Spin Kinetic Theory From Algebra

Outline

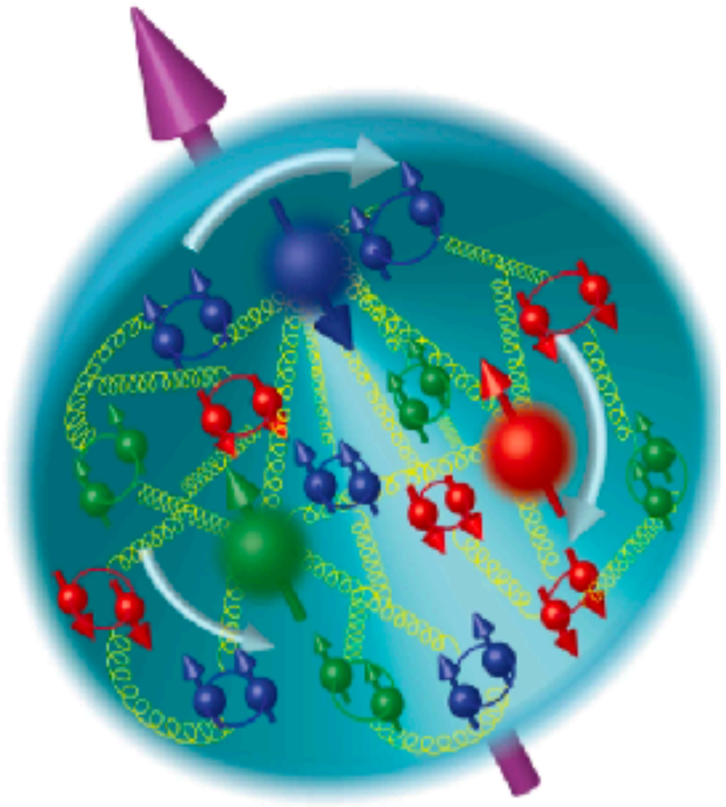
- Introduction
- Poisson Bracket and Spin Kinetic Theory
- Dissipation and spin
- Discussion and outlook

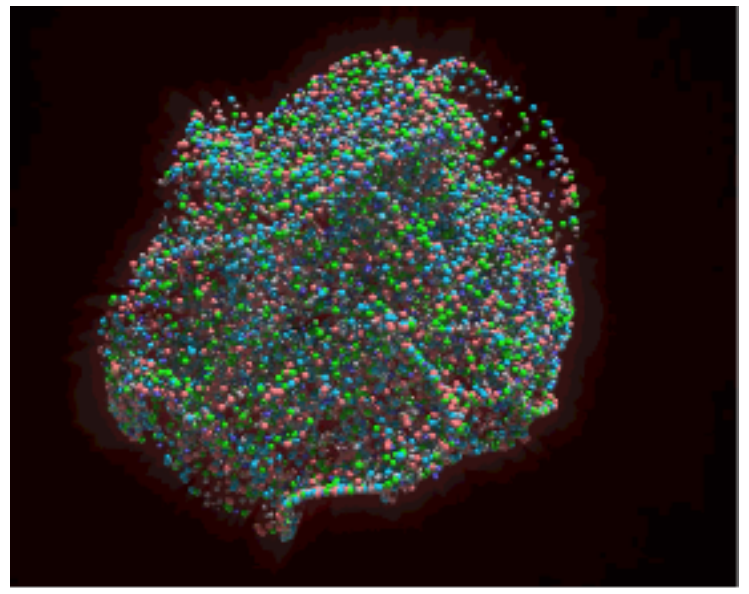
Yi Yin

Ref: Zonglin Mo, YY, 2512.23960;
Jeremy Hansen, YY, in preparation

Spin Workshop, Hefei, May. 12, 2026

QCD Spin Structure





- Intriguing for proton (confined)
- Rich phenomenon in quark matter (deconfined)

Spin Kinetic Equation

- Powerful tool to describe spin dynamics in medium
- Significant progress in derivation from microscopic theory (top-down)

  works by many

- This talk: "bottom-up" construction based on algebra
  - Clarifying algebraic structure underlying spin dynamics
  - Building EFT for spin with symmetry principle

Proposal

Zonglin Mo, YY 2512.23960

- D.o.fs: $f(t, \vec{x}, \vec{p})$, $\vec{g}(t, \vec{x}, \vec{p})$ parametrizing spin-averaged and spin-dependent distribution

- E.o.M. in collision-less regime from Poisson Brackets (P.B.)

$$\partial_t f = \{\epsilon_p, f\}$$

$$\partial_t \vec{g} = \text{spin dependent P.B.?}$$

- Constitutive relation for observables (to be matched with field theory),

$$\text{e.g: } \overrightarrow{\mathscr{A}}[\vec{g}, f] = c_1 \vec{g}_{\parallel} + c_2 \vec{g}_{\perp} + c_3 \vec{v} \times \vec{\partial} f + \ldots \ ,$$

Poisson Bracket and Canonical Transformation (C.T)

Delacrétaz et. al (U. Chicago group), 2022

- Related by the change of distribution (classically)

$$f(\vec{x},\vec{p}) \to f(\vec{x}-\nabla_{\vec{p}}\phi, \vec{p}+\nabla_{\vec{x}}\phi) = f + \{f,\phi\} + \dots \ ,$$

- Precisely reproduced by the commutator of field generator for C.T :

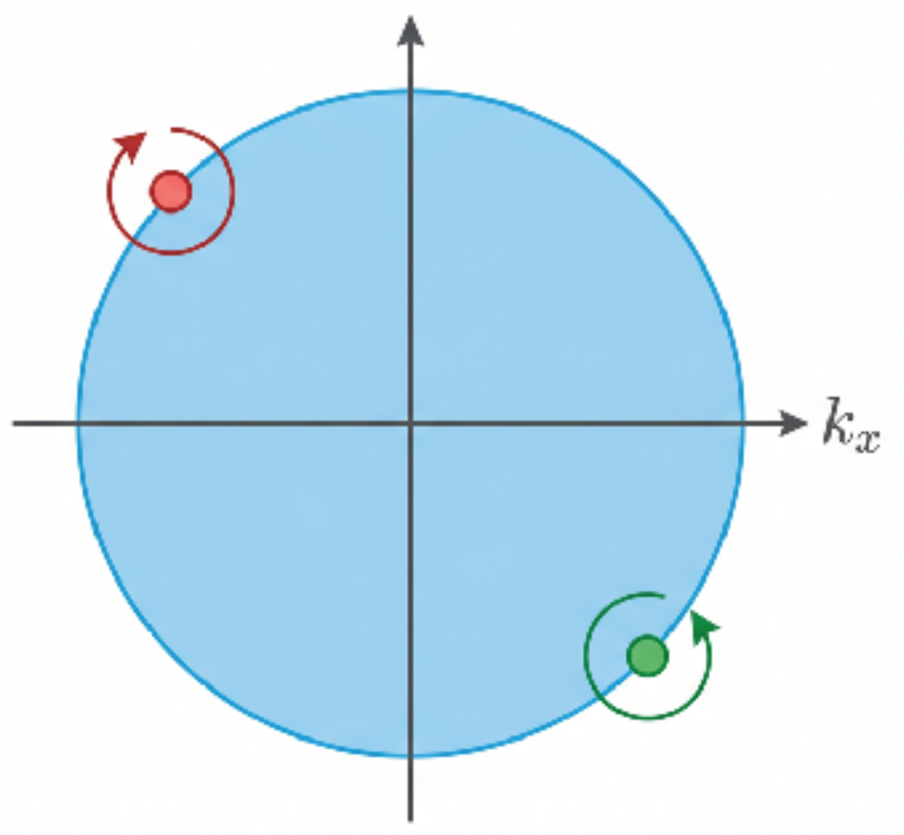
$$\hat{O}_\phi = \int_{\vec{x},\vec{p}} \phi(\vec{x},\vec{p})\hat{T}_0(\vec{x},\vec{p}) \qquad \hat{T}_0(\vec{x},\vec{p}) \equiv \int_{\vec{y}} e^{i\vec{y}\cdot\vec{p}}\hat{\psi}^\dagger(\vec{x}+\vec{y}/2)\hat{\psi}(\vec{x}-\vec{y}/2)$$

$$[\hat{O}_{\phi_1}, \hat{O}_{\phi_2}] = \int_{\vec{x},\vec{p}} \{\phi_1,\phi_2\}_{P.B.}\hat{T}_0$$

Kinetic Theory and Symmetry

Else et al 2402.14066;
Delacretaz et al, 2203.05004

$\hat{j}^0(x) = \int_{\vec{p}} \hat{T}_0(\vec{x}, \vec{p})$ → C.T. correspond to generating U(1) phase in momentum space



- Covariance under C.T. means particles can choose their phase independently

- Vector Wigner function is the Noether current of C.T.

$\mathscr{V}^\mu(x, \vec{p}) \sim p^\mu f$ Collisionless kinetic eqn $\Longleftrightarrow$ conservation of $\mathscr{V}^\mu$;

- Liouville's theorem is a consequence of symmetry

Nonlinear Bosonization of Fermi Surfaces: The Method of Coadjoint Orbits

Luca V. Delacrétaz,[1,2] Yi-Hsien Du,[1] Umang Mehta,[1,3] and Dam Thanh Son[1,2,4]

[1]*Kadanoff Center for Theoretical Physics,*
University of Chicago, Chicago, Illinois 60637, USA

Collisionless dynamics of general non-Fermi liquids from hydrodynamics of emergent conserved quantities

Dominic V. Else
Perimeter Institute for Theoretical Physics, Waterloo, ON N2L 2Y5, Canada

Spin-dependent P.B.

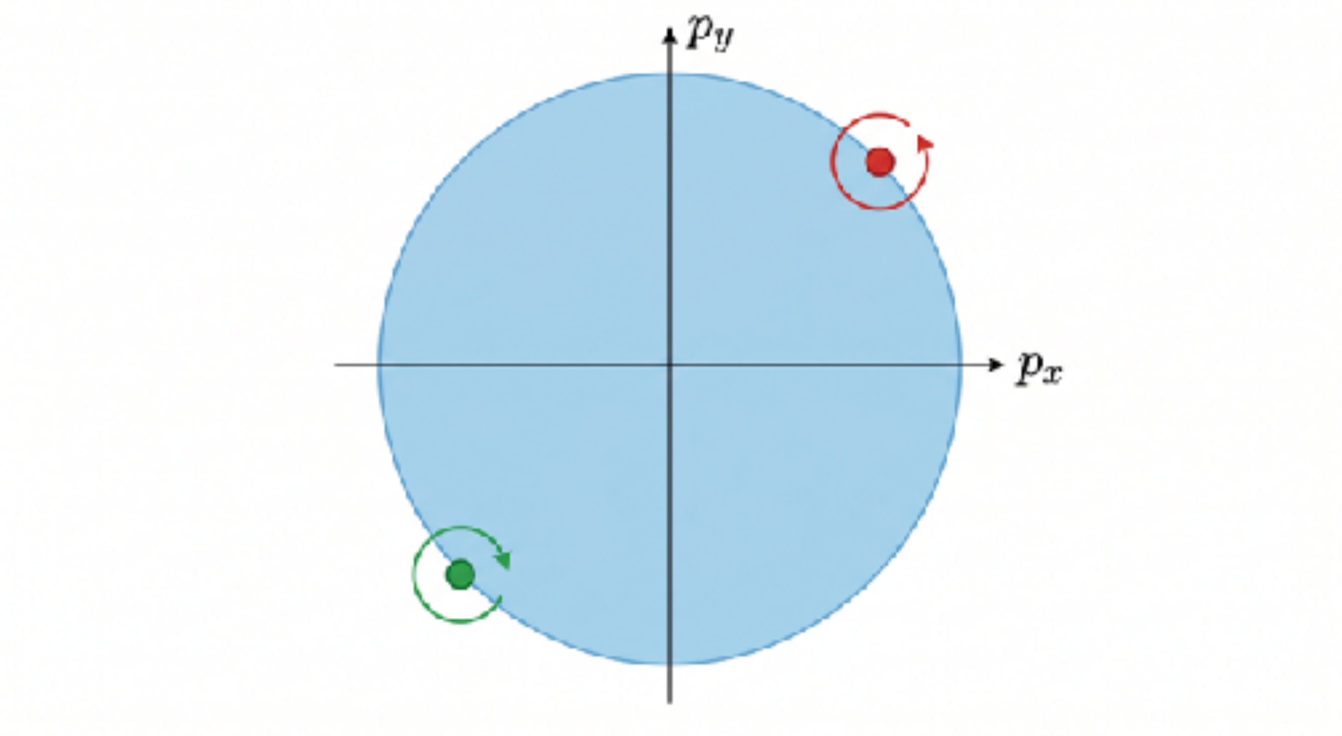
Delacrétaz et. al (U. Chicago group), 2022

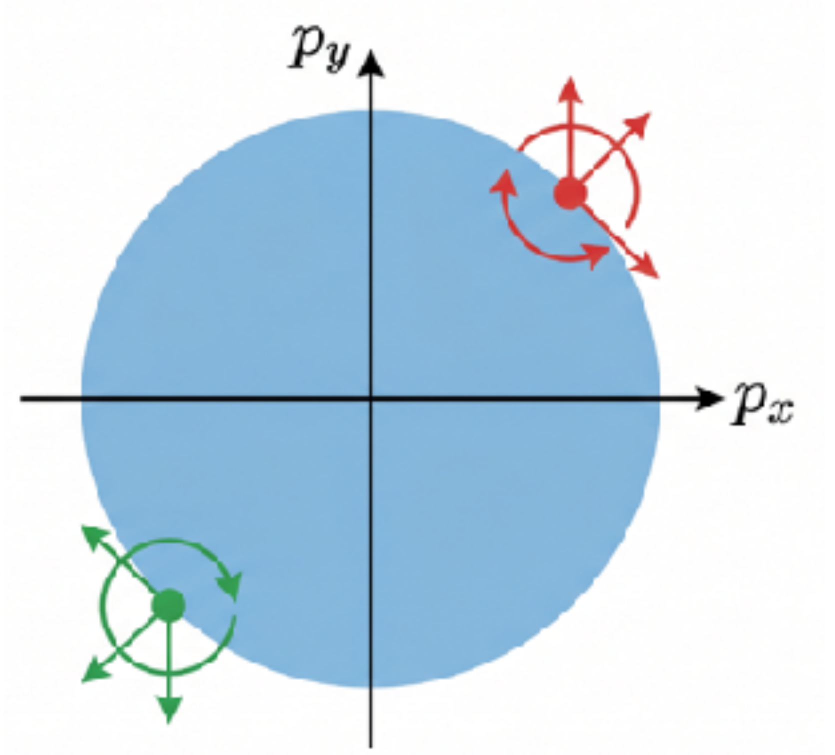
- Generator of spin-dependent C.T.

$$\hat{O}_\varphi = \int_{\vec{x},\vec{p}} \varphi^a(\vec{x},\vec{p})\hat{T}_a(\vec{x},\vec{p})\,, \qquad \hat{T}_a(\vec{x},\vec{p}) \sim \hat{\psi}^\dagger \sigma_a \hat{\psi}$$

- Symmetry: freedom in choosing spin quantization axis in phase space

- Commutator among $\hat{O}_\phi, \hat{O}_\varphi$ leads to spin-dependent P.B. non-Abelian





Kinetic Equation from Algebra

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- Spin dependent P.B. between $(f, \vec{g})$ and energy $(\epsilon[f, \vec{g}], \zeta^a[f, \vec{g}])$

$$\partial_t f + \{f, \epsilon\} = 0 \qquad \boxed{\partial_t \vec{g} + \{\vec{g}, \epsilon\} + \{f, \vec{\zeta}\} - \vec{g} \times \vec{\zeta} = 0}$$

- Energy functional

$$\epsilon_{\vec{p}}[f, \vec{g}] = E_{\vec{p}} + \int_{\vec{p}'} \mathscr{F}(\vec{p}, \vec{p}')\delta f(\vec{p}') + \ldots$$

off-equili.

- Magnetic field functional $\vec{\zeta}$ (c.f. $\Delta E \propto \hat{S} \cdot \hat{B}$)

What Drives Spin Dynamics

$$\partial_t \vec{g} + \{\vec{g}, \epsilon\} + \{f, \vec{\zeta}\} - \vec{g} \times \vec{\zeta} = 0$$

- $\{g^a, \epsilon\} \sim \vec{v} \cdot \partial_{\vec{x}} g^a$: drift term
- $\{f, \vec{\zeta}\} = (\partial_{\vec{p}} f) \cdot (\partial_{\vec{x}} \zeta^a) - (\partial_{\vec{x}} f) \cdot (\partial_{\vec{p}} \zeta^a)$
  - Stern-Gerlach force (gradient of magnetic field pulls on spin)
  - Gradient of distribution flips spin with magnetic field (connected to EM form factor) $\langle p', s' | \vec{j}_\perp(0) | p, s \rangle \big|_{\mathrm{EF}} = e\, (\sigma_z)_{s's}\, (\vec{e}_z \times i\vec{\Delta}_\perp)\, G_M(Q^2)$
- $\vec{g} \times \vec{\zeta}$: precession around $\vec{\zeta}$ (c.f. BMT)

Matching to field theory

- Linear response theory, e.g.: $\overrightarrow{\mathscr{A}} \sim \langle (\bar{\psi}\gamma^5\vec{\gamma}\psi)(x)\, J^\mu(0)\rangle\, A_\mu$

- Wigner function expressible in terms of $f, \vec{g}$ with $\vec{\zeta} = -(\overrightarrow{B})/\epsilon_{\vec{p}}$

$$\vec{\mathcal{A}} = \underbrace{\vec{g}_{\|} + (1-\vec{v}^{\,2})\,\vec{g}_{\perp} + \frac{1}{\epsilon_{\vec{p}}}\,\vec{v}\times\vec{\partial} f}_{\text{dynamical (non-analytic)}} \; - \underbrace{\frac{f_0'\,\vec{v}^{\,2}}{\epsilon_p}\,\vec{B}_{\perp} - \frac{f_0}{\epsilon_p^2}\,(\vec{v}\times\vec{E})}_{\text{external (analytic)}}$$

c.f. $\vec{j} = -D\vec{\partial}n + \sigma\overrightarrow{E}$

- Identical expression for dynamical part regardless of source (EM, gravity)

- Validating "bottom-up" approach

Proper dynamical d.o.f. …

- Response contains both non-local and local terms

$$\mathscr{A} \sim \left(\frac{1}{\omega - \vec{q}\cdot\vec{v} + i\epsilon}\right) + \text{local piece}$$

- $f, \vec{g}$ precisely capture the non-analytic response, proving they are the correct slow/infrared d.o.f.

- Isolating the dynamical part simplifies the E.o.M.

$$\partial_t \vec{g} + \{\vec{g}, \epsilon\} + \{f, \vec{\zeta}\} - \vec{g} \times \vec{\zeta} = 0$$

$$0 = \delta(q^2 - m^2)\left[q \cdot \Delta f_V + \hbar\left(\frac{E_\mu S^{\mu\nu}_{a(n)}}{q \cdot n}\Delta_\nu + S^{\mu\nu}_{a(n)}(\partial_\mu F_{\rho\nu})\partial^\rho_q + (\partial_\mu S^{\mu\nu}_{a(n)})\Delta_\nu\right) f_A\right] - \frac{\delta'(q^2 - m^2)}{q \cdot n} B^\mu \Box_{\mu\nu} \tilde{a}^\nu$$
$$+ \frac{\hbar}{2}\delta(q^2 - m^2)\epsilon^{\mu\nu\alpha\beta}\left[\Delta_\mu\left(\frac{n_\beta}{q \cdot n}\right)\left[(\Delta_\nu a_\alpha) + F_{\nu\alpha}\right] + \frac{n_\beta}{q \cdot n}\left((\partial_\mu F_{\rho\nu})(\partial^\rho_q a_\alpha) + \left[(\Delta_\nu a_\alpha) - F_{\rho\nu}(\partial^\rho_q a_\alpha)\right]\Delta_\mu\right)\right] f_A$$

E.o.M for $\mathscr{A}$

Polarization Probes Quark-Medium Interaction

- Fermion self-energy Σ_μ plays the role of vector potential

$$\zeta^i_{ind} \propto B^i + \epsilon^{ijk}\partial_j \Sigma_k[\delta f]$$

Also works by Shuo Fang, Shu Lin, Shi Pu, Naoki Yamamoto, and Di-Lun Yang

$\Sigma[\delta f] \sim$ spin carrier-medium coupling

$$\sim g^2 \int_{p'} D(p - p')\, \delta f_{\text{quark}} + \ldots \; .$$

Gluon distribution

Unique opportunity to measure quark self-energy in medium

Action for Spin Kinetic Theory

- We constructed Schwinger-Keldysh action with spin-dependent P.B. algebra

  Zonglin Mo, YY 2512.23960

  - Spinless part equivalent to hard thermal loop action with cleaner power counting structure
  - Spin part: starting point for analyzing the spin structure of plasma with RG (spin structure depending on resolution scales)
  - Action of fermionic theory in terms of Bosonic d.o.f. : hints for solving sign problem?
- Broad view: action for non-hydrodynamic with emergent symmetry

Spin and Dissipation

Dissipation in Spin Transport?

- Collisions generally generate entropy but how does "entropy" transfer between spin and orbital part?
  - Shear-induced polarization: shear effect is typically dissipative but spin is inherently quantum

Wang et al, 2507.15238;
Becattini and YY, in preparation.

$$\mathscr{A}^i \sim \underbrace{c_\sigma \epsilon^{ikj} \sigma^{jl}}_{\text{shear}} + \underbrace{c_\mu \epsilon^{ikj} \partial_j \mu_B}_{\text{baryonic Hall}}$$

- Entropy production for spin kinetic theory is needed!

$$\partial \cdot s[f, \mathscr{A}] = ?$$

Massless limit: Chen-Stephanov-Son, PRL 2015

Berry Connection and AM transfer in a Collision

- Scattering at the same position preserves orbital AM

$$\vec{x} \times (\vec{p}_1 + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') = 0$$

- Berry connection is exp. value of position operator in spin space

$$\langle u, s | (-i\frac{\partial}{\partial \vec{p}}) | u, s' \rangle = \overrightarrow{A}_{ss'}(\vec{p})$$

- Matrix-valued collision kernel $\hat{C}[\rho_{ss'}]$ depends on Berry connection modified distribution

Weickgenannt et al, PRD2021 ; Xin-Li Sheng et al, PRD 2022

$$\hat{\rho}_{ss'}(\vec{x}, \vec{p}) = mf(\vec{x}, \vec{p})\delta_{ss'} + \overrightarrow{\mathscr{A}}^{PRF}(\vec{x}, \vec{p}) \cdot \vec{\sigma}_{ss'} + m\overrightarrow{A}_{ss'} \cdot \vec{\partial} f$$

Quantum shift in position transfer AM between spin and orbital

Entropy Production with Spin

Jeremy Hansen, YY, in preparation

- We propose the following expression consistent with 2nd law

$$\partial \cdot s = \int_{\vec{p}} \mathrm{tr} A(\log A - \log B) \geq 0\,, \quad \Longrightarrow \quad \partial \cdot s[f, \mathscr{A}]$$

- determined by relative entropy between the two-particle state
  - after transition: $A = t(\hat{\rho}'_1 \otimes \hat{\rho}'_2)t^\dagger$; t is transition amplitude
  - imbalance between loss and gain: $B = -i(t^\dagger - t)(\hat{\rho}_1 \otimes \hat{\rho}_2)$
- Obtained from entropy current and kinetic theory

$$s^\mu = \int_{\vec{p}} \underbrace{p^\mu \mathrm{tr}(\hat{\rho}\log(\hat{\rho}) - \hat{\rho})}_{\text{Von Neumann}} + \text{Berry contribution}$$

Is Shear-Induced Polarization (SIP) Dissipative?

- Consider a fluid with shear only: $\delta\hat{\rho} = \delta f \delta_{ss'} + \delta\mathscr{A}\sigma_{ss'}$, evaluating entropy production rate $\partial \cdot s[f, \mathscr{A}]$
- Matching to 2nd-order hydro. expectation:

$$T\partial \cdot s = \frac{\eta}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} + \lambda_1\sigma_{\mu\nu}\sigma^{\mu\rho}\sigma_\rho^\nu$$

- Two possibilities concerning SIP's contribution to entropy production
  - SIP is non-dissipative
  - Or it contributes to λ_1

2nd order hydro. and spin structure

Second order hydrodynamic coefficients from kinetic theory

Mark Abraao York and Guy D. Moore

McGill University Dept. of Physics,
3600 rue University, Montréal QC H3A 2T8 Canada
(Dated: October 2008: v2 February 2009)

we only need to determine its contribution to the stress-energy tensor, which at leading order in coupling is determined in terms of f by

$$T_{\mu\nu}(x) = \sum_a \int_{\mathbf{p}} 2p_\mu p_\nu f(x,\mathbf{p})\,. \qquad (2.7)$$

Missing spin contributions?

- 2nd order hydro might be sensitive to spin distribution but has been poorly appreciated

Summary

Summary

- Spin observables probe in-medium properties of quarks/gluons
- Algebra method is powerful in studying spin dynamics
    - Extension: spin hydro. ; phonons; gluons
- Spin dissipation opens new window in understanding quark medium

Back-up

Symmetry, Algebra and Dynamics

E.o.M from algebra

- Elegant, non-linear, robust w.r.t coupling
  - Spin hydro. for superfluid Helium — *Dzyaloshinskii, Volovick 1978*
  - Hydro. of nuclear matter — *Son PRL2000*
  - Neutral charm mesons near threshold — *Bratten-Hammer PRL2022*
- Applying similar idea to kinetic theory

Chiral Perturbation Theory

- Pion fields $\pi_A(x)$: conjugate to symmetry generator

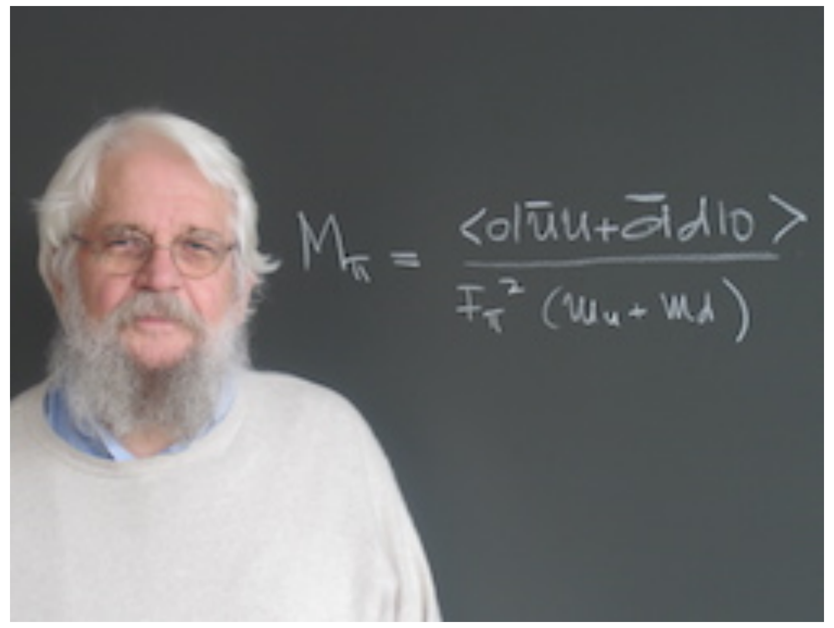
$$SU(2) \quad \Sigma \sim e^{i\pi_A \rho_A}$$

- Dynamics fixed by current algebra
- Algebra method has broad application

[Submitted on 25 Sep 1996]

Phonons as Goldstone Bosons

H. Leutwyler (University of Bern and CERN)



Leutwyler