Polarization distribution from nucleons to matter: some analytical perspectives

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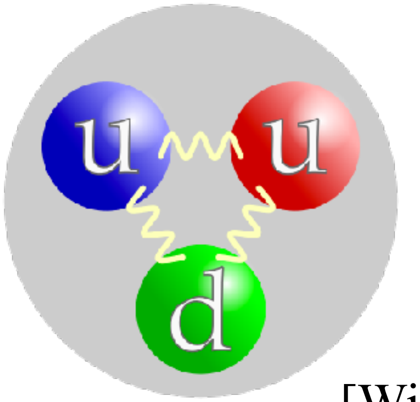
— Int. Symp. on Spin Polarization
in Relativistic Heavy Ion Collisions 2026 —

Polarized Nucleons

Polarized Proton

Proton has spin-1/2… that means:



[Wikipedia]

[BNL]

Jaffe-Manohar decomposition

~30% ~40%

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_{\rm can}^{q} + L_{\rm can}^{g}$$

based on the canonical EMT

Polarized Proton

$$\langle p', s'|T^{(\mu\nu)}(x)|p, s\rangle$$

$$=: \bar{u}(p', s')\left[A(t)\gamma^{(\mu}P^{\nu)} + B(t)\frac{P^{(\mu}\mathrm{i}\sigma^{\nu)\rho}\Delta_\rho}{2M} + D(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M}\right]u(p, s)\,\mathrm{e}^{\mathrm{i}(p'-p)x}$$

$J(0) = \frac{1}{2}\left[A(0) + B(0)\right] = \frac{1}{2}$ in Ji's decomposition

$$\langle p', s'|T^{[\mu\nu]}(x)|p, s\rangle = \frac{\mathrm{i}}{2}\Delta_\lambda\,\langle p', s'|S^{\lambda\mu\nu}(x)|p, s\rangle$$

$$=: \bar{u}(p', s')\left[\mathcal{G}(t)\frac{P^{[\mu}\mathrm{i}\sigma^{\nu]\rho}\Delta_\rho}{2M}\right]u(p, s)\,\mathrm{e}^{\mathrm{i}(p'-p)x}$$

cf. spin hydrodynamics

Spin form factor only in the canonical decomposition

Polarized Proton

$$T^{\mu\nu}_{A,\text{can}} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu A^\alpha)}\partial^\nu A^\alpha - g^{\mu\nu}\mathcal{L}_A = -F^\mu_{\ \alpha}\partial^\nu A^\alpha + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

$$T^{\mu\nu}_{\psi,\text{can}} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\partial^\nu \psi - g^{\mu\nu}\mathcal{L}_\psi = \bar{\psi}i\gamma^\mu\partial^\nu\psi - g^{\mu\nu}\bar{\psi}(i\gamma^\alpha D_\alpha - m)\psi$$

Belinfante improved form $T^{\mu\nu}_{\text{Bel}} := T^{\mu\nu}_{\text{can}} + \partial_\lambda K^{\lambda\mu\nu}_{\text{Bel}}$

$$K^{\lambda\mu\nu}_{\text{Bel}} = -F^{\lambda\mu}A^\nu + \frac{i}{4}\bar{\psi}\left(-i\varepsilon^{\lambda\mu\nu\rho}\gamma_5\gamma_\rho + 2g^{\mu\nu}\gamma^\lambda - 2g^{\lambda\nu}\gamma^\mu\right)\psi$$

$$\tilde{T}^{\mu\nu}_{A,\text{Bel}} = -F^\mu_{\ \alpha}F^{\nu\alpha} - \bar{\psi}\gamma^\mu eA^\nu\psi + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

$$\tilde{T}^{\mu\nu}_{\psi,\text{Bel}} = \bar{\psi}i\gamma^\mu\overleftrightarrow{\partial}^\nu\psi + \frac{1}{4}\varepsilon^{\mu\nu\lambda\rho}\partial_\lambda(\bar{\psi}\gamma_5\gamma_\rho\psi)$$

Polarized Proton

Analyzing the pseudogauge problem with gluons is technically complicated (with subtlety of gauge invariance).

Quark sector is better defined, but less nontrivial…

What if… we have a theory described by pions (ChEFT) with vector mesons ?   → pseudogauge problem!

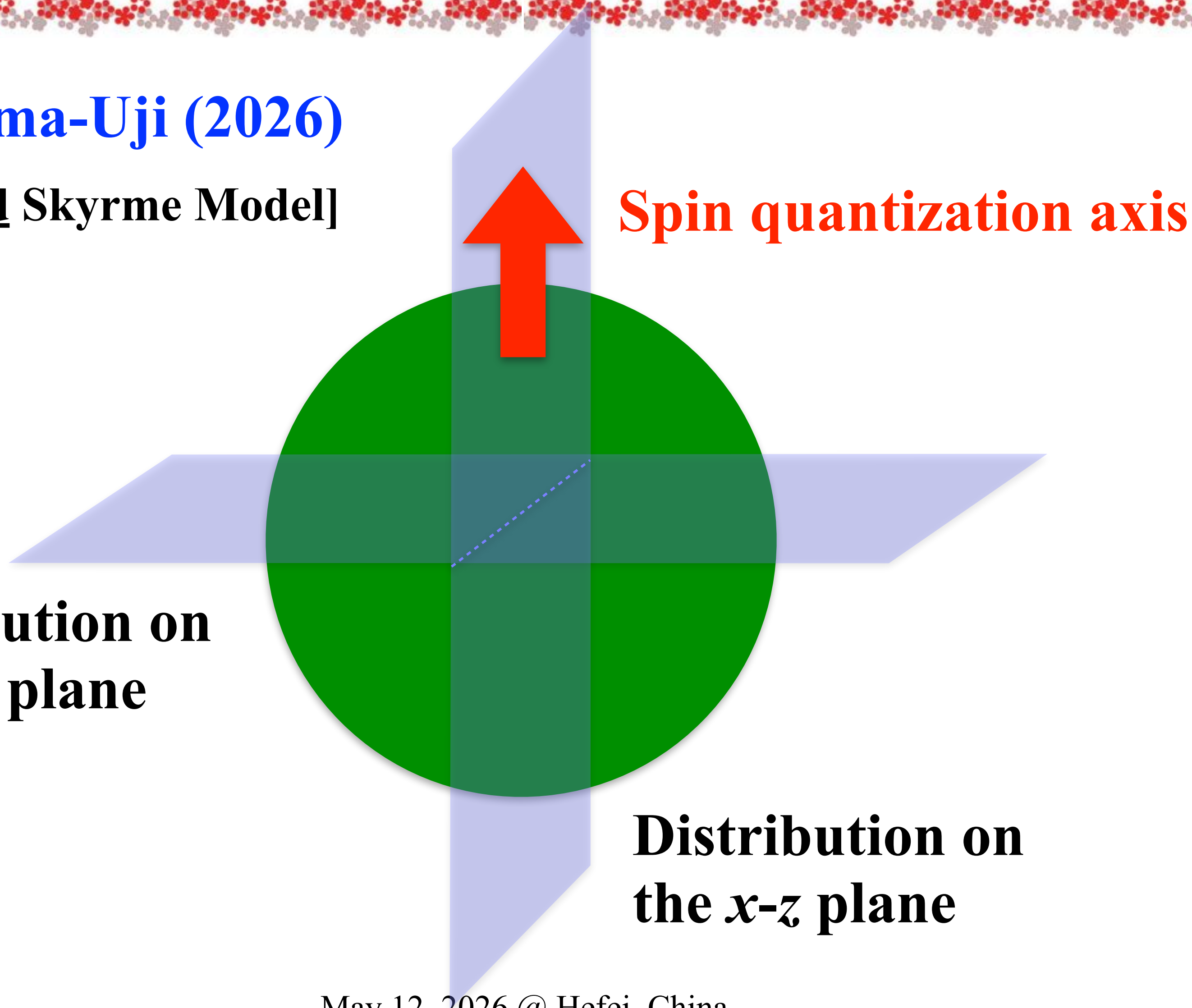
Adking-Nappi (1984)
Igarashi-Johmura-Kobayashi-Otsu-Sato-Sawada (1985)
Meissner-Kaiser-Wirzba-Weise (1986)
Meissner-Kaiser-Weise (1987)

No need to introduce an ad-hoc Skyrme term but the WZW term makes the solitonic baryon stable!

Polarized Proton

Fukushima-Uji (2026)

[<u>Vectorized</u> Skyrme Model]



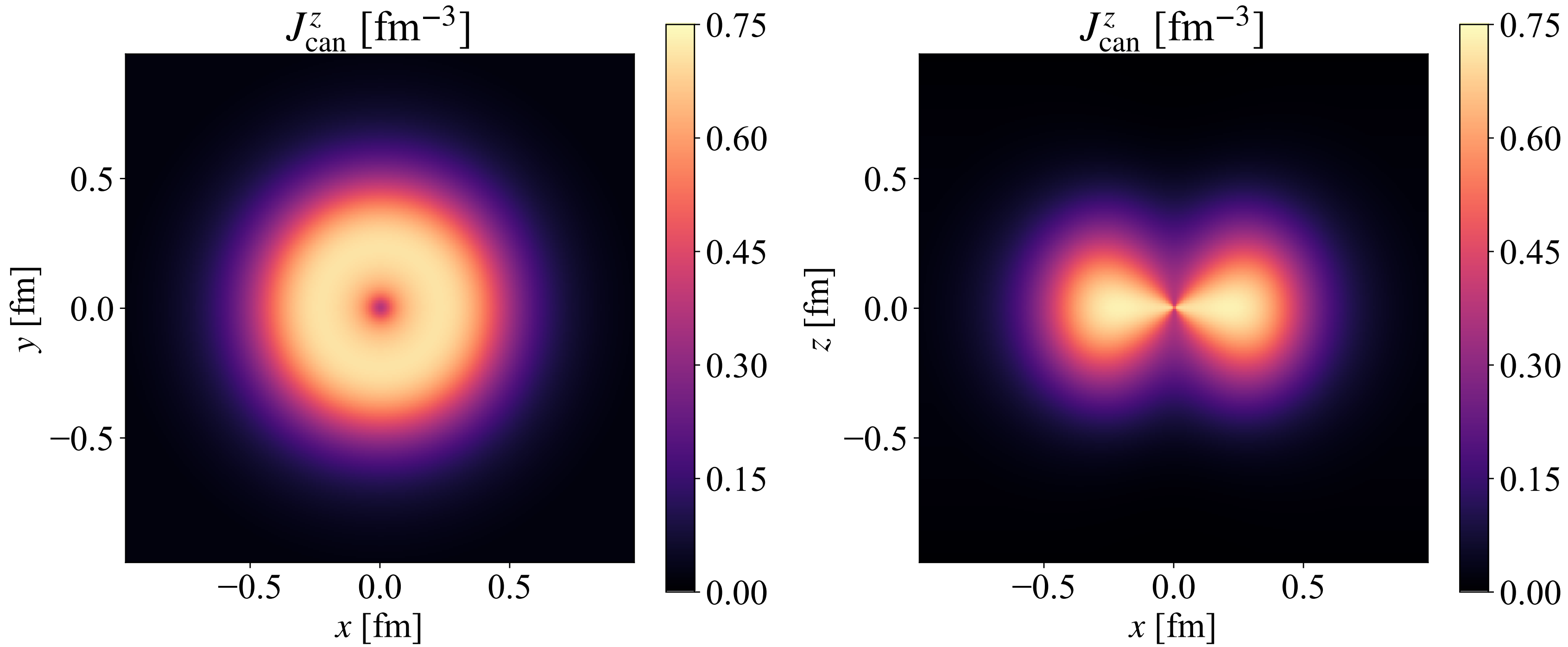
Distribution on the *x*-*y* plane

Distribution on the *x*-*z* plane

Polarized Proton

Fukushima-Uji (2026) $L_{\text{can}} \simeq 0.36$ $S_{\text{can}} \simeq 0.14$

"Canonical" total angular momentum distribution

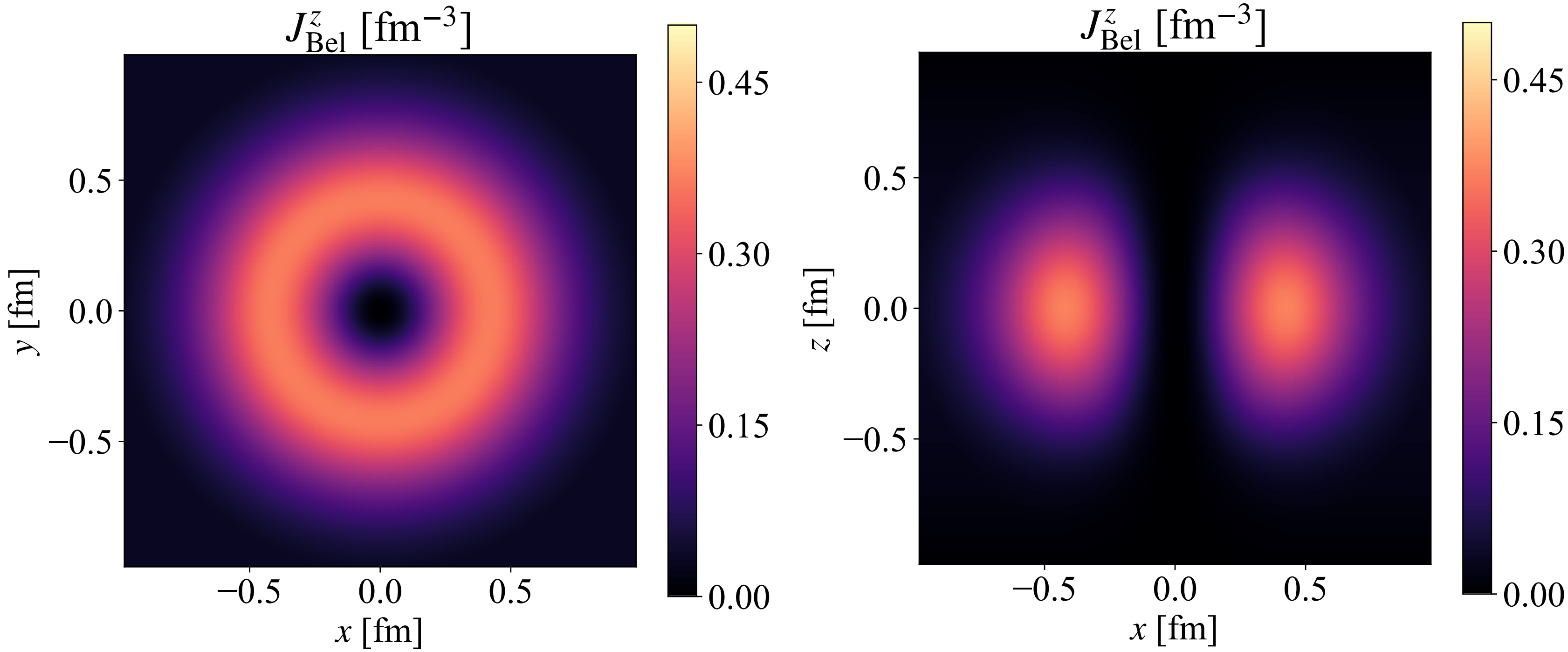


OAM suppressed at the center naturally… but…?

Polarized Proton

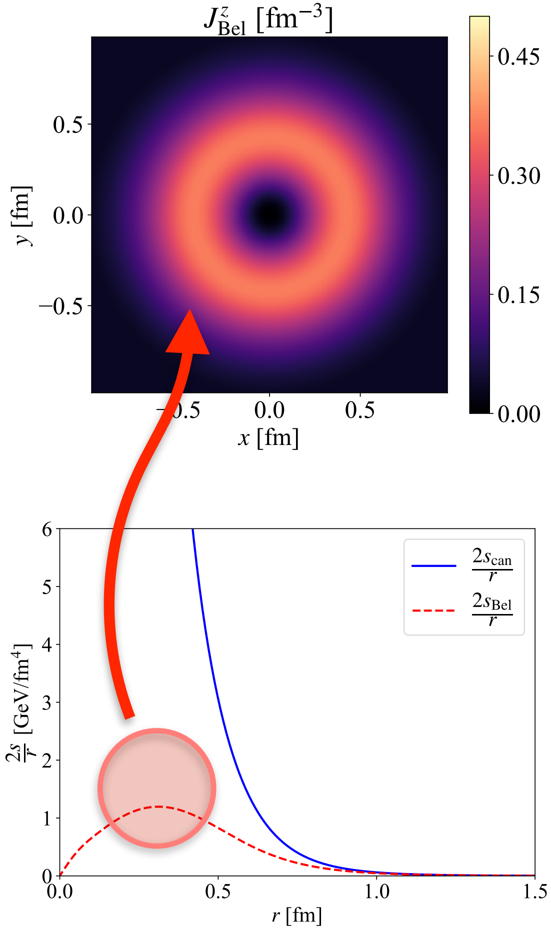
Fukushima-Uji (2026) $L_{\rm Bel} = J_{\rm Bel} = \frac{1}{2}$

"Belinfante" total angular momentum distribution



Why the "total" AM has a hole at the center? Puzzling…

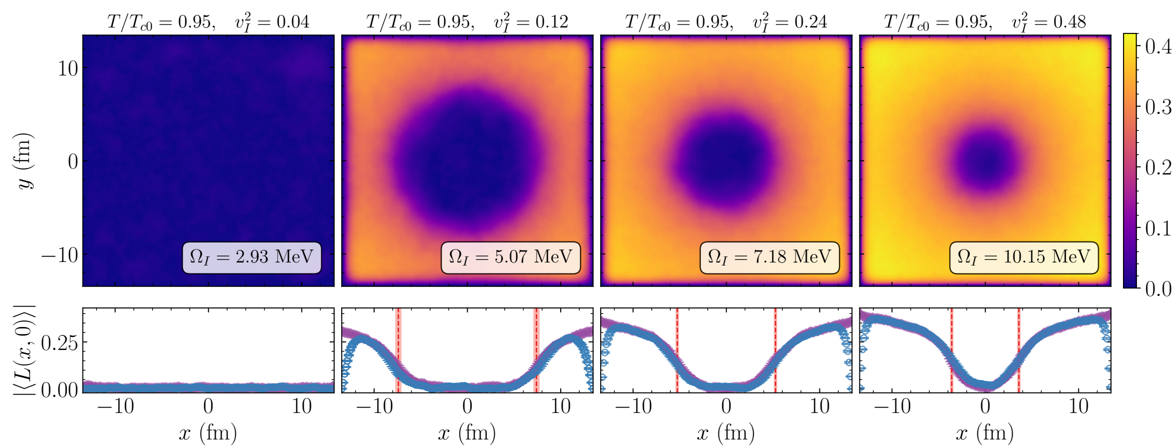
Polarized Proton



Fukushima-Uji (2025)

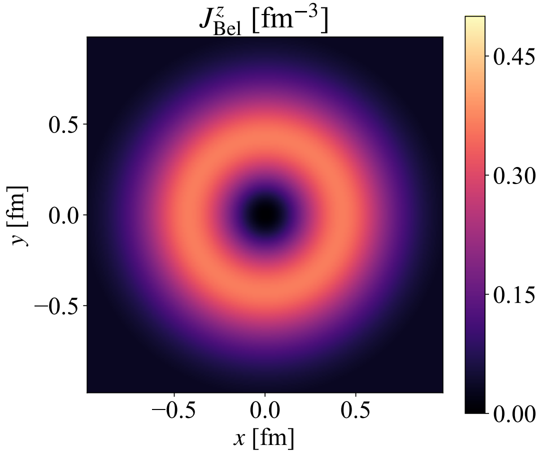
Has something to do with the "rotation controversy" ?

Braguta-Chernodub-Roenko (2023)



Larger AM density
~ Stronger confining (shear) force?

Polarized Proton



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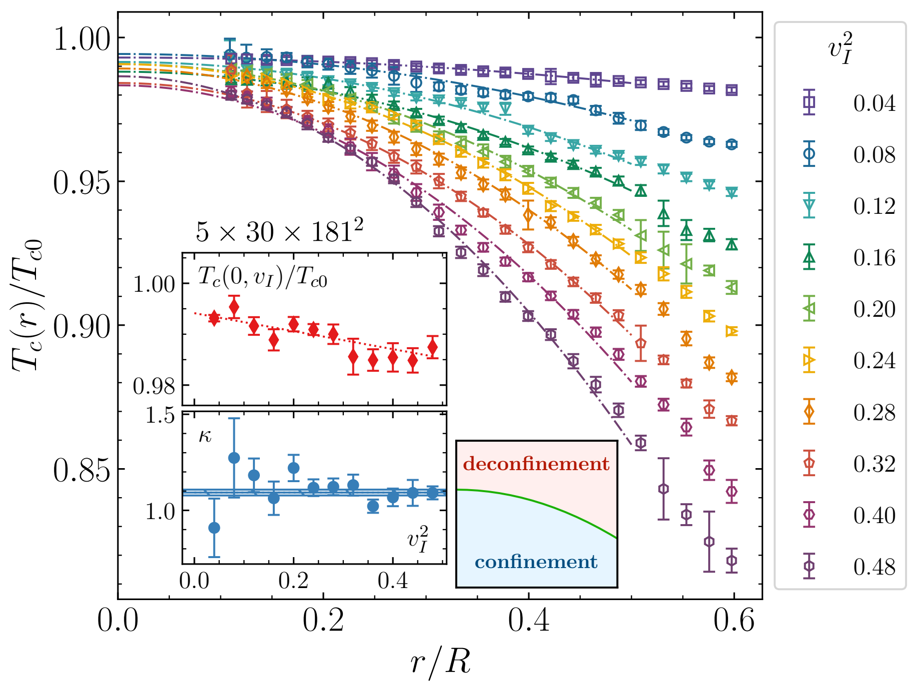
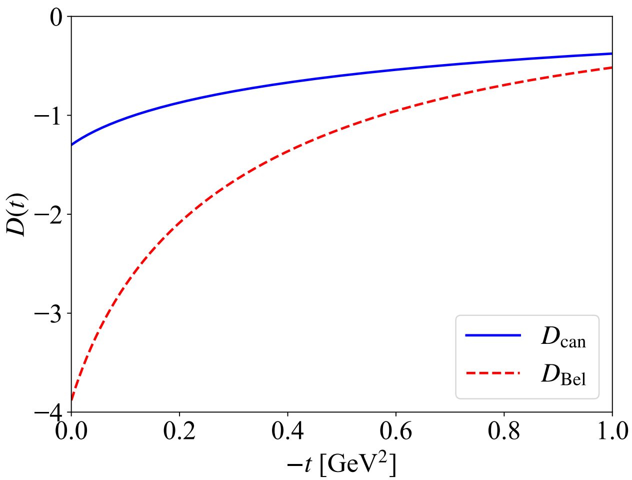


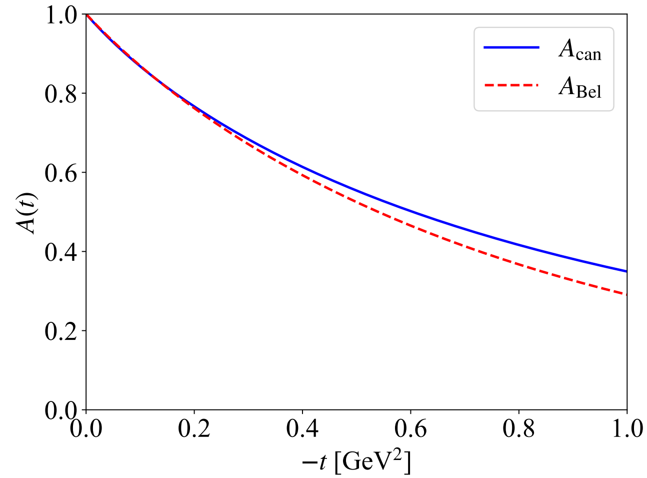
Rotation seems to have no direct coupling to spin at the center ?

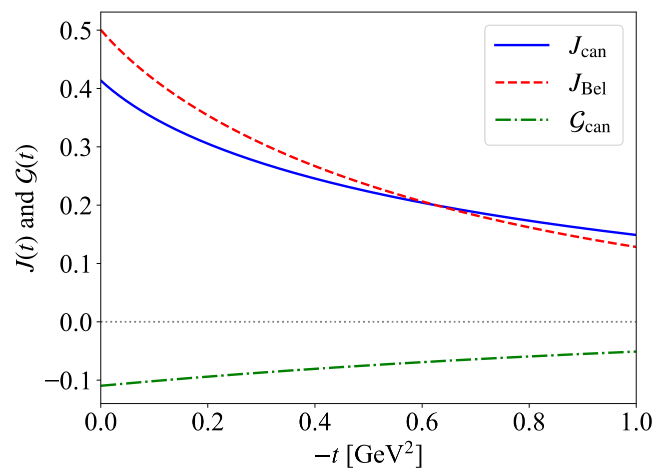
Spin is eaten by many body interactions ??

Polarized Proton

Fukushima-Uji (2026)



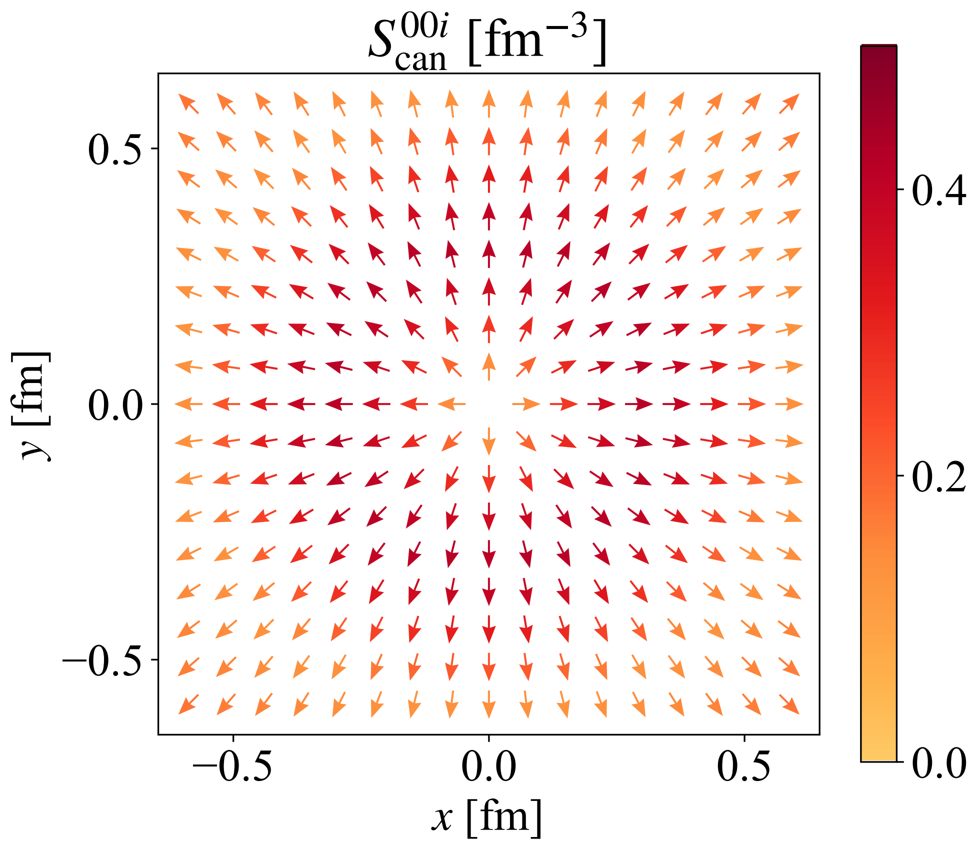




Form factors have milder dependence on pseudogauge except for the D-term…
Is this really physical?
See a recent work by Ji-Yang (2025)

No Ideas...

Fukushima-Uji (2026)



Nonzero spin boost charge density inside the polarized nucleon.
cf. $S^{++i} = 0$

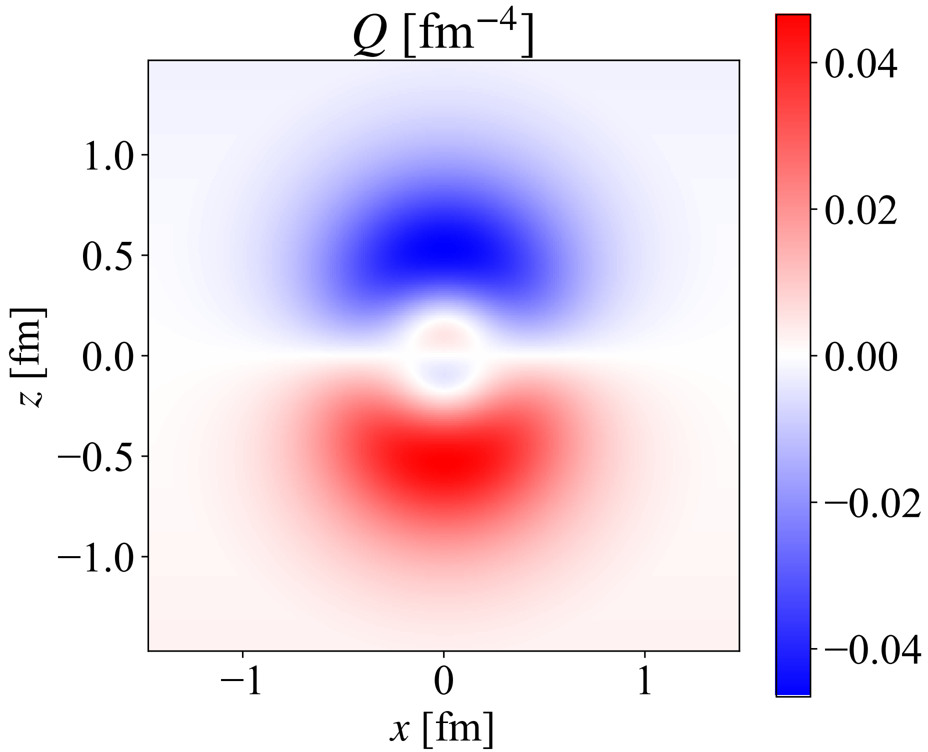
$\int d^3x L^{00i}$ **give the center of energy weighted by** T^{00}**.**

Makes contrast to the 3dof spin hydrodynamics?

Fang-Fukushima-Pu-Wang (2025)

Some New Ideas…

Fukushima-Uji (2026) coming soon!



Such a model is so useful for thinking experiments…

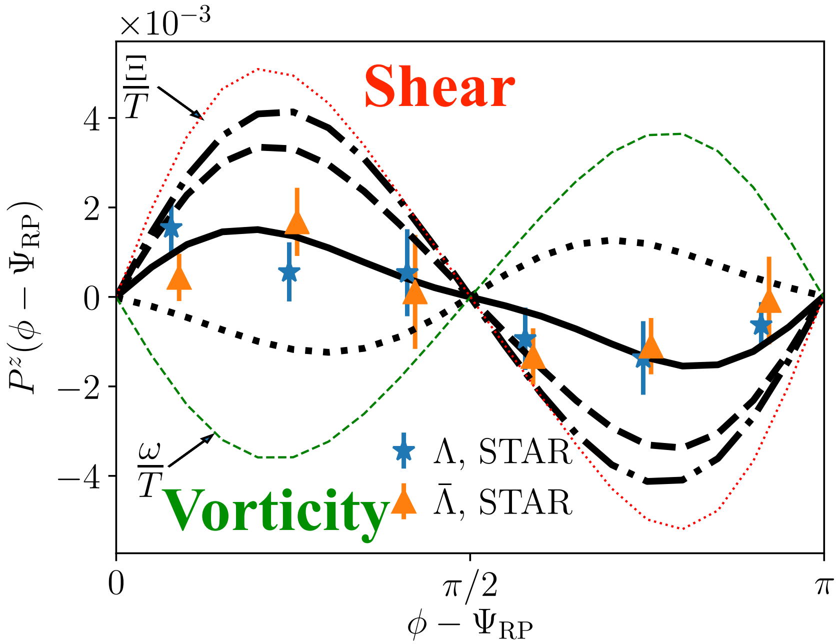
Topological dipole is induced by polarization!

CME or CVE counterpart at the nucleon level may be detectable at EIC.

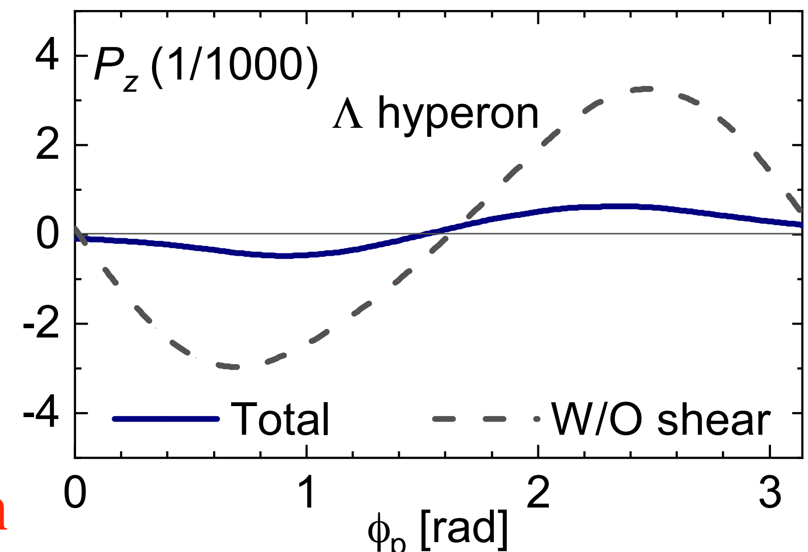
Polarized Matter

Local Polarization Sign Problem

Becattini-Buzzegoli-Palermo-Inghirami-Karpenko (2021)



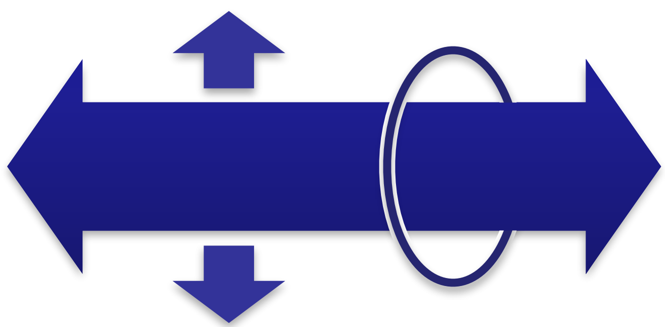
Importance of shear discussed by:
Liu-Yin (2021)
Fu-Liu-Pang-Song-Yin (2021)



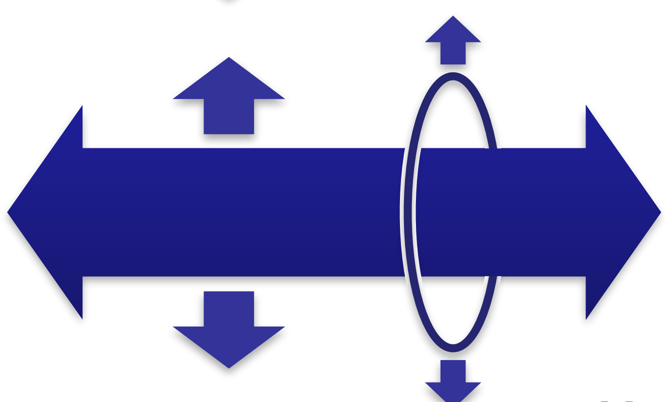
Why such nearly cancellation seen between vorticity and shear?

Anisotropic Gubser Flow

Bjorken flow: 1D conformal sol.
Infinite homogeneous in transverse

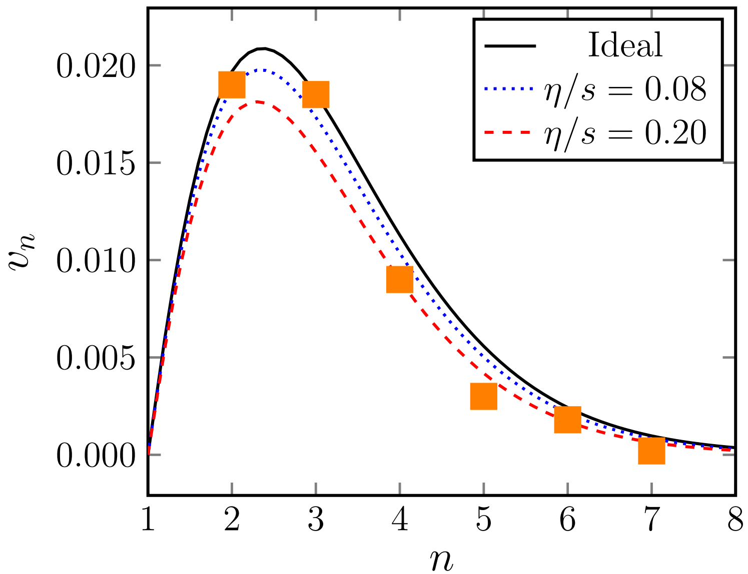


Gubser flow: 1+1D conformal sol.
Transversely localized and expanding
No elliptic anisotropy



Anisotropic Gubser flow:
Ellipticity introduced perturbatively

Anisotropic Gubser Flow



Higher harmonics from anisotropic Gubser flow

Hatta-Noronha-Torrieri-Xiao (2014)

Hatta-Monnai-Xiao (2015)

$$\varepsilon \;\to\; \varepsilon\left(1 + \epsilon_n A \cos n\phi\right)$$

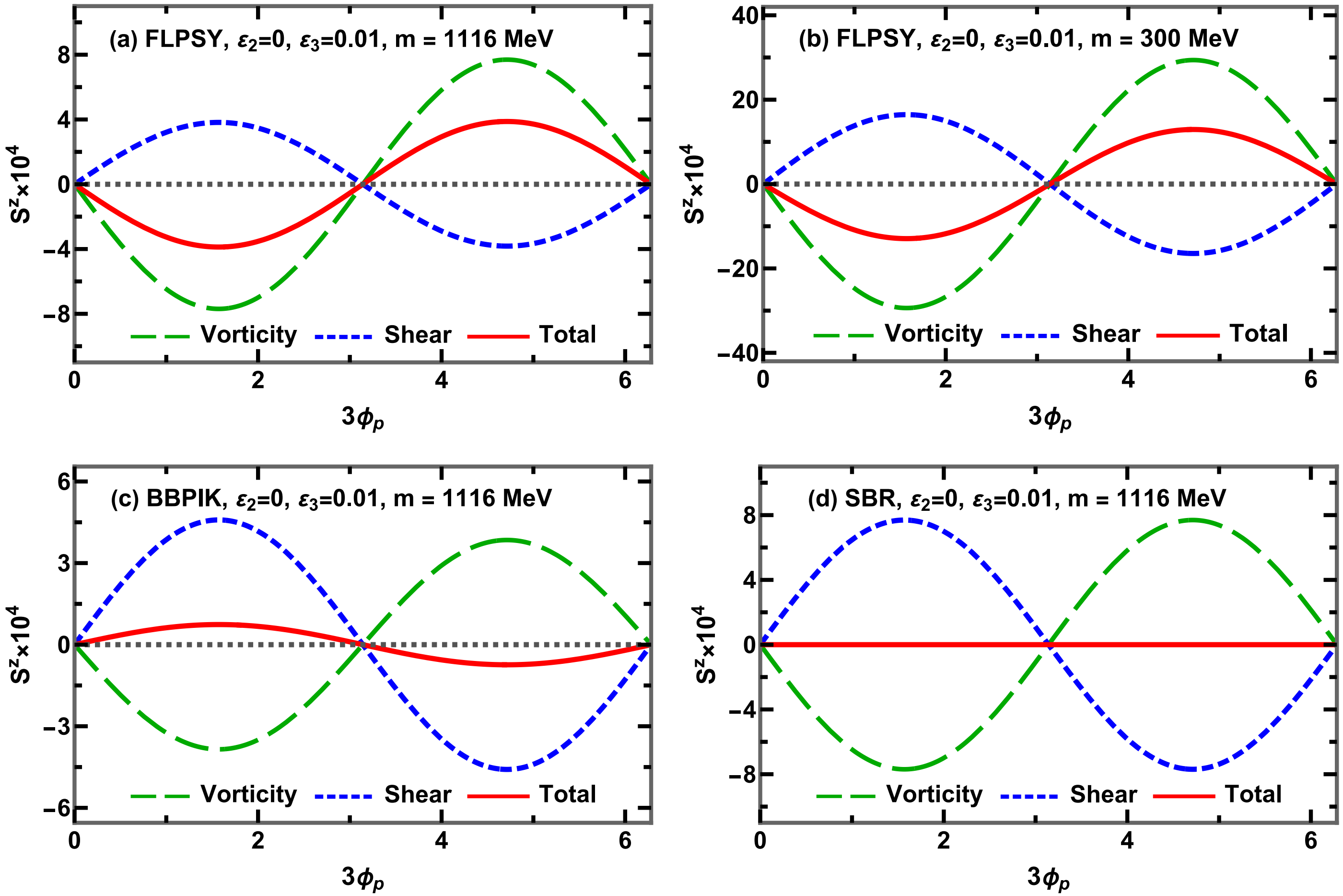
$$u_\perp \;\to\; u_\perp + \epsilon_n B \cos n\phi$$

A, *B* determined by (viscous) hydrodynamical equations

Let us follow this prescription with dominant perturbation at *n*=2 and small correction at *n*=3.

Local Polarization

Cao-Fukushima-Pu-Wang (2026) coming soon!



Local Polarization

Spin polarization from the thermal vorticity

$$S^\mu_\varpi(p) = \frac{-1}{8m\mathcal{N}(p)} \int_{\Sigma_f} [\mathrm{d}\Sigma]\epsilon^{\mu\nu\rho\sigma} p_\nu \varpi_{\rho\sigma}$$

$[d\Sigma] = d\Sigma \cdot p\, n_F(1 - n_F)$

Spin polarization from the thermal shear

$$S^\mu_\xi(p) = \frac{-1}{4m\mathcal{N}(p)} \int_{\Sigma_f} [\mathrm{d}\Sigma]\epsilon^{\mu\nu\rho\sigma} \frac{p_\nu n_\rho}{p \cdot n} p^\lambda \xi_{\sigma\lambda}$$

FLPSY: $n^\mu \propto u^\mu$

BBPIK: $n^\mu \propto (1,0,0,0)$ in (t, x, y, z) dropping $\partial^\mu \beta$

SBR: $n^\mu \perp \Sigma_f$ [$n^\mu \propto \partial^\mu \beta$ for isothermal Σ_f]

Sheng-Becattini-Roselli (2025)

Local Polarization

BBPIK

$$\varpi_{\mu\nu} \to \varpi^K_{\mu\nu} = -\frac{1}{2}\beta(\partial_\mu u_\nu - \partial_\nu u_\mu)$$

$$\xi_{\mu\nu} \to \xi^K_{\mu\nu} = \frac{1}{2}\beta(\partial_\mu u_\nu + \partial_\nu u_\mu)$$

With anisotropic Gubser flow giving $T(\rho)$:

$$\varpi_{\mu\nu} = -\frac{\beta}{2}(\partial_\rho u_\sigma - \partial_\sigma u_\rho) - \frac{1}{2}(u_\sigma \partial_\rho \beta - u_\rho \partial_\sigma \beta)$$

$$\simeq -\beta(\partial_\rho u_\sigma - \partial_\sigma u_\rho) + (\text{higher order})$$

***T*-gradient not negligible but simply doubled.**

Local Polarization

BBPIK

Perturbed Gubser flow has no non-accel. terms.

$$\frac{1}{2}\epsilon^{z\nu\rho\sigma}p_\nu \varpi^K_{\rho\sigma} = p_x \varpi^K_{ty} - p_y \varpi^K_{tx} + \ldots$$

Nonzero elliptic flows not necessarily lead to nonzero vorticities…

$$\epsilon^{z\nu\rho\sigma}\frac{p_\nu n_\rho}{p\cdot n}p^\lambda \xi^K_{\sigma\lambda} = p_x \xi^K_{ty} - p_y \xi^K_{tx} + \ldots$$

Only remaining parts

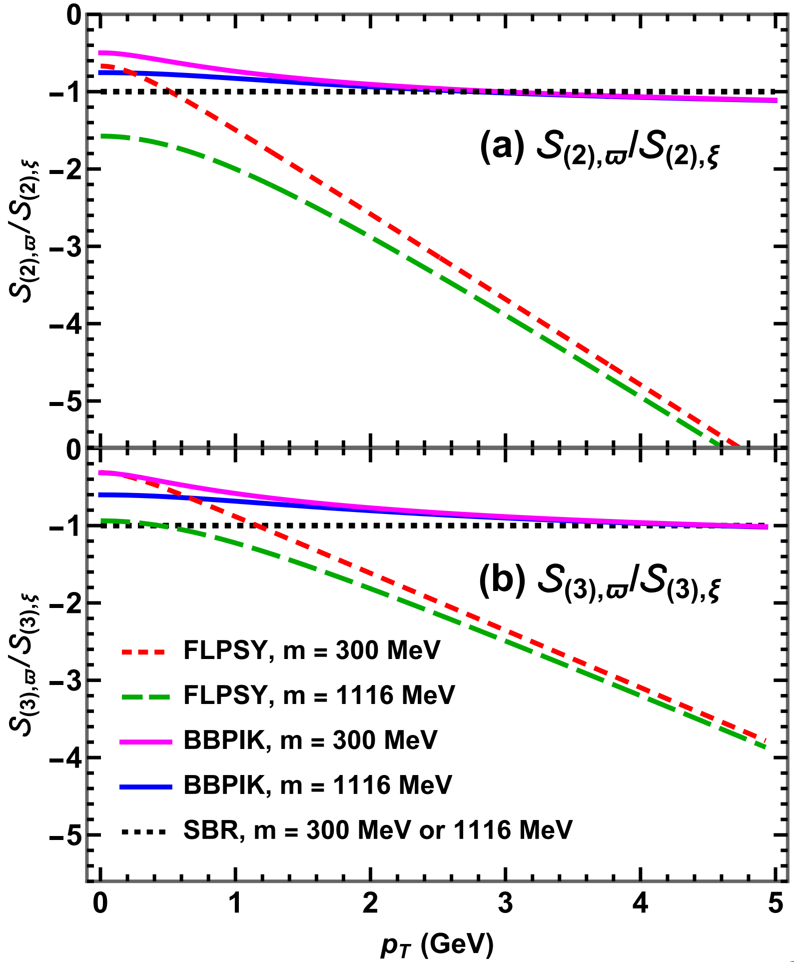
Accelerations $\partial_t u_x$, $\partial_t u_y$ exactly cancel out!

SBR

Choice of $n^\mu \propto \partial^\mu \beta$ makes cancellation between the T-gradient terms of $\varpi_{\mu\nu}$ and $\xi^{\mu\nu}$ justifying the BBPIK replacement. Why no remaining parts???

Local Polarization

Cao-Fukushima-Pu-Wang (2026) coming soon!



To reproduce the sign inverted behavior, small and positive would be preferable.

Anisotropic Gubser flow is a (over)simplified model but seems to grasp the nature of previous studies.

Summary

- **Our theory technologies can be applied to the spin polarization problem of nucleons and the GPD (moments) measurable at EIC.**
- **Spin-related observables are calculable in hadronic models and in lattice-QCD simulations, including topological contents inside the nucleons.**
- **Simple analytical modeling is useful to understand physical mechanisms underlying the local spin polarization phenomena. Anisotropic Gubser flow gives an intuitive insight!**