理论物理中心年终汇报

龚明

量子信息重点实验室，中国科学技术大学（USTC）

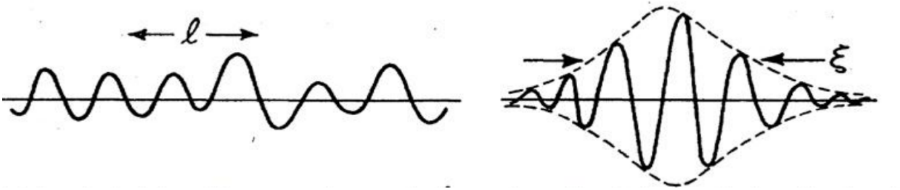
2025 年 12 月 20 日

Outline

1. **Anderson localization and mobility edges**
2. **Brownian motion in periodic and quasi-periodic systems**
3. **Perfect state transfer and remote entanglement generation with SC qubits**
4. Conclusion

(I) Anderson Localization (AL)

$$(\frac{p^2}{2m} + U(\mathbf{x}))\psi_\alpha = \epsilon_\alpha \psi_\alpha.$$

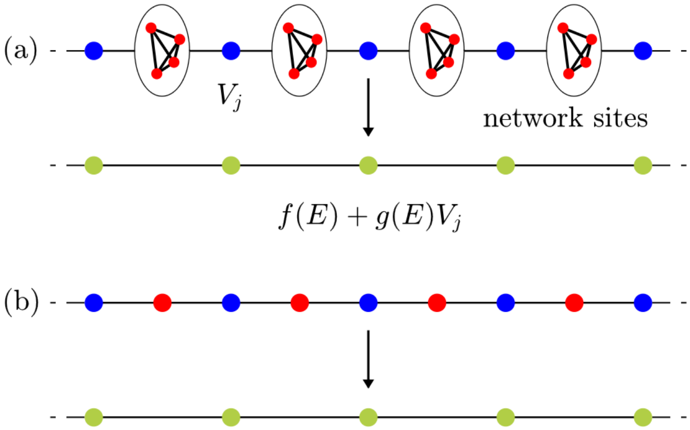


1. The wave function density $|\psi(\mathbf{r})| \sim e^{-|\mathbf{r}|/\xi}$ from the coherent backscattering.
2. Localization in 1d, 2d ($W_c = 0$), and finite W_c in 3d with random potentials.
3. Ubiquitous in all physical models.

Anderson, Phys. Rev. (1957); Lee and Ramakrishnan, Rev. Mod. Phys. (1985).

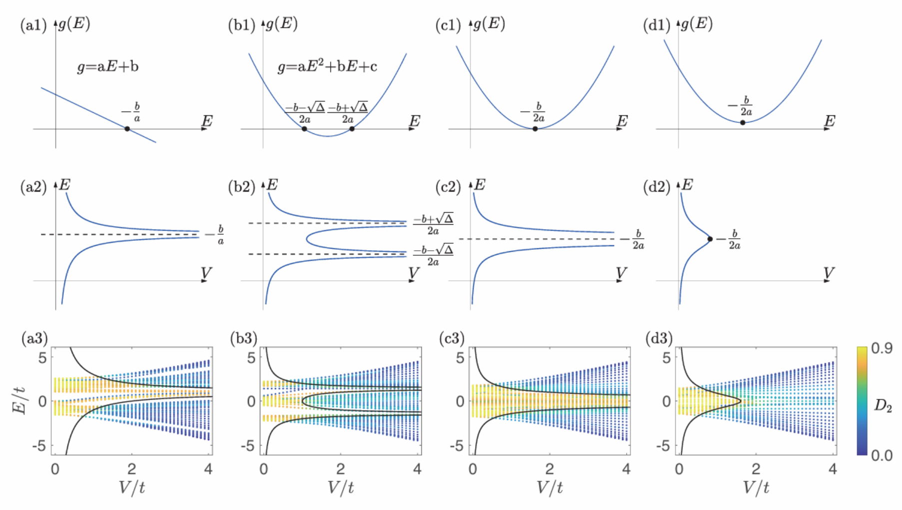
Refs: Xiaoshui Lin et al, PRA, 2024; PRB, 2023; Haitao Hu et al, PRL, 2025; PRB, 2025; PRB, 2025; Yang Chen et al, arXiv:2504.12595.

Basic idea



From lattice to network ⇒ Localization and Topological phase transition, with energy-dependent parameters. For example

$$H_{\text{eff}} = \sum_i g(E) V \cos(\alpha i) c_i^\dagger c_i + t^\kappa c_i^\dagger c_{i+1} + \text{h.c.}, \quad H\psi = f(E)\psi.$$



The simplest picture to understand ME, Anderson transition and resonant state in 1d quasi-periodic network models. Haitao Hu, PRL, 2025.

The future of MEs

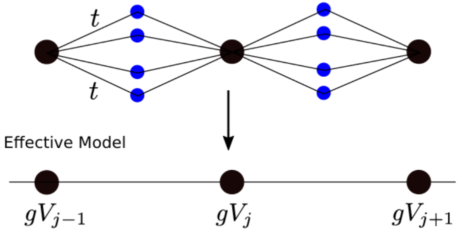
Now the searching of MEs is almost **completely** resolved.

$$H_{\text{eff}} = \sum_i V_i(E) c_i^\dagger c_i + t_i(E) c_i^\dagger c_{i+1} + \text{h.c.}, \quad H\psi = f(E)\psi.$$

In terms of

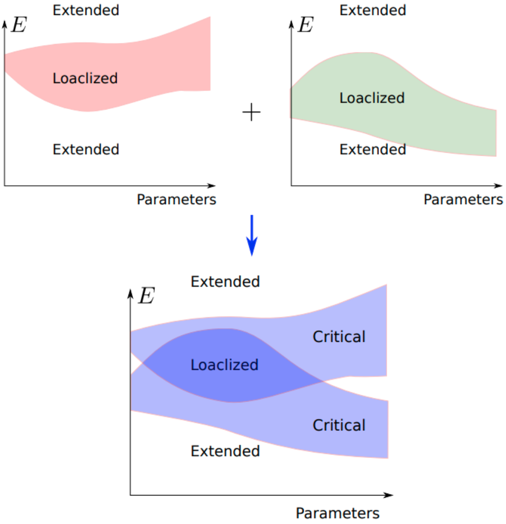
$$V_i \to V_i(E), \quad t \to t_i(E).$$

for various MEs in Anderson localization.



Haitao Hu et al, PRB, 2025, for quasi-periodic network model and Anderson transition.

Fate of overlap wave functions



The overlapped regime may become a critical phase (XS Lin et al, 2024; 2023)
$\longrightarrow$ an unresolved question, which needs to be explored in future.

(I) Conclusion

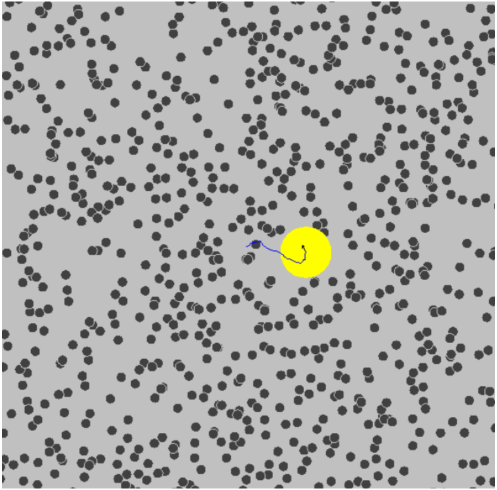
1. A simple yet straightforward way for EMs from H_{eff}, in which we can realize any types of MEs, using

$$H_{\text{eff}} = \sum_i V_i(E) c_i^\dagger c_i + t_i(E) c_i^\dagger c_{i+1} + \text{h.c.}, \quad H\psi = f(E)\psi,$$

   in which $V_i(E)$, $t_i(E)$ and $f(E)$ can be designed arbitrary.

2. A possible general way for critical phase by interaction between localized phase and extended phase with quasi-periodic potentials.

(II) Brown motion in statistical physics



Our question: Relation between Gaussian distribution and

$$D = \frac{\langle x^2 \rangle}{2t}.$$

Einstein Relation and Gaussian PDF

For the following Brown motion

$$\dot{x} = \xi,$$

the related diffusion function and Gaussian distribution are

$$\frac{p(x,t)}{\partial t} = D\frac{\partial^2 p(x,t)}{\partial x^2}, \quad p(x,t) = \frac{1}{\sqrt{4\pi Dt}}\exp\left(-\frac{x^2}{4Dt}\right).$$

A direct calculation yields

$$\langle x^2 \rangle = \int p(x,t) x^2 dx = 2Dt.$$

Einstein Relation and Gaussian PDF

For the following Brown motion

$$\dot{x} = \xi,$$

the related diffusion function and Gaussian distribution are

$$\frac{p(x,t)}{\partial t} = D\frac{\partial^2 p(x,t)}{\partial x^2}, \quad p(x,t) = \frac{1}{\sqrt{4\pi Dt}}\exp\left(-\frac{x^2}{4Dt}\right).$$

A direct calculation yields

$$\langle x^2 \rangle = \int p(x,t) x^2 dx = 2Dt.$$

Our question: If $\langle x^2 \rangle = 2Dt$, what is the possible properties of $p(x,t)$, and its relation to Gaussian PDF? We may expect

$$p(x,t) \sim f(x,t) \times \text{Gaussian},$$

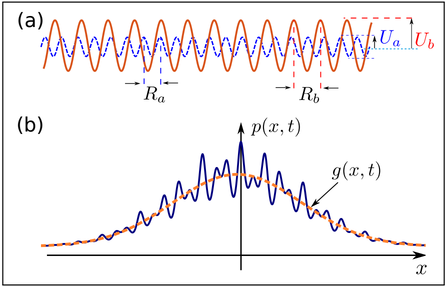
which may be *implied* in a lot of literature.

Brownian motion in quasi-periodic potentials

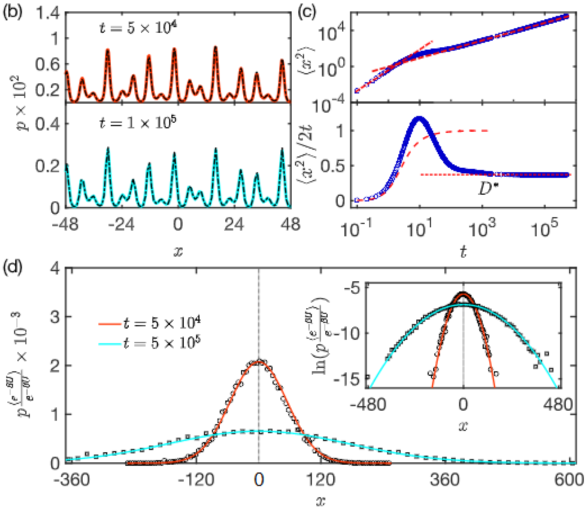
We find that the Lifson-Jackson formula in periodic system should be modified

$$D^* = \lim_{t\to\infty} \frac{\langle x^2 \rangle}{2t} = \frac{D_0}{\langle \exp(\beta U)\rangle \langle \exp(-\beta U)\rangle},$$

$$\langle \exp(-\beta U)\rangle = \lim_{L\to\infty} \frac{1}{L}\int_0^L dx e^{-\beta U(x)}, \quad p(x,t) \sim e^{-\beta U(x)} \times \exp\left(-\frac{x^2}{4D^* t}\right).$$



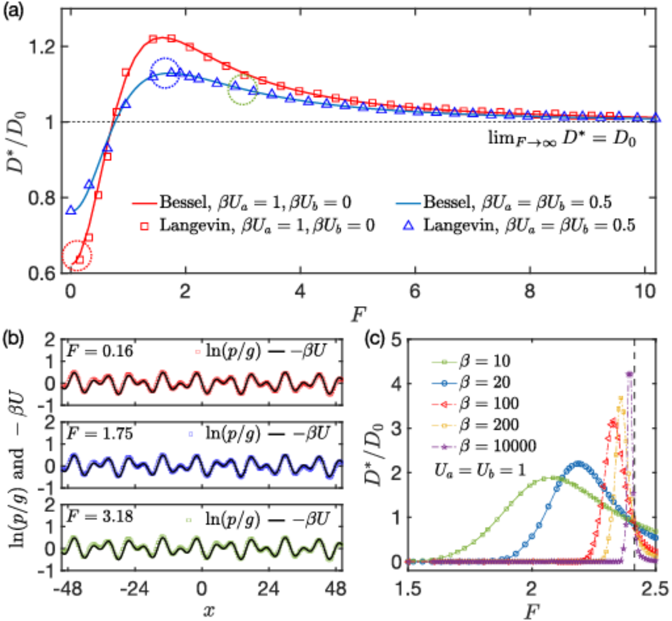
Yang et al, 2025; Ming Gong, 2025. This formula, which is elegant and simple, will find applications in levitated particles and active cells.



Demonstration of the PDF with high accuracy, showing that this result is exact (Ming Gong, 2025) that (when t is large)

$$p(x,t) \sim e^{-\beta U(x)} \times \exp\left(-\frac{x^2}{4D^* t}\right).$$

Giant diffusion with quasi-periodic potential



Exact proof based on first-passage time

Proof of the exact diffusion constant via first passage time in quasi-periodic potentials

Ming Gong[1,2,3,4]

[1] *Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei 230026, China*
[2] *Anhui Province Key Laboratory of Quantum Network, University of Science and Technology of China, Hefei 230026, China*
[3] *Hefei National Laboratory, University of Science and Technology of China, Hefei 230088, China*
[4] *Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei 230026, China*

(Dated: October 14, 2025)

Brownian motion in terms of Lifson and Jackson (LJ) formula has been widely explored in periodic systems and it has been believed for a long time that the LJ formula only applies to periodic potentials. Recently we show that for the following Brownian motion $\gamma \dot{x} = -U'(x) + \xi$, where $U(x)$ is the quasi-periodic potential, the effective diffusion constant can still be described by the LJ formula $D^* = D/(\langle \exp(\beta U)\rangle \langle \exp(-\beta U)\rangle)$, where the average is redefined as $\langle \exp(\beta U)\rangle = \lim_{L\to\infty} L^{-1}\int_0^L \exp(\beta U(x))dx$. In this manuscript we prove this result exactly using the mean first passage time $\tau(x)$, with boundary conditions $\tau(\pm L) = 0$, and show that the effective diffusion constant can be determined using $D^* = \lim_{L\to\infty} L^2/(2\tau(0))$, where $\pm L$ is the two positions of the absorbing boundary. We exactly solve the equation of motion of $\tau(x)$ and obtain the above result with the aid of Jacobi-Anger expansion method. Our result can be generalized to the other potentials and even higher dimensions, which can greatly broaden our understanding of Brownian motion in more general circumstances. The requirement for a well-defined effective diffusion constant D^* in more general potentials is also discussed.

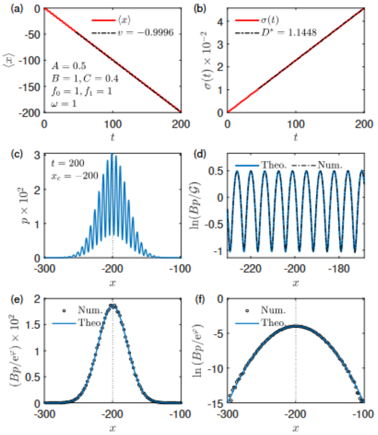
Solve the equation of first-passage time and obtain D^*

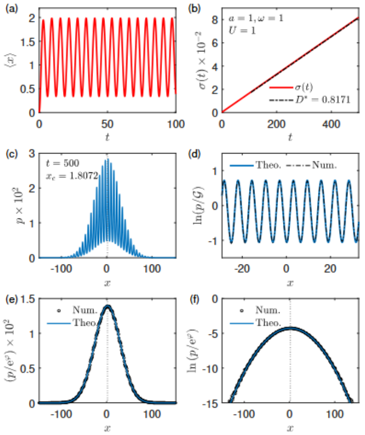
$$De^{-\Phi(x)}\frac{\partial}{\partial x}e^{\Phi(x)}\frac{\partial}{\partial x}\tau(x) = 0, \quad D^*\frac{\partial^2 \tau(x)}{\partial x^2} = -1.$$

PDF with time-dependent periodic potentials

For the following Brownian motion with a time-dependent potential

$$\gamma \dot{x} = -\frac{\partial U(x,t)}{\partial x} + \xi, \quad p(x,t) \sim \exp(\varphi) \times \text{Gaussian}.$$





In which both $U(x,t)$ and φ are periodic function of x; Boxuan Han et al, 2025.

Solution of the Boltzman weight

For the following solution

$$p(x,t) \sim \underbrace{\exp(\varphi)}_{w,\ \text{Boltzman weight}} \times \text{Gaussian}, \quad \phi = \beta U(x,t),$$

we provide a way to determine φ, that

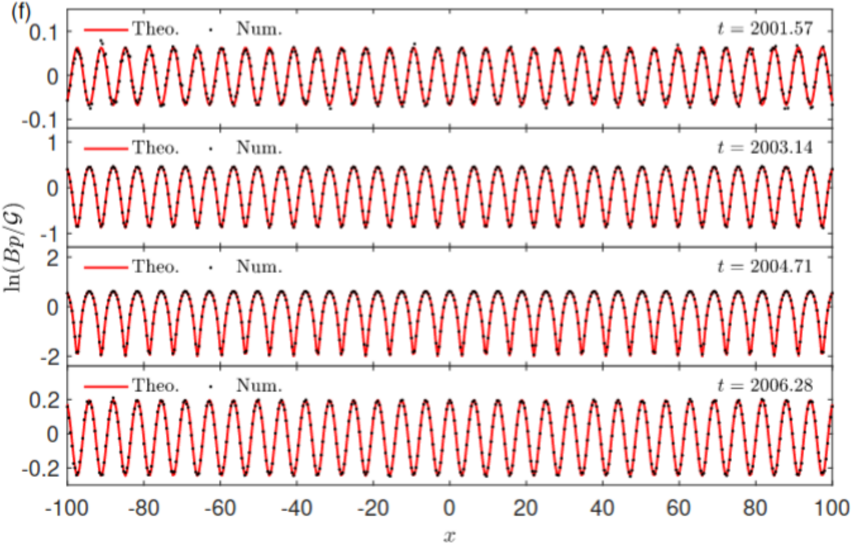
$$\frac{\partial w}{\partial t} + \phi_x w = f(t),$$

for any function $f(t)$. Thus

$$\phi_x = \frac{f(t)}{w(x,t)} - \partial_x \ln w(x,t).$$

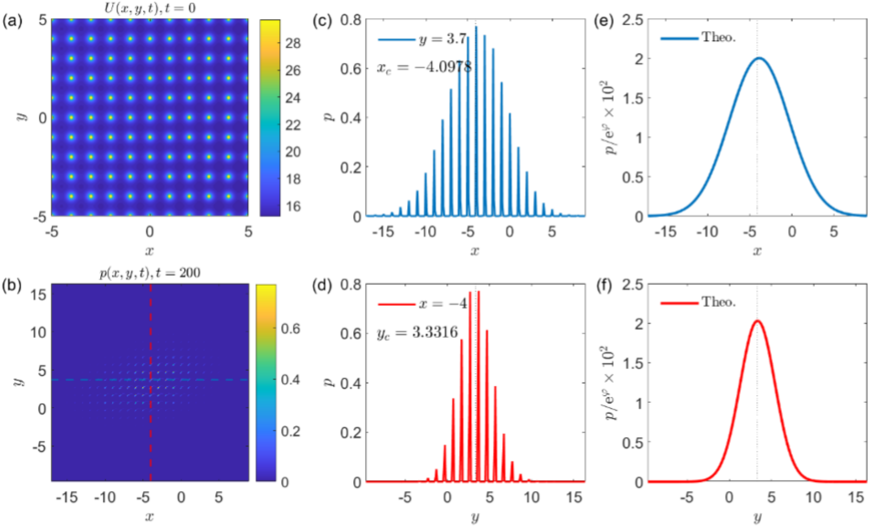
- A lot of potentials $\phi(x,t)$ can yield the same Boltzmann weight by changing of $f(t)$.
- $w(x)$ can be time-independent, while $\phi(x,t)$ can be time-dependent, which is new to this community.

Numerical confirmation



$$w(x,t) = B + A\cos(\omega t)\cos(x) + C\cos(x).$$

Brownian motion in 2D periodic potentials



$$p(x,y,t) \sim \exp(\varphi(x,y,t)) \times \text{Gauss.}$$

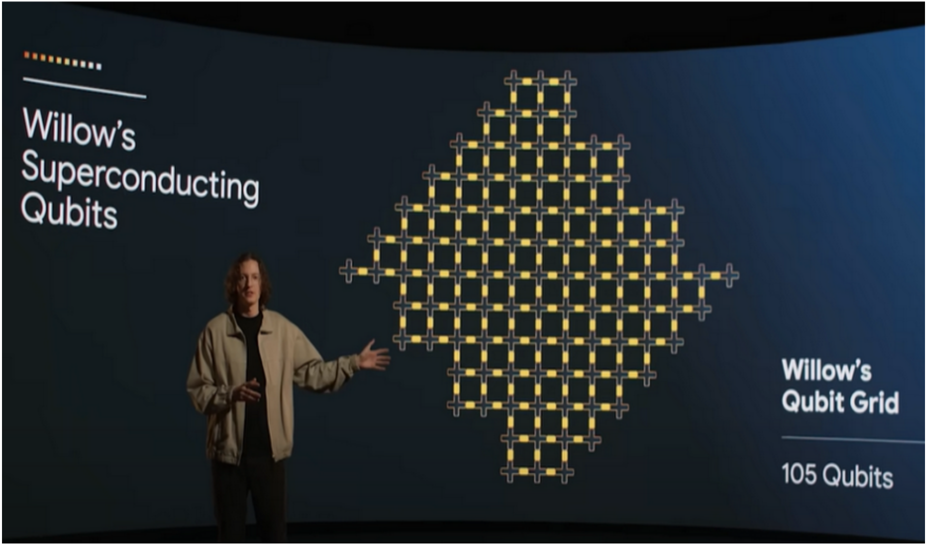
(II) Conclusion

1. Generalize the Brownian motion from periodic potentials to quasi-periodic potentials and then to more general bounded potentials.
2. Generalize the Brownian motion to time-dependent potentials, show that

$$p(x, t) \sim \exp(\varphi) \times \text{Gaussian.}$$

3. Demonstrate the similar solution in 2D (including time-dependent) periodic potentials.

(III) Google Willow with 105 SC qubits, 2025



Our question: How to realize various quantum states (especially entanglement) in these physical systems?

PST with Spin model in quantum simulations

VOLUME 92, NUMBER 18 PHYSICAL REVIEW LETTERS week ending 7 MAY 2004

Perfect State Transfer in Quantum Spin Networks

Matthias Christandl,[1,*] Nilanjana Datta,[2] Artur Ekert,[1,3] and Andrew J. Landahl[4,5]

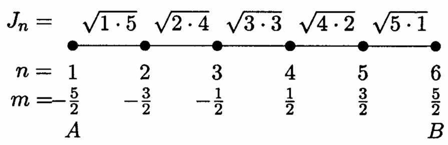
Consider the following **Spin model**

$$H = \sum_n J_n(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y) = \frac{1}{2}\sum_n J_n(\sigma_n^+ \sigma_{n+1}^- + \text{h.c.}) = \frac{J}{2} S_x,$$

with

$$J_n = \sqrt{n(N-n)}, \quad n = 1, 2, \cdots, N-1.$$

When $N = 6$



Spin ladder operator vs PST

For S=5/2

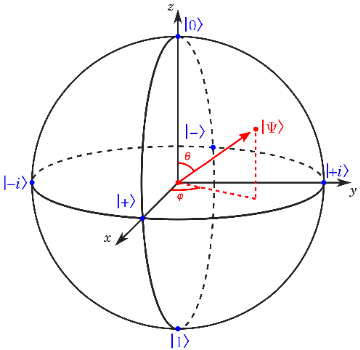
$$S_x = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{5} & 0 & 0 & 0 & 0 \\ \sqrt{5} & 0 & \sqrt{8} & 0 & 0 & 0 \\ 0 & \sqrt{8} & 0 & \sqrt{9} & 0 & 0 \\ 0 & 0 & \sqrt{9} & 0 & \sqrt{8} & 0 \\ 0 & 0 & 0 & \sqrt{8} & 0 & \sqrt{5} \\ 0 & 0 & 0 & 0 & \sqrt{5} & 0 \end{pmatrix} \qquad S_y = \frac{1}{2i}\begin{pmatrix} 0 & \sqrt{5} & 0 & 0 & 0 & 0 \\ -\sqrt{5} & 0 & \sqrt{8} & 0 & 0 & 0 \\ 0 & -\sqrt{8} & 0 & \sqrt{9} & 0 & 0 \\ 0 & 0 & -\sqrt{9} & 0 & \sqrt{8} & 0 \\ 0 & 0 & 0 & -\sqrt{8} & 0 & \sqrt{5} \\ 0 & 0 & 0 & 0 & -\sqrt{5} & 0 \end{pmatrix}$$

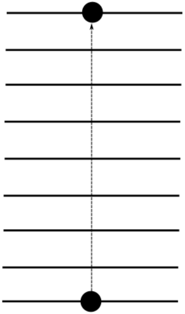
$$S_z = \begin{pmatrix} 5/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5/2 \end{pmatrix}$$

$$S_+ = \begin{pmatrix} 0 & \sqrt{5} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad S_- = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{5} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{5} & 0 \end{pmatrix}$$

For the spin ladder operator

$$S^\dagger |J, m\rangle = \sqrt{J(J+1) - m(m+1)}\,|J, m+1\rangle.$$





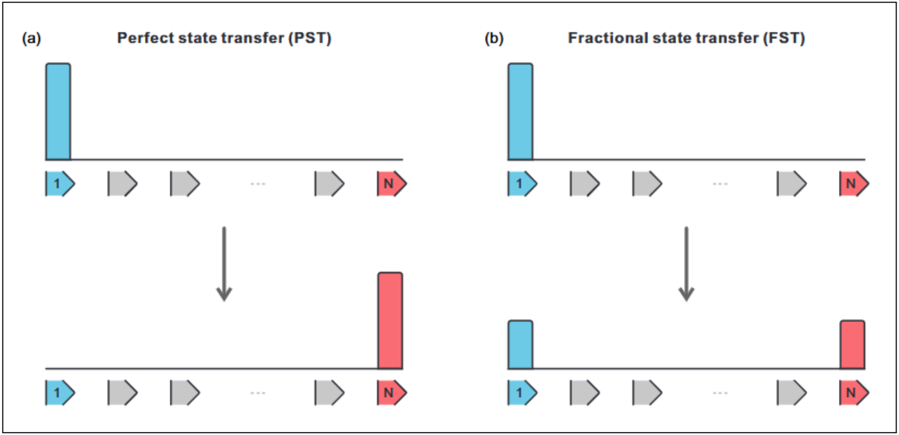
The PST is equivalent to transfer the ground state $|J, -J\rangle$ to the highest energy state $|J, J\rangle$. A beautiful idea by Christandl et al, 2004, with $N = 2J + 1$.

In the subspace A_G with one spin excitations: $S_i^\dagger |\downarrow\rangle^{\otimes N}$, $i = 1, 2, 3, \cdots, N$, the spin model H is reduced to

$$H = \frac{1}{2} \sum_n J_n c_n^\dagger c_{n+1} + \text{h.c.}, \quad \Rightarrow \quad H = S_x.$$

With matrix elements the same as the spin ladder operator.

PST, FST and remote entanglement generation

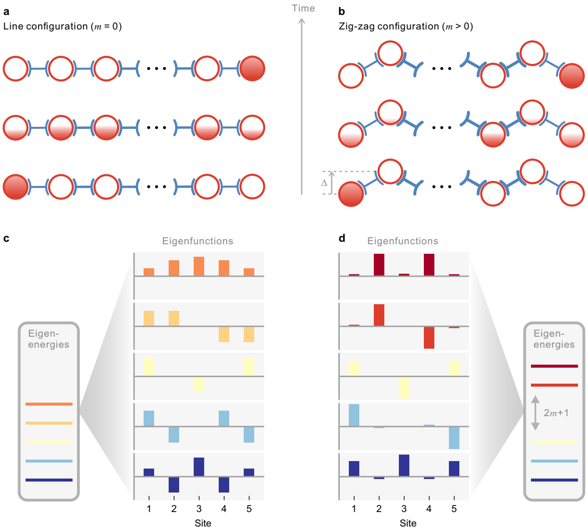


Based on FST we can realize entanglement

$$U|00000001\rangle = \frac{1}{\sqrt{2}}(|00000001\rangle + |10000000\rangle).$$

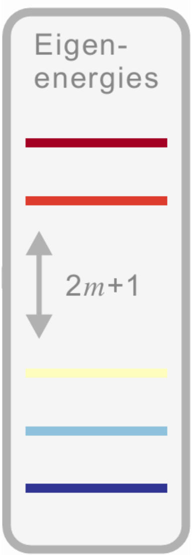
We realize this physic using 1x3, 1x5 and 3x3 systems with SC qubits.

Our New scheme



Tianle Wang et al, arXiv:2506.06669 (experiment), arXiv:2510.13584 (theory).

Hamiltonian and spectra



$$H = \sum_n \omega_n \sigma_n^z + \frac{1}{2} J_{n,n+1}(\sigma_n^\dagger \sigma_{n+1} + \text{h.c.}), \quad \text{where}$$

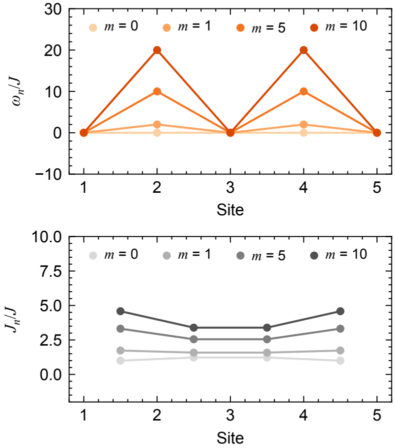
$$\begin{aligned}
\omega_n &= \mu_n \cdot 2m \cdot J, \\
J_n &= \frac{J}{2}\sqrt{[n + \mu_n 2m][N - n + \mu_{n+1} 2m]}, \\
\mu_n &= \{1, 0, 1, 0, 1, 0, 1, 0, \cdots\}.
\end{aligned}$$

The eigenvalues of this model is, with m,

$$E_n = -\frac{N-1}{2}, -\frac{N-1}{2} + 1, -\frac{N-1}{2} + 2, \cdots, -1, 0,$$
$$2m + 1, 2m + 2, \cdots, 2m + \frac{N-1}{2},$$

with an energy gap Δ above the zero point

$$\Delta = (2m + 1), \quad m \in \mathbb{Z}, \text{ or } , \mathbb{Q}.$$



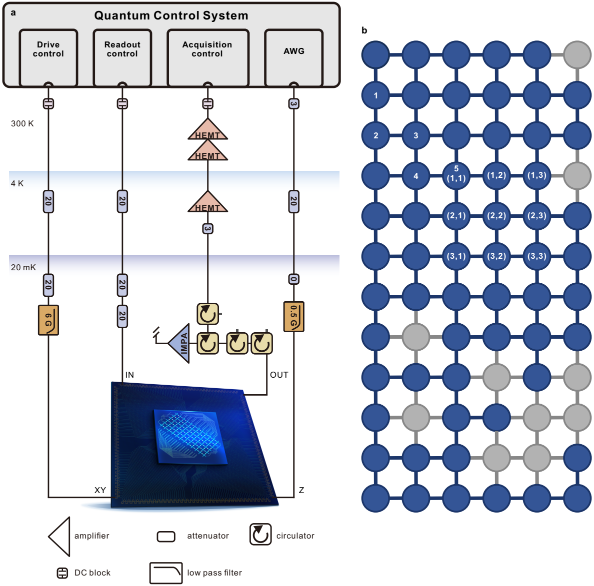
- Zig-zag configuration for any $m \in \mathbb{Z}^+$.
- When $m = 0$, the model reduces to the line configuration.
- When $m \to \infty$, the occupation on even sites vanishes

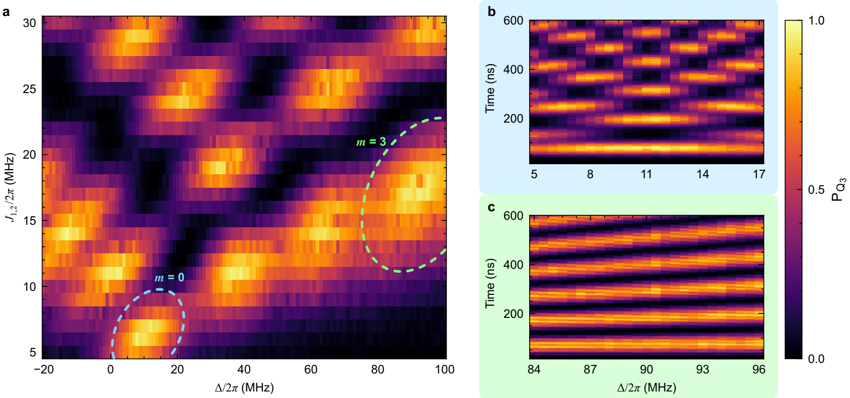
$$\omega_n^{\text{eff}} = -\frac{J(N-1)}{4} = \text{const},$$

$$J_n^{\text{eff}} = -\frac{J}{2}\sqrt{\frac{n+1}{2}\left(\frac{N+1}{2} - \frac{n+1}{2}\right)},$$

the model reduces to the line configuration with $(N+1)/2$ sites.

Qubits in the SC chip



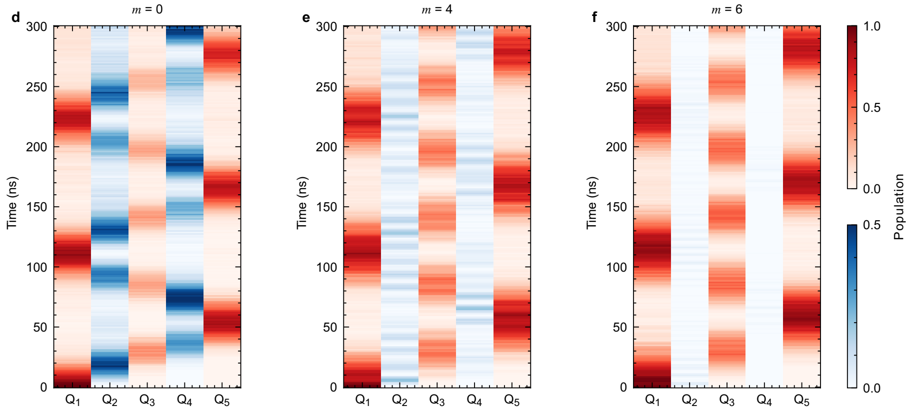


Analytical solution for a 1×3 qubit chain, yielding the population of Q_3 at time τ as

$$P_{Q_3} = \frac{1}{4}(\cos\frac{\Omega\tau}{2} - \cos\frac{\Delta\tau}{2})^2 + \frac{1}{4}(\frac{\Delta}{\Omega}\sin\frac{\Omega\tau}{2} - \sin\frac{\Delta\tau}{2})^2,$$

where $\Omega^2 = \Delta^2 + 8J_{1,2}^2$, with J_1 and J_2 depending on m. $P_{Q_3} = 1$ is met when the ω_n and $J_{n,n+1}$ conditions are satisfied.

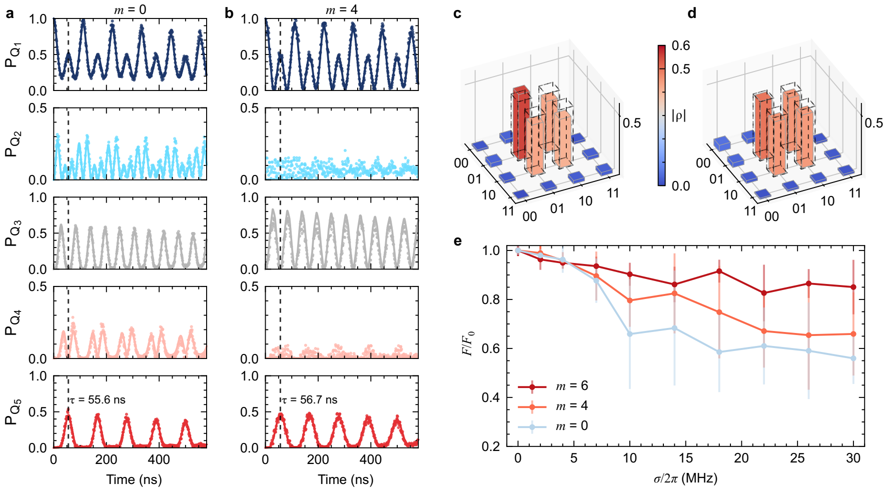
PST versus m in a 1×5 qubit chain

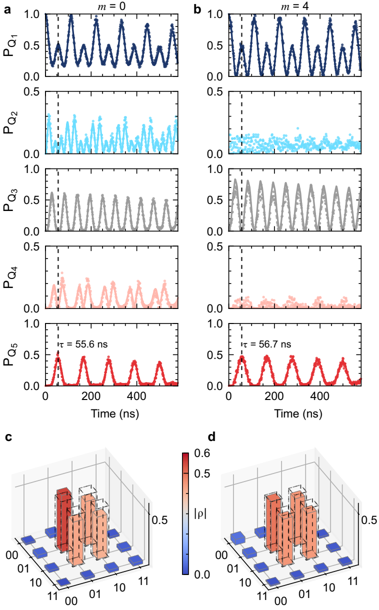


For $m = 6$, almost no visible population remains on qubits Q_2 and Q_4, indicating that the noise channels and the associated dissipation of the even qubits have been effectively switched off.

Expected that m increases $\Rightarrow$ More robust against noise and dissipation .

Effect of m


a
m = 0
b
m = 4
c
d
e
P_{Q_1}
P_{Q_2}
P_{Q_3}
P_{Q_4}
P_{Q_5}
τ = 55.6 ns
τ = 56.7 ns
Time (ns)
|ρ|
00
01
10
11
F/F_0
m = 6
m = 4
m = 0
$\sigma/2\pi$ (MHz)



1. An isospectral deformation is required from PST to FST

$$\tilde{J}_{\frac{N-1}{2}} = (\cos\theta + \sin\theta) J_{\frac{N-1}{2}},$$
$$\tilde{J}_{\frac{N+1}{2}} = (\cos\theta - \sin\theta) J_{\frac{N+1}{2}}.$$

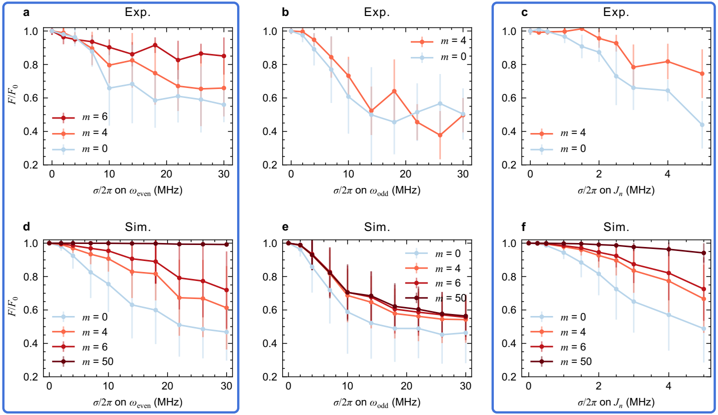
2. FST at half period ($\tau = 55.6$ ns) generates a Bell state between Q_1 and Q_5,

$$|\uparrow_1\downarrow_5\rangle + |\downarrow_1\uparrow_5\rangle.$$

with fidelities of $F = 0.909$ for $m = 0$ and 0.925 for $m = 4$.

3. F increases with increasing of m due to the suppression of noise/dissipation.

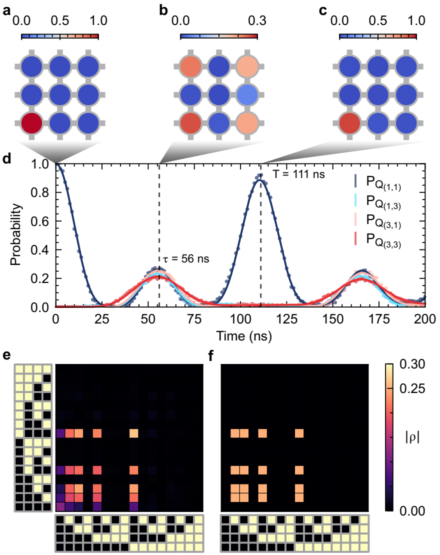
Robustness against noise



Gaussian noise applied to: (a) ω_{even}, (b) ω_{odd}, and (c) J_n.

Robustness against ω_{even} and J_n is enhanced with increasing m, as the even sites are effectively integrated out in this case.

Remote entanglement in 2D (3×3)



1. In the 2D case, $\omega_n^{x,y}$ and $J_n^{x,y}$ must satisfy the FST condition along each dimension.
2. FST at half period ($\tau = 56$ ns) generates a four-qubit W state

$$\frac{1}{2}(|1000\rangle + |0100\rangle + |0010\rangle + |0001\rangle),$$

with a fidelity of 0.85 and probability

$$P = 1/4.$$

3. This is similar to, but not a time-reversal process.
4. The total time

$$\tau \sim 1/g,$$

which is independent of N.

The second Hamiltonian

Question: Can we have some new models for PST?

$$H = \sum_n \omega_n c_n^\dagger c_n + J_n c_n^\dagger c_{n+1} + \text{h.c.}.$$

Eigenvalues

$$\lambda_n = n - \frac{N+1}{2} + (n-2)(n-1)\frac{m}{2}.$$

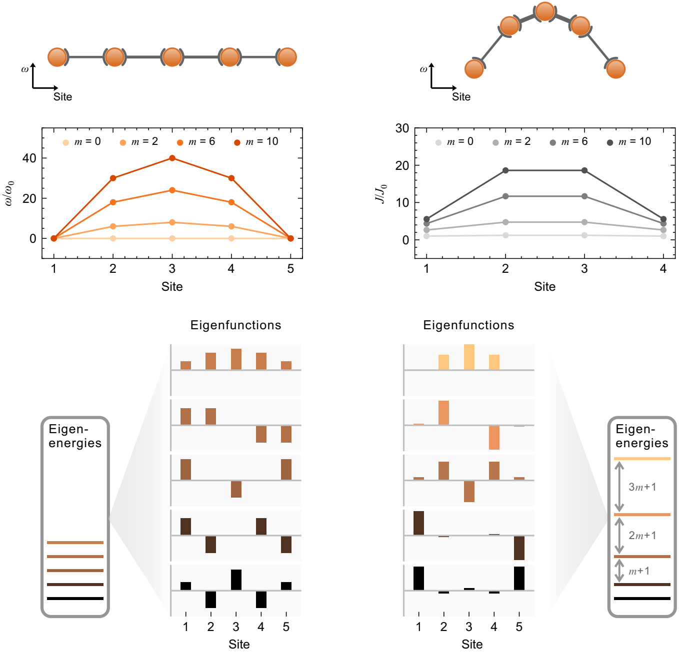
With frequency and coupling terms

$$\omega_n = (N-n)(n-1)m,$$

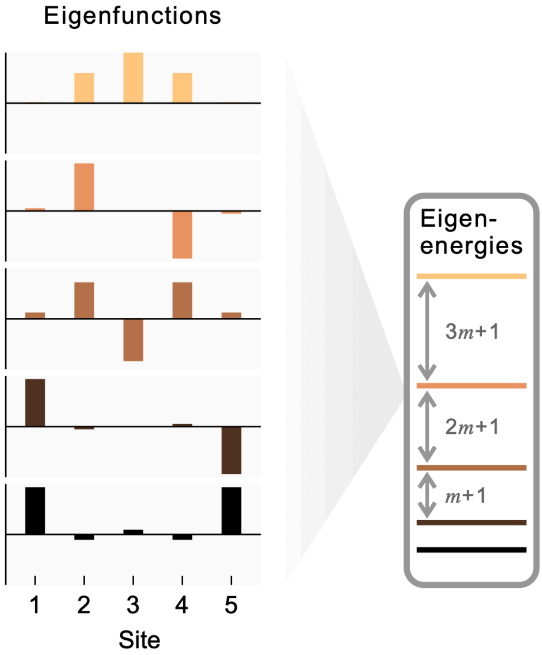
forming a dome-like profile, and

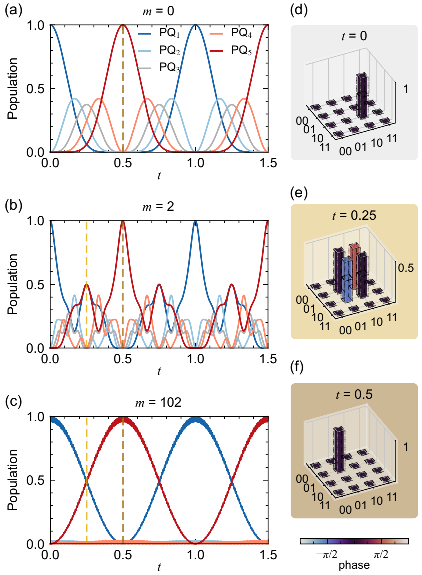
$$J_n = \frac{1}{2}\sqrt{((N-(n+1))nm+n)} \times \sqrt{((N-n)(n-1)m+N-n)}.$$

The low-energy subspace contains only two eigenlevels with energies of -2 and -1, the eigenvectors show significant amplitude suppression at specific sites.



ω
Site
m = 0
m = 2
m = 6
m = 10
40
20
0
ω/ω0
1
2
3
4
5
Site
30
20
10
0
J/J0
1
2
3
4
Site
Eigenfunctions
Eigenfunctions
Eigen-
energies
Eigen-
energies
3m+1
2m+1
m+1
1
2
3
4
5
Site
1
2
3
4
5
Site


Eigenfunctions
1
2
3
4
5
Site
Eigen-
energies
3m+1
2m+1
m+1

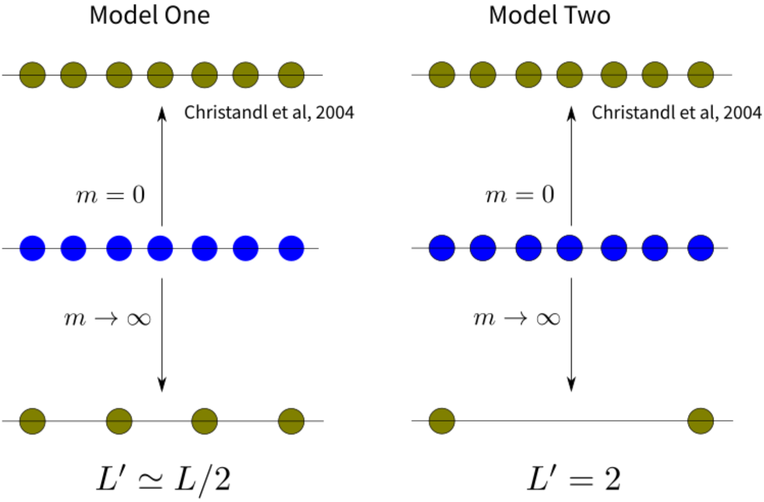


1. When $m = 0$, the excitation evolves from the first qubit to the last qubit after half a period, achieving a PST.
2. When $m = 2$ in the dome model, in addition to PST at $t = 0.5$, FST occurs at $t = 0.25$, generating a remote entanglement.
3. When $m \to \infty$, the population on intermediate qubits are significantly suppressed.
4. As compared with the first scheme with zig-zag

$$\omega_n = \mu_n \cdot 2m \cdot J$$

this new scheme may further suppress the noise effect and dissipation effect.

Limits in these two different models



Using the same idea by Haitao Hu et al, PRL, 2025.

(III) Conclusion

1. Proposed two new physical models for PST and FST, and remote entanglement generation.
2. Realize the FST and PST using 3, 5 and 9 (3×3) SC qubits in experiment.
3. Demonstrate the robust of PST and FST by controlling the parameter m, which control the level spacing between the occupied and unoccupied bands.

谢 谢 大 家

Summary of this talk:

1. Anderson localization and mobility edges.
2. Brownian motion in periodic and quasi-periodic systems.
3. Perfect state transfer and remote entanglement generation with SC qubits.