Study of $D^0/\overline{D}^0 \to K_S^0/K_L^0\pi^0\pi^+\pi^-$ at BESIII and Simulation and Reconstruction of STCF DTOF

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Thesis Proposal Talk
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Outline

- Works on BESIII
  - Introduction
  - Study of $D^0/\bar{D}^0 \to K_S^0\pi^0\pi^+\pi^-$
  - Study of $D^0/\bar{D}^0 \to K_L^0\pi^0\pi^+\pi^-$
- Works on STCF
  - Introduction
  - Simulation and Reconstruction of STCF DTOF
- Summary and Prospect

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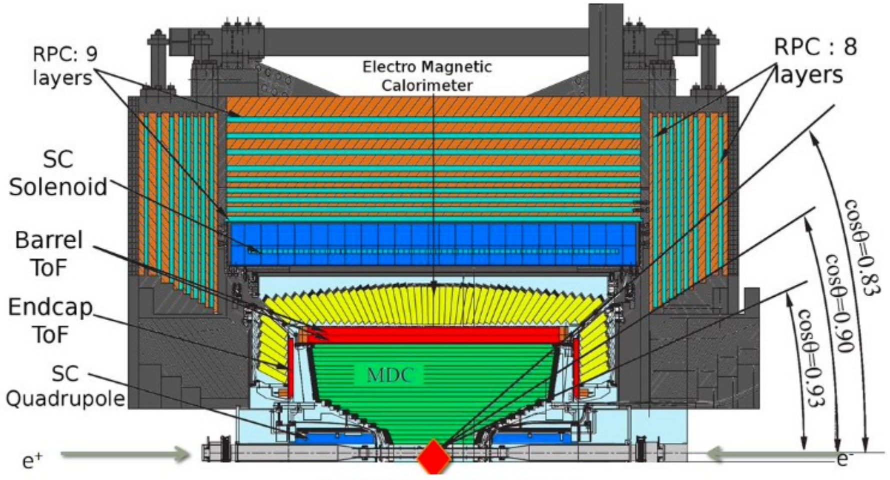
BEPCII and BESIII

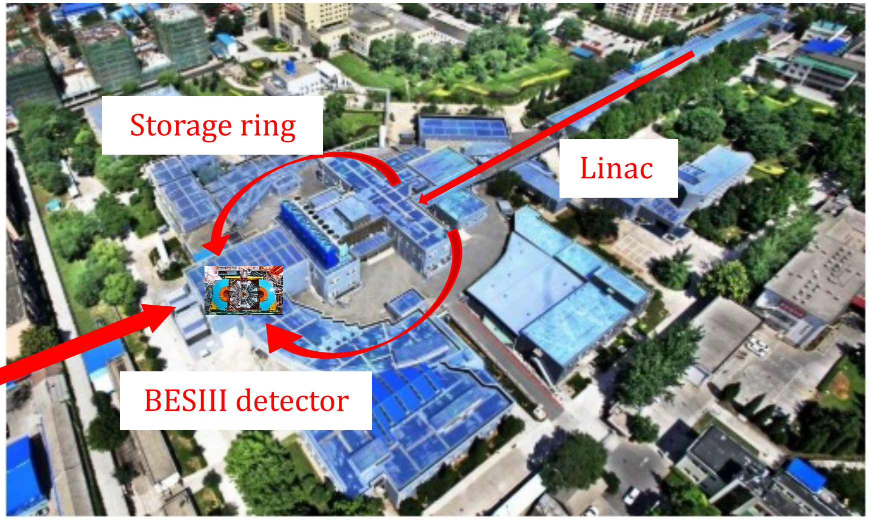
Electromagnetic Calorimeter
CsI(Tl): L = 28 cm
Barrel $\sigma_E = 2.5\%$
Endcap $\sigma_E = 5.0\%$

Muon Counter RPC
Barrel: 9 layers
Endcaps: 8 layers
$\sigma_{spatial}$=1.48 cm



Main Drift Chamber
Small cell, 43 layer
σ_{xy}=130 μm
$dE/dx \sim 6\%$
$\sigma_p/p = 0.5\%$ at 1GeV

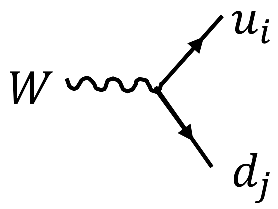
Time Of Flight
Plastic scintillator
σ_T(barrel) = 68 ps
σ_T(endcap) = 110 ps
(update to 60 ps with MRPC)



$E_{cm} = 1.84 - 4.95$ GeV
Peak luminosity @$E_{cm} = 3.773$ GeV: 1.1×10^{33} cm^{-2}s^{-1}
Circumference: 237.53 m
Crossing angle: 2×11 mrad

Motivation

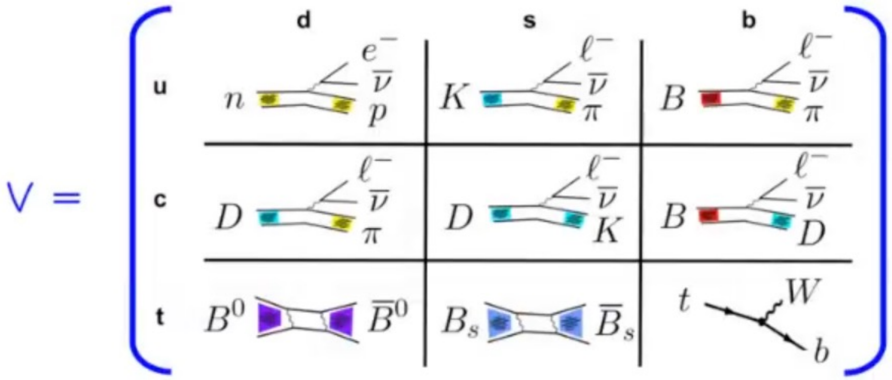
- $\mathcal{L} \propto -\frac{g}{\sqrt{2}}(\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^\mu W_\mu^+ V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$

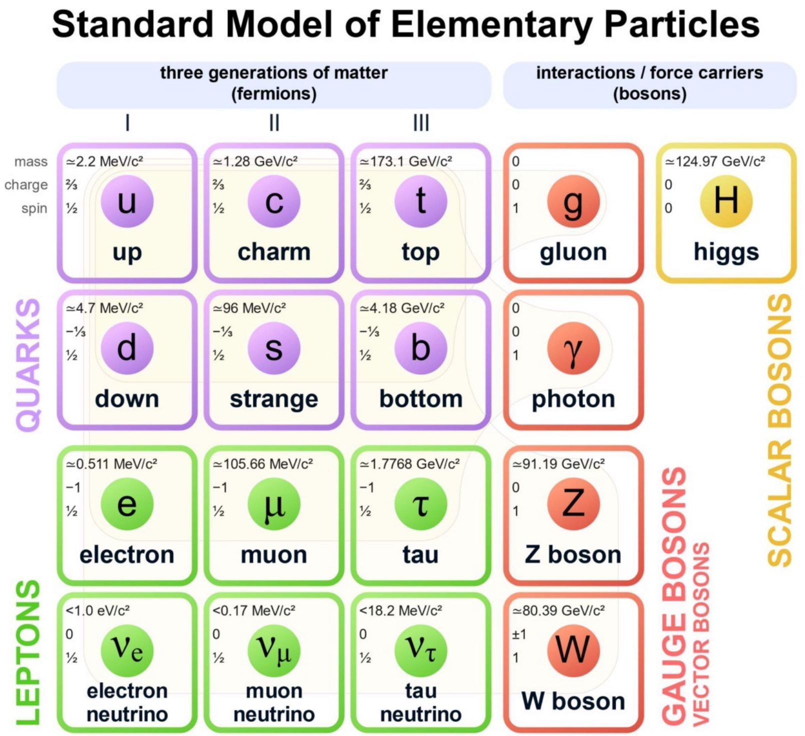


$$V_{CKM} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

3×3 unitary complex matrix

Mass eigenstates are not weak interaction eigenstates.





Motivation

- Different parameterization of CKM matrix

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

  - ✓ 3 mixing angles: $s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij}$
  - ✓ 1 CPV KM phase: δ is the phase for all CPV phenomena in flavor-changing processes in SM.

- Wolfenstein parameterization

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2+|V_{us}|^2}}, \qquad s_{23} = A\lambda^2 = \lambda\left|\frac{V_{cb}}{V_{us}}\right|,$$

$$s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar{\rho}+i\bar{\eta})\sqrt{1-A^2\lambda^4}}{\sqrt{a-\lambda^2}[1-A^2\lambda^4(\bar{\rho}+i\bar{\eta})]}$$

Motivation

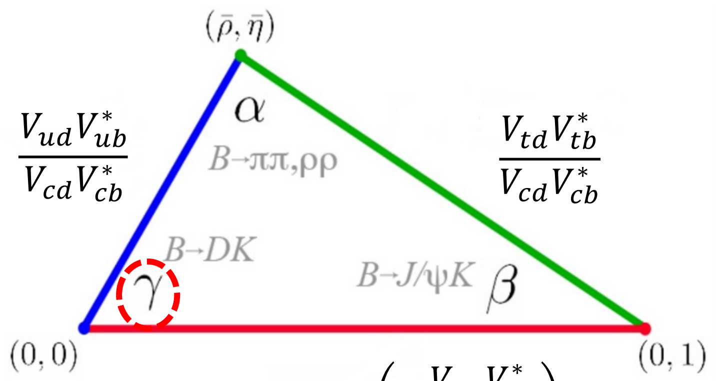
- $V_{CKM}^{\dagger} V_{CKM} = V_{CKM} V_{CKM}^{\dagger} = 1 \Rightarrow \Sigma_i V_{ij} V_{ik}^* = \delta_{jk}, \quad \Sigma_j V_{ij} V_{kj}^* = \delta_{ik}$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \Rightarrow \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} + 1 = 0$$

$$\alpha \equiv \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta \equiv \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma \equiv \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$



$$\gamma \equiv \varphi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

✓ Involving $b \to c$ and $b \to u$:

γ can be measured by $B \to D$ + hardron

Motivation

➤ Measurement of γ/φ_3

$$A(B^- \to (f)_D K^-) = A_B\left(A_f + r_B e^{i(\delta_B-\gamma)}\bar{A}_f\right)$$
$$= A_B\left(a_f e^{i\delta_D} + r_B e^{i(\phi_B-\gamma)}\bar{a}_f e^{i\bar{\delta}_D}\right)$$

r_B: ratio of two amplitude,

δ_B: strong phase difference.

$$\frac{A(B^-\to\bar{D}^0K^-)}{A(B^-\to D^0K^-)} = r_B e^{i(\delta_B-\gamma)}$$

- Decay rate:

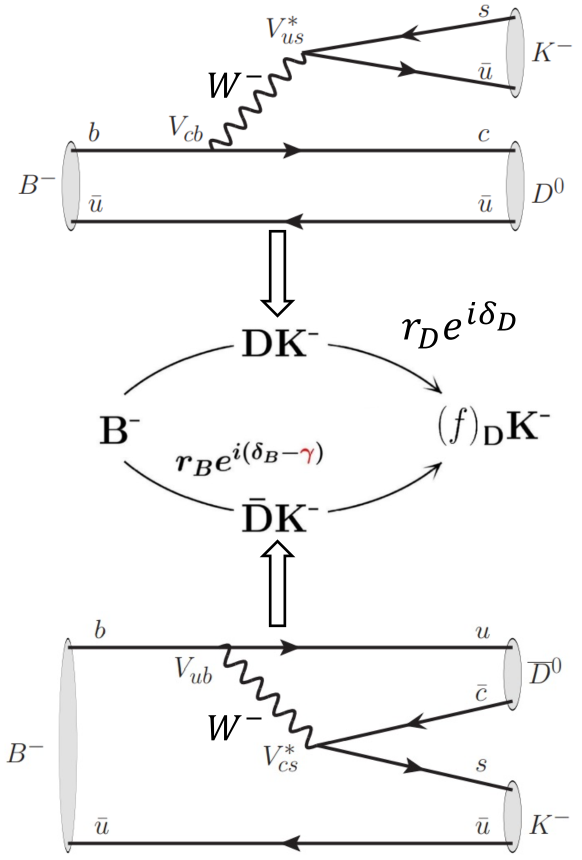
$$\Gamma(B^- \to [f]_D K^-) = |A_B|^2\left[1 + r_B^2 r_D^2 + 2R_f r_D r_B \cos(\delta_B - \gamma - \Delta\delta_D)\right]$$

Inputs from $D^0 \to f$ and $\bar{D}^0 \to f$ decays are needed

$$\left(r_D^f\right)^2 = \frac{\int |\bar{A}_f|^2 \mathrm{d}\Phi}{\int |A_f|^2 \mathrm{d}\Phi} \quad R_f e^{-i\delta_D^f} = \frac{\int A_f^* \bar{A}_f \mathrm{d}\Phi}{\sqrt{\int |A_f|^2 \mathrm{d}\Phi \int |\bar{A}_f|^2 \mathrm{d}\Phi}} \quad \underline{c_i(s_i)} = \frac{\int |A_f||\bar{A}_f| \cos\Delta\delta_D^f\, \underline{(\sin\Delta\delta_D^f)} \mathrm{d}\Phi_i}{\sqrt{\int |A_f|^2 \mathrm{d}\Phi_i \int |\bar{A}_f|^2 \mathrm{d}\Phi_i}}$$

- Different D decay models:
  - ✓ ADS: CF and DCS decays (e.g. $K\pi, K\pi\pi^0$) $\leftarrow R_f, \delta_D^f$
  - ✓ GLW: (Quasi-)CP eigenstates (e.g. $KK, \pi^+\pi^-\pi^0$) $\leftarrow F_+$
  - ✓ GGSZ: Multi-body Self-conjugate decay (e.g. $K_S^0\pi^+\pi^-\pi^0$) $\leftarrow c_i, s_i$

$V_{cb}V_{us}^* \sim A\lambda^3$



$V_{ub}V_{cs}^* \sim A\lambda^3(\rho - i\eta)$

$D^0\bar{D}^0$ Data Sample

- Quantum correlated (QC) $D^0\bar{D}^0$ decay at $\psi(3770)$

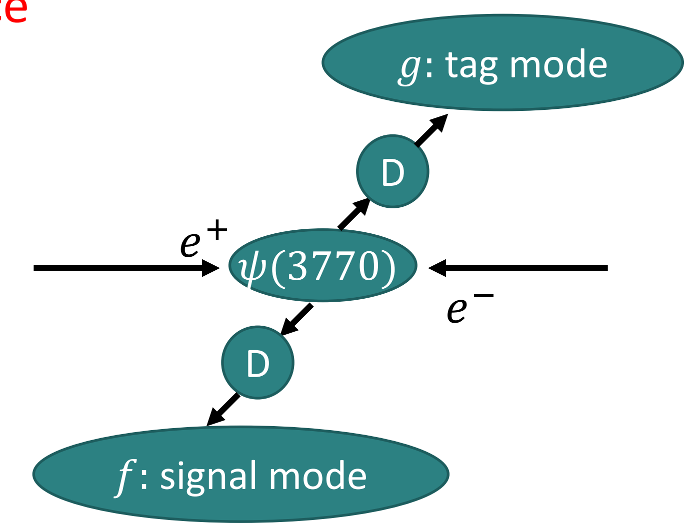
$$e^+e^- \to \psi(3770) \to D^0\bar{D}^0 \to fg \xrightarrow{C_{\psi(3770)}=-1} |\psi(3770)\rangle \to \frac{1}{\sqrt{2}}(|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle)$$

No CPV $\downarrow$

$$\boxed{\Gamma(f|g)} \propto \left(1+\frac{y^2-x^2}{2}\right)\left[\left(r_D^f\right)^2+\left(r_D^g\right)^2-2r_D^f r_D^g R_f \boxed{R_g}\cos\left(\boxed{\Delta\delta_D^f}-\Delta\delta_D^g\right)\right]+\frac{y^2+x^2}{2}\left[1+\left(r_D^f r_D^g\right)^2-2r_D^f r_D^g R_f R_g\cos\left(\Delta\delta_D^f+\Delta\delta_D^g\right)\right]$$

$\propto$ Number of events coherence factor strong phase difference

- QC $D^0\bar{D}^0$ produced at BESIII
  - ✓ $20.3\ \mathrm{fb}^{-1}$ @ $\sqrt{s} = 3.773$ GeV ~73.4 M $D^0\bar{D}^0$ pairs
- Double Tag Method
  - ✓ Single Tag (ST): reconstruct one $D/\bar{D}$
  - ✓ Double Tag(DT): reconstruct both $D\bar{D}$



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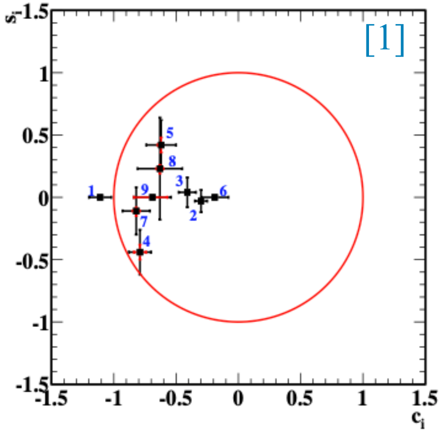
Motivation

- Multi-body self-conjugate *D* decay with large branching fraction
  - Provide strong phase parameters as input of GGSZ method.
  - Only Rough Results by CLEO[1] (Bin the PHSP with resonance mass region.) $\sim 25\times$ CLEO
- Many potential processes $D \to VV, VP$ and AP.
  - Study for the features in the polarization of $D^0 \to K^*(892)\rho(770)$.
  - Lack of accurate experimental measurements.

TABLE VIII. Results for $D^0 \to \bar{K}^0\pi^+\pi^-\pi^0$.

Amplitude	Fraction (%)	Phase	Branching fraction (%)
Four-body nonresonant	21.0±14.7±15.0	−0.45±0.55	2.2±1.6±1.7
$K^{*-}\rho^+$ longitudinal	19.3±7.4	0	
$K^{*-}\rho^+$ transverse	21.1±12.0	2.0±0.48	
$K^{*-}\rho^+$ total	40.4±12.5±8.4		6.2±2.3±2.0
$\bar{K}^0\omega$	19.5±4.3±1.4		2.3±0.7±0.6
$\bar{K}^{*0}\rho^0$ transverse	4.2±3.7		1.3±1.2
$K_1(1270)^-\pi^+$	4.8±1.5		1.0±0.4
$\bar{K}^{*0}\pi^+\pi^-$	12.7±7.0		3.9±2.3

Reference: D. Coffman *et al.* (Mark III Collaboration), Phys. Rev. D **45**, 2196 (1992)



Double Tag Method

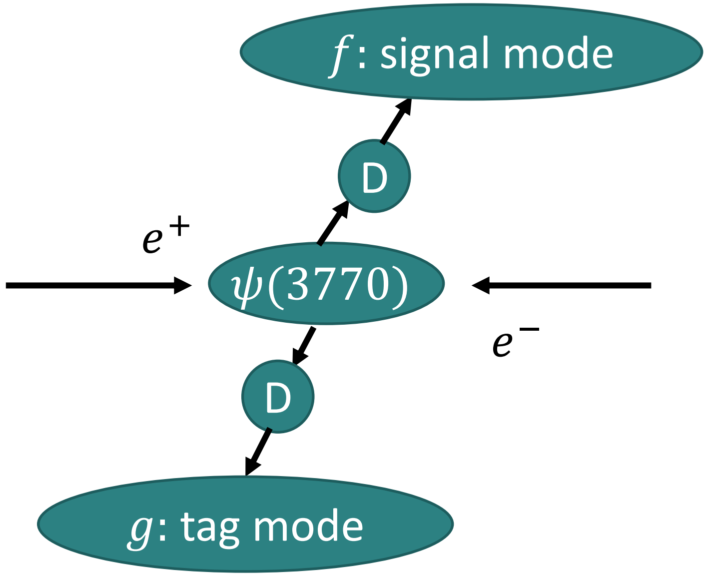
- Tag Mode

Type	Tag mode
(quasi-)flavor	$\boxed{K^+\pi^-, K^+\pi^-\pi^0, K^+\pi^-\pi^+\pi^-}$, $K^+e^-\bar{\nu}_e$
CP-even	K^+K^-, $\pi^+\pi^-$, $K_S^0\pi^0\pi^0$, $K_L^0\pi^0$
CP-odd	$K_S^0\pi^0$, $K_S^0\eta_{\gamma\gamma}$, $K_S^0\eta'_{\gamma\pi^+\pi^-}$, $K_S^0\eta'_{\pi^+\pi^-\eta_{\gamma\gamma}}$, $K_L^0\pi^0\pi^0$
CP-mixed	$K_{S,L}^0\pi^+\pi^-$, $K_S^0\pi^+\pi^-\pi^0$, $\pi^+\pi^-\pi^0$

$\square$: tag modes used in amplitude analysis

- Signal Mode

Signal $D^0 \to K_S^0\pi^0\pi^+\pi^-$



Event Selection

Initial Selection

20.3 fb^{-1} $\psi(3770)$ Data Samples in BOSS version 7.1.2 used for $D^0 \to K_S^0\pi^0\pi^+\pi^-$ with decay mode $K_S^0 \to \pi^+\pi^-, \pi^0 \to \gamma\gamma$.

- Good Charged tracks
  - ✓ $|\mathrm{Rz}| < 10.0$ cm, $|\mathrm{Rxy}| < 1.0$ cm
  - ✓ $|\cos\theta| < 0.93$
- Good Showers
  - ✓ $E > 25$ MeV for barrel ($|\cos\theta| < 0.8$)
  - ✓ $E > 50$ MeV for endcap ($0.86 < |\cos\theta| < 0.92$)
  - ✓ $0 \leq t_{TDC} \leq 14$ ($\times$ 50 ns)
  - ✓ $\theta_{\gamma-chrg} > 10°$
- Particle Identification (dE/dx and TOF)

  Use SimplePID
  - ✓ π: $CL_\pi > CL_K$, $CL_\pi > 0$
  - ✓ K: $CL_K > CL_\pi$, $CL_K > 0$
  - ✓ e : $CL_e > CL_K$, $CL_e > CL_\pi$, $CL_K > 0$
- $\pi^0 \to \gamma\gamma$ candidate
  - ✓ Reject γ both in endcap
  - ✓ $[0.115, 0.150]$ GeV/c^2, $\chi^2_{1C} < 50$
- $\eta \to \gamma\gamma$ candidate
  - ✓ $[0.505, 0.575]$ GeV/c^2, $\chi^2_{1C} < 50$
- $K_S^0 \to \pi^+\pi^-$ candidate
  - ✓ $|\mathrm{Rz}| < 20.0$ cm, $|\cos\theta| < 0.93$
  - ✓ $L/\sigma_L > 2$, $\chi^2_{vtx,\,svtx} < 100$
  - ✓ $[0.487, 0.511]$ GeV/c^2
- η' candidate
  - ✓ $\pi^+\pi^-\eta$: $[0.940, 0.976]$ GeV/c^2
  - ✓ $\rho\gamma$: $[0.940, 0.976]$ GeV/c^2,
    $\rho \to \pi^+\pi^-$: $[0.626, 0.924]$ GeV/c^2

Event Selection

Initial Selection

- ST Selection
  - Cosmic ray rejection for $K^+\pi^-$ tag mode
  - K_S^0 rejection
    - ✓ $K^+\pi^-\pi^+\pi^-$: $|M_{\pi^+\pi^-} - 0.4976| > 0.03\ \mathrm{GeV/c^2}$
    - ✓ $\pi^+\pi^-\pi^0$: $M_{\pi^+\pi^-} \in [0.48, 0.52]\ \mathrm{GeV/c^2}$
  - Minimum $|\Delta\mathrm{E}|$ to select the best ST D^0
  - Cut of $\Delta\mathrm{E}$ and $\mathrm{M_{BC}}$ are defined as:

$$\Delta E = E_{\bar{D}^0} - E_{beam}$$

$$M_{BC} = \sqrt{E_{beam}^2 - |\vec{p}_{\bar{D}^0}|^2}$$

Decay modes	Data ΔE cut (GeV)	MC ΔE cut (GeV)
$\bar{D}^0 \to K^+\pi^-$	$(-0.027, 0.027)$	$(-0.027, 0.027)$
$\bar{D}^0 \to K^+\pi^-\pi^0$	$(-0.062, 0.049)$	$(-0.062, 0.049)$
$\bar{D}^0 \to K^+\pi^+\pi^-\pi^-$	$(-0.026, 0.024)$	$(-0.026, 0.024)$
$\bar{D}^0 \to K_S^0\pi^+\pi^-$	$(-0.024, 0.024)$	$(-0.024, 0.024)$
$\bar{D}^0 \to K_S^0\pi^+\pi^-\pi^0$	$(-0.050, 0.038)$	$(-0.050, 0.038)$
$\bar{D}^0 \to \pi^+\pi^-\pi^+\pi^-$	$(-0.031, 0.028)$	$(-0.031, 0.028)$
$\bar{D}^0 \to \pi^+\pi^-\pi^0$	$(-0.063, 0.050)$	$(-0.058, 0.045)$
$\bar{D}^0 \to K^+K^-$	$(-0.026, 0.027)$	$(-0.027, 0.027)$
$\bar{D}^0 \to \pi^+\pi^-$	$(-0.037, 0.038)$	$(-0.032, 0.032)$
$\bar{D}^0 \to K_S^0\pi^0\pi^0$	$(-0.084, 0.060)$	$(-0.069, 0.040)$
$\bar{D}^0 \to K_S^0\pi^0$	$(-0.069, 0.053)$	$(-0.066, 0.049)$
$\bar{D}^0 \to K_S^0\eta_{\gamma\gamma}$	$(-0.045, 0.045)$	$(-0.044, 0.043)$
$\bar{D}^0 \to K_S^0\eta'_{\pi^+\pi^-\eta_{\gamma\gamma}}$	$(-0.030, 0.030)$	$(-0.030, 0.029)$
$\bar{D}^0 \to K_S^0\eta'_{\gamma\pi^+\pi^-}$	$(-0.033, 0.027)$	$(-0.034, 0.029)$

Event Selection

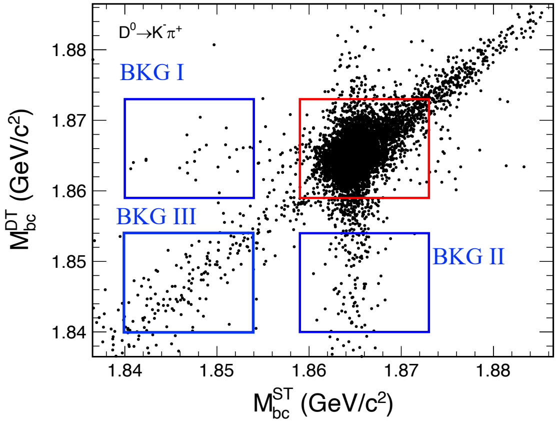
For Amplitude Analysis Modes

$\bar{D}^0 \to K^+\pi^-, K^+\pi^-\pi^0, K^+\pi^-\pi^+\pi^-$

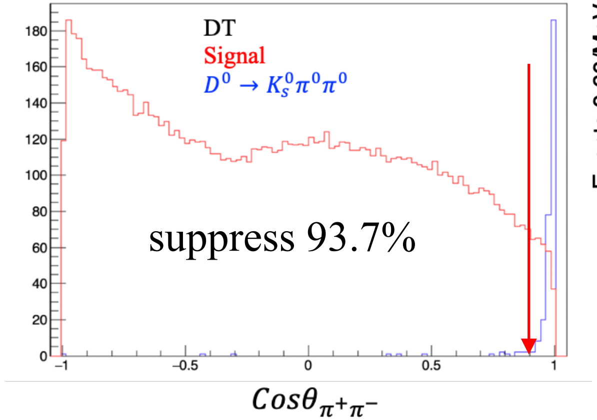
- DT Selection
  - 3C Kinematic fit
    Constrain: π^0, K_S^0, D^0 mass
  - ΔE requirement for DT tag $D^0 \to K_S^0\pi^+\pi^-\pi^0$
    $(\mu - 3.5\sigma, \mu + 3.5\sigma)$: $(-0.050, 0.038)$ [GeV]

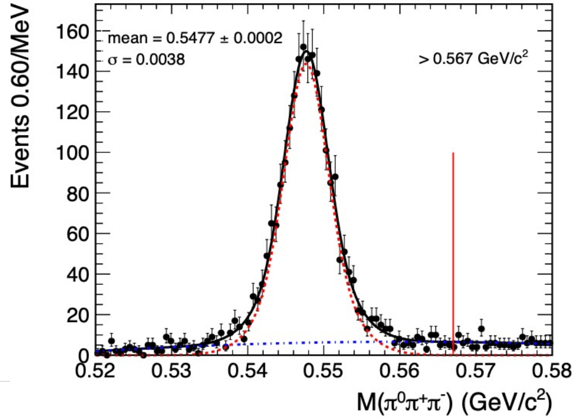
In $\mathrm{M_{BC}}$ spectrum:

- Signal region $(\bar{\mu} \pm 3.5\bar{\sigma}) \in (1.859, 1.873)$
- Sideband region $6\sigma \in (1.840, 1.854)$



- Veto $D^0 \to \pi^0\pi^0K_S^0$ Peaking BKG
  $\cos\theta_{\pi^+\pi^-} < 0.9$
- Veto η:
  $M(\pi^0\pi^+\pi^-) > 0.567\ \mathrm{GeV/c^2}$





Amplitude Analysis
Likelihood Fit

➢ PDF Function:

$$P(p) = \boxed{\epsilon(p)} \left[\boxed{\omega} \frac{\boxed{f_s(p)}}{\underline{\int f_s(p)\epsilon(p)\mathrm{d}\Phi}} + (1-\omega) \frac{\boxed{f_b(p)/\epsilon(p)}}{\underline{\int f_b(p)/\epsilon(p) \times \epsilon(p)\mathrm{d}\Phi}} \right]$$

efficiency (PHSP MC)

Purity (M_{bc} 2D fit)

Signal PDF (QC amplitude)

background PDF (efficiency correction)

MC integral

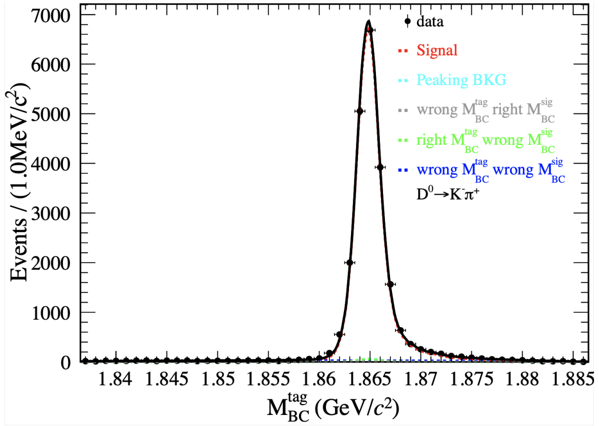
➢ Log-likelihood in Amplitude Fit:

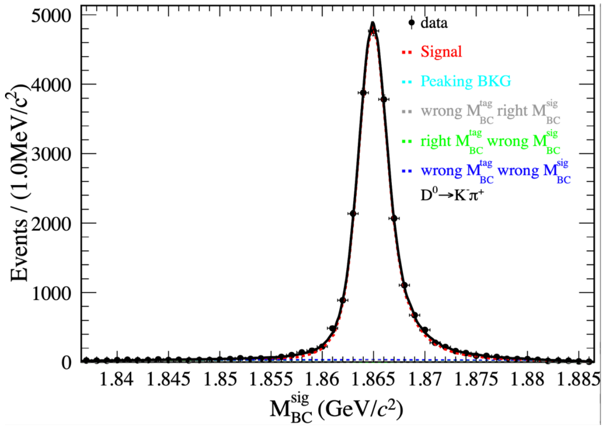
$$\ln L = \sum_i^{N_{data}} \ln \left[f \frac{f_s(p_i)}{\int f_s(p)\epsilon(p)\mathrm{d}\Phi} + (1-f) \frac{f_b(p_i)/\epsilon(p_i)}{\int f_b(p)/\epsilon(p) \times \epsilon(p)\mathrm{d}\Phi} \right] + \sum_i^{N_{data}} \ln \epsilon(p_i)$$

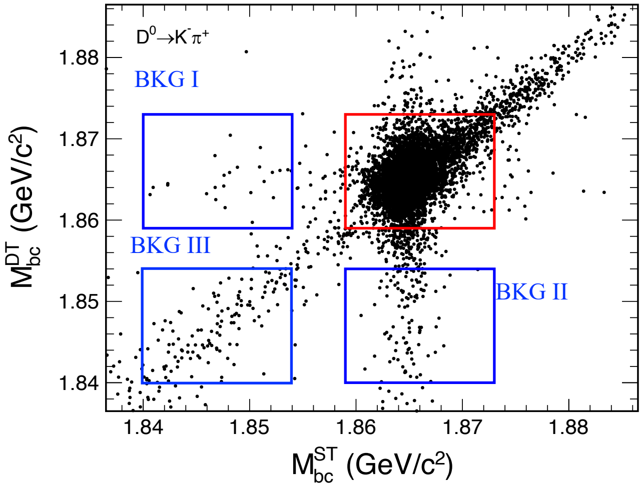
Amplitude Analysis

Signal Yields & Purity

- Fit Method
  - Sig: MC Shape $s(x,y) \otimes \mathrm{Gauss}(x,y)$
  - BKG I: $\mathrm{Argus}(x) \times \mathrm{MC\ Shape}\ s(y)$
  - BKG II: $\mathrm{Argus}(y) \times \mathrm{MC\ Shape}\ s(x)$
  - BKG III: $\mathrm{T}(x-y; \mu, \sigma(x+y), n) \times \mathrm{Argus}(x) \times \mathrm{Argus}(y)$
  - Peaking BKG: Estimated and modeled by Inc MC







$$T(y;\mu,\sigma,n) = \frac{\Gamma(n/2+0.5)}{\sigma\sqrt{n\pi}\Gamma(n/2)}\left[1+\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right]^{-\frac{n+1}{2}}$$

Signal region $(\bar{\mu}-3.5\bar{\sigma}, \bar{\mu}+3.5\bar{\sigma}) \in (1.859, 1.873)$

ST mode	Signal Yield	Signal Purity
$\bar{D}^0 \to K^+\pi^-$	25222.2 ± 25.9	$(97.5 \pm 0.1)\%$
$\bar{D}^0 \to K^+\pi^-\pi^0$	43992.5 ± 46.9	$(93.9 \pm 0.1)\%$
$D^0 \to K^+\pi^-\pi^-\pi^+$	24045.0 ± 52.1	$(92.3 \pm 0.2)\%$

~ 93259.7

Amplitude Analysis

Background PDF

- BKG PDF $f_b(p)$ is described by Inclusive MC, using ML

Sample 0 — BKG MC

Sample 1 — PHSP MC

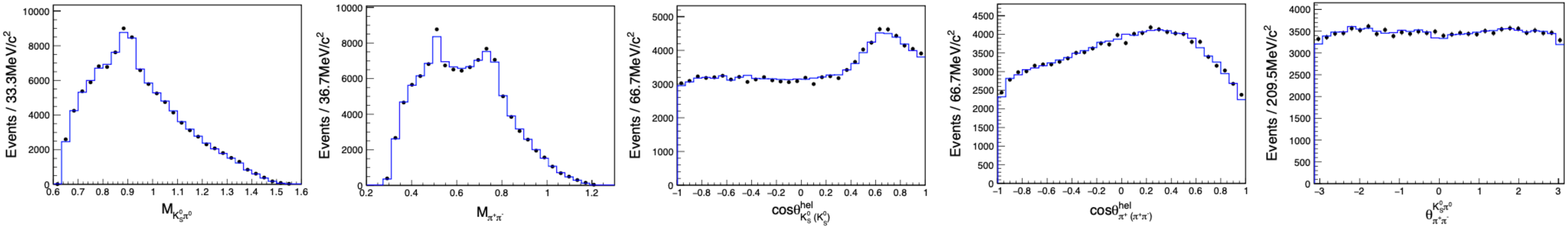
Train a classification model
Based on XGBoost

Input event $\vec{f_i}$ → Classification model → output

probability ratio

$$\frac{Prob_0(\vec{f_i})}{Prob_1(\vec{f_i})} \sim \frac{f_B(p_i)}{\epsilon(p_i)}$$

Features: $\vec{f} = \{M_{K_S^0\pi^0}, M_{\pi^+\pi^-}, \cos\theta_{K_S^0}^{K_S^0\pi^0}, \cos\theta_{\pi^+}^{\pi^+\pi^-}, \phi_{\pi^+\pi^-}^{K_S^0\pi^0}\}$



Data Point: Evts of BKG; Blue Hist: PHSP Signal MC sampled by weight obtained from ML

$\chi = -0.5$

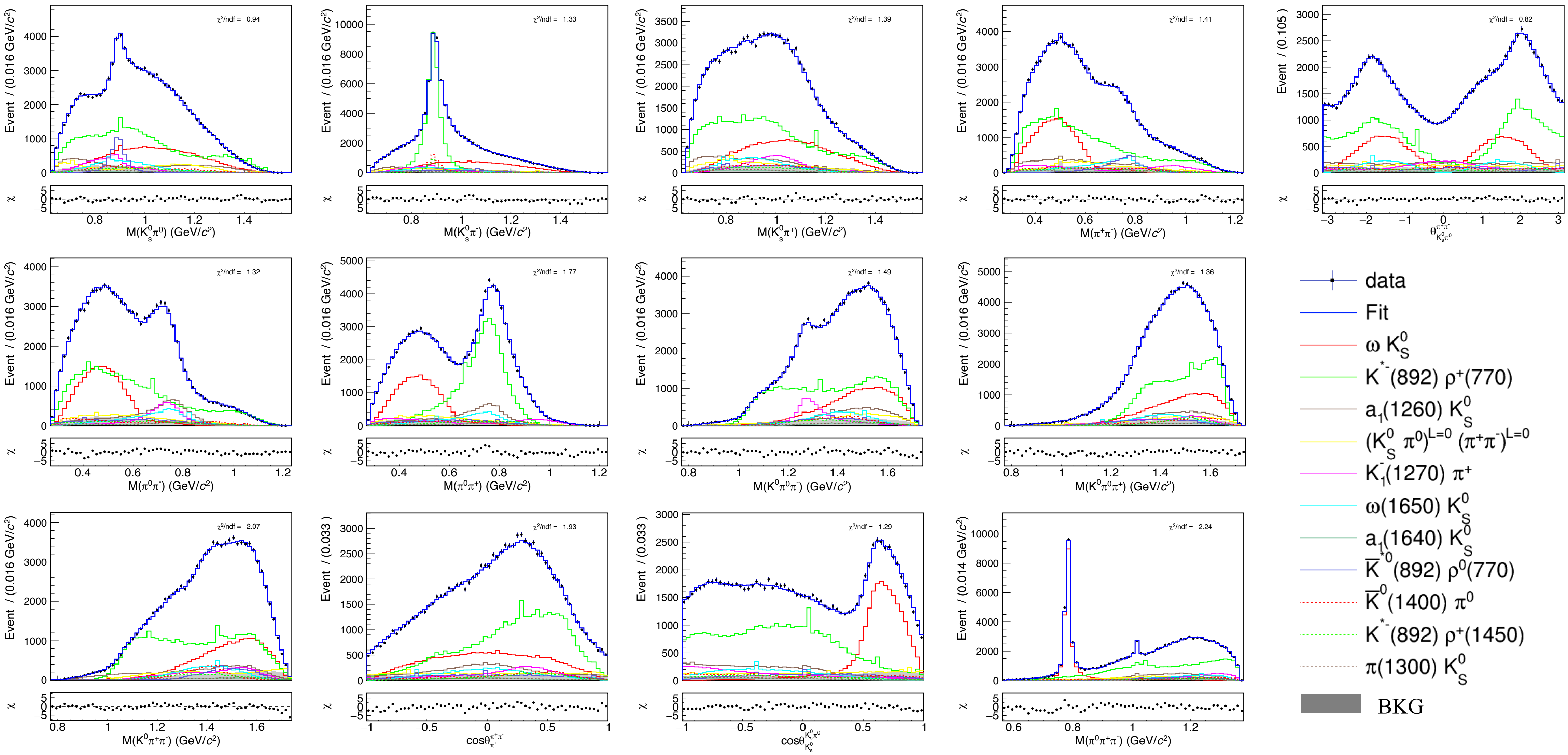
Amplitude Analysis

Signal PDF

- QC signal amplitude
$$\frac{\mathrm{d}\Gamma(D^0 \to f)}{\mathrm{d}\Phi_f} \propto |A_f|^2 + r_{\bar{g}}^2|\bar{A}_f|^2 - r_{\bar{g}}R_{\bar{g}}e^{i\delta_{\bar{g}}}A_f\bar{A}_f^* - r_{\bar{g}}R_{\bar{g}}e^{-i\delta_{\bar{g}}}A_f^*\bar{A}_f$$
Neglect DDbar mixing
$$= |A_f - r_{\bar{g}}R_{\bar{g}}e^{-i\delta_{\bar{g}}}\bar{A}_f|^2 + r_{\bar{g}}^2(1 - R_{\bar{g}}^2)|\bar{A}_f|^2 \,.$$
- D^0 amplitude: $A_f(p) = \sum_i \Lambda_i U_i(p)$, Λ_i: complex coupling factor
- Partial wave amplitude: $U_i(p) = S_i(p)B_{L_D}(p)P_{R_1}(p)B_{L_{R_1}}(p)P_{R_2}(p)B_{L_{R_2}}(p)$
- B_L : Blatt-Weisskopf barrier factor; P_R: Propagator
- S_i : spin factor constructed in covariant Zemach tensor;
- $\bar{D}^0$ amplitude: $\bar{A}_f(p) = \sum_i \Lambda_i \bar{U}_i(p) = \sum_i \Lambda_i U_i(\bar{p})$, suppose CP conservation
  $\bar{p}$: CP –conjugate of phase space point p;
- Amplitude Consider the Isospin symmetry in strong decay (CG coefficients)

Amplitude Analysis

Fit Result



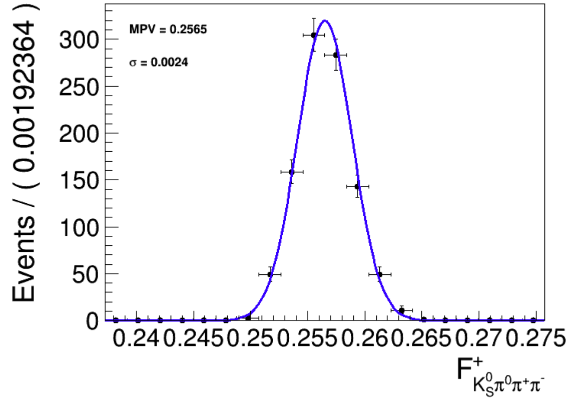
Amplitude Analysis

CP-even Fraction & Longitudinal Polarization Fraction

- CP-even Fraction

$$F_+ = \frac{\int |A_+(p)|^2 \, d\Phi}{\int |A_+(p)|^2 + |A_-(p)|^2 \, d\Phi} \quad , \qquad A_\pm(p) = \frac{1}{\sqrt{2}}[A_{D^0}(p) \pm A_{\bar{D}^0}(p)]$$

$$F_+ = 0.257 \pm 0.002 \pm 0.006 \ (\mathbf{non-\eta})$$



- Longitudinal Polarization Fraction

$$f_L = \frac{|H_{00}|^2}{|H_{11} + H_{-1-1} + H_{00}|^2}$$

Process	$D^0 \to K^{*-}(892)\,\rho(770)^+$	$D^0 \to \bar{K}^{*0}(892)\,\rho(770)^0$
$D \to V_1V_2 \to (P_1P_2)(P_3P_4)$	$0.509 \pm 0.005 \pm 0.012$	$0.108 \pm 0.008 \pm 0.013$
$D \to V_1V_2$	$0.468 \pm 0.005 \pm 0.011$	$0.077 \pm 0.007 \pm 0.010$

Amplitude Analysis

Systematic Uncertainty

I. Background Estimation

II. Detection Efficiency

III. Resonance line shape

IV. Radius of the Centrifugal Barrier

V. Quantum Correlation Correction

VI. Extra Partial Waves

VII. Fit Bias

(in units of stat. error)

Parameters	I	II	III	IV	V	VI	VII	Total
$M_{K^*(892)^-}$	0.07	0.13	0.31	1.55	0.41	0.19	0.00	1.65
$\Gamma_{K^*(892)^-}$	0.27	0.43	0.73	1.01	0.05	0.19	0.00	1.36
$M_{\bar{K}^*(892)^0}$	0.03	0.13	0.77	1.18	0.03	0.37	0.00	1.46
$\Gamma_{\bar{K}^*(892)^0}$	0.28	0.02	1.03	0.95	0.21	0.13	0.00	1.45
$M_{a_1(1260)}$	0.27	0.19	0.62	0.60	0.11	0.03	0.00	0.93
$\Gamma_{a_1(1260)}$	0.76	0.80	2.07	1.24	0.68	0.63	0.00	2.80
$M_{K_1(1270)}$	0.36	0.13	0.96	0.64	0.45	0.45	0.00	1.37
$\Gamma_{K_1(1270)}$	0.05	0.07	0.16	0.13	0.02	0.08	0.00	0.24
$M_{\pi(1300)}$	1.75	1.83	2.65	2.30	1.34	1.78	0.00	4.87
$\Gamma_{\pi(1300)}$	0.58	0.49	1.03	1.03	0.52	0.05	0.00	1.72
LASS a	0.14	0.24	0.91	0.66	0.17	1.07	0.00	1.59
LASS r	0.13	0.26	0.56	0.67	0.40	0.53	0.00	1.14
$\pi\pi_S\ s_0^{prod}$	0.00	0.10	0.23	0.55	0.40	0.15	0.00	0.74
$F_+^{K_S^0\pi^0\pi^+\pi^-}$	0.50	0.51	2.59	0.65	0.12	0.22	0.03	2.78
$f_L\ (D \to VV)$								
$D^0 \to K^*(892)^-\rho(770)^+$	0.00	0.30	1.00	1.88	0.16	0.50	0.01	2.22
$D^0 \to \bar{K}^*(892)^0\rho(770)^0$	0.17	0.06	0.12	1.32	0.04	0.45	0.16	1.43
$f_L\ (D \to VV \to PPPP)$								
$D^0 \to K^*(892)^-\rho(770)^+$	0.00	0.30	1.01	2.03	0.16	0.50	0.02	2.35
$D^0 \to \bar{K}^*(892)^0\rho(770)^0$	0.18	0.07	0.07	1.59	0.06	0.46	0.00	1.67

Fit Fraction	I	II	III	IV	V	VI	VII	Total
$D^0 \to \omega K_S^0$	0.53	0.46	0.50	3.00	0.21	0.12	0.03	3.13
$D^0 \to \omega(1420)K_S^0$	0.27	0.03	3.93	5.00	0.79	0.63	0.23	6.45
$D^0 \to \omega(1650)K_S^0$	0.01	0.46	5.14	4.68	0.18	0.67	0.14	7.00
$D^0 \to a_1(1260)K_S^0$	0.04	0.02	2.11	1.81	0.01	0.06	0.15	2.78
$D^0 \to a_2(1320)K_S^0$	0.11	0.02	0.17	0.30	0.14	1.14	0.07	1.21
$D^0 \to a_1(1640)K_S^0$	0.26	0.14	2.01	2.10	0.04	0.08	0.16	2.93
$D^0 \to \phi K_S^0$	0.12	0.06	0.11	0.29	0.08	0.01	0.06	0.35
$D^0 \to h_1(1170)K_S^0$	0.34	0.08	1.39	0.73	0.10	0.09	0.09	1.62
$D^0 \to \pi(1300)K_S^0$	0.18	0.01	0.72	2.22	0.22	0.12	0.28	2.37
$D^0 \to K(1410)^-\pi^+$	0.06	0.08	0.70	1.07	0.09	1.74	0.02	2.17
$D^0 \to K^*(892)^-\rho(770)^+$	0.40	0.23	0.36	4.05	0.12	0.11	0.15	4.10
$D^0 \to K^*(892)^-\rho(770)^+[S]$	0.44	0.19	0.13	1.52	0.00	0.15	0.04	1.61
$D^0 \to K^*(892)^-\rho(770)^+[P]$	0.20	0.06	0.08	1.90	0.16	0.30	0.13	1.95
$D^0 \to K^*(892)^-\rho(770)^+[D]$	0.05	0.22	0.50	3.20	0.01	0.01	0.02	3.24
$D^0 \to \bar{K}^*(892)^0\rho(770)^0$	0.17	0.22	0.23	1.59	0.06	0.38	0.35	1.71
$D^0 \to \bar{K}^*(892)^0\rho(770)^0[S]$	0.18	0.02	0.41	0.28	0.08	0.03	0.27	0.60
$D^0 \to \bar{K}^*(892)^0\rho(770)^0[P]$	0.05	0.11	0.16	1.33	0.05	0.60	0.03	1.47
$D^0 \to \bar{K}^*(892)^0\rho(770)^0[D]$	0.21	0.25	0.18	1.77	0.01	0.03	0.25	1.83
$D^0 \to K^*(892)^-\rho(1450)^+$	0.33	0.05	0.79	0.74	0.06	0.31	0.19	1.19
$D^0 \to K^*(892)^-\rho(1450)^+[S]$	0.36	0.07	0.77	1.30	0.04	0.04	0.11	1.56
$D^0 \to K^*(892)^-\rho(1450)^+[P]$	0.07	0.12	0.01	0.12	0.05	0.56	0.05	0.59
$D^0 \to K^*(892)^-\rho(1450)^+[D]$	0.03	0.43	0.65	2.70	0.00	0.01	0.05	2.81
$D^0 \to \bar{K}^*(892)^0\rho(1450)^0$	0.00	0.39	0.55	0.46	0.05	0.04	0.06	0.82
$D^0 \to \bar{K}^*(892)^0\rho(1450)^0[D]$	0.00	0.39	0.55	0.46	0.05	0.04	0.06	0.82
$D^0 \to K^*(1680)^-\rho(770)^+$	0.29	0.12	2.15	2.26	0.07	0.20	0.02	3.15
$D^0 \to K^*(1680)^-\rho(770)^+[S]$	0.29	0.12	2.15	2.26	0.07	0.20	0.02	3.15
$D^0 \to K^*(892)^- f_2(1270)$	0.25	0.16	0.88	0.63	0.05	0.46	0.14	1.22
$D^0 \to (K_S^0\pi^-)^{L=0}\rho(770)^+$	0.49	0.15	1.12	1.05	0.00	0.12	0.13	1.63
$D^0 \to (K_S^0\pi^0)^{L=0}\rho(770)^0$	0.73	0.10	0.27	1.93	0.01	0.03	0.05	2.08
$D^0 \to \bar{K}^*(892)^0(\pi^+\pi^-)^{L=0}$	0.14	0.44	1.49	1.51	0.26	0.03	0.15	2.19
$D^0 \to (K_S^0\pi^0)^{L=0}(\pi^+\pi^-)^{L=0}$	0.08	0.28	2.38	2.50	0.19	0.10	0.15	3.48
$D^0 \to \bar{K}^*(1680)^0(\pi^+\pi^-)^{L=0}$	0.10	0.10	0.45	1.24	0.07	0.07	0.06	1.33
$D^0 \to (K_S^0\pi^0)^{L=1}\rho(1450)^0$	0.36	0.16	0.52	0.32	0.09	1.15	0.06	1.36
$D^0 \to K_1(1270)^-\pi^+$	0.31	0.18	1.06	0.83	0.05	0.04	0.14	1.40
$D^0 \to K_1(1270)^0\pi^0$	0.04	0.24	0.16	0.40	0.02	0.02	0.19	0.53
$K_1(1270) \to \rho(770)K_S^0[S]$	0.32	0.30	1.45	1.21	0.05	0.12	0.00	1.95
$K_1(1270) \to \rho(1450)K_S^0[D]$	0.12	0.09	0.48	1.04	0.02	0.02	0.11	1.16
$K_1(1270) \to K^*(892)\pi[S]$	0.07	0.01	0.48	0.84	0.00	0.04	0.05	0.98
$K_1(1270) \to K^*(892)\pi[D]$	0.02	0.23	0.79	0.79	0.02	0.07	0.07	1.14
$K_1(1270) \to (K_S^0\pi)^{L=0}\pi[P]$	0.16	0.11	0.73	0.42	0.05	0.12	0.24	0.91
$D^0 \to K_1(1400)^0\pi^0$	0.24	0.21	0.30	0.71	0.02	0.10	0.03	0.84
$D^0 \to K_1(1400)^-\pi^+$	0.25	0.03	0.81	0.57	0.03	0.09	0.02	1.03
$K_1(1400) \to K^*(892)\pi[S]$	0.13	0.09	0.19	1.49	0.15	0.15	0.10	1.53
$K_1(1400) \to \rho(770)K_S^0[D]$	0.16	0.22	1.95	1.01	0.01	0.01	0.12	2.22
$D^0 \to K_2(1430)^-\pi^+$	0.05	0.02	0.13	0.15	0.01	0.35	0.17	0.44
$D^0 \to K_2(1430)^0\pi^0$	0.11	0.14	0.08	0.66	0.01	0.13	0.09	0.71
$K_2(1430) \to K^*(892)\pi[S]$	0.11	0.18	0.13	1.18	0.03	0.17	0.19	1.23
$K_2(1430) \to \rho(770)K_S^0$	0.12	0.11	0.33	1.12	0.01	0.09	0.26	1.21
$D^0 \to K(1460)^-\pi^+$	0.13	0.03	1.11	1.38	0.02	0.08	0.12	1.78
$D^0 \to K(1460)^0\pi^0$	0.43	0.54	1.73	1.14	0.16	0.12	0.01	2.20
$K(1460) \to K^*(892)\pi[S]$	0.41	0.30	2.05	1.68	0.18	0.09	0.08	2.71
$K(1460) \to (K_S^0\pi)^{L=0}\pi$	0.41	0.30	2.03	1.67	0.18	0.09	0.08	2.69
$D^0 \to K_1(1650)^0\pi^0$	0.86	0.17	0.54	0.74	0.10	0.00	0.27	1.30
$D^0 \to K_1(1650)^-\pi^+$	0.18	0.10	0.42	0.86	0.11	0.00	0.14	0.99
$K_1(1650) \to \rho(770)K_S^0[S]$	0.22	0.06	0.54	0.51	0.01	0.04	0.05	0.78
$K_1(1650) \to K^*(892)\pi[S]$	0.02	0.05	0.96	0.87	0.09	0.06	0.06	1.30
$K_1(1650) \to (K_S^0\pi)^{L=0}\pi$	0.25	0.14	0.89	1.30	0.04	0.06	0.06	1.60
$D^0 \to K^*(1680)^0\pi^0$	0.23	0.05	0.17	1.54	0.21	0.14	0.08	1.59
$D^0 \to K^*(1680)^-\pi^+$	0.04	0.06	0.07	1.79	0.23	0.43	0.18	1.86
$D^0 \to K^*(1680)^+\pi^-$	0.08	0.09	0.66	1.43	0.01	0.10	0.24	1.61
$K^*(1680) \to \rho(770)K_S^0[P]$	0.22	0.31	0.67	1.06	0.73	0.18	0.29	1.54
$K^*(1680) \to K^*(892)\pi[P]$	0.22	0.32	0.71	1.12	0.73	0.22	0.18	1.59

Phase (ϕ)	I	II	III	IV	V	VI	VII	Total
$D^0 \to \phi K_S^0$	0.03	0.09	0.48	0.46	0.24	0.31	0.05	0.78
$D^0 \to \pi(1300)K_S^0$	0.23	0.13	1.15	1.58	0.08	0.19	0.07	1.98
$D^0 \to \omega(1420)K_S^0$	0.17	0.91	2.69	2.51	2.80	0.20	0.10	4.72
$D^0 \to \omega(1650)K_S^0$	0.47	0.90	1.64	1.48	3.78	0.27	0.17	4.51
$D^0 \to h_1(1170)K_S^0$	0.46	0.27	0.24	1.80	0.72	0.11	0.00	2.03
$D^0 \to a_1(1260)K_S^0$	0.15	0.24	1.03	2.09	0.33	0.56	0.13	2.44
$D^0 \to a_1(1640)K_S^0$	0.07	0.06	8.06	1.16	0.11	0.46	0.15	8.16
$D^0 \to a_2(1320)K_S^0$	0.09	0.01	0.42	0.31	0.31	0.04	0.02	0.61
$D^0 \to K^*(892)^-\rho(770)^+[S]$	0.24	0.12	0.86	2.50	1.20	0.70	0.09	2.99
$D^0 \to K^*(892)^-\rho(770)^+[P]$	0.06	0.12	2.34	1.58	1.67	0.47	0.11	3.31
$D^0 \to K^*(892)^-\rho(770)^+[D]$	0.29	0.32	0.68	5.01	1.13	0.71	0.08	5.24
$D^0 \to \bar{K}^*(892)^0\rho(770)^0[S]$	0.30	0.05	0.90	1.49	1.02	0.10	0.20	2.05
$D^0 \to \bar{K}^*(892)^0\rho(770)^0[P]$	0.05	0.05	0.40	1.86	0.25	0.25	0.25	1.96
$D^0 \to \bar{K}^*(892)^0\rho(770)^0[D]$	0.02	0.15	0.34	2.84	1.01	0.25	0.06	3.05
$D^0 \to K^*(892)^-\rho(1450)^+[S]$	0.15	0.07	1.21	2.17	0.65	0.02	0.11	2.58
$D^0 \to K^*(892)^-\rho(1450)^+[P]$	0.10	0.21	0.66	0.46	0.22	1.14	0.11	1.44
$D^0 \to K^*(892)^-\rho(1450)^+[D]$	0.07	0.01	2.78	3.62	0.28	0.11	0.05	4.57
$D^0 \to \bar{K}^*(892)^0\rho(1450)^0[D]$	0.06	0.02	2.39	2.57	0.21	0.06	0.01	3.52
$D^0 \to K^*(892)^- f_2(1270)[P]$	0.23	0.10	0.55	0.23	0.24	0.04	0.14	0.70
$D^0 \to K^*(892)^- f_2(1270)[P]$	0.13	0.02	0.19	0.69	0.43	1.44	0.25	1.69
$D^0 \to K_1(1270)^-\pi^+(K_1(1270) \to \rho(770)K_S^0[S])$	0.28	0.07	3.89	1.70	0.97	0.29	0.01	4.37
$D^0 \to K_1(1270)^0\pi^0(K_1(1270) \to \rho(770)K_S^0[S])$	0.36	0.04	1.00	1.68	1.04	0.35	0.21	2.28
$K_1(1270) \to \rho(1450)K_S^0[D]$	0.50	0.57	1.79	1.07	0.56	0.53	0.18	2.36
$K_1(1270) \to K^*(892)\pi[S]$	0.20	0.02	1.58	1.18	0.16	0.08	0.07	1.99
$K_1(1270) \to K^*(892)\pi[D]$	0.11	0.13	3.55	0.73	0.11	0.11	0.09	3.64
$K_1(1270) \to (K_S^0\pi)^{L=0}\pi[P]$	0.02	0.02	0.11	0.07	0.04	0.02	0.01	0.14
$D^0 \to K_1(1400)^-\pi^+(K_1(1400) \to K^*(892)\pi[S])$	0.30	0.30	2.98	0.98	0.97	0.37	0.07	3.33
$D^0 \to K_1(1400)^0\pi^0(K_1(1400) \to K^*(892)\pi[S])$	0.31	0.15	3.02	0.66	0.18	0.16	0.02	3.12
$K_1(1400) \to \rho(770)K_S^0[D]$	0.04	0.07	0.40	0.19	0.10	0.11	0.10	0.48
$D^0 \to K(1460)^0\pi^0(K(1460) \to K^*(892)\pi[S])$	0.27	0.12	3.98	0.73	0.54	0.11	0.03	4.09
$D^0 \to K(1460)^-\pi^+(K(1460) \to K^*(892)\pi[S])$	0.15	0.43	1.58	1.33	0.73	0.19	0.07	2.25
$K(1460) \to (K_S^0\pi)^{L=0}\pi$	0.01	0.06	1.22	1.38	0.05	0.02	0.10	1.85
$D^0 \to K_1(1650)^0\pi^0(K_1(1650) \to \rho(770)K_S^0[S])$	0.05	0.11	2.56	0.08	0.47	0.27	0.22	2.63
$D^0 \to K_1(1650)^-\pi^+(K_1(1650) \to \rho(770)K_S^0[S])$	0.14	0.17	1.19	0.45	0.14	0.16	0.01	1.30
$K_1(1650) \to K^*(892)\pi[S]$	0.18	0.34	4.21	0.88	0.06	0.04	0.20	4.32
$D^0 \to K^*(1680)^0\pi^0(K^*(1680) \to \rho(770)K_S^0[P])$	0.69	0.24	5.46	1.80	0.58	2.01	0.28	6.17
$D^0 \to K^*(1680)^+\pi^-(K^*(1680) \to \rho(770)K_S^0[P])$	0.56	0.11	6.79	0.43	0.14	0.01	0.18	6.83
$D^0 \to K^*(1680)^-\pi^+(K^*(1680) \to \rho(770)K_S^0[P])$	0.28	0.08	5.72	1.70	0.25	3.14	0.27	6.76
$K^*(1680) \to K^*(892)\pi[P]$	0.39	0.04	7.41	1.79	0.08	1.79	0.33	7.85
$D^0 \to K_2(1430)^-\pi^+(K_2(1430) \to K^*(892)\pi[S])$	0.03	0.05	1.21	0.94	0.30	0.37	0.08	1.61
$D^0 \to K_2(1430)^0\pi^0(K_2(1430) \to K^*(892)\pi[S])$	0.10	0.10	0.66	0.64	0.37	0.56	0.14	1.16
$K_2(1430) \to \rho(770)K_S^0$	0.16	0.15	2.14	0.53	0.04	0.64	0.07	2.30
$D^0 \to K^*(1680)^-\rho(770)^+[S]$	0.16	0.04	3.59	0.79	0.23	0.24	0.15	3.70
$D^0 \to K(1410)^-\pi^+$	0.05	0.02	2.61	0.56	0.99	2.62	0.13	3.88
$D^0 \to (K_S^0\pi^0)^{L=1}\rho(1450)^0$	0.06	0.24	0.54	0.48	0.69	0.26	0.01	1.06
$D^0 \to (K_S^0\pi^-)^{L=0}\rho(770)^+$	0.00	0.01	0.13	0.05	0.02	0.00	0.14	0.20
$D^0 \to (K_S^0\pi^0)^{L=0}\rho(770)^0$	0.01	0.01	0.45	0.02	0.02	0.00	0.01	0.45
$D^0 \to \bar{K}^*(892)^0(\pi^+\pi^-)^{L=0}$								
β_3	0.33	0.07	0.22	0.87	0.33	0.02	0.04	1.01
$f_{\pi\pi}$	0.02	0.04	0.15	0.40	0.29	0.10	0.01	0.53
$K_1(1650) \to (K_S^0\pi)^{L=0}\pi$								
β_1	0.14	0.96	0.81	1.76	0.95	0.21	0.05	2.37
β_2	0.29	0.65	1.36	1.95	0.12	0.25	0.13	2.51
$f_{\eta\eta}$	0.16	0.32	0.01	0.47	0.17	0.16	0.07	0.64
$D^0 \to \bar{K}^*(1680)^0(\pi^+\pi^-)^{L=0}$								
β_1	0.51	1.01	0.34	2.20	0.97	0.24	0.00	2.69
β_3	0.34	1.67	0.74	2.34	1.35	0.47	0.40	3.33
$f_{\eta\eta}$	0.36	0.11	0.21	2.25	0.22	0.03	0.05	2.30
$D^0 \to (K_S^0\pi^0)^{L=0}(\pi^+\pi^-)^{L=0}$								
β_2	0.00	0.02	0.00	0.07	0.03	0.00	0.18	0.20
β_4	0.00	0.01	0.04	0.02	0.02	0.00	0.16	0.17
β_5	0.00	0.03	0.07	0.07	0.11	0.01	0.01	0.15
f_{KK}	0.01	0.02	0.03	0.03	0.04	0.00	0.12	0.14
$f_{\pi\pi}$	0.03	0.02	1.50	1.21	0.11	0.00	0.02	1.93
$f_{\eta\eta}$	0.00	0.01	0.02	0.02	0.02	0.00	0.10	0.10

Branching Fraction Measurement

- BFs are calculated by $$\mathcal{B}_f = \frac{\Sigma_g N^{\rm DT}_{fg}}{\Sigma_g N^{\rm ST}_g(\epsilon^{\rm DT}_{fg}/\epsilon^{\rm ST}_g)[1 - \frac{2r_gR_g\cos\delta_g}{1+r_g^2}(2F^f_+ - 1)]}.$$

Using $\mathcal{B}(K^0_S \to \pi^+\pi^-) = (69.20 \pm 0.05)\%$ and $\mathcal{B}(\pi^0 \to \gamma\gamma) = (98.823 \pm 0.034)\%$ as PDG

	This Work	PDG
$\mathcal{B}(\bar{D}^0 \to K^0_S\pi^0\pi^+\pi^-)$ (non-η)	$(5.586 \pm 0.019_{\rm stat.} \pm 0.037_{\rm sys.})\%$	$(5.083 \pm 0.6)\%$

- Branching Fraction of the component (FFs > 1%) in $\bar{D}^0 \to K^0_S\pi^0\pi^+\pi^-$

Components	Branching Fractions (%)
$D^0 \to \omega K^0_S$	$(1.178 \pm 0.009_{\rm stat.} \pm 0.007_{\rm sys.} \pm 0.028_{\rm model})\%$
$D^0 \to \phi K^0_S$	$(0.058 \pm 0.002_{\rm stat.} \pm 0.000_{\rm sys.} \pm 0.001_{\rm model})\%$
$D^0 \to K_1(1270)^-\pi^+$	$(0.378 \pm 0.008_{\rm stat.} \pm 0.002_{\rm sys.} \pm 0.011_{\rm model})\%$
$D^0 \to \bar{K}_1(1270)^0\pi^0$	$(0.082 \pm 0.003_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.002_{\rm model})\%$
$D^0 \to \bar{K}_1(1400)^0\pi^0$	$(0.227 \pm 0.007_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.006_{\rm model})\%$
$D^0 \to \bar{K}(1460)^0\pi^0$	$(0.120 \pm 0.009_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.020_{\rm model})\%$
$D^0 \to K(1460)^-\pi^+$	$(0.092 \pm 0.005_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.009_{\rm model})\%$
$D^0 \to a_1(1260)K^0_S$	$(0.798 \pm 0.020_{\rm stat.} \pm 0.005_{\rm sys.} \pm 0.056_{\rm model})\%$
$D^0 \to \pi(1300)K^0_S$	$(0.122 \pm 0.006_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.014_{\rm model})\%$
$D^0 \to a_1(1640)K^0_S$	$(0.090 \pm 0.008_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.023_{\rm model})\%$
$D^0 \to \omega(1420)K^0_S$	$(0.124 \pm 0.009_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.058_{\rm model})\%$
$D^0 \to \omega(1650)K^0_S$	$(0.335 \pm 0.017_{\rm stat.} \pm 0.002_{\rm sys.} \pm 0.119_{\rm model})\%$

Components	Branching Fractions (%)
$D^0 \to K^*(892)^-\rho(770)^+$	$(2.189 \pm 0.023_{\rm stat.} \pm 0.013_{\rm sys.} \pm 0.094_{\rm model})\%$
$D^0 \to \bar{K}^*(892)^0\rho(770)^0$	$(0.232 \pm 0.006_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.010_{\rm model})\%$
$D^0 \to K^*(892)^-\rho(1450)^+$	$(0.174 \pm 0.008_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.010_{\rm model})\%$
$D^0 \to (K^0_S\pi^0)^{L=1}\rho(1450)^0[P]$	$(0.069 \pm 0.004_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.005_{\rm model})\%$
$D^0 \to (K^0_S\pi^-)^{L=0}\rho(770)^+$	$(0.084 \pm 0.006_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.010_{\rm model})\%$
$D^0 \to \bar{K}^*(892)^0(\pi^+\pi^-)^{L=0}$	$(0.070 \pm 0.004_{\rm stat.} \pm 0.001_{\rm sys.} \pm 0.009_{\rm model})\%$
$D^0 \to (K^0_S\pi^0)^{L=0}(\pi^+\pi^-)^{L=0}$	$(0.380 \pm 0.010_{\rm stat.} \pm 0.002_{\rm sys.} \pm 0.035_{\rm model})\%$

Branching Fraction Measurement
Systematic Uncertainty

- Summation of all sources of systematic uncertainty.

Source	Relative Systematic Uncertainty (%)
$\pi^{\pm}$ Tracking	0.3
$\pi^{\pm}$ PID	0.1
π^{0} Reconstruction	0.1
K_S^0 Reconstruction	0.2
ST Fit	0.1
ΔE Cut in Signal Side	0.2
Signal Range in $M_{\rm BC}^{\rm sig}$	0.2
DT Fit	0.4
Amplitude Model	0.1
Quantum Correlation Correction	0.2
MC Statistics	0.1
Total	0.7

Event Selection

For Strong Phase Measurement

- **Partial Reconstruction**

  Tag Modes with missing track K_L^0/ν_e.

  $E_{miss} = E_{\bar{D}^0} - \Sigma_i E_i$,

  $\vec{p}_{miss} = \vec{p}_{\bar{D}^0} - \Sigma_i \vec{p}_i$.

  - $M_{BC}^{sig} \in [1.86,\ 1.87]\ \mathrm{GeV/c^2}$
  - $K_L^0 + X$ $\quad M_{miss}^2 = E_{miss}^2 - |\vec{p}_{miss}|^2$
    - ✓ No additional charged tracks, no matter good charged track or not for $K^0_{S\ \to\pi^+\pi^-} + X$ BKG.
    - ✓ No additional π^0 or η.
  - $\nu_e + X$ $\quad U_{miss} = E_{miss} - |\vec{p}_{miss}|$
    - ✓ Only 2 good charged tracks (identified as K, e with opposite charge).
    - ✓ $E/p > 0.8$ (to reject misidentified $\mu^\pm/\pi^\pm$).

- **Kinematic Fit**
  - $K_S^0\pi^+\pi^-(\pi^0)$: Constrain final states to D^0 mass, no χ^2 cut.
  - $K_L^0\pi^+\pi^-$: Constrian K_L^0 missing mass $+2D^0$ mass + total 4-momentum.

Strong Phase Measurement

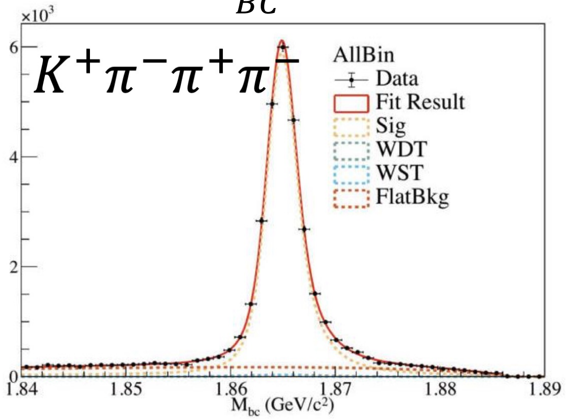
Signal Yields

- **Fully Reconstruction Mode**
  - Signal: MC shape ⊗ Gauss
  - Flat BKG: Argus
  - Peaking BKG: MC shape and fix ratio between Signal by Inc MC
  - Wrong DT and wrong ST BKG: Fix shape and yields by Inc MC
- **Partial Reconstruction Mode**

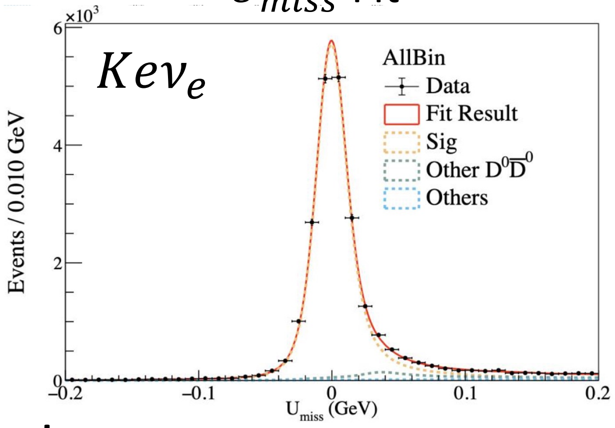
  $Ke\nu_e$
  - Signal: MC shape ⊗ Gauss
  - Other $D^0\bar{D}^0$ BKG: Inc MC shape
  - Flat BKG: 2st Chebychev function

  $K_L^0 + X$
  - Signal: MC shape ⊗ Gauss
  - Peak BKG: Inc MC shape
  - Other $D^0\bar{D}^0(D^+D^-, q\bar{q}, \dots)$ BKG: Inc MC shape
  - Flat BKG: 2st Chebychev function

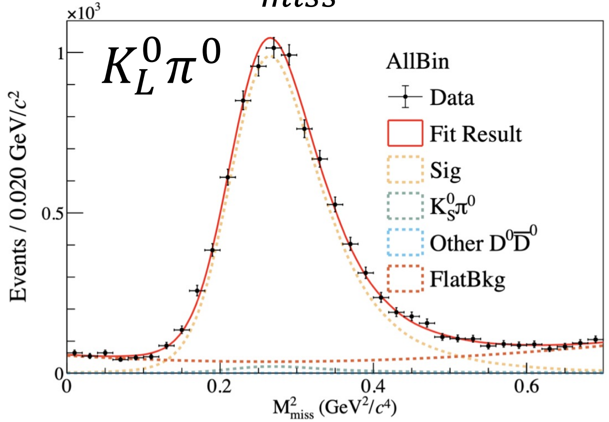
M_{BC}^{sig} Fit



U_{miss} Fit



M_{miss}^2 Fit



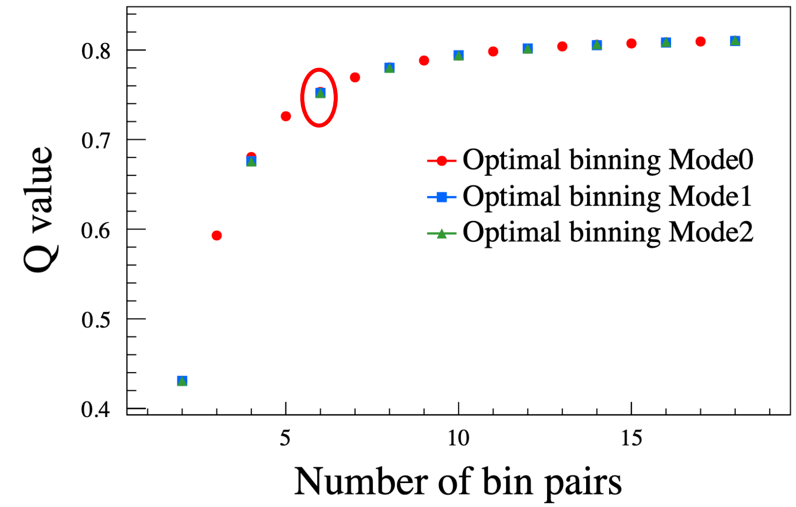
Strong Phase Measurement
Binning Scheme

- PHSP is divided by $\ln r_D$ and δ_D, $r_D e^{-i\delta_D} = A_{\bar{D}}/A_D$
- Optimization with Q value

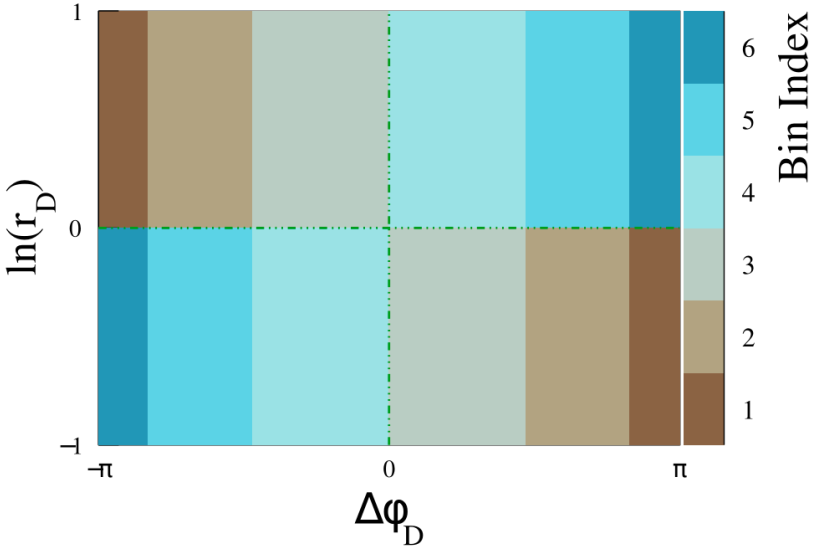
$$Q_{\pm}^2 = \frac{\sum_i \left(\frac{1}{\sqrt{N_{B\pm}^i}}\frac{\mathrm{d}N_{B\pm}^i}{\mathrm{d}x_\pm}\right)^2 + \left(\frac{1}{\sqrt{N_{B\pm}^i}}\frac{\mathrm{d}N_{B\pm}^i}{\mathrm{d}y_\pm}\right)^2}{\int_D \left[\left(\frac{1}{\sqrt{\Gamma_{B\pm}(\mathbf{p})}}\frac{\mathrm{d}\Gamma_{B\pm}(\mathbf{p})}{\mathrm{d}x_\pm}\right)^2 + \left(\frac{1}{\sqrt{\Gamma_{B\pm}(\mathbf{p})}}\frac{\mathrm{d}\Gamma_{B\pm}(\mathbf{p})}{\mathrm{d}y_\pm}\right)^2\right]\mathbf{dp}}$$

$$= \frac{\sum_i \left[(2x_\pm K_i + 2\sqrt{K_i\bar{K}_i}c_i)^2 + (2y_\pm K_i + 2\sqrt{K_i\bar{K}_i}s_i)^2\right]/N_{B\pm}^i}{4\sum_i K_i}$$

$$x_\pm = r_B\cos(\delta_B \pm \gamma)$$
$$y_\pm = r_B\sin(\delta_B \pm \gamma)$$



- Optimization way (Mode2):
  - ✓ Keep CP symmetry
  - ✓ Bin boundaries vary symmetrically around $\delta_D = 0$



Strong Phase Measurement
Parameters Measurement

- Expected DT yields

flavor
$$N_i^{fg} + N_i^{f\bar{g}} = \frac{S^g + S^{\bar{g}}}{\epsilon^g(1+r_{\bar{g}}^2)} B_f \epsilon_{ij}^{fg} \left[T_j^f + r_{\bar{g}}^2 \bar{T}_j^f - 2 r_{\bar{g}} R_{\bar{g}} \sqrt{T_j^f \bar{T}_j^f} (c_j^f \cos\delta_{\bar{g}} - s_j^f \sin\delta_{\bar{g}}) \right]$$

CP
$$N_i^{fg} = \frac{S^g}{\epsilon^g \left[2 - 2y(2F_+^g - 1) \right]} B_f \epsilon_{ij}^{fg} \left[T_j^f + \bar{T}_j^f - (2F_+^g - 1) 2 \sqrt{T_j^f \bar{T}_j^f} c_j^f \right]$$

$K^0_{S,L}\pi^+\pi^-(\pi^0)$
$$N_{ij}^{fg} = \frac{S^g}{\epsilon^g \left[2 - 2y(2F_+^g - 1) \right]} B_f \epsilon_{ij,kl}^{fg} \left[T_k^f \bar{T}_l^g + \bar{T}_k^f T_l^g - 2 \sqrt{T_k^f \bar{T}_k^f T_l^g \bar{T}_l^g} (c_k^f c_l^g + s_k^f s_l^g) \right]$$

Input parameters	Reference	Fit Parameters
$N_{D\bar{D}}, B_g$ (for $K_L^0 X$ and $Ke\nu_e$)	CPC 48, 123001 (2024), PDG	B_f, R_i, c_i, s_i
ST yields (S^g) and efficiency (ϵ^g)	BESIII Doc-1504 and Doc-1600	
r and δ of Kpi, y	HFLAV	$R_i = \begin{cases} T_i, & i = -6 \\ T_i \Big/ \sum_{j\le i} T_j, & -6 < i < 6 \\ 1, & i = 6 \end{cases}$
r, R and δ of Kpipi0 and K3pi	JHEP05(2021)164	
Ki/ci/si of $K^0_{S,L}\pi^+\pi^-$	JHEP06(2025)086	
Ki/ci/si of $K^0_{S,L}\pi^+\pi^-\pi^0$	BESIII Doc-1540 Model prediction	

Strong Phase Measurement

Likelihood Construction

- Likelihood fit: $-2\ln L = -2\sum_i \ln G(N^{exp}_{obs_i}; p^{fit}_{obs_i})$
  - $N^{exp}_{obs_i}$: The expected event number
  - $G(x;p)$: The likelihood function of N_{sig} in DT fit
  - $p^{fit}_{obs_i}$: parameters of likelihood distribution

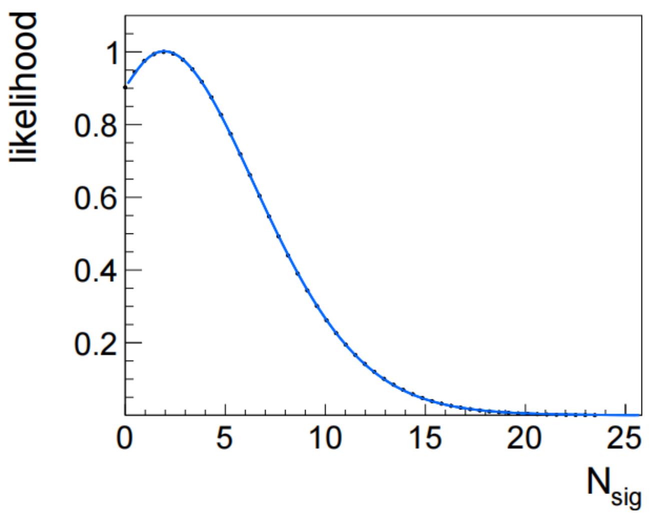
Nominal fit to obtain the N^{obs}_{sig} and σ^{obs}_{sig}

$N^{obs}_{sig} / \sigma^{obs}_{sig}$ < 3

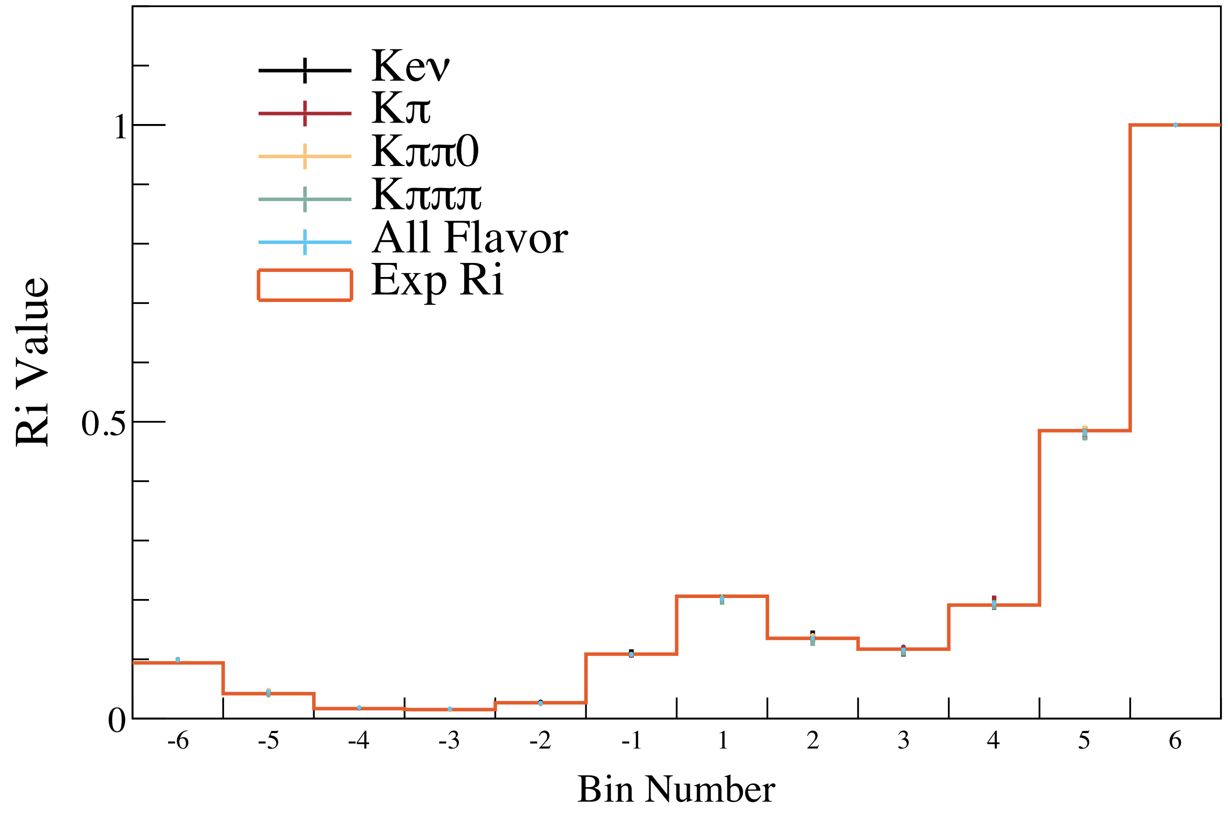
$N^{obs}_{sig} / \sigma^{obs}_{sig}$ >= 3

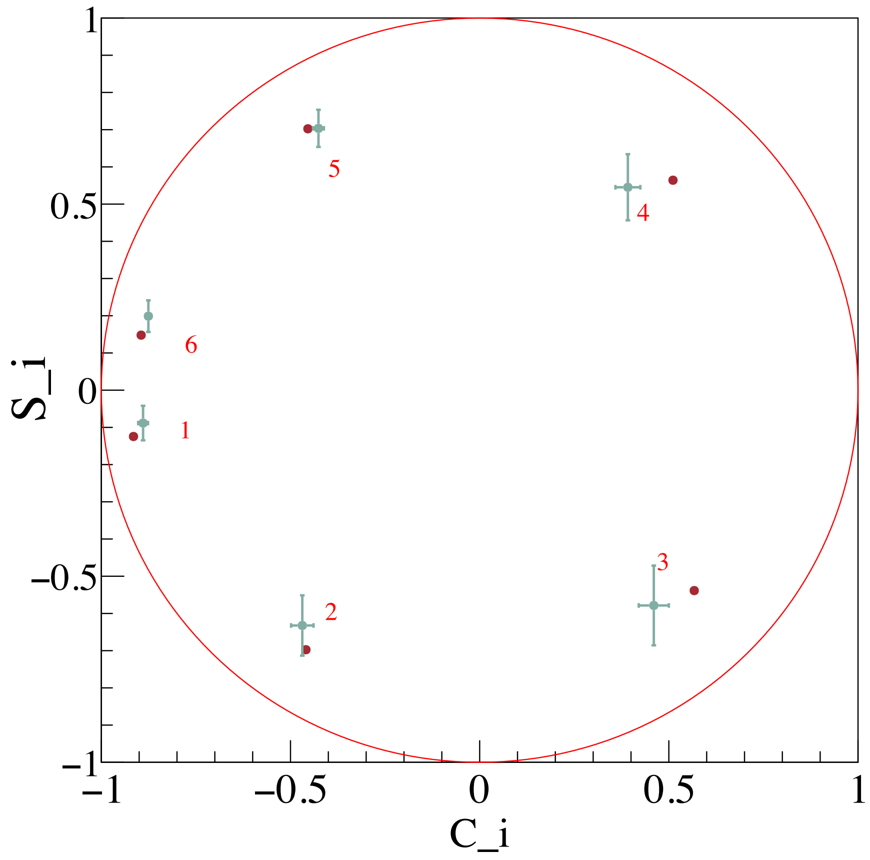
$$G(N;p) = Gaus(N; N^{obs}_{sig}, \sigma^{obs}_{sig})$$

Fit to N^{obs}_{sig} v.s. likelihood



Strong Phase Measurement
Fit Result





Red Line: Prediction by Amplitude Mode

Summary

Based on $20.3\ \mathrm{fb}^{-1}\ \psi(3770)$ data collected by BESIII

- ✓Finish the amplitude analysis of $D^0 \to K_S^0\pi^0\pi^+\pi^-$, and predict the CP-even fraction and longitudinal polarization fraction by model.
- ✓Obtain the binning strong phase parameters under 2×6 bins scheme optimized by the amplitude model.

☐ Next to do

- I/O check of the binning strong phase parameters.
- Systematic uncertainty study of the strong phase parameters.

Outline

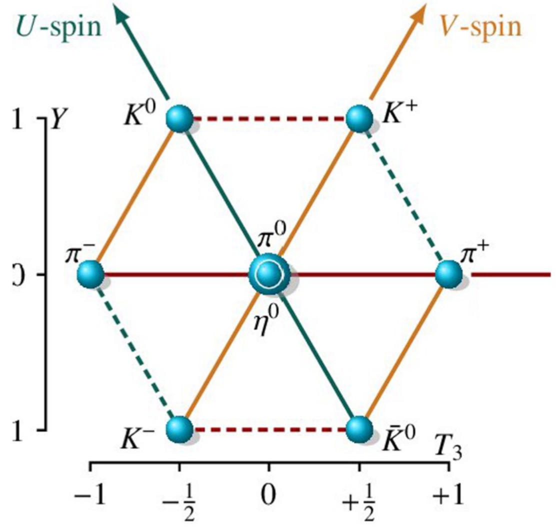
- Works on BESIII
  - Introduction
  - Study of $D^0/\bar{D}^0 \to K_S^0\pi^0\pi^+\pi^-$
  - Study of $D^0/\bar{D}^0 \to K_L^0\pi^0\pi^+\pi^-$
- Works on STCF
  - Introduction
  - Simulation and Reconstruction of STCF DTOF
- Summary and Prospect

Motivation

- The amplitude model can provide more information for the subsequent strong phase difference measurement.

- The CF $D^0 \to \bar{K}^0 X$ and DCS $D^0 \to K^0 X$ transitions coherently contribute to $D^0 \to K^0_{S/L} X$ decays, which leads to interference that induces asymmetries between $D^0 \to K^0_{S/L} X$ decay rates.

  The U-spin breaking features can be determined with the amplitude of $D^0 \to K^0_{L/S}\pi^+\pi^-\pi^0$.



The octet of 0^- mesons. Unitarity of hadrons, U-spin and V-spin.

Double Tag Method

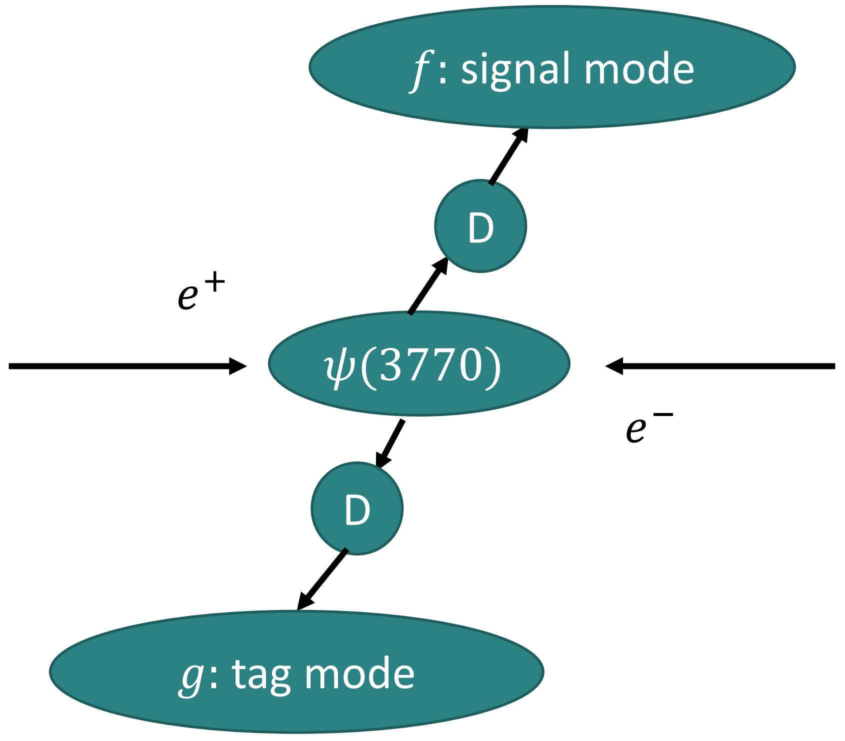
$20.3\ \mathrm{fb}^{-1}\ \psi(3770)$ Data Samples in BOSS version 7.1.2 used for $D^0 \to K_L^0\pi^0\pi^+\pi^-$ with decay mode $\pi^0 \to \gamma\gamma$.

- Tag Mode

  $\overline{D}^0 \to K^+\pi^-$
  $\overline{D}^0 \to K^+\pi^-\pi^0$
  $\overline{D}^0 \to K^+\pi^-\pi^+\pi^-$

- Signal Mode

  Signal $D^0 \to K_L^0\pi^0\pi^+\pi^-$



Event Selection

Initial Selection

- Good Charged tracks
  - ✓ |Rz| < 10.0 cm, |Rxy| < 1.0 cm
  - ✓ |cos θ| < 0.93
- Good Showers
  - ✓ $E > 25$ MeV for barrel ($|\cos\theta| < 0.8$)
  - ✓ $E > 50$ MeV for endcap ($0.86 < |\cos\theta| < 0.92$)
  - ✓ $0 \leq t_{TDC} \leq 14$ (× 50 ns)
  - ✓ $\theta_{\gamma-chrg} > 10°$
- Particle Identification (dE/dx and TOF)

  Use SimplePID
  - ✓ π: $CL_\pi > CL_K$, $CL_\pi > 0$
  - ✓ K: $CL_K > CL_\pi$, $CL_K > 0$
- $\pi^0 \to \gamma\gamma$ candidate
  - ✓ Reject γ both in endcap
  - ✓ $[0.115, 0.150]$ GeV/c^2, $\chi^2_{1C} < 50$

➢ Signal D^0 meson

$$E_{D^0} = E_{beam}, \qquad M_{D^0} = M_{D^0}^{PDG}$$

$\vec{P}_{D^0}$ direction is opposite to ST $\vec{P}_{\bar{D}^0}$

- $N_{\pi^+} = 1, N_{\pi^-} = 1$
- Select π^0 with $\theta_{\gamma-\mathrm{trk}_{\mathrm{miss}}} > 15°$ and least χ^2_{1C}
- Recoil K_L^0 : $\vec{P}_{miss} = \vec{P}_{D^0} - \vec{P}_{\pi^0\pi^+\pi^-}$
- Number of $\pi^\pm$ from K_S^0 candidate as 0
  - ✓ |Rz| < 20.0 cm
  - ✓ $|\cos\theta| < 0.93$
- 4C to obtain P_μ used for PWA
  - ✓ 4-momentum constrain

$$P_\mu^{miss,\pi^0,\pi^+,\pi^-} = P_\mu^{D^0}$$

Event Selection
Further Selection

ST Selection

- Cosmic ray rejection for $K^{\pm}\pi^{\mp}$ tag mode
- K_S^0 rejection for $K\pi\pi\pi$ tag mode
  $|M_{\pi^+\pi^-} - 0.4976| > 0.03\ \mathrm{GeV/c^2}$
- Minimum $|\Delta E|$ to select best ST D^0
- Cut of ΔE and M_{BC} are defined as:

$$\Delta E = E_{\bar{D}^0} - E_{beam}. \quad (\mu - 3.5\sigma, \mu + 3.5\sigma) \qquad M_{BC} = \sqrt{E_{beam}^2 - |\vec{p}_{\bar{D}^0}|^2}$$

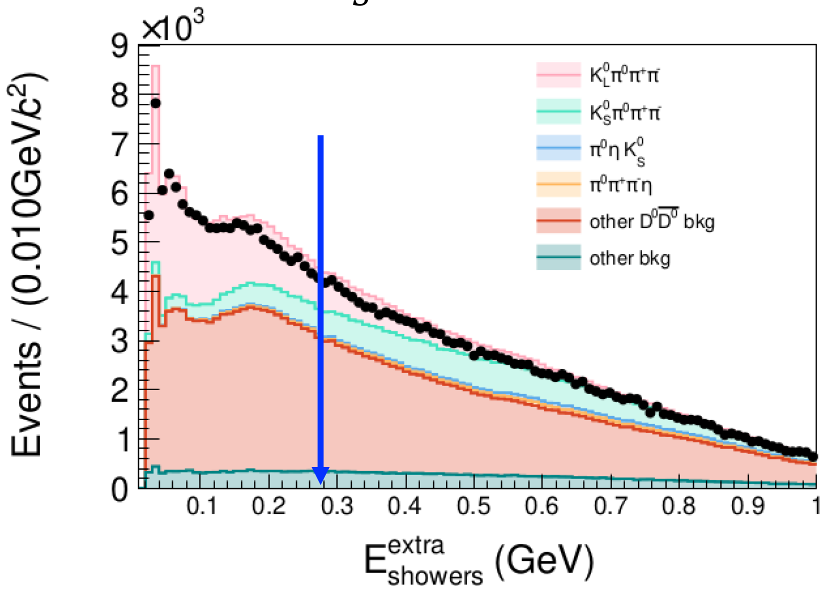
Tag mode	$\Delta \mathrm{E}\ [\mathrm{GeV}]$	$M_{BC}\ [\mathrm{GeV/c^2}]$
$\bar{D}^0 \to K^+\pi^-$	(-0.027, 0.027)	(1.8365, 1.8865)
$\bar{D}^0 \to K^+\pi^-\pi^0$	(-0.062, 0.049)	(1.8365, 1.8865)
$\bar{D}^0 \to K^+\pi^-\pi^+\pi^-$	(-0.026, 0.024)	(1.8365, 1.8865)

DT Selection

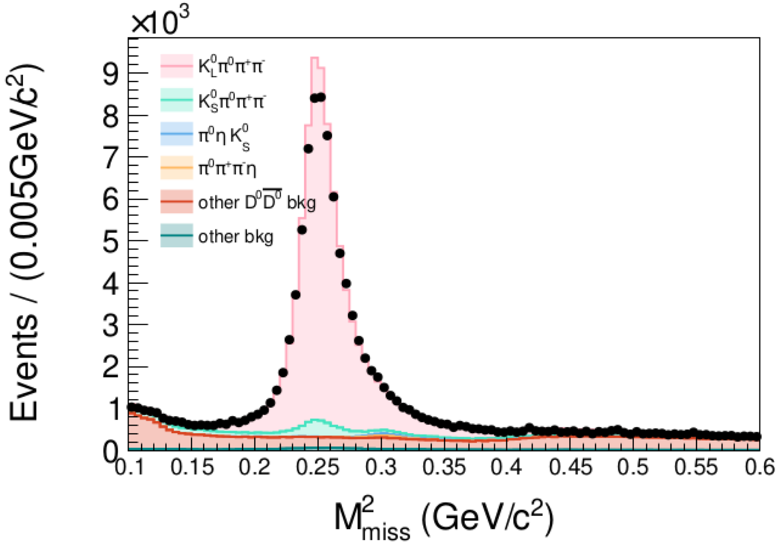
- $E_{showers}^{extra} < 0.27\ \mathrm{GeV}$

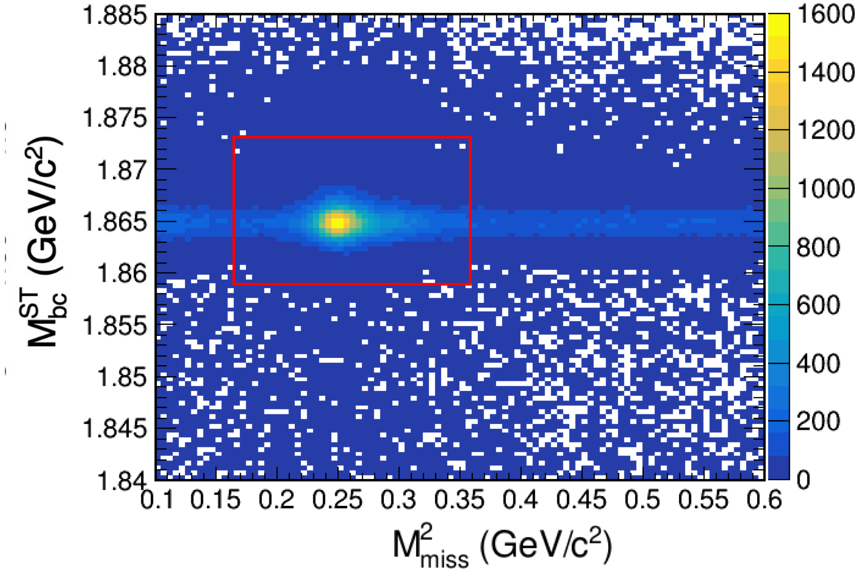
(showers with $\theta > 15°$ with $\vec{P}_{miss}$)

For $K_S^0\pi^0\pi^+\pi^-$ BKG



Background Analysis





- Signal region $(\mu - 3.5\sigma, \mu + 3.5\sigma)$
- $\mathrm{M}^2_{\mathrm{miss}}$: $(0.164, 0.359)$ [GeV]
- M^{ST}_{BC} : $(1.859, 1.873)$ [GeV]
- BKG in signal region: **~20%** (peak BKG~8%)

Signal yields ~ 290 k

- $\bar{D}^0 \to K^+\pi^-$: 76 k
- $\bar{D}^0 \to K^+\pi^-\pi^0$: 140 k
- $\bar{D}^0 \to K^+\pi^-\pi^0$: 75 k

Summary

✓ Finish the initial signal event selection of $D^0 \to K_L^0\pi^0\pi^+\pi^-$.

✓ Study the Background by the 40x Inclusive MC.

☐ Next to do

- K_L^0 tag method to further suppress the BKG.
- Amplitude analysis of $D^0 \to K_L^0\pi^0\pi^+\pi^-$.

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STCF Detector

Super Tau-Charm Facility is a natural extension for a post-BEPCII.

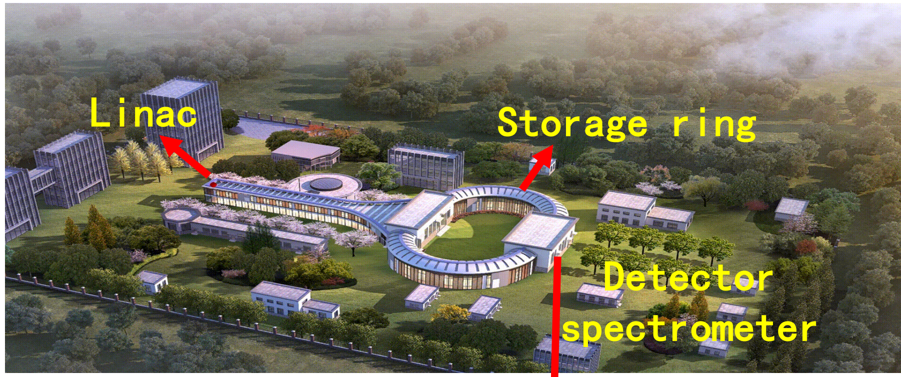
➤ Parameters of STCF:

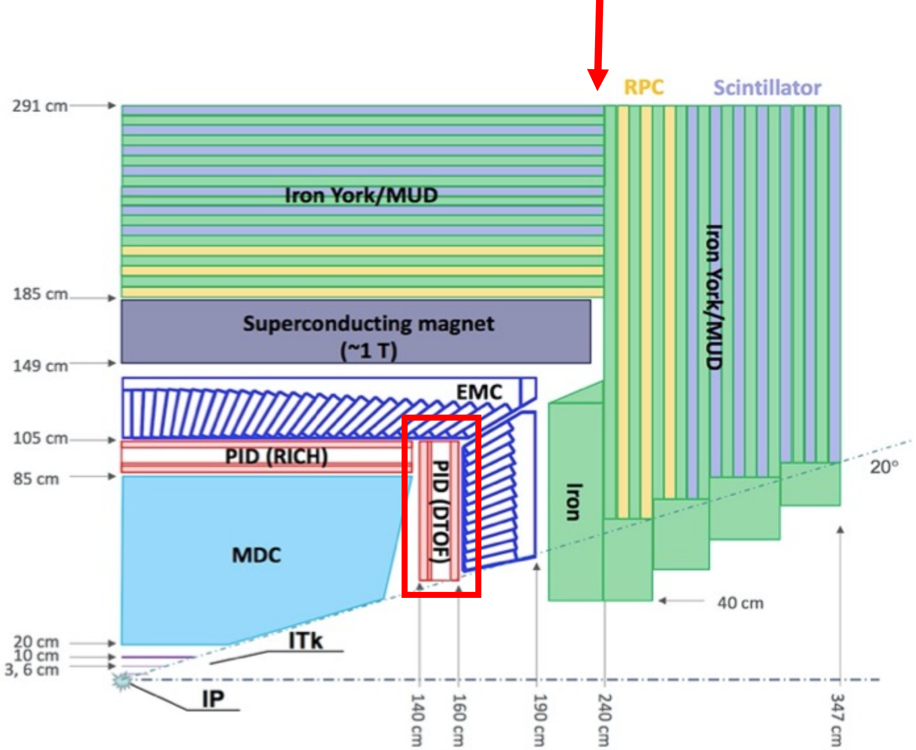
- $E_{c.m.} = 2 - 7$ GeV   Broader energy range
- Peak luminosity: $0.5\sim1\times10^{35}\mathrm{cm}^{-2}\mathrm{s}^{-1}$

2 orders larger than BEPCII

➤ DTOF Detector

- PID Detect in end-cap
  - ✓ Compact structure
  - ✓ Lower amount of substance
  - ✓ Large momentum working range
  - ✓ High effective photon yields ~ 30
  - ✓ PID requirement: π/K 4σ separation $p \leq 2$ GeV/c





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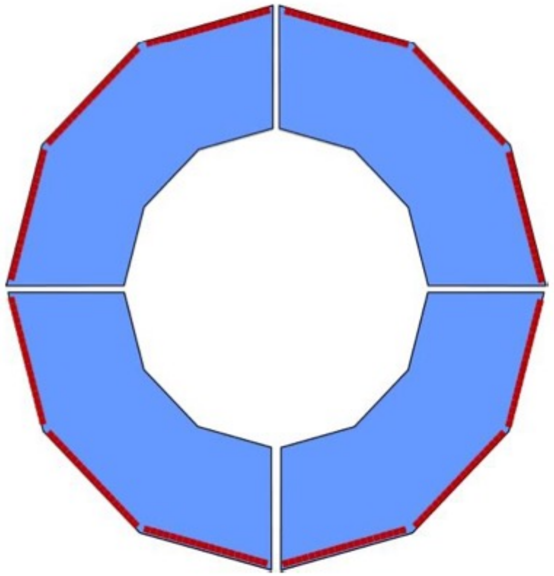
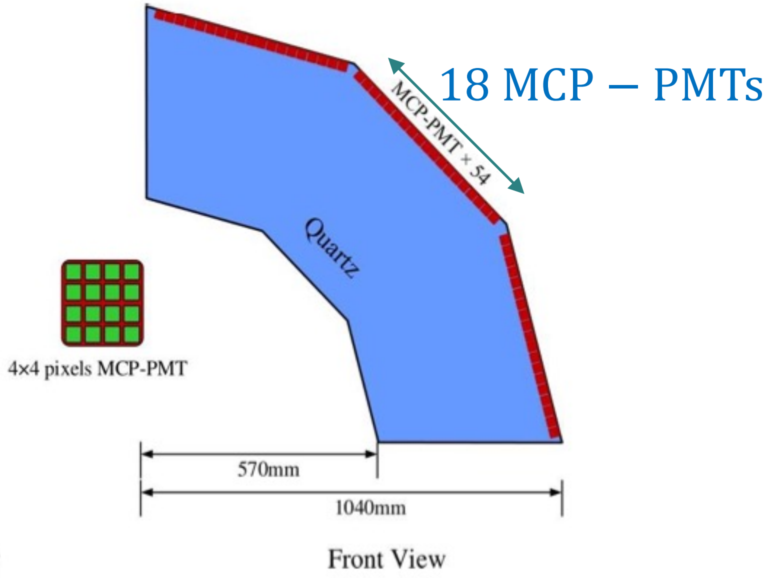
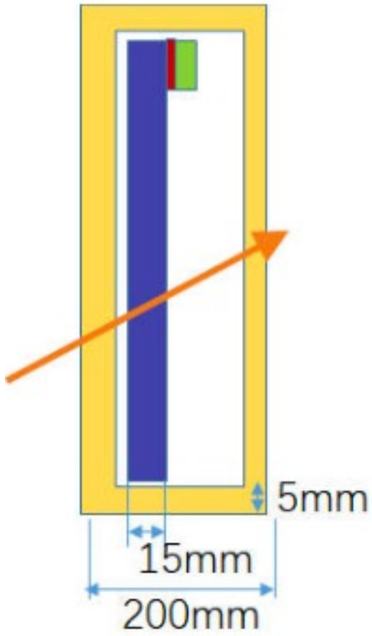
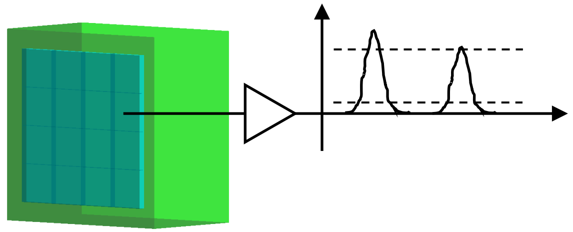
DTOF Geometry Configuration

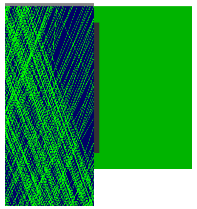
- Two identical endcap discs, $\sim \pm 1400$ mm away from the collision point along the beam direction.
- Each disc: 4 sectors, $R_{min} = 570$ mm , $R_{max} = 1050$ mm.
- Covering polar angles $\theta \in (22° - 36°)$.









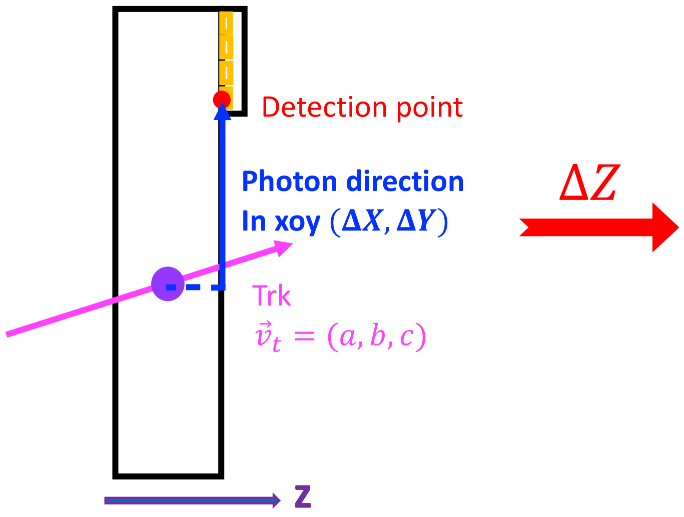


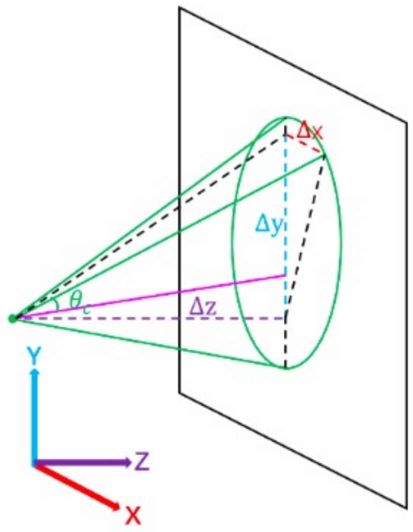
DTOF Reconstruction
Timing Method

- TOF Reconstruction

$$\cos\theta_c = \frac{1}{n\beta} = \frac{\vec{v}_t \cdot \vec{v}_p}{|\vec{v}_t| \cdot |\vec{v}_p|} \qquad \begin{cases} \vec{v}_t = (a, b, c) \\ \vec{v}_p = (\Delta X, \Delta Y, \Delta Z) \end{cases}$$

$$LOP = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2} \Longrightarrow TOF_{rec} = T - \frac{LOP \cdot \overline{n_g}}{c} - T_0$$

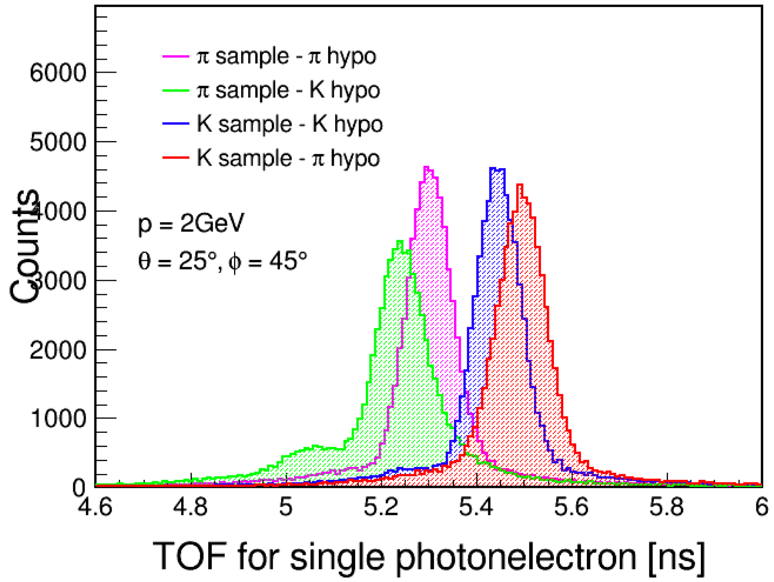




- Likelihood construction

$$\mathcal{L}_h = \prod_{i=1}^{N_{p.e.}} \overbrace{N_h S_h\left(TOF_{rec} \mid TOF_{hypo}\right)}^{\text{signal}} + \overbrace{0.05}^{\text{bkg}}$$

where $TOF_{hypo} = \frac{LOF}{c\beta_{hypo}}$



DTOF Reconstruction

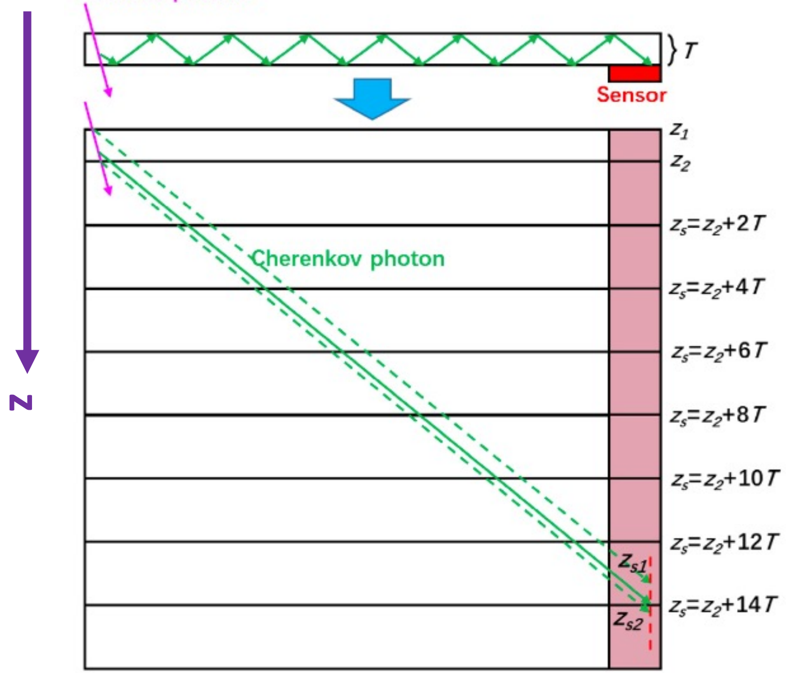
Imaging Method

- Photon TOA v.s. (x_s, y_s) Reconstruction

$$cos\theta_c = \frac{1}{n\beta} = \frac{\vec{v}_t \cdot \vec{v}_p}{|\vec{v}_t| \cdot |\vec{v}_p|} \qquad \begin{cases} \vec{v}_t = (a, b, c) \\ \vec{v}_p = (x_s - x_0, y_s - y_0, z_s - z_0) \end{cases}$$

$$z_s = z_2 + 2mT$$

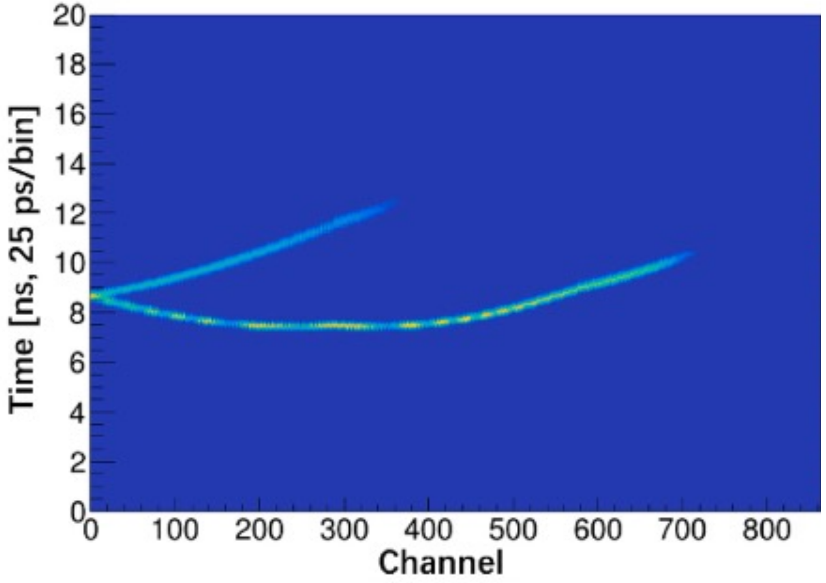
$$(x_s, y_s) \Longrightarrow z_e, \phi_c \Longrightarrow TOA = TOF + \frac{\Delta \mathrm{LOF_e}}{\beta c} + TOP$$



- Likelihood construction

$$\mathcal{L}_h = \prod_{i=1}^{N_{p.e.}} f_h(ch_i, t_i) = \prod_{i=1}^{N_{p.e.}} \bar{N}_h S_h(ch_i, t_i) + B$$

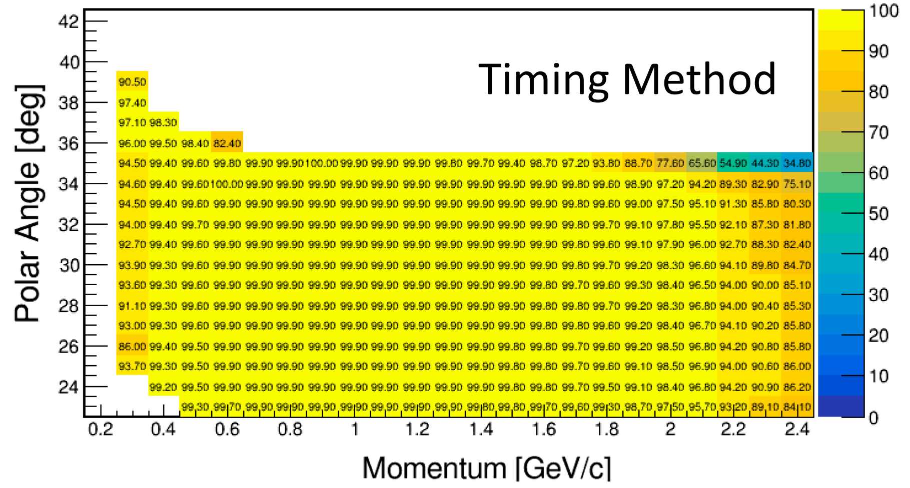
$$\sum_{ch_i, t_i} \bar{N}_h S_h(ch_i, t_i) = N_h$$

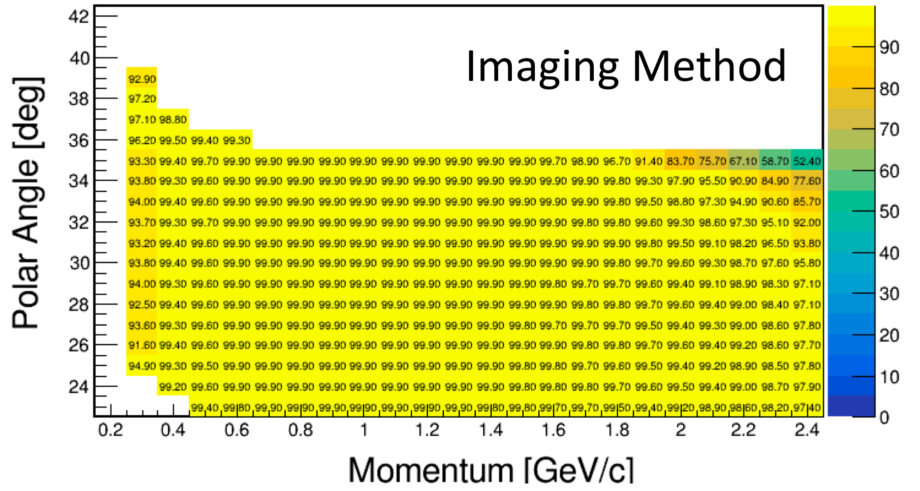


DTOF Reconstruction under the OSCAR

- Reconstruction Performance
  - Timing Method
    - $\sigma_t \sim$ 69 ps by single photon-electron
    - $\sigma_t \sim$ 50 ps by multi-photon-electrons
    - π efficiency $\sim$ 98% at p = 2.0 GeV/c
  - Imaging Method
    - π efficiency $\sim$ 99% at p = 2.0 GeV/c
    - Imaging method performed better at $p > 2$ GeV/c

π efficiency after mixing BKG in different $(|\vec{p}|, \theta)$: $(K\ \text{mis} - \text{ID} = 2\%)$





Summary

✓ Finish the simulation software of DTOF under the OSCAR.

✓ Establish the reconstruction algorithm of DTOF under the OSCAR

Timing Method

- π efficiency $\sim 98\%$, at $\mathrm{p} = 2.0\ \mathrm{GeV/c}$.
- Overall reconstructed TOF time resolution $\sim 50\ \mathrm{ps}$.

Imaging Method

- π efficiency $\sim 99\%$, at $p = 2.0\ \mathrm{GeV/c}$.

☐ Next to do

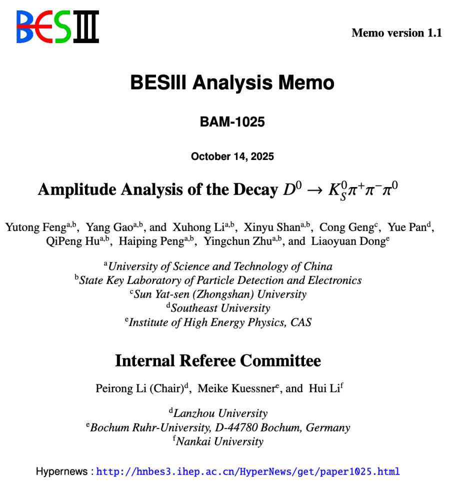
- Test the tracking PID efficiency in the physics process under the OSCAR.

Outline

- Works on BESIII
  - Introduction
  - Study of $D^0/\overline{D}^0 \to K_S^0\pi^0\pi^+\pi^-$
  - Study of $D^0/\overline{D}^0 \to K_L^0\pi^0\pi^+\pi^-$
- Works on STCF
  - Introduction
  - Simulation and Reconstruction of STCF DTOF
- Summary and Prospect

Summary and Prospect

- Study of $D^0/\bar{D}^0 \to K_S^0\pi^0\pi^+\pi^-$
  - ✓ Amplitude analysis and branching fraction Memo is reviewed in BESIII.
  - ✓ 2×6 strong phase difference result report at BESIII collaboration meeting.
  - ☐ I/O check and the systematic uncertainty of strong phase parameters.



BESIII Memo version 1.1

BESIII Analysis Memo

BAM-1025

October 14, 2025

Amplitude Analysis of the Decay $D^0 \to K_S^0\pi^+\pi^-\pi^0$

Yutong Feng[a,b], Yang Gao[a,b], and Xuhong Li[a,b], Xinyu Shan[a,b], Cong Geng[c], Yue Pan[d], QiPeng Hu[a,b], Haiping Peng[a,b], Yingchun Zhu[a,b], and Liaoyuan Dong[e]

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[c]*Sun Yat-sen (Zhongshan) University*
[d]*Southeast University*
[e]*Institute of High Energy Physics, CAS*

Internal Referee Committee

Peirong Li (Chair)[d], Meike Kuessner[e], and Hui Li[f]

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[f]*Nankai University*

Hypernews : http://hnbes3.ihep.ac.cn/HyperNews/get/paper1025.html

- Study of $D^0/\bar{D}^0 \to K_L^0\pi^0\pi^+\pi^-$
  - ✓ Preliminary selection and background study.
  - ☐ K_L^0 tag method to further suppress background, and the amplitude analysis.

- Simulation and Reconstruction of STCF DTOF
  - ✓ Simulation and reconstruction established under OSCAR.
  - ☐ Prepare the paper about the performance report of DTOF under OSCAR.

The 2024 International Workshop on Future Tau Charm Facilities， 2024.1.14-1.18

- Others Work

  I. Cross section measurement of $e^+e^- \to \eta J/\psi$ at center of mass energy between 3.81 and 4.95 GeV at BESIII. Published in Phys. Rev. D 109 (2024) 9, 092012.

  II. Update of the strong phase difference in $D^0 \to \pi^+\pi^-\pi^+\pi^-$. Amplitude analysis and strong phase are finished, and the I/O check and sys. uncertainty are ongoing...

Backup

Motivation

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \arg\left(\frac{c_{12}c_{13}s_{13}e^{i\delta}}{(s_{12}c_{23}+c_{12}s_{23}s_{13}e^{i\delta})(s_{23}c_{13})}\right)$$

$s_{13} \ll s_{23} \ll s_{12} \ll 1$, remain leading order

$$\gamma \approx \arg\left(\frac{c_{12}c_{13}s_{13}}{s_{12}c_{23}s_{23}c_{13}}e^{i\delta}\right) \approx \delta$$

$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = 91.58° \approx \frac{\pi}{2}$$

$$\therefore \beta = \pi - \alpha - \gamma \approx \frac{\pi}{2} - \delta$$

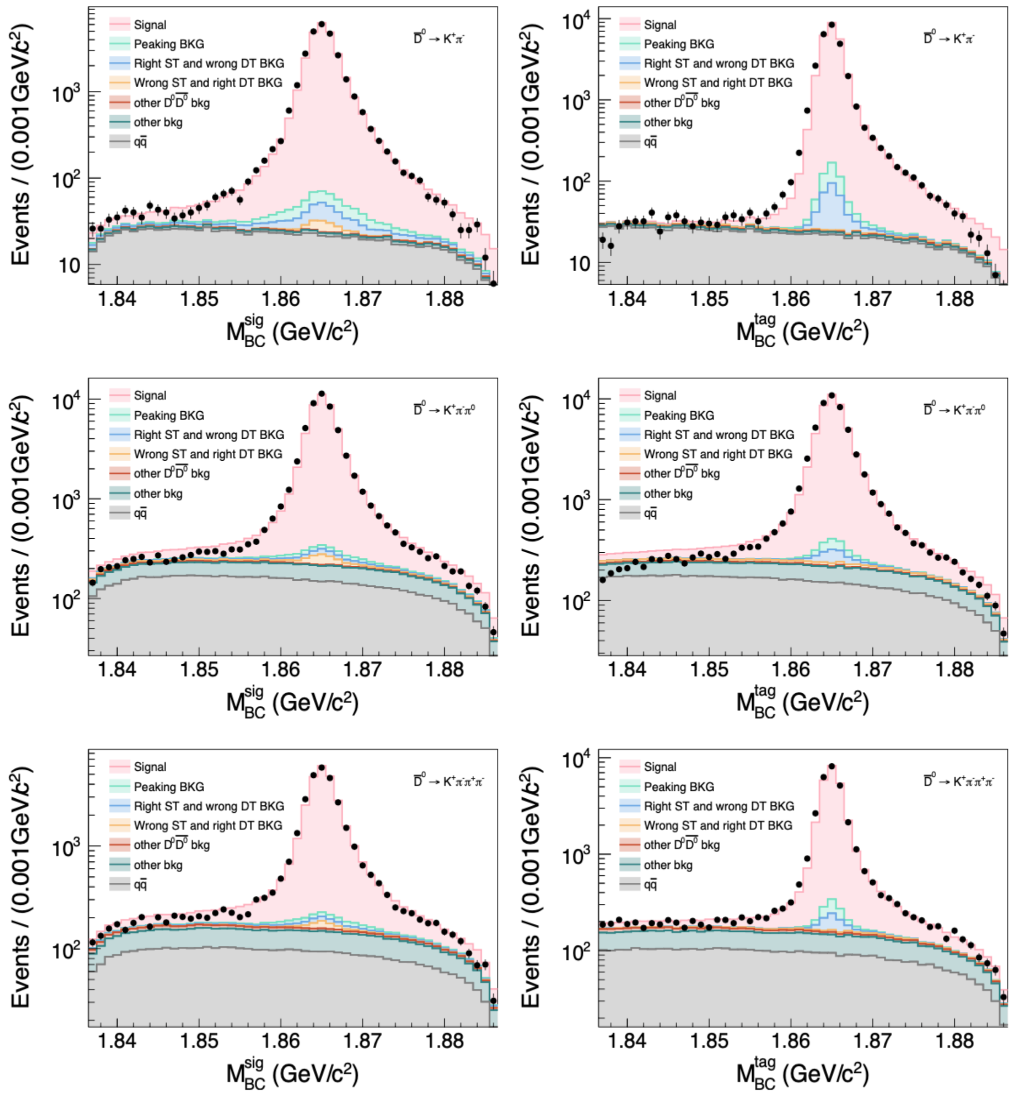
Background Analysis

Main Peaking BKG in DT:

- ✓ $D^0 \to K_S^0\eta',\ \eta' \to \gamma\pi^+\pi^-$
- ✓ $D^0 \to K^-\pi^+\pi^-\pi^+$
- ✓ $D^0 \to \pi^0 K_S^0\eta,\ \ \eta \to \gamma\pi^+\pi^-$

Peaking BKG have the corresponding Model, can be described well.

- BKG I:
  From $\psi(3770) \to D^0\,\bar{D}^0$ with wrong ST D^0 and right signal D^0.
- BKG II:
  From $\psi(3770) \to D^0\,\bar{D}^0$ with right ST D^0 and wrong signal D^0.
- BKG III:
  From $e^+e^- \to q\bar{q}$, distributed along the diagonal in the M_{sig}^{BC} v.s. M_{tag}^{BC} distribution.
- Peaking BKG:
  BKG with Peaking in both ST and signal M^{BC} spectrum.



Isospin Amplitude

Isospin states for hadrons: $|u\rangle : |\frac{1}{2},\frac{1}{2}\rangle,\ |\bar{u}\rangle : -|\frac{1}{2},-\frac{1}{2}\rangle,\ |d\rangle : |\frac{1}{2},-\frac{1}{2}\rangle,\ |\bar{d}\rangle : |\frac{1}{2},\frac{1}{2}\rangle$

	π^+,ρ^+	π^0,ρ^0,a^0	π^-,ρ^-
I=1	$\|u\bar{d}\rangle : \|1,1\rangle$	$\|\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}\rangle : -\|1,0\rangle$	$\|d\bar{u}\rangle : -\|1,-1\rangle$
	η,h,ω,ϕ,f		
I=0	$\|(u\bar{u}+d\bar{d})+g(s\bar{s})\rangle : -\|0,0\rangle$		

	K^+	K^0	$\bar{K}^0$	K^-
I=$\frac{1}{2}$	$\|u\bar{s}\rangle : \|\frac{1}{2},+\frac{1}{2}\rangle$	$\|d\bar{s}\rangle : \|\frac{1}{2},-\frac{1}{2}\rangle$	$\|\bar{d}s\rangle : \|\frac{1}{2},+\frac{1}{2}\rangle$	$\|\bar{u}s\rangle : -\|\frac{1}{2},-\frac{1}{2}\rangle$

Isospin symmetry in strong decay (CG Coefficients):

- $D^0 \to K\pi$:

$$A(D^0 \to K^-\pi^+) = A\left(\rho^-_{\pi^0\pi^-}K^0_S,\pi^+\right) + c\left[A\left(K^-_{K^0_S\pi^-}\pi^0,\pi^+\right) - A\left(\bar{K}^0_{K^0_S\pi^0}\pi^-,\pi^+\right)\right]$$

$$A(D^0 \to K^+\pi^-) = A\left(\rho^+_{\pi^0\pi^+}K^0_S,\pi^-\right) + c\left[A\left(K^+_{K^0_S\pi^+}\pi^0,\pi^-\right) - A\left(K^0_{K^0_S\pi^0}\pi^+,\pi^-\right)\right]$$

$$A(D^0 \to \bar{K}^0\pi^0) = A\left(\rho^0_{\pi^+\pi^-}K^0_S,\pi^0\right) + c2A\left(K^-_{K^0_S\pi^-}\pi^+,\pi^0\right)$$

$$A(D^0 \to K^0\pi^0) = A\left(\rho^0_{\pi^+\pi^-}K^0_S,\pi^0\right) - c2A\left(K^+_{K^0_S\pi^+}\pi^-,\pi^0\right)$$

- $D^0 \to a_1\pi$:

$$A(D^0 \to a_1^0\pi^0) = -A\left(\rho^+_{\pi^+\pi^0}\pi^-,K^0_S\right) + A\left(\rho^-_{\pi^0\pi^-}\pi^+,K^0_S\right) - 2cA(f_{\pi^+\pi^-}\pi^0,K^0_S)$$

- $D^0 \to h_1\pi$:

$$A(D^0 \to h_1\pi^0) = -A\left(\rho^+_{\pi^+\pi^0}\pi^-,K^0_S\right) - A\left(\rho^-_{\pi^0\pi^-}\pi^+,K^0_S\right) + A(\rho^0_{\pi^+\pi^-}\pi^0,K^0_S)$$

Spin Factor

Decay Chain	Spin Factor
$D[S] \to P\ P_1,\ P[S] \to S\ P_2,\ S[S] \to P_3\ P_4$	1
$D[S] \to P\ P_1,\ P[P] \to V\ P_2,\ V[P] \to P_3\ P_4$	$\tilde{t}_\mu(P)\tilde{t}^\mu(V)$
$D[S] \to P\ P_1,\ P[D] \to T\ P_2,\ T[D] \to P_3\ P_4$	$\tilde{t}_{\mu\nu}(P)\tilde{t}^{\mu\nu}(T)$
$D[P] \to A\ P_1,\ A[S] \to V\ P_2,\ V[P] \to P_3\ P_4$	$\tilde{t}_\mu(D)P^{\mu\nu}(A)\tilde{t}_\nu(V)$
$D[P] \to A\ P_1,\ A[D] \to V\ P_2,\ V[P] \to P_3\ P_4$	$\tilde{t}_\mu(D)\tilde{t}^{\mu\nu}(A)\tilde{t}_\nu(V)$
$D[P] \to A\ P_1,\ A[P] \to S\ P_2,\ S[S] \to P_3\ P_4$	$\tilde{t}_\mu(D)\tilde{t}^\mu(A)$
$D[P] \to A\ P_1,\ A[P] \to T\ P_2,\ T[D] \to P_3\ P_4$	$\tilde{t}_\mu(D)\tilde{t}_\nu(A)P^{\mu\nu\rho\sigma}(A)\tilde{t}_{\rho\sigma}(T)$
$D[P] \to A\ P_1,\ A[F] \to T\ P_2,\ T[D] \to P_3\ P_4$	$\tilde{t}_\mu(D)\tilde{t}^{\mu\nu\rho}(A)\tilde{t}_{\nu\sigma}(T)$
$D[P] \to V_1\ P_1,\ V_1[P] \to V_2\ P_2,\ V_2[P] \to P_3\ P_4$	$\tilde{t}^\mu(D)\epsilon_{\mu\nu\rho\sigma}p^\sigma_{V_1}\tilde{t}^\rho(V_1)\tilde{t}^\nu(V_2)$
$D[D] \to PT\ P_1,\ PT[S] \to T\ P_2,\ T[D] \to P_3\ P_4$	$\tilde{t}_{\mu\nu}(D)P^{\mu\nu\alpha\beta}(PT)\tilde{t}_{\alpha\beta}(T)$
$D[D] \to PT\ P_1,\ PT[D] \to T\ P_2,\ T[D] \to P_3\ P_4$	$\tilde{t}_{\mu\nu}(D)P^{\mu\nu\rho\sigma}(PT)\tilde{t}^\alpha_\rho(PT)P_{\alpha\sigma\beta\gamma}(PT)\tilde{t}^{\beta\gamma}(T)$
$D[D] \to PT\ P_1,\ PT[P] \to V\ P_2,\ V[P] \to P_3\ P_4$	$\tilde{t}_{\mu\nu}(D)P^{\mu\nu\alpha\beta}(PT)\tilde{t}_\alpha(PT)\tilde{t}_\beta(V)$
$D[D] \to PT\ P_1,\ PT[F] \to V\ P_2,\ V[P] \to P_3\ P_4$	$\tilde{t}_{\mu\nu}(D)\tilde{t}^{\mu\nu\rho}(PT)\tilde{t}_\rho(V)$
$D[D] \to PT\ P_1,\ PT[D] \to S\ P_2,\ S[S] \to P_3\ P_4$	$\tilde{t}_{\mu\nu}(D)\tilde{t}^{\mu\nu}(PT)$
$D[D] \to T\ P_1,\ T[D] \to V\ P_2,\ V[P] \to P_3\ P_4$	$\tilde{t}^{\mu\alpha}(D)\epsilon_{\mu\nu\rho\sigma}p^\sigma_T\tilde{t}^\nu_\alpha(T)\tilde{t}^\rho(V)$
$D[D] \to T_1\ P_1,\ T_1[P] \to T_2\ P_2,\ T_2[D] \to P_3\ P_4$	$\tilde{t}_{\mu\nu}(D)P^{\mu\nu\alpha\beta}(T_1)\epsilon_{\alpha\lambda\sigma\rho}p^\rho_{T_1}\tilde{t}^\lambda(T_1)\tilde{t}^\sigma_\beta(T_2)$
$D[S] \to S_1\ S_2,\ S_1[S] \to P_1\ P_2,\ S_2[S] \to P_3\ P_4$	1
$D[P] \to V\ S,\ V[P] \to P_1\ P_2,\ S[S] \to P_3\ P_4$	$\tilde{t}_\mu(D)\tilde{t}^\mu(V)$
$D[S] \to V_1\ V_2,\ V_1[P] \to P_1\ P_2,\ V_2[P] \to P_3\ P_4$	$P^{\mu\nu}(D)\tilde{t}_\mu(V_1)\tilde{t}^\nu(V_2)$
$D[P] \to V_1\ V_2,\ V_1[P] \to P_1\ P_2,\ V_2[P] \to P_3\ P_4$	$\epsilon_{\mu\nu\alpha\beta}p^\beta_D\tilde{t}^\mu(D)\tilde{t}^\nu(V_1)\tilde{t}^\alpha(V_2)$
$D[D] \to V_1\ V_2,\ V_1[P] \to P_1\ P_2,\ V_2[P] \to P_3\ P_4$	$\tilde{t}_{\mu\nu}(D)\tilde{t}^\mu(V_1)\tilde{t}^\nu(V_2)$
$D[D] \to T\ S,\ T[D] \to P_1\ P_2,\ S[S] \to P_3\ P_4$	$\tilde{t}_{\mu\nu}(D)\tilde{t}^{\mu\nu}(T)$
$D[P] \to T\ V,\ T[D] \to P_1\ P_2,\ V[P] \to P_3\ P_4$	$\tilde{t}_\mu(D)P^{\mu\nu\rho\sigma}(D)\tilde{t}_{\rho\sigma}(T)\tilde{t}_\nu(V)$
$D[D] \to T\ V,\ T[D] \to P_1\ P_2,\ V[P] \to P_3\ P_4$	$\epsilon_{\mu\nu\alpha\beta}p^\beta_D\tilde{t}^{\mu\rho}(D)\tilde{t}^\nu_\rho(T)\tilde{t}^\alpha(V)$
$D[S] \to T_1\ T_2,\ T_1[D] \to P_1\ P_2,\ T_2[D] \to P_3\ P_4$	$\tilde{t}_{\mu\nu}(T_1)P^{\mu\nu\rho\sigma}(D)\tilde{t}_{\rho\sigma}(T_2)$
$D[P] \to T_1\ T_2,\ T_1[D] \to P_1\ P_2,\ T_2[D] \to P_3\ P_4$	$\epsilon_{\mu\nu\rho\sigma}p^\sigma_D\tilde{t}^\rho(D)P_{\alpha\beta}(D)\tilde{t}^{\mu\alpha}(T_1)\tilde{t}^{\nu\beta}(T_2)$
$D[D] \to T_1\ T_2,\ T_1[D] \to P_1\ P_2,\ T_2[D] \to P_3\ P_4$	$\tilde{t}^\nu_\mu(D)P^{\mu\alpha\beta\gamma}(D)\tilde{t}_{\beta\gamma}(T_1)P_{\nu\alpha\rho\sigma}(D)\tilde{t}^{\rho\sigma}(T_2)$

- **Blatt-Weisskopf barrier factor**

$$B_0(q) = 1$$

$$B_1(q) = \sqrt{\frac{2}{q^2 + q_0^2}}$$

$$B_2(q) = \sqrt{\frac{13}{q^4 + 3q^2q_0^2 + 9q_0^4}}$$

q: breakup momentum

$q_0 = 0.197321/R\ GeV/c$

$R = 5 * 0.197321 \approx 1\ fm$ for D0

$R = 3 * 0.197321 \approx 0.6\ fm$ for other resonance

- **Spin projection operator:**

$$P^{(0)}(p_a) = 1$$

$$P^{(1)}_{\mu\mu'}(p_a) = -g_{\mu\mu'} + \frac{p_{a\mu}p_{a\mu'}}{p_a^2}$$

$$P^{(2)}_{\mu\nu\mu'\nu'}(p_a) = \frac{1}{2}[P^{(1)}_{\mu\mu'}(p_a)P^{(1)}_{\nu\nu'}(p_a) + P^{(1)}_{\mu\nu'}(p_a)P^{(1)}_{\nu\mu'}(p_a)] - \frac{1}{3}P^{(1)}_{\mu\nu}(p_a)P^{(1)}_{\mu'\nu'}(p_a)$$

- **Pure orbital angular momentum**

$$\tilde{t}^{(L)}_{\mu_1\cdots\mu_l} = (-1)^L P^{(L)}_{\mu_1\cdots\mu_L\mu'_1\cdots\mu'_L}(p_a)r^{\mu'_1}\cdots r^{\mu'_L}$$

Amplitude Analysis

Fit Result

Amplitude	Magitude	Phase (rad)	Fit Fraction(%)	Significance (σ)	Decay Chain
ωK_S^0	2.000 (fixed)	0.000 (fixed)	21.10 ± 0.15	$> 10.0\sigma$	
ϕK_S^0	0.103 ± 0.002	1.009 ± 0.024	1.04 ± 0.04	$> 10.0\sigma$	
$\omega(1420)$	2.852 ± 0.098	2.946 ± 0.033	2.84 ± 0.19	6.0σ	
$\omega(1650)$	9.355 ± 0.231	0.747 ± 0.025	6.73 ± 0.35	8.5σ	VP
$K(1410)^-\pi^+$ [$K(1410) \to K_S^0\rho(770)$]	0.584 ± 0.057	0.365 ± 0.102	0.10 ± 0.02	6.7σ	
$\bar{K}^*(1680)^0\pi^0$	1.518 ± 0.064	−1.426 ± 0.042	0.91 ± 0.06	$> 10.0\sigma$	
$K^*(1680)^-\pi^+$	1.592 ± 0.077	1.442 ± 0.052	0.70 ± 0.06	$> 10.0\sigma$	
$K^*(1680)^+\pi^-$	0.889 ± 0.059	−0.434 ± 0.064	0.22 ± 0.03	$> 10.0\sigma$	
$\pi(1300)K_S^0$	6.036 ± 0.171	3.700 ± 0.030	2.03 ± 0.11	$> 10.0\sigma$	
$\bar{K}(1460)^0\pi^0$	1.927 ± 0.057	2.159 ± 0.029	2.03 ± 0.16	$> 10.0\sigma$	PP
$K(1460)^-\pi^+$	3.602 ± 0.098	0.174 ± 0.027	1.61 ± 0.09	$> 10.0\sigma$	
$h_1(1170)K_S^0$	2.446 ± 0.090	−1.427 ± 0.037	0.69 ± 0.05	$> 10.0\sigma$	
$a_1(1260)K_S^0$	12.484 ± 0.185	1.464 ± 0.016	10.49 ± 0.30	$> 10.0\sigma$	
$a_1(1640)K_S^0$	10.964 ± 0.509	−0.146 ± 0.043	1.55 ± 0.15	9.2σ	
$K_1(1270)^-\pi^+$	6.498 ± 0.097	4.183 ± 0.019	6.78 ± 0.15	$> 10.0\sigma$	
$\bar{K}_1(1270)^0\pi^0$	2.760 ± 0.062	1.281 ± 0.027	1.47 ± 0.06	$> 10.0\sigma$	AP
$\bar{K}_1(1400)^0\pi^0$	2.201 ± 0.036	0.616 ± 0.018	4.04 ± 0.12	$> 10.0\sigma$	
$K_1(1400)^-\pi^+$	0.756 ± 0.052	3.444 ± 0.068	0.23 ± 0.03	$> 10.0\sigma$	
$\bar{K}_1(1650)^0\pi^0$	1.974 ± 0.102	0.104 ± 0.052	0.52 ± 0.04	$> 10.0\sigma$	
$K_1(1650)^-\pi^+$	1.966 ± 0.118	3.376 ± 0.060	0.26 ± 0.03	$> 10.0\sigma$	
$a_2(1320)K_S^0$	0.103 ± 0.008	−1.300 ± 0.080	0.09 ± 0.01	$> 10.0\sigma$	
$K_2^*(1430)^-\pi^+$	0.058 ± 0.002	1.766 ± 0.046	0.35 ± 0.03	$> 10.0\sigma$	TP
$\bar{K}_2^*(1430)^0\pi^0$	0.036 ± 0.002	4.457 ± 0.050	0.20 ± 0.02	$> 10.0\sigma$	
$K^*(892)^-\rho(770)^+$	-	-	38.49 ± 0.39	-	
$K^*(892)^-\rho(770)^+[S]$	3.320 ± 0.039	0.895 ± 0.014	7.63 ± 0.17	$> 10.0\sigma$	
$K^*(892)^-\rho(770)^+[P]$	1.415 ± 0.012	4.706 ± 0.011	16.38 ± 0.25	$> 10.0\sigma$	
$K^*(892)^-\rho(770)^+[D]$	0.772 ± 0.008	0.022 ± 0.013	11.22 ± 0.22	$> 10.0\sigma$	
$\bar{K}^*(892)^0\rho(770)^0$	-	-	4.09 ± 0.10	-	
$\bar{K}^*(892)^0\rho(770)^0[S]$	1.370 ± 0.029	2.601 ± 0.024	1.33 ± 0.05	$> 10.0\sigma$	
$\bar{K}^*(892)^0\rho(770)^0[P]$	0.440 ± 0.009	−1.526 ± 0.023	1.62 ± 0.07	$> 10.0\sigma$	
$\bar{K}^*(892)^0\rho(770)^0[D]$	0.319 ± 0.007	−0.094 ± 0.024	1.93 ± 0.08	$> 10.0\sigma$	VV
$K^*(892)^-\rho(1450)^+[S]$	17.424 ± 0.482	3.932 ± 0.030	2.45 ± 0.14	$> 10.0\sigma$	
$K^*(892)^-\rho(1450)^+[P]$	3.189 ± 0.151	2.127 ± 0.045	0.89 ± 0.08	$> 10.0\sigma$	
$K^*(892)^-\rho(1450)^+[D]$	1.478 ± 0.080	0.456 ± 0.060	0.49 ± 0.05	$> 10.0\sigma$	
$\bar{K}^*(892)^0\rho(1450)^0$ [D]	1.157 ± 0.064	0.806 ± 0.059	0.30 ± 0.03	$> 10.0\sigma$	
$K^*(1680)^-\rho(770)^+$ [S]	19.533 ± 0.990	−0.272 ± 0.049	0.57 ± 0.06	$> 10.0\sigma$	
$(K_S^0\pi^0)^{L=1}\rho(1450)^0$ [P]	36.597 ± 1.242	1.418 ± 0.033	1.24 ± 0.08	$> 10.0\sigma$	

Amplitude	Magitude	Phase (rad)	Fit Fraction(%)	Significance (σ)
$(K_S^0\pi^-)^{L=0}\rho(770)^+$	197.265 ± 0.582	4.051 ± 1.611	1.48 ± 0.11	$> 10.0\sigma$
R	−0.005 ± 0.100		-	-
ϕ_R	−0.178 ± 0.395		-	-
ϕ_F	−0.066 ± −0.003		-	-
$(K_S^0\pi^0)^{L=0}\rho(770)^0$	579.148 ± 2.074	−0.906 ± 1.626	0.45 ± 0.04	$> 10.0\sigma$
R	−0.063 ± 1.336		-	-
ϕ_R	2.719 ± 0.013		-	-
ϕ_F	−3.167 ± −0.001		-	-
$\bar{K}^*(892)^0(\pi^+\pi^-)^{L=0}$	-	-	1.31 ± 0.07	$> 10.0\sigma$
β_5	260.436 ± 8.910	1.332 ± 0.033	-	-
$f_{\pi\pi}^{\rm prod}$	6.534 ± 0.193	3.751 ± 0.031	-	-
$\bar{K}^*(1680)^0(\pi^+\pi^-)^{L=0}$	-	-	0.89 ± 0.06	$> 10.0\sigma$
β_1	207.487 ± 4.158	3.876 ± 0.018	-	-
β_3	2355.190 ± 47.293	0.426 ± 0.017	-	-
$f_{\eta\eta}^{\rm prod}$	94.244 ± 10.926	0.229 ± 0.108	-	-
$(K_S^0\pi^0)^{L=0}(\pi^+\pi^-)^{L=0}$	-	-	6.87 ± 0.18	$> 10.0\sigma$

Amplitude	Magitude	Phase (rad)	Relative Fit Fraction(%)	Significance (σ)
$K_1(1270) \to \rho(770)K_S^0$ [S]	1 (fixed)	0 (fixed)	76.84 ± 1.84	$> 10.0\sigma$
$K_1(1270) \to \rho(1450)K_S^0$ [D]	0.650 ± 0.052	2.659 ± 0.086	1.89 ± 0.31	8.8σ
$K_1(1270) \to K^*(892)\pi$ [S]	0.134 ± 0.005	0.628 ± 0.034	11.93 ± 0.68	$> 10.0\sigma$
$K_1(1270) \to K^*(892)\pi$ [D]	0.036 ± 0.002	0.184 ± 0.058	2.59 ± 0.28	$> 10.0\sigma$
$K_1(1270) \to (K_S^0\pi)^{L=0}\pi$ [P]	123.600 ± 0.550	2.743 ± 1.623	10.29 ± 0.95	$> 10.0\sigma$
R	−0.002 ± 0.028		-	-
ϕ_R	0.731 ± 0.262		-	-
ϕ_F	−0.007 ± 0.000		-	-
$K_1(1400) \to K^*(892)\pi$ [S]	1 (fixed)	0 (fixed)	102.28 ± 0.22	$> 10.0\sigma$
$K_1(1400) \to \rho(770)K_S^0$ [D]	0.184 ± 0.016	−0.420 ± 0.088	1.83 ± 0.32	8.4σ
$K_2^*(1430) \to K^*(892)\pi$	1 (fixed)	0 (fixed)	65.45 ± 3.10	$> 10.0\sigma$
$K_2^*(1430) \to \rho(770)K_S^0$	1.382 ± 0.118	0.112 ± 0.089	19.25 ± 2.49	$> 10.0\sigma$
$K(1460) \to K^*(892)\pi$	1 (fixed)	0 (fixed)	45.61 ± 3.16	$> 10.0\sigma$
$K(1460) \to (K_S^0\pi)^{L=0}\pi$	108.928 ± 2.564	−1.077 ± 1.696	54.56 ± 3.18	$> 10.0\sigma$
R	0.249 ± 0.477		-	-
ϕ_R	0.962 ± 0.039		-	-
ϕ_F	3.073 ± −0.032		-	-
$K_1(1650) \to \rho(770)K_S^0$ [S]	1 (fixed)	0 (fixed)	32.77 ± 2.79	$> 10.0\sigma$
$K_1(1650) \to K^*(892)\pi$ [S]	0.371 ± 0.029	0.335 ± 0.074	32.47 ± 3.64	8.2σ
$K_1(1650) \to K_S^0(\pi^+\pi^-)^{L=0}$	-	-	44.85 ± 4.45	8.7σ
β_1	2.995 ± 0.896	0.622 ± 0.299	-	-
β_2	15.620 ± 2.046	−1.201 ± 0.139	-	-
$f_{\eta\eta}^{\rm prod}$	34.540 ± 2.983	3.962 ± 0.086	-	-
$K^*(1680) \to \rho(770)K_S^0$ [P]	1 (fixed)	0 (fixed)	20.07 ± 1.21	$> 10.0\sigma$
$K^*(1680) \to K^*(892)\pi$ [P]	0.613 ± 0.025	2.684 ± 0.042	63.74 ± 1.51	$> 10.0\sigma$

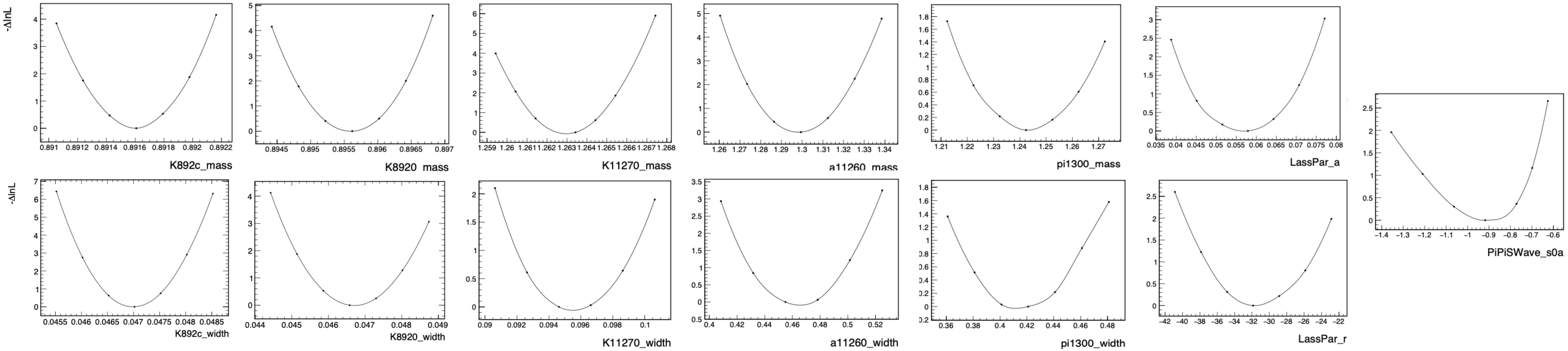
Amplitude Analysis

Fit Result

- The parameters of $a_1(1260), \pi(1300), K_1(1270)$ and $K\pi$ S-wave will be determined in our fit due to their dominant contribution in the channel and inaccurate results from PDG.

	This Work		PDG [49]	
Resonance	Mass(GeV/c^2)	Width(GeV)	Mass(GeV/c^2)	Width(GeV)
$a_1(1260)$	1.298 ± 0.013	0.465 ± 0.024	1.230 ± 0.040	$0.250 \sim 0.600$
$\pi(1300)$	1.243 ± 0.017	0.412 ± 0.034	1.300 ± 0.100	$0.200 \sim 0.600$
$K_1(1270)$	1.263 ± 0.001	0.096 ± 0.003	1.253 ± 0.007	0.090 ± 0.020
$K^*(892)^-$	0.8916 ± 0.0002	0.0470 ± 0.0004	0.8917 ± 0.0003	0.0514 ± 0.0008
$\bar{K}^*(892)^0$	0.8956 ± 0.0004	0.0467 ± 0.0008	0.8956 ± 0.0002	0.0473 ± 0.0005

Parameter	Value
$K\pi$ S-wave LASS model	
a	0.058 ± 0.009
r	-31.795 ± 4.254
$\pi\pi$ S-wave K-matrix model	
s_0^{Prod}	$-0.8956^{+0.1399}_{-0.8956}$



Amplitude Analysis
Longitudinal Polarization fraction

Covariant Helicity Amplitude for $D \to V_1 V_2 \to (P_1 P_2)(P_3 P_4)$

$$H_{\lambda_{V_1},\lambda_{V_2}} = F^{J_D}_{\lambda_{V_1},\lambda_{V_2}} D^{J_D}_{\lambda_D,\lambda_{V_1}-\lambda_{V_2}}(\Omega_D) \cdot T_{V_1}(m_{V_1})\, F^{J_{V_1}}_{\lambda_{P_1},\lambda_{P_2}}\, D^{J_{V_1}}_{\lambda_{V_1},\lambda_{P_1}-\lambda_{P_2}}(\Omega^{\{D\}}_{V_1}) \cdot T_{V_2}(m_{V_2})\, F^{J_{V_2}}_{\lambda_{P_3},\lambda_{P_4}}\, D^{J_{V_2}}_{\lambda_{V_2},\lambda_{P_3}-\lambda_{P_4}}(\Omega^{\{D\}}_{V_2})$$

$$J_D = 0 \implies \lambda_{V_1} = \lambda_{V_2} \quad \therefore |A|^2 = \left| \sum_{\lambda_{V_1}\lambda_{V_2}} H_{\lambda_{V_1}\lambda_{V_2}} \right|^2 = |H_{11} + H_{-1-1} + H_{00}|^2$$

$$F^{J_D}_{\pm1\pm1} = \sqrt{\frac{1}{3}} g_{00} \pm \sqrt{\frac{1}{2}} W r g_{11} + \sqrt{\frac{1}{6}} r^2 g_{22} \quad , \quad F^{J_D}_{00} = \gamma_1 \gamma_2 (-\sqrt{\frac{1}{3}} g_{00} + \sqrt{\frac{2}{3}} r^2 g_{22}) \qquad r = 2|q|$$

$|q|$: momentum of V in D rest frame $\quad$ γ: Lorentz factor of V in D rest frame $\quad$ W: mass of D (m)

- $D \to V_1 V_2$, fix at mass of $V_1 V_2$
  - Fix mass of $V_1 V_2$ at PDG value, their momentum can be calculated.
  - $V_1 V_2$ is considered as final state.

$$f_L = \frac{|H_{00}|^2}{|H_{11} + H_{-1-1} + H_{00}|^2}$$

Process	$D^0 \to K^{*-}(892)\,\rho(770)^+$	$D^0 \to \bar{K}^{*0}(892)\,\rho(770)^0$
$D \to V_1 V_2 \to (P_1 P_2)(P_3 P_4)$	$0.509 \pm 0.005 \pm 0.012$	$0.108 \pm 0.008 \pm 0.013$
$D \to V_1 V_2$	$0.468 \pm 0.005 \pm 0.011$	$0.077 \pm 0.007 \pm 0.010$

Strong Phase Measurement

Binning Scheme

- **Mode I**: The bin boundaries are allowed to vary independently, as shown in Figure 2d. The number of bin pairs of this mode could be odd.
- **Mode II**: The middle bin boundaries 0 is fixed, and the left and right bin boundaries are allowed to vary independently, as shown in Figure 2e.
- **Mode III**: The middle bin boundaries 0 is fixed, and the left and right bin boundaries are allowed to vary symmerically, as shown in Figure 2f.

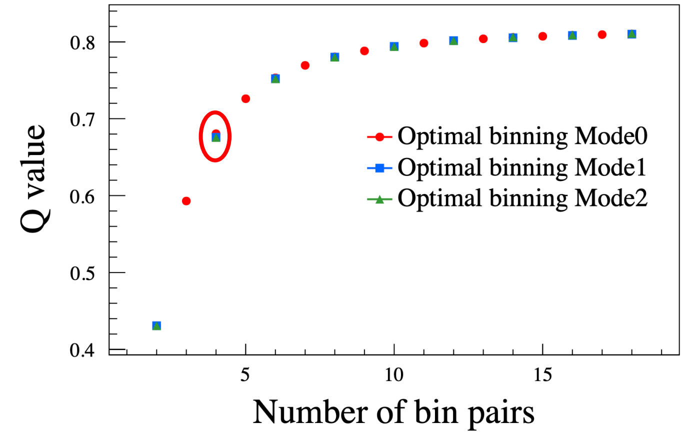
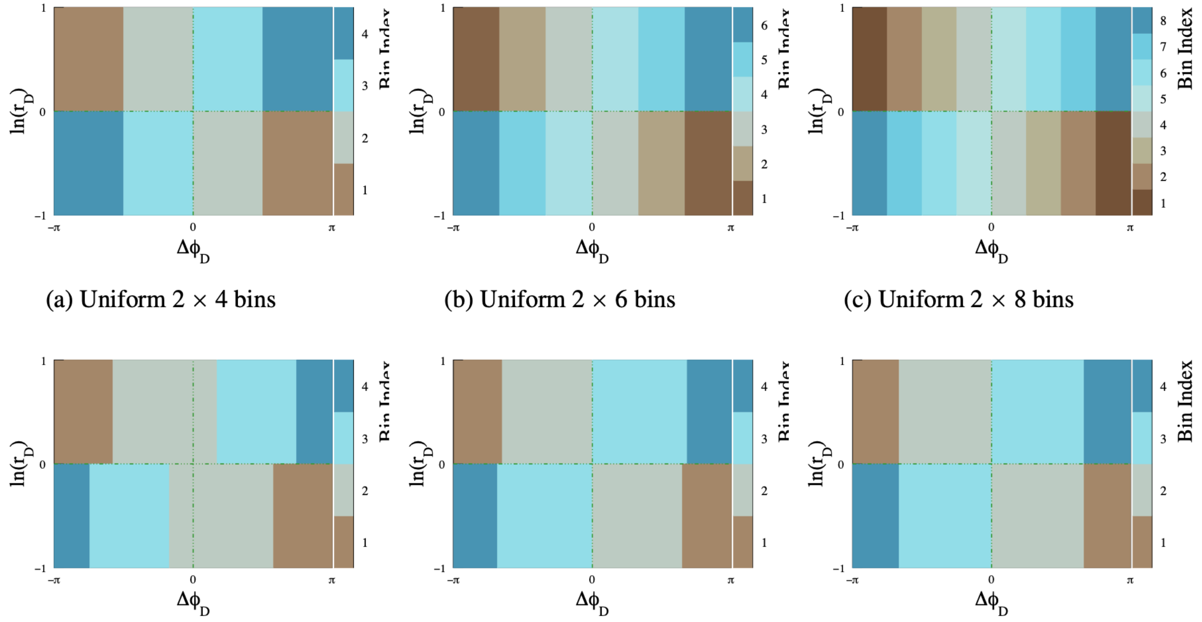


Table 2: c_i, s_i, and T_i for the optimized 2×4 binning scheme.

Bin	c_i	s_i	T_i	$\bar{T}_i$
1	-0.8579	-0.2089	0.1951	0.1028
2	0.2738	-0.6499	0.0919	0.0216
3	0.1596	0.6920	0.1594	0.0295
4	-0.8105	0.2691	0.2823	0.1172



(a) Uniform 2×4 bins (b) Uniform 2×6 bins (c) Uniform 2×8 bins

(d) Mode I optimized 2×4 bins (e) Mode II optimized 2×4 bins (f) Mode III optimized 2×4 bins

Strong Phase Measurement

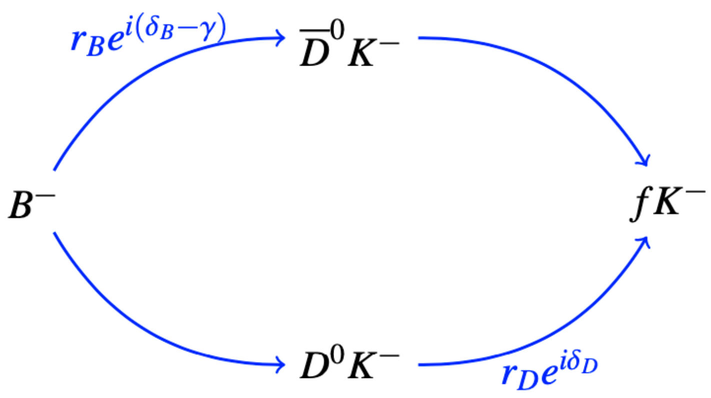
$$A_f = \langle f| H |D^0\rangle = a_f e^{i\phi_f}, \quad \bar{A}_f = \langle f| H |\bar{D}^0\rangle = \bar{a}_f e^{i\bar{\phi}_f}, \qquad A_{f\bar{g}} = A_f \bar{A}_g - \bar{A}_f A_g,$$

$$A_g = \langle g| H |D^0\rangle = a_g e^{i\phi_g}, \quad \bar{A}_g = \langle g| H |\bar{D}^0\rangle = \bar{a}_g e^{i\bar{\phi}_g}. \qquad A_{fg} = \frac{p}{q} A_f A_g - \frac{q}{p} \bar{A}_f \bar{A}_g.$$

$$A_\pm = \frac{(A_g \pm \bar{A}_g)}{\sqrt{2}}. \quad F_+^g = \frac{\int |A_+|^2 \, d\Phi_g}{\int |A_+|^2 + |A_-|^2 \, d\Phi_g} = \frac{\int |A_g|^2 + |\bar{A}_g|^2 + 2 |A_g| |\bar{A}_g| \cos \Delta\phi_f \, d\Phi_f}{\int 2 \left(|A_g|^2 + |\bar{A}_g|^2\right) d\Phi_g}.$$

According to this definition, it is easy to know that $F_+^g = 1$ for CP even mode, and $F_+^g = 0$ for CP odd mode. And the CP parity can be expressed as $\eta_{CP} = (2F_+^g - 1)$.

Strong Phase Measurement



$$\mathcal{A}\left(B^- \to (f)_D K^-\right) = A_B\left(A_f + r_B e^{i(\delta_B-\gamma)}\bar{A}_f\right)$$
$$= A_B\left(a_f e^{i\phi_f} + r_B e^{i(\delta_B-\gamma)}\bar{a}_f e^{i\bar{\phi}_f}\right)$$

$$A(B^- \to \bar{D}^0 K^-)/A(B^- \to D^0 K^-) = r_B e^{i(\delta_B-\gamma)}$$

where r_B is the magnitude, and δ_B is the strong phase difference.

With the decay amplitudes, the decay rates are then written as

$$\Gamma\left[B^- \to DK^-, D \to f_{\mathbf{p}}\right] \propto \left|\bar{A}_{\mathbf{p}}^f\right|^2 r_B^2 + \left|A_{\mathbf{p}}^f\right|^2 + 2\left|A_{\mathbf{p}}^f\right|\left|\bar{A}_{\mathbf{p}}^f\right|\left[x_- \cos\left(\Delta\phi_{\mathbf{p}}^f\right) + y_- \sin\left(\Delta\phi_{\mathbf{p}}^f\right)\right]$$

where the index word $\mathbf{p}$ represents the point in the PHSP of $D \to f$, $x_\pm = r_B\cos(\delta_B \pm \gamma)$ and $y_\pm = r_B\cos(\delta_B \pm \gamma)$. For the B^+ decay, the decay rate is similar as follows:

$$\Gamma\left[B^+ \to DK^+, D \to f_{\mathbf{p}}\right] \propto \left|\bar{A}_{\mathbf{p}}^f\right|^2 r_B^2 + \left|A_{\mathbf{p}}^f\right|^2 + 2\left|A_{\mathbf{p}}^f\right|\left|\bar{A}_{\mathbf{p}}^f\right|\left[x_+ \cos\left(\Delta\phi_{\mathbf{p}}^f\right) - y_+ \sin\left(\Delta\phi_{\mathbf{p}}^f\right)\right]$$

$$T_i^f \equiv \int_i \left|A_{\mathbf{p}}^f\right|^2 \mathrm{d}\Phi(\mathbf{p}),$$
$$\bar{T}_i^f \equiv \int_i \left|\bar{A}_{\mathbf{p}}^f\right|^2 \mathrm{d}\Phi(\mathbf{p}),$$
$$c_i^f \equiv \frac{1}{\sqrt{T_i^f \bar{T}_i^f}} \int_i \left|A_{\mathbf{p}}^f\right|\left|\bar{A}_{\mathbf{p}}^f\right| \cos\left(\Delta\phi_{\mathbf{p}}^f\right) \mathrm{d}\Phi(\mathbf{p}),$$
$$s_i^f \equiv \frac{1}{\sqrt{T_i^f \bar{T}_i^f}} \int_i \left|A_{\mathbf{p}}^f\right|\left|\bar{A}_{\mathbf{p}}^f\right| \sin\left(\Delta\phi_{\mathbf{p}}^f\right) \mathrm{d}\Phi(\mathbf{p}).$$

If the event number N^f of $D \to f$ has been measured, then the event number in each bin is given by $K_i^f = N^f T_i^f, \bar{K}_i^f = N^f \bar{T}_i^f$. Therefore, T_i^f and $\bar{T}_i^f$ can also be defined as $T_i^f = \frac{K_i^f}{\sum_i K_i^f}$ and $\bar{T}_i^f = \frac{\bar{K}_i^f}{\sum_i \bar{K}_i^f}$.

After binning the PHSP, the decay rates of $B^- \to DK^-$ is re-written as

$$\Gamma\left[B^- \to DK^-, D \to f_i\right] \propto \bar{K}_i^f r_B^2 + K_i^f + 2\sqrt{K_i^f \bar{K}_i^f}\left(c_i^f x_- + s_i^f y_-\right)$$

with our measured c_i^f, s_i^f, T_i^f, $x_\pm$ and $y_\pm$ can be obtained from fit to B meson decay, and γ can be extracted !

U-spin

- The U-spin symmetry forms a SU(2) doublet under the exchange of d and s quarks.

In addition to the pure CF and DCS sub-transitions, a mixture of these, where the two pions are produced in a CP eigenstate resonance, is also a viable transition process to the $K_S^0\pi^+\pi^-$ final state. Using the phase convention $CP\left|K^0\right\rangle = -\left|\bar{K}^0\right\rangle$, the partial amplitude of intermediate processes involving only neutral CP eigenstate resonances i.e. $D^0 \to K_S^0(\pi^+\pi^-)_r$ can be written as a superposition of $D^0 \to \bar{K}^0\pi^+\pi^-$ and $D^0 \to K^0\pi^+\pi^-$ as follows:

$$A\left(D^0 \to K_S^0(\pi\pi)_{k_{CP}}\right) = \frac{1}{\sqrt{2}}\left(A\left(D^0 \to \bar{K}^0(\pi^+\pi^-)\right) - A\left(D^0 \to K^0(\pi^+\pi^-)\right)\right)$$

$$= \frac{1}{\sqrt{2}}A\left(D^0 \to \bar{K}^0(\pi^+\pi^-)\right)\left(1 - \frac{A\left(D^0 \to K^0(\pi^+\pi^-)\right)}{A\left(D^0 \to \bar{K}^0(\pi^+\pi^-)\right)}\right),$$

with $\left[\frac{A(D^0\to K^0(\pi^-\pi^+))}{A(D^0\to \bar{K}^0(\pi^-\pi^+))} = -\tan^2\theta_C\hat{\rho}\right]$, where θ_C was defined as the Cabibbo angle and $\hat{\rho}$ was defined as "normalized ratio of transition amplitudes" by Bigi and Yamamoto and may also be understood as "U-spin breaking" parameter, the $K_S^0\pi^-\pi^+$ CP transition amplitude can be written as,

$$A\left(D^0 \to K_S^0(\pi\pi)_{k_{CP}}\right) = \frac{1}{\sqrt{2}}A\left(D^0 \to \bar{K}^0(\pi^+\pi^-)\right)\left(1 + \hat{\rho}_{k_{CP}}\tan^2\theta_C\right).$$

The U-spin breaking parameter, $\hat{\rho}(= re^{i\delta})$ is a complex and purely empirical quantity. For a three-body decay with multiple exclusive CP eigenstate resonant contributions (k_{CP}), $\hat{\rho}$ factors for each are denoted by $\hat{\rho}_{k_{CP}}$. The $\hat{\rho}$ parameters primarily carry the phase-shifts generated as a result of DCS interference, and naively, the magnitudes ($|\hat{\rho}|$) are expected to be $\sim O(1)$. However, the magnitudes are to be empirically measured for the possibility of deviation from the nominal $\tan^2\theta_C$ Cabibbo factor. A similar treatment to the decay process $D^0 \to K_L^0\pi^-\pi^+$ yields,

$$A\left(D^0 \to K_L^0(\pi\pi)_{k_{CP}}\right) = \frac{1}{\sqrt{2}}A\left(D^0 \to \bar{K}^0(\pi^+\pi^-)\right)\left(1 - \hat{\rho}_{k_{CP}}\tan^2\theta_C\right).$$

$$\frac{A\left(D^0 \to K_L^0(\pi\pi)_{k_{CP}}\right)}{A\left(D^0 \to K_S^0(\pi\pi)_{k_{CP}}\right)} = \frac{1 - \tan^2\theta_C\hat{\rho}_{k_{CP}}}{1 + \tan^2\theta_C\hat{\rho}_{k_{CP}}} \approx 1 - 2\tan^2\theta_C\hat{\rho}_{k_{CP}} + O\left(\tan^4\theta_C\right),$$

$$A\left(D^0 \to K_L^0(\pi\pi)_{k_{CP}}\right) = \left(1 - 2\tan^2\theta_C\hat{\rho}_{k_{CP}}\right) \times A\left(D^0 \to K_S^0(\pi\pi)_{k_{CP}}\right).$$

$$\boxed{\begin{aligned} A\left(D^0 \to K_S^0\pi^+\pi^-\right) &= \sum_r A^{CF}_{\bar{K}^0\pi\pi} + \sum_{r'} A^{DCS}_{K^0\pi\pi} + \sum_{k_{CP}} A^{CP}_{K_S^0(\pi^+\pi^-)_{k_{CP}}}, \\ A\left(D^0 \to K_L^0\pi^+\pi^-\right) &= \sum_r A^{CF}_{\bar{K}^0\pi\pi} - \sum_{r'} A^{DCS}_{K^0\pi\pi} + \sum_{k_{CP}} \left(1 - 2\tan^2\theta_C\hat{\rho}_{k_{CP}}\right) A^{CP}_{K_S^0(\pi^+\pi^-)_{k_{CP}}}. \end{aligned}}$$

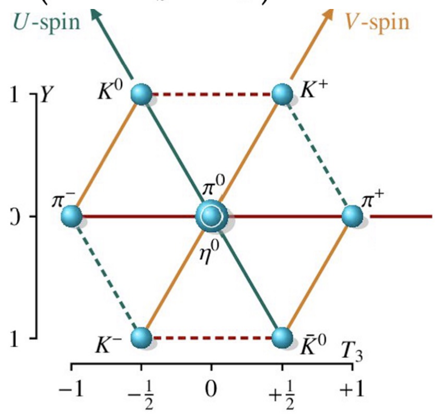


Figure 1: The octet of 0^- mesons. Unitarity of hadrons, U-spin and V-spin.

CP-even fraction & Strong Phase Difference

- What is F_+? ➡ The fraction of CP + to f.

$$F_+ = \frac{\Gamma_+}{\Gamma_+ + \Gamma_-} = \frac{\int |A_+|^2 d\phi_f}{\int |A_+|^2 d\phi_f + \int |A_-|^2 d\phi_f}$$

- Define CP eigenstates and ignore CPV:

$$|D^0_+ > = |D^0 > + |\bar{D}^0 >$$
$$|D^0_- > = |D^0 > - |\bar{D}^0 >$$

- We have before:

$$A(D^0 \to f) = A_f = a_f e^{i\delta_f}$$
$$A(\bar{D}^0 \to f) = A_{-f} = a_{-f} e^{i\delta_{-f}}$$

- So we have:

$$F_+ = \frac{\int |a_f|^2 + |a_{-f}|^2 + 2|a_f||a_{-f}|\cos(\Delta\delta_f) d\phi_f}{2\int |a_f|^2 + |a_{-f}|^2 d\phi_f}$$

$$\Downarrow$$

$$2F_+ - 1 = \frac{2}{\int |a_f|^2 + |a_{-f}|^2 d\phi_f} \int |a_f||a_{-f}|\cos(\Delta\delta_f) d\phi_f$$

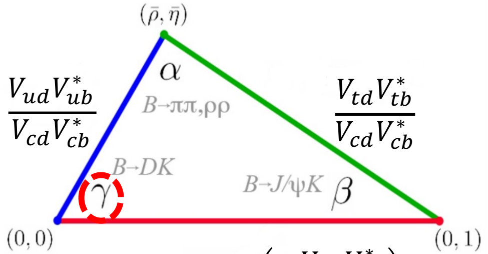
Strong Phase Measurement

- Used in LHCb-PUB-2016-025

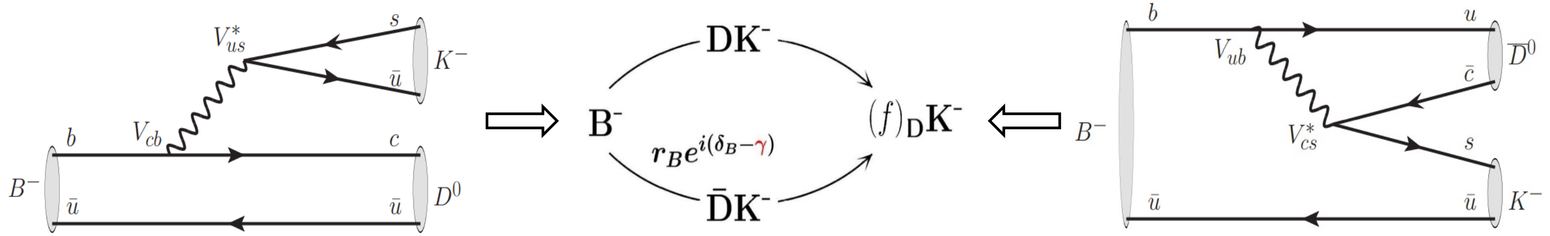
Decay Mode	Parameters	Status@BESIII ($2.93\ \text{fb}^{-1}$)
$K_S^0\pi^+\pi^-$	c_i, s_i	Finished
$K_S^0K^+K^-$	c_i, s_i	Finished
$K^-\pi^+\pi^+\pi^-$	R, δ	Finished
$K^+K^-\pi^+\pi^-$	F_+ or c_i, s_i	On Going
$\pi^+\pi^-\pi^+\pi^-$	F_+ or c_i, s_i	On Going
$K^-\pi^+\pi^0$	R, δ	Finished
$K_S^0K^\pm\pi^\mp$	R, δ	……
$\pi^+\pi^-\pi^0$	F_+	On Going
$K_S^0\pi^+\pi^-\pi^0$	F_+ or c_i, s_i	On Going
$K^+K^-\pi^0$	F_+	On Going
$K^-\pi^+$	δ	Finished (Update)

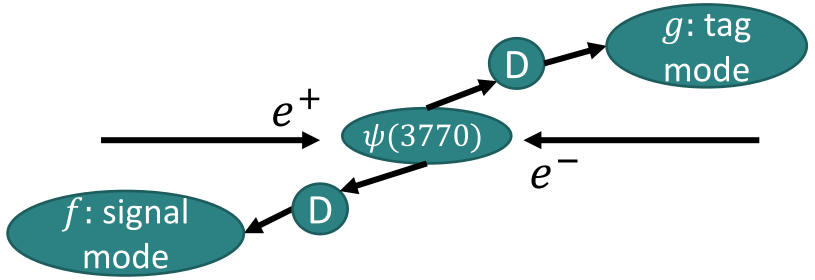
学术能力｜北京谱仪 BESIII 物理分析

$D^0 \to K_S^0\pi^0\pi^+\pi^-$ 过程的分波分析与分支比测量



$\gamma \equiv \varphi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$





- CKM 矩阵的幺正性是检验标准模型的重要方法之一。测量 γ 角需要中性粲介子的衰变信息作为输入参数，其实验精度是高精度检验三角形幺正性的重大挑战。
- 中性粲介子量子关联性的研究是深入探索非微扰 QCD 本质的重要途径，也是精密测量粲介子衰变的强相位差、CP 本征态成分比例等参数最理想的场所。
- 味物理的研究是寻找新物理重要途径；对于 $B \to VV$ 过程的预测与测量均表明其纵向极化率 $f_L = 1$，但 $D \to VV$ 过程具有明显不同的衰变性质，有助于寻找 $D^0 - \bar{D}^0$ 混合中的新物理。

- BESIII 实验具有 20/fb 的 $\psi(3770)$ 数据，具有较高的重建效率，可提供大量且干净的量子关联 $D^0/\bar{D}^0$ 样本。
- 本过程具有较大的衰变分支比，可获得大量的信号样本，给出目前**最精确**的分波分析结果，进而可得到相关衰变参数及 $D^0 \to VV, VP$ 和 AP 等过程分支比。

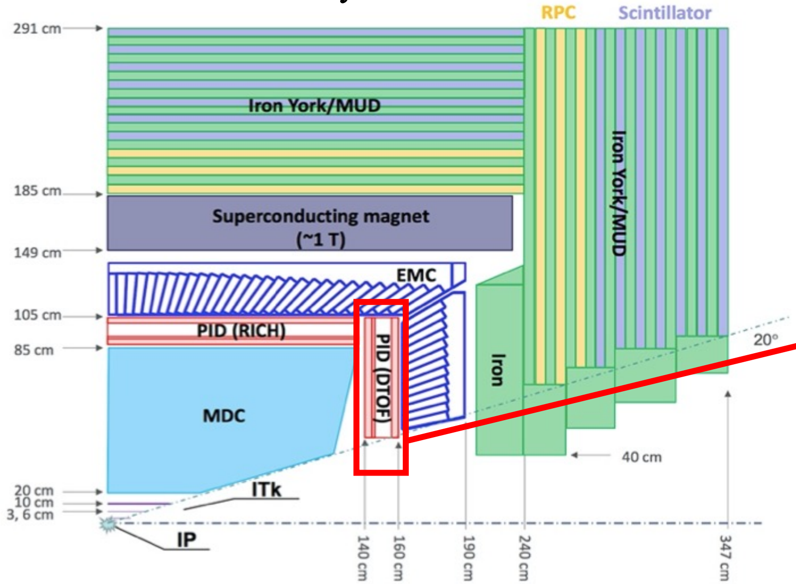
学术能力 | STCF 粒子模拟软件开发

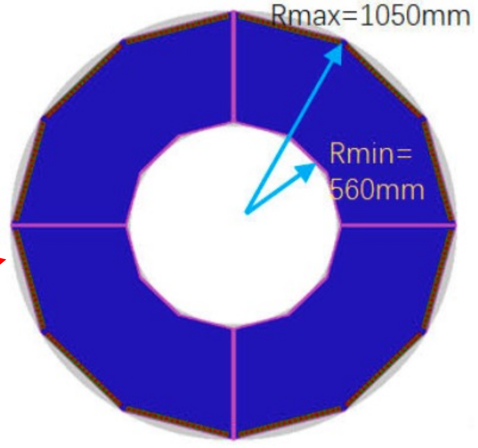
STCF 端盖粒子鉴别探测器 DTOF(DIRC-like TOF) 在全模拟框架下的软件开发

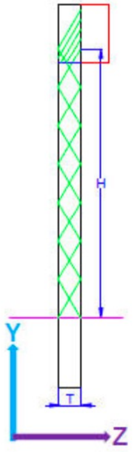
- STCF 是我国预研的工作在 τ-c 能区具有更高亮度的新一代正负电子对撞机，强大的粒子鉴别能力是支撑丰富的物理分析工作的基础
- DIRC 已被广泛应用至国际其他高能物理实验中，其紧致的结构、较低的物质量、优秀的位置和时间分辨在保证了较高的粒子分辨能力

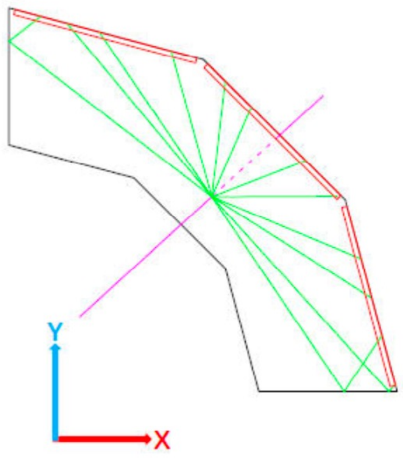
c.m. energy: 2 – 7GeV

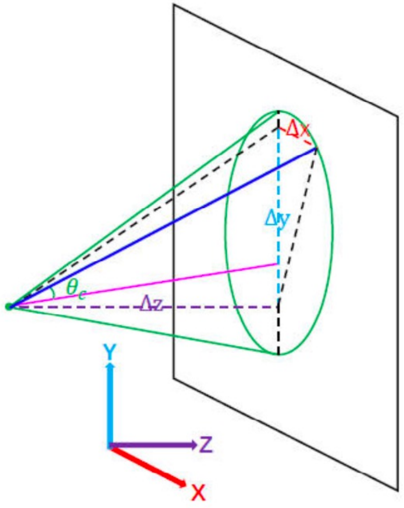
Peak luminosity: $0.5\sim1\times10^{35}\mathrm{cm}^{-2}\mathrm{s}^{-1}$







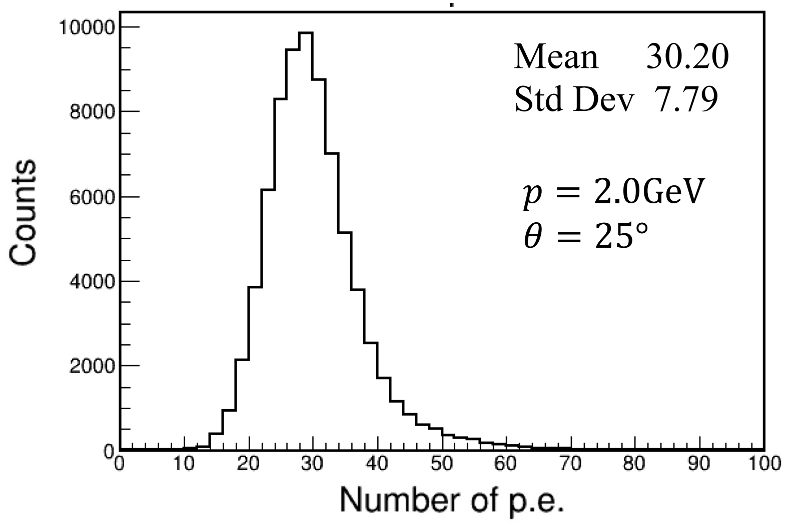


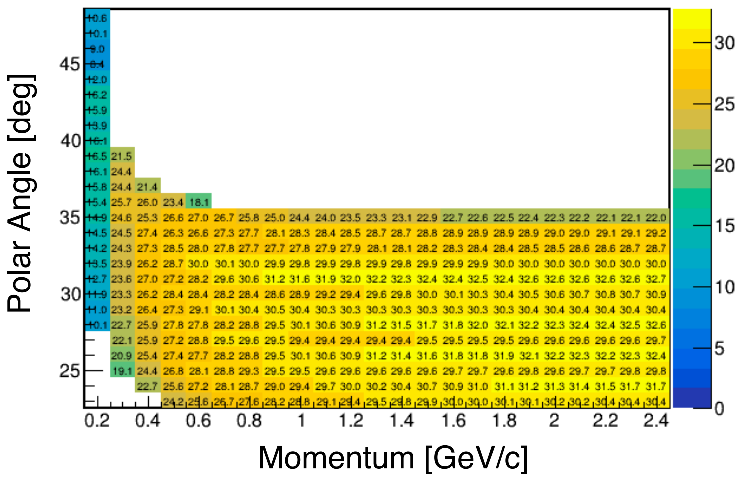


DTOF Simulation

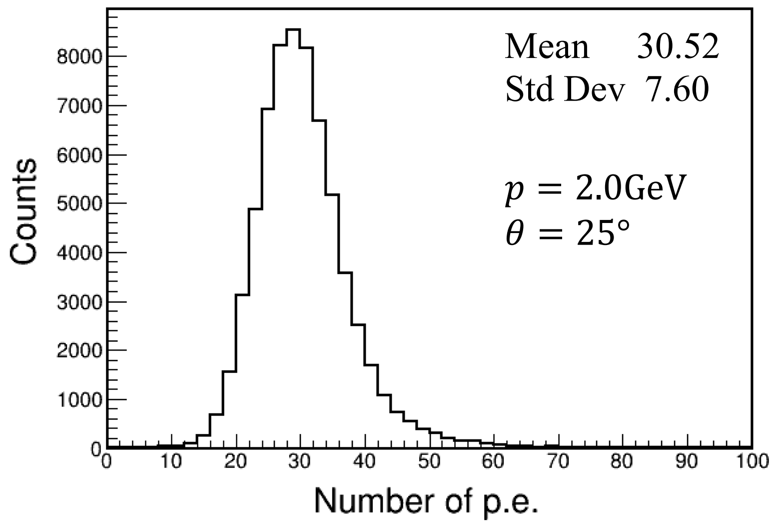
Photon Yield

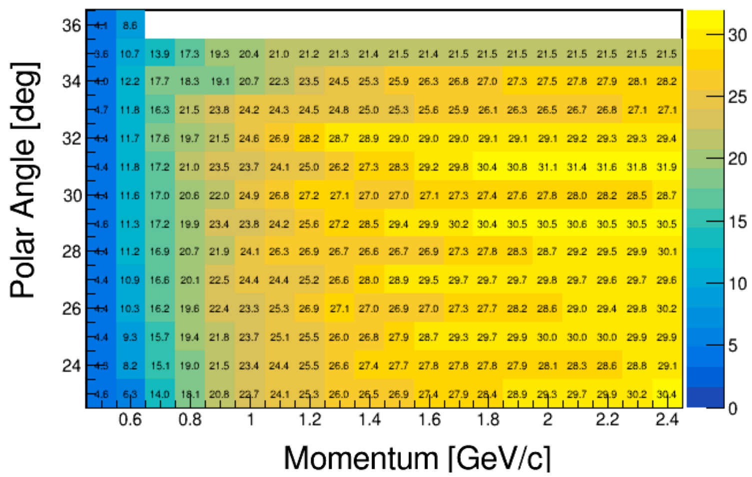
π Sample





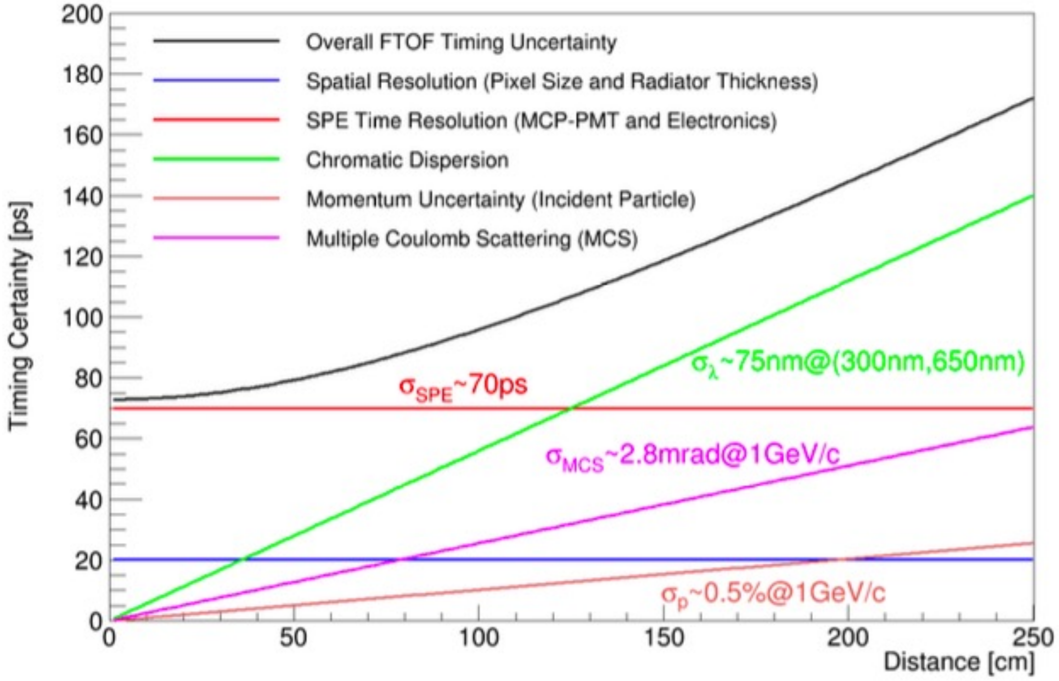
K Sample





DTOF Reconstruction
Single Timing Uncertainties

$$\sigma_t = \sigma_{T_0} \oplus \sigma_{t_{MCS}} \oplus \sigma_{t_{ext(\vec{r},\vec{p})}} \oplus \frac{\sigma_{t_\lambda}}{\sqrt{N_{p.e.}}} \oplus \frac{\sigma_{t_D}}{\sqrt{N_{p.e.}}} \oplus \frac{\sigma_{TTS}}{\sqrt{N_{p.e.}}}$$



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- Single photon timing uncertainty:

@2GeV

Overall σ_t ~50 ps

π	σ_{T_0}	$\sigma_{t_{MCS}}$	$\sigma_{t_{ext}}$	σ_{t_λ}	σ_{t_D}	σ_{TTS}	Total
/ps	40	9.8	16.5	40.7	14.4	30	68.8