Hadron structures on the light cone

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- IGFAE, Spain: Meijian Li, Wenyang Qian (→ CCNU)
- Jacksonville U: G. Chen
- IMP, Lanzhou/Huizhou: Xingbo Zhao, C. Mondal, Weijie Du
- NUAA: Chao Shi
- USTC: Qun Wang, Yang Li, Chen Chen, B. Gurjar

+ Students ...

* BLFQ collaboration

Biggest problem 130 yr. ago: electron structure

- Contemporary view: a spherical corpuscles of the size $r_e \sim 10^{-15}$m, moving in high speeds
- Origin of mass from the interaction (electromagnetism)?
- By product: electromagnetism of the moving body, aka. relativity
- Wrong problem, right (and great) answers

Thomson Lorentz Abraham Einstein Minkowski Laub

Ting

Hadron structures: a biased history

- Proton magnetic moment (1930s)
- Elastic scattering of proton (1950s)
- Quark model (early 1960s)
- Chiral symmetry breaking (1960s)
- Deep inelastic scattering (late 1960s)
- Quantum chromodynamics (1970s)
- ……

Tremendous progress, but many puzzles remain

See, F. Gross and E. Klempt (eds.), *50 Years of quantum chromodynamics*, EPJC, 2023

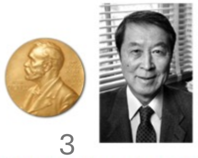

Nobel prize 1943


Nobel prize 1951


Nobel prize 1969


Nobel prize 1990


Nobel prizes 1999 & 2004

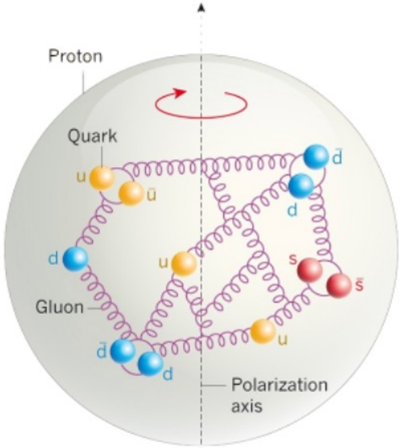

Nobel prize 2008

Parton picture

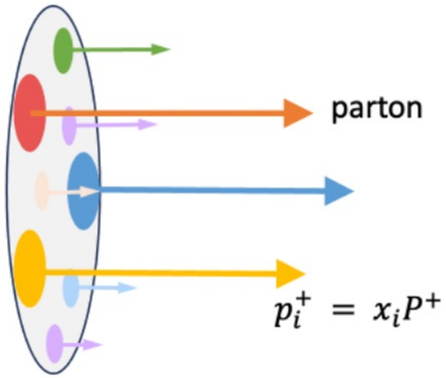
- Quark model: quarks and gluons are strongly coupled
- Parton model: the strong interaction is effectively frozen due to relativity

quark model, Gell-Mann 1964

parton model, Feynman 1969



$P \to \infty$



$T_{\text{int}} \sim \Delta t_{QCD} = \Lambda_{QCD}^{-1} = 10^{-23}\text{s}$

quarks are strongly coupled

$T_{\text{int}} \sim \gamma \Delta t_{QCD} \gg \Lambda_{QCD}^{-1} = 10^{-23}\text{s}$

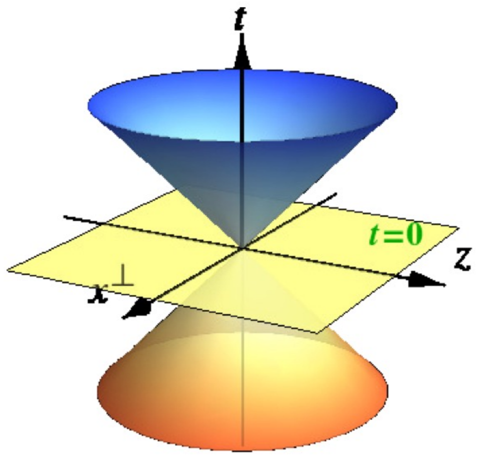
partons are free!

Light cone quantization

- Parton model can be formalized as light-cone quantization

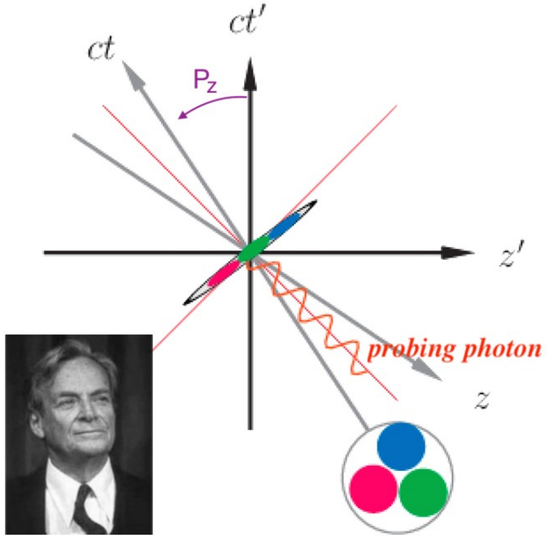
Polyzou, 2021

equal-time quantization
$t = 0$

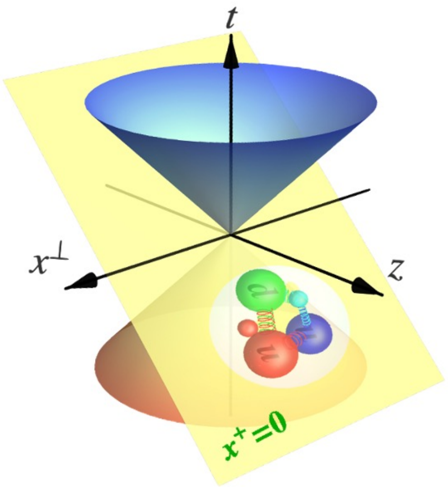


$$i\frac{\partial}{\partial t}|\Psi\rangle = P^0|\Psi\rangle$$

infinite momentum frame
$P_z \to \infty$



light-front quantization
$x^+ = t + z/c = 0$

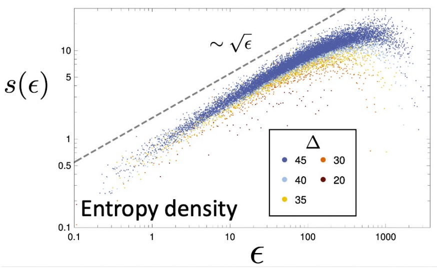


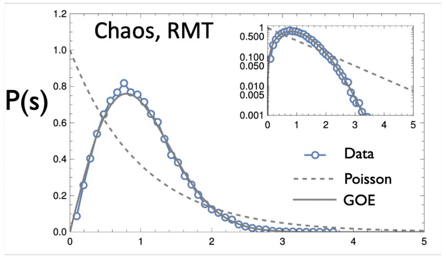
$$i\frac{\partial}{\partial x^+}|\Psi\rangle = P_+|\Psi\rangle$$

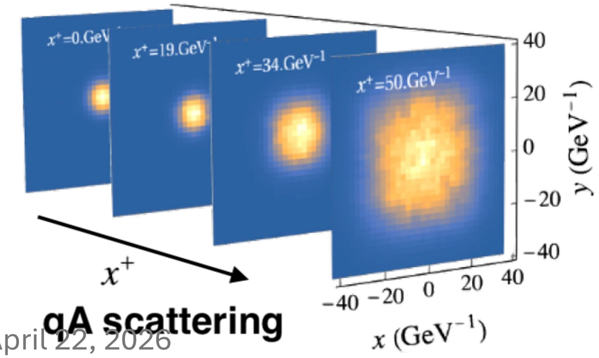
$$i\frac{\partial}{\partial x^\mu}|\Psi\rangle = P_\mu|\Psi\rangle$$

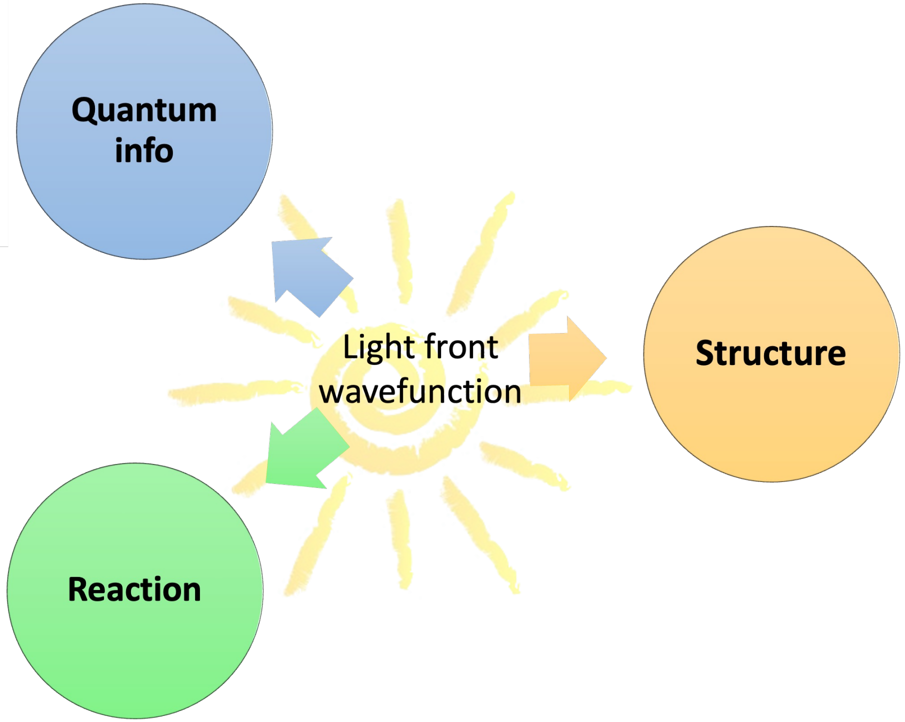
Light-cone wave functions

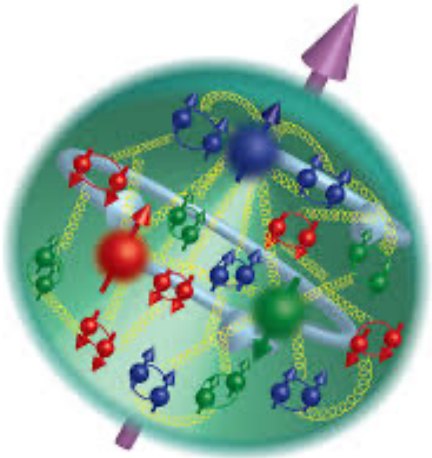
- Hadronic wave function $|\Psi\rangle$ contains the full quantum information of the system











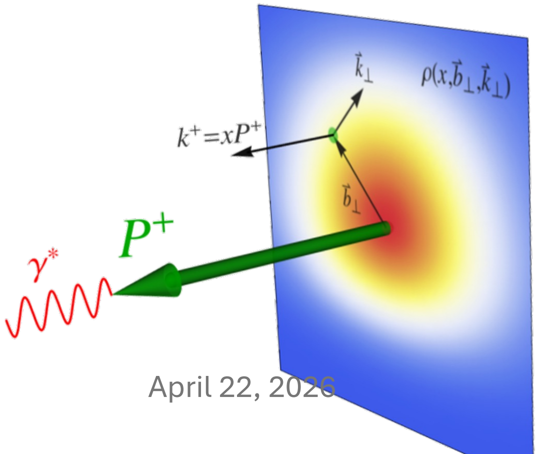
Transverse density

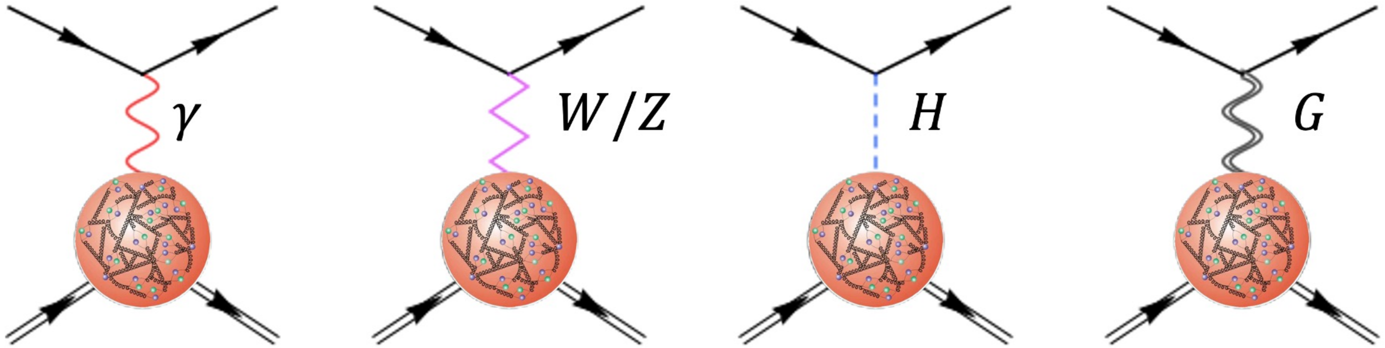
- Light-cone picture is essential for defining the hadronic densities

Miller, 2018

- Form factor $\longleftrightarrow$ one-body densities (OBDs):
  - Drell-Yan-West: $\rho_{ch}(r_\perp) = \langle \sum_i e_i \delta^2(r_\perp - r_{i\perp}) \rangle$
  - Brodsky-Hwang-Ma-Schmidt: $T^{++}(r_\perp) = 2P^+P^+\langle \sum_i x_i \delta^2(r_\perp - r_{i\perp}) \rangle$
  - Cao-Li-Vary: $T^{12}(r_\perp) = \left\langle \sum_i \frac{\nabla_\perp^1 \nabla_\perp^2 - \overleftrightarrow{\nabla}_{i\perp}^1 \overleftrightarrow{\nabla}_{i\perp}^2}{2x_i} \delta^2(r_\perp - r_{i\perp}) \right\rangle$

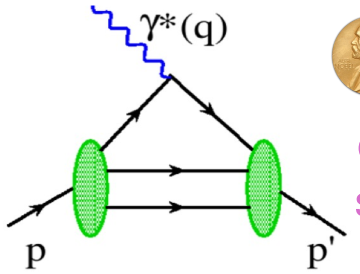
$$\langle O \rangle = \sum_n \int [dx_i d^2 r_{i\perp}] |\psi_n|^2 \times O_n(\{x_i, r_{i\perp}\})$$





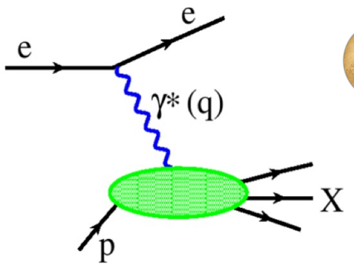
Hadronic densities

- Generalized parton distributions play a key role in hadron densities



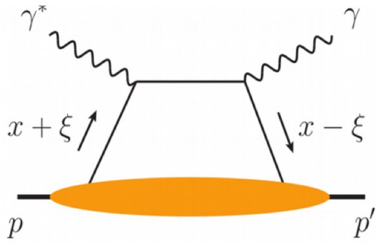
1961

elastic eA scattering



1990

DIS

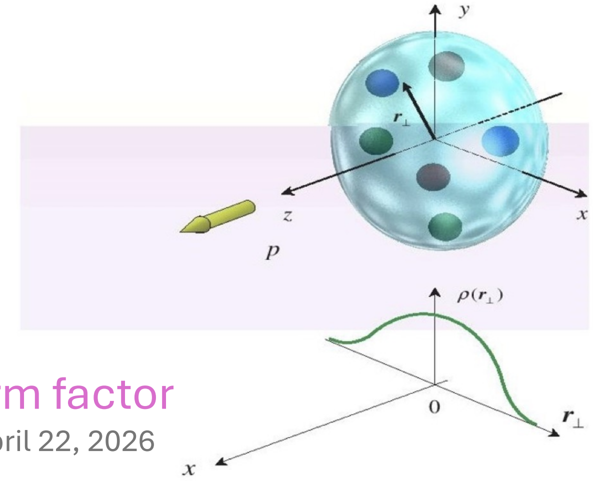


DVCS

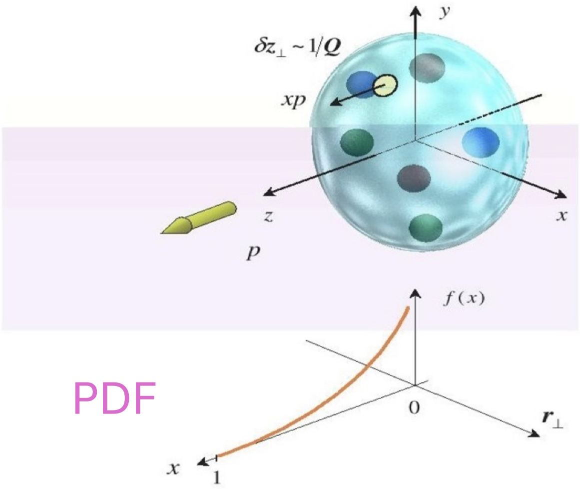
Established extended nature of nucleon

discovered the existence (quarks) inside the nucleon

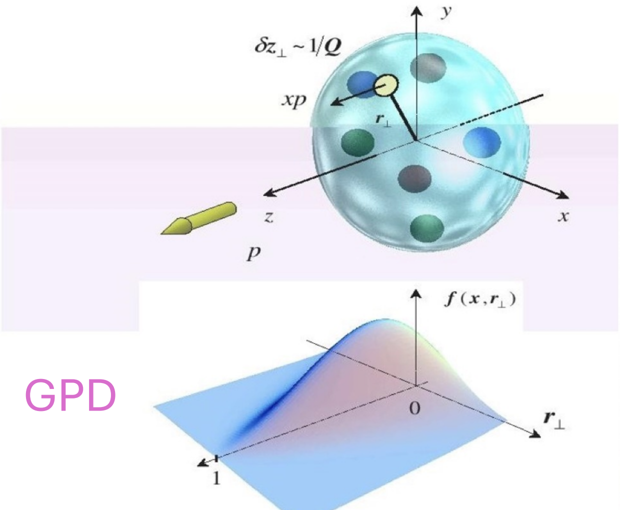
provides 3D spatial structure of the nucleon



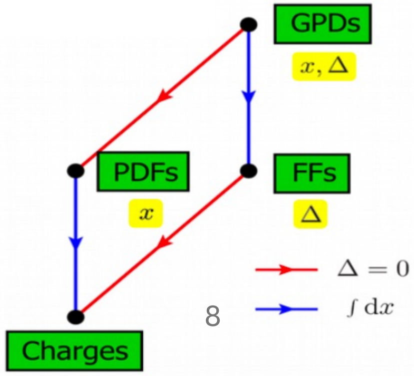
form factor



PDF



GPD



$$\Phi^{q[\gamma^+]}_{\rm GPD}(P,x,\xi,\vec{\Delta}_\perp) = \frac{1}{2P^+}\,\bar{u}(p',S')\left[\gamma^+ H^q(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}\,E^q(x,\xi,t)\right]u(p,S)$$

Connection of GPDs:

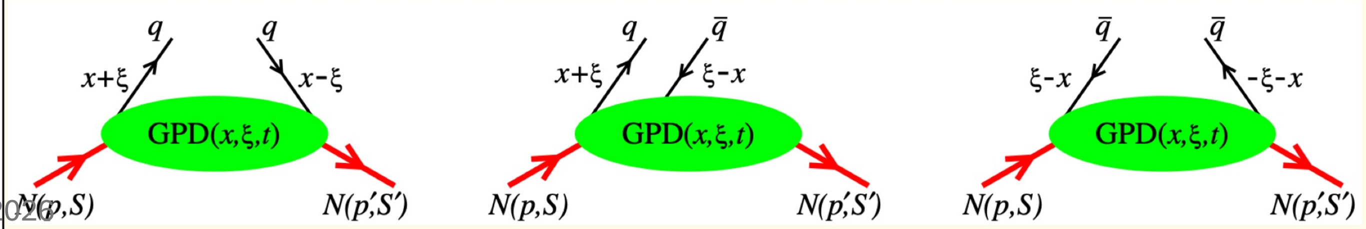
- **PDFs :** $H^q(x,0,0) = f_1^q(x)\,, \qquad \tilde{H}^q(x,0,0) = g_1^q(x)\,, \qquad H_T^q(x,0,0) = h_1^q(x)\,,$
- **EMFFs :** $F_1^q(t) = \int_{-1}^{1} dx\, H^q(x,\xi,t)\,, \qquad F_2^q(t) = \int_{-1}^{1} dx\, E^q(x,\xi,t)\,,$
- **Axial-vector FFs :** $G_A^q(t) = \int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t)\,, \qquad G_P^q(t) = \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t)$
- **GFFs :** $\int dx\, x\, H^q(x,\xi,t) = A_q(t) + \xi^2 D_q(t)\,, \qquad \int dx\, x\, E^q(x,\xi,t) = B_q(t) - \xi^2 D_q(t)$
- **Total Angular Momentum :** $J^q = \frac{1}{2}\int_{-1}^{1} dx\, x\,[H^q(x,0,0) + E^q(x,0,0)]$

  $\frac{1}{2} = \sum_q J^q + J^g$

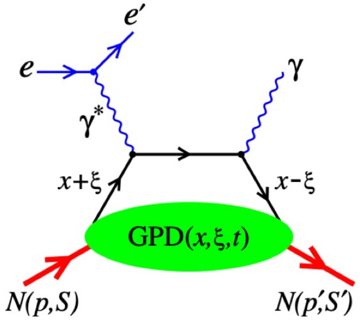
  $J^g = \frac{1}{2}\int_{0}^{1} dx\, x\,[H^g(x,0,0) + E^g(x,0,0)]$

Regions of GPDs:

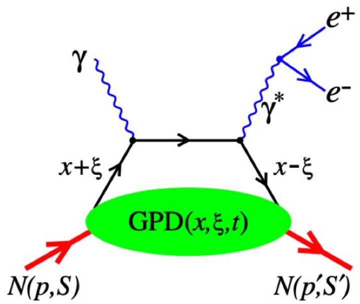
$x \in [\xi,\,1] \qquad x \in [-\xi,\,\xi] \qquad x \in [-1,-\xi]$ for $\xi > 0$



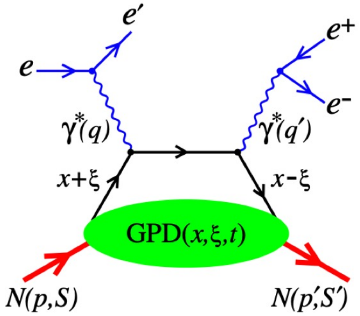
Access to GPDs:



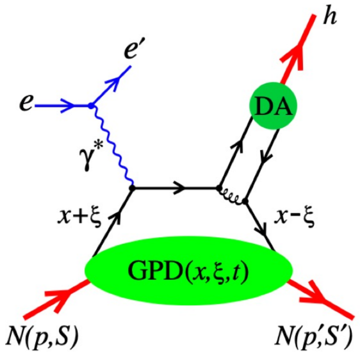
DVCS



Timelike DVCS



Double DVCS



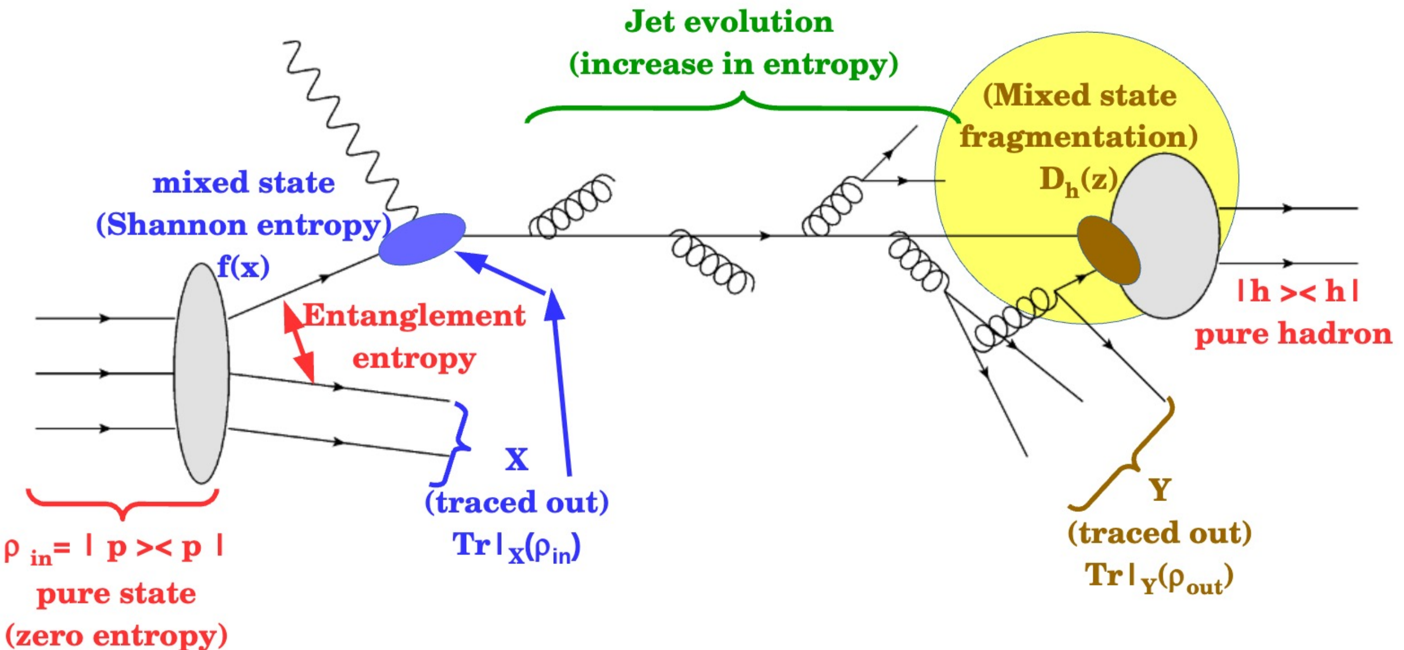
HEMP

Quantum entanglement

Zhang, Qian, Zhou, YL, Wang, 2512.21228

- LC formalism allows analytic evaluation of entanglement entropy
- We show that parton entanglement entropy is closely related to Shannon entropy of parton distribution

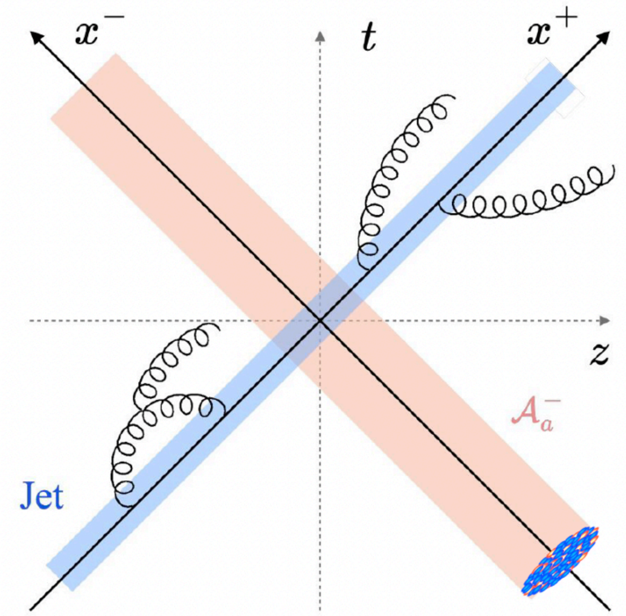
$$S(\rho_i) = H\big(f_i(x, k_\perp)\big) + \text{quantum correc.}$$



Time-dependent processes

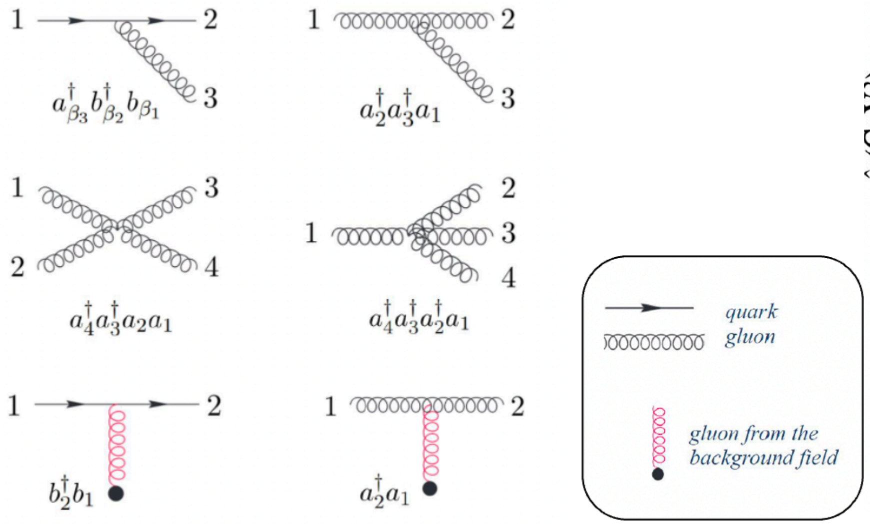
- Jet propagation, strong external fields, hadron reactions
- Naturally suitable for quantum computing

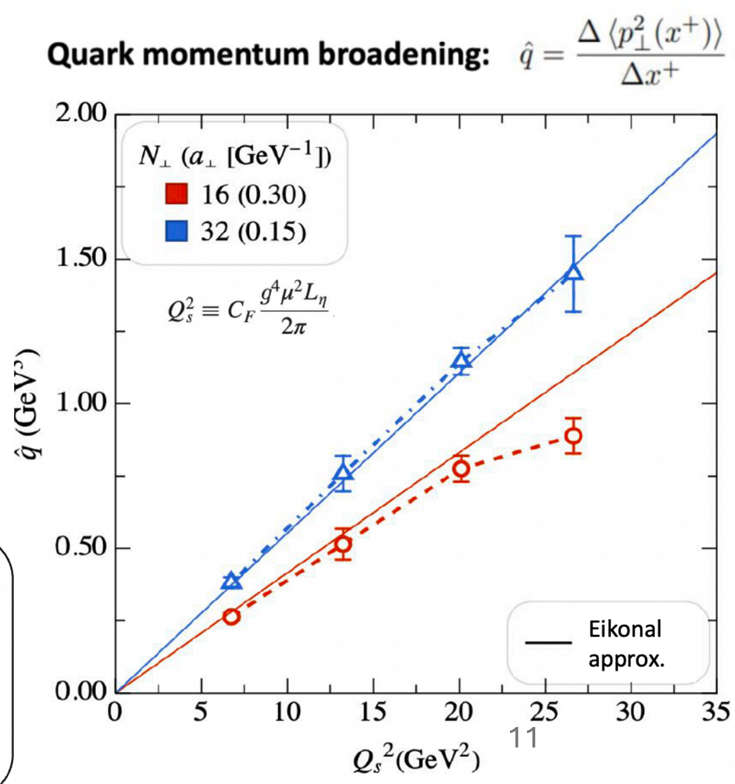
Qian & Li et al.



$$i\frac{\partial}{\partial x^+}|\Psi\rangle = H_{LF}|\Psi\rangle$$

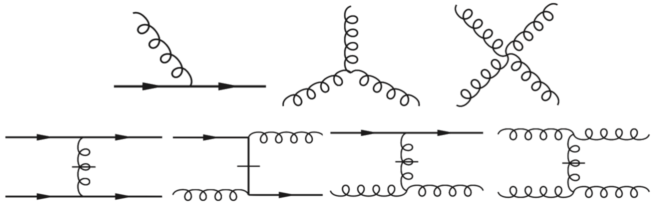
where, $H_{LF} = H_{LFQCD} + J_\mu \mathcal{A}^\mu$





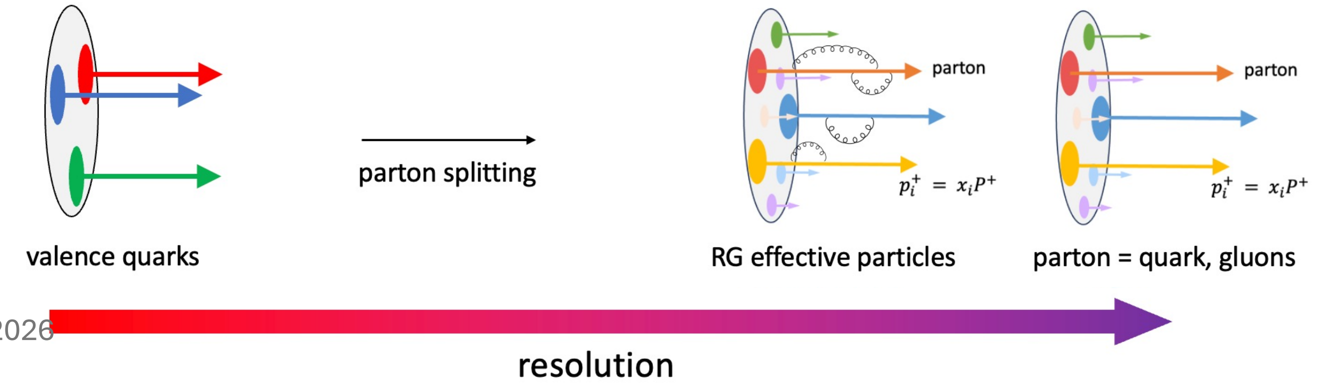
Light cone Hamiltonian method

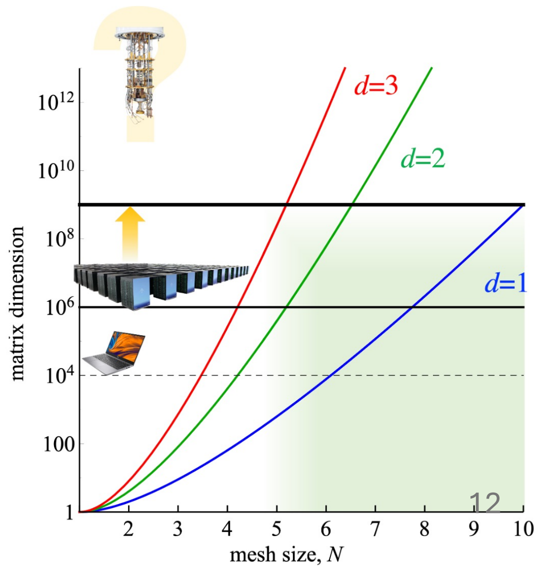
$$H_{LC}|\psi_h\rangle = M_h^2|\psi_h\rangle$$



where, $H_{LC} = \sum_i \frac{\vec{p}_{i\perp}^2 + m_i^2}{x_i} + V$

- Analogy to non-relativistic quantum many-body problems
- Implementation: basis light-front quantization (BLFQ)
- Exponential wall
- Hamiltonian RG





Building the LF Hamiltonian matrix using BLFQ

Step-1: Expand the system's state in the Fock space

Fock state expansion of the meson, baryon and deuteron bound state

$$|\Psi\rangle_M = \psi_{\{q\bar{q}\}}|q\bar{q}\rangle + \psi_{\{q\bar{q}g\}}|q\bar{q}g\rangle + \psi_{\{q\bar{q}gg\}}|q\bar{q}gg\rangle + \psi_{\{q\bar{q}q\bar{q}\}}|q\bar{q}q\bar{q}\rangle + \ldots$$

$$|\Psi\rangle_B = \psi_{\{qqq\}}|qqq\rangle + \psi_{\{qqqg\}}|qqqg\rangle + \psi_{\{qqqq\bar{q}\}}|qqqq\bar{q}\rangle + \ldots$$

$$|\Psi\rangle_D = \psi_{\{qqq\ qqq\}}|qqq\ qqq\rangle + \psi_{\{qqq\ qqq\ g\}}|qqq\ qqq\ g\rangle + \ldots$$

Step-2: Decide how to describe each particle's motion and quantum numbers.

Here comes the **basis** (**Heart of the BLFQ**)

Single-particle basis state → **to identify a particle**

$$\psi_{rs}(x, \vec{\kappa}^\perp) = \sum_{n,m,l} \langle n, m, l, r, s|\psi\rangle \times \phi_{nm}\left(\vec{\kappa}^\perp\right)\chi_l(x)$$

$\langle n, m, l, r, s|\psi\rangle$ → Eigenvector of the Hamiltonian

$\phi_{nm}\left(\vec{\kappa}^\perp\right)$ → Single particle motion in **transverse direction**

$\chi_l(x)$ → Single particle motion in **longitudinal direction**

Build **many-body basis states** for each Fock sector

Step-3: Compute the Hamiltonian matrix $\langle \text{basis}_i| H |\text{basis}_j\rangle$

Hamiltonian = K.E. terms + QCD interactions

Positronium

[Wiecki:2014ola]

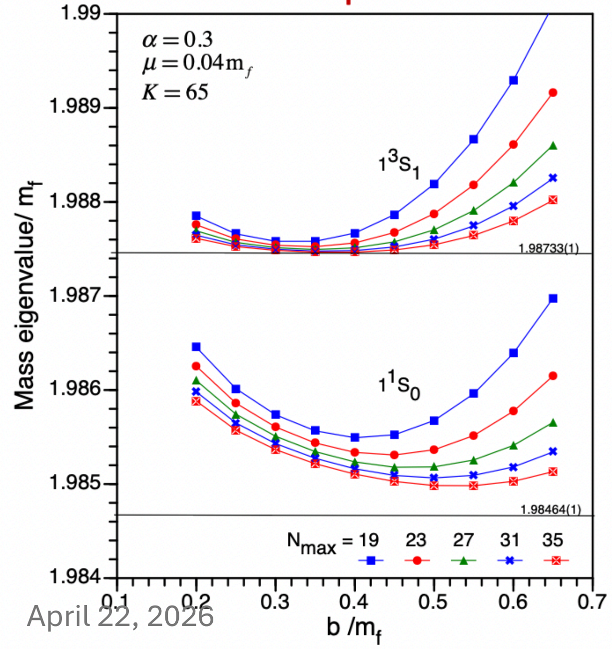
- Bloch-Wilson interaction: perturbative solution to the OSL effective Hamiltonian [Krautgartner:1991xz]

$$V = \frac{4\pi\alpha}{Q^2}\bar{u}_{s'}(k')\gamma_\mu u_s(k)\bar{v}_{\bar{s}}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}') + O(\alpha^2)$$

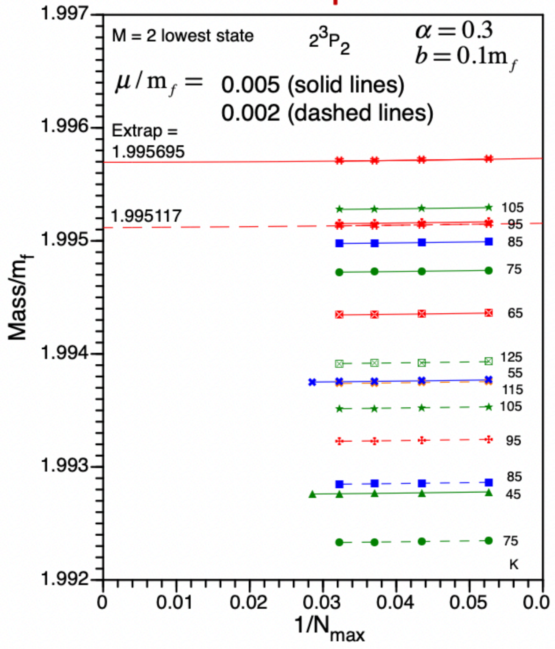
- Comparison with perturbative QED: [Bethe & Salpeter, 1977 Springer; cf. Lamm:2016djr]

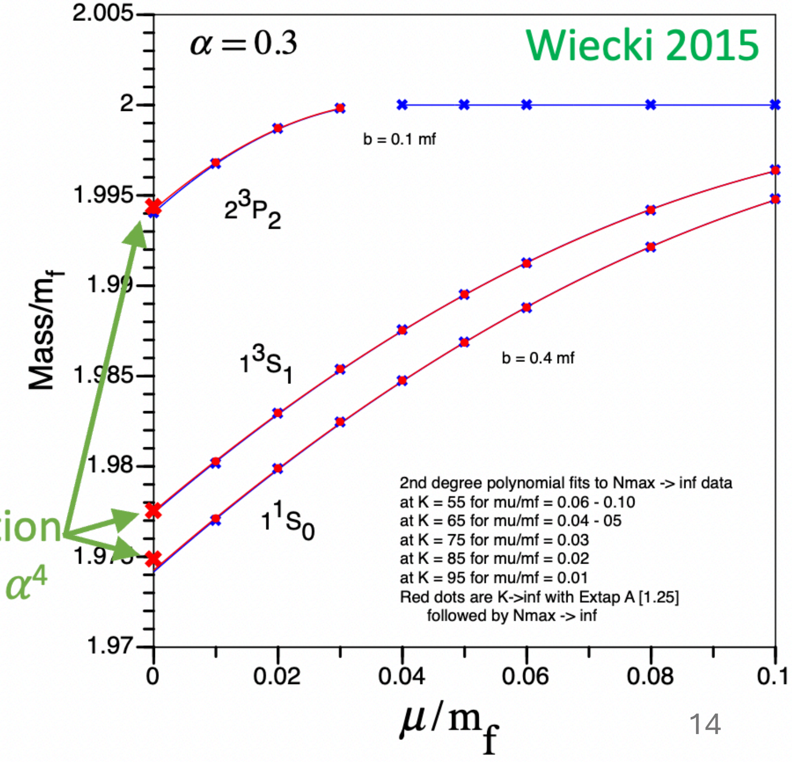
$$H = T + H_{\rm Col} + H_{\rm Dar} + H_{\rm rel} + H_{\rm LS} + H_{\rm SS}$$

variational minima wrt basis scale parameter b



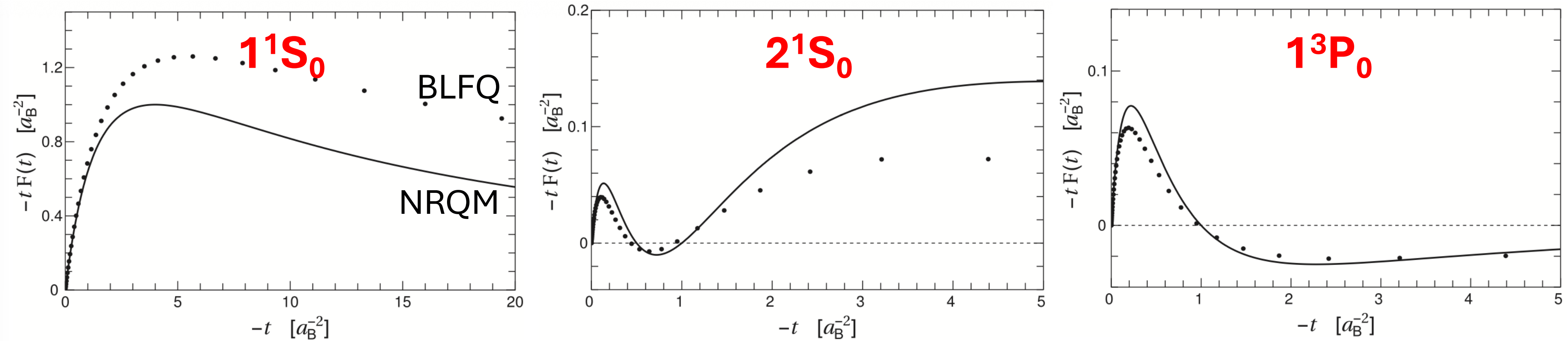
basis extrapolations



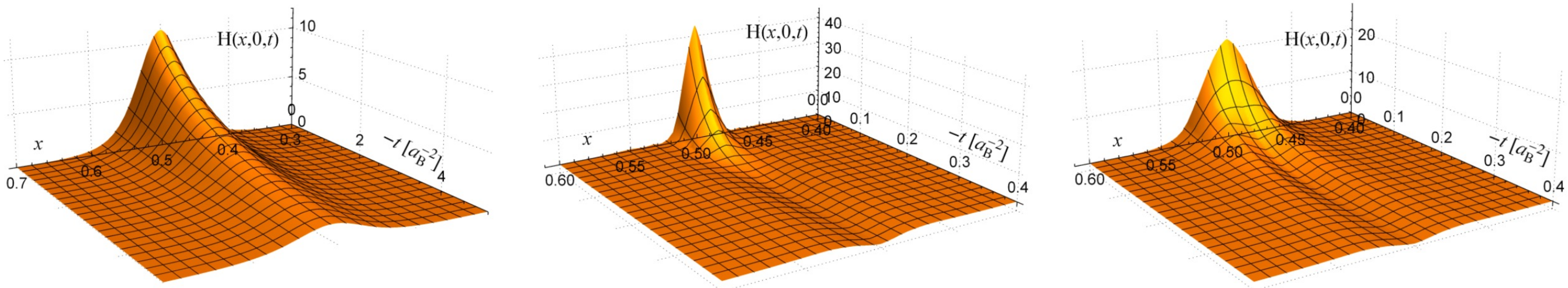


perturbation theory at α^4

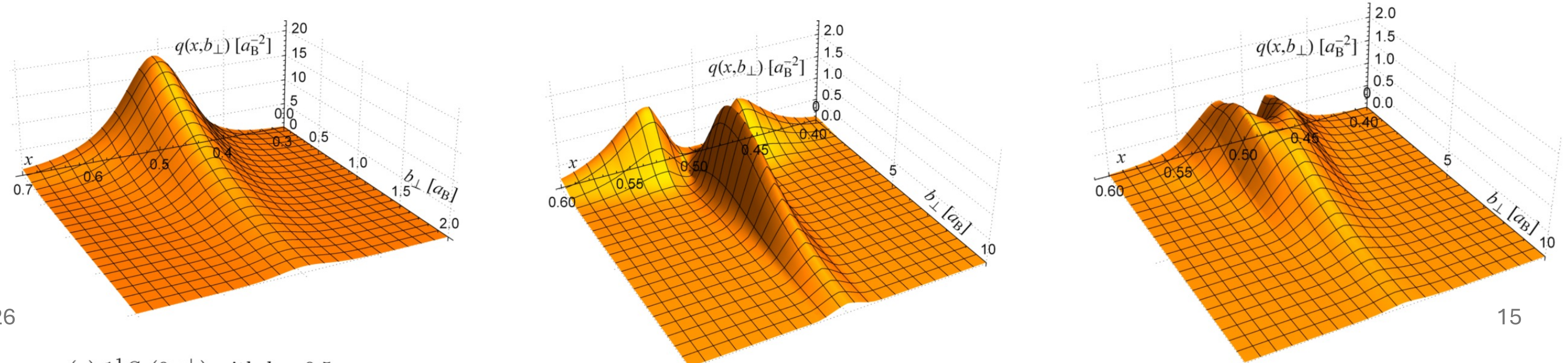
Form factor



GPD H
($\xi = 0$)

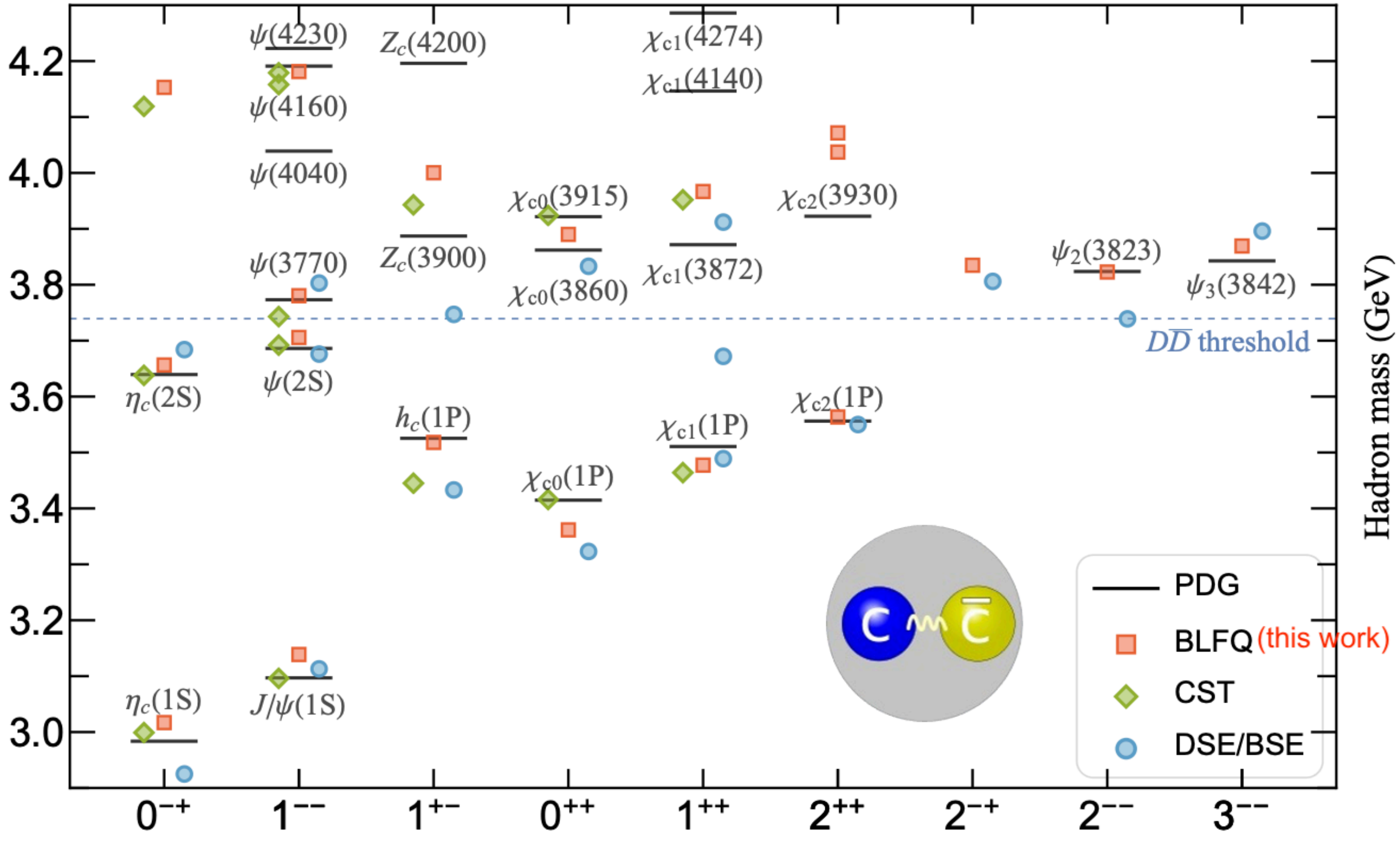


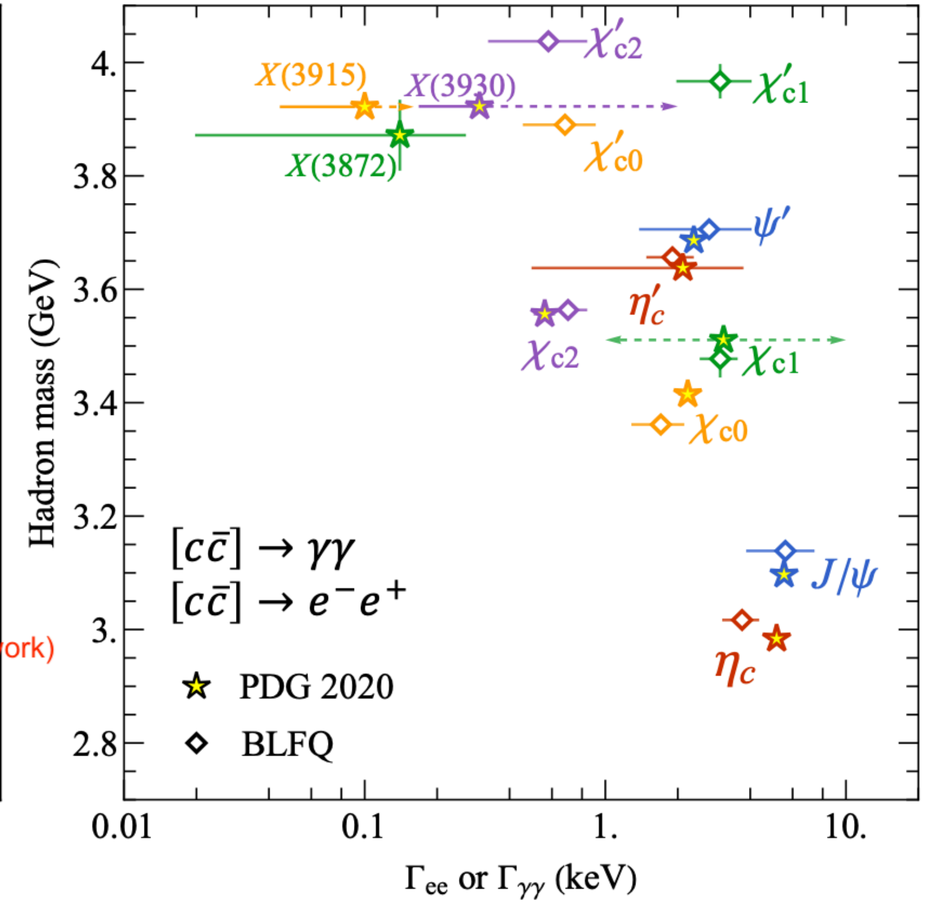
GPD q



Charmonium: hydrogen atom of QCD

[Li:2015zda, Li:2017mlw, Li:2021ejv]



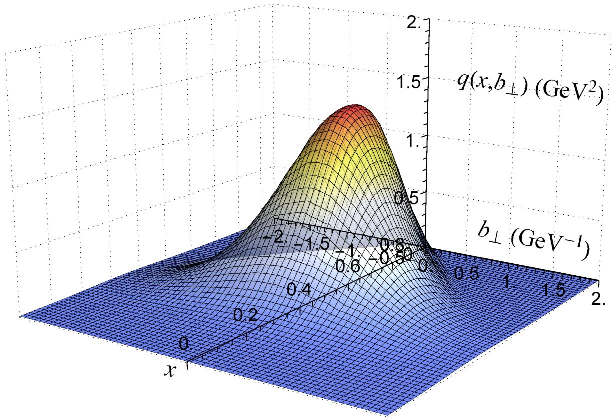


- Two free parameters (m_c, κ), rms deviation: 30 MeV
- Good agreement with the PDG data for both the masses and the widths

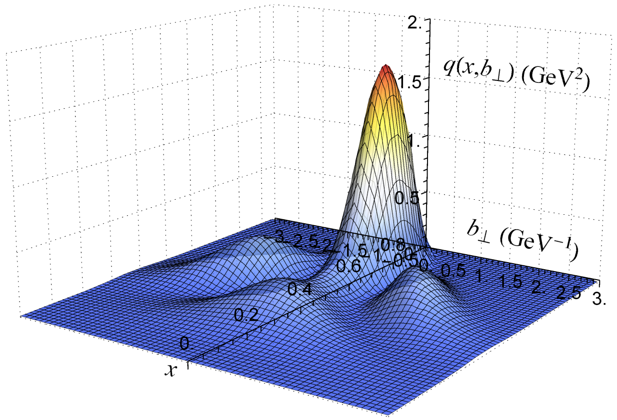
[Gross:2022hyw]

[LFWFs published on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2]

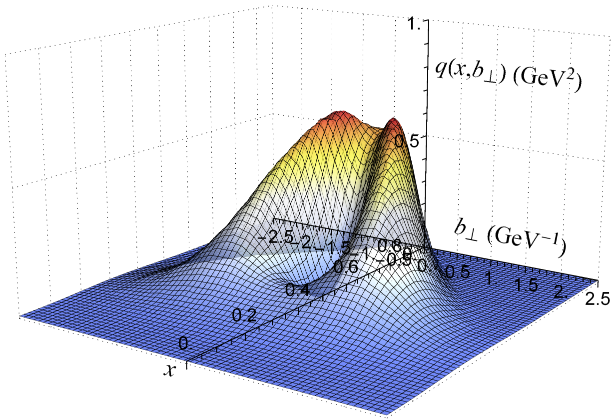
Quarkonium impact-parameter space GPDs



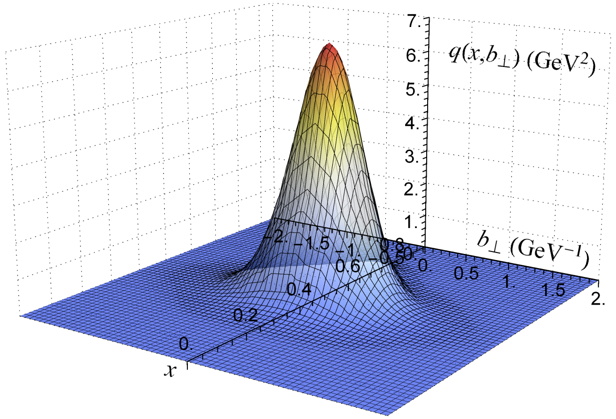
η_c



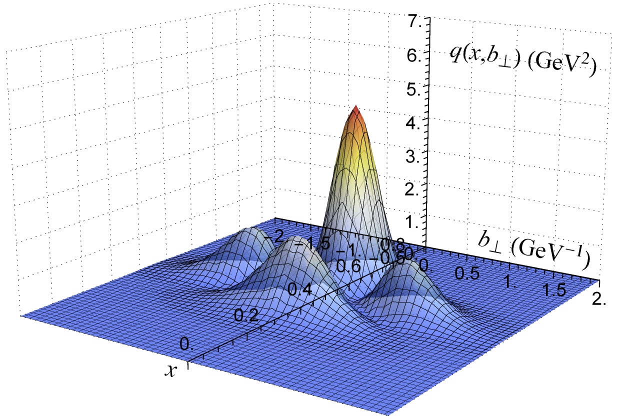
η_c'



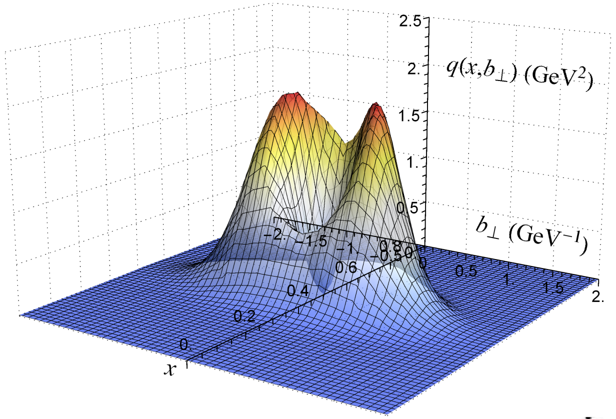
χ_c



η_b



η_b'



χ_b

GPDs of J/psi

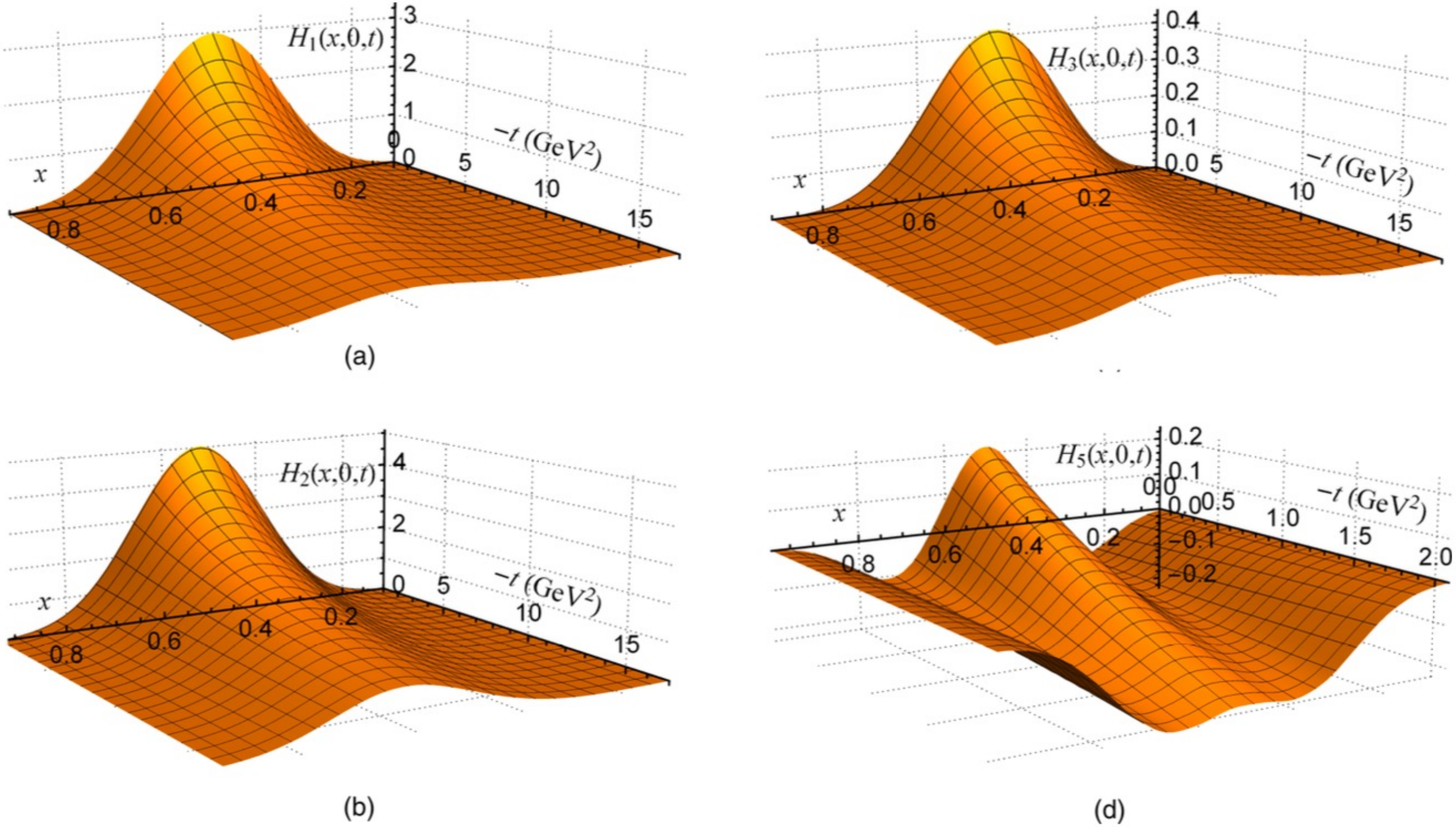


FIG. 8. 3D plot of helicity nonflip vector meson GPDs, $H_i(x, \xi = 0, t = -\Delta_\perp^2)$, $i = 1, 2, 3, 5$ [Eqs. (11), (12), (13), and (15)] for $J/\psi\ (1^3S_1)$ in the BLFQ approach.

Nucleon with Valence Fock Component $|\text{proton}\rangle = \psi_{uud}|uud\rangle$

- The LF eigenvalue equation: $H_{\text{eff}}|\Psi\rangle = M^2|\Psi\rangle$

$$H_{\text{eff}} = \sum_a \frac{\vec{p}^{\,2}_{\perp a} + m_a^2}{x_a} + \frac{1}{2}\sum_{a\neq b}\kappa^4\left[x_a x_b(\vec{r}_{\perp a} - \vec{r}_{\perp b})^2 - \frac{\partial_{x_a}(x_a x_b \partial_{x_b})}{(m_a + m_b)^2}\right]$$
$$+ \frac{1}{2}\sum_{a\neq b}\frac{C_F 4\pi\alpha_s}{Q^2_{ab}}\bar{u}_{s'_a}(k'_a)\gamma^\mu u_{s_a}(k_a)\bar{u}_{s'_b}(k'_b)\gamma^\nu u_{s_b}(k_b)g_{\mu\nu}$$

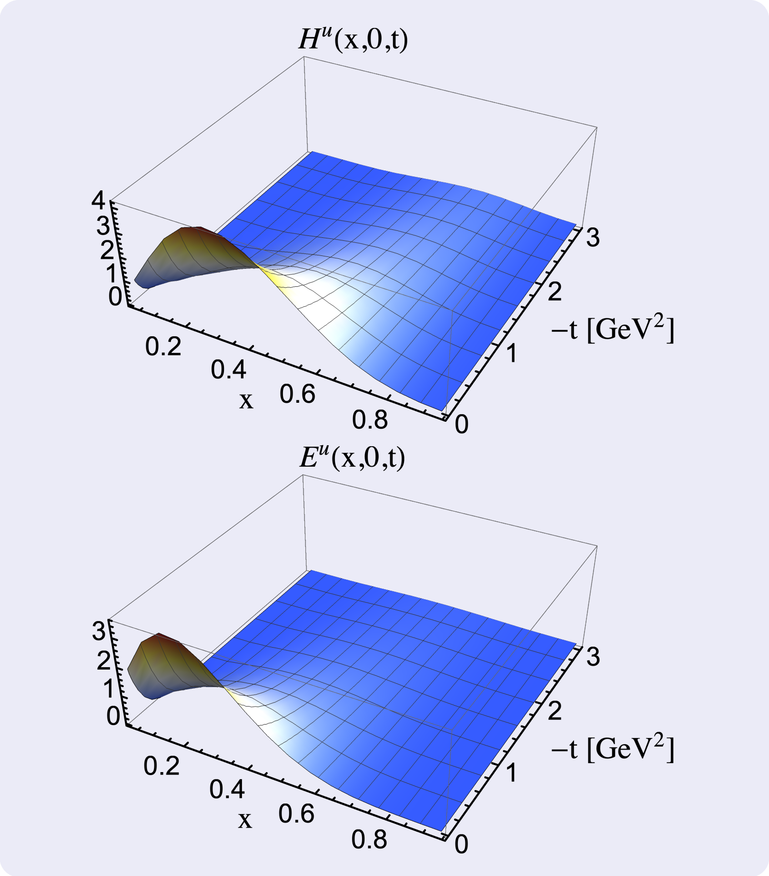
- For the first Fock sector:

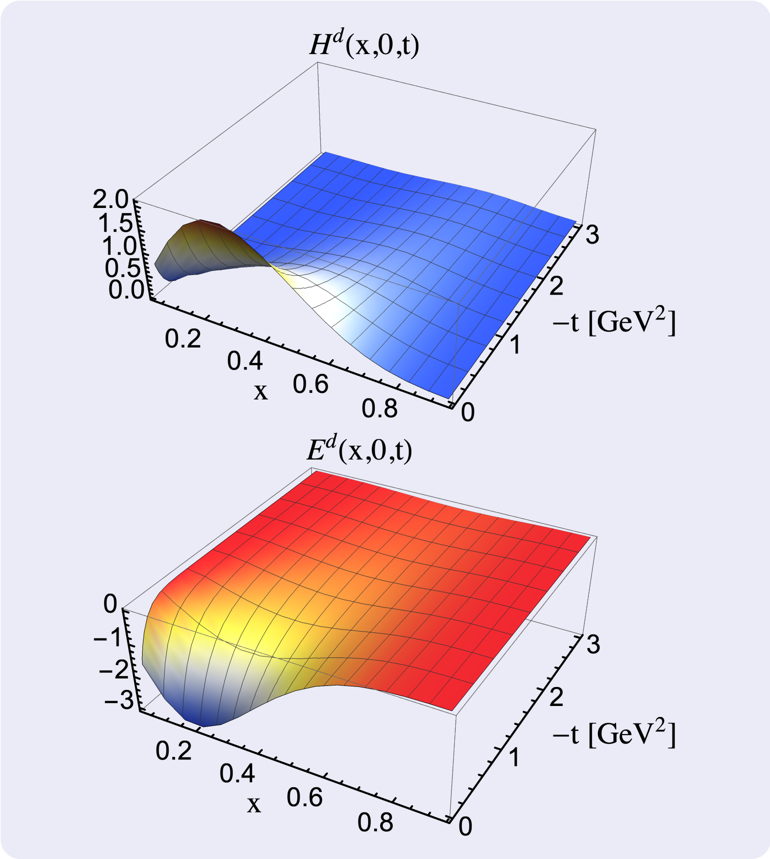
$$|qqq\rangle = |n_{q1}, m_{q1}, k_{q1}, \lambda_{q1}\rangle \otimes |n_{q2}, m_{q2}, k_{q2}, \lambda_{q2}\rangle \otimes |n_{q3}, m_{q3}, k_{q3}, \lambda_{q3}\rangle$$

- Transverse : 2D harmonic oscillator basis $\phi_{nm}(\vec{p}_\perp)$;
  Plane wave basis in longitudinal direction.
- The valence wavefunction in momentum space:

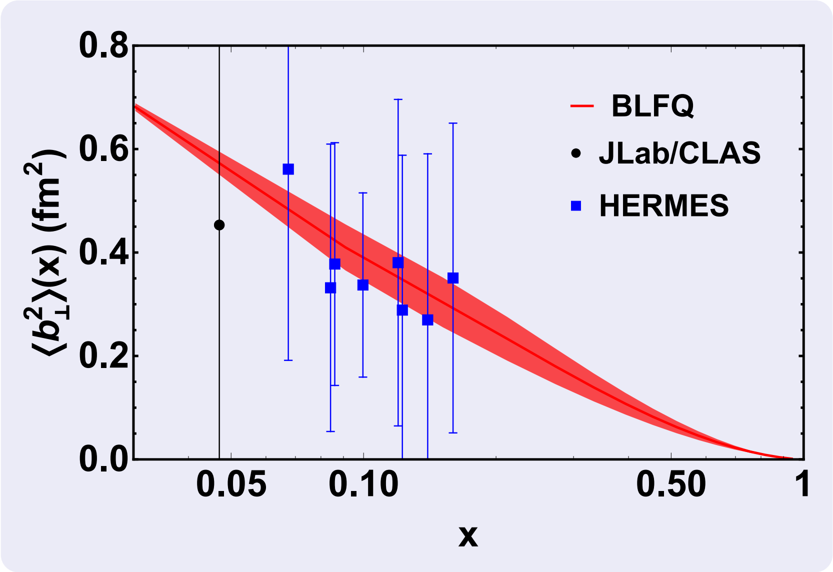
$$\Psi^{M_J}_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}} = \sum_{n_i, m_i}\left[\psi(\alpha_i)\prod_{i=1}^{3}\phi_{n_i m_i}(\vec{p}_{\perp i})\right]$$

Valence Quark Leading Twist GPDs





$$\langle b_\perp^2 \rangle^q(x) = \frac{\int d^2\vec{b}_\perp b_\perp^2 H^q(x, b_\perp)}{\int d^2\vec{b}_\perp H^q(x, b_\perp)}$$



- Qualitative nature consistent with phenomenological models [2]

Proton with One Dynamical Gluon

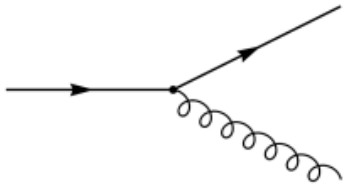
$$P^+ P^- |\Psi\rangle = M^2 |\Psi\rangle$$

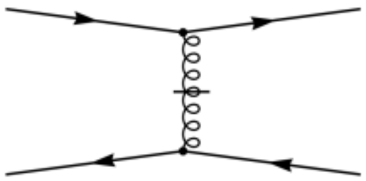
$$|\text{proton}\rangle = \psi_{uud}|uud\rangle + \psi_{uudg}|uudg\rangle$$

$$P^- = P^-_{\text{QCD}} + P^-_C$$

QCD Interaction:

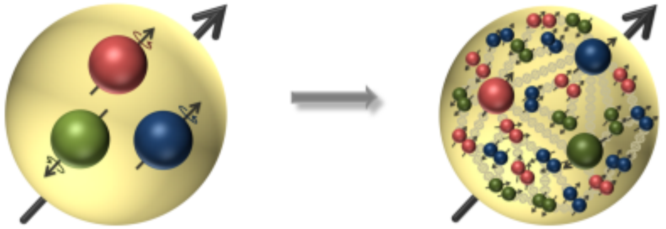
$$P^-_{\text{QCD}} = \int \mathrm{d}x^- \mathrm{d}^2 x^\perp \Big\{ \frac{1}{2}\bar{\psi}\gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+}\psi - \frac{1}{2} A_a^i \left[m_g^2 + (i\partial^\perp)^2\right] A_a^i + g_s \bar{\psi}\gamma_\mu T^a A_a^\mu \psi + \frac{1}{2} g_s^2 \bar{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi}\gamma^+ T^a \psi \Big\},$$





Confinement only in leading Fock:

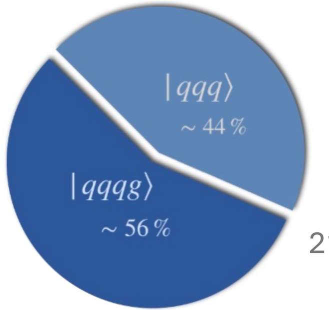
$$P^-_C P^+ = \frac{\kappa^4}{2} \sum_{i \neq j} \left\{ \vec{r}^{\,2}_{ij\perp} - \frac{\partial_{x_i}(x_i x_j \partial_{x_j})}{(m_i + m_j)^2} \right\}$$



Parameters:

Truncation: Nmax=9, K=16.5
HO parameters: b=0.7GeV, b_{inst}=3GeV

m_u	m_d	m_g	κ	m_f	g
0.31GeV	0.25GeV	0.50GeV	0.54GeV	1.80GeV	2.40

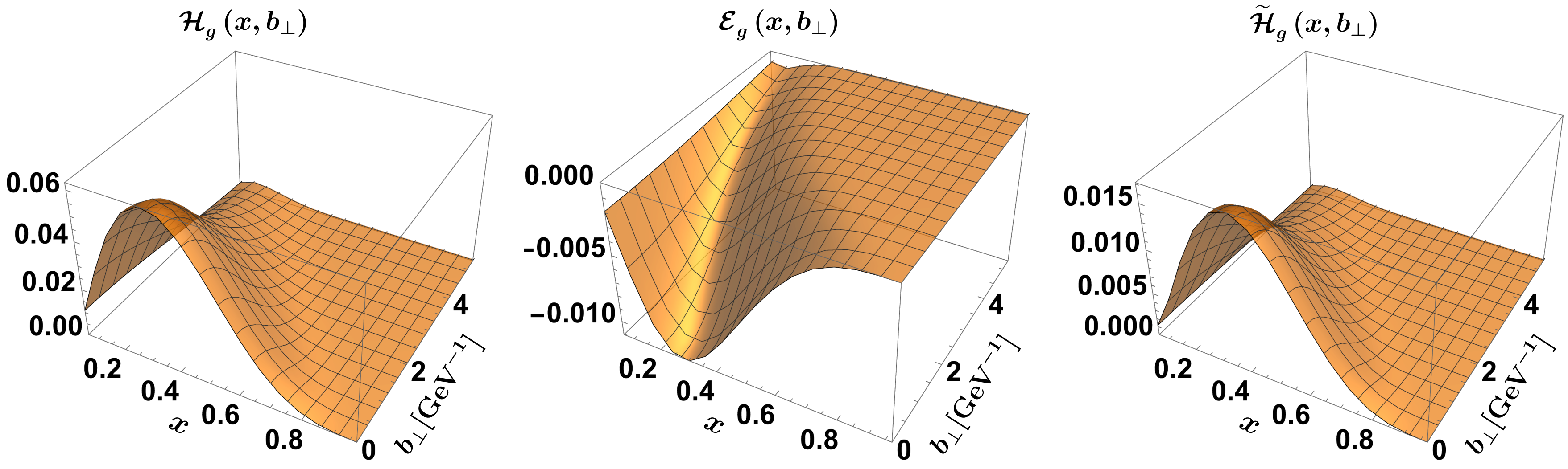


Gluon GPDs

$$F^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left(\gamma^+ H^g(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E^g(x,\xi,t)\right) u(p,\lambda)\,,$$

$$\tilde{F}^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left(\gamma^+\gamma_5\, \tilde{H}^g(x,\xi,t) + \frac{\Delta^+\gamma_5}{2M} \tilde{E}^g(x,\xi,t)\right) u(p,\lambda)\,.$$

Non-skewed GPDs $(x, 0, t)$ → FT → GPDs $(x, b_\perp)$



- Model scale : $\mu_0^2 = 0.23 - 0.25$ GeV^2 (by matching $\langle x \rangle$ with global fit)
- Total Angular Momentum: $J = \frac{1}{2}\int dx\, x[H(x,0) + E(x,0)]$;
  $J_g = 0.066$, 13.2% of the proton TAM.

Effective Hamiltonian with Dynamical Gluon and Sea Quarks

Fock expansion:

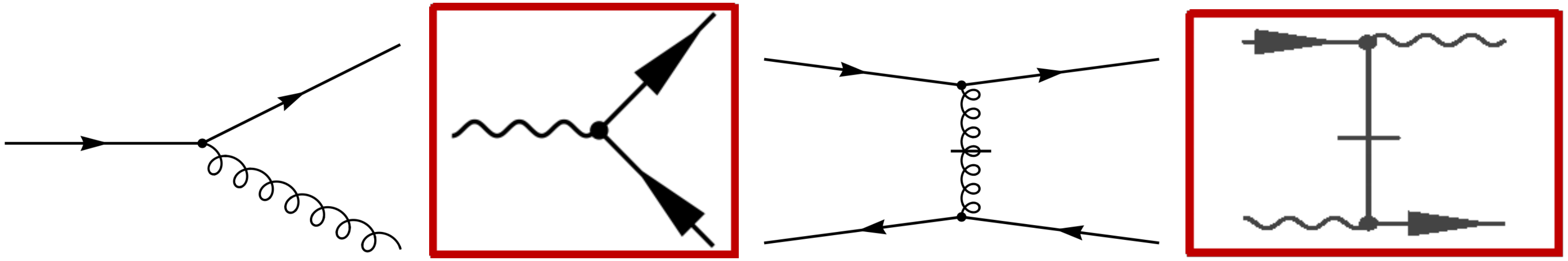
$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + c_1\,|\,uudu\bar{u}\rangle + c_2\,|\,uudd\bar{d}\rangle + c_3\,|\,uuds\bar{s}\rangle + \ldots$$

Light-front QCD Hamiltonian :

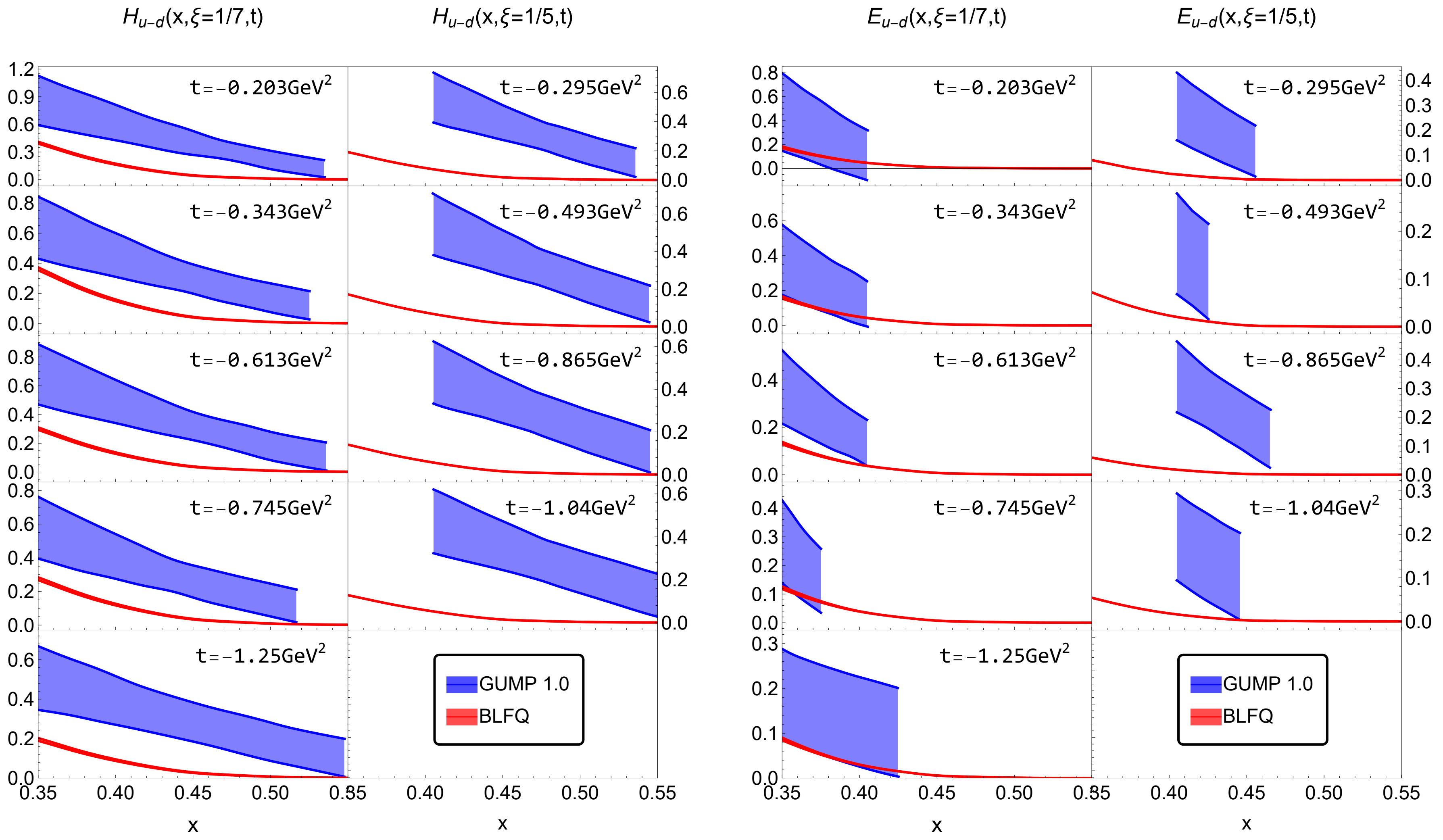
$$H_{\text{LF}} = \sum_a \frac{\vec{p}_{\perp a}^{\,2} + m_a^2}{x_a} + H_{\text{confinement}} + H_{\text{vertex}} + H_{\text{inst}}$$

$$H_{\text{vertex}} + H_{\text{inst}} = g_s \bar{\psi}\gamma_\mu T^a A_a^\mu \psi + \frac{1}{2} g_s^2 \bar{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi}\gamma^+ T^a \psi$$

$$+ \frac{1}{2} g_s^2 \bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{(i\partial^+)} A_\nu \gamma^\nu \psi$$



Comparison with Global Analysis



- Qualitative agreement at low skewness with the first global GPD extraction (GUMP1.0) using experimental and lattice data at NLO.

BLFQ Road-map toward First Principles

Vary et al. (BLFQ Collaboration), Eur. Phys. J. Spec. Top. (2025)

Basis Light-Front Quantization (BLFQ)

2020

$|qqq\rangle$ → K.E. + Confinement + One Gluon Exchange interaction

- EMFFs, PDFs & Axial FFs →
  - PRD **102**, 016008 (2020)
  - PRD **104**, 094036 (2021)
- GPDs (Twist-2, -3) →
  - PRD **105**, 094018 (2022)
  - PRD **109**, 014015 (2024)
  - PRD **109**, 034031 (2024)
  - PLB **855**, 138809 (2024)
- TMDs →
  - PLB **833**, 137360 (2022)
  - PRD **109**, 014015 (2024)
- GFFs →
  - PRD **110**, 056027 (2024)
- DPDs →
  - PRD **112**, 014018 (2025)

2023

$|qqq\rangle + |qqqg\rangle$ → K.E. + Confinement in $|qqq\rangle$ + **QCD interactions (Instantaneous gluon interaction + quark-gluon vertex interaction)**

- PDFs & Spin Decomposition →
  - PRD **108**, 094002 (2023)
- GPDs (Twist-2, -3) →
  - PLB **847**, 138305 (2023)
  - PLB **860**, 139153 (2025)
  - PLB **866**, 139584 (2025)
- TMDs (Twist-2, -3) →
  - PLB **855**, 138831 (2024)
  - PLB **855**, 138829 (2024)
- GFFs & Mass Decomposition →
  - arXiv-hep: 2506.07554

2025

$|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$ → K.E. + **QCD interactions (Instantaneous gluon interaction + Instantaneous fermion interaction + quark-gluon vertex interaction)**

- EMFFs, PDFs & Spin Decomposition →
  - PLB **867**, 139599 (2025)

⋮

First Principles of LF QCD → K.E. + **All QCD interactions**

Summary

- The parton picture on the light cone is vital for describing hadron structures relevant to the EIC

- Light-cone Hamiltonian formalism offers a natural framework for understanding parton distributions within the strong interaction

- We utilize BLFQ to access various GPDs, achieving qualitative agreement with available experimental and theoretical data

Thank you!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CELEBRATING 30 YE
HAPPY 30TH BIRTHDAY
GPD
30