



上海交通大学
SHANGHAI JIAO TONG UNIVERSITY

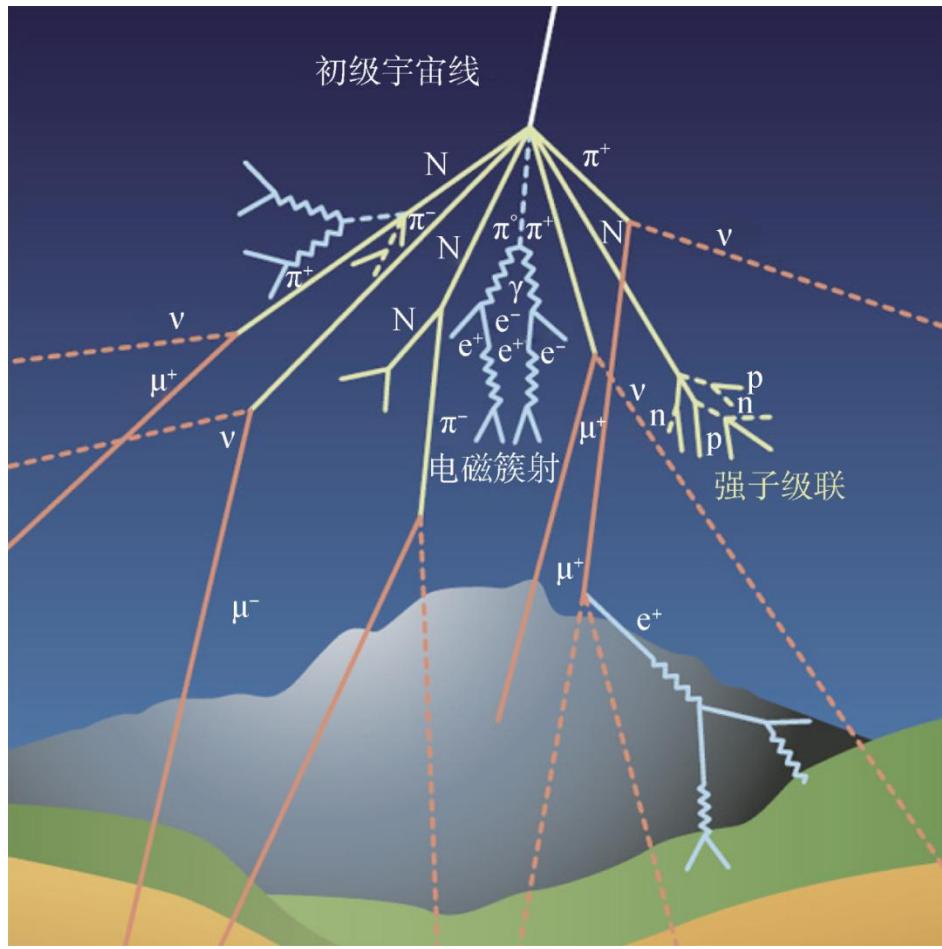
基于气体时间投影室技术 的缪子动量重建算法研究

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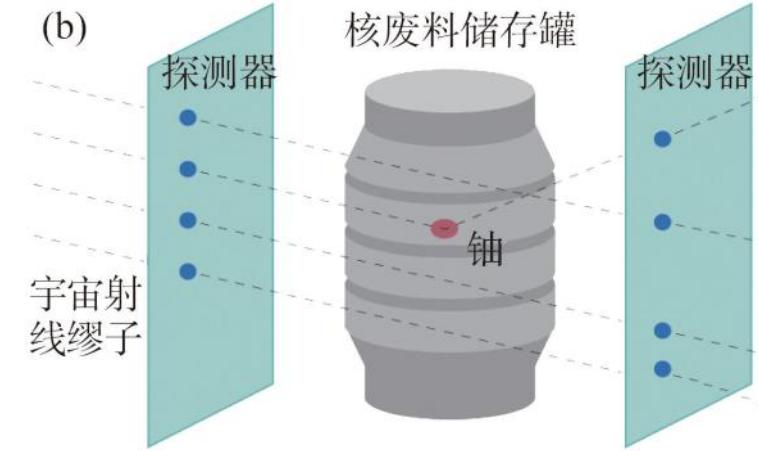
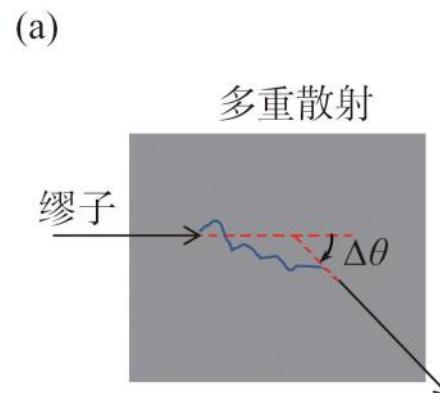
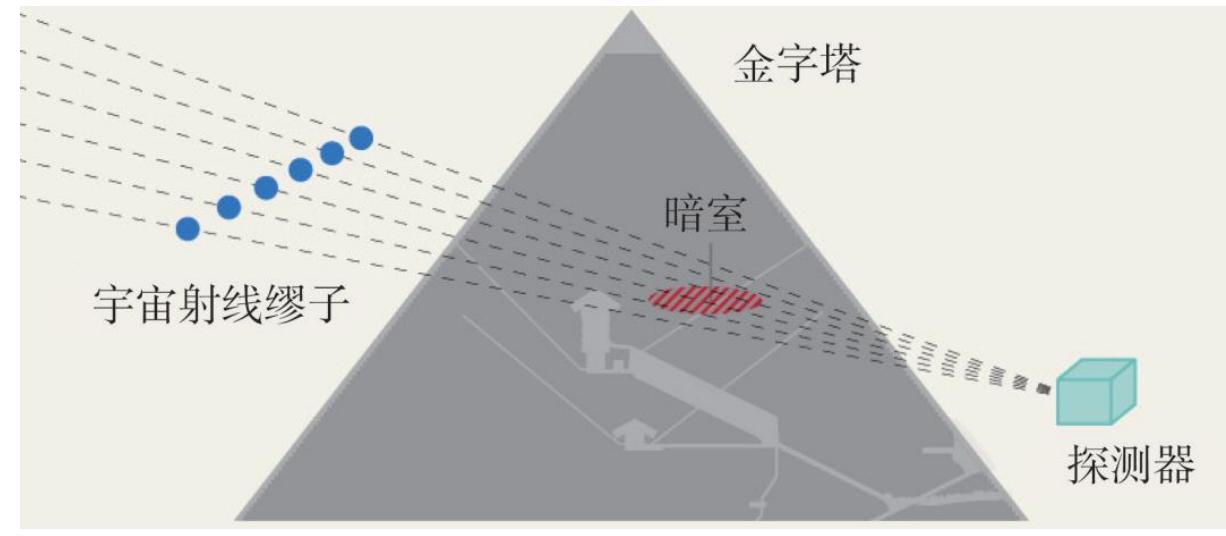


▶ 背景：缪子成像



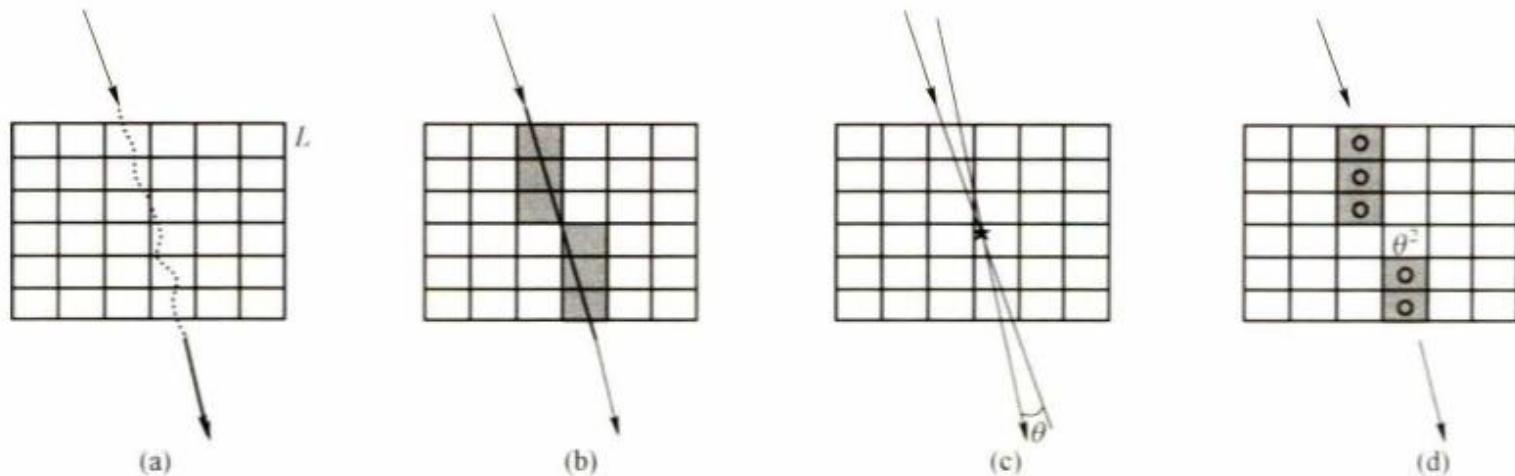
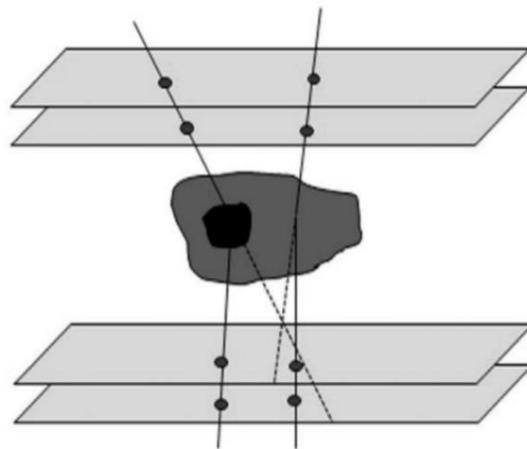
□ 平均能量： $3\text{GeV} \sim 4\text{GeV}$

□ 通量： $1\text{ muon}(\text{cm}^{-2} \cdot \text{minute}^{-1})$



□ 穿透性强，会发生多重散射

▶ 背景：多次库伦散射



□ 最近邻点算法 (PoCA) : 规定单个散射事件

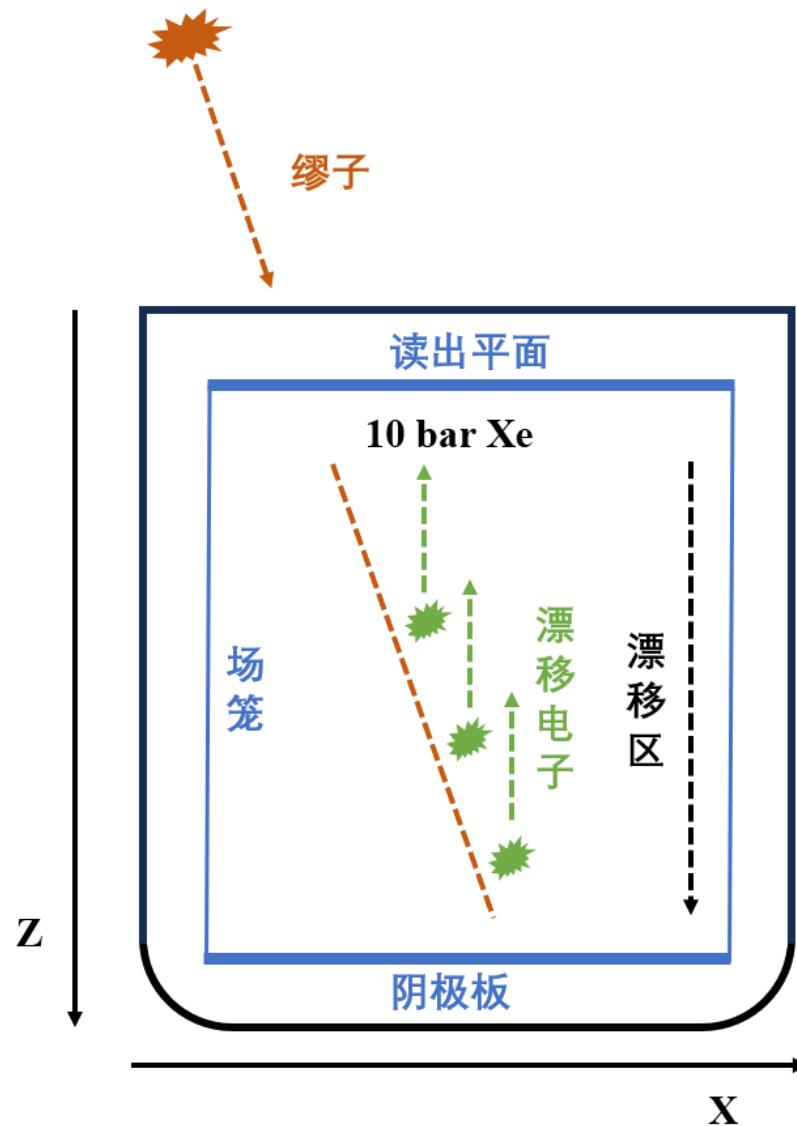
□ 将延长线的交点记为散射点

$$\sigma_\theta = \frac{13.6 \text{MeV}}{\beta cp} \sqrt{\frac{L}{L_0}} \left[1 + 0.038 \ln\left(\frac{L}{L_0}\right) \right]$$

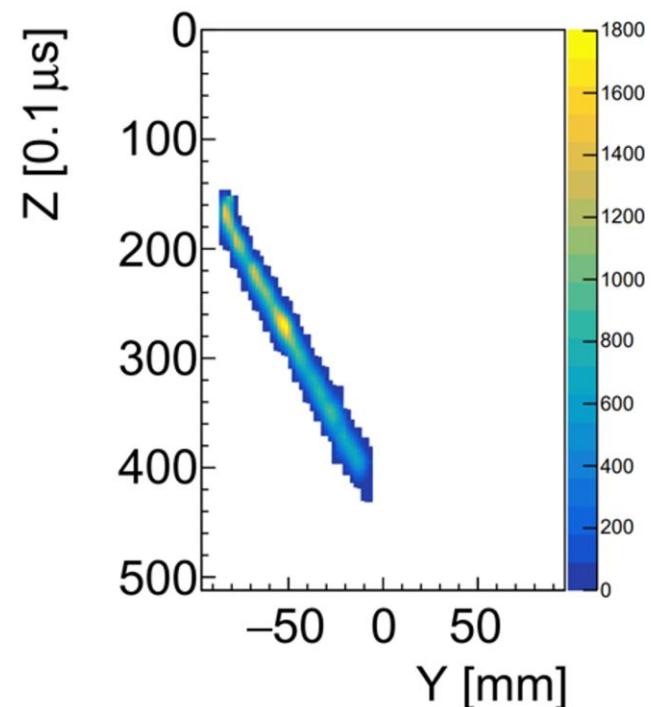
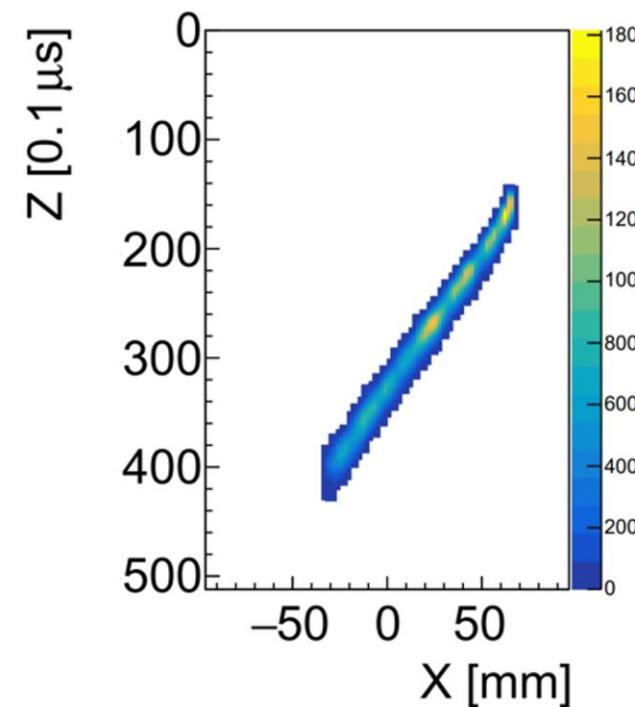
□ 莫里哀理论：散射角分布展宽与缪子能量、材料辐射长度关系

获得动量信息能够更好重建散射过程

气体时间投影室



- 同时记录带电粒子的能量沉积和运动轨迹
- 缪子径迹很直
- 高精度测量仍然能够评估弯曲程度



卡尔曼滤波算法



状态向量

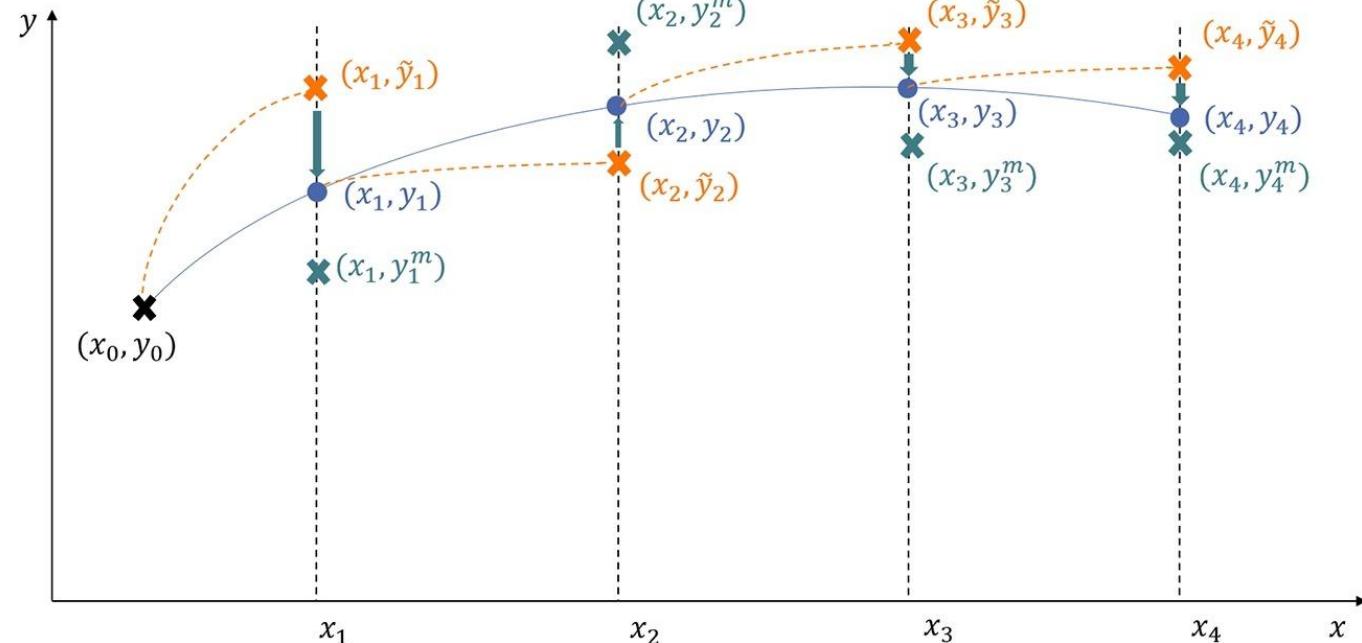
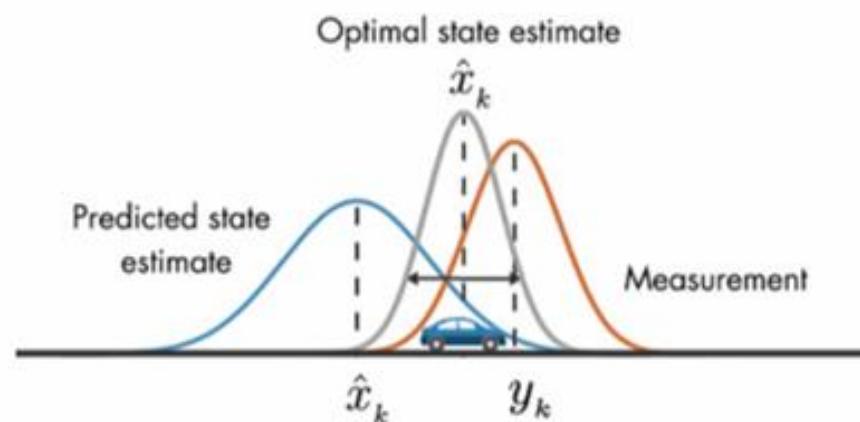
$$x_k = F_k x_{k-1} + \omega_k \quad (\text{系统动态变化})$$

观测向量

$$z_k = H_k x_k + v_k \quad (\text{系统观测})$$

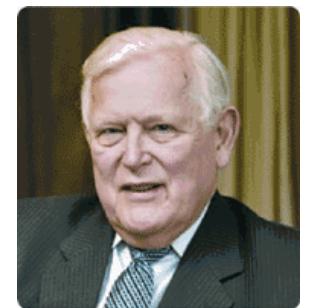
过程不确定性

测量噪声



- 输入：过程噪声和测量噪声
- 时间步迭代线性系统估计

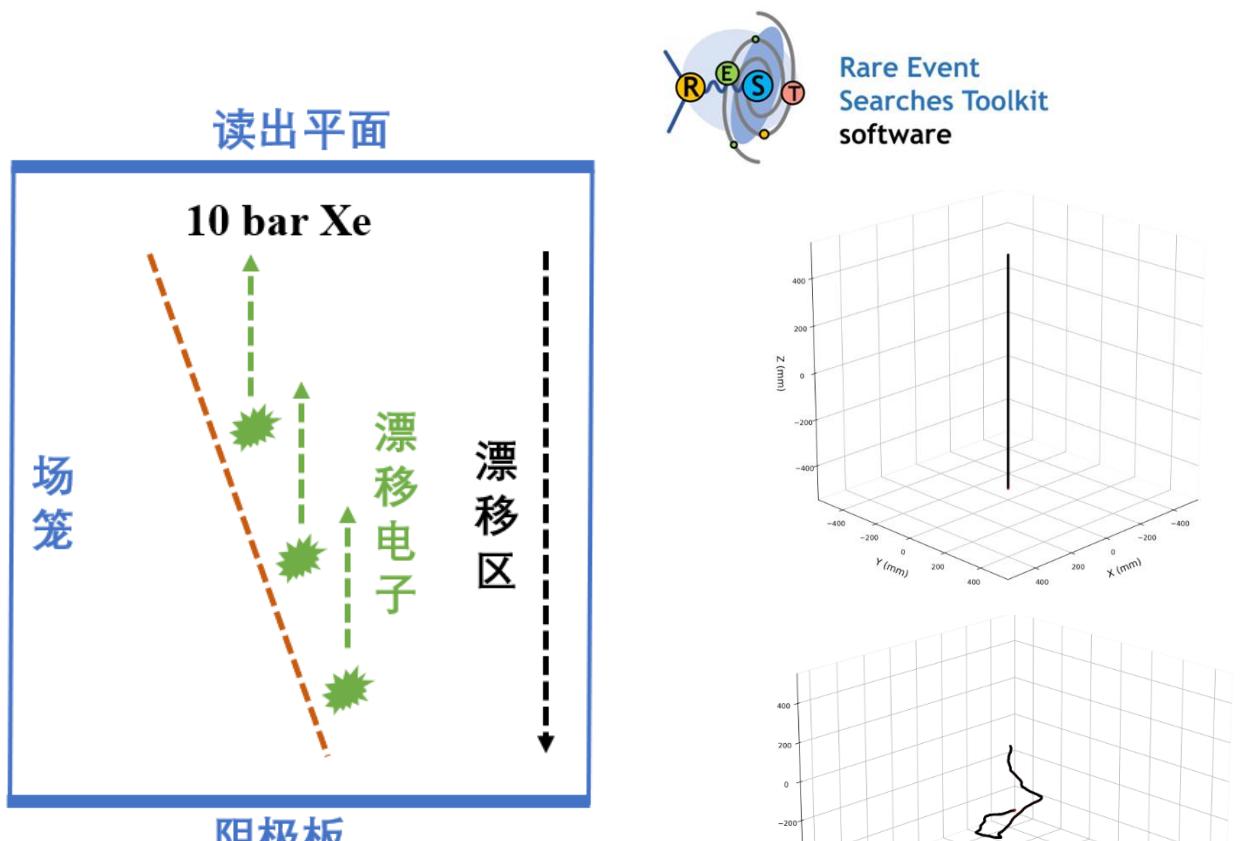
目标：在卡尔曼的基础上对过程噪声完成筛选



▶ 数据模拟

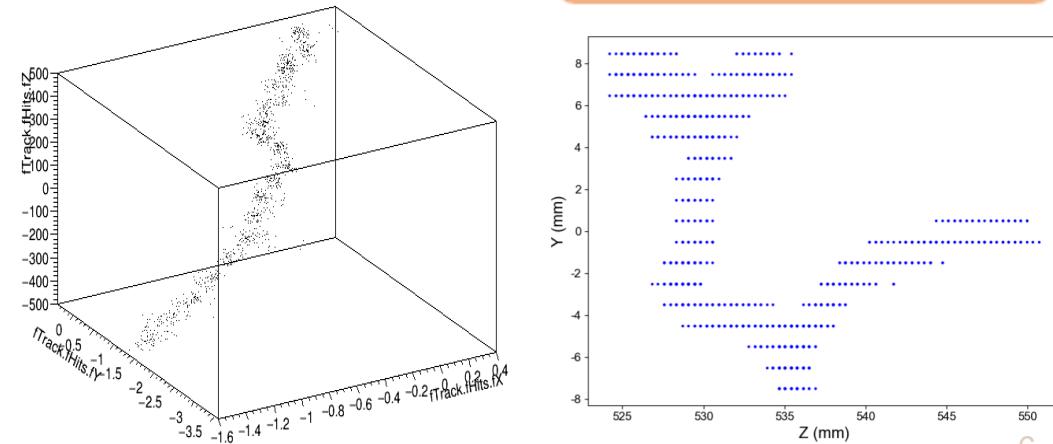


REST-for-Physics, a ROOT-based framework for event oriented data analysis and combined Monte Carlo response 

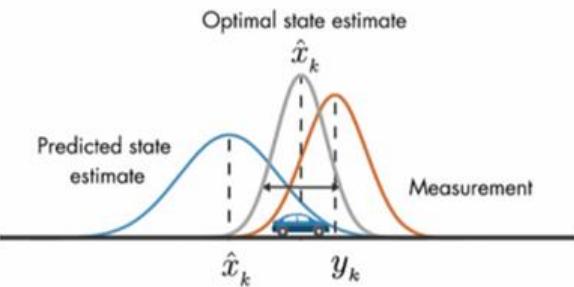


- REST: 稀有事件搜索工具包
- 基于ROOT和Geant4
- 电场: 1000V/cm
- 径迹笔直, 小尺度上有波动
- 信号展宽扩散
- 读出栅格化

PRELIMINARY



算法建模



$$x_k = F_k x_{k-1} + \omega_k$$

(系统动态变化)

状态向量

$$\begin{bmatrix} [x]_k \\ [y]_k \\ [z]_k \\ [u_x]_k \\ [u_y]_k \\ [u_z]_k \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 1 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 1 & 0 & 0 & \lambda \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} [x]_{k-1} \\ [y]_{k-1} \\ [z]_{k-1} \\ [u_x]_{k-1} \\ [u_y]_{k-1} \\ [u_z]_{k-1} \end{bmatrix} +$$

测量值

$$\begin{bmatrix} [x^m]_k \\ [y^m]_k \\ [z^m]_k \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} [x]_k \\ [y]_k \\ [z]_k \end{bmatrix} +$$

$$+ \begin{bmatrix} G(0, [\sigma_x]_k) \\ G(0, [\sigma_y]_k) \\ G(0, [\sigma_z]_k) \end{bmatrix}$$

$$\sigma_\theta = \frac{13.6 MeV}{\beta cp} \sqrt{\frac{L}{L_0} \left[1 + 0.038 \ln\left(\frac{L}{L_0}\right) \right]}$$

过程噪声

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ G(0, [\theta_x]_k) \\ G(0, [\theta_y]_k) \\ G(0, [\theta_z]_k) \end{bmatrix}$$

测量噪声

$$z_k = H_k x_k + \epsilon_k$$

(系统观测)

似然值筛选

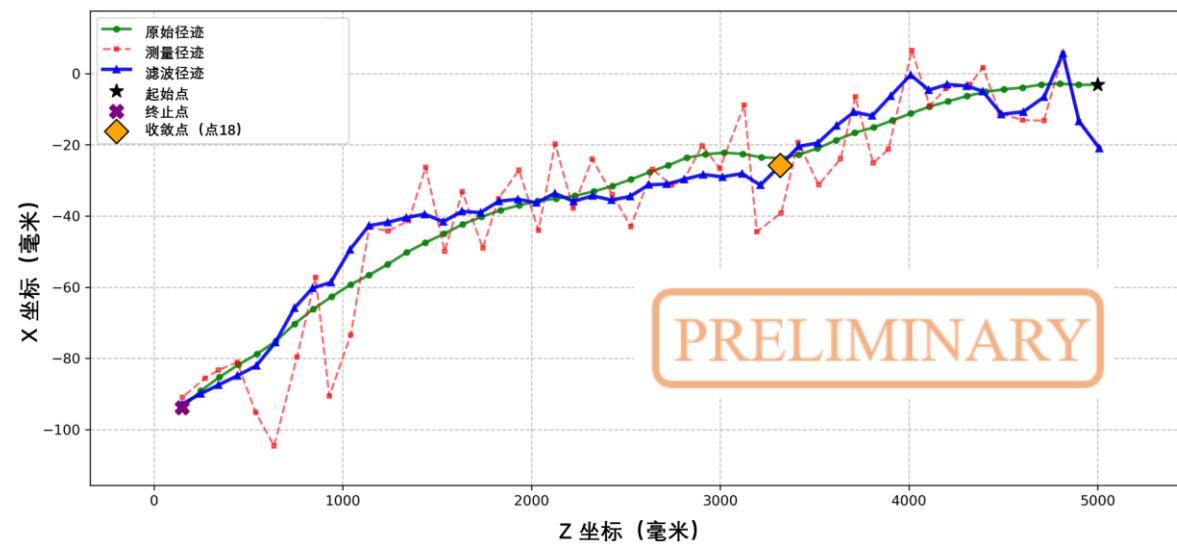
滤波过程



□ 状态的不确定性用协方差来表示

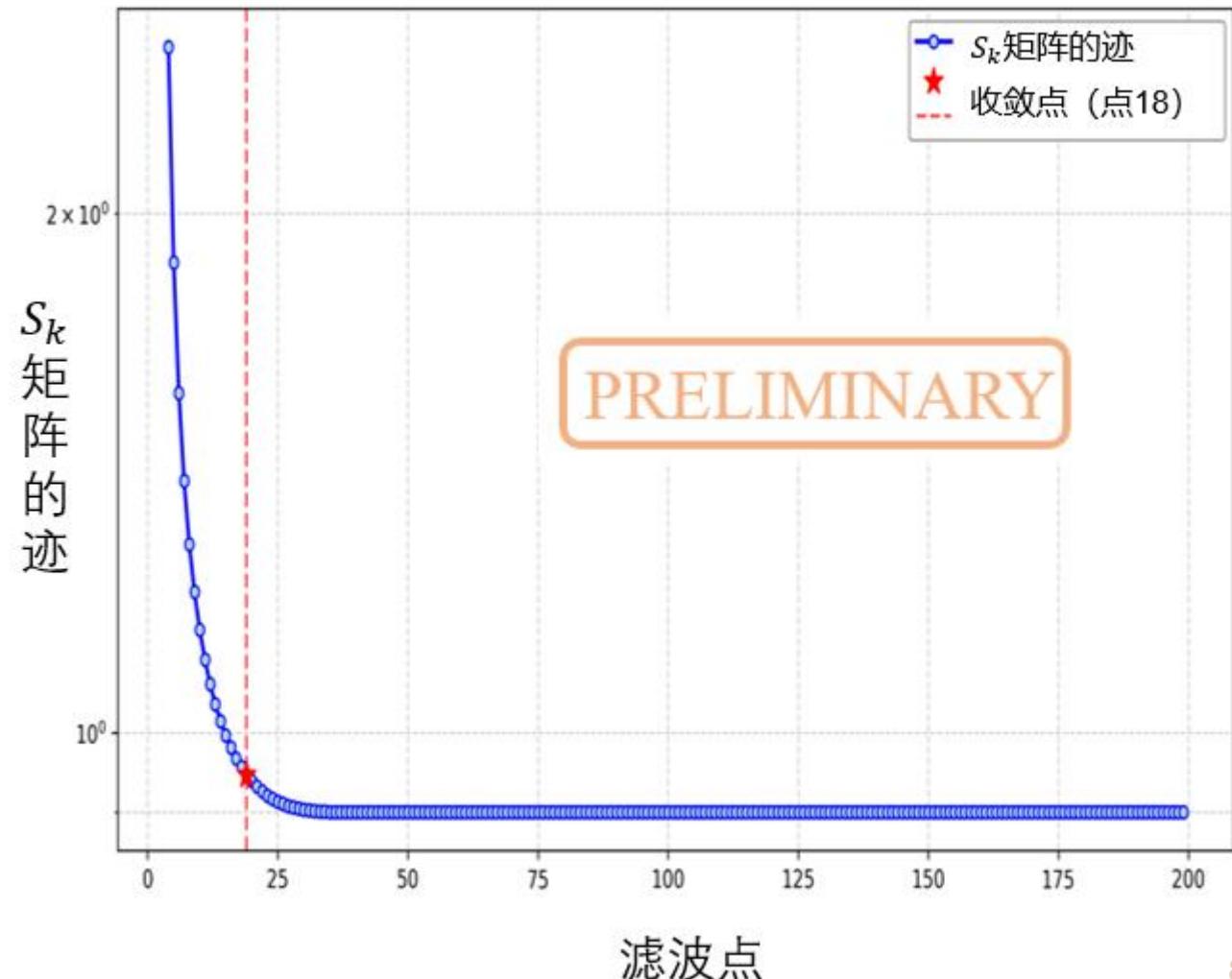
□ 残差 \tilde{y}_k 以及对应的协方差矩阵 S_k

$$\tilde{y}_k = z_k - H_k \hat{x}_{k|k-1}$$



□ 单条径迹滤波对比

□ 残差协方差矩阵的迹趋于稳定说明滤波收敛

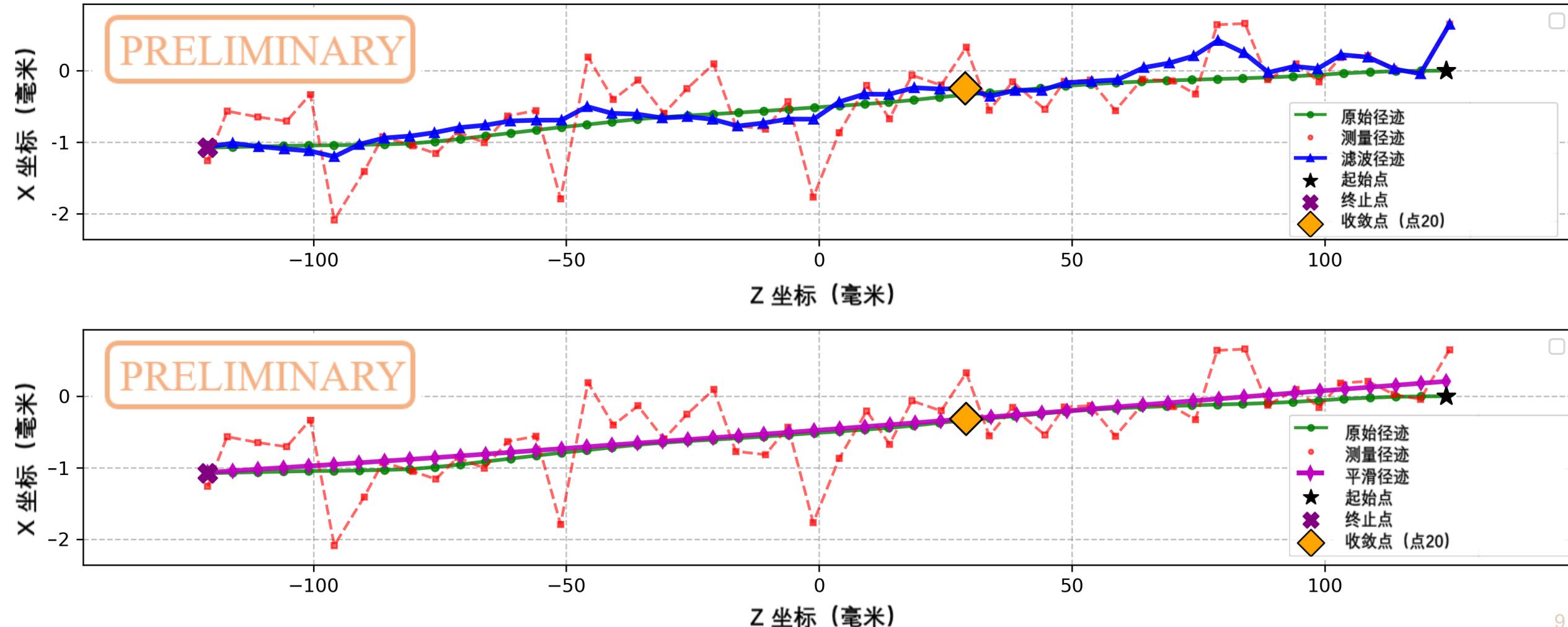


RTS平滑算法



$$(m_1, m_2, \dots, m_k) \Rightarrow \hat{x}_{k|k}$$

$$(m_1, m_2, \dots, m_n) \Rightarrow \hat{x}_{n|k}$$



似然值筛选



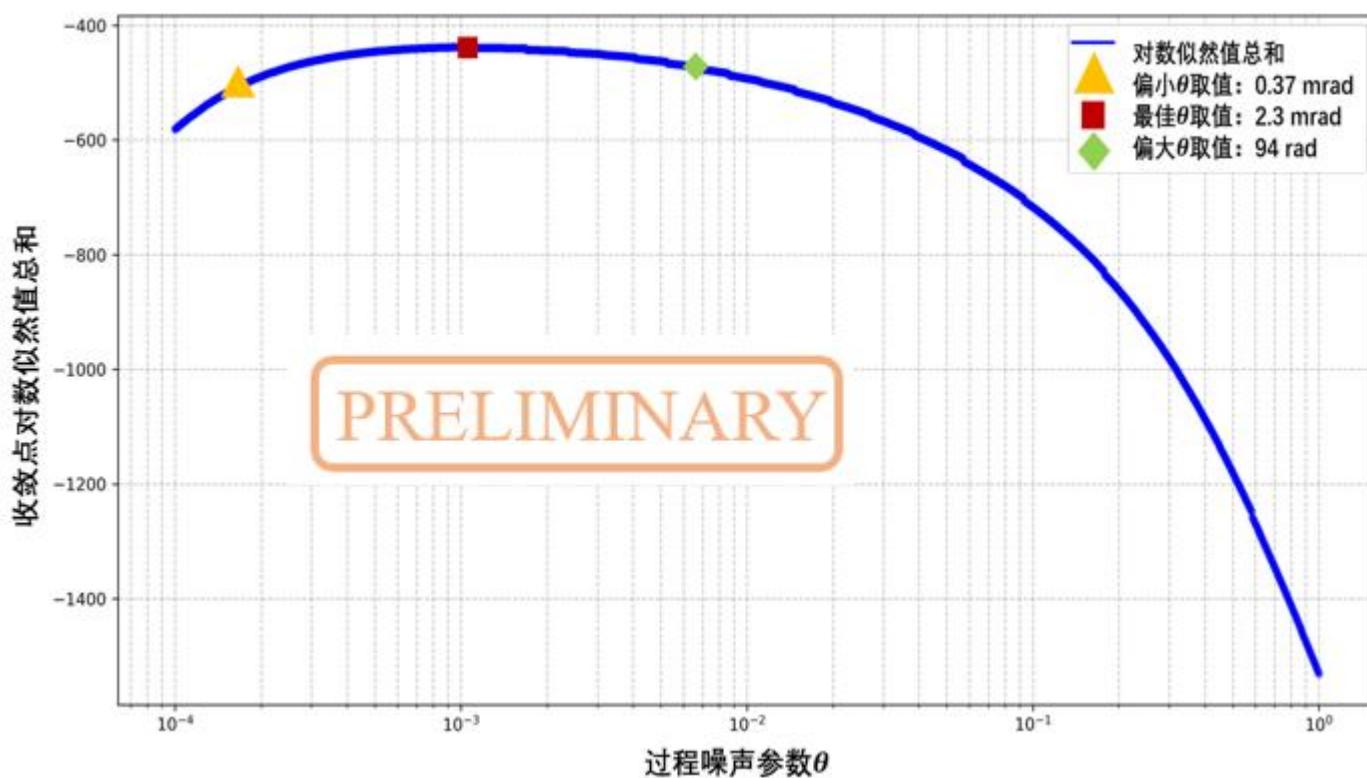
- 当前估计得到当前测量的概率
- 残差 \tilde{y}_k 以及对应的协方差矩阵 S

$$\tilde{y}_k = z_k - H_k \hat{x}_{k|k-1}$$

$$L_k = \frac{1}{\sqrt{(2\pi)^3 |S_k|}} \times e^{-0.5 \times (\tilde{y}_k^T S_k \tilde{y}_k)}$$

$$L = \sum_{k=m}^{k=n} \log(L_k)$$

- 单条径迹的似然值筛选案例



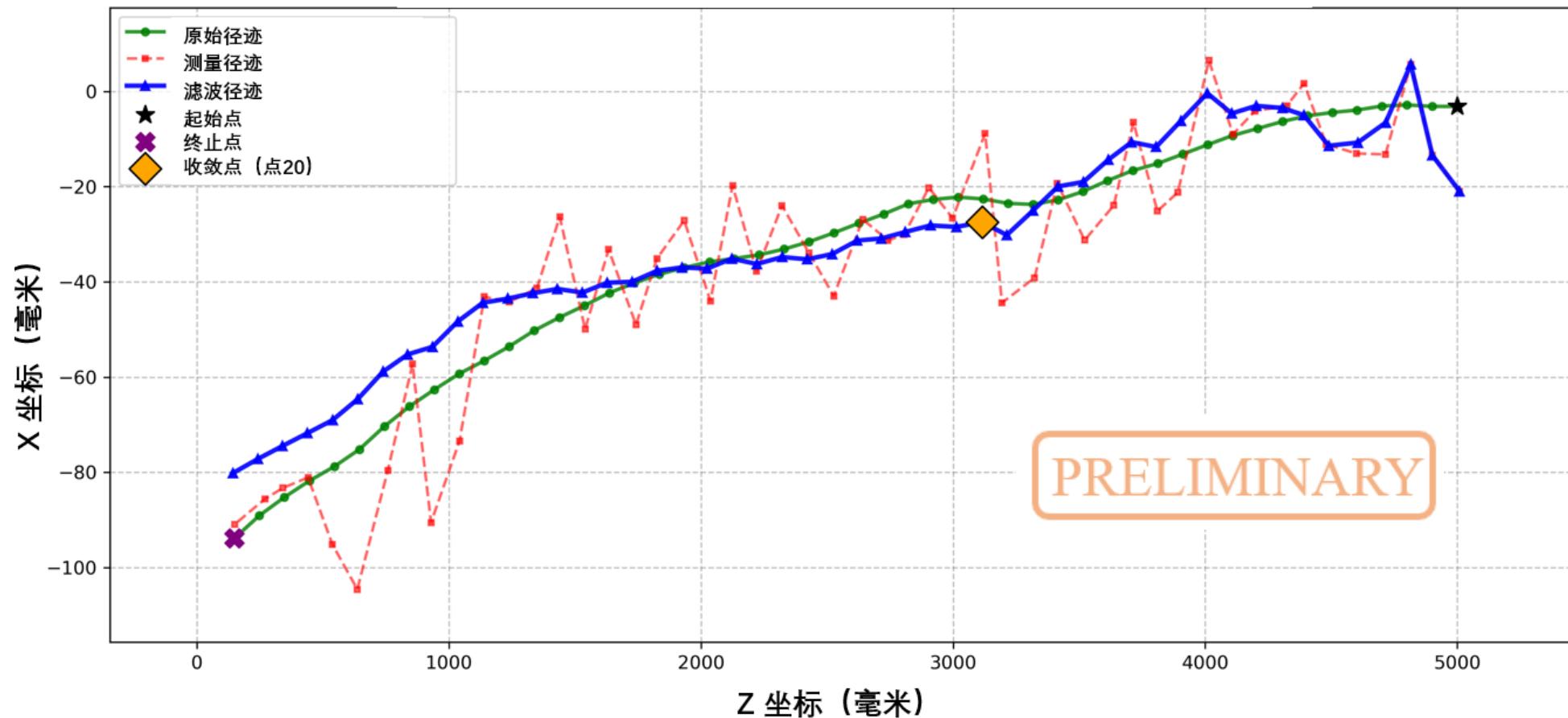
▶ 径迹对比示意图



□ 散射角参数偏小

□ 偏小 θ 取值：0.37mrad

□ 500MeV单条径迹示例



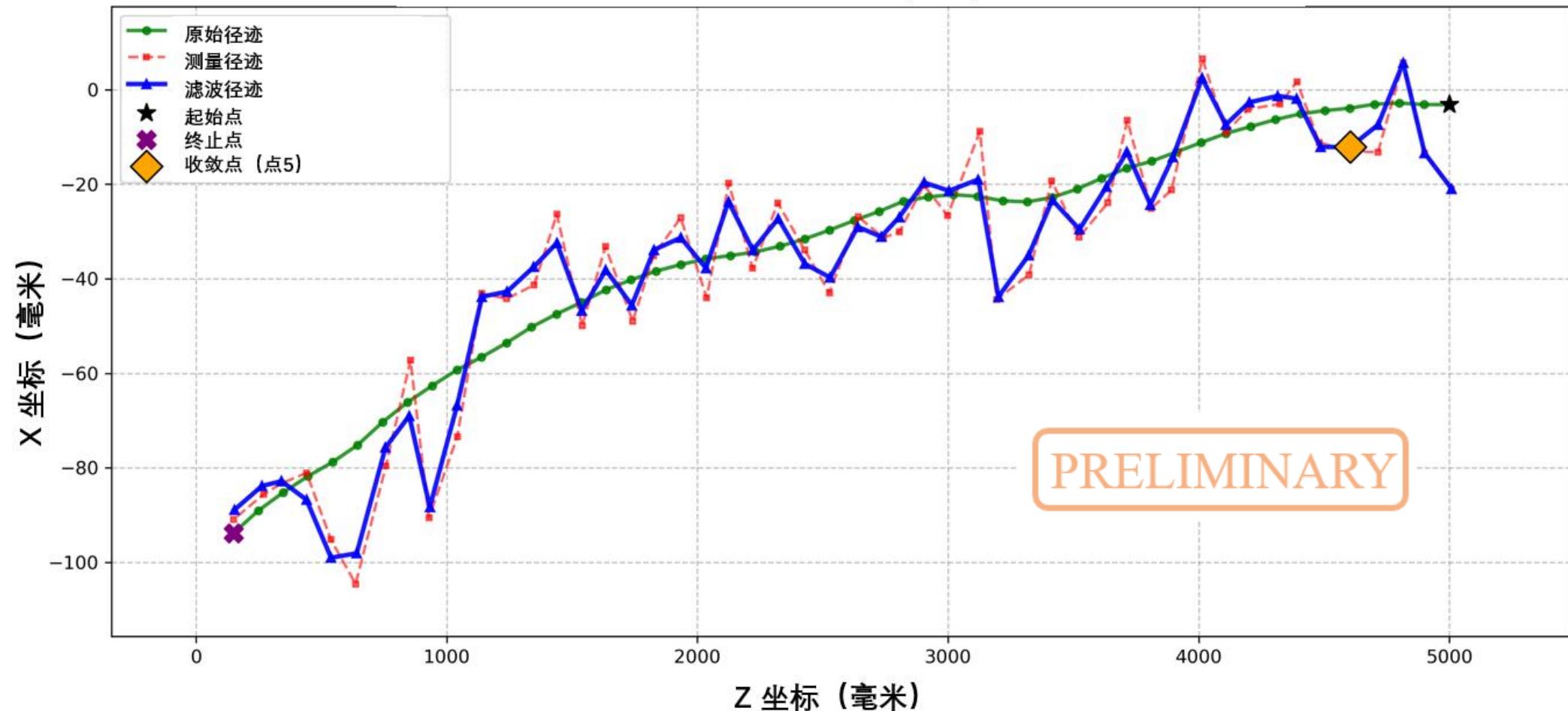
▶ 径迹对比示意图



□ 散射角参数偏大

□ 偏大 θ 取值: 94mrad

□ 500MeV单条径迹示例



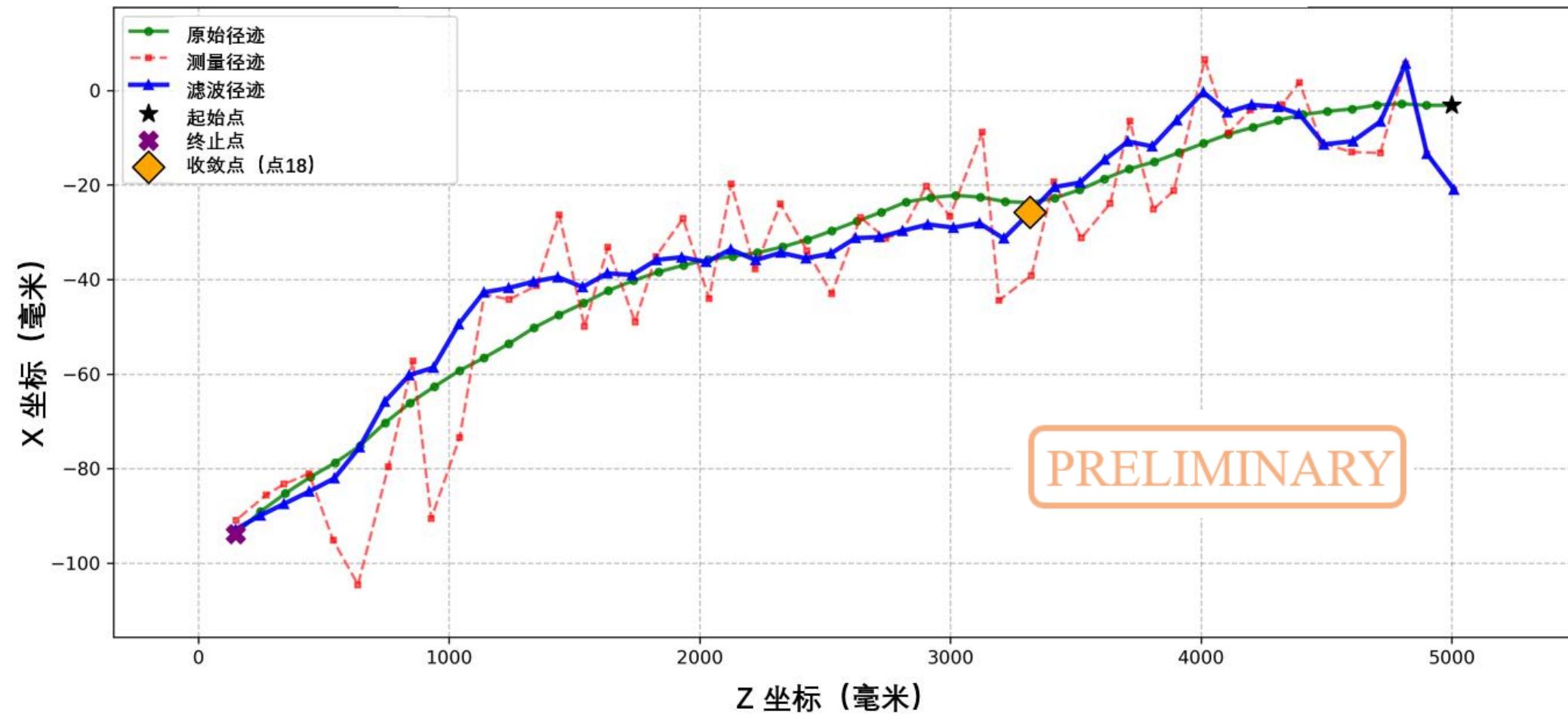
▶ 径迹对比示意图



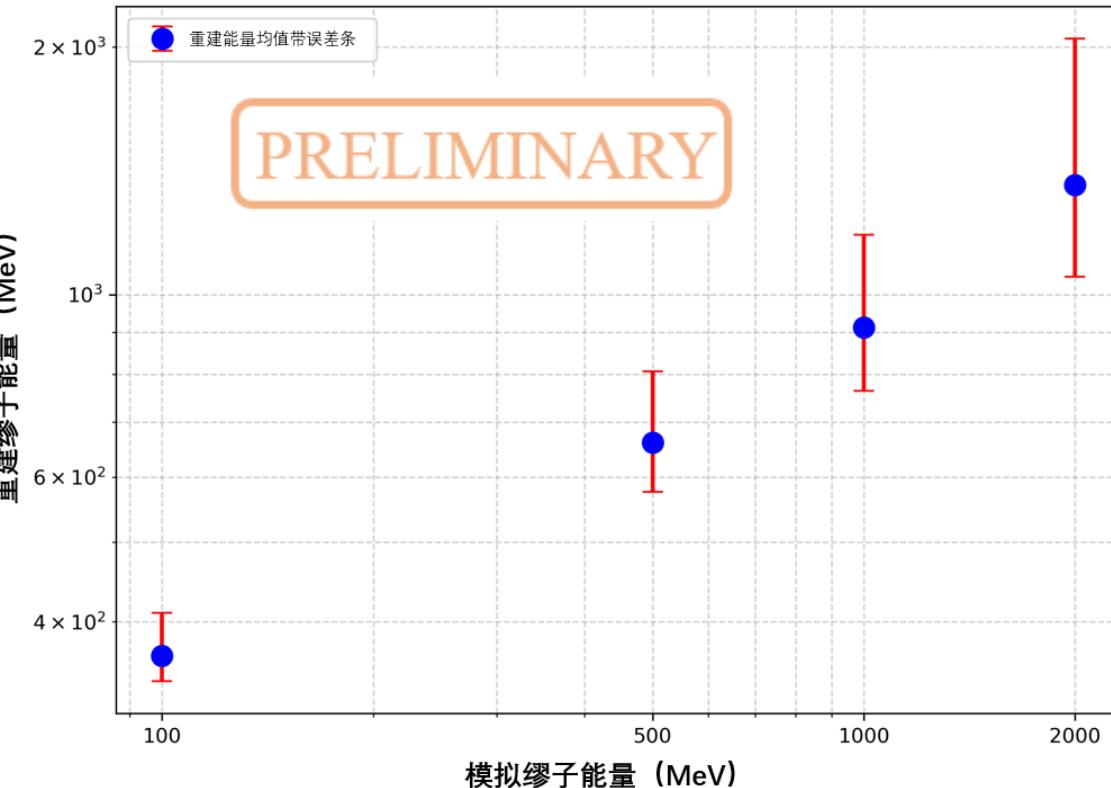
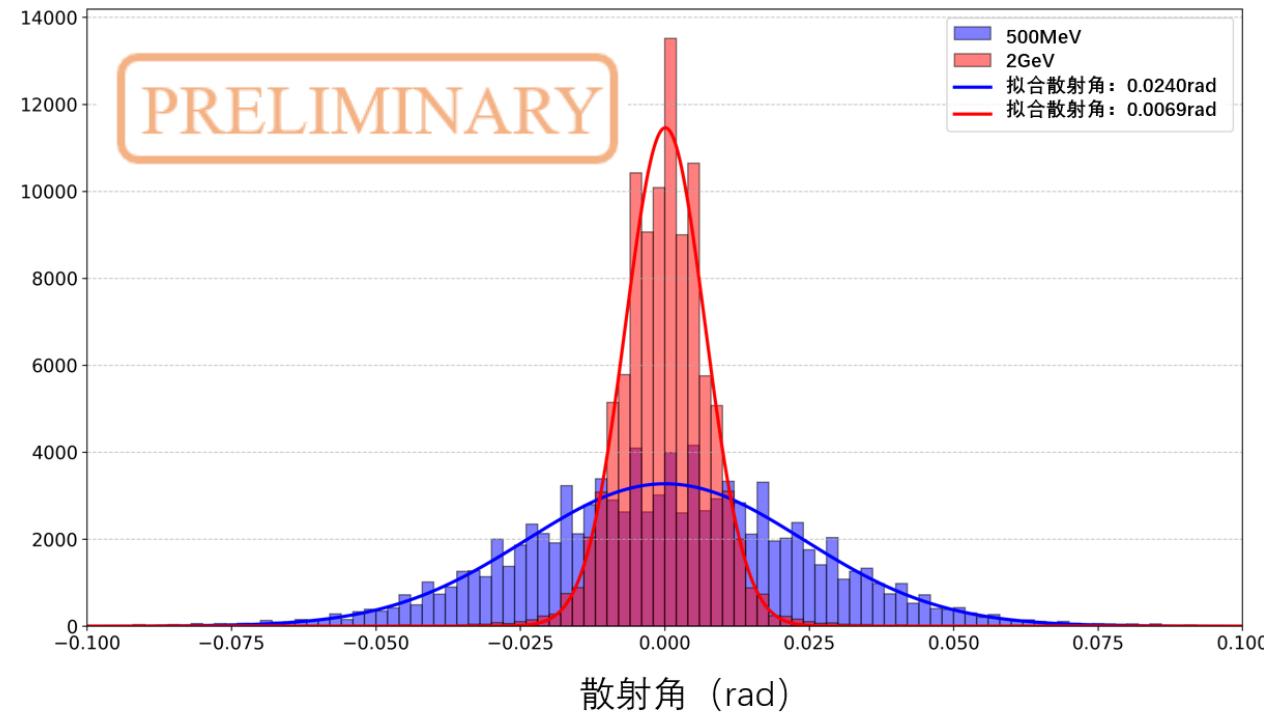
□ 散射角参数合适

□ 合适 θ 取值: 2.3 mrad

□ 500MeV单条径迹示例



特征散射角



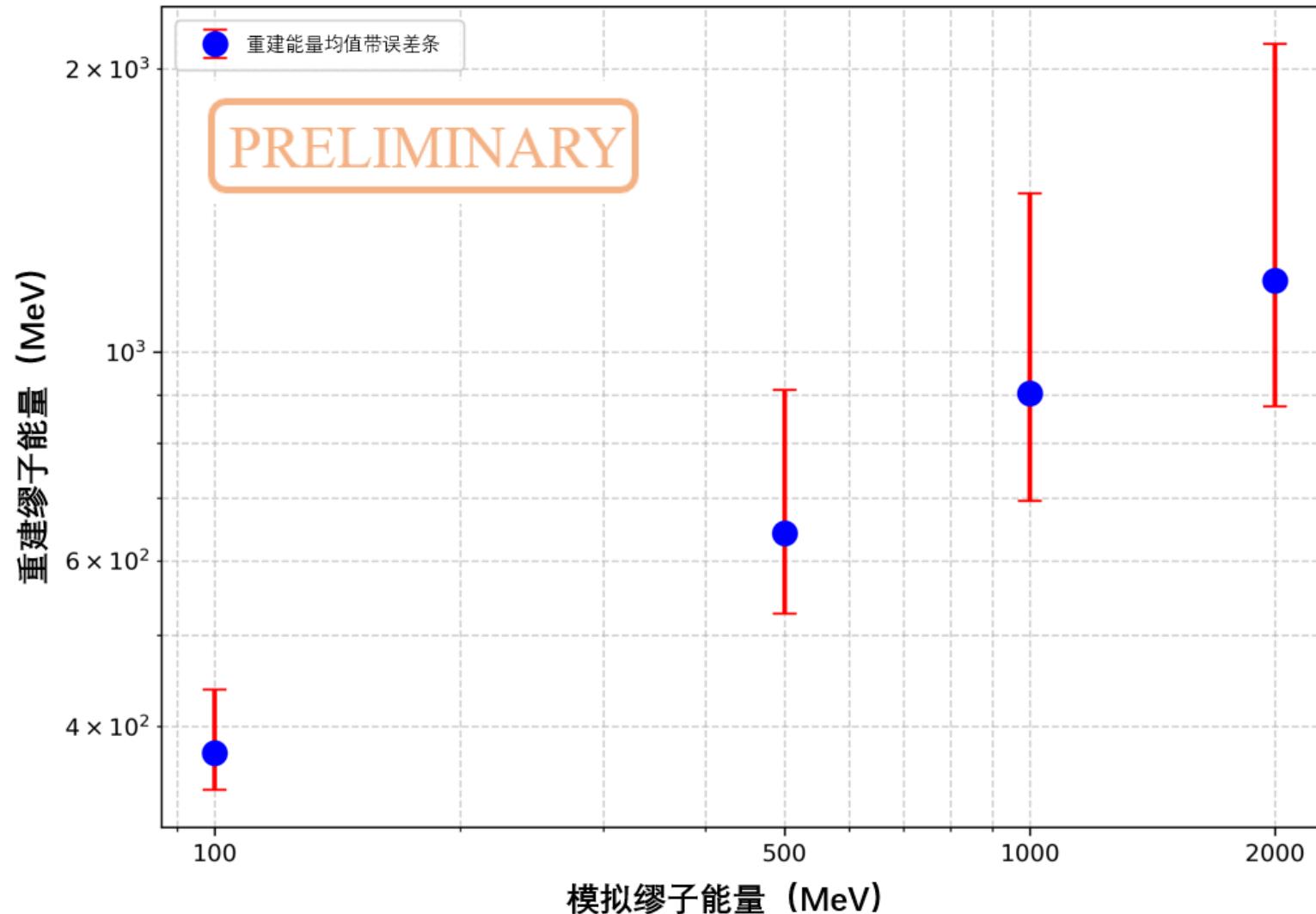
- 模拟数据的散射角统计
- 500MeV和2GeV差别为1个数量级

- 四种单一能量的重建统计结果

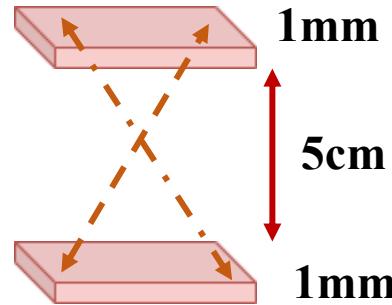
重建结果



□ 工作气体从Xe换成Ar □ 重建效果不变

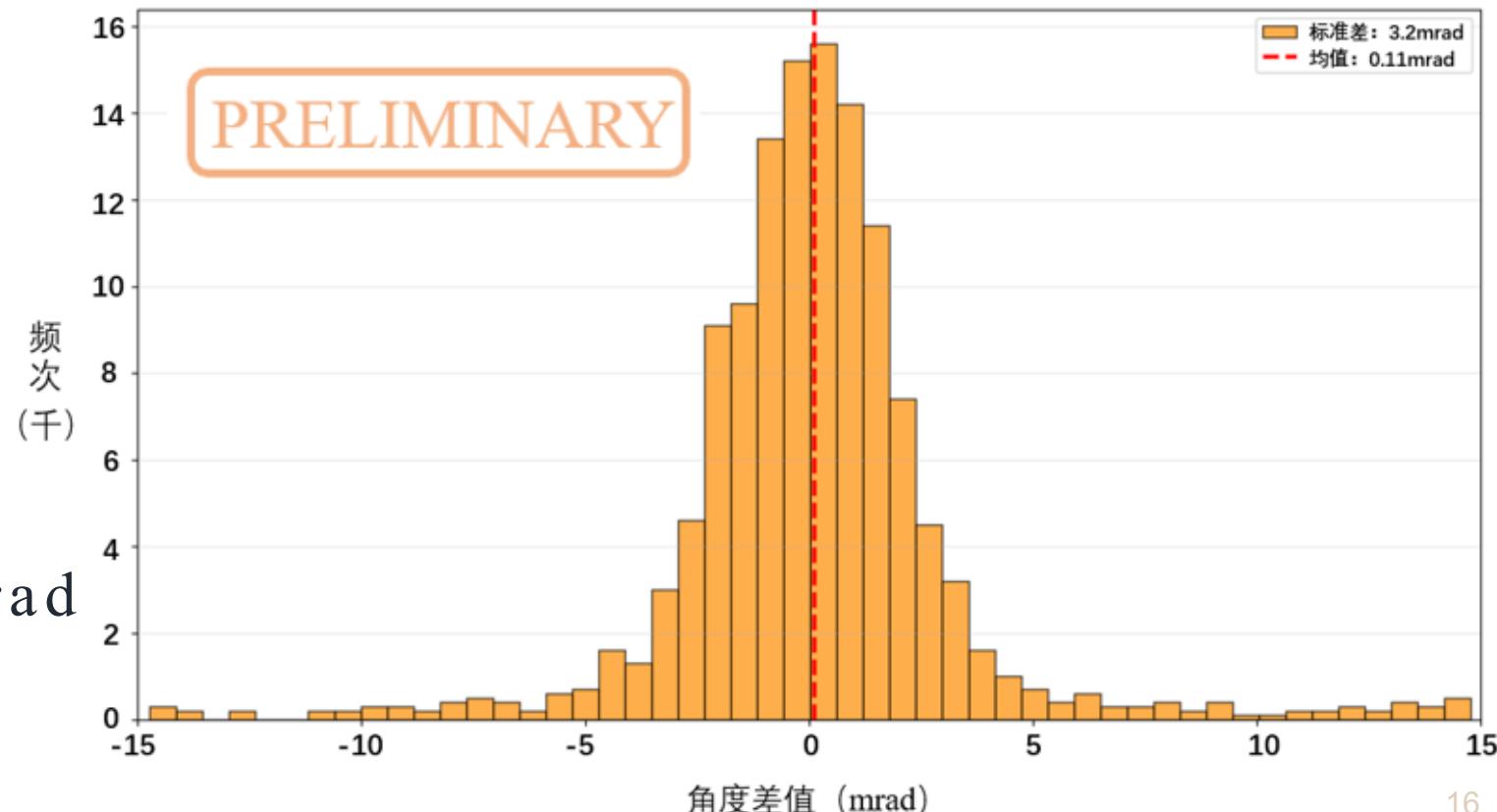


▶ 方向重建



- 2000MeV能量径迹
- 重建方向和原始方向的差值
- 样本量：100,000条
- 出射方向差值标准差：3.2mrad

- 上下距离5cm的 $1\text{ mm} \times 1\text{ mm}$ 像素块
- 方向分辨误差：56mrad





- 气体时间投影室（TPC）能够有效记录缪子径迹和能量沉积
 - 利用径迹信息能用对缪子的动量进行估算
 - 此方法对于缪子射线方向上的优化也能够提升成像精度
-
- 下一步将探测器的长度纳入研究范围，进行优化
 - 未来计划在实验数据上进行验证和应用