Validating Wilson Flow Renormalization on the Lattice: Quark Bilinear and Glue Operators

梁 剑

华南师范大学 量子物质研究院

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Outline

- Wilson flow & SFTX
- Gluon momentum fraction
- Quark bilinear

Wilson flow

$$B_\mu\Big|_{t_f=0} = A_\mu \qquad \chi\Big|_{t_f=0} = \psi \qquad \bar{\chi}\Big|_{t_f=0} = \bar{\psi}$$

$$\partial_{t_f} B_\mu = D_\nu G_{\nu\mu} \qquad \partial_{t_f}\chi = \Delta\chi \qquad \partial_{t_f}\bar{\chi} = \bar{\chi}\overleftarrow{\Delta}$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu] \qquad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

$$\Delta = D_\mu D_\mu \qquad D_\mu = \partial_\mu + B_\mu$$

After renormalization of the fundamental parameters of QCD, the flowed gluon field is finite and does not require renormalization

The flowed quark fields require additional multiplicative field renormalization

M. Lüscher, JHEP08 071 (2010)
M. Lüscher and P. Weisz, JHEP02 051 (2011)
M. Lüscher, JHEP04 123 (2013)

Short flow time expansion (SFTX)

$$\tilde{O}_g(t_f) \equiv \lim_{a\to 0}[O_g^L(t_f, a)] = \zeta_O(\mu, t_f) O_g^R(\mu) + \mathcal{O}(t_f)$$

$$\tilde{O}_q(t_f) \equiv \lim_{a\to 0}[\mathring{Z}_\chi(t_f, a) O_q^L(t_f, a)] = \zeta_O(\mu, t_f) O_q^R(\mu) + \mathcal{O}(t_f)$$

$$O^R(\mu) = \lim_{t_f\to 0} \frac{\lim_{a\to 0}[\mathring{Z}_\chi(t_f, a) O^L(t_f, a)]}{\zeta_O(\mu, t_f)} = \lim_{t_f\to 0}\lim_{a\to 0}\left[\frac{\mathring{Z}_\chi(t_f, a) O^L(t_f, a)}{\zeta_O(\mu, t_f)}\right]$$

The 1st limit $a \to 0$ should be taken with all t_f satisfying $\sqrt{8t_f} \gg a$. The 2nd limit $t_f \to 0$ is usually done linearly.

The golden window: $a \ll \sqrt{8t_f} \ll \Lambda_{\rm QCD}^{-1}$. Similarly, the golden window of RI/MOM is $\Lambda_{\rm QCD} \ll p \ll \pi/a$.

Gauge invariant, chiral symmetry, no mixing, smearing...

M. Lüscher and P. Weisz, JHEP02 051 (2011)
H. Suzuki, PTEP 083B03 (2013)

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Wilson flow & SFTX

⇨ Gluon momentum fraction

Quark bilinear

Formalism

$$T_{\mu\nu}(x) = c_i(t)\tilde{\mathcal{O}}_{i,\mu\nu}(t,x)$$

R. V. Harlander, Y. Kluth and F. Lange, EPJC78 944 (2018)

$$T^g_{\mu\nu} = \frac{1}{g_0^2}\left[O_1 - \frac{1}{4}O_2\right]$$

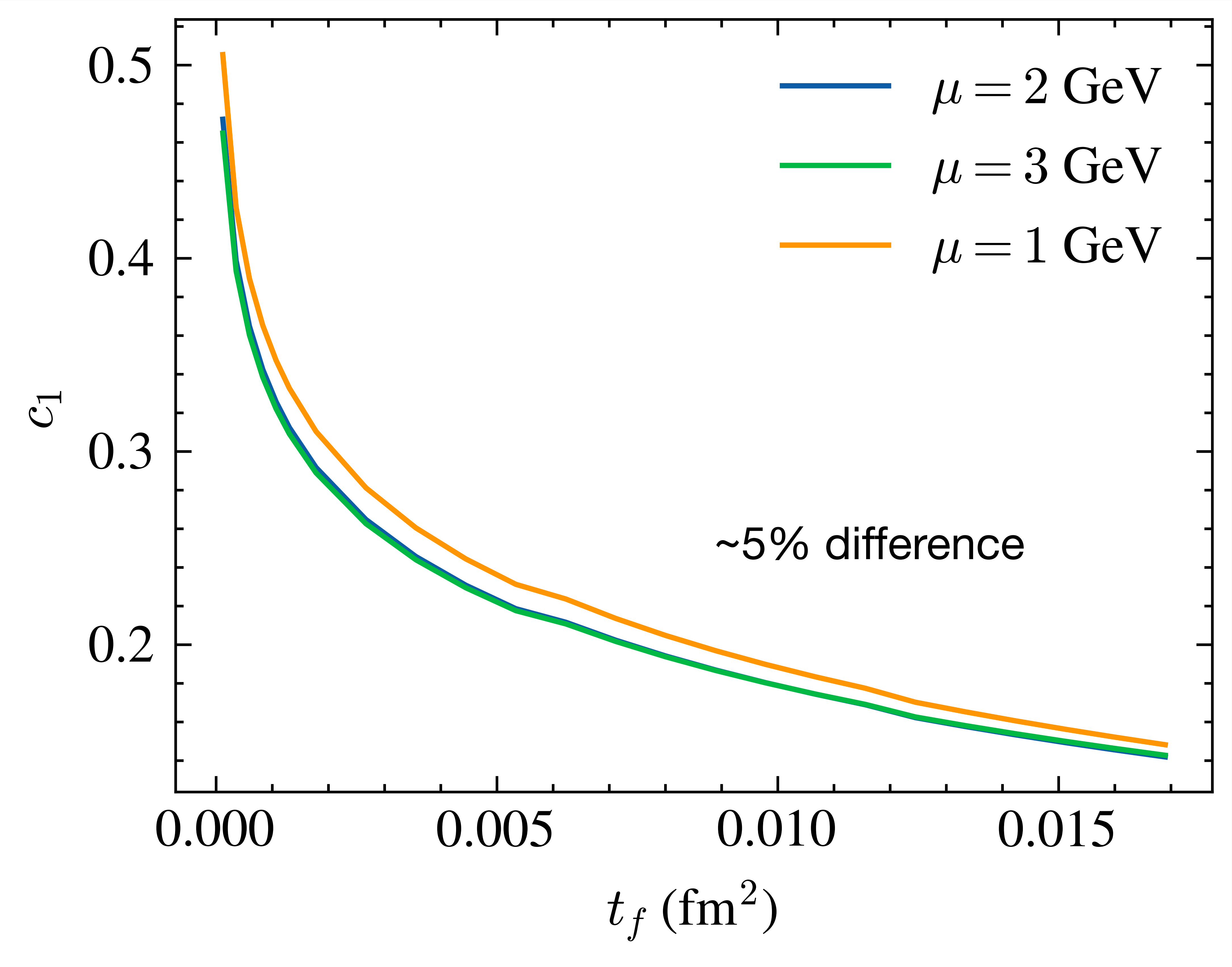
$$O_{1,\mu\nu} = G^a_{\mu\rho}G^a_{\nu\rho}$$

$$O_{2,\mu\nu} = \delta_{\mu\nu}G^a_{\rho\sigma}G^a_{\rho\sigma}$$

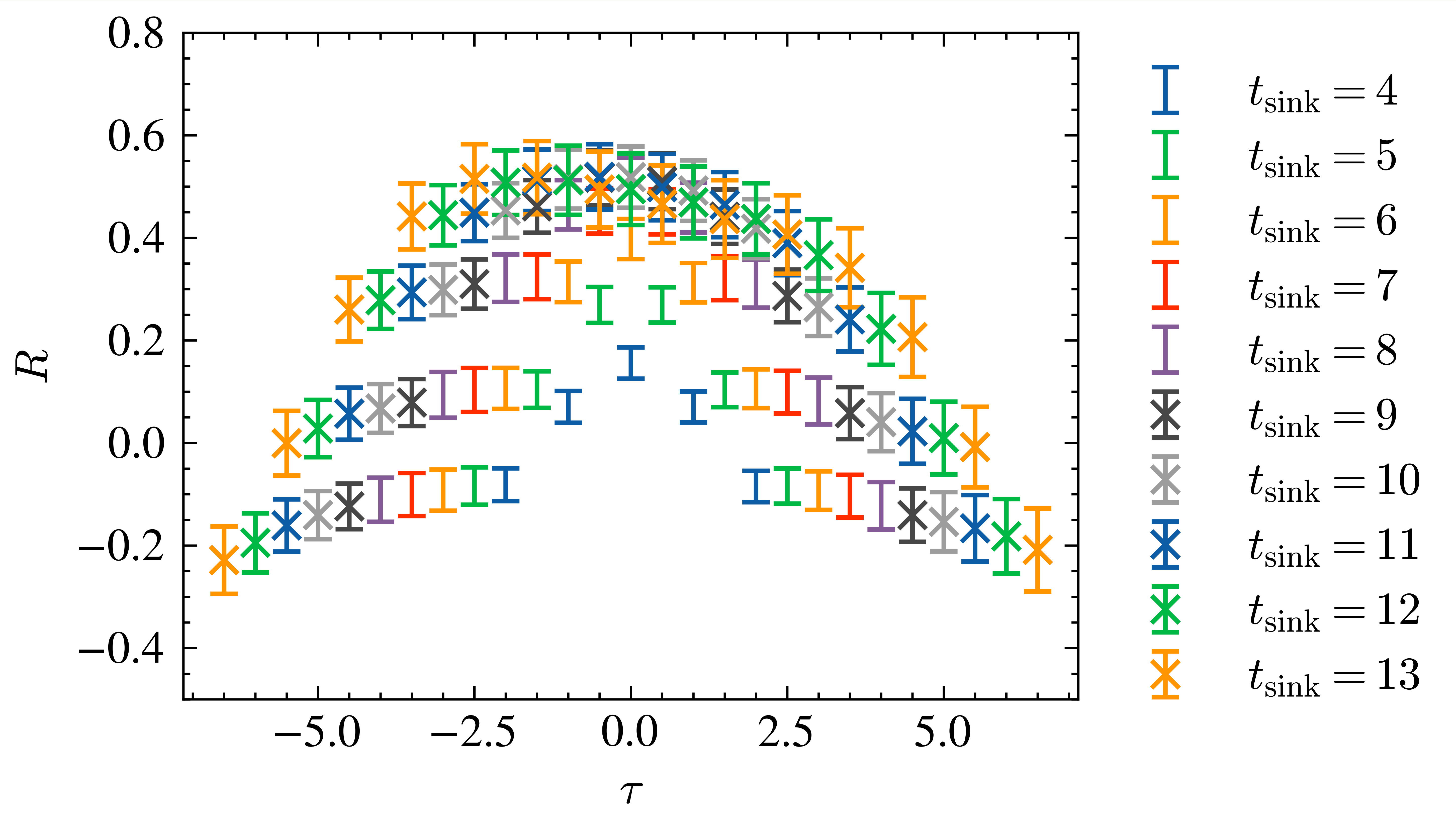
$$O^R_i(\mu) = \lim_{t_f\to 0}\lim_{a\to 0} M_{ij}(\mu, t_f)O^L_j(t_f),$$

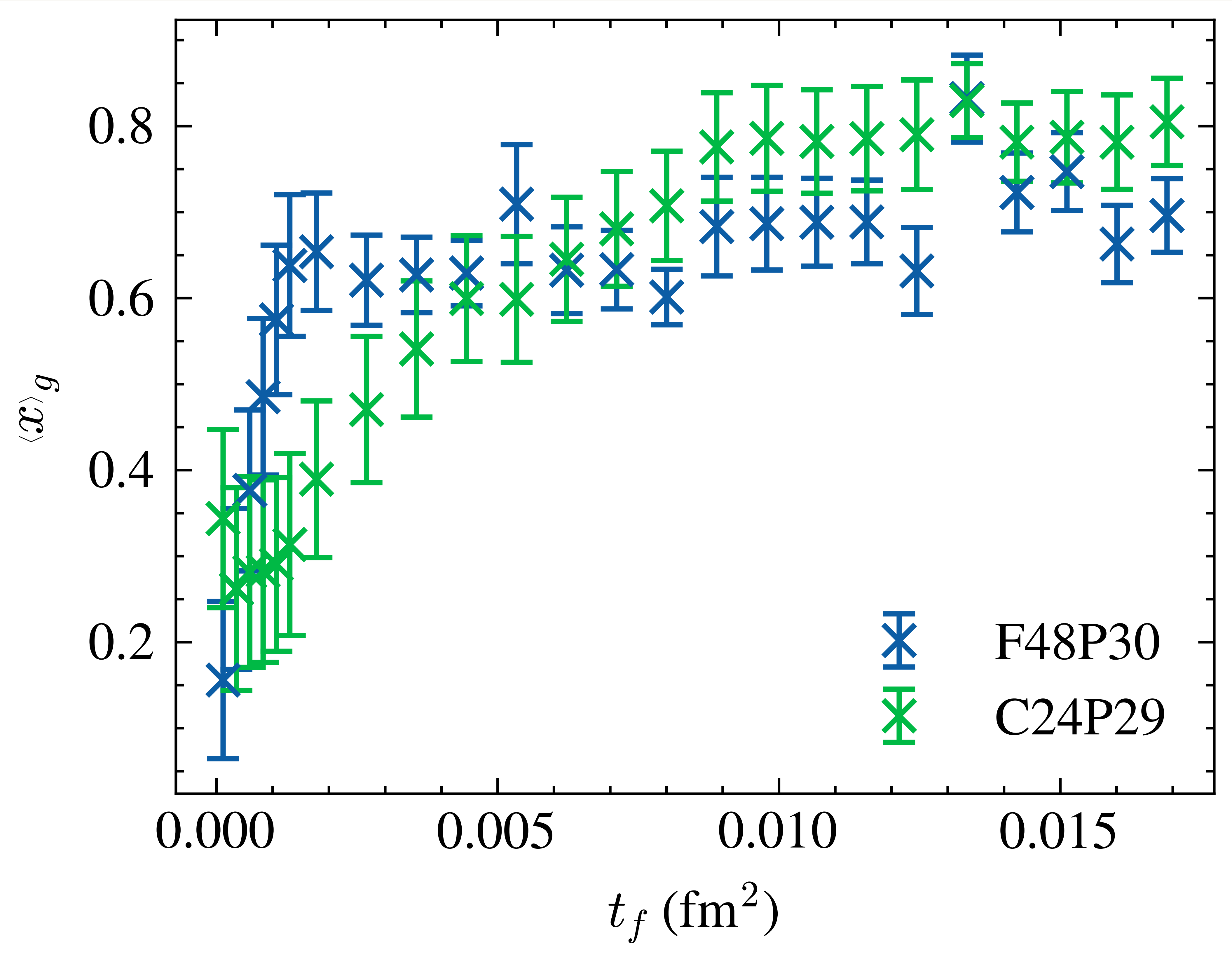
$$O = O_{1,44} - \frac{1}{3}O_{1,ii}$$

$$\langle N|O|N\rangle = 4E^2\langle x\rangle_g$$

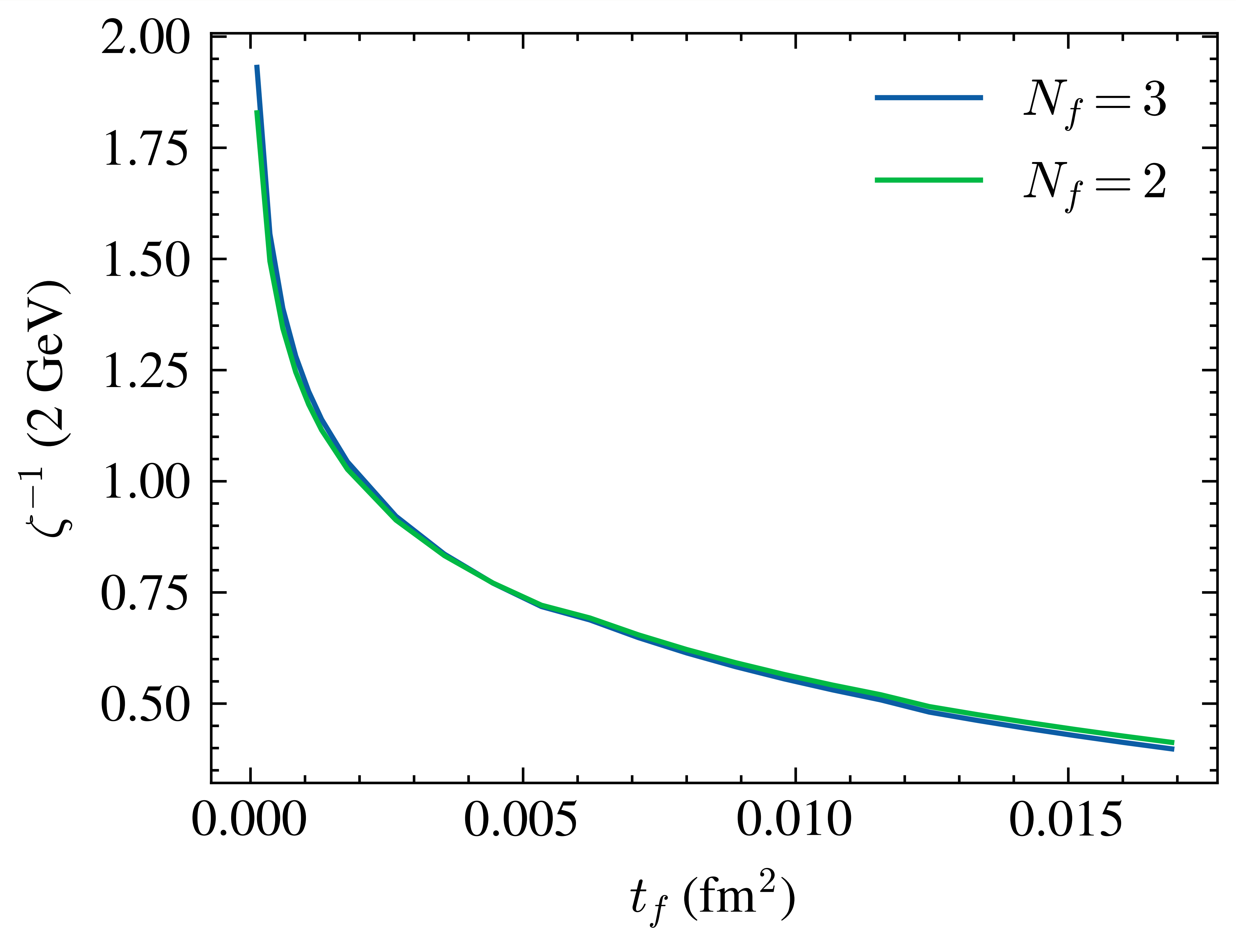


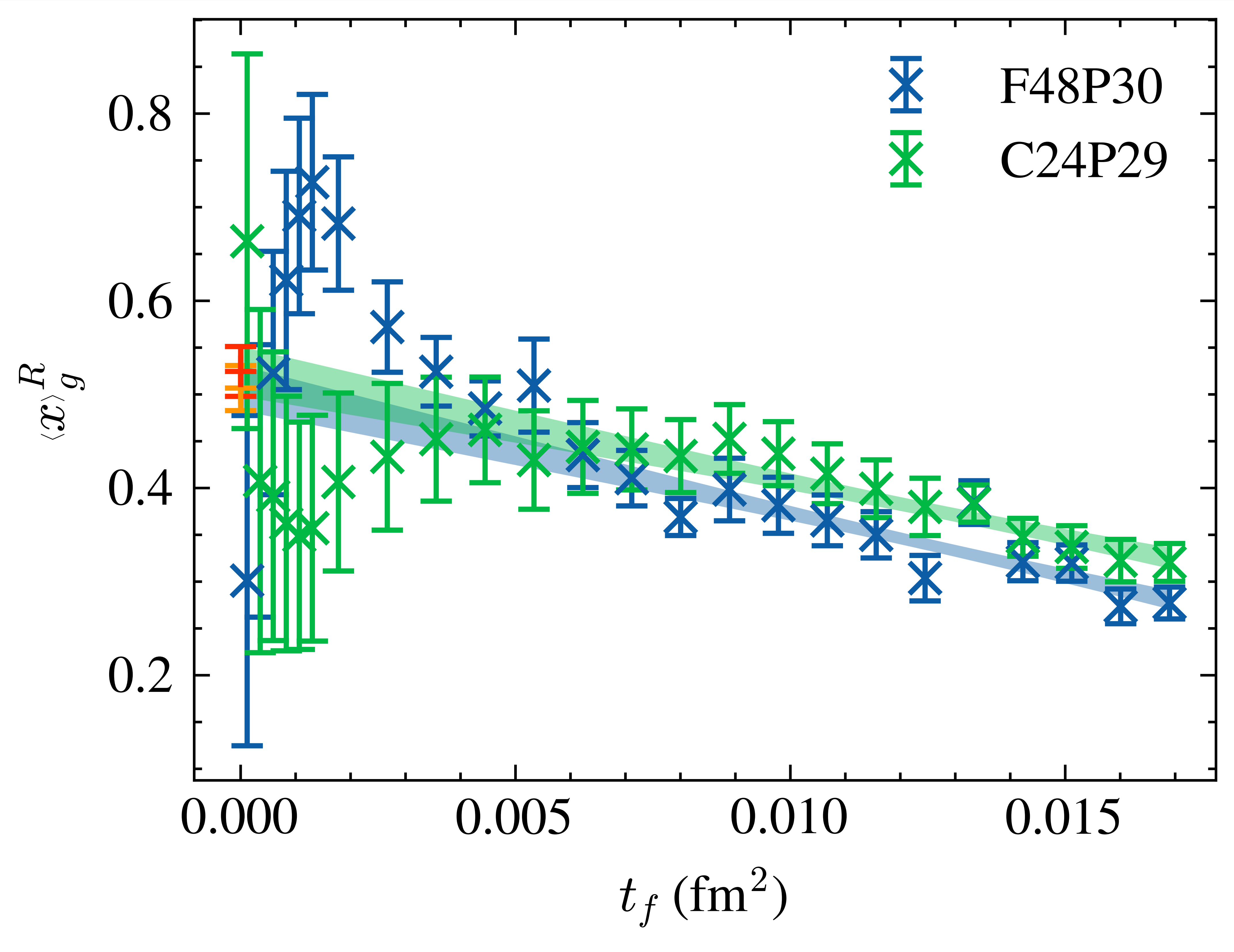
Lattice matrix elements





Preliminary results





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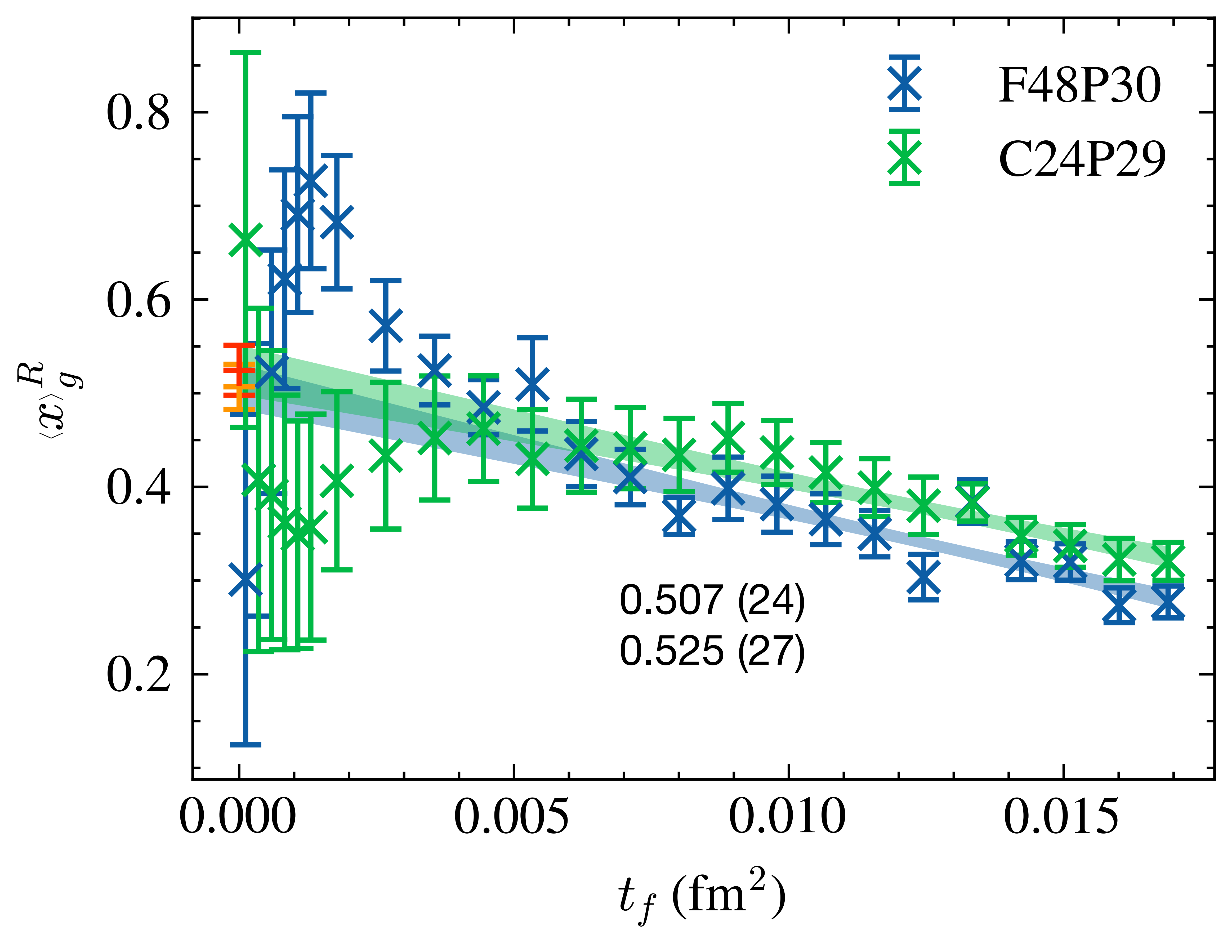

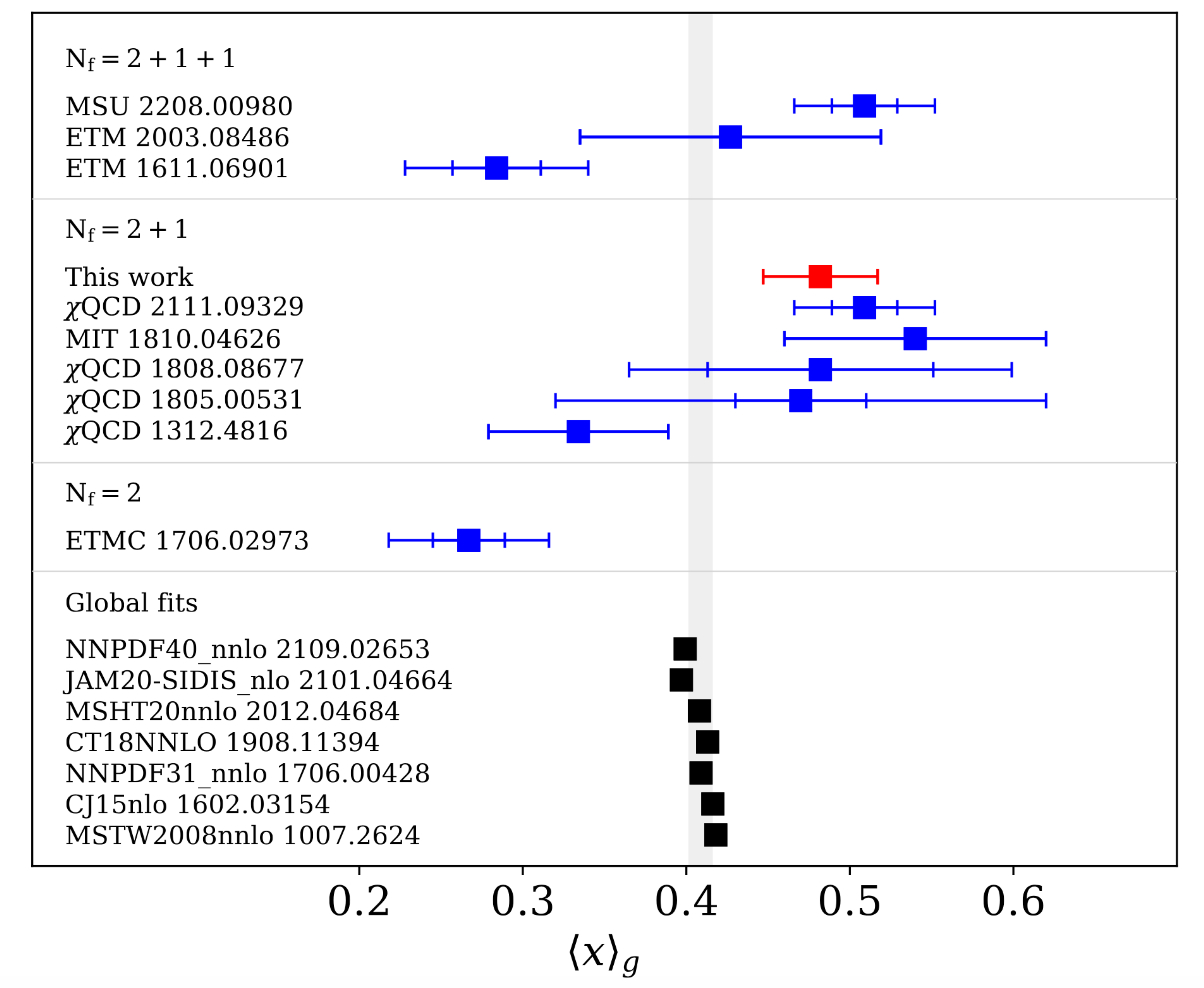

arXiv:2602.14260

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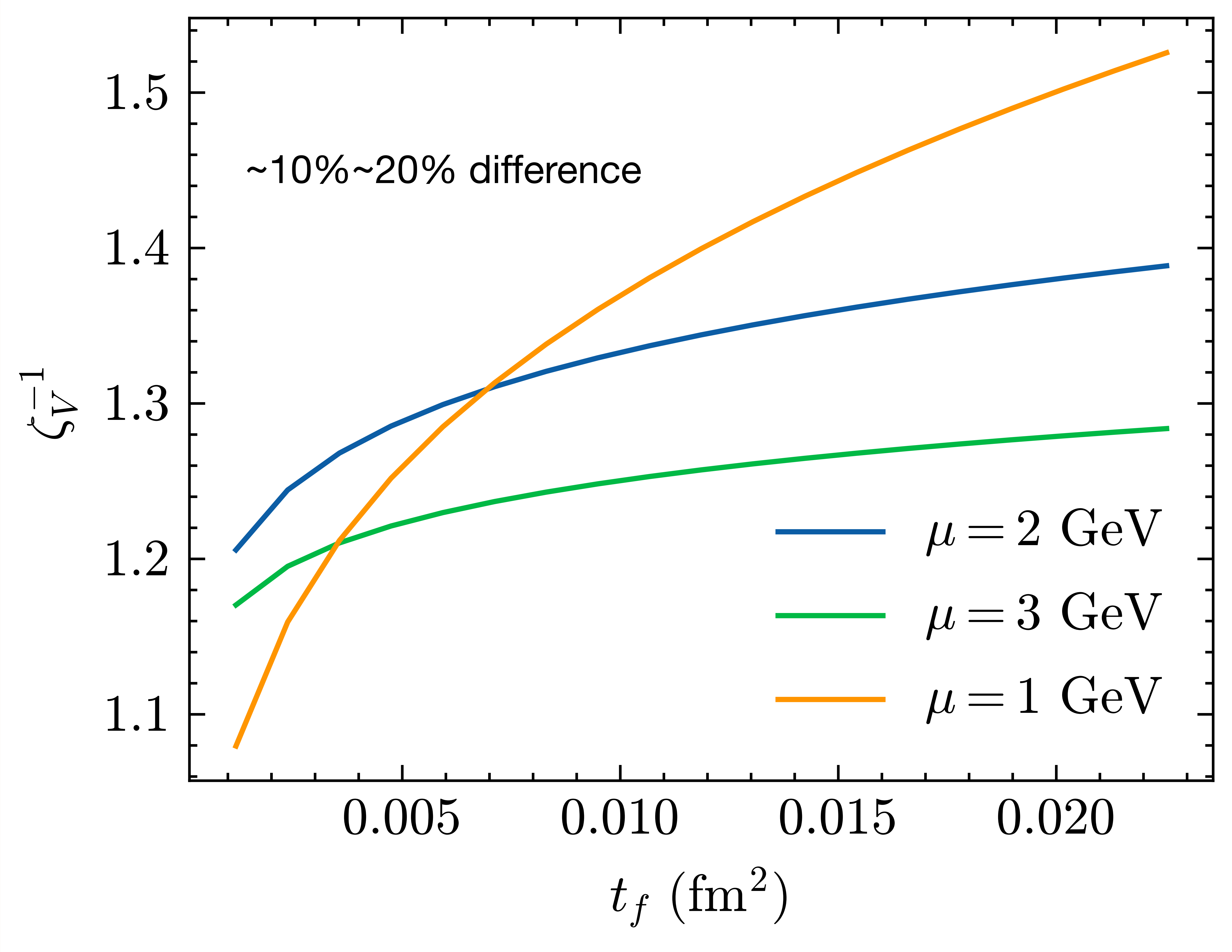
Check on matching

$$\tilde{O}_q(t_f) \equiv \lim_{a\to 0}[\mathring{Z}_\chi(t_f, a) O_q^L(t_f, a)] = \zeta_O(\mu, t_f) O_q^R(\mu) + \mathcal{O}(t_f)$$

J. Borgulat, R. V. Harlander, J. T. Kohnen and F. Lange, JHEP05 179 (2024)



Renormalization of flowed quarks

$$\mathring{Z}_\chi(t_f, a) = \frac{-2N_c}{(4\pi t_f)^2 \left\langle \bar{\chi}(t_f, x) \overleftrightarrow{\not{D}} \chi(t_f, x) \right\rangle}$$

$$\mathring{Z}_\chi(t_f, a)\langle V_\mu^L(t_f, a)\rangle = \zeta_V(\mu, t_f)\langle V_\mu^R(\mu)\rangle + \mathcal{O}(t_f, a)$$

$$Z_V(\mu, a)\langle V_\mu^L(t_f = 0,\ a)\rangle = \langle V_\mu^R(\mu)\rangle$$

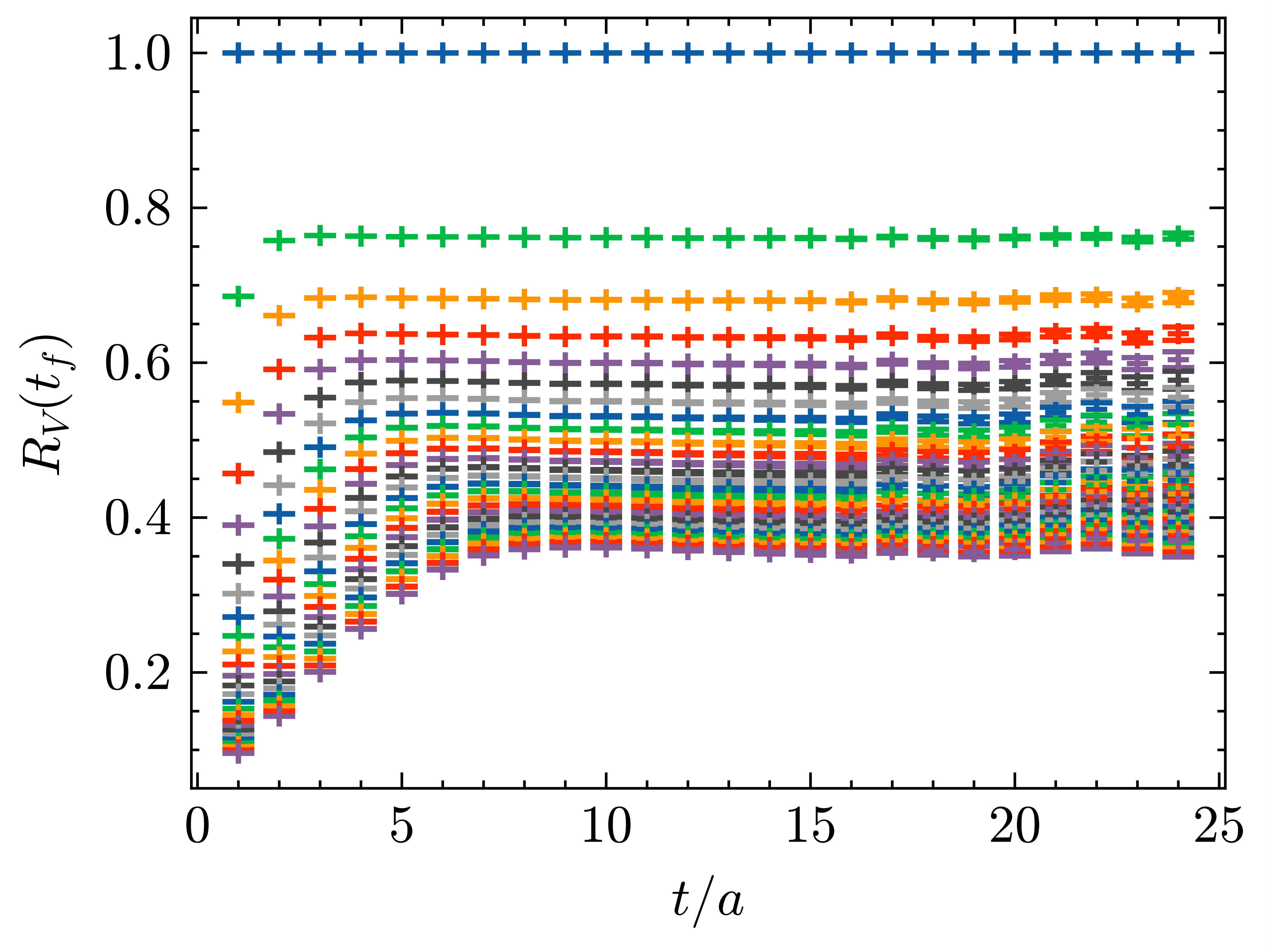
$$\frac{\mathring{Z}_\chi(t_f, a)\langle V_\mu^L(t_f, a)\rangle}{Z_V(\mu, a)\langle V_\mu^L(t_f = 0, a)\rangle} = \zeta_V(\mu, t_f) + \mathcal{O}(t_f, a)$$

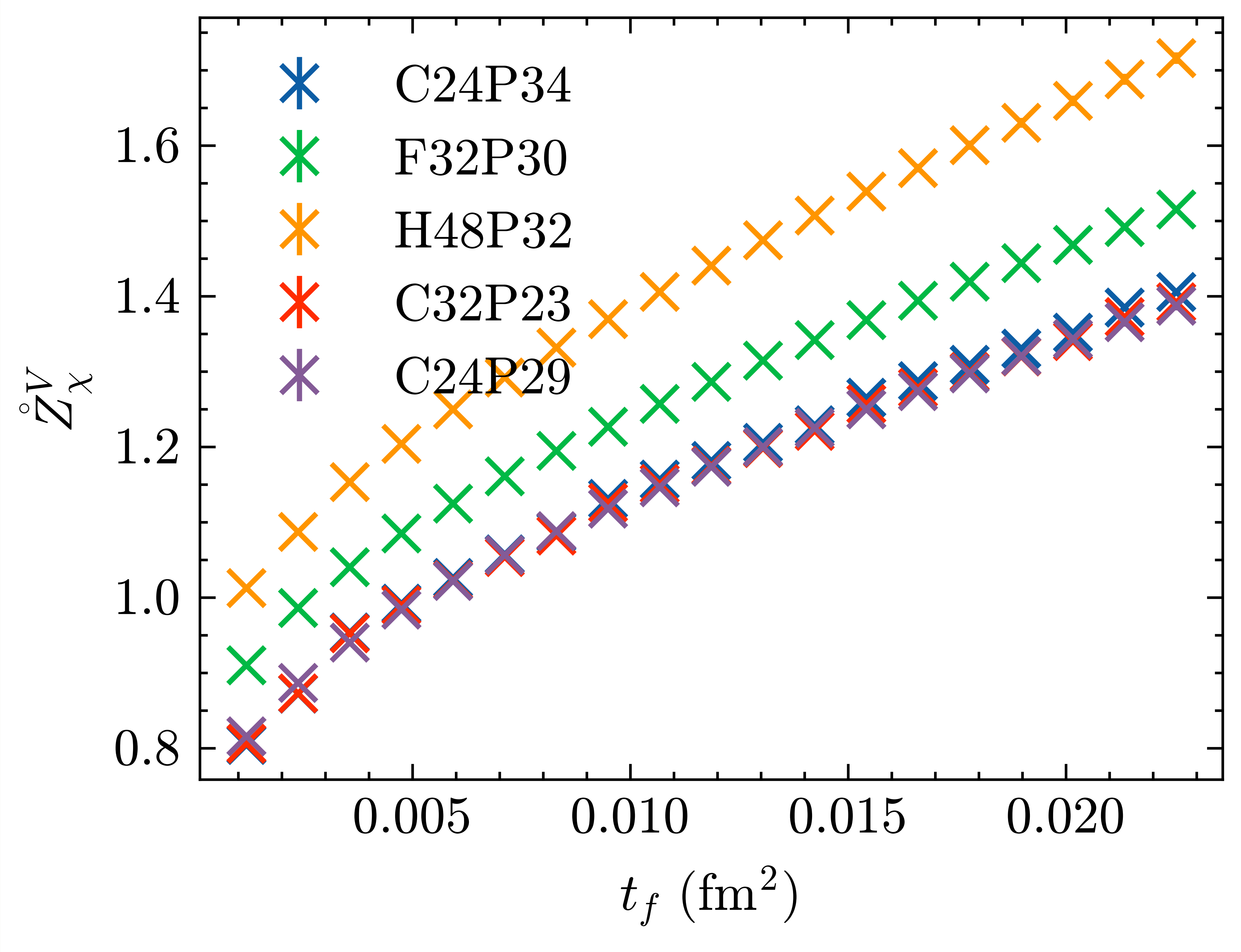
$$\mathring{Z}_\chi^V(t_f, a) \equiv \frac{\zeta_V(\mu, t_f)\langle V_\mu^L(t_f = 0,\ a)\rangle}{\langle V_\mu^L(t_f, a)\rangle} Z_V(\mu, a) \equiv \frac{\zeta_V(\mu, t_f)}{\langle R_{V,\mu}^L(t_f, a)\rangle} Z_V(\mu, a)$$

Similar idea of fixing Z_V: for the vector case, $\mathring{Z}_\chi^V(t_f, a)$ absorbs all the discrete effects.

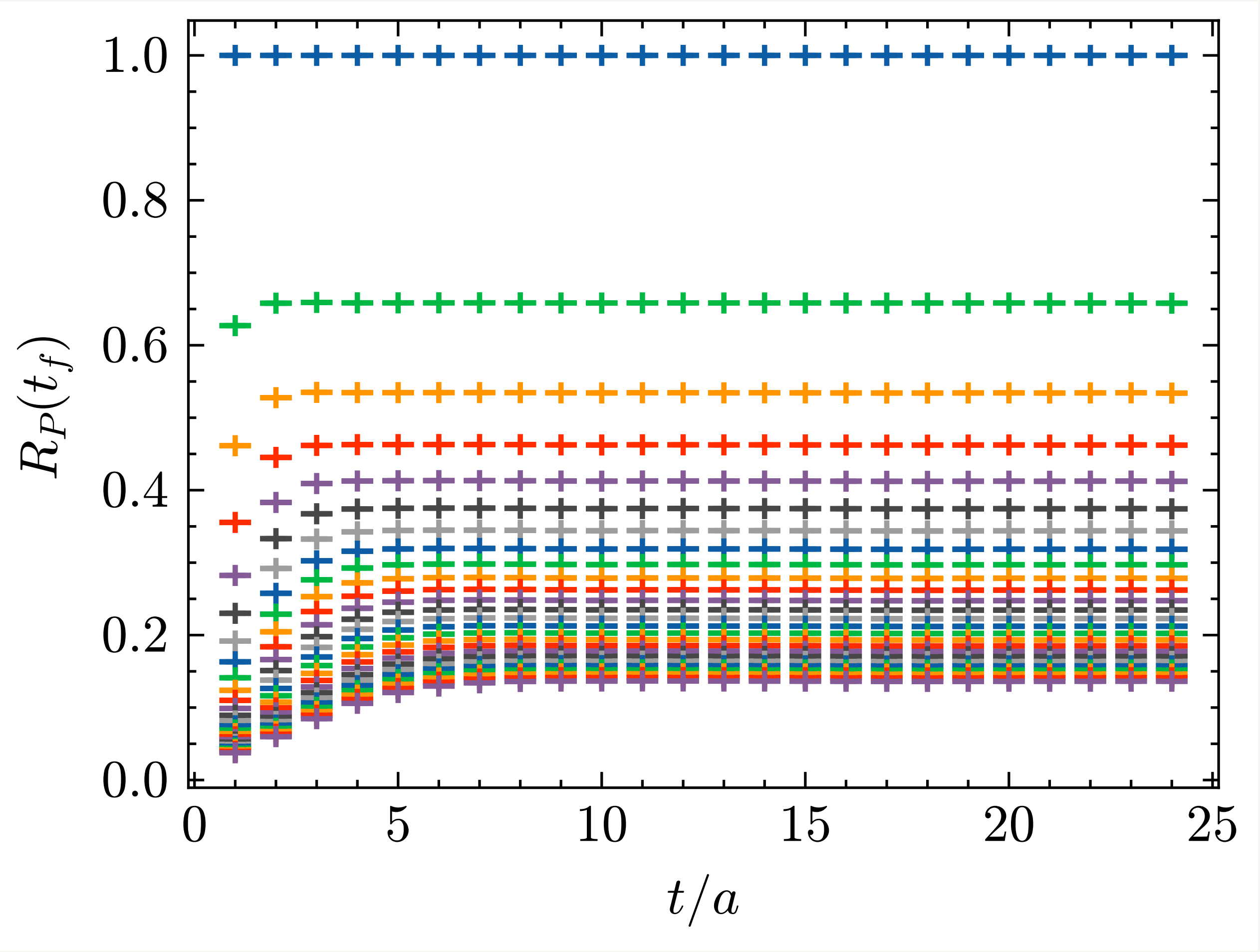
Renormalization of flowed quarks

$$\mathring{Z}^V_\chi(t_f, a) \equiv \frac{\zeta_V(\mu, t_f)\langle V^L_\mu(t_f = 0,\ a)\rangle}{\langle V^L_\mu(t_f, a)\rangle} Z_V(\mu, a) \equiv \frac{\zeta_V(\mu, t_f)}{\langle R^L_{V,\mu}(t_f, a)\rangle} Z_V(\mu, a)$$



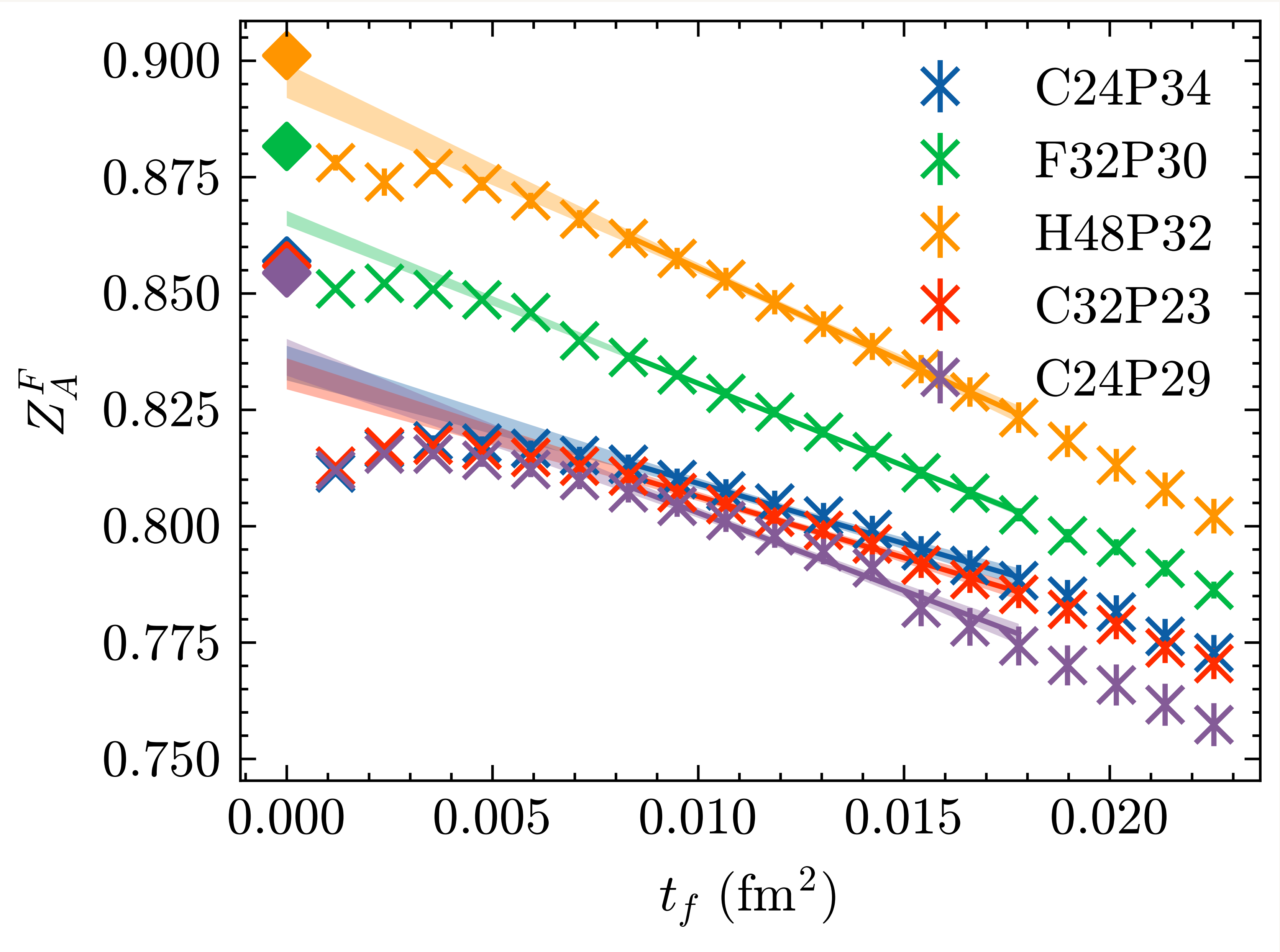


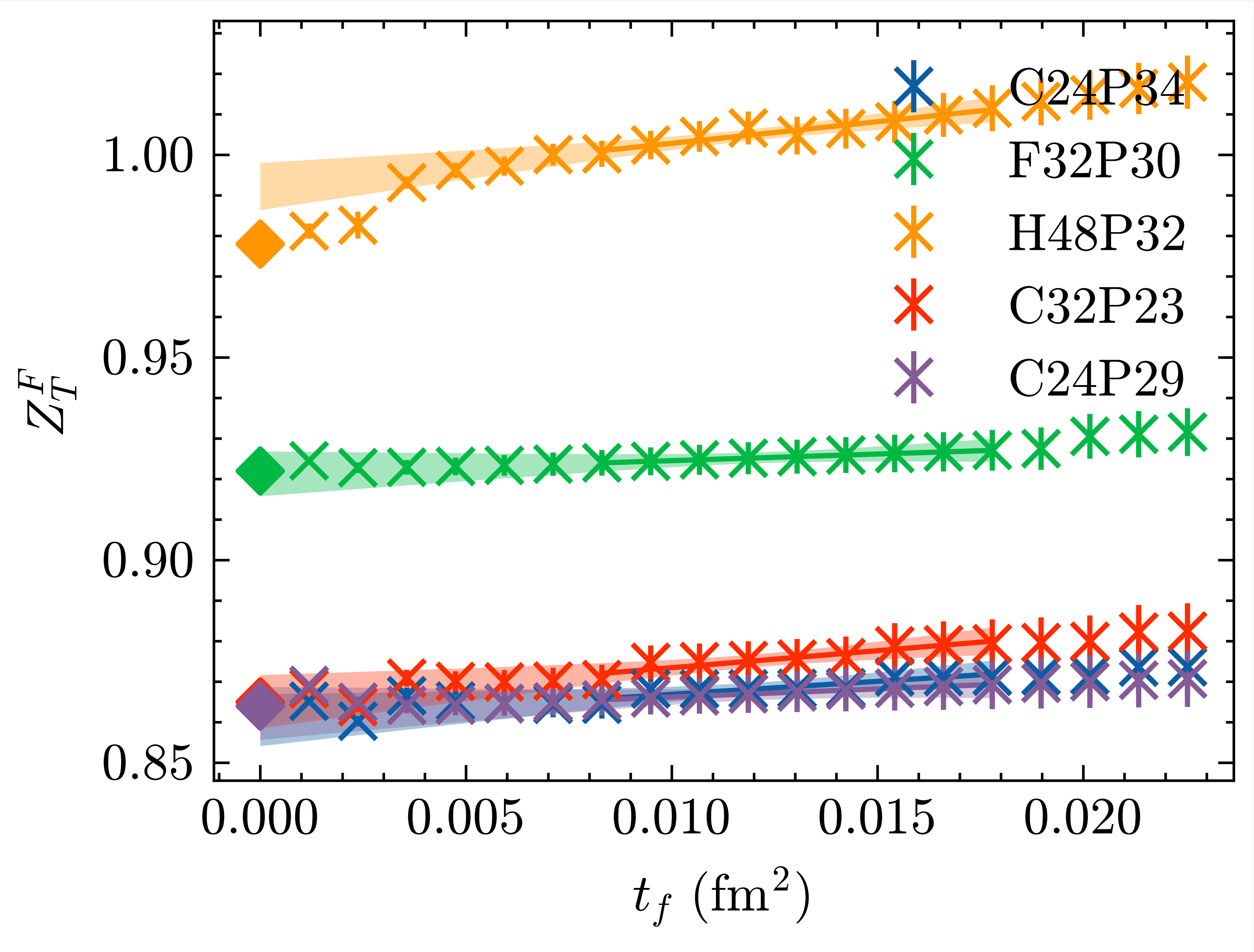
Check against RI/MOM

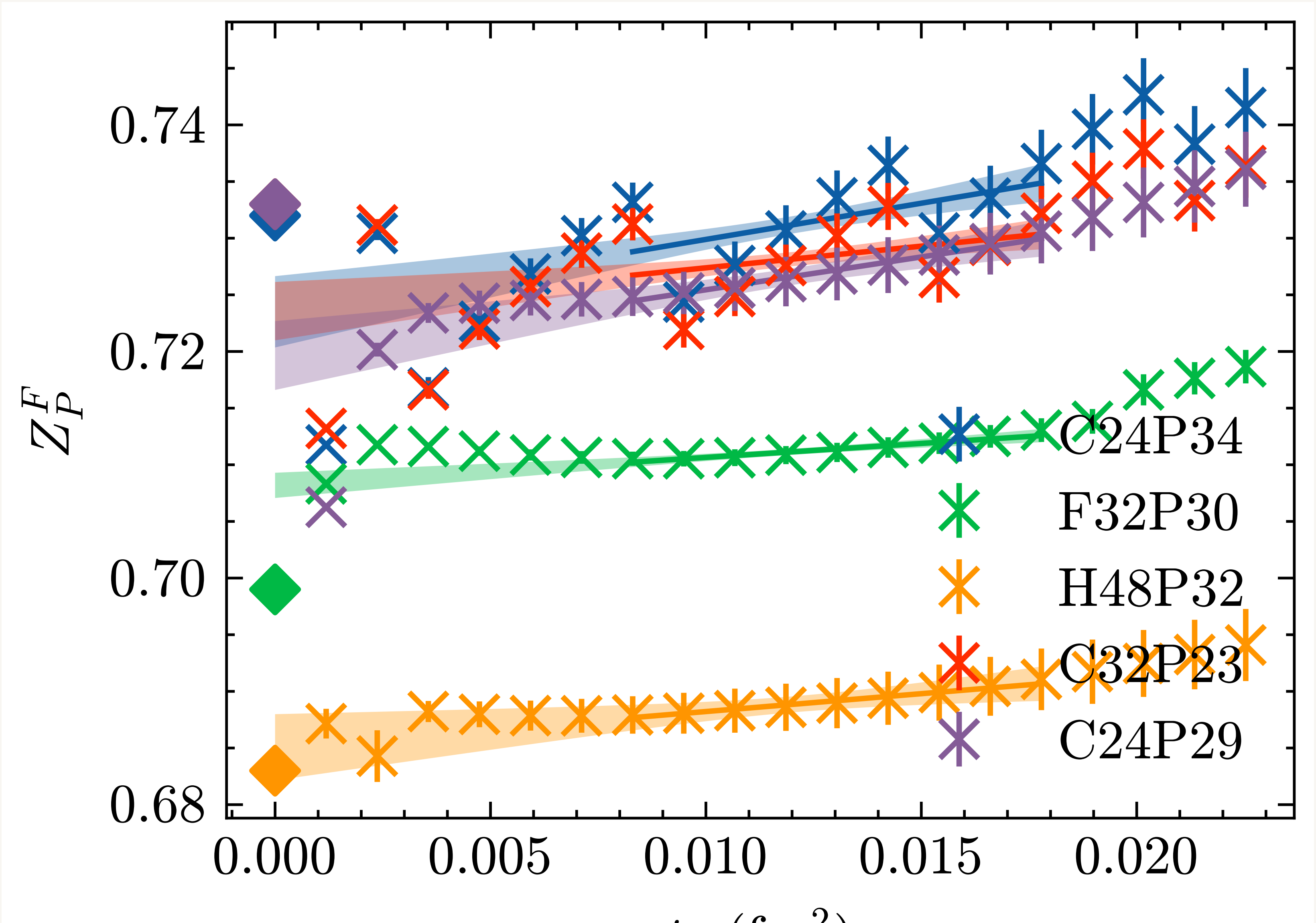


$$Z_O^{\text{RI/MOM}}(a,\mu) \sim \frac{O^R(\mu)}{O^L(t_f=0,a)}$$

$$Z_O^{\text{F}}(a,\mu) = \frac{\lim_{t_f\to 0}\lim_{a\to 0}\left[\dfrac{\mathring{Z}_\chi^V(t_f,a)\,O^L(t_f,a)}{\zeta_O(\mu,t_f)}\right]}{O^L(t_f=0,a)} \sim \lim_{t_f\to 0}\left[\frac{\mathring{Z}_\chi^V(t_f,a)\,R_O^L(t_f,a)}{\zeta_O(\mu,t_f)}\right] + \mathcal{O}(a)$$







Summary and Take-home

- **Validation:** Rigorous numerical tests confirm the viability of Wilson flow for renormalizing quark bilinear and gluonic operators.
- **Performance:** The scheme yields consistent results within current systematic uncertainties.
- **Outlook:** Refinement of systematic controls and extensions to physical observables are currently in progress.

Thank You!