Exploring Nucleon structure and the proton mass from holographic QCD

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In collaborate with: Jiali Deng

Based on: Jiali Deng and Defu Hou, Phys.Rev.D 112 (2025) 3, 036011;
arXiv:2512.17554，arXiv:2603.04794

Significance of Nucleon Structure Studies

1. Understanding the Origin of Mass

To answer the ultimate question: **"Where does most of our mass come from?"**
The answer lies not in the Higgs mechanism, but primarily in the **dynamical energy of QCD** and the non-perturbative structure of the nucleon.

2. Understanding the Origin of Spin

Similar to the mass puzzle: How does the proton's spin of **1/2 emerge** from the spins and orbital angular momenta of its constituent quarks and gluons?
This remains the central question of the **"spin crisis"** in particle physics.

3. Testing Fundamental Theory (QCD)

The nucleon serves as the most important **laboratory** for studying strong interactions in the non-perturbative regime.
It provides a crucial testing ground for validating and developing our understanding of QCD.

How to "See" the Internal Structure?

Probe Type	Process	Revealed Physical Information	Key Observables
Electroweak Probe	DIS	Parton momentum distribution	Structure Functions → PDFs
Electroweak Probe	Elastic scattering	Electromagnetic distribution	EM FFs
Gravitational Probe	(Theoretical concept)	Mechanical distribution	GFFs
Hadron Spectrum	–	Global properties: Mass of nucleon as QCD bound state	Mass spectrum

表: Experimental probes for studying nucleon internal structure

S. Kumano,Qin-Tao Song， and O. V. Teryaev Phys.Rev.D 97 (2018) 1, 014020

Outline

- Nucleon Structure in Holographic VQCD Model
- Nucleon Structure in Holographic Soft Model
- From form factors to GPD
- Trace Anomaly Contribution to the Proton Mass

- **Nucleon Structure in Holographic VQCD Model**
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VQCD Model

Large-N_c Limits Comparison

Matti Jarvinen and Elias Kiritsis, JHEP 03 (2012) 002
U. G¨ursoy, E. Kiritsis, F. Nitti, JHEP 02 (2008) 019

- **Standard 't Hooft limit**: $N_c \to \infty$, $\lambda = g_{\mathrm{YM}}^2 N_c$ and N_f finite,
- **Veneziano limit**: $N_c \to \infty$, $N_f \to \infty$, $\frac{N_f}{N_c} = x$ fixed
- Preserves important quark effects while maintaining large-N_c simplifications

Holographic Construction

- **Gluon sector**: 5D Einstein-dilaton gravity

$$S_g = M^3 N_c^2 \int d^5x \sqrt{g}[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)]$$

- **Flavor sector**: Tachyonic Dirac-Born-Infeld action

$$S_{DBI} = -M^3 N_c^2 \int d^5x \sqrt{g} V_f(\lambda, \tau) \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)F_{ab})}$$

- **Metric ansatz**:

$$ds^2 = e^{2A(z)}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$$

Mass Spectrum

By solving the equations of motion, we obtain a Schrödinger-like equation:

$$-\varphi''_{R/L}(z) + (m_5^2 e^{2A_S(z)} \pm m_5 e^{A_S(z)} A'_S(z))\varphi_{R/L}(z) = M_n^2 \varphi_{R/L}(z)$$

	proton	M_{exp}/Gev	other/Gev	Our/Gev	%M
n=1	N(939)	0.938	0.987	0.939	0.107
n=2	N(1440)	1.360 to 1.380	1.264	1.333	2.701
n=3	N(1710)	1.680 to 1.720	1.531	1.653	2.764
n=4	N(1880)	1.820 to 1.900	1.791	1.893	1.774
n=5	N(2100)	2.050 to 2.150	2.046	2.097	0.143
n=6	N(2300)	2.300	2.296	2.273	1.174

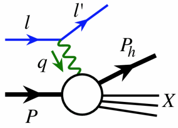
图: Mass spectrum results from numerical solution of the Schrödinger equation

Jiali Deng and Defu Hou, Phys.Rev.D 112 (2025) 3, 036011

Navas, S. and others. ParticleDataGroup:2024cfk

Structure Function

The hadronic tensor is defined as:



$$W^{\mu\nu} = 4\pi \int d^4x \, e^{iq\cdot x} \langle P | J^\mu(x) J^\nu(0) | P \rangle$$

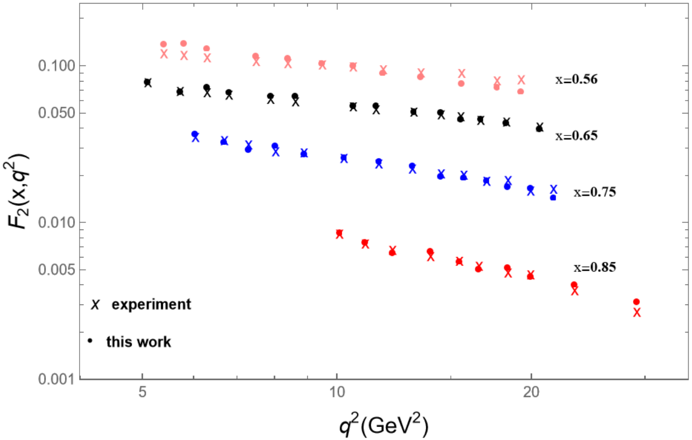
which can be decomposed into structure functions:

$$W^{\mu\nu} = F_1 \left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + \frac{2x}{q^2} F_2 (P^\mu + \frac{q^\mu}{2x})(P^\nu + \frac{q^\nu}{2x})$$

where $x = \frac{Q^2}{2P.q}$ is the Bjorken scaling variable.
The interaction term in the holographic description:

$$\eta_\mu \langle P + q, s_X | J^\mu(0) | P, s \rangle = S_{\text{int}} = g_V \int dz \, d^4y \sqrt{-g} \, \Phi(z, y) A_\mu(z, y) J^\mu(y)$$

Structure Function



Jiali Deng and Defu Hou, Phys.Rev.D 112 (2025) 3, 036011
L. W. Whitlow and E. M. Riordanr, et al, Whitlow:1991uw

Electromagnetic form factors

Hadron matrix element

$$\langle p', s' | J^\mu(0) | p, s\rangle = e\bar{u}(p', s')\Gamma^\mu(p, p')u(p, s)$$

Electric charge:
$q_e = F_1(0) = G_E(0)$
Magnetic moment:
$\mu = q + F_2(0) = G_M(0)$

Vertex function:

$$\Gamma^\mu(p, p') = \gamma^\mu \underbrace{F_1(Q^2)}_{\text{Dirac}} + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}\underbrace{F_2(Q^2)}_{\text{Pauli}}, \quad \Delta = p' - p, P = (p' + p)/2$$

$$= \frac{MP^\mu}{P^2}\underbrace{G_E(Q^2)}_{\text{Electric}} + \frac{i\epsilon^{\mu\alpha\beta\lambda}\Delta_\alpha P_\beta \gamma_\lambda \gamma^5}{2P^2}\underbrace{G_M(Q^2)}_{\text{Magnetic}}$$

Sachs EM FFs

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2), G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Electromagnetic form factors

In the holographic description, spin-conserving matrix elements can be represented as:

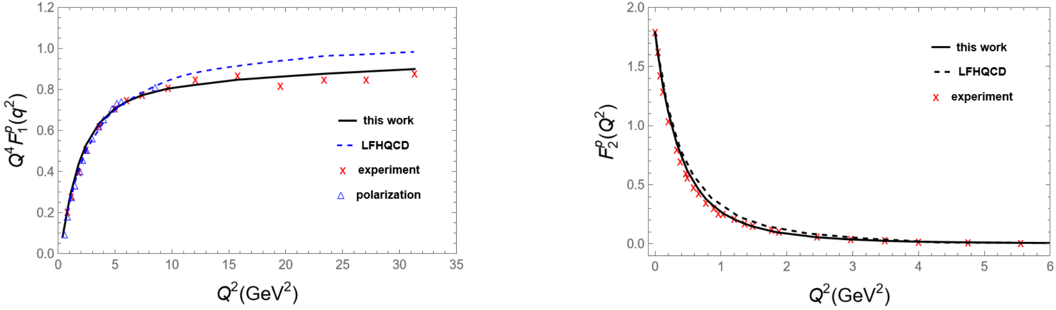
$$\int d^4xdz\sqrt{-g}\bar{\Psi}_{p'}(x,z)e^M_A\Gamma^A\phi_M(x,z)\Psi_p(x,z)\sim(2\pi)^4\delta^4(p'-p-q)\eta_\mu\phi(z)\bar{u}(p')\gamma^\mu F_1(q^2)u(p)$$

The spin-flip matrix element can be written as:

$$\int d^4xdz\sqrt{-g}\bar{\Psi}_{p'}(x,z)e^M_Ae^N_B[\Gamma^A,\Gamma^B]F_{MN}(x,z)\Psi_p(x,z)\quad\sim(2\pi)^4\delta^4(p'-p-q)\eta_\mu\phi(z)\bar{u}(p')\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)u(p).$$

Raza Sabbir Sufian,Guy F. de T´eramond, Stanley J, et al. Brodsky, Phys.Rev.D 95 (2017) 1, 014011

Electromagnetic form factors



Raza Sabbir Sufian,Guy F. de T´eramond, Stanley J, et al. Brodsky, Phys.Rev.D 95 (2017) 1, 014011
Jiali Deng and Defu Hou, Phys.Rev.D 112 (2025) 3, 036011

Gravitational Form Factors

Hadron matrix element

Xiong-Hui Cao and Feng-Kun Guo, et al. Cao:2025dkv

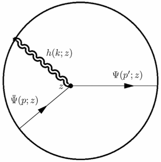
$$\langle p', s' | T^{\mu\nu}(0) | p, s \rangle = e\bar{u}(p', s')\Gamma^{\mu\nu}(p, p')u(p, s)$$

Vertex function:

$$\Gamma^{\mu\nu}(p, p') = \gamma^{(\mu}P^{\nu)}A(Q^2) + \frac{iP^{(\mu}\sigma^{\nu)\alpha}q_\alpha}{2M}B(Q^2) + \frac{q^\mu q^\nu - \eta^{\mu\nu}q^2}{2M}D(Q^2), \quad q = p' - p, P = (p' + p)/2$$

The interaction term in the holographic description:

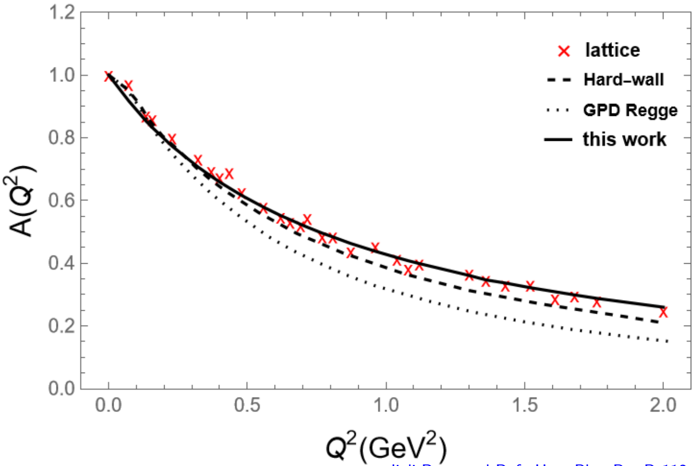
$$\int d^5x\sqrt{g}h_{\mu\nu}T^{\mu\nu}_F \sim \langle p', s' | T^{\mu\nu}(0) | p, s \rangle$$



The equation of motion for graviton

$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{mn}\partial_N h_{\mu\nu}) + m^2 h_{\mu\nu} = 0$$

Gravitational Form Factors



Jiali Deng and Defu Hou, Phys.Rev.D 112 (2025) 3, 036011
Daniel C. Hackett, Dimitra A. Pefkou and Phiala E. Shanahan, Phys.Rev.Lett. 132 (2024) 25, 251904
Zainul Abidin and Carl E. Carlson, Phys.Rev.D 79 (2009) 115003

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- **Nucleon Structure in Holographic Soft Model**
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- Trace Anomaly Contribution to the Proton Mass

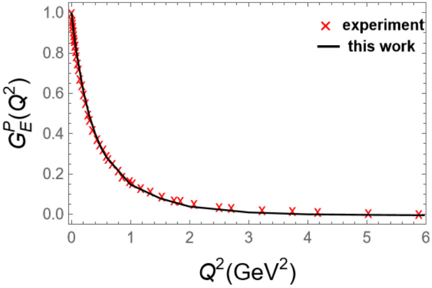
Mass Spectrum

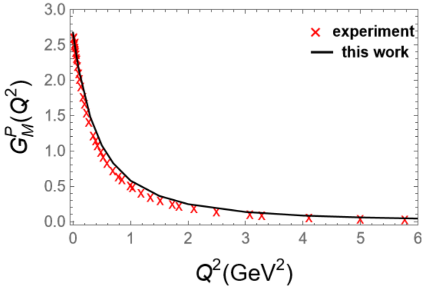
	Proton	M_{exp}/Gev	SW/Gev	Our/Gev	%M
n=1	N(939)	0.938	0.987	0.936	0.21
n=2	N(1440)	1.36 to 1.38	1.264	1.366	0.31
n=3	N(1710)	1.68 to 1.72	1.531	1.663	2.17
n=4	N(1880)	1.82 to 1.9	1.791	1.903	2.33
n=5	N(2100)	2.05 to 2.15	2.046	2.112	0.58
n=6	N(2300)	2.30	2.296	2.301	0.05

图: Mass spectrum results from numerical solution of the Schrödinger equation

Jiali Deng and Defu Hou, arXiv:2512.17554
Navas, S. and others. ParticleDataGroup:2024cfk
FolcoCapossoli, Miguel Angel Martín Contreras, Danning Li, et al. Chin.Phys.C 44 (2020) 6, 064104

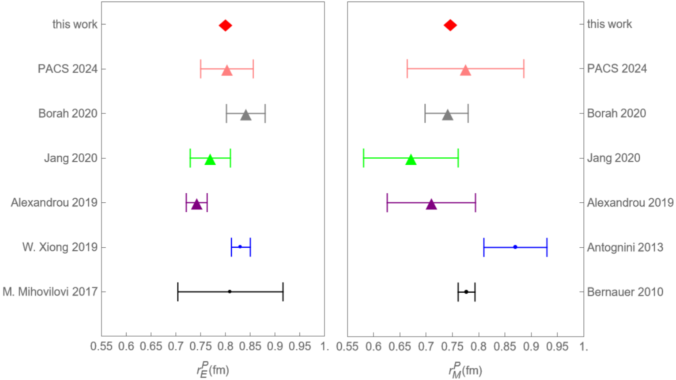
Electromagnetic Form Factors





Jiali Deng and Defu Hou, arXiv:2512.17554
Navas, S. and others. ParticleDataGroup:2024cfk

Electromagnetic radius



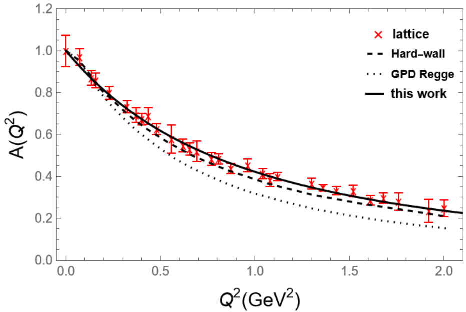
$$\langle r_E^2 \rangle = \frac{\int d^3\mathbf{r}\, r^2 G_E(r)}{\int d^3\mathbf{r}\, G_E(r)} = 6\frac{dG_E(t)}{dt}|_{t=0}.$$

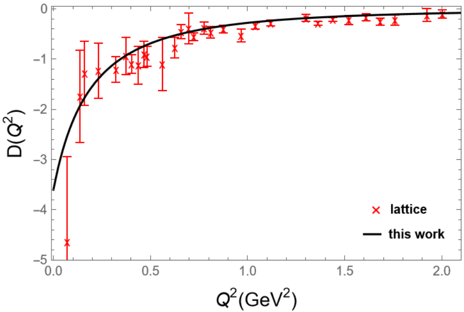
$$\langle r_M^2 \rangle = \frac{\int d^3\mathbf{r}\, r^2 G_M(r)}{\int d^3\mathbf{r}\, G_M(r)} = \frac{6}{\mu}\frac{dG_M(t)}{dt}|_{t=0}$$

$$Our\ results: r_E = 0.80fm,\ \ r_M = 0.75fm$$

Jiali Deng and Defu Hou, arXiv:2512.17554

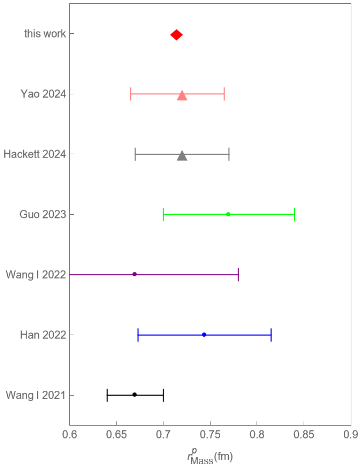
Gravitational Form Factors

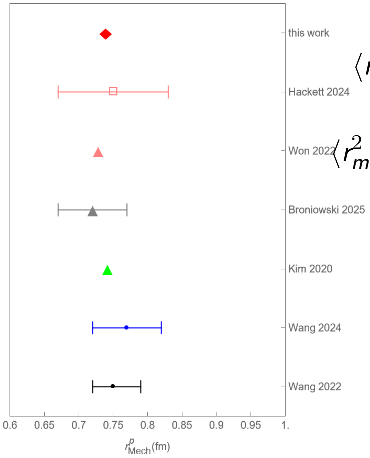




Jiali Deng and Defu Hou, arXiv:2512.17554

Gravitational radius





$$\langle r^2_{mass}\rangle = \frac{\int d^3\mathbf{r}\, r^2 \epsilon(r)}{\int d^3\mathbf{r}\, \epsilon(r)}$$

$$\langle r^2_{mech}\rangle = \frac{\int d^3\mathbf{r}\, r^2 F_{||}(r)}{\int d^3\mathbf{r}\, F_{||}(r)}$$

Our results :

$$r_{mass} = 0.714 fm,$$

$$r_{mech} = 0.74 fm$$

Jiali Deng and Defu Hou, arXiv:2512.17554

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Electromagnetic form factors

In LFHQCD, the electromagnetic form factors of protons can be expressed as

$$F_1^p(t) = F_{\tau=3}(t), F_2^p(t) = \chi_p[(1-\gamma_p)F_{\tau=4}(t) + \gamma_p F_{\tau=6}(t)]$$

The form factor of any twist can be expressed as

$$F_\tau(t) = \frac{1}{N(\tau)} B(\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}) = \frac{1}{(1-\frac{t}{M_0^2})(1-\frac{t}{M_1^2})(1-\frac{t}{M_{\tau-2}^2})}$$

where $M_n^2 = 4\lambda(n + \frac{1}{2})$, $n = 0, 1, 2 ... \tau - 2$. It can also be written as $B(\tau - 1, 1 - \alpha(t)$ with Regge trajectory

$$\alpha(t) = \frac{1}{2} + \frac{t}{4\lambda}.$$

This is the trajectory of the vector meson ρ and λ is fixed by the ground state mass of the ρ meson: $\lambda = (0.548 GeV)^2$.

Stanley J. Brodsky,Guy F. de T´eramond and Hans G¨unter Dosch, et al. Phys.Rept. 584 (2015) 1-105
Guy F. de T´eramond, Tianbo Liu and Raza Sabbir Sufian, et al. Phys.Rev.Lett. 120 (2018) 18, 182001

The GPDs of quark

The integral form of the form factor is

$$F_\tau(t) = \frac{1}{N(\tau)} \int_0^1 \frac{dw(x,t)}{dx} w(x,t)^{-t/4\lambda - 1/2} [1 - w(x,t)]^{\tau-2} dx,$$

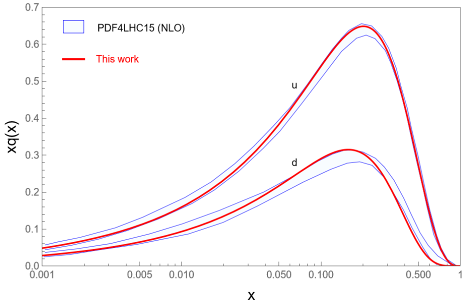
where x represents the longitudinal momentum fraction with $x \in (0,1)$ and w satisfies the constraint conditions $w(0,t) = 0$ and $w(1,t) = 1$.

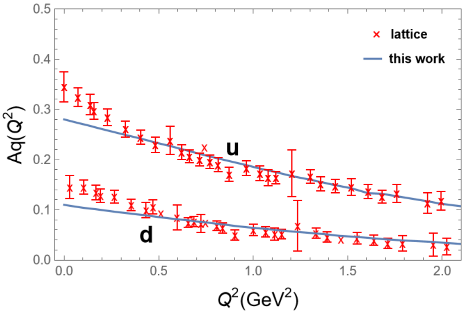
$$F_\tau(t) = \int_0^1 H_\tau(x,t) dx, \;\; H_\tau(x,t) = q_\tau(x,t) e^{tf(x,t)},$$

$$q_\tau(x,t) = \frac{1}{N(\tau)} \frac{dw(x,t)}{dx} [1 - w(x,t)]^{\tau-2} w(x,t)^{-1/2}, f(x,t) = \frac{1}{4\lambda} Log(\frac{1}{w(x,t)}),$$

where $q_\tau(x,0)$ and $f(x,t)$ represent the PDF and the profile function, respectively.

The PDF and GFF of quark



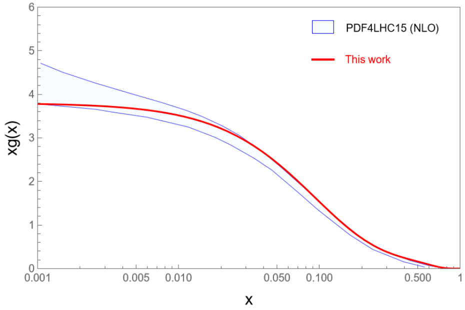


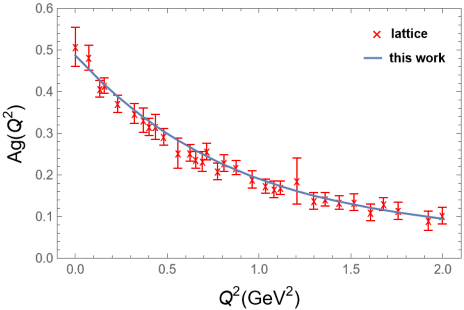
Jiali Deng and Defu Hou, arXiv:2603.04794

The GPDs of gluon

The effective Reggie trajectory for Pomeron is

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t, \qquad M_n^2 = 4\lambda_g n \Rightarrow M_0 = 0\,GeV,\ \ M_1 = (2 - 2.24)\,GeV$$





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Nucleon mass decomposition

Following Lorentz symmetry, one can parameterize their matrix elements as

Yoshitaka Hatta and Di-Lun Yang,Hatta:2018ina
Bigeng Wang and Yi-Bo Yang, et al. He:2021bof

$$\langle P| T_{q,kin}^{\mu\nu} |P\rangle = 2a(P^\mu P^\nu - \frac{\eta^{\mu\nu}}{4} M^2),$$

$$\langle P| T_{g,kin}^{\mu\nu} |P\rangle = 2(1-a)(P^\mu P^\nu - \frac{\eta^{\mu\nu}}{4} M^2),$$

$$\langle P| T_{m}^{\mu\nu} |P\rangle = \frac{1}{2} b\eta^{\mu\nu} M^2, \langle P| T_{a}^{\mu\nu} |P\rangle = \frac{1}{2}(1-b)\eta^{\mu\nu} M^2.$$

Then the proton mass can be expressed as

$$M = M_q + M_g + M_m + M_a,$$

$$M_q = \frac{3a}{4}M, \quad M_g = \frac{3(1-a)}{4}M, \quad M_m = \frac{b}{4}M, \quad M_a = \frac{1-b}{4}M.$$

Cross section computation via holography

The cross section of the photoproduction process takes the form

$$\sigma(\gamma p \to pJ/\psi) = \frac{e^2}{64\pi MKW|P_{cm}|}\int dt \langle P|\epsilon \cdot J(q)|P' k\rangle\langle P' k|\epsilon^* \cdot J(q)|P\rangle,$$

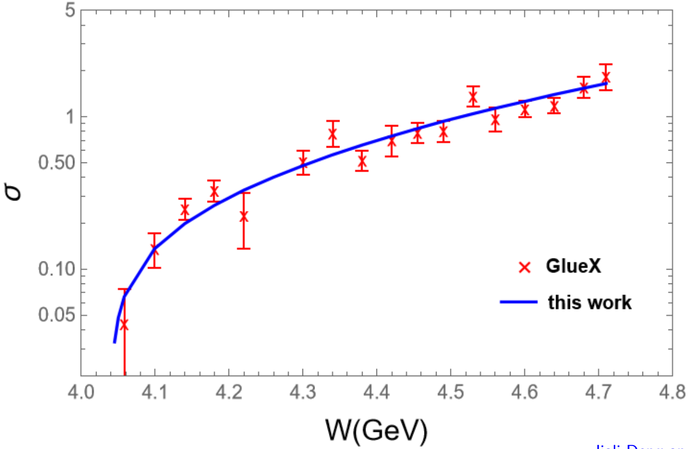
This matrix element can be represented by boundary operators $T^{\mu\nu}$ and $F^{\mu\nu}F_{\mu\nu}$

$$\langle P|\epsilon\cdot J(q)|P' k\rangle \approx \frac{-2k^2}{f_\psi R^3}\int_0^{z_m} dz \frac{\delta S_{D7}(q,k,z)}{\delta g_{\mu\nu}}\frac{z^2}{R^2}\langle P|T^{gTT}_{\mu\nu}|P'\rangle + \frac{6k^2}{8 f_\psi R^3}\int_0^{z_m} dz \frac{\delta S_{D7}(q,k,z)}{\delta\phi}\frac{z^4}{4}\langle P|\frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}|P'\rangle$$

We incorporate charm quarks into the theory through the addition of a D7-brane with the action

$$S_{D7} = -T_{D7}\int d^8\xi e^{-\phi}\sqrt{-det(G_{ab}+2\pi\alpha' F_{ab})}$$

Cross section



Jiali Deng and Defu Hou, arXiv:2603.04794

We choose the most suitable set of parameters for the experiment: $b = 0.05$. we can obtain that the contribution of trace anomaly to proton mass is approximately 23.75%

Summary and outlook

1. We computed the proton mass spectrum, structure Fs ,EMFFs,G in GFFs, in good agreement with experiments and lattice QCD determinations.
2. In the soft-wall hQCD , we computed the proton mass spectrum, EMFFs ,GFFs and the proton radius, in good agreement with experiments and lattice QCD
3. We extract the **quark GPDs** from the EMFFs using LFHQCD , extend the approach to obtain the **gluon GPDs** and derive a glueball mass of $(2-2.24)GeV$
4. By computing the scattering cross-section, we determine that **the trace anomaly contributes 23.75% to the proton mass**.

Next, we will calculate the D-term, the mechanical properties, and the three-dimensional structure of the proton.

Thank you very much !