Generalized Parton Distributions and Nucleon Spin Structure from Lattice QCD

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Generalized parton distributions

Gauge-Invariant Decomposition of Nucleon Spin #3

Xiang-Dong Ji (MIT, LNS and Washington U., Seattle) (Mar, 1996)

Published in: *Phys.Rev.Lett.* 78 (1997) 610-613 • e-Print: hep-ph/9603249 [hep-ph]

Deeply virtual Compton scattering #4

Xiang-Dong Ji (Maryland U. and MIT, LNS and Washington U., Seattle) (Sep, 1996)

Published in: *Phys.Rev.D* 55 (1997) 7114-7125 • e-Print: hep-ph/9609381 [hep-ph]

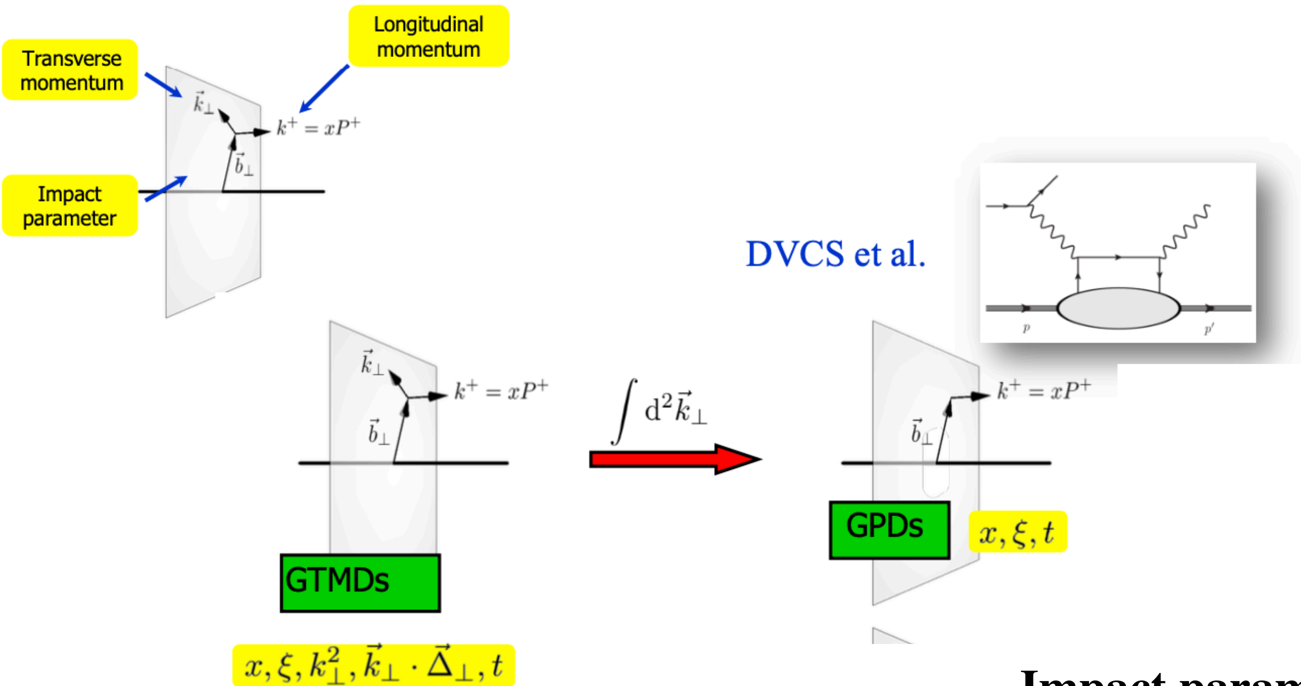
Example: Unpol. quark GPD

$$F(x,\xi,t) = \frac{1}{2\bar{P}^+}\int \frac{d\lambda}{2\pi} e^{-ix\lambda}\langle P'|O_{\gamma^+}(\lambda n)|P\rangle = \frac{1}{2\bar{P}^+}\bar{u}(P')\left[H(x,\xi,t)\gamma^+ + E(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right]u(P)$$

$$O_{\gamma^+}(\lambda n) = \bar{\psi}\left(\frac{\lambda n}{2}\right)\gamma^+ W\left(\frac{\lambda n}{2}, -\frac{\lambda n}{2}\right)\psi\left(-\frac{\lambda n}{2}\right), \quad \bar{P} = \frac{P'+P}{2}, \quad \Delta = P' - P, \quad t = \Delta^2, \quad \xi = -\frac{\Delta^+}{2\bar{P}^+}$$

See also Mueller et al, FP 94', Radyushkin, PRD 99'

Generalized parton distributions



Generalized parton distributions

- Link PDFs and FFs
- Correlate transverse coordinate and longitudinal momentum of partons
- Shed light on nucleon spin and mass structure

Impact parameter distribution

Burkhardt, PRD 00'

$$\mathsf{q}(x, b) = \int \frac{d\mathbf{q}}{(2\pi)^2} H(x, \xi = 0, t = -\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{b}}$$

Angular momentum (Ji sum rule)

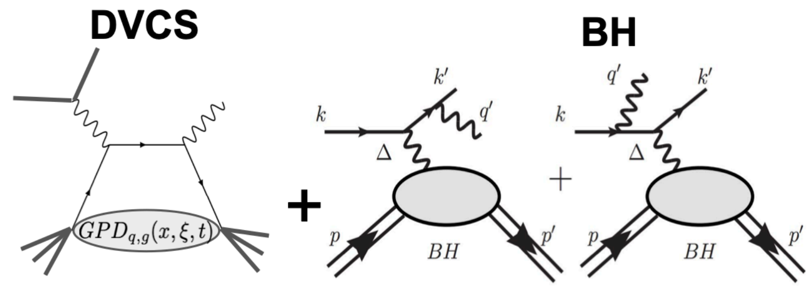
Ji, PRL 97' & PRD 97'

$$\mathbf{J_q} = \frac{1}{2}\int_{-1}^{1} dx\, x[H(x,\xi,0) + E(x,\xi,0)]$$

GPDs from phenomenology

- Extraction of GPDs from DVCS

$$\sigma = \sigma_{BH} + \sigma_{DVCS} + \sigma_I$$



$$\sigma_{BH}(x_{Bj},t,Q^2,E_b,\phi) = \frac{\Gamma}{t}\left[A^{BH}_{UU}\left(F_1^2+\tau F_2^2\right) + B^{BH}_{UU}\tau G_M^2(t)\right]$$

No CFFs

$$\begin{aligned}\sigma_{\mathcal{I}}(x_{Bj},t,Q^2,E_b,\phi) = \frac{\Gamma}{Q^2 t}\Big[&A_{\mathcal{I}}(x_{Bj},t,Q^2,E_b,\phi)\Big(F_1(t)\,\Re e\mathcal{H}(x_{Bj},t,Q^2) + \tau F_2(t)\,\Re e\mathcal{E}(x_{Bj},t,Q^2)\Big)\\ &+ B_{\mathcal{I}}(x_{Bj},t,Q^2,E_b,\phi)G_M(t)\Big(\Re e\mathcal{H}(x_{Bj},t,Q^2) + \Re e\mathcal{E}(x_{Bj},t,Q^2)\Big)\\ &+ C_{\mathcal{I}}(x_{Bj},t,Q^2,E_b,\phi)G_M(t)\,\Re e\widetilde{\mathcal{H}}(x_{Bj},t,Q^2)\Big]\end{aligned}$$

Linear CFFs: 3

$$\begin{aligned}\sigma_{DVCS}(x_{Bj},t,Q^2,E_b,\phi) = \frac{\Gamma}{Q^2}\frac{2}{1-\epsilon}\Big[&(1-\xi^2)\Big[(\Re e\mathcal{H})^2 + (\Im m\mathcal{H})^2 + (\Re e\widetilde{\mathcal{H}})^2 + (\Im m\widetilde{\mathcal{H}})^2\Big]\\ &+ \frac{t_o - t}{4M^2}\Big[(\Re e\mathcal{E})^2 + (\Im m\mathcal{E})^2 + \xi^2(\Re e\widetilde{\mathcal{E}})^2 + \xi^2(\Im m\widetilde{\mathcal{E}})^2\Big]\\ &- 2\xi^2\Big(\Re e\mathcal{H}\,\Re e\mathcal{E} + \Im m\mathcal{H}\,\Im m\mathcal{E} + \Re e\widetilde{\mathcal{H}}\,\Re e\widetilde{\mathcal{E}} + \Im m\widetilde{\mathcal{H}}\,\Im m\widetilde{\mathcal{E}}\Big)\Big]\end{aligned}$$

Quadratic CFFs: 8

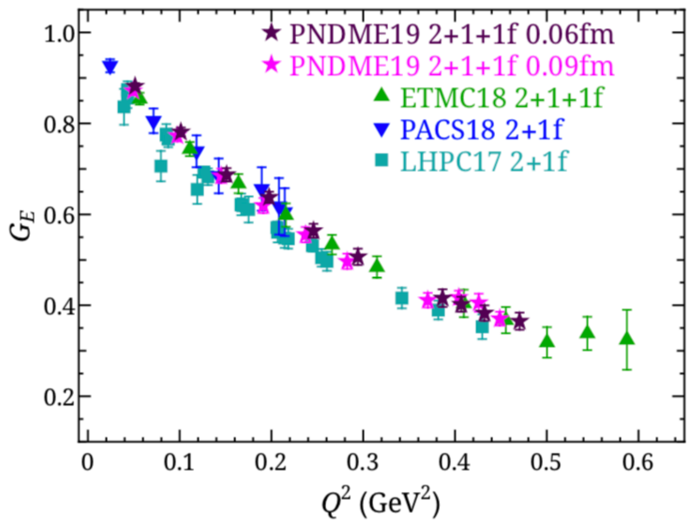
Very complicated to disentangle all these pieces

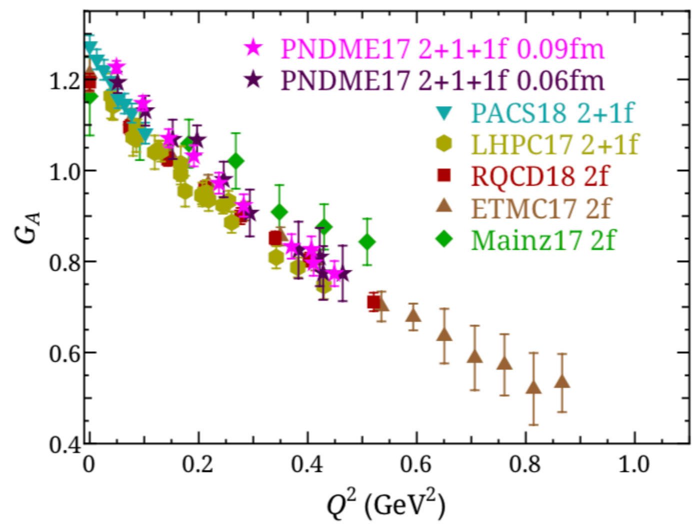
- Moreover,

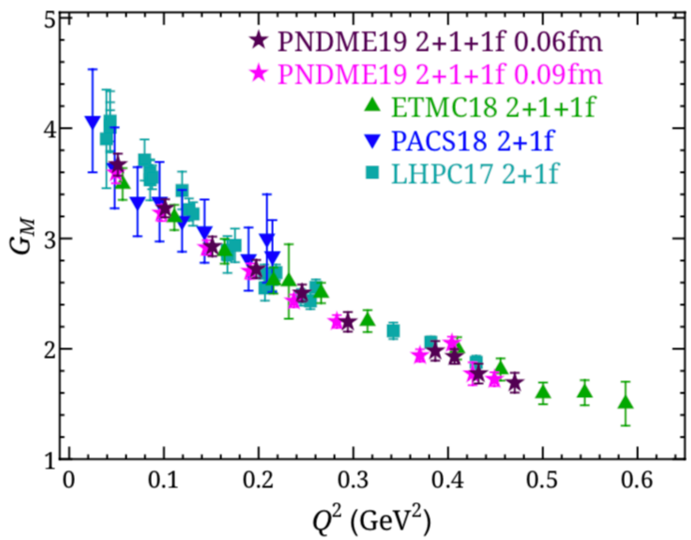
$$\mathcal{H}\left(\xi,t,Q^2\right) = \int_{-1}^{1}\frac{\mathrm{d}x}{\xi}\sum_{a=g,u,d,\ldots} C^a\left(\frac{x}{\xi},\frac{Q^2}{\mu_F^2},\alpha_S\left(\mu_F^2\right)\right)H^a\left(x,\xi,t,\mu_F^2\right)$$

GPDs from lattice

- Complementary inputs from lattice QCD - **moments**







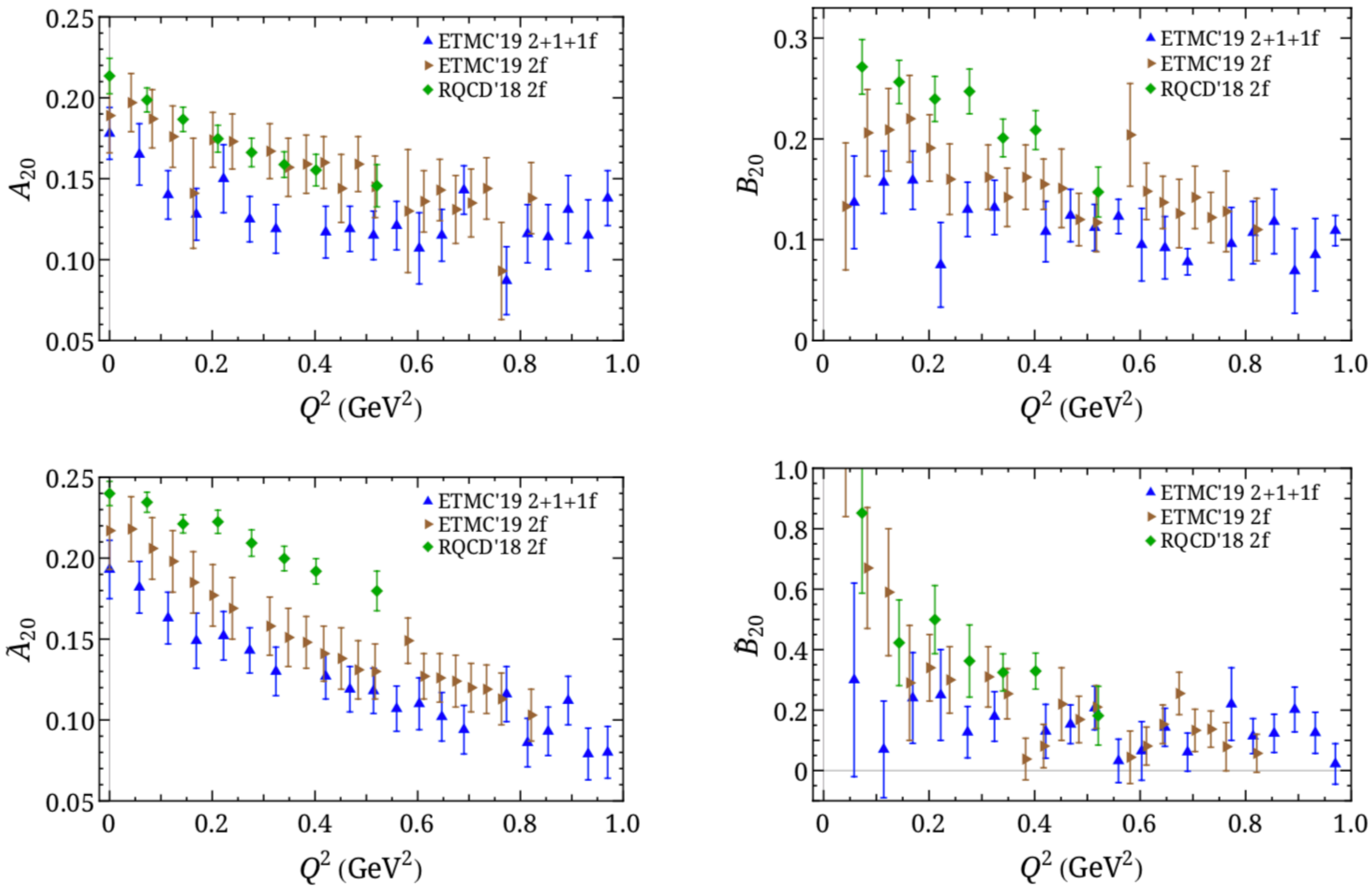
$$\langle N(p_f)|V_\mu^+(x)|N(p_i)\rangle = \bar{u}^N \left[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{2M_N}F_2(q^2)\right] u_N e^{iq\cdot x}$$

$$\langle N(p_f)|A_\mu^+(x)|N(p_i)\rangle = \bar{u}_N \left[\gamma_\mu\gamma_5 G_A(q^2) + iq_\mu\gamma_5 G_P(q^2)\right] u_N e^{iq\cdot x}$$

Constantinou, JHZ et al, PPNP 21’

GPDs from lattice

- Complementary inputs from lattice QCD - **moments**



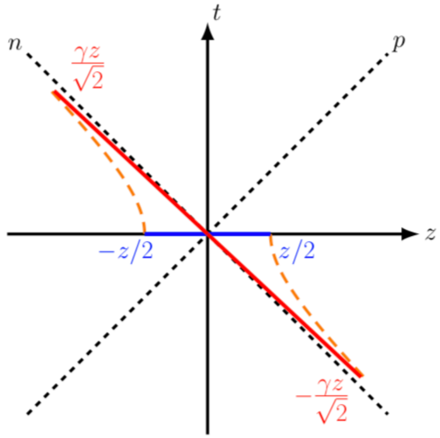
$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\left[A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m_N} + C_{20}(q^2)\,\frac{1}{m_N}q^{\{\mu}q^{\nu\}}\right]u_N(p,s),$$

$$\langle N(p',s')|\mathcal{O}_A^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{i}{2}\left[\tilde{A}_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}}\gamma^5 + \tilde{B}_{20}(q^2)\,\frac{q^{\{\mu}P^{\nu\}}}{2m_N}\gamma^5\right]u_N(p,s),$$

GPDs from lattice

- Complementary inputs from lattice QCD - **distributions**

Large-Momentum Effective Theory



Ji, PRL 13' & SCPMA 14',
Ji, JHZ et al, RMP 21'

Example:

Quark PDF: $$q^{[\Gamma]}(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle PS|\bar{\psi}(0)\Gamma U(0;\xi^-)\psi(\xi^-)|PS\rangle$$

From $$\tilde{q}^{[\Gamma]}(x) = \int \frac{dz}{4\pi} e^{-ixzP_z} \langle PS|\bar{\psi}(0)\Gamma U(0;z)\psi(z)|PS\rangle$$

Factorization:

Xiong, JHZ et al, PRD 14', Izubuchi et al, PRD 18', Yao, Ji, JHZ, JHEP 23'

$$q^{[\Gamma]}(x) = C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) \otimes \tilde{q}(y) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{x^2P_z^2}, \frac{\Lambda^2_{\rm QCD}}{(1-x)^2P_z^2}\right)$$

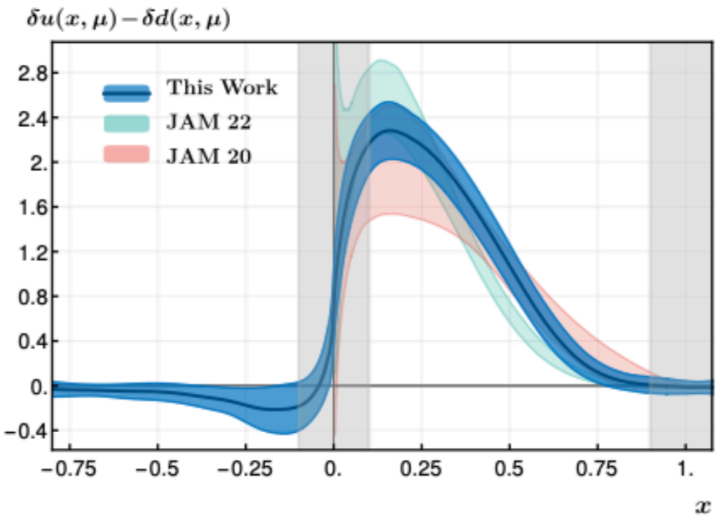
GPDs from lattice

- Complementary inputs from lattice QCD - **distributions**

Large-Momentum Effective Theory

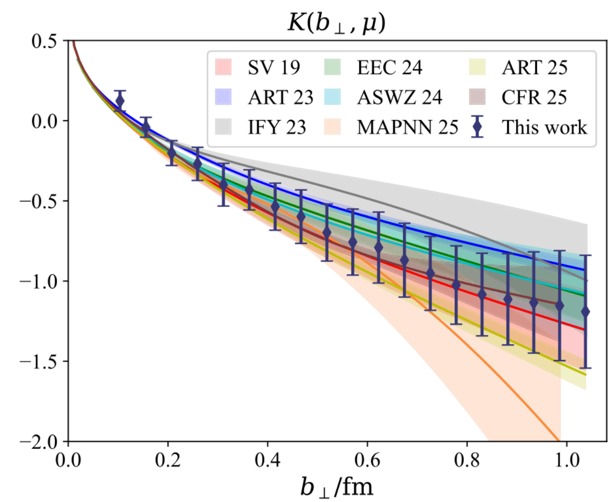
Some state-of-the-art results

Nucleon quark transversity PDF



Yao, JHZ et al (LPC), PRL 23'

Collins-Soper kernel



Tan et al (LPC), PRD 26'

See Peng Sun's talk

See talks by Qi-An Zhang, Jin-Xin Tan

GPDs from lattice

- Complementary inputs from lattice QCD - **distributions**

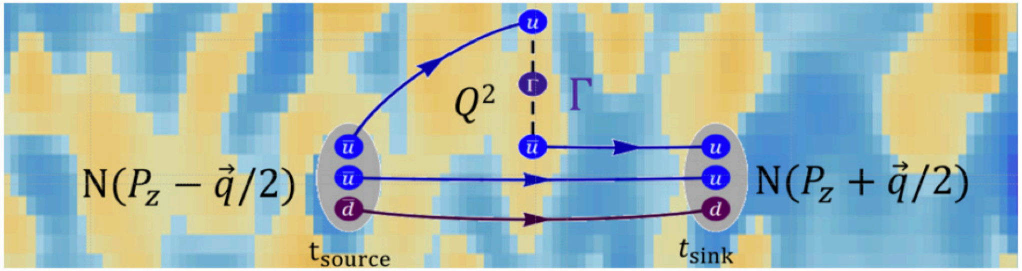
Factorization for GPDs

Ji, JHZ et al, PRD 15' & Xiong, JHZ, PRD 15'

$$H^{\pi}_{u-d}(x,\xi,t,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\xi}{y},\frac{yP^z}{\mu}\right) \tilde{H}^{\pi}_{u-d}(y,\xi,t,P^z) + h.t.$$

$$\tilde{h}_{\rm lat}(z,P^z,t,a) = \frac{P^z}{P_0}\langle N(\vec{P}+\frac{\vec{\Delta}}{2})\,|\,\bar{q}(z)\Gamma(\prod_n U_z(n\hat{z}))\tau_3 q(0)\,|\,N(\vec{P}-\frac{\vec{\Delta}}{2})\rangle$$

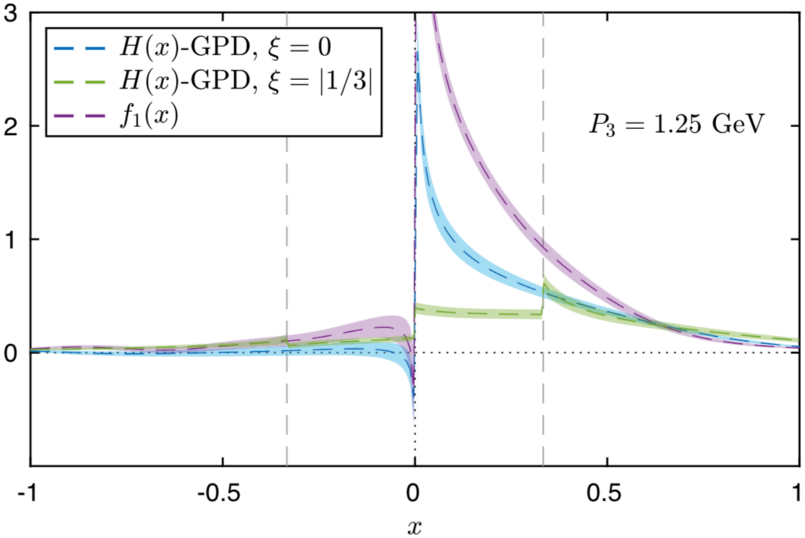
$$\tilde{H}^{\pi}_{u-d}(x,\xi,t,P^z) = \int \frac{dz}{4\pi} e^{ixzP^z}\tilde{h}^R_{\rm lat}(z,P^z,t)$$



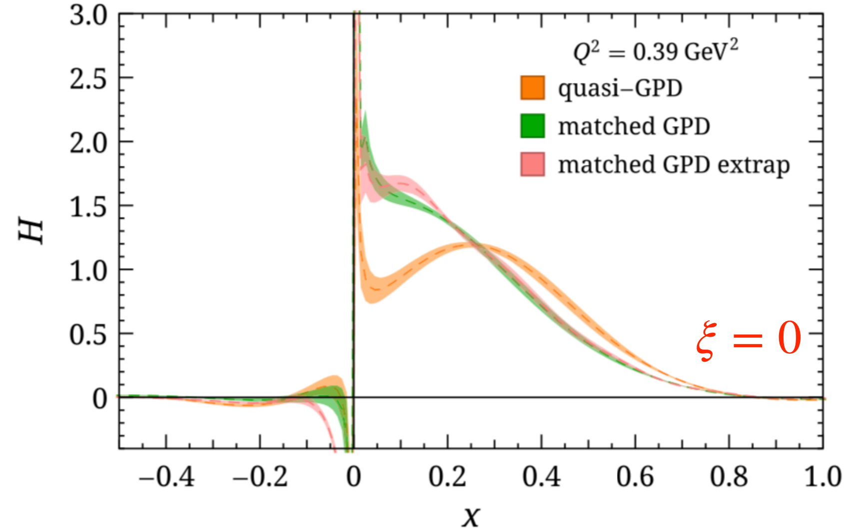
Chen, JHZ et al, NPB 20'

Lattice results on GPDs

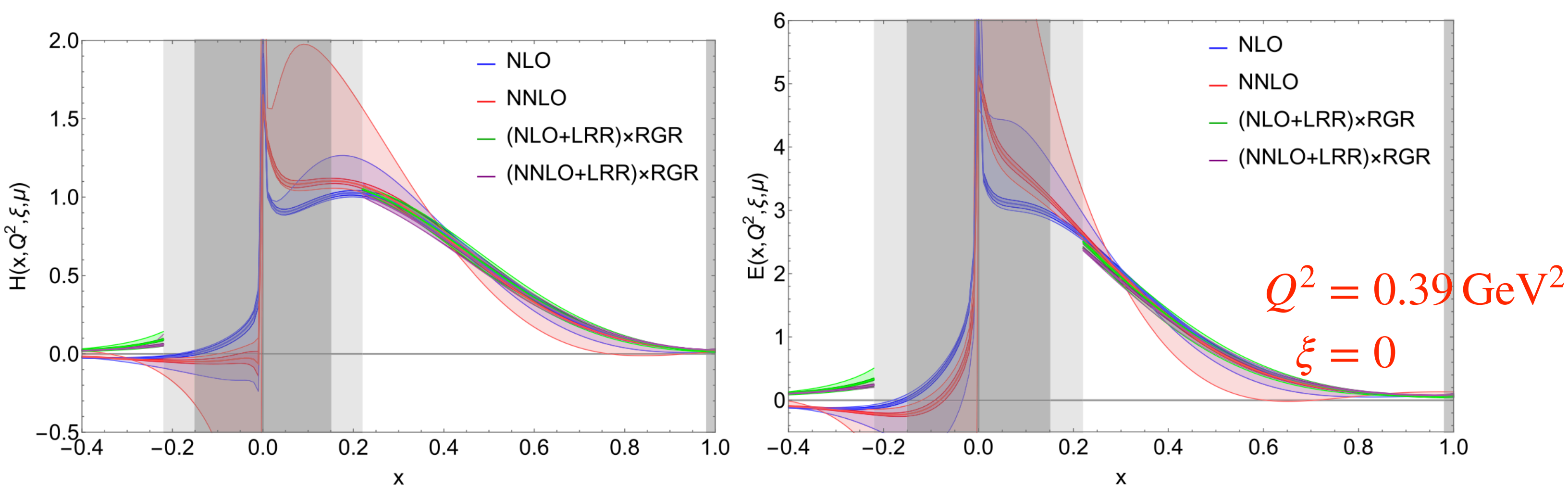
Quark GPDs of proton



Alexandrou et al, PRL 20’



Lin, PRL 21’



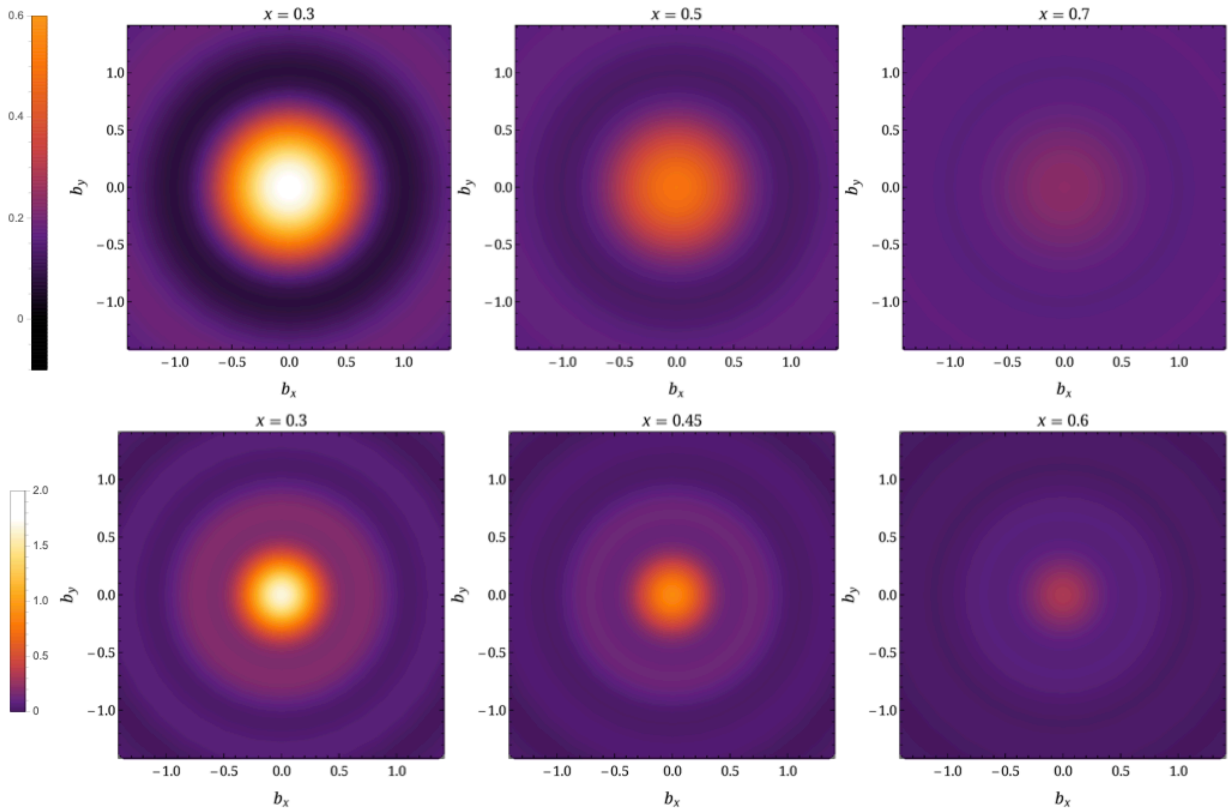
Holligan et al, PRD 24’

Lattice results on GPDs

Quark GPDs of proton

Impact parameter distribution

$$\mathsf{q}(x,b) = \int \frac{d\mathbf{q}}{(2\pi)^2} H(x,\xi=0,t=-\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{b}}$$

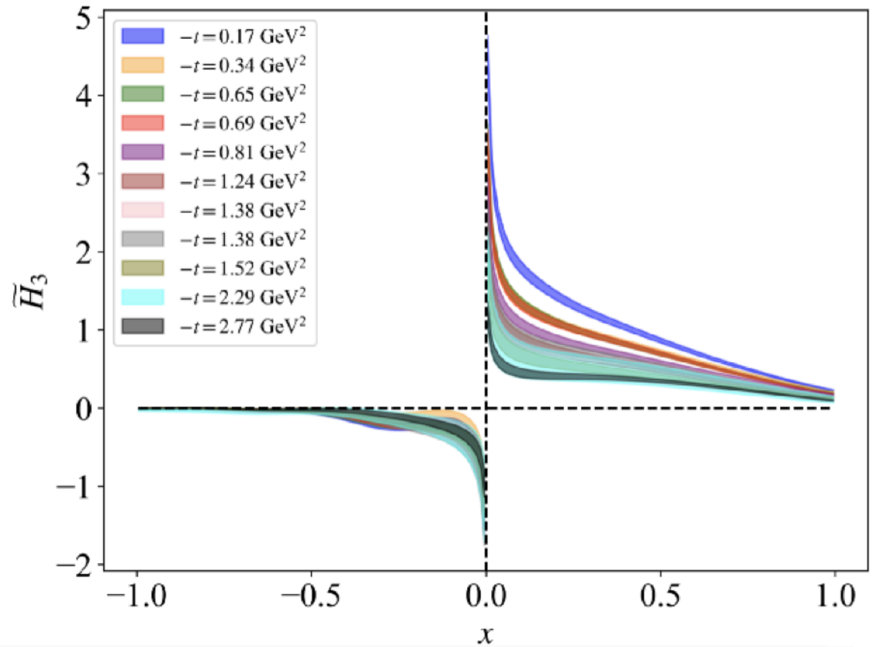


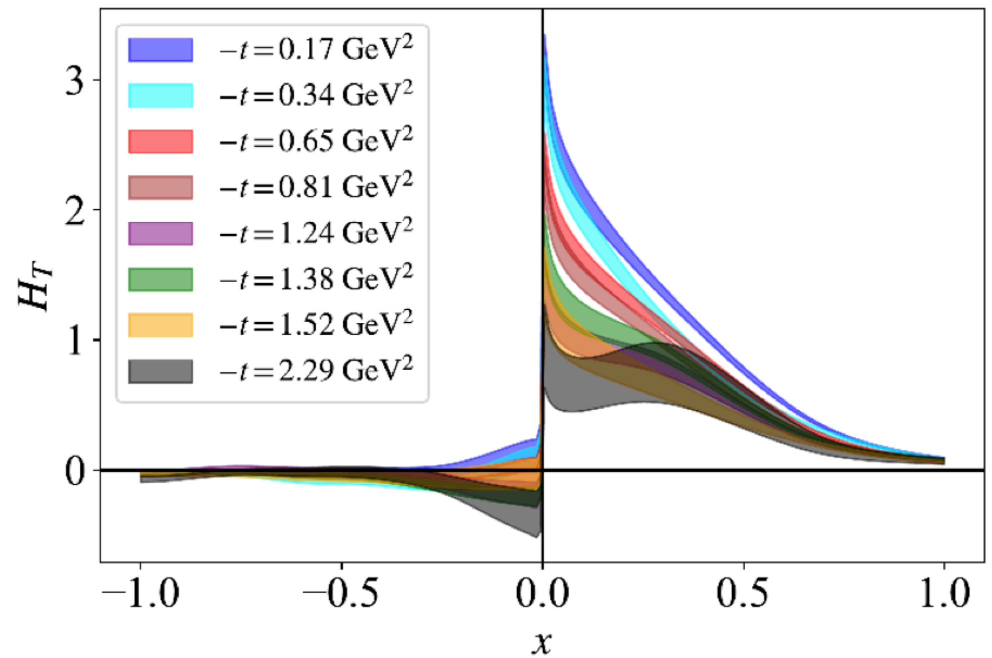
Lin, PRL 21' & PLB 22'

Lattice results on GPDs

Quark GPDs of proton

Results from asymmetric frame



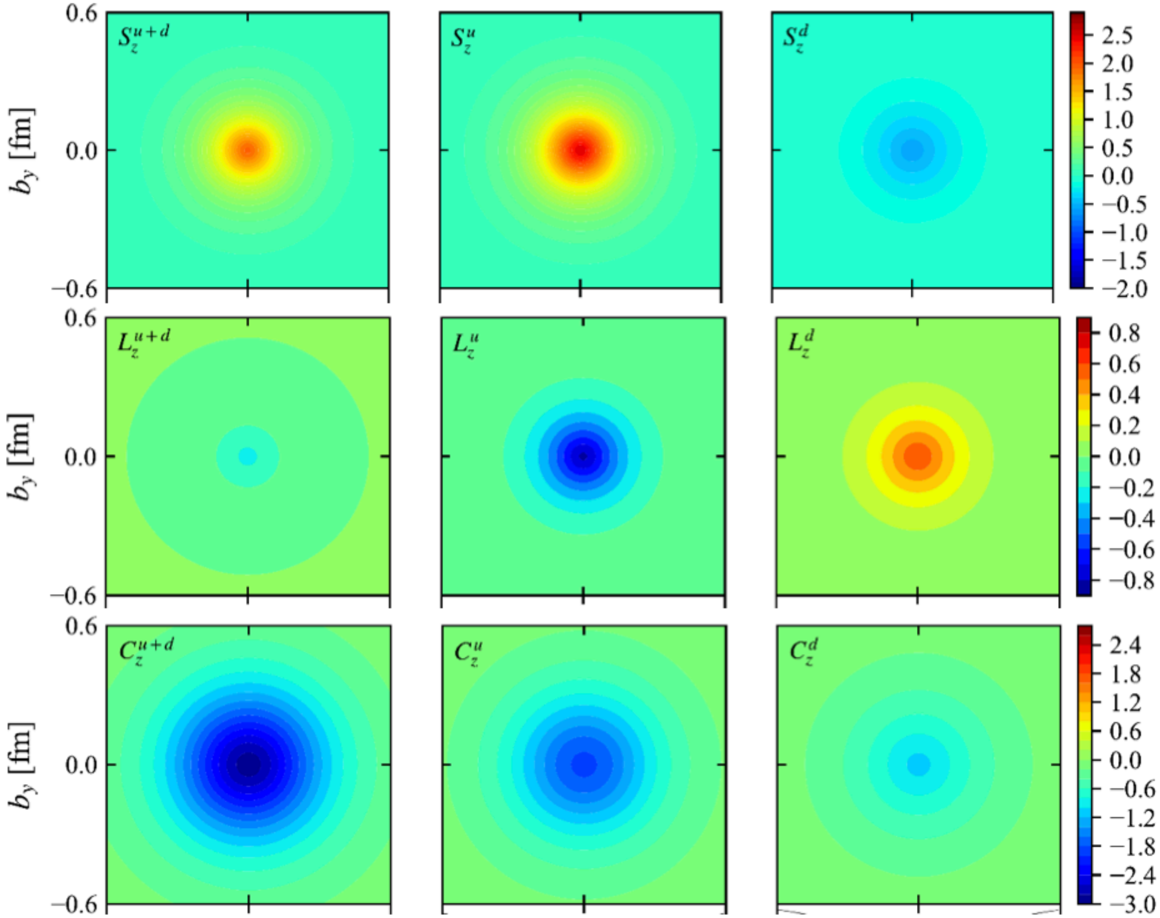


Bhattacharya et al, PRD 22', PRD 24' & PRD 25'

Lattice results on GPDs

Quark GPDs of proton

Results from asymmetric frame: moments from short-distance factorization

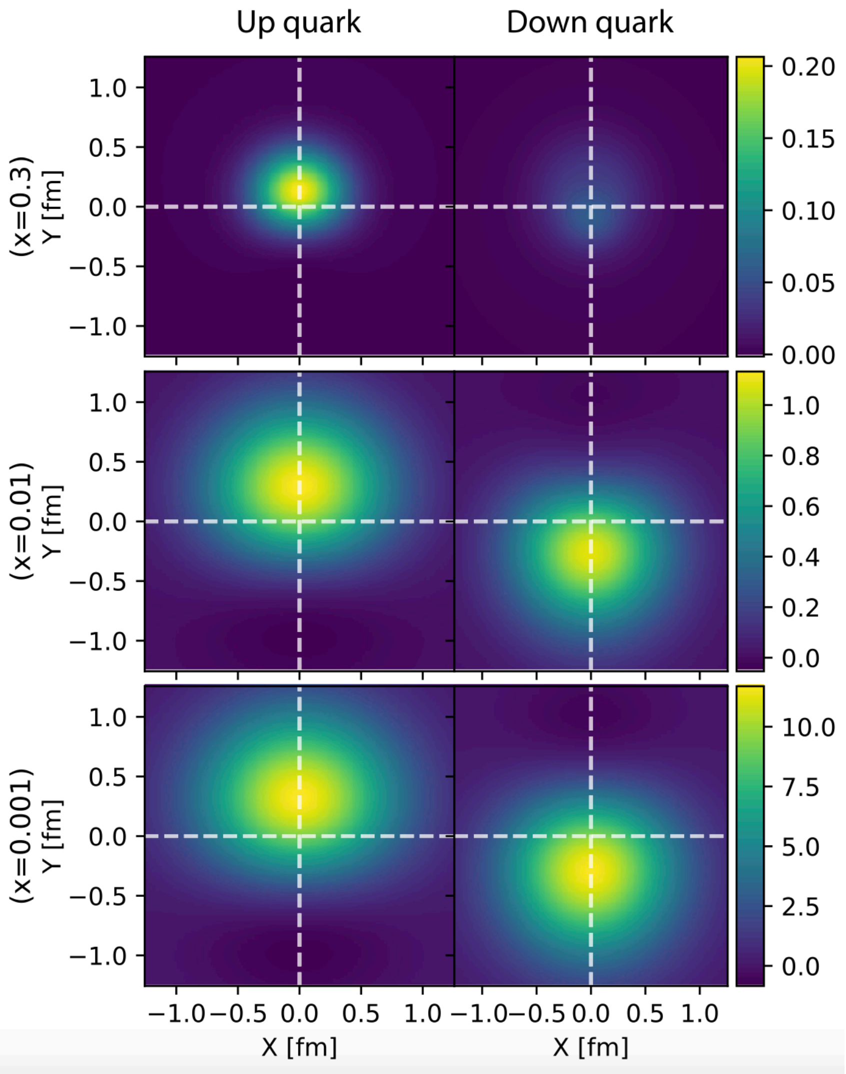


Bhattacharya et al, JHEP 25'

Lattice results on GPDs

Quark GPDs of proton

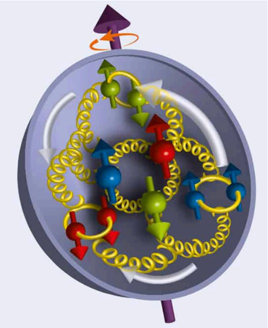
Experimental and lattice inputs combined



See Yuxun Guo's talk

Guo et al, PRL 25'

Proton spin structure



Jaffe-Manohar sum rule Jaffe and Manohar, NPB 90'

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

$$\mathbf{J} = \int d^3\mathbf{x}\left[\psi_f^\dagger \frac{\mathbf{\Sigma}}{2}\psi_f + \psi_f^\dagger(\mathbf{x}\times\frac{1}{i}\nabla)\psi_f + \mathbf{E}_a\times\mathbf{A}_\mathbf{a} + E_a^i(\mathbf{x}\times\nabla)A_a^i\right]$$

- Complete decomposition into quark and gluon spin & OAM
- Gauge-dependent, but with clear partonic interpretation

Ji sum rule Ji, PRL 97'

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

$$\mathbf{J} = \int d^3\mathbf{x}\left[\psi_f^\dagger \frac{\mathbf{\Sigma}}{2}\psi_f + \psi_f^\dagger(\mathbf{x}\times\frac{1}{i}\mathbf{D})\psi_f + \mathbf{x}\times(\mathbf{E}\times\mathbf{B})\right]$$

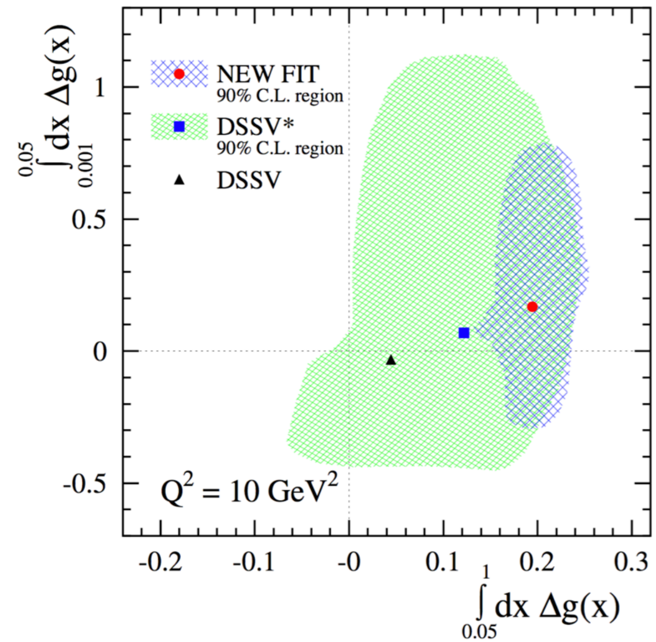
- Frame- and gauge-independent
- Quark and gluon contributions related to the moments of GPDs

Proton spin structure

- The total gluon helicity ΔG can be measured by probing the spin-dependent gluon helicity distribution in polarized high-energy scattering experiments

$$\Delta g(x) = \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \langle PS | F_a^{+\mu}(\xi^-) \mathcal{L}_{ab}(\xi^-, 0) \tilde{F}_{b,\mu}^+(0) | PS \rangle$$

$$\Delta G = \int dx \Delta g(x) \text{ is still } \textbf{nonlocal}$$



- Complicated nonlocal lightcone correlation, and reduces to $\mathbf{E_a} \times \mathbf{A_a}$ in the lightcone gauge $\mathbf{A^+ = 0}$
- Difficult to calculate from theory

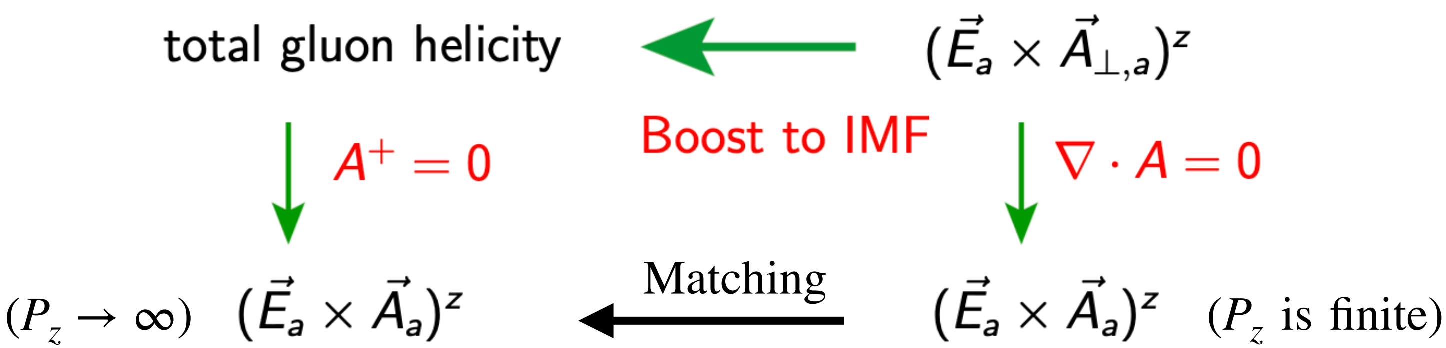
Proton spin structure

- ΔG can be obtained by boosting the matrix element of the static operator $\mathbf{E_a} \times \mathbf{A_{\perp,a}}$ to the infinite momentum frame **Ji, JHZ, Zhao, PRL 13'**
  - $\mathbf{A_\perp}$ takes a nonlocal form in general, but reduces to $\mathbf{A}$ in the Coulomb gauge
  - Coulomb gauge approaches lightcone gauge under large Lorentz boost

- ΔG can be calculated by studying the matrix element

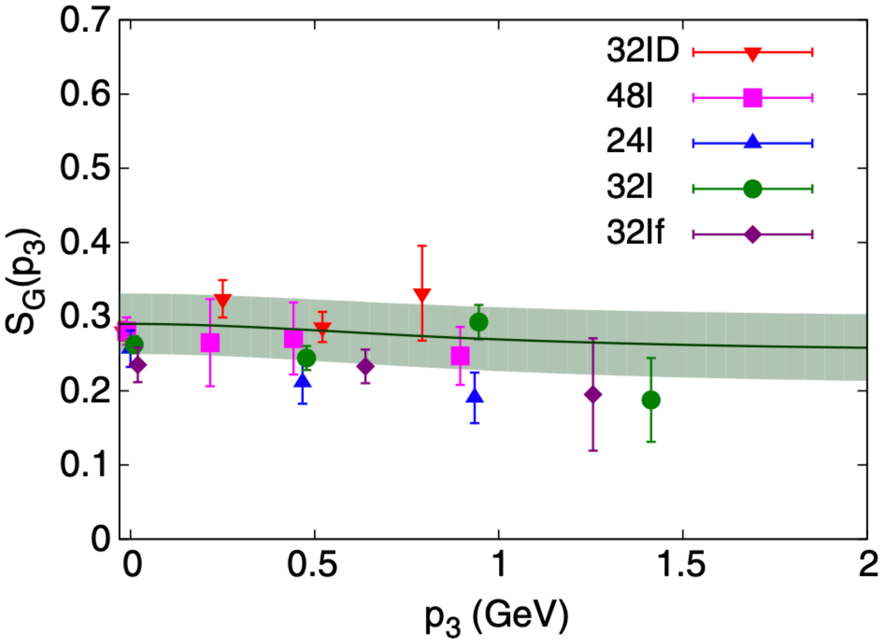
$$\Delta\tilde{G} = < PS|\,\vec{E}\times\vec{A}\,|PS >_{\text{C.G./univ.class}}$$ **Hatta et al, PRD 14'**

in a large momentum nucleon state



Proton spin structure

- Lattice calculation **Yang et al, PRL 17'**



Potential improvements:

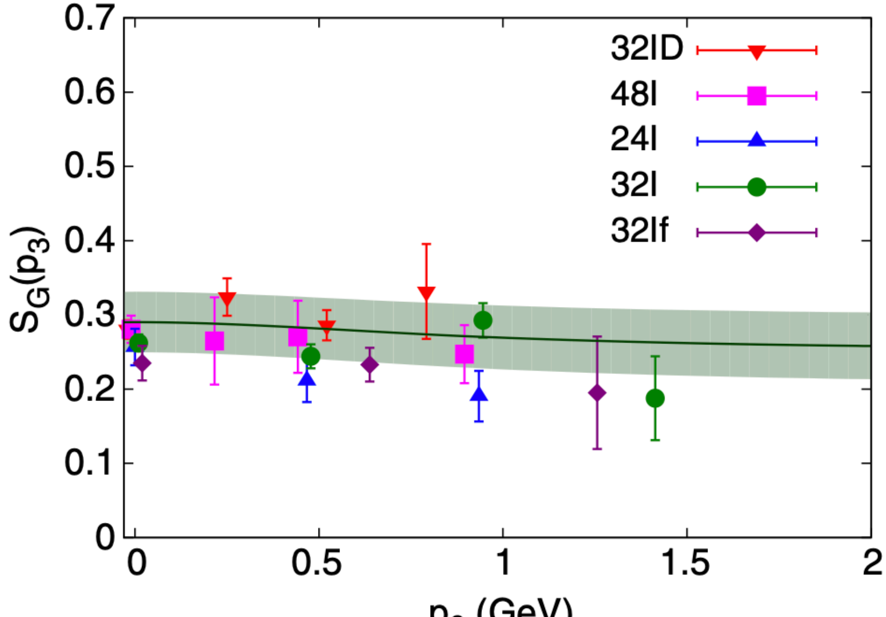
- Nonperturbative renormalization
- Inconsistency between factorizations for ΔG and $\Delta g(x)$

Both issues can be resolved simultaneously for appropriate choices of gluon operators

Pang, Yao, JHZ, JHEP 24'

Proton spin structure

- Lattice calculation Yang et al, PRL 17'



Potential in

- Nonpertur
- Inconsiste

Two approaches to access ΔG:

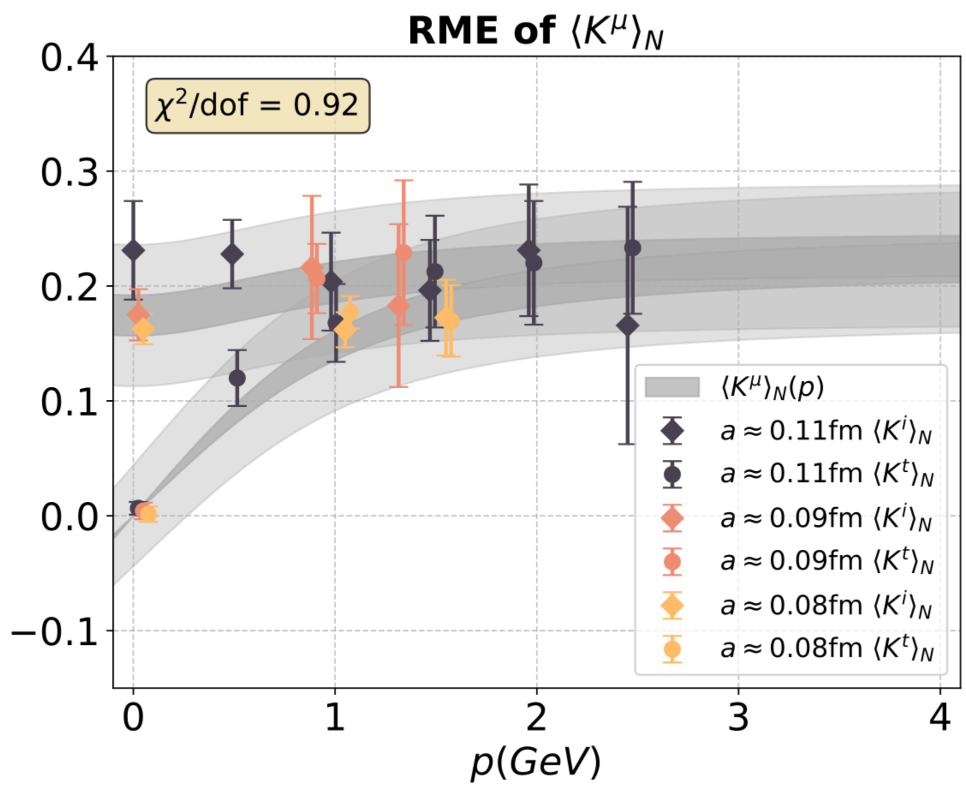
- Calculate the nucleon matrix element of K^0/K^z in Coulomb gauge
- Calculate polarized gluon helicity correlation and integrate over z

Both issues can be resolved simultaneously for appropriate choices of gluon operators

Pang, Yao, JHZ, JHEP 24'

Proton spin structure

- From topological current matrix element in a fixed gauge



See Dian-Jun Zhao's talk

Zhao, JHZ et al (LPC), arXiv: 2512.24315

Summary and outlook

- 3D tomography is an important goal of the EIC/EicC program

- In addition to phenomenological analyses, lattice QCD can provide valuable complementary information on GPDs

- Validation of spin sum rule
  - The total gluon helicity ΔG can be accessed through two different types of approaches on the lattice
    - From local operator matrix element in an appropriately fixed gauge
    - From the nonlocal correlation function defining the gluon helicity distribution $\Delta g(x)$
  - Extension to quark and gluon OAM