Strong interaction origin of hadron masses

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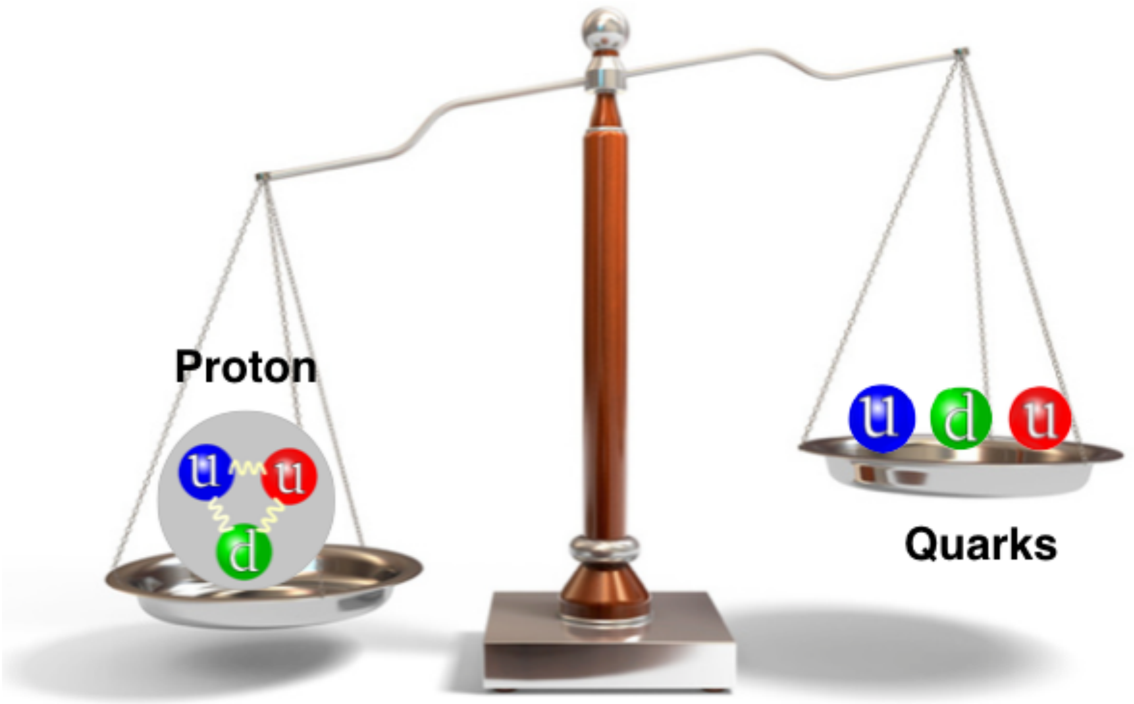
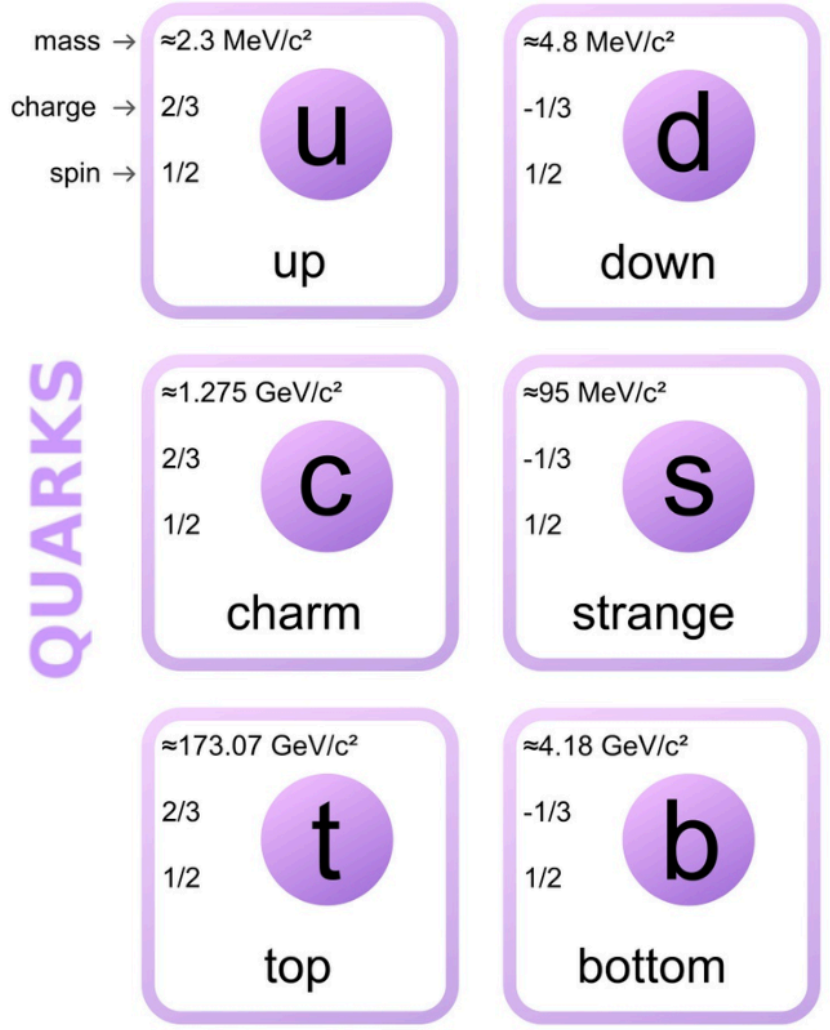
Baryon masses

and their strong interaction origin

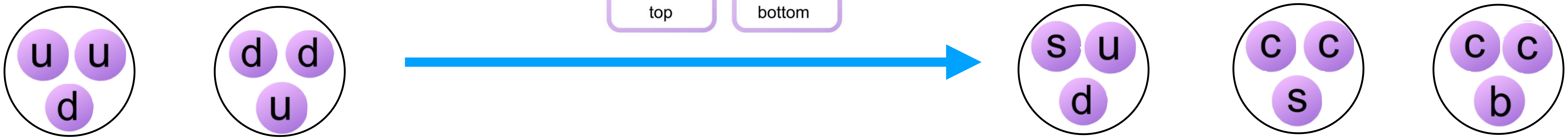


$m_p = 0.9383$ GeV $m_n = 0.9396$ GeV



?

$m_\Lambda = 1.116$ GeV, $m_{\Omega_{cc}} = 3.519$ GeV, $m_{\Omega_{ccb}} = ?$



Quark mass **Quark energy** **Gluon energy** **Trace anomaly**

- Hamiltonian: $H \equiv T^{00} = \sum_q \bar{q}\gamma^0 D^0 q + \frac{1}{2}\mathrm{Tr}[G^0{}_\mu G^{0\mu} - \frac{1}{4}G^2] + \frac{1}{4}H_a = \sum_q m_q \bar{q}q + \sum_{q,i} \bar{q}\gamma^i D^i q + \frac{1}{2}\mathrm{Tr}[G^0{}_\mu G^{0\mu} - \frac{1}{4}G^2] + \frac{1}{4}H_a$

  X. Ji, PRL74(1995)1071

- “Pressure” $\sum_i T^{ii} = \sum_{q,i} \bar{q}\gamma^i D^i q + \frac{1}{2}\mathrm{Tr}[\sum_i G^i{}_\mu G^{i\mu} + \frac{3}{4}G^2] - \frac{3}{4}H_a = \sum_{q,i} \bar{q}\gamma^i D^i q + \frac{1}{2}\mathrm{Tr}[G^0{}_\mu G^{0\mu} - \frac{1}{4}G^2] - \frac{3}{4}H_a$

- EMT trace $T^\mu_\mu = T^{00} - \sum_i T^{ii} = \sum_q m_q \bar{q}q + H_a$

  J. C. Collins et al., PRD16(1977)438
  N.K. Nielsen, NPB120(1977)212,
  M.A. Shifman et al, PLB78(1987)443

Trace anomaly

and different decompositions

$$T^\mu_\mu = H_m + H_a,\ H_m = \sum_q m_q \bar{q} q,\ H_a \equiv H_a^q + H_a^g,$$

$$H_a^q \equiv -\frac{\mu^2}{m_q}\frac{\partial m_q}{\partial \mu^2} H_m = \left[\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\right] H_m,$$

$$H_a^g \equiv \frac{\mu^2}{\alpha_s}\frac{\partial \alpha_s}{\partial \mu^2} G^2 = \left[(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)\right] G^2$$

J. C. Collins et al., PRD16(1977)438
N.K. Nielsen, NPB120(1977)212,
M.A. Shifman et al, PLB78(1987)443

$$\langle H_m \rangle_H \propto \alpha_s^0 \qquad \langle H_a^q \rangle_H \propto \alpha_s^1 \qquad \langle H_a^g \rangle_H \propto \alpha_s^2$$

$$T^\mu_\mu = H^q_{\mathrm{HRK}} + H^g_{\mathrm{HRK}},$$

$$H^q_{\mathrm{HRK}} \equiv \left(1 + \frac{4}{9\pi}\alpha_s\right) H_m + \frac{\alpha_s}{12\pi} N_f G^2 + \mathcal{O}(\alpha_s^2),\ H^{q=c,b,t}_{\mathrm{HRK}} \propto \alpha_s^2$$

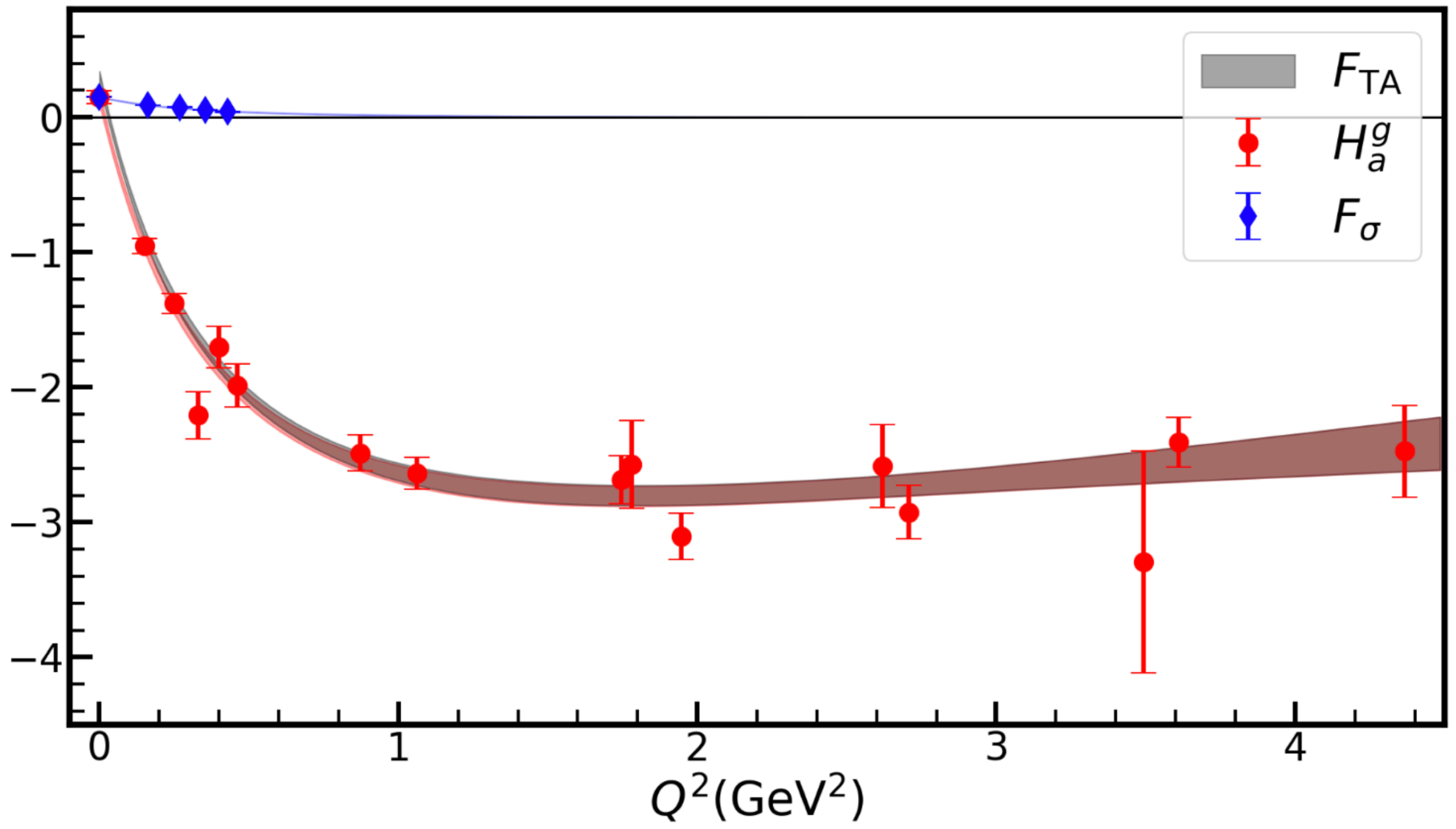
$$H^g_{\mathrm{HRK}} \equiv \left(-\frac{11}{8\pi}\right) G^2 + \frac{14}{9\pi}\alpha_s H_m + \mathcal{O}(\alpha_s^2)$$

Y. Hatta, A Rajan, K. Tanaka, JHEP12(2018),008

- The traditional trace anomaly decomposition suggests a clear α_s power counting which expected to be valid in the weak coupling limit (and also heavy quark limit?);
- The HRK decomposition provides better cancellation of the heavy quark contributions;
- Quantitative understanding of trace anomaly can only be accessed from Lattice QCD.

Trace anomaly

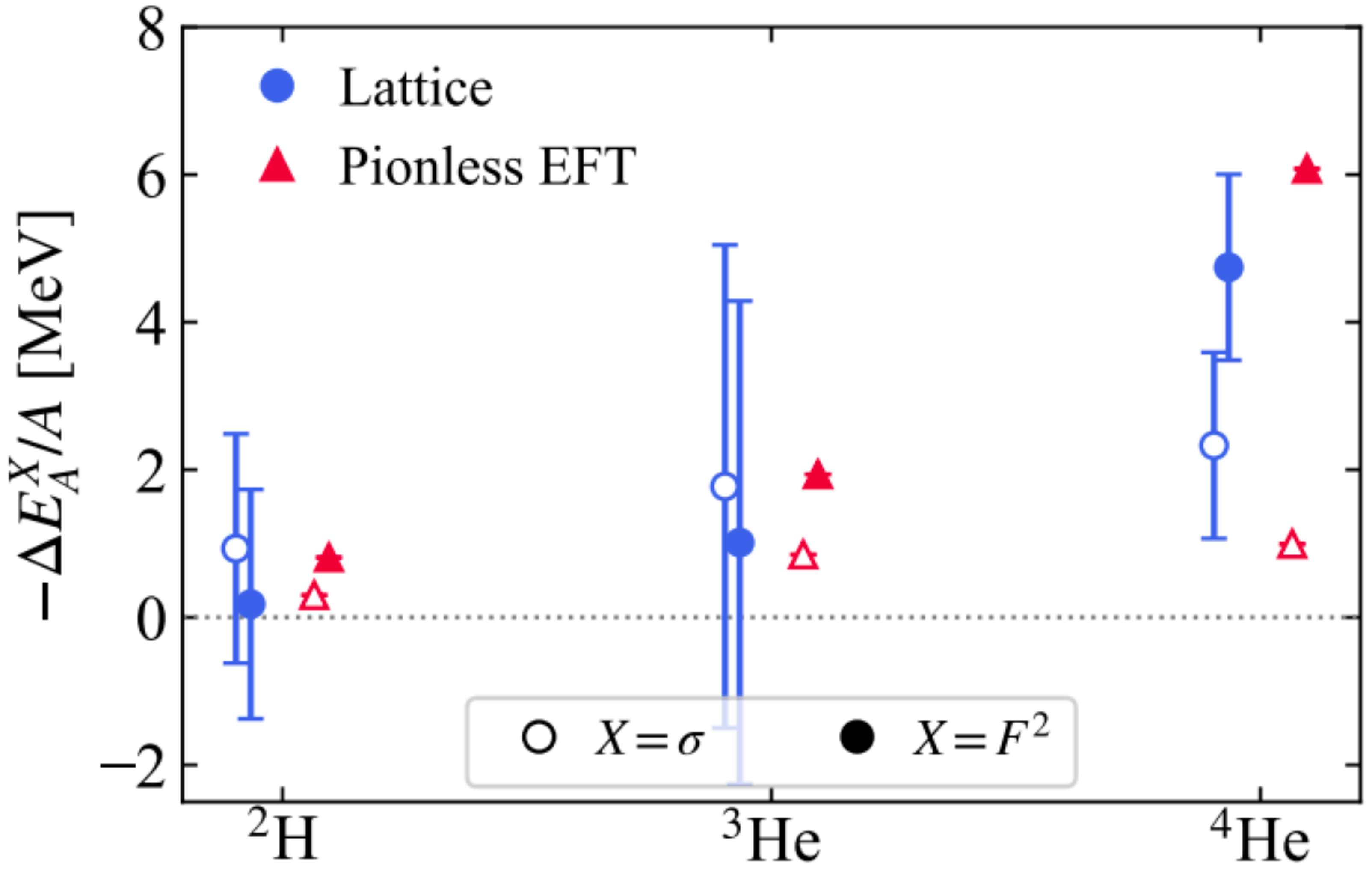
and nuclear physics



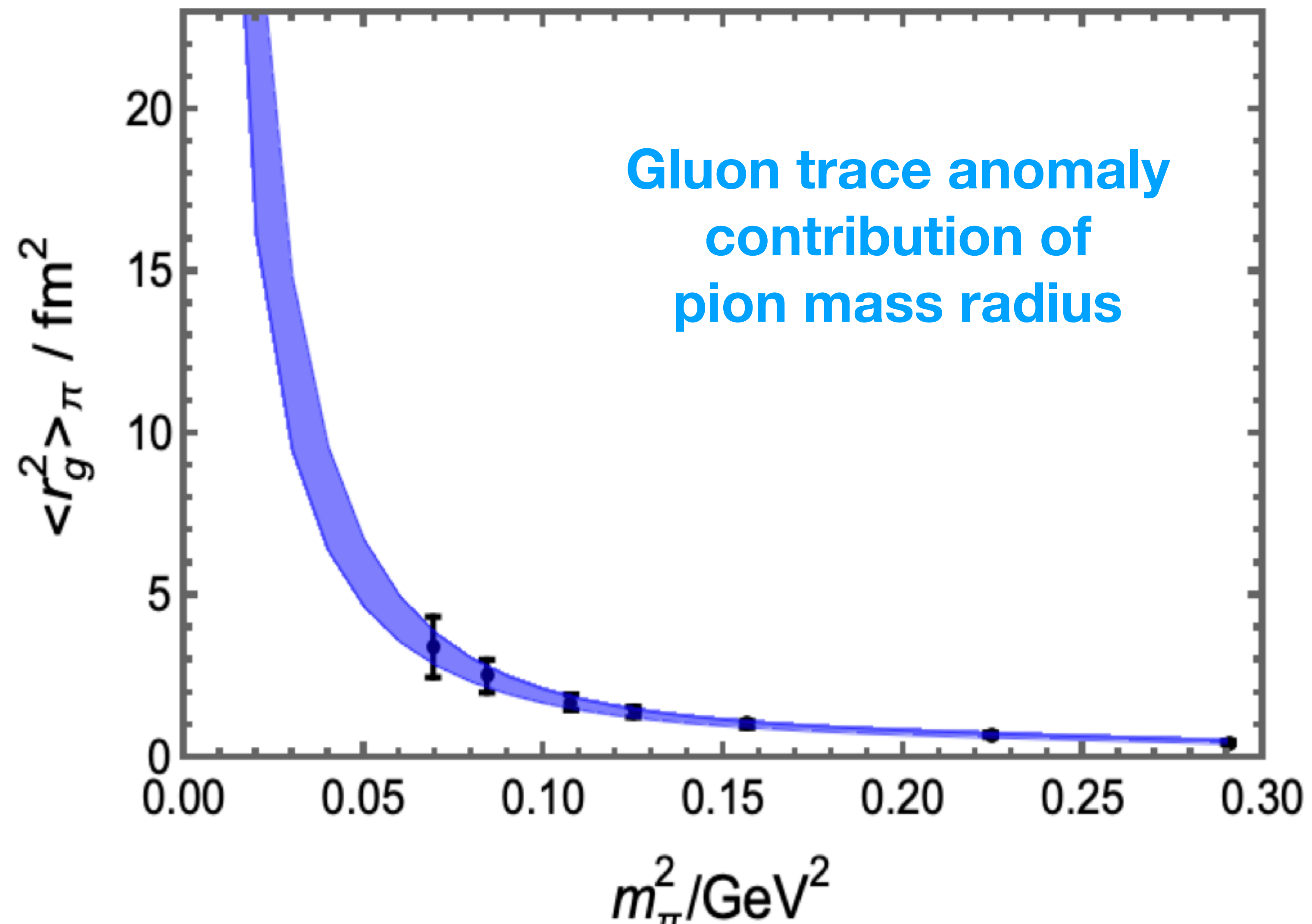
F. He, P. Sun, **YBY**, χQCD, PRD 104(2021)074507

B. Wang. et.al., χQCD, PRD 109(2024)094504

Z.H Hu. et.al., CLQCD, in preparation



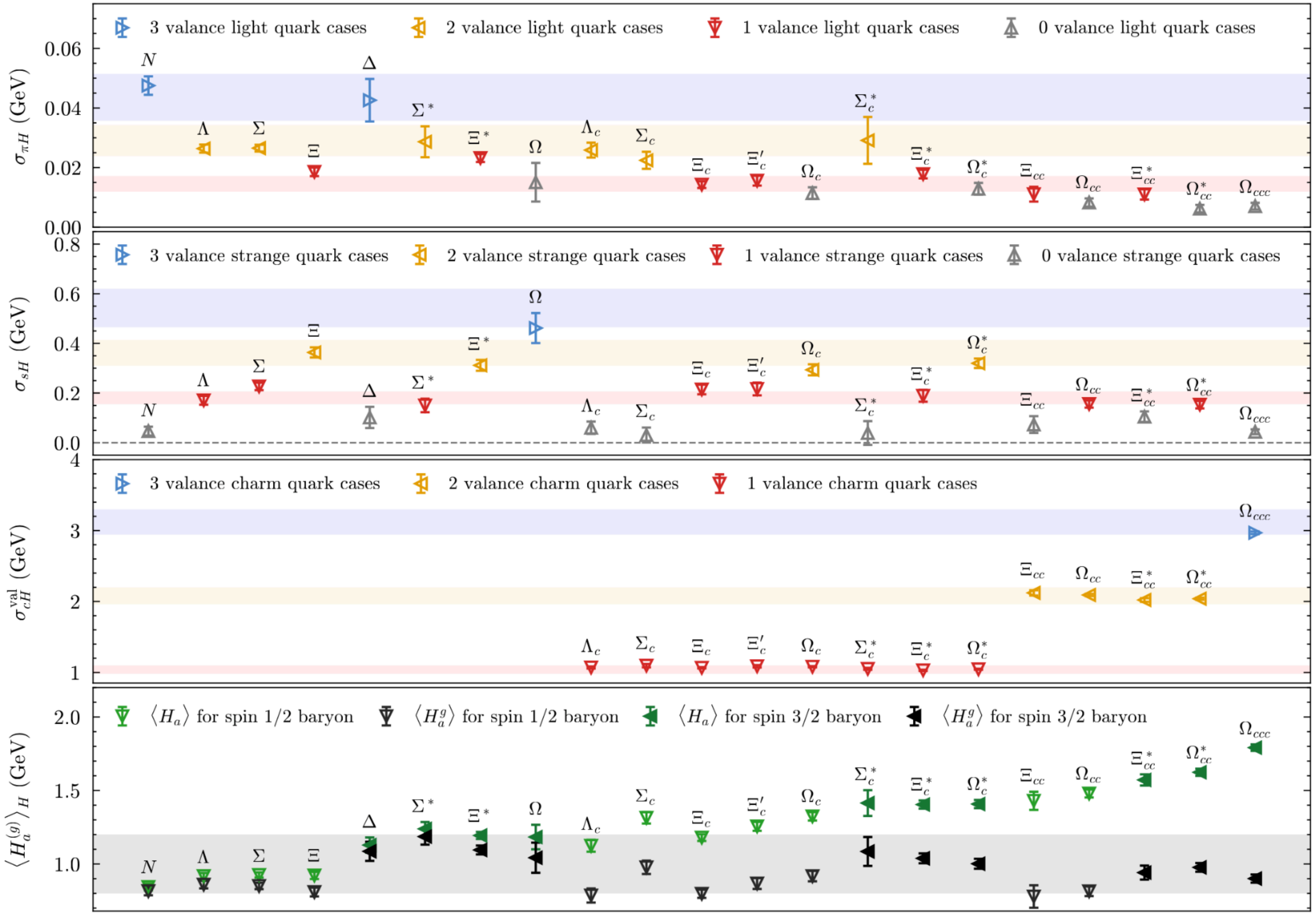
D. Chakraborty et.al., arXiv:2603.28872



Gluon trace anomaly contribution of pion mass radius

- The mass radius of pion can be an order of magnitude larger than the charge radius, due to the trace anomaly;
- Effective interactions between nuclear through pion would relate to such a large radius.
- The nuclear banding energy would also majorly come from the trace anomaly.

Strong interaction origin of the baryon masses



B.-L. Hu, et. al., CLQCD, arXiv: 2411.18402

$$\bar{q}(x)\gamma_4 q(x) = \int \frac{\mathrm{d}^3 p}{(2\pi^3)}\left(a^\dagger_{+,p}a_{+,p} - a^\dagger_{-,p}a_{-,p}\right)$$

$$\bar{q}(x)q(x) = \int \frac{\mathrm{d}^3 p}{(2\pi^3)}\frac{m}{E}\left(a^\dagger_{+,p}a_{+,p} + a^\dagger_{-,p}a_{-,p}\right)$$
$$\leq \frac{\mathrm{d}^3 p}{(2\pi^3)}\left(a^\dagger_{+,p}a_{+,p} + a^\dagger_{-,p}a_{-,p}\right)$$

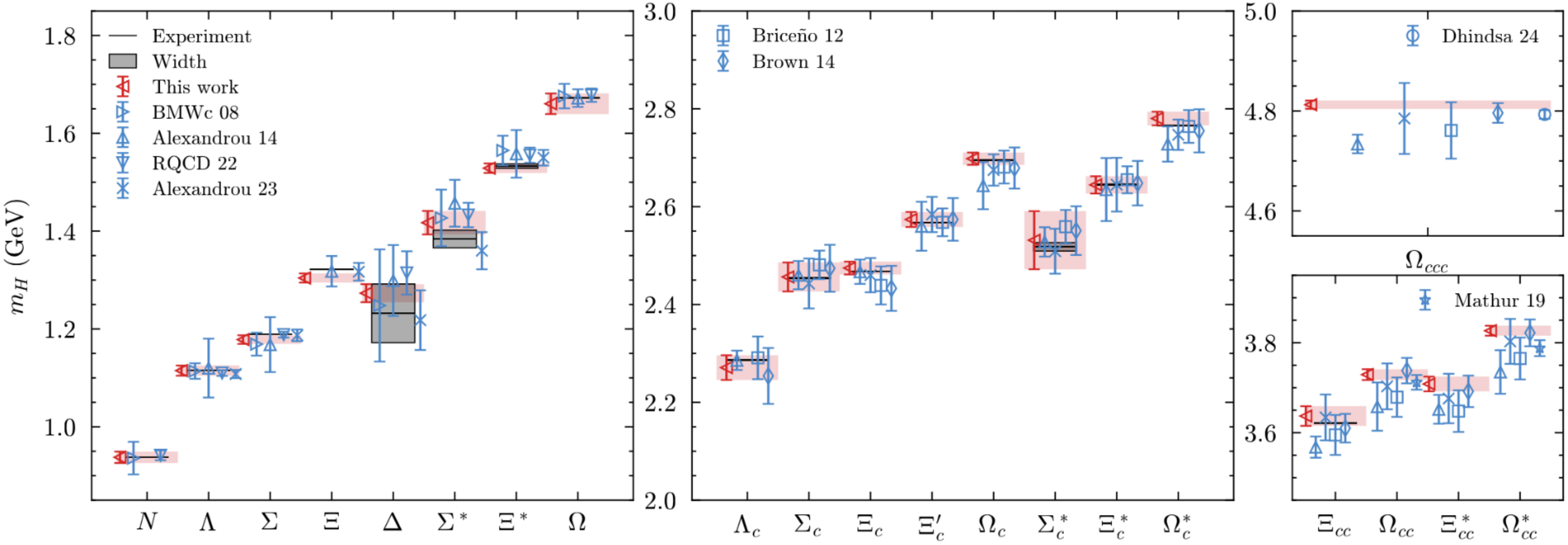
At $\overline{\mathrm{MS}}$ 2 GeV:

- Light quark mass contributions are enhanced by a factor of 4-8;
- Strange quark mass contributions are enhanced by a factor of 2-3;
- Charm quark mass contribution only have minor enhancement from γ_m.
- Gluon trace anomaly is insensitive to flavor after the quark trace anomaly is subtracted.

What will happen in the cases with bottom quark?

Baryon masses

Systematical uncertainties

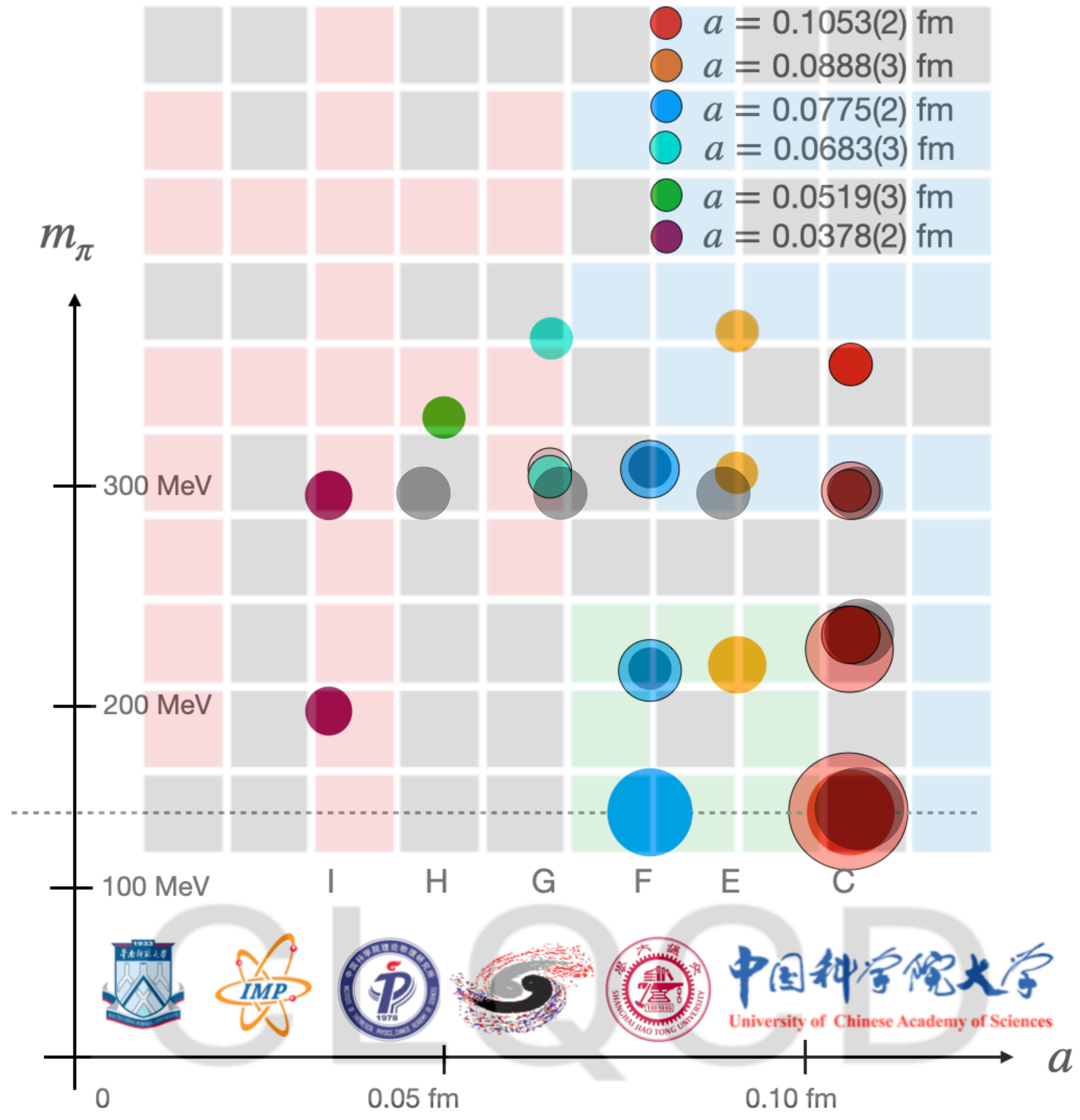


B.-L. Hu, et. al., CLQCD, arXiv: 2411.18402

- The uncertainty of the pure QCD predictions can be much smaller than 1% for the heavy flavor baryon masses.
- How large is the effects from the QED correction?
- How about the missing charm quark loop?

CLQCD ensembles

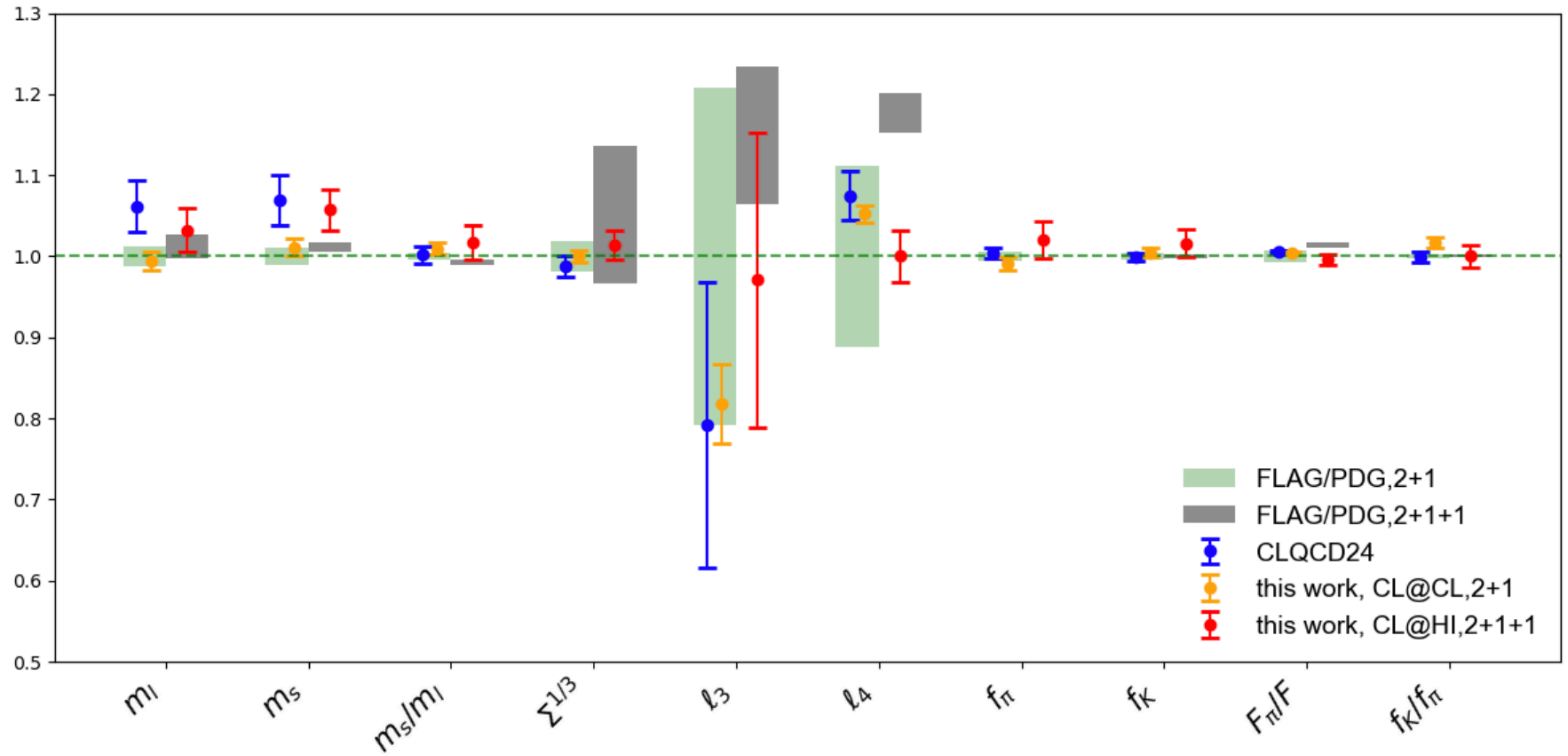
	Country/ Region	Smallest lattice spacing	No. of physical point ensembles	Largest spacial size	No. of fermion discretization
MILC	US	0.03 fm	5	5.8 fm	1
RBC	US	0.06 fm	3	5.5 fm	1
BMW	EN	0.05 fm	15	10 fm	2
CLS	EN	0.04 fm	2	5.5 fm	1
ETM	EN	0.05 fm	5	6.3 fm	1
PACS	JP	0.06 fm	3	10 fm	1
CLQCD	**CN**	**0.04 fm**	**4**	**6.7 fm**	**2**

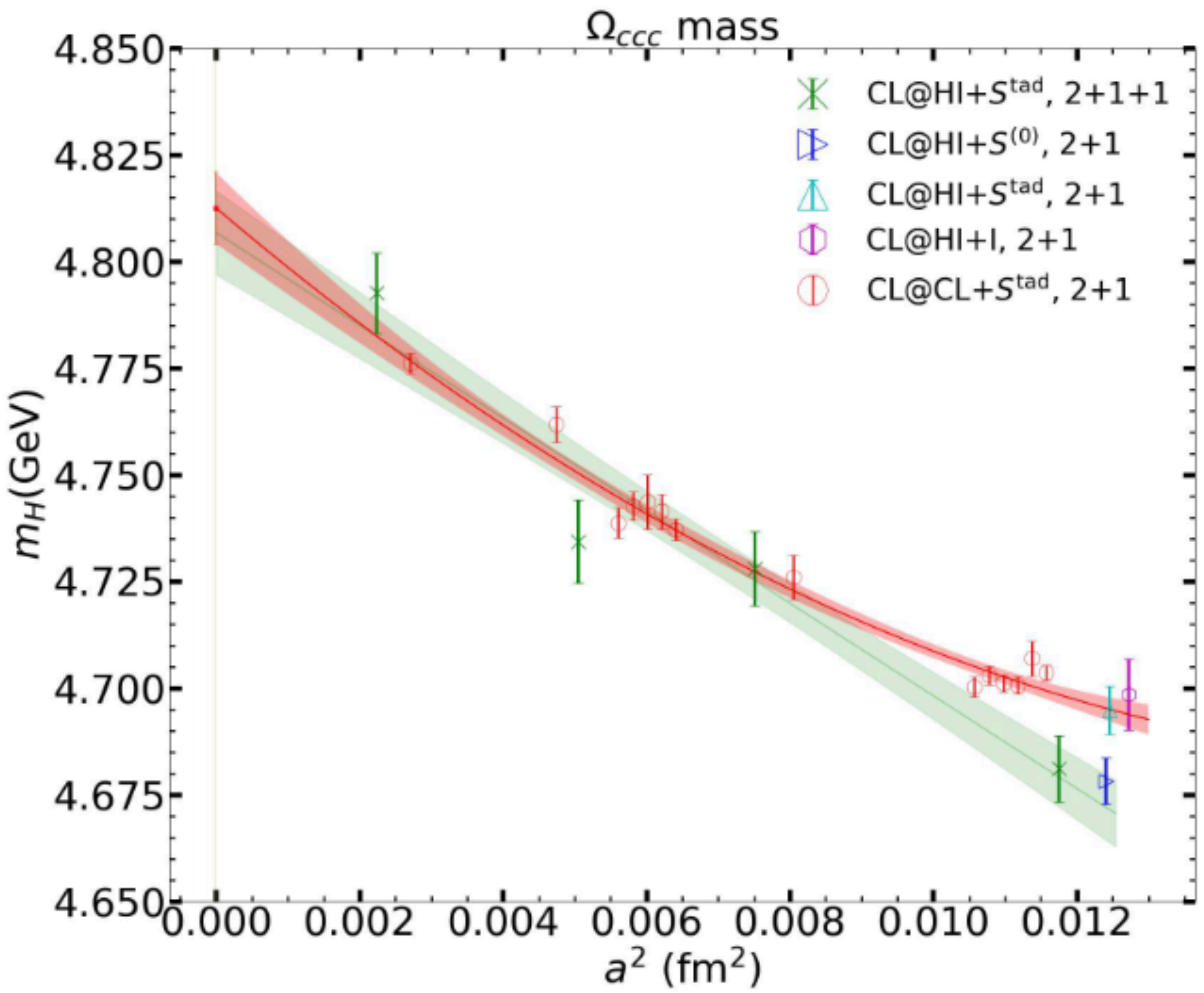


Fermion Discretization	No. of ensembles	Sea fermion flavors	Lattice spacing range (fm)	No. of ensembles at physical pion mass	Device for data generation	Device for data anslysis
Clover	20	2+1	0.038-0.105	3	CUDA GPU	Sugon DCU
HISQ	10	2+1+1	0.048-0.108	1	Sugon DCU	Sugon DCU

Systematical uncertainties

Charm sea effects





- The second generation of the CLQCD ensembles uses Nf=2(light)+1(strange)+1(charm) setup and further suppresses the discretization errors;
- Good consistency between the 2+1 and 2+1+1 results for the light quark physics, using the same clover fermion action.
- Also good agreement for $m_{\Omega_{ccc}}$ within 0.2% uncertainty.

Z.X. Zhang et al.,[CLQCD],arXiv:2512.19265
T.W Lin et al. [CLQCD], arXiv:2603.04230

Systematical uncertainties

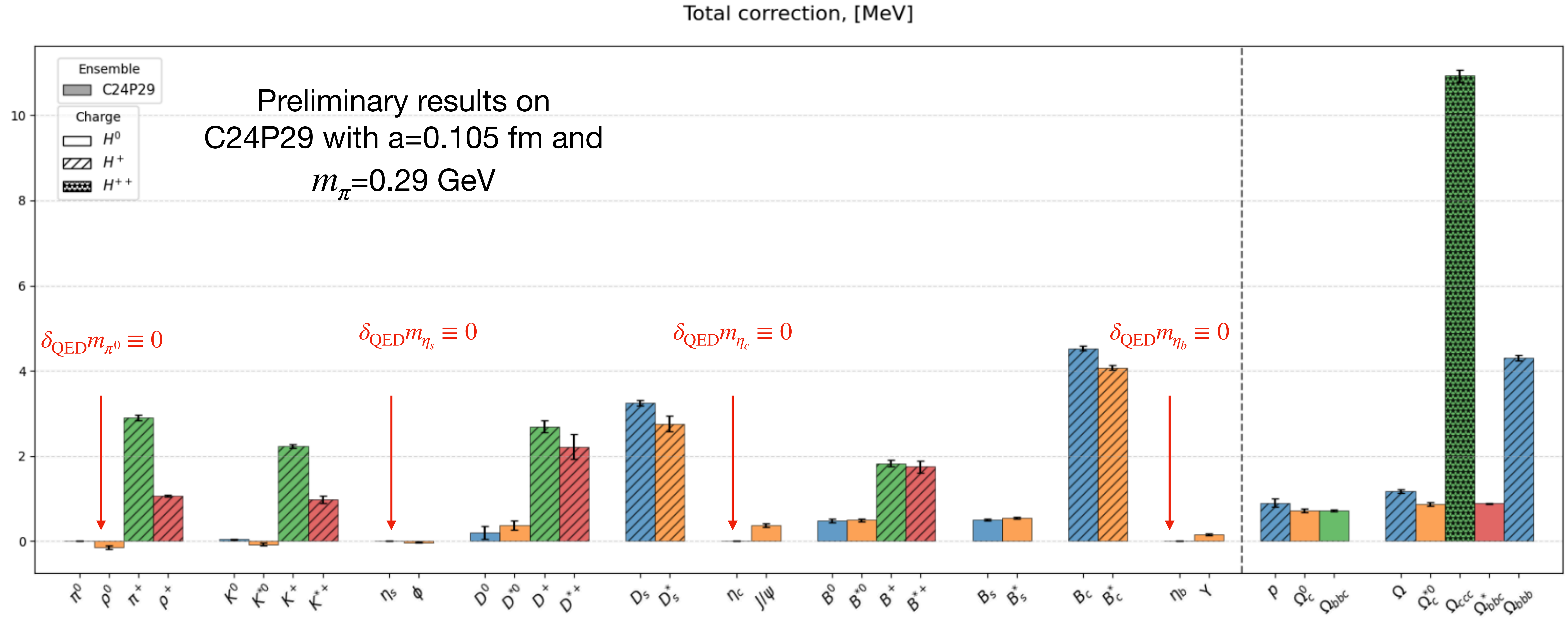
QED corrections

$$\delta_{\rm QED} m_q = e_q^2 \frac{\delta^{\rm int,\bar{q}q}_{\rm QED} m_{\eta_q} - \delta^{\rm self,q}_{\rm QED} m_{\eta_q} - \delta^{\rm self,\bar{q}}_{\rm QED} m_{\eta_q}}{2\langle \bar{q}q\rangle_{\eta_q}},$$

$$\delta_{\rm QED} m_H = \sum_q e_q^2 (\delta^{\rm self,q}_{\rm QED} m_H + \delta_{\rm QED} m_q \langle \bar{q}q\rangle_H) + \sum_q \sum_{q' \neq q} e_q e_{q'} \delta^{\rm int,qq'}_{\rm QED} m_H$$

- Positive/negative hadron mass correction for attractive/repulsive QED interaction between quarks;
- $\sim 2e_H^2$ MeV in the Deshen's scheme, for all the hadron masses we investigated.

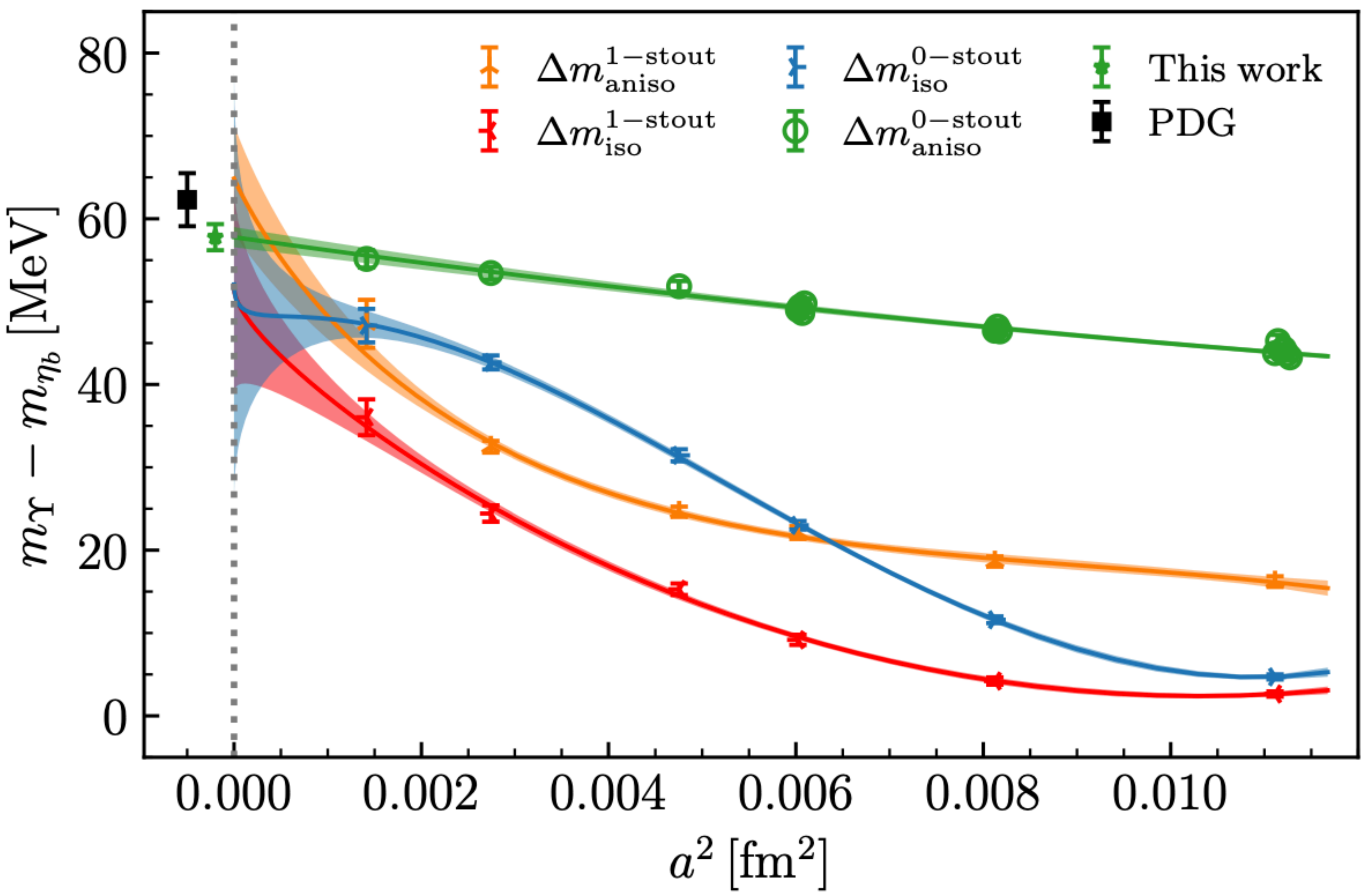
Y.Y. Liu et al. [CLQCD], in preparation



Preliminary results on C24P29 with a=0.105 fm and m_π=0.29 GeV

Bottom quark system

Calibration



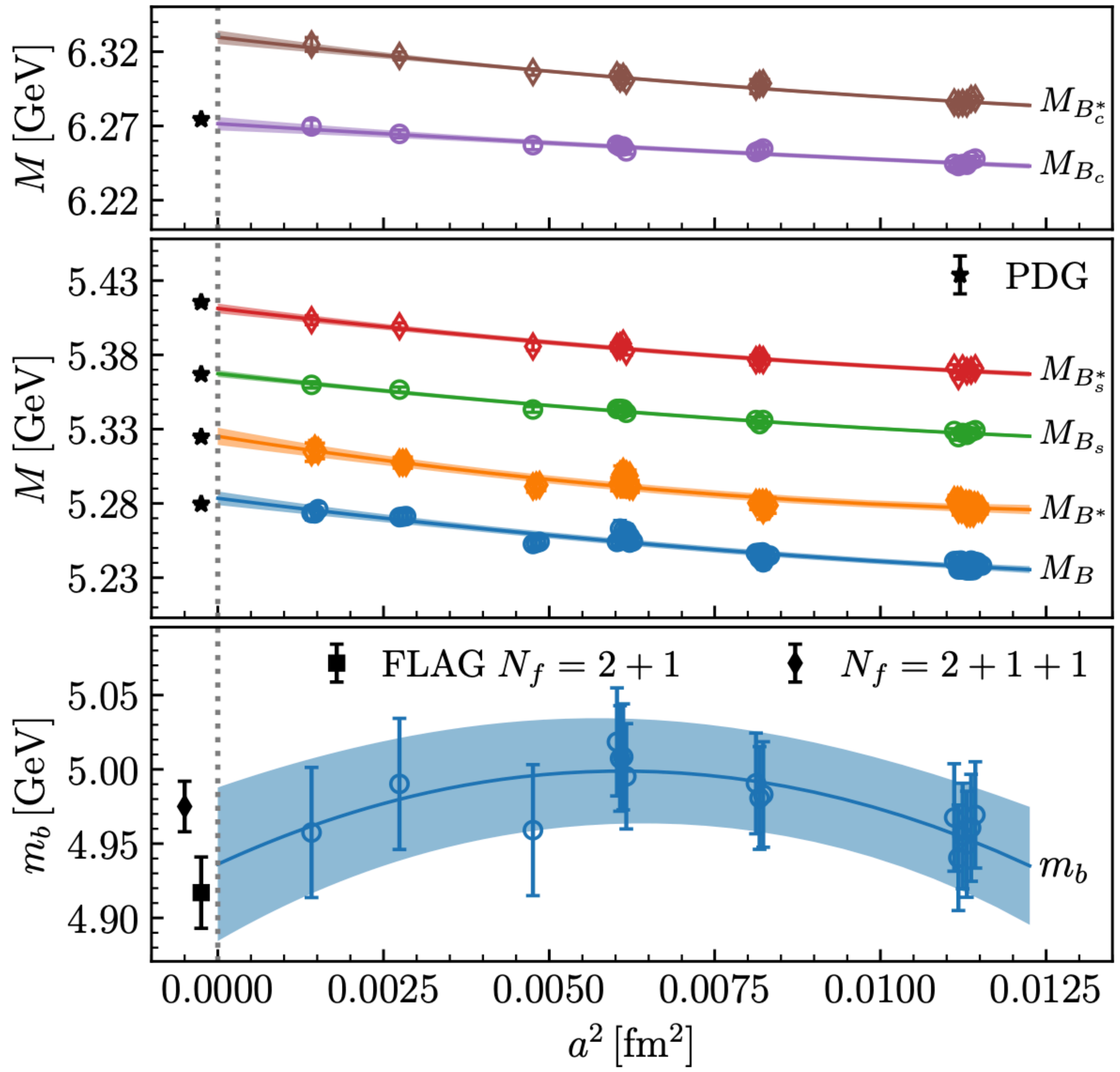
- Anisotropic heavy quark action can significantly suppress the discretization error of the bottom quark;

L.M. Liu et al. PRD81(2010)094505

- Combining the ISB and QED corrections gives

$$m^{\mathrm{phys}}_{B^0} = 5283.5(4.6) + 1.1(0.1)_{\mathrm{ISB}} + 0.2(0.1)_{\mathrm{QED}} = 5284.8(4.6)\ \mathrm{GeV}$$
$$m^{\mathrm{phys}}_{B^+} = 5283.5(4.6) - 1.1(0.1)_{\mathrm{ISB}} + 1.7(0.2)_{\mathrm{QED}} = 5284.1(4.6)\ \mathrm{GeV}$$

and $m^{\mathrm{phys}}_{B_s} = 5367.3(2.3) + 0.2(0.2)_{\mathrm{QED}} = 5367.5(2.3)\ \mathrm{GeV}$.

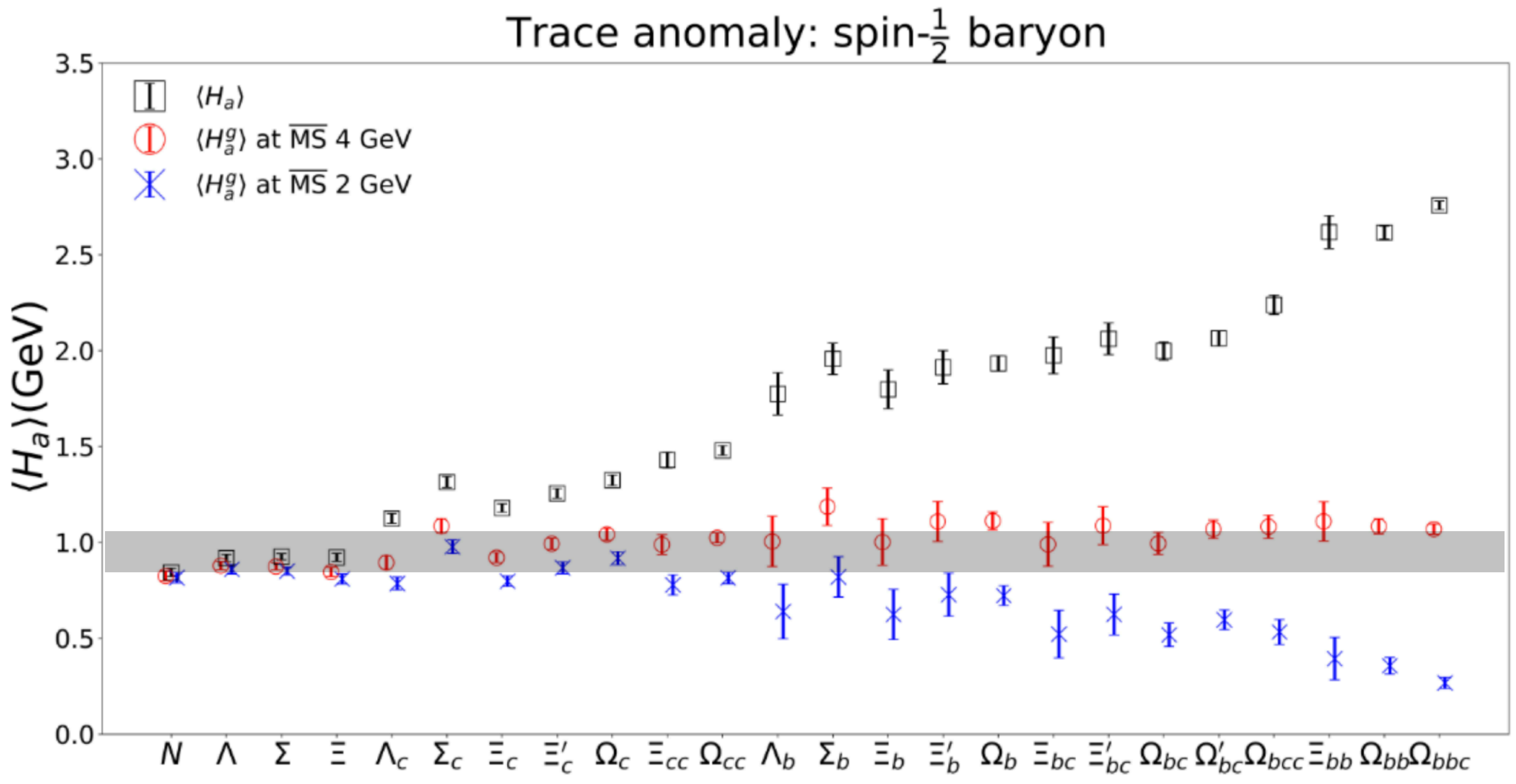


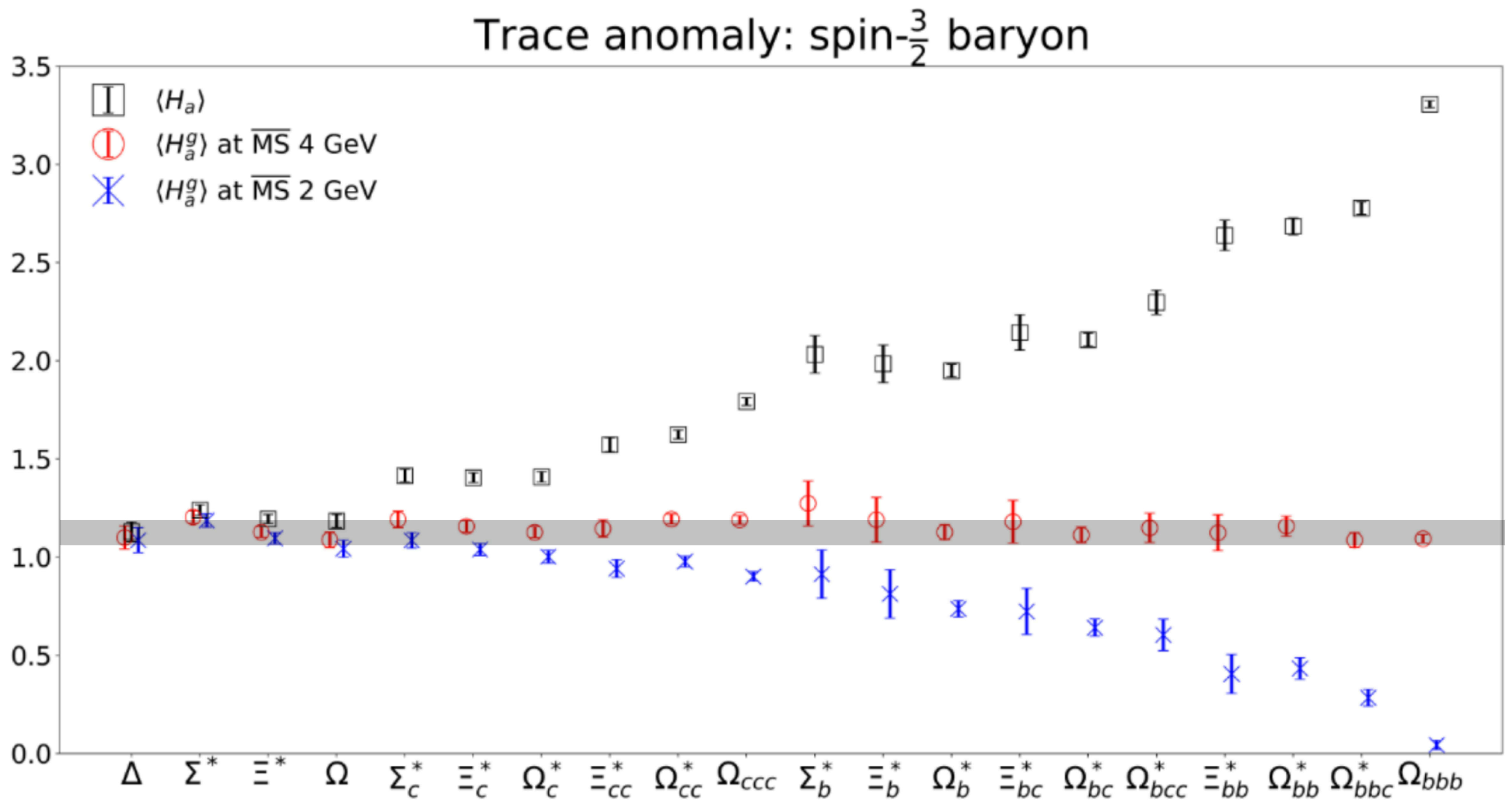
- $m_b^{\overline{\mathrm{MS}}}(m_b) = 4.185(37)$ GeV based on CLQCD ensembles.

M.C. Cai et al. [CLQCD], arXiv:2603.01846

Gluon trace anomaly

Updated results





C.Y. Zhang, H.Y Du et al. [CLQCD], in preparation

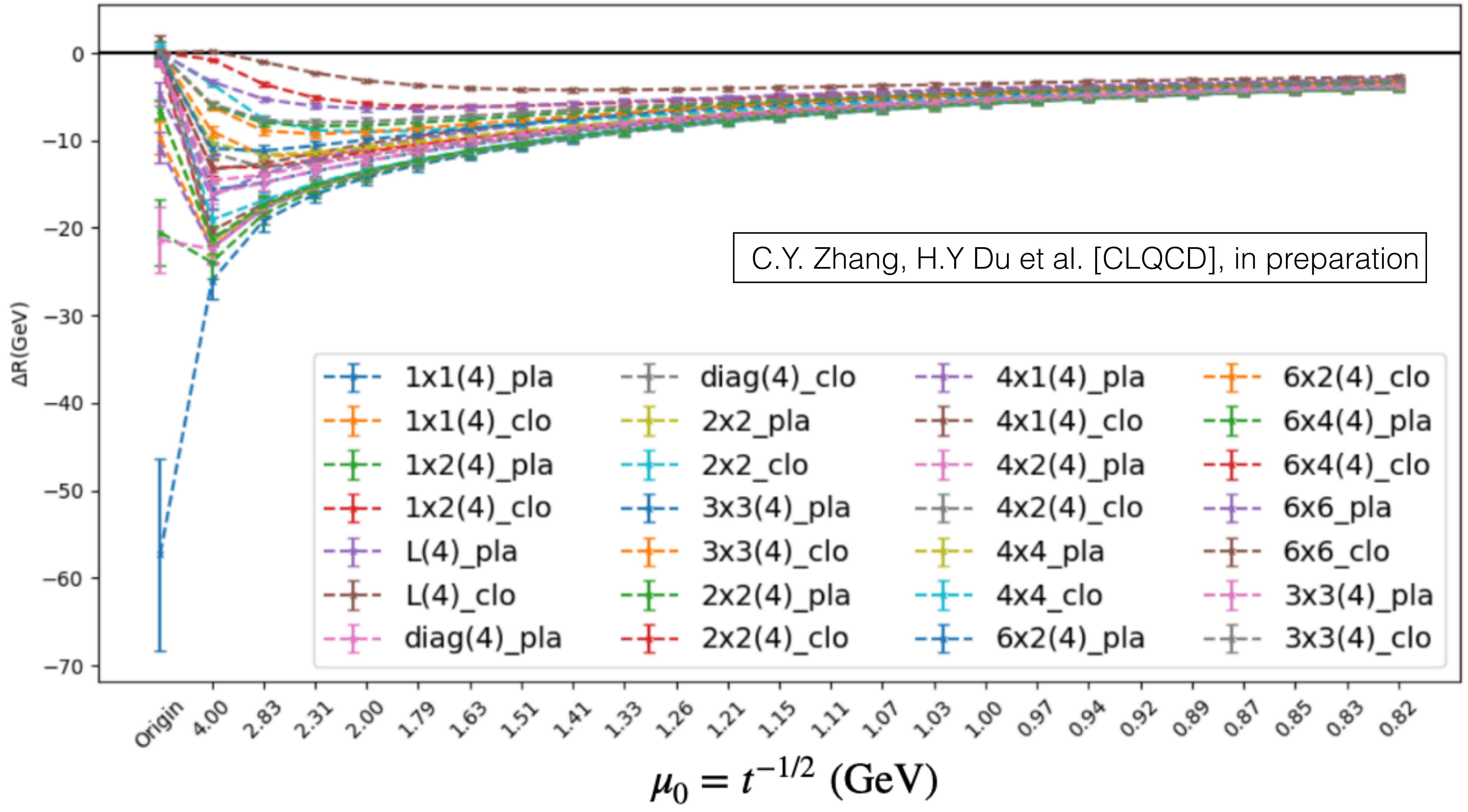
- $m_b\langle \bar{b}b \rangle \sim n_b \times 3.7$ GeV for all the bottomed ground state baryons and also S-wave botttominum;
- $H_a^g \sim 1$ GeV for all the ground state baryons from ~1 GeV (nucleon) to 15 GeV Ω_{bbb}, at $\overline{\text{MS}}$ 4 GeV.
- $\langle H_m \rangle_{\Omega_{bbb}} \gg \langle H_a^q \rangle_{\Omega_{bbb}} \gg \langle H_a^g \rangle_{\Omega_{bbb}}$ at ~2 GeV as predicted by the naive power counting.

Trace anomaly

Direct calculation

- On the lattice, the trace anomaly should come from the breaking of the scale invariance,
$$\langle P_{1\times1}\rangle_H - \frac{1}{4}\langle P_{2\times2}\rangle_H = \mathcal{O}(a^2) + \mathcal{O}(\alpha_s).$$
- The gradient flow introduces a “soft” UV cutoff which can remove the impact of arbitrary regularization effect with large enough t;
- Taking $\langle g^2G^2\rangle_{\Omega_{ccc}}$ at $a = 0.052$ fm as example, the matrix element becomes independent of the definition of G^2 at large enough t.

$$\int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} \frac{1}{(\frac{4}{a^2}\sum_\mu \sin^2\frac{p_\mu a}{2} + m^2)^n} \rightarrow \int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} \frac{e^{-p^2 t}}{(\frac{4}{a^2}\sum_\mu \sin^2\frac{p_\mu a}{2} + m^2)^n}$$
$$= \int \frac{d^4p}{(2\pi)^4} \frac{e^{-p^2 t}}{(p^2+m^2)^n}\Big|_{t\gg a^2/\pi^2}$$

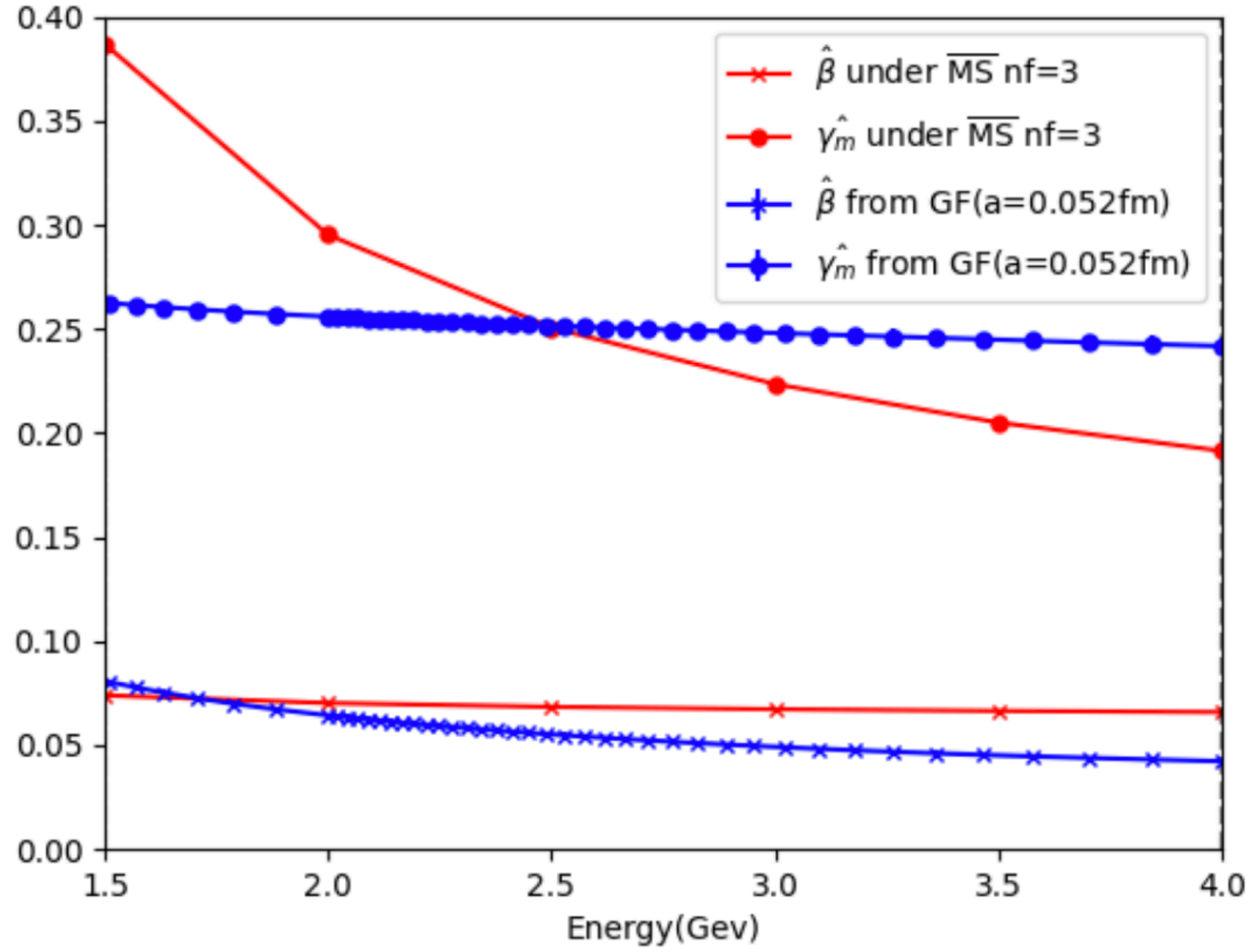


C.Y. Zhang, H.Y Du et al. [CLQCD], in preparation

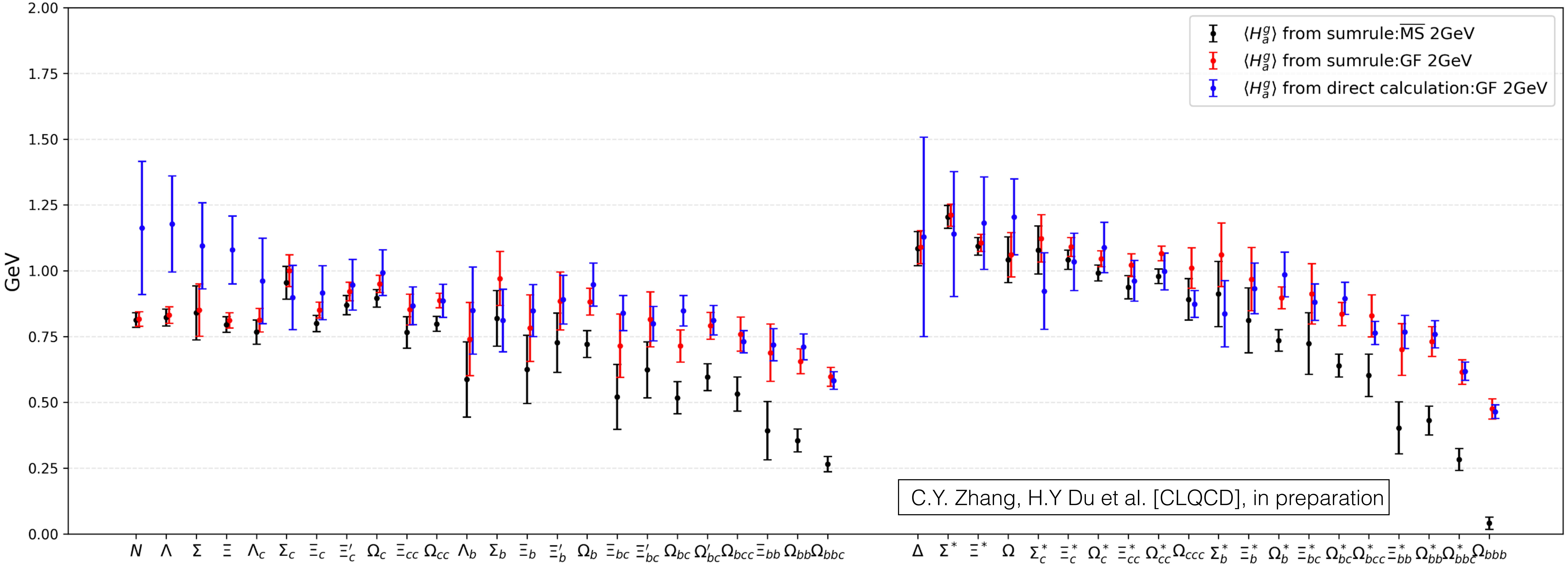
Trace anomaly

$\overline{\mathrm{MS}}$ v.s. LO gradient flow

Naively defining $\mu^{\mathrm{GF}} = t^{-1/2}$ and neglecting the $\mathcal{O}(\alpha_s)$ matching between $\overline{\mathrm{MS}}$ and gradient flow, and solve γ_m and $\hat{\beta}$ though $\langle T^\mu_\mu \rangle_H = (1+\gamma_m)\sum_q \langle m_q \bar{q} q\rangle_H + \hat{\beta}\langle g^2 G^2\rangle_H$ for all the studied hadrons:



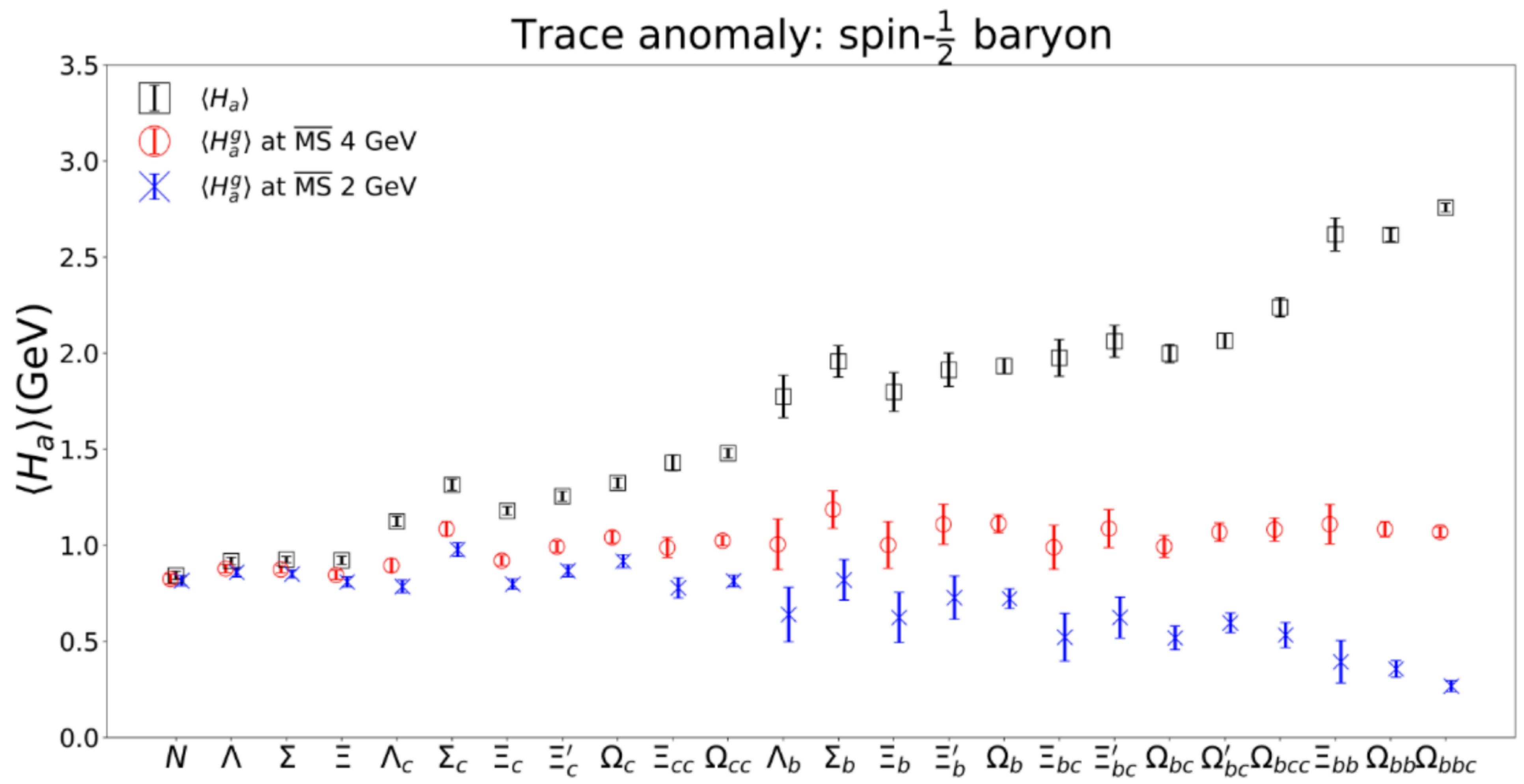
- γ_m and $\hat{\beta}$ agree with the $\overline{\mathrm{MS}}$ values up to $\mathcal{O}(\alpha_s)$ corrections;
- (Preliminarily) verify the trace sum rule in the entire mass region.

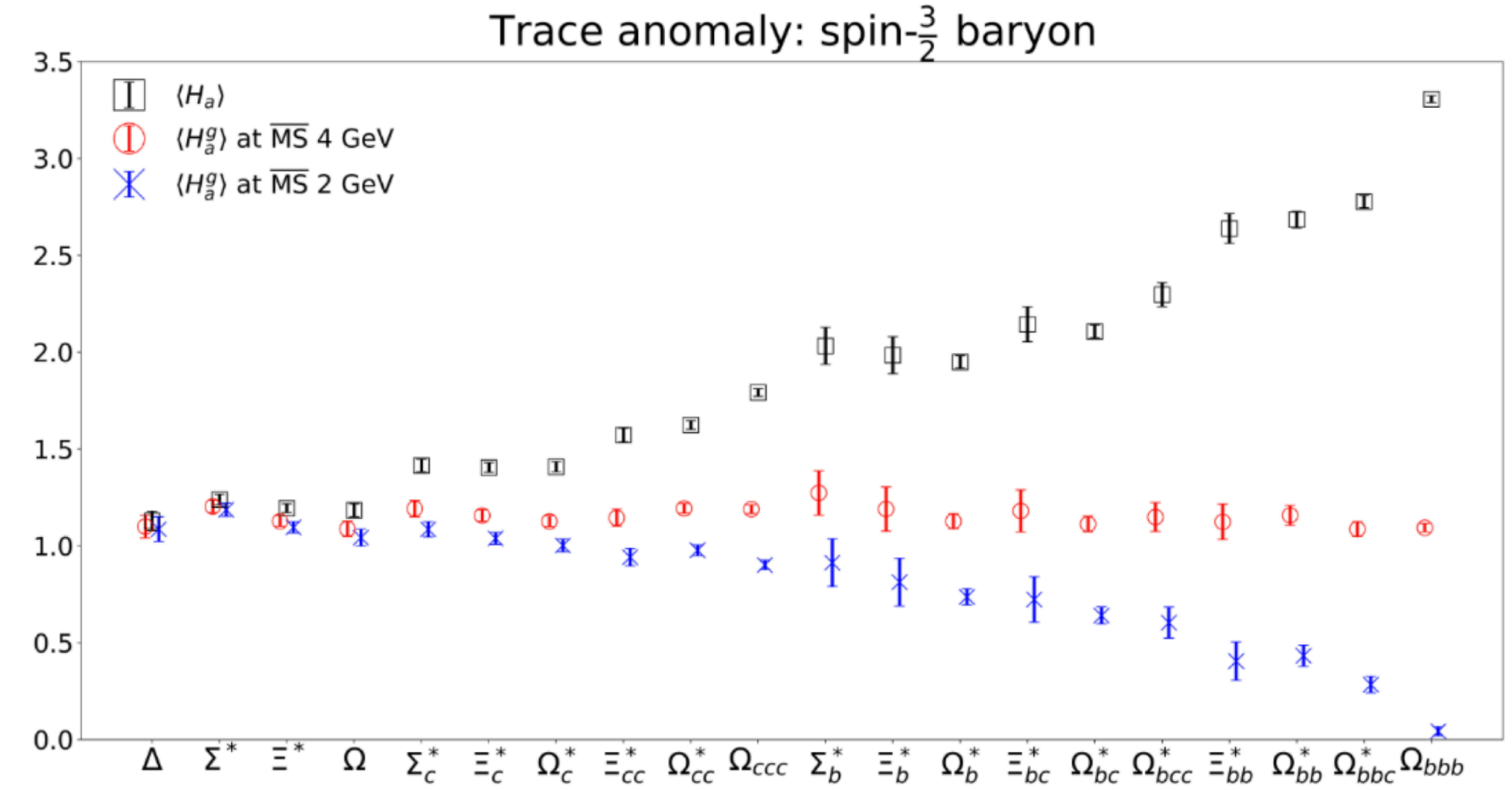


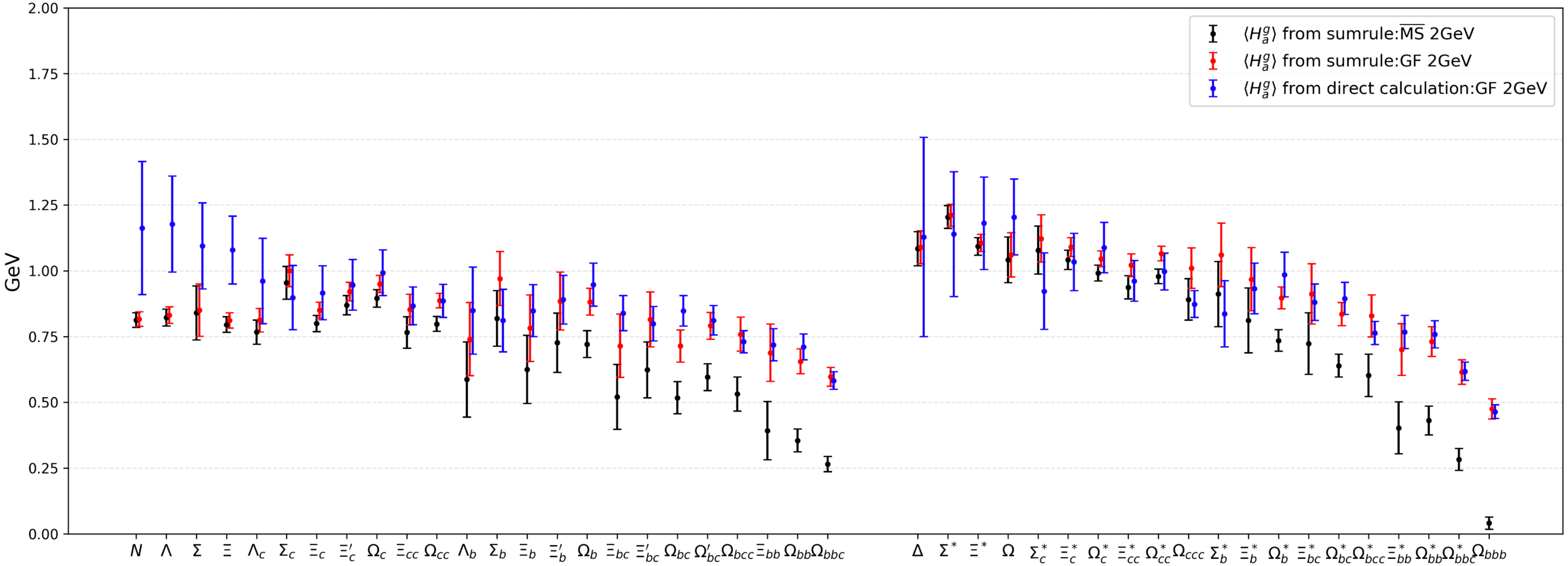
C.Y. Zhang, H.Y Du et al. [CLQCD], in preparation

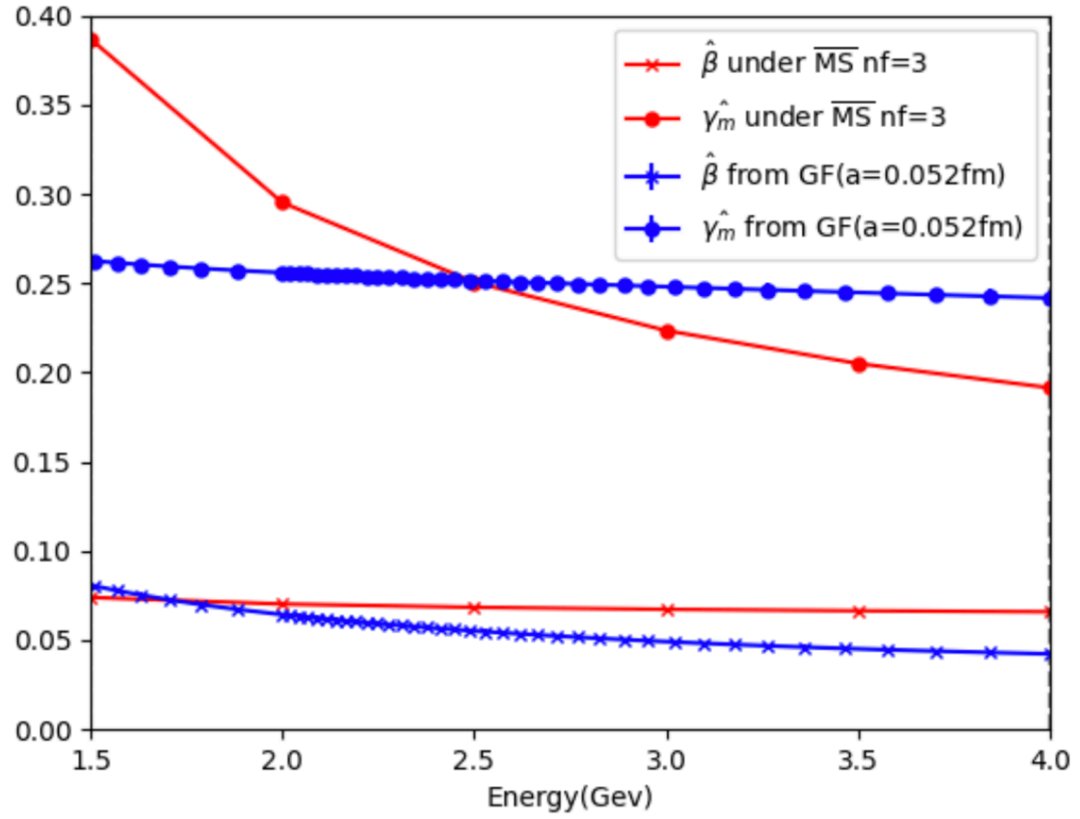
Summary

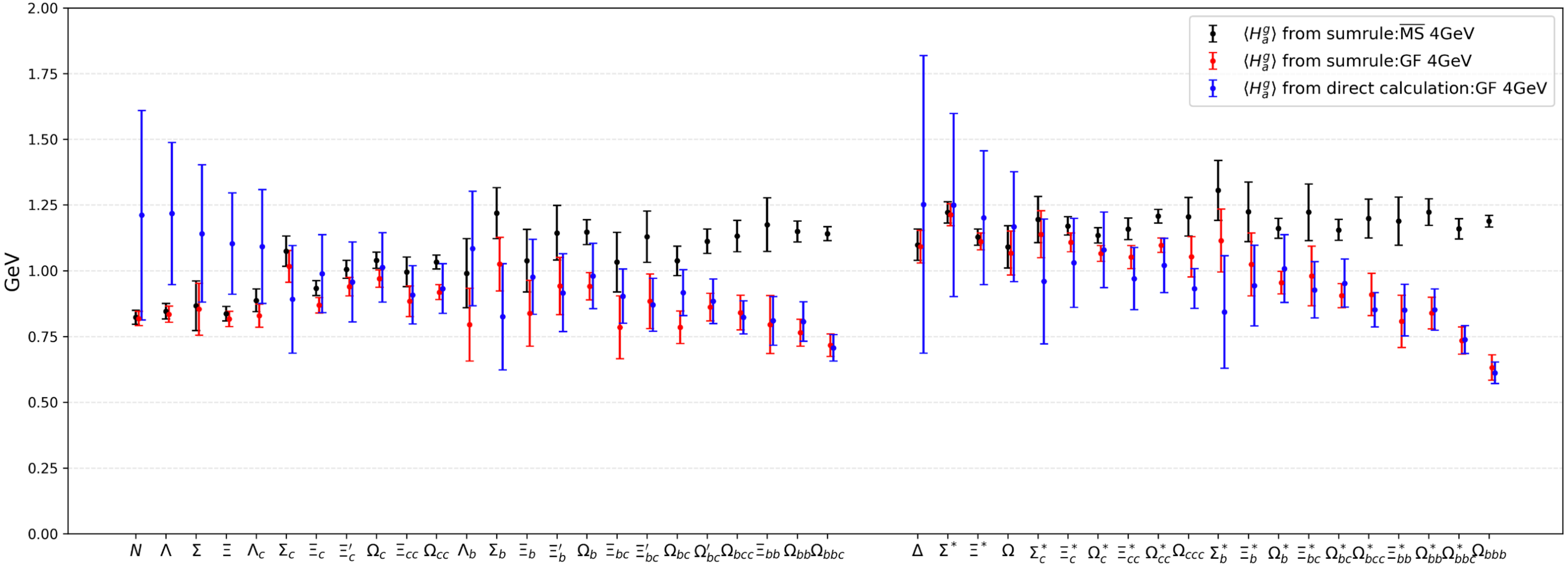
- Strong interaction contributes to the hadron masses in two ways:
- Convert the scale dependent quark mass into scale independent contribution with flavor dependence enhancements;
- Sizable contribution from the trace anomaly, and its gluon component can be flavor insensitive at $\overline{\mathrm{MS}}$ 4 GeV;
- Systematic studies of the direct calculation of the gluon trace anomaly are in progress.






$\langle H_a^g \rangle$ from sumrule:$\overline{\text{MS}}$ 2GeV
$\langle H_a^g \rangle$ from sumrule:GF 2GeV
$\langle H_a^g \rangle$ from direct calculation:GF 2GeV
GeV
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00
N Λ Σ Ξ Λ_c Σ_c Ξ_c Ξ'_c Ω_c Ξ_{cc} Ω_{cc} Λ_b Σ_b Ξ_b Ξ'_b Ω_b Ξ_{bc} Ξ'_{bc} Ω_{bc} Ω'_{bc} Ω_{bcc} Ξ_{bb} Ω_{bb} Ω_{bbc} Δ Σ^* Ξ^* Ω Σ_c^* Ξ_c^* Ω_c^* Ξ_{cc}^* Ω_{cc}^* Ω_{ccc} Σ_b^* Ξ_b^* Ω_b^* Ξ_{bc}^* Ω_{bc}^* Ω_{bcc}^* Ξ_{bb}^* Ω_{bb}^* Ω_{bbc}^* Ω_{bbb}


$\hat{\beta}$ under $\overline{\text{MS}}$ nf=3
$\hat{\gamma_m}$ under $\overline{\text{MS}}$ nf=3
$\hat{\beta}$ from GF(a=0.052fm)
$\hat{\gamma_m}$ from GF(a=0.052fm)
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40
1.5 2.0 2.5 3.0 3.5 4.0
Energy(Gev)


$\langle H_a^g \rangle$ from sumrule:$\overline{\text{MS}}$ 4GeV
$\langle H_a^g \rangle$ from sumrule:GF 4GeV
$\langle H_a^g \rangle$ from direct calculation:GF 4GeV
GeV
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00
N Λ Σ Ξ Λ_c Σ_c Ξ_c Ξ'_c Ω_c Ξ_{cc} Ω_{cc} Λ_b Σ_b Ξ_b Ξ'_b Ω_b Ξ_{bc} Ξ'_{bc} Ω_{bc} Ω'_{bc} Ω_{bcc} Ξ_{bb} Ω_{bb} Ω_{bbc} Δ Σ^* Ξ^* Ω Σ_c^* Ξ_c^* Ω_c^* Ξ_{cc}^* Ω_{cc}^* Ω_{ccc} Σ_b^* Ξ_b^* Ω_b^* Ξ_{bc}^* Ω_{bc}^* Ω_{bcc}^* Ξ_{bb}^* Ω_{bb}^* Ω_{bbc}^* Ω_{bbb}