The generalized parton distribution functions and their determinations

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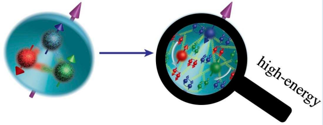
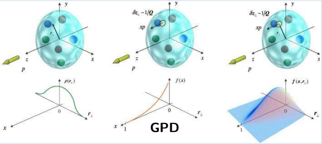
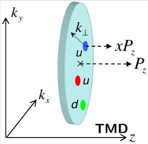
Internal structures of hadrons

- Hadrons are composite particles with complicated internal structures

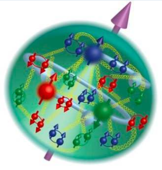


- PDFs only encode longitudinal information; GPDs and TMDs include transverse information





- GPDs are crucial for understanding the spin structure of hadrons

$\sum_{s,j}$  $\overset{?}{=} \frac{1}{2}$

Unsolved problem in physics:

? *How do the quarks and gluons carry the spin of protons?*

Spin sum rules

- **Proton is a fermion and is the lowest energy baryon state** $\implies$ $s_p = 1/2$
- **It is also a composite particle made of quarks and gluons: reductionism** $s_p \stackrel{?}{=} s_q + s_g + \cdots$
- **spin sum rules:**

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

[A. Jaffe, A. Manohar (1990)] **intuitive, but gauge-dependent**

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

$$\frac{1}{2}\Delta\Sigma + L_q = \frac{1}{2}\int_{-1}^{1} dx\, x\,[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)]$$

$$J_g = \frac{1}{2}\int_{0}^{1} dx\,[H_g(x,\xi,t=0) + E_g(x,\xi,t=0)]$$

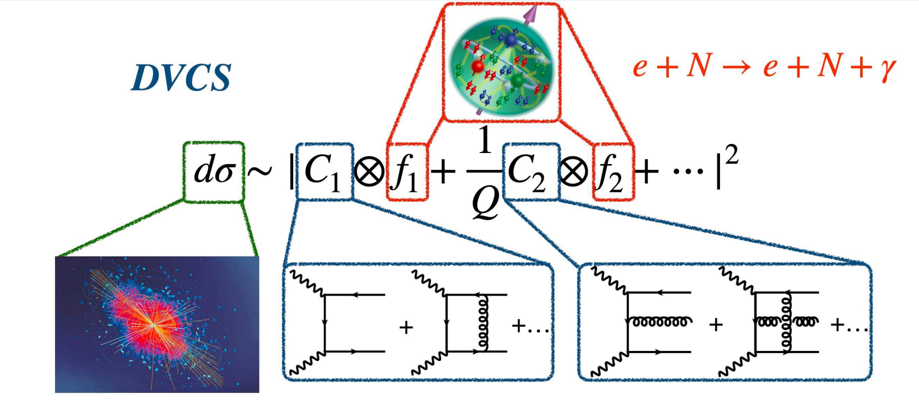
[X. Ji (1996)] **from gauge-independent Belinfante-improved EMT, each is μ-dependent**

- **GPDs and PDFs**

$H_{q,g}(x,0,0) = f_{q,g}(x)$ **GPD $\mapsto$ PDF,** $E_{q,g}$ **is genuine nonforward function**

Extracting structure information of hadrons from high-energy experiment

- **Theoretical foundation for accessing structural information of hadrons: factorization**



[X. Ji (1996), Herman Feshbach Prize (2016)]

- **One of the main scientific goal of JLAB 12, EIC, EicC**

Deeply Virtual Compton Scattering (DVCS)

- **Golden channel for extracting the Generalized Parton Distributions (GPDs)**

$$\gamma^*(q) + N(p) \to \gamma(q') + N(p') \qquad \gamma^*(q') \text{ for DDVCS process}$$

Kinematics: $q'^2 = 0,\ q^2 = -Q^2,\ t = \Delta^2 = (p' - p)^2,\ p^2 = p'^2 = m^2,\quad P_\mu = (p_\mu + p'_\mu)/2$

- **DVCS amplitude defined as (hadronic)**

$$\delta(p + q - p' - q')\mathcal{A}_{\mu\nu}(q, q', p) = i \int \frac{d^4x d^4y}{(2\pi)^4}\, e^{-iq\cdot x + iq'\cdot y}\, \langle p'|\, T\{j_\mu^{\rm em}(x) j_\nu^{\rm em}(y)\}\, |p\rangle$$

decomposable into helicity amplitudes [V. Braun, A. Manashov, B. Pirnay, (2012)]

$$\mathcal{A}_{\mu\nu}(q, q', p) = \varepsilon_\mu^+ \varepsilon_\nu^{*+} \mathcal{A}^{++} + \varepsilon_\mu^- \varepsilon_\nu^{*-} \mathcal{A}^{--} + \varepsilon_\mu^0 \varepsilon_\nu^{*+} \mathcal{A}^{0+} + \varepsilon_\mu^0 \varepsilon_\nu^{*-} \mathcal{A}^{0-}$$
$$+ \varepsilon_\mu^+ \varepsilon_\nu^{*-} \mathcal{A}^{+-} + \varepsilon_\mu^- \varepsilon_\nu^{*+} \mathcal{A}^{-+} + q'_\nu \mathcal{A}_\mu^{(3)}$$

Parity conservation dictates:

$$\mathcal{A}^{++} = \mathcal{A}^{--} \equiv \mathcal{A}^{(0)}, \qquad \mathcal{A}^{0\pm} \equiv (\varepsilon_\mu^\pm P^\mu)\mathcal{A}^{(1)}, \qquad \mathcal{A}^{\mp\pm} \equiv (\varepsilon_\mu^\pm P^\mu)^2 \mathcal{A}^{(2)},$$

- **Other parameterizations available, exact relations between them known**

[A. Belitsky, D. Müller, YJ, (2014)], [V. Braun, A. Manashov, D. Müller, B. Pirnay, (2014)]

Higher α_s corrections at leading-power

- **The helicity amplitudes $\mathcal{A}^{(0)}$ contributes at the lending-power in $1/Q$**

$$\mathcal{A}^{(0)} = \mathcal{A}^{(0)}_{\rm LP} + \mathcal{O}(1/Q)\,, \quad \mathcal{A}^{(1)} = \mathcal{O}(1/Q)\,, \quad \mathcal{A}^{(2)} = \mathcal{O}(1/Q^2)$$

- **$\mathcal{A}^{(0)}_{\rm LP}$ factorizes into a convolution of perturbative (Wilson) coefficient function and GPD**

$$\mathcal{A}^{(0)}_{\rm LP}(\xi, Q^2) \sim \mathcal{H}^{\rm LP}_{V/A,i} = \int_{-1}^{1} \frac{dx}{\xi}\, C_{V/A,i}\left(\alpha_s, \frac{x}{\xi}, \frac{\mu^2}{Q^2}\right) F_{V/A,i}(x,\xi,\mu)\,, \quad t\text{-dependence} \sim \frac{t}{Q^2}\,,$$

$$\langle p|' \mathcal{O}_q(z_1, z_2)\,|p\rangle = 2P_+ \int_{-1}^{1} dx\, \mathrm{e}^{-iP_+\xi(z_1+z_2)+iP_+x(z_1-z_2)} F_q(x,\xi,t)\,,$$

$$F_q(x,\xi,t) = \frac{1}{2P_+}\left[H_q(x,\xi,t)\bar{u}(p')\not{n}u(p) + E_q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2m_N}u(p)\right]\,,$$

- **$\mathcal{A}^{(0)}_{\rm LP}$ is the main contribution in JLAB-12 and EIC DVCS process (BH as noise)**
- **$C_{V/A,i}$ captures dynamics at $\sim Q$ scale, whereas $F_{V/A}$ describes physics at $\sim \Lambda_{\rm QCD}$**
- **Both $C_{V/A}$ and $\sim \Lambda_{\rm QCD}$ are factorization scale μ-dependent**

$$\mu\frac{d\mathcal{H}}{d\mu} = 0\,, \quad \xrightarrow{\text{resummation/RGE evolution}} \quad \mu\frac{d}{d\mu}\mathcal{H}\Big|_{\alpha_s^n} = \mathcal{O}(\alpha_s^{n+1})$$

Scale dependence of GPDs

- The GPD satisfies the RGE
  - Flavor nonsinglet case

$$\left(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial\alpha_s}+\mathbb{H}_{\rm NS}\right)F_q(x,\xi,\mu)=0\,,$$

$\mathbb{H}$ is called evolution kernel. In position space $\mathbb{H}_{\rm NS}\Leftrightarrow\mathbb{H}_{\rm ERBL}$, partially known to three-loops

[S. Mikhailov and A. Radyushkin (1985), G. Katz (1985)], [V. Braun, A. Manashov, S. Moch, M. Strohmaier (2017)]

  - Flavor singlet case known to two-loops

$$\left[\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial\alpha_s}+\begin{pmatrix}\mathbb{H}_{\rm S}^{qq} & \mathbb{H}_{\rm S}^{qq}\\ \mathbb{H}_{\rm S}^{gq} & \mathbb{H}_{\rm S}^{gg}\end{pmatrix}\right]\begin{pmatrix}F_q(x,\xi,\mu)\\ F_g(x,\xi,\mu)\end{pmatrix}=0$$

[A. Belitsky, D. Müller (2000)], [V. Braun, A. Manashov, S. Moch, M. Strohmaier (2019)]

- $\mathbb{H}$ obtainable by computing conformal anomaly Δ of one-loop less, a general quantity
- $\mathbb{H}_{\rm pos.}\overset{\rm F.T}{\mapsto}\mathbb{H}_{\rm mom.}$ is easy, but NOT vice versa
- Evolution kernels for higher-twist GPDs known to one-loop

[V. Braun, A. Manashov, J. Rohrwild (2009)], [A. Belitsky, YJ (2014)]

Perturbative Wilson coefficient function in DVCS

- The complete Wilson coefficients $C_{V/A,i}$ for DVCS at LP are known to $\mathcal{O}(\alpha_s^2)$
  - Flavor nonsinglet vector case computed via two-loop conformal anomaly

    [V. Braun, A. Manashov, S. Moch, J. Schoenleber (2021)]
  - Flavor nonsinglet axial-vector case computed
    require evanescent operators $\Leftrightarrow$ renormalization scheme

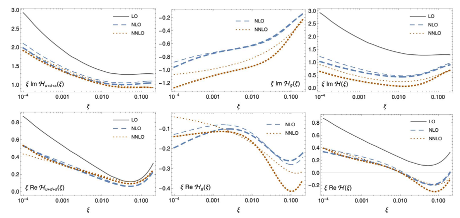
    [V. Braun, A. Manashov, S. Moch, J. Schoenleber (2021)] (by $\Delta^{(2)}$), [J. Gao, T. Huber, YJ, Y.-M. Wang (2021)] (IBP, MI, evan. operator)
  - Flavor singlet vector contribution computed using IBP, MI technique

    [V. Braun, YJ, J. Schoenleber (2022)]
  - Flavor singlet axial-vector and transversity contribution known

    [YJ, J. Schoenleber (2023)]
- All coefficient functions are expressible using HPL functions of weight 0 and 1

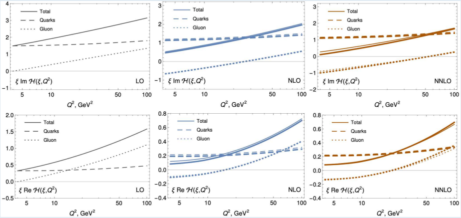
Numerical impact of NNLL’ corrections

- **The gluonic contribution at two-loop is significant compared to one-loop**



CFF $\mathcal{H}$ as a function of ξ at $\mu^2 = Q^2 = 4$ GeV^2.

- **In total, the two-loop correction is smaller, but non-negligible** [V. Braun, YJ, J. Schoenleber (2022)]



CFF $\mathcal{H}$ as a function of Q^2 at $\xi = 0.005$. The growth of $\mathcal{H}$ is almost entirely driven by gluon contribution.

Full helicity amplitudes in DVCS

- **Helicity amplitudes $\mathcal{A}^{(k)}$ can be expanded in $1/Q$,**

$$\boxed{\begin{aligned}\mathcal{A}^{(0)} &= \mathcal{A}^{(0)}_{0,1} + \mathcal{A}^{(0)}_{2,t} + \mathcal{A}^{(0)}_{2,m^2} + \mathcal{A}^{(0)}_{4,t^2} + \mathcal{A}^{(0)}_{4,m^2t} + \mathcal{A}^{(0)}_{4,m^4} + \cdots \\ \mathcal{A}^{(1)} &= \mathcal{A}^{(1)}_{1,1} + \mathcal{A}^{(1)}_{3,t} + \mathcal{A}^{(1)}_{3,m^2} + \mathcal{A}^{(1)}_{5,t^2} + \mathcal{A}^{(1)}_{5,m^2t} + \mathcal{A}^{(1)}_{5,m^4} + \cdots \\ \mathcal{A}^{(2)} &= \mathcal{A}^{(2)}_{2,1} + \mathcal{A}^{(2)}_{4,t} + \mathcal{A}^{(2)}_{4,m^2} + \mathcal{A}^{(2)}_{6,t^2} + \mathcal{A}^{(2)}_{6,m^2t} + \mathcal{A}^{(2)}_{6,m^4} + \cdots\end{aligned}}$$

with

$$\mathcal{A}^{(i)}_{j,z} \sim \frac{z}{Q^j} \,,$$

- **subleading corrections are generated from two sources:**

$$\boxed{\mathcal{A}^{(i)}_{j>0,z} = \mathcal{A}^{(i),\mathrm{kin}}_{j,z} + \mathcal{A}^{(i),\mathrm{dyn}}_{j,z}}$$

- **dynamical corrections carries genuine higher-twist information of the hadron, while kinematic ones are induced by twist-two GPDs**
- **Each term is a convolution of coefficient function $\mathcal{C}^{(i)}_{j,z}$ and GPD H_k, schematically*:**

$$\mathbb{A}^{(i),\mathrm{kin}}_{j,z} = C^{(i)}_{j,z} \otimes H_2 \,, \qquad \mathcal{A}^{(i),\mathrm{dyn}}_{j,z} = C^{(i,k)}_{j,z} \otimes H_k \,, k > 2 \,,$$

***if factorization holds**

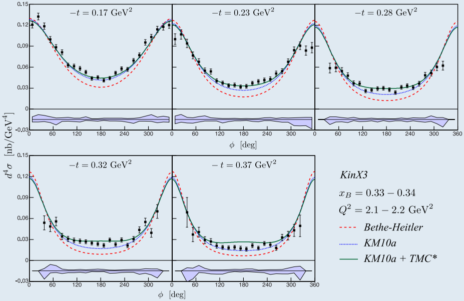
- **DIS case: $t = 0 \mapsto$ Nachtmann correction, (1973)**

Motivation

- **Theory side:**
  - necessary to remove frame dependence, restore EM gauge and translation symmetry
  - a systematic framework for higher-power kinematic contribution
  - verify factorization at NNLP order
  - application of conformal symmetry to higher-power corrections
- **Phenomenological side:**
  - provide theory support for nuclei DVCS measurements (significant m^2 corrections)

[M. Hattawy et al. [CLAS], Phys. Rev. Lett. 119, no.20, 202004 (2017)]

  - mismatch between theory and experiment, resolvable by higher-power corrections?



[M. Defurne et al. [Hall A Collaboration], (2015)]

Kinematic corrections: local form

- **OPE expansion of current correlator (schematically,scalar current for simplicity):**

[V. Braun, YJ, A. Manashov (2021)]

$$T\{j(x_1)j(x_2)\} = \sum_N \Big\{ A_N^{\mu_1\cdots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\cdots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\cdots\mu_N}}_{\text{descendants of twist-2}}$$

$$+ C_N^{\mu_1\cdots\mu_N} \partial^2 \underbrace{\mathcal{O}^N_{\mu_1\cdots\mu_N}} + D_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\cdots\mu_N}}_{\text{descendants}} + \cdots \Big\}$$

$$+ \text{ genuine higher-twist contributions}$$

$$\equiv \sum_N C_N^{\mu_1\cdots\mu_N}(x,\partial)\mathcal{O}^N_{\mu_1\cdots\mu_N} + \text{genuine higher-twist contributions}$$

- **employ conformally covariant OPE:** [S. Ferrara, A. F.Grillo, and Gatto, 1971-1973]

CFs of descendants are related to the CFs of the twist-2 operators by conformal symmetry

$$A_N^{\mu_1\cdots\mu_N} \overset{O(4,2)}{\mapsto} C_N^{\mu_1\cdots\mu_N}(x,\partial)$$

CFs for kinematic corrections: local OPE of leading twist and descendants

- **A glimpse of local coefficient functions ($\otimes$ with $\mathcal{O}^{(k)}_{N,V}(x^u_{12}), \mathcal{O}^{(k)}_{N,A}(x^u_{12}), k = 0, 1, 2$)**

[V. Braun, YJ, A. Manashov (2021)]

  - **vector case:**

$$
\begin{aligned}
\mathbb{C}^{\mu\nu,(0)}_{N,\mathrm{V}} = \frac{r_{N,\mathrm{V}}}{(-x_{12}^2+i0)^2}\Big[&(N+1)g_{\mu\nu}\Big(1-\frac{1}{4}\frac{u\bar{u}}{N+1}x_{12}^2\partial^2\Big)\\
&+\frac{1}{2N}x_{12}^2(\partial_1^\mu\partial_2^\nu-\partial_1^\nu\partial_2^\mu)+\Big(1-\frac{1}{4}\frac{u\bar{u}}{N}x_{12}^2\partial^2\Big)\Big(\frac{\bar{u}}{u}x_{21}^\mu\partial_1^\nu+\frac{u}{\bar{u}}x_{12}^\nu\partial_2^\mu\Big)\\
&-\frac{1}{4}\frac{u\bar{u}}{N(N+1)}x_{12}^2\partial^2\big(x_{21}^\nu\partial_1^\mu+x_{12}^\mu\partial_2^\nu\big)\\
&-\frac{x_{12}^\mu x_{12}^\nu}{N+1}u\bar{u}\partial^2\Big(1-\frac{1}{4}\frac{u\bar{u}}{N+2}x_{12}^2\partial^2\Big)\Big]+\cdots
\end{aligned}
$$

  - **axial-vector case:**

$$
\begin{aligned}
\mathbb{C}^{\mu\nu,(0)}_{N,\mathrm{A}} = \frac{r_{N,\mathrm{A}}}{(-x_{12}^2+i0)^2}\Big\{&\epsilon^{\mu\nu}{}_{\beta\gamma}x_{12}^\beta\Big[N\Big(\frac{u}{\bar{u}}\partial_2^\gamma-\frac{\bar{u}}{u}\partial_1^\gamma\Big)\\
&-\frac{1}{4}\frac{u\bar{u}x_{12}^2\partial^2}{(N+1)}\Big(\partial_2^\gamma-\partial_1^\gamma+(N+1)\Big(\frac{u}{\bar{u}}\partial_2^\gamma-\frac{\bar{u}}{u}\partial_1^\gamma\Big)\Big)\Big]\\
&-\big(x_{12}^\nu\epsilon^\mu{}_{\alpha\beta\gamma}+x_{12}^\mu\epsilon^\nu{}_{\alpha\beta\gamma}\big)x_{12}^\alpha\Big(1-\frac{1}{4}\frac{u\bar{u}x_{12}^2\partial^2}{N+1}\Big)\partial_1^\beta\partial_2^\gamma\Big\}+\cdots
\end{aligned}
$$

Kinematic corrections: nonlocal form

- **resummning back into nonlocal operator $\mapsto$ GPD/DD** [V. Braun, YJ, A. Manashov (2022)]

$$
\begin{aligned}
\mathbb{A}_{\rm V}^{\mu\nu} &= \mathrm{T}\{j^\mu(x)j^\nu(0)\} \\
&= \frac{1}{i\pi^2}\Bigg\{\frac{1}{(-x^2+i0)^2}\int_0^1 dv\Big[\big[g^{\mu\nu}(x\partial)-x^\mu\partial^\nu\big]O(v,0)-x^\nu(\partial^\mu-i\Delta^\mu)O(1,v)\Big] \\
&\quad -\frac{1}{(-x^2+i0)}\int_0^1 du\int_0^u dv\left[\frac{i}{2}(\Delta^\nu\partial^\mu-\Delta^\mu\partial^\nu)O(u,v)-\frac{\Delta^2\,\bar{u}}{4}x^\mu\partial^\nu O(u,v)\right] \\
&\quad +\frac{\Delta^2}{2}\frac{x^\mu x^\nu}{(-x^2+i0)^2}\int_0^1 du\,u\int_0^u dv\,O(u,v)+\cdots\Bigg\},
\end{aligned}
$$

$O(z_1,z_2)$ **non-local twist-2 light-ray operators $\mapsto$ GPD/DD**, also $\Delta\cdot\partial_x O(z_1,z_2)\,,\ \cdots$

- **many highly nontrivial cancellations must happen to resum local $\mapsto$ nonlocal**
  - **need "intertwining operator" to relate operators of different conformal spin**
- **similar for axial-vector case**

Phenomenology: nucleon DVCS

- **Fourier transform to momentum space: final prediction to NNLP***

$$\mathcal{A}_{\mu\nu} = -g^{\perp}_{\mu\nu}\mathcal{A}^{(0)} + \frac{1}{Q}\left(q_\mu + q'_\mu \frac{Q^2}{q\cdot q'}\right) g_{\nu\rho}P^\rho \mathcal{A}^{(1)} + \frac{1}{2}\left(g^{\perp}_{\mu\rho}g^{\perp}_{\nu\rho} - \epsilon_{\mu\rho}\epsilon_{\nu\sigma}\right) P^\rho P^\sigma \mathcal{A}^{(2)} + q'_\nu A^{(3)}_\mu \,,$$

$$g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q'_\nu}{(qq')} - q'_\mu q'_\nu \frac{Q^2}{(qq')^2}\,, \qquad \epsilon^{\perp}_{\mu\nu} = \frac{1}{(qq')}\epsilon_{\mu\nu\alpha\beta}q^\alpha q'^\beta\,, \qquad P^\mu = \frac{1}{2}(p+p')_\mu\,,$$

- **Fourier transform to momentum space: final prediction to NNLP***

$$\mathcal{A}^{(0)} \sim 1 + \frac{1}{Q} + \frac{1}{Q^2} + \cdots\,,$$
$$\mathcal{A}^{(1)} \sim \frac{1}{Q} + \frac{1}{Q^3} + \cdots\,,$$
$$\mathcal{A}^{(2)} \sim \frac{1}{Q^2} + \frac{1}{Q^4} + \cdots\,,$$

- **Further contributions can be calculated if necessary**

A glimpse to the final analytic results

- **structure of $\mathbb{A}^{(i)}_{j,k}$, an example:** [V. Braun, YJ, A. Manashov (2024), (2025)]

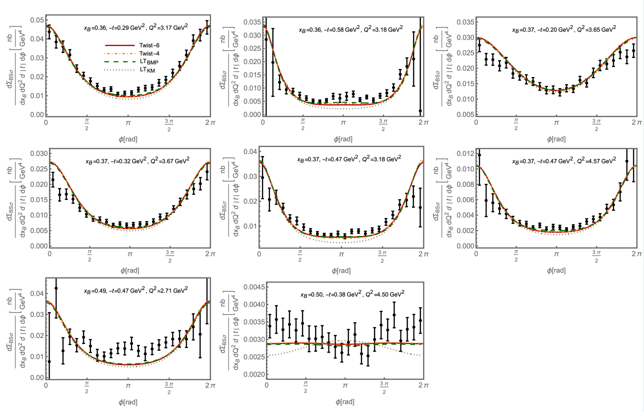
$$\mathcal{A}^{(2)}_{1/Q^4} = -\frac{\varkappa}{(q\cdot q')^2}\left(4|P_\perp|^2 D_\xi^2 + \frac{3}{\xi} t D_\xi - 4t\right) \times D_\xi^2 \int_{-1}^{1} \frac{dx}{\xi} \mathcal{H}(x,\xi,t)\left(\frac{1}{\bar{x}_\xi}\left(\mathrm{Li}_2(x_\xi) - \zeta_2\right) - \ln \bar{x}_\xi\right) + \cdots$$

$$D_\xi = \xi^2 \partial_\xi\,,\; x_\xi = \frac{x+\xi}{2\xi} + i0\,,\; \bar{x}_\xi = 1 - x_\xi\,,\; |P_\perp|^2 = -m^2 - t(1-\xi^2)/(4\xi^2).$$

- $\mathbb{A}^{(1)}$ **and** $\mathbb{A}^{(2)}$ **relatively simple,** $\mathbb{A}^{(0)}_{4,k}$ **complicated; factorization holds!**
  - **factorization violating** $1/q'^2\,, \ln q'^2$ **cancel in final result, highly nontrivial for** $\mathcal{A}^{(0)}$
  - **only** $\mathrm{Li}_2,\ \ln^2$ **appears in final result**
  - **agree with previous lower-order result** [V. Braun, A. Manashov, B. Pirnay, Phys. Rev. D 86 (2012) 014003]

Numerics

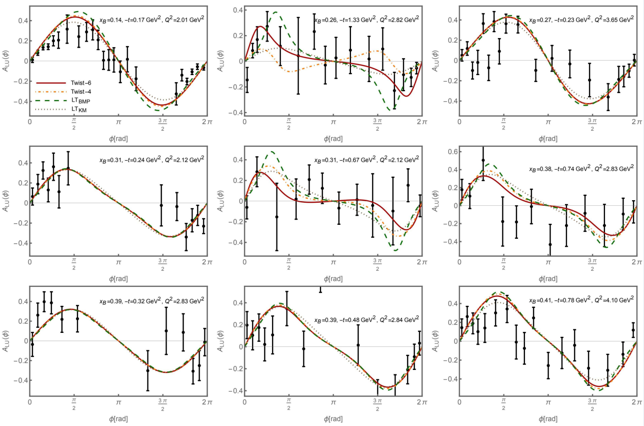
- **Expansion parameter $1/Q^2 \to 1/(qq') = 2/(Q^2+t)$!** [V. Braun, YJ, A. Manashov (2025)]



GPD model: GK12; angular dependence of the spin-averaged total cross section

Numerics

[V. Braun, YJ, A. Manashov (2025)]



GPD model: GK12; angular dependence of beam spin asymmetry

GPDs from LaMET

- **GPDs are complicated functions:**
  - **experimental data are limited, involving inverse-problem**
  - **direct calculations of GPDs from first principles?**
- **Large Momentum Effective Theory (LaMET) provides a concrete theoretical framework**

[X. Ji (2013)]

OPE $\Longrightarrow \quad \mathcal{O}^{\mathrm{LC}}_{\mathrm{tw2}}(z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \, C(\alpha,\beta)\, \mathcal{O}^{\mathrm{SL}}_{\mathrm{tw2}}(z_{12}^{\alpha}, z_{21}^{\beta}) + \cdots\,, \quad z_{12}^{x} = \bar{x} z_1 + x z_2\,,$

$\langle p' |\, \mathcal{O}^{\mathrm{LC}}_{\mathrm{tw2}}(z_1, z_2)\, | p \rangle \sim H; \qquad \langle p' |\, \mathcal{O}^{\mathrm{SL}}_{\mathrm{tw2}}(z_{12}^{\alpha}, z_{21}^{\beta})\, | p \rangle \sim \mathbb{H}$ **can be simulated on lattice**,

$H(x,\xi,\mu) = \int_0^1 dy\, \mathbb{C}(y,\xi,\mu/P_z)\mathbb{H}(y,\xi,\mu/P_z)\,, \quad t = 0$ for leading-twist

- $\mathbb{C}$ **known in full at one-loop, partially at two-loops**

[X. Ji. A. Schäfer, X. Xiong, J. Zhang (2015), · · · , J. Ma, Z. Pang, C. Zhang, G. Zhang (2022), F. Yao, Y. Ji, J. Zhang (2022)]

[Y. Ji, F. Yao, J. Zhang (2024)]

- **Simulations of spacelike correlators ($\mathbb{H}$) have been carried out with promising results**

[C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen (2020), H. Lin (2020), · · · , A. NieMiera, W. Good, H. Lin, F. Yao (2025)]

Conclusions and outlooks

- **Conclusions**
  - complete one-loop Wilson coefficient functions available
  - indispensable for analyzing JLab12, EIC, EIcC data
  - complete LO NNLP kinematic corrections are obtained, tree-level factorization holds
  - kinematic corrections can become large in certain kinematic regions
  - result respects all symmetries to $1/Q^5$ order
- **Outlooks**
  - three-loop GPD evolution kernels
  - three-loop DVCS coefficient functions: convergence of gluon contribution
  - two-loop DDVCS and DVMP coefficient functions: accessible in JLAB and EIC data
  - more extensive numerical analysis for JLAB, EIC, EIcC data
  - embedding into public codes
  - α_s corrections, kinematic corrections from gluon GPD
  - apply to other exclusive processes: target mass correction
  - complete NNLO matching coefficient for quasi-GPDs
  - $\cdots$