How Can We Reverse the Arrow of Time ?

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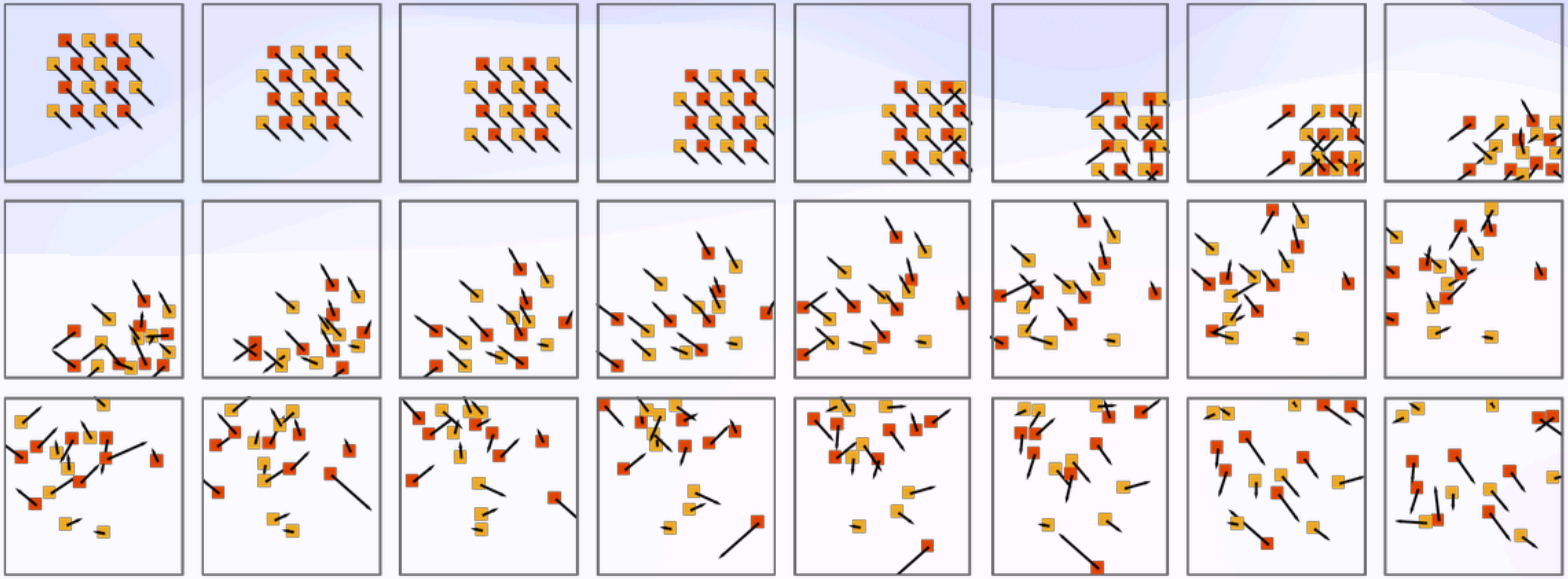
Institute for Advanced Study
Tsinghua University

安徽省基础科学中心2025学术年会
合肥
2025. 12

Second Law of Thermodynamics

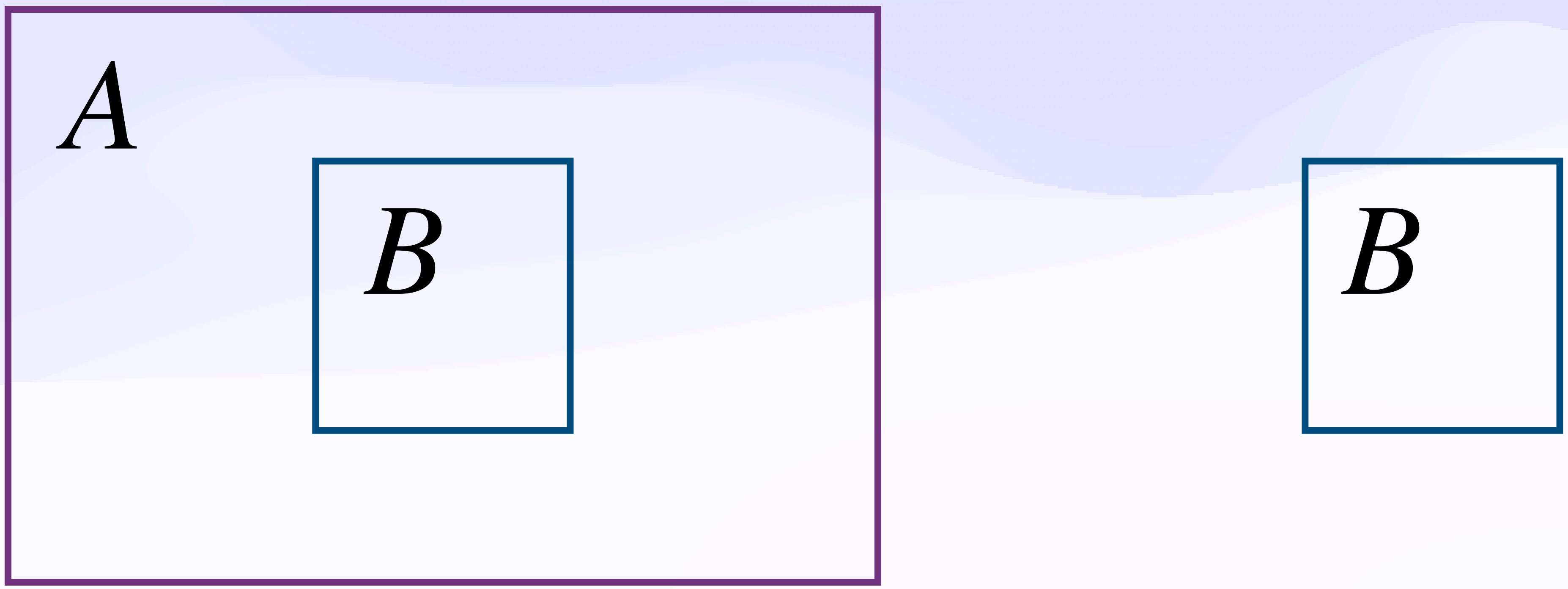
The total entropy of an interacting thermodynamic system never decreases



Irreversibility: the arrow of time

Thermalization of a Quantum Many-body State

Entanglement Entropy = Thermal Entropy



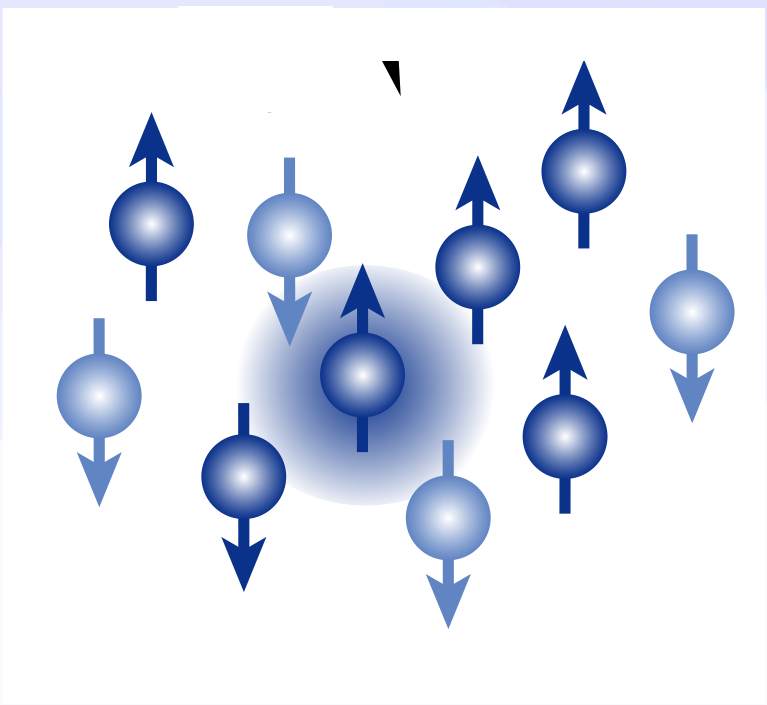
$$\rho_B^{ent} = Tr_A(|\Psi\rangle\langle\Psi|)$$

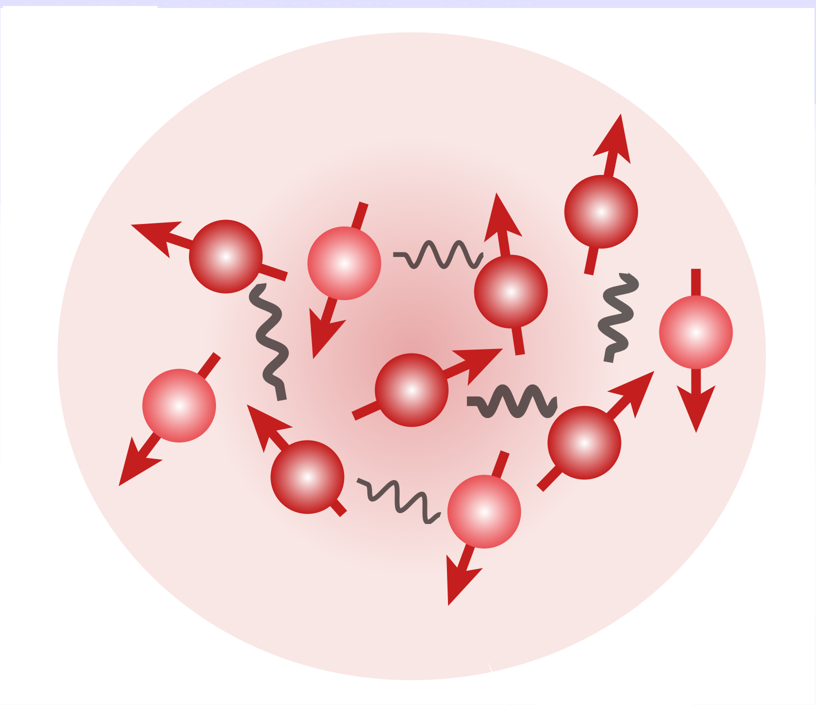
$$\rho_B^{the} = e^{-\beta H}$$

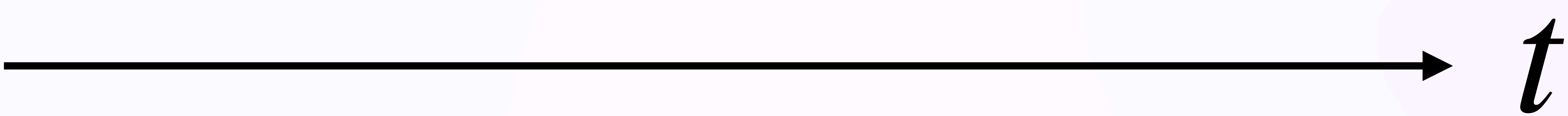
$$\rho_B^{ent} = \rho_B^{the}$$

Arrow of Time in Quantum Many-Body System

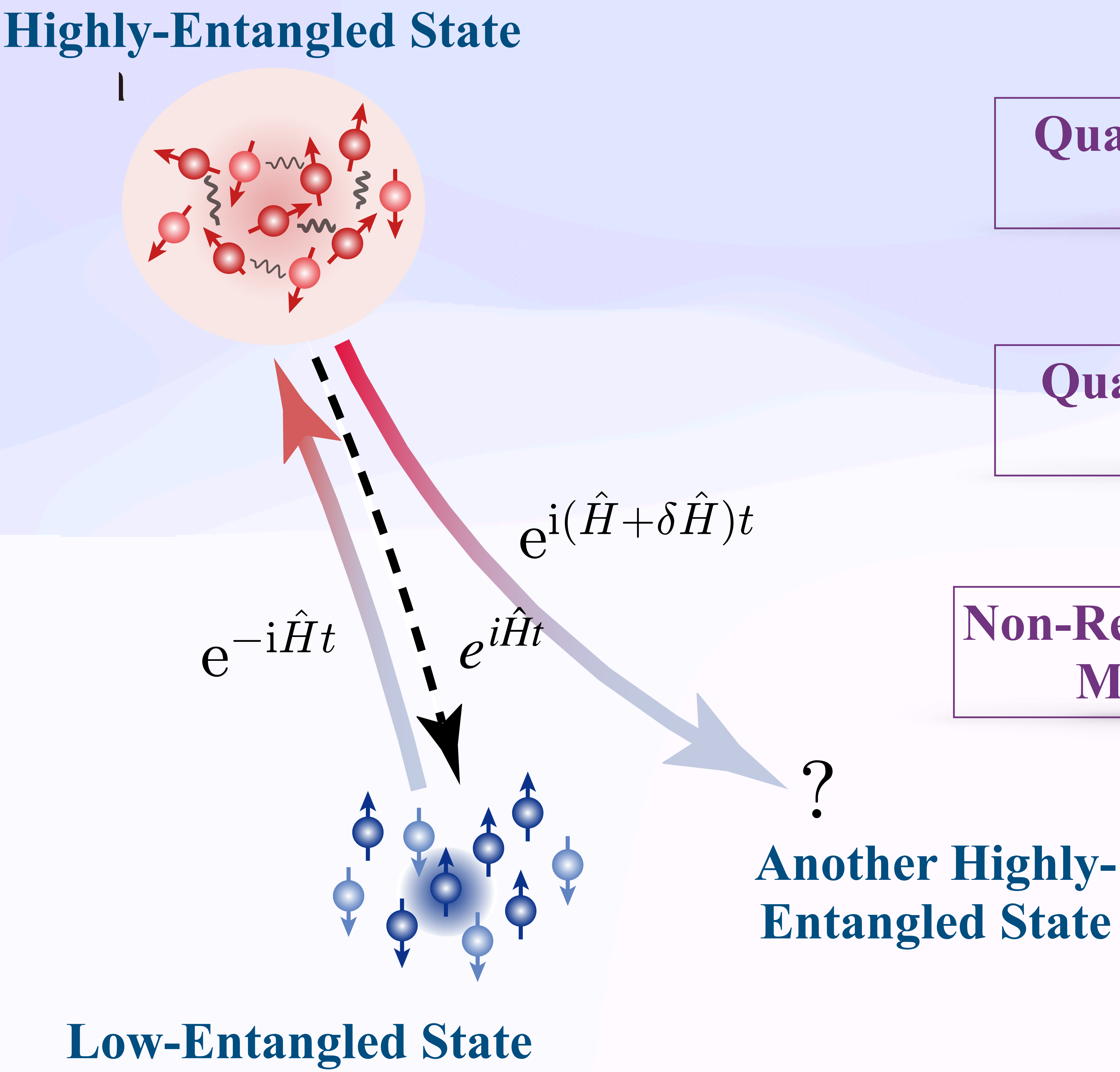
Entanglement increases under a Hamiltonian evolution







Reverse Quantum Many-Body Dynamics



Quantum Information Scrambling

Quantum Many-body Chaos

Non-Reversibility of Quantum Many-body Systems

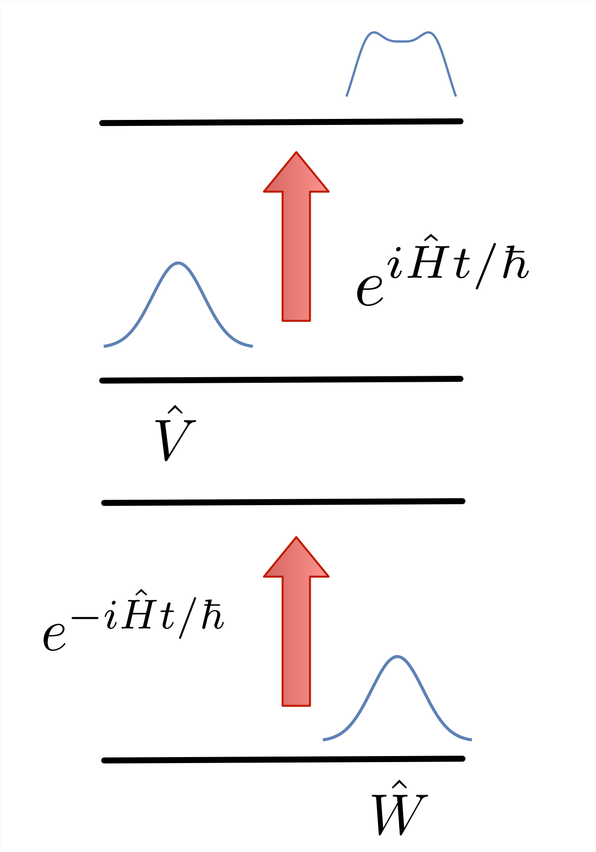
OTOC

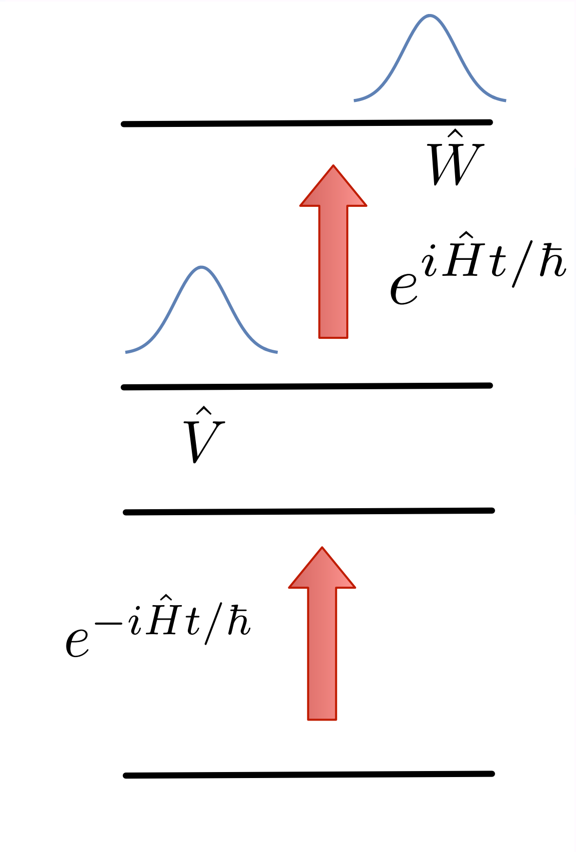
- **Out-of-Time-Ordered Correlator**

$$Tr[\hat{W}(0)\hat{V}(t)\hat{W}(0)\hat{V}(t)]$$

The overlap between these two outcomes:

$$\hat{V}(t)\hat{W}\,|\Psi\rangle \qquad \hat{W}\hat{V}(t)\,|\Psi\rangle$$





OTOC

- **Out-of-Time-Ordered Commutator**

$$Tr[[\hat{W}(0), \hat{V}(t)]^2]$$

containing two correlates with normal order and two correlators with out-of-time-order

$$\hat{V}(t) = e^{i\hat{H}t}\hat{V}e^{-i\hat{H}t}$$ **operator complexity increases**

$$= \hat{V} + [\hat{H}, \hat{V}](it) + \frac{1}{2}[\hat{H}, [\hat{H}, \hat{V}](it)^2 + \ldots$$

$$Tr[[\hat{W}(0), \hat{V}(t)]^2] \sim e^{\lambda t}$$

Lyapunov exponent

OTOC

Contents lists available at ScienceDirect

Science Bulletin

journal homepage: www.elsevier.com/locate/scib

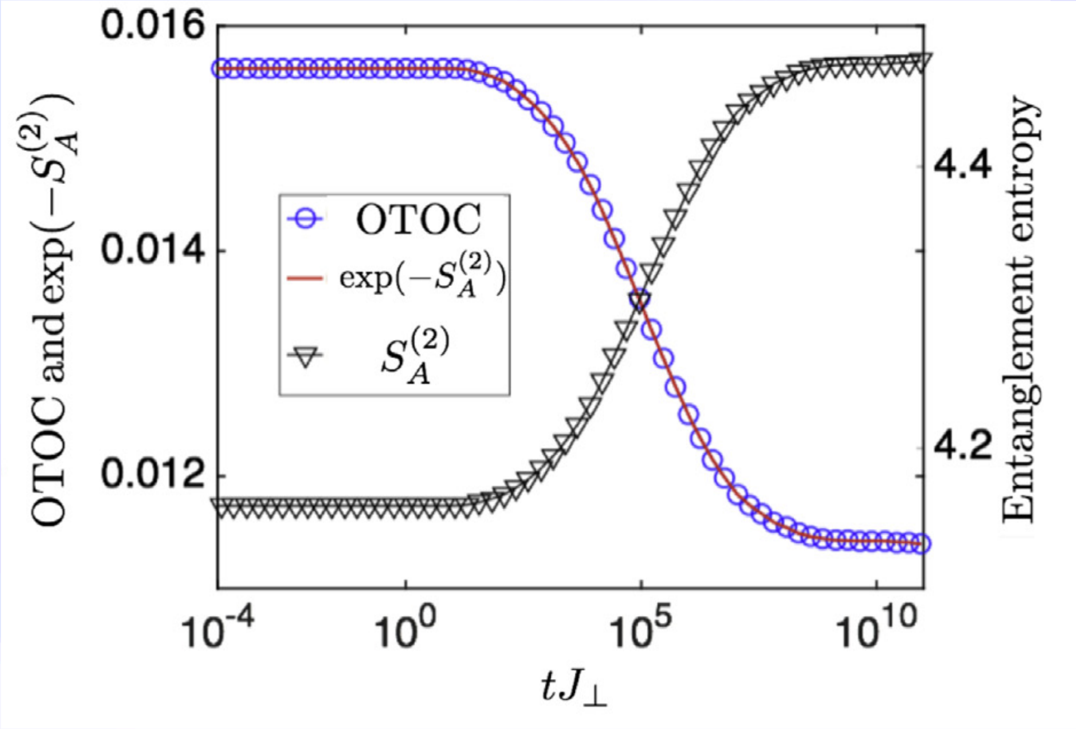
Article

Out-of-time-order correlation for many-body localization

Ruihua Fan[a,b,1], Pengfei Zhang[a,1], Huitao Shen[c], Hui Zhai[a,d,*]

[a] Institute for Advanced Study, Tsinghua University, Beijing 100084, China
[b] Department of Physics, Peking University, Beijing 100871, China
[c] Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[d] Collaborative Innovation Center of Quantum Matter, Beijing 100084, China

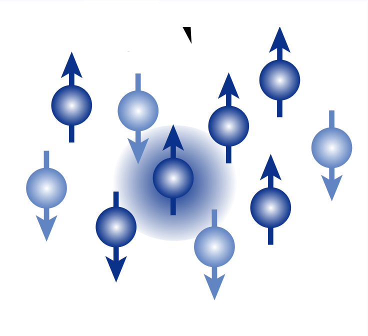
Citation >350

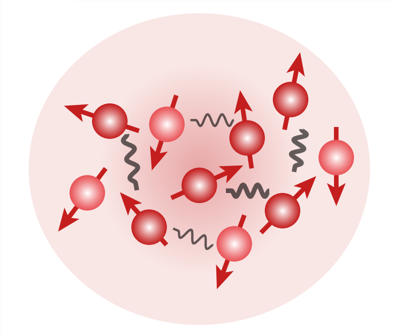


$$\exp(-S_A^{(2)}) = \sum_{\hat{M}\in B} \langle \hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)\rangle_{\beta=0}$$

linear growth

exponential decrease





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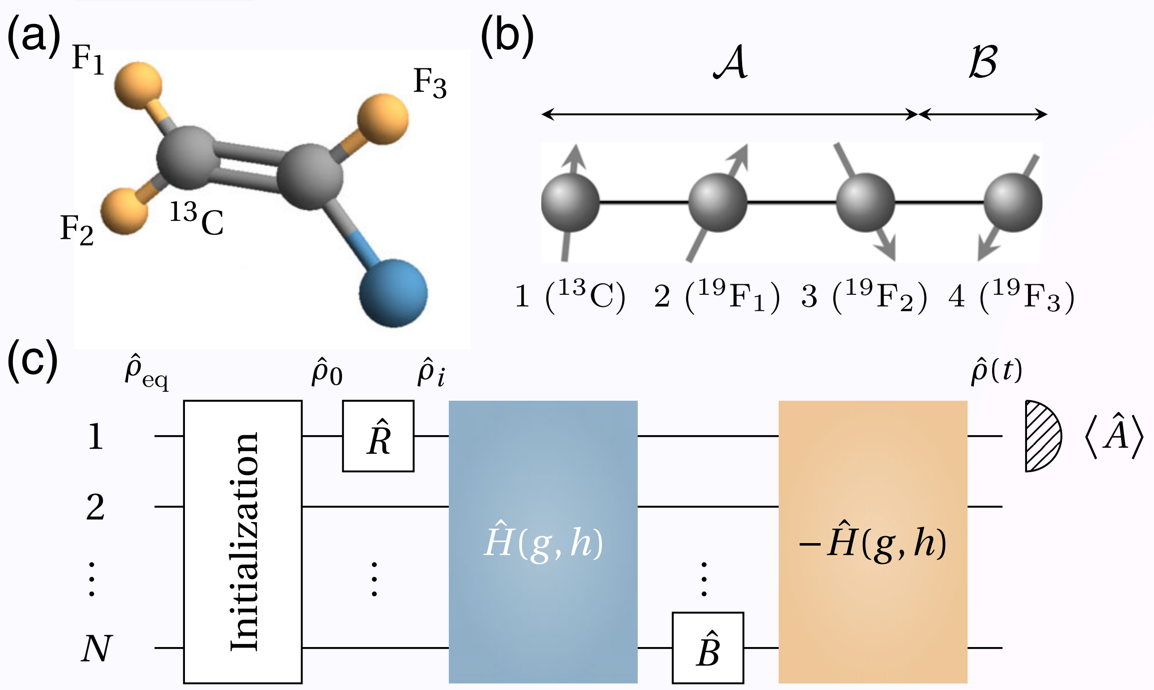
First Experimental Measurements of OTOC

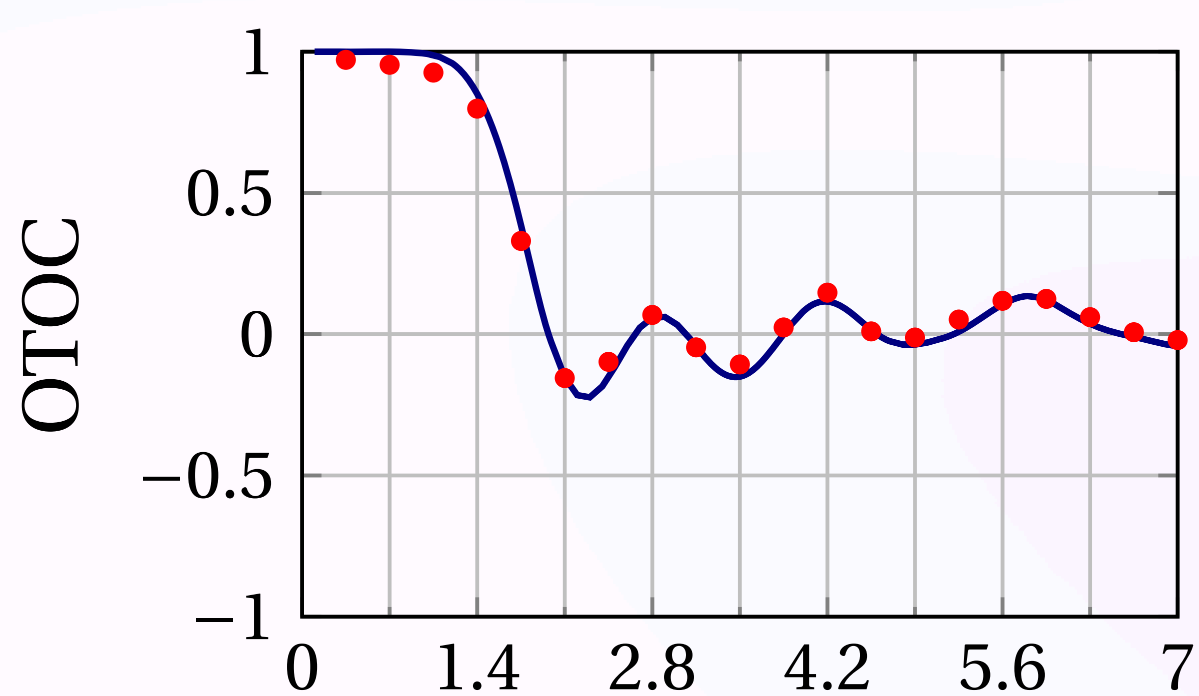
Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X **7,** 031011 (2017)

Measuring Out-of-Time-Order Correlators on a Nuclear Magnetic Resonance Quantum Simulator

Jun Li,[1] Ruihua Fan,[2,3] Hengyan Wang,[3] Bingtian Ye,[3] Bei Zeng,[4,5,2,*] Hui Zhai,[2,6,†] Xinhua Peng,[7,8,9,‡] and Jiangfeng Du[7,8]

Citation >500





First Experimental Measurements of OTOC

nature physics

ARTICLES

PUBLISHED ONLINE: 22 MAY 2017 | DOI: 10.1038/NPHYS4119

Measuring out-of-time-order correlations and multiple quantum spectra in a trapped-ion quantum magnet

Martin Gärttner[1†], Justin G. Bohnet[2†], Arghavan Safavi-Naini[1], Michael L. Wall[1], John J. Bollinger[2] and Ana Maria Rey[1*]

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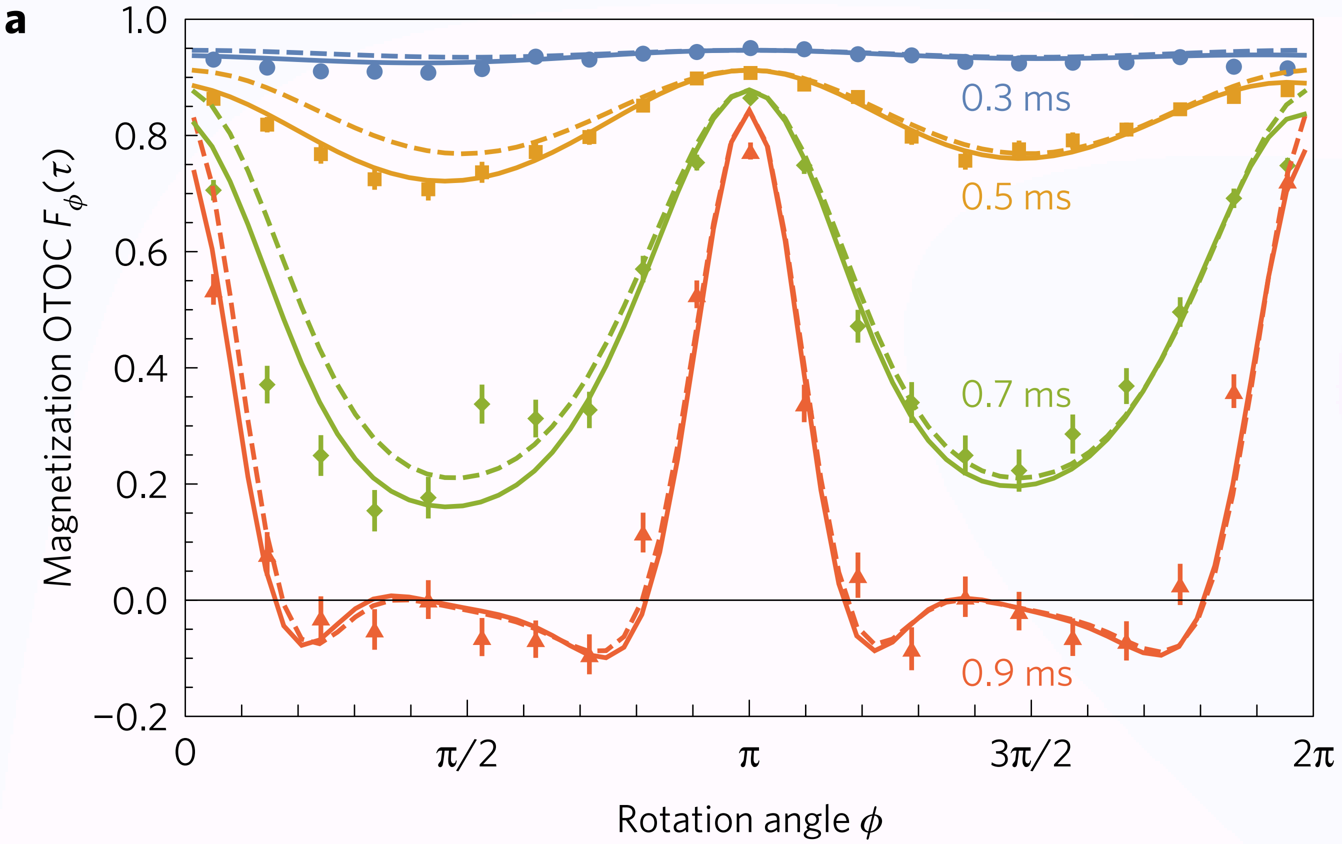


First Experimental Measurements of OTOC

Physics

VIEWPOINT

Seeing Scrambled Spins

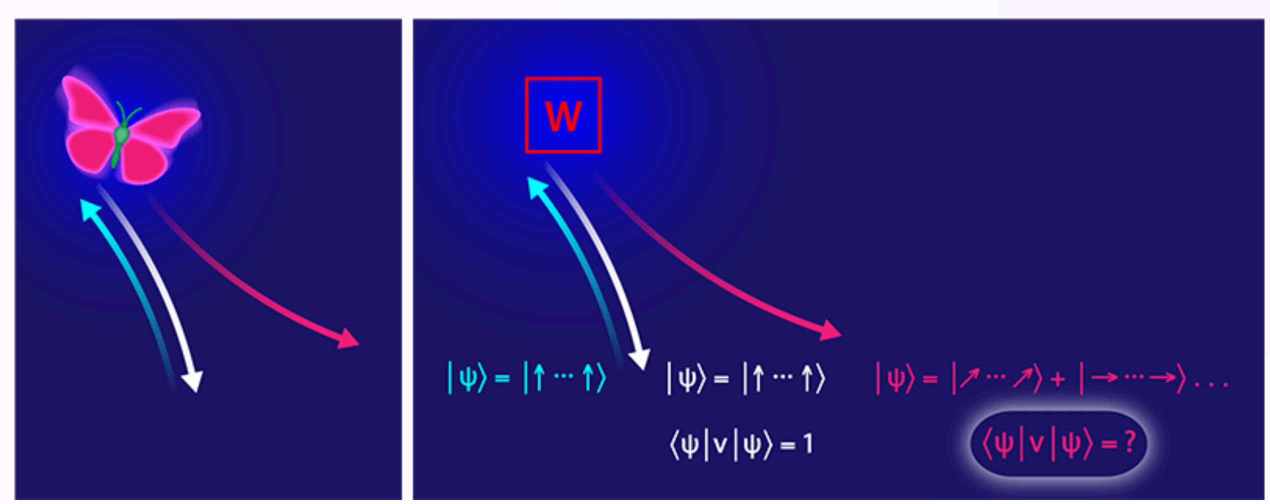
Two experimental groups have taken a step towards observing the "scrambling" of information that occurs as a many-body quantum system thermalizes.

by Brian Swingle* and Norman Y. Yao†

mation. Two groups, one in China [5] and one in the US [6], have taken a step towards tracking this scrambling of information in systems of quantum spins.

[5] J. Li, R. Fan, H. Wang, B. Ye, B. Zeng, H. Zhai, X. Peng, and J. Du, "Measuring Out-of-Time-Order Correlators on a Nuclear Magnetic Resonance Quantum Simulator," Phys. Rev. X **7**, 031011 (2017).

[6] M. Gärttner, J. G. Bohnet, A. Safavi-Naini, M. L. Wall, J. J. Bollinger, and A. M. Rey, "Measuring Out-of-time-order Correlations and Multiple Quantum Spectra in a Trapped-ion Quantum Magnet," Nat. Phys. (2017).



First Experimental Measurements of OTOC

news & views

QUANTUM SIMULATION

Probing information scrambling

Quantum information encoded in one of many interacting particles quickly becomes scrambled. A set of tools for tracking this process is on its way.

Monika Schleier-Smith

Analogous time-reversal schemes have a long history in nuclear magnetic resonance (NMR) experiments[9,10]. Gärttner and colleagues' work, together with recent related work on nuclear spins[11,12], rediscovers a decades-old NMR protocol and recognizes its significance for measuring out-of-time-order correlations. The protocol begins by

11. Wei, K. X., Ramanathan, C. & Cappellaro, P. Preprint at http://arxiv.org/abs/1612.05249 (2016).
12. Li, J. *et al.* Preprint at http://arxiv.org/abs/1609.01246 (2016)

Following-up Experiments on OTOC

PHYSICAL REVIEW LETTERS **123,** 090605 (2019)

Emergent Prethermalization Signatures in Out-of-Time Ordered Correlations

Ken Xuan Wei,[1] Pai Peng (彭湃),[2] Oles Shtanko,[1] Iman Marvian,[3] Seth Lloyd,[4]
Chandrasekhar Ramanathan,[5] and Paola Cappellaro[6,*]

NMR Experiment by MIT group, PRL 2019

LETTER

https://doi.org/10.1038/s41586-019-0952-6

Verified quantum information scrambling

K. A. Landsman[1*], C. Figgatt[1,6], T. Schuster[2], N. M. Linke[1], B. Yoshida[3], N. Y. Yao[2,4] & C. Monroe[1,5]

Trap Ion Experiment by Maryland group, Nature, 2019

RESEARCH

QUANTUM SENSING

Improving metrology with quantum scrambling

Zeyang Li (李泽阳)[1], Simone Colombo[1], Chi Shu[1,2], Gustavo Velez[1,3], Saúl Pilatowsky-Cameo[4], Roman Schmied[5], Soonwon Choi[4], Mikhail Lukin[2], Edwin Pedrozo-Peñafiel[1], Vladan Vuletić[1*]

Cold Atom Experiment by MIT Group, Science 2023

Following-up Experiments on OTOC

RESEARCH

QUANTUM SIMULATION

Information scrambling in quantum circuits

Superconducting Qubit Experiment by Google Group, Science 2021

Article

Observation of constructive interference at the edge of quantum ergodicity

https://doi.org/10.1038/s41586-025-09526-6

Google Quantum AI and Collaborators*

Received: 3 November 2024

Superconducting Qubit Experiment by Google Group, Nature 2025

Comments on Current Experimental Status

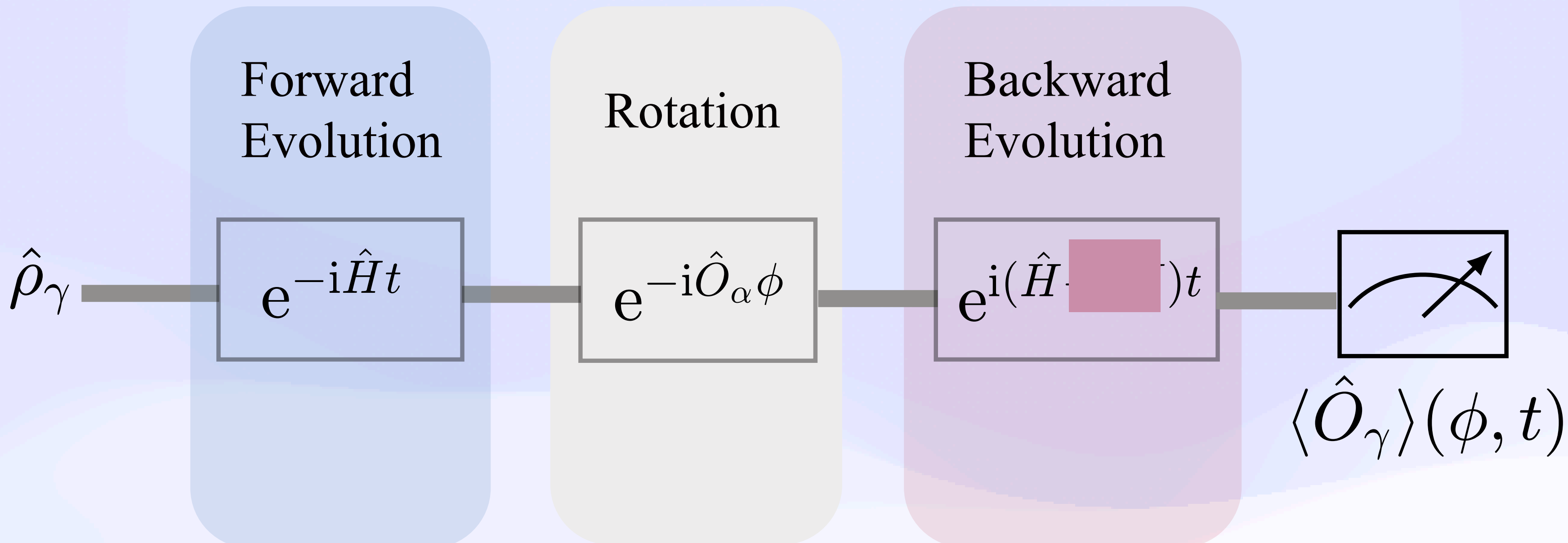
$$Tr[[\hat{W}(0), \hat{V}(t)]^2] \sim e^{\lambda t}$$

Lyapunov exponent

- **None of these experiments sees well-defined exponential behavior of OTOC and is able to extract the Lyapunov exponent.**

- **The reason is that measuring OTOC always involves backward time evolution, which is always imperfect.**

Experimental Protocol

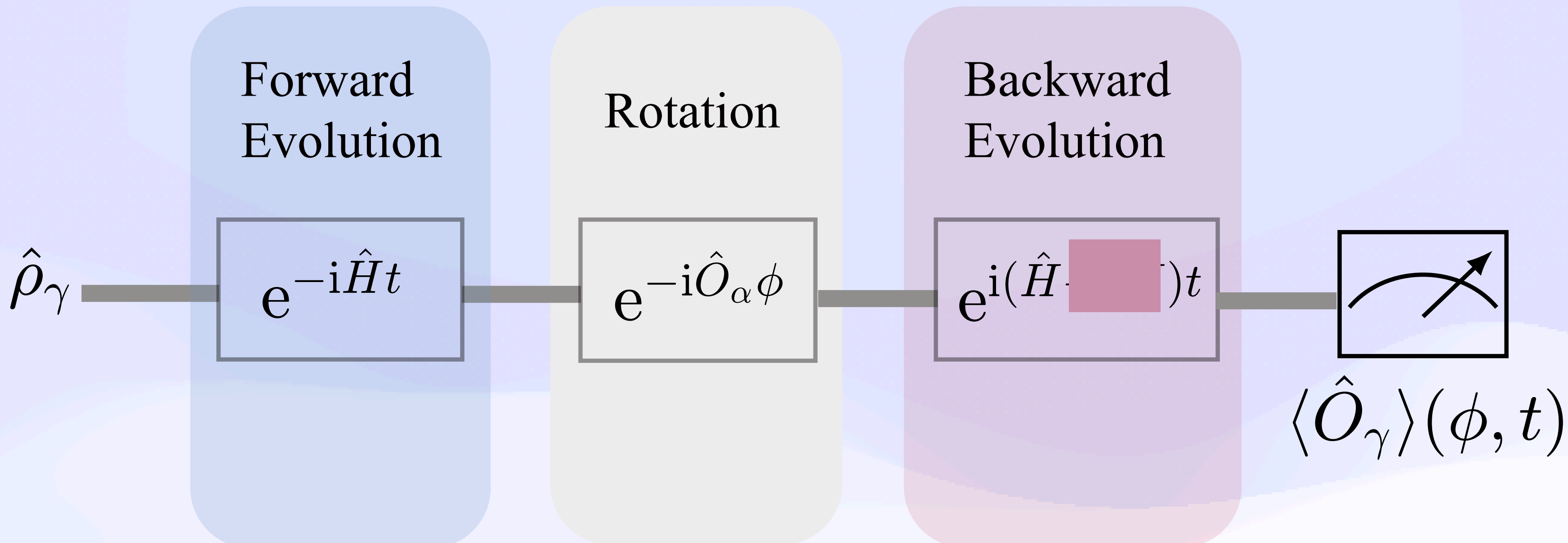


$$\hat{\rho}_\gamma = 1 + \epsilon \hat{O}_\gamma$$

$$\hat{O}_\gamma = \sum_i s_\gamma^i$$

$$\gamma = x, y, z$$

Experimental Protocol



$$Tr[\hat{O}_\gamma e^{i\hat{H}t}e^{-i\hat{O}_\alpha\phi}e^{-i\hat{H}t}\hat{O}_\gamma e^{i\hat{H}t}e^{i\hat{O}_\alpha\phi}e^{-i\hat{H}t}]$$

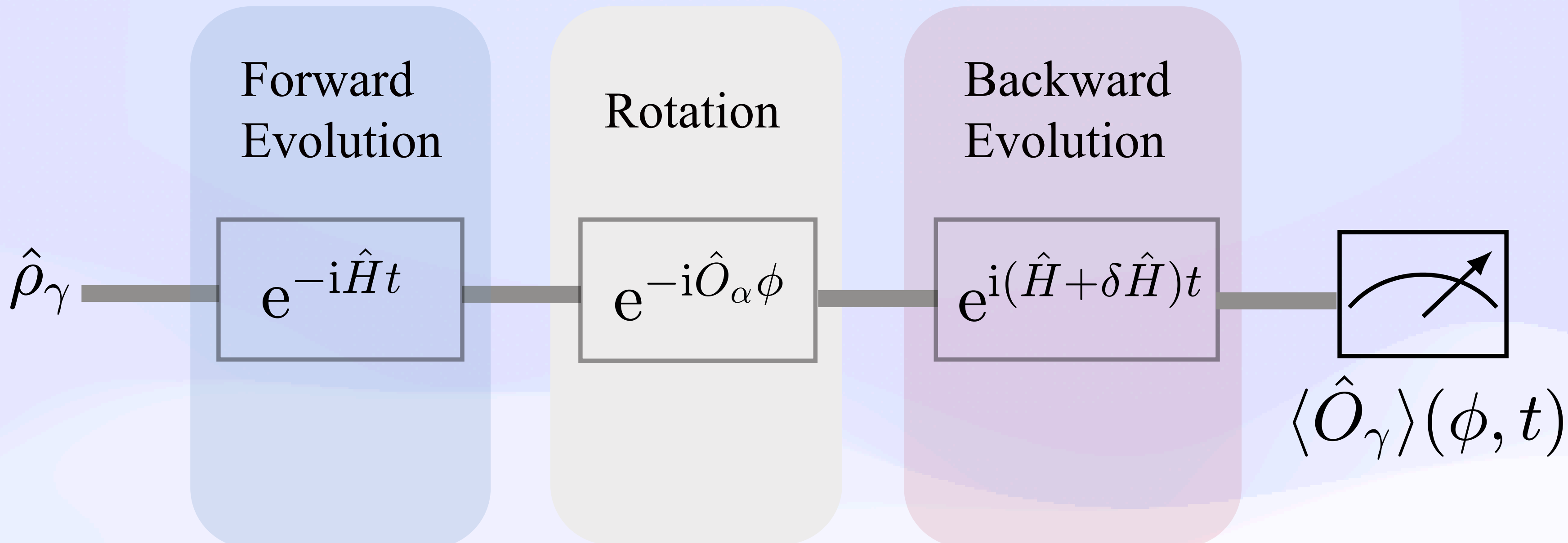
OTO Correlator

$$F(\phi,t)=Tr[\hat{O}_\gamma e^{-i\hat{O}_\alpha(t)\phi}\hat{O}_\gamma e^{i\hat{O}_\alpha(t)\phi}]$$

OTO Commutator

$$-\frac{\partial^2 F(\phi,t)}{\partial\phi^2}=Tr[[\hat{O}_\alpha(t),\hat{O}_\gamma]^2]$$

Experimental Protocol



$$F(\phi,t) = Tr[\hat{O}_\gamma \hat{V}(t) e^{-i\hat{O}_\alpha(t)\phi} \hat{O}_\gamma e^{i\hat{O}_\alpha(t)\phi} \hat{V}(t)]$$

$$\hat{V}(t) = e^{-iT\int_0^t dt'\delta\hat{H}(t')}$$

Imperfection in the backward evolution

Consequence: $F(0,t)$ **is not a constant.**

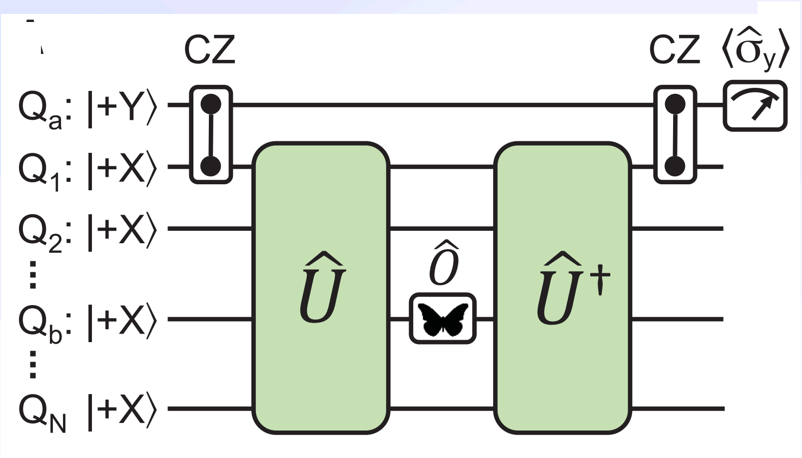
Out-of-Time-Ordered Correlator

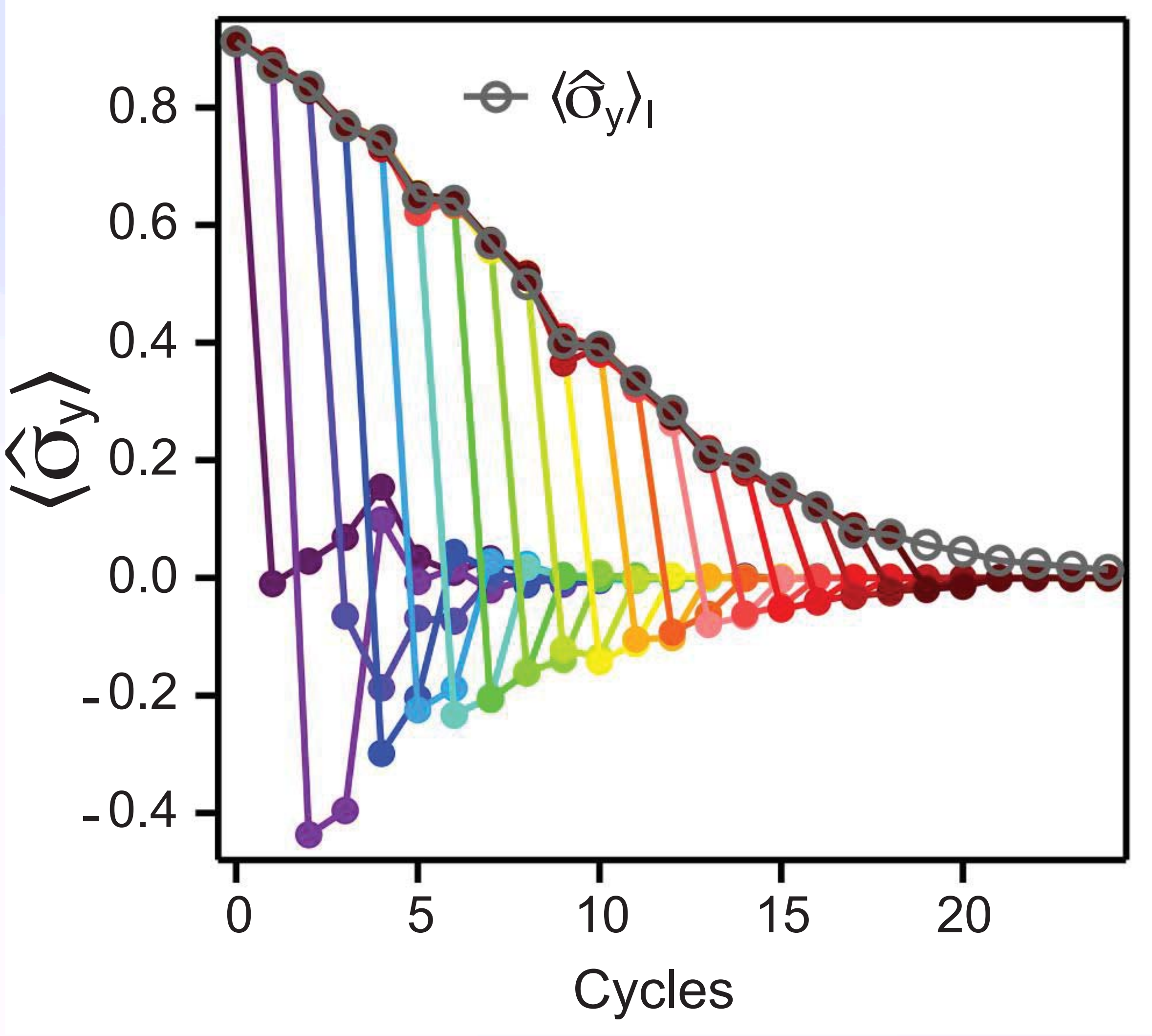
Consequence: $F(0,t)$ **is not a constant.**

RESEARCH

QUANTUM SIMULATION

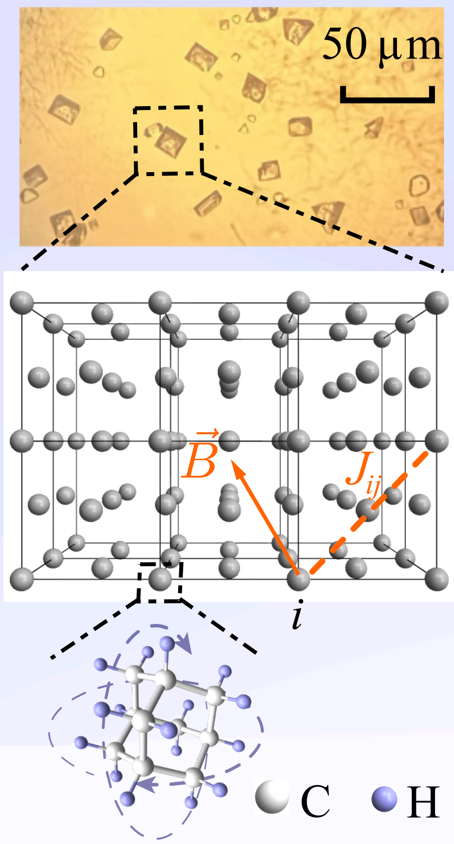
Information scrambling in quantum circuits

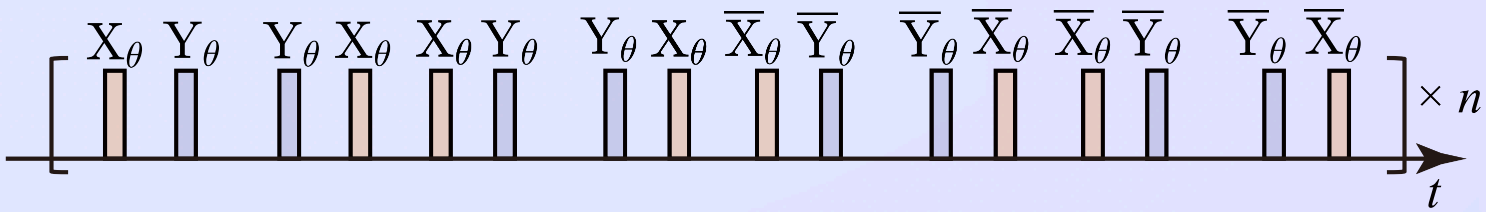


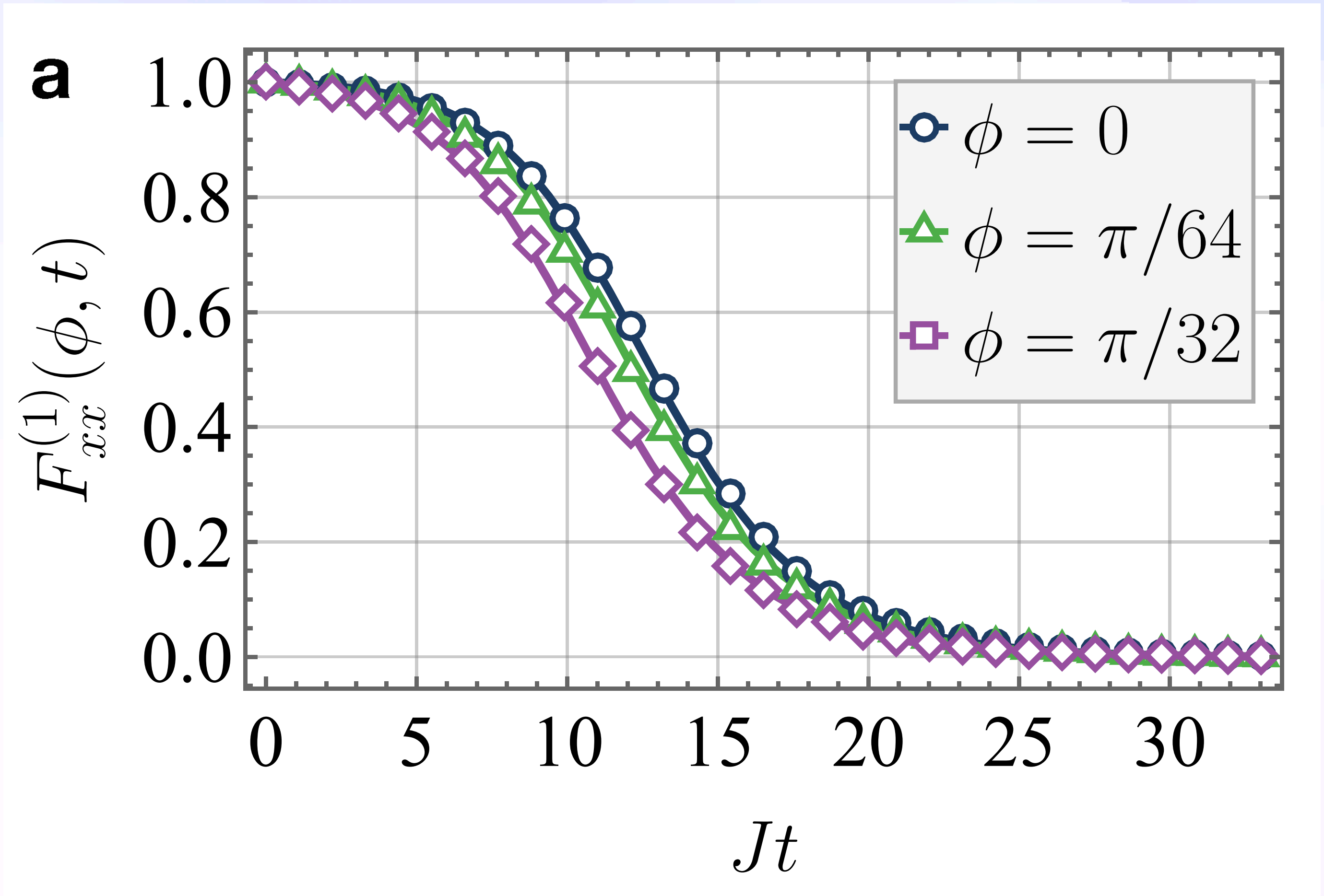


Error-Mitigation: $$\frac{F(\phi, t)}{F(0,t)}$$

Out-of-Time-Ordered Correlator





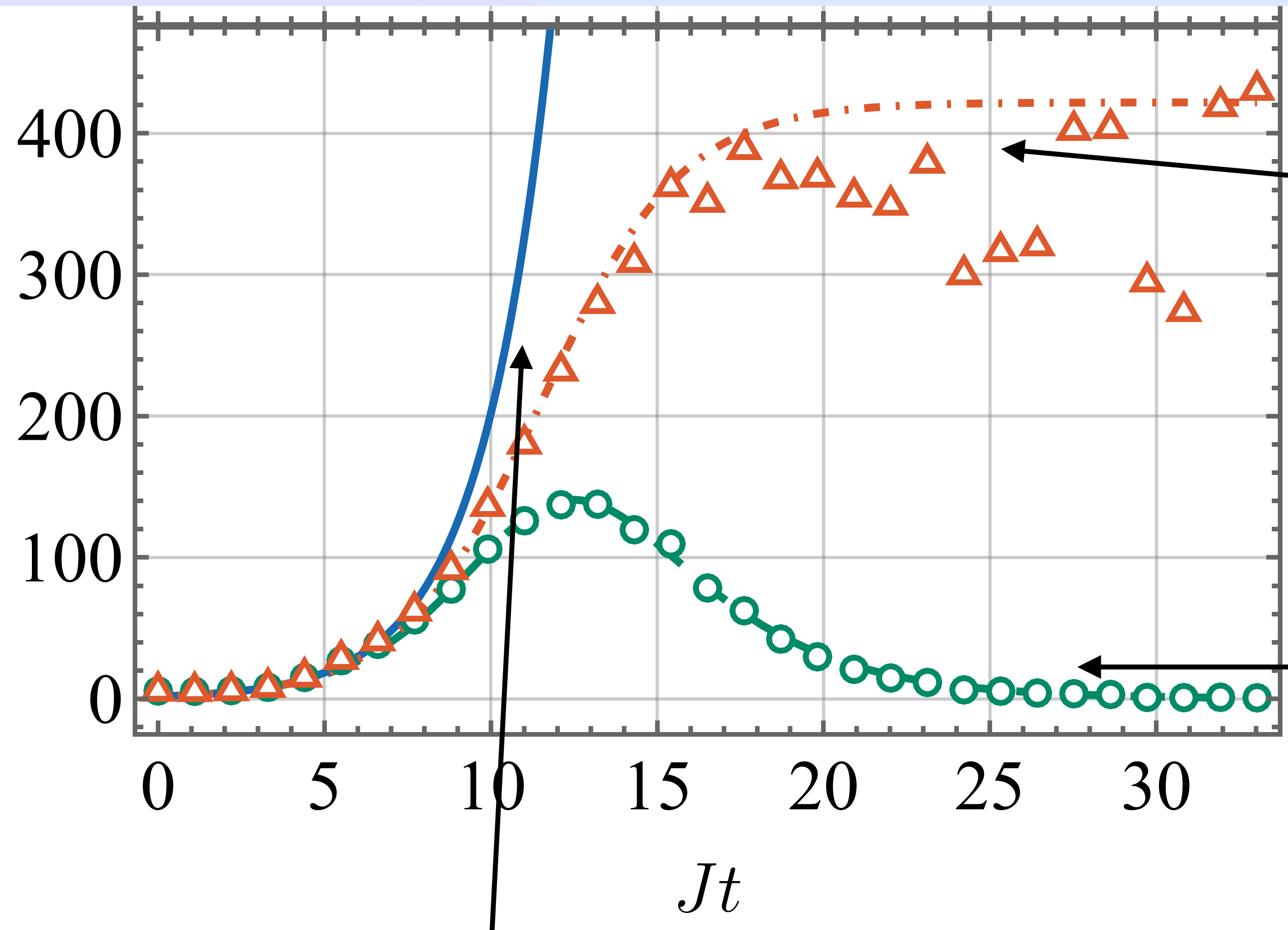


$$\hat{H}_0 = \sum_{i<j,m,n} \sum_{\mu,\nu} J_{ij} \xi_{\mu\nu} \hat{S}_{im}^{\mu} \hat{S}_{jn}^{\nu}$$

Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, arXiv: 2506.19915, PRL, accepted

Mitigating Error in Reversed Dynamics

OTO Commutator $Tr[[\hat{O}_\alpha(t), \hat{O}_\gamma]^2]$



Error-Mitigation:

$$\frac{F(\phi, t)}{F(0,t)}$$

No Mitigation

How do we obtain this one?

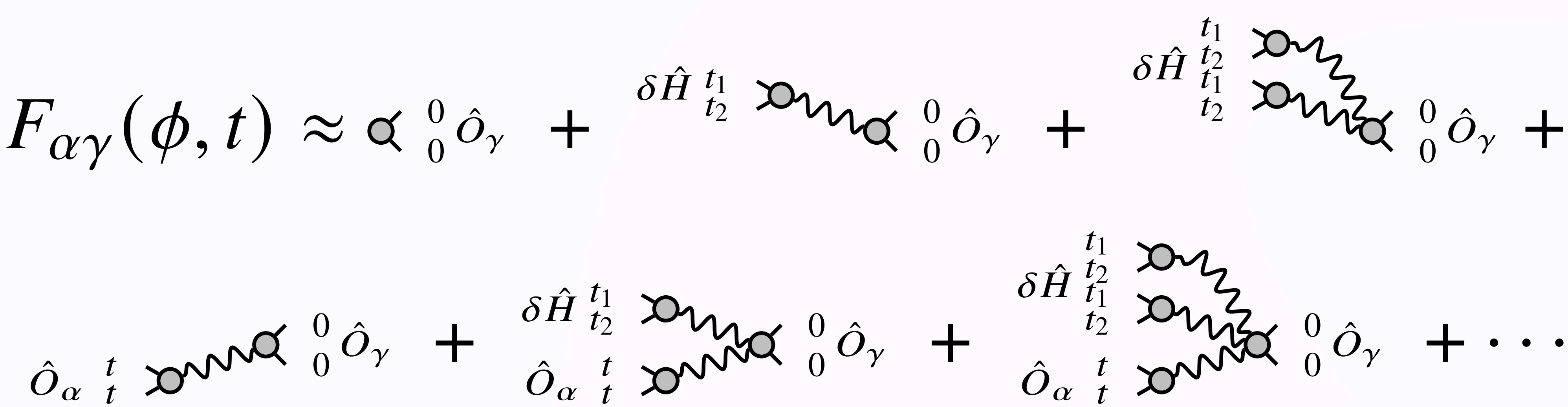
Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, arXiv: 2506.19915, PRL, accepted

Scramblon Theory for OTOC

$$F(\phi, t) = Tr[\hat{O}_\gamma \hat{V}(t) e^{-i\hat{O}_\alpha(t)\phi} \hat{O}_\gamma e^{i\hat{O}_\alpha(t)\phi} \hat{V}(t)]$$

- $\delta\hat{H} = 0$ **OTOC between** $\hat{O}_\gamma$ **and** $e^{i\phi\hat{O}_\alpha(t)}$
- $\phi = 0$ **OTOC between** $\hat{O}_\gamma$ **and** $\hat{V}(t)$

Scramblon is a new collective mode mediating information scrambling



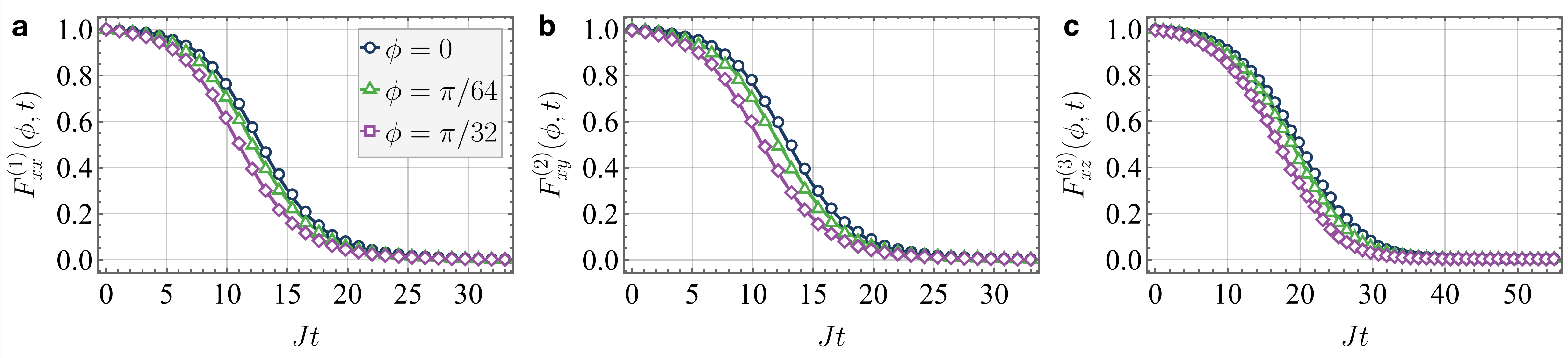
Yingfei Gu, A. Kitaev and Pengfei Zhang, JHEP 2022

Scramblon Ansatz for OTOC with Imperfections

$$F(\phi, t) = Tr[\hat{O}_\gamma \hat{V}(t) e^{-i\hat{O}_\alpha(t)\phi} \hat{O}_\gamma e^{i\hat{O}_\alpha(t)\phi} \hat{V}(t)]$$

Scramblon Ansatz

$$F_{\alpha\gamma}(\phi, t) = \frac{1}{\left(1 + a e^{\varkappa t} + b\phi^2 e^{\varkappa t}\right)^{2\Delta}}$$



Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, arXiv: 2506.19915, PRL, accepted

Verification of the Scarmblon Ansatz

$$F(\phi, t) = Tr[\hat{O}_\gamma \hat{V}(t) e^{-i\hat{O}_\alpha(t)\phi} \hat{O}_\gamma e^{i\hat{O}_\alpha(t)\phi} \hat{V}(t)]$$

Scramblon Ansatz

$$F_{\alpha\gamma}(\phi, t) = \frac{1}{(1 + a e^{\varkappa t} + b\phi^2 e^{\varkappa t})^{2\Delta}}$$

- κ is independent of α and γ
- a and Δ is independent of α
- $M_{\alpha\gamma} = b_{\alpha\gamma}\Delta_\gamma$ must be symmetric and rank-1

Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, arXiv: 2506.19915, PRL, accepted

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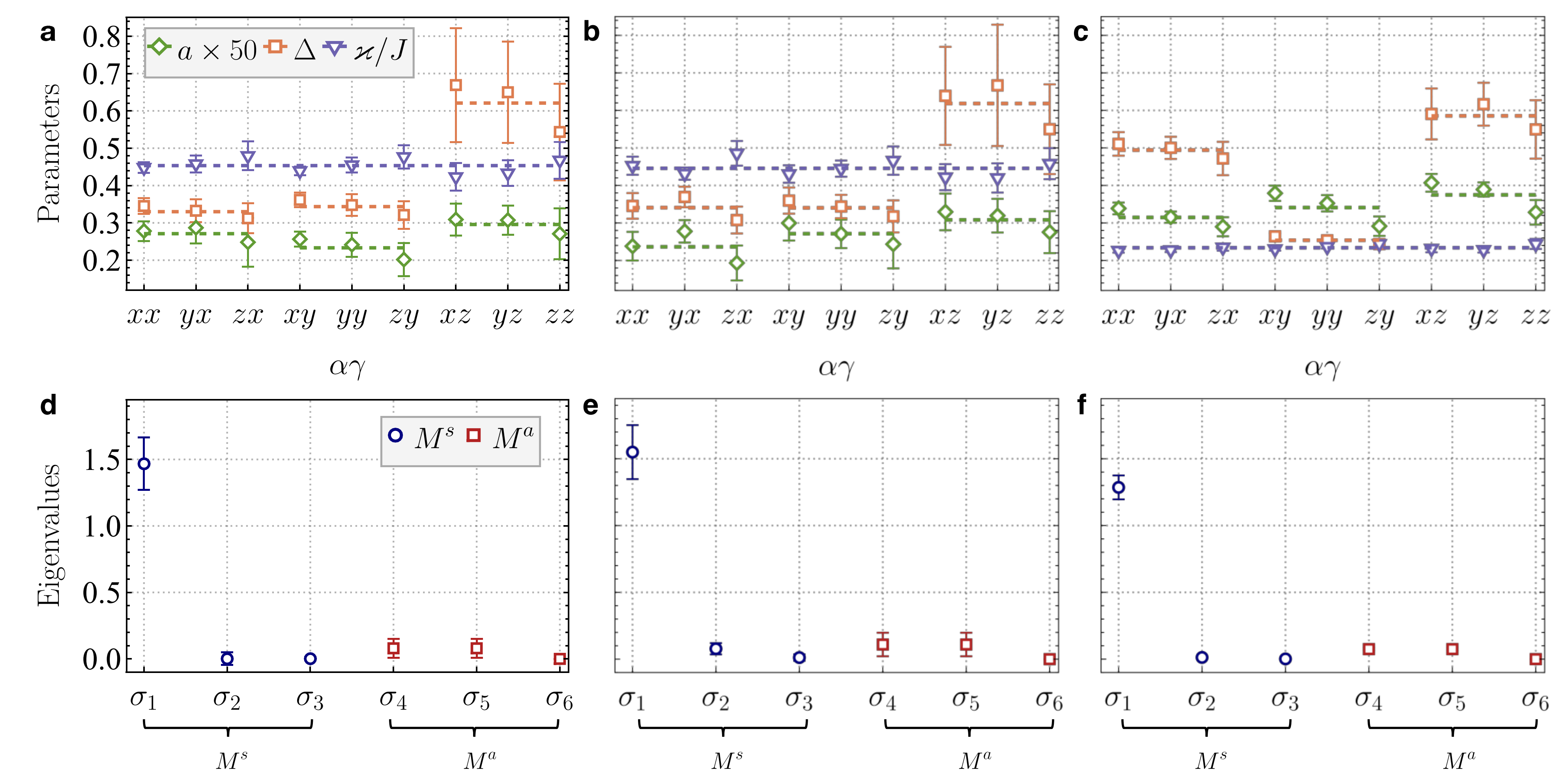

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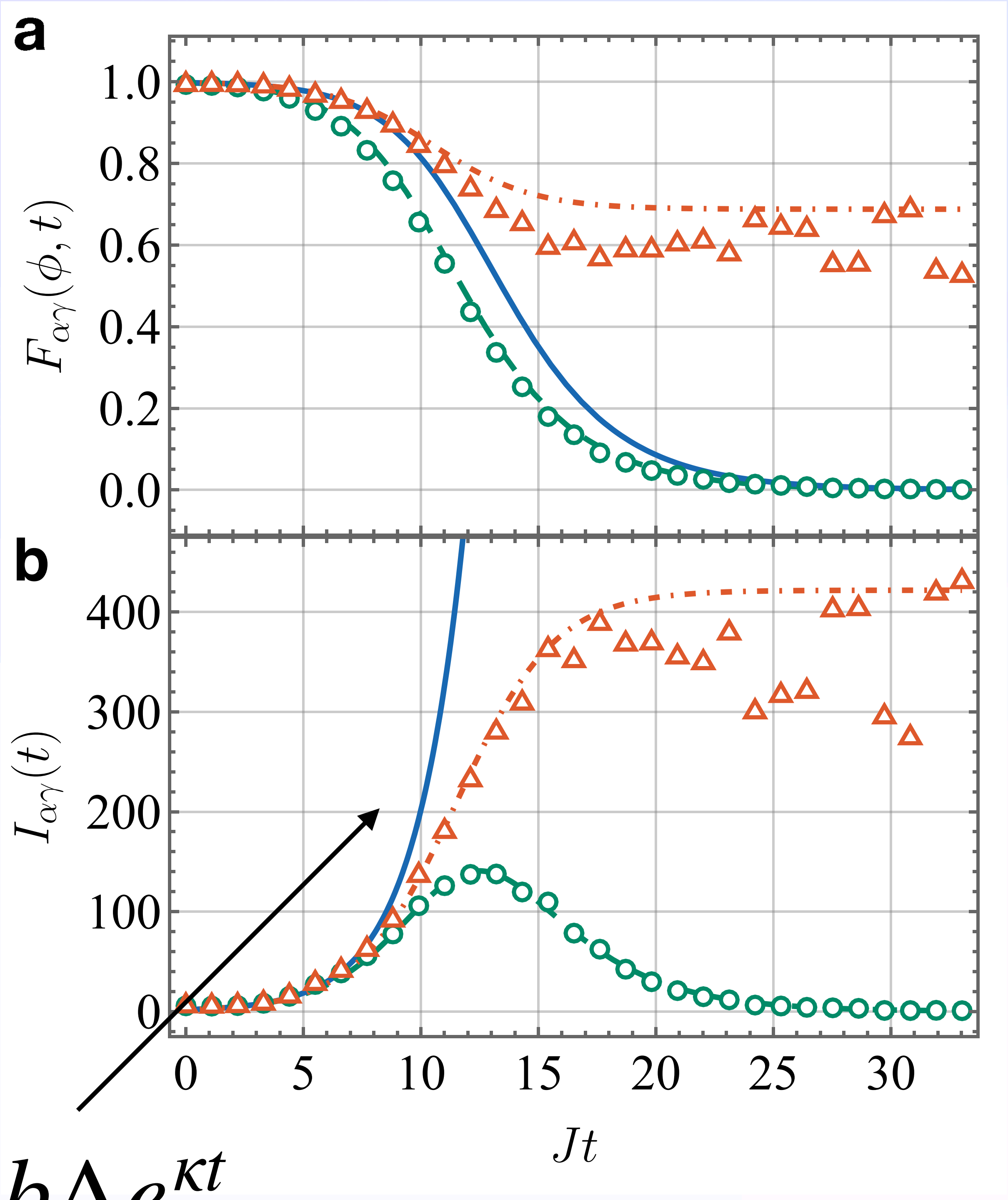
Error Mitigation with Scramblon Theory



$2b\Delta e^{\kappa t}$

Error-Mitigation: $\dfrac{F(\phi, t)}{F(0,t)}$

Scramblon Ansatz with

$a = 0$

No Mitigation

Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, arXiv: 2506.19915, PRL, accepted

Summary

- The **first** experimental validation of the scramblon theory for quantum information scrambling
- The **first** experimental extrapolation of the Lyapunov exponent for the quantum many-body case

Of course, one cannot reverse the arrow of time in nature, but perhaps one can reverse the arrow of time by error mitigation with the help of a proper theory.

Yu-Chen Li, Tian-Gang Zhou, Sengyu Zhang, Ze Wu, Liqiang Zhao, Haochuan Yin, Xiaoxue An, Hui Zhai, Pengfei Zhang, Xinhua Peng and Jiangfeng Du, arXiv: 2506.19915, PRL, accepted

九年
从第一次测量OTOC到第一次得到Lyapunov指数
是理论上对信息扩散理解的不断深入
从液体核磁到固体核磁
是实验上量子模拟操控能力的不断提升

Thank you very much for your attention !