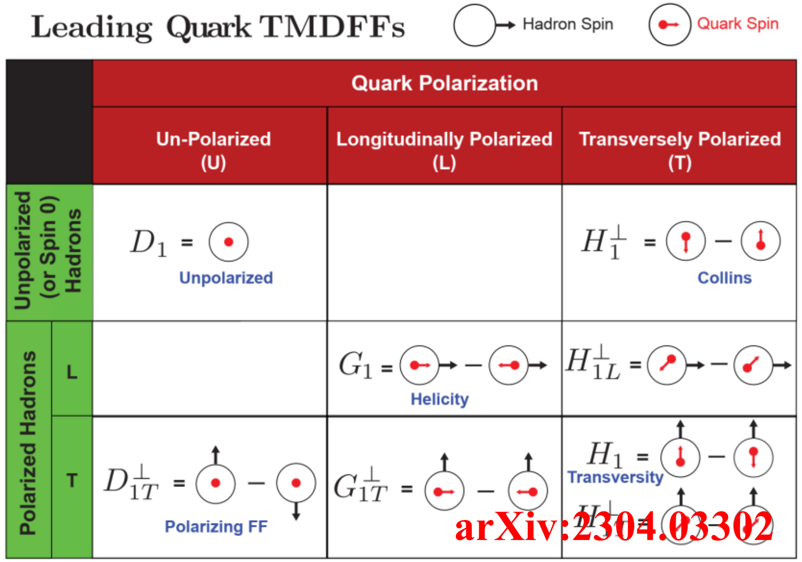
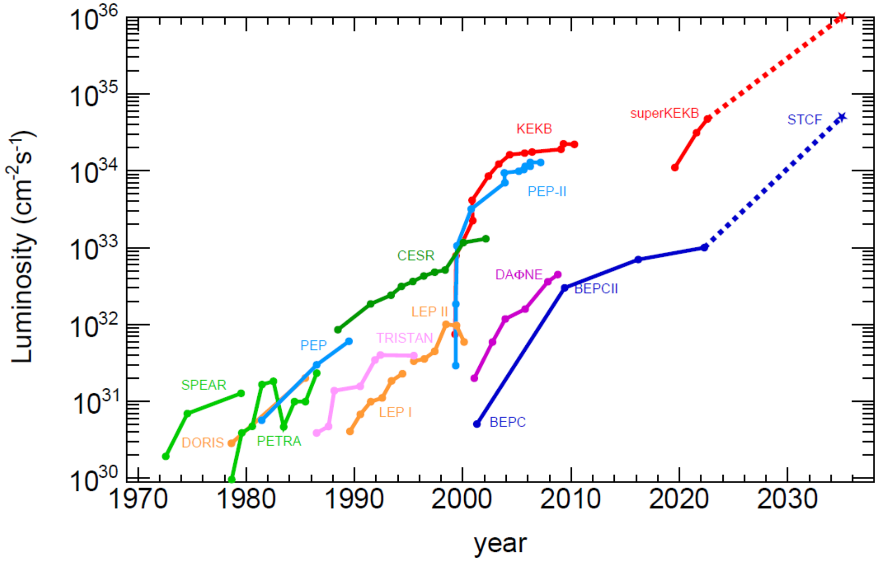
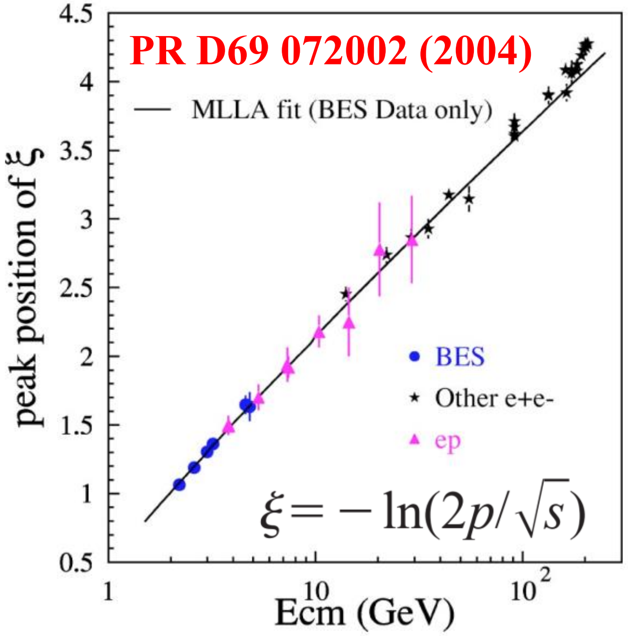
Inclusive hadron production at BESIII

鄢文标(中国科学技术大学)



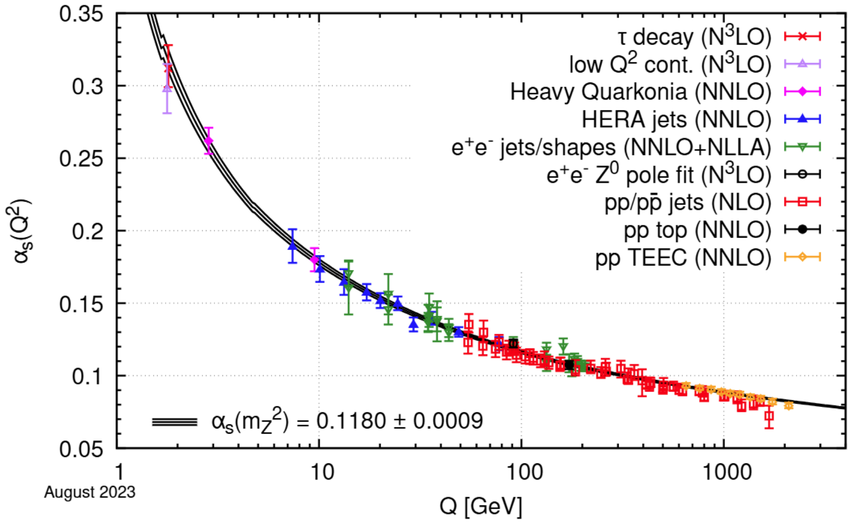
arXiv:2304.03302

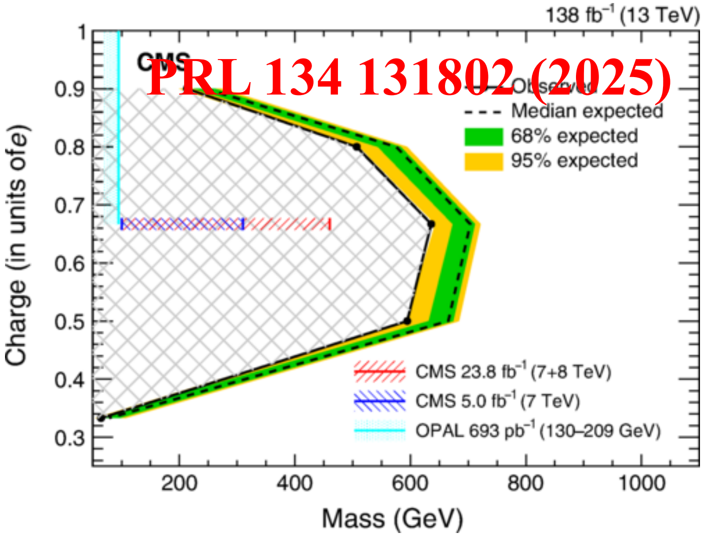




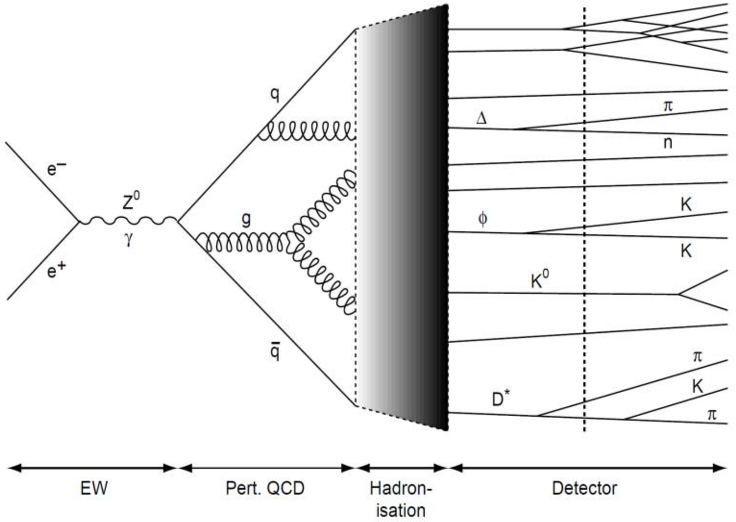
第一届中国电子离子对撞机相关物理年会，2026.04.21，青岛

QCD: Asymptotic freedom & confinement

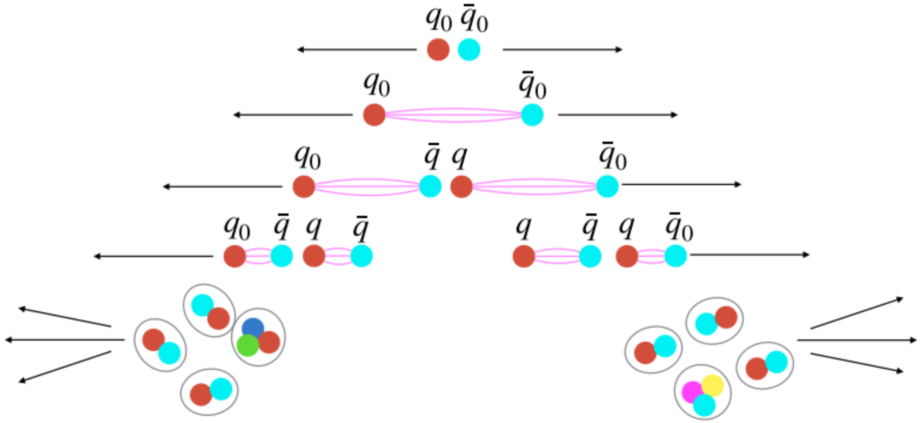




PRL 134 131802 (2025)



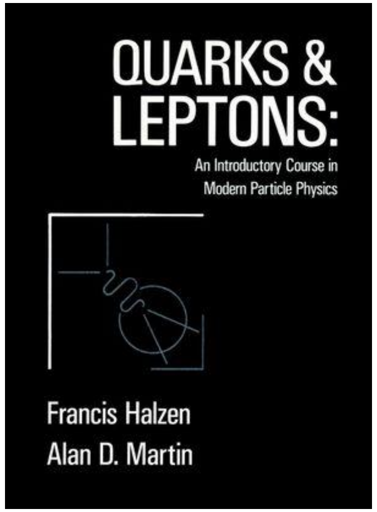
- QCD coupling constant $\alpha_s(Q)$
  - ✓ high Q: asymptotic freedom, perturbative QCD
  - ✓ low Q: non-perturbative QCD
- Confinement: partons do not exist as free particles, but are always confined in hadron
- Essence of confinement ? why & how ?
  - ✓ hadronization models & Fragmentation function
  - ✓ LPHD: Local Parton Hadron Duality hypothesis
  - ✓ machine learning for hadronization

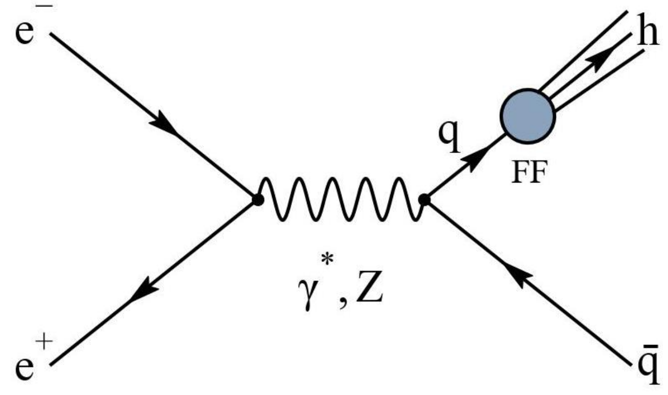


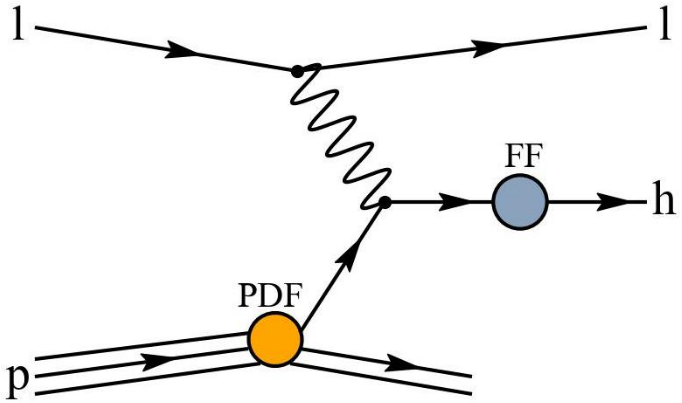
Eur. Phys. J. C85 16(2025)

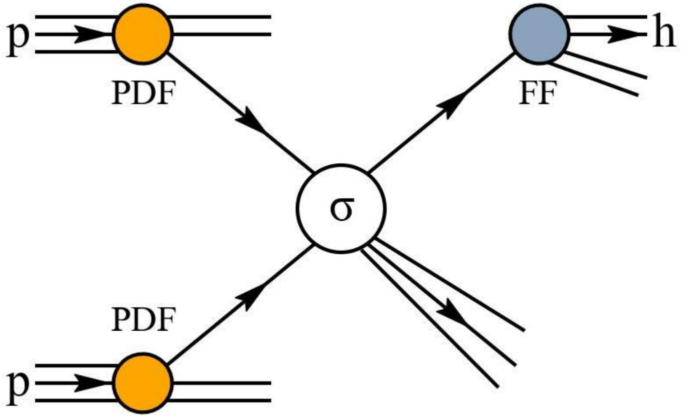
Fragmentation function: integrated $D_1^h(z)$

- Fragmentation function $D_q^h(z)$: probability that hadron h is found in the debris of a parton carrying a fraction z of parton's energy
- Consequence of confinement
- FF: QCD first principle (NOT YET)
  - ✓ FF evolution function: DGLAP
  - ✓ fitting: parametrization & experimental data
  - ✓ universality: e^+e^-, DIS, pp, $p\bar{p}$ data
- FFs contribute to virtually all processes
  - ✓ e^+e^-: the cleanest process for FF study









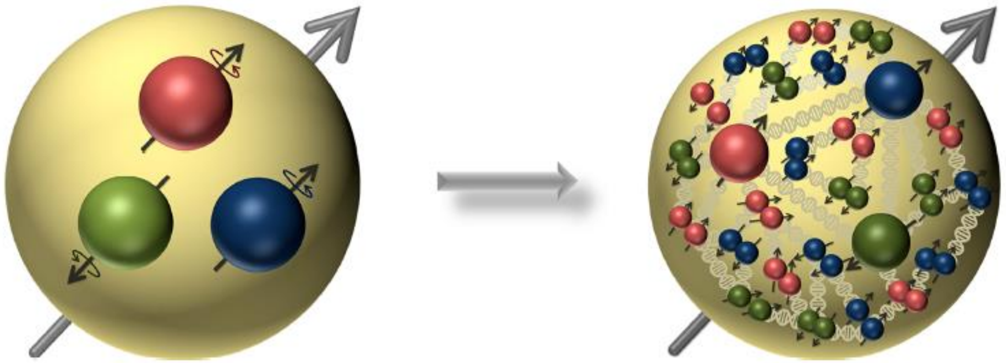
FFs with quark/hadron polarization

Hadron polarization	Quark polarizatiom @ PPNP 91 136 (2016)		
	Unpolarized	Longitudinally	Transversely
Unpolarized	D_1^h		$H_1^{\perp h}$
Longitudinally		G_1^h	$H_{1L}^{\perp h}$
Transversely	$D_{1T}^{\perp h}$	G_{1T}^h	H_1^h $H_{1T}^{\perp h}$

$$D_{hq^\uparrow}(z, P_{h\perp}) = D_1^q(z, P_{h\perp}^2) + H_1^{\perp q}(z, P_{h\perp}^2)\frac{(\hat{\mathbf{k}} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{zM_h}$$

- **Theoretically many more, in particular with polarized hadrons in the final state and transverse momentum dependence (TMD)**

FFs for EIC & EicC

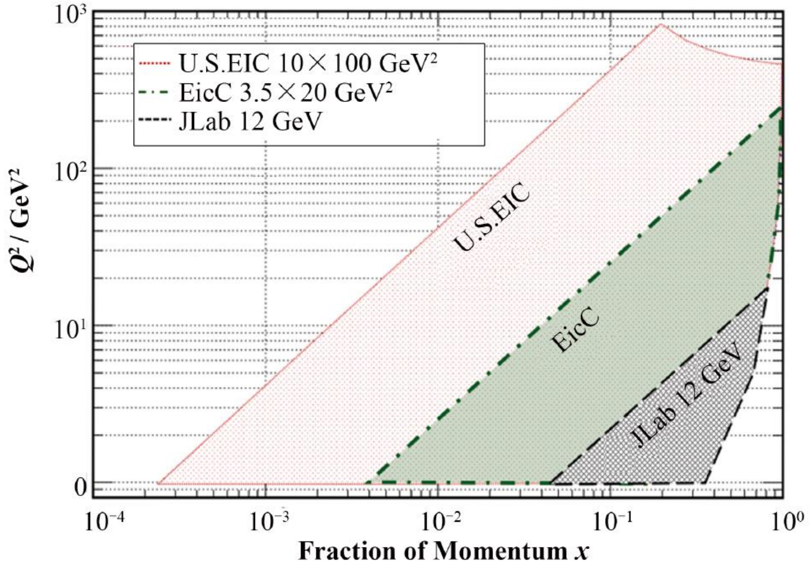


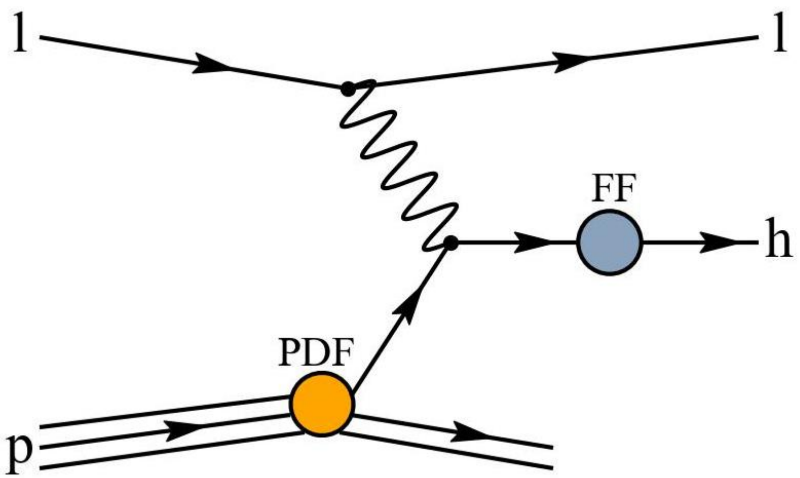
Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

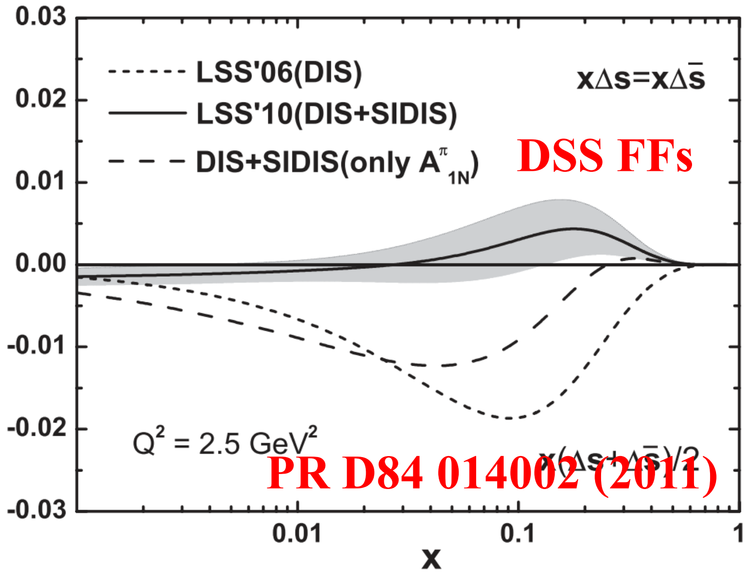
arXiv:2304.03302

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

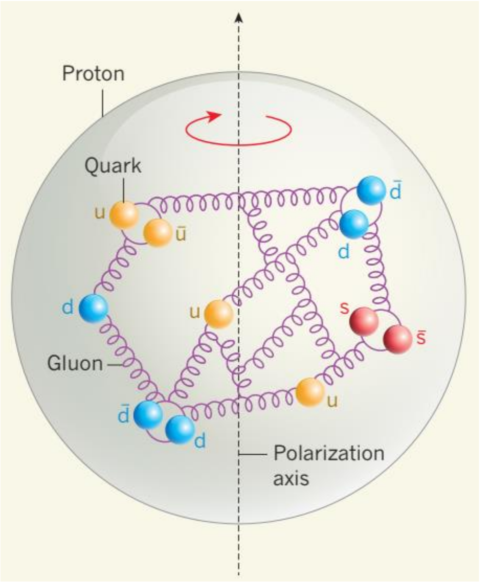


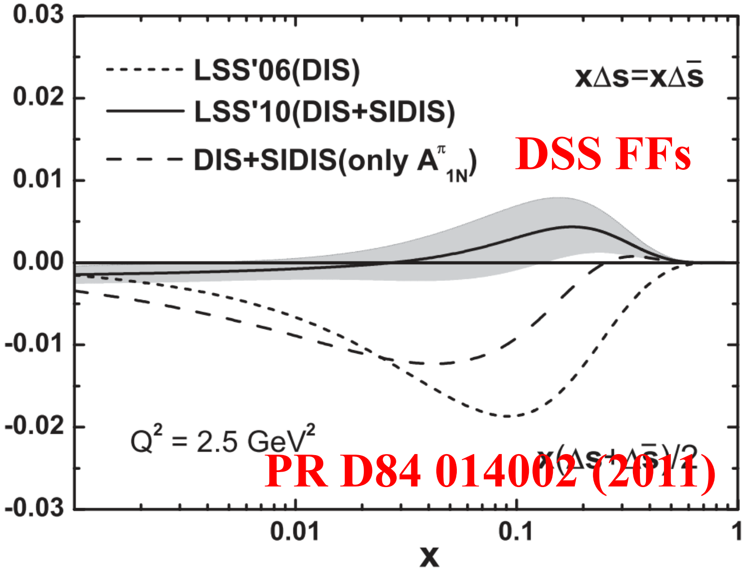


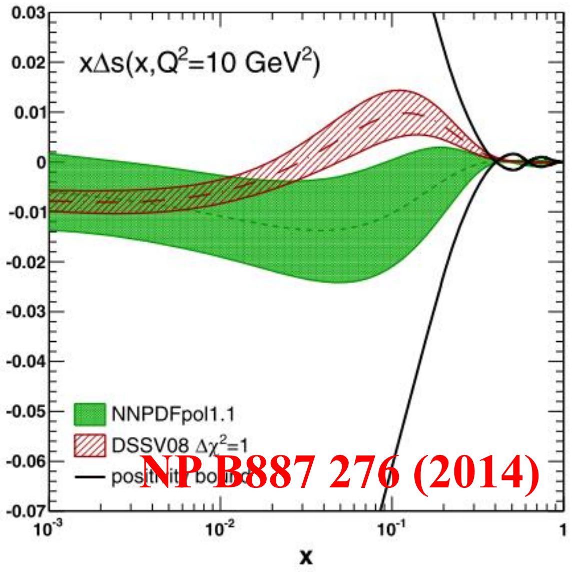


Precise knowledge of FFs will be crucial

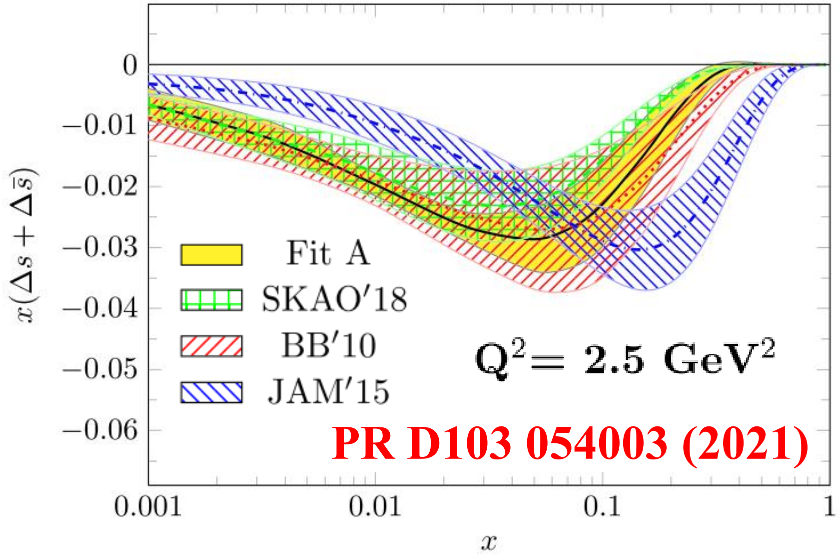
Strange quark polarization puzzle



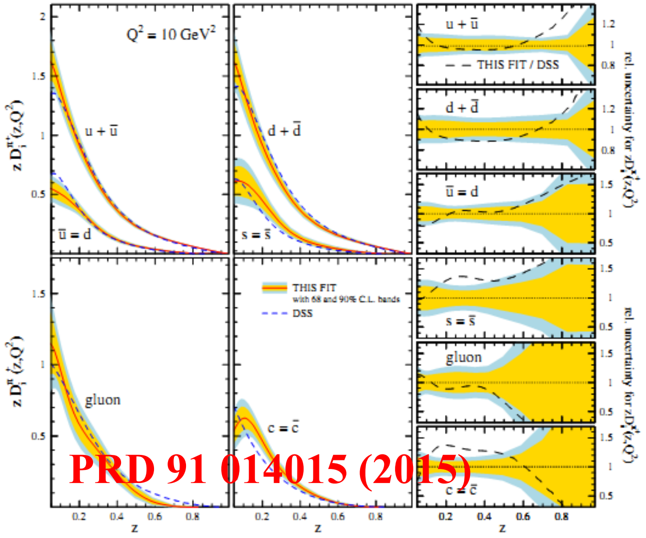


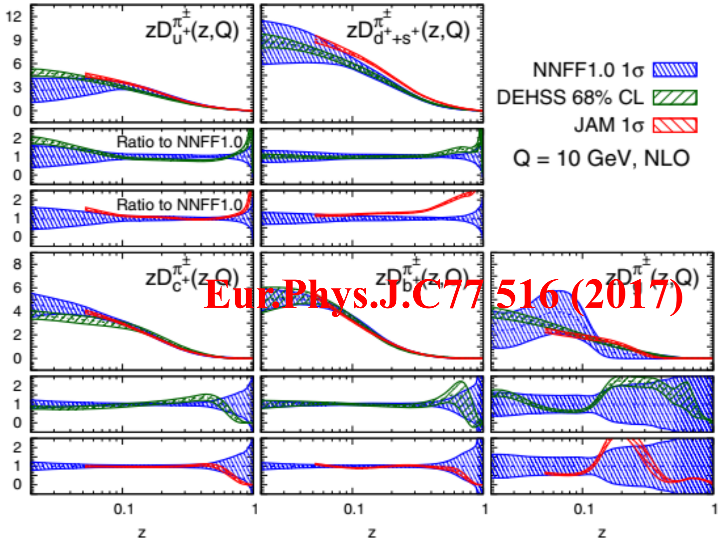


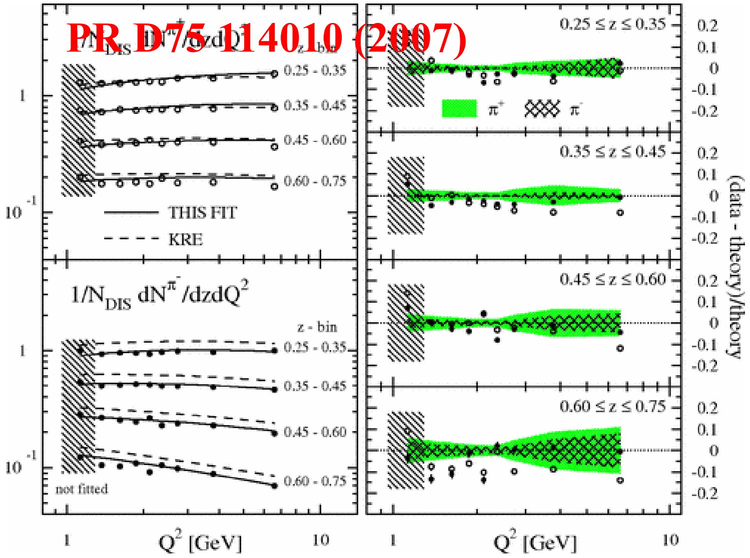
- Strange quark density function: $\Delta s(x)+\Delta\bar{s}(x)$
  - ✓ inclusive DIS: only proton PDF
    - a. negative for all values of x
  - ✓ semi-inclusive DIS: proton PDF & kaon FF
    - a. DSS FFs: positive for most of measured x
    - b. NNPDF FF: negative
    - c. JAM FFs: negative
- Reliable FFs knowledge ? important !!!



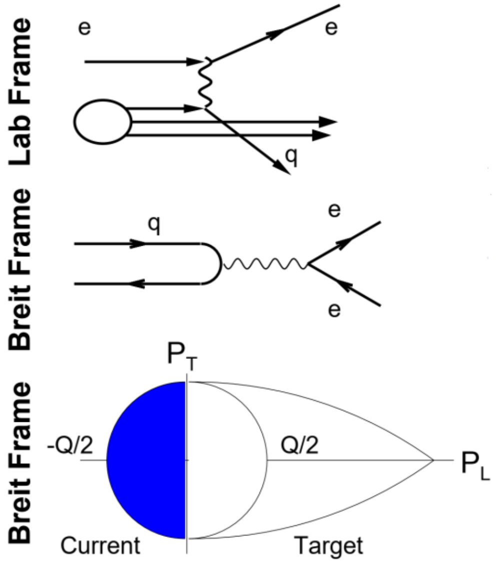
Pion FF: Best known FF



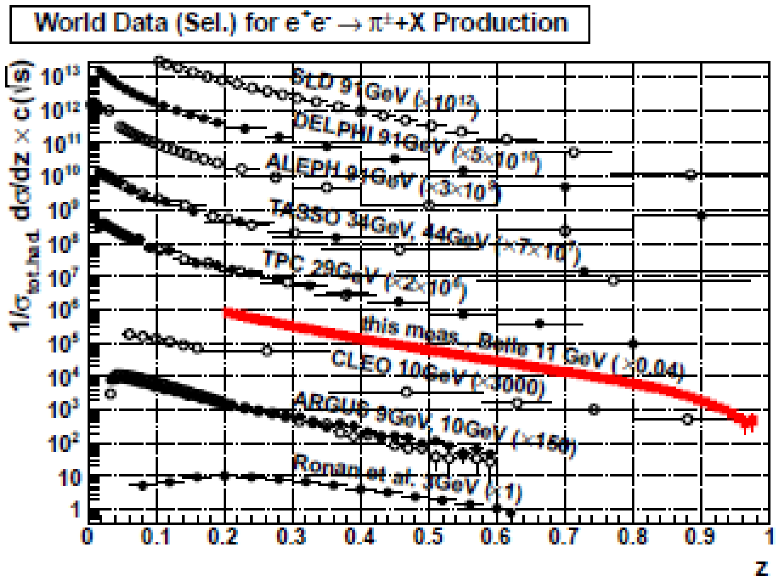


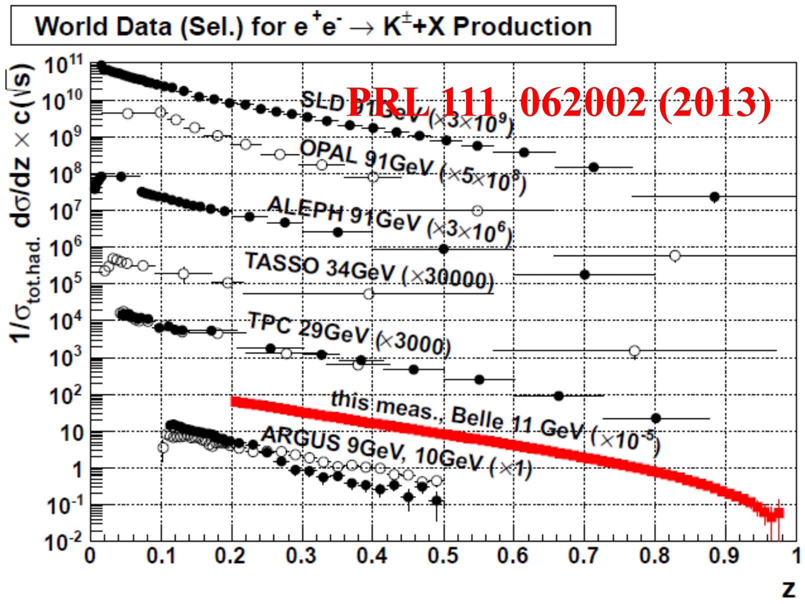


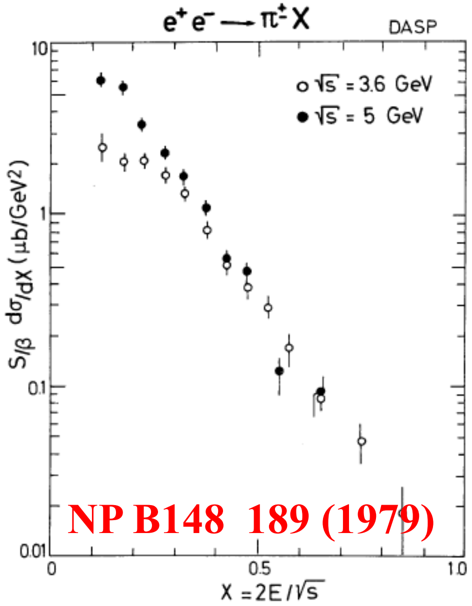
- For z≥0.8, uncertainty rapidly increase because of lack of experimental data
- Breit frame @ DIS: incoming quark scatters off photon and returns along same axis
  - ✓ current region is analogous to half of e^+e^- collision
  - ✓ Born level: DIS Q = e^+e^- $\sqrt{s}$
- DSS FFs: HERMES ep pion data at 10% level
  - ✓ describe BESIII data at ?% level



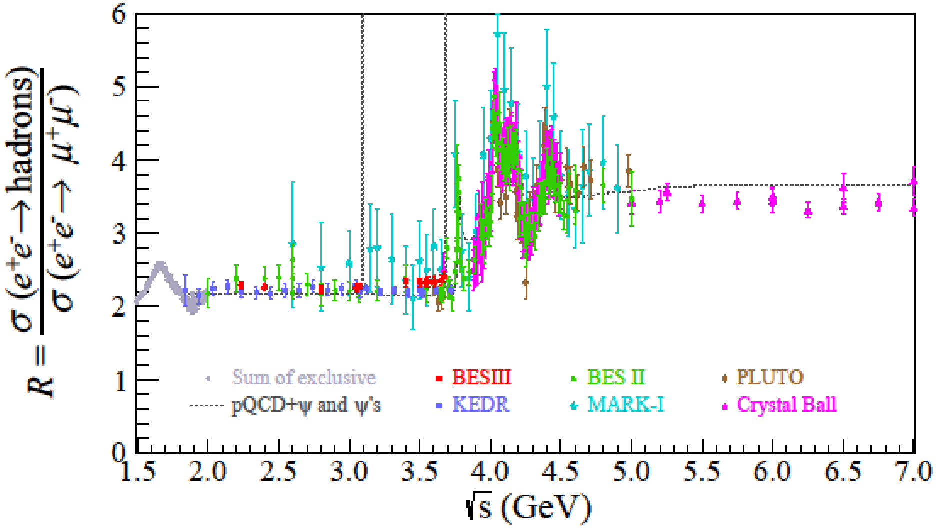
World π & K data @ e^+e^- collision



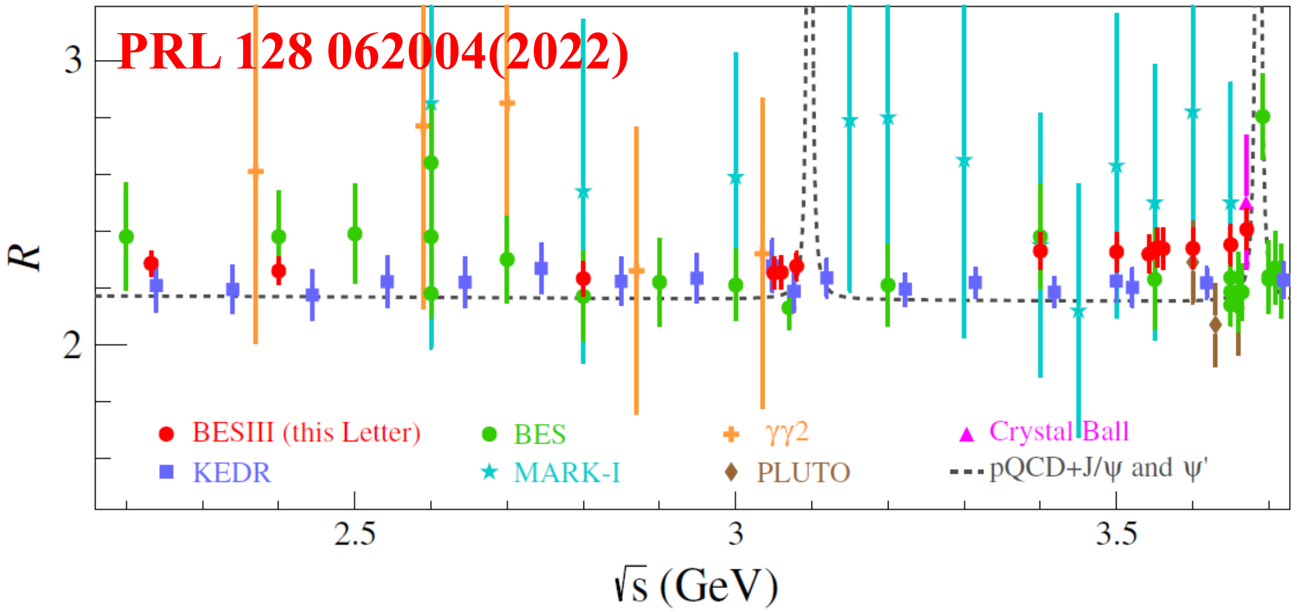


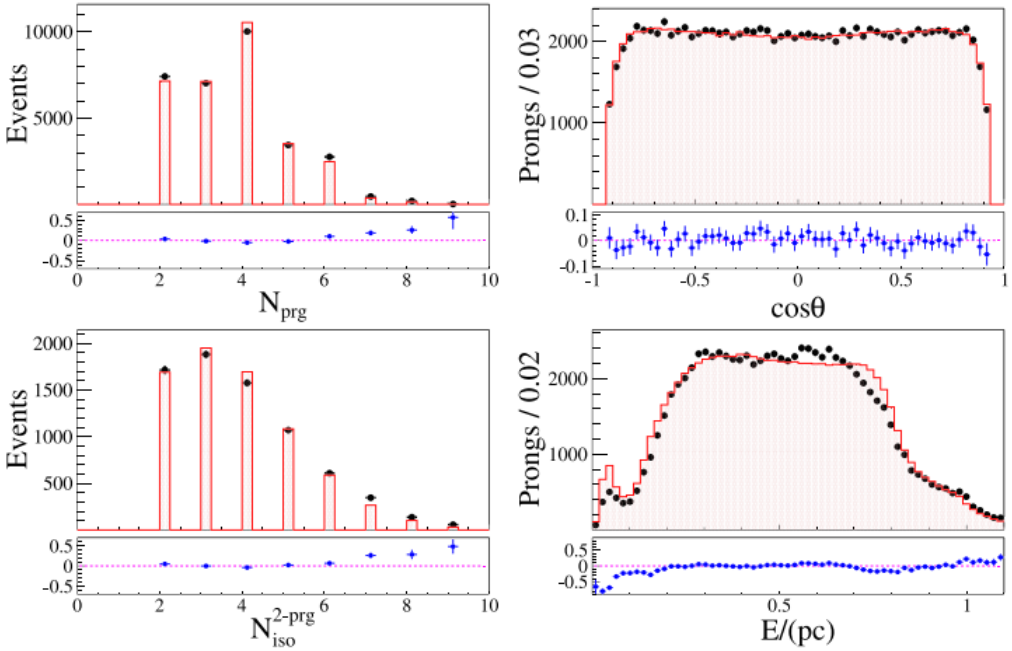


- Data sets at high z ?
- Data sets at $\sqrt{s} < 10$ GeV e^+e^- collision ?
- Charged π data by DASP
  - ✓ stat. uncertainty: 18%
- BESIII: opportunities & challenges
  - ✓ e^+e^-: the cleanest process for FF study



The challenge @ $e^+e^- \to q\bar{q} \to$ hadrons

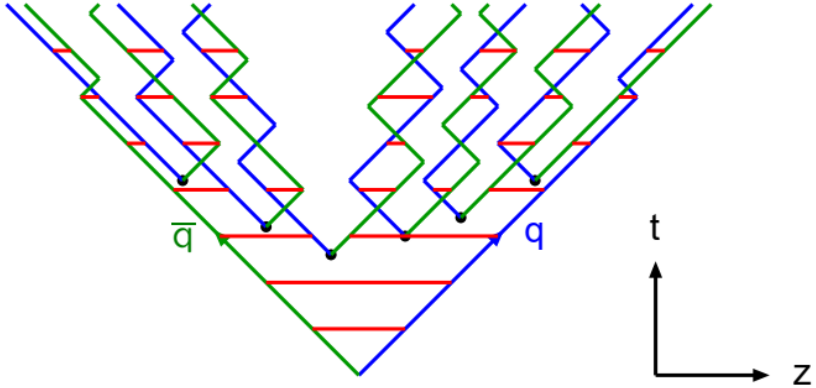




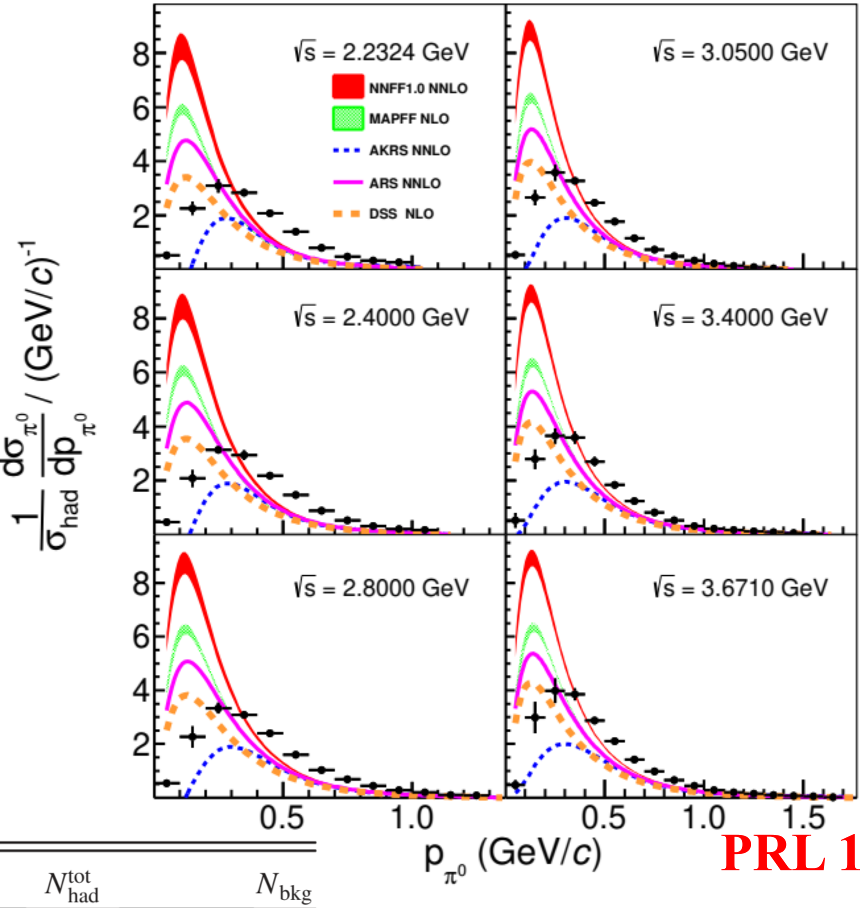
- **MC generator of $e^+e^- \to q\bar{q} \to$ hadrons**
- **Reliable hadronic event generator @ low $\sqrt{s}$**
  - ✓ **left-right asymmetry ⇒ scaling function**

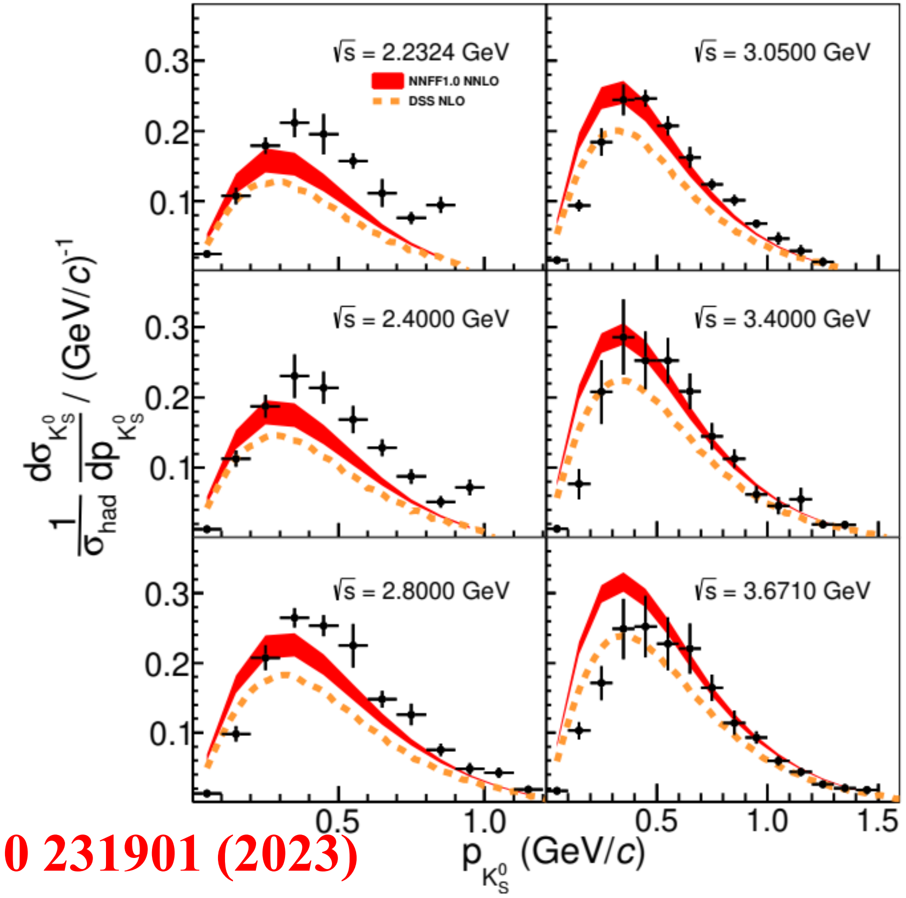
$$f(z) = \frac{N}{z}(1-z)^a \exp\left(-\frac{bm^2}{z}\right)$$

  - ✓ **special at low $\sqrt{s}$: Lund area law & ConExc**
  - ✓ **tuning MC parameters**



$e^+e^- \rightarrow \pi^0/K^0_S + X$ @ BESIII



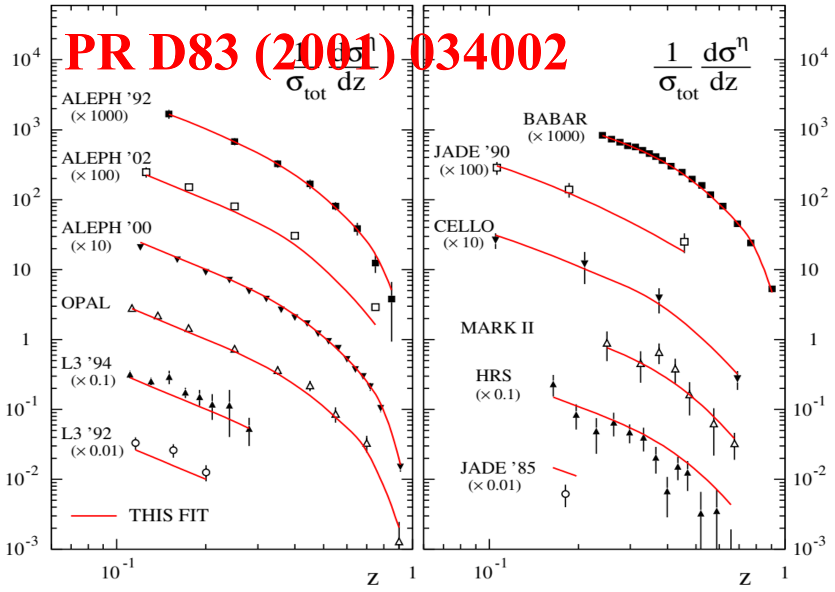


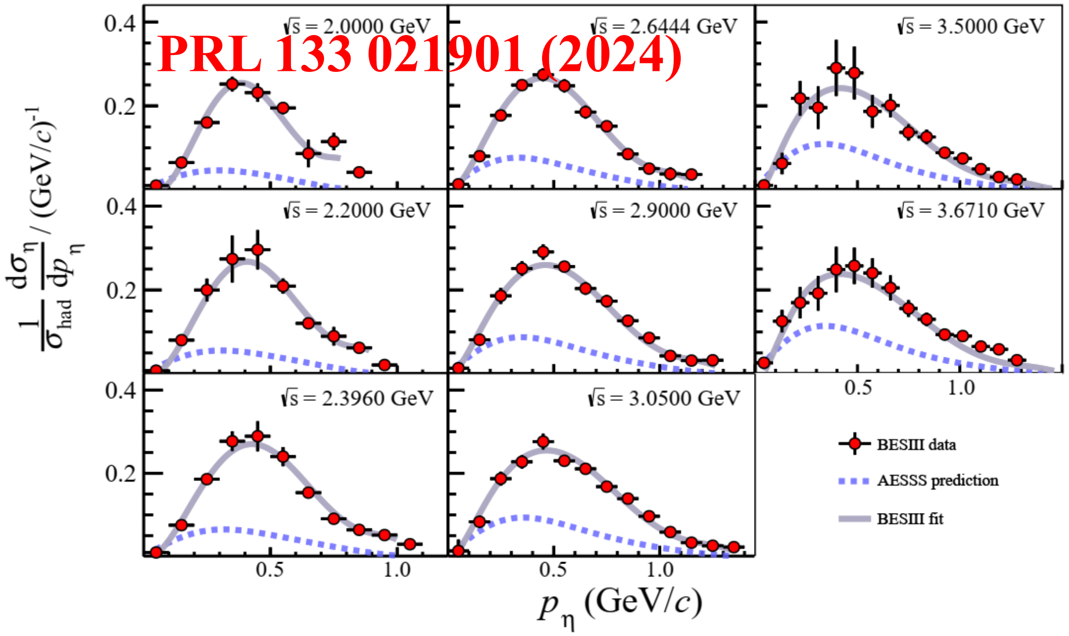
PRL 130 231901 (2023)

$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb^{-1})	N^{tot}_{had}	N_{bkg}
2.2324	2.645	83227	2041
2.4000	3.415	96627	2331
2.8000	3.753	83802	2075
3.0500	14.89	283822	7719
3.4000	1.733	32202	843
3.6710	4.628	75253	6461

- **Dominated uncertainty: MC generator**
- **Inclusive π^0 production: surprise , disagreement !!!**
- **Inclusive K^0_S production: not so bad**

$e^+e^- \to \eta + X$ @ BESIII



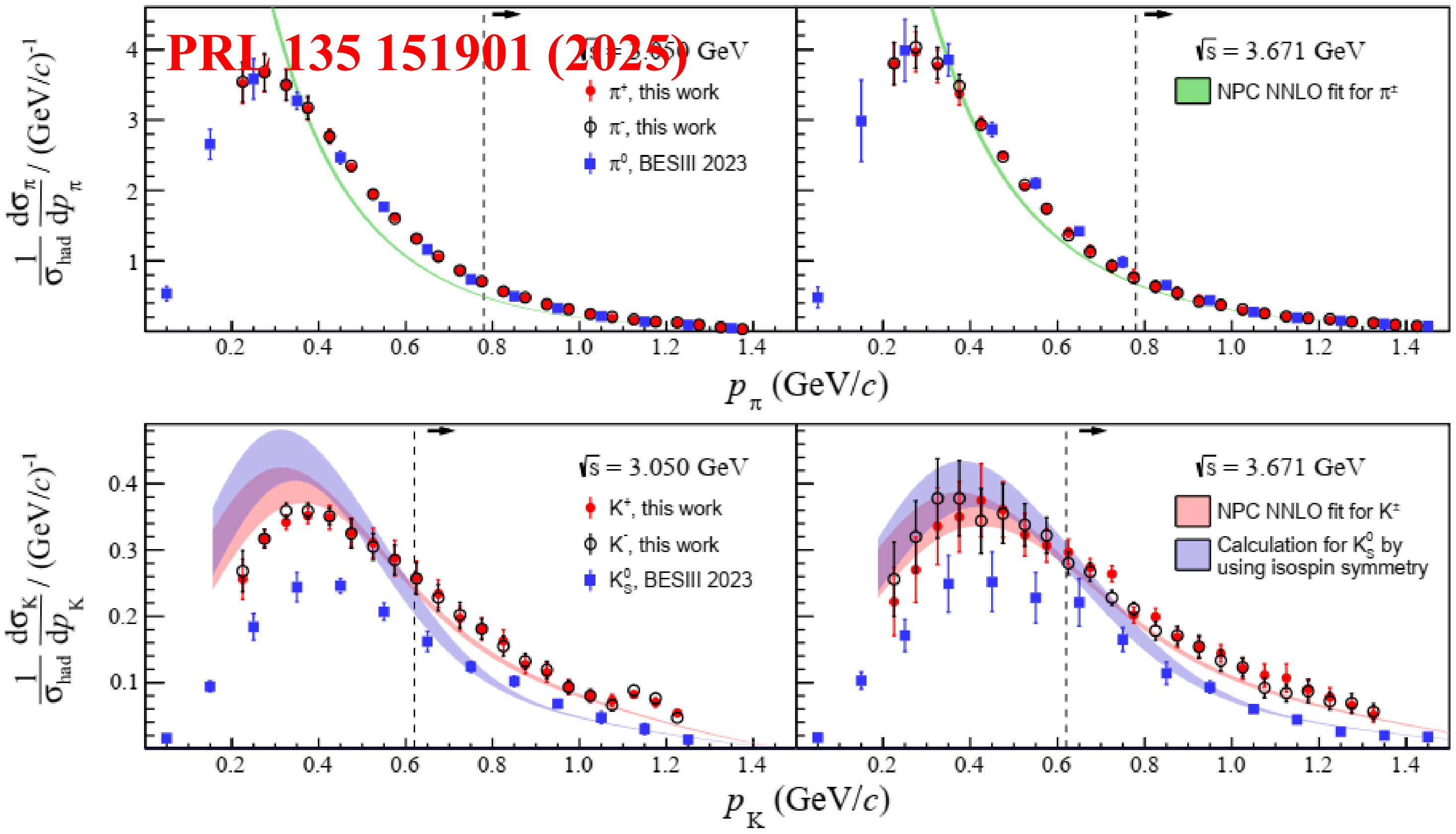


- η FF @ NLO: data at $\sqrt{s} > 10$ GeV e^+e^-
  - ✓ missing theory uncertainty
- PRD83 (2001) 034002 vs. BESIII data
  - ✓ underestimate !
- BESIII fit: PR D111 034030 (2025)
  - ✓ NNLO accuracy, hadron mass & higher twist
  - ✓ $\sqrt{s} > 10$ GeV e^+e^- data + BESIII data

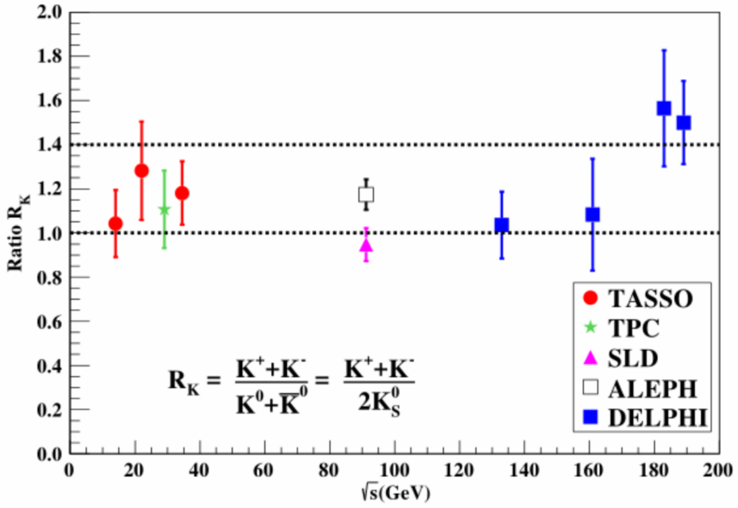
$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb^{-1})	$N_{\rm had}^{\rm tot}$	$N_{\rm bkg}$
2.0000	10.074	350 298 ± 591	8722 ± 93
2.2000	13.699	445 019 ± 666	10 737 ± 103
2.3960	66.869	1 869 906 ± 1365	47 550 ± 218
2.6444	33.722	817 528 ± 902	21 042 ± 145
2.9000	105.253	2 197 328 ± 1478	56 841 ± 238
3.0500	14.893	283 822 ± 531	7719 ± 87
3.5000	3.633	62 670 ± 249	1691 ± 41
3.6710	4.628	75 253 ± 273	6461 ± 80

Inclusive $\pi^{\pm}$ & $K^{\pm}$ vs. pQCD

PRL 135 151901 (2025)



- Inclusive pion cross section
  - ✓ $\pi^+ \approx \pi^- \approx \pi^0$
  - ✓ Isospin symmetry
- Inclusive kaon cross section
  - ✓ $K^+ \approx K^- \approx 1.4\ K_S$
  - ✓ Large isospin violation ?



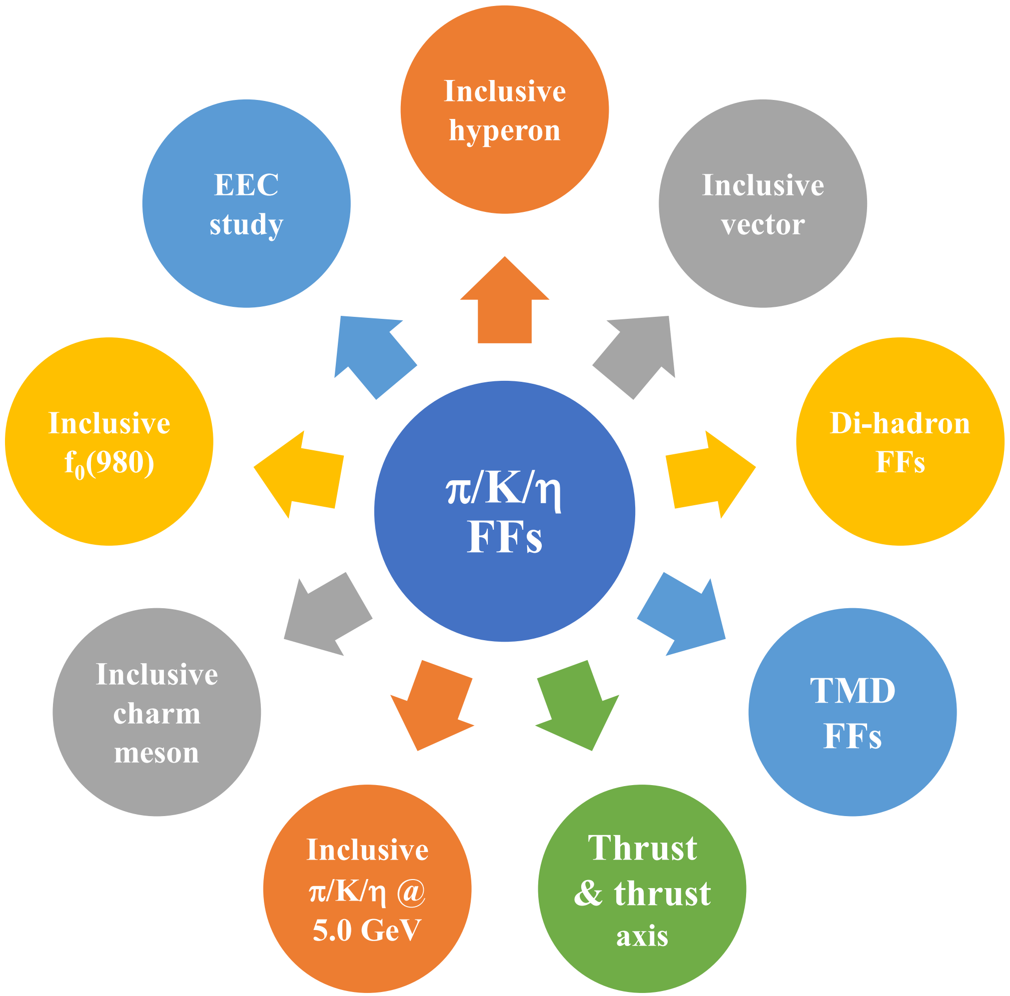
$D_i^{\pi^0}(z,Q) = \frac{1}{2} D_j^{\pi^\pm}(z,Q)$ @NPB 803 42 (2008)

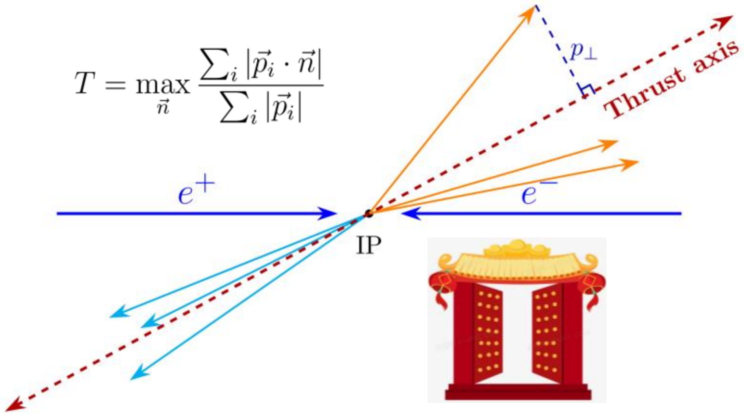
$D_i^{K_S^0}(z,Q) = \frac{1}{2} D_j^{K^\pm}(z,Q)$ $\begin{cases} \text{if i = d(u), j = u(d)} \\ \text{otherwise i = j} \end{cases}$

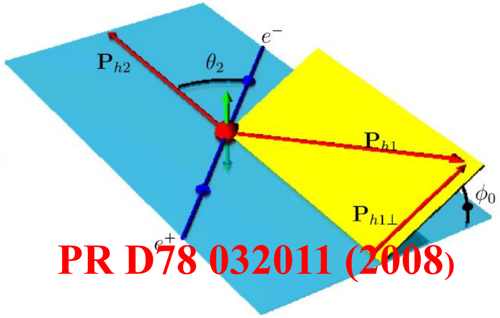
- NPC NNLO: $\sqrt{s}$ > 3.0 GeV & E_h > 0.8 GeV @ BESIII
- PRL 135 041902 (2025): Validity of QCD factorization and pQCD calculation at energy scales down to 3 GeV
- Test of isospin symmetry with FF fitting

To do list @ BESIII

Theoretically many more, in particular with polarized hadrons in the final state and transverse momentum dependence (TMD)







PR D78 032011 (2008)

PR D96 032005 (2017)

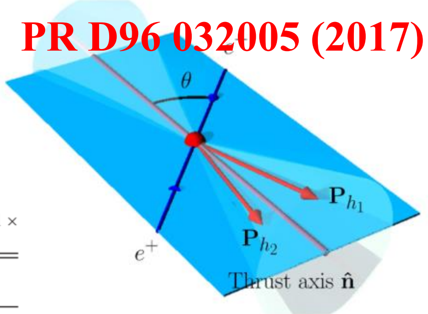


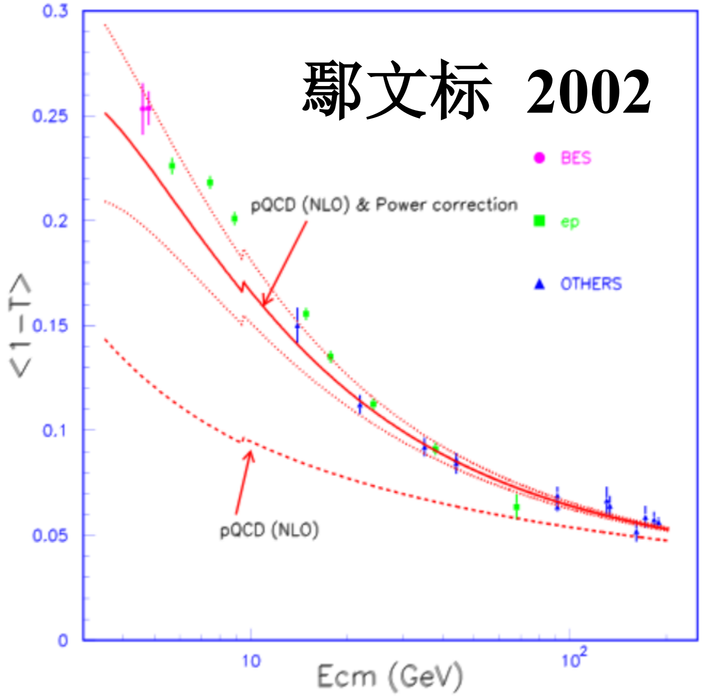
TABLE II. The 18 leading twist components of the FFs for quark fragments to spin-1 hadrons. The symbol × means that the corresponding FF disappears after the integration over transverse momentum.

Quark polarization	Hadron polarization	TMD FFs	Integrated over $\vec{k}_{F\perp}$	Name
U	U	$D_1(z, k_{F\perp})$	$D_1(z)$	Number density
	T	$D^{\perp}_{1T}(z, k_{F\perp})$	×	
	LL	$D_{1LL}(z, k_{F\perp})$	$D_{1LL}(z)$	Spin alignment
	LT	$D^{\perp}_{1LT}(z, k_{F\perp})$	×	
	TT	$D^{\perp}_{1TT}(z, k_{F\perp})$	×	
L	L	$G_{1L}(z, k_{F\perp})$	$G_{1L}(z)$	Spin transfer (longitudinal)
	T	$G^{\perp}_{1T}(z, k_{F\perp})$	×	
	LT	$G^{\perp}_{1LT}(z, k_{F\perp})$	×	
	TT	$G^{\perp}_{1TT}(z, k_{F\perp})$	×	
T	U	$H^{\perp}_{1}(z, k_{F\perp})$	×	Collins function
	$T(\parallel)$	$H_{1T}(z, k_{F\perp})$	$H_{1T}(z)$	Spin transfer (transverse)
	$T(\perp)$	$H^{\perp}_{1T}(z, k_{F\perp})$		
	L	$H^{\perp}_{1L}(z, k_{F\perp})$	×	
	LL	$H^{\perp}_{1LL}(z, k_{F\perp})$	×	
	[illegible]	[illegible]	$H_{1LT}(z)$	
	TT	$H^{\perp}_{1TT}(z, k_{F\perp}), H'^{\perp}_{1TT}(z, k_{F\perp})$	×, ×	

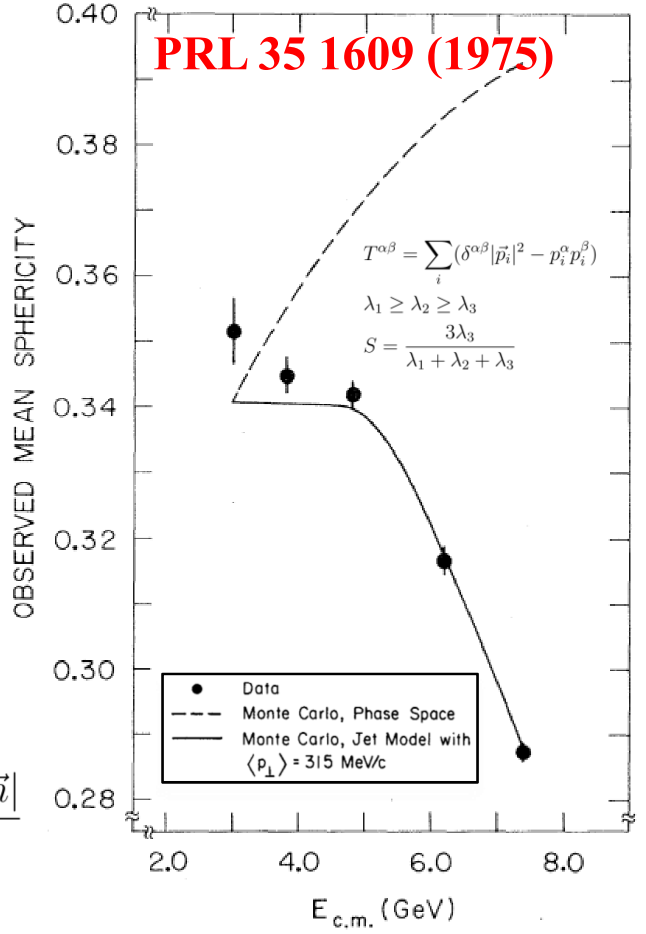
PR D94 034003 (2016)

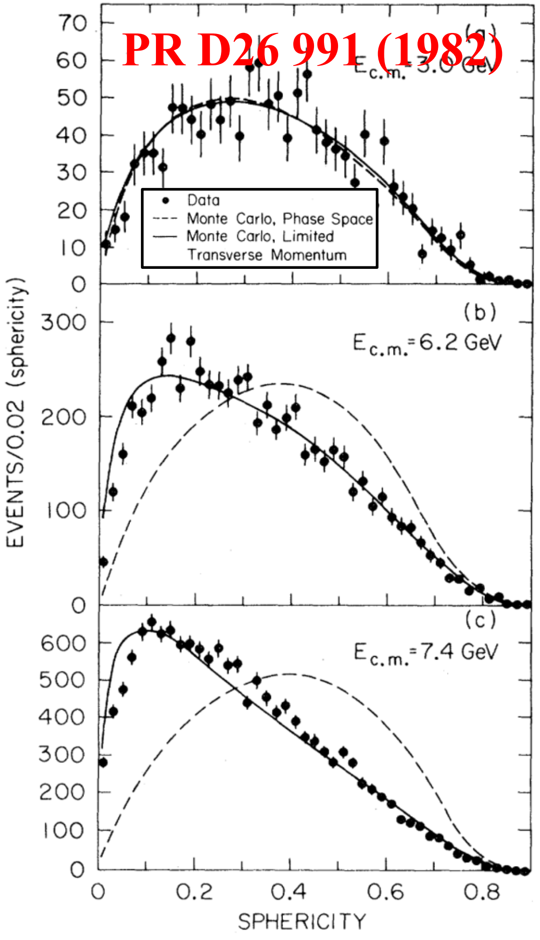
Thrust axis: more is different

- **BESIII: correctly thrust axis ?**
- **Phase space model vs. jet model**
  - ✓ **reliable @ $\sqrt{s} \geq 5$ GeV ?**
- **Usage of thrust axis**
  - ✓ **two hadron $\in / \notin$ of a quark**
  - ✓ **TMD physics topics @ BESIII**



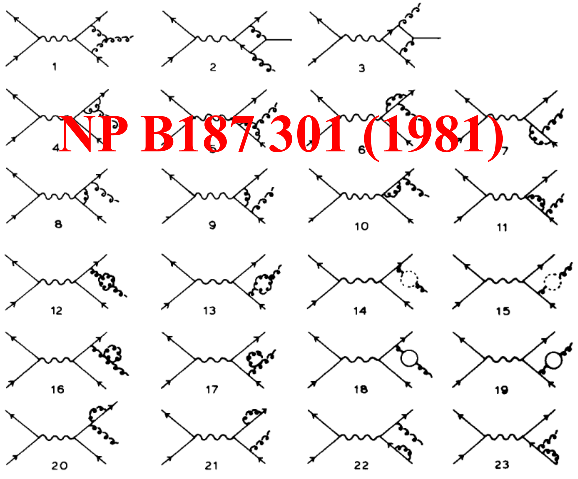
$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$





Edward Farhi @ PRL 39 1587 (1977): Hadronic jets have been observed in electron positron annihilation experiments at energies above 5 or 6 GeV

Thrust $\tau \equiv 1 - T$: exp. data @ low $\sqrt{s}$???



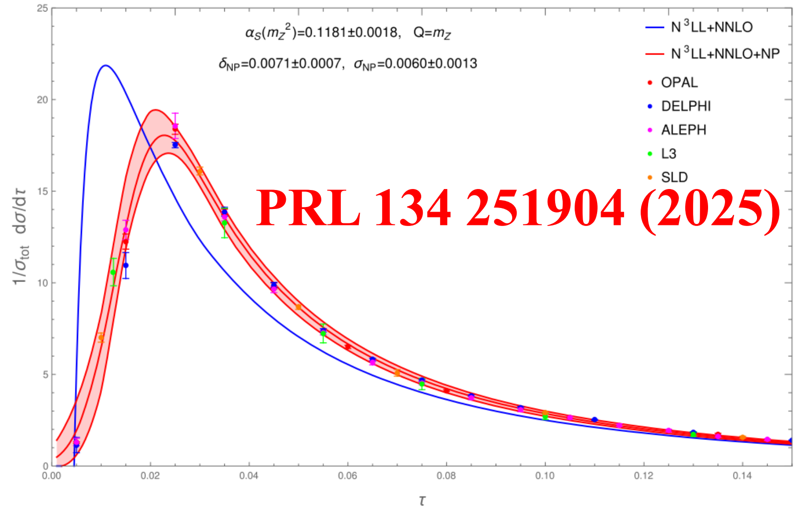
NP B187 301 (1981)

Fig. 2. Feynman diagrams for the virtual contributions in order α_s^2.

TABLE I. Partonic contributions to the thrust distribution in perturbative QCD. PRL 99 132002 (2007)

LO	$\gamma^* \to q\bar{q}g$	tree-level
NLO	$\gamma^* \to q\bar{q}g$	one-loop
	$\gamma^* \to q\bar{q}gg$	tree-level
	$\gamma^* \to q\bar{q}q\bar{q}$	tree-level
NNLO	$\gamma^* \to q\bar{q}g$	two-loop
	$\gamma^* \to q\bar{q}gg$	one-loop
	$\gamma^* \to q\bar{q}q\bar{q}$	one-loop
	$\gamma^* \to q\bar{q}q\bar{q}g$	tree-level
	$\gamma^* \to q\bar{q}ggg$	tree-level

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$



PRL 134 251904 (2025)

Eur. Phys. J. C78 498 (2018)

Experiment	$\sqrt{s}$, GeV, data	$\sqrt{s}$, GeV, MC	Events
SLD [47]	91.2 (91.2)	91.2	60,000
OPAL [50]	91.2 (91.2)	91.2	336,247
OPAL [51]	91.2 (91.2)	91.2	128,032
L3 [48]	91.2 (91.2)	91.2	169,700
DELPHI [49]	91.2 (91.2)	91.2	120,600
TOPAZ [52]	59.0–60.0 (59.5)	59.5	540
TOPAZ [52]	52.0–55.0 (53.3)	53.3	745
TASSO [53]	38.4–46.8 (43.5)	43.5	6434
TASSO [53]	32.0–35.2 (34.0)	34.0	52,118
PLUTO [58]	34.6 (34.6)	34.0	6964
JADE [54]	29.0–36.0 (34.0)	34.0	12,719
CELLO [57]	34.0 (34.0)	34.0	2600
MARKII [56]	29.0 (29.0)	29.0	5024
MARKII [56]	29.0 (29.0)	29.0	13,829
MAC [55]	29.0 (29.0)	29.0	65,000
TASSO [53]	21.0–23.0 (22.0)	22.0	1913
JADE [54]	22.0 (22.0)	22.0	1399
CELLO [57]	22.0 (22.0)	22.0	2000
TASSO [53]	12.4–14.4 (14.0)	14.0	2704
JADE [54]	14.0 (14.0)	14.0	2112

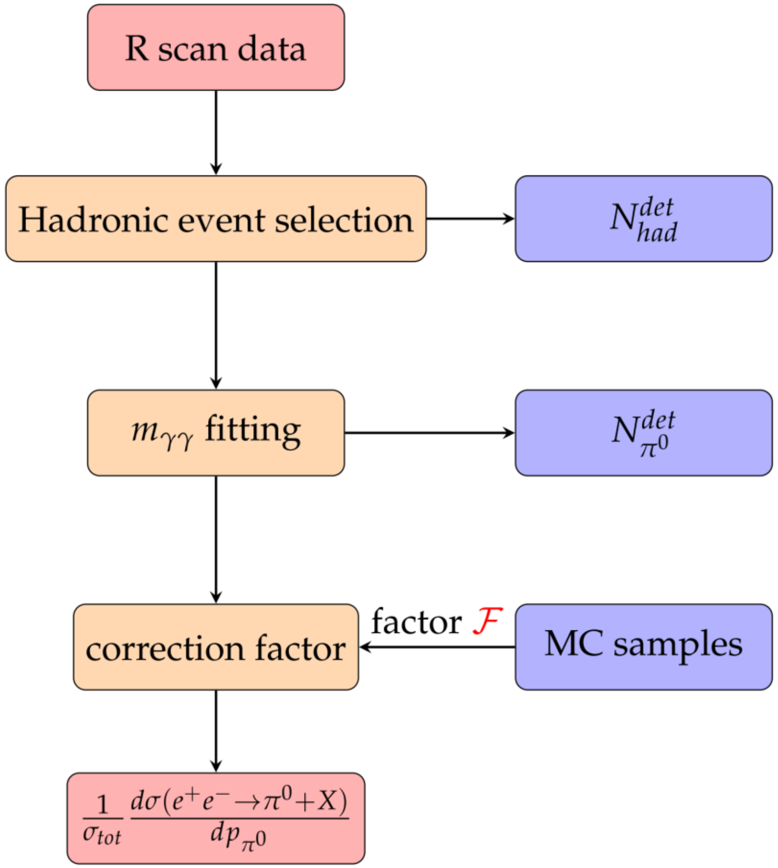
- **Fixed order calculation, NLO @ 1981 ⇨ NNLO @ 2007**
- **Small τ (τ << 1) due to soft & collinear gluons**
  - ✓ **re-summation of large $\ln(1/\tau)$ to all orders in α_s**
  - ✓ **N^{4}LL in full QCD @ PRL 134 251904 (2025)**
- **NNLO & N^{4}LL vs. NNLO & N^{3}LL @ Z^0: few permille**
  - ✓ **higher order: more complex & difficulty**
- **Experiment results @ low $\sqrt{s}$: sensitive to higher order of α_s**
- **BESIII/Belle/Belle II: O(1-100M) $q\bar{q}$ events, not yet !**

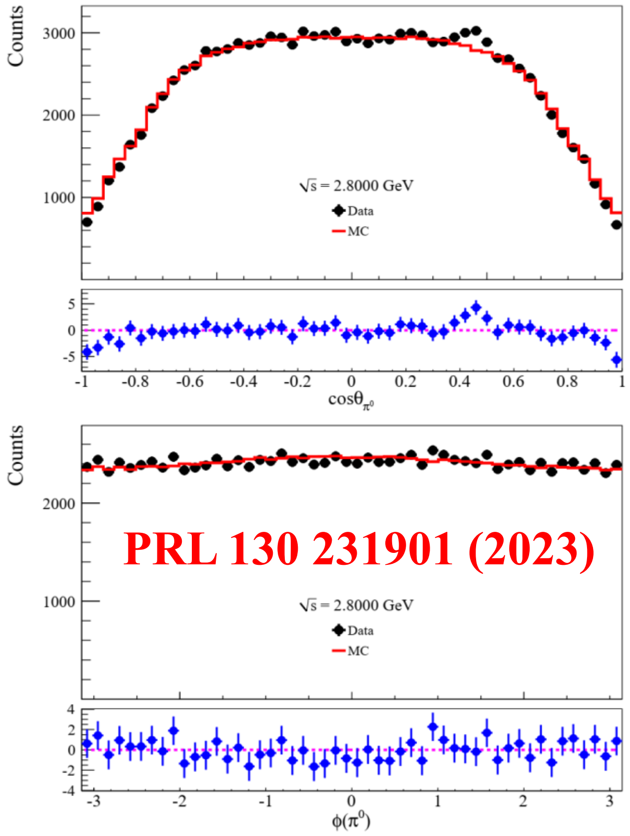
Summary

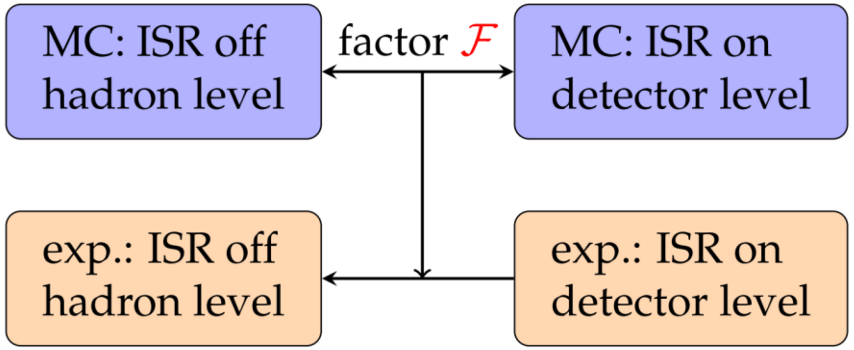
- The FF play a crucial role in describing hadronization process, and understanding of non-perturbative QCD dynamics.
- Inclusive π^0 & K_s: Large discrepancy with theory calculation. PRL 130 231901 (2023)
- Inclusive η: FF fitting with NNLO accuracy, hadron mass correction & higher twist, could describe BESIII data. PRL 133 021901 (2024)
- Inclusive π & K: Validity of QCD factorization and pQCD calculation at energy scales down to 3 GeV, isospin symmetry with FF fitting. PRL 135 151901 (2025)
- To do list @ BESIII
  - ✓ FFs with hadron: beyond pion/kaon/eta
  - ✓ TMD FFs & Di-hadrons FFs: possible
  - ✓ spin effect at hadronization: any surprises ?
  - ✓ thrust & EEC: old & new topics

Measurment of $e^+e^- \to \pi^0 + X$ @ BESIII

$$\frac{1}{\sigma_{total}}\frac{d\sigma(e^+e^- \to \pi^0 + X)}{dp_{\pi^0}} \Rightarrow \frac{1}{N_{had}}\frac{N_{\pi^0}}{\Delta p_{\pi^0}} = \mathcal{F}\frac{1}{N_{had}^{det}}\frac{N_{\pi^0}^{det}}{\Delta p_{\pi^0}}$$







$$\mathcal{F} = \frac{N_{\pi^0}^{true}(off)}{N_{had}^{true}(off)} \Big/ \frac{N_{\pi^0}^{det}(on)}{N_{had}^{det}(on)}$$

- **Bin-by-bin unfolding method**
- **Correction factor $\mathcal{F}$**
  - ✓ **event selection efficiency**
  - ✓ **radiative correction**