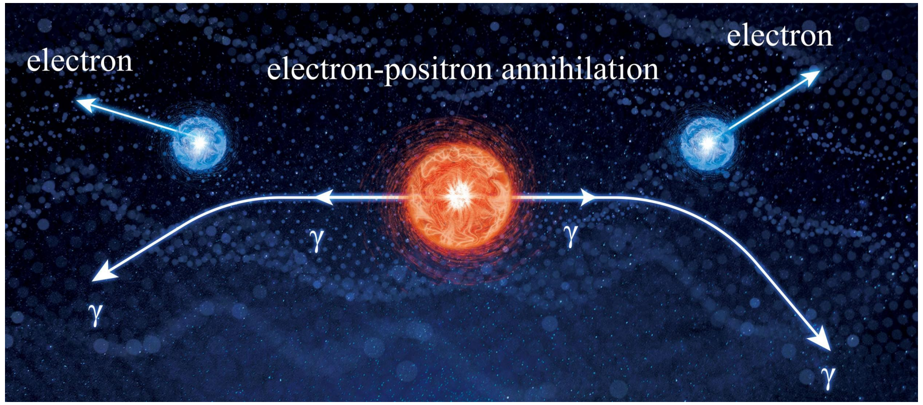
Quantum Entanglement in Particle Physics



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安徽省基础学科研究中心 2025 学术年会，20/12/2025

Outline

- Historic role of particle physics in entanglement studies.
  - Contributions by CS Wu, CN Yang, TD Lee (all heroes of parity revolution, all connected with Yangtz Delta).
- Our work towards quantum HEP
  - Using entanglement for HEP .
- Our work towards HE quantum information.
  - Quantum teleportation.
  - Violation of nonlocal realism (Leggett Inequalities)

EPR 1935 (今年QM100+EPR90)

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

Quantum Entanglement

- In 1935, Einstein-Podolsky-Rosen demonstrated the conflict between local realism and quantum mechanics using quantum entangled states. They claimed that quantum mechanics is incomplete.
- Einstein first unearthed the importance of quantum entanglement (conflicts with local realism), deserves the largest credit on this subject (contrary to the usual say "Einstein was wrong"). [YS, Road to quantum entanglement, Chinese Journal of Nature, 2022]

Schrödinger 1935

- In the same year, Schrödinger coined quantum entanglement:
  - Another way of expressing the peculiar situation is: the best possible knowledge of the a whole does not necessarily include the best possible knowledge of its parts…I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought . By the interaction the two representatives become entangled.

W. Furry 1936

- Schrödinger (and EPR) thought entanglement unreasonable, speculated it might become nonentangled when particles are separated.
- W. Furry thought such a speculation unreasonable.
- People unreasonably called it Furry hypothesis.

MARCH 1, 1936 PHYSICAL REVIEW VOLUME 49

Note on the Quantum-Mechanical Theory of Measurement

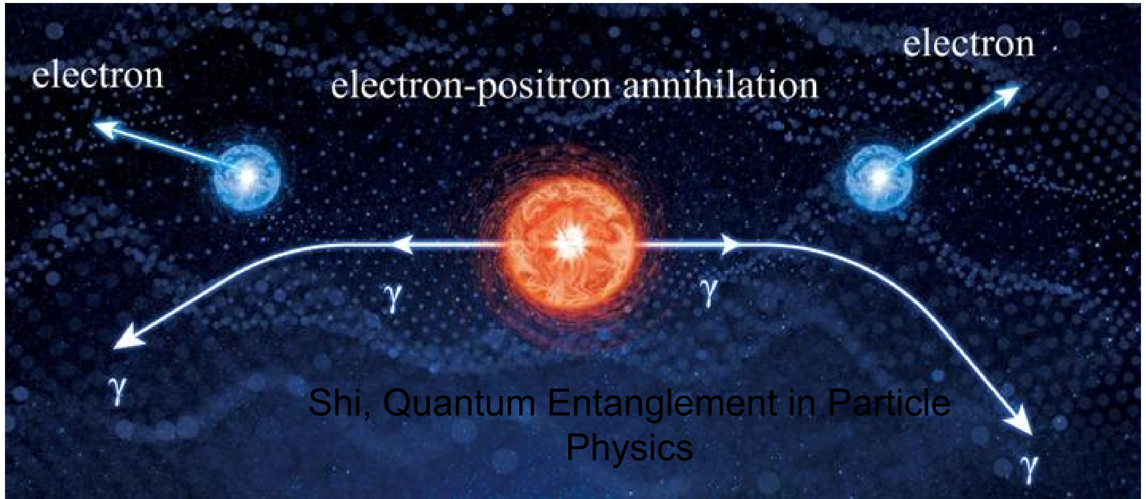
W. H. FURRY, *Department of Physics, Harvard University*

(Received November 12, 1935)

In recent notes by Einstein, Podolsky and Rosen and by Bohr, attention has been called to the fact that certain results of quantum mechanics are not to be reconciled with the assumption that a system has independently real properties as soon as it is free from mechanical interference. We here investigate in general, and in abstract terms, the extent of this disagreement. When suitably formulated such an assumption gives to certain types of questions the same answers as does quantum mechanics; this is true of the formulas usually given in discussions of the theory of measurement. There exists, however, a general class of cases in which contradictions occur. That such contradictions are not restricted to the abstract mathematical theory, but can be realized in the commonest physical terms, is shown by the working out of an example.

A different context: Testing QED

- 1946, Wheeler proposed: asymmetry in coincident measurement of photons produced from the e+e- annihilation.
- Quantitatively corrected by other theorists.
- Two photons from e-e+ annihilation: polarizations are always perpendicular, scattered by electrons respectively.
- Rate symmetries between moving directions being perpendicular and parallel, as a function of scattering angle.



Shi, Quantum Entanglement in Particle Physics

Picture by YS 2023

Wu, Shaknov, 1950

The Angular Correlation of Scattered Annihilation Radiation*

C. S. Wu and I. Shaknov
Pupin Physics Laboratories, Columbia University, New York, New York
November 21, 1949

- Previous experiments not satisfactory.
- Sensitivity of the γ detector by Wu and Shaknov was 10 times that of previous ones.
- Positron source Cu64 was activated by deuterium bombardment in Columbia cyclotron.
- Result: 2.04∓0.08. (Theory: 2.00)

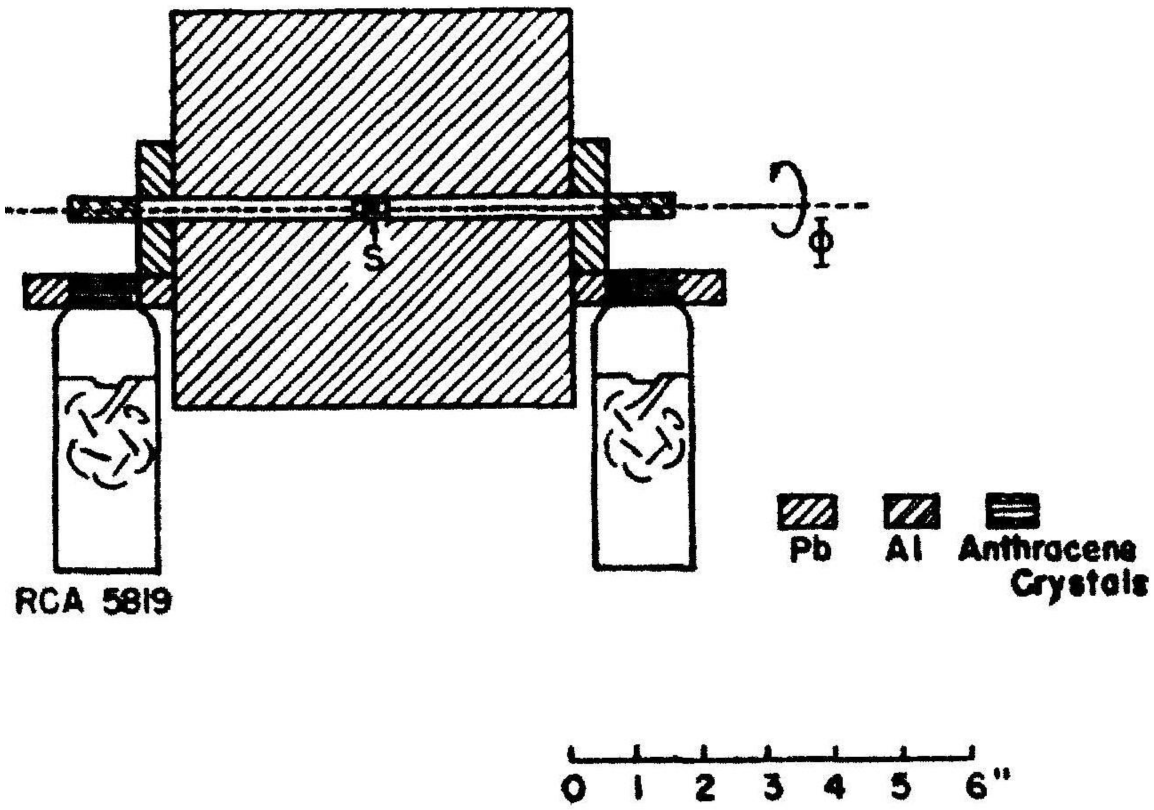
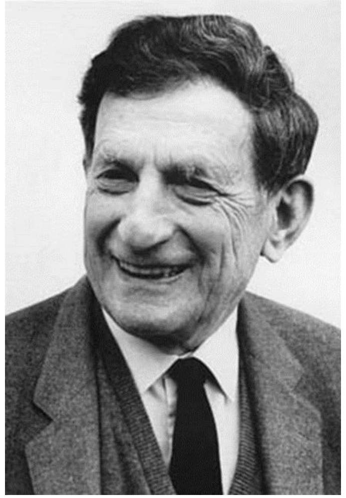


FIG. 1. Schematic diagram of experiment.

Yang's Selection rule (1950)

- Wu-Shaknov experiment was consistent with Yang's selection rule (a particle of spin 1 cannot decay into two photons).
- Based on the invariance of rotation and inversion.
- The first sentence of Yang's paper says that Wheeler had pointed out that a positronium in the triplet state cannot decay through annihilation with the emission of two photons.

David Bohm: Spin Entanglement and Wu-Shaknov Experiment



- 1951, Bohm: discrete (spin-1/2) version of the EPR paradox. Looked for experiments.
- In 1957, Bohm-Aharonov noted that 1950 Wu-Shaknov achieved photon polarization correlation (did not use the word "entanglement").
- B-A proved that the non-entangled state could not give the experimental results of Wu-Shaknov.

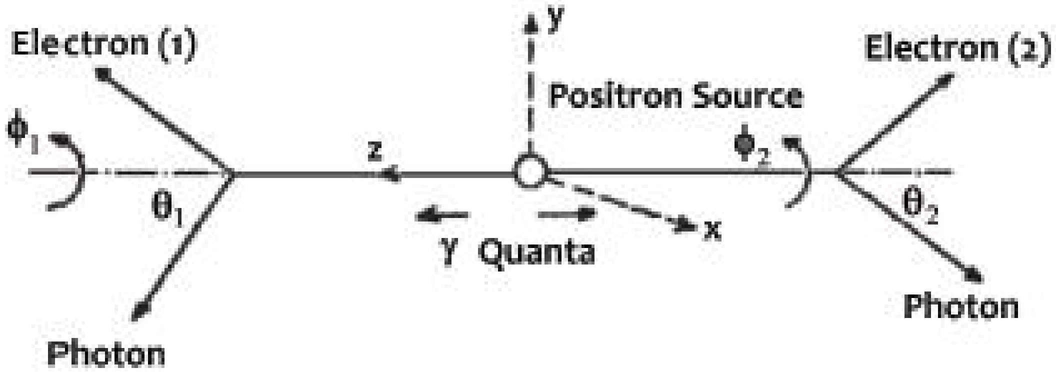
C. N. Yang's appraisal at 2015

C. N. Yang

II

In the same year, she published another paper,[2] "*The Angular Correlation of Scattered Annihilation Radiation*", studying the polarizations of the two γ in

$$e^+ + e^- \to \gamma + \gamma\,.$$



That was the first experiment on *quantum entanglement*, which is a very hot new area of research in the 21st century.

My history talks/articles

- Encouraged by Prof Yang, I had systematically studied Wu's contribution.
- 2022 comment (before Nobel Prize 2022, whose official doc did not mention Wu): "For the first time, Wu-Shaknov experiment achieved a clearly spatially separated quantum entangled state in a controlled experiment".
- YS, CS Wu as the experimental pioneer in quantum entanglement: a 2022 note, Mod. Phys. Lett. A 40, 253000 (2025); arXiv:2502.06458
- Invited talk "Scientific Spirit of Chien-Shiung Wu: From Quantum Entanglement to Parity Nonconservation", Int. Sym. Comm. 110th Birth Ann. of Chien-Shiung Wu Southeast University, 2022.5.31. Chinese transcript: 2023.6.2, English translation: arXiv:2504.16978
- Historical origins of quantum entanglement in particle physics, in Chinese, 2023.3.17; English translation: arXiv:2507.13582

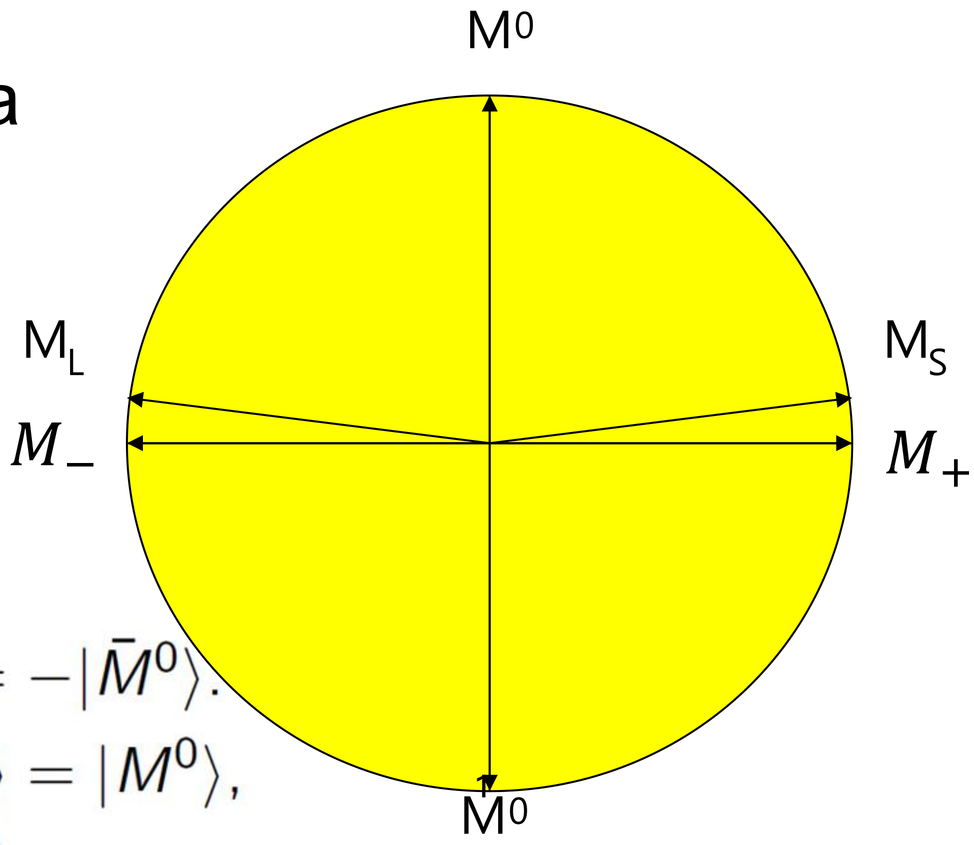
1975, Wu's group attempted to test Bell's inequality

- 1964, Bell inequality. For a Bell test, polarization needs to be measured in a direction that is neither parallel nor perpendicular.
  In 1950 Wu-Shaknov experiment, the azimuth angles of the two photons detected were always perpendicular.
- Clauser visited Wu to confirm this and caused Wu's interest in Bell test.
  In 1975, Chien-Shiung Wu and his students Kasday and Ullman measured the coincidence probability of two photons in a wide range of polar and azimuthal angles.
  ANGULAR-CORRELATION OF COMPTON-SCATTERED ANNIHILATION PHOTONS AND HIDDEN VARIABLES, KASDAY, LR; ULLMAN, JD, WU, CS, 1975 | NUOVO CIMENTO B 25 (2) , pp.633-661.
  The results are consistent with the entangled state and contradict the non-entangled state. But they admitted that it could exclude hidden variables.

Meson in a two dimensional Hilbert space

- Particle and antiparticle form a 2D Hilbert space.
- similar to qubit.

M=K,B,D.



- $\mathcal{F}|M^0\rangle = |M^0\rangle$, $\mathcal{F}|\bar{M}^0\rangle = -|\bar{M}^0\rangle$.
- $CP|M^0\rangle = |\bar{M}^0\rangle$, $CP|\bar{M}^0\rangle = |M^0\rangle$,
- $|M_\pm\rangle = \frac{1}{\sqrt{2}}(|M^0\rangle \pm |\bar{M}^0\rangle)$.
- $CP|M_\pm\rangle = \pm|M_\pm\rangle$.

Lee-Oehme-Yang 1957

- LOY considered every discrete symmetry may be broken, so there exist coherent superpositions of particle and antiparticle. Truly made it analogous to spin ½.
- 8 May 2014, my email to Yang: kaon decay and neutrino oscillation can be described as simple quantum mechanical two-state or three-state systems, under Wigner-Weisskopf approximation, are these approaches started by you?'
- In 12 hours, Yang: 'Yes, the whole mixing matrix idea was initiated by the LOY paper. We used the Weisskopf-Wigner formalism to describe the time evolution of a system in which all 3 discrete symmetries may be broken. At the time, this description was not really needed, since it was believed by everybody that K1 and K2 did not mix, (because of Gell-Mann-Pais). We developed the general case of mixing for completeness. After1964, our formalism became THE FORMALISM. It was generalized later to the 3 neutrino case.'

Historical origins of quantum entanglement in particle physics, in Chinese, 2023.3.17;
English translatiopm: arXiv:2507.13582

Goldhaber-Lee-Yang 1958: the earliest written meson entangled states

- Entangled states of mesons first appeared in a paper by them together with Goldhaber, though did not pay attention to the entanglement.,
- Discussed the quantum state of a pair of K mesons. However, they considered that each particle can be in four basis states, two neutral states and a positive and negative unit charge states, all entangled states.
- Method was similar to Yang's selection .
- I told Prof Yang on 10 Feb 2012.

Historical origins of quantum entanglement in particle physics, in Chinese, 2023.3.17; English translatiopm: arXiv:2507.

Lee-Yang 1961 unpublished work on entangled neutral kaons

- In a meeting of the ZGS Users Group at Argonne National Lab on 28 May 1960, Lee discussed the possibility of correlated kaons similar to EPR question, resulted from proton-antiproton annihilation.
- A paper by Inglis 1961 includes a chapter on the unpublished work of Lee and Yang, which gave the entangled state of neutral kaons similar to the spin singlet state.
- Lee-Yang noted that it is impossible for the two neutral kaons to be observed as both K0 or both K0-bar at a same time.
- They also calculated the probability that the two particles are observed to be both K0-bar at different moments.

Our Work towards quantum high energy physics

1. Using Quantum Entanglement to Study CP and CPT Violations

- Z. Huang, YS, Euro. Phys. J. C 72, 1900 (2012).
- YS, Euro. Phys. J. C 73, 2506 (2013).
- Z. Huang, YS, Phys. Rev. D 89, 016018 (2014).

Time evolution and joint decay of $|\Psi_\mp\rangle$

- $$|\Psi_\mp(t_a,t_b)\rangle = \frac{1}{\sqrt{2}}(|M^0(t_a)\rangle_a|\bar{M}^0(t_b)\rangle_b \mp |\bar{M}^0(t_a)\rangle_a|M^0(t_b)\rangle_b)$$
- For state $|\Psi_\mp(t_a,t_b)\rangle$, the joint rate that Alice decays to $|\psi_a\rangle$ at t_a while Bob decays to $|\psi_b\rangle$ at t_b is

$$I(\psi_a,t_a;\psi_b,t_b) = |\langle\psi_a,\psi_b|\mathcal{H}_a\mathcal{H}_b|\Psi_\mp(t_a,t_b)\rangle|^2.$$

$$\begin{aligned}\langle\psi_a,\psi_b|\mathcal{H}_a\mathcal{H}_b|\Psi_\mp(t_a,t_b)\rangle & \\ &= \frac{1}{\sqrt{2}}(\langle\psi_a|\mathcal{H}_a|M^0(t_a)\rangle\langle\psi_b|\mathcal{H}_b|\bar{M}_0(t_b)\rangle \\ &- \langle\psi_a|\mathcal{H}_a|\bar{M}_0(t_a)\rangle\langle\psi_b|\mathcal{H}_b|M^0(t_b)\rangle).\end{aligned}$$

Integrated rates and asymmetries

Integrated rate

$$I'(\psi^1, \psi^2, \Delta t) = \int_0^\infty I(\psi^1, t_a; \psi^2, t_a + \Delta t) dt_a,$$

which is simply $I(\psi^1, t_a; \psi^2, t_a + \Delta t)$ with $e^{-(\Gamma_S+\Gamma_L)t_a}$ replaced as $1/(\Gamma_S + \Gamma_L)$.

$$A(\psi^1\psi^2, \psi^3\psi^4, \Delta t) \equiv \frac{I'(\psi^1, \psi^2, \Delta t) - I'(\psi^3, \psi^4, \Delta t)}{I'(\psi^1, \psi^2, \Delta t) + I'(\psi^3, \psi^4, \Delta t)}.$$

We use notation $A_\mp$ for $|\Psi_\mp\rangle$.

Theorem 1: For $\Delta t = 0$, any unequal-state asymmetry $A_\mp(\psi^a\psi^b, \psi^b\psi^a; \Delta t = 0)$ always vanishes no matter whether there is CP or CPT violation.

Could be used to confirm the entanglement of the pair.

Semileptonic decays of $|\Psi_-\rangle$ into flavor eigenstates $|l^\pm\rangle$

- Examples for $|l^+\rangle$:
  $M^- \bar{l}\nu$, $D^- D_s^+$, $D^- K^+$, $\pi^- D_s^+$, $\pi^- K^+$ from B^0;
  $D_s^- \pi^+$, $D_s^- D^+$, $K^- \pi^+$, $K^- D^+$ from B_S^0.
- Examples for $|l^-\rangle$:
  $M^+ l\bar{\nu}$, $D^+ D_s^-$, $D^+ K^-$, $\pi^+ D_s^-$, $\pi^+ K^-$ from $\bar{B}^0$;
  $D_s^+ \pi^-$, $D_s^+ D^-$, $K^+ \pi^-$, $K^+ D^-$ from $\bar{B}_s^0$.
- Define:
  $R^+ \equiv \langle l^+|\mathcal{H}|M^0\rangle = a + b$,
  $R^- \equiv \langle l^-|\mathcal{H}|\bar{M}^0\rangle = a^* - b^*$,
  $S^+ \equiv \langle l^+|\mathcal{H}|\bar{M}_0\rangle = c^* - d^*$,
  $S^- \equiv \langle l^-|\mathcal{H}|M_0\rangle = c + d$.
  a, b, c, d: quantities defined in literature.
  - Direct CP conservation $\Longrightarrow R^+ = R^-$ and $S^+ = S^-$.
  - Direct CPT conservation $\Longrightarrow (R^+)^* = R^-$ and $(S^+)^* = S^-$.
  - $\Delta\mathcal{F} = \Delta Q$ rule $\Longrightarrow S^\pm = 0$.

Equal-flavor asymmetry for $|\Psi_-\rangle$

$$A_-(++,--,\Delta t) = \frac{|x_S(S^+)^2 - x_L^{-1}(R^+)^2 + (1-\Omega)R^+S^+|^2 - |x_S(R^-)^2 - x_L^{-1}(S^-)^2 + (1-\Omega)R^-S^-|^2}{|x_S(S^+)^2 - x_L^{-1}(R^+)^2 + (1-\Omega)R^+S^+|^2 + |x_S(R^-)^2 - x_L^{-1}(S^-)^2 + (1-\Omega)R^-S^-|^2}.$$

- **Theorem 2:** $A_-(++,--,\Delta t)$ is always a constant independent of Δt.
- **Theorem 3:** *If $A_-(++,--,\Delta t) \neq 0$, then there exists one or two of the following violations: (1) CP is violated indirectly, (2) both CP and CPT are violated directly.*
- **Theorem 4:** *If $A_-(++,--,\Delta t) \neq 0$ while CPT is assumed to be conserved both directly ($(R^+)^* = R^-$, $(S^+)^* = S_-$) and indirectly ($x_S = x_L = q/p$, $\Omega = 1$), then in addition to indirect CP violation ($q/p \neq 1$), we can draw the following conclusions:*
  (1) $|q/p| \neq 1$, i.e. T must also be violated indirectly;
  (2) $|R^\pm| \neq |S^\pm|$.

Unequal-flavor asymmetry for $|\Psi_-\rangle$

$$A_-(+-,-+,\Delta t) = \frac{(|U|^2-|V|^2)(e^{-\Gamma_L \Delta t}-e^{-\Gamma_S \Delta t})+4\Im(U^* V)\sin(\Delta m \Delta t)}{(|U|^2+|V|^2)(e^{-\Gamma_L \Delta t}+e^{-\Gamma_S \Delta t})+4\Re(U^* V)\cos(\Delta m \Delta t)}.$$

- **Theorem 5** $A_-(+-,-+,\Delta t = 0) = 0$.
- **Theorem 6** *If* $A_-(+-,-+,\Delta t) \neq 0$, *then CP must be violated, directly or indirectly or both.*
- **Theorem 7** *If* $A_-(+-,-+,\Delta t) \neq 0$ *for* $\Delta t \neq 0$ *while CPT is assumed to be conserved both directly and indirectly, then we can draw the following conclusions:*
  (1) $|R^+| = |R^-| \neq |S^+| = |S^-|$;
  (2) $S^{\pm} \neq 0$, *i.e.* $\Delta \mathcal{F} = \Delta Q$ *rule must be violated.*

Hadronic decays of $|\Psi_-\rangle$ into CP eigenstates $|h^\pm\rangle$

- Examples of $|h^+\rangle$: $\pi^+\pi^-$, $\pi^0\pi^0$.
- Examples of $|h^-\rangle$: $\pi^0\pi^0\pi^0$.
- Define:
  $Q^\pm \equiv \langle h^\pm|\mathcal{H}|M_\pm\rangle$,
  $X^\pm \equiv \langle h^\pm|\mathcal{H}|M_\mp\rangle$,
  $\xi_\pm \equiv \frac{X^\pm}{Q^\pm}$.
- They are related to $\eta's$ usually defined:

$$\eta_{h^+} \equiv \frac{\langle h^+|\mathcal{H}|M_L\rangle}{\langle h^+|\mathcal{H}|M_S\rangle} = \frac{\xi^+ + \epsilon_L}{1+\epsilon_S\xi^+},$$
$$\eta_{h^-} \equiv \frac{\langle h^-|\mathcal{H}|M_S\rangle}{\langle h^-|\mathcal{H}|M_L\rangle} = \frac{\xi^- + \epsilon_S}{1+\epsilon_L\xi^-}.$$

  - If CP is conserved directly $\Rightarrow X^\pm = 0$.
  - If CPT is conserved directly $\Rightarrow X^\pm$ is purely imaginary, i.e. $X^\pm = -X^{\pm *}$.
- One can, of course, calculate using basis $|M^0\rangle$ and $|\bar{M}^0\rangle$, rather than $M_\pm\rangle$.

Equal-CP asymmetry for $|\Psi_-\rangle$

Asymmetry:

$$B_-(h_a^x h_b^y, h_a^y h_b^x, \Delta t) = \frac{I[h_a^x,t_a;h_b^y,t_a+\Delta t]-I[h_a^y,t_a;h_b^x,t_a+\Delta t]}{I[h_a^x,t_a;h_b^y,t_a+\Delta t]+I[h_a^y,t_a;h_b^x,t_a+\Delta t]} = \frac{I'[h_a^x,h_b^y,\Delta t]-I'[h_a^y,h_b^x,\Delta t]}{I'[h_a^x,h_b^y,\Delta t]+I'[h_a^y,h_b^x,\Delta t]}.$$

Equal-CP asymmetry: $B_-(++,--,\Delta t) = \frac{P(Q^+,X^+)-P(X^-,Q^-)}{P(Q^+,X^+)+P(X^-,Q^-)}$,

$P(\beta,\gamma) \equiv$
$|-(1-x_L^{-1}+x_S-\Omega)\beta^2+2(x_L^{-1}+x_S)\beta\gamma+(1+x_L^{-1}-x_S-\Omega)\gamma^2|^2.$

Theorem 9: $B_-(++,--,\Delta t)$ *is always a constant independent of* Δt.

Simple results concerning joint decays of $|\Psi_+\rangle$

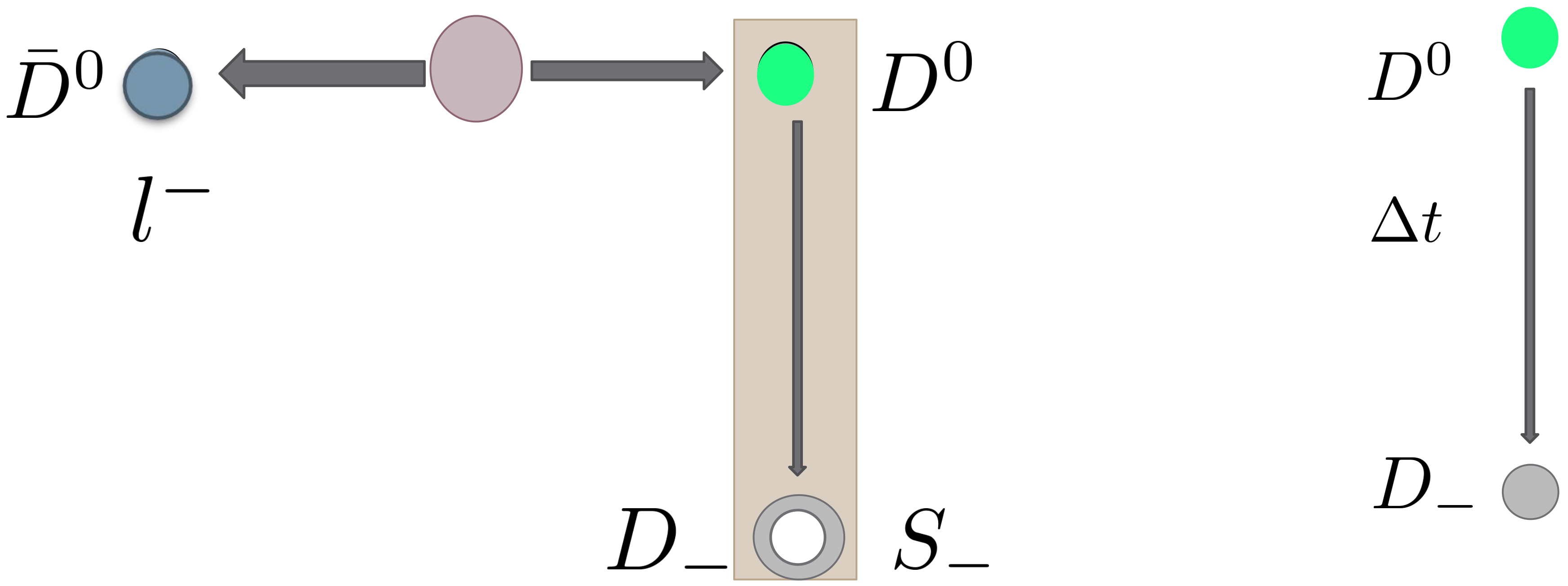
We have also calculated joint decays of $|\Psi_+\rangle$. Here are some simple results:

- **Theorem 10:** If $A_+(l^+l^-, l^-l^+; \Delta t)$ is nonzero, then CP must be violated indirectly.
- **Theorem 11:** If $A_+(l^+l^-, l^-l^+; \Delta t)$ depends on the first order of ϵ_M, then CP must be violated directly and the $\Delta\mathcal{F} = \Delta Q$ rule must be violated.
- **Theorem 12:** The deviation of $I_+(h^+, t_a; h^-, t_a)$ or $I_+(h^-, t_a; h^+, t_a)$ from zero implies CP violation, direct or indirect or both. [Immediately implied by $|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|M_+\rangle|M_+\rangle - |M_-\rangle|M_-\rangle)$.]
- **Theorem 13:** If $I_+(h^+, t_a; h^-, t_a)$ or $I_+(h^-, t_a; h^+, t_a)$ or both are of the order of $O(\Delta_M)$ and $O(\epsilon_M)$, then CP is violated directly.

2. Genuine T Violation Signal in terms of entangled Dmesons

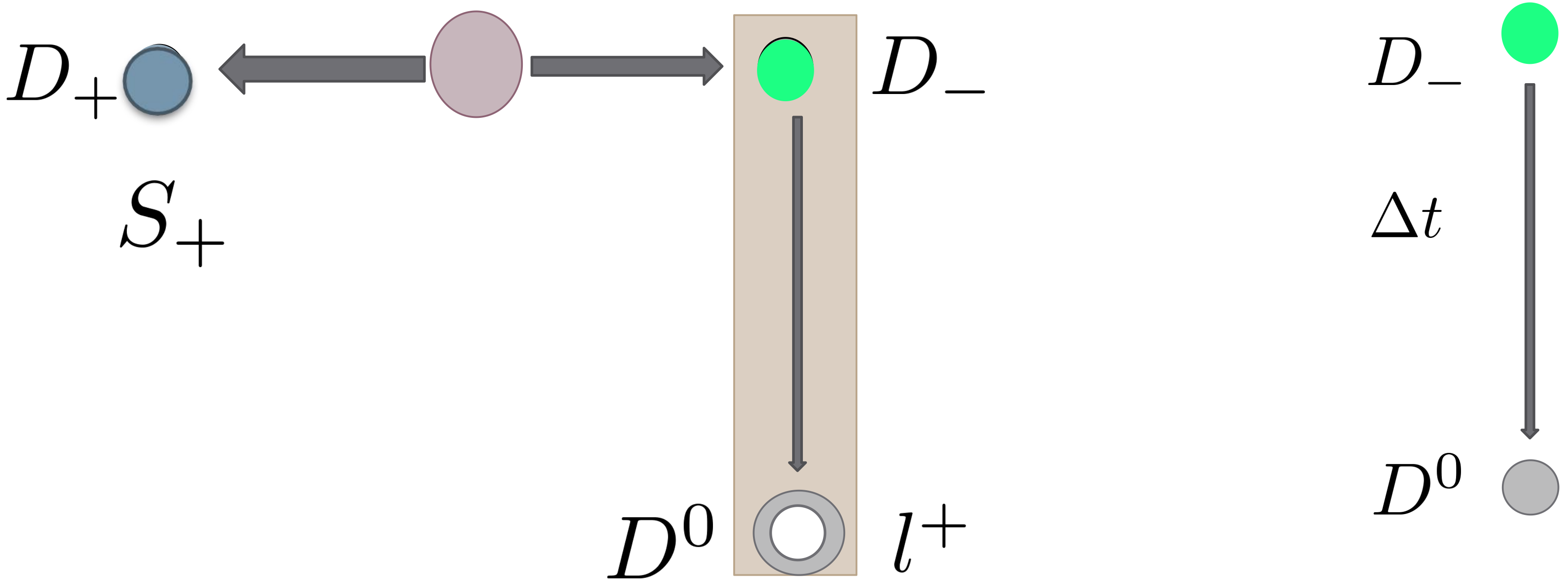
YS, JC Yang, Time reversal symmetry violation in entangled pseudoscalar neutral charmed mesons, Phys. Rev. D 98, 075079 (2018).

T-conjugate processes from C=-1 entangled state (1)



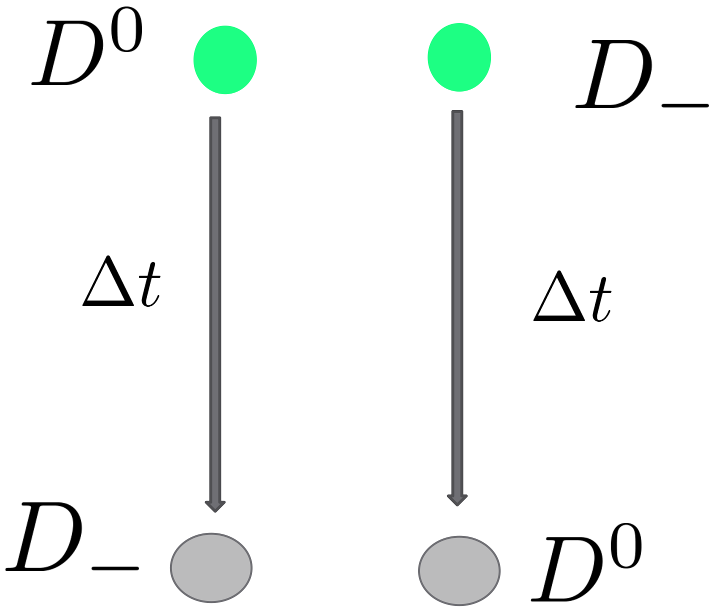
$$|\Psi_-\rangle = |M^0\rangle \otimes |\bar{M}^0\rangle - |\bar{M}^0\rangle \otimes |M^0\rangle$$

T-conjugate processes from C=-1 entangled state (2)



$$|\Psi_-\rangle = |M_+\rangle \otimes |M_-\rangle - |M_-\rangle \otimes |M_-$$

T-conjugate processes from C=-1 entangled state



Transition rates can be calculated from joint decay rates of the entangled state.

transition of meson b	final state of meson a	final state of meson b	T-conjugate transition	final state of meson a	final state of meson b
$D^0 \to D_-$	l^-	S_-	$D_- \to D^0$	S_+	l^+
$D^0 \to D_+$	l^-	S_+	$D_+ \to D^0$	S_-	l^+
$\bar{D}^0 \to D_-$	l^+	S_-	$D_- \to \bar{D}^0$	S_+	l^-
$\bar{D}^0 \to D_+$	l^+	S_+	$D_+ \to \bar{D}^0$	S_-	l^-

T-violation signals

$$A^1_-(\Delta t) = \frac{R_-(l^-, S_-, \Delta t) - R_-(S_+, l^+, \Delta t)}{R_-(l^-, S_-, \Delta t) + R_-(S_+, l^+, \Delta t)},$$

$$A^2_-(\Delta t) = \frac{R_-(l^-, S_+, \Delta t) - R_-(S_-, l^+, \Delta t)}{R_-(l^-, S_+, \Delta t) + R_-(S_-, l^+, \Delta t)},$$

$$A^3_-(\Delta t) = \frac{R_-(l^+, S_-, \Delta t) - R_-(S_+, l^-, \Delta t)}{R_-(l^+, S_-, \Delta t) + R_-(S_+, l^-, \Delta t)},$$

$$A^4_-(\Delta t) = \frac{R_-(l^+, S_+, \Delta t) - R_-(S_-, l^-, \Delta t)}{R_-(l^+, S_+, \Delta t) + R_-(S_-, l^-, \Delta t)}.$$

$$\hat{A}^1_- = \frac{R_-(l^-, S_-) - R_-(l^+, S_+)}{R_-(l^-, S_-) + R_-(l^+, S_+)}, \quad \hat{A}^2_- = \frac{R_-(l^-, S_+) - R_-(l^+, S_-)}{R_-(l^-, S_+) + R_-(l^+, S_-)}.$$

Results within Standard model

- Calculations come down to a few parameters:

$$x \equiv \frac{\Delta m}{\Gamma} = 0.0037, \quad y \equiv \frac{\Delta\Gamma}{2\Gamma} = 0.0066,$$
$$|q/p| = 0.91, \quad arg(q/p)/2 = 0.91.$$

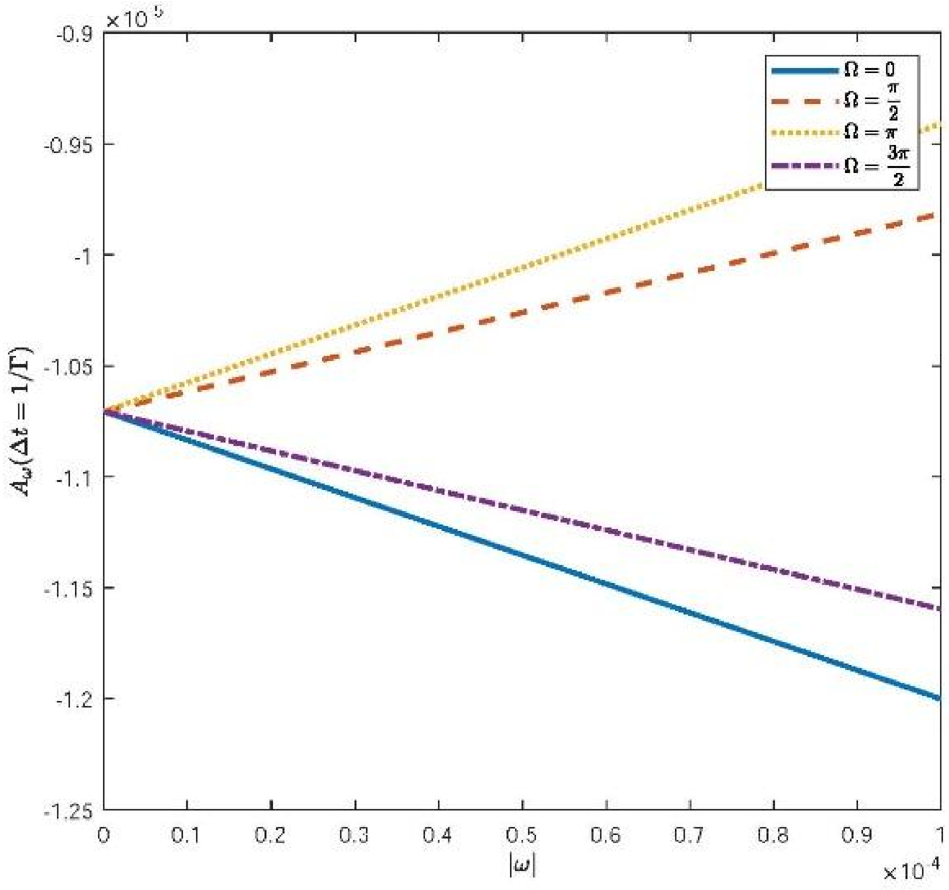
- The time-dependent T-asymmetries for $\Delta t = 1/\Gamma$, and time-independent T-asymmetries are found to be of the order of 10^{-5}

CPT violation

- Local quantum field theory (Lorentz invariance, local interaction,unitarity) $\Rightarrow$ CPT.
- In some quantum gravity theories, because of objects unaccessible to low energy observers, CPT may be violated.
- Such CPTV leads to "omega effect"(Bernabeu, Ellis, Mavromatos, etc.)

$$|\Psi_\omega\rangle = |\Psi_-\rangle + \omega|\Psi_+\rangle$$

T-violating signals in presence of omega effect



$$\omega \equiv |\omega| \exp i\Omega.$$

For $|\omega| = 10^{-4}$, T-violation signals in D system are changed as large as 27%

Our Work towards high energy quantum information

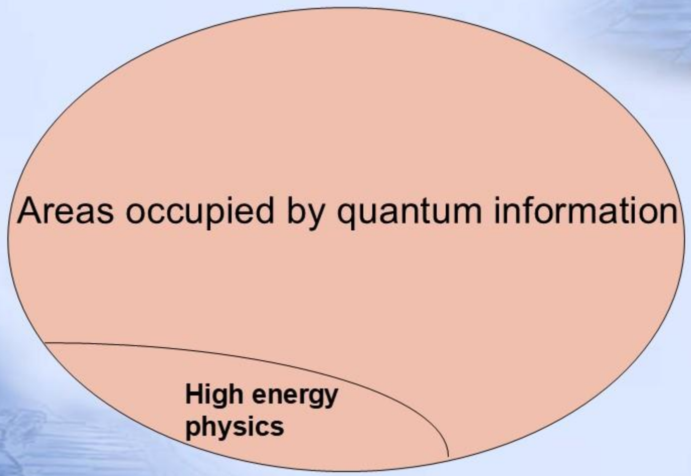
Towards high energy quantum information: quantum teleportation using neutral kaons

Yu Shi
(Fudan University)

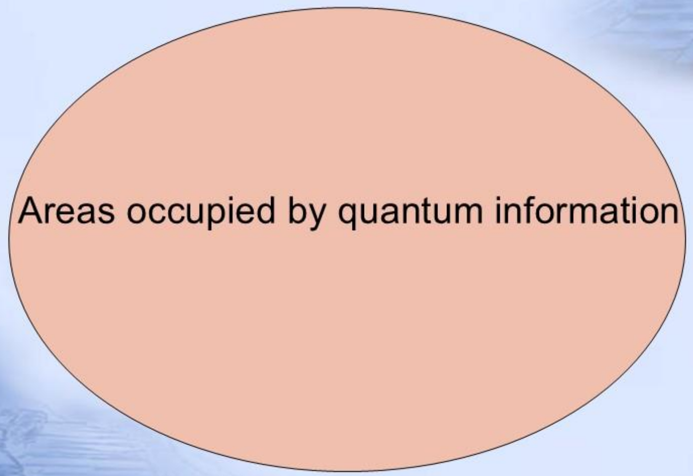
Ref: quant-ph/0605070; appearing in Phys. Lett. B

International Conference on Quantum Foundation and Technology: Frontier and Future, Hangzhou, 27/8/2006

Map of Quantum Physics



New Map of Quantum Physics



1. Proposal for a High Enegy Quantum Information Process: quantum teleportation in terms of entangled mesons

- YS, Phys. Lett. B 641, 75 (06); 641 (2006) 492.
- YS and Y. L. Wu, EPJC 55, 477 (08).

Generation of an EPR pair

- At t=0, a and b is created as

$$\begin{aligned} |\Psi_{ab}(0)\rangle &= \frac{1}{\sqrt{2}}(|K^0\rangle_a|\bar{K}^0\rangle_b - |\bar{K}^0\rangle_a|K^0\rangle_b) \\ &= \frac{1}{\sqrt{2}}(|K_2\rangle|K_1\rangle - |K_1\rangle|K_2\rangle) \\ &= \frac{r}{\sqrt{2}}(|K_L\rangle_a|K_S\rangle_b - |K_S\rangle_a|K_L\rangle_b), \end{aligned}$$

$$r = (|p|^2 + |q|^2)/2pq = (1 + |\epsilon|^2)/(1 - \epsilon^2)$$

- Decay under weak interaction:

$$|\Psi_{ab}(t)\rangle = M(t)|\Psi_-\rangle_{ab},$$

$$M(t) = \exp[-i(\lambda_S + \lambda_L)\Lambda_b t],\ \Lambda_b = 1/\sqrt{1 - v_b^2}$$

The third koan

- A third koan c is generated at t_z in an unknown state

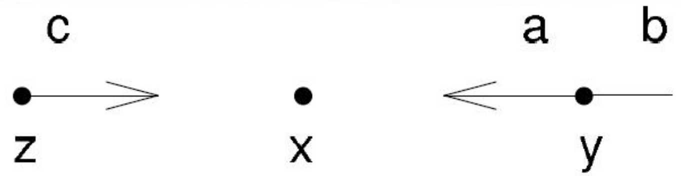
$$|\Psi_c(t_z)\rangle = \alpha|K^0\rangle_c + \beta|\bar{K}^0\rangle_c.$$

- Naturally decays

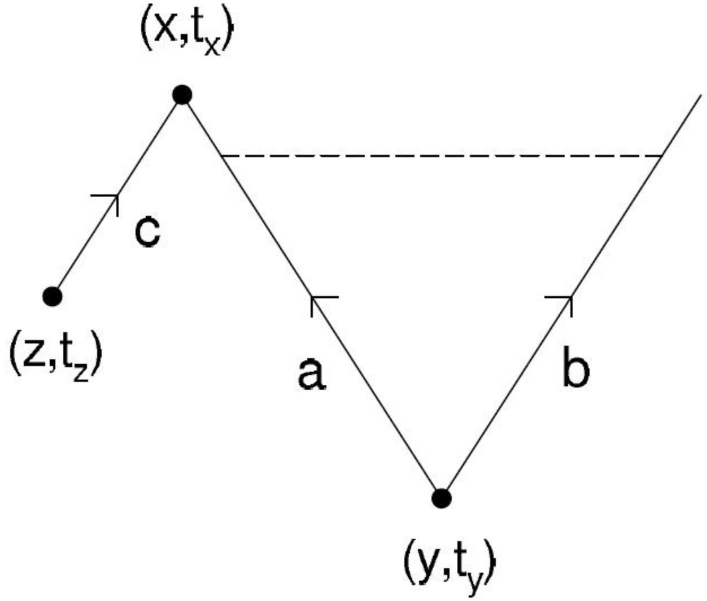
$$|\Psi_c(t)\rangle = F(t)|K^0\rangle_c + G(t)|\bar{K}^0\rangle_c,$$

$$F(t) = [(\alpha + \beta p/q)e^{-i\lambda_S \Lambda_c (t-t_z)} + (\alpha - \beta p/q)e^{-i\lambda_L \Lambda_c (t-t_z)}]/2,$$
$$G(t) = [(\alpha q/p + \beta)e^{-i\lambda_S \Lambda_c (t-t_z)} - (\alpha q/p - \beta)e^{-i\lambda_L \Lambda_c (t-t_z)}]/2.$$

Let c and a collide!



Spacetime diagram:



Eigenstates of P, S and I of c+a

- P, S, I are conserved by strong interaction, which governs c-a collision.

$$|\phi_1\rangle_{ca} = |K^0K^0\rangle \text{ with } P=1,\ S=2,\ I=1;$$
$$|\phi_2\rangle_{ca} = |\bar{K}^0\bar{K}^0\rangle \text{ with } P=1,\ S=-2,\ I=1;$$
$$|\phi_3\rangle_{ca} = |\Psi_+\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle|\bar{K}^0\rangle + |\bar{K}^0\rangle_c|K^0\rangle), \text{ with } P=1,\ S=0,\ I=1;$$
$$|\phi_4\rangle_{ca} = |\Psi_-\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle_c|K^0\rangle \text{ with } P=-1,\ S=0,\ I=0.$$

Three-kaon state before collision,decomposed in the strong interaction basis

$$
\begin{aligned}
|\Psi_{cab}(t)\rangle &= |\Psi_c(t)\rangle \otimes |\Psi_{ab}(t)\rangle \\
&= \frac{M(t)}{2}\{\sqrt{2}F(t)|\phi_1\rangle_{ca}|\bar{K}^0\rangle_b \\
&\quad -\sqrt{2}G(t)|\phi_2\rangle_{ca}|K^0\rangle_b \\
&\quad -|\phi_3\rangle_{ca}[F(t)|K^0\rangle_b - G(t)|\bar{K}^0\rangle_b] \\
&\quad -|\phi_4\rangle_{ca}[F(t)|K^0\rangle_b + G(t)|\bar{K}^0\rangle_b]\}.
\end{aligned}
$$

It is not a Bell basis. This basis is physical, while Bell basis is not.

Collision

- The collision effects a unitary transformation $\mathcal{S}$ in a negligible time interval

$$
\begin{aligned}
|\Psi_{cab}(t_x)\rangle \quad = \quad & \frac{M(t_x)}{2}\{\sqrt{2}F(t_x)\mathcal{S}|\phi_1\rangle_{ca}|\bar{K}^0\rangle_b \\
& -\sqrt{2}G(t_x)\mathcal{S}|\phi_2\rangle_{ca}|K^0\rangle_b \\
& -\mathcal{S}|\phi_3\rangle_{ca}[F(t_x)|K^0\rangle_b - G(t_x)|\bar{K}^0\rangle_b] \\
& -\mathcal{S}|\phi_4\rangle_{ca}[F(t_x)|K^0\rangle_b + G(t_x)|\bar{K}^0\rangle_b]\}.
\end{aligned}
$$

- As $\mathcal{S}$ is governed by strong interaction, $\mathcal{S}|\phi_i\rangle_{ca}$ $(i = 1, 2, 3, 4)$ is still an eigenstate of S, P and I, with the same eigenvalues as for $|\phi_i\rangle_{ca}$.

Detection of outgoing particles of c-a collision

- Using strong interaction with nuclear matter, the detection completes Alice's two-particle projection in the basis $\{\mathcal{S}|\phi_i\rangle_{ca}\}$.
- Probabilities: $$|M(t_x)|^2|F(t_x)|^2/2,$$ $$|M(t_x)|^2|G(t_x)|^2/2,$$ $$|M(t_x)|^2[|F(t_x)|^2+|G(t_x)|^2]/4,$$ $$|M(t_x)|^2[|F(t_x)|^2+|G(t_x)|^2]/4.$$
- Corresponding projected state of b:

$$|\bar{K}^0\rangle_b,\ |K^0\rangle_b,\ [F(t_x)|K^0\rangle_b - G(t_x)|\bar{K}^0\rangle_b]/\sqrt{|F(t_x)|^2+|G(t_x)|^2},$$
$$[F(t_x)|K^0\rangle_b + G(t_x)|\bar{K}^0\rangle_b]/\sqrt{|F(t_x)|^2+|G(t_x)|^2}.$$

- Decay affects the probabilities.

Adopt a stochastic strategy

- This is because it is hard to implement subsequent unitary transformation on b.
- Bob chooses to retain or abandon b particle, based on the projection result of c-a.
- Teleportation of $F(t_x)|K^0\rangle_b + G(t_x)|\bar{K}^0\rangle_b$

  is made if projection result of c-a is $\mathcal{S}|\Psi_-\rangle_{ca}$.

Verification scheme (1)

- Different projection results lead to different values of strangeness ratio ξ.
- For $|\Psi(t)\rangle_b = f(t)|K^0\rangle_b + g(t)|\bar{K}^0\rangle_b$, $\xi(t) = |f(t)|^2/g(t)|^2$.
- No projection/teleportation: $\xi(t) = 1.$
- Successful teleportation:

$$\xi(t) = \frac{|F(t_x)(e^{-\Gamma_S \tau/2}+e^{-\Gamma_L \tau/2})+G(t_x)(e^{-\Gamma_S \tau/2}-e^{-\Gamma_L \tau/2})|^2}{|F(t_x)(e^{-\Gamma_S \tau/2}-e^{-\Gamma_L \tau/2})+G(t_x)(e^{-\Gamma_S \tau/2}+e^{-\Gamma_L \tau/2})|^2},$$
$$\tau = \Lambda_b(t - t_x - 0).$$

very different from 1

Verification scheme (2)

- Different projection results lead to different values of CP ratio .
- For $|\Psi(t)\rangle_b = u_1(t)|K_1\rangle_b + u_2(t)|K_2\rangle_b$, $\zeta(t) = |u_1(t)|^2/u_2(t)|^2$.
- No projection/teleportation: $\zeta(t) = 1$.
- Successful teleportation: significantly different from 1.
- Advantage 1: valid no matter whether CP is violated.
- Advantage 2: easy experimental implementation, using non-leptonic decays (CP=1: decay to 2 pions; CP=-1: decay to 3 pions).

Another informational process: Entanglement swapping

- A and B are entangled, C and D are entangled.
- A and C are subject to a measurement (projection).
- Then B and D become entangled, though they never meet.
- Entangling partners are swapped.

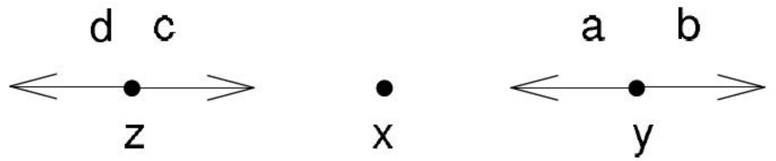
Preparation

In addition to $|\Psi_-\rangle_{ab}$ generated at $t=0$,
another kaon pair d and c is generated as $|\Psi_-\rangle_{dc}$ at time t_z.
$|\Psi_{dc}(t)\rangle = M'(t-t_z)|\Psi_-\rangle_{dc}$,
$M'(t-t_z) = \exp[-i(\lambda_S+\lambda_L)\Lambda_d(t-t_z)]$.

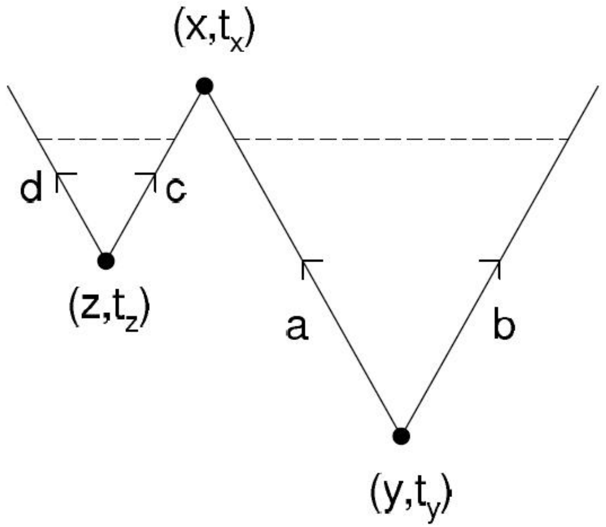
$$\begin{aligned}
|\Psi_{dcab}(t)\rangle &= M'(t-t_z)M(t)|\Psi_-\rangle_{dc}|\Psi_-\rangle_{ab} \\
&= \tfrac{M'(t-t_z)M(t)}{2}(|\Psi_+\rangle_{ca}|\Psi_+\rangle_{db} - |\Psi_-\rangle_{ca}|\Psi_-\rangle_{db} \\
&\quad -|K^0\bar{K}^0\rangle_{ca}|\bar{K}^0\bar{K}^0\rangle_{db} - |\bar{K}^0\bar{K}^0\rangle_{ca}|K^0K^0\rangle_{db}).
\end{aligned}$$

Collision

Let c and a collide at t_x



Spacetime diagram:



Detection

Collision:

$$|\Psi_{dcab}(t_x+0)\rangle = \frac{M'(t_x-t_z)M(t_x)}{2}(\mathcal{S}|\Psi_+\rangle_{ca}|\Psi_+\rangle_{db} \\ -\mathcal{S}|\Psi_-\rangle_{ca}|\Psi_-\rangle_{db} \\ -\mathcal{S}|K^0K^0\rangle_{ca}|\bar{K}^0\bar{K}^0\rangle_{db} \\ -\mathcal{S}|\bar{K}^0\bar{K}^0\rangle_{ca}|K^0K^0\rangle_{db}).$$

In detecting outgoing particles from the collision, c and a are projected to: $\mathcal{S}|\Psi_+\rangle_{ca}$, $\mathcal{S}|\Psi_-\rangle_{ca}$, $\mathcal{S}|K^0K^0\rangle_{ca}$ or $\mathcal{S}|\bar{K}^0\bar{K}^0\rangle_{ca}$. Correspondingly d and b are projected to: $|\Psi_+\rangle_{ca}$, $|\Psi_-\rangle_{ca}$, $|K^0K^0\rangle_{ca}$ and $|\bar{K}^0\bar{K}^0\rangle_{ca}$, respectively, each with probability $|M'(t_x-t_z)M(t_x)|^2/4$. .

The projection result is revealed by P, S and I of the outcomes of $c-a$ collision, according to which b and d are retained or abandoned.

Verification scheme (1)

- Measure the S asymmetry of b and d

$$A(t) = [p_{diff}(t) - p_{same}(t)]/[p_{diff}(t) + p_{same}(t)]$$

$p_{diff}(t)$ ($p_{same}(t)$):
probability to have different (same) strangeness values

- Many runs are needed.
- No entanglement swapping (all runs are considered): A(t)=0
- Entanglement swapping succeeds (consider those runs in which c-a project to $\mathcal{S}|\Psi_-\rangle_{ca}$: A(t)=1

Verification scheme (2)

- Measure the CP asymmetry of b and d

$$A(t) = [p_{diff}(t) - p_{same}(t)]/[p_{diff}(t) + p_{same}(t)]$$

$p_{diff}(t)$ ($p_{same}(t)$):
probability to have different (same) values of CP.

- Many runs (events) are needed.
- No entanglement swapping (all runs are considered): A(t)=-1
- Entanglement swapping succeeds (consider those runs in which c-a project to $\mathcal{S}|\Psi_-\rangle_{ca}$: A(t)=1

Question

- Can we do single-copy quantum systems in high energy experiments?
- Quantum weirdness mostly comes from such a circumstance.

2. Entangled mesons violate a generalized Leggett Inequality， in which decays also depend on hidden variables

- YS and JC Yang, Particle physics violating crypto-nonlocal realism, European Physical Journal C 80, 861 (2020).

Crypto-nonlocal realism and Leggett Inequalities

- BI Violation: Which should be abandoned, locality or realism?
- Leggett consider crypto-nonlocal realism and derived Leggett Inequality (violated by QM) :
  - The physical state is a statistical mixture of sub-ensembles of various definite polarization directions.
  - With the hidden variables, the measured quantity also depends on the polarizer axis on the other side (non-local).
  - Local HVT can be regarded as a special case.
  - For each sub-ensemble, the measured quantity obeys Malus Law (which is local) for the average of the hidden parameters.

$$\bar{A}(\mathbf{u}) = \int d\lambda \rho_{\mathbf{u},\mathbf{v}}(\lambda) A(\mathbf{a},\mathbf{b},\lambda) = \mathbf{u}\cdot\mathbf{a}$$

Using high energy particles for Bell/Leggett tests (Not Yet Tested)

- Relativistic, massive, involving strong, weak and electromagnetic interactions.
- Decays are similar to measurements.
- However, the decay modes, products, times are possibly dependent on the hidden variables at the source, thus the decays of different particles may be correlated.
- It is a kind of loophole of measurement setup.

Our work

- Assume that the measurement setting possibly depends on hidden variables： decay (analogous to a rotating polaroid) is determined by the hidden variables at the particle source. [particularly suitable for particle phys]
- Generalized crypto-nonlocal realism, Leggett inequalities, local realism, Bell inequalities.

Assume the measurement setup also depends on hidden variables

Average over distribution of nonlocal hidden variables:

$$\int d\lambda\ \rho_{\boldsymbol{u},\boldsymbol{v},\boldsymbol{\alpha},\boldsymbol{\beta}} A(\boldsymbol{u},\boldsymbol{v},\boldsymbol{\alpha}(\boldsymbol{\lambda}),\boldsymbol{\beta}(\boldsymbol{\lambda}),\lambda) = \bar{A}(\boldsymbol{u},\boldsymbol{a}) = \boldsymbol{u}\cdot\boldsymbol{a}$$

$$\int d\lambda\ \rho_{\boldsymbol{u},\boldsymbol{v},\boldsymbol{\alpha},\boldsymbol{\beta}} B(\boldsymbol{u},\boldsymbol{v},\boldsymbol{\alpha}(\boldsymbol{\lambda}),\boldsymbol{\beta}(\boldsymbol{\lambda}),\lambda) = \bar{B}(\boldsymbol{v},\boldsymbol{b}) = \boldsymbol{v}\cdot\boldsymbol{b}$$

$$\int d\lambda\ \rho_{\boldsymbol{u},\boldsymbol{v},\boldsymbol{\alpha},\boldsymbol{\beta}} A(\boldsymbol{u},\boldsymbol{v},\boldsymbol{\alpha}(\boldsymbol{\lambda}),\boldsymbol{\beta}(\boldsymbol{\lambda}),\lambda) B(\boldsymbol{u},\boldsymbol{v},\boldsymbol{\alpha}(\boldsymbol{\lambda}),\boldsymbol{\beta}(\boldsymbol{\lambda}),\lambda) = \overline{AB}(\boldsymbol{u},\boldsymbol{v},\boldsymbol{a},\boldsymbol{b})$$

Statistical average over polarizations:

$$E(\mathbf{a},\mathbf{b}) = \int d\mathbf{u} d\mathbf{v} F(\mathbf{u},\mathbf{v}) \overline{AB}(\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}),$$

Measuring after evolution, equivalent to measurement in a time-dependent basis, like a rotating polaroid

$$U(\theta_{\mathbf{a}}, \rho_{\mathbf{a}}) = \left(\cos\frac{\theta_{\mathbf{a}}}{2} - i\sin\frac{\theta_{\mathbf{a}}}{2}\left(\cos(\rho_{\mathbf{a}})\sigma^x + \sin(\rho_{\mathbf{a}})\sigma^y\right)\right). \tag{10}$$

Suppose that following the evolution $U(\theta_{\mathbf{a}}, \rho_{\mathbf{a}})$, a signal A is recorded as $A = +1$ if $|0\rangle$ is detected, while $A = -1$ if $|1\rangle$ is detected. The QM expectation value of A is

$$\bar{A}(\mathbf{u}) = \frac{|\langle 0|U|\mathbf{u}\rangle|^2 - |\langle 1|U|\mathbf{u}\rangle|^2}{|\langle 0|U|\mathbf{u}\rangle|^2 + |\langle 1|U|\mathbf{u}\rangle|^2} = \mathbf{u}\cdot\mathbf{a}, \tag{11}$$

Semileptonic decay on the flavor basis, satisfies Malus Law, thus the measurement "direction" can be determined.

For a B_d meson, the measurement in the flavor basis $\{|B^0\rangle, |\bar{B}^0\rangle\}$, corresponding to $A_l = \pm 1$, can be made in the semileptonic decay channel, as the direct CP violation or wrong sign decay is negligible [58],

$$\bar{A}_l(\mathbf{u}) = \frac{|\langle B^0|U(t)|\mathbf{u}\rangle|^2 - |\langle \bar{B}^0|U(t)|\mathbf{u}\rangle|^2}{|\langle B^0|U(t)|\mathbf{u}\rangle|^2 + |\langle \bar{B}^0|U(t)|\mathbf{u}\rangle|^2} = \mathbf{u} \cdot \mathbf{a}^l(t), \qquad (12)$$

where $\mathbf{a}^l(t) = (\sin(2\beta)\sin(x\Gamma t), -\cos(2\beta)\sin(x\Gamma t), \cos(x\Gamma t))$.

Use B meson, as the decay width are close to each other.

Indirect CP violation effects in terms of phase β。

xΓ is the difference of mass-decay width.

Shi, Quantum Entanglement in Particle Physics

Decay on CP basis also satisfies Malus Law, from this the measuring "direction" can be determined

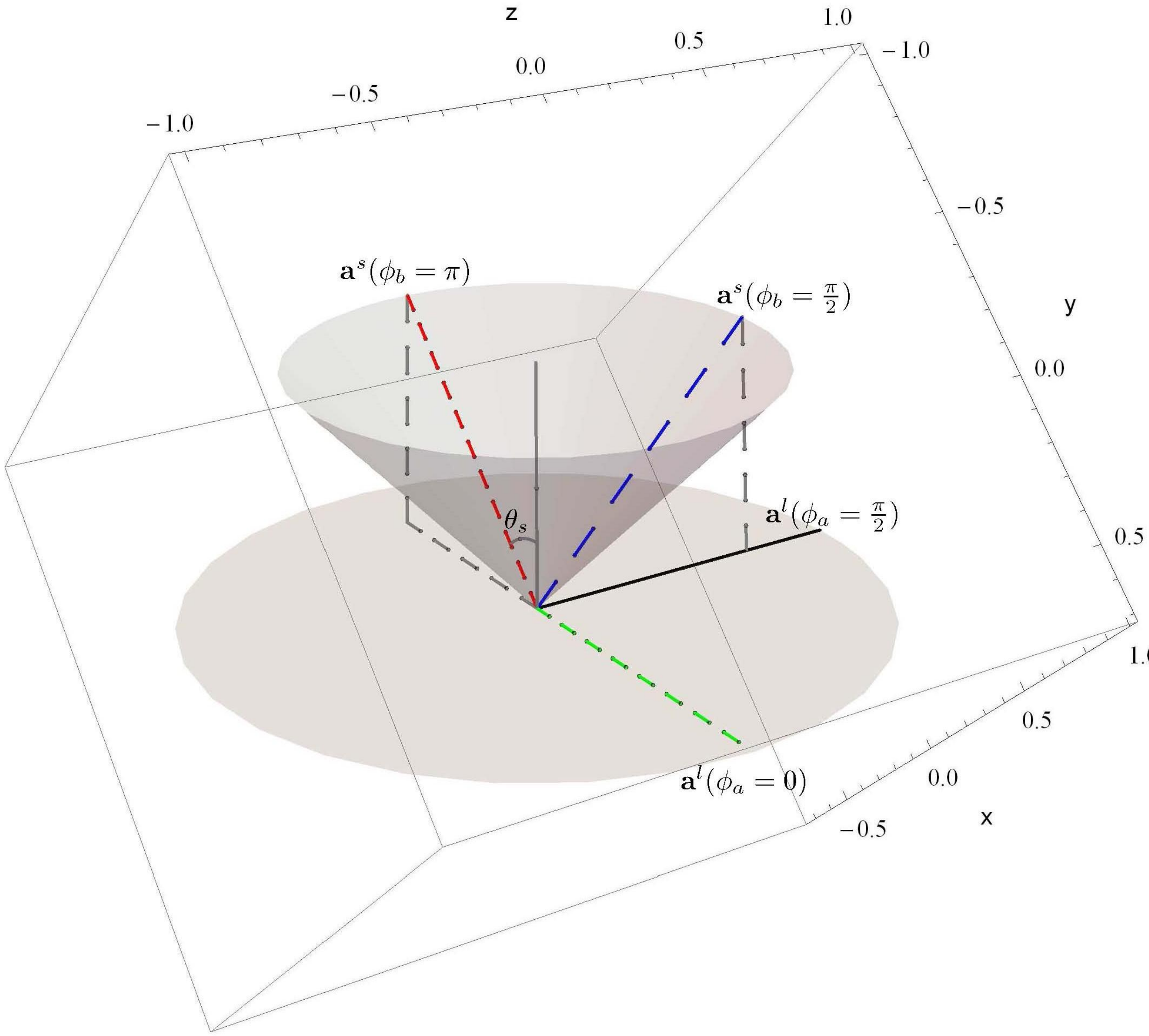
Likewise, observing the decay product to be CP eigenstates $S_\pm$ effectively measures the meson to be $|B_\pm\rangle \equiv \left(|B^0\rangle \pm |\bar{B}^0\rangle\right)/\sqrt{2}$, as the direct CP violation is negligible [58]. With $B_\pm$ corresponding to $A_s = \pm 1$,

$$\bar{A}_s(\mathbf{u}) = \frac{|\langle B_+|U(t)|\mathbf{u}\rangle|^2 - |\langle B_-|U(t)|\mathbf{u}\rangle|^2}{|\langle B_+|U(t)|\mathbf{u}\rangle|^2 + |\langle B_-|U(t)|\mathbf{u}\rangle|^2} = \mathbf{u}\cdot\mathbf{a}^s(t), \tag{13}$$

where $\mathbf{a}^s(t) = \left(\sin^2(2\beta)\cos(x\Gamma t) + \cos^2(2\beta), \sin(4\beta)\sin^2\left(x\Gamma t/2\right), -\sin(2\beta)\sin(x\Gamma t)\right).$

Shi, Quantum Entanglement in Particle Physics

The direction of "polaroid" corresponding to the decay



$\theta_s = 2\beta$

FIG. 1: The effective measuring directions $\mathbf{a}^l$ and $\mathbf{a}^s$. In a certain coordinate system, $\mathbf{a}^l(\phi_l)$ is on xy plane, $\mathbf{a}^s(\theta_s, \phi_s)$ is on a cone. For B_d mesons, $\phi_l = x\Gamma t$,

Correlation function

We first consider correlation functions of various combinations of $\mathbf{a}^l$ and $\mathbf{a}^s$. Define $\hat{E}^{\pm}(\mathbf{a},\mathbf{b}) \equiv E(\mathbf{a},\mathbf{b})+E(\mathbf{b},\pm\mathbf{b})$, and rewrite $\hat{E}^{\pm}(\mathbf{a}^s(\theta_s,\phi_a),\mathbf{a}^l(\phi_b))$ as $\hat{E}^{\pm}_{sl}(\theta_s,\xi,\varphi)$, where $\xi \equiv (\phi_a+\phi_b)/2$, $\varphi \equiv \phi_a-\phi_b$. $\hat{E}^{\pm}_{ll}(\theta_s,\xi,\varphi)$ and $\hat{E}^{+}_{ss}(\theta_s,\xi,\varphi)$ are similarly defined. Furthermore, we consider the averages over ξ, $\hat{E}^{-}_{sl}(\theta_s,\varphi) \equiv \int \frac{d\xi}{2\pi}\hat{E}^{-}_{sl}(\theta_s,\xi,\varphi)$ and so on.

Upper bound

$$\hat{E}_{sl}^{-}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1)}{4\cos(\frac{\varphi_2}{2})}\hat{E}_{ll}^{-}(\theta_s,\varphi_2)$$
$$\leq 2\left(1+\frac{\pi\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1)}{4\cos(\frac{\varphi_2}{2})}\right)-\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1), \tag{26}$$

where $L_1(\theta_s,\varphi)\equiv|\mathbf{a}^s+\mathbf{a}^l|=\sqrt{2+2\cos(\varphi)\sin(\theta_s)}$, $\theta_1(\theta_s,\varphi)=\cos^{-1}\frac{\cos(\theta_s)}{\sqrt{2+2\cos(\varphi)\sin(\theta_s)}}$. With $0<\theta_s<\pi/2$, we have $\sin(\theta_1)>0$, $\cos(\theta_1)>0$.

Lower bound

We find two lower bounds. The first is given as

$$\hat{E}^{+}_{sl}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\left|\sin(\frac{\varphi_2}{2})\right|}\hat{E}^{+}_{ll}(\theta_s,\varphi_2)$$
$$\geq -2\left(1+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\left|\sin(\frac{\varphi_2}{2})\right|}\right)+\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1).$$

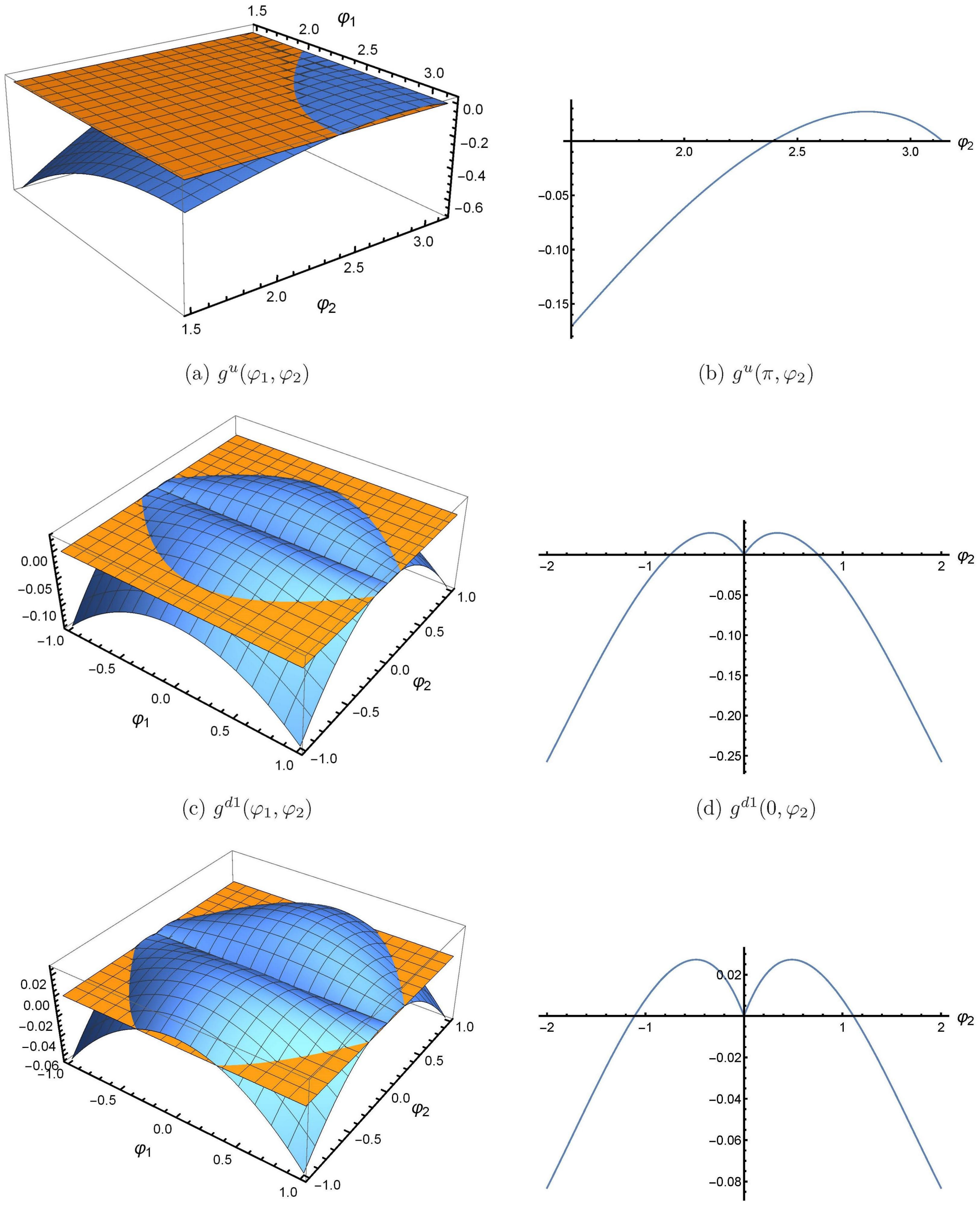
where $L_2(\theta_s,\varphi)=\sqrt{2-2\cos(\varphi)\sin(\theta_s)}$, $\theta_2(\theta_s,\varphi)=\cos^{-1}\frac{\cos(\theta_s)}{\sqrt{2-2\cos(\varphi)\sin(\theta_s)}}$.

The second lower bound is given as

$$\hat{E}^{+}_{sl}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\sin(\theta_s)\left|\sin(\frac{\varphi_2}{2})\right|}\hat{E}^{+}_{ss}(\varphi_2)$$
$$\geq -2\left(1+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\sin(\theta_s)\left|\sin(\frac{\varphi_2}{2})\right|}\right)+\cos(\theta_2)L_2(\theta_s,\varphi_1).$$

LI is violated standard mode

- g^u=(LHS−RHS)/|LHS|.
- $g^{d1,2}$=(LHS−RHS)/|LHS|.
- condition:θs≠0 (CP violation)
- Consistent with the real world (CP is indeed violated)!
- QM (LI violation) is consistent with CP violation.



(a) $g^u(\varphi_1, \varphi_2)$ (b) $g^u(\pi, \varphi_2)$

(c) $g^{d1}(\varphi_1, \varphi_2)$ (d) $g^{d1}(0, \varphi_2)$

(e) $g^{d2}(\varphi_1, \varphi_2)$ (f) $g^{d2}(0, \varphi_2)$

3. Generalized Bell inequalities for hyperon entanglement
(decay also depends on hidden variables)

YS and JC Yang, Entangled baryons: violation of inequalities based on local realism assuming dependence of decays on hidden variables, European Physical Journal C 80, 116 (2020).

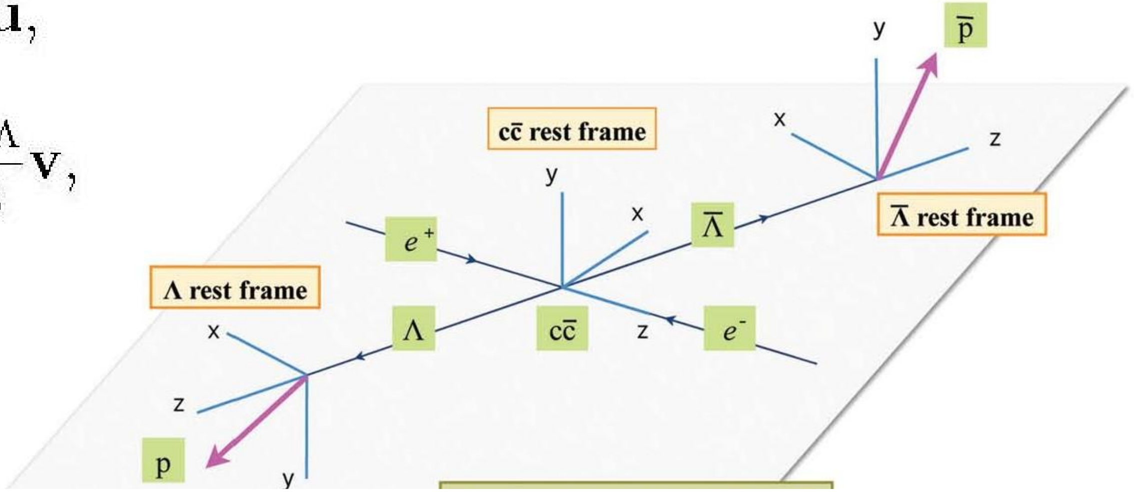
- Hyperon entanglement.
- Momentum direction of proton from the hyperon decay is correlated with hyperon spin, and plays a role similar to measurement agent. (scattering as “measurement”)
- The angular distribution plays a role similar to Malus Law.
- In the hidden variable theory, consider statistical mixture of polarizations.
- Study the correlation function of momenta directions of proton and antiproton.

$$\frac{d\sigma(\Lambda \to p\pi^-)}{d\Omega_p} = \frac{1}{4\pi}(1 + \alpha_\Lambda \mathbf{s}_\Lambda \cdot \mathbf{n}_p)\,, \quad \frac{d\sigma(\Lambda \to \bar{p}\pi^+)}{d\Omega_{\bar{p}}} = \frac{1}{4\pi}(1 - \alpha_\Lambda \mathbf{s}_{\bar{\Lambda}} \cdot \mathbf{n}_{\bar{p}})$$

In HV theory, A and B are momentum direction of proton and antiproton, required to give the same averages as QM

$$\bar{\mathbf{A}} = \int d\lambda_A d\lambda_B \rho_A(\lambda_A)\rho_B(\lambda_B)\mathbf{A}(\lambda_A) = \frac{\alpha_\Lambda}{3}\mathbf{u},$$

$$\bar{\mathbf{B}} = \int d\lambda_A d\lambda_B \rho_A(\lambda_A)\rho_B(\lambda_B)\mathbf{B}(\lambda_B) = -\frac{\alpha_\Lambda}{3}\mathbf{v},$$



$$E(\mathbf{a},\mathbf{b}) \equiv -\langle \overline{\mathbf{A}\cdot\mathbf{a}\mathbf{B}\cdot\mathbf{b}}\rangle = -\sum_{ij} a_i b_j \langle \overline{A_i B_j}\rangle$$

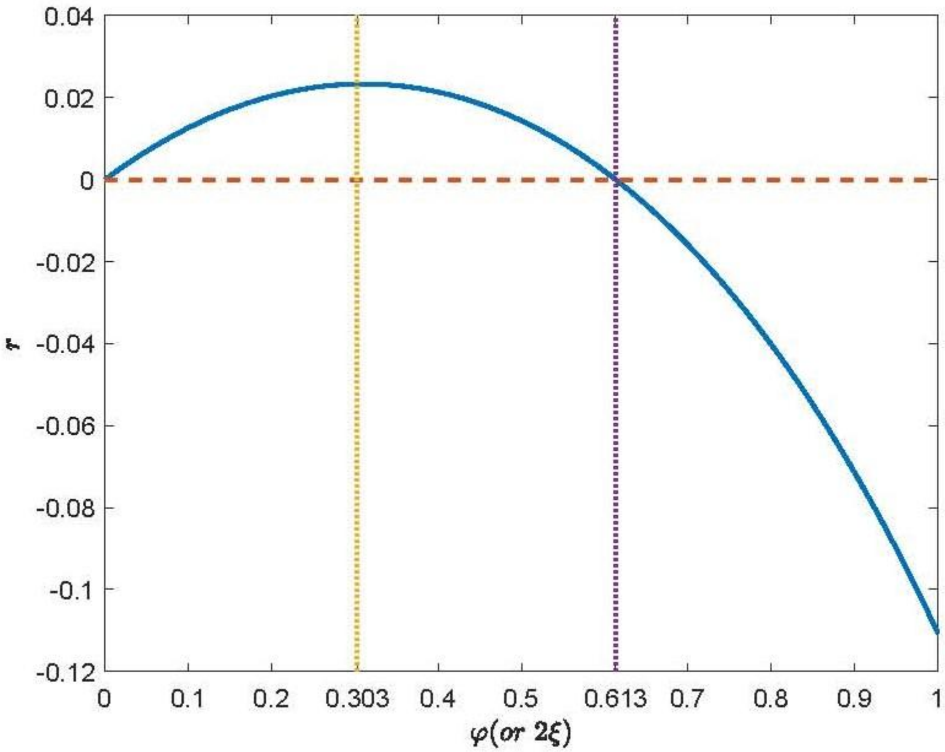
$$= -\int d\mathbf{u}d\mathbf{v}d\lambda_A d\lambda_B F(\mathbf{u},\mathbf{v})\rho_A(\lambda_A,\mathbf{u},\mathbf{v})\rho_B(\lambda_B,\mathbf{u},\mathbf{v})\mathbf{A}(\lambda_A,\mathbf{u},\mathbf{v})\cdot\mathbf{a}\mathbf{B}(\lambda_B,\mathbf{u},\mathbf{v})\cdot\mathbf{b}$$

$$= \frac{\alpha_\Lambda^2}{9}\int d\mathbf{u}d\mathbf{v}F(\mathbf{u},\mathbf{v})\mathbf{u}\cdot\mathbf{a}\mathbf{v}\cdot\mathbf{b},$$

$$\left|E_N^{\mathbf{ab}}(\varphi)+E_N^{\mathbf{ab}}(0)\right|+\left|E_N^{\mathbf{cd}}(\varphi)+E_N^{\mathbf{cd}}(0)\right|\leq\frac{\alpha_\Lambda^2}{9}\left(4-2u_N\left|\sin\frac{\varphi}{2}\right|\right)$$

$$\left|\hat{E}_N^{\mathbf{ab}}(\xi)+\hat{E}_N^{\mathbf{ab}}(0)\right|+\left|\hat{E}_N^{\mathbf{cd}}(\xi)+\hat{E}_N^{\mathbf{cd}}(0)\right|\leq\frac{\alpha_\Lambda^2}{9}\left(4-2u_N\left|\sin\xi\right|\right)\ u_N=\cot\left(\pi/2N\right)/N$$

- r=(L-R)/|L|, L is calculated from QM.



The violation ratio r for the first (second) inequality, as a function of φ (2ξ) for η_c (χ_{c_0}).

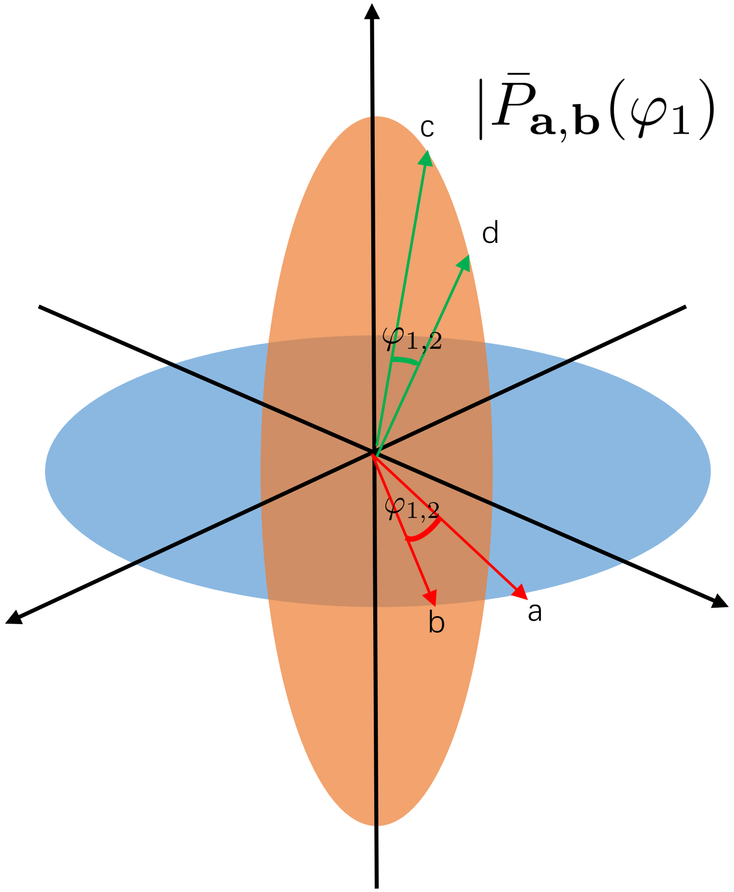
4. Minimal Entanglement to Violate Leggett Inequalities

- Every pure bipartite entangled state violate CHSHI.
- But not every pure entangled state violate LI
- We have derived the minimum entanglement to violate LI.
- So LI violation is stronger constraint.

JC Yang and YS, to be submitted.

Original Leggett Inequality

- For cryptical nonlocal hidden variable theories (with time-independent measuring directions):

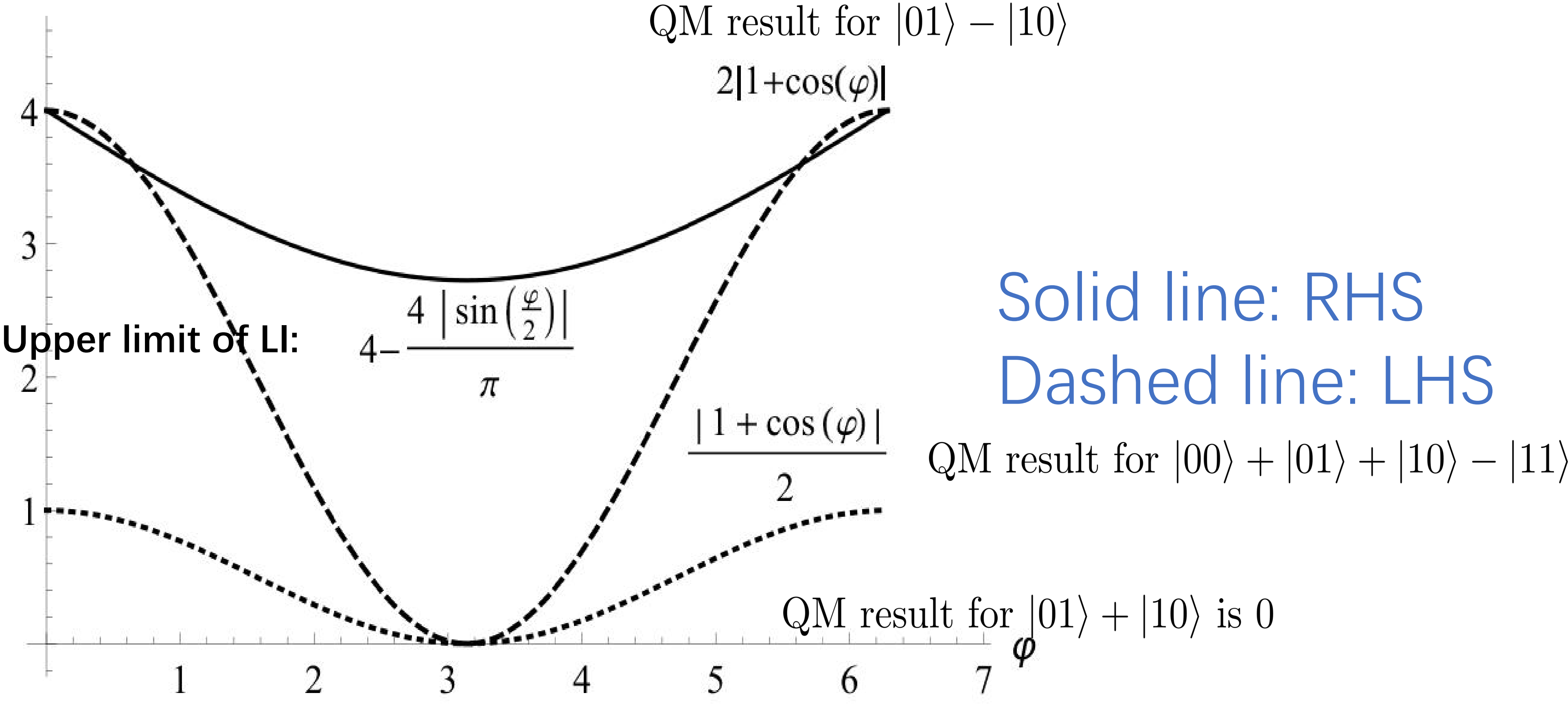


$$|\bar{P}_{\mathbf{a},\mathbf{b}}(\varphi_1) + \bar{P}_{\mathbf{a},\mathbf{b}}(\varphi_2)| + |\bar{P}_{\mathbf{c},\mathbf{d}}(\varphi_1) + \bar{P}_{\mathbf{c},\mathbf{d}}(\varphi_2)| \leq 4 - \frac{4}{\pi}\left|\sin(\frac{\varphi_1 - \varphi_2}{2})\right|,$$

$$\mathbf{a} - \mathbf{b} \perp \mathbf{c} - \mathbf{d}$$

- Is there a **condition on entanglement** to violate Leggett nequality?

Maximally entangled states



Solid line: RHS
Dashed line: LHS

If one can freely choose measurement basis, all states with the same entanglement can be transformed by local unitaries to (|01>-|10>)-like, violating LI.

Sufficient condition

- If one can freely choose the measurement basis, whether LI can be broken is solely dependent on concurrence (entanglement).
- Proof:
- For any real number X:

$$4-\frac{4}{\pi}\left|\sin\left(\frac{\varphi_1-\varphi_2}{2}\right)\right|-\left|X\left(\cos(\varphi_1)+\cos(\varphi_2)\right)\right|\geq 4-\frac{2\sqrt{\pi^2X^2+4}}{\pi}.$$

$$4-\frac{4}{\pi}\left|\sin\left(\frac{\varphi_1-\varphi_2}{2}\right)\right|-\left|(y-z)^2\left(\cos(\varphi_1)+\cos(\varphi_2)\right)\right|\geq 4-\frac{2\sqrt{\pi^2((y-z)^2)^2+4}}{\pi}.$$

Sufficient condition

QM result for $y|01\rangle + z|10\rangle$

$$\left|4-\frac{4}{\pi}\left|\sin\left(\frac{\varphi_1-\varphi_2}{2}\right)\right|\right| - \left|(y-z)^2\left(\cos(\varphi_1)+\cos(\varphi_2)\right)\right| \geq 4 - \frac{2\sqrt{\pi^2((y-z)^2)^2+4}}{\pi}.$$

$(1+C)^2$ for $y|01\rangle + z|10\rangle,\ yz<0$

- Leggett Ineq:

$$|\bar{P}_{\mathbf{a},\mathbf{b}}(\varphi_1)+\bar{P}_{\mathbf{a},\mathbf{b}}(\varphi_2)| + |\bar{P}_{\mathbf{c},\mathbf{d}}(\varphi_1)+\bar{P}_{\mathbf{c},\mathbf{d}}(\varphi_2)| \leq 4-\frac{4}{\pi}\left|\sin\left(\frac{\varphi_1-\varphi_2}{2}\right)\right|,$$

- For large enough C($\frac{\sqrt{\pi^2-1}}{\pi}-1\approx 0.8960$): $\cdots$ > $\cdots$, LI is broken

$y|01\rangle + z|10\rangle,\ yz<0$

Sufficient condition

- For $y|01\rangle + z|10\rangle,\ yz < 0$, when C>0.8960, LI can be violated.
- For $x|00\rangle + y|01\rangle + z|10\rangle + w|11\rangle$?

- Choose different bases for polarizers is equivalent to apply two local unitary transformations,

$$p^{\pm\pm}(\mathbf{a},\mathbf{b}) = \left|\langle f_L^{\pm} f_R^{\pm}|U_{\rm L}(\mathbf{a}) \otimes U_{\rm R}(\mathbf{b})|\Psi\rangle\right|^2, \quad f_{L,R}^{+} = U'|\uparrow\rangle_{L,R}, \quad f_{L,R}^{-} = U''|\downarrow\rangle_{L,R}.$$

- It can be proved for **any** two particle state: (the detail is tedious)

$$x|00\rangle + y|01\rangle + z|10\rangle + w|11\rangle \Rightarrow y|01\rangle + z|10\rangle, \quad y,z \in R, \quad yz < 0$$

Is it also the necessary condition?

- An optimization problem： Find the smallest concurrence for the violation of LI

$$\mathbf{p} = \{x, y, z, w, \varphi_1, \varphi_2\}$$

S_{QM} : *result of QM for* $x|00\rangle + y|01\rangle + z|10\rangle + w|11\rangle$

$$\begin{cases} \min_{\mathbf{p}} f(\mathbf{p}) = C^2, \\ \text{subject to } g(\mathbf{p}) = S_{\text{LI}}(\mathbf{p}) - S_{\text{QM}}(\mathbf{p}) < 0. \end{cases}$$

$$|\bar{P}_{\mathbf{a},\mathbf{b}}(\varphi_1) + \bar{P}_{\mathbf{a},\mathbf{b}}(\varphi_2)| + |\bar{P}_{\mathbf{c},\mathbf{d}}(\varphi_1) + \bar{P}_{\mathbf{c},\mathbf{d}}(\varphi_2)| \le 4 - \frac{4}{\pi}\left|\sin\left(\frac{\varphi_1 - \varphi_2}{2}\right)\right|,$$

Numerical result is: **0.8960**

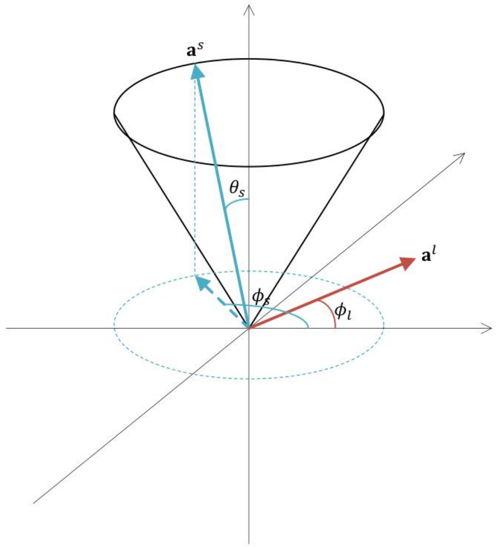
$$\frac{\sqrt{\pi^2 - 1}}{\pi} - 1 \approx 0.8960$$

Same as the sufficient condition

Leggett inequality with one vector on a cone

- For a special case: the polarizer vectors are not on two planes but one on a plane, the other on a cone (these equalities are for the case of entangled meson pairs)

$$\mathbf{a}^l(\phi_l) = (\cos\phi_l, \sin\phi_l, 0)$$
$$\mathbf{a}^s(\theta_s, \phi_s) = (\sin\theta_s\cos\phi_s, \sin\theta_s\sin\phi_s, \cos\theta_s)$$



$$\hat{E}_{sl}^-(\varphi_1) + \frac{\pi\cos(\theta_1)L_1(\theta_s,\varphi_1)}{4\cos(\frac{\varphi_2}{2})}\hat{E}_{ll}^-(\varphi_2) \le 2\left(1+\frac{\pi\cos(\theta_1)L_1(\theta_s,\varphi_1)}{4\cos(\frac{\varphi_2}{2})}\right) - \cos(\theta_1)L_1(\theta_s,\varphi_1),$$

$$\hat{E}_{sl}^+(\varphi_1) + \frac{\pi\cos(\theta_2)L_2(\theta_s,\varphi_1)}{4\left|\sin(\frac{\varphi_2}{2})\right|}\hat{E}_{ll}^+(\varphi_2) \ge -2\left(1+\frac{\pi\cos(\theta_2)L_2(\theta_s,\varphi_1)}{4\left|\sin(\frac{\varphi_2}{2})\right|}\right) + \cos(\theta_2)L_2(\theta_s,\varphi_1).$$

$$\hat{E}_{sl}^+(\varphi_1) + \frac{\pi\cos(\theta_2)L_2(\theta_s,\varphi_1)}{4\sin(\theta_s)\left|\sin(\frac{\varphi_2}{2})\right|}\hat{E}_{ss}^+(\varphi_2) \ge -2\left(1+\frac{\pi\cos(\theta_2)L_2(\theta_s,\varphi_1)}{4\sin(\theta_s)\left|\sin(\frac{\varphi_2}{2})\right|}\right) + \cos(\theta_2)L_2(\theta_s,\varphi_1).$$

$$L_1(\theta_s,\varphi) = \sqrt{2+2\cos(\varphi)\sin(\theta_s)}$$
$$\theta_1 = \cos^{-1}\frac{\cos(\theta_s)}{\sqrt{2+2\cos(\varphi)\sin(\theta_s)}},$$
$$L_2(\theta_s,\varphi) = \sqrt{2-2\cos(\varphi)\sin(\theta_s)}$$
$$\theta_2 = \cos^{-1}\frac{\cos(\theta_s)}{\sqrt{2-2\cos(\varphi)\sin(\theta_s)}}.$$

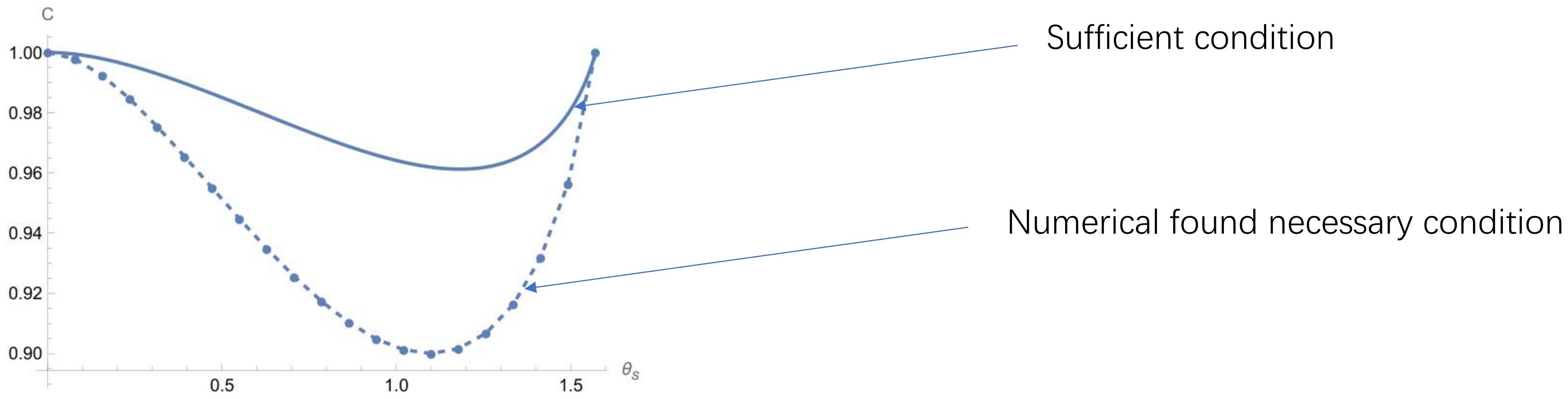
$$E^{\pm}(\mathbf{a},\mathbf{b}) = E(\mathbf{a},\mathbf{b}) + E(\mathbf{b},\pm\mathbf{b})$$

- Similar arguments applies:

- Sufficient condition:

$$C > C_{CLI_{1,2}}(\theta_s)$$

$$C_{CLI_{1,2}}(\theta_s) = \frac{1}{2\left((\sin(\theta_s)+1)^2 + \pi^2\cos^2(\theta_s)\right)} \{4(\sin(\theta_s)+1) + \cos(\theta_s)\left[-2(\sin(\theta_s)+1)\right.$$
$$\left. + \pi\left(\pi\cos(\theta_s) + \sqrt{8\sin(\theta_s) + \cos(\theta_s)\left(-4\sin(\theta_s) + (\pi^2-4)\cos(\theta_s) + 12\right) - 8}\right)\right]\}.$$



Summary

1. Particle physics played some crucial role in early histories of entanglement study.
2. Using Quantum Entanglement to Study CP and CPT Violations.
3. Genuine T Violation Signal in terms of entangled mesons. A Proposal for a High Energy quantum.
4. Entangled mesons violate a generalized Leggett Inequality, in which decays also depend on hidden variables. Effectively the measuring directions are time-dependent.
5. Entangled hyperons violated a generalized inequality, for the correlations between momenta directions of the decay products proton/antiproton.
6. A minimum entanglement is found for violation of the usual Leggett Inequality.

Thank you!