Baryon Construction with η' Meson Field

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Quantum Chromodynamics (QCD)

- Lagrangian of QCD

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f \right) \psi_f - \frac{1}{2g^2} G^a_{\mu\nu} G^{\mu\nu,a},$$

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2} \mathrm{tr}\, \tilde{G}^{\mu\nu} G_{\mu\nu} = \frac{\theta}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \partial_\mu \mathrm{tr} \left(G_\nu \partial_\rho G_\sigma - \frac{2i}{3} G_\nu G_\rho G_\sigma \right).$$

- Properties: asymptotic freedom, color confinement, Yang-Mills mass gap...
- The global symmetry of QCD with N_f massless quark flavors

$$U(N_f) \times U(N_f) \longleftrightarrow U_V(1) \times U_A(1) \times SU_V(N_f) \times SU_A(N_f).$$

- In the quantum level, QCD has axial anomaly η' and chiral symmetry breaking $\pi, K \ldots$

Chiral Effective Field Theory (χEFT)

- The symmetry of QCD with N_f massless quark flavors

$$U(N_f) \times U(N_f) \longleftrightarrow U_V(1) \times U_A(1) \times SU_V(N_f) \times SU_A(N_f).$$

- If energy scale $< \Lambda_\chi \sim 1\mathrm{GeV}$, the chiral symmetry $SU_A(N_f)$ suffers spontaneously symmetry breaking, then generating Nambu-Goldstone (NB) bosons, such as $\pi, K, \ldots$

$$\mathcal{L}_\chi = \frac{f_\pi^2}{4}\mathrm{tr}(\partial^\mu U^\dagger \partial_\mu U) + \cdots \quad \text{with} \quad U(x) = \exp(2i\pi^a T^a/f_\pi) \in SU(N_f).$$

- The Wess-Zumino-Witten (WZW) term $N_f \geq 3$

$$S_{\mathrm{WZW}} = -\frac{N_c}{240\pi^2}\int_D d^5y \; i\epsilon^{\mu\nu\rho\sigma\tau}\mathrm{tr}\left(U^\dagger\partial_\mu U U^\dagger\partial_\nu U U^\dagger\partial_\rho U U^\dagger\partial_\sigma U U^\dagger\partial_\tau U\right).$$

[S. Weinberg, Wess, Zumino, E. Witten, K. C. Chou, K. Wu, et al.]

Skyrme model

- Skyrme model

$$\mathcal{L}_{\mathsf{Sky}} = \frac{f_\pi^2}{4}\,\mathrm{tr}(\partial^\mu U^\dagger \partial_\mu U) + \frac{1}{32e^2}\,\mathrm{tr}([U^\dagger\partial_\mu U, U^\dagger\partial_\nu U][U^\dagger\partial^\mu U, U^\dagger\partial^\nu U]).$$

- Topological soliton solution $U(\mathbf{x}) : \mathbb{R}^3 \to SU(N_f), N_f \geq 2$ labeled by Homotopy group class:

$$\pi_3(SU(N_f)) = \mathbb{Z}, \quad \longleftrightarrow \quad B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\sigma\rho}\,\mathrm{tr}(U^\dagger\partial_\nu U U^\dagger\partial_\sigma U U^\dagger\partial_\rho U).$$

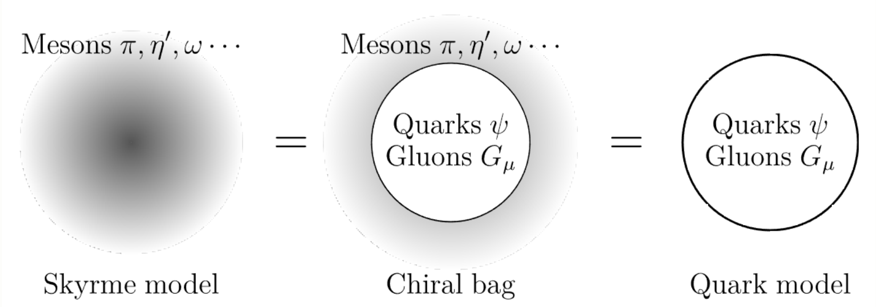
- Skyrmion shares the same N_c behavior with baryon on mass, size, amplitudes with meson, and the topological current is identified with baryon current by symmetry and anomaly matching or direct fermion number calculation.

[Callan, Coleman, Wess, Zumino, S. Weinberg, Skyrme and et al.]

Chiral Bag Model

- Chiral bag model

$$\mathcal{L}_{\mathrm{CB}} = \left(\bar{\psi} i \partial\!\!\!/ \psi - B\right) \theta_V - \frac{1}{2} \bar{\psi} \exp(\mathrm{i} \boldsymbol{\sigma} \cdot \boldsymbol{\pi} \gamma^5 / f_\pi) \psi \delta_V + \theta_{\bar{V}} \mathcal{L}_{\mathsf{Sky}}.$$



- Cheshire Cat Principle (CCP): The physics is independent of the size of bag.

[Inoue, Chodos, Hosaka, Thomas, et al.]

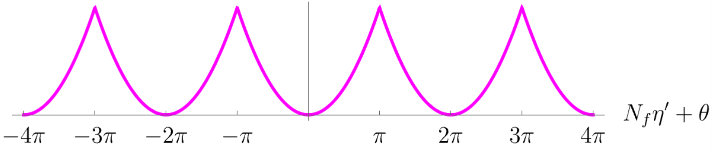
One-flavor Baryon

- Skyrmion: for $N_f \geq 2$ case $\pi_3(SU(N_f)) = \mathbb{Z}$, non-trivial topological soliton structure of Nambu-Goldstone bosons exists.
- For $N_f = 1$ case, no Nambu-Goldstone boson exists, no non-trivial topological soliton structure exists.
- Even under the both chiral and large N_c limits, one can get massless η' meson, but homotopy group $\pi_3(U_A(1)) = 0$ is trivial.
- Homotopy group $\pi_1(U_A(1)) = \mathbb{Z}$ is non-trivial, use it to construct $N_f = 1$ baryon with high spin $N_c/2$.

Baryons as Quantum Hall Droplets

- In the large N_c limit, the low-energy effective theory of $\eta' \simeq \eta' + 2\pi$

$$\mathcal{L}^{\text{eff}}_{\eta'} = \frac{N_f\, f_\pi^{\,2}}{8}\partial_\mu \eta' \partial^\mu \eta' + \frac{f_\pi^{\,2}}{8N_f} m^2_{\eta'} \min_{n\in\mathbb{Z}} \left(N_f \eta' + \theta - 2\pi n\right)^2 .$$



- The η' domain wall supports Level -1 and Rank N_c Chern-Simons (CS) theory $\bar{a} \in \mathfrak{su}(N_c)$, which is dual to a abelian CS theory $a \in \mathfrak{u}(1)$ by Level/Rank duality

$$\frac{-1}{4\pi}\int_{\mathcal{M}_3} \text{Tr}\left(\bar{a}\wedge \mathrm{d}\bar{a} - \frac{2i}{3}\bar{a}^3\right), \quad \longleftrightarrow \quad \int_{\mathcal{M}_3} \frac{N_c}{4\pi} a\wedge \mathrm{d}a + \frac{1}{2\pi} a \wedge \mathrm{d}A^B.$$

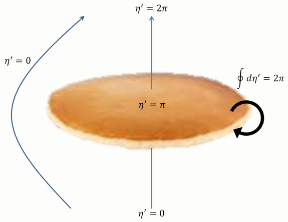
$$SU(N_c)_{-1}, \quad \longleftrightarrow \quad U(1)_{N_c}.$$

[E. Witten, G. Veneziano, Z. Komargodski, N. Seiberg, D. Gaiotto, A. Kapustin and et al.]

Baryons as Quantum Hall Droplets

- One can cut a patch from the domain wall to get a sheet. The sheet actually is a droplet realizing the fractional quantum hall effect with filling number $1/N_c$

$$U(1)_{N_c} : \int_{\mathcal{M}_3} \frac{N_c}{4\pi} a \wedge \mathrm{d}a, \quad \longrightarrow \quad \frac{N_c}{4\pi} \int \mathrm{d}x\mathrm{d}t \left(\partial_t \omega \partial_x \omega - v(\partial_x \omega)^2 \right).$$



$$B = \frac{1}{2\pi} \int \partial_\varphi \omega.$$

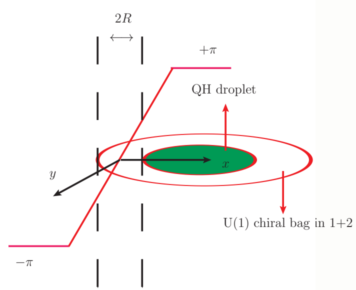
- To maintain the gauge invariance of Chern-Simons theory on spacetime manifold with boundary. A boundary and chiral mode ω emerges, carrying one baryon number.
- Two point function shows the spin of droplet

$$\langle e^{iN\omega}(x) e^{-iN\omega}(0) \rangle \sim x^{-N_c} \longrightarrow \text{spin} = \frac{N_c}{2}.$$

[Z. Komargodski, N. Seiberg, D. Gaiotto, A. Kapustin and et al.]

Baryons as Chiral Bags by CCP

- The boundary of Quantum Hall Droplet can be extended as a chiral bag in a 1+2 dimensional strip using the Cheshire cat principle (CCP).
- The chiral bag carries quark field with charge 1, and is clouded by η' field. In the limit of the zero bag radius is a quantum Hall droplet.



- The quark charge, which interacts with $SU(N_c)$ gluon field inside quantum Hall droplet, can leak through the anomaly. When the radius is zero, after bosonizaton, an emergent action on the vortex line is achieved

$$\frac{N_c}{4\pi}\int_{2+1} a \wedge \mathrm{d}a.$$

[Yong-Liang Ma, Maciej A. Nowak, Mannque Rho, and Ismail Zahed et al.]

Chern-Simons-Higgs Theory

- For one-flavor case, with quark field taken into account, a complete duality on the η' domain wall

$$SU(N_c)_{-1} + \psi \quad \longleftrightarrow \quad U(1)_{N_c} + \phi,$$

  the $U(1)$ baryon number symmetry is gauged and becomes an abelian gauge theory on the right hand.
- For one-flavor case, the theory $U(1)_{N_c} + \phi$ on η' domain wall should be

$$\mathcal{L}_A[a,\phi] = |\partial_\mu\phi - \mathrm{i}a_\mu\phi|^2 - V(\phi^*\phi) + \frac{N_c}{4\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho,$$

  with Higgs-type potential

$$V(\phi^*\phi) = N_c\sum_{I=2}^{\infty} c_I(\frac{\phi^*\phi}{N_c} - v^2)^I, v > 0,$$

  which ensures a non-zero vacuum expectation value of $\phi^*\phi/N_c = v^2$ with $v > 0$.

[PS Hsin, N Seiberg, SG Naculich, HJ Schnitzer, SG Naculich, HA Riggs, HJ Schnitzer et al.]

Vortices of Chern-Simons-Higgs Theory

- The non-trivial vacuum $\phi^*\phi = N_c v^2$, ensures the theory has non-trivial topological soliton solutions labeled with winding number $n \in \mathbb{N}$

$$\phi(\mathbf{r}) = \sqrt{N_c}\mathrm{e}^{in\theta} f(r),\ a_0(\mathbf{r}) = A_0(r),\ \mathbf{a}(\mathbf{r}) = \frac{A(r)}{r}(\sin\theta, -\cos\theta).$$

- One can observe $\mathbf{a}(\mathbf{r}) \neq 0$, In essence, the Chern-Simons term attaches flux to vortices

$$N_c \frac{1}{2\pi} f_{12} = -\phi^\dagger \phi.$$

- The Chern-Simons term also bring a topological current

$$j^\mu = \frac{N_c}{2\pi}\epsilon^{\mu\nu\rho}\partial_\nu a_\rho,\ \mathcal{L}_A \supset j^\mu A^B_\mu.$$

[DP Jatkar, A Khare R. Banerjee, P. Mukherjee et al.]

Vortices are Anyons

- The vortex solutions of this theory carry flux quanta and topological charge

$$\Phi = \int \epsilon^{0\nu\rho}\partial_\nu a_\rho dxdy = 2\pi n, \quad \longrightarrow \quad Q = \int j^{\,0} dxdy = nN_c.$$

- The object with both charge and flux is famous as anyon. If two anyons with charge q and flux Φ exchange, the wave-functions generates an Aharonov-Bohm phase and induces spin

$$\psi(\boldsymbol{r}_2, \boldsymbol{r}_2) = e^{2\pi s}\psi(\boldsymbol{r}_2, \boldsymbol{r}_2), \quad \longrightarrow \quad s = \frac{q\Phi}{4\pi}.$$

- These vortices of Chern-Simons-Higgs theory have spin

$$s = \frac{Q\Phi}{4\pi} = \frac{N_c}{2} n^2, n \in \mathbb{Z}.$$

[F. Wilczek, A. Zee, Laughlin, Jain, S. Rao et al.]

Baryons as Vortices

- The spin of simplest vortex with $n = \pm 1$ is

$$s = \frac{N_c}{2}.$$

- The ϕ become edge mode of the Chern-Simons theory far away the vortex center, the topological charge can be defined as baryon number

$$\text{Edge mode} : a_\mu \sim -i\phi^{-1}\partial_\mu \phi, \quad \longrightarrow \quad \text{baryon number} : B = \frac{Q}{N_c}.$$

- The vortices with $n = \pm 1$ can be seen as (anti)baryons, and $|n| \geq 2$ as multi-baryon structures, just like skyrmion in $3 + 1$ dimension.
- Comparing with the quantum Hall droplet picture, the vortices have natural boundary for the ϕ and a_μ get mass from Higgs potential.

Large N_c Behavior

- In the large N_c limit, the quark density of every color $\phi_c^*\phi_c = \phi^*\phi/N_c$ should keep finite. The Lagrangian density can be expressed as

$$\mathcal{L}_A = N_c\left[|\partial_\mu\phi_c - \mathrm{i}a_\mu\phi_c|^2 - V_c(\phi_c^*\phi_c) + \frac{1}{4\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho\right],$$

where

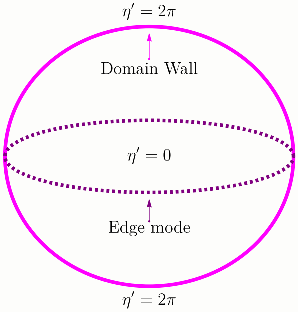
$$V_c(\phi_c^*\phi_c) = V(\phi^*\phi)/N_c = \sum_{I=2}^{\infty} c_I\left(\frac{\phi^*\phi}{N_c} - v^2\right)^I = \sum_{I=2}^{\infty} c_I(\phi_c^*\phi_c - v^2)^I, v > 0.$$

- All large N_c behavior of the vortices are the same as those of baryons

$$\begin{aligned}
\text{Size} &\sim \max\{m_\phi^{-1}, m_{a_\mu}^{-1}\} \sim N_c^0;\\
\text{Energy(Mass)} &\sim \mathcal{H}_A \times \text{Size}^2 \sim \mathcal{L}_A \times \text{Size}^2 \sim N_c;\\
\text{Vortices scattering } \mathcal{M} &\sim e^{S_A} \sim S_A \sim \mathcal{L}_A \times \text{Size}^2 \times \text{Time} \sim N_c.
\end{aligned}$$

From Domain Wall to Chiral Bag

- To construct one-flavor baryon in 3+1 dimension, We can glue two η' domain wall patches into a sphere. Inside the sphere $\eta' = 0$ and outside $\eta' = 2\pi$.



- The sphere actually is the boundary of a chiral bag.
- The chiral bag model combines MIT bag model and Skyrme model: a MIT bag surrounded by a cloud of chiral mesons, such as pions. Quark-meson field interactions occur on the bag surface.
- Now, according to Wu-Yang description of monopole, the edge mode measures the quantity of monopoles inside the bag, indicating monopoles condensate causes confinement.

Monopoles Condensation Causes Confinement

- Assuming N_c monopoles condensate inside the bag causes confinement, which is described by the Abelian-Higgs model

$$\Lambda^2 \int |(\mathrm{d} - iN_c\tilde{A})\Phi|^2, \quad \xrightarrow{\langle\Phi\rangle = ve^{i\varphi}} \quad \Lambda^2 \int |(\mathrm{d}\varphi - iN_c\tilde{A})|^2.$$

- In low-energy limit $\Lambda^2 \to \infty$, this is a discrete $\mathbb{Z}_{N_c}$ gauge theory

$$\tilde{A} = \frac{\mathrm{d}\varphi}{N_c}, \quad \longrightarrow \quad \exp\left\{ i \oint_\gamma \tilde{A} \right\} = e^{2\pi i n/N_c} \in \mathbb{Z}_{N_c}.$$

- Dualize the magnetic gauge field $\tilde{A}$ to return the usual electric gauge field A,

$$\Lambda^2 \int |(\mathrm{d}\varphi - iN_c\tilde{A})|^2, \quad \longrightarrow \quad \frac{i}{2\pi} \int \mathrm{d}\tilde{A} \wedge (\mathrm{d}A - N_c B).$$

This has functional form of a BF-theory in and where B is a 2-form field.

[GT Horowitz, M Dierigl, A Pritzel, T Banks, N Seiberg, S Gukov, A Kapustin et al.]

Bag Surface

- Under the principle of gauge invaraince, one can take the theta term and vacuum branches into account

$$S = \frac{i}{2\pi}\int \mathrm{d}\tilde{A}\wedge(A - N_c B) + \frac{N_c\theta}{4\pi}B\wedge B + \frac{N_c N_f \eta'}{4\pi}B\wedge B - \frac{N_c n}{2}B\wedge B,$$

$$\frac{1}{2\pi}\oint \mathrm{d}\lambda \in \mathbb{Z},\ B\to B+\mathrm{d}\lambda,\ A\to A+N_c\lambda,\ \tilde{A}\to\tilde{A}+\frac{\theta+N_f\,\eta'}{2\pi}\lambda.$$

- This action possesses a 1-form gauge symmetry parametrized by a 1-form λ, which fulfills the quantization condition for any surface Σ, The charge objects of 1-form symmetry is Wilson line and corresponding conserved charge is magnetic flux.
- Generally, if set η'_{in} inside the bag and η'_{Σ} outside, the bag surface is an η' domain wall. The effective action is not invariant under 1-form transformation when cross the bag surface

$$\Delta S_{\mathrm{surface}} = -\left(\frac{\eta'_{\Sigma}-\eta'_{\mathrm{in}}}{2\pi}\right)\times\frac{i}{4\pi}\int_{\Sigma} N_c(2\lambda\wedge B+\lambda\wedge\mathrm{d}\lambda).$$

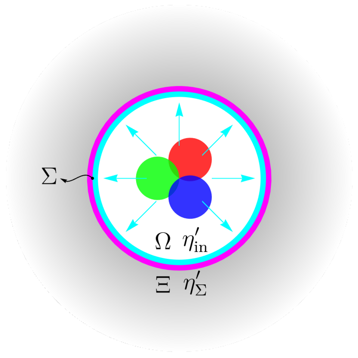
[GT Horowitz, M Dierigl, A Pritzel, T Banks, N Seiberg, S Gukov, A Kapustin et al.]

Topological Action on the Bag Surface

- To maintain gauge invariance, it is predicted there exist an $U(N_f)_{-N_c}$ Chern-Simons theory on the η' domain wall

$$S_\Sigma = -\left(\frac{\eta'_\Sigma - \eta'_{\text{in}}}{2\pi}\right) \times \frac{i}{4\pi}\int_\Sigma \left[N_c \text{tr}\left(\mathbb{A}\text{d}\mathbb{A} - \frac{2i}{3}\mathbb{A}^3\right) + 2\text{tr}(\mathbb{A})\text{d}A\right],$$

where $\mathbb{A}$ is a dynamical $U(N_f)$ gauge field.



- The gauge field in the bulk A interacts with the dynamical field $\mathbb{A}$ on the bag surface, just as a background field. The flux of $\mathbb{A}$ is induced by A from Gauss' law

$$\frac{1}{2\pi N_f}\int_\Sigma \text{tr}(\text{d}\mathbb{A}) = -\frac{1}{2\pi N_c}\int_\Sigma \text{d}A.$$

Block Baryon Number Leak

- The monopoles inside the bag carry the magnetic field $B_A = \mathrm{d}A/N_c$ and Witten effect induces the baryon charge Q_A

$$\frac{\mathrm{d}Q_A}{\mathrm{d}\eta'_\Sigma} \equiv \frac{\mathrm{d}}{\mathrm{d}\eta'_\Sigma}\oint_\Sigma \mathrm{d}S\boldsymbol{E}_A\cdot\boldsymbol{n} = \frac{1}{8\pi^2}\oint_\Sigma \mathrm{d}S\boldsymbol{B}_A\cdot\boldsymbol{n}.$$

- The dynamical field $\mathbb{A}$ on the bag surface also possesses non-zero flux due to Gauss's law, leading to the corresponding baryon charge $Q_\mathbb{A}$

$$\frac{\mathrm{d}Q_\mathbb{A}}{\mathrm{d}\eta'_\Sigma} \equiv \frac{\mathrm{d}}{\mathrm{d}\eta'_\Sigma}\oint_\Sigma \mathrm{d}S\boldsymbol{E}_\mathbb{A}\cdot\boldsymbol{n} = \frac{1}{8\pi^2}\oint_\Sigma \mathrm{d}S\boldsymbol{B}_\mathbb{A}\cdot\boldsymbol{n}.$$

- By the flux relation, one can find the leak of baryon number is blocked

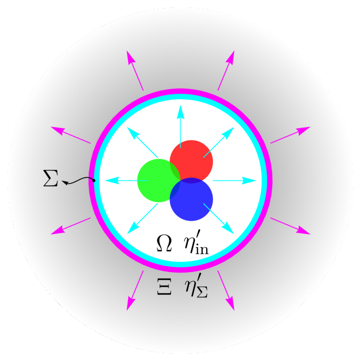
$$\frac{1}{2\pi N_f}\int_\Sigma \mathrm{tr}(\mathrm{d}\mathbb{A}) = -\frac{1}{2\pi N_c}\int_\Sigma \mathrm{d}A, \quad\longrightarrow\quad \frac{\mathrm{d}Q_A}{\mathrm{d}\eta'_\Sigma} + \frac{\mathrm{d}Q_\mathbb{A}}{\mathrm{d}\eta'_\Sigma} = 0.$$

Block Color Charge Leak

- The leak of baryon number is accompanied by the color charge leak

$$\frac{\mathrm{d}Q_G^a}{\mathrm{d}\eta'_\Sigma} \equiv \frac{\mathrm{d}}{\mathrm{d}\eta'_\Sigma}\oint_\Sigma \mathrm{d}S\boldsymbol{E}_G^a\cdot\boldsymbol{n} = \frac{1}{8\pi^2}\oint_\Sigma \mathrm{d}S\boldsymbol{B}_G^a\cdot\boldsymbol{n},$$

The dual $U(N_f)_{-N_c} \longleftrightarrow SU(N_c)_{N_f}$ Chern-Simons theory block the color charge leak, but with zero baryon number.



- On the bag surface, the Chern-Simons $U(N_f)_{N_c}$ field of vector meson $\mathbb{V}$ should couple with A to maintain 1-form symmetry

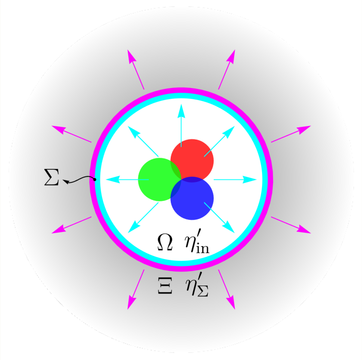
$$S_\Sigma = -\left(\frac{\eta'_\Sigma - \eta'_{\mathrm{in}}}{2\pi}\right)\times\frac{i}{4\pi}\int_\Sigma\Big[N_c\,\mathrm{tr}\left(\mathbb{A}\mathrm{d}\mathbb{A}-\frac{2i}{3}\mathbb{A}^3\right) -N_c\mathrm{tr}\left(\mathbb{V}\mathrm{d}\mathbb{V}-\frac{2i}{3}\mathbb{V}^3\right)+2(\mathrm{tr}\mathbb{V}+\mathrm{tr}\mathbb{A})\mathrm{d}A\Big].$$

Baryon Number of the Chiral Bag

- Baryon number inside the bag equates

$$Q_{\text{in}} = -\left(\frac{\eta'_\Sigma - \eta'_{\text{in}}}{2\pi}\right) \times \frac{1}{2\pi}\int_\Sigma \text{tr}(\text{d}\mathbb{V}) = N_f\left(\frac{\eta'_\Sigma - \eta'_{\text{in}}}{2\pi}\right) \times \frac{1}{2\pi N_c}\int_\Sigma \text{d}A.$$

For one-flavor chiral bag, one can choose $\eta'_{\text{in}} = 0, \eta'_\Sigma = 2\pi$ and $\frac{1}{2\pi}\int_\Sigma dA = N_c$.



- Baryon number outside the bag should be carried by the vector meson field

$$Q_{\text{out}} = -\int_\Xi \left(\frac{\text{d}\eta'}{2\pi}\right) \times \frac{1}{2\pi}\text{dtr}(\mathbb{V}).$$

- Therefore, the total baryon number of chiral bag is

$$Q = Q_{\text{in}} + Q_{\text{out}}.$$

Chern-Simons-Higgs Theory

- Since $\mathbb{V}$ describes the monopole density inside the bag by interacting with A in a topological manner, restrict the complex scalar field Φ on the bag surface

$$\phi = \Phi|_{\Sigma},\ \langle\Phi\rangle \neq 0, \quad \longrightarrow \quad \langle\phi\rangle \neq 0.$$

- For the one-flavor case, the effective theory on the bag surface

$$\int_{\Sigma} |\mathrm{d}\phi - \mathrm{i}\omega\phi|^2 + \frac{N_c}{4\pi}\omega \mathrm{d}\omega - V(\phi^*\phi).$$

- For the multi-flavor case, the theory generalizes to a non-Abelian one

$$\int_{\Sigma} |\mathrm{d}\boldsymbol{\phi} - \mathrm{i}\mathbb{V}\boldsymbol{\phi}|^2 + \frac{N_c}{4\pi}\mathrm{tr}\left(\mathbb{V}\mathrm{d}\mathbb{V} - i\frac{2}{3}\mathbb{V}^3\right) - V(\boldsymbol{\phi}^\dagger\boldsymbol{\phi}),$$

where $\boldsymbol{\phi} = (\phi^1, \phi^2, \ldots, \phi^{N_f})^T$ is a complex field with N_f components.

Hidden Local Symmetry and Vector Mesons

- Hidden local symmetry

$$U = [h(x)\xi_L(x)]^\dagger[h(x)\xi_R(x)], \quad \mathbb{V}_\mu \to h\mathbb{V}_\mu h^\dagger + ih\partial_\mu h^\dagger, \quad D_\mu\xi_{L,R} = (\partial_\mu - i\mathbb{V}_\mu)\xi_{L,R}.$$

- The gauge-invariant Lagrangian is constructed as:

$$\mathcal{L} = \frac{f_\pi^2}{4}\,\mathrm{tr}\left[\partial_\mu(\xi_R^\dagger\xi_L)\,\partial^\mu(\xi_L^\dagger\xi_R)\right] - a\frac{f_\pi^2}{4}\,\mathrm{tr}\left[(D_\mu\xi_L\,\xi_L^\dagger + D_\mu\xi_R\,\xi_R^\dagger)^2\right] - \frac{1}{4g_{\mathbb{V}}^2}\,\mathrm{tr}(\mathbb{V}_{\mu\nu}\mathbb{V}^{\mu\nu}).$$

  where vector meson is identified as $\rho_\mu = \mathbb{V}_\mu/g_{\mathbb{V}}$ and $a = 2$.

- The WZW term and the hidden Wess-Zumino terms (contracted with ϵ tensor)

$$S_{\mathrm{WZW}}\left[U \to \xi_L(x)^\dagger\xi_R(x)\right], \quad \mathcal{L}_{\mathrm{hWZ}} = \frac{N_c}{16\pi^2}\sum_{i=1}^{3} c_i\mathcal{L}_i.$$

[T. Kugo, K. Yamawaki, T. Yanagida et al.]

Gluon and Vector Meson

- The hidden Wess-Zumino terms introduce η' couples with vector meson

$$\mathcal{L}_{\mathsf{hWZ}} = \frac{N_c}{16\pi^2}\sum_{i=1}^{3} c_i \mathcal{L}_i, \quad \supset \quad \frac{1}{2\pi N_f}\mathrm{d}\eta' \wedge \frac{N_c}{4\pi}\left(\mathbb{V}\mathrm{d}\mathbb{V} - \frac{2i}{3}\mathbb{V}^3\right).$$

- On the domain wall with η' jump $2\pi N_f$, the level/rank duality is a duality between gluon field G and vector meson field $\mathbb{V}$

$$SU(N_c)_{-N_f} \quad \longleftrightarrow \quad U(N_f)_{N_c},$$
$$\frac{-N_f}{4\pi}\left(G\mathrm{d}G - \frac{2i}{3}G^3\right) \quad \longleftrightarrow \quad \frac{N_c}{4\pi}\left(\mathbb{V}\mathrm{d}\mathbb{V} - \frac{2i}{3}\mathbb{V}^3\right).$$

- Vector meson fields inherit the topological information from gluon field.

[T. Kugo, K. Yamawaki, T. Yanagida, Avner Karasik et al.]

Summary

- We construct single-flavor baryons on η' domain walls, establishing a correspondence with quantum Hall liquids, and demonstrating that vortex solutions carry topological charges consistent with single-flavor baryons in QCD.
- The introduction of a topological field theory mechanism within the chiral bag model to effectively describe the confinement of color flux, along with a proposal for characterizing color confinement via magnetic monopole condensation.
- We clarify the crucial role played by vector mesons in the construction of baryonic topological solitons, elucidating the level-rank duality between gluons and vector mesons and extending the applications of topological field theory in hadronic physics.

Acknowledgment

THANKS!