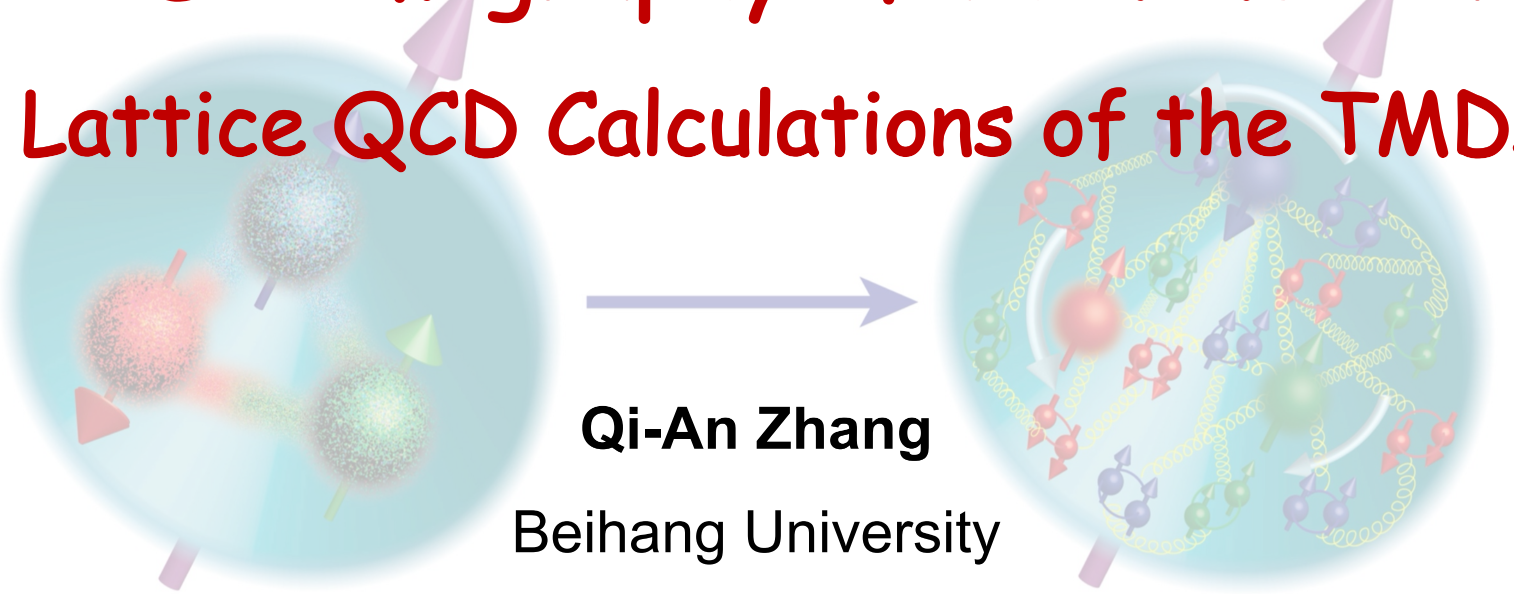
Lattice Parton Collaboration

北京航空航天大学 BEIHANG UNIVERSITY

3D Tomography of the Nucleon

Lattice QCD Calculations of the TMDs



Qi-An Zhang

Beihang University

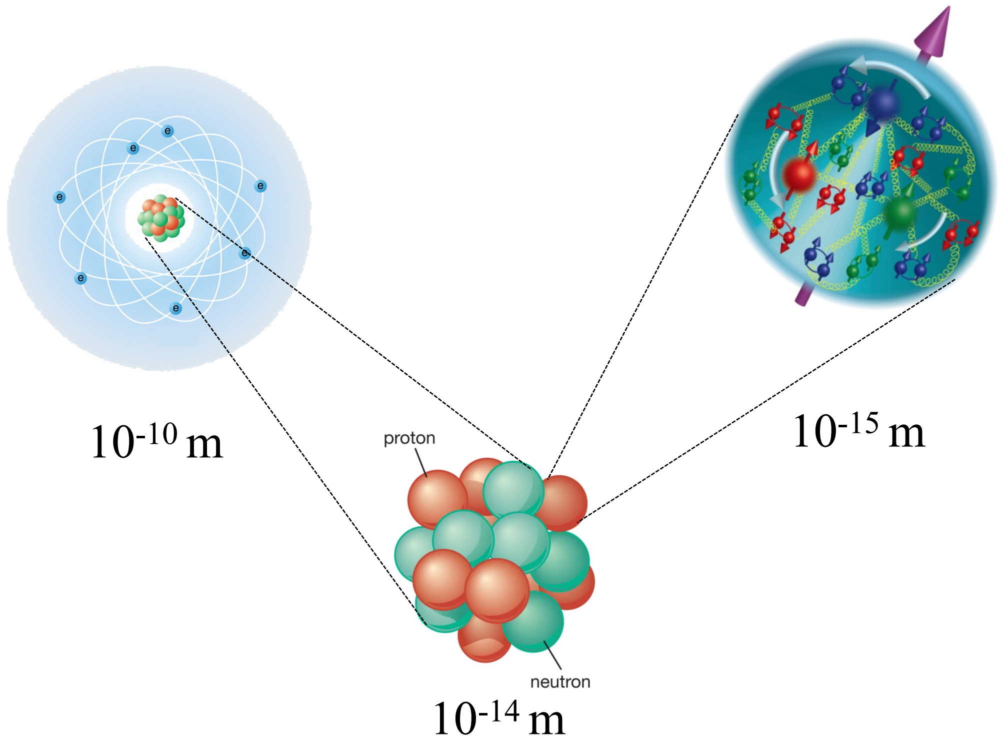
On behalf of Lattice Parton Collaboration (LPC)

Apr. 21, 2026 @ 第一届中国电子离子对撞机相关物理年会, 青岛

Based on ***PRD109, 114513 (2024); PRD111, 094507 (2025); JHEP08, 086 (2025); PRD113, 054505 (2026)***

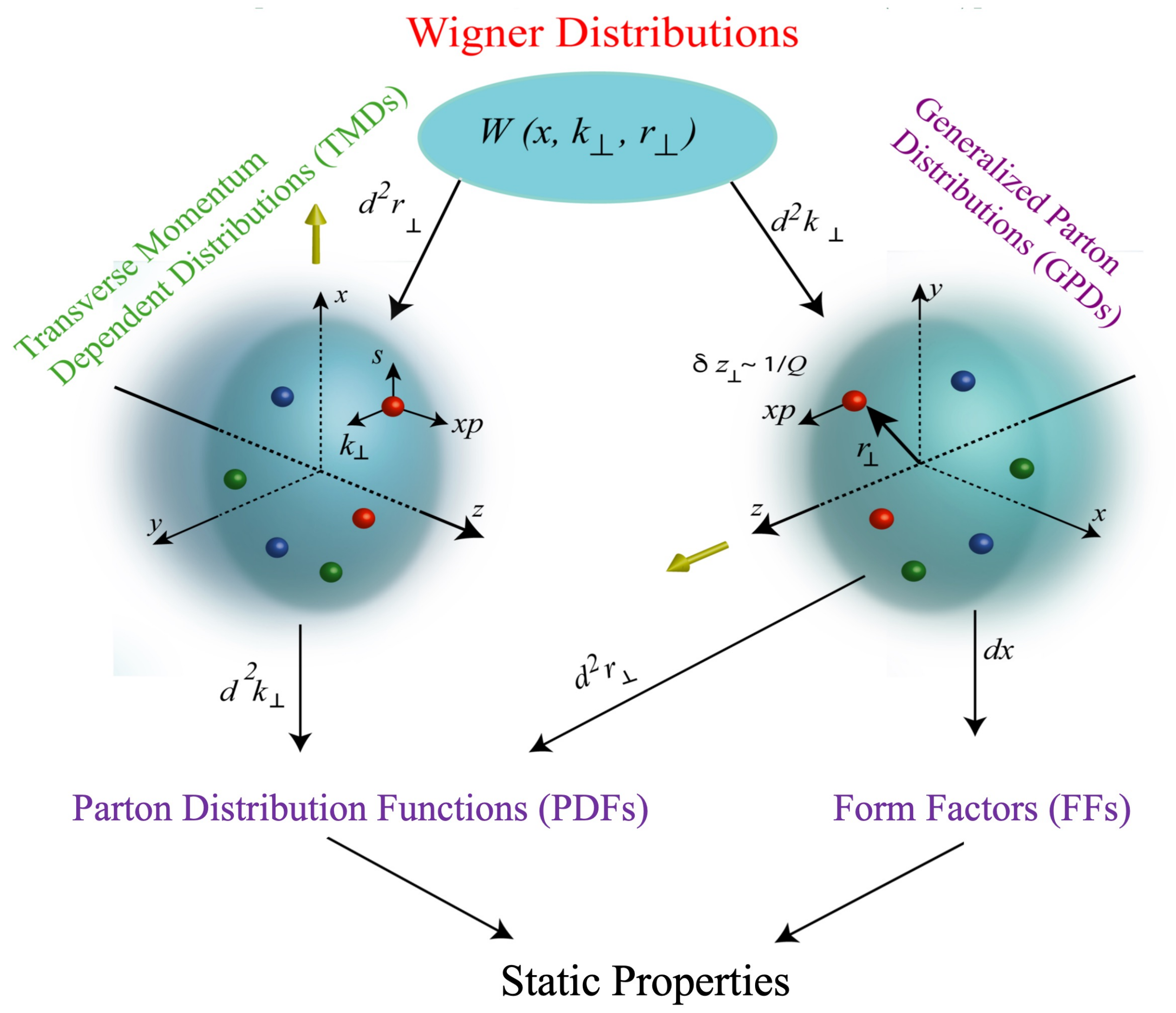
Nucleons: Building Blocks of the Visible Universe

Despite more than half a century of exploration, the nucleon is still a very mysterious object, and the most abundant piece of matter in the visible Universe



- How does quarks & gluons forms nucleon, how do they distribute?
- How does nucleon mass & spin emerge from strong interactions?

The Landscape of Nucleon Structure



- Wigner distribution (5-D)

  Not directly accessible in experiments

- TMDs and GPDs (3-D)

  Still limited in experiments, with relatively low precision

- PDFs and FFs (1-D)

  Precisely measured over a broad kinematic range

- Spins, charge, magnetic moment, charge radius, … (0-D)

  Have been determined with high preciseion

The Landscape of Nucleon Structure

- Wigner distribution (5-D)

Not directly accessible in experiments

- TMDs and GPDs (3-D)

Still limited in experiments, with relatively low precision

and FFs (1-D)

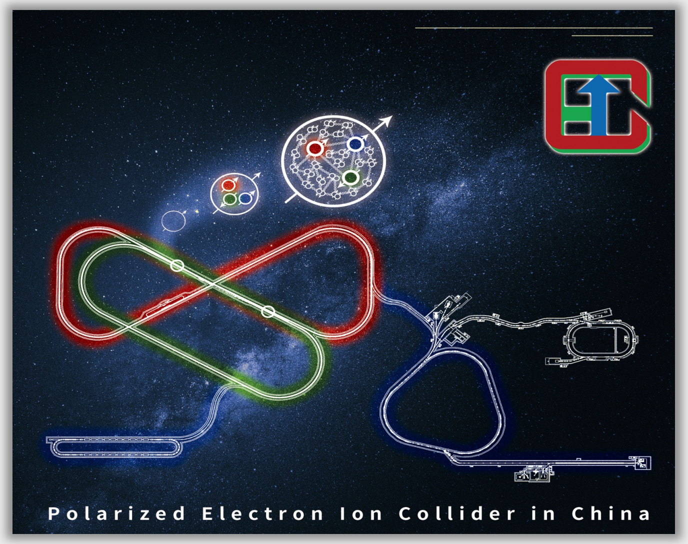
isely measured over a broad kinematic range

, charge, magnetic moment, charge radius, …

Have been determined with high preciseion

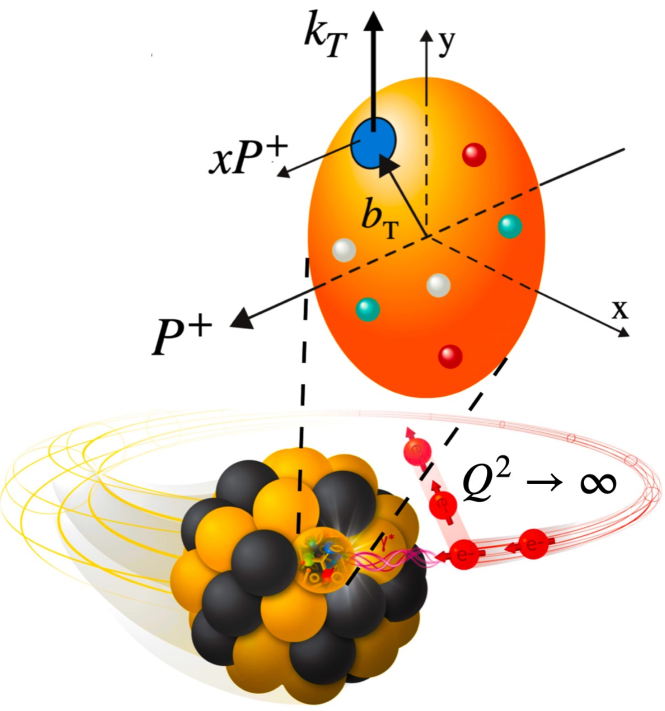
The Next Experiment Frontier

EicC (China)



EIC (US)

TMDs: 3D Momentum Tomography of Nucleon



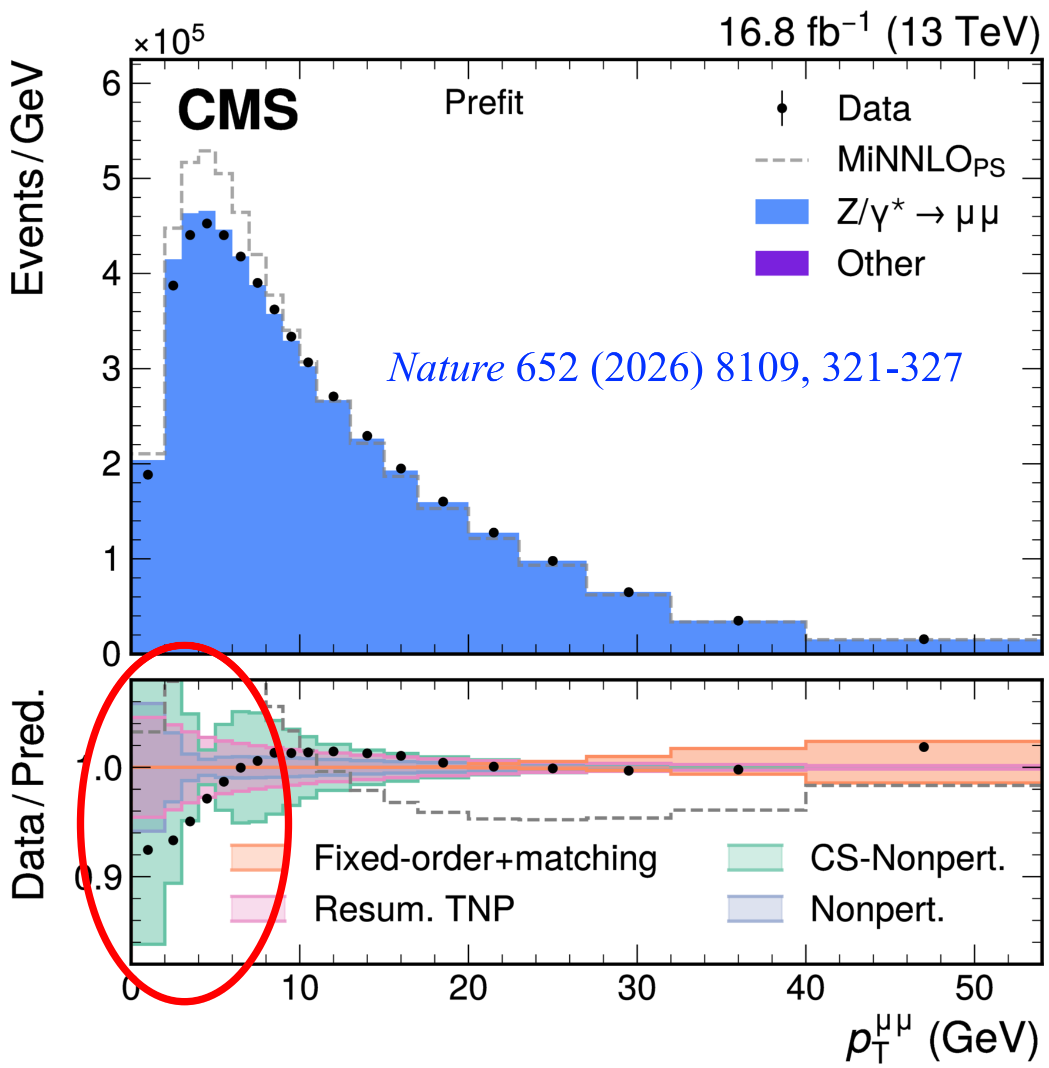
TMDs: Spin-dependent 3D momentum space images from semi-inclusive scattering

Leading Quark TMDPDFs

Nucleon Spin — Quark Spin

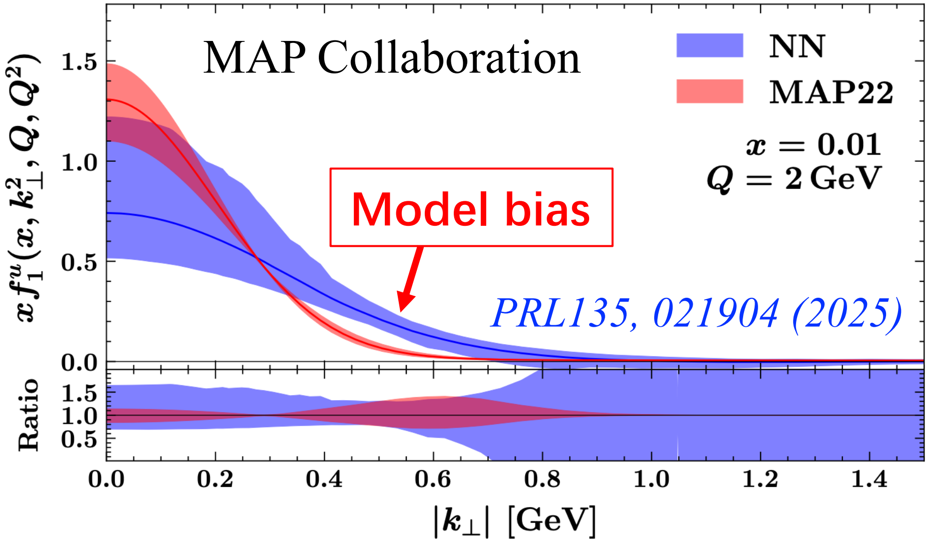
Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity; $h_{1T}^\perp$ = − Pretzelosity

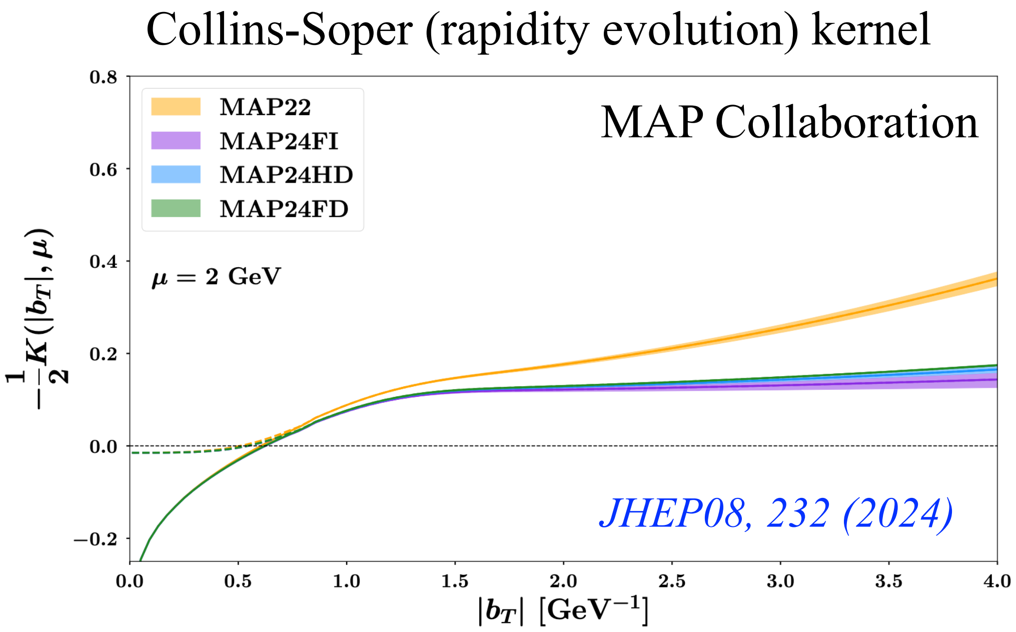
TMDs at the Precision Frontier

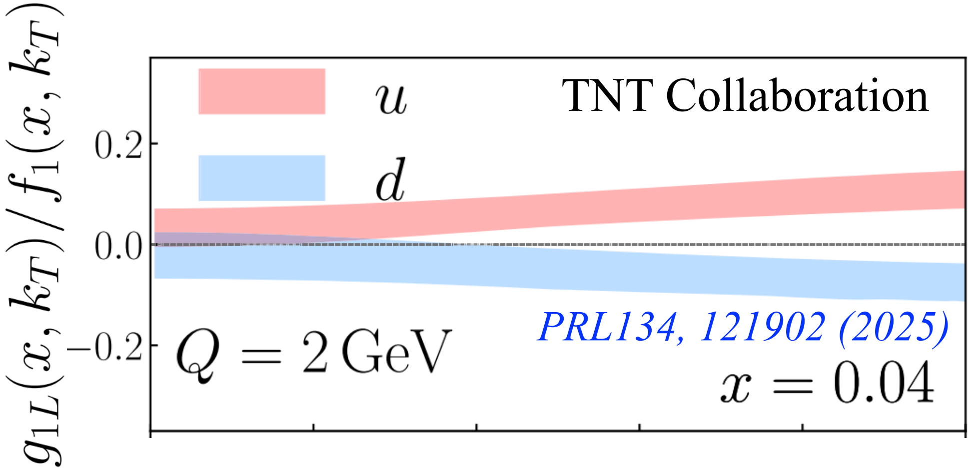


- TMD uncertainties dominate, which can cause an $\mathcal{O}$(10MeV) measurement bias in m_W.
- Precision frontier: hadronic systematics are becoming the limiting uncertainty.
- Lattice QCD provides systematically improvable calculations directly from QCD.

TMDs from Global Analysis of Experimental Data

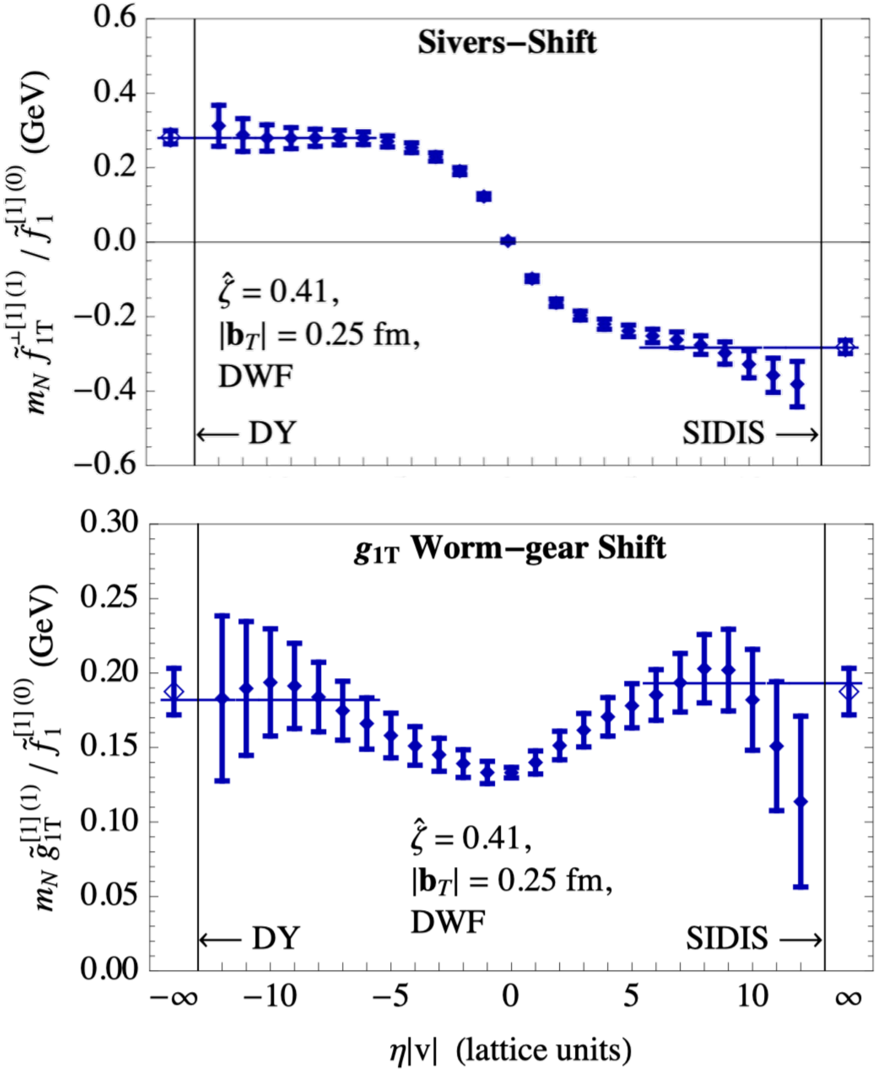


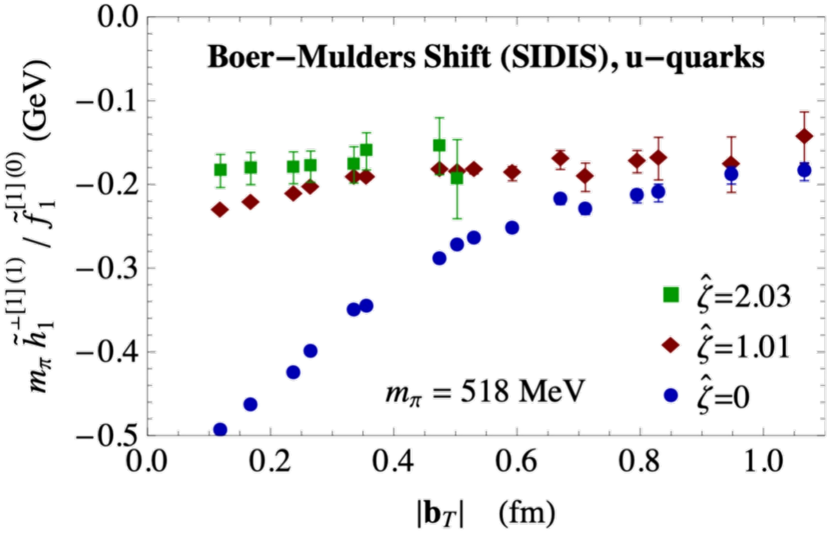




- Global fit is limited by sparse data.
- Rely on model assumptions, and produce noticeable inconsistencies.

TMDs from Lattice: OPE Moments



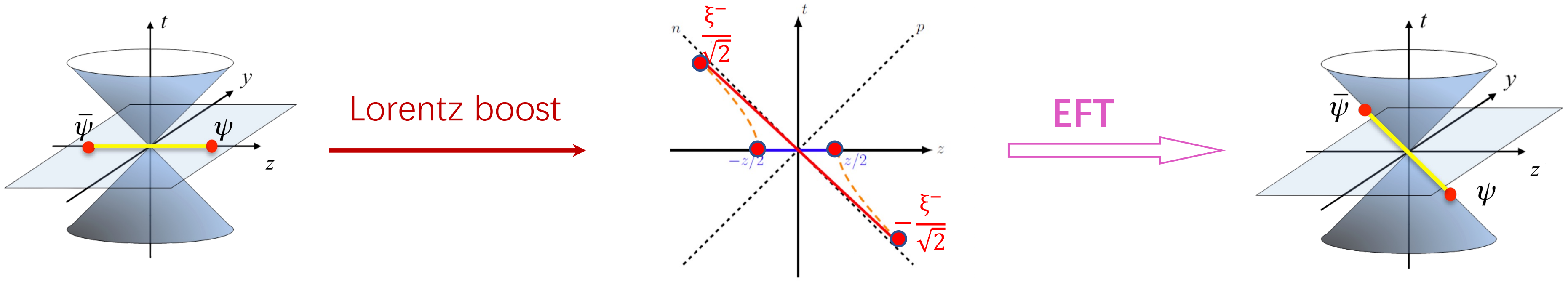


- Without soft function, only ratios can be accessed.
- Only the ratio of lowest order moments.

PRD93, 054501 (2016), PRD96, 094508 (2017)

LaMET: A New Horizon for Nucleon Structure from Lattice

Large-momentum effective theory (LaMET): connecting Euclidean lattice and physical observables



Ji, PRL110, 262002 (2013), see Rev.Mod.Phys.93, 035005 (2021) for review

- Equal-time distributions can be directly computed in lattice QCD:

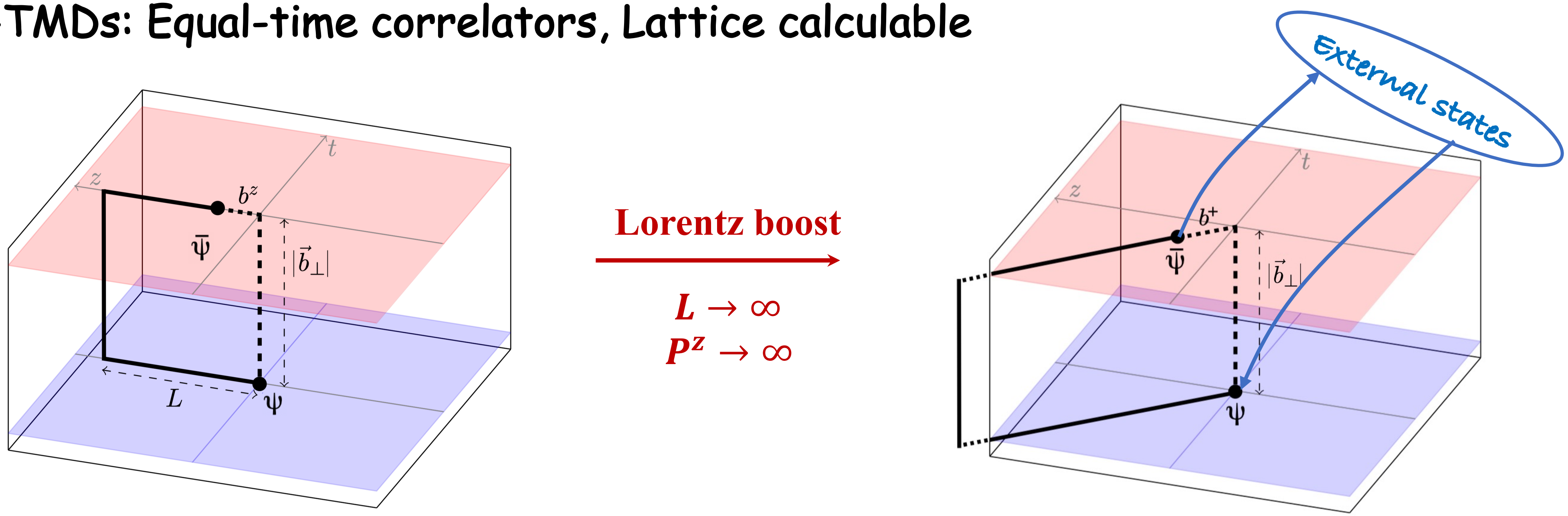
$$\tilde{q}(x, P^z) = \int \frac{dz}{2\pi} e^{-izP^z} \langle P^z | \bar{\psi}(z) \Gamma \mathcal{U}(z, 0) \psi(0) | P^z \rangle$$

- Recovering light-cone distributions in the large-momentum limit:

$$q(y, \mu) = \int_{-\infty}^{+\infty} \frac{dx}{|x|} \mathcal{C}\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{q}(x, P^z) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(yP^z, (1-y)P^z)^2}\right)$$

TMDs from Lattice QCD within LaMET

Quasi-TMDs: Equal-time correlators, Lattice calculable



Factorization in LaMET:

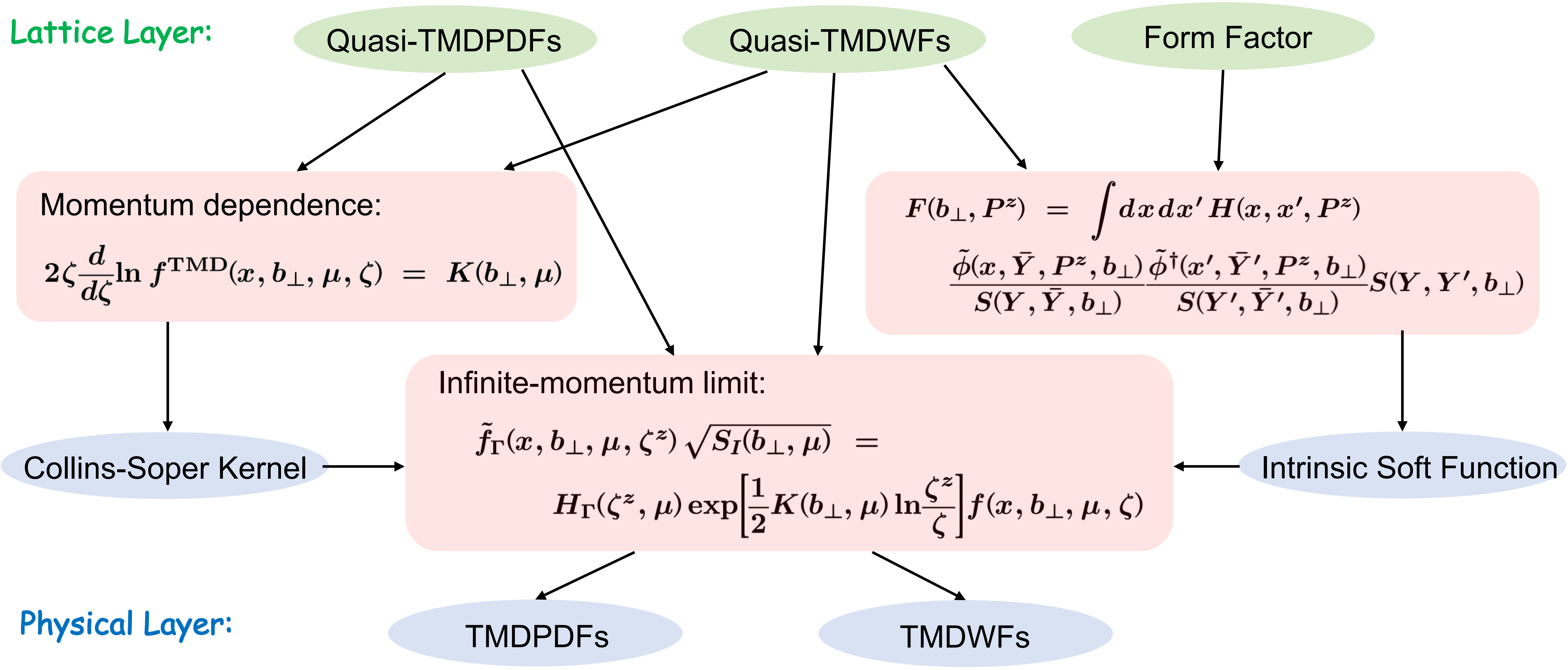
PLB811, 135946 (2020); JHEP04, 178, (2022)

Perturbative matching kernel

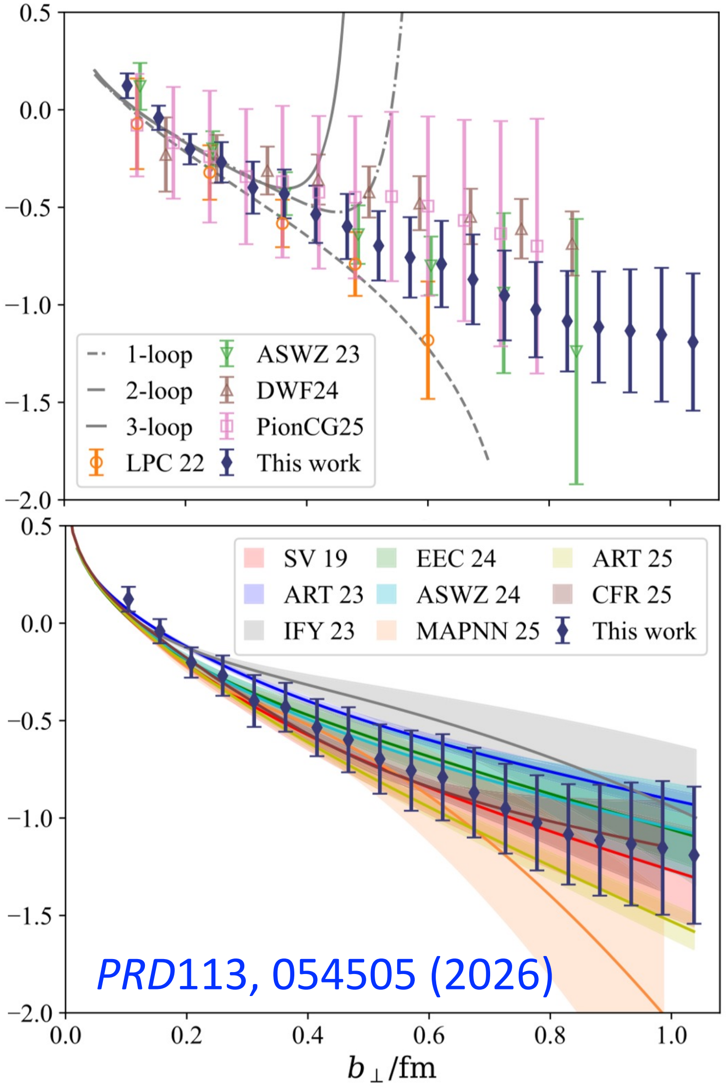
$$\tilde{f}_\Gamma(x, b_\perp, \mu, \zeta^z)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma(\zeta^z, \mu)\exp\left[\frac{1}{2}K(b_\perp, \mu)\ln\frac{\zeta^z}{\zeta}\right]f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{\zeta^z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2\zeta^z}\right)$$

Quasi TMDs — Intrinsic Soft Function — Collins-Soper kernel — Light-cone TMDs

TMDs from Lattice QCD within LaMET

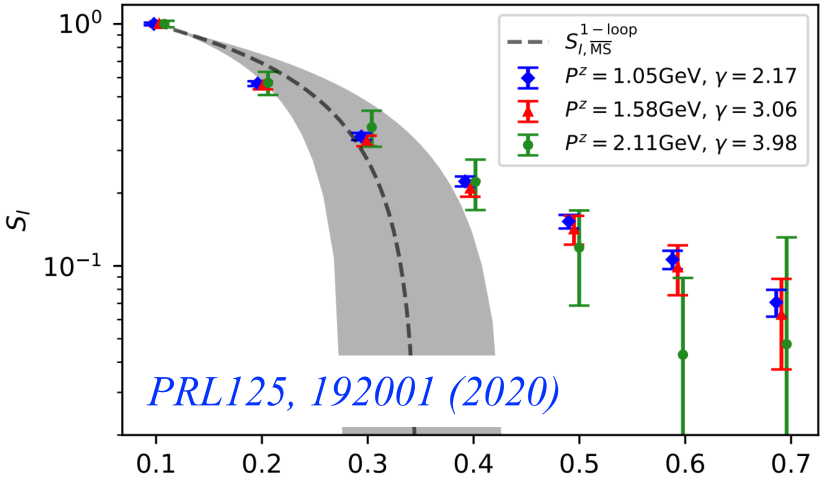


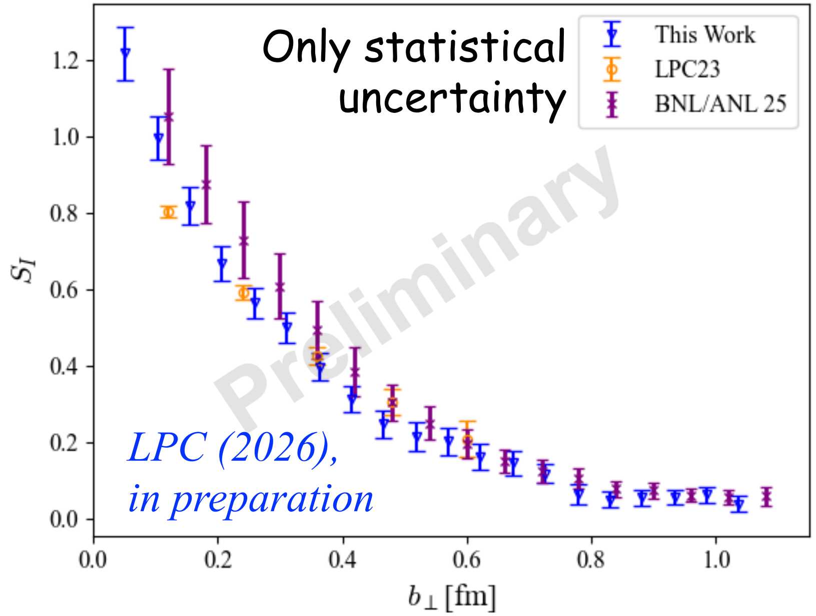
Collins-Soper Kernel from Lattice QCD



- Universal nonperturbative input for TMD evolution
- A benchmark quantity for lattice TMD studies
  - Continuum, physical pion mass, and large-momentum limits are now being addressed
  - Lattice QCD provides first-principles constraints up to $b_T \sim 1$fm
  - Agreement with global fits provides mutual validation
- A precise CS kernel is a key step toward first-principles predictions of TMD observables

Intrinsic Soft Function from Lattice QCD

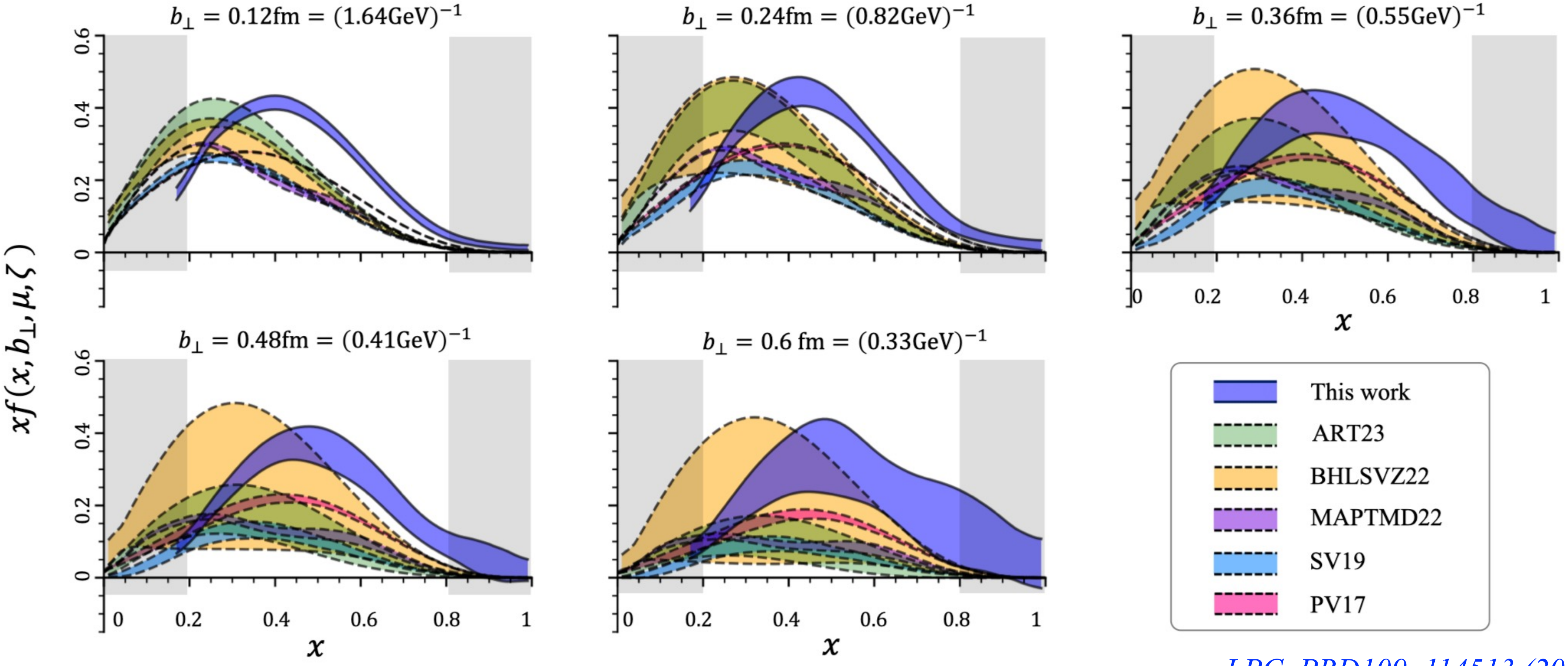




- Universal nonperturbative input in TMD factorization:
  - Encodes nonperturbative soft gluon radiation
  - Required for the absolute normalization of TMDPDFs
  - Without the soft function, lattice calculations only access ratios of TMDs
- Toward physical predictions
  - Continuum, physical-pion-mass, and infinite-momentum limits are being addressed
  - Provides essential input for precision calculations

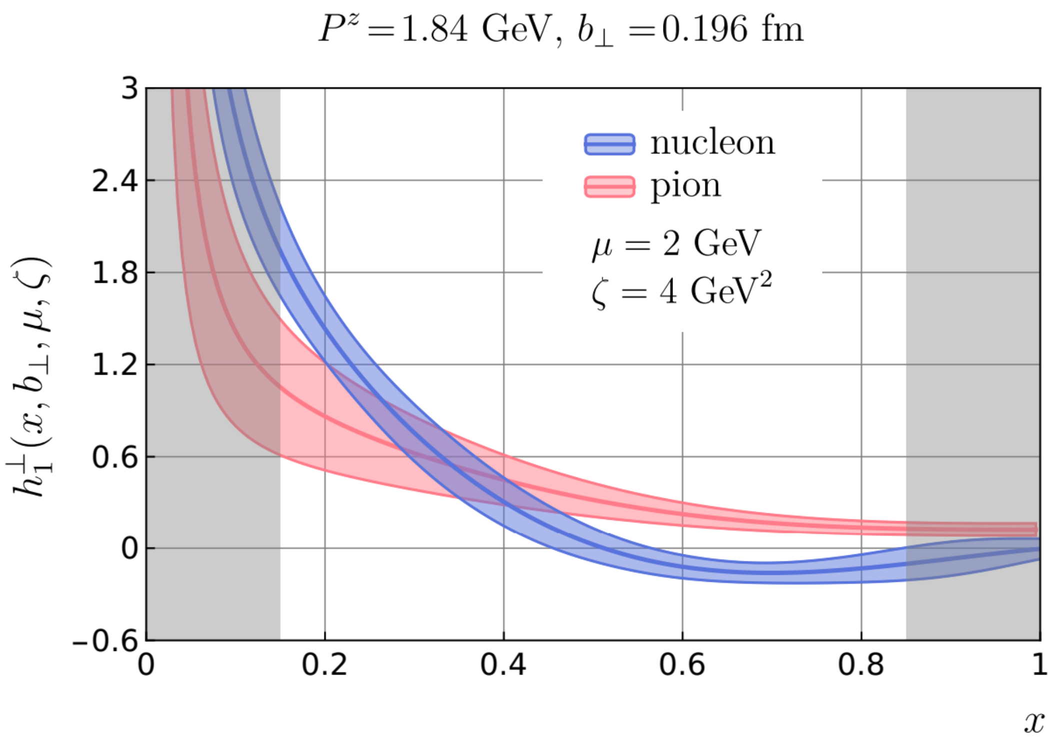
Nucleon Unpolarized TMDPDF from Lattice QCD

First exploratory calculation, still limited, but already enables comparison with global fits



LPC, PRD109, 114513 (2024)

Boer-Mulders Function from Lattice QCD



LPC, PRD111, 094507 (2025); JHEP 08, 086 (2025)

- T-odd distribution, encodes the correlation between the quark spin and its transverse momentum.
- Sensitive to spin–orbit correlations in QCD
- First-principles lattice calculations are now available for both nucleon and pion Boer–Mulders functions
- Pion vs. nucleon reveals hadron-dependent spin–orbit correlations.

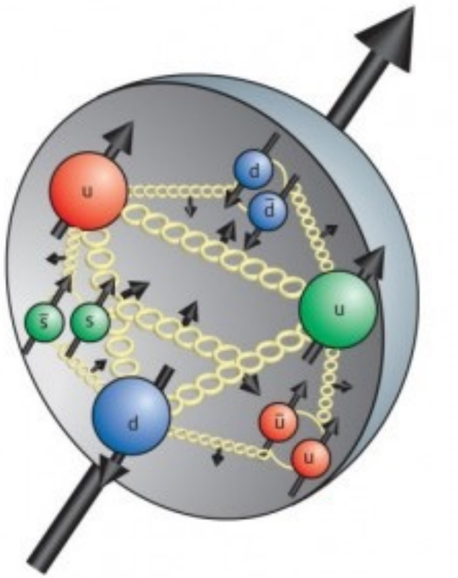
A Unified Lattice Framework for Leading-Twist TMDs

Quasi-TMD correlators with different Dirac structures and spin projectors map onto the leading-twist TMDs:

$$\int \frac{d\xi^z}{2\pi}\frac{d^2\xi_\perp}{(2\pi)^2} e^{-ixP^z\xi^z - ik_\perp\cdot\xi_\perp}\mathrm{Tr}\langle P^z, S|\bar{\psi}(0)\Gamma U(0,\xi^z)\psi(\xi^z)|P^z, S\rangle \sim (f_1, g_{1L}, h_1, f_{1T}^\perp, h_1^\perp, \ldots)$$

Quark polarization:

- Unpolarized: $\Gamma = \gamma^z$ or γ^t
- Helicity: $\Gamma = \gamma^z\gamma_5$ or $\gamma^t\gamma_5$
- Transversity: $\Gamma = i\sigma_{\perp z}\gamma_5$ or $i\sigma_{\perp t}\gamma_5$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$



Nucleon polarization:

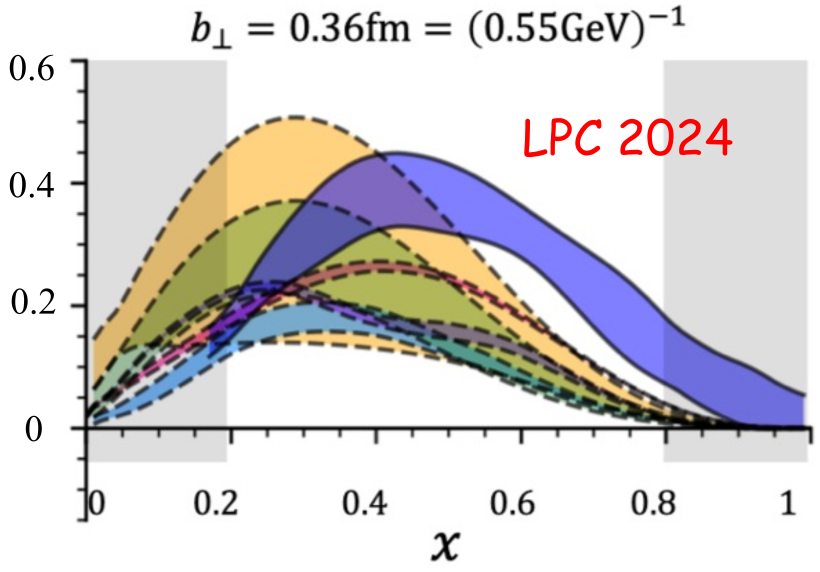
- Unpolarized: $T_{\text{unpol}} = \frac{1+\gamma_4}{2}$
- Helicity: $T_{\text{hel}}^{\pm} = \frac{1+\gamma_4}{2}\frac{1\pm\gamma_5\gamma_3}{2}$
- Transversity: $T_{\text{trans}}^{(\pm,x)} = \frac{1+\gamma_4}{2}\frac{1\pm\gamma_5\gamma_1}{2}$

  $T_{\text{trans}}^{(\pm,y)} = \frac{1+\gamma_4}{2}\frac{1\pm\gamma_5\gamma_2}{2}$

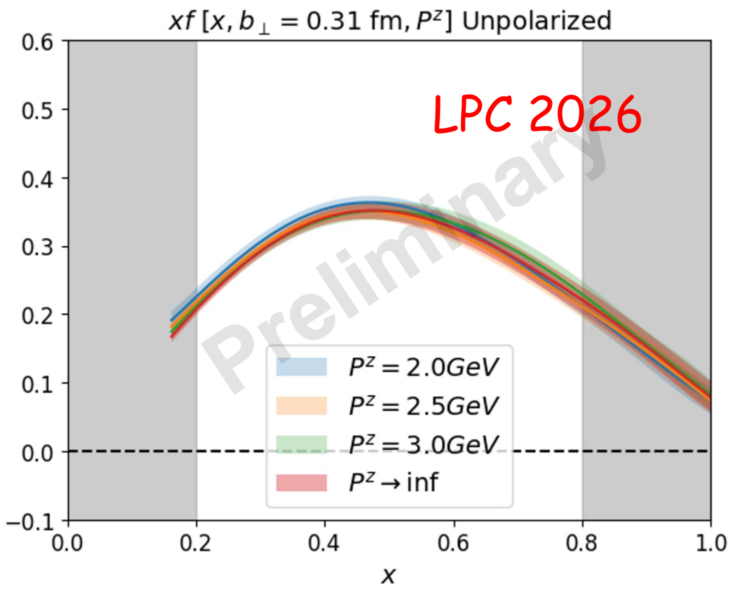
A Unified Lattice Framework for Leading-Twist TMDs

Nucleon unpolarized TMDPDF

More details see Jin-Xin Tan's talk (4/22, 14:20-14:45)



- Exploratory study
- Single lattice spacing
- Unphysical pion mass



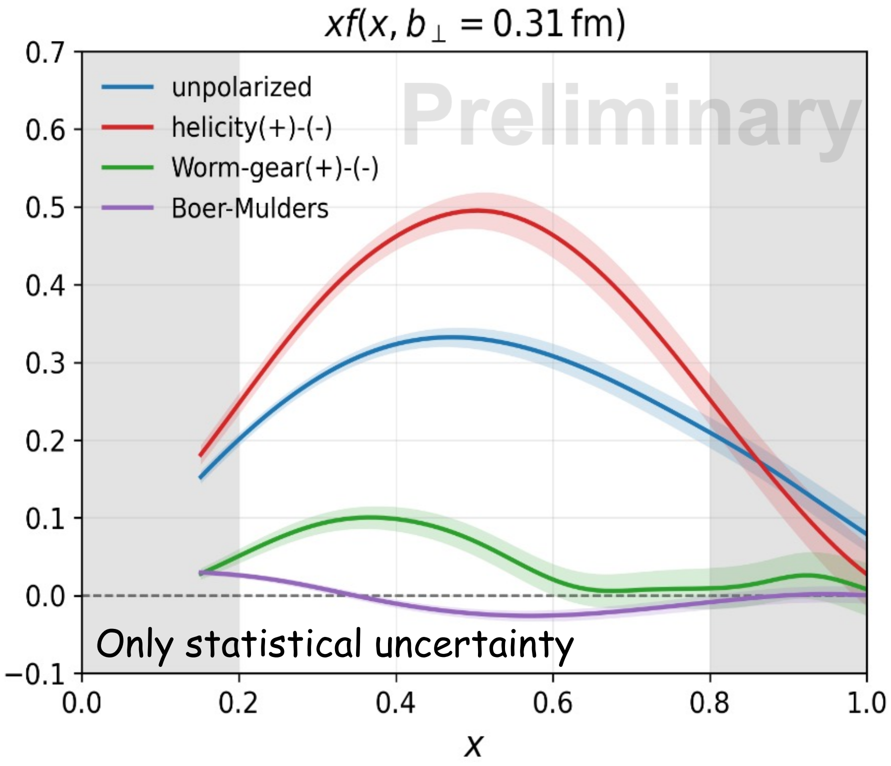
Only statistical uncertainty

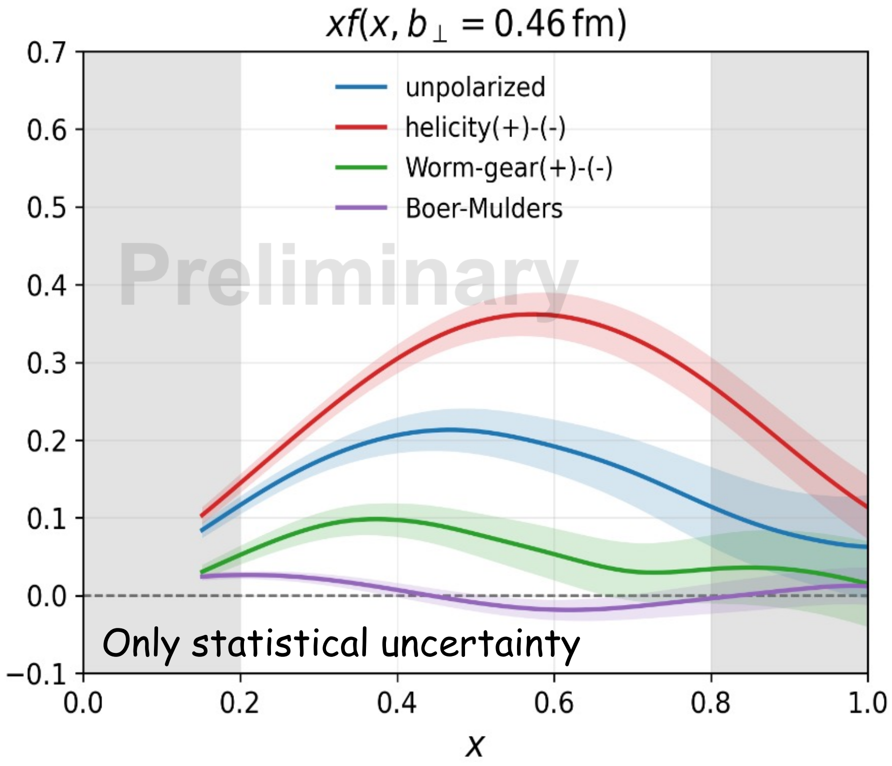
- Precision calculation
- Continuum extrapolation
- Physical mass extrapolation

A Unified Lattice Framework for Leading-Twist TMDs

TMDPDFs for unpolarized and longitudinal polarized nucleon:

More details see Jin-Xin Tan's talk (4/22, 14:20-14:45)

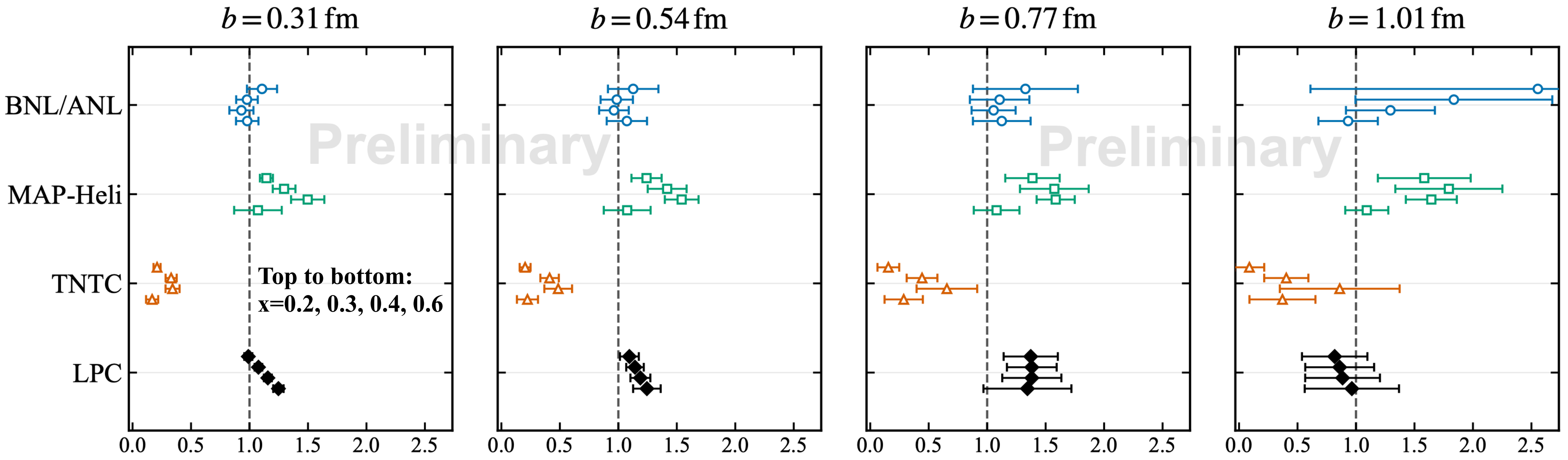




A Unified Lattice Framework for Leading-Twist TMDs

BNL/ANL (LQCD): PRL135, 20 (2025)
MAP (Global fit): PRL134, 121901 (2025)
TNTC (Global fit): PRL134, 121902 (2025)

Ratios between nucleon helicity and unpolarized TMDPDF:



- The ratios show no significant b_T dependence
- Providing first-principles benchmark for the nucleon 3D structure.

Summary and Outlook

TMDs of the nucleon are part of the future frontier of nuclear science, especially in the Electron-Ion Collider and first-principles QCD calculations.

Lattice QCD + LaMET provides a new perspective on exploring the 3-D structure of hadrons.

With controlled continuum, physical-mass, and large-momentum limits, lattice calculations will deliver essential nonperturbative inputs for precision TMD phenomenology.