香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen

CLQCD

1st Anniversary on EicC

Total Gluon Helicity Contribution to the Proton Spin from Lattice QCD

2026.04.21 Beijing/PM 2:40-3:00

Speaker: Dian-Jun Zhao

DJZ et al., arXiv: 2512.24315

In Collaboration with Long Chen, Hongxin Dong, Xiangdong Ji, Liuming Liu, Zhuoyi Pang, Andreas Schäfer, Peng Sun, Yi-Bo Yang, Jianhui Zhang, Shiyi Zhong

Introduction on spin structure

$$E_{q,g}(x,\xi,t)\,|_{\xi=t=0}$$

Moment of twist-2 GPD

Ji spin decomposition:

$$\frac{1}{2} = J_q^{\text{Ji}} + J_g^{\text{Ji}} = \frac{1}{2}\Big(\int dxx[H_q(x,0,0) + E_q(x,0,0)] + \int dxx[H_g(x,0,0) + E_g(x,0,0)]\Big)$$

Jaffe−Manohar spin decomposition: $\frac{1}{2} = \underbrace{L_q^{\text{JM}} + L_g^{\text{JM}}} + \frac{1}{2}\Delta\Sigma_q + \Delta G$

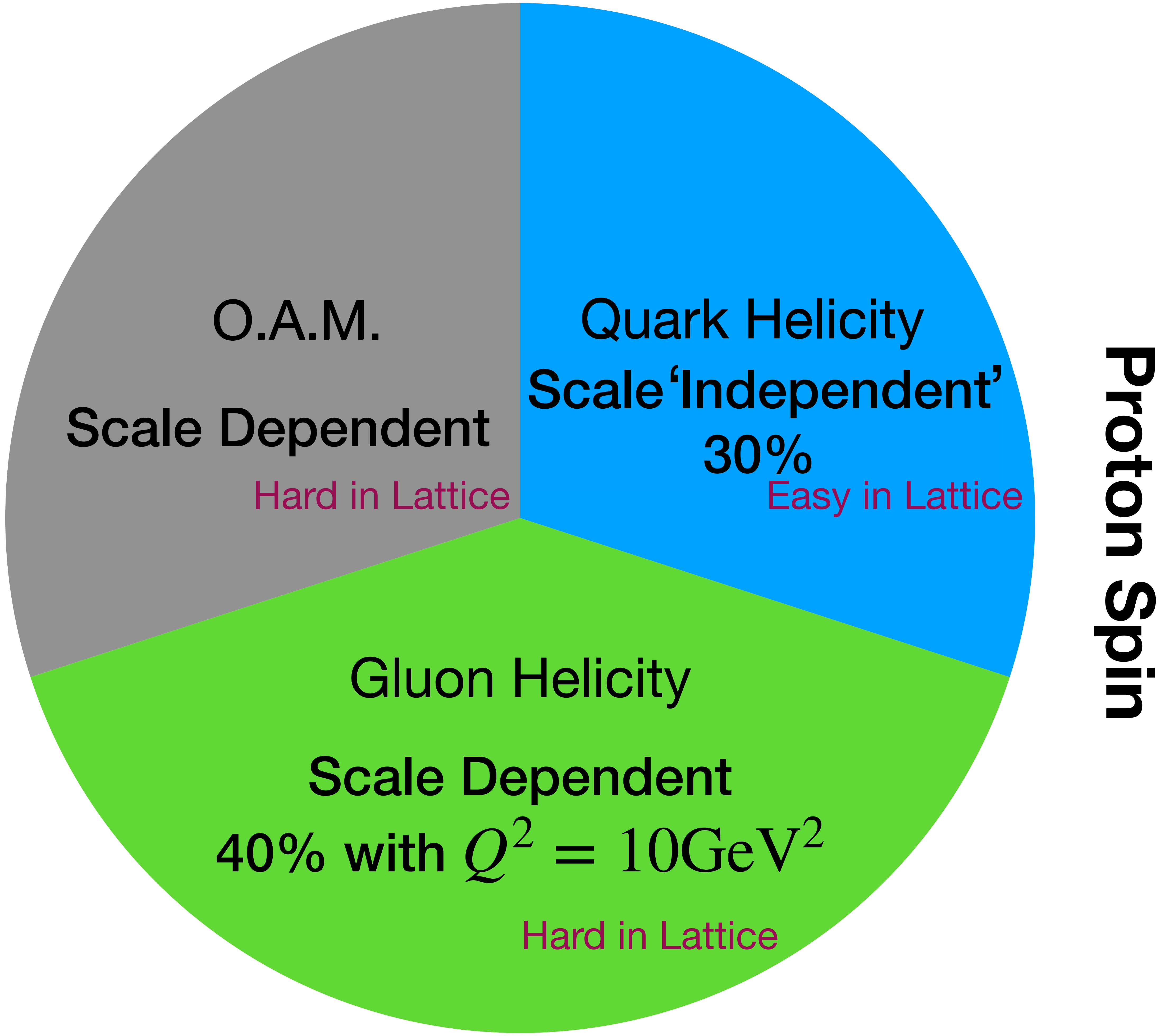
Moment of polarized PDF

Moment of twist-3 GTMD, WD

Helicity can answer 'how much longitudinal spin', but to answer 'where the OAM comes from and what the three-dimensional structure looks like', it must enter **GPD**.

X.-D. Ji, Phys. Rev. Lett. 78, 610 (1997), arXiv:hep-ph/9603249.
R. L. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990).
Y.-B. Yang, arXiv:hep-lat/1904.04138.

Spin structure under Jaffe — Manohar decomposition



Quark helicity in lattice QCD

$$\Delta q = \langle \mathrm{PS} | \bar{q}\gamma_5 \vec{\gamma} \cdot \vec{S} q | \mathrm{PS} \rangle$$

$$\int d^3x (\bar{q}\gamma_\mu\gamma_5 q)(x) = 2m_f \int d^3x x \vec{x} P(x) - 2i \int d^3x x \vec{x} q(x)$$

$$\Delta u = 0.835(15), \quad \Delta d = -0.435(15),$$
$$\Delta s = -0.095(15), \quad \Delta c \simeq 0.00$$

$$\boxed{\Delta\Sigma = \frac{1}{2}\sum \Delta q = 0.155(25)}$$

C. Adolph et al. Phys. Lett., B753:18–28, 2016.
D. Florian et al. Phys. Rev. D, 80:034030, 2009.
E. R. Nocera, et al. Nucl. Phys., B887:276–308, 2014.
D. Florian, et al. Phys. Rev. Lett., 113(1):012001, 2014.

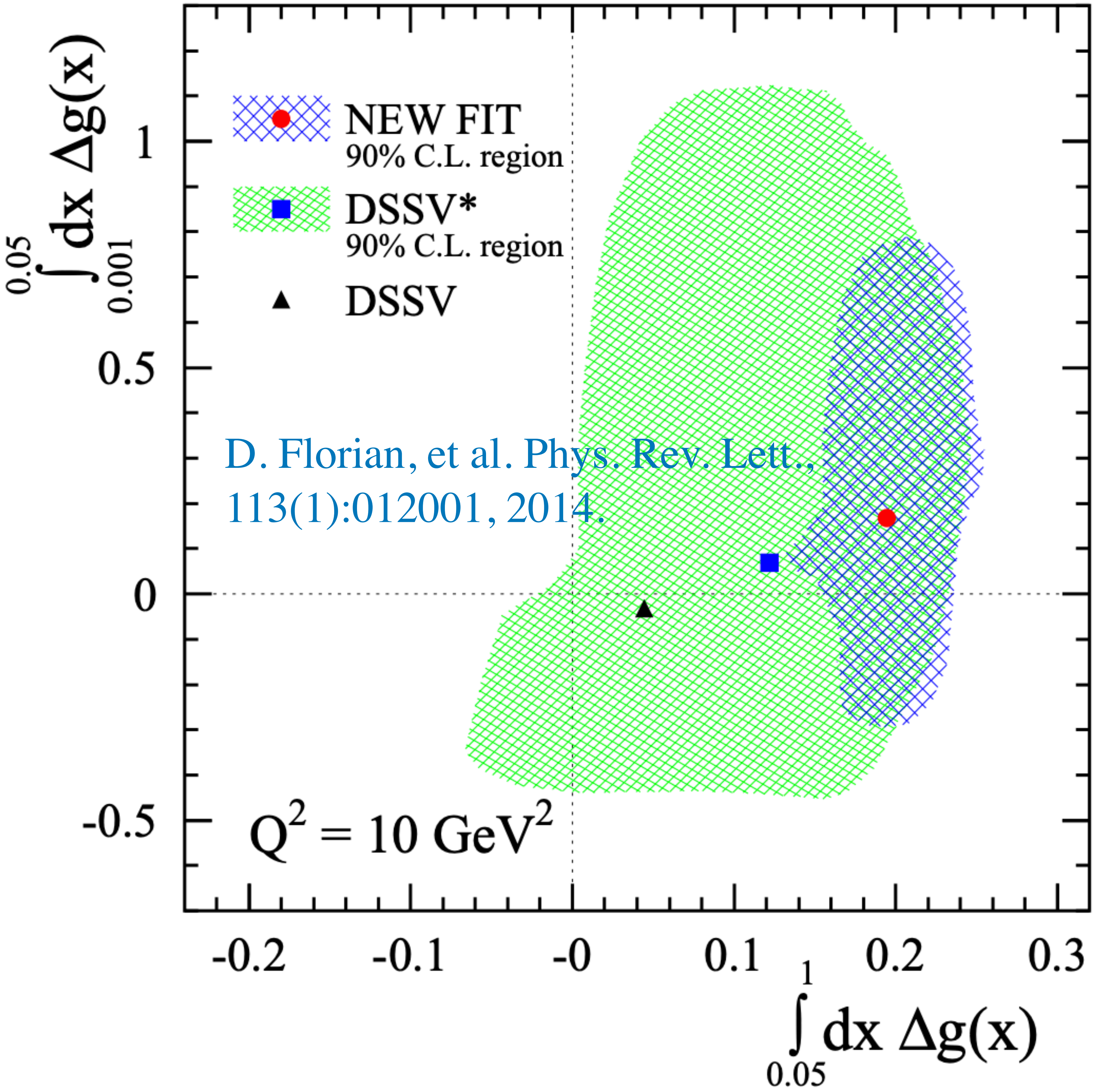
J. Liang, el al. Phys. Rev. D, 98(7):074505, 2018.
H.-W. Lin, et al. Phys. Rev. D, 98:094512, 2018.
C. Alexandrou, et al. Phys. Rev. Lett., 119(14):142002, 2017.
M. Gong, et al. Phys. Rev., D95(11):114509, 2017.

Lattice interpretation of gluon helicity

Gluon helicity in lattice QCD

$$\Delta G = \int dx \Delta g(x) = \int dx \frac{i}{2xP^+} \int \frac{d\varepsilon^-}{2\pi} e^{-ix\varepsilon^- P^+} \langle \mathrm{PS} | F_a^{+\mu}(\varepsilon^-) \mathscr{L}_{ab}(\varepsilon^-, 0) \tilde{F}^+_{b,\mu}(0) | \mathrm{PS} \rangle$$

Integrating LCPDF



ΔG is difficult to calculate in LQCD because light-cone gauge (L.G.) $\not\longleftrightarrow$ Euclidean space!

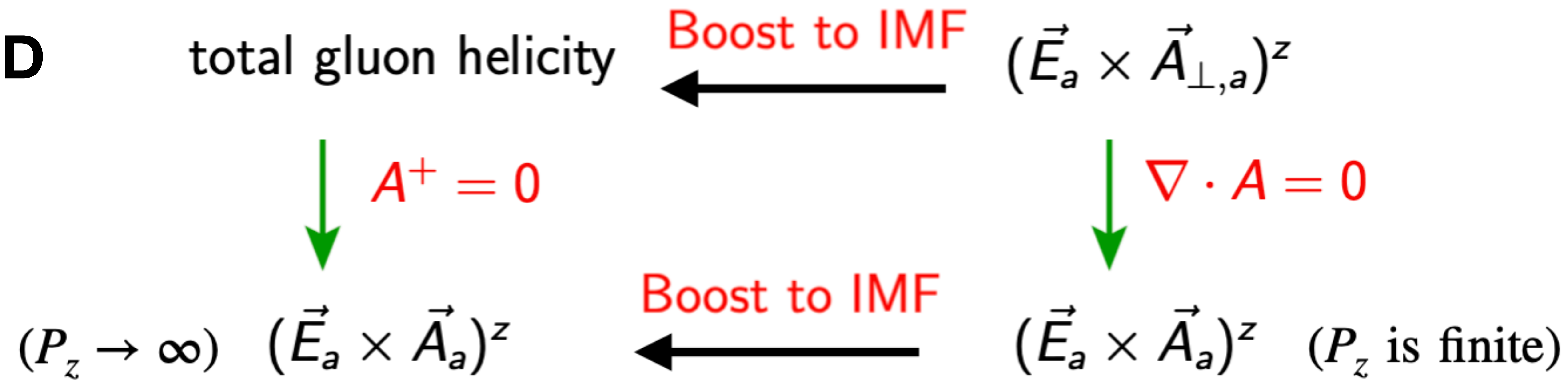
Gauge invariant non-local $\longrightarrow$ Coulomb local

LaMET suggests that Coulomb gauge fixing (C.G.) condition become to L.G. when nucleon to IMF, then $\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\mathrm{C.G.}} = S_G|_{\mathrm{IMF}}$ with matching for their difference in UV behavior.

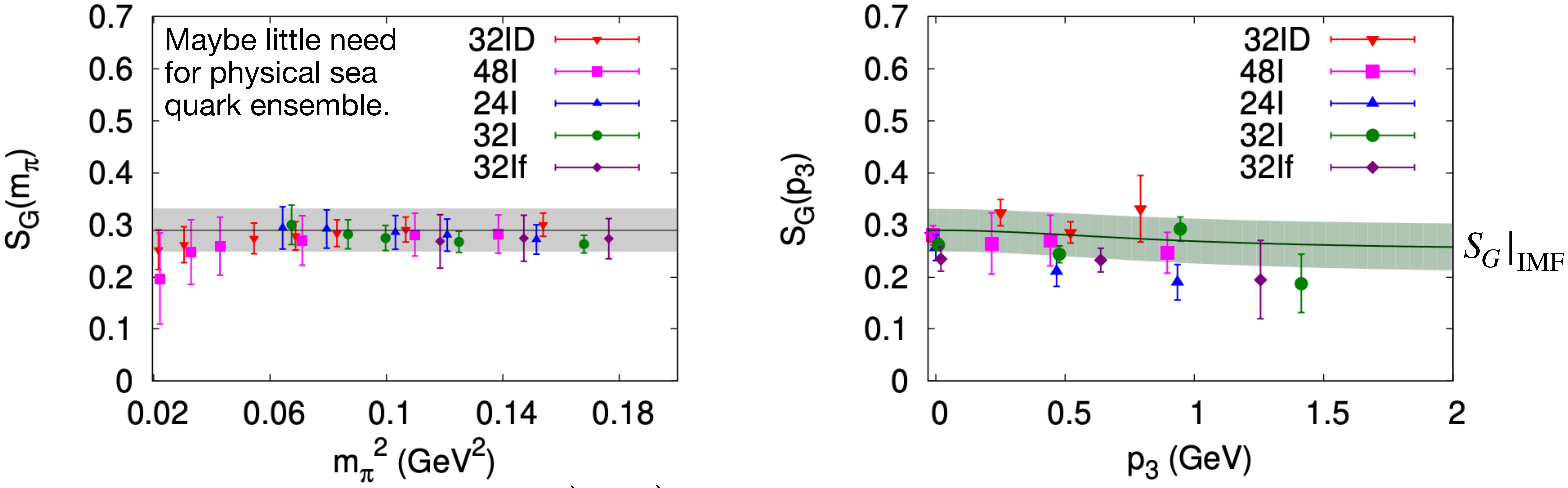
X.-J. Ji et al. 10.1103/PhysRevLett.111.112002

Lattice interpretation of gluon helicity

Gluon helicity in lattice QCD



Y.-B. Yang et al., 10.1103/PhysRevLett.118.102001



$$\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\mathrm{C.G.}} = S_G|_{\mathrm{IMF}} = 0.251(47)(16)$$

Lattice interpretation of gluon helicity

Z.-Y. Pang et al., 10.1007/JHEP07(2024)222

Potential problems with ΔG extraction using $\langle \vec{E} \times \vec{A} \rangle_{\mathrm{C.G.}}$

1. Large momentum external state is not large enough;

2. Should be non-perturbative renormalization + perturbative matching;

3. Inconsistency of $\int \frac{dy}{|y|} C'_{gg}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) \Delta g(y) + \cdots \neq C_{gg} \Delta G + \cdots$.

Intrinsic momentum scale in matching shall be parton momentum, not proton momentum!

Gluon helicity ΔG from topological current

We propose a scheme that relates ΔG to **local topological current** $K^{\mu}_{\mathrm{C.G.}}$. This scheme solves all of these problems and doesn't require lattice perturbative theory.

$$\boxed{\text{Topological Current } K^{\mu}(x) = \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}[A_{\nu} F_{\rho\sigma} - 2ig_s A_{\nu} A_{\rho} A_{\sigma}/3](x)}$$

$$\text{Target Three-PT } \langle \mathrm{PS}_{\mathrm{Proj.i}} | K^{t/i} | \mathrm{PS}_{\mathrm{Proj.i}} \rangle_{\mathrm{C.G.}} \propto S^{t/i} \Delta G + \mathrm{h.t.}$$

Lattice Setup

Z.-C. Hu et al. 10.1103/PhysRevD.109.054507

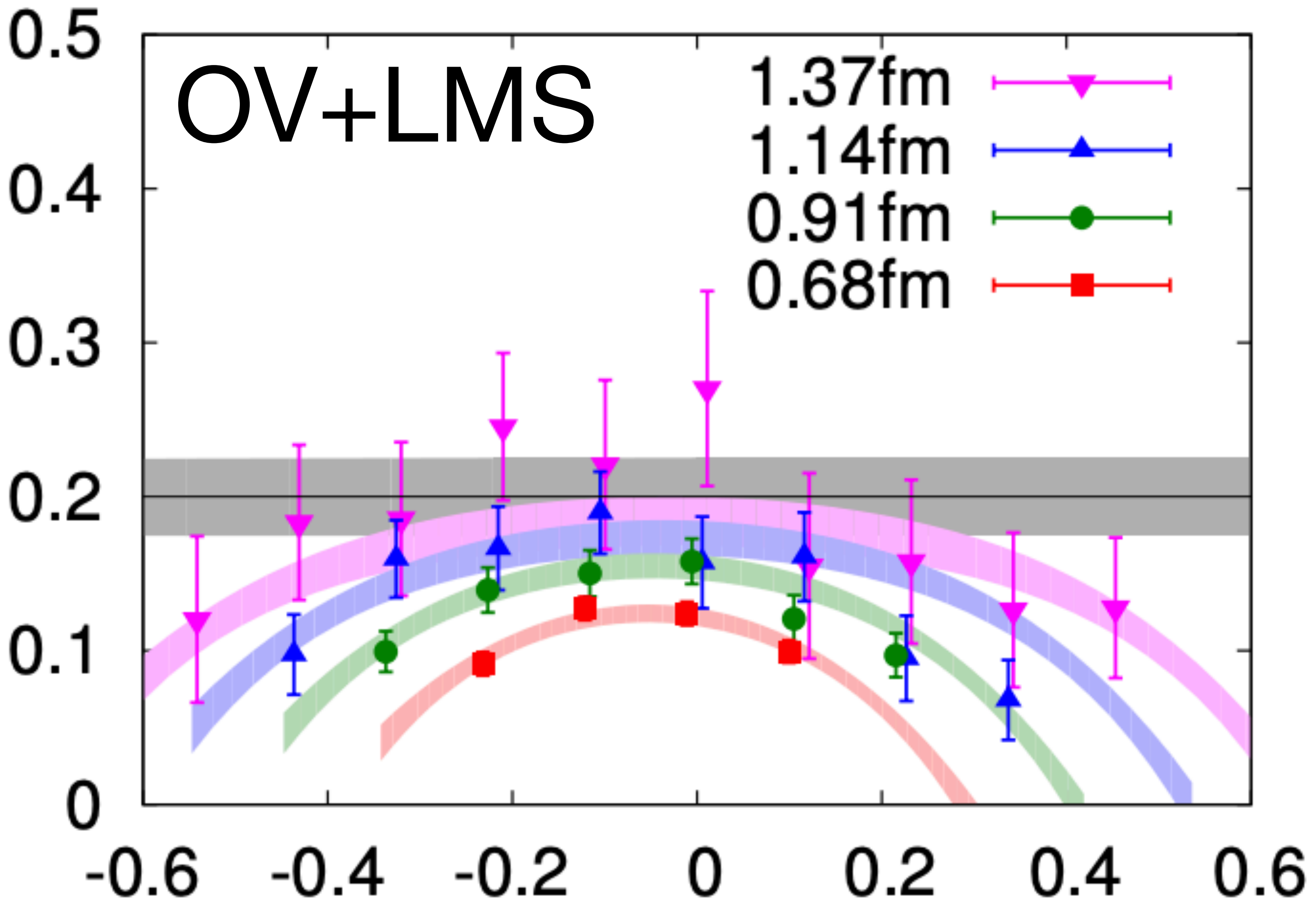
CLQCD	Sea Type	$N_L^3 \times N_T$	Lattice Spacing(fm)	$m_\pi^{(s)}$(MeV)	Valence Type	Ncfg	Nsource
C24P29	Clover (2+1) + TITLS	$24^3 \times 72$	0.1052	292.3(1.0)	Clover	880	72
C48P23		$48^3 \times 96$		224.1(1.2)		380	Volume
E32P29		$32^3 \times 64$	0.0897	287.3(2.5)		890	64
F32P30		$32^3 \times 96$	0.0775	300.4(1.2)		780	96

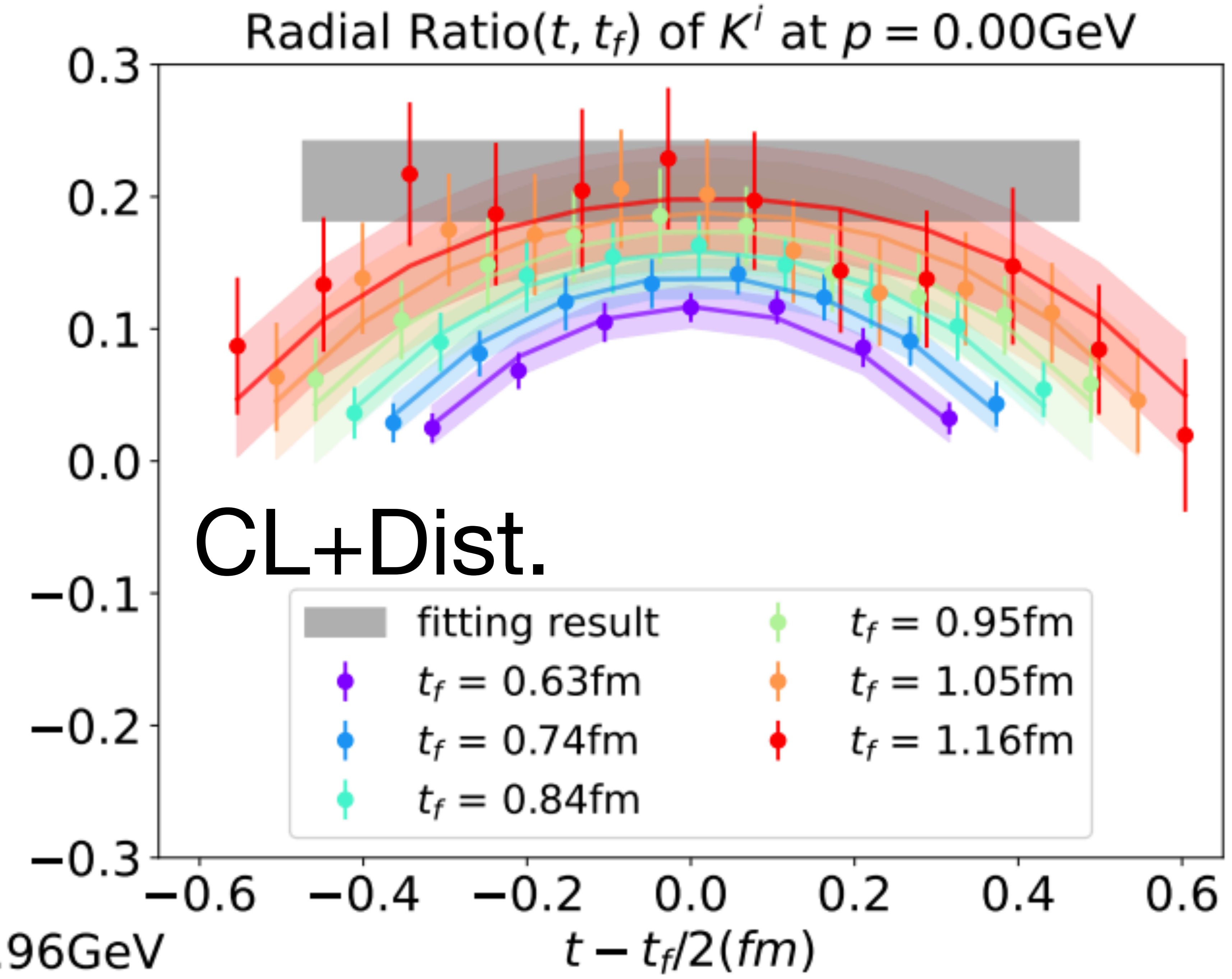
Ensemble used for renormalization has same UV properties as ensemble used for bare matrix element calculations.

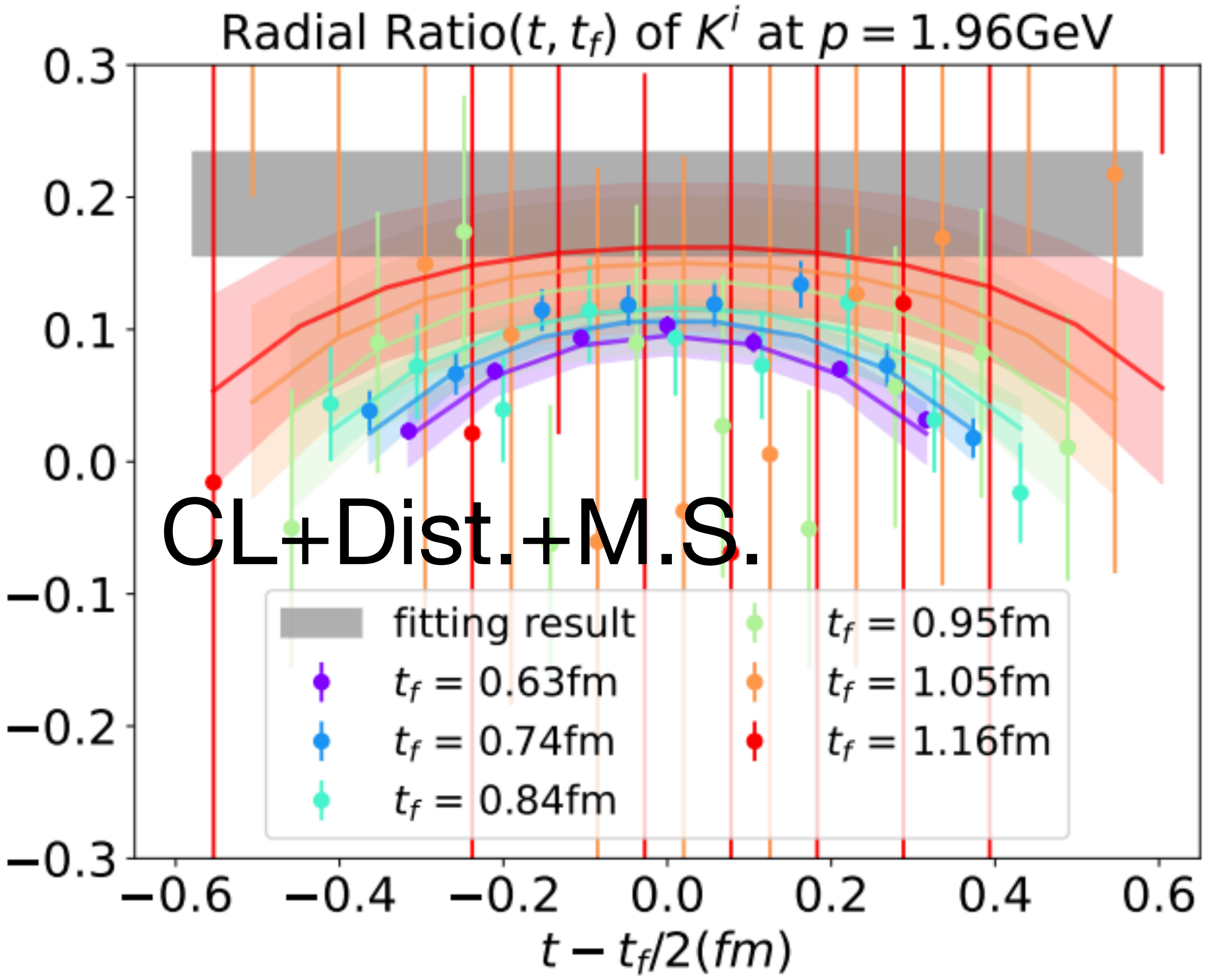
BME with optimization smear methods

a≈0.1fm

Y.-B. Yang et al. [10.1103/PhysRevLett.118.102001](https://doi.org/10.1103/PhysRevLett.118.102001)







D.-J. Zhao et al., arXiv:hep-lat/2512.24315

Appropriate smearing scheme can improve SNR of bare matrix elements.

Fitting Form:

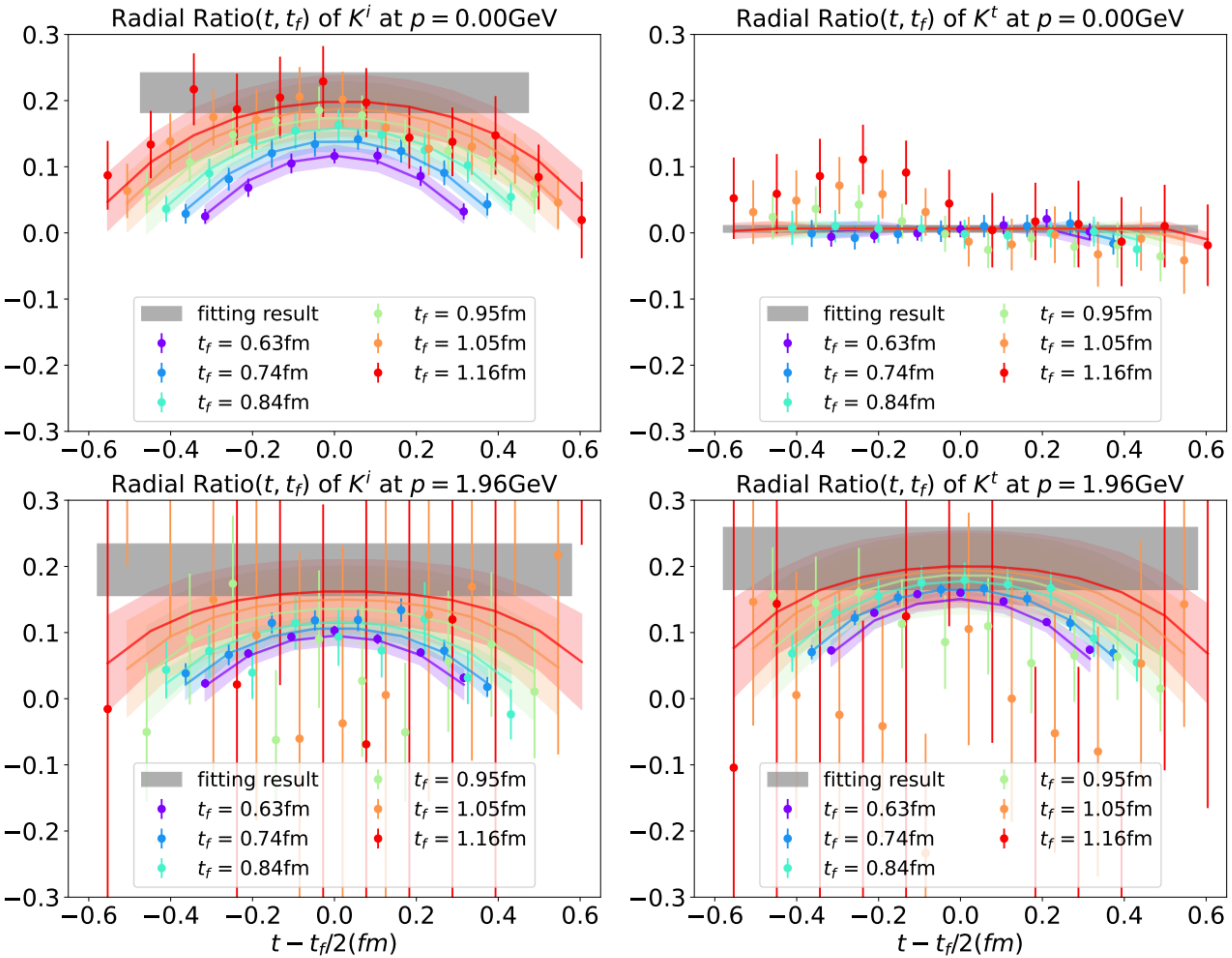
$$R(p;t_f,t_i) = \langle \{K^\mu, E\times A\}\rangle_N^{\mathrm{B.}}(p) + c_1 e^{-\Delta E(t_f-t_i)} + c_2 e^{-\Delta E t_i}$$

Dist. for small p_z, Dist. + M.S. for large p_z.

BME Performance under different momentum

a≈0.1fm

D.-J. Zhao et al., arXiv:hep-lat/2512.24315



Fitting Form:

$$R_{K^\mu}(p; t_f, t_i) = \langle K^\mu \rangle_N^{\text{B.}}(p) + c_1 e^{-\Delta E(t_f - t_i)} + c_2 e^{-\Delta E t_i}$$

Almost all of $\langle K^\mu \rangle$'s signal comes from $\epsilon^{\mu\nu\rho\sigma}Tr[A_\nu F_{\rho\sigma}]$.

Renormalization of topological current K^μ

RI/MOM is non-perturbative renormalization scheme on LQCD.

Z.-Y. Pang et al. 10.1007/JHEP07(2024)222
D.-J. Zhao et al., arXiv:hep-lat/2512.24315

Lattice to RI/MOM

$$\langle PS|\{\overset{\frown}{K^\rho, J^{5\rho}}\}|PS\rangle^{\text{tree.}} = Z^{\text{RI}}_{\{1,2\}1}\langle PS|K^\rho|PS\rangle^{\text{lat.}} + Z^{\text{RI}}_{\{1,2\}2}\langle PS|J^{5\rho}|PS\rangle^{\text{lat.}}$$

Coupled

$$\langle K^\mu\rangle^{\text{RI}}_N = Z^{\text{RI}}_{11}\left(\langle K^\mu\rangle^{\text{B.}}_N - \frac{K^{\text{lat.,q}}}{J_5^{\text{lat.,q}}}\langle J^{5\rho}\rangle^{\text{B.}}_N\right)$$

RG Running

$$R^{3\text{loop}}_{ij}(\mu^{\text{RI}}, \mu = e^{-\frac{5}{6}}\mu^{\text{RI}}) \to R^{3\text{loop}}_{ij}(\mu^{\text{RI}}, \mu = \sqrt{10}\text{GeV})$$

S. J. Brodsky, et al., Phys. Rev. D 28, 228 (1983).

Improve the first few terms' convergence of perturbative series.

RI/MOM to $\overline{\text{MS}}$

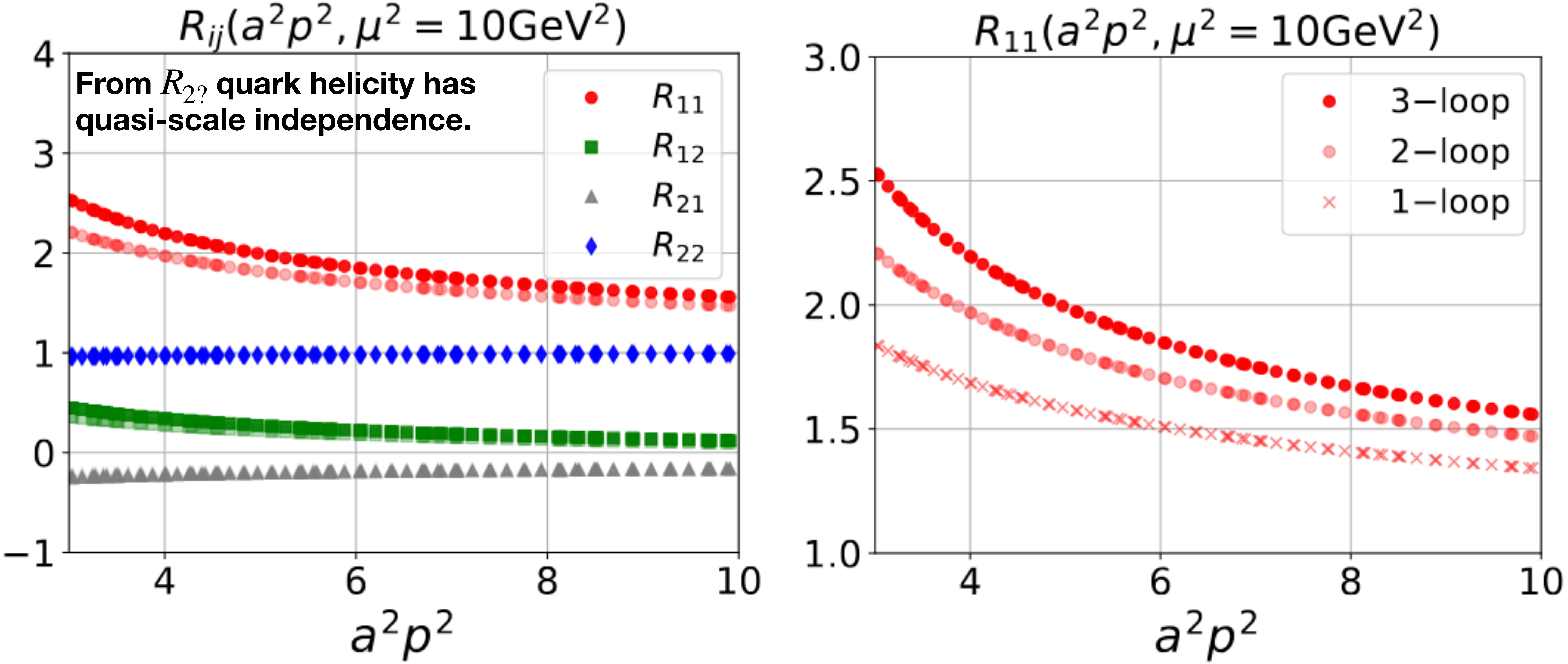
$$\langle K^\mu\rangle^{\overline{\text{MS}}}_N = R^{3\text{loop}}_{11}\langle K^\mu\rangle^{\text{RI}}_N + R^{3\text{loop}}_{12}\langle J^{5\rho}\rangle^{\text{RI}}_N) \approx \bar{Z}^{\text{lat}}_{11}\langle K^\mu\rangle^{\text{B.}}_N$$

$$\langle J^{5\rho}\rangle^{\overline{\text{MS}}}_N \approx \bar{Z}^{\text{lat}}_{22}\langle J^{5\rho}\rangle^{\text{B.}}_N$$

where $\underbrace{\bar{Z}^{\text{lat}}_{11}}_{3} \equiv \underbrace{R^{3\text{loop}}_{11}}_{1}\underbrace{Z^{\text{RI}}_{11}}_{2}$

Convergence of perturbation matching

$$R_{ij}(\mu^{\mathrm{RI}}, \mu = e^{-\frac{5}{6}}\mu^{\mathrm{RI}}) \rightarrow R_{ij}(\mu^{\mathrm{RI}}, \mu = \sqrt{10}\mathrm{GeV})$$



The convergence of perturbation matching is not quite good, and higher loop correction may be required in future calculations.

CDER and a^2p^2 poly-subtraction of $\bar{Z}_{11}^{\text{lat}}$

a≈0.1fm

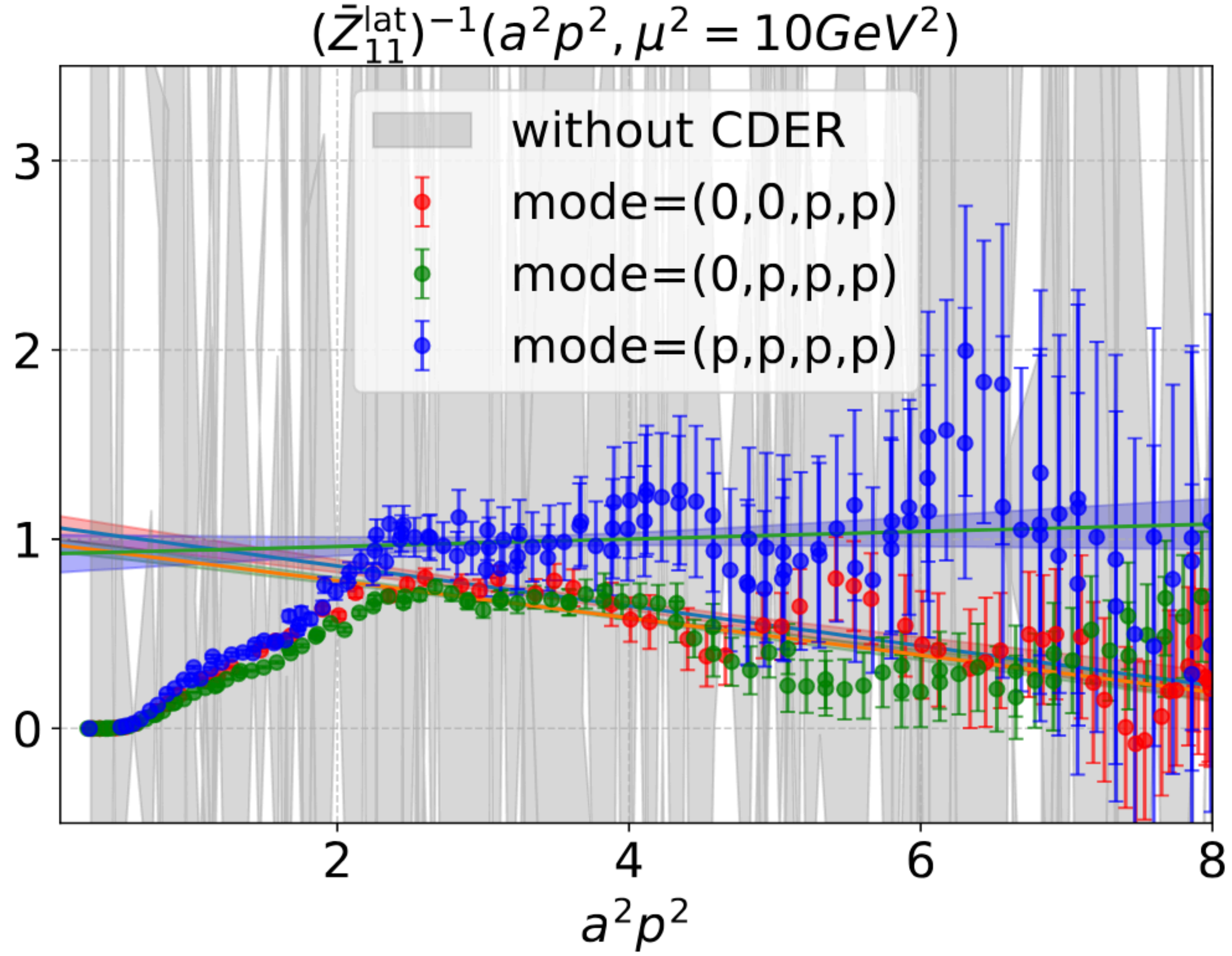
$$Z_{11}^{\text{RI}}(\mu^{\text{RI}}) = \frac{Z_g(\mu^{\text{RI}})}{\text{Tr}[S_g^{-1}(p_1)G_K(p_1,p_2)S_g^{-1}(p_2)(K^{\text{tree,g}})^{-1}]}$$

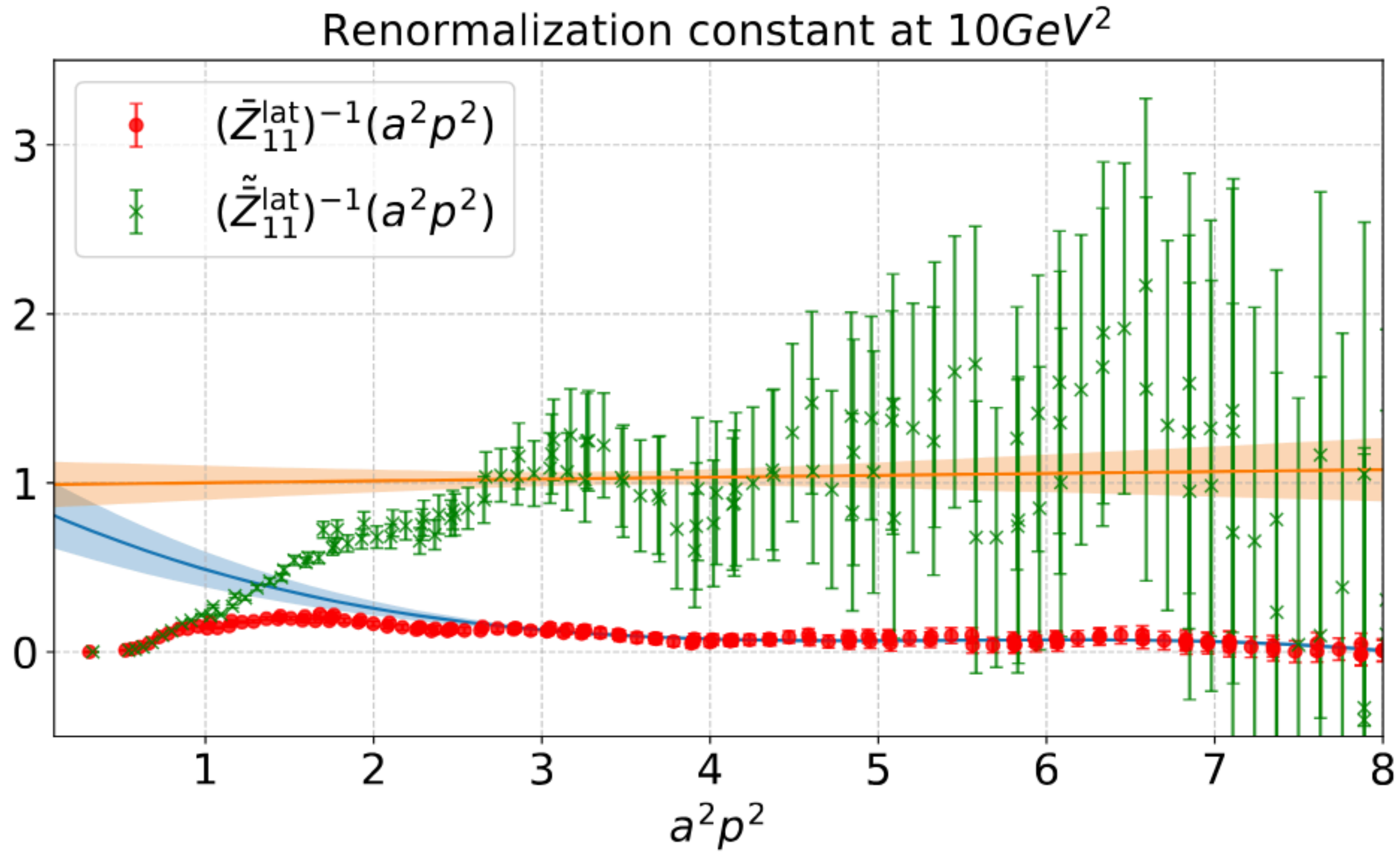
Z_{11}^{RI} contributed by gluon Green functions is difficult to see signal, so we use **CDER** scheme to enhance SNR.

Y.-B. Yang et al., 10.1103/PhysRevD.98.074506
Y.-B. Yang, et al., Phys. Rev. Lett. 121, 212001

$$\tilde{\bar{Z}}_{11}^{\text{lat.}}(\mu^2, a^2p^2) = \bar{Z}_{11}^{\text{lat.}}(\mu^2, a^2p^2)f(a^2p^2 \to 0)/f(a^2p^2)$$

where $f(a^2p^2) = \dfrac{\langle \text{Tr}[A_\mu^{n-\text{HYP}}(p)A_\mu^{n-\text{HYP}}(-p)]\rangle}{\langle \text{Tr}[A_\mu(p)A_\mu(-p)]\rangle}$



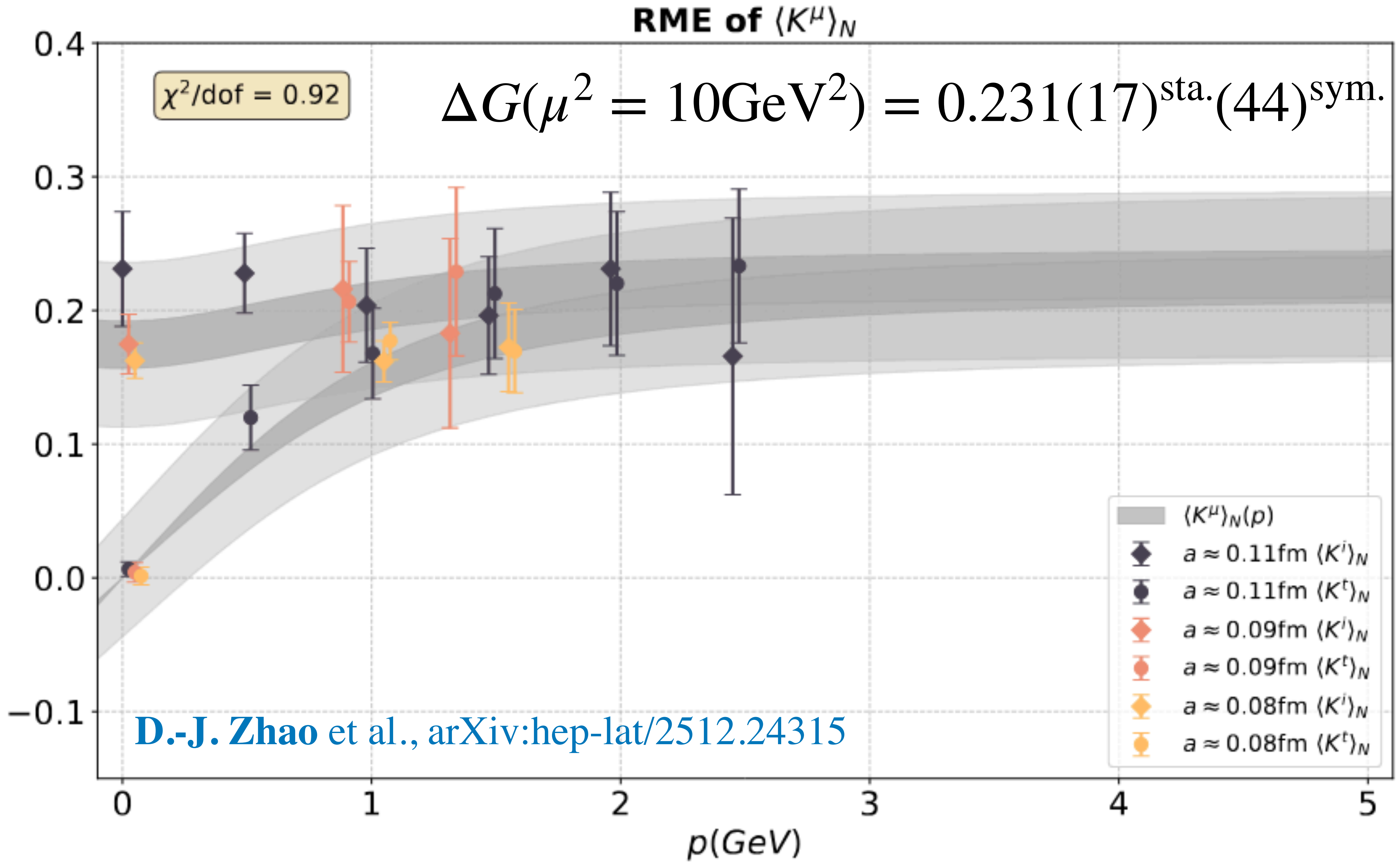


Continuous limit extrapolation

$$\langle K^{\mu}\rangle_N^{\overline{\text{MS}}} = \bar{Z}_{11}^{\text{lat.}}\langle K^{\mu}\rangle_N^{\text{B.}} = \frac{S^{\mu}}{E}\left(\Delta G + \frac{c_{\text{h.t.}}}{E^2}\right) + c_a a^2$$

Systematic error sources:

1. Zero-momentum external state data points
2. Finite lattice spacing
3. Choice of the UV window in the RI/MOM scale
4. Perturbative inputs



If we really take '**spin structure**' as the goal instead of '**helicity**' as the endpoint, then the next step is not another helicity observable, but **GPD**.

Summary and Outlook

Summary

1. **Distillation + Momentum smear** (for B.M.E.) and **CDER** (for Renorm.) scheme.

2. After non-perturbative renormalization, $\Delta G(\mu^2 = 10\mathrm{GeV}^2) = 0.231(17)^{\mathrm{sta.}}(44)^{\mathrm{sym.}}$, which accounts for $46(9)\ \%$ of proton spin.

Outlook

1. Need to calculate dependence on off-diagonal renormalization.

2. Lattice methodology for O.A.M calculations is required to give accurate results for proton spin components.

3. Combine $\Delta\Sigma$ using Blending to provide octet spin decomposition.

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