Precision calculations for SIDIS and determination of light-hadron Fragmentation Functions

Jun Gao

INPAC, Shanghai Jiao Tong University

based on works of the NPC collaboration, and works in collaboration with L. Dong, S. Fang, HT Li, DY Shao, HX Zhu, YJ Zhu

第一届中国电子离子对撞机相关物理年会 青岛 2026/4/20

上海交通大学 Shanghai Jiao Tong University

SHANGHAI JIAO TONG UNIVERSITY Department of Physics

INPAC INSTITUTE OF NUCLEAR AND PARTICLE PHYSICS

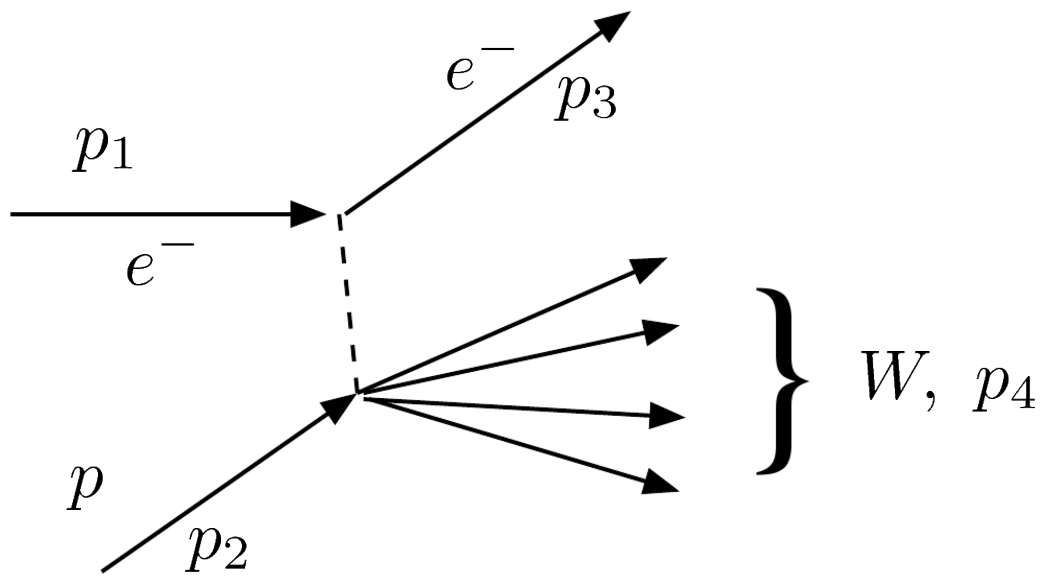
Outline

- 1. Introduction
- 2. NPC23 analysis on FFs of light charged/neutral hadrons
- 3. Precision calculations for SIDIS up to N3LO
- 4. Summary

Deep inelastic scattering

- Deep inelastic scattering on fixed-target reveals the internal structure of nucleons, consisting of quarks and gluons (QPM) as described by parton distribution functions (PDFs), and lead to the establishment of QCD

DIS kinematics



$$W^2 = M^2 + 2M\nu + q^2 = M^2 + Q^2(1/x - 1)$$

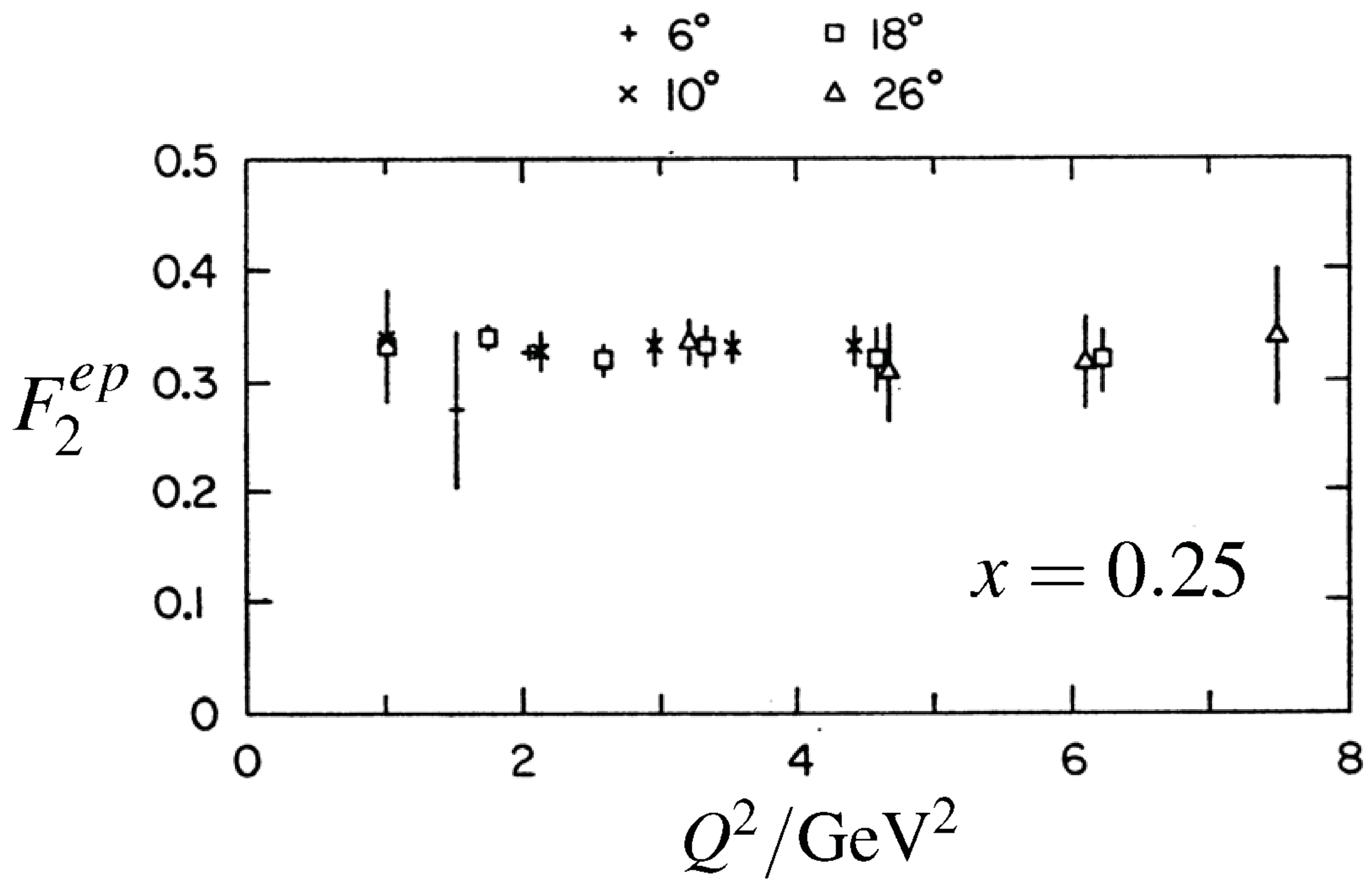
W=M → elastic scattering

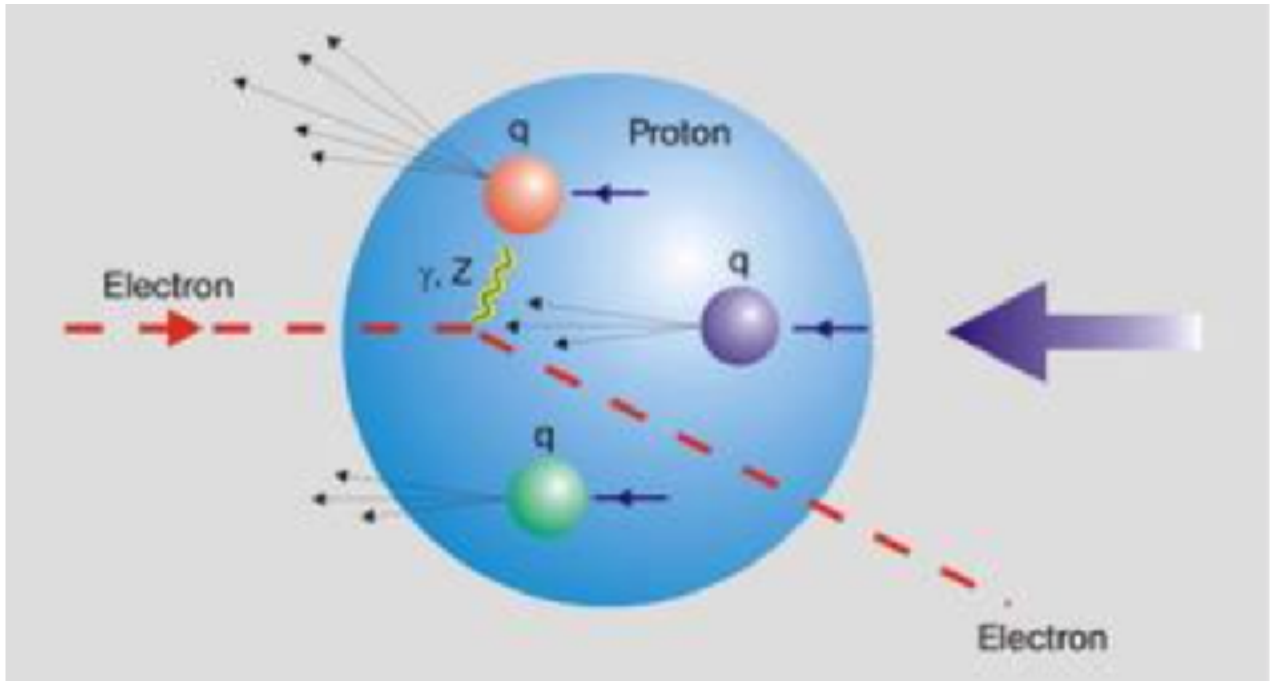
1 < W < 2 GeV → inelastic scattering (exitation of resonances)

W > 2 GeV → deep inelastic scattering

	lab frame - proton at rest before collision: $p_2 = (M, 0, 0, 0)$		lorentz invariant form
energy loss of incoming particle	$\nu = E_1 - E_3$		$\nu = \frac{p_2 q}{M}$
Bjorken x	$x = \frac{Q^2}{2M\nu}$	x in [0,1]	$x = \frac{Q^2}{2p_2 q}$
fractional energy loss of incoming particle	$y = 1 - \frac{E_3}{E_1}$	y in [0,1]	$y = \frac{p_2 q}{p_2 p_1}$
4-momentum transver	$q^2 = (p_1 - p_3)^2$	$Q^2 = -q^2$	$q^2 = (p_1 - p_3)^2$

Bjorken scaling and QPM



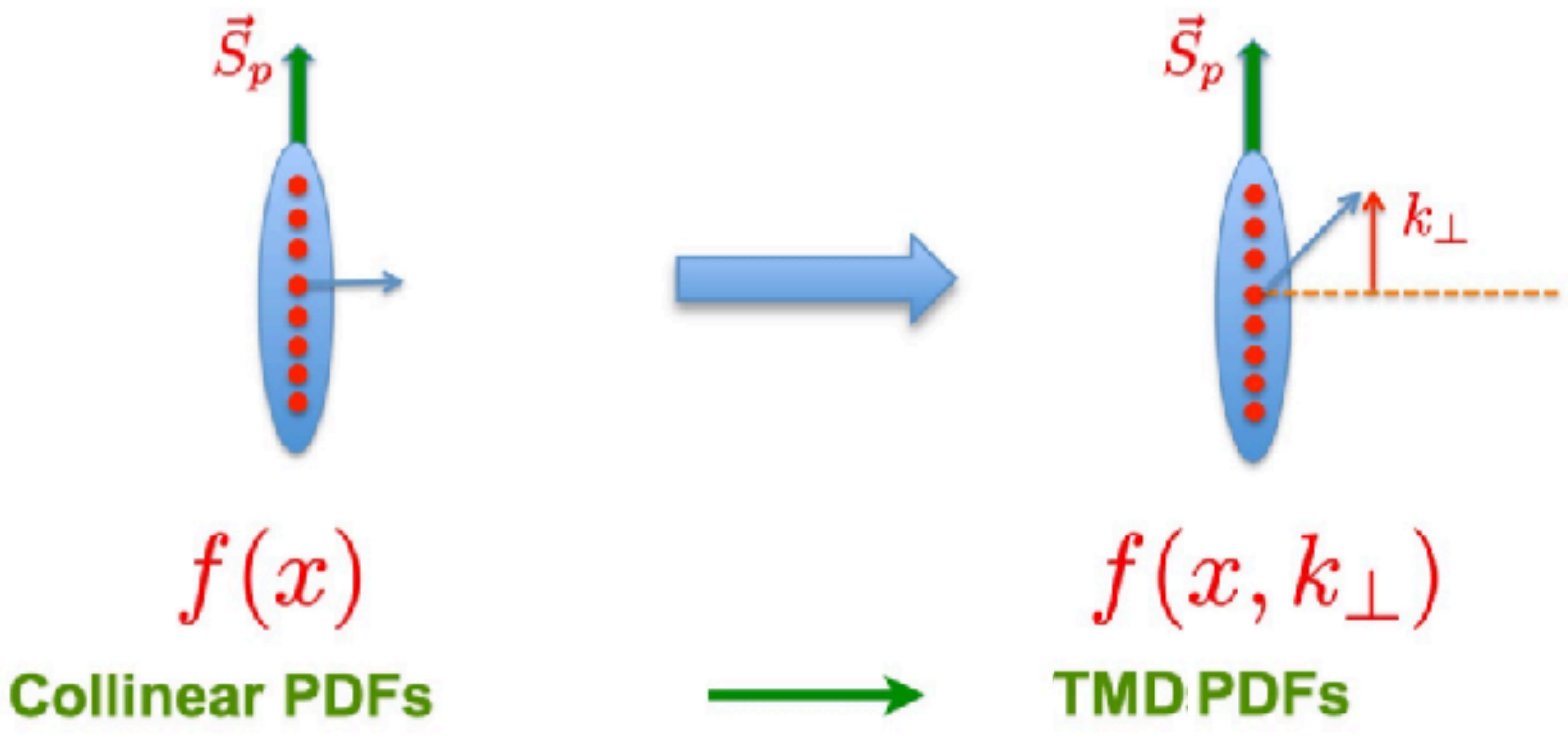


Parton distribution functions (PDFs)

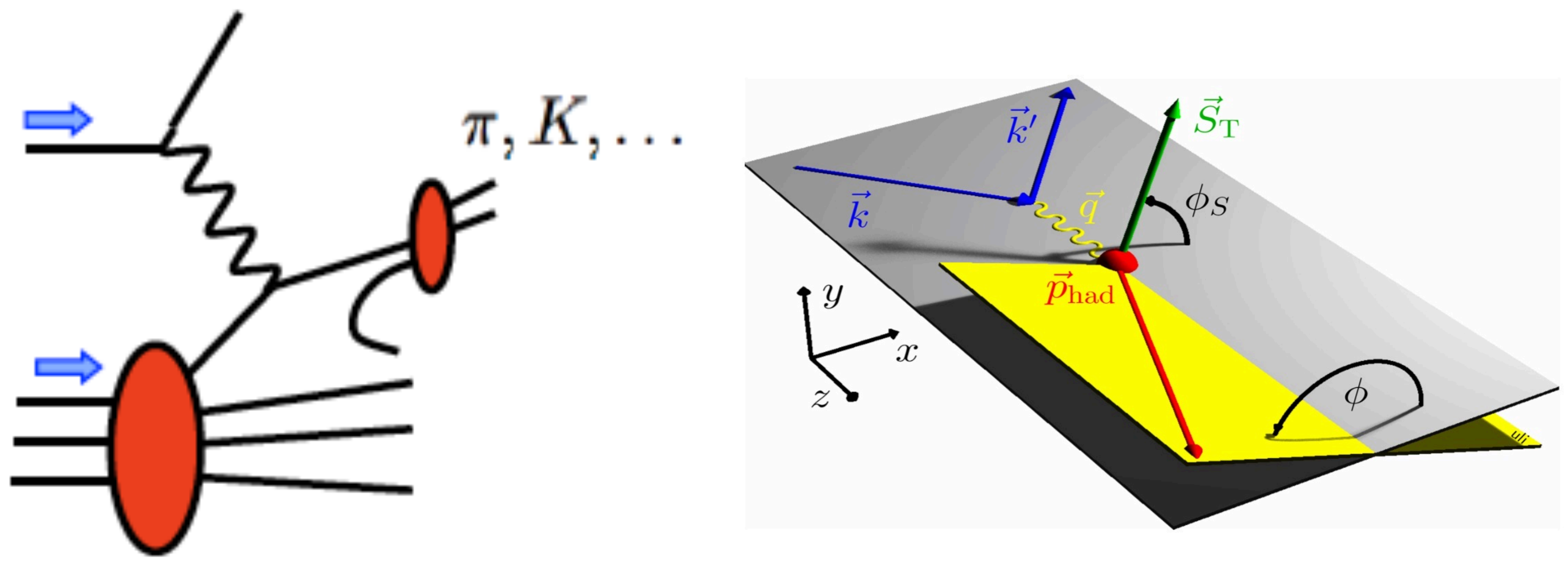
Nucleon tomography and SIDIS

- Semi-inclusive DIS (SIDIS) play an essential role in study of multi-dimensional structure of nucleon and dynamics of hadronization which rely on complementary information from additional hadrons or jets

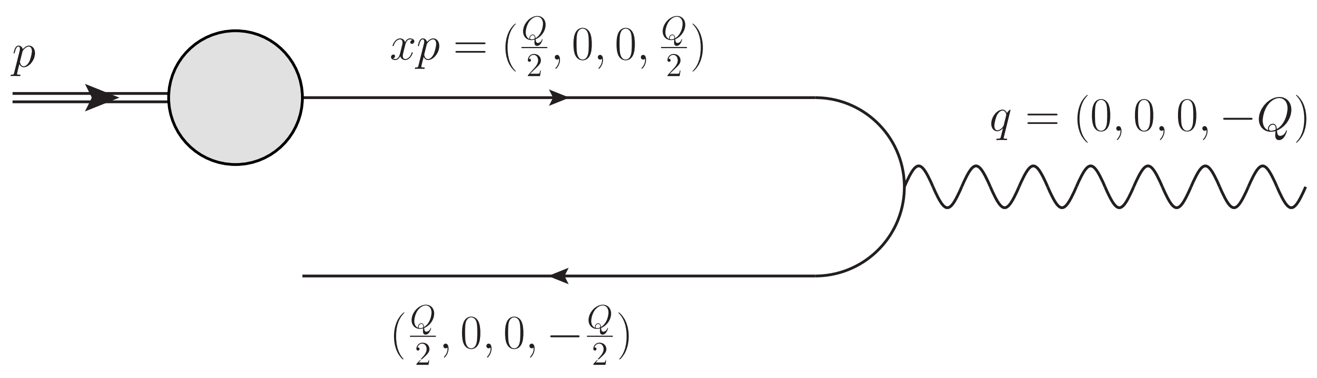
Transverse momentum dependent PDFs

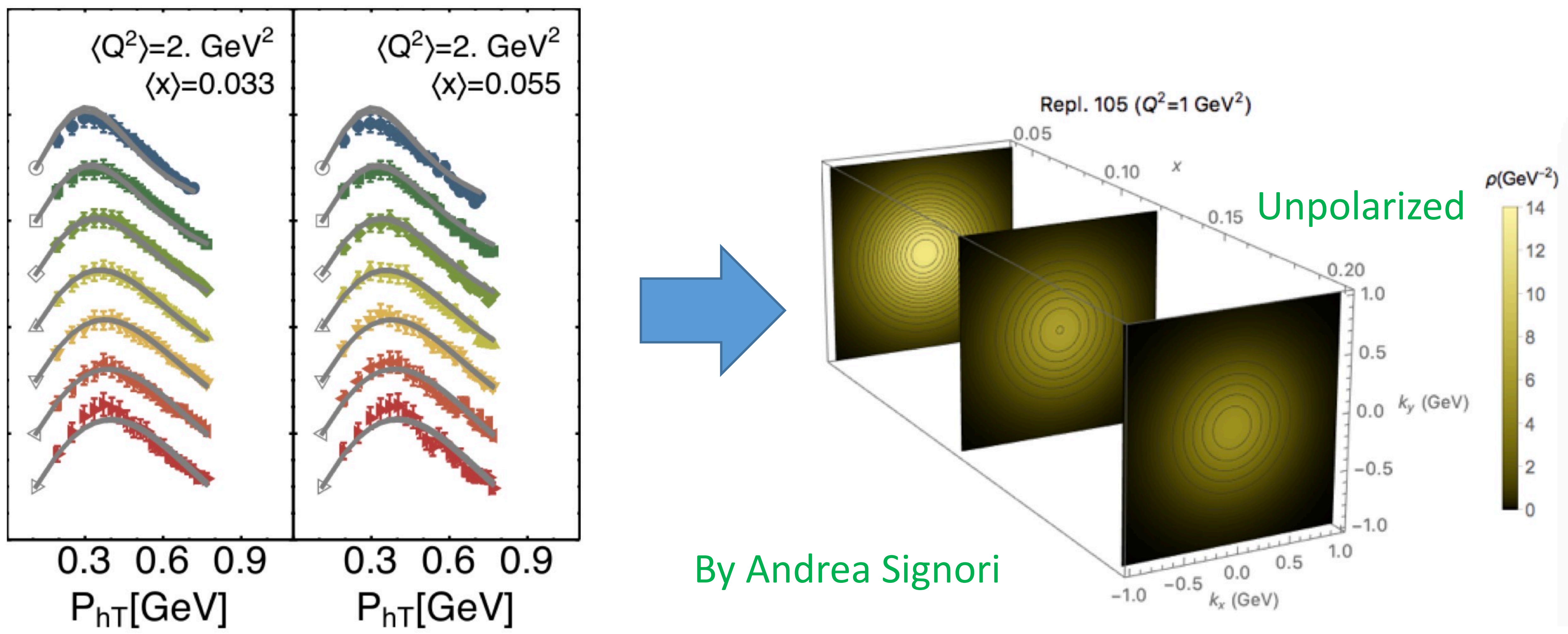


semi-inclusive DIS



- Transverse momentum of final state hadrons provide us the access to the initial parton's TMD PDFs; in addition tagged hadron also help with flavor separation of initial partons



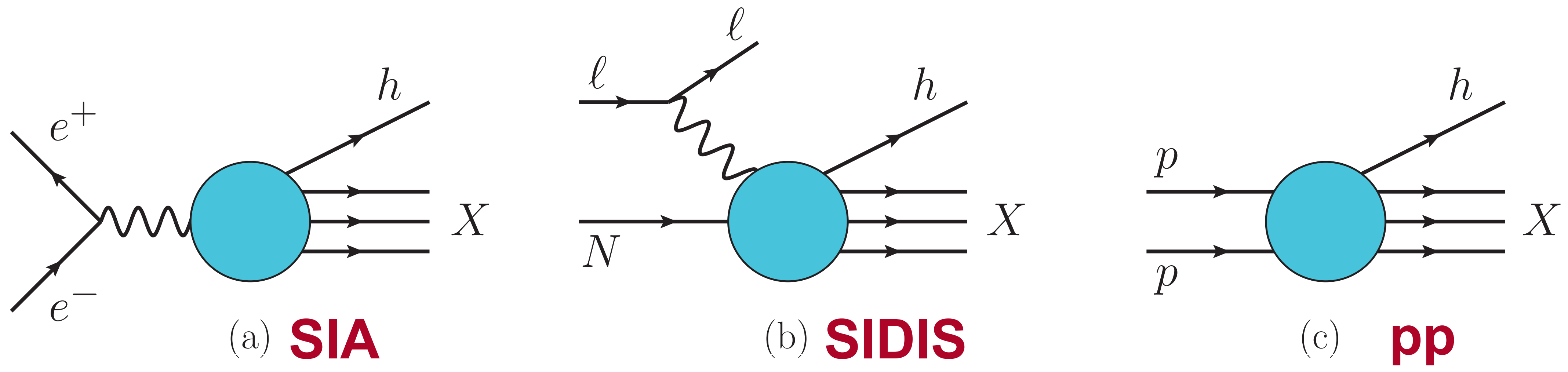


Single inclusive hadron production

- In its simplest form, fragmentation functions (FFs) describe number density of the identified hadron wrt the fraction of momentum of the initial parton it carries, as measured in single inclusive hadron production, e.g., from single-inclusive annihilation (SIA), SIDIS, and pp collisions

single inclusive hadron production/observable

[1607.02521]

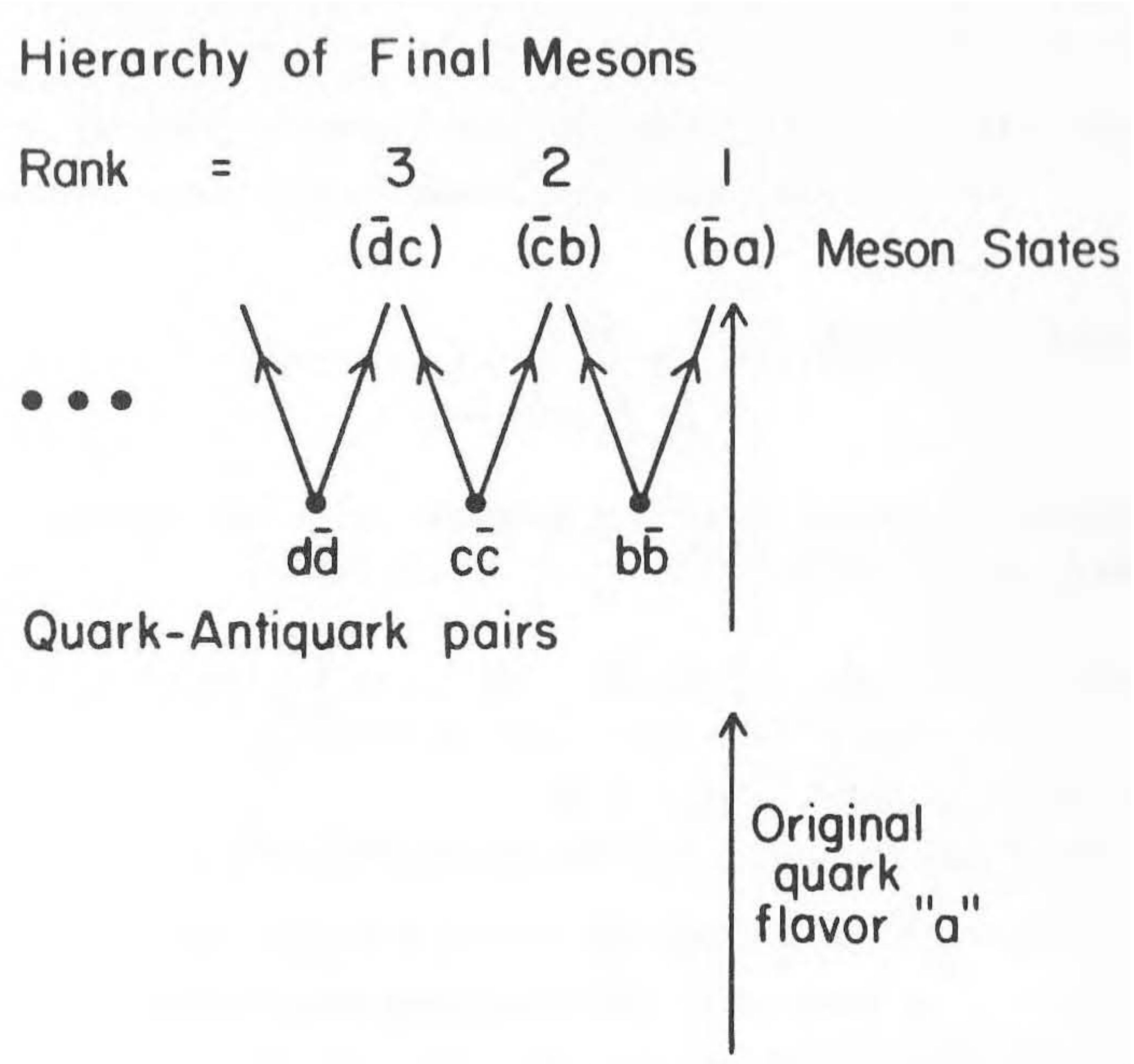


$$\frac{1}{\sigma_{\rm tot}} \frac{d\sigma^{e^+e^-\to hX}}{dz} = F^h(z,Q^2)\,, \quad z = \frac{2\,E_h}{\sqrt{s}}$$

$$\frac{d^3\sigma^h(x,z,Q^2)/dxdQ^2dz}{d^2\sigma^{DIS}(x,Q^2)/dxdQ^2} = D^h(z,Q^2), \quad z = \frac{2E_h^{BF}}{Q}$$

exp. definition of Fragmentation Functions (FFs)

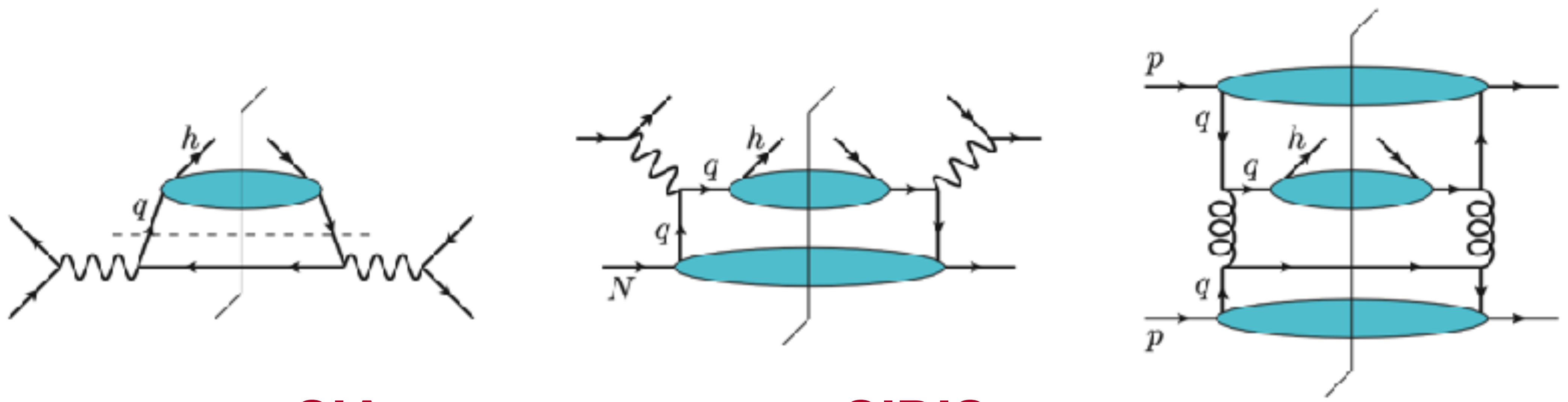
parton model **[Field&Feynman]**



quark decaying functions to mesons via creation of quark-antiquark pairs in cascade

QCD collinear factorization

- QCD collinear factorization ensures universal separation of long-distance and short-distance contributions in high energy scatterings involving initial/final state hadrons, and enables predictions on cross sections



$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma^{e^+e^-\to hX}}{dz} = \frac{1}{\sum_q e_q^2}\left(2F_1^h(z,Q^2)+F_L^h(z,Q^2)\right)$$

$$2F_1^h(z,Q^2) = \sum_q e_q^2\left(D_1^{h/q}(z,Q^2)+\frac{\alpha_s(Q^2)}{2\pi}\left(C_1^q\otimes D_1^{h/q}+C_1^g\otimes D_1^{h/g}\right)(z,Q^2)\right)$$

$$\frac{d^3\sigma^{\ell p\to\ell hX}}{dx\,dy\,dz} = \frac{2\pi\alpha_{\text{em}}^2}{Q^2}\left(\frac{1+(1-y)^2}{y}\,2F_1^h(x,z,Q^2)+\frac{2(1-y)}{y}\,F_L^h(x,z,Q^2)\right)$$

$$2F_1^h(x,z,Q^2) = \sum_q e_q^2\Big(f_1^{q/p}D_1^{h/q}+\frac{\alpha_s(Q^2)}{2\pi}\Big(f_1^{q/p}\otimes C_1^{qq}\otimes D_1^{h/q} + f_1^{q/p}\otimes C_1^{gq}\otimes D_1^{h/g}+f_1^{g/p}\otimes C_1^{qg}\otimes D_1^{h/q}\Big)\Big],$$

- coefficient functions, hard scattering; infrared (IR) safe, calculable in pQCD, independent of the hadron
- FFs/PDFs, reveal inner structure of hadrons or parton-hadorn transition; NP origin, universal, e.g. DIS vs. pp collisions; fitted from data
- runnings of FFs/PDFs with scales μ_D/μ_f are governed by the DGLAP equation

unpolarized collinear FFs, operator definition

$$D_1^{h/q}(z) = \frac{z}{4}\sum\!\!\!\!\!\!\int_X \int\frac{d\xi^+}{2\pi}e^{ik^-\xi^+}\,\text{Tr}\Big[\langle 0|\,\mathcal{W}(\infty^+,\xi^+)\,\psi_q(\xi^+,0^-,\vec{0}_T)\,|P_h,S_h;X\rangle \times \langle P_h,S_h;X|\,\bar{\psi}_q(0^+,0^-,\vec{0}_T)\,\mathcal{W}(0^+,\infty^+)\,|0\rangle\,\gamma^-\Big].$$

$$\frac{d}{d\ln\mu^2}D_1^{h/i}(z,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi}\sum_j\int_z^1\frac{du}{u}P_{ji}(u,\alpha_s(\mu^2))\,D_1^{h/j}\left(\frac{z}{u},\mu^2\right)$$

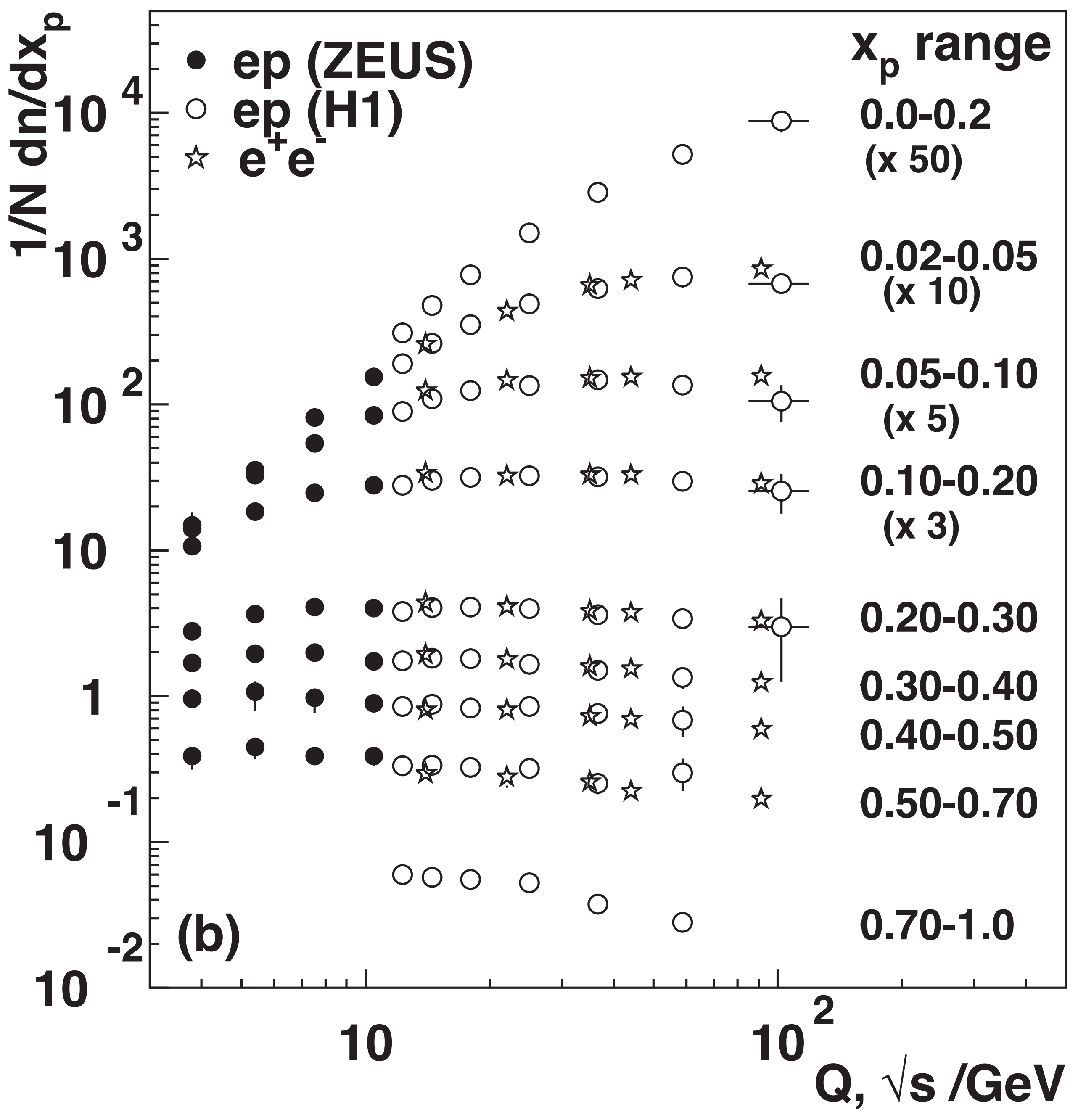
[Collins, Soper, Sterman]

- 2. NPC23 analysis on FFs of light charged/neutral hadrons

Global analysis of FFs

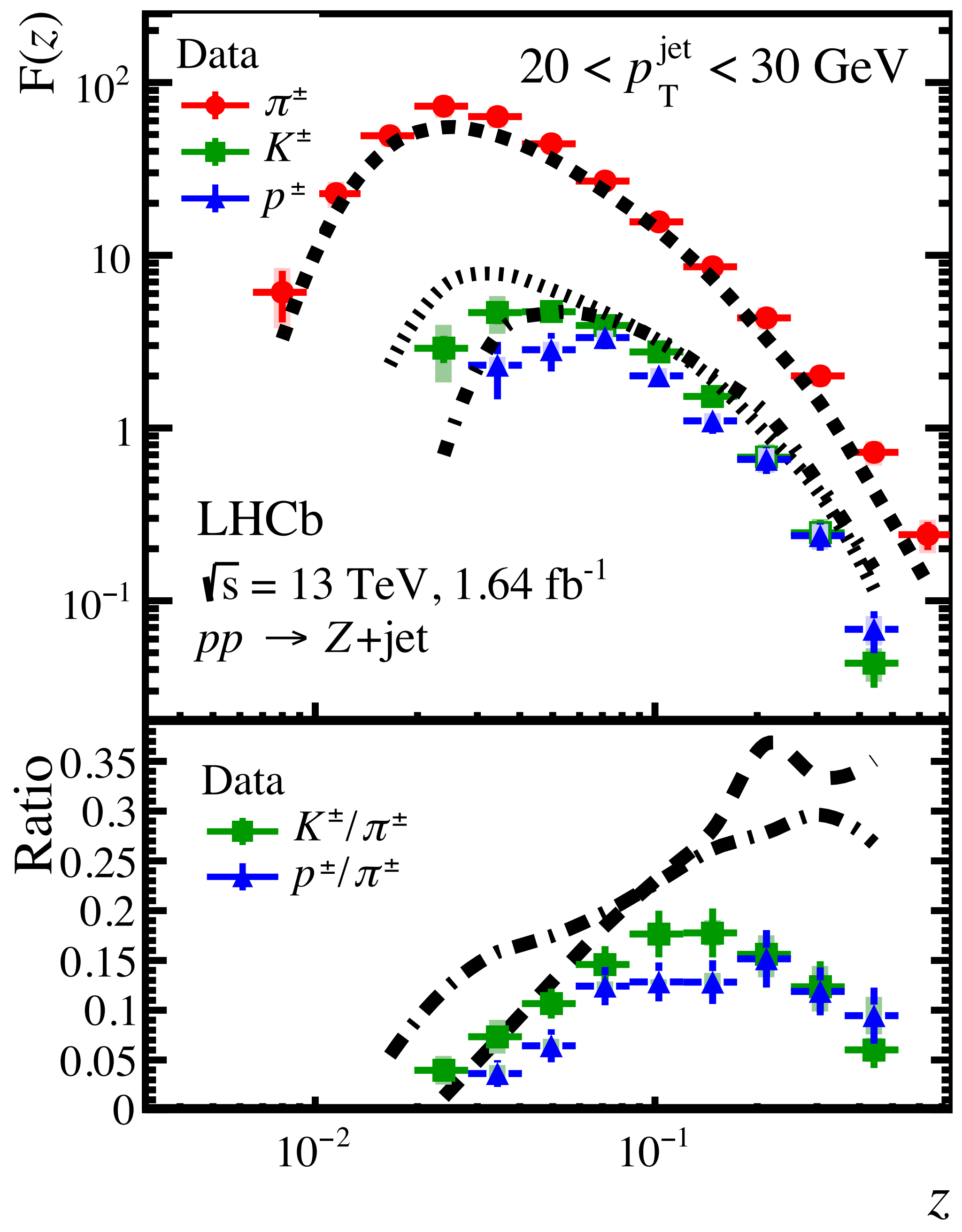
- Measurements are available from colliders SLAC, LEP, HERA, RHIC, LHC and fixed-target HERMES, COMPASS experiments for various light charged hadrons as well as neutral hadrons; many groups provide phenomenological FFs from global analysis at NLO/NNLO in QCD

single incl. production of unidentified charged hadrons (SIA & SIDIS)



[Particle data group]

jet fragmentation to light charged hadrons (LHCb)



[2208.11691]

$$z = \frac{\mathbf{p}_{\rm had} \cdot \mathbf{p}_{\rm jet}}{|\mathbf{p}_{\rm jet}|^2}, \quad F(z) = \frac{1}{N_{Z+\rm jet}} \frac{dN_{\rm had}(z)}{dz}$$

global analysis

[1607.02521 for a review]

- major groups/families include BKK, AKK, HKNS, DSS, NNFF, MAPFF, JAM, SAK etc.
- mostly done at NLO in QCD since exact NNLO coefficient functions only known recently for SIDIS
- different determination can be quite different due to selection of data sets as well as theory treatments, not converge as well as the case of PDF fits

pinning down uncertainties of FFs is crucial for EIC(c) physics !

Overview of the NPC23 analysis of FFs

- Establishing a new framework on global analysis of fragmentation functions to identified charged hadrons, including charged pion, kaon and proton, using most recent data from SIA, SIDIS, and pp collisions

parametrization of FFs to charged pion/kaon/ proton at an initial scale (Q=5 GeV):

$$zD_i^h(z, Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^{m} a_{i,n}^h (\sqrt{z})^n\right)$$

parton-to-π^+	favored	α	β	a_0	a_1	a_2	d.o.f.
u	Y						5
$\bar{d} \simeq u$	Y	-	-		-	-	1
$\bar{u} = d$	N					x	4
$s = \bar{s} \simeq \bar{u}$	N	-				x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F				4

parton-to-K^+	favored	α	β	a_0	a_1	a_2	d.o.f.
u	Y					x	4
$\bar{s} \simeq u$	Y	-	-		-	x	1
$\bar{u}=d=\bar{d}=s$	N					x	4
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F			x	3

parton-to-p	favored	α	β	a_0	a_1	a_2	d.o.f.
$u = 2d$	Y					x	4
$\bar{u} = \bar{d} = s = \bar{s}$	N				x	x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F			x	3

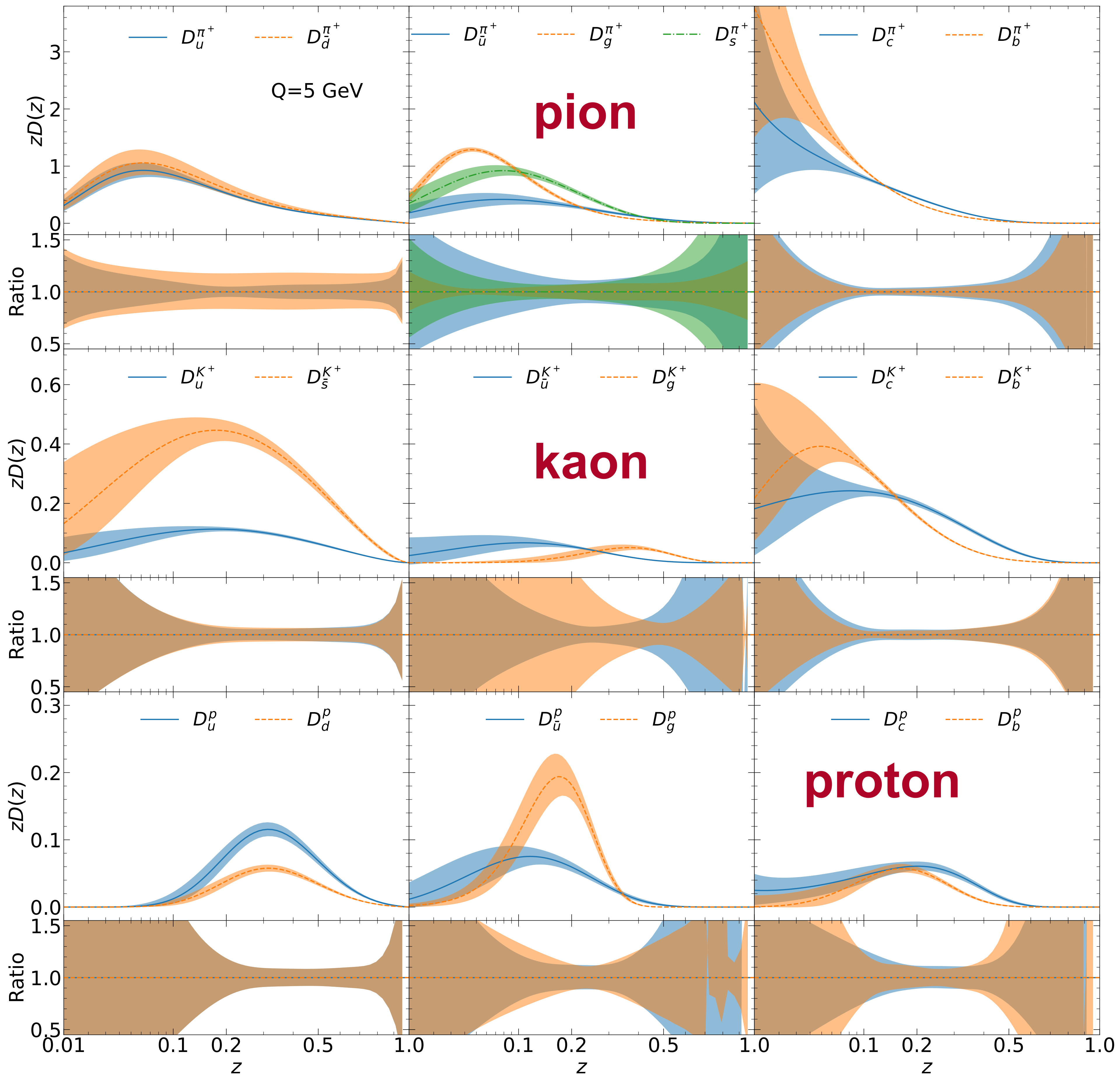
- a joint determination of FFs to charged pion, kaon and proton (via ratios or sum) at NLO in QCD (63 parameters) including estimation of uncertainties with Hessian sets
- apply a strong selection criteria on kinematics of the fragmentations to ensure validity of LT factorization and perturbative calculations (E_h/ $p_{T,h}$>4 GeV, and z>0.01)
- including theory uncertainties (residual scale variations) into the covariance matrix
- use fast interpolation techniques as in FMNLO which largely increase efficiency of the fit

[JG, Liu, Shen, Xing, Zhao, 2401.02781 (PRL), 2407.04424 (PRD Editor's suggestion)]

most complete FFs of light hadrons with inclusion of extensive data sets at high energies!

NPC23 FFs to charged hadrons

- We arrive at a best-fit of the charged pion, kaon and proton FFs together with 126 Hessian error FFs, two for each of the eigenvector direction; FFs are generally well constrained in the region with z~0.1-0.7

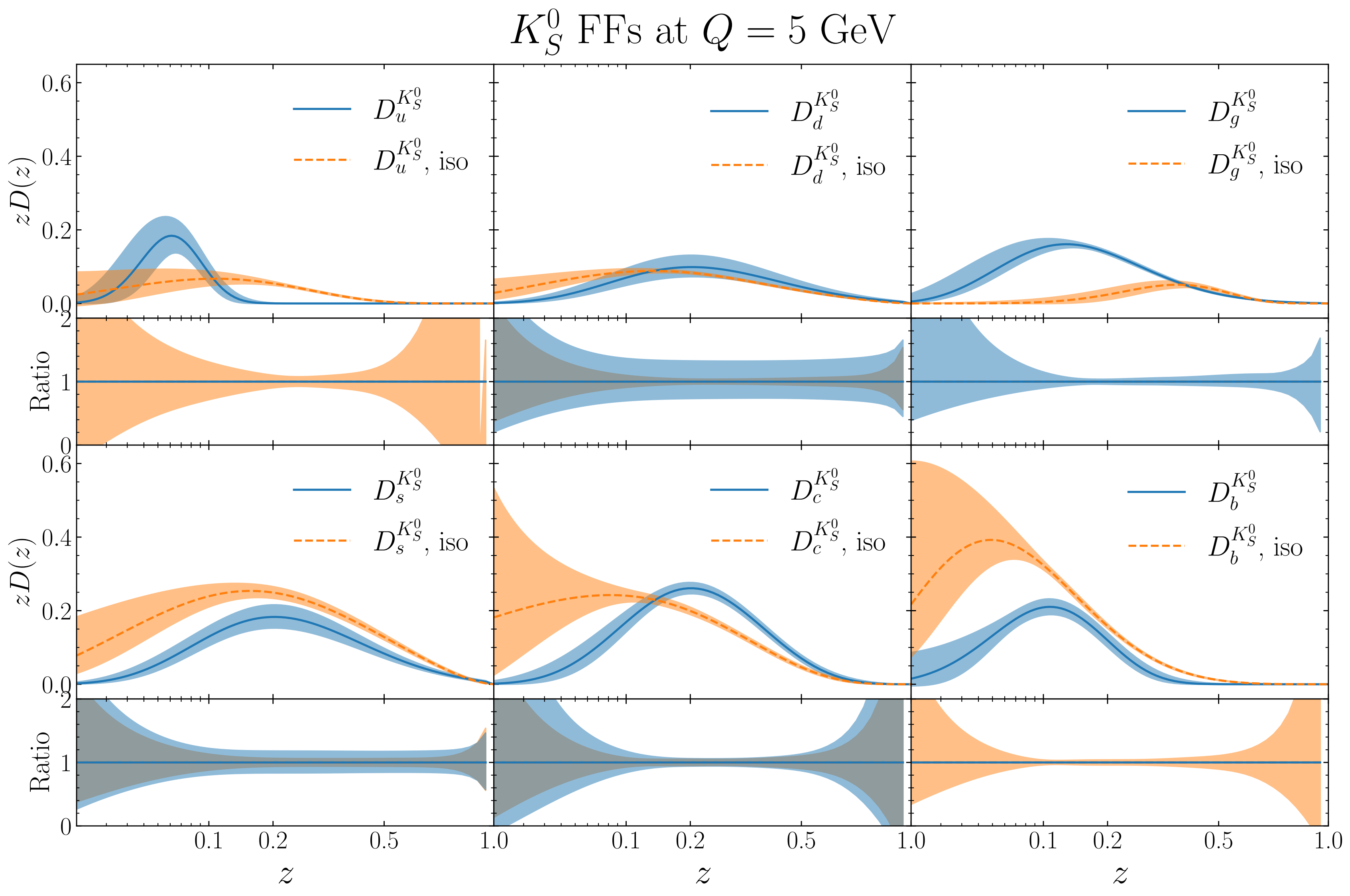


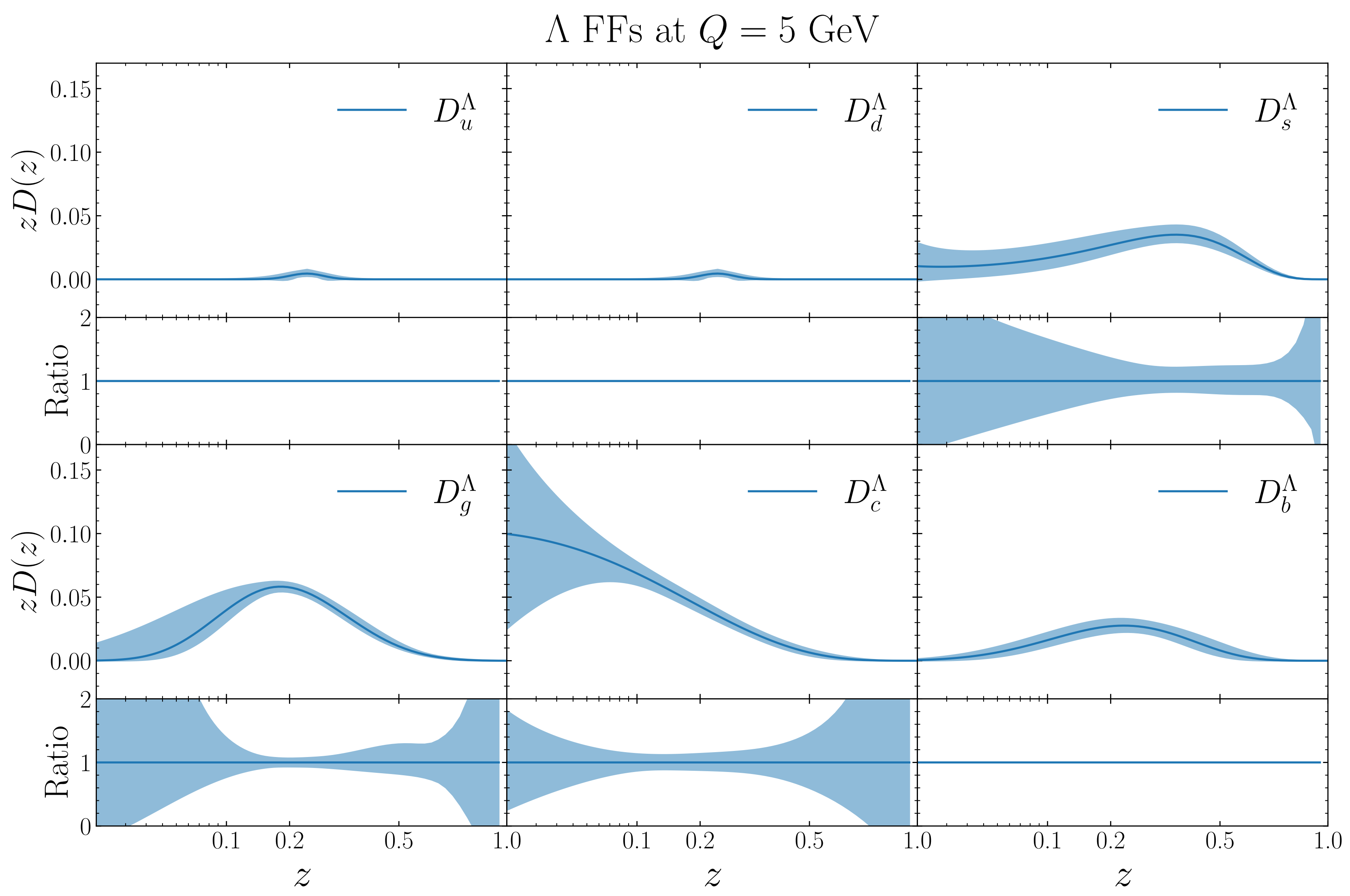
FFs (positively charged) vs. momentum fraction

- our results show an uncertainty of 3%, 4% and 8% for FFs of gluon to pion at z=0.05, 0.1 and 0.3, respectively
- similarly an uncertainty of 4%, 4% and 7% for FFs of u-quark to pion, kaon and proton at z=0.3, respectively
- FFs of heavy-quarks are well constrained for z between 0.1~0.5 due to the tagged SIA events of Z-pole measurements
- a preference for larger FFs of s quark to pion due to pulls from SIA data
- high precision of gluon FFs is mostly due to the data of jet fragmentation from the LHC

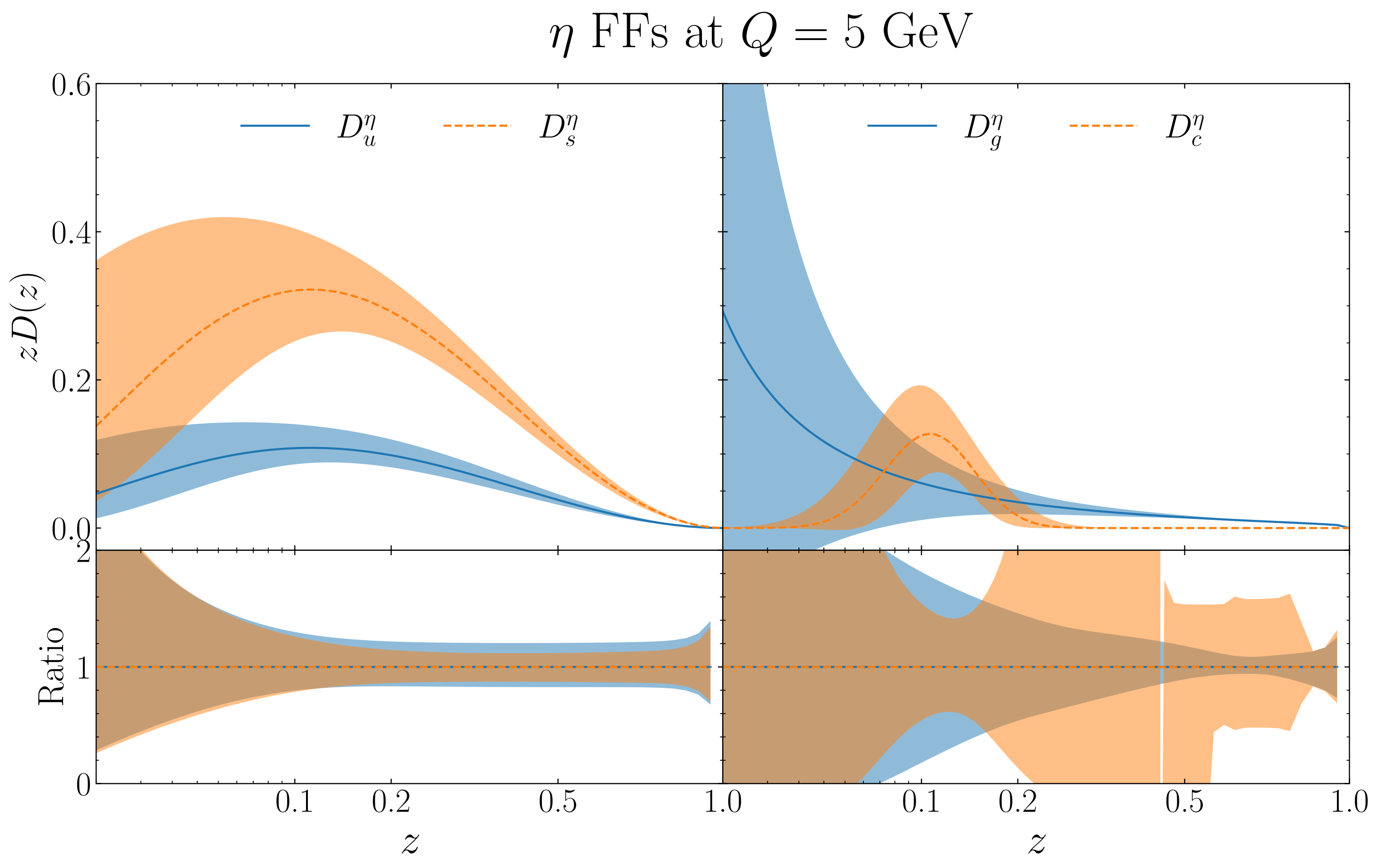
NPC23 FFs to neutral hadrons

- Global analysis of FFs to light neutral hadrons at NLO with high-energy data from SIA, SIDIS and pp collisions, inlcluding kaon, eta, lambda; FFs of neural pion are constructed via isospin symmetry and find very good agreement with high-energy data **[JG, Li, Shen, Xing, Zhao, Zhou, Zhou, 2503.21311 (PRD)]**





- similar setup as the charged hadron fit with high-energy data; first inclusion of SIA jet and SIDIS data
- FFs of K_S from direct fit are compared to those constructed via isospin symmetry **[Chen, Liang, Pan, Song, Wei]**; further measurements are needed for testing isospin symmetry in kaons FFs



NPC23 analysis at NNLO

- Global analysis of FFs to light charged hadrons at full NNLO with SIA and SIDIS data, especially with the recent BESIII measurements, focusing on low momentum transfer region and test of collinear factorization

parametrization form (pi+/K+)

$$zD_i^h(z,Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h}\exp\left(\sum_{n=0}^{m} a_{i,n}^h z^{n/2}\right)$$

flavor	favored	a_0	α	β	a_1	a_2
$u=\bar{d}$	✓	✓	✓	✓	✓	✓
$d=\bar{u}$	✗	✓	✓	✓	✓	✓
$s=\bar{s}$	✗	✓	$=\alpha_d$	✓	✓	✓
$c=\bar{c}$	✗	✓	✓	✓	✓	✓
$b=\bar{b}$	✗	✓	✓	✓	✓	✓
g	✗	✓	✓	✓	✓	✗

flavor	favored	a_0	α	β	a_1	a_2
u	✓	✓	✓	✓	✓	✓
$\bar{s}$	✓	✓	$=\alpha_u$	$=\beta_u$	✓	✓
$s=\bar{u}=d=\bar{d}$	✗	✓	✓	✓	✓	✗
$c=\bar{c}$	✗	✓	✓	✓	✓	✓
$b=\bar{b}$	✗	✓	✓	✓	✓	✓
g	✗	✓	✓	✓	✓	✗

BESIII charged pion/kaon **[2502.16084]**

$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb^{-1})	$N_{\rm had}^{\rm tot}$	$N_{\rm bkg}$
2.0000	10.074	350298 ± 592	8722 ± 94
2.2000	13.699	445019 ± 668	10737 ± 104
2.3960	66.869	1869906 ± 1368	47550 ± 219
2.6444	33.722	817528 ± 905	21042 ± 146
2.9000	105.253	2197328 ± 1483	56841 ± 239
3.0500	14.893	283822 ± 533	7719 ± 88
3.5000	3.633	62670 ± 251	1691 ± 42
3.6710	4.628	75253 ± 275	6461 ± 81

[JG, Shen, Xing, Zhao, Zhou, 2502.17837 (PRL)]

- a joint determination of FFs to charged pion, kaon at NNLO in QCD (54 parameters) with Hessian uncertainties
- parametrization at Q_0=1.4 GeV for light flavors and at mass threshold for heavy quarks; 3-loop DGLAP evolution from **[Mitov, Moch, Vogt, Almasy; Chen, Yang, Zhu, Zhu]** implemented in HOPPET
- SIDIS coefficient functions from **[Bonino, Gehrmann, Stagnitto; Goyal, Moch, Pathak, Rana, Ravindran];** SIA with hadron mass corrections; include residual scale variations into the covariance matrix
- select SIA and SIDIS data with z>0.01, Q>2 GeV, and E_h> $E_{h,min}$ (0.8 GeV default) with later varied

first FFs of light hadrons at full NNLO ensuring test of factorization at low Q2!

Non-singlet FFs from charge asymmetries

- We extract non-singlet FFs of charged pion and kaon at NNLO in QCD from charge asymmetries measured in hadron fragmentation, and provide a benchmark for various QCD models, MC generators and global analyses **[JG, Liu, Zhou, 2507.14637, CPC cover story & editors' suggestion]**

charge asymmetries in SIDIS and SIA

$$\sigma_{h^+} - \sigma_{h^-}|_{NC} = \sum_{i=u,d,s} \Big(f_{i/H}(x) - f_{\bar{i}/H}(x)\Big) \otimes D_{i^-}^{h^+}(z) \otimes \mathcal{A}_{NS}^{NC}(z,x,i),$$

$$\sigma_{h^+} - \sigma_{h^-}|_{\nu CC} = \sum_{i=d,s} \Big(f_{i/H}(x) \otimes D_{i'^-}^{h^+}(z) \otimes \mathcal{A}_{NS}^{\nu CC,q}(z,x) - f_{\bar{i}'/H}(x) \otimes D_{i^-}^{h^+}(z) \otimes \mathcal{A}_{NS}^{\nu CC,\bar{q}}(z,x)\Big),$$

$$\sigma_{h^+} - \sigma_{h^-}|_{SIA} = \sum_{i=u,d,s} D_{i^-}^{h^+}(z) \otimes \Big(\mathcal{A}_{NS}^{SIA,q}(z,\cos\theta,i) - \mathcal{A}_{NS}^{SIA,\bar{q}}(z,\cos\theta,i)\Big),$$

nominal parametrization of NS FFs

$$zD_{i^-}^h(z,Q_0) \equiv z(D_i^h - D_{\bar{i}}^h)(z,Q_0) = z^\alpha(1-z)^\beta \exp(a_0)$$

- NCSIDIS data from COMPASS with iso-scalar **[2016]** and proton targets **[2024]**, HERMES with proton/deuteron targets **[2013]**
- (anti-)neutrino CCSIDIS data from ABCMO **[1983]**, and NNLO coefficient functions from **[Bonino, Gehrmann, Lochner, Schonwald, Stagnitto]**
- SIA data from SLD **[1999; 2004]** quark-tagged jet measurements, and NNLO coefficient functions from **[Zhou, JG]**
- **3(5)** free parameters in total for pion-only fit (joint fit of pion and kaon), including NS FFs of u to pi+, u to K+, and s to K-

Non-singlet FFs from charge asymmetries

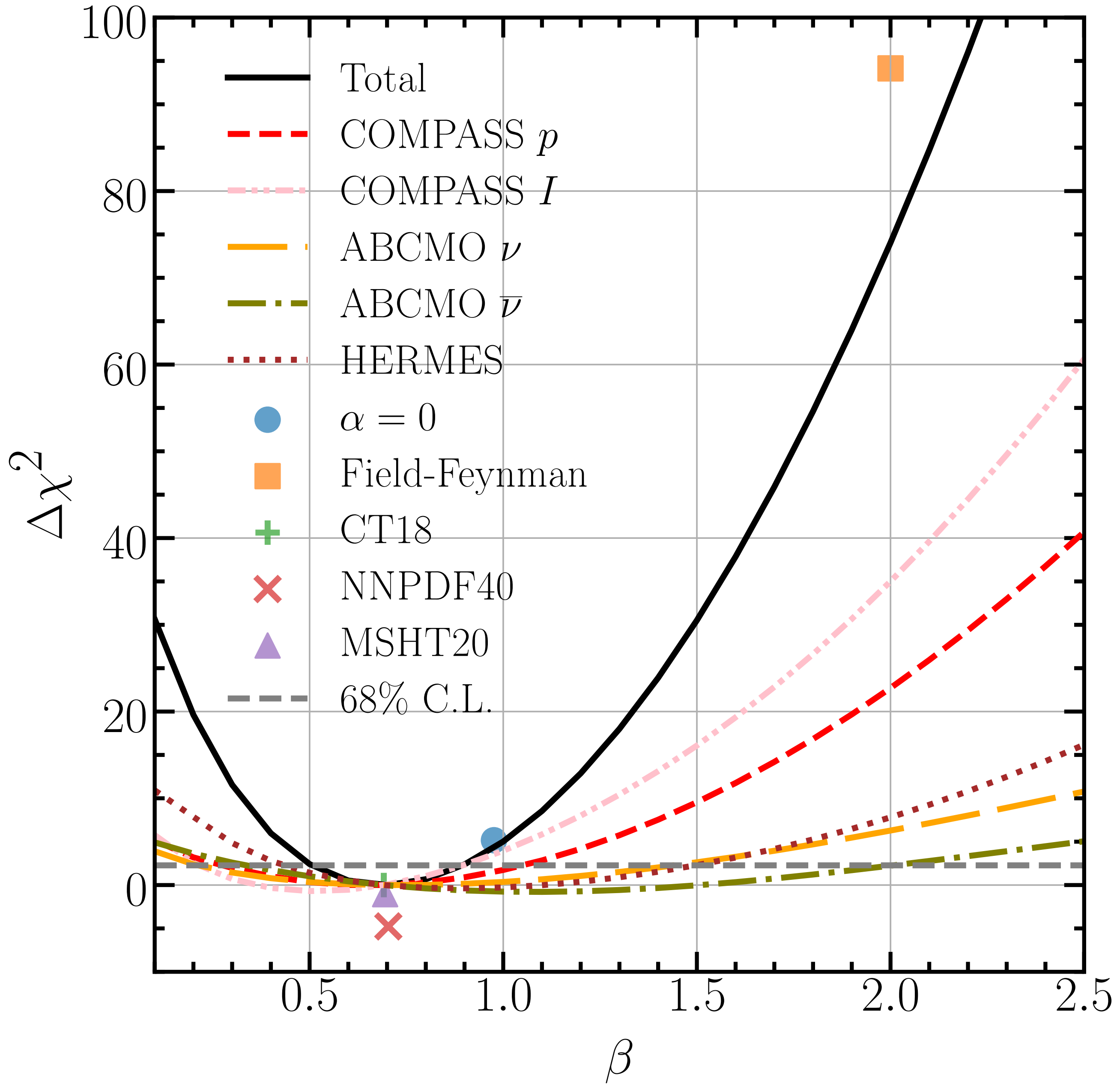
- In the pion-only fit we find a preference on the scaling index β close to 1 rather than 2, at Q=1.3 GeV, despite of various systematic effects; several models are tested including Field-Feynman parametrization and CSM FFs **[Xing, Bian, Cui, Roberts, 2025]**

various fits to the pion NS FFs

$zD^{\pi+}_{u^-}$	α	β	$a_0(a_1)$	χ^2 (N_{pt})
nominal	$-0.335^{+0.22}_{-0.21}$	$0.692^{+0.20}_{-0.19}$	$-1.891^{+0.30}_{-0.28}$	252.4 (249)
mod.1	**0**	0.976	-1.438	257.5 (249)
mod.2	-0.035	**1.0**	-1.460	257.4 (249)
mod.3	0.893	**2.0**	-0.091	326.4 (249)
mod.4	**1.0**	**2.0**	**0.054**	346.5 (249)
mod.5	**0.681**	**2.298**	**-1.352(2.444)**	563.2 (249)
sys.1	-0.323	0.703	-1.846	247.6 (249)
sys.2	-0.476	0.665	-1.984	152.7 (144)
sys.3	-0.355	0.732	-1.828	153.8 (136)
sys.4	-1.000	0.835	-3.919(2.350)	252.2 (249)
sys.5	-0.349	0.568	-1.885	252.4 (249)
sys.6	-0.373	0.534	-1.954	243.3 (249)

- 3-parameter nominal fit (z>0.3) shows a very good chi2 and a scaling index of about 0.7±0.2; changing PDFs, Q_0 , z cut, using only proton data, or switch to 4-parameter fit or NLO fit, all return a scaling index within uncertainties

a scan on the scaling index



- a surprisingly consistent preference of β close to 1 than 2 from all SIDIS data studied; see **[Aichier, Schafer, Volgesang, 2010]** for related discussions on pion PDFs

- 3. Precision calculations for SIDIS up to N3LO

Hadron production cross sections at NLO

- Generic algorithms on NLO calculations of jet production cross sections have been developed for long times, based on local subtraction or phase-space slicing method; especially automation of NLO jet cross sections exists, e.g., in MG5 and Sherpa

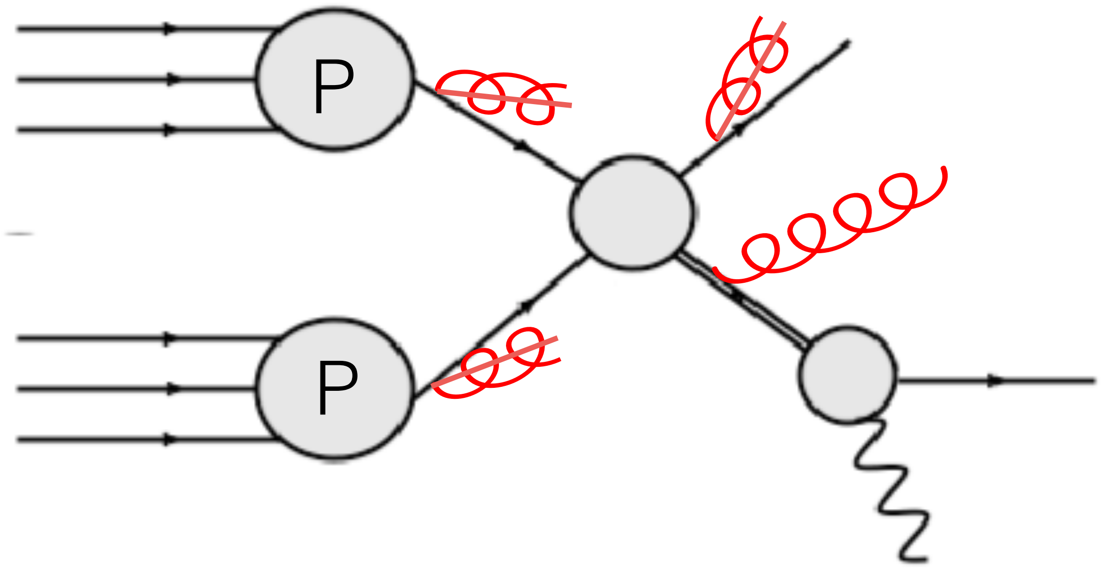
local subtraction

$$\frac{d\sigma}{dF} = \int dPS_m \Big[|M|^2_{B,m} + |M|^2_{V,m} + |\tilde{\mathcal{I}}|^2_m \Big] \delta(\hat{F}(p_m; f_m) - F) + \int dPS_{m+1} \Big[|M|^2_{R,m+1} \delta(\hat{F}(p_{m+1}; f_{m+1}) - F) - |\mathcal{I}|^2_{m+1} \delta(\hat{F}(\tilde{p}_m; \tilde{f}_m) - F) \Big]$$

phase-space slicing

$$\frac{d\sigma}{dF} = \int dPS_m \Big[|M|^2_{B,m} + |M|^2_{V,m} \Big] \delta(\hat{F}(p_m; f_m) - F) + \int dPS_{m+1} (\Theta(C - \lambda) + \Theta(\lambda - C)) \Big[|M|^2_{R,m+1} \delta(\hat{F}(p_{m+1}; f_{m+1}) - F) \Big]$$

QCD radiations



- FKS subtraction (jet), **[Frixione, Kunszt, Signer]**, as implemented in Madgraph5
- Dipole subtraction (jet & hadron), **[Catani, Seymour]**, as implemented in MCFM
- Two-cutoff slicing (jet & hadron), **[Harris, Owens]**
- Antenna subtraction (jet & hadron), **[2406.09925]**; alternative subtraction **[2403.14574]**; semi-analytical calculations, **[1903.01529, BigkT]**
- STRIPPER (jet & hadron at NNLO), **[2503.11489]**

public tools and automation on the hadron cross sections calculations are very much limited!

The story of higher-order calculations for SIDIS

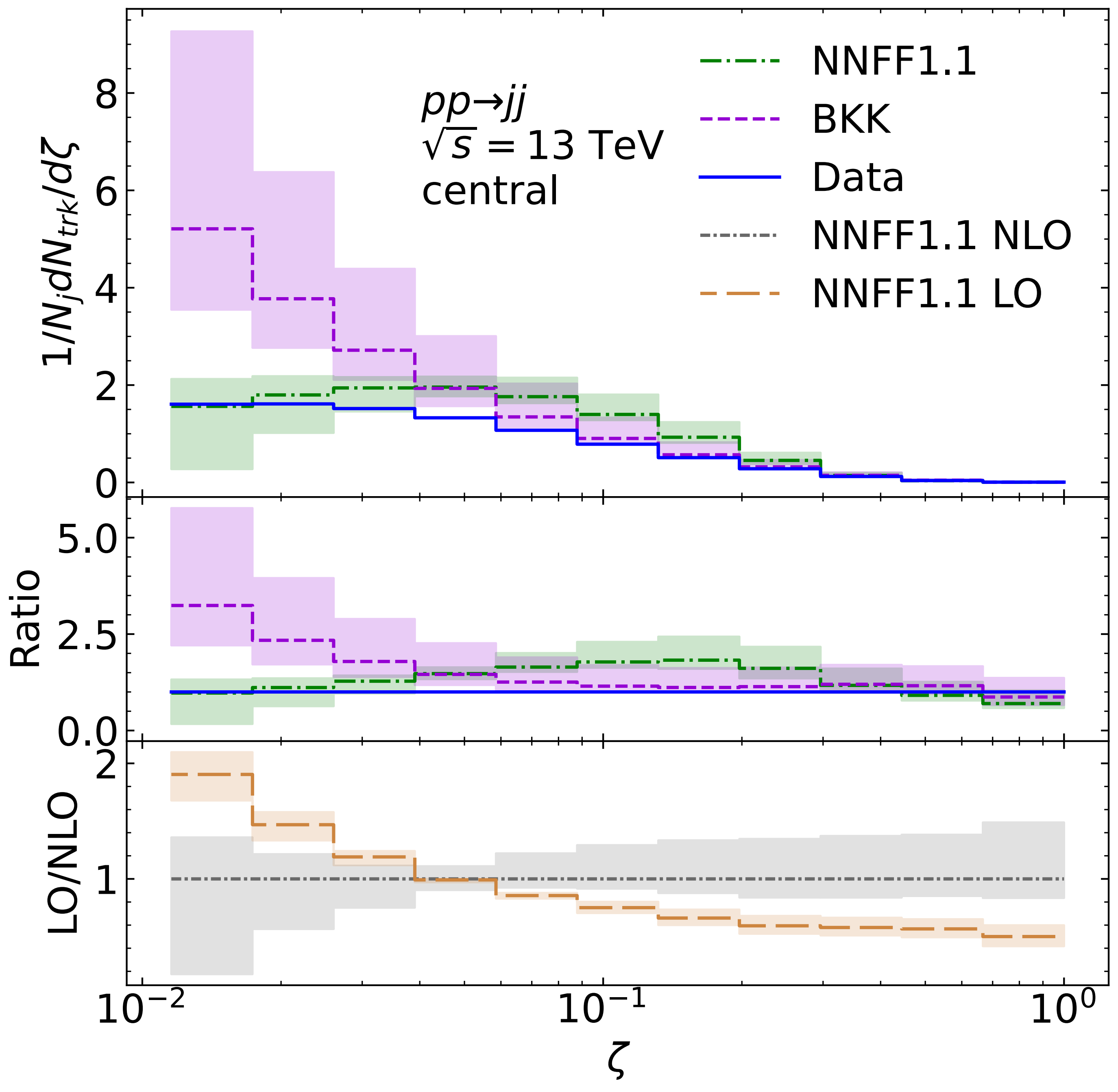
- In early **2023**, I collaborated with HuaXing on a NLO calculation for hard functions of Energy-energy correlations. Soon we realized that the same method can be applied to hadron production as well.

FMNLO (fragmentation at NLO in QCD)

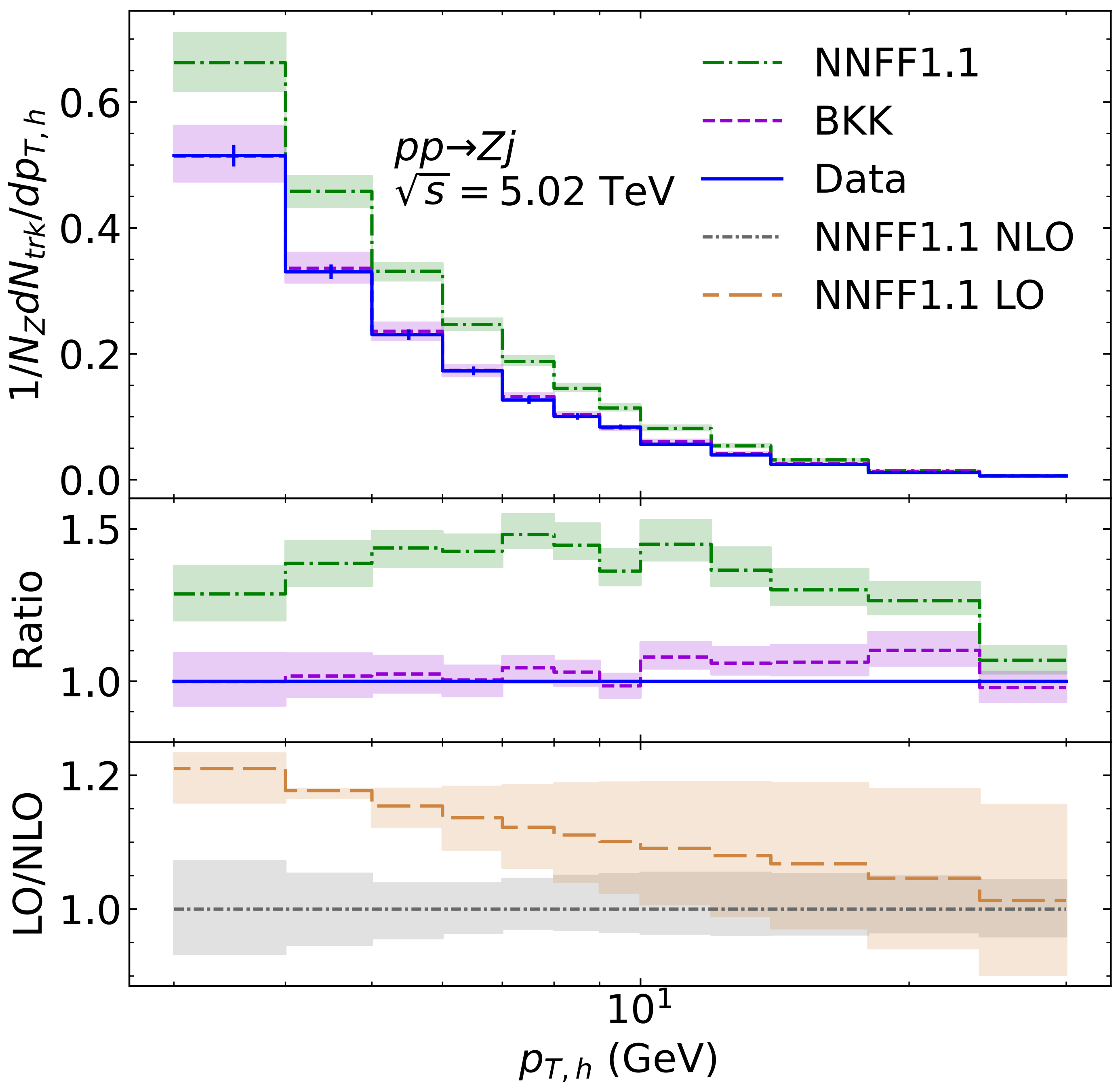
- FMNLO is a program for automated and fast calculations of fragmentation cross sections of arbitrary hard processes. It is based on a hybrid scheme of phase-space slicing method and local subtraction method, accurate to NLO in QCD

  - automation of fragmentation calculations for arbitrary hard processes up to NLO, within SM and BSMs via MG5_aMC@NLO
  - fast convolution of partonic cross sections with FFs without repeating the time consuming MC integrations using interpolation grids
  - generalizations: transverse observables, **NNLO corrections**

QCD inclusive dijets at LHC



Z-boson tagged jet



🔥 **News**

2024.07: 🎉🎉 FMNLOv2.1 NNLO calculations are available for limited cases, SIA, decay of the Higgs boson to gluons, and SIDIS.

2024.07: 🎉🎉 FMNLOv2.0 include a SIDIS module for calculations of SIDIS at NLO.

2023.05: 🎉🎉 FMNLOv1.0 first release of FMNLO interfaced with MG5_aMC@NLO.

https://fmnlo.sjtu.edu.cn/~fmnlo/

[JG, Liu, Shen, Zhou, 2305.14620 (JHEP)]

The story of higher-order calculations for SIDIS

- In early **2023**, I collaborated with HuaXing on a NLO calculation for hard functions of Energy-energy correlations. Soon we realized that the same method can be applied to hadron production as well.

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PHYSICAL REVIEW LETTERS **132,** 251901 (2024)

Editors' Suggestion

Semi-Inclusive Deep-Inelastic Scattering at Next-to-Next-to-Leading Order in QCD

Leonardo Bonino and Thomas Gehrmann
Physik-Institut, Universität Zürich, Winterthurerstrasse 190, 8057 Zürich, Switzerland

Giovanni Stagnitto
Università degli Studi di Milano-Bicocca & INFN, Piazza della Scienza 3, 20216 Milano, Italy

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Next-to-Next-to-Leading Order QCD Corrections to Semi-Inclusive Deep-Inelastic Scattering

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[1]*The Institute of Mathematical Sciences, A CI of Homi Bhabha National Institute, Taramani, 600113 Chennai, India*
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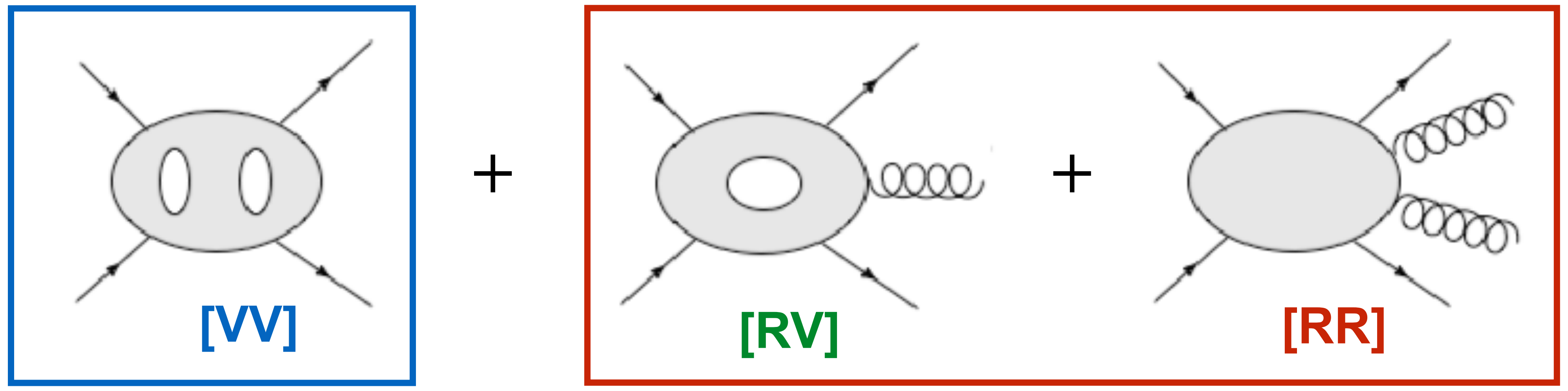
- In late **2025**, after a long cooling down, we finally find a bug in our numerical code and immediately we are able to reproduce the known NNLO corrections

Transverse momentum subtraction for SIDIS

- Generalizing the qT-subtraction formalism (Catani–Grazzini) from Drell-Yan to SIDIS using hadron transverse momentum qT (Breit frame) as the slicing variable, validated against existing analytic NNLO results

components of NNLO corrections



qT subtraction/slicing method

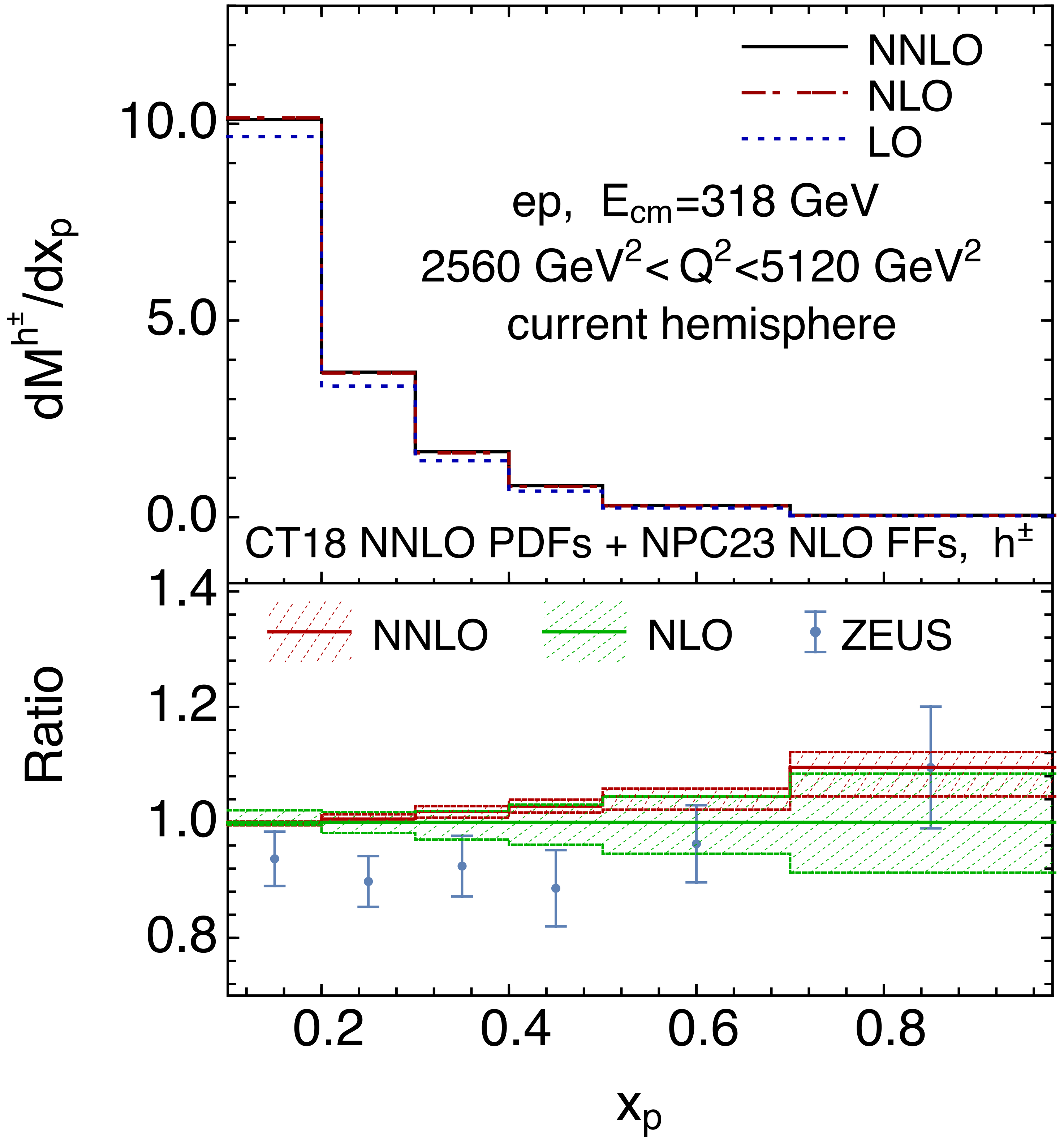
$$\frac{d\sigma}{d\mathcal{O}} = \int_{q_{\mathrm{T,cut}}}^{q_{\mathrm{T,max}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}} + \int_0^{q_{\mathrm{T,cut}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}}$$

TMD factorization of the unresolved part

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\ell+N\to\ell'+h+X}}{\mathrm{d}^2\vec{q}_\perp \mathrm{d}x\mathrm{d}y\mathrm{d}z} \simeq \sum_q H_q(Q^2,\mu) \times \int \frac{\mathrm{d}^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{q}_\perp} f_1^q(x, b_\perp, \xi_0^n, \mu_0^n) \times D_1^q\left(z, \frac{b_\perp}{z}, \xi_0^{\bar{n}}, \mu_0^{\bar{n}}\right)$$
$$\times \prod_i e^{-2K^i_{\mathrm{cusp}}(\mu_0^i,\mu)+A^i_H(\mu_0^i,\mu)} \times \left(\frac{\xi^i}{{\mu_0^i}^2}\right)^{A^i_{\mathrm{cusp}}(\mu_0^i,\mu)} \times \left(\frac{\sqrt{\xi^i}}{\sqrt{\xi_0^i}}\right)^{K^i(b_\perp,\mu_0^i)},$$

[JG, Li, Zhu, Zhu, 2602.06364]

charged hadron multiplicity at HERA



- excellent agreements with NNLO analytical results for all partonic channels; our calculations are fully differential and can be extended to N3LO

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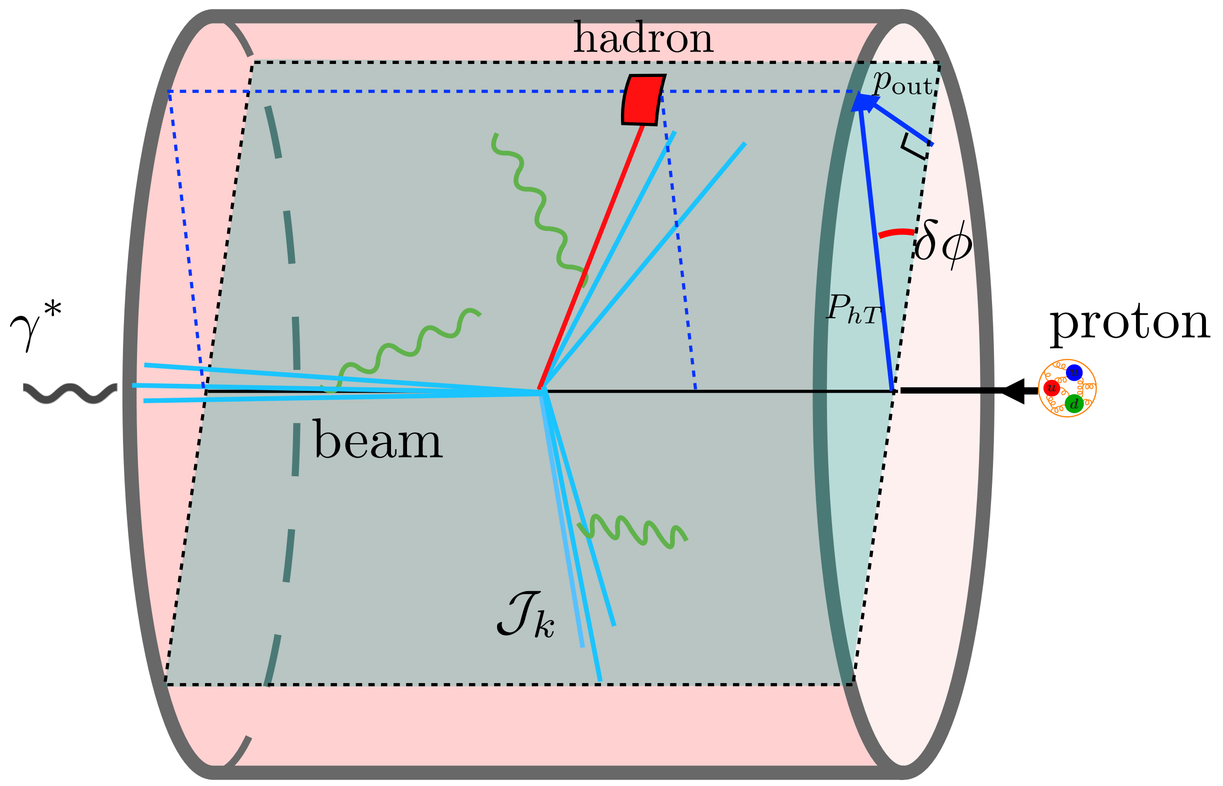
- In late **2025**, after a long cooling down, we finally find a bug in our numerical code and immediately we are able to reproduce the known NNLO corrections

- Running to **2026**, through a series of works, we now made a new breakthrough of leveraging the perturbative precision to N3LO in QCD for SIDIS

SIDIS at finite transverse momentum

✦ First complete NNLO calculation for identified hadron production in DIS at finite hadron PT, based on qT-subtraction with out-of-plane momentum defined with WTA (Winner-Take-All) recombination scheme

out-of-plane momentum/azimuthal decorrelation



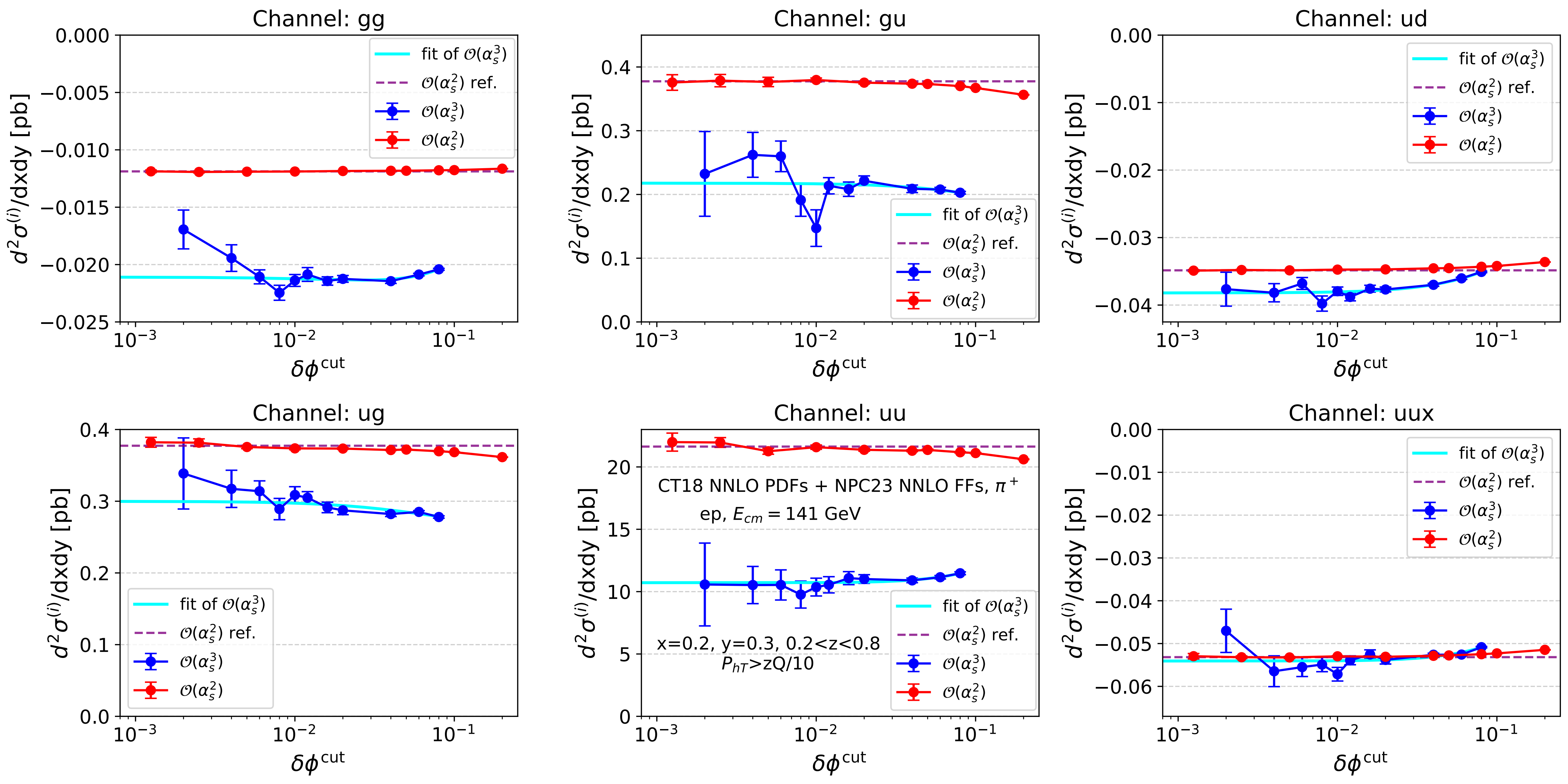
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}} = \int_0^{\delta\phi^{\rm cut}} \mathrm{d}\delta\phi \frac{\mathrm{d}\sigma}{\mathrm{d}\delta\phi\,\mathrm{d}\mathcal{O}} + \int_{\delta\phi^{\rm cut}}^{\delta\phi^{\rm max}} \mathrm{d}\delta\phi \frac{\mathrm{d}\sigma}{\mathrm{d}\delta\phi\,\mathrm{d}\mathcal{O}},$$

TMD factorization of the unresolved part

$$\frac{\mathrm{d}\sigma_{\rm LP}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}\,\mathrm{d}p_{\rm out}} = \int \frac{\mathrm{d}b}{2\pi} e^{ip_{\rm out}b/\zeta} \times \sum_{ijk} \int \mathrm{d}\xi\, H_{ei\to ejk}(Q,\xi,\zeta) \times \mathcal{B}_{i/p}(\xi,b)\,\mathcal{D}_{h/j}(\zeta,b)\,\mathcal{J}_k(b)\,S_{ijk}(b)$$

[Dong, Fang, JG, Li, Shao, Zhu, 2602.22972]

pion cross sections vs. resolution parameter



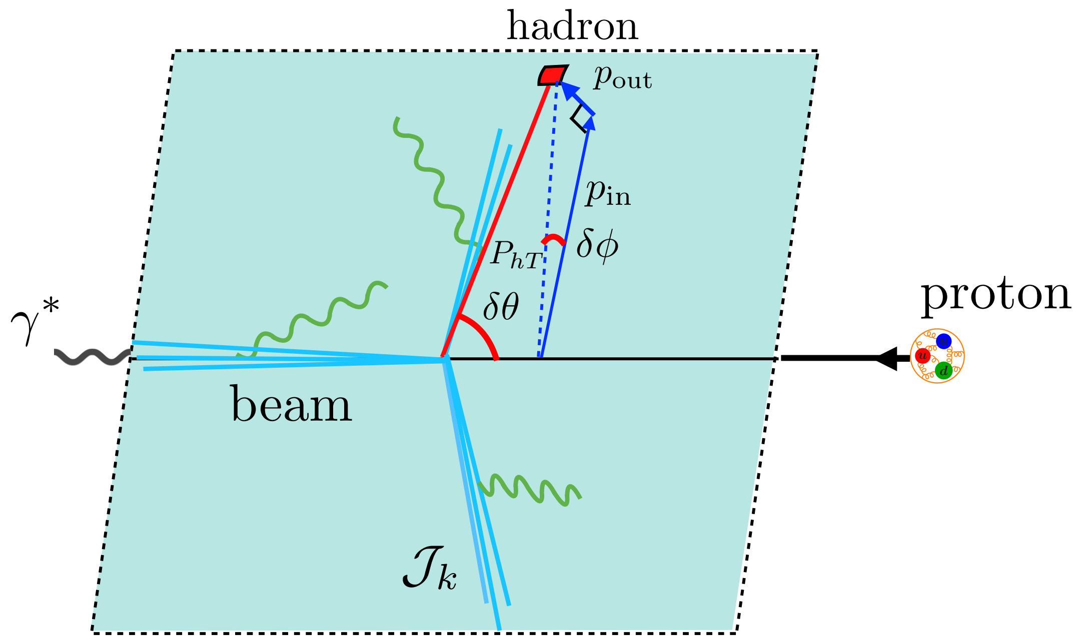
❖ excellent convergence of the QCD corrections wrt. the cutoff of slicing variable for all partonic channels

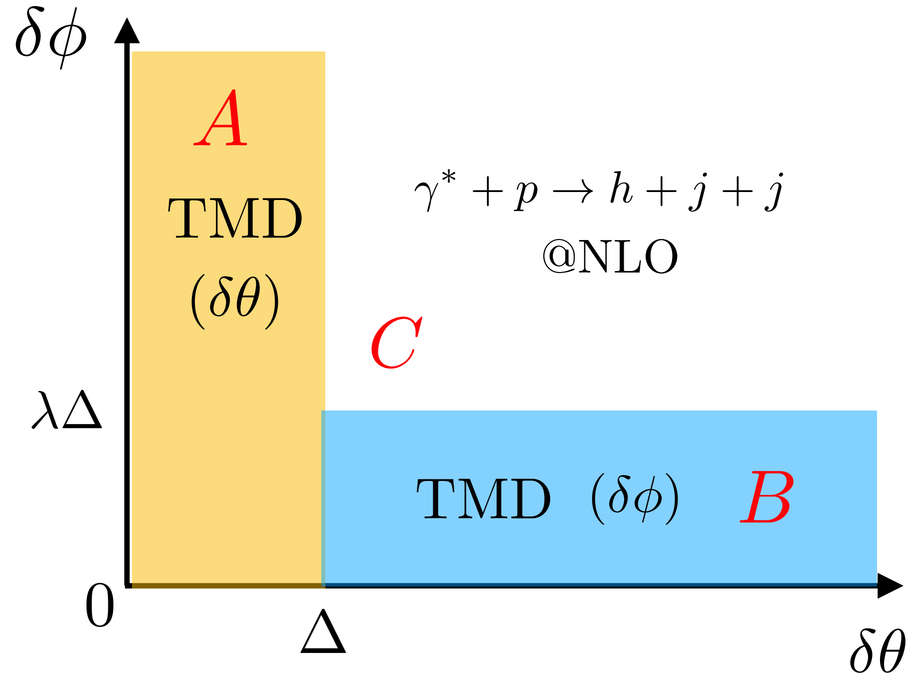
❖ NNLO corrections (blue) are about half of the NLO ones (red) for the two major channels gu and uu

Two-dimensional transverse momentum subtraction

- Introduction of the two-dimensional transverse momentum subtraction formalism for identified hadron production, using both conventional and out-of-plane qT as slicing variables, resulting a first calculation of SIDIS at N3LO in QCD

hadron kinematics and slicing variables



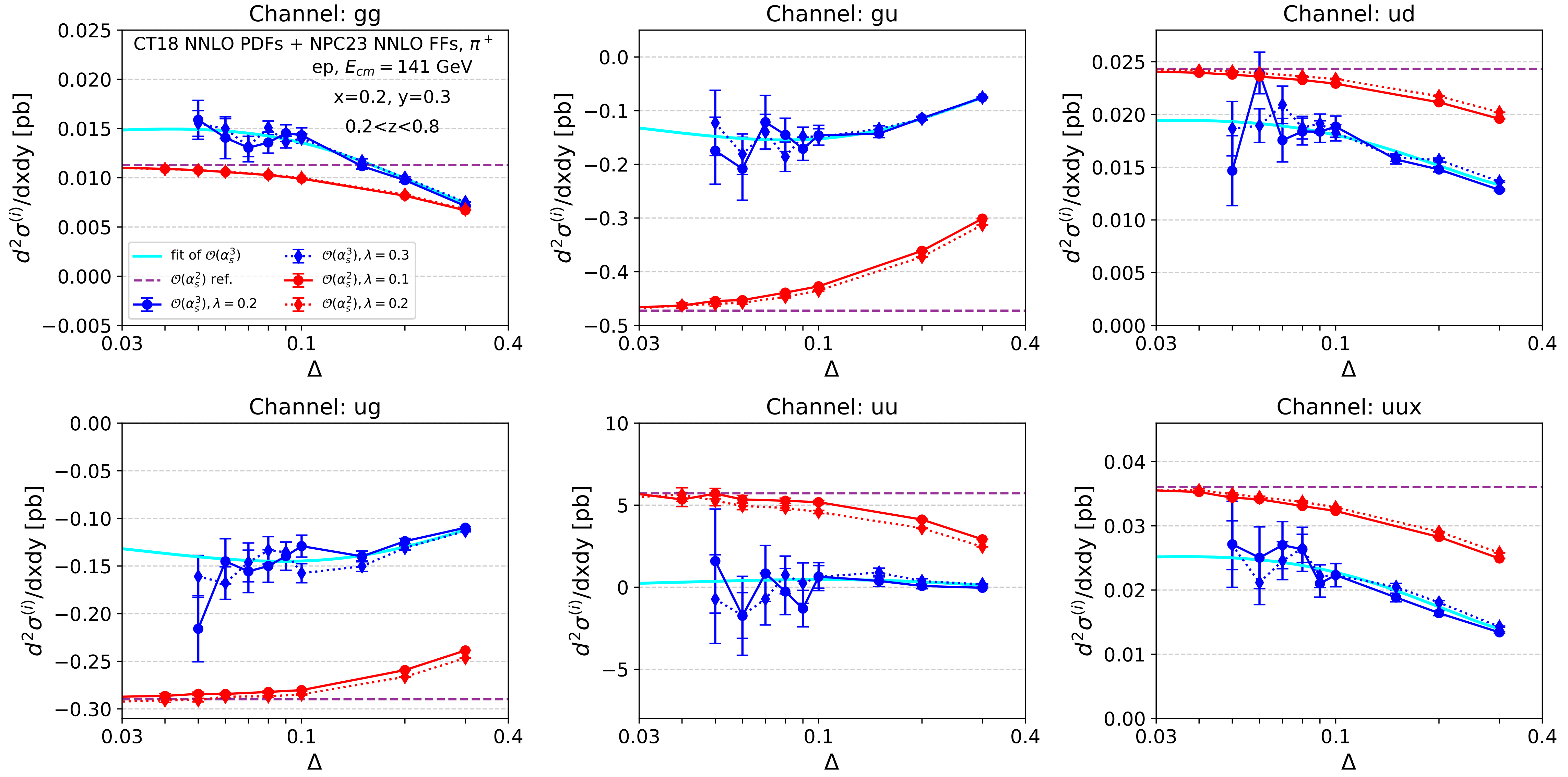


double slicing and two cutoffs

$$\frac{d\sigma}{d\mathcal{O}} = \int_0^{\Delta} d\delta\theta \frac{d\sigma^A}{d\delta\theta d\mathcal{O}} + \int_{\Delta}^{\delta\theta^{\rm max}} d\delta\theta$$
$$\times \left(\int_0^{\lambda\Delta} d\delta\phi \frac{d\sigma^B}{d\delta\theta d\delta\phi d\mathcal{O}} + \int_{\lambda\Delta}^{\delta\phi^{\rm max}} d\delta\phi \frac{d\sigma^C}{d\delta\theta d\delta\phi d\mathcal{O}} \right)$$

[Dong, Fang, JG, Li, Shao, Zhu, Zhu, 2603.29673]

pion cross sections vs. resolution parameter



❖ good convergence of the QCD corrections wrt. both cutoffs for all partonic channels, numerically challenging for small cutoffs

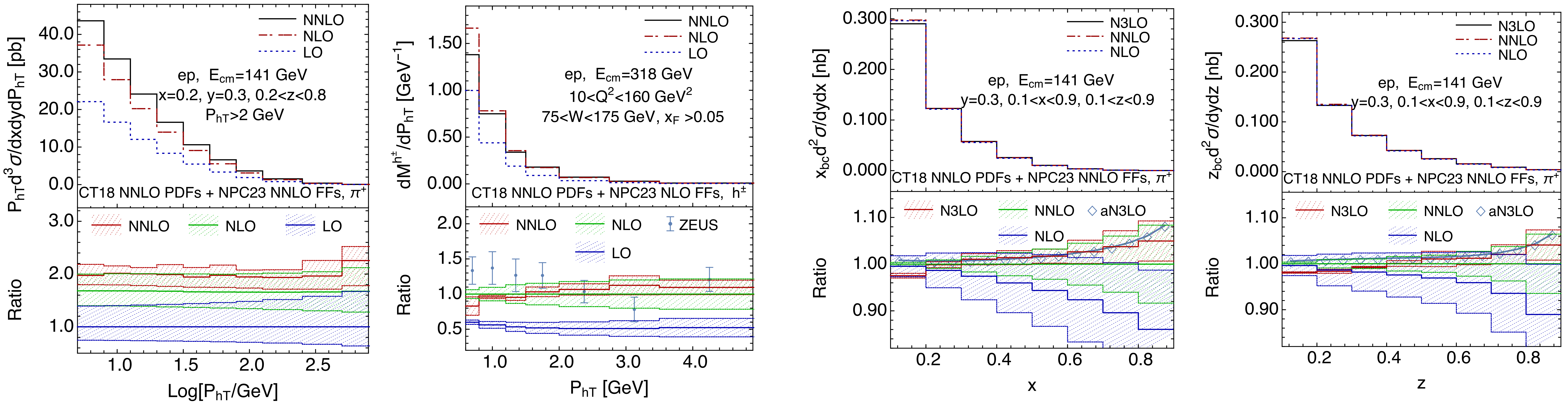
❖ N3LO corrections (blue) are much smaller than NNLO ones (red) for three major channels uu, gu and ug

Implication for Electron-ion colliders

- New perturbative QCD corrections for SIDIS exhibit good convergence and reduced scale variations for differential cross sections at future electron-ion collider, ensuring precision hadron tomography for study of nucleon structure and hadronizations

hadron transverse momentum spectrum at EIC and HERA upto NNLO

distribution of Bjorken-x and z for SIDIS at EIC upto N3LO



[Dong, Fang, JG, Li, Shao, Zhu, Zhu, 2602.22972, 2603.29673]

Summary

- Identified hadron production is essential for the study of nucleon structure and QCD hadronization at high energies, especially SIDIS for determination of PDFs and FFs

- We developed a systematic routine of subtraction/slicing methods for identified hadron production, and leveraged perturbative precision of SIDIS to N3LO in QCD

- The NPC collaboration provides most comprehensive/advanced global analyses on FFs of light hadrons, including on hadron species, data selections, and theoretical precisions (to NNLO)

- With the advancements on ingredients of both non-perturbative and perturbative QCD we are on the way of building a precision framework for theoretical predictions of SIDIS at future electron-ion colliders

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Thank you for your attention!