Discrete flavor and CP symmetries in light of JUNO and neutrino global fit

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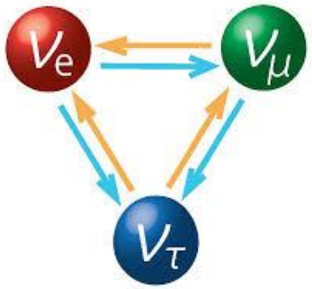
In collaboration with Gui-Jun Ding, Cai-Chang Li and S. T. Petcov,
based on arXiv:2512.03809

Parameters in the 3-flavor Neutrino Paradigm

Neutrino oscillation indicates **neutrino mixing** and **non-zero neutrino mass**

- Neutrino flavor states and mass states are connected by a unitary matrix U_{PMNS}:



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

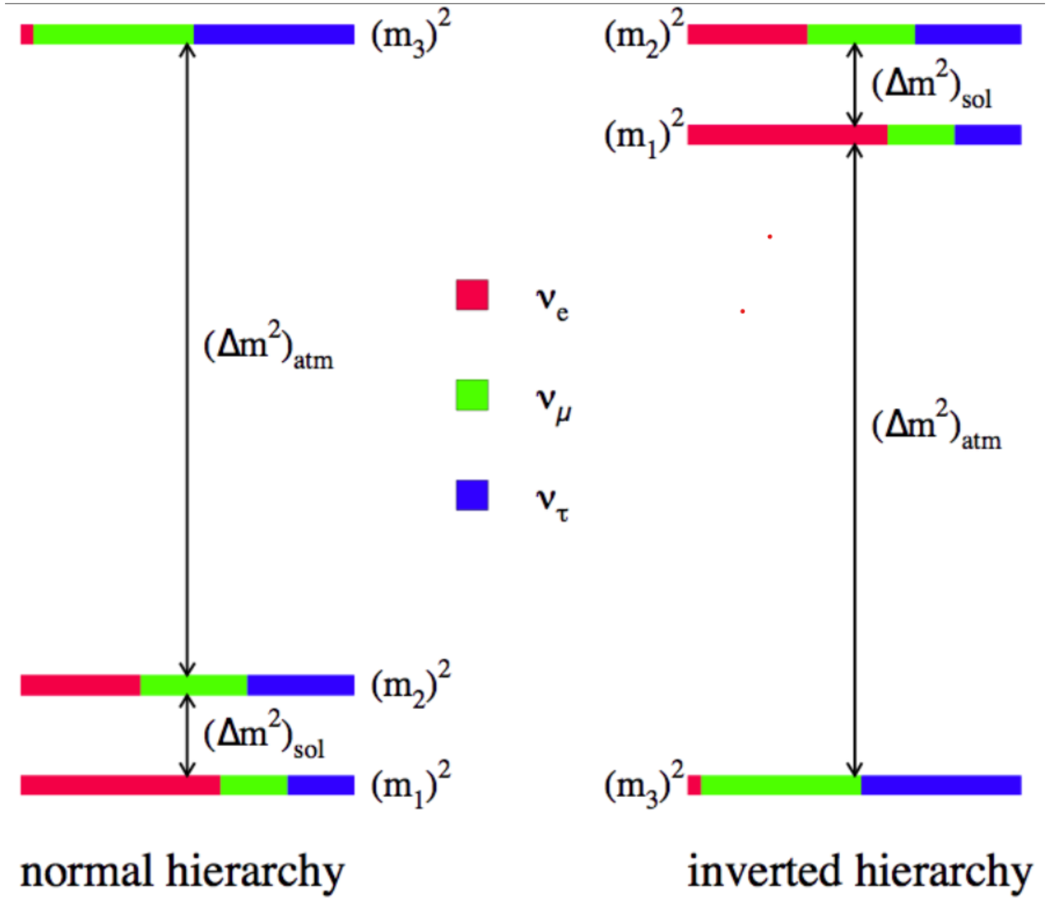
[PDG]

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij}, s_{ij} \equiv \sin\theta_{ij}$$

3 mixing angles: $\theta_{12}, \theta_{13}, \theta_{23}$
1 Dirac CPV phase: δ_{CP}
2 Majorana CPV phases: α_{21}, α_{31}
2 mass-squared differences: $\Delta m^2_{21}, \Delta m^2_{31}$



$m_1 < m_2 < m_3$

$m_3 < m_1 < m_2$

The global fit result of neutrino mixing parameters

NuFIT 6.0 (2024)

[Estenban et al.,2410.05380]

IC24 with SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2\theta_{12}$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.470^{+0.017}_{-0.013}$	$0.435 \to 0.585$	$0.550^{+0.012}_{-0.015}$	$0.440 \to 0.584$
	$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$41.3 \to 49.9$	$47.9^{+0.7}_{-0.9}$	$41.5 \to 49.8$
	$\sin^2\theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \to 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \to 0.02409$
	$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.59^{+0.11}_{-0.11}$	$8.25 \to 8.93$
	$\delta_{CP}/^\circ$	212^{+26}_{-41}	$124 \to 364$	274^{+22}_{-25}	$201 \to 335$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.513^{+0.021}_{-0.019}$	$+2.451 \to +2.578$	$-2.484^{+0.020}_{-0.020}$	$-2.547 \to -2.421$

NO: $m_1 < m_2 < m_3$

IO: $m_3 < m_1 < m_2$

www.nu-fit.org

- Unknow:

① Octant of θ_{23} :

θ_{23} >45° or θ_{23} <45°

② CP-violation phases:

δ_{CP}, α_{21} and α_{31}

③ Mass ordering:

The sign of Δm^2_{32}

The result of JUNO 59.1 days data

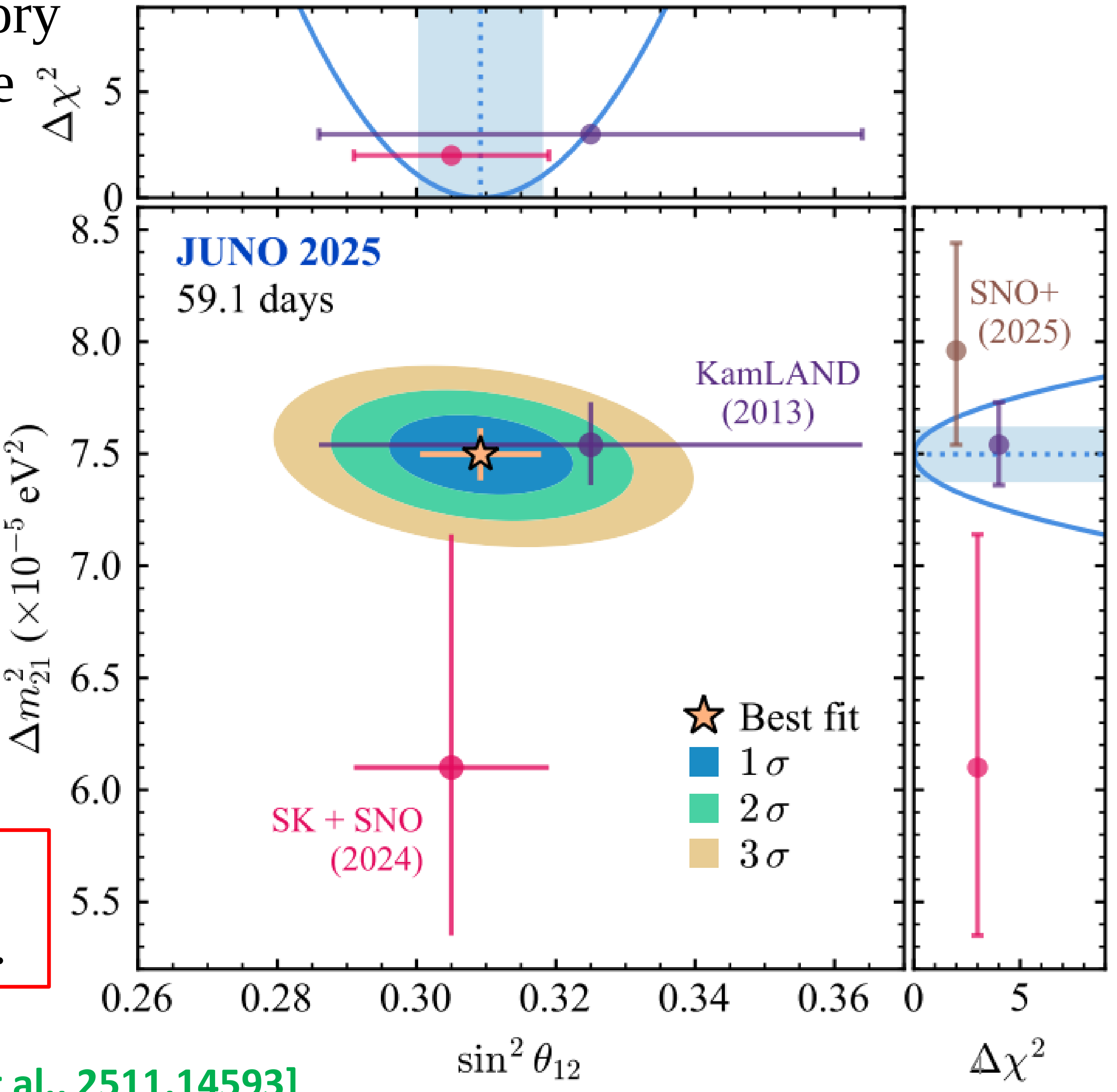
- The Jiangmen Underground Neutrino Observatory (JUNO) achieves world-leading precision on the solar oscillation parameters:

$$\boxed{\begin{aligned} \sin^2\theta_{12} &= 0.3092 \pm 0.0087, \\ \Delta m^2_{21} &= (7.5 \pm 0.12) \times 10^{-5}\,\mathrm{eV}^2 \end{aligned}}$$

- Comparing with PDG precision:

	PDG 2025	JUNO 59.1 days
$\sin^2\theta_{12}$	3.9%	2.81%
Δm^2_{21}	2.5%	1.55%

The result of JUNO 59.1-days data improved the precision of $\sin^2\theta_{12}$ and Δm^2_{21} by a factor of 1.6.



[Abusleme et al., 2511.14593]

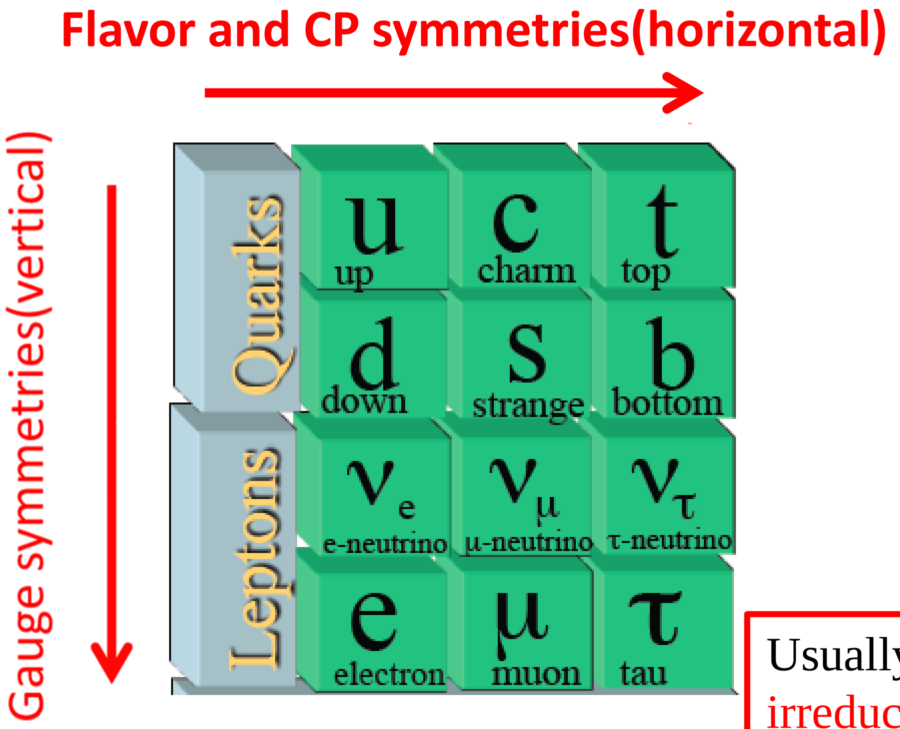
Flavor and CP symmetries to lepton mixing

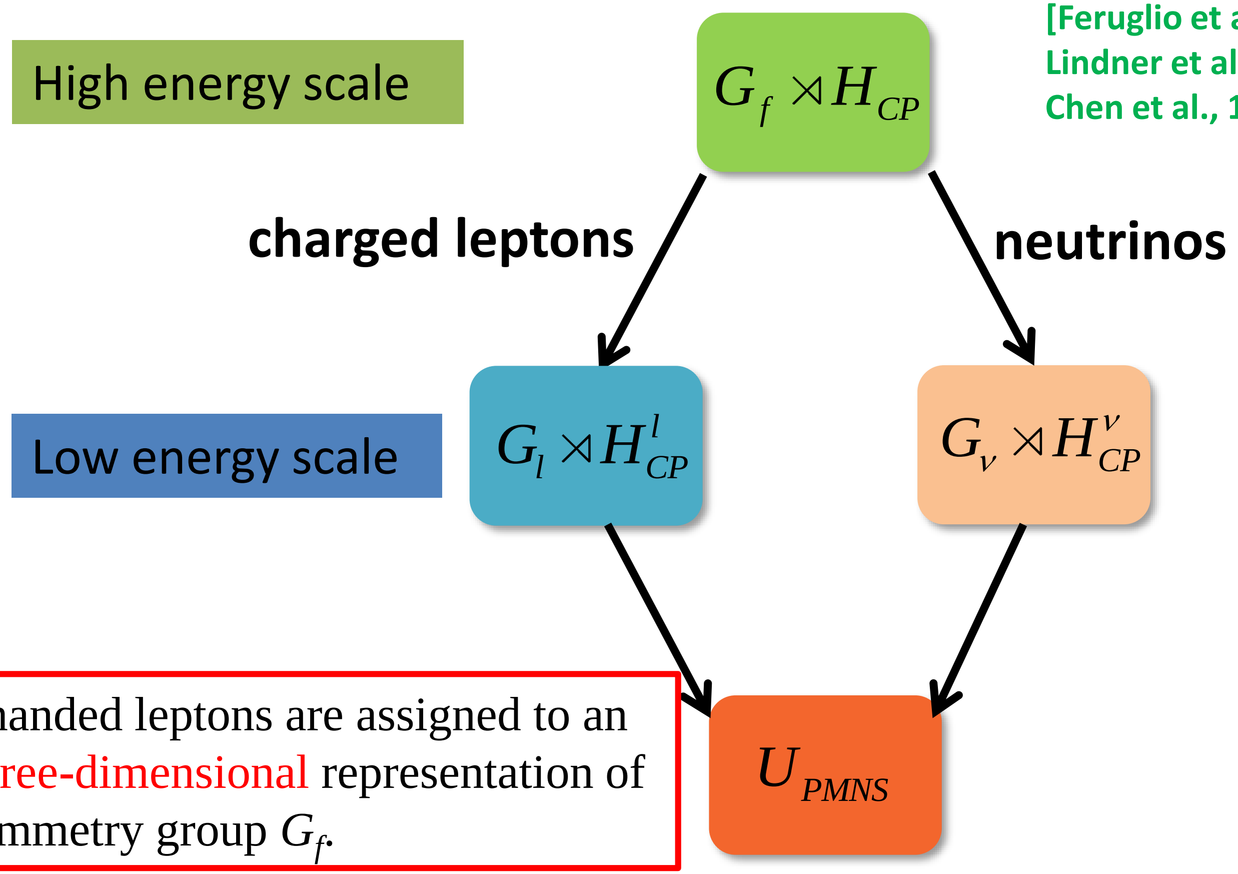
- **Basic idea:**

Relate lepton mixing to how are G_f and generalized CP broken

Lepton mixing matrix arises from mismatch of the different subgroups $G_l \rtimes H^l_{CP}$ and $G_\nu \rtimes H^\nu_{CP}$

$$\begin{tikzcd}
\varphi(x) \arrow[r, "\mathcal{CP}"] \arrow[d, "g'"'] & X\varphi^*(\mathcal{P}x) \arrow[d, "g"] \\
\rho(g')\varphi(x) = X\rho^*(g)X^{-1}\varphi(x) \arrow[r, phantom, "\boxed{X\rho^*(g)X^{-1} = \rho(g')}"] & X\rho^*(g)\varphi^*(\mathcal{P}x) \arrow[l, "\mathcal{CP}^{-1}"']
\end{tikzcd}$$





[Feruglio et al., 1211.5560; Lindner et al., 1211.6953; Chen et al., 1402.0507]

Usually left-handed leptons are assigned to an irreducible three-dimensional representation of the flavor symmetry group G_f.

Possible choices of residual symmetries

- The breaking of flavor and generalized CP symmetries can lead to the lepton mixing matrix depends on few free parameters:

[Ding et al., 2402.16963]

$\mathcal{G}_l$	$\mathcal{G}_\nu$	U	number of parameters
Z_n	$K_4 \times CP$	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	0
Z_n	$Z_2 \times CP$	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta) P_\nu Q_\nu$	1
$Z_2 \times CP$	$K_4 \times CP'$	$Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	
$Z_2 \times CP$	$Z_2 \times CP'$	$Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$	2
Z_2	$K_4 \times CP$	$Q_l^\dagger P_l^T U_{23}^\dagger(\theta_l, \delta_l) \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	
Z_n	CP	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu O_3(\theta_1, \theta_2, \theta_3) Q_\nu$	3
CP	$K_4 \times CP'$	$Q_l^\dagger O_3^T(\theta_1, \theta_2, \theta_3) \Sigma_l^\dagger \Sigma_\nu Q_\nu$	
Z_2	$Z_2 \times CP$	$Q_l^\dagger P_l^T U_{23}^\dagger(\theta_l, \delta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$	

large groups required

low predictivity

We focus on the mixing patterns with 1 and 2 free parameters.

The lepton mixing angles as well as Dirac and Majorana CP phases can be predicted by residual symmetry, neutrino masses are not constrained except in concrete models.

Breaking pattern with one parameter

$$G_f \rtimes H_{CP}$$

charged lepton

neutrino

$Z_n^{g_l}, n \geq 3$ or K_4

$Z_2^{g_\nu} \times X_\nu$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T,$$
$$\Sigma_\nu^\dagger G_\nu \Sigma_\nu = \pm \mathrm{diag}(1,-1,-1)$$

$$U_l = \Sigma_l P_l Q_l \qquad U_\nu = \Sigma_\nu R_{23}(\theta) P_\nu Q_\nu$$

$$U_{PMNS}(\theta) = Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta) P_\nu Q_\nu$$

- All mixing angles and CP phases are expressed in term of one free parameter $\theta \in [0, \pi)$.
- One column of the lepton mixing matrix is fixed.

The choices of flavor and CP symmetries

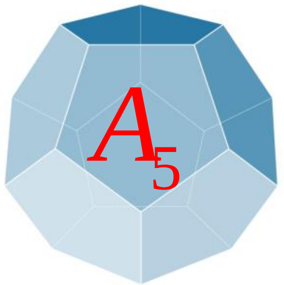
- A systematical analysis of the discrete flavor group G_f up to order 2000 find that all viable mixing patterns with one parameter can be obtained by considering:

$$\boxed{A_5 \rtimes H_{CP}, \quad \Sigma(168) \rtimes H_{CP}, \quad \Delta(6n^2) \rtimes H_{CP}}$$

[Yao et al.,1606.05610]

- A_5 is the group of even permutations of five objects, it has two generators S and T:

Dodecahedron



$$S^2 = T^5 = (ST)^3 = 1$$

- 3-dimensional representation:

$$\mathbf{3}: \quad S = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & -\phi_g & \phi_g - 1 \\ -\sqrt{2} & \phi_g - 1 & -\phi_g \end{pmatrix}, \quad T = \mathrm{diag}(1, \omega_5, \omega_5^4),$$

$$\phi_g = (1+\sqrt{5})/2, \quad \omega_5 = e^{2\pi i/5}$$

Golden ratio

- $\Sigma(168)$ is a non-Abelian subgroup of SU(3) of order 168, it has two generators S and T:

$$S^2 = (ST)^3 = T^7 = (ST^3)^4 = 1$$

- 3-dimensional representation:

$$\mathbf{3}: \quad S = \frac{2}{\sqrt{7}} \begin{pmatrix} -s_2 & -s_1 & s_3 \\ -s_1 & s_3 & -s_2 \\ s_3 & -s_2 & -s_1 \end{pmatrix}, \quad T = \mathrm{diag}(\omega_7, \omega_7^2, \omega_7^4),$$

$$\omega_7 = e^{2\pi i/7}, \quad s_n = \sin\frac{2n\pi}{7}$$

$\boxed{H_{CP}}$ CP and flavor symmetry transformations are of the same form in the chosen basis.

$\Delta(6n^2)$ group and CP

- $\Delta(6n^2)$ is an infinite series of non-Abelian finite subgroup of SU(3), it has four generators a, b, c and d:

$$a^3=b^2=(ab)^2=1,\quad c^n=d^n=1,\qquad cd=dc,$$
$$aca^{-1}=c^{-1}d^{-1},\quad ada^{-1}=c,\quad bcb^{-1}=d^{-1},\quad bdb^{-1}=c^{-1}$$

- 3-dimensional irreducible representations:

$$a=\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix},\quad b=(-1)^k\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\quad c=\begin{pmatrix}\eta^l&0&0\\0&\eta^{-l}&0\\0&0&1\end{pmatrix},\quad d=\begin{pmatrix}1&0&0\\0&\eta^l&0\\0&0&\eta^{-l}\end{pmatrix},$$
$$\eta=e^{2\pi i/n},\quad k=0,1,\quad l=1,2,\ldots,n-1$$

H_{CP} CP and flavor symmetry transformations are of the same form in the chosen basis.

Lepton mixing matrix from $\Delta(6n^2)$ and CP

- Lepton mixing matrix from $\Delta(6n^2)\rtimes H_{CP}$

Case I: $\boxed{G_\ell=\langle ac^s d^t\rangle, G_\nu=Z_2^{bc^x d^x}, X_{\nu r}=\rho_r(c^\gamma d^{-2x-\gamma})}$ $\quad s,t,x,\gamma=0,1,\cdots n-1$

$$U^I=\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2}\sin\varphi_1 & e^{i\varphi_2} & \sqrt{2}\cos\varphi_1\\ \sqrt{2}\cos\left(\varphi_1-\frac{\pi}{6}\right) & -e^{i\varphi_2} & -\sqrt{2}\sin\left(\varphi_1-\frac{\pi}{6}\right)\\ \sqrt{2}\cos\left(\varphi_1+\frac{\pi}{6}\right) & e^{i\varphi_2} & -\sqrt{2}\sin\left(\varphi_1+\frac{\pi}{6}\right)\end{pmatrix}R_{23}(\theta)Q_\nu,$$

fixed column

$$\boxed{\varphi_1=\frac{s-x}{n}\pi,\quad \varphi_2=\frac{2t-s-3(\gamma+x)}{n}\pi,}$$

$$R_{23}(\theta)=\begin{pmatrix}1&0&0\\0&\cos\theta&\sin\theta\\0&-\sin\theta&\cos\theta\end{pmatrix}$$

fixed by residual symmetry

Case II: $\boxed{G_\ell=\langle ac^s d^t\rangle,\quad G_\nu=Z_2^{c^{n/2}},\quad X_{\nu r}=\rho_r(c^\gamma d^\delta),}$ $\quad s,t,\gamma,\delta=0,1,\cdots n-1$

$$U^{II}=\frac{1}{\sqrt{3}}\begin{pmatrix}e^{i\varphi_1} & 1 & e^{i\varphi_2}\\ \omega e^{i\varphi_1} & 1 & \omega^2 e^{i\varphi_2}\\ \omega^2 e^{i\varphi_1} & 1 & \omega e^{i\varphi_2}\end{pmatrix}R_{13}(\theta)Q_\nu,$$

$$\boxed{\varphi_1=\frac{\gamma+\delta+2s}{n}\pi,\quad \varphi_2=\frac{2\delta-\gamma+2t}{n}\pi.}$$

$$R_{13}(\theta)=\begin{pmatrix}\cos\theta&0&\sin\theta\\0&1&0\\-\sin\theta&0&\cos\theta\end{pmatrix}$$

up to permutation of rows

TM2

Lepton mixing matrix from A_5 and $\Sigma(168)$ with CP

- Lepton mixing matrix from $A_5 \rtimes H_{CP}$

Case III: $G_\ell = Z_5^T, G_\nu = Z_2^{T^3ST^2ST^3}, X_{\nu r} = \rho_r(S),$

Case IV: $G_\ell = K_4^{(ST^2ST^3S, TST^4)}, G_\nu = Z_2^S, X_{\nu r} = \rho_r(T^3ST^2ST^3)$

$$U^{III} = \begin{pmatrix} -i\sqrt{\frac{\phi_g}{\sqrt{5}}} & \sqrt{\frac{1}{\sqrt{5}\phi_g}} & 0 \\ i\sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ i\sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta)Q_\nu,$$

GR2

Golden ratio: $\phi_g = (1+\sqrt{5})/2$

$$U^{IV} = \frac{1}{2}\begin{pmatrix} \phi_g & 1 & \phi_g - 1 \\ \phi_g - 1 & -\phi_g & 1 \\ 1 & 1-\phi_g & -\phi_g \end{pmatrix} R_{23}(\theta)Q_\nu,$$

up to permutation of rows

- Lepton mixing matrix from $\Sigma(168) \rtimes H_{CP}$

TM2

Case V: $G_\ell = Z_3^{ST^4ST^2S}, G_\nu = Z_2^S, X_{\nu r} = \rho_r(1).$

$$U^{V} = \frac{1}{2\sqrt{3}}\begin{pmatrix} \sqrt{3}-1 & 2e^{-i\varphi} & -(\sqrt{3}+1)e^{\frac{3\pi i}{4}} \\ -\sqrt{3}-1 & 2e^{-i\varphi} & (\sqrt{3}-1)e^{\frac{3\pi i}{4}} \\ 2 & 2e^{-i\varphi} & 2e^{\frac{3\pi i}{4}} \end{pmatrix} R_{13}(\theta)Q_\nu,$$

up to permutation of rows

Viable lepton mixing with one parameter

- Best-fit results of mixing angles and CP phases for all discrete groups with order ≤ 168

- 3σ interval of JUNO:

$$\boxed{0.2831 \leq \sin^2\theta_{12} \leq 0.3353}$$

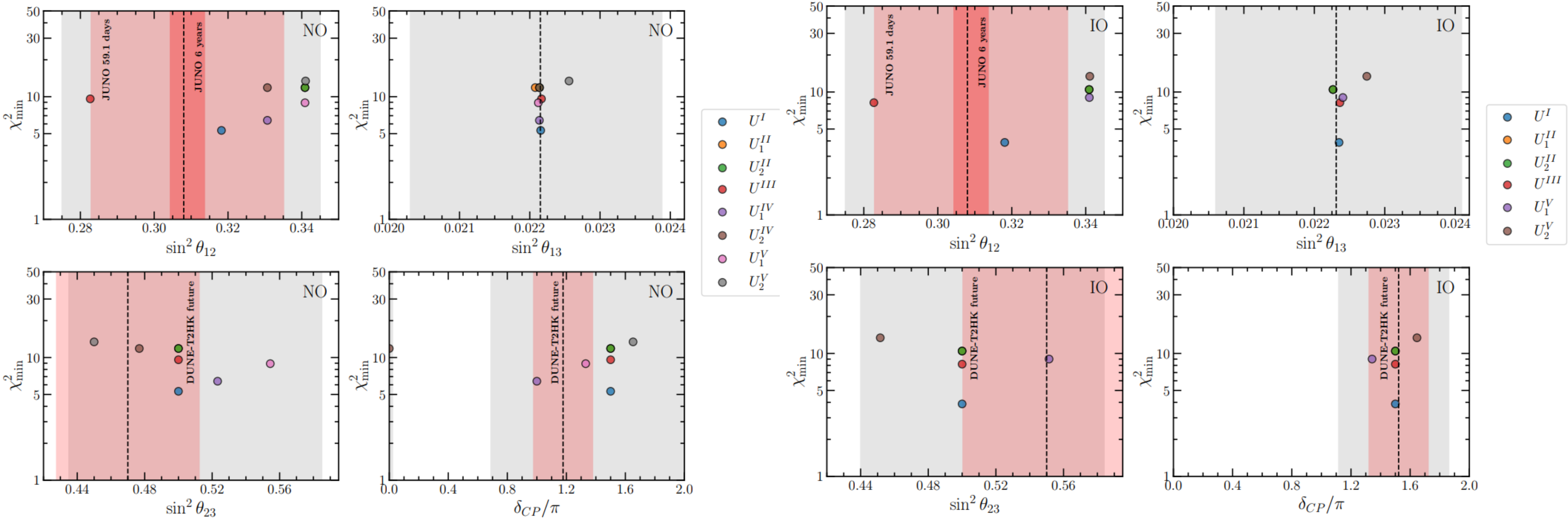
Order	Case	(φ_1, φ_2)	$\theta_{\rm bf}/\pi$	$\chi^2_{\rm min}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	α_{21}/π (mod 1)	α_{31}/π (mod 1)
	U^{I}	$(-\frac{\pi}{2}, \frac{\pi}{2})$	0.9170	5.308	0.318	0.02215	0.5	1.5	0	0
TM2	U_1^{II}	$(0, \pi)$	0.1918	11.862	0.341	0.02208	0.5	1.5	0	0
	U_2^{II}	$(0, \pi)$	0.3083	11.859	0.341	0.02214	0.5	1.5	0	0
GR2 →	U^{III}	—	0.0560	9.600	0.283	0.02217	0.5	1.5	0	0
NO	U_1^{IV}	—	0.9053	6.414	0.331	0.02214	0.523	1	0	0
	U_2^{IV}	—	0.9053	11.856	0.331	0.02214	0.477	0	0	0
	U_1^{V}	—	0.5715	8.921	0.341	0.02212	0.554	1.331*	0.161*	0.559*
TM2	U_2^{V}	—	0.5746	13.427	0.341	0.02256	0.450	1.652	0.837	0.439
IO	U^{I}	$(-\frac{\pi}{2}, \frac{\pi}{2})$	0.9166	3.887	0.318	0.02235	0.5	1.5	0	0
	U_1^{II}	$(0, \pi)$	0.1915	10.471	0.341	0.02227	0.5	1.5	0	0
	U_2^{II}	$(0, \pi)$	0.3085	10.471	0.341	0.02227	0.5	1.5	0	0
	U^{III}	—	0.0563	8.188	0.283	0.02236	0.5	1.5	0	0
	U_1^{V}	—	0.5736	9.012	0.341	0.02241	0.551	1.343*	0.162*	0.561*
	U_2^{V}	—	0.5758	13.416	0.341	0.02274	0.452	1.646	0.837	0.438

- ***U^I*** can accommodate experiment data at 3σ for both NO and IO
- ***$U^{IV}_{1,2}$*** can accommodate experiment data at 3σ for NO
- **TM2** and **GR2** are disfavored at more than 3σ

[Ding et al.,2512.03809]

Viable lepton mixing with one parameter

➢ Precise measurements of mixing angles and CP phase [Ding et al.,2512.03809]

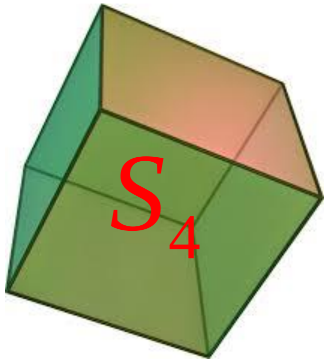


- After 6 years of data collection, JUNO will measure $sin^2\theta_{12}$ with 1σ uncertainty of 0.5% (current 2.8%).

Future precision measurements can test these one-parameter mixing patterns.

Example of viable lepton mixing with one parameter

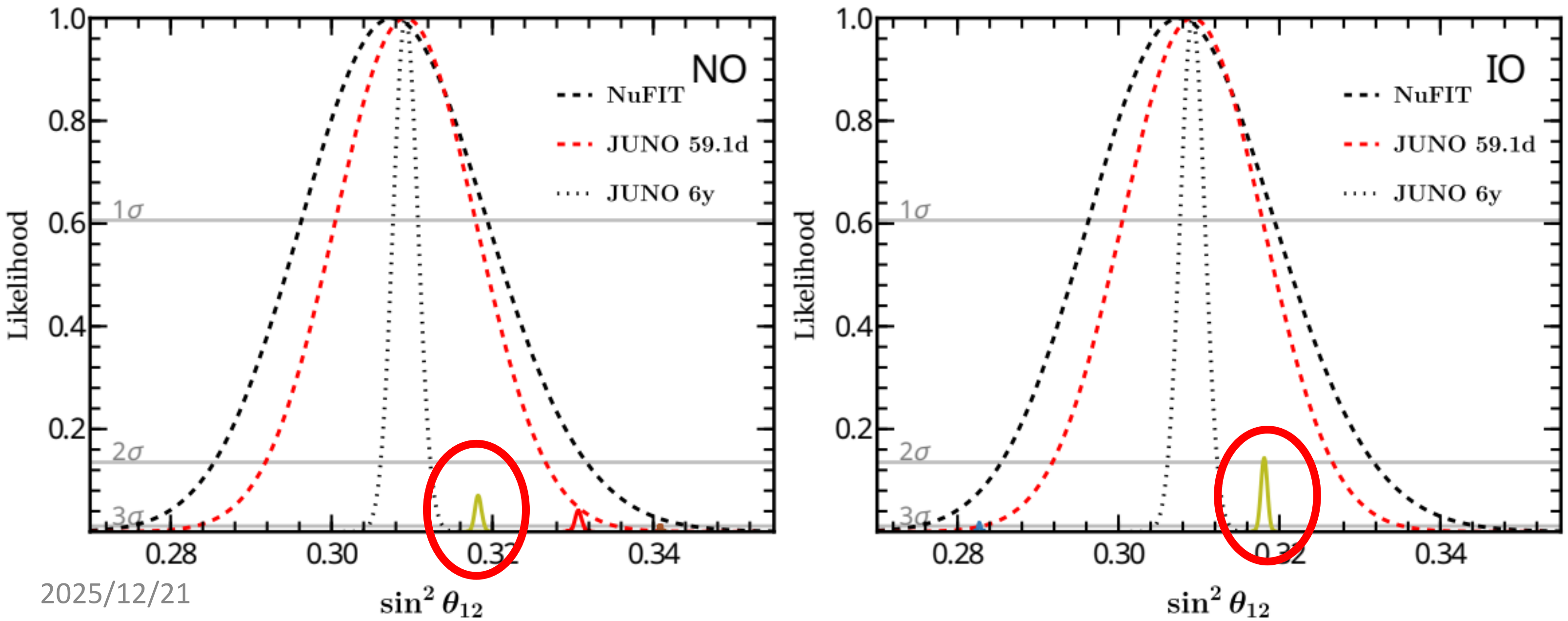
- The minimal group that can realized viable U^I is: $\Delta(24) \cong S_4$



with $(\varphi_1, \varphi_2) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$U^I = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2}\sin\varphi_1 & e^{i\varphi_2} & \sqrt{2}\cos\varphi_1 \\ \sqrt{2}\cos\left(\varphi_1 - \frac{\pi}{6}\right) & -e^{i\varphi_2} & -\sqrt{2}\sin\left(\varphi_1 - \frac{\pi}{6}\right) \\ \sqrt{2}\cos\left(\varphi_1 + \frac{\pi}{6}\right) & e^{i\varphi_2} & -\sqrt{2}\sin\left(\varphi_1 + \frac{\pi}{6}\right) \end{pmatrix} R_{23}(\theta) Q_\nu, \Rightarrow U^I = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \frac{i}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{i}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{i}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{23}(\theta) Q_\nu$$

TM1



- Sum rules

$$3\cos^2\theta_{12}\cos^2\theta_{13} = 2,$$
$$\cos\delta_{CP} = \frac{(5\sin^2\theta_{13} - 1)\cot 2\theta_{23}}{2\sin\theta_{13}\sqrt{2 - 6\sin^2\theta_{13}}}$$

$\sin^2\theta_{12} \approx 0.318$

$\sin^2\theta_{23} = \frac{1}{2} \quad \Rightarrow \quad \cos\delta_{CP} = 0$

Maximal CP phase !

Breaking pattern with two parameters

[Lu, Ding, 1610.05682, 1806.02301; Li, Lu, Ding, 1706.04576]

$$G_f \rtimes H_{CP}$$

charged lepton

neutrino

$$Z_2^{g_l} \times X_l$$

$$Z_2^{g_\nu} \times X_\nu$$

$$X_l = \Sigma_l \Sigma_l^T, \quad \Sigma_l^\dagger \rho_{\mathbf{3}}(g_l)\Sigma_l = \pm\mathrm{diag}(1,-1,-1)$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T, \quad \Sigma_\nu^\dagger \rho_{\mathbf{3}}(g_\nu)\Sigma_\nu = \pm\mathrm{diag}(1,-1,-1)$$

$$U_l = \Sigma_l R_{23}(\theta_l) P_l Q_l \qquad U_\nu = \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$$

$$U_{PMNS}(\theta_l, \theta_\nu) = Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$$

- All mixing angles and CP phases are expressed in terms of two free parameters θ_l and $\theta_\nu \in [0,\pi)$
- One element of the lepton mixing matrix is fixed.

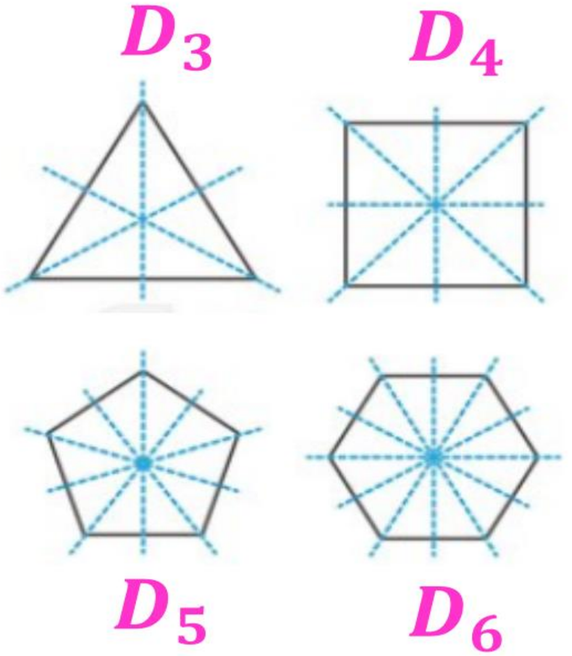
The flavor and CP symmetries

- Similar as in case of one parameter

$$\boxed{\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \sim \mathbf{3}} \qquad \boxed{A_5 \rtimes H_{CP}, \quad \Sigma(168) \rtimes H_{CP}, \quad \Delta(6n^2) \rtimes H_{CP}} \quad + \quad \boxed{D_n \rtimes H_{CP}} \qquad \boxed{L_e \sim \mathbf{1}, \quad \begin{pmatrix} L_\mu \\ L_\tau \end{pmatrix} \sim \mathbf{2}}$$

- The dihedral group D_n is the symmetry group of a n-sided regular polygon, it has two generators R and S

$$R^n = S^2 = (RS)^2 = 1$$



- Irreducible representations : 1-dim, 2-dim

$$1_1: \quad R = S = 1, \qquad 1_2: \quad R = 1, \quad S = 1,$$

$$1_3: \quad R = -1, \ S = 1, \qquad 1_4: \quad R = S = -1 \quad (even \quad n),$$

$$2_j: \ R = \begin{pmatrix} e^{2\pi i j/n} & 0 \\ 0 & e^{2\pi i j/n} \end{pmatrix}, \qquad S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad j = 0, 1, \ldots, \left[\frac{n-1}{2}\right]$$

$$\boxed{H_{CP}} \qquad \boxed{X = \rho(g), \ g \in D_n}$$

Lepton mixing matrix from A_5 and CP

➢ Breaking patterns from $A_5 \rtimes H_{CP}$

Case VI:

The independent breaking patterns from $A_5 \rtimes H_{CP}$ and the corresponding Σ matrix			
$(Z_2^{g_\ell},\, Z_2^{g_\nu})$	$(X_{\ell r},\, X_{\nu r})$	$\Sigma = \Sigma_\ell^\dagger \Sigma_\nu$	**Fixed element**
$(Z_2^S,\, Z_2^{TST^4})$	$(\rho_r(1),\, \rho_r(T^3ST^2ST^3S))$	$\Sigma_1^{VI} = \text{diag}(i,-i,1)\, U_{RC}\, \text{diag}(-1,1,-1)$	$\frac{\phi_g}{2}$
	$(\rho_r(T^3ST^2ST^3),\, \rho_r(T^3ST^2ST^3S))$	$\Sigma_2^{VI} = \text{diag}(i,1,1)\, U_{RC}\, \text{diag}(1,-1,1)$	
$(Z_2^S,\, Z_2^{T^2ST^3})$	$(\rho_r(1),\, \rho_r(ST^2S))$	$\Sigma_3^{VI} = \text{diag}(1,-1,-i)\, P_{312}U_{RC}\, \text{diag}(i,1,-1)$	$\frac{1}{2}$
	$(\rho_r(T^3ST^2ST^3),\, \rho_r(ST^2S))$	$\Sigma_4^{VI} = \text{diag}(i,1,1)\, P_{312}U_{RC}\, \text{diag}(i,1,-1)$	
$(Z_2^S,\, Z_2^{T^4(ST^2)^2})$	$(\rho_r(1),\, \rho_r(T^3ST^2ST^3S))$	$\Sigma_5^{VI} = \text{diag}(1,-1,-i)\, U_{RC}P_{213}\, \text{diag}(i,1,1)$	$\frac{1}{2\phi_g}$
	$(\rho_r(T^3ST^2ST^3),\, \rho_r(T^3ST^2ST^3S))$	$\Sigma_6^{VI} = \text{diag}(i,1,1)\, U_{RC}P_{213}\, \text{diag}(i,1,1)$	

➢ The lepton mixing matrices reads:

$$U^{VI} = Q_l^\dagger P_l^T R_{23}^T(\theta_l)\Sigma^{VI} R_{23}(\theta_\nu) P_\nu Q_\nu$$

$$U_{RC} = \frac{1}{2}\begin{pmatrix} \phi_g & 1/\phi_g & 1 \\ 1/\phi_g & 1 & -\phi_g \\ 1 & -\phi_g & -1/\phi_g \end{pmatrix}$$

Golden ratio: $\phi_g = (1+\sqrt{5})/2$

Lepton mixing matrix from Σ(168) and CP

- Breaking patterns from $\Sigma(168)\rtimes H_{CP}$

Case VII:

$$\boxed{\begin{aligned} &G_\ell = Z_2^S, \quad G_\nu = Z_2^{T^2ST^5}, \quad X_{lr} = \rho_r(1), \quad X_{\nu r} = \rho_r(T^4),\\ &G_\ell = Z_2^S, \quad G_\nu = Z_2^{T^2ST^5}, \quad X_{lr} = \rho_r(T^2ST^5ST^2), \quad X_{\nu r} = \rho_r(T^4ST^5ST^4) \end{aligned}}$$

- Two lepton mixing patterns:

$$\boxed{\Sigma = \Sigma_l^\dagger \Sigma_\nu}$$

$$\left.\begin{aligned} &\Sigma_1^{VII} = \mathrm{diag}(1, e^{-\frac{\pi i}{4}}, e^{\frac{\pi i}{4}})\Sigma_2^{VII}\,\mathrm{diag}(1, e^{\frac{\pi i}{4}}, e^{-\frac{\pi i}{4}}),\\ &\Sigma_2^{VII} = \frac{1}{4}\begin{pmatrix} \boxed{-2} & \sqrt{2\left(\sqrt{7}+3\right)} & \sqrt{2\left(3-\sqrt{7}\right)} \\ \sqrt{2\left(\sqrt{7}+3\right)} & \sqrt{7}-1 & \sqrt{2} \\ \sqrt{2\left(3-\sqrt{7}\right)} & \sqrt{2} & -\sqrt{7}-1 \end{pmatrix}. \end{aligned}\right\}$$

Fixed element

$$\frac{1}{2}$$

Lepton mixing matrix from $\Delta(6n^2)$ and CP

- 3 Breaking patterns from $\Delta(6n^2)\rtimes H_{CP}$

	The independent breaking patterns from $\Delta(6n^2)\rtimes H_{CP}$ and the corresponding Σ matrix			Fixed element
	$(Z_2^{g_\ell},\ Z_2^{g_\nu})$	$(X_{\ell\boldsymbol{r}},\ X_{\nu\boldsymbol{r}})$	$\Sigma=\Sigma_\ell^\dagger\Sigma_\nu$	
Case VIII:	$(Z_2^{bc^xd^x},\ Z_2^{bc^yd^y})$	$(\rho_{\boldsymbol{r}}(c^\alpha d^{-2x-\alpha}),\ \rho_{\boldsymbol{r}}(c^\beta d^{-2y-\beta}))$	$\Sigma^{VIII}=\begin{pmatrix}\boxed{\cos\varphi_1} & -i\sin\varphi_1 & 0\\ -i\sin\varphi_1 & \cos\varphi_1 & 0\\ 0 & 0 & e^{i\varphi_2}\end{pmatrix}$	$\cos\varphi_1$
Case IX:	$(Z_2^{bc^xd^x},\ Z_2^{abc^y})$	$(\rho_{\boldsymbol{r}}(c^\alpha d^{-2x-\alpha}),\ \rho_{\boldsymbol{r}}(c^\beta d^{2y+2\beta}))$	$\Sigma^{IX}=\frac{1}{2}\begin{pmatrix}\boxed{1} & -\sqrt{2}e^{i\varphi_2} & -1\\ -1 & -\sqrt{2}e^{i\varphi_2} & 1\\ -\sqrt{2}e^{i\varphi_1} & 0 & -\sqrt{2}e^{i\varphi_1}\end{pmatrix}$	$\frac{1}{2}$
Case X:	$(Z_2^{bc^xd^x},\ Z_2^{c^{n/2}})$	$(\rho_{\boldsymbol{r}}(c^\alpha d^{-2x-\alpha}),\ \rho_{\boldsymbol{r}}(c^\beta d^\gamma))$	$\Sigma^{X}=\frac{1}{\sqrt{2}}\begin{pmatrix}\boxed{e^{i\varphi_2}} & 0 & -1\\ e^{i\varphi_2} & 0 & 1\\ 0 & \sqrt{2}e^{i\varphi_1} & 0\end{pmatrix}$	$\frac{1}{\sqrt{2}}$

$\alpha,\beta,\gamma,x,y=0,1,\cdots n-1$

- Discrete parameters:

$$\left.\begin{aligned}
&VIII:\quad \varphi_1=\frac{x-y}{n}\pi,\quad \varphi_2=\frac{3(x-y+\alpha-\beta)}{n}\pi,\\
&IX:\quad \varphi_1=\frac{3\alpha+2(x+y)}{n}\pi,\quad \varphi_2=-\frac{3\beta+2(x+y)}{n}\pi,\\
&X:\varphi_1=\frac{2x+3\alpha-2\beta+\gamma}{n}\pi,\quad \varphi_2=-\frac{2x+\beta+\gamma}{n}\pi.
\end{aligned}\right\}$$

fixed by residual symmetry

Lepton mixing matrix from D_n and CP

- Breaking patterns from $D_n \rtimes H_{CP}$

$L_e \sim \mathbf{1}, \quad \begin{pmatrix} L_\mu \\ L_\tau \end{pmatrix} \sim \mathbf{2}$

Residual symmetry: $G_\ell = Z_2^{SR^{z_e}}, \quad X_\ell = \{R^{-z_e+x}, SR^x\}, \qquad G_\nu = Z_2^{SR^{z_\nu}}, \quad X_\nu = \{R^{-z_\nu+y}, SR^y\}$

$z_e, z_\nu = 0, 1, \ldots, n-1$

x=y=0, **odd n**

x,y=0, n/2. **even n**

$\boxed{\Sigma = \Sigma_l^\dagger \Sigma_\nu}$

$$\Sigma^{VIII} = \begin{pmatrix} \cos\varphi_1 & -i\sin\varphi_1 & 0 \\ -i\sin\varphi_1 & \cos\varphi_1 & 0 \\ 0 & 0 & e^{i\varphi_2} \end{pmatrix}$$

same as in case of $\Delta(6n^2)$ and CP

- The same mixing patterns U^{VIII}, but with different possible values of discrete parameters φ_1 and φ_2

$\Delta(6n^2) \rtimes H_{CP}$:

$$\varphi_1 (\mathrm{mod}\, 2\pi) = 0, \frac{1}{n}\pi, \frac{2}{n}\pi, \ldots, \frac{2n-1}{n}\pi,$$
$$\varphi_2 (\mathrm{mod}\, 2\pi) = 0, \frac{3}{n}\pi, \frac{6}{n}\pi, \ldots, \frac{2n-3}{n}\pi, \quad 3 \mid n,$$
$$\varphi_2 (\mathrm{mod}\, 2\pi) = 0, \frac{1}{n}\pi, \frac{2}{n}\pi, \ldots, \frac{2n-1}{n}\pi, \quad 3 \nmid n.$$

$D_n \rtimes H_{CP}$:

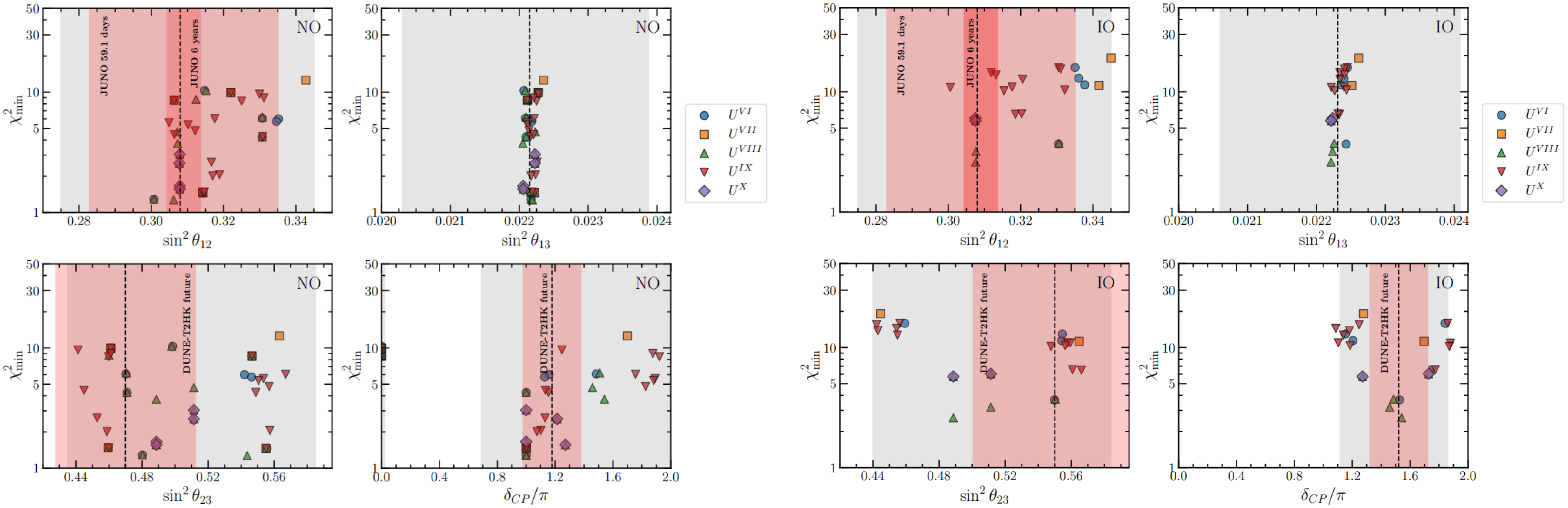
$$\varphi_1 (\mathrm{mod}\, 2\pi) = 0, \frac{1}{n}\pi, \frac{2}{n}\pi, \ldots, \frac{2n-1}{n}\pi,$$
$$\varphi_2 (\mathrm{mod}\, 2\pi) = 0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi, \quad 2 \mid n,$$
$$\varphi_2 (\mathrm{mod}\, 2\pi) = 0, \pi, \quad 2 \nmid n.$$

The group order of D_n is much smaller than that of $\Delta(6n^2)$: $2n \ll 6n^2$

Numerical results of lepton mixings with two parameters

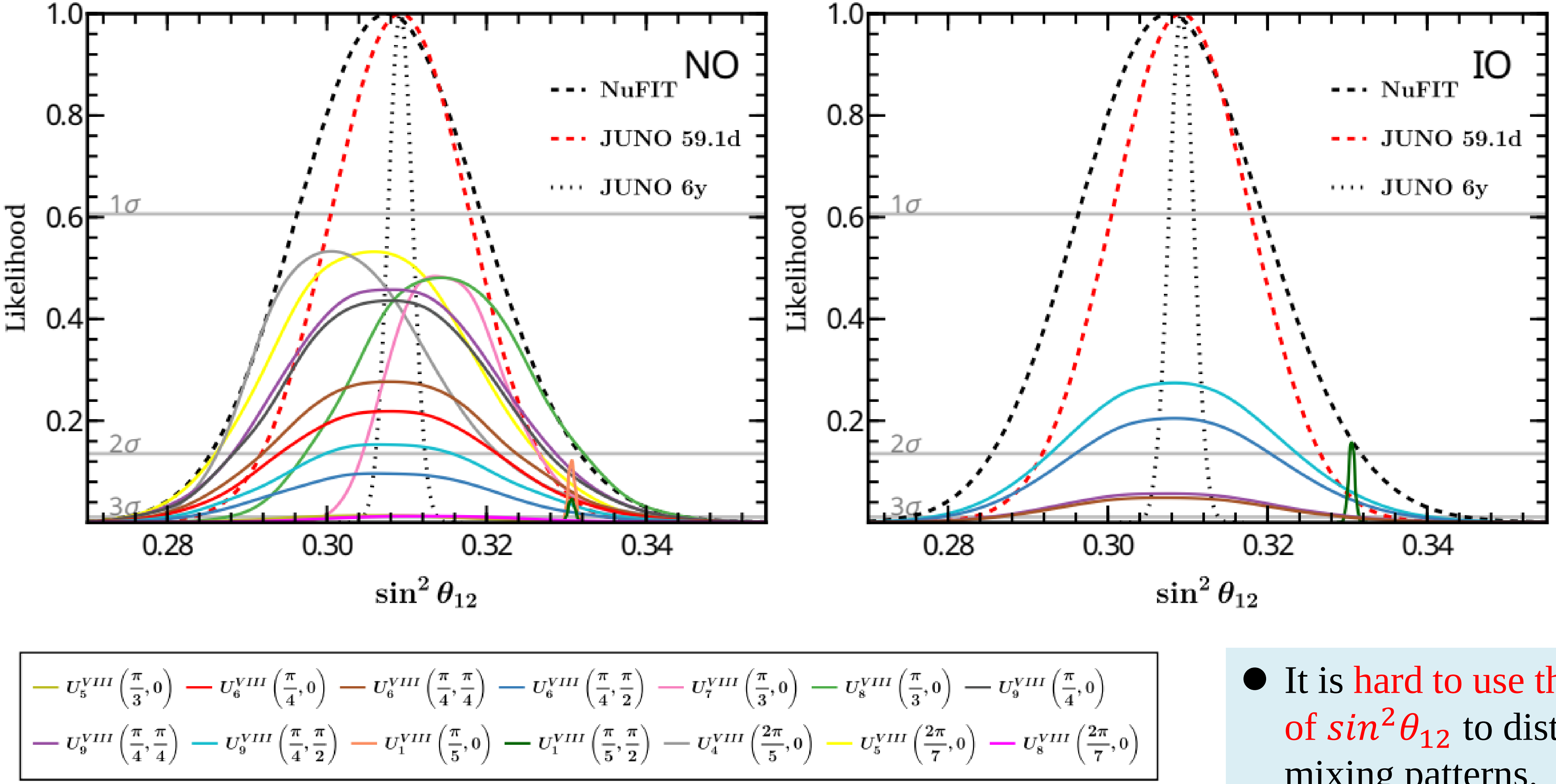
➢ Precise measurements of mixing angles and CP phase

[Ding et al.,2512.03809]



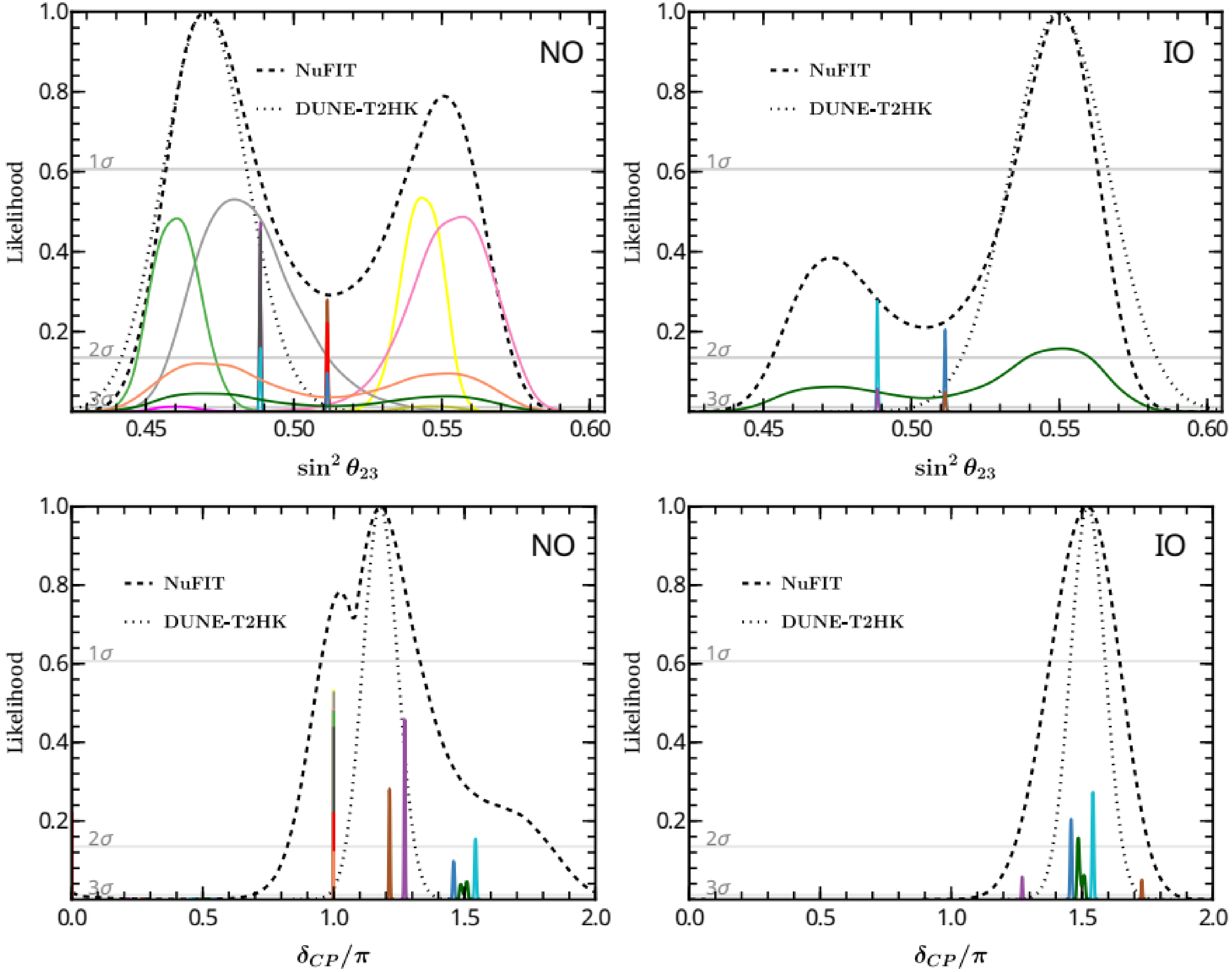
➢ Nearly all two-parameter mixing patterns remain consistent with current NuFIT and JUNO 59.1-day data.

Predictions of solar neutrino mixing angle



- It is hard to use the predictions of $sin^2\theta_{12}$ to distinguish these mixing patterns.

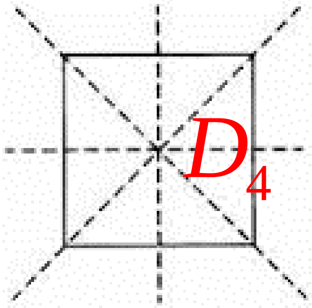
Predictions of atmospheric neutrino mixing angle and CP phase



- The predictions of $sin^2\theta_{23}$ and δ_{CP} are narrow and different among these mixing patterns.

- Future DUNE and T2HK experiments can help to test these mixing patterns.

Example of viable lepton mixing with two parameters



- Minimal group that can realized viable mixing with non-trivial CP phases is ***D_4***

with $(\varphi_1, \varphi_2) = \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ **Fixed** $\boxed{\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}}$

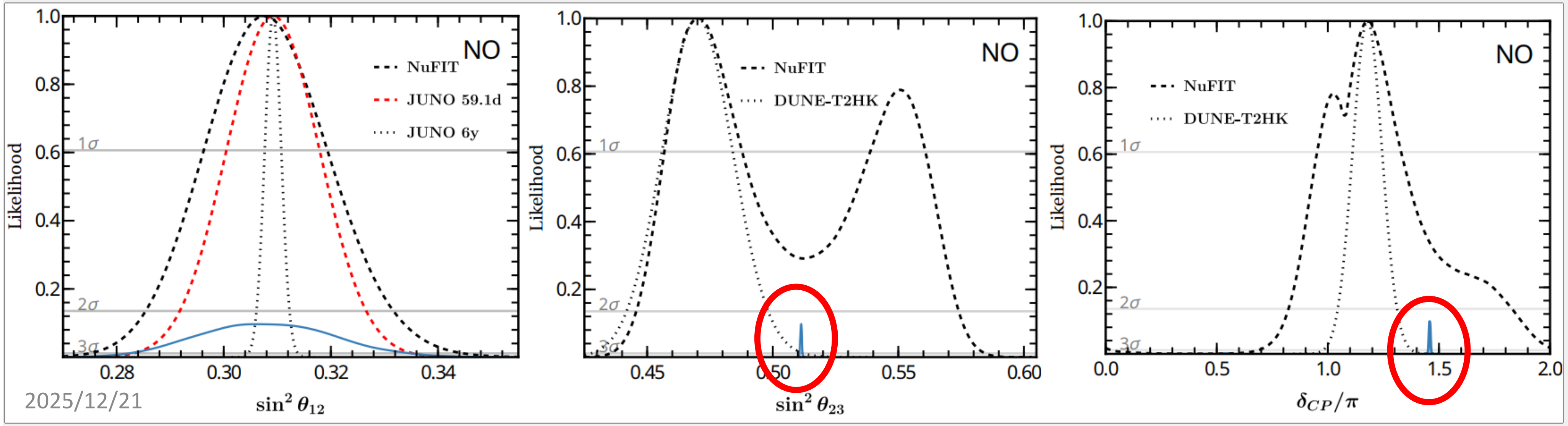
$$U_6^{VIII} = Q_l.\begin{pmatrix} c_\ell s_\nu e^{i\varphi_2} - c_\nu s_\ell \cos\varphi_1 & c_\ell c_\nu e^{i\varphi_2} + s_\ell s_\nu \cos\varphi_1 & -s_\ell \sin\varphi_1 \\ -c_\nu \sin\varphi_1 & s_\nu \sin\varphi_1 & \boxed{\cos\varphi_1} \\ s_\ell s_\nu e^{i\varphi_2} + c_\ell c_\nu \cos\varphi_1 & c_\nu s_\ell e^{i\varphi_2} - c_\ell s_\nu \cos\varphi_1 & c_\ell \sin\varphi_1 \end{pmatrix}.Q_\nu$$

- Sum rule:

$$\cos^2\theta_{13}\sin^2\theta_{23} = \cos^2\varphi_1$$

Order	Case	φ_1	φ_2	$\theta_\ell^{\mathrm{bf}}/\pi$	$\theta_\nu^{\mathrm{bf}}/\pi$	$\chi^2_{\min}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	α_{21}/π (mod 1)	α_{31}/π (mod 1)
NO	U_6^{VIII}	$\frac{\pi}{4}$	$\frac{\pi}{2}$	0.0676	0.6838	4.680	0.307	0.02223	0.511	1.458*	0.791*	0.854*

mixing angles and CP phase can be accommodated



$\sin^2\theta_{23} \approx 0.51,$
$\delta_{CP} \approx 1.46\pi.$

Summary

Case	Fixed column	Case	Fixed entry
U^{I}	$\sqrt{\frac{2}{3}}\left(\sin\varphi_1, \cos(\varphi_1-\frac{\pi}{6}), \cos(\varphi_1+\frac{\pi}{6})\right)^T$	U^{VI}	$\frac{\phi_g}{2}, \frac{1}{2}, \frac{1}{2\phi_g}$
U^{II}	$\frac{1}{\sqrt{3}}(1,1,1)^T$	U^{VII}	$\frac{1}{2}$
U^{III}	$\left(\sqrt{\frac{1}{\sqrt{5}\phi_g}}, \sqrt{\frac{\phi_g}{2\sqrt{5}}}, \sqrt{\frac{\phi_g}{2\sqrt{5}}}\right)^T$	U^{VIII}	$\cos\varphi_1$
U^{IV}	$\frac{1}{2}(\phi_g, \phi_g-1, 1)^T$	U^{IX}	$\frac{1}{2}$
U^{V}	$\frac{1}{\sqrt{3}}(1,1,1)^T$	U^{X}	$\frac{1}{\sqrt{2}}$

- We confront the lepton mixing predictions derived using discrete flavor and CP symmetries with the first JUNO data and the results of latest global neutrino data analysis.

- The JUNO 59.1-day dataset has ruled out part of mixing patterns with one parameter, and the minimal group that can generate viable predictions is S_4.

- Almost all two-parameter mixing patterns that can accommodate NuFIT and current JUNO results, the minimal group that can generate viable predictions is D_4.

- Future high precision JUNO (6 years), DUNE and T2HK results can test these mixing patterns.

Thanks !

Backup

Breaking pattern with one parameter

$$G_f \rtimes H_{CP}$$

charged lepton

neutrino

$$Z_n^{g_l}, n \geq 3 \quad \text{or} \quad K_4$$

$$Z_2^{g_\nu} \times X_\nu$$

$$U_l = \Sigma_l P_l Q_l \qquad U_\nu = \Sigma_\nu R_{23}(\theta) P_\nu Q_\nu$$

$$U_{PMNS}(\theta) = Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta) P_\nu Q_\nu$$

- Flavor and CP transformation:

$$l_L \to G_l l_L, \quad \nu_L \to G_\nu \nu_L, \quad \nu_L(x) \to i X_\nu \gamma^0 C \bar{\nu}_L^T(x')$$

- The invariant of Lagragian:

$$\mathcal{L} = -\bar{l}_R m_l l_L - \frac{1}{2} \nu_L^T C m_\nu \nu_L + h.c.$$

$$G_l^\dagger m_l^\dagger m_l G_l = m_l^\dagger m_l$$

$$G_\nu^T m_\nu G_\nu = m_\nu, \quad X_\nu^T m_\nu X_\nu = m_\nu^*$$

$$U_l^\dagger m_l^\dagger m_l U_l = \hat{m}_l^2$$
$$U_l^\dagger G_l U_l = \text{diag}\left(e^{i\alpha_e}, e^{i\alpha_\mu}, e^{i\alpha_\tau}\right)$$

$$U_\nu^T m_\nu U_\nu = \hat{m}_\nu$$
$$U_\nu^\dagger G_\nu U_\nu = \text{diag}\left(\pm1, \pm1, \pm1\right)$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T,$$
$$\Sigma_\nu^\dagger G_\nu \Sigma_\nu = \pm\text{diag}(1, -1, -1)$$

Takagi factorization

$$U_{\text{PMNS}} \equiv U_l^\dagger U_\nu$$

- All mixing angles and CP phases are expressed in term of one free parameter $\theta \in [0, \pi)$.
- One column of the lepton mixing matrix is fixed.

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \mathrm{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}) , \tag{1.1}$$

NuFIT 6.0 (2024)

$$|U|_{3\sigma}^{\text{IC19 w/o SK-atm}} = \begin{pmatrix} 0.801 \to 0.842 & 0.519 \to 0.580 & 0.142 \to 0.155 \\ 0.248 \to 0.505 & 0.473 \to 0.682 & 0.649 \to 0.764 \\ 0.270 \to 0.521 & 0.483 \to 0.690 & 0.628 \to 0.746 \end{pmatrix}$$

$$U|_{3\sigma}^{\text{IC24 with SK-atm}} = \begin{pmatrix} 0.801 \to 0.842 & 0.519 \to 0.580 & 0.142 \to 0.155 \\ 0.252 \to 0.501 & 0.496 \to 0.680 & 0.652 \to 0.756 \\ 0.276 \to 0.518 & 0.485 \to 0.673 & 0.637 \to 0.743 \end{pmatrix}$$