

Predicting isovector charmonium-like states from $X(3872)$ properties

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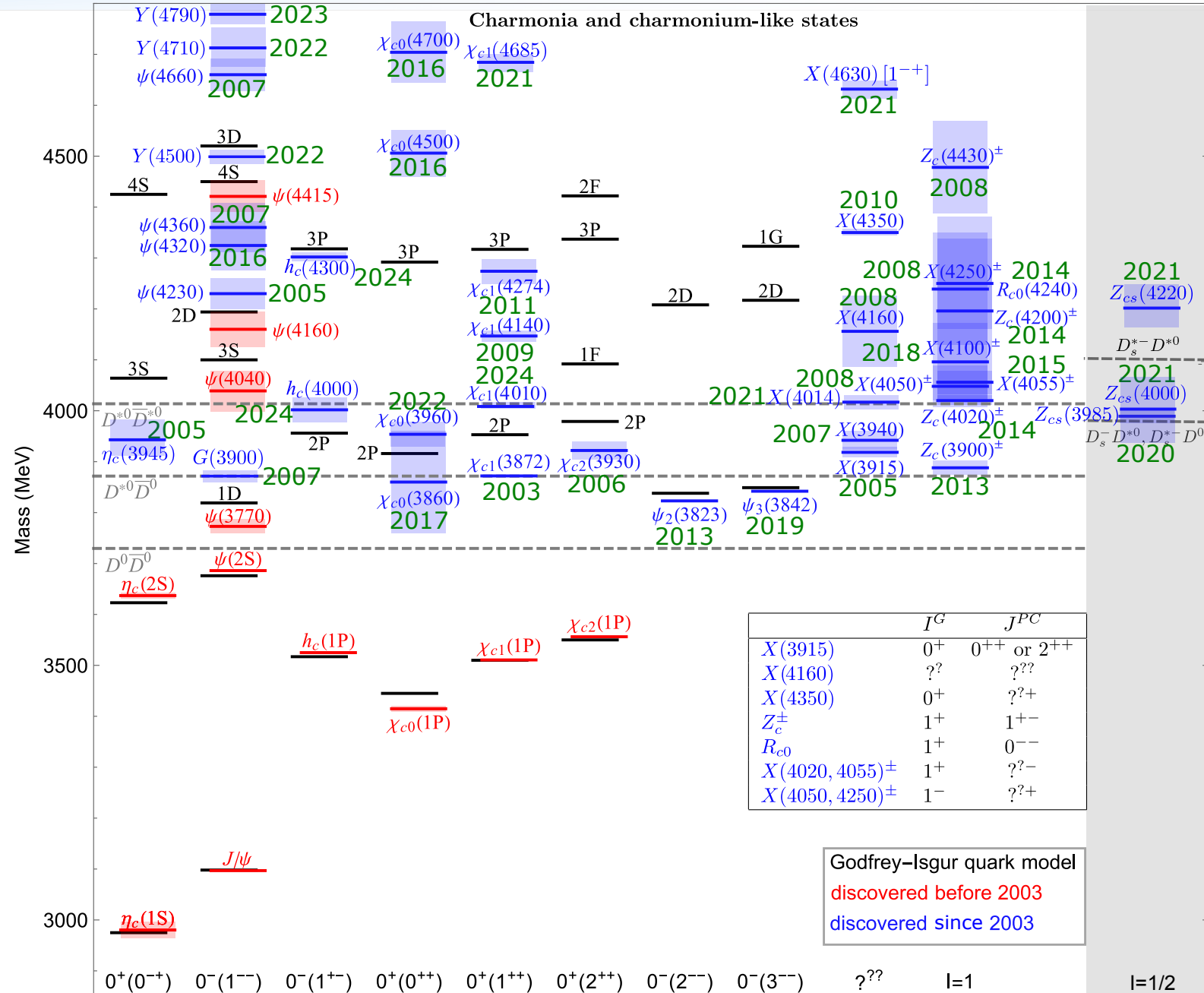
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Zhen-Hua Zhang, T. Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, A. Rusetsky, [Predicting isovector charmonium-like states from \$X\(3872\)\$ properties](#), JHEP 08 (2024) 130;

Jorgivan Dias, Teng Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, Y. Zhang, Z.-H. Zhang, [Dispersive analysis of the isospin breaking in the \$X\(3872\) \rightarrow J/\psi \pi^+ \pi^-\$ and \$X\(3872\) \rightarrow J/\psi \pi^+ \pi^0 \pi^-\$ decays](#), Phys. Rev. D 111 (2025) 014031;

Teng Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, [Precise determination of the properties of \$X\(3872\)\$ and of its isovector partner \$W_{c1}\$](#) , arXiv:2502.04458

Charmonia and charmonium-like states



X(3872) and possible isospin-1 partners

- X(3872) has been discovered by Belle for more than 20 years, debates are still ongoing!
- Excellent observable for distinguishing models: Isospin-1 partners!

❑ No, in charmonium model

L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, PRD 71 (2005) 014028

❑ Quark bound states, in compact tetraquark model

- With isospin-independent quark interactions, isoscalar and isovector tetraquarks must coexist

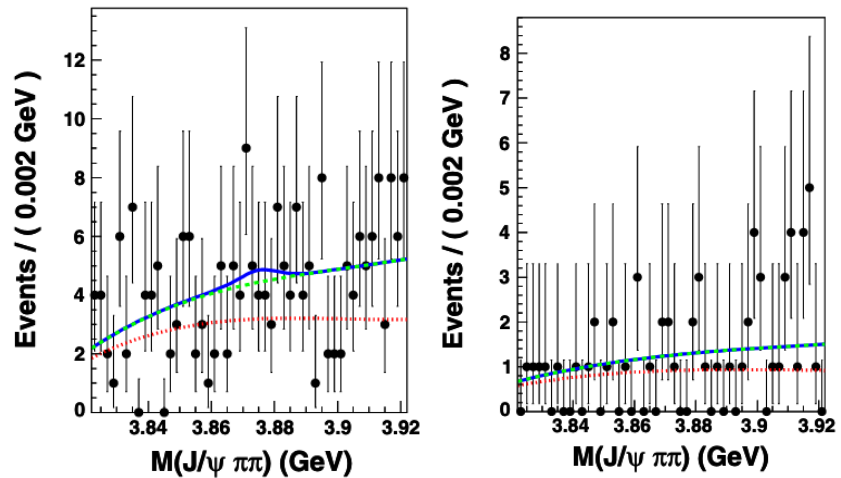
$$I = 1 \text{ multiplet: } [cu][\bar{c}\bar{d}], \frac{1}{\sqrt{2}}([cu][\bar{c}\bar{u}] - [cu][\bar{c}\bar{d}]), [cd][\bar{c}\bar{u}]$$

❑ How about hadronic molecular picture?

- Thought to be non-existing, but never carefully investigated
- Will be shown to exist as virtual states in this talk

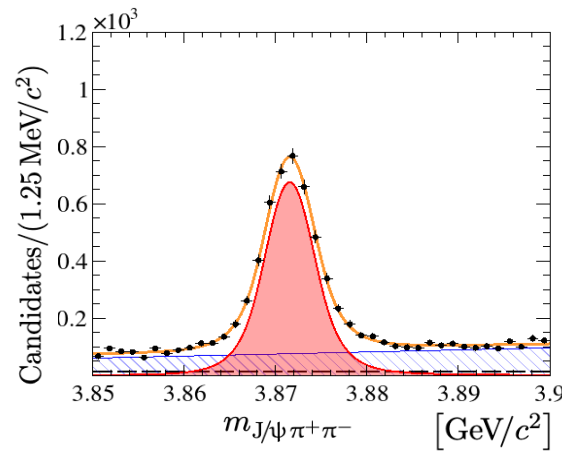
So far negative signal

- No signal in the charged channel so far
- No signal around the $D^+ D^{*-}$ threshold



$$\bar{B}^0 \rightarrow K^- \pi^+ \pi^0 J/\psi \quad B^+ \rightarrow K^0 \pi^+ \pi^0 J/\psi$$

Belle, PRD 84 (2011) 052004

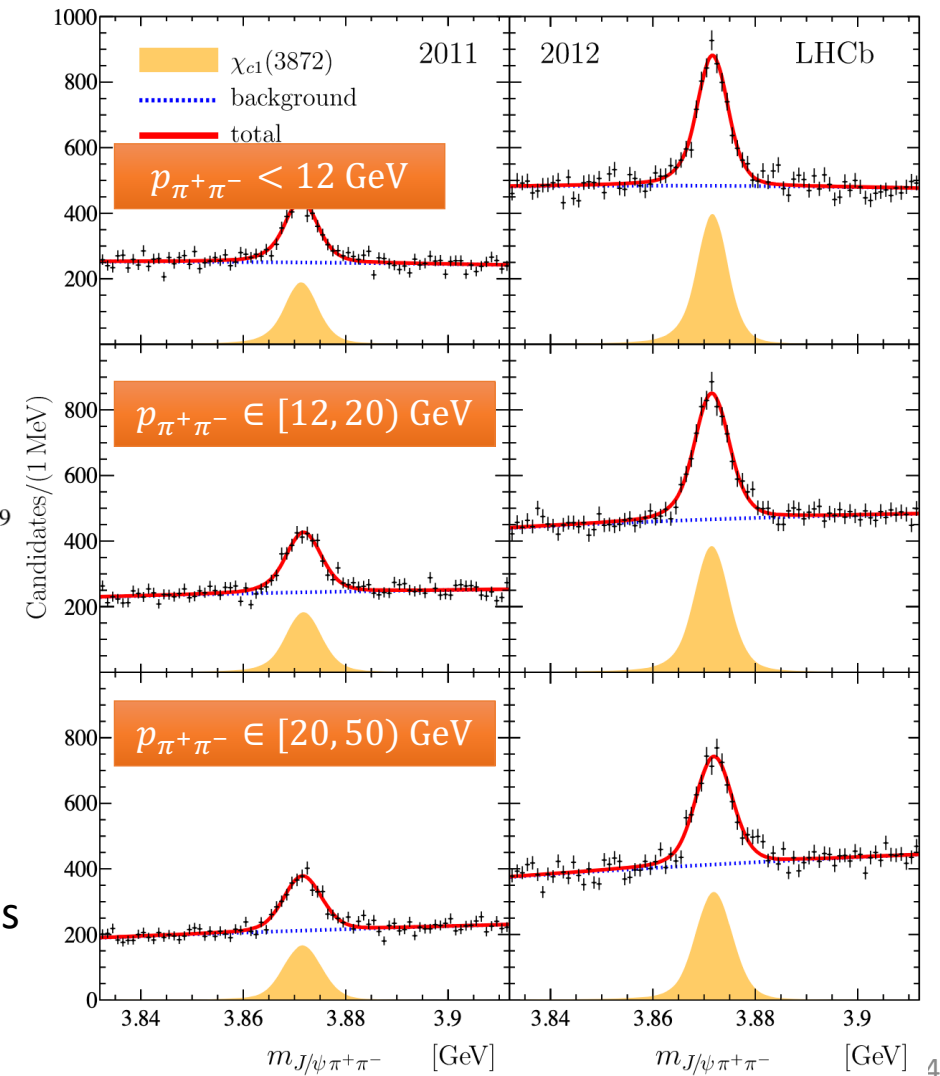


$$B^+ \rightarrow K^+ \pi^+ \pi^- J/\psi$$

LHCb, JHEP 08 (2020) 123

$$\pi^+ \pi^- J/\psi \text{ from } b\text{-hadrons}$$

LHCb, PRD 102 (2020) 092005



$J^{PC} = 1^{++}$ sector

- **Hadronic molecules**: consider S-wave interactions between charm and anti-charm mesons
- For each isospin, only **two low-energy constants (LECs) at LO in nonrelativistic expansion** for S-wave interactions of 6 meson pairs
- For the $J^{PC} = 1^{++}$ sector, also **two LECs at LO**:
 - $I = 0: C_{0X}; I = 1: C_{1X}$
- Two inputs from $X(3872)$ properties :

➤ Mass

$$M_X = 3871.69^{+0.00+0.05}_{-0.04-0.13} \text{ MeV} \quad \text{LHCb, PRD 102 (2020) 092005}$$

$$M_{D^0} + M_{D^{*0}} = 3871.69(7) \text{ MeV} \quad \text{PDG 2024}$$

➤ Isospin breaking in decays

LHCb, PRD 108 (2023) L011103

$$R_X = \left| \frac{\mathcal{M}_{X(3872) \rightarrow J/\psi \rho^0}}{\mathcal{M}_{X(3872) \rightarrow J/\psi \omega}} \right| = 0.29 \pm 0.04$$

Extracted using BW for resonances;

Updated to **0.26 ± 0.03** using **Omnes repr.** for $\pi\pi$ P-wave $\mathcal{M}_{X \rightarrow J/\psi \pi\pi} = \mathcal{N} \varepsilon_{ijk} \varepsilon_{\psi}^i \varepsilon_X^j q_{\pi}^k \underbrace{P(s)\Omega(s)}_{\text{Omnes}} [1 + \underbrace{\kappa_X G_{\omega}(s)}_{\rho\omega \text{ mixing}}]$
 J. Dias et al., PRD 111 (2025) 014031

➤ Neutral systems X and W_{c1}^0 : coupled channels

$$\checkmark \quad (D\bar{D}^*)_0 \equiv (D^0\bar{D}^{*0} - \bar{D}^0D^{*0})/\sqrt{2}$$

$$\checkmark \quad (D\bar{D}^*)_{\pm} \equiv (D^+D^{*-} - D^-D^{*+})/\sqrt{2}$$

➤ Charged systems $W_{c1}^{\pm}$: single channel

Lippmann-Schwinger equation (LSE)

● Coupled channels: $D^0\bar{D}^{*0}, D^+D^{*-}$ with $C = +$

● T matrix is given by the LSE:

$$T(E; p', p) = V(E; p', p) + \int \frac{l^2 dl}{2\pi^2} V(E; p', l) G(E; l) T(E; l; p)$$

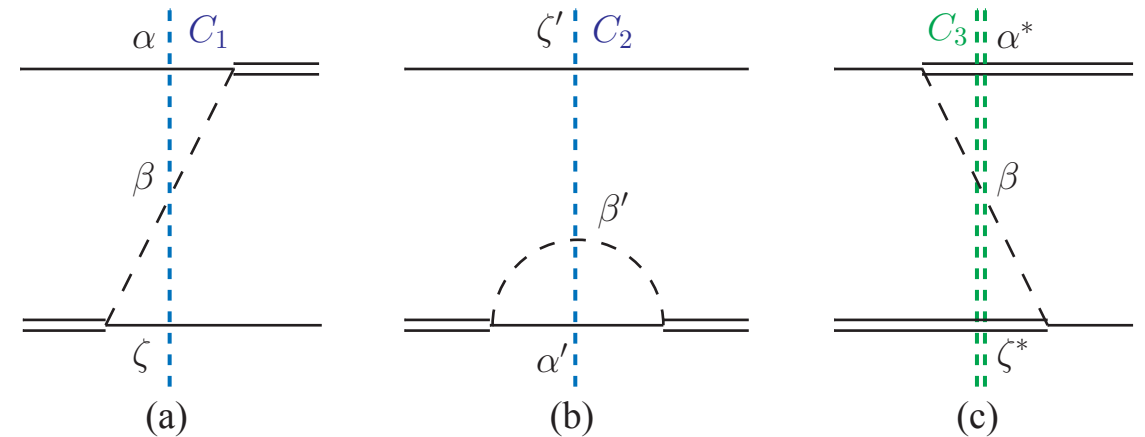
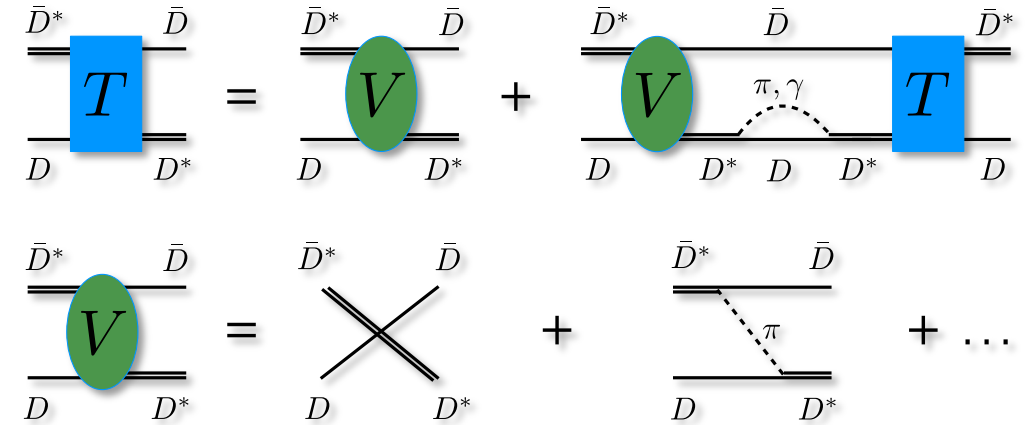
Potential: contact term (C_{0X}, C_{1X}) + one-pion exchange (OPE)

$$V_{\text{ct}} = \frac{1}{2} \begin{pmatrix} C_{0X} + C_{1X} & C_{0X} - C_{1X} \\ C_{0X} - C_{1X} & C_{0X} + C_{1X} \end{pmatrix}$$

➤ 3-body effects: OPE, D^* selfenergy

$$V_{\pi}(E; p', p) = \frac{g^2}{6F_{\pi}^2} \begin{pmatrix} \frac{1}{2} V_{D^0\pi^0\bar{D}^0}^{SS} & V_{D^0\pi^+D^-}^{SS} \\ V_{D^+\pi^-\bar{D}^0}^{SS} & \frac{1}{2} V_{D^+\pi^0D^-}^{SS} \end{pmatrix}$$

$$V_{\alpha\beta\zeta}^{SS} = \frac{1}{2} \int_{-1}^1 dz \frac{q^2}{2E_{\beta}(q)} \left(\frac{1}{D_{\alpha\beta\zeta}^R} + \frac{1}{D_{\alpha^*\beta\zeta^*}^A} \right)$$



retarded OPE

 D^* selfenergy

advanced OPE

Molecular line shapes at LO

- Scattering length approx.: $k \cot \delta = -\frac{1}{a} + \dots$

- Poles: **bound or virtual state** ($\kappa = 1/|a|$)

- Bound and virtual state can hardly be distinguished above threshold ($E > 0$)

$$|T_{\text{NR}}(E)|^2 \propto \left| \frac{1}{\pm\kappa + i\sqrt{2\mu E}} \right|^2 = \frac{1}{\kappa^2 + 2\mu E}$$

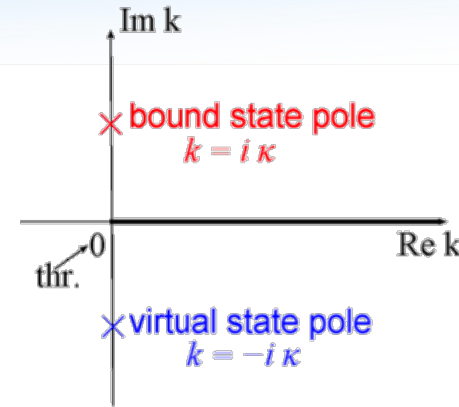
- Different below threshold ($E < 0$)

- bound state: peaked below threshold

$$|T_{\text{NR}}(E)|^2 \propto \frac{1}{(\kappa - \sqrt{-2\mu E})^2}$$

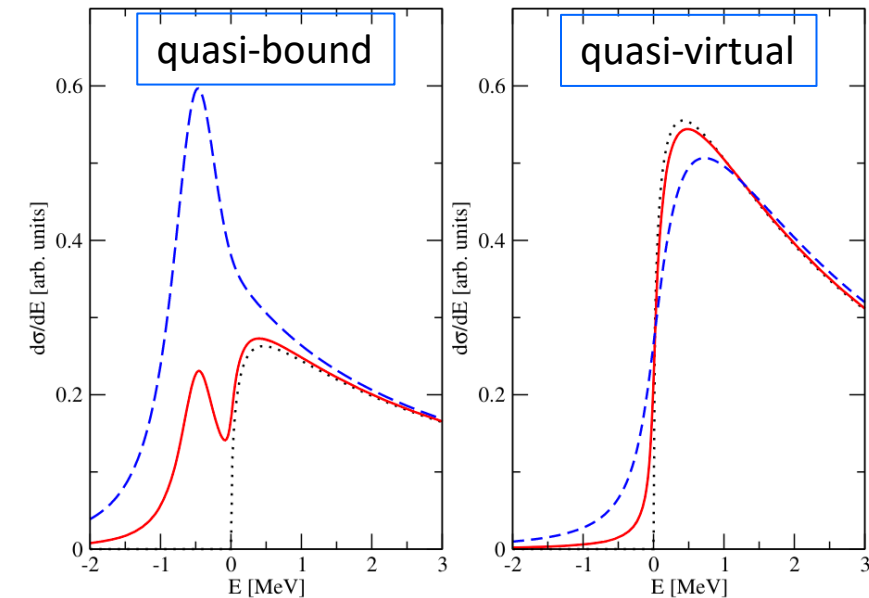
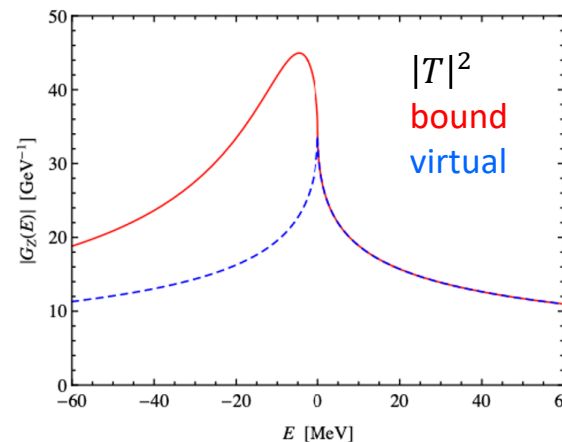
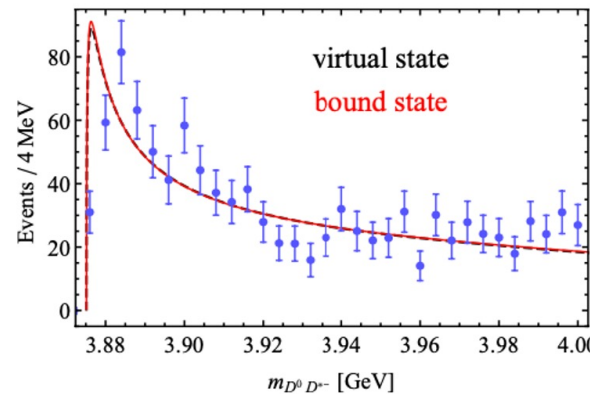
- virtual state: sharp cusp at threshold

$$|T_{\text{NR}}(E)|^2 \propto \frac{1}{(\kappa + \sqrt{-2\mu E})^2}$$



FKG, et al., RMP 90 (2018) 015004;
N. Brambilla et al., Phys.Rept. 873 (2020) 1

line shapes w/ phase space;
one unstable constituent:



$\Gamma = 0$	0.1 MeV	1 MeV
dotted	dashed	solid

Lippmann-Schwinger equation (LSE)

Z.-H. Zhang et al., JHEP 08 (2024) 130

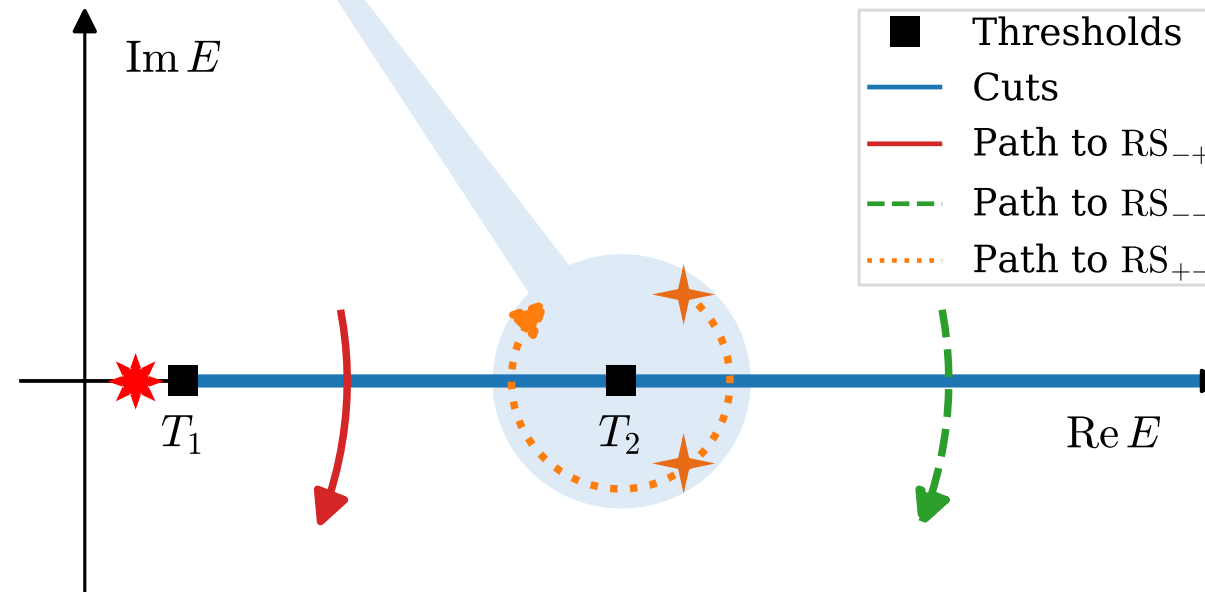
- Two poles of the T -matrix for the $(D\bar{D}^*)_0 - (D\bar{D}^*)_{\pm}$ scattering amplitudes (4 Riemann sheets)

□ $X(3872)$ pole on the 1st RS (RS_{++})

□ $W_{c1}(3880)$ pole on the 4th RS (RS_{+-})

➤ Shaded by D^+D^{*-} threshold

➤ Cusp at the D^+D^{*-} threshold!!!



Prediction of an isospin vector partner of $X(3872)$

- There must be near-threshold isovector W_{c1} states

❑ **Virtual state** pole in the stable D^* limit

- W_{c1}^+ in $D^+ \bar{D}^{*0}$ single-channel scattering amplitude: pole on the 2nd Riemann sheet (RS),

8_{-5}^{+8} MeV below $D^0 D^{*-}$ threshold

$$W_{c1}^{\pm}: 3866.9_{-7.7}^{+4.6} - i(0.07 \pm 0.01) \text{ MeV}$$

- W_{c1}^0 in $(D \bar{D}^*)_0 - (D \bar{D}^*)_{\pm}$ scattering amplitudes: pole on the 4th RS (RS₊₋),

$1.3_{-0.0}^{+0.8}$ MeV above $D^+ D^{*-}$ threshold

$$W_{c1}^0: 3881.2_{-0.0}^{+0.8} + i1.6_{-0.9}^{+0.7} \text{ MeV}$$

❑ Must appear as **threshold cusps!!!**

❑ **Compact tetraquarks** (Maiani et al. (2005)) **cannot be virtual states** as they do not feel the thresholds

Cutoff insensitivity checked: poles relative to thresholds varied within 5% for $\Lambda \in [0.5, 1.0]$ GeV

- **Virtual state W_{c1} was confirmed in lattice QCD calculation** with $M_{\pi} = 280$ MeV

M. Sadl et al., PRD 111 (2025) 054513

J^{PC}	Interpolators	$1/a_0 [\text{fm}^{-1}]$	$r_0 [\text{fm}]$	χ^2/N_{dof}	$\Delta m_V [\text{MeV}]$
1^{+-}	All	$0.46_{-0.45}^{+1.16}$	$0.96_{-0.73}^{+0.43}$	0.13	$-3.0_{-31.1}^{+3.0}$
	$\eta_c \rho$ excl	$0.54_{-0.44}^{+1.07}$	$2.23_{-1.08}^{+0.95}$	0.24	$-2.8_{-17.1}^{+2.6}$
1^{++}	All	$0.62_{-0.51}^{+1.30}$	$1.78_{-2.44}^{+0.25}$	0.18	$-3.8_{-19.5}^{+3.6a}$
	$J/\psi \rho, \eta_c a_0$ excl	$0.96_{-0.91}^{+1.42}$	$2.19_{-1.00}^{+0.36}$	0.15	$-6.7_{-19.5}^{+6.7}$

^aUncertainty is so large that it is unbounded from below.

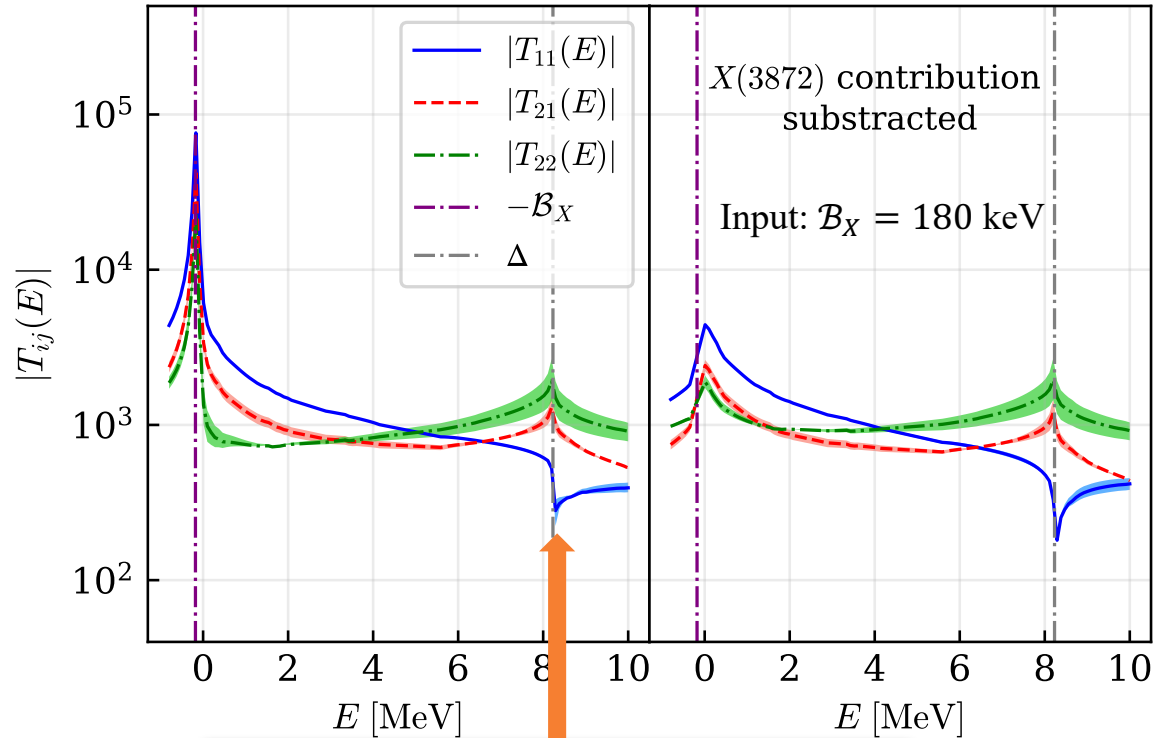
sign convention different from ours

Also obtained in one-boson exchange model
in X.-X. Chen, Z.-M. Ding, J. He, PRD 111 (2025) 114008

Why have they not been observed?

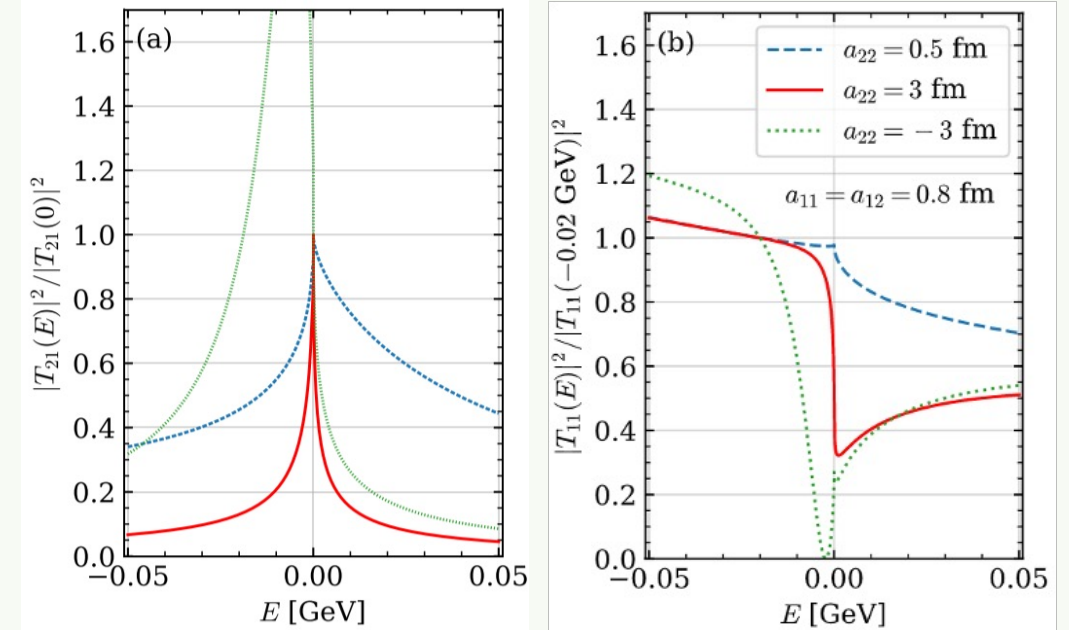
- W_{c1}^0 lives in the same amplitudes as the $X(3872)$, effects shielded by X

➤ W_{c1}^0 in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes



Threshold cusp!
peak or dip depends on processes

- Universality of dip for large scattering length
 - ✓ For strongly interacting channel-2 (large a_{22}), there must be a dip around threshold



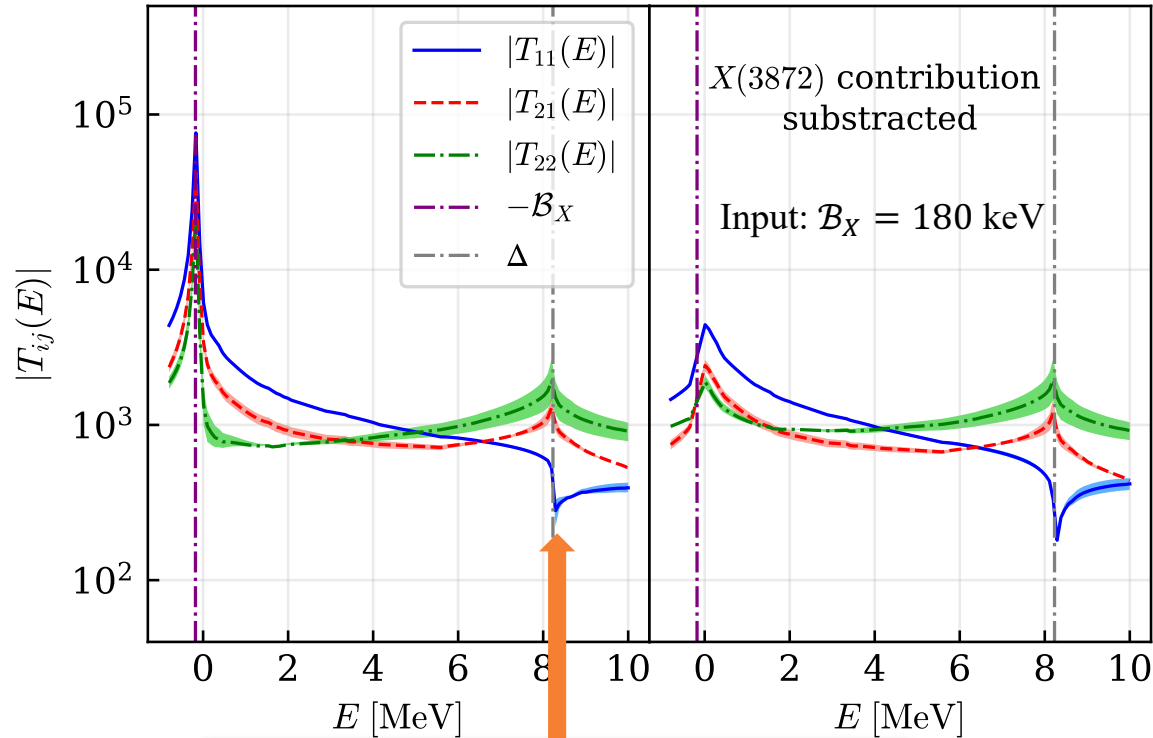
$$T_{11}(E) = \frac{-8\pi\Sigma_2 \left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E} \right)}{\left(\frac{1}{a_{11}} - ik_1 \right) \left[\frac{1}{a_{22, \text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E) \right]}$$

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

Why have they not been observed?

- W_{c1}^0 lives in the same amplitudes as the $X(3872)$, effects shaded by X

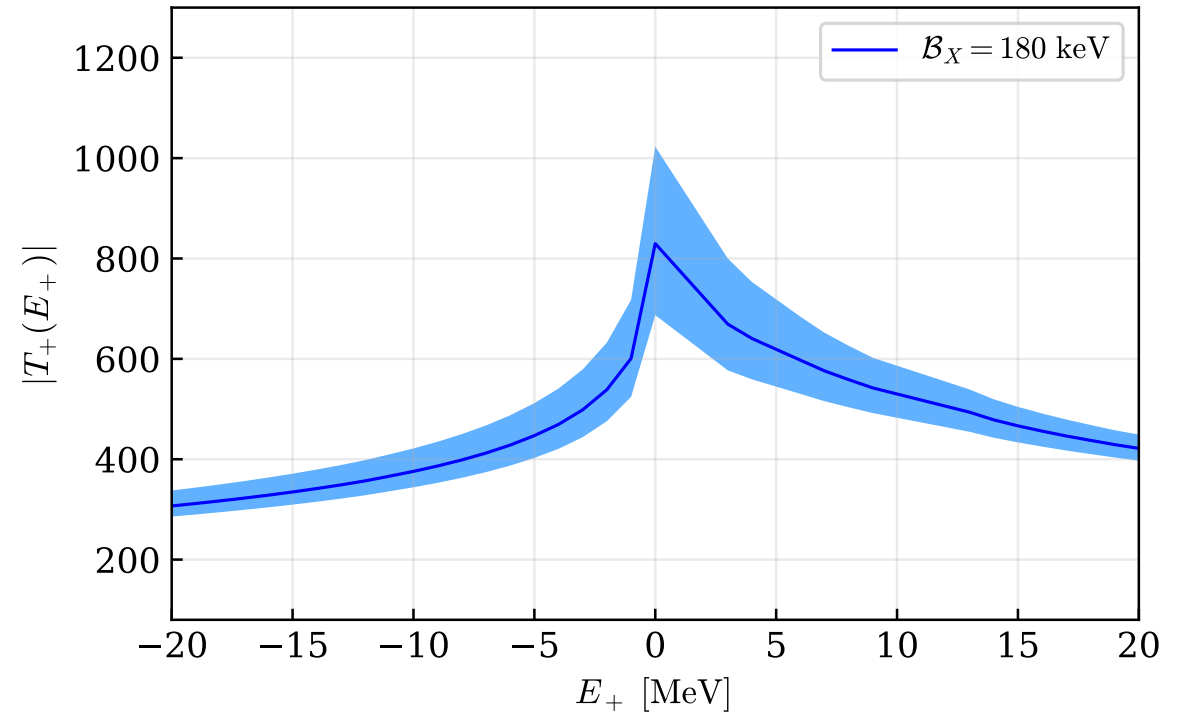
➤ W_{c1}^0 in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes



Threshold cusp!
peak or dip depends on processes

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

➤ Charged W_{c1}^+ in $D^+\bar{D}^{*0}$ scattering amplitude: height much lower than the X peak



➤ should be searched for in high-statistic $J/\psi\pi^\pm\pi^0$ data

- The observed $X(3872)$ signals should contain the W_{c1}^0 contribution as well $\Rightarrow$ combined analysis !!

Combined analysis of BESIII and LHCb data for $X(3872)$

Teng Ji et al., arXiv:2502.04458

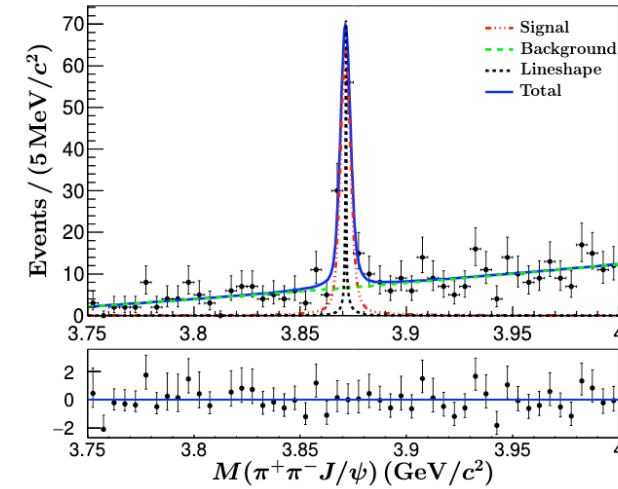
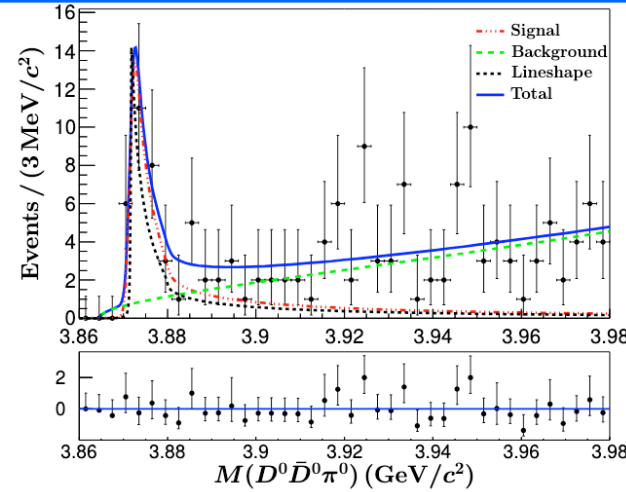
- $X(3872)$ line shapes $\Rightarrow X(3872)$ + possible $W_{c1}(3880)^0$
- $\pi^+\pi^-$ invariant mass distribution $\Rightarrow$ isospin breaking, information on $I = 1$

BESIII:

$$e^+e^- \rightarrow \gamma[D^0\bar{D}^0\pi^0]$$

$$e^+e^- \rightarrow \gamma[J/\psi\pi^+\pi^-]$$

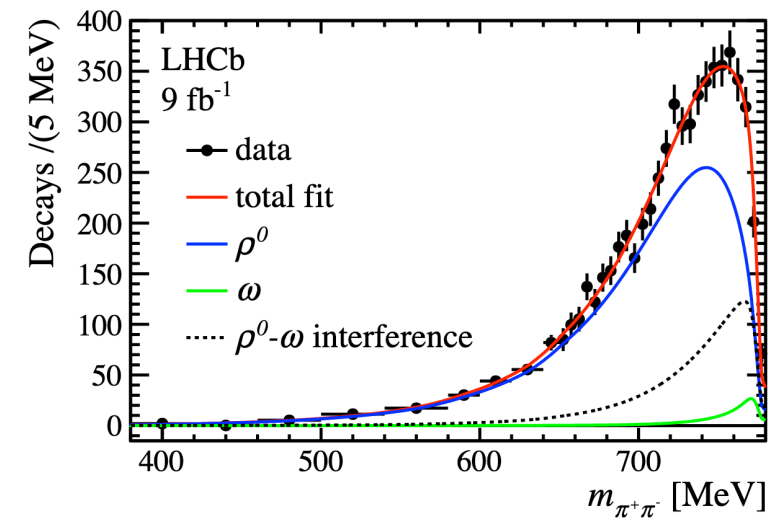
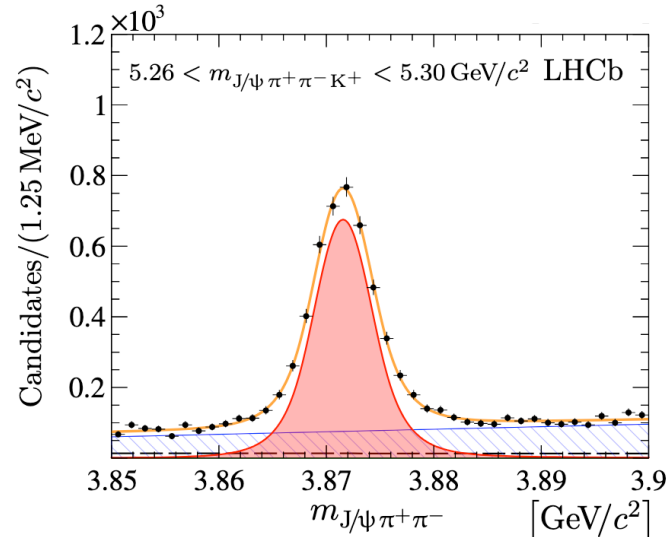
BESIII, PRL 132 (2024) 151903



LHCb:

$$B^+ \rightarrow K^+[J/\psi\pi^+\pi^-]$$

LHCb, JHEP 08 (2020) 123;
PRD 108 (2023) L011103



Combined analysis of BESIII and LHCb data for $X(3872)$

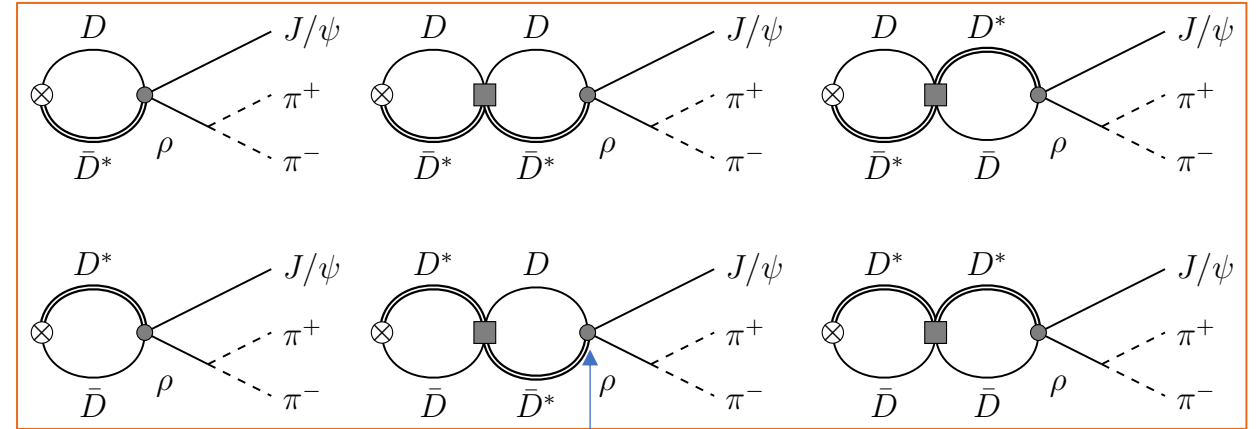
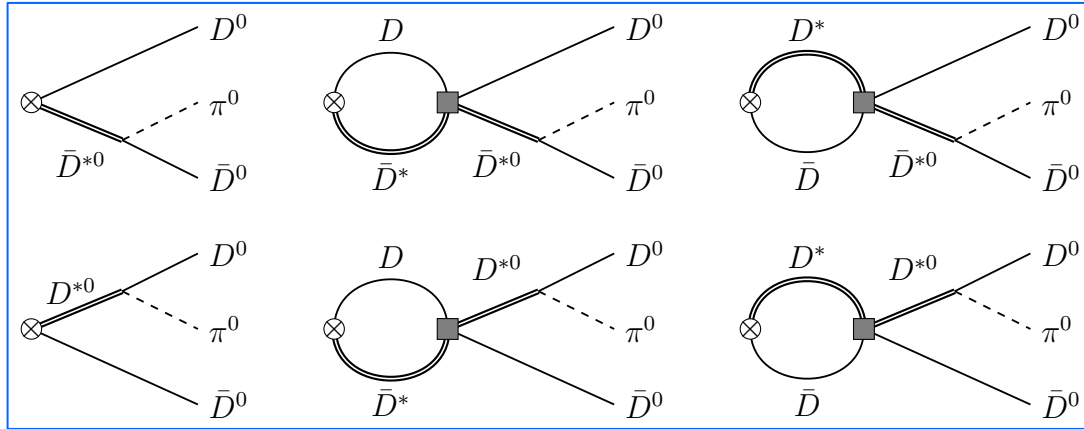
● Coupled channels

□ $(D\bar{D}^*)_0$, $(D\bar{D}^*)_{\pm}$: contact terms + OPE, $D\bar{D}\pi$ three-body effects considered

□ Inelastic channels:

➤ $J/\psi\rho, J/\psi\omega$: ρ included using the Omnes dispersive approach, ρ - ω mixing considered

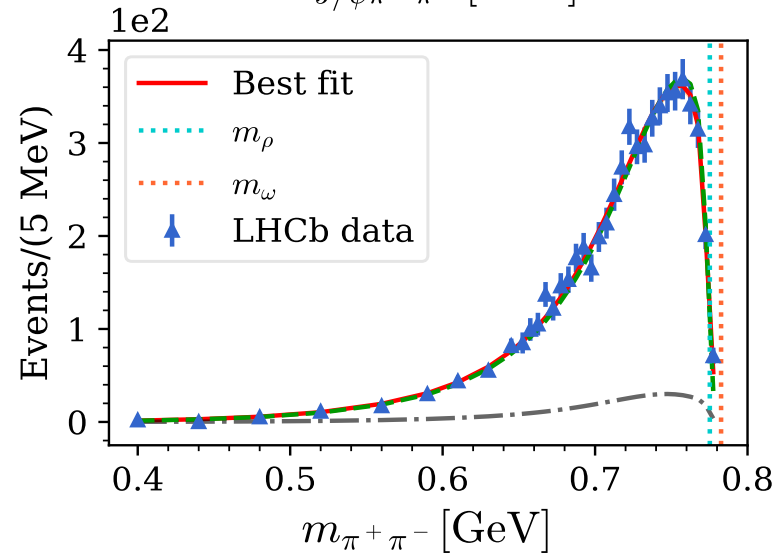
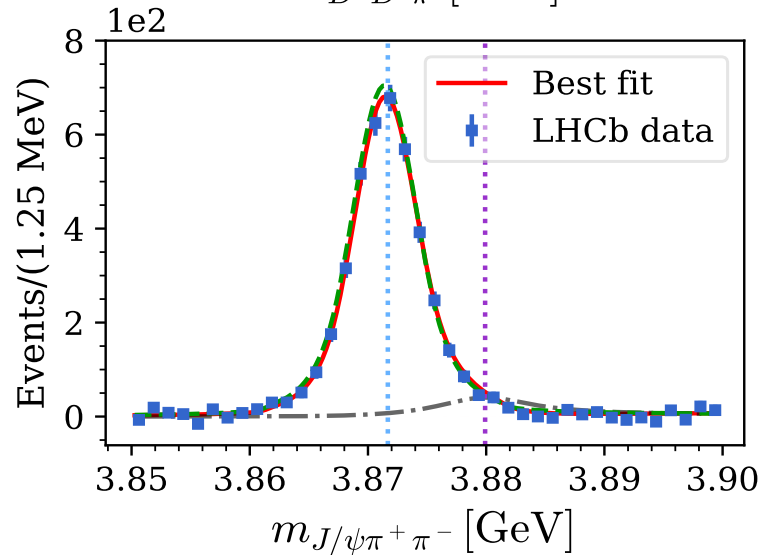
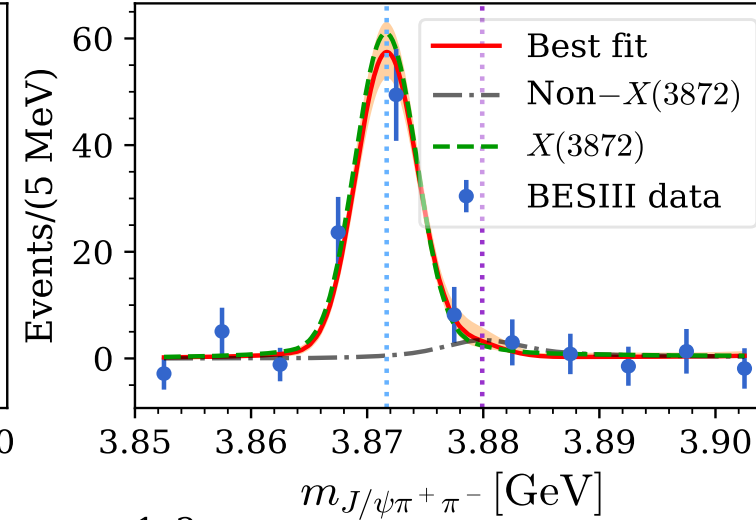
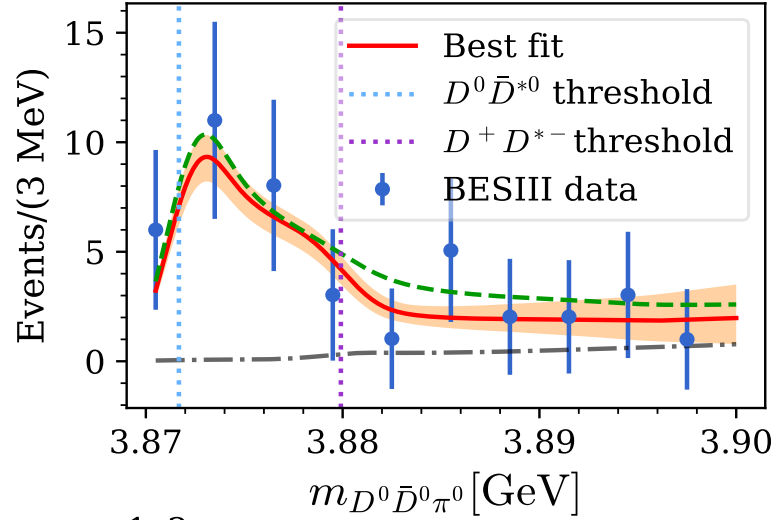
➤ $J/\psi\gamma, \psi'\gamma, \chi_{cJ}(1P)\pi^0$: neglected in the baseline fit, included in uncertainty analysis



ρ - ω mixing included

Combined analysis of BESIII and LHCb data for $X(3872)$

Results updated



Best fit: $\chi^2/\text{dof} = 57/(96 - 10) = 0.66$

Pole positions: $X(3872)$ and isovector $W_{c1}(3880)$

● Poles

□ $X(3872)$ as a bound state below $D^0\bar{D}^{*0}$ threshold (2.7σ)

$$E_X = (-160_{-38}^{+28+32+40} - 125_{-13}^{+12+14+15}i) \text{ keV}$$

$$\delta m_{D^0} = \delta m_{D^{*0}} = 50 \text{ keV}, \delta(m_{D^{*0}} - m_{D^0}) = 30 \text{ keV}$$

$$M_X = (3871.53_{-0.08}^{+0.06} - 0.13_{-0.04}^{+0.02}i) \text{ MeV}$$

□ $W_{c1}(3880)^0$ pole on RS_{+-} , relative to the D^+D^{*-} threshold:

$$E_W = (3.1 \pm 0.7 + 1.3_{-0.6}^{+1.9}i) \text{ MeV}$$

● Residues:

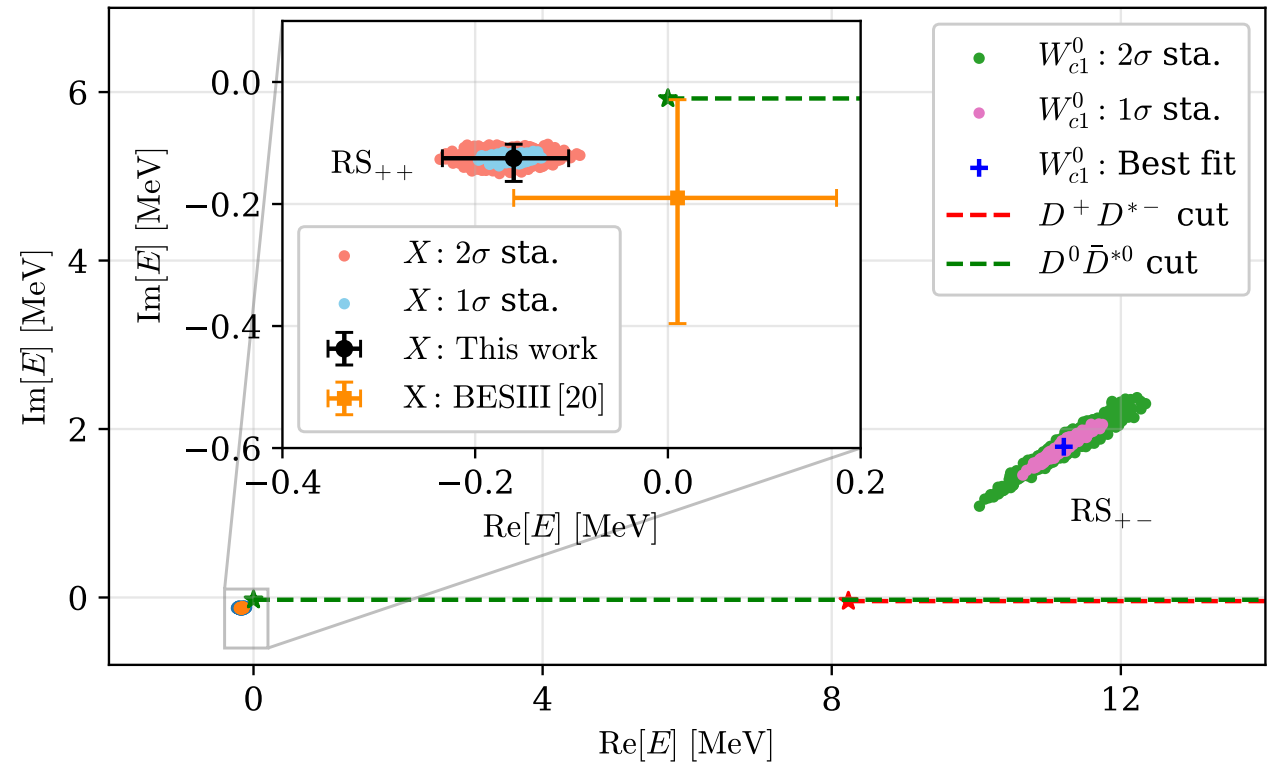
$$g_{X,0} = (0.35 \pm 0.03)e^{0.12_{-0.04}^{+0.08}i} \text{ GeV}^{-\frac{1}{2}},$$

$$g_{X,\pm} = (0.22 \pm 0.02)e^{0.09_{-0.05}^{+0.09}i} \text{ GeV}^{-\frac{1}{2}},$$

$$g_{W,0} = (0.44 \pm 0.07)e^{-1.1_{-0.09}^{+0.05}i} \text{ GeV}^{-\frac{1}{2}},$$

$$g_{W,\pm} = (0.70_{-0.06}^{+0.03})e^{-2.18_{-0.05}^{+0.16}i} \text{ GeV}^{-\frac{1}{2}}.$$

➤ Much more precise than previous determination



$X(3872)$ couples more strongly to $D^0\bar{D}^{*0}$;

$W_{c1}(3880)$ couples more strongly to D^+D^{*-}

Other properties of X(3872)

Results updated

- Width (twice of the imaginary part of the pole): 250_{-62}^{+48} keV

- Branching fractions computed using the method in L.A. Heuser, G. Chanturia, FKG, C. Hanhart, M. Hoferichter, B. Kubis, EPJC 84 (2024) 599

Mode	$D^0 \bar{D}^0 \pi^0$	$D^0 \bar{D}^0 \gamma$	$J/\psi \pi^+ \pi^-$	$J/\psi \pi^+ \pi^- \pi^0$	others
BR(%)	41_{-4}^{+3}	22 ± 2	5_{-1}^{+2}	16_{-3}^{+4}	16 ± 2

- Isospin breaking ratio $R_X \equiv \left| \frac{g_{XJ/\psi \rho}}{g_{XJ/\psi \omega}} \right| = 0.26(2)$

- Compositeness using a formula including range corrections Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

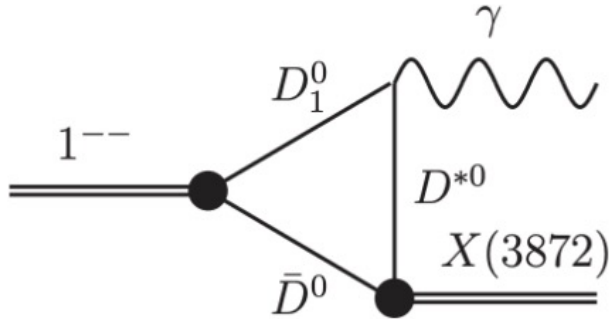
$$X = 1 - \exp \left(\frac{1}{\pi} \int_0^\infty dE \frac{\text{Re } \delta(E)}{E - \text{Re } E_X} \right) = 0.97(2)$$

Signal of $W_{c1}(3880)$?

- Signal of $W_{c1}(3880)^0$ almost invisible in the current data, reasons:

- Virtual state, threshold cusp

- $(D\bar{D}^*)_0$ easier produced than $(D\bar{D}^*)_{\pm}$ for both $e^+e^- \rightarrow \gamma D\bar{D}^*$ @ $\sqrt{s} \approx 4.23$ GeV and $B^+ \rightarrow K^+ D\bar{D}^*$



FKG et al., PLB 725 (2013) 127

$$\Gamma(D_1^0 \rightarrow \gamma D^{*0}) \gg \Gamma(D_1^+ \rightarrow \gamma D^{*+})$$

J.G. Korner et al., PRD 47 (1993) 3955;
Fayyazuddin et al., PRD 50 (1994) 2329

- For B^+ decays, fit parameters (ratio of production vertices): $P_{\pm}/P_0 = 0.5 \pm 0.1$

Data:

PDG 2024

$$\frac{\text{Br}[B^+ \rightarrow K^+ (D^+ D^{*-} + D^- D^{*+})]}{\text{Br}[B^+ \rightarrow K^+ (D^0 \bar{D}^{*0} + \bar{D}^0 D^{*0})]} = 0.14 \pm 0.02$$

- Switching $u \leftrightarrow d$, situation should be different for B^0 decays

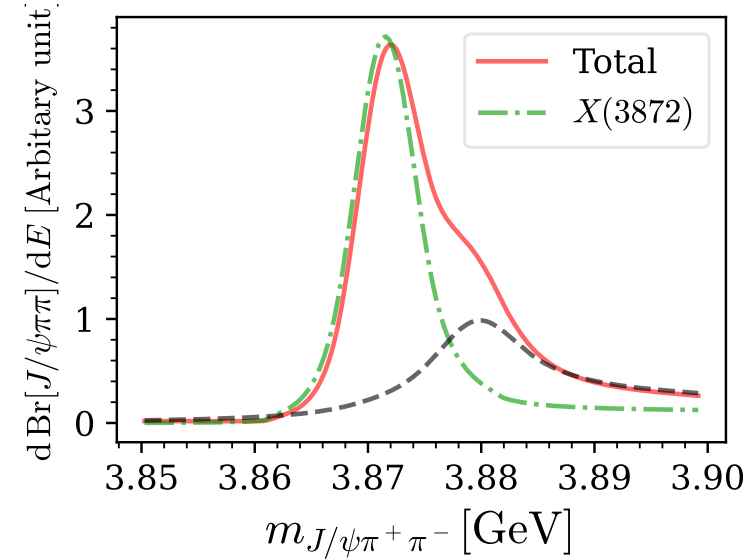
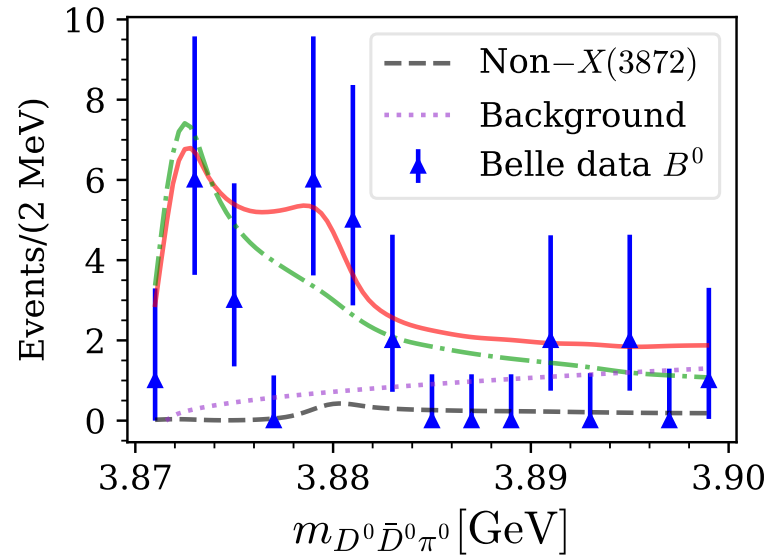
Data:

PDG 2024

$$\frac{\text{Br}[B^0 \rightarrow K^0 (D^+ D^{*-} + D^- D^{*+})]}{\text{Br}[B^0 \rightarrow K^0 (D^0 \bar{D}^{*0} + \bar{D}^0 D^{*0})]} = 5.8 \pm 2.7$$

Implications of the existence of $W_{c1}(3880)$

- $W_{c1}(3880)^0$ signal should be stronger in $B^0 \rightarrow K^0[D^0\bar{D}^0\pi^0, J/\psi\pi^+\pi^-]$ decays, to be checked @ LHCb, Belle II



- Cusp at $D^+\bar{D}^{*0}$ threshold in $J/\psi\pi^\pm\pi^0$

Other implications to be explored

- $\text{Br}(X \rightarrow \psi' \gamma) / \text{Br}(X \rightarrow J/\psi \gamma)$: could be different at different experiments

$$\Gamma(\chi_{c1}(3872) \rightarrow \gamma \psi(2S)) / \Gamma(\chi_{c1}(3872) \rightarrow \gamma J/\psi)$$

PDG 2024					
VALUE	CL%	EVTS	DOCUMENT ID	TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •					
< 0.59	90		ABLIKIM	2020W BES3	$e^+ e^- \rightarrow \gamma \chi_{c1}(3872)$
$2.46 \pm 0.64 \pm 0.29$		36 ± 9	¹ AAIJ	2014AH LHCb	$B^+ \rightarrow \gamma \psi(2S) K^+$
< 2.1	90		BHARDWAJ	2011 BELL	$B^+ \rightarrow \gamma \psi(2S) K^+$
3.4 ± 1.4			AUBERT	2009B BABR	$B^+ \rightarrow \gamma c \bar{c} K'$

¹ From 36.4 ± 9.0 events of $\chi_{c1}(3872) \rightarrow J/\psi \gamma$ decays with a statistical significance of 4.4σ .

- $\Gamma(X \rightarrow \chi_{cJ} \pi^0)$ with iso-vector final states: could receive sizable contribution from $W_{c1}(3880)^0$

$\pi^0 \chi_{c2}$	< 5%
$\pi^0 \chi_{c1}$	$(3.8^{+1.9}_{-1.7})\%$
$\pi^0 \chi_{c0}$	< 16%

Summary and outlook

- Pole position of the $X(3872)$ determined to be (relative to the $D^0\bar{D}^{*0}$ threshold)

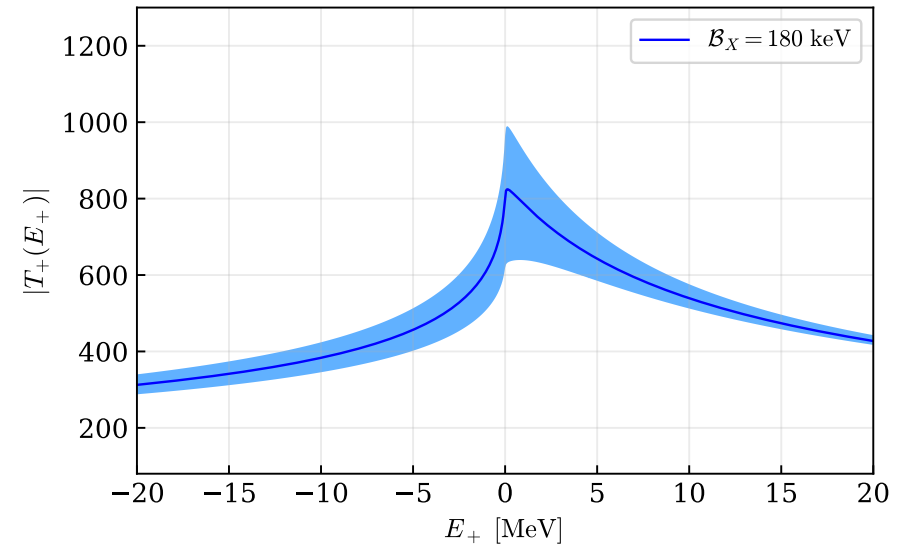
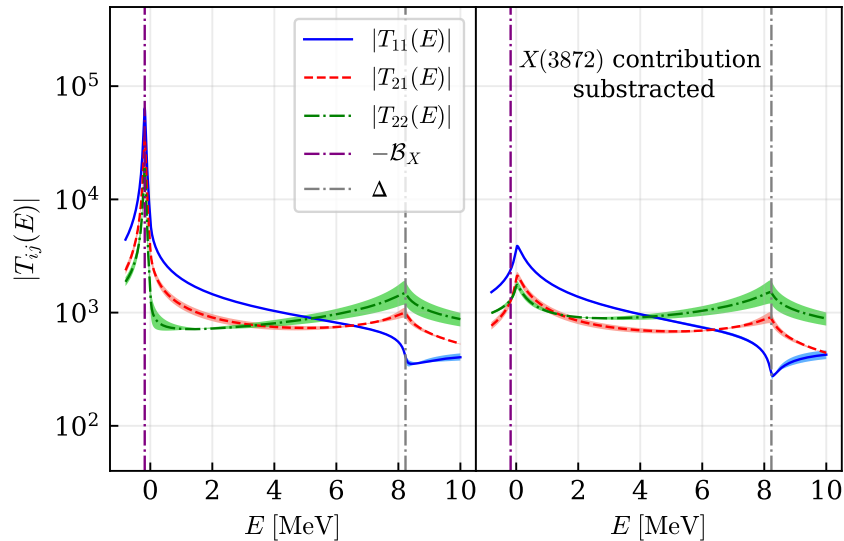
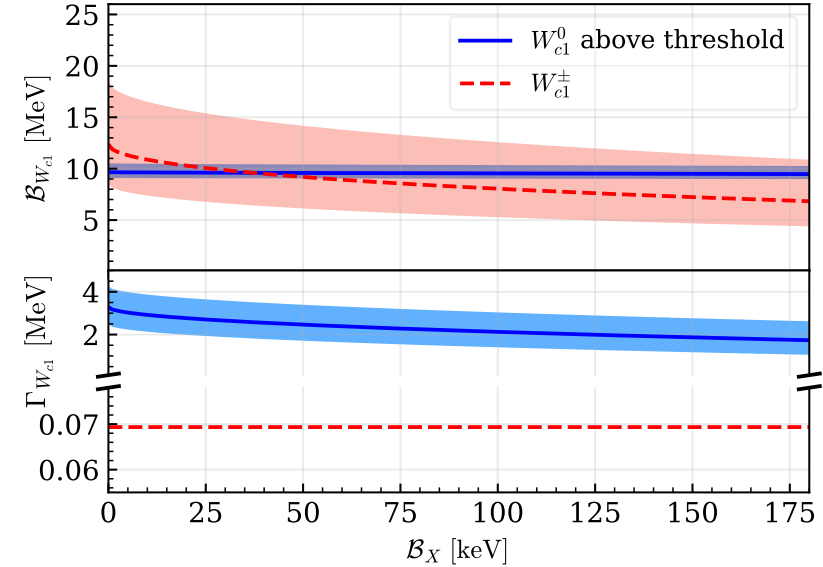
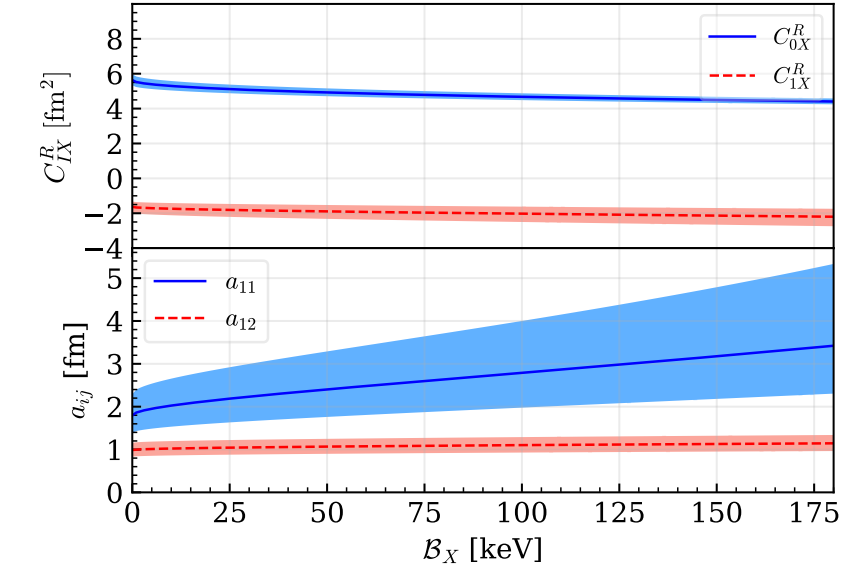
$$E_X = (-160_{-47-57}^{+43+38} - 125_{-25-28}^{+18+15}i) \text{ keV}$$

- Compositeness of $X(3872)$: $0.97(2)$
- Existence of an isovector $W_{c1}(3880)$
 - ▣ Signal of $W_{c1}(3880)^0$ predicted to be more visible in $B^0 \rightarrow K^0[D^0\bar{D}^0\pi^0, J/\psi\pi^+\pi^-]$
 - ▣ Signal of $W_{c1}(3880)^\pm$: threshold cusp at $D^+\bar{D}^{*0}$ threshold in $J/\psi\pi^\pm\pi^0$
- Some $X(3872)$ decays need to be reanalyzed

Thank you for your attention!

Results in the pionless theory

- All the qualitative features in the pion-full theory persist in the much simpler pionless theory



X(3872) line shapes

- Line shapes of a near-threshold resonance depend on reaction mechanism!

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- Peak for $|T_{21}|$ (1: lower inelastic channel; 2: elastic channel)

$$T_{21}(E) = \frac{-8\pi\Sigma_2}{a_{12}(1/a_{11} - ik_1)} \left[\frac{1}{a_{22,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E) \right]^{-1}$$

- Dip for $|T_{11}|$ if scattering length for channel-2 is large

$$T_{11}(E) = \frac{-8\pi\Sigma_2 \left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E} \right)}{\left(\frac{1}{a_{11}} - ik_1 \right) \left[\frac{1}{a_{22,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E) \right]}$$

$$= -8\pi E_2^{\text{thr}} \left(\underbrace{\frac{1}{a_{11}^{-1} - ik_1}}_{\text{background}} + \underbrace{\frac{a_{12}^{-2}(a_{11}^{-1} - ik_1)^{-2}}{a_{22,\text{eff}}^{-1} - ik_2}}_{\text{pole term}} \right)$$

background

pole term

The interfering phase is fixed by unitarity!

- $X(3872)$ shows up as a dip in $e^+e^- \rightarrow X \rightarrow J/\psi\pi\pi$ direct production

