



NNLO精度的共线碎裂函数 (Collinear Fragmentation Functions at NNLO)

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Based on: Jun Gao, **XS**, Hongxi Xing, Yuxiang Zhao, Bin Zhou (NPC Collaboration)

PRL 135, 041902, 2025

Outline

➤ Introduction

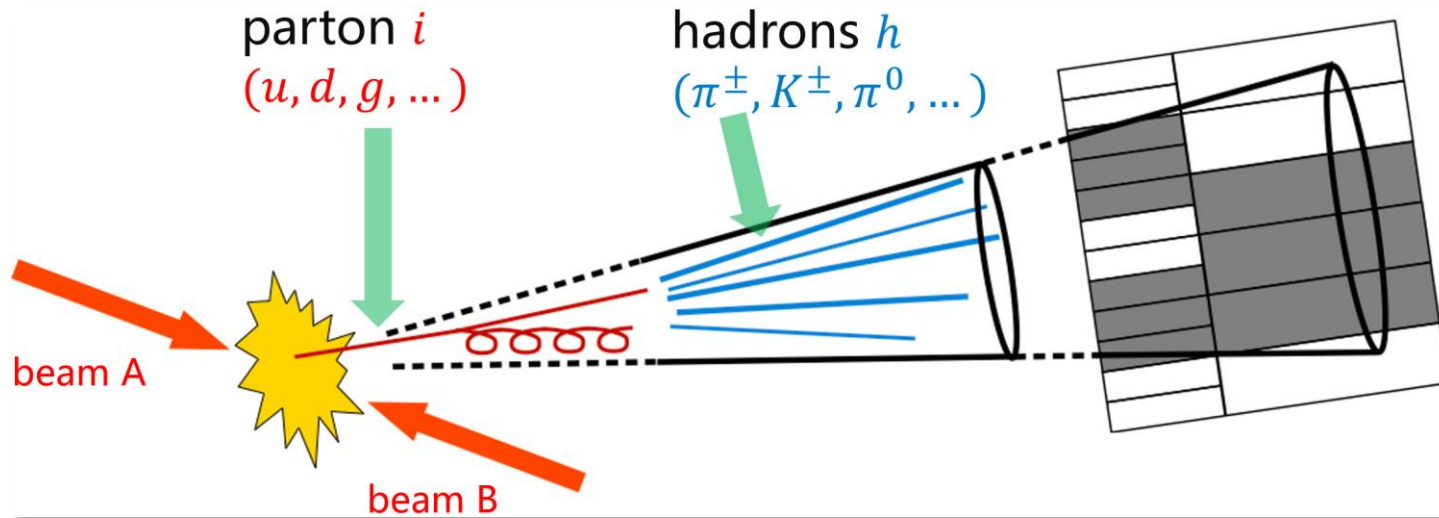
- Fragmentation Functions (FFs) in parton model and QCD
- Determination of collinear FFs

➤ NPC NNLO FFs analysis

- Determination of NNLO FFs
- Constraints on PDFs

➤ Summary

Fragmentation Functions in parton model



FF = number density of finding

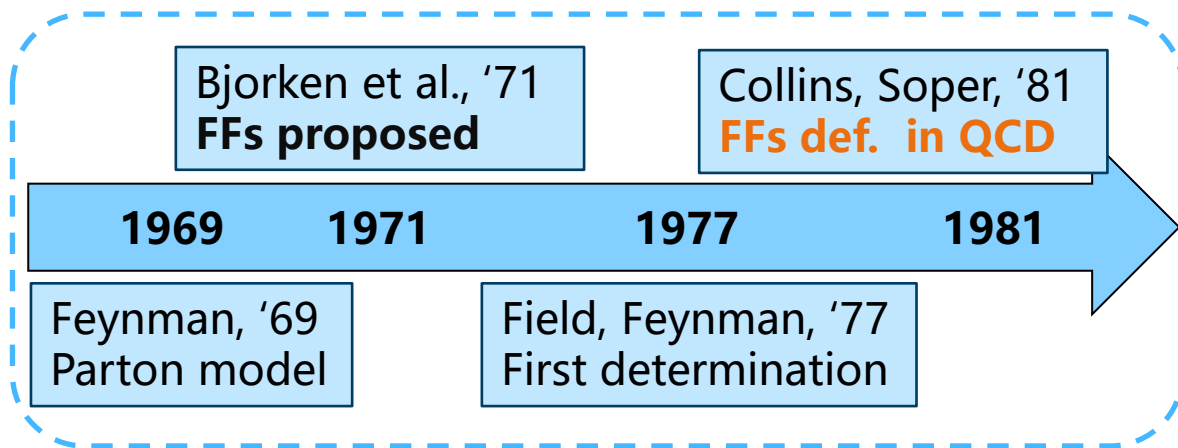
- a specific hadron h
- with momentum fraction z "in" parton i

$$D_{h/i} \left(z = \frac{p_h^+}{p_i^+} \right) \longleftrightarrow f_{i/h} \left(x = \frac{p_i^+}{p_h^+} \right)$$

碎裂函数

部分子分布函数

Fragmentation Functions (FFs) in QCD



$$D_{h/q}(z) = \frac{z}{4} \sum_X \int \frac{d\xi^+}{2\pi} e^{iP_h^- \xi^+ / z} \text{Tr} \left[\langle 0 | \mathcal{W}(\infty^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \right. \\ \left. \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle \gamma^- \right]$$

$$D(z, Q_0)$$

$$D(z, Q)$$

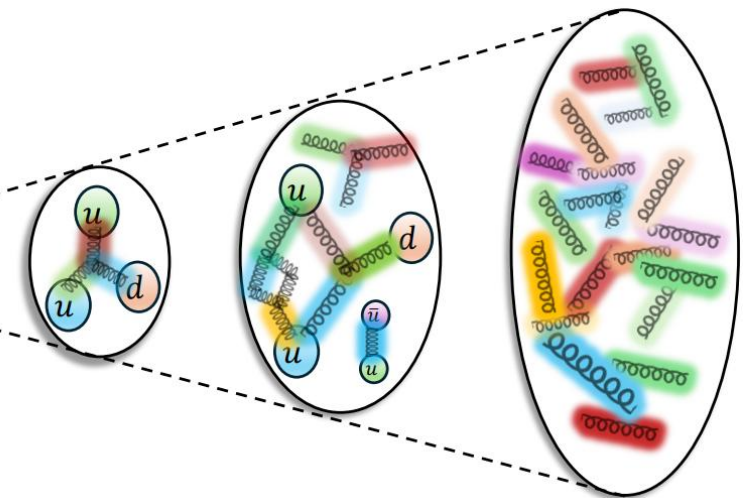
$$\frac{dD_{h/i}(z, Q)}{d \ln^2 Q} = P_{ji}(y) \otimes D_{h/j}\left(\frac{z}{y}, Q\right)$$

timelike DGLAP evolution

PDFs

Hadron

Parton distribution function describes the probability of finding a quark or gluon

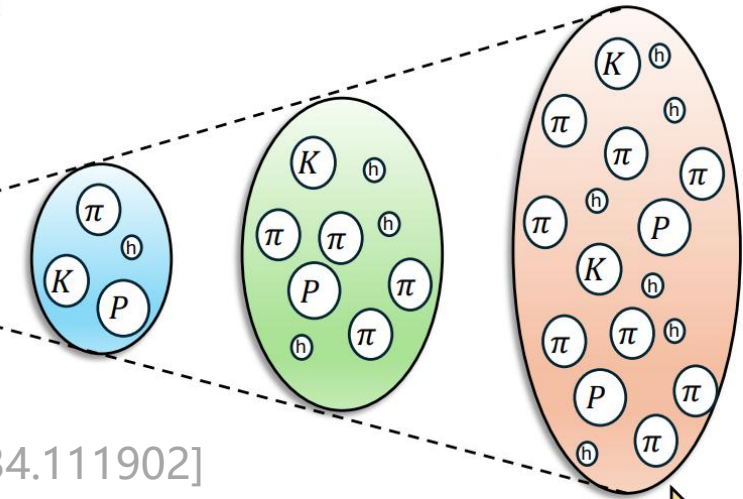


Crossing Symmetry $\left\{ \begin{array}{l} e^- + h \rightarrow e^- + X \\ e^- + e^+ \rightarrow h + X \end{array} \right\}$

FFs

Parton

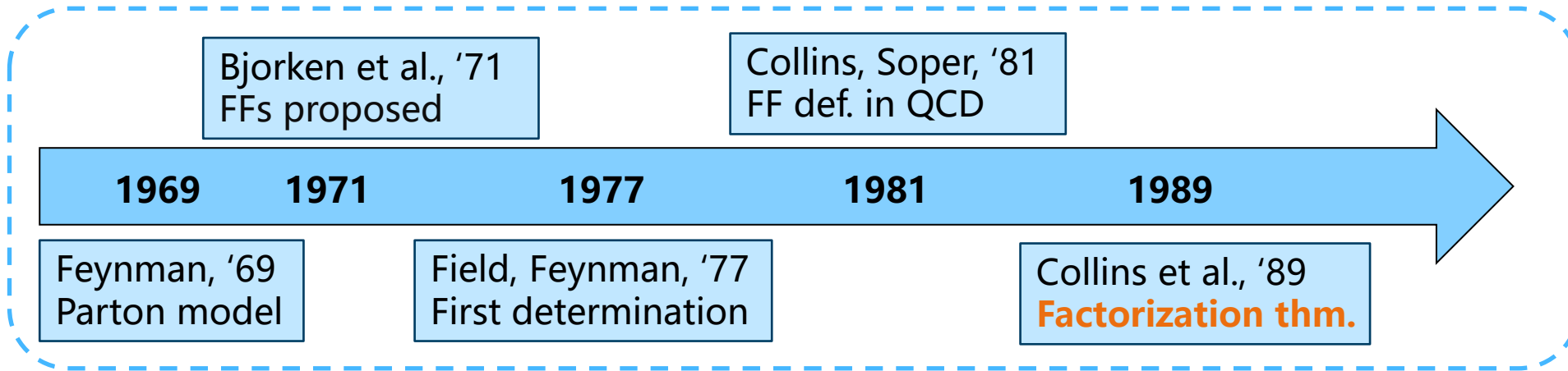
Fragmentation function describes the probability of producing a specific hadron.



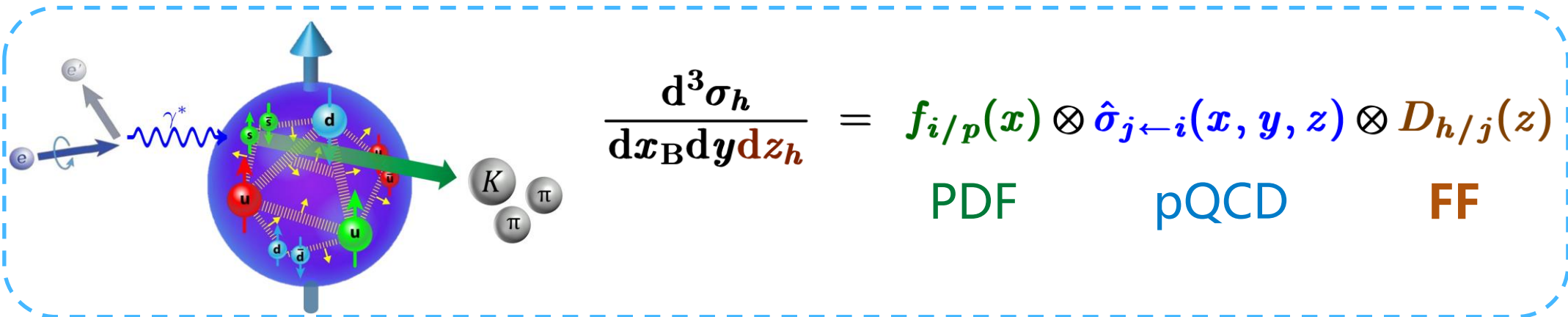
[Figure from PRL.134.111902]



FFs are key ingredients of QCD factorization framework



➤ Semi-Inclusive DIS (SIDIS) : $e + N \rightarrow e + \mathbf{h} + X$

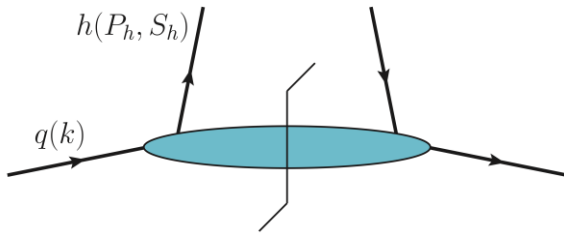


➤ $e^+e^- \rightarrow h + X$ (SIA) $\Rightarrow \sigma = \text{pQCD} \otimes \mathbf{FF}$

➤ $pp \rightarrow h + X \Rightarrow \sigma = \mathbf{PDF} \otimes \mathbf{PDF} \otimes \text{pQCD} \otimes \mathbf{FF}$



Determination of FFs

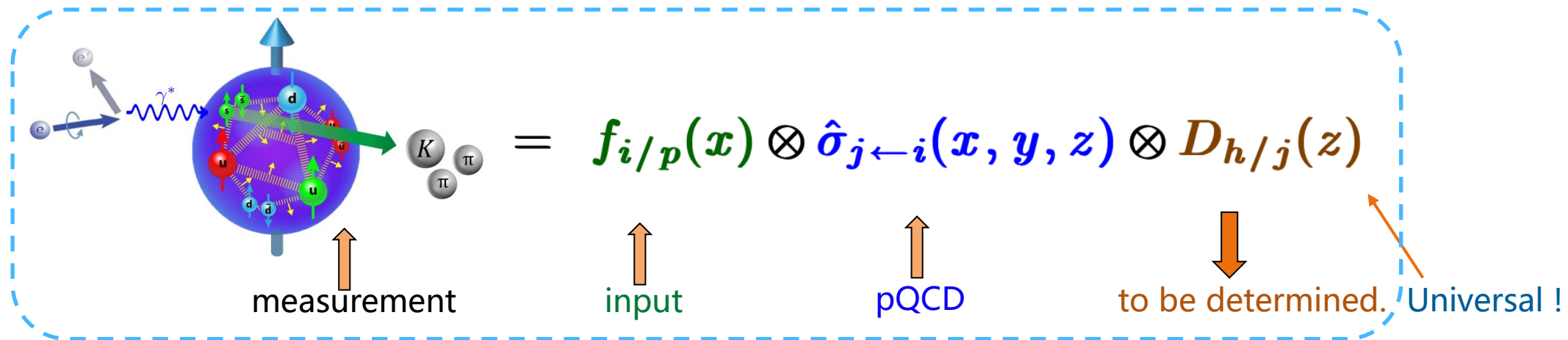


$$D_{h/q}(z) = \frac{z}{4} \sum_X \int \frac{d\xi^+}{2\pi} e^{iP_h^- \xi^+ / z} \text{Tr} \left[\langle 0 | \mathcal{W}(\infty^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \right. \\ \left. \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle \gamma^- \right]$$

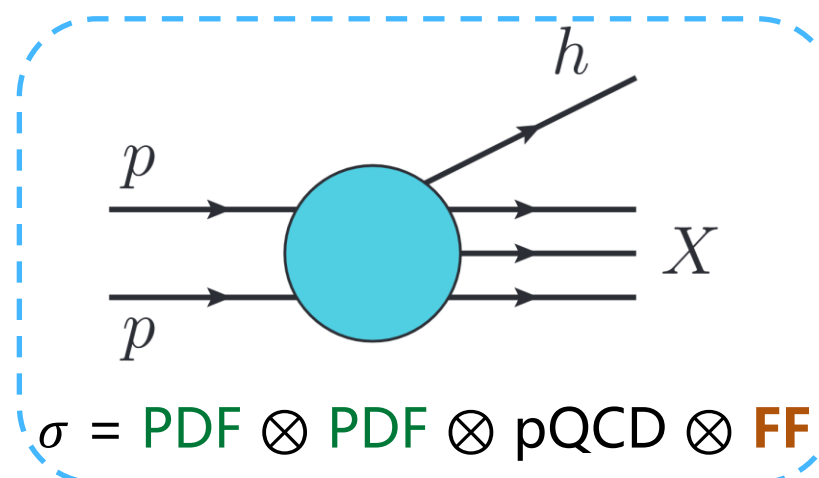
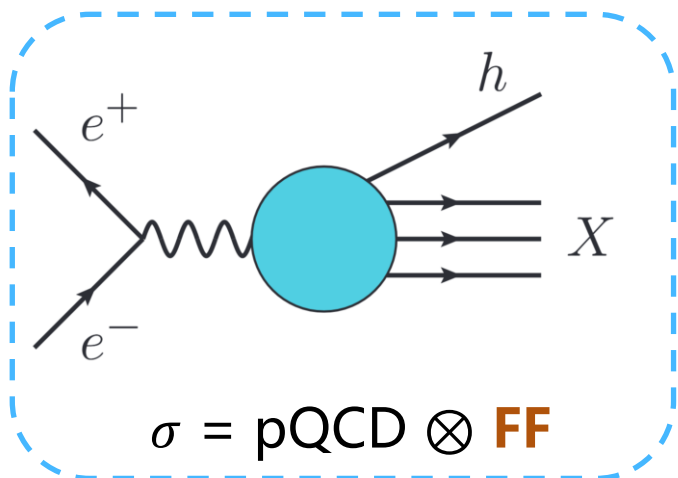
- ❖ No lattice QCD calculation yet.
- ❖ Using quantum computers [Li, Xing, Zhang, 2406.05683]
[Galvez-Viruet et al. 2510.18869]
- ❖ In simpler theory: 't Hooft model [Jia, Mo, Xiong, 2310.17640]

Determination of FFs: global data fits

❖ e p collision:



❖ ee and pp



Global analyses of collinear FFs to light hadrons

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓
hadron in jet	✗	✗	✗	✗	✗	✓
FFs (charged h)	π^\pm, K^\pm, p	π^\pm, K^\pm	π^\pm, K^\pm, p	π^\pm	π^\pm, K^\pm	π^\pm, K^\pm, p
FFs (neutral h)			η			K^0, η, Λ
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO

➤ **NLO** analyses from **Non-perturbative Physics Collaboration (NPC)** [See also Jun Gao's talk on Monday]

- NPC23 FFs to light **charged** hadrons:

Gao, Liu, **XS**, Xing, Zhao, *PRL* 132, 261903, '24

Gao, Liu, **XS**, Xing, Zhao, *PRD* 110, 114019, '24 (Editors' suggestion)

- NPC23 FFs to light **neutral** hadrons:

Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD* 112, 054045, '25



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➤ Introduction

- Fragmentation Functions (FFs, 碎裂函数) in parton model and QCD
- Determination of collinear FFs

➤ **NPC NNLO FFs analysis**

- Determination of NNLO FFs
- Constraints on PDFs

➤ Summary

FFs play a key role in the era of high-precision physics

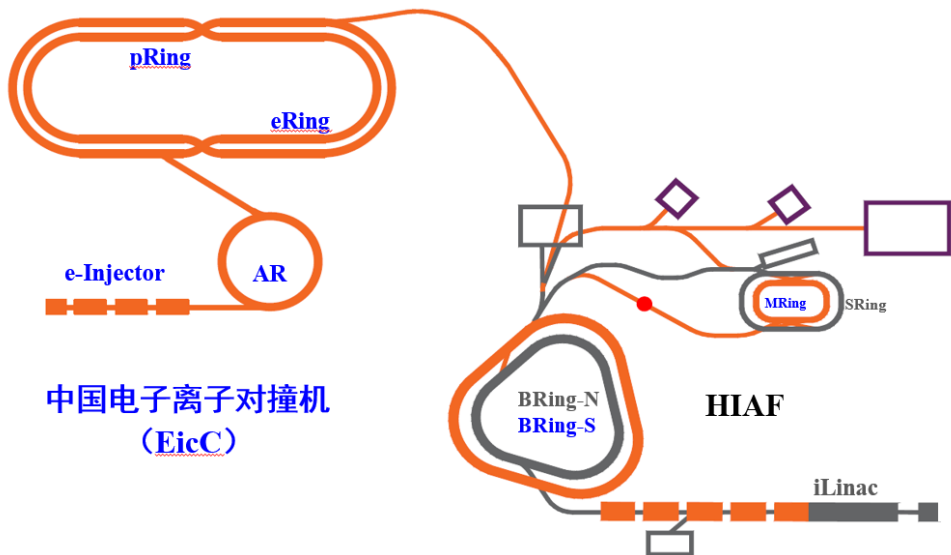


Figure and table from Jiancheng Yang's talk on Monday

	HERA	EIC	EicC
峰值亮度 ($10^{33}\text{cm}^{-2}\text{s}^{-1}$)	0.05	10	6
积分亮度 (fb^{-1})	0.21	100	150
电子极化率	60%	70%	80%
质子极化率	无	70%	70%

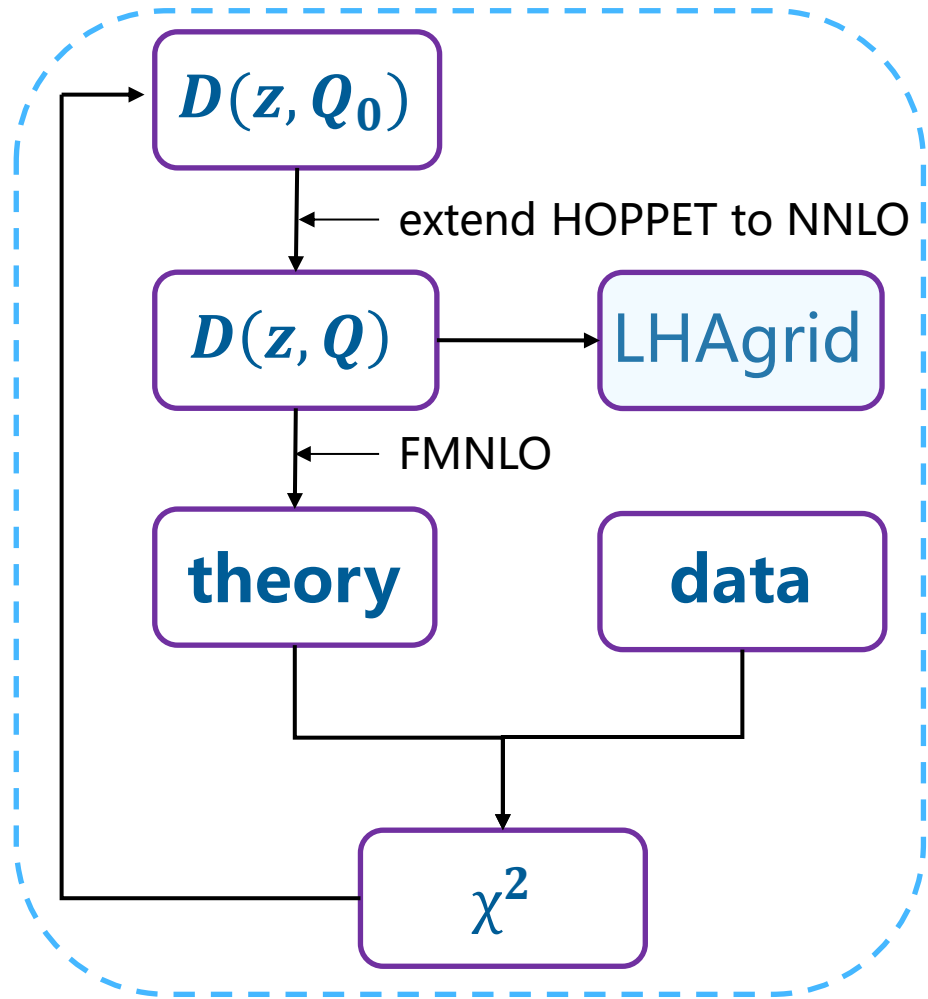
The unprecedented high-precision measurements require high-precision FFs input:

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓	✗
hadron in jet	✗	✗	✗	✗	✗	✓	✗
FFs	π^\pm, K^\pm, p	π^\pm, K^\pm	π^\pm, K^\pm, p, h^\pm η	π^\pm	π^\pm, K^\pm	π^\pm, K^\pm, p, h^\pm K^0, η, Λ	π^\pm, K^\pm
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO	NNLO

[Gao, **XS**, Xing, Zhao, Zhou, *PRL* 135, 041902, 2025]



The framework



- FFs at starting scale $Q_0 = 1.4 \text{ GeV}$

$$zD_i^h(z, Q_0) = z^{\alpha_i^h} (1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^m a_{i,n}^h z^{n/2}\right)$$

+ charge/isospin symmetries

- FFs at arbitrary energy scale Q

3-loop timelike DGLAP evolution

[Mitov, Moch, Vogt, Almasy]

[Chen, Yang, Zhu, Zhu, '20]

- SIA/SIDIS coefficient functions at NNLO

[Bonino+, '24], [Goyal+, '24]

[See Jun Gao's talk for N3LO]

The **first** global FF fit (ee+SIDIS) at full NNLO accuracy



The datasets

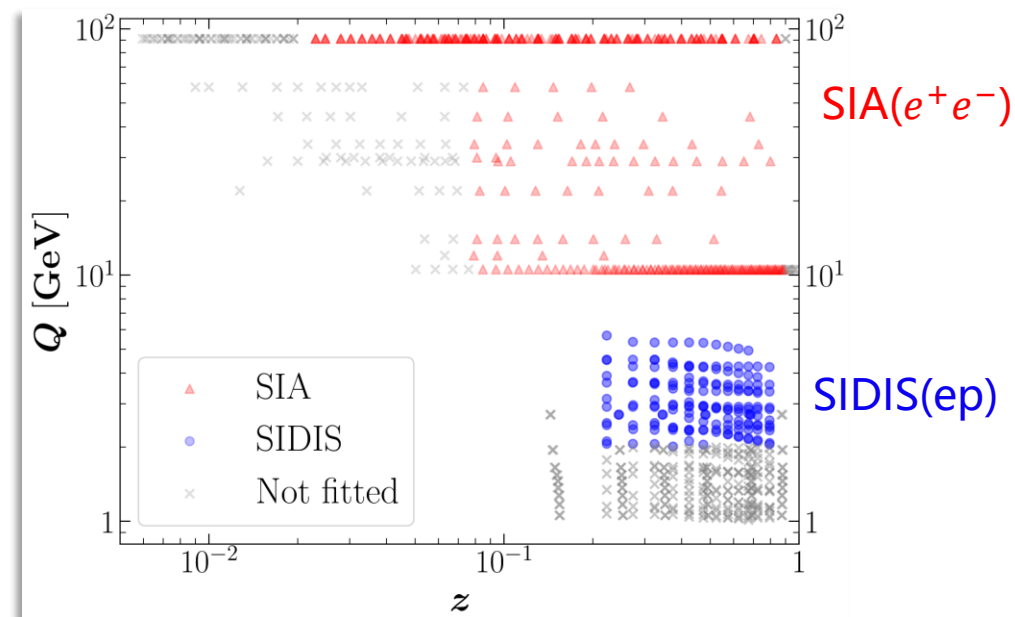
➤ SIA(e^+e^-) data used in the fit:

exp.	\sqrt{s}/GeV	lum. (n_Z)	year	final states	hadrons
DELPHI	189	157.7 pb ⁻¹	2002	inc. had.	π^\pm, K^\pm
OPAL	m_Z	780 000	1994	$Z \rightarrow q\bar{q}$	π^\pm, K^\pm
ALEPH	m_Z	520 000	1995	$Z \rightarrow q\bar{q}$	π^\pm, K^\pm
DELPHI	m_Z	1 400 000	1998	$Z \rightarrow q\bar{q}$	π^\pm, K^\pm
				$Z \rightarrow b\bar{b}$	π^\pm, K^\pm
SLD	m_Z	400 000	2004	$Z \rightarrow q\bar{q}$	π^\pm, K^\pm
				$Z \rightarrow b\bar{b}$	π^\pm, K^\pm
				$Z \rightarrow c\bar{c}$	π^\pm, K^\pm
TASSO	44	34 pb ⁻¹	1989	inc. had.	π^\pm, π^0
TASSO	34	77 pb ⁻¹	1989	inc. had.	π^\pm, K^\pm
TPC/2 γ	29	70 pb ⁻¹	1988	inc. had.	π^\pm, K^\pm
Belle	10.52	68 fb ⁻¹	2013	inc. had.	π^\pm, K^\pm
BaBar	10.54	0.91 fb ⁻¹	2013	inc. had.	π^\pm, K^\pm
BESIII	2.0-3.671	253 pb ⁻¹	2025	inc. had.	π^\pm, K^\pm

[BESIII, PRL135, 151901, 2025]

[See Wenbiao's talk on Tuesday]

➤ separated kinematic region of e^+e^- and ep data



Data sets used by MAP group

Khalek, Bertone, Nocera PRD.104.034007, '21

➤ This work uses both ee and ep data at $Q \sim 3\text{GeV}$

The first test on universality of FFs at $Q \sim 3\text{ GeV}$ (using ee + ep)

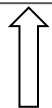


The results

➤ our FFs describe both ee and ep data well

➤ test of **leading-twist** factorization

$E_{h,min}$ [GeV]	BESIII		COMPASS		B-factories		HE-SIA		global		
	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2	χ^2/N_{pt}
0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77



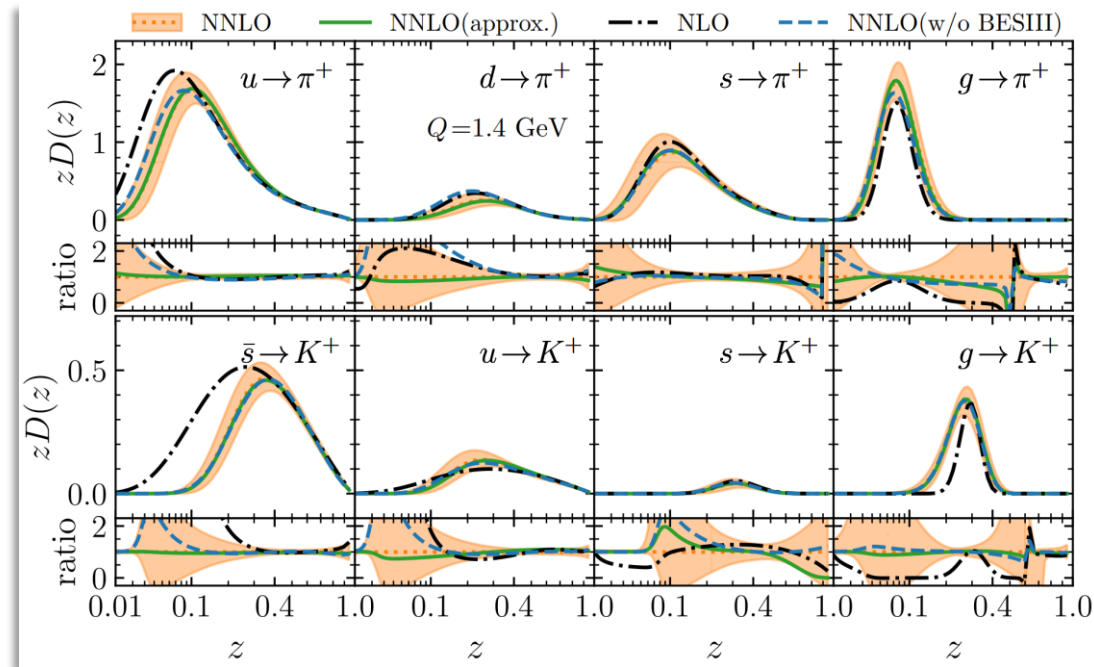
energy cut of the identified hadron

➤ This work: $\chi^2_{NNLO} < \chi^2_{NLO}$

Previous studies: $\chi^2_{appr.NNLO} > \chi^2_{NLO}$

[PRL.129.012002, PLB.2022.137456]

[Gao, XS, Xing, Zhao, Zhou, *PRL* 135, 041902, 2025]



LHAg grids of our FFs have been submitted to the LHAPDF repository.

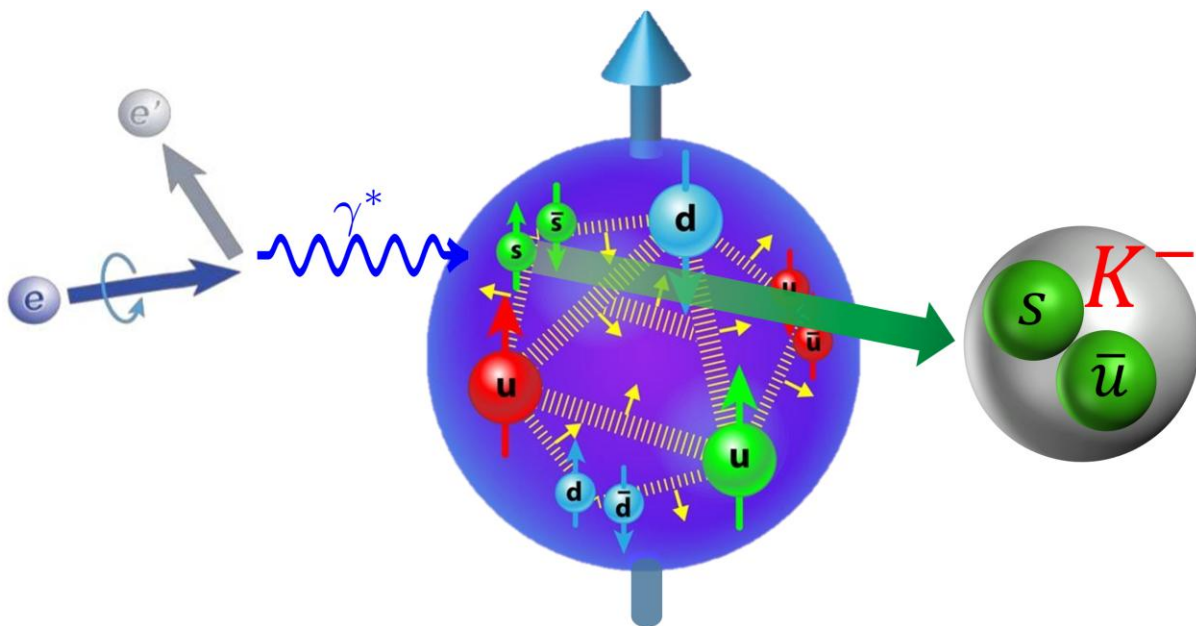
<https://www.lhapdf.org/pdfsets.html>



Application: constraining proton PDFs at NNLO

SIDIS may also constrain PDFs:

$$\frac{d^3\sigma_h}{dx_B dy dz_h} = \underbrace{f_{i/p}(x)}_{\text{unpolarized PDF}} \otimes \hat{\sigma}_{j \leftarrow i}(x, y, z) \otimes \underbrace{D_{h/j}(z)}_{\text{FF}}$$



➤ LO xsec of SIDIS off an isoscalar target (COMPASS)

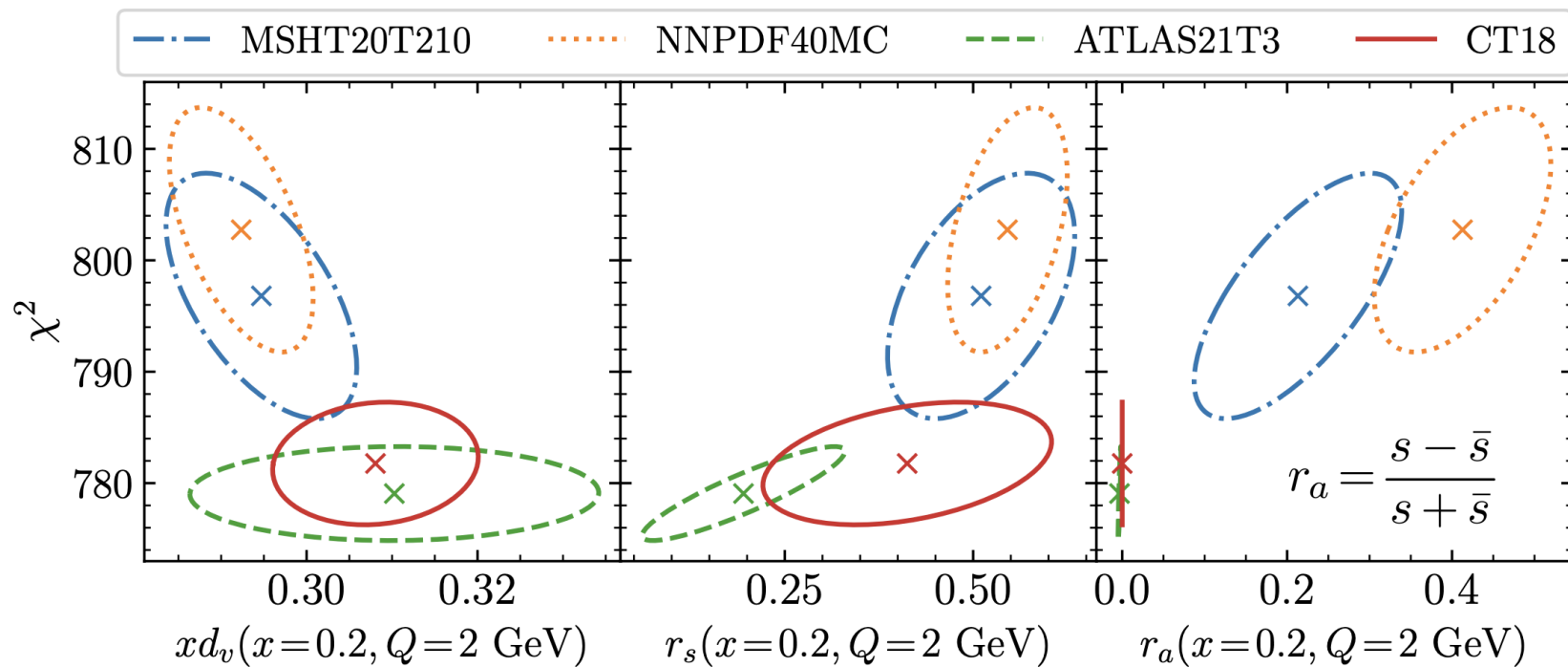
$$\begin{aligned} & \frac{d\sigma^{K^+}}{dx dy dz} - \frac{d\sigma^{K^-}}{dx dy dz} \\ & \sim 2 \left(u_v(x) + d_v(x) \right) \left(D_u^{K^+}(z) - D_{\bar{u}}^{K^+}(z) \right) \\ & + \underbrace{\left(s(x) - \bar{s}(x) \right)}_{\text{PDF}} \underbrace{\left(D_s^{K^+}(z) - D_{\bar{s}}^{K^+}(z) \right)}_{\text{FF}} + \dots \end{aligned}$$

is sensitive to **strangeness asymmetry**

$$r_a = \frac{s - \bar{s}}{s + \bar{s}}$$

Application: constraining proton PDFs at NNLO

➤ Correlation between χ^2 and PDFs



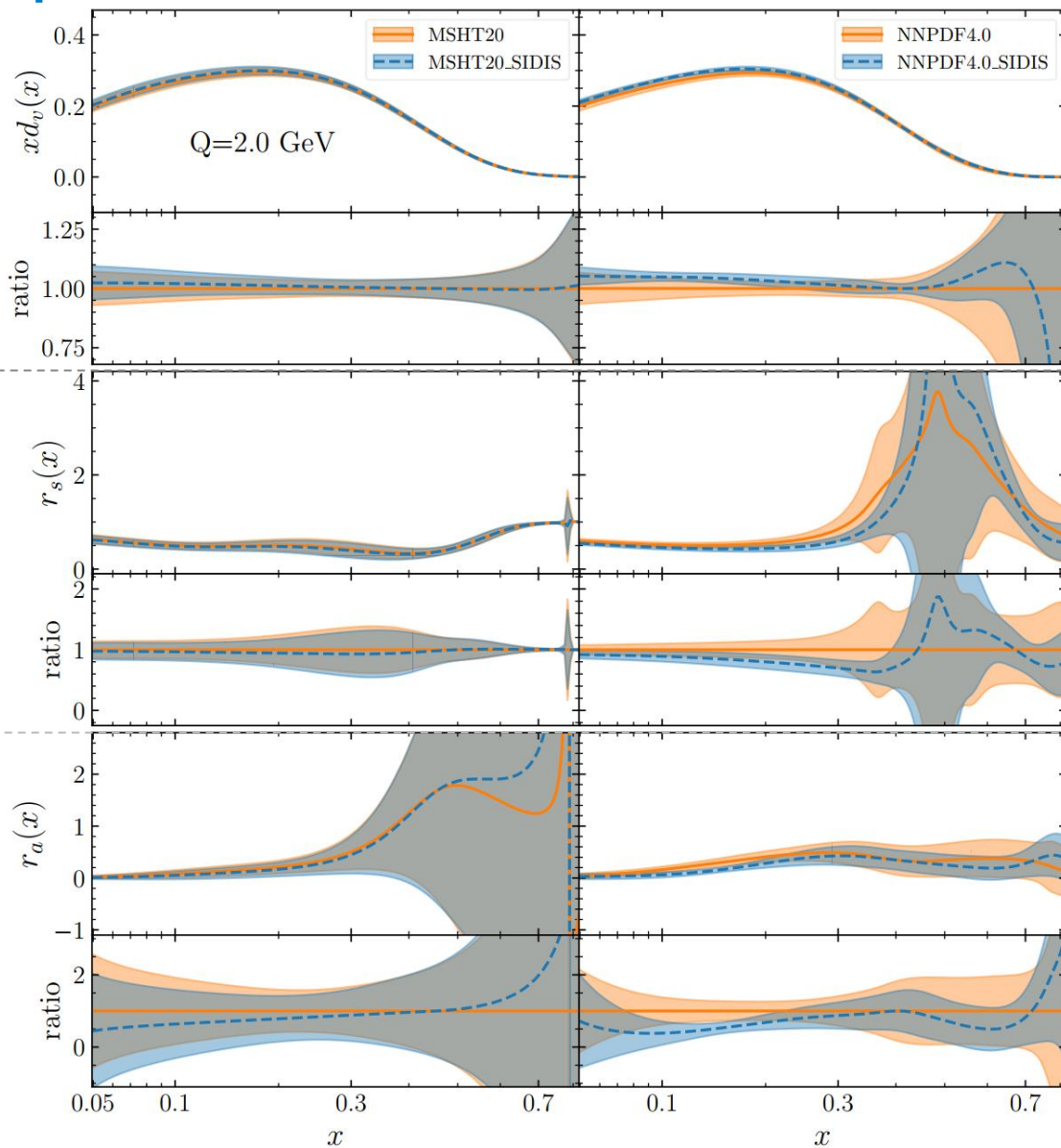
➤ We generate **modified PDFs** which reflect the impact of SIDIS data

- Reweighting of the **NNPDF4.0** PDF set
- Profiling of the **MSHT20** PDF set



Application: PDF sets before and after reweighting/profiling

[Gao, **XS**, Xing, Zhao, Zhou, *PRL* **135**, 041902, 2025]



$$d_v := d(x, Q) - \bar{d}(x, Q)$$

$$r_s := \frac{s(x, Q) + \bar{s}(x, Q)}{\bar{u}(x, Q) + \bar{d}(x, Q)}$$

$$r_a := \frac{s(x, Q) - \bar{s}(x, Q)}{s(x, Q) + \bar{s}(x, Q)}$$



Summary

- FFs are key inputs for calculations of hadron production rate from first principles.
- We present the **first** global (ee+SIDIS) FFs determination at full **NNLO**.
- Our FFs are in good agreement with low-Q data from **both ee and SIDIS**.
- We find a preference for a reduced asymmetry in the strange (anti-)quark PDFs.

NPC FF sets		
SIA (<i>ee</i>)	✓	✓
SIDIS (<i>ep</i>)	✓	✓
<i>pp</i> incl. hadron	✓	
hadron in jet	✓	
hadrons	π^\pm, K^\pm, p K^0, η, Λ	π^\pm, K^\pm
pQCD order	NLO	NNLO
Q_{\min}	4GeV	1.3GeV

NPC FF sets are available at <https://www.lhapdf.org/pdfsets.html>

NLO charged hadron:

Gao, Liu, **XS**, Xing, Zhao, *PRL* 132, 261903, 2024

Gao, Liu, **XS**, Xing, Zhao, *PRD* 110, 114019, 2024

NLO neutral hadron:

Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD* 112, 054045, 2025

NNLO:

Gao, **XS**, Xing, Zhao, Zhou, *PRL* 135, 041902, 2025



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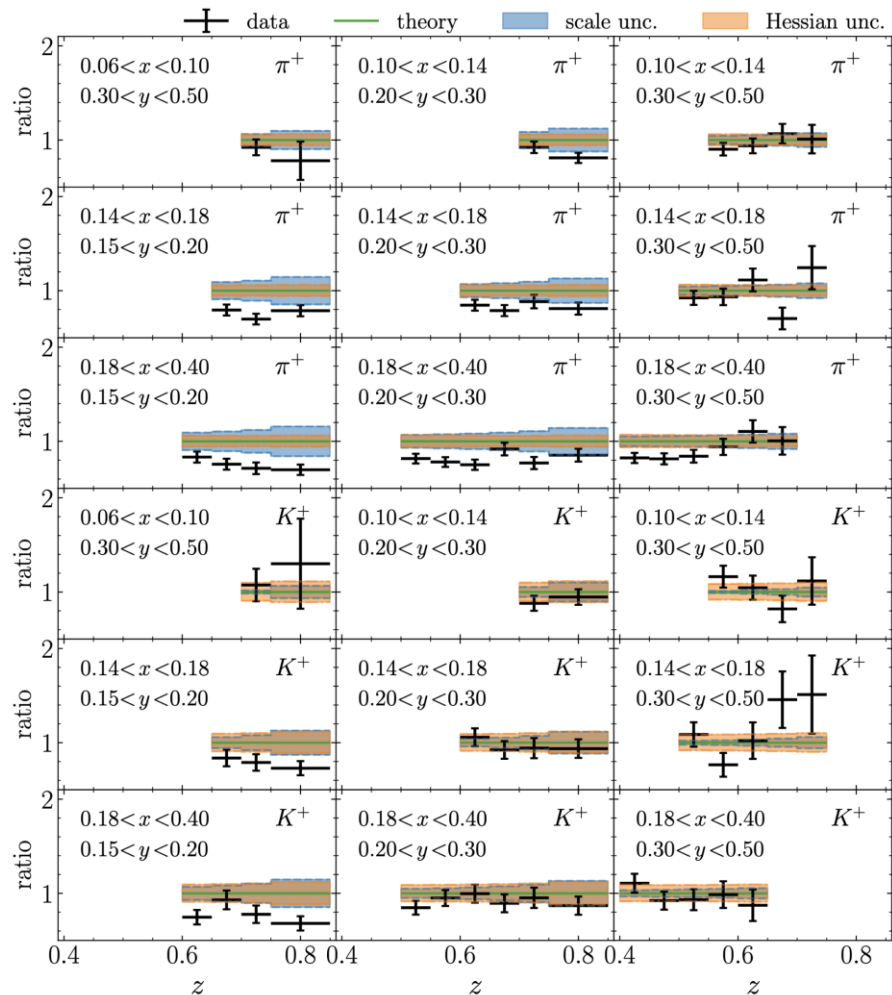
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Thank you!

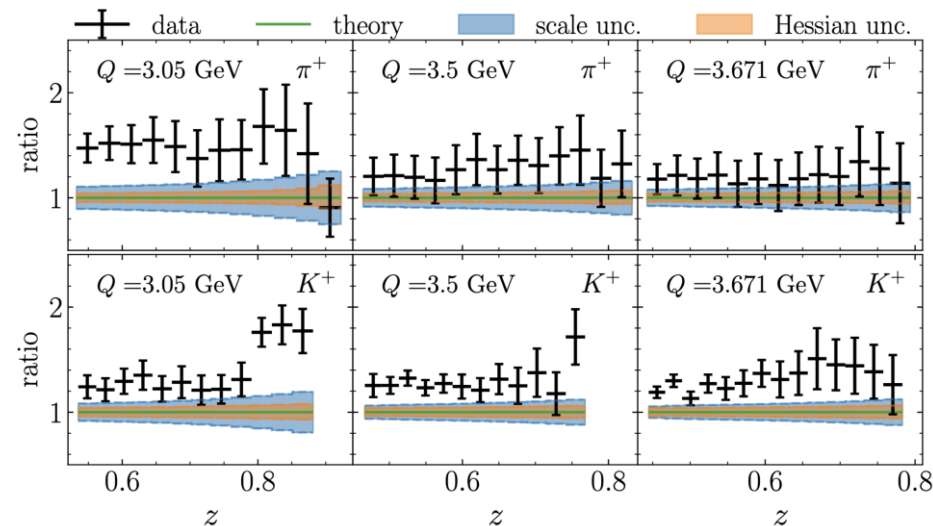


Backup

Theory v.s. data for COMPASS06 (SIDIS: 2~5GeV)



Theory v.s. data for BESIII (SIA: ~3GeV)



➤ Kinematic cuts in our analyses:

- $Q > 3 \text{ GeV}$ (SIA)
- $Q > 2 \text{ GeV}$ (SIDIS)
- $z > 0.01, E_h > 0.8 \text{ GeV}$

Backup

NNPDF40MC

