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Lattice-QCD validation of hadron mass and trace-anomaly decomposition sum rules

Hadron mass decomposition in a common $\overline{\text{MS}}$ scheme for η_c and J/ψ

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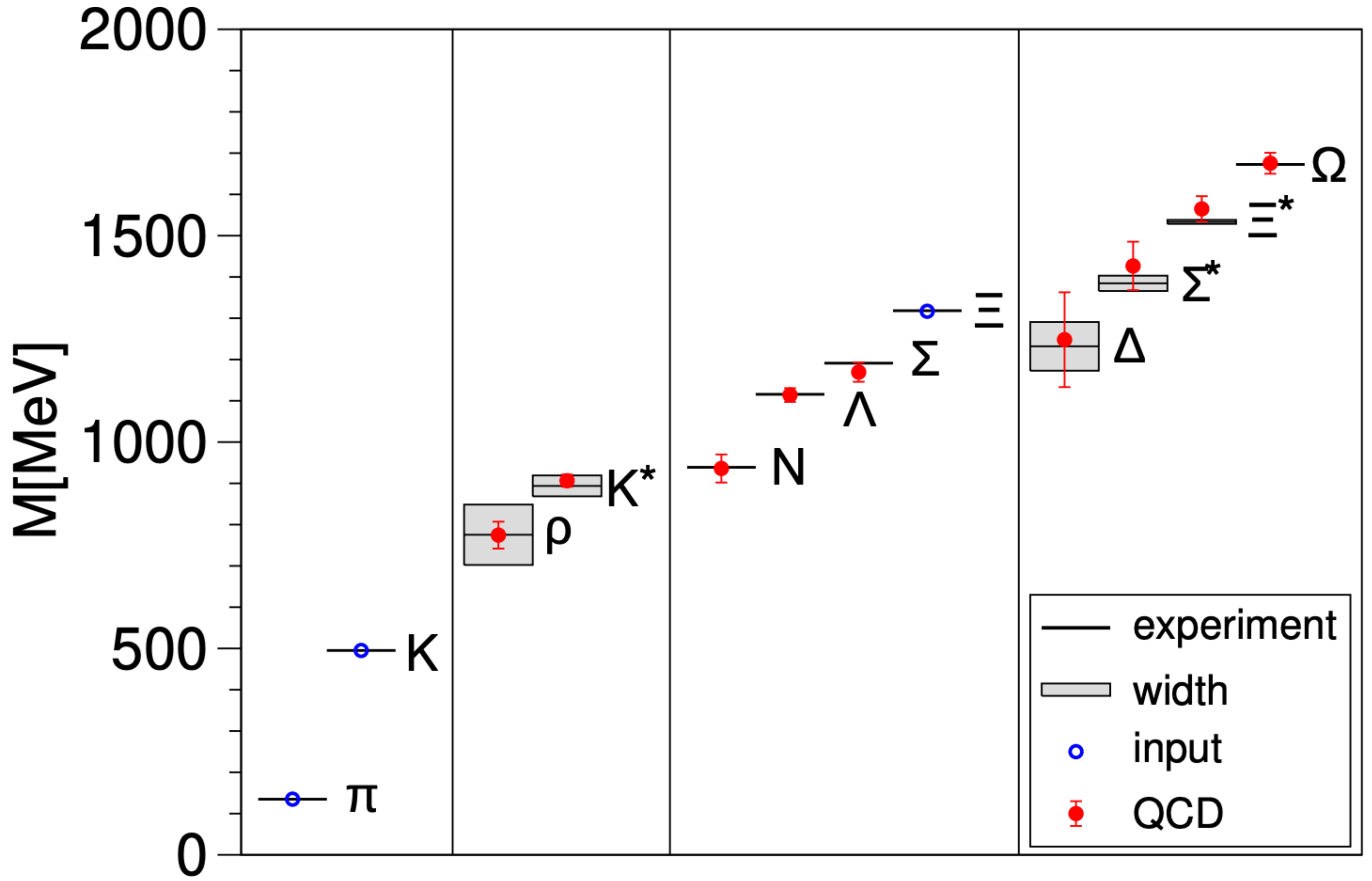
Dennis Bollweg, HTD, Xiang Gao, Ran Luo, Swagato Mukherjee, arXiv: 2601.13070

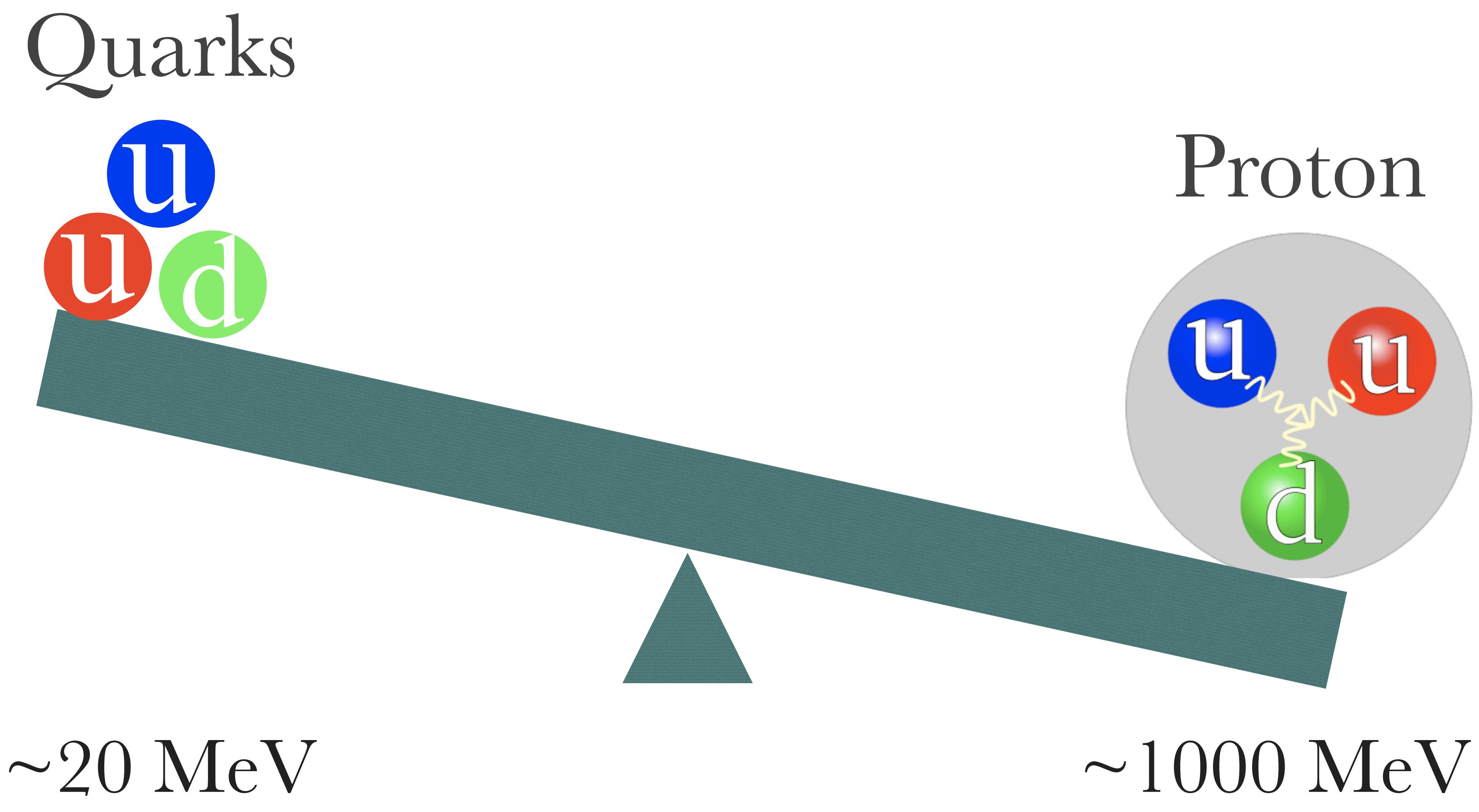
第一届中国电子离子对撞机相关物理年会

April 19-22, 2026@青岛

What generates the hadron mass?



Dürr et al., Science 322:1224-1227,2008



Energy-Momentum Tensor (EMT): from QCD operator to hadron-mass sum rules

$$T^{\mu\nu}_{\mathrm{QCD}} = T^{\mu\nu}_q + T^{\mu\nu}_g$$

$$\langle H(p) \,|\, T^{\mu\nu} \,|\, H(p) \rangle = 2p^\mu p^\nu$$

$$T^{\mu\nu} = \left(T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^\alpha_{\ \alpha} \right) + \frac{1}{4} g^{\mu\nu} T^\alpha_{\ \alpha}$$

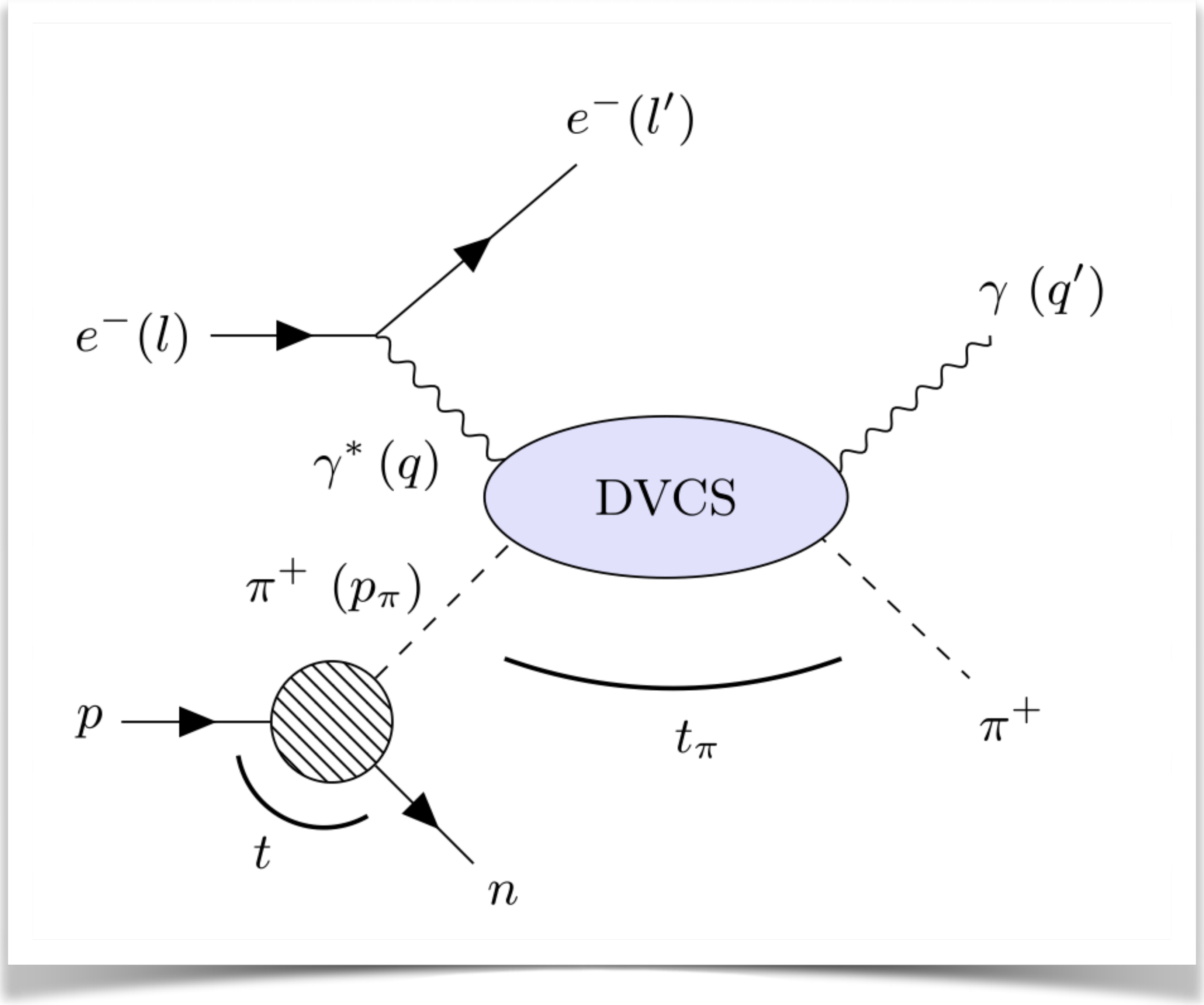
same EMT information ⟶ different organizations ⟶ different hadron-mass sum rules

Ji | HRT | Lorcé | MPR

Ji, PRL 74 (1995) 1071 ;Hatta, Rajan, & Tanaka, JHEP 12 (2018),008 ; Lorcé, EPJC 78 (2018)120; Metz, Pasquini, & Rodini, PRD 102 (2020) 114042

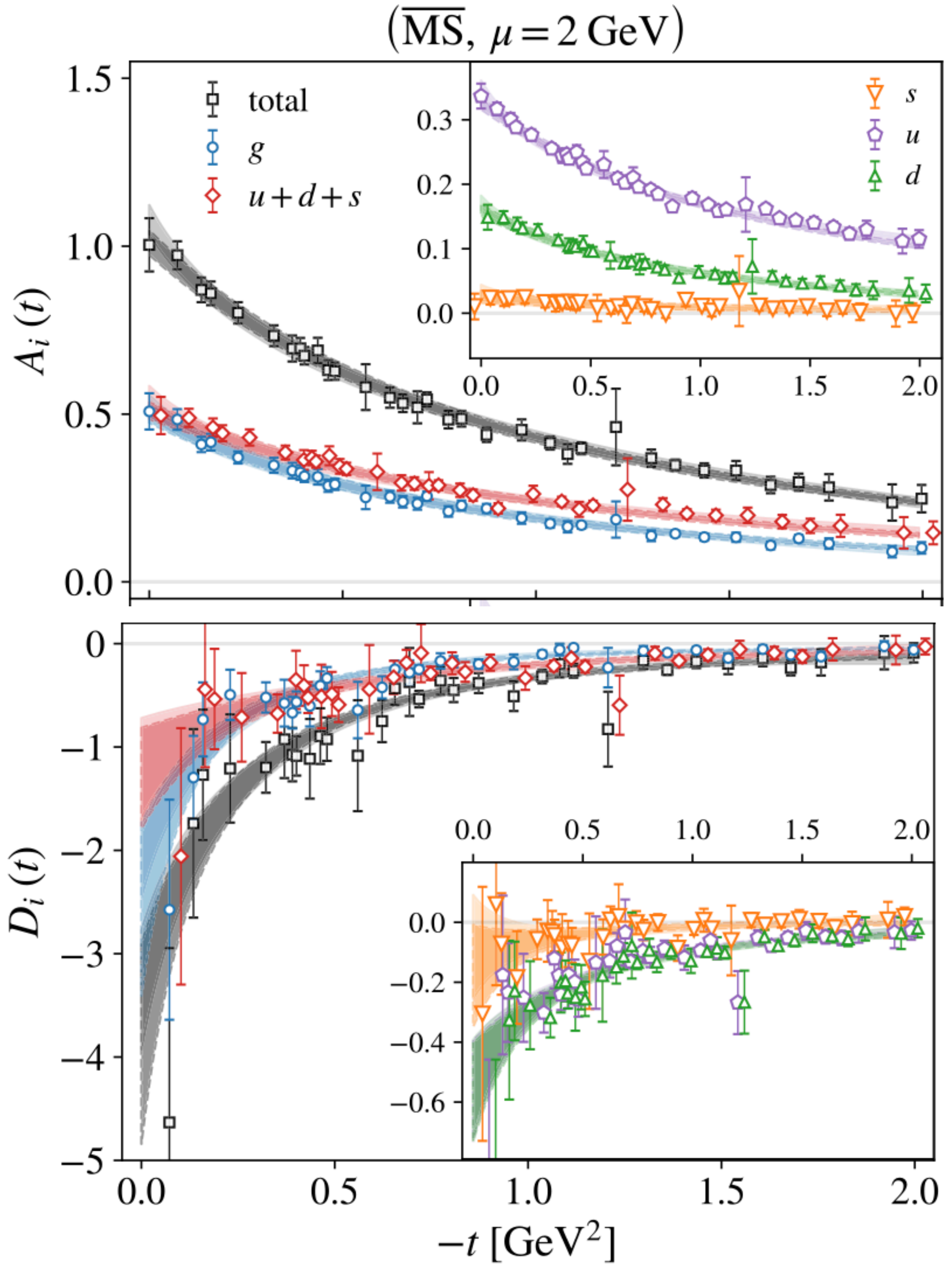
Current status: partial EMT access from experiment and lattice QCD

Deeply virtual exclusive processes
⇒ partial GFF / traceless EMT access



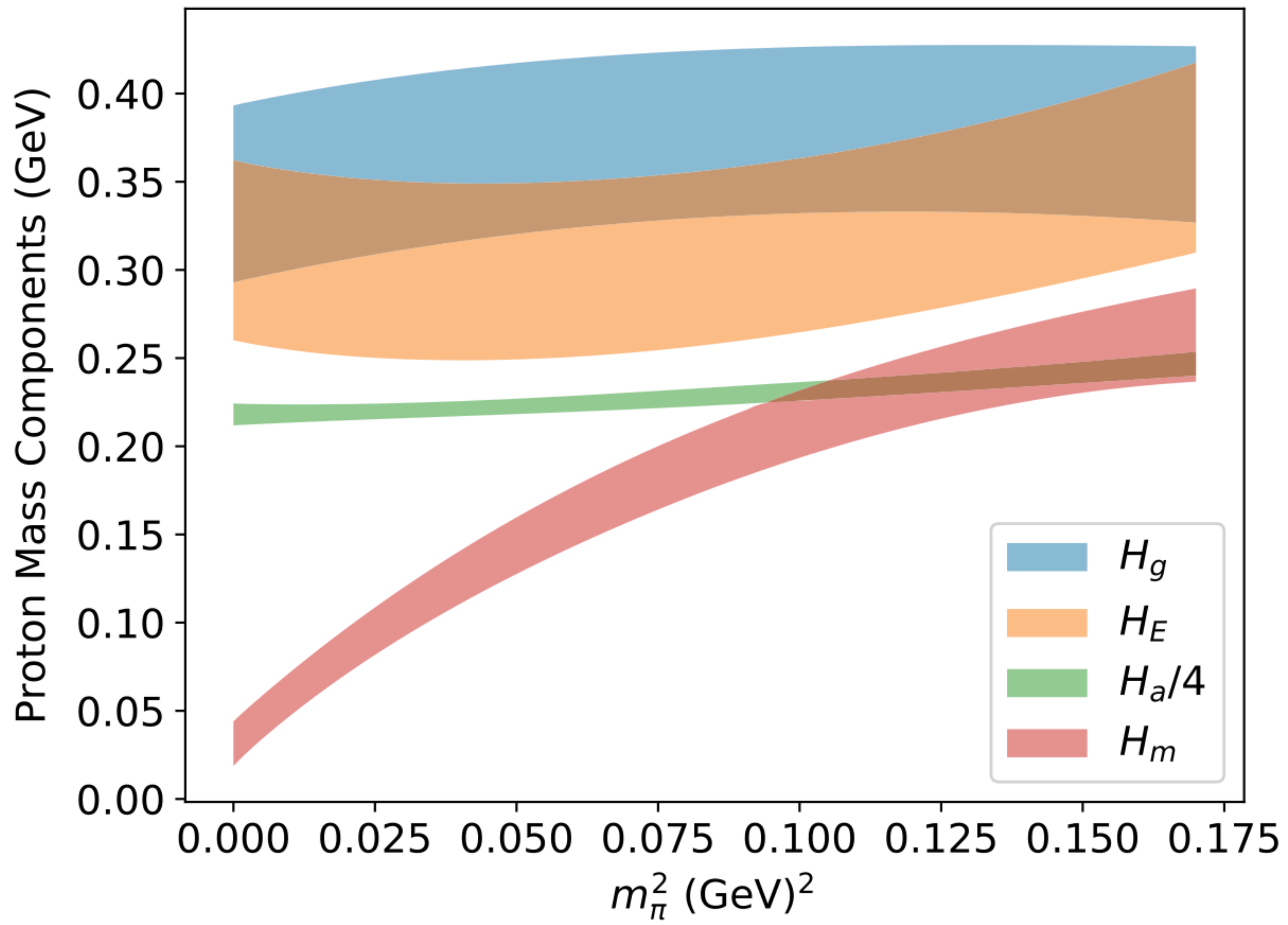
See EXP extracted pressure distribution in e.g.
Burkert, Elouadrhiri & Girod, Nature 557 (2018)396

Recent lattice QCD:
GFFs and spatial structure accessible
but the trace sector remained difficult



Hackett et al., PRL. 132 (2024) 251904

Earlier lattice QCD
proton mass decompositions:
relied on partial EMT input or
indirect closure



Yang, et al.,[χQCD], PRL 121 (2018) 212001

See also Yi-Bo Yang's talk

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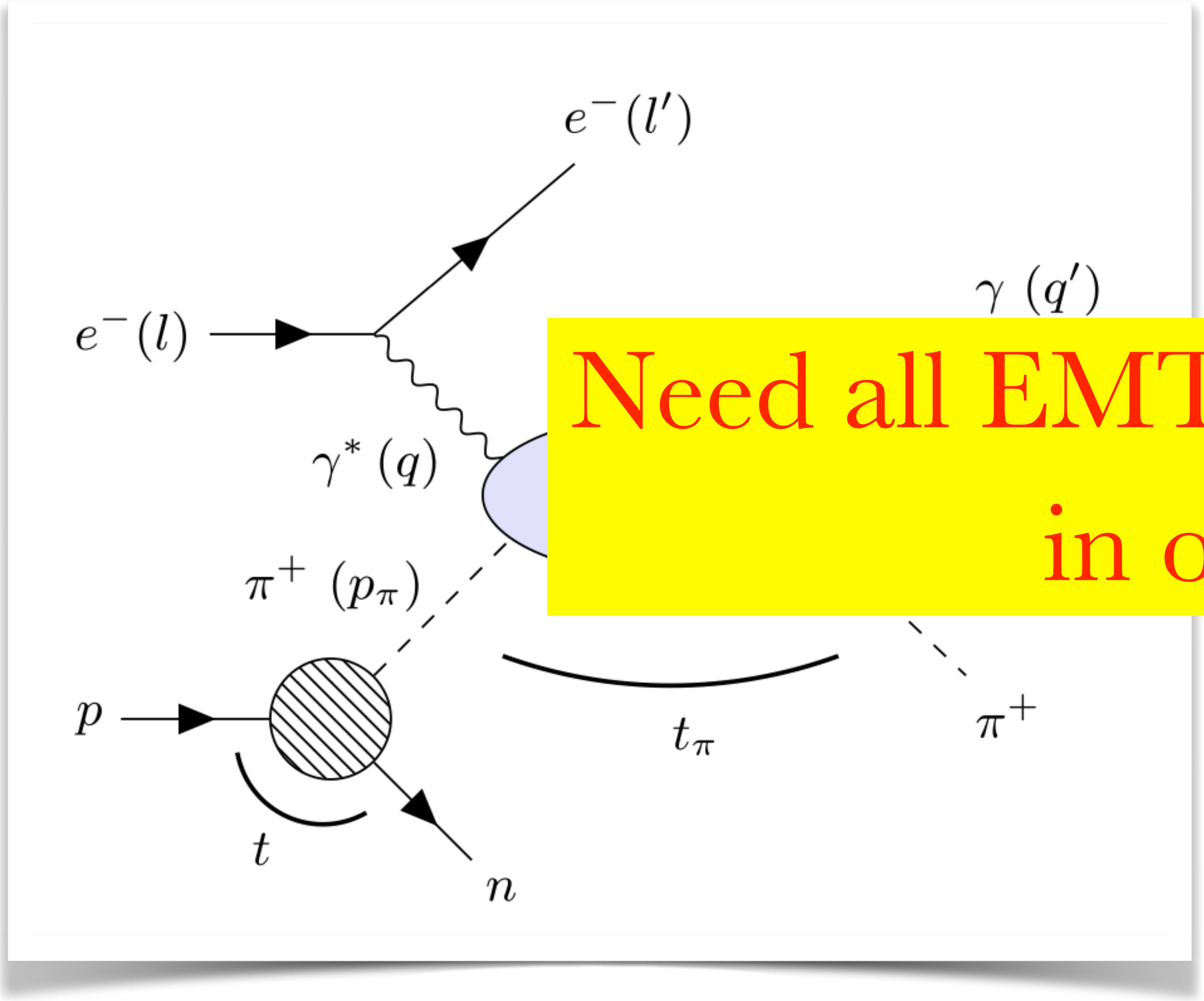

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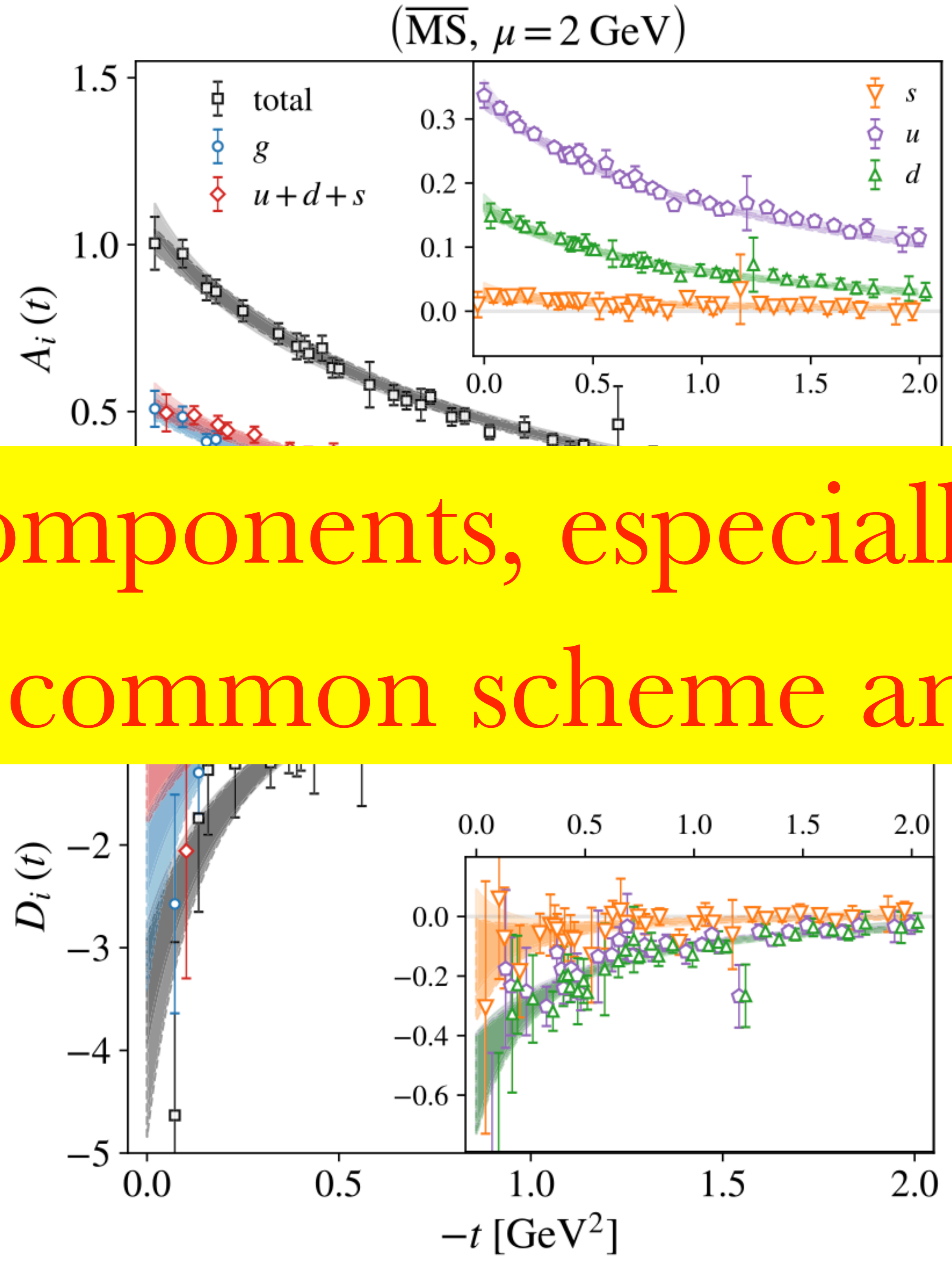

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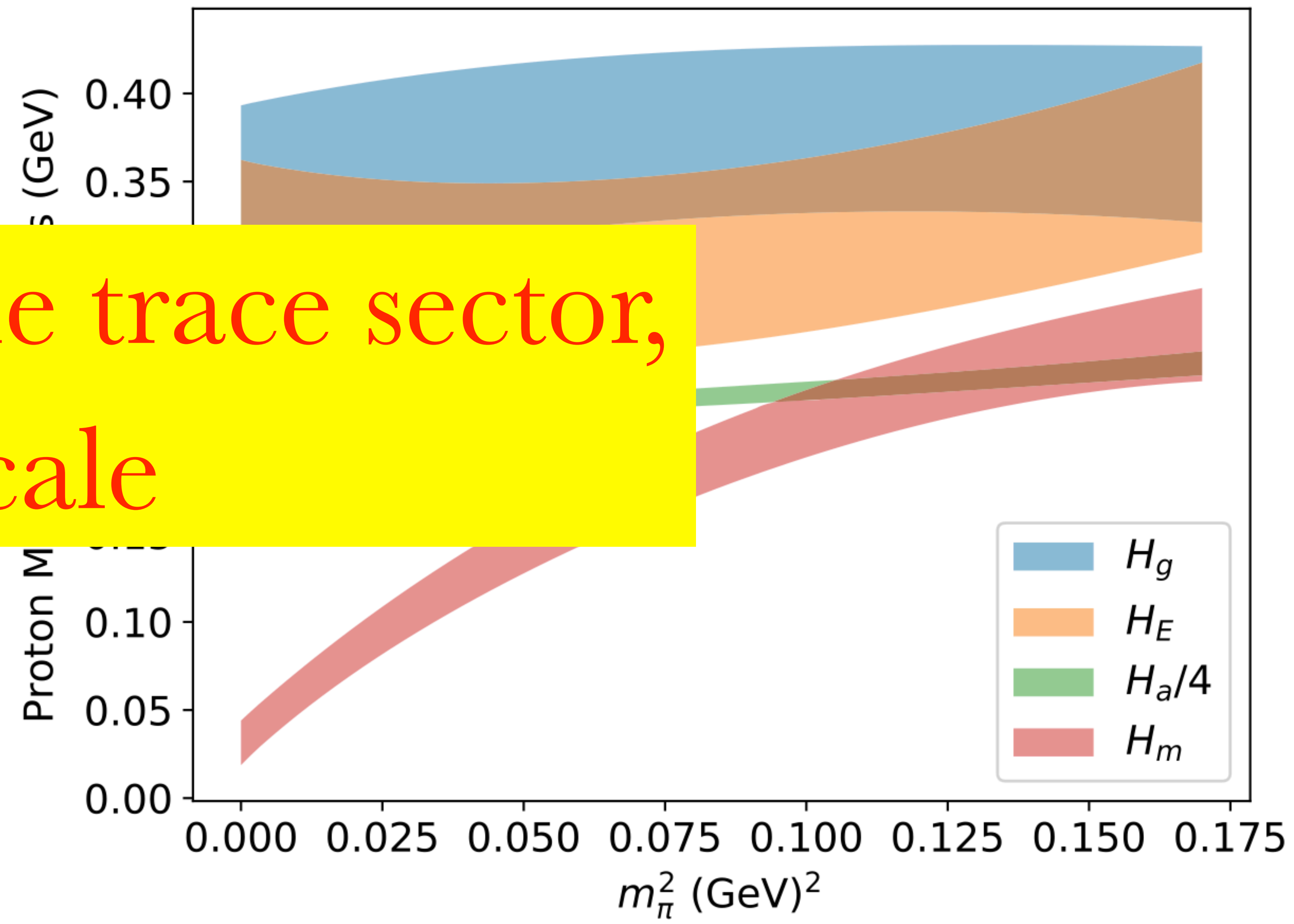

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Need all EMT components, especially the trace sector, in one common scheme and scale

What was difficult before

$$T_{\rho\sigma}(x) = \mathcal{O}_{1,\rho\sigma}(x) - \frac{1}{4}\mathcal{O}_{2,\rho\sigma}(x) + \frac{1}{4}\mathcal{O}_{3,\rho\sigma}(x) - \frac{1}{2}\mathcal{O}_{4,\rho\sigma}(x) - \mathcal{O}_{5,\rho\sigma}(x)$$

Gluon part: $T_{g,\rho\sigma}$

$$\mathcal{O}_{1,\rho\sigma} = \frac{2}{g_0^2}\mathrm{Tr}^{\,c}\left[F_{\rho\omega}F_{\sigma\omega}\right],\quad \mathcal{O}_{2,\rho\sigma} = \frac{2}{g_0^2}\delta_{\rho\sigma}\mathrm{Tr}^{\,c}\left[F_{\omega\xi}F_{\omega\xi}\right]$$

Quark part: $T_{q,\rho\sigma}$

$$\mathcal{O}_{3,\rho\sigma} = \sum_f \mathcal{O}_{3,\rho\sigma,f} = \sum_f \bar{\psi}_f\left(\gamma_\rho \overleftrightarrow{D}_\sigma + \gamma_\sigma \overleftrightarrow{D}_\rho\right)\psi_f$$

$$\mathcal{O}_{4,\rho\sigma} = \delta_{\rho\sigma}\sum_f \bar{\psi}_f \overleftrightarrow{\not{D}}\psi_f\,,\qquad \mathcal{O}_{5,\rho\sigma} = \delta_{\rho\sigma}\sum_f m_f\bar{\psi}_f\psi_f$$

Tensor operators: $\mathcal{O}_1, \mathcal{O}_3$ Pure-trace operators: $\mathcal{O}_2, \mathcal{O}_4, \mathcal{O}_5$

- EMT trace lies in the scalar channel under lattice hypercubic symmetry $\Rightarrow$ mixing with lower-dimensional operators
- power-divergent subtraction is required
- conventional RI/MOM-type renormalization: gauge dependent/IR sensitive/ noisy

Belinfante, Phys. Rev. 128 (1962) 2832; Belitsky & Radyushkin, Phys. Rept. 418 (2005) 1; Freese, Phys. Rev. D 106 (2022) 125012,…

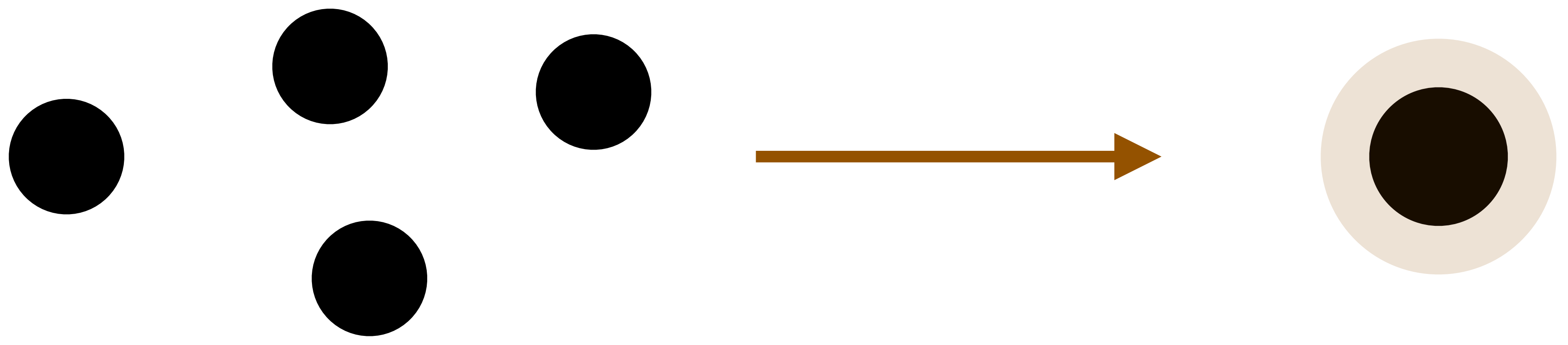
Our strategy: from gradient flow to a common $\overline{\text{MS}}$ EMT scheme

Gradient Flow (GF): Lüscher et al., 10', 11', 14'

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(0,x) = A_\mu(x)$$

Flow time t_f smooth UV fluctuations

over a radius of $\sqrt{8t_f}$



Small-flow-time EMT reconstruction

$$T^{\overline{\text{MS}}}_{\rho\sigma}(x,\mu) = \lim_{t_f\to 0} \Sigma_i c_i(t_f,\mu)\bar{\mathcal{O}}_{i,\rho\sigma}(t_f,x)$$

flowed operators are matched to the physical EMT;
the coefficients c_i are known perturbatively

- controls mixing and avoids power-divergent subtractions
- puts traceless and trace sectors into one common framework

Operationally in this work

Flowed EMT at fixed t_f ⇨ Take $a \to 0$ at each fixed t_f ⇨ 2-loop matching GF $\to$ MSbar ⇨ Zero-Flow-time extrapolation $t_f \to 0$ ⇨ All EMT ingredients in one common scheme

common scheme-and-scale determination of traceless and trace contributions

Lattice setup and extracted matrix elements

Ensembles

- 2+1 flavor HISQ, HotQCD ensembles
- $a \approx 0.06, 0.08, 0.12$ fm
- $64 \times 48^3, 64^4, 64^4$
- physical strange, near-physical light quarks
- physical charmonium: η_c & J/ψ

Measured quantities

- flowed EMT operators $\mathcal{O}_1, \cdots \mathcal{O}_5$
- flowed quark and gluon matrix elements
- both traceless & trace sectors

Extraction

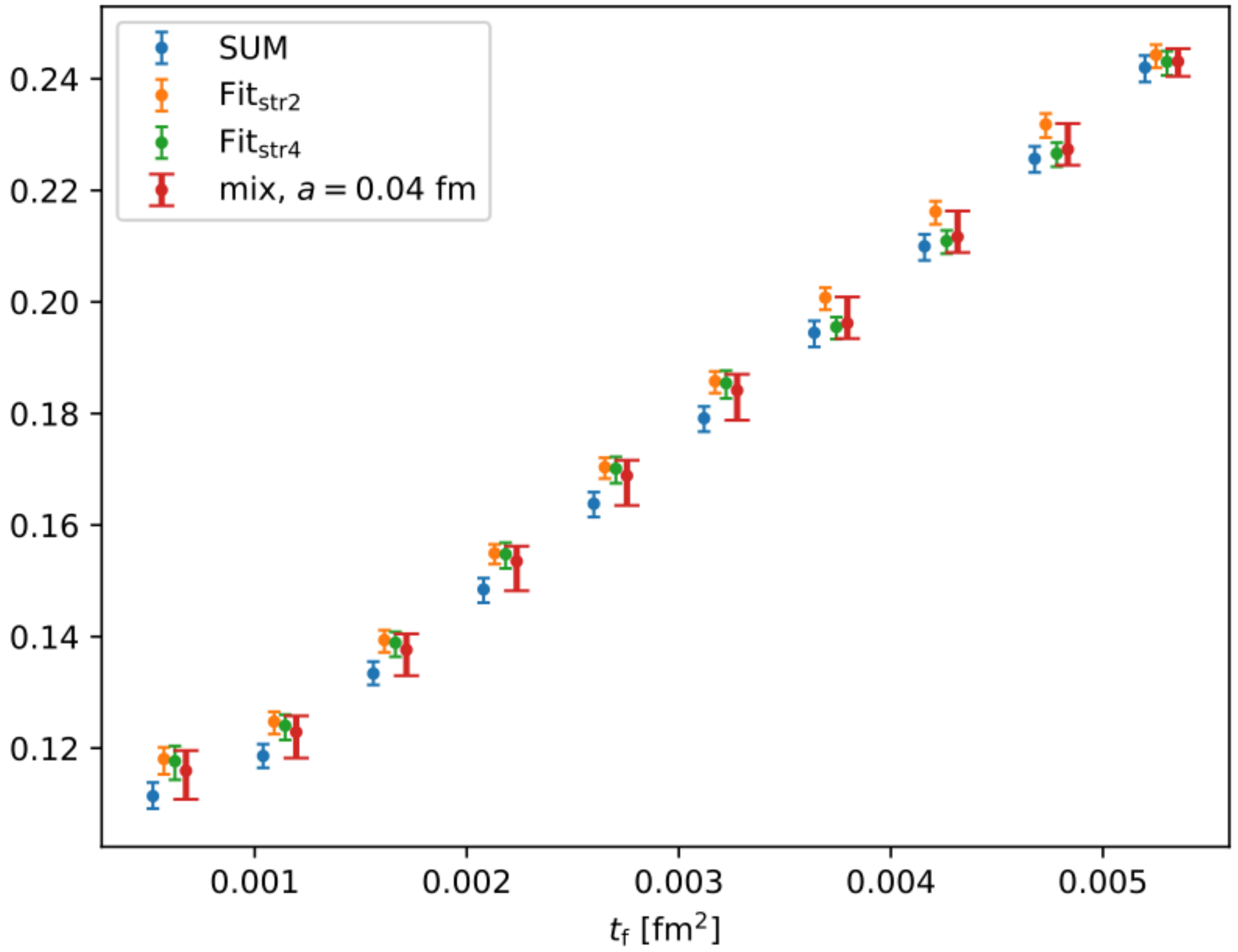
- 2-point & 3-point correlators $\rightarrow < H | \mathcal{O}_i | H >$
- $a \rightarrow 0$ at fixed t_f
- GF $\rightarrow \overline{\mathrm{MS}}$ matching
- $t_f \rightarrow 0$

Dennis Bollweg, HTD, Xiang Gao, Ran Luo, Swagato Mukherjee, arXiv: 2601.13070

Stable extraction of flowed EMT matrix elements

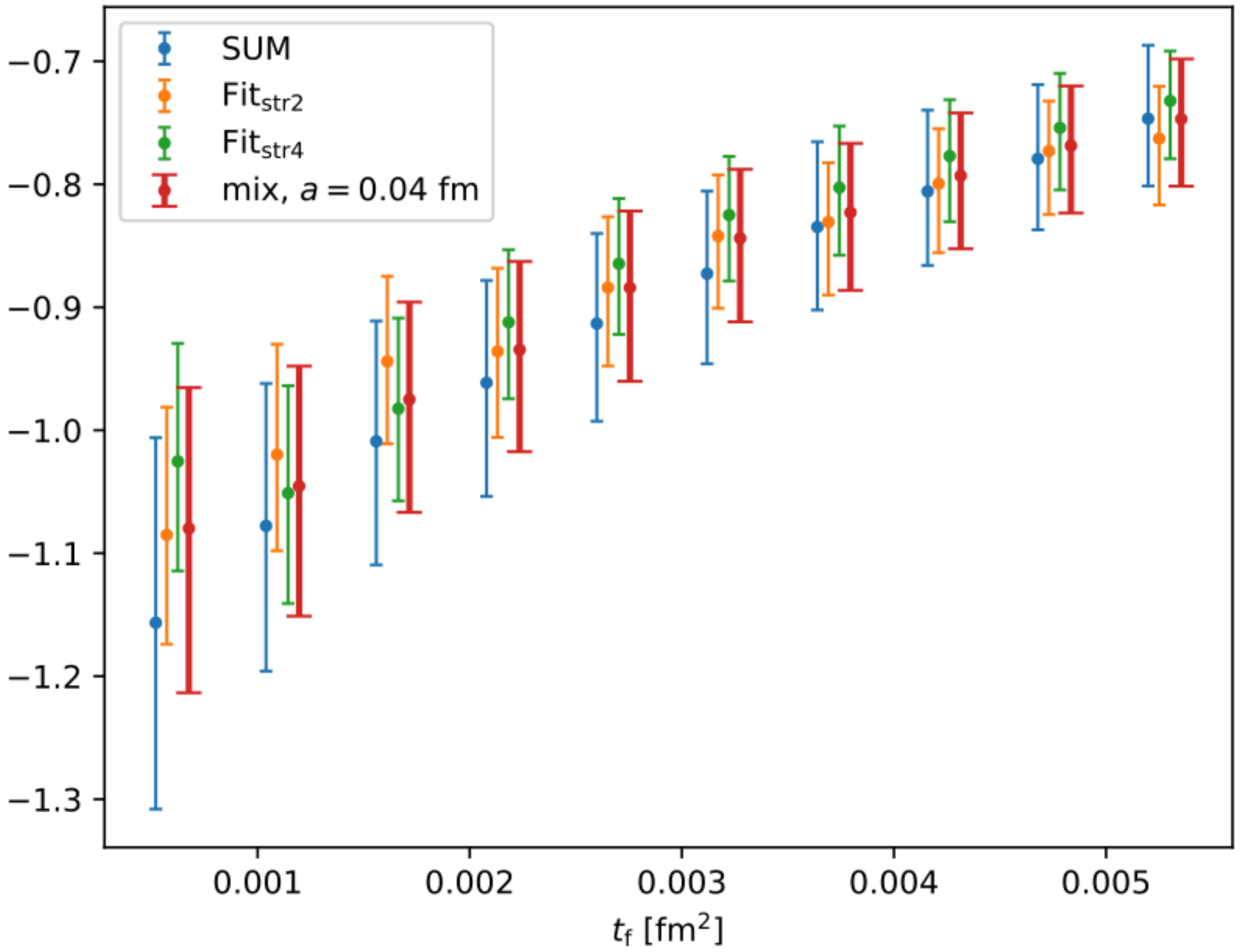
quark matrix element

$\frac{1}{2M}\sum_{k=1}^{3}\langle\eta_c|\widetilde{\mathcal{O}}_{3,kk}/3|\eta_c\rangle$



gluon matrix element

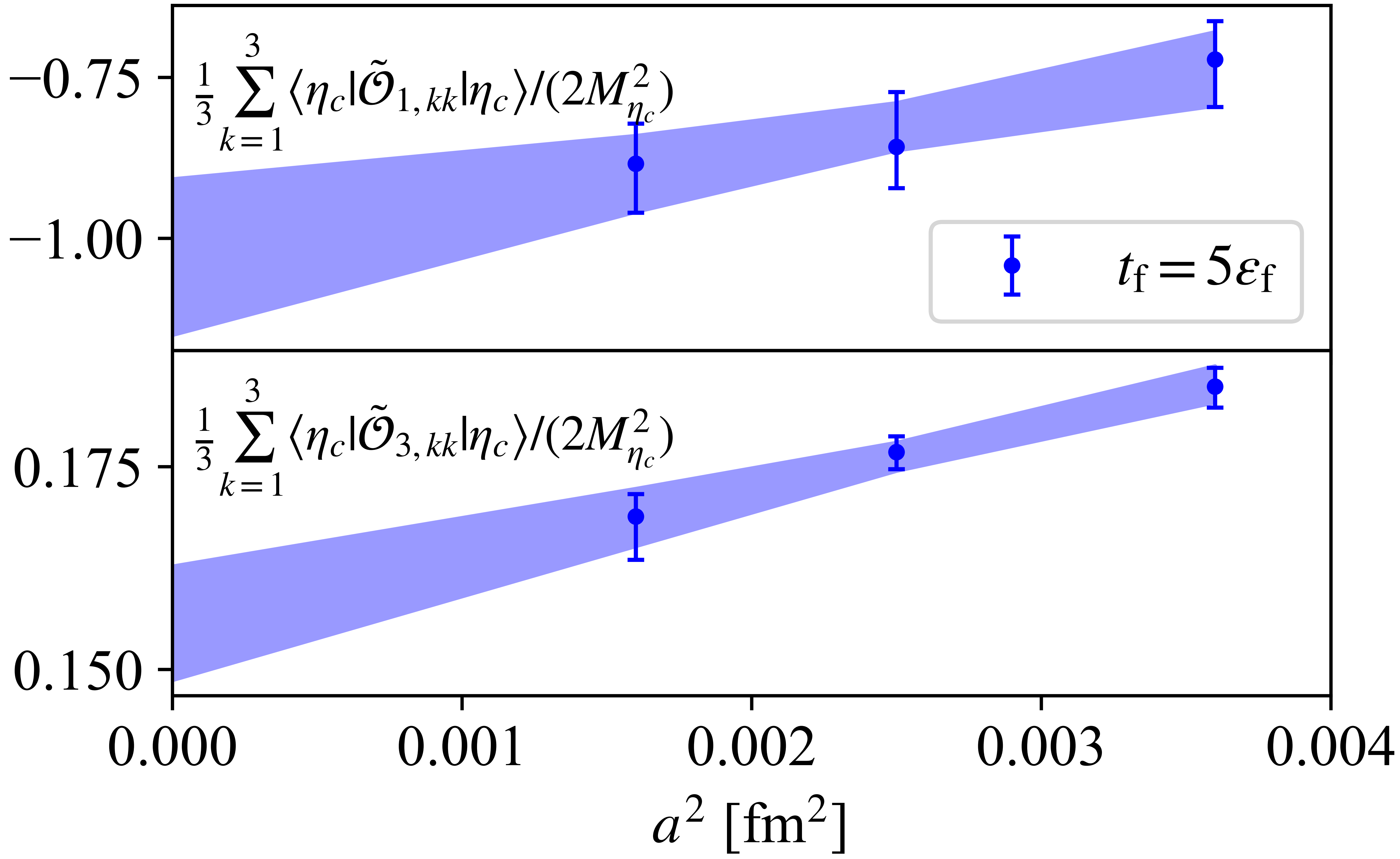
$\frac{1}{2M}\sum_{k=1}^{3}\langle\eta_c|\widetilde{\mathcal{O}}_{1,kk}/3|\eta_c\rangle$



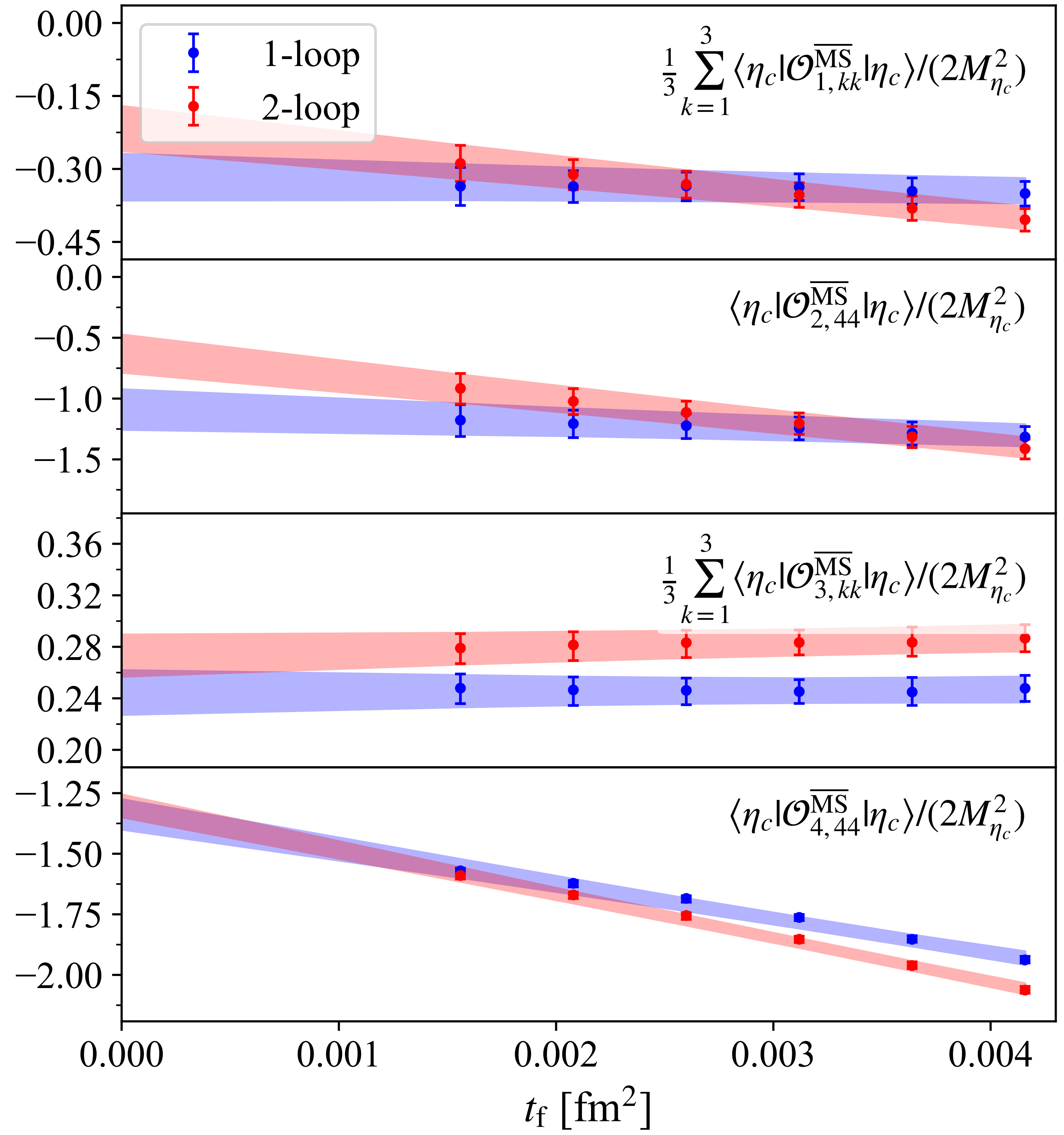
Flowed quark and gluon EMT matrix elements are extracted stably from standard 3pt/2pt ratios

From flowed matrix elements to the physical EMT

Continuum extrapolation at fixed flow time t_f



GF → $\overline{\text{MS}}$ matching and $t_f \to 0$ extrapolation



Pert. matching from: Suzuki, 14'15'; Harlander et al., 18'

Smooth continuum, matching and $t_f \to 0$ behavior enables a common determination of all EMT components

Forward-limit GFF ingredients at rest for η_c and J/ψ

$A(0)$: momentum fraction type term; $\bar{C}(0)$: trace-related GFF ingredient; $\bar{f}(0)$: extra spin-1 structure for J/ψ

	η_c		J/ψ		
	$A(0)$	$\bar{C}(0)$	$A(0)$	$\bar{C}(0)$	$\bar{f}(0)$
gluon(g)	0.084(45)	0.058(18)	0.077(35)	-0.146(29)	-0.012(32)
charm(c)	0.918(17)	-0.068(4)	0.926(34)	0.176(87)	0.023(5)
total	1.000(46)	−0.010(18)	0.997(43)	0.030(30)	0.010(32)

$$\langle \eta_c | T^{\overline{\rm MS}}_{X,00} | \eta_c \rangle = 2M^2_{\eta_c}[A_X(0) + \bar{C}_X(0)]$$

$$\sum_{k=1}^{3} \langle \eta_c | T^{\overline{\rm MS}}_{X,kk} | \eta_c \rangle = -6M^2_{\eta_c}\bar{C}_X(0)$$

$$\langle J/\psi | T^{\overline{\rm MS}}_{X,11} | J/\psi \rangle = 2M^2_{J/\psi}\left[\frac{1}{2}\bar{C}_X(0) + \frac{3}{4}\bar{f}_X(0)\right]$$

$$\langle J/\psi | T^{\overline{\rm MS}}_{X,00} | J/\psi \rangle = 2M^2_{J/\psi}[A_X(0) - \frac{1}{2}\bar{C}_X(0) + \frac{1}{4}\bar{f}_X(0)]$$

$$\sum_{k=2,3} \langle J/\psi | T^{\overline{\rm MS}}_{X,kk} | J/\psi \rangle = 4M^2_{J/\psi}\left[\frac{1}{2}\bar{C}_X(0) - \frac{1}{4}\bar{f}_X(0)\right]$$

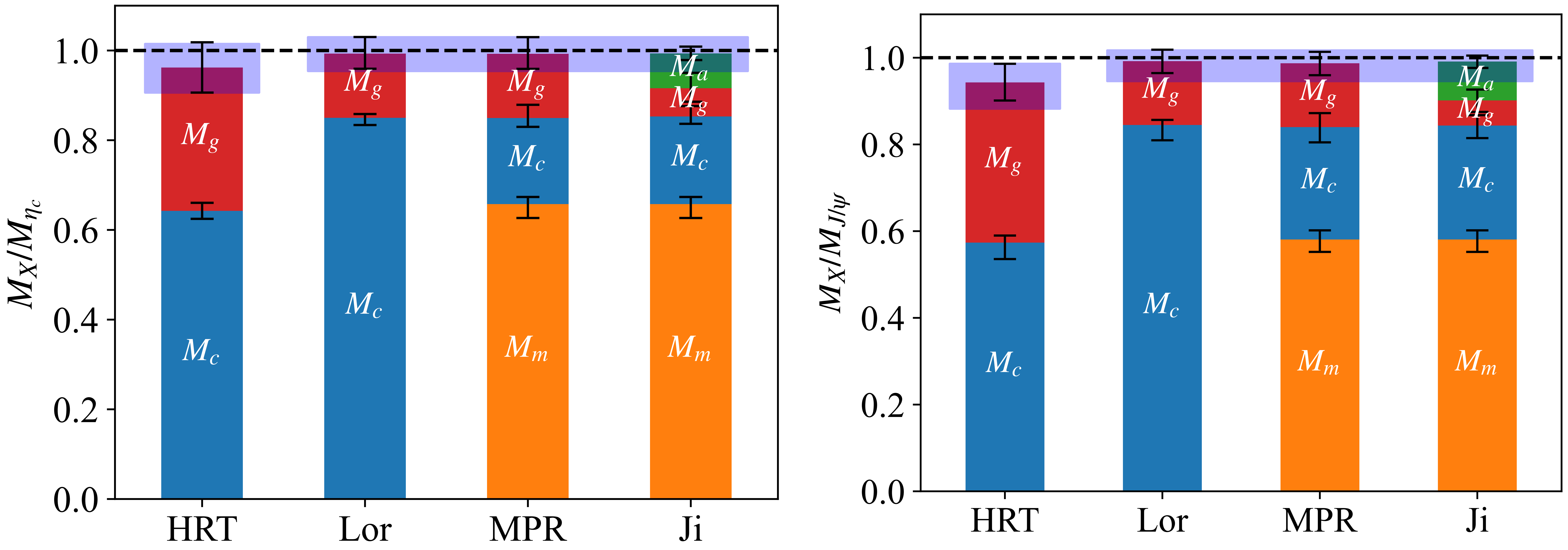
- $\bar{C}(0)$ **determined directly** for η_c and J/ψ
- $A_g(0) + A_c(0) = 1$ for both states
- $\bar{C}_g(0) + \bar{C}_c(0) = 0$ within uncertainties
- $\bar{f}_g + \bar{f}_c(0) = 0$ within uncertainties for J/ψ

Consistent with the forward EMT sum rules

Mass decompositions of η_c and J/ψ : four sum rules

Same EMT input; HRT / Lorcé: 2-part organization; MPR /Ji: explicit trace-sector split

dashed line / purple band: closure to 1 within uncertainties



M_c: charm energy ; M_g: gluon; M_m: explicit charm-mass term; M_a: trace-anomaly

Dennis Bollweg, HTD, Xiang Gao, Ran Luo, Swagato Mukherjee, arXiv: 2601.13070

Summary

- Constructed the full QCD EMT, including the trace sector, in a common $\overline{\text{MS}}$ framework
- Provides a general first-principles framework for hadron mass, spin, and GFFs

- Determined forward-limit GFF ingredients for η_c and J/ψ, including $\bar{C}(0)$, for the first time
- HRT, Lorcé, MPR and Ji mass sum rules are mutually consistent within uncertainties
- Gluon, explicit charm-mass, and anomaly contributions are all nontrivial in charmonium mass decomposition

Same EMT input, different organizations - one consistent first-principles picture

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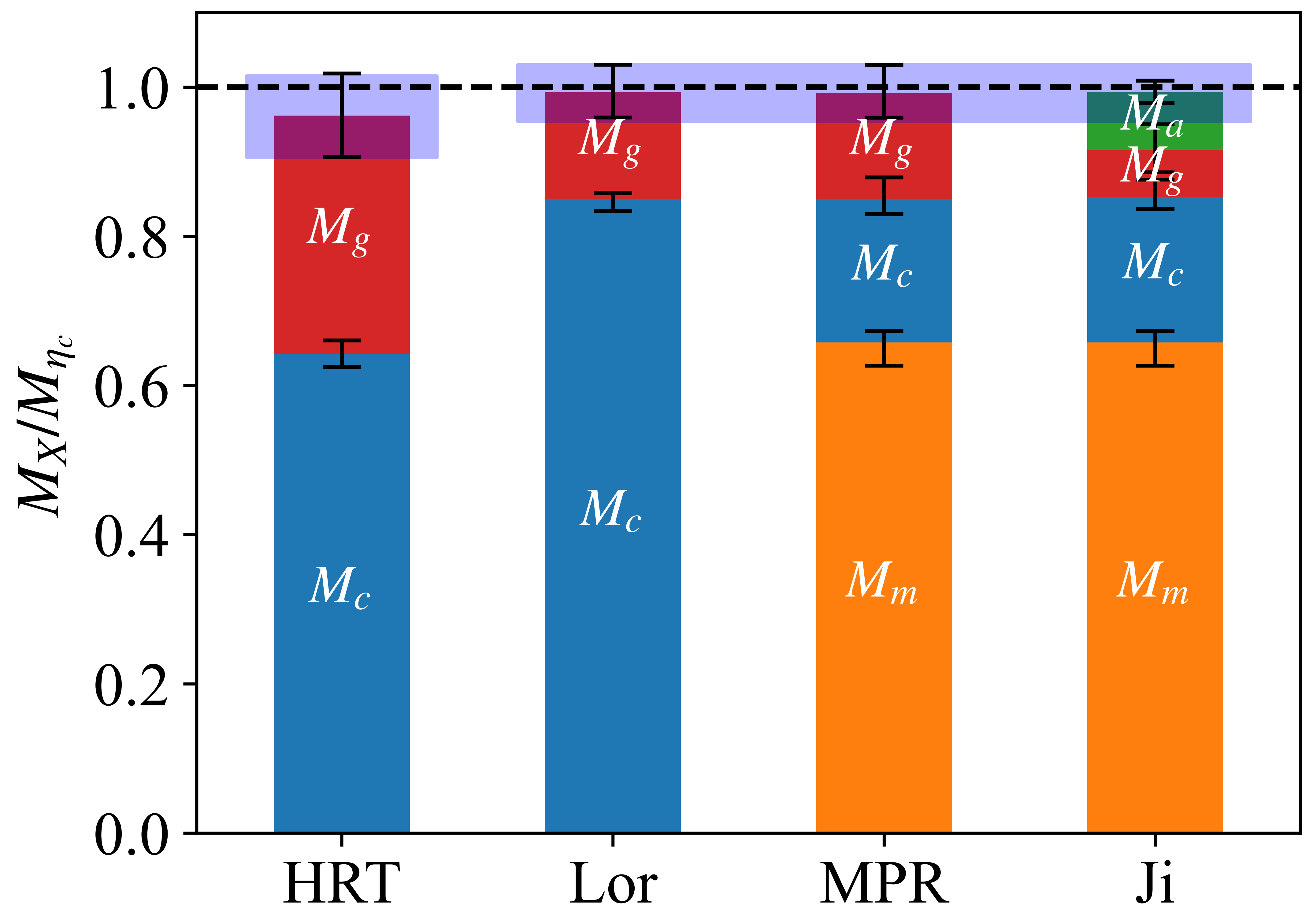
Four sum rules



$$M_H = \sum_{X=q,g} M_X^{\mathrm{HRT}}, \text{ with } M_X^{\mathrm{HRT}} = \frac{\langle H| \left(T_X^{\overline{\mathrm{MS}}}\right)^{\rho}_{\rho} |H\rangle}{2M_H}.$$

$$M_H = \sum_{X=q,g} M_X^{\mathrm{Lor}}, \quad \text{with} \quad M_X^{\mathrm{Lor}} = \frac{\langle H|T_{X,00}^{\overline{\mathrm{MS}}}|H\rangle}{2M_H}.$$

$$M_H = \sum_{X=q,g,m} M_X^{\mathrm{MPR}}, \quad \text{with} \quad M_m^{\mathrm{MPR}} = \sum_f \sigma_f,$$

$$M_q^{\mathrm{MPR}} = M_q^{\mathrm{Lor}} - \sum_f \sigma_f, \quad M_g^{\mathrm{MPR}} = M_g^{\mathrm{Lor}}.$$

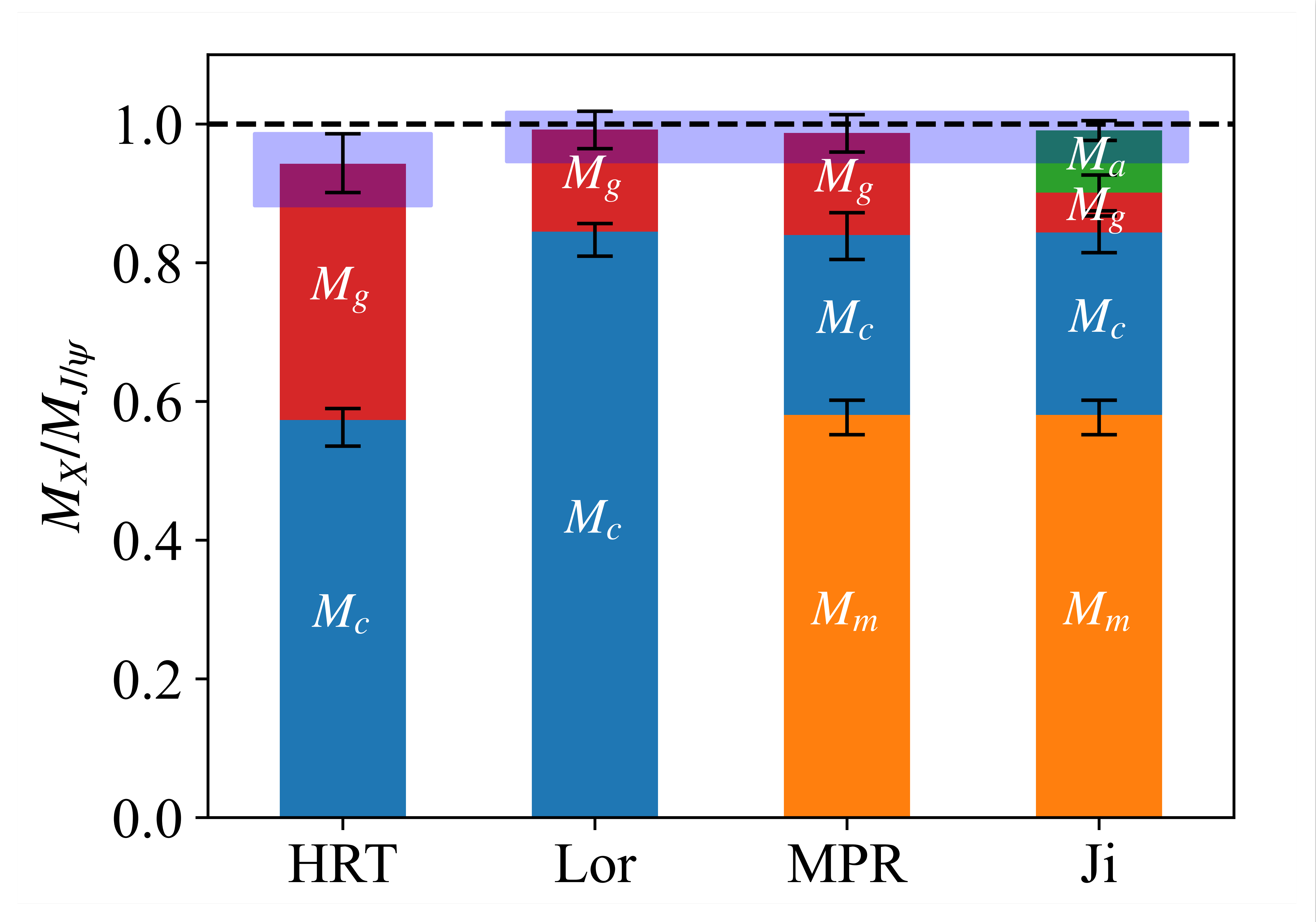
$$M_H = \sum_{X=q,g,m,a} M_X^{\mathrm{Ji}}, \quad \text{with} \quad M_m^{\mathrm{Ji}} = M_m^{\mathrm{MPR}},$$

$$M_q^{\mathrm{Ji}} = M_q^{\mathrm{Lor}} - \frac{1}{4} M_q^{\mathrm{HRT}} - \frac{3}{4} M_m^{\mathrm{MPR}},$$

$$M_g^{\mathrm{Ji}} = M_g^{\mathrm{Lor}} - \frac{1}{4} M_g^{\mathrm{HRT}},$$

$$M_a^{\mathrm{Ji}} = \frac{1}{4}\left[M_q^{\mathrm{HRT}} + M_g^{\mathrm{HRT}} - M_m^{\mathrm{MPR}}\right].$$

Four sum rules



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$$M_H = \sum_{X=q,g,m,a} M_X^{\mathrm{Ji}}, \quad \text{with} \quad M_m^{\mathrm{Ji}} = M_m^{\mathrm{MPR}},$$

$$M_q^{\mathrm{Ji}} = M_q^{\mathrm{Lor}} - \frac{1}{4}M_q^{\mathrm{HRT}} - \frac{3}{4}M_m^{\mathrm{MPR}},$$

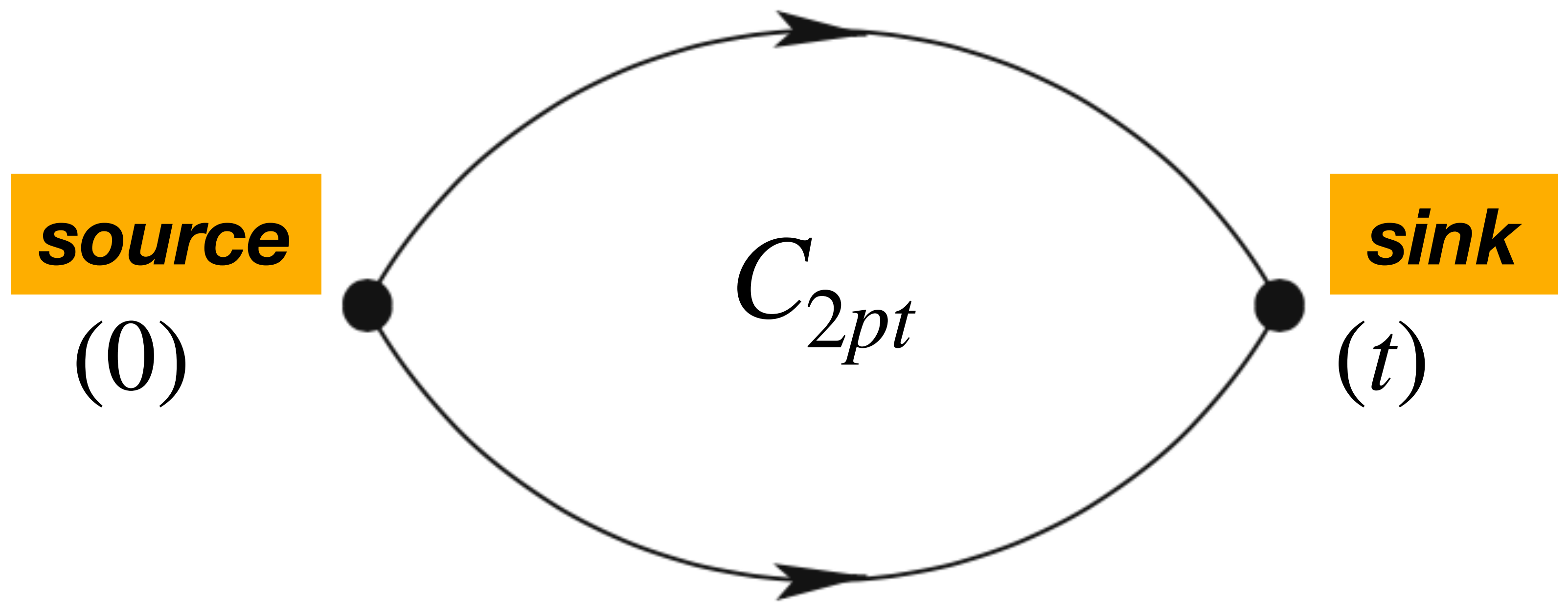
$$M_g^{\mathrm{Ji}} = M_g^{\mathrm{Lor}} - \frac{1}{4}M_g^{\mathrm{HRT}},$$

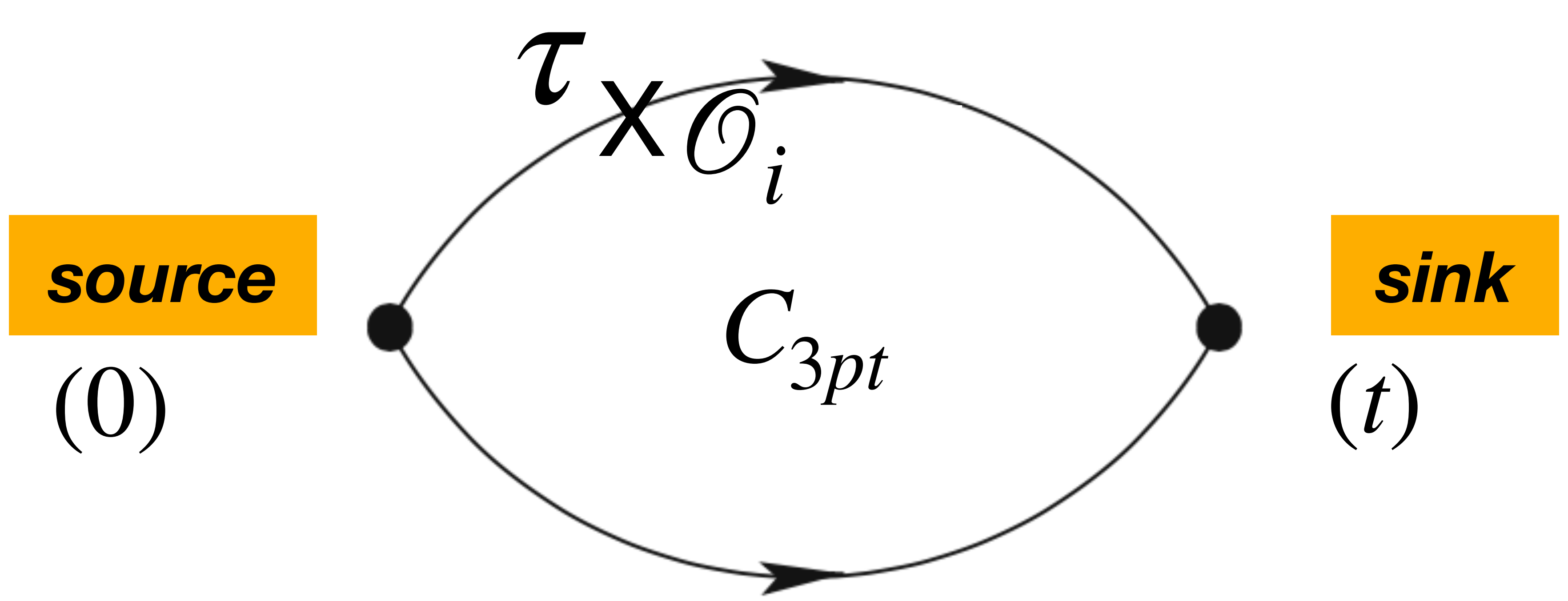
$$M_a^{\mathrm{Ji}} = \frac{1}{4}\left[M_q^{\mathrm{HRT}} + M_g^{\mathrm{HRT}} - M_m^{\mathrm{MPR}}\right].$$

$$M_H = \sum_{X=q,g,m} M_X^{\mathrm{MPR}}, \quad \text{with} \quad M_m^{\mathrm{MPR}} = \sum_f \sigma_f,$$

$$M_q^{\mathrm{MPR}} = M_q^{\mathrm{Lor}} - \sum_f \sigma_f, \quad M_g^{\mathrm{MPR}} = M_g^{\mathrm{Lor}}.$$

Extraction of EMT matrix elements





$$C^{2\text{pt}}(t) = \sum_{n=0}^{n_{\max}} |Z_n|^2 \left[e^{-E_n t} + e^{-E_n(N_t a - t)}\right]$$

$$C^{\rho\sigma}_{X,\Gamma}(t,\tau) = \sum_{m,n} Z_m^* Z_n \frac{\langle m|\widetilde{\mathcal{O}}_{i,\rho\sigma}|n\rangle}{2M} e^{-\tau E_n} e^{-(t-\tau)E_m}$$

Ratio of 3pt- to 2pt- correlator

$$R^{\rho\sigma}_{X,\Gamma}(t,\tau) = \frac{1}{2M} \frac{\langle 0|\widetilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle - \frac{\mathrm{Re}(Z_1^* Z_0 \langle 0|\widetilde{\mathcal{O}}_{i,\rho\sigma}|1\rangle)}{|Z_0|^2}\left[e^{-\Delta E \tau} + e^{-\Delta E(t-\tau)}\right]}{1 + (|Z_1|^2/|Z_0|^2)e^{-\Delta E t}}$$

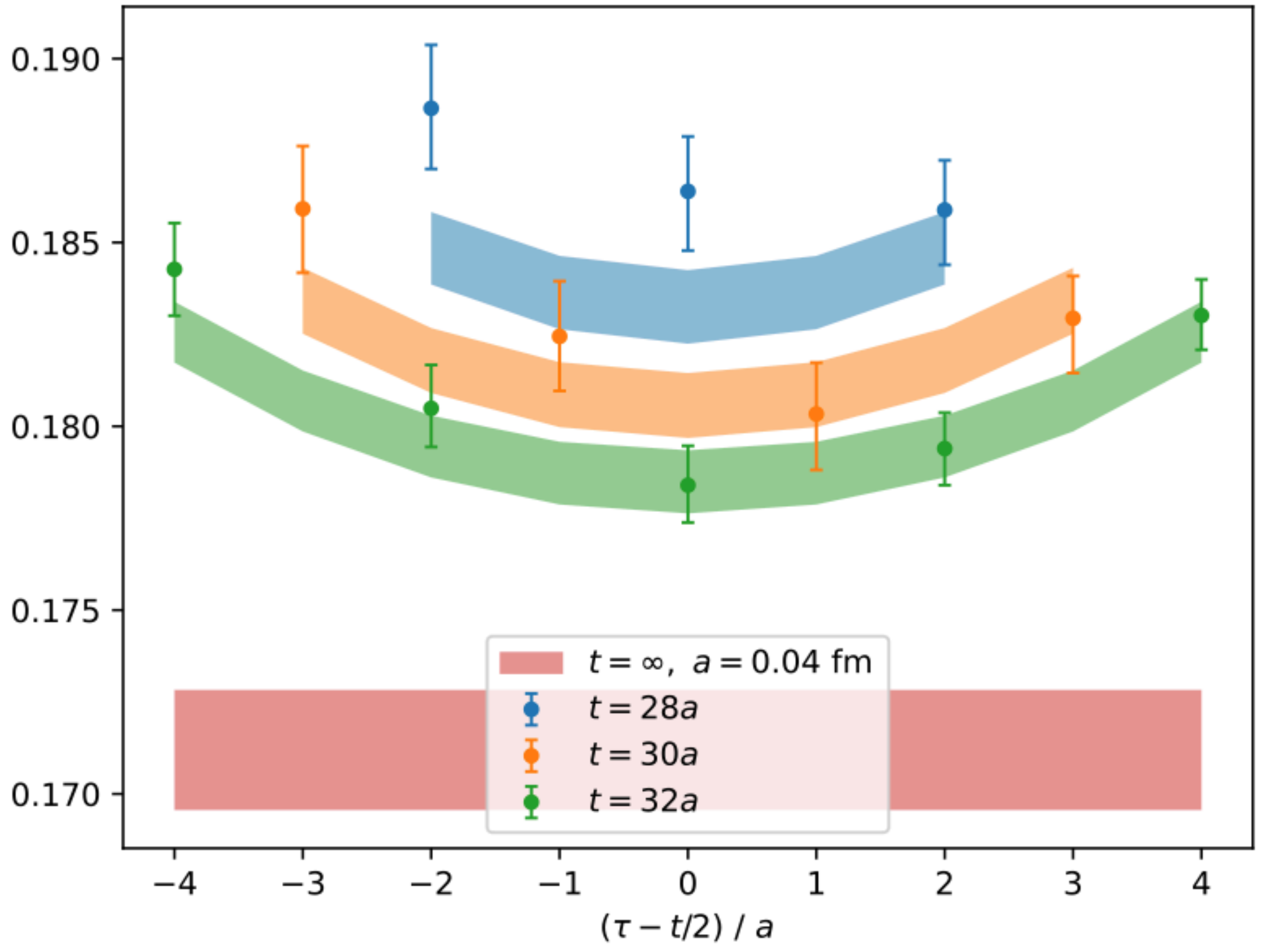
EMT element in the quark sector

$$\frac{1}{3}\sum_{k=1}^{3} \frac{1}{2M} \langle \eta_c|\widetilde{\mathcal{O}}_{3,kk}|\eta_c\rangle = \lim_{t \gg \tau \gg 0} \frac{1}{3} \sum_{k=1}^{3} R^{kk}_{q,\gamma_5}$$

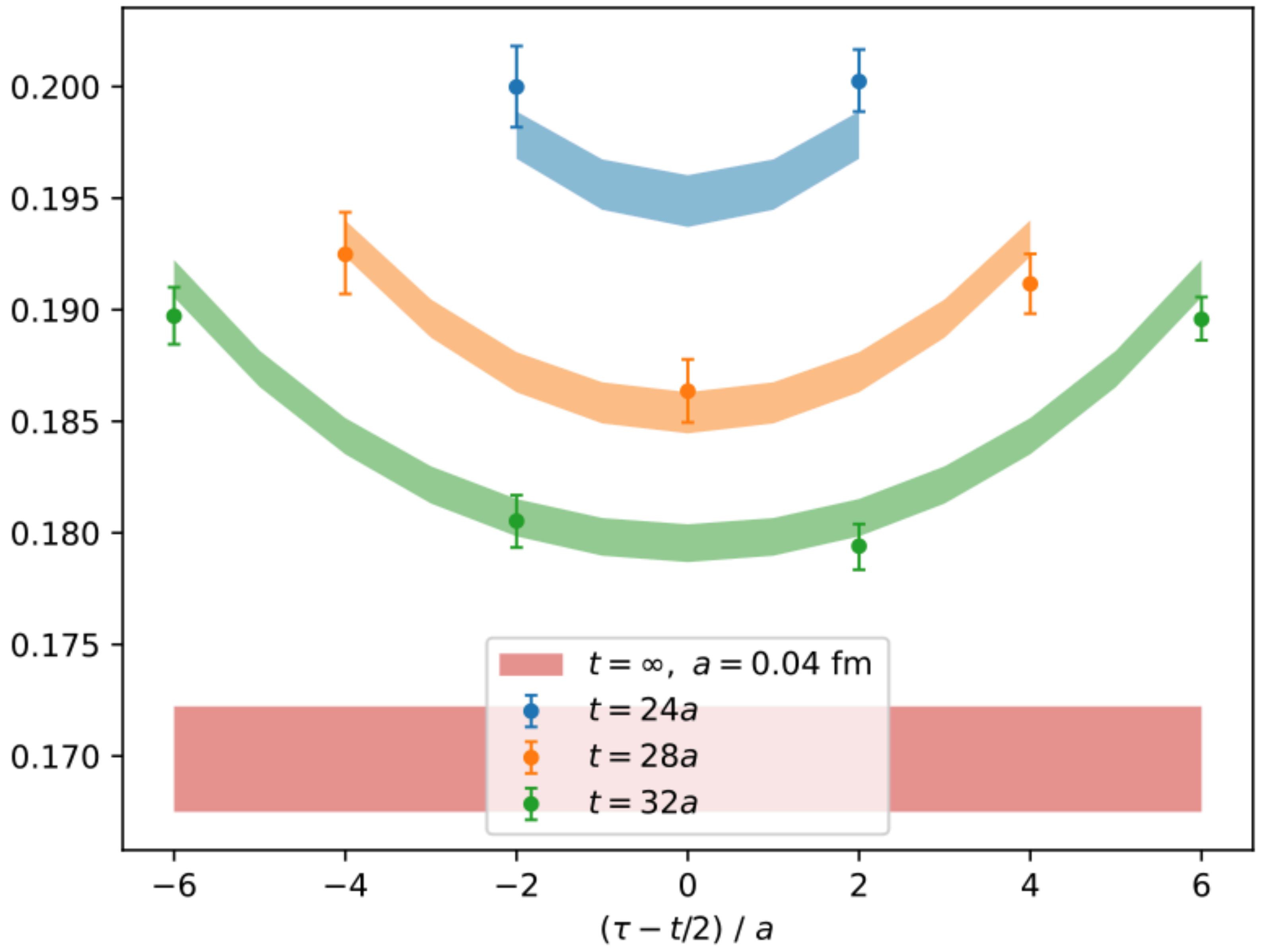
$$\mathcal{O}_{3,\rho\sigma} = \sum_f \mathcal{O}_{3,\rho\sigma,f} = \sum_f \bar{\psi}_f \left(\gamma_\rho \overleftrightarrow{D}_\sigma + \gamma_\sigma \overleftrightarrow{D}_\rho\right) \psi_f$$

Demonstration of Extraction of EMT matrix elements $< \eta_c | \tilde{\mathcal{O}}_{3,kk} | \eta_c >$

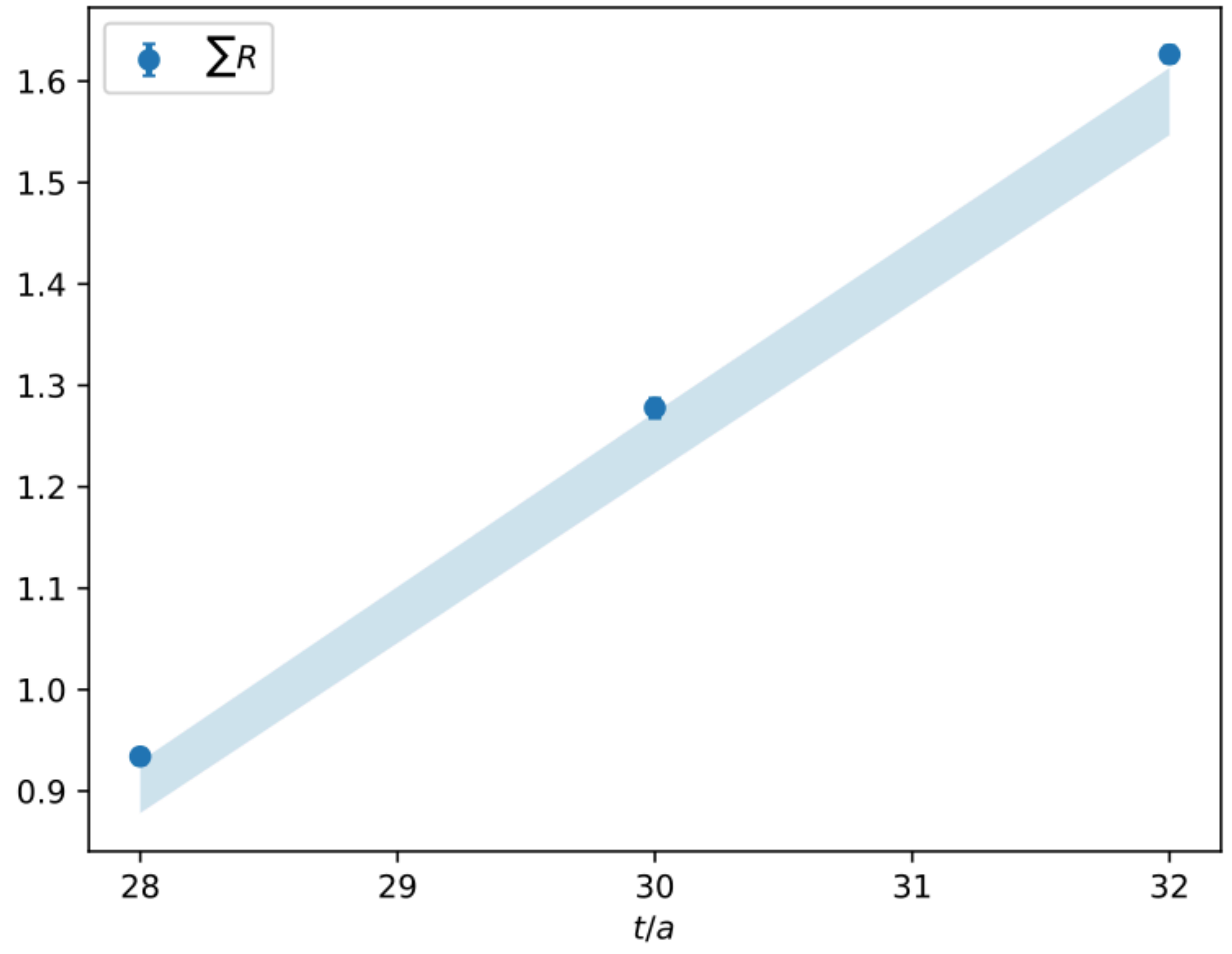
$$R^{\rho\sigma}_{q,\gamma_5} = C^{\text{3pt},\rho\sigma}_{X,\gamma_5}(t,\tau)/C^{\text{2pt}}_{\gamma_5}(t)$$



2-stride fit



4-stride fit



Summation fit