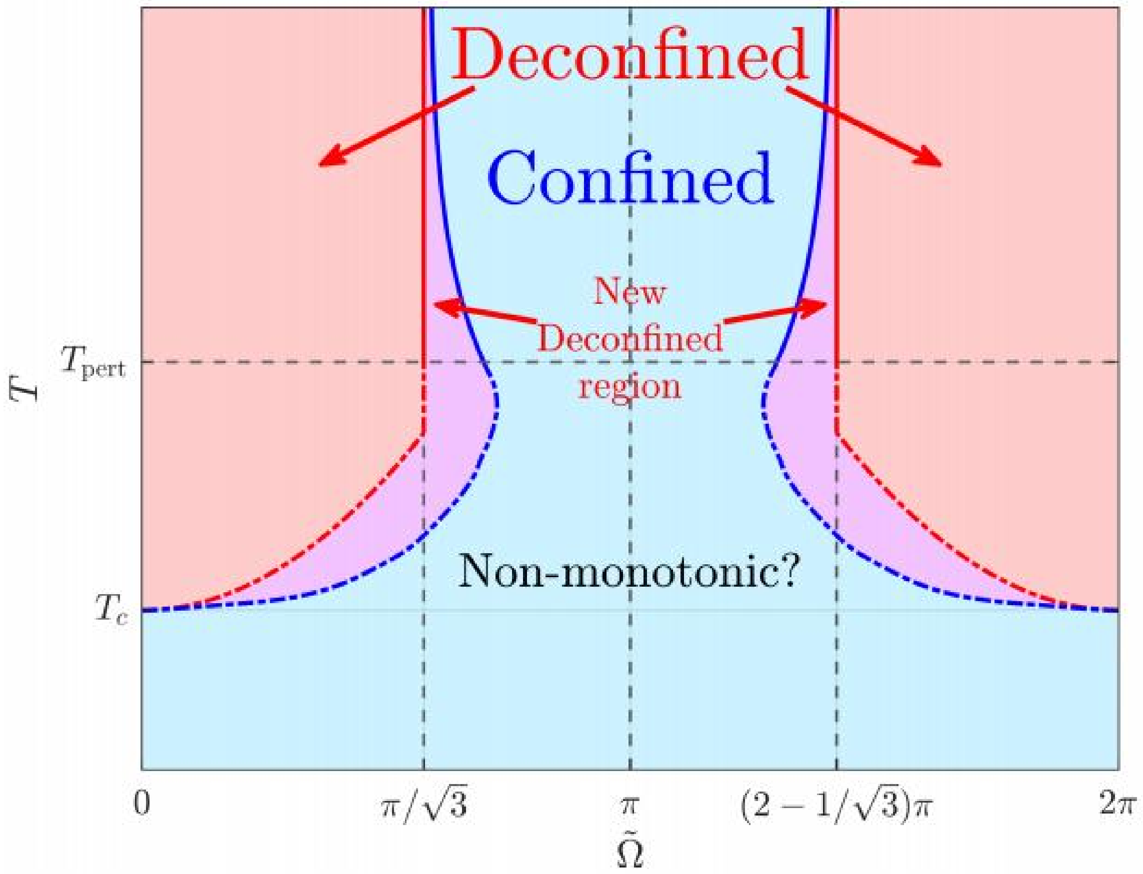


Main message

The chromomagnetic background significantly enriches the $\tilde{\Omega}$-T phase structure.

Implications

- perturbative confinement transition:
  - first-order
  - temperature dependent
- high-temperature deconfined region:
  - significantly enlarged
- possible non-perturbative implications:
  - non-monotonic phase boundary
  - possible critical end point



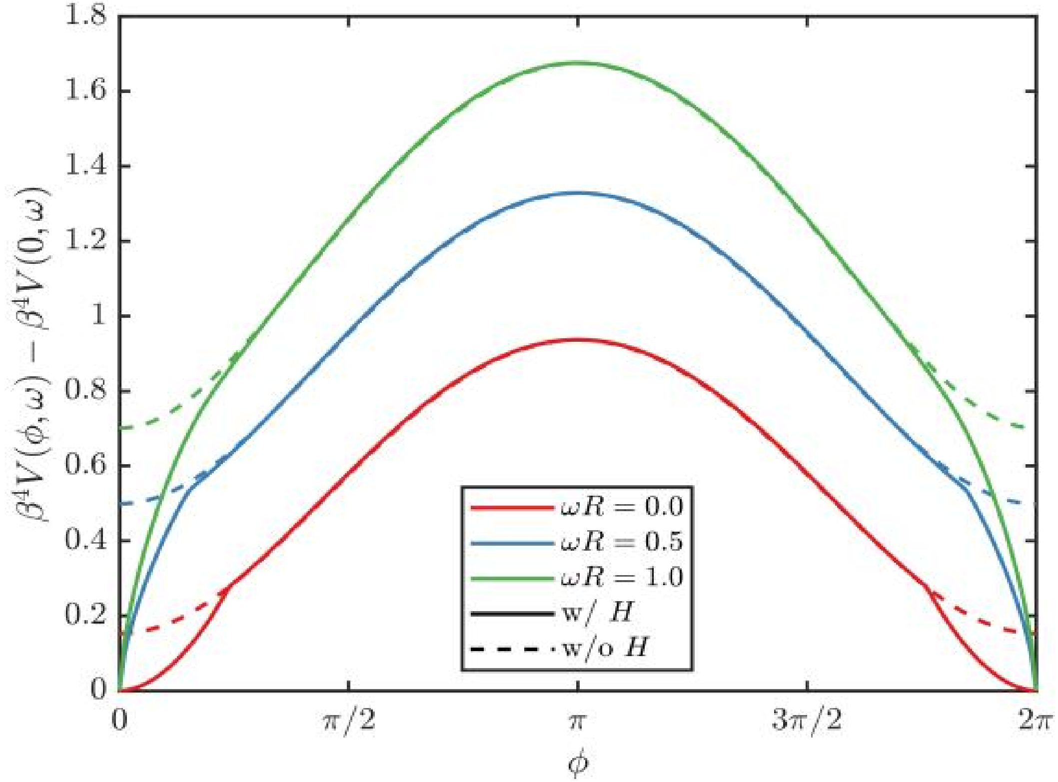
Treatment

Impose boundary conditions to preserve causality:

$$F(-\lambda_k^l, |l|+1, X) = 0, \qquad X = \frac{1}{2} g H R^2$$

Physical implication of the cusp

- local thermodynamic instability / singular behavior
- a constant chromomagnetic background is not stable under real rotation
- **not** caused by an unbalanced radial drift force



Summary

- We studied the perturbative confinement transition of SU(2) Yang–Mills theory in the presence of
  - a chromoelectric background,
  - a chromomagnetic background,
  - and rotation.
- Imaginary rotation can induce chromomagnetic condensation and reshape the route toward perturbative confinement.
- The resulting $\tilde{\Omega}$-T phase structure becomes richer.

Outlook

- lattice simulations:
  - test the induced chromomagnetic condensation
  - verify the first-order boundary and the phase diagram
- include dynamical quarks:
  - explore whether imaginary rotation can also induce chiral symmetry breaking
  - investigate a possible perturbative chiral phase transition
- extend to SU(3) gauge theory for a closer connection to QCD

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