A Microscopic Transport Framework for Heavy-Ion Collisions from High to Low Energies—PHSD/PHQMD

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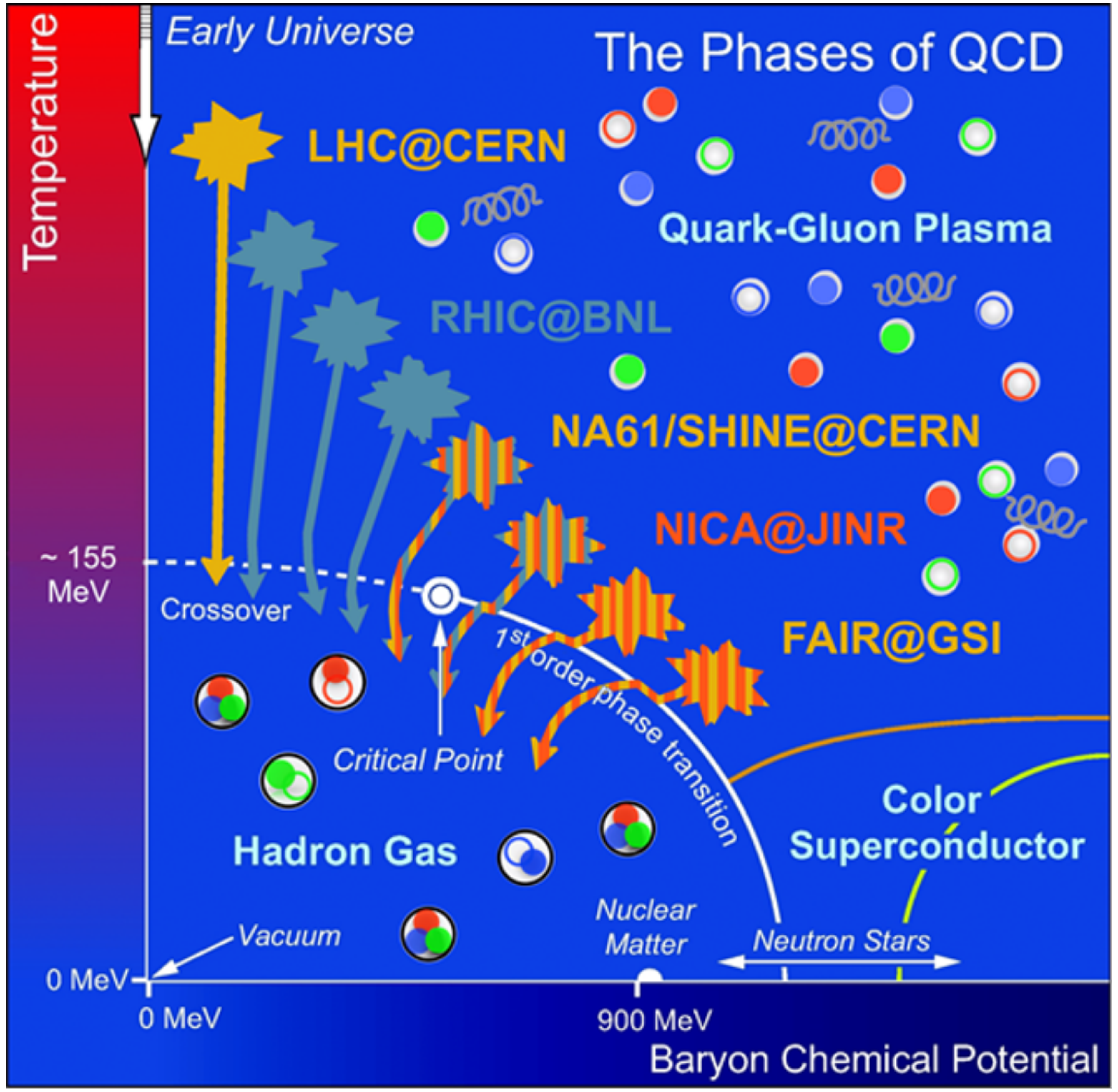
CRC-TR 211 Strong-interaction matter under extreme conditions

Heavy ion collisions

QCD phase diagram in the (T, μ_b) plane



- **Equation-of-State of hot and dense matter;**
- **Study of the phase transition from hadronic to Quark-Gluon Plasma (QGP);**
- **Search for a critical end point (CEP);**
- **Search for signatures of chiral symmetry restoration;**
- **Study of the in-medium properties of hadrons at high baryon density and temperature;**
- **…**

Dynamical description of Heavy ion collisions

~ 20 fm/c ~∞ fm/c

The goal:

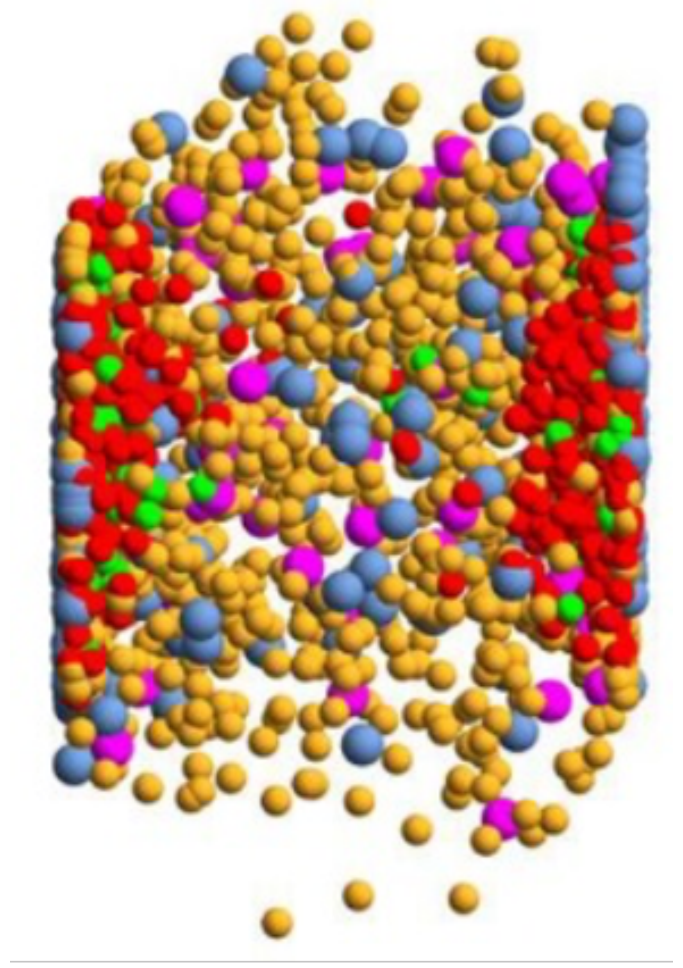
to develop a dynamical microscopic transport approach to study the evolution of HIC

➡ PHSD (Parton-Hadron-String-Dynamics)

➡ PHQMD(Parton-Hadron-Quantum-Molecular-Dynamics)

PHSD:

I. Development fo the microscopic transport theory: from BUU to Kadanoff-Baym dynamics



History: semi-classical BUU equation

Boltzmann-Uehling-Uhlenbeck equation (non-relativistic formulation)—propagation of particles in the self-generated Hartree-Fock mean-field potential *U(r,t)*:

Ludwig Boltzmann
1844-1906

$$\frac{\partial}{\partial t} f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m}\vec{\nabla}_{\vec{r}}\, f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}} U(\vec{r},t)\, \vec{\nabla}_{\vec{p}} f(\vec{r},\vec{p},t) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

collision term:
elastic and inelastic

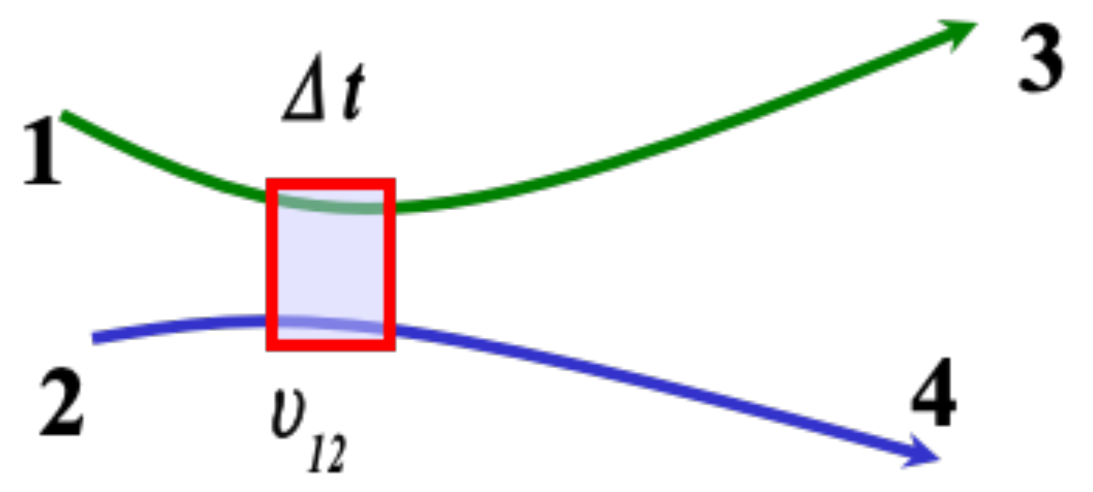
$f(\vec{r}, \vec{p}, t)$ **is the single particle phase-space distribution function —probability to find the particle at position *r* with momentum *p* at time *t***

- **self-generated Hartree-Fock mean-field potential:**

$$U(\vec{r},t) = \frac{1}{(2\pi\hbar)^3}\sum_{\beta_{occ}}\int d^3r'\, d^3p\;\; V(\vec{r}-\vec{r}\,',t)\;\; f(\vec{r}\,',\vec{p},t) \;+\; (Fock\;\; term)$$

- **Collision term for 1+2→3+4 (let's consider fermions) :**

$$I_{coll} = \frac{4}{(2\pi)^3}\int d^3p_2\, d^3p_3 \int d\Omega\;\; |\upsilon_{12}|\,\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\cdot\frac{d\sigma}{d\Omega}(1+2\rightarrow 3+4)\cdot P$$



Including Pauli blocking: $P = f_3 f_4(1-f_1)(1-f_2) - f_1 f_2(1-f_3)(1-f_4)$

Gain term: 3+4→1+2

Loss term: 1+2→3+4

History: developments of relativistic transport models

Low energy HIC ⟹ High energy HIC

Non-relativistic semi-classical BUU

⟹ Relativistic transport models

‘Relativistic Vlasov-Uehling-Uhlenbeck model for heavy-ion collisions’
Che-Ming Ko, Qi Li, Phys.Rev. C37 (1988) 2270

‘Covariant Boltzmann-Uehling-Uhlenbeck approach for heavy-ion collisions’
Bernhard Blaettel, Volker Koch, Wolfgang Cassing, Ulrich Mosel, Phys.Rev. C38 (1988) 1767;

‘Relativistic BUU approach with momentum dependent mean fields’
T. Maruyama, B. Blaettel, W. Cassing, A. Lang, U. Mosel, K. Weber, Phys.Lett. B297 (1992) 228

‘The Relativistic Landau-Vlasov method in heavy ion collisions’
C. Fuchs, H.H. Wolter, Nucl.Phys. A589 (1995) 732

….

DJBUU, GiBUU, IBL, IBUU, LBUU, pBUU, RBUU, RVUU, SMASH, BAMPS,…

History: Covariant transport equation

❑ Covariant BUU equation :

$$\left\{ \left(\Pi_\mu - \Pi_\nu (\partial^p_\mu U^\nu_V) - m^* (\partial^p_\mu U^\nu_S) \right) \partial^\mu_x + \left(\Pi_\nu (\partial^x_\mu U^\nu_V) + m^* (\partial^x_\mu U^\nu_S) \right) \partial^\mu_p \right\} f(x,p) = I_{coll}$$

$$I_{coll} \equiv \sum_{2,3,4} \int d2\, d3\, d4\ [G^+G]_{1+2\to3+4}\ \delta^4(\Pi + \Pi_2 - \Pi_3 - \Pi_4)$$

$$d2 \equiv \frac{d^3 p_2}{E_2}$$

$$\times \{ f(x,p_3)\ f(x,p_4)\ (1 - f(x,p))\ (1 - f(x,p_2))$$

Gain term
3+4 → 1+2

$$- f(x,p)\ f(x,p_2)\ (1 - f(x,p_3))\ (1 - f(x,p_4))\ \}$$

Loss term
1+2 → 3+4

$m^*(x,p) = m + U_S(x,p)$ - effective mass

$\Pi_\mu(x,p) = p_\mu - U_\mu(x,p)$ - effective momentum

➔ coupled set of BUU equations for different hadrons:

$$Df_N = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ... \right]$$

$$Df_\Delta = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ... \right]$$

...

$$Df_\pi = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ... \right]$$

...

From weakly to strongly interacting systems

Properties of matter (on hadronic and partonic levels) in heavy-ion collisions:
strongly interacting system! Degrees of freedom – dressed partons/Hadronic matter
– modification of hadron properties at finite T, μ_B (vector mesons, strange mesons)

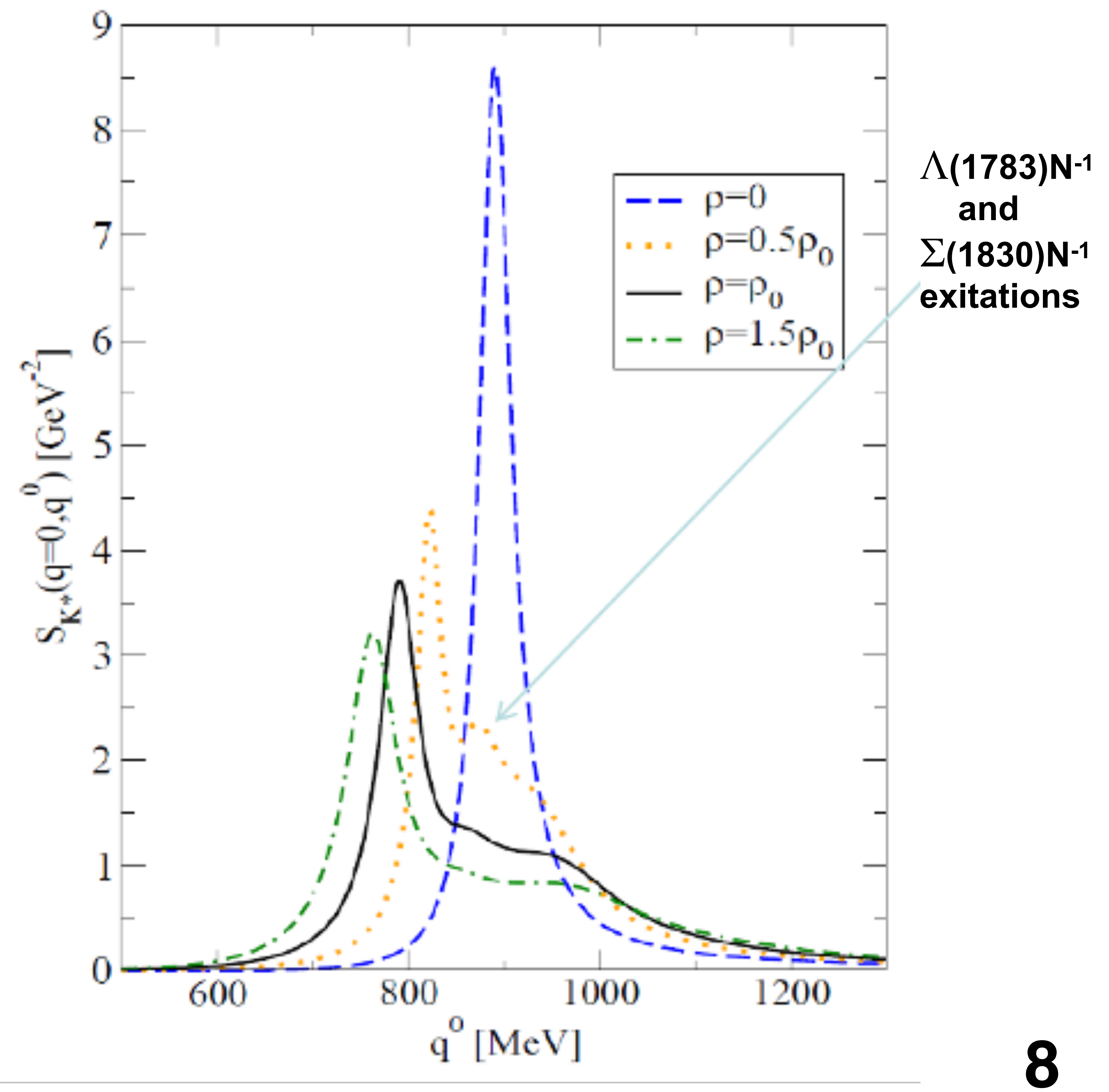
Many-body theory:
Strong interaction ➔ large width = short life-time
➔ broad spectral function ➔ quantum object

- **How to describe the dynamics of broad strongly interacting quantum states in transport theory?**

- [ ] **Semi-classical BUU**

- [x] **Quantum Kadanoff-Baym equation**

K^* spectral function



Dynamical description of strongly interacting systems

- Quantum field theory → **Kadanoff-Baym dynamics** for resummed single-particle **Green functions** $S^<$

[Dyson-Schwinger equation on the closed-time-path]

$$\hat{S}_{0x}^{-1}\, S_{xy}^{<} = \Sigma_{xz}^{ret} \odot S_{zy}^{<} + \Sigma_{xz}^{<} \odot S_{zy}^{adv}$$

convolution integral over the closed time-path

$$\hat{S}_{0x}^{-1} \equiv -(\partial_x^\mu \partial_\mu^x + M_0^2) \quad \textit{boson}$$
$$= i\gamma^\mu \partial_\mu^x - M \quad \textit{fermion}$$

$$iS_{xy}^{<} = \eta \langle \{ \Phi^{+}(y) \Phi(x) \} \rangle$$

$$iS_{xy}^{>} = \langle \{ \Phi(y) \Phi^{+}(x) \} \rangle$$

$$iS_{xy}^{c} = \langle T^{c} \{ \Phi(x) \Phi^{+}(y) \} \rangle \quad - causal$$

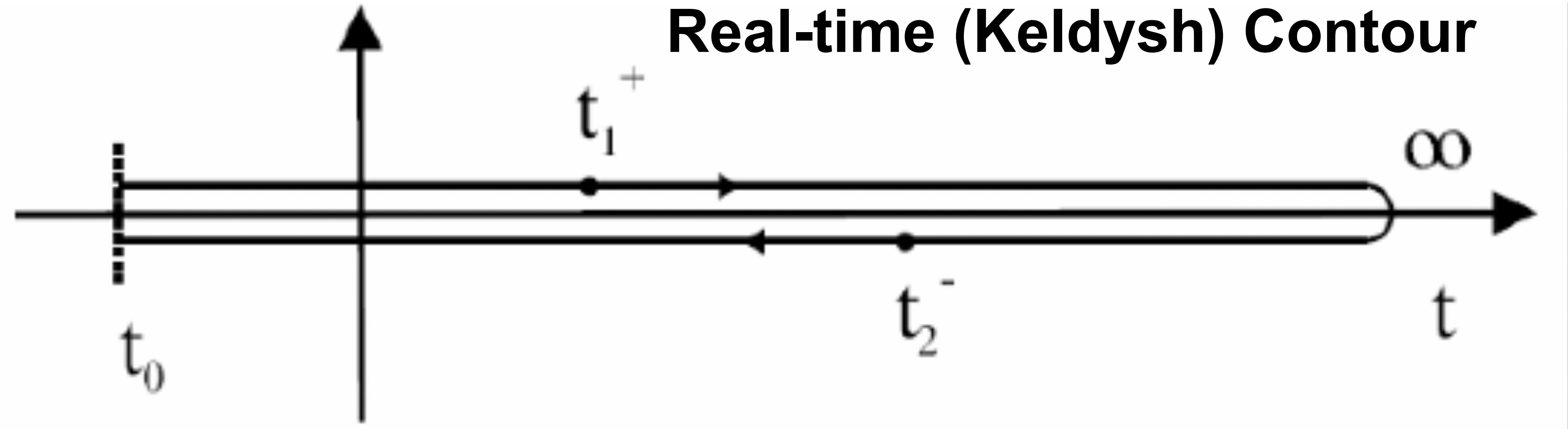
$$iS_{xy}^{a} = \langle T^{a} \{ \Phi(x) \Phi^{+}(y) \} \rangle \quad - anticausal$$

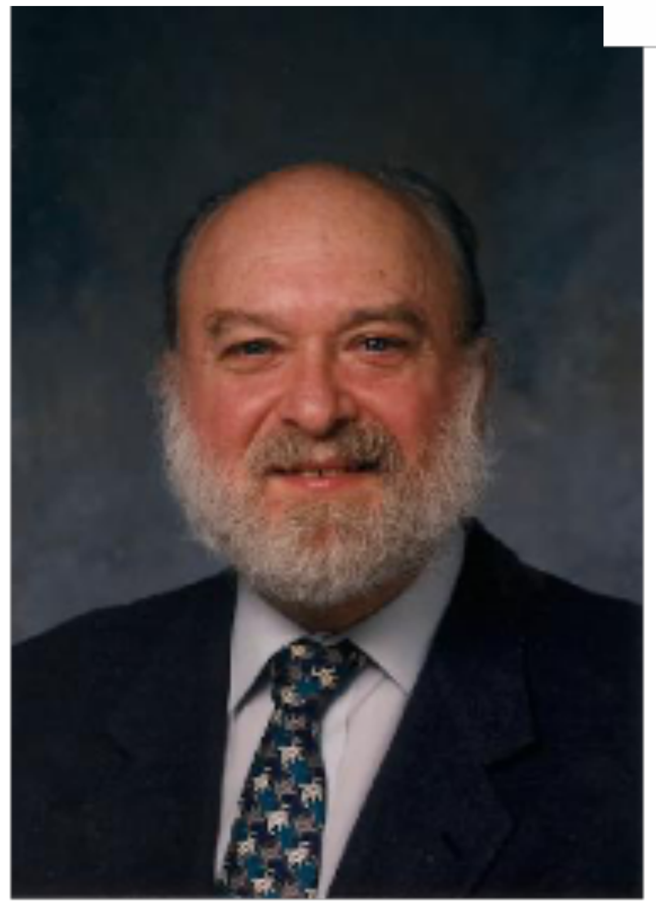
$$S_{xy}^{ret} = S_{xy}^{c} - S_{xy}^{<} = S_{xy}^{>} - S_{xy}^{a} \quad - retarded$$

$$S_{xy}^{adv} = S_{xy}^{c} - S_{xy}^{>} = S_{xy}^{<} - S_{xy}^{a} \quad - advanced$$

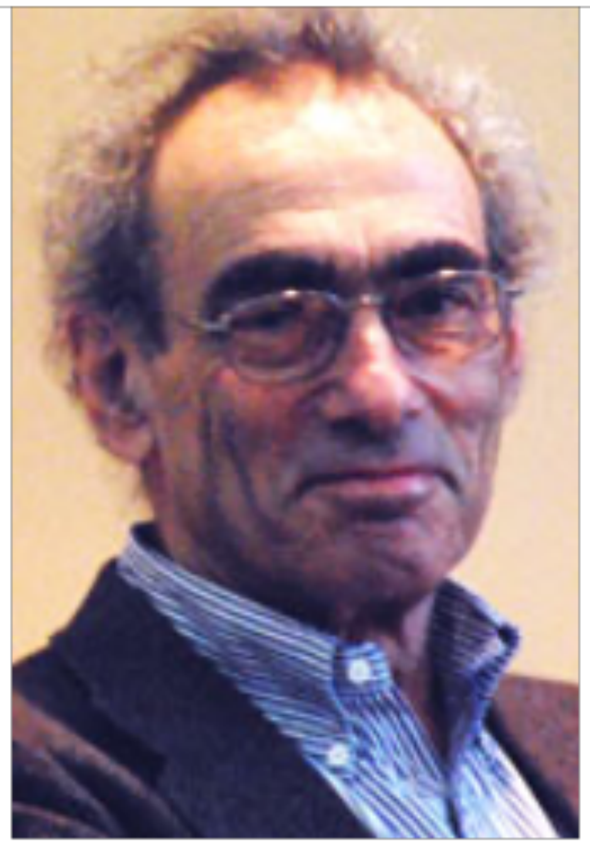
$$\eta = \pm 1 (bosons\,/\,fermions)$$

$$T^{a}(T^{c}) - (anti-)time-ordering\ operator$$





Leo Kadanoff



Gordon Baym

Can be solved exactly for Φ^4 – theory !

S. Juchem, W. Cassing, and C. Greiner, Phys. Rev. D 69 (2004) 025006; Nucl. Phys. A 743 (2004) 92

From Kadanoff-Baym Eq. to generalized transport Eq.

Do the Wigner transformed Kadanoff-Baym equations and next gradient expansion:

W. Botermans, R. Malfliet, Phys. Rep. 198 (1990) 115

0-order—> Generalized mass-shell equations:

$$[P^2 - M_0^2 - Re\Sigma^{ret}_{XP}]\, S^{<}_{XP} - \Sigma^{<}_{XP}\, ReS^{ret}_{XP} = \frac{1}{2}\diamond\{\Sigma^{<}_{XP}\}\{A_{XP}\} - \frac{1}{2}\diamond\{\Gamma_{XP}\}\{S^{<}_{XP}\}$$

4-dimentional of the Poisson Bracket:

$$\diamond\{F_1\}\{F_2\} := \frac{1}{2}\left(\frac{\partial F_1}{\partial X_\mu}\frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu}\frac{\partial F_2}{\partial X^\mu}\right)$$

1-order—> Generalized transport equations:

drift term — Vlasov — backflow term — collision term = ‚loss' term - ‚gain'

$$\diamond\{P^2 - M_0^2 - Re\Sigma^{ret}_{XP}\}\{S^{<}_{XP}\} - \diamond\{\Sigma^{<}_{XP}\}\{ReS^{ret}_{XP}\} = \frac{i}{2}\,[\Sigma^{>}_{XP}\, S^{<}_{XP} - \Sigma^{<}_{XP}\, S^{>}_{XP}]$$

Backflow term incorporates the off-shell behavior in the propagation

- **Propagation of the Green function** $iS^{<}_{XP} = A_{XP}N_{XP}$**, which carries information not only on the number of particles (N_{XP}), but also on their properties, interactions and correlations (A_{XP})**
- **Spectral function:** $A_{XP} = \dfrac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$ $\qquad \Gamma_{XP} = -\,Im\Sigma^{ret}_{XP} = 2p_0\Gamma$

Generalized testparticle off-shell equations of motion

- Employ testparticle Ansatz for the real valued quantity $iS^{<}_{XP}$

$$F_{XP} = A_{XP} N_{XP} = i\, S^{<}_{XP} \sim \sum_{i=1}^{N} \delta^{(3)}(\vec{X} - \vec{X}_i(t))\ \delta^{(3)}(\vec{P} - \vec{P}_i(t))\ \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine equations of motion !

➔ Generalized testparticle Cassing-Juchem off-shell equations of motion for particles:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

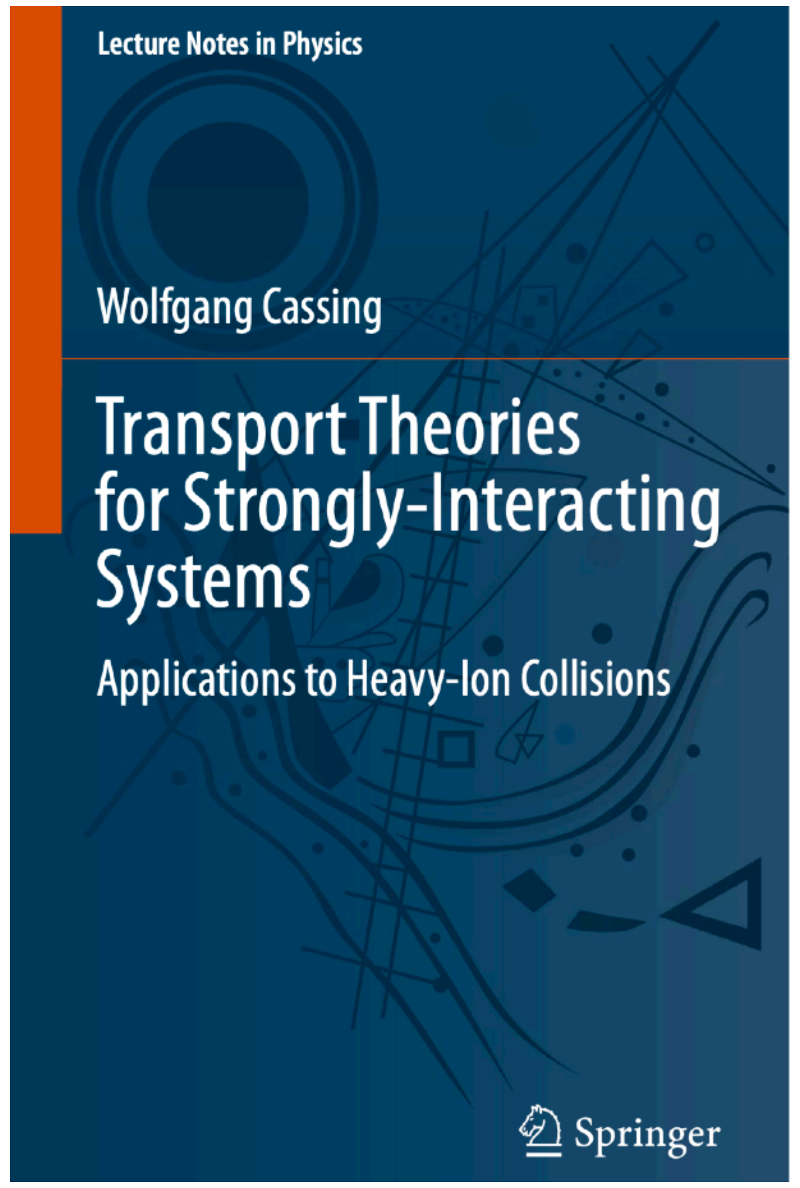
$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_{i} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

Realized in PHSD:



On-shell limits: from KB to BUU

- **$\Gamma(X,P) \rightarrow 0$**

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma_{XP}^{ret})^2 + \Gamma_{XP}^2/4}$$

quasiparticle approximation:

$$A_{XP} = 2p\delta(P^2 - M_0^2)$$

- **$\Gamma(X,P)$ such that**

E.g.: Γ = const. or $\Gamma = \Gamma_{\text{vacuum}}(M)$

Vacuum spectral function with constant or mass dependent width Γ

In on-shell limits the "backflow term" - which incorporates the off-shell behavior in the particle propagation - vanishes:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma_{(i)}^{ret} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

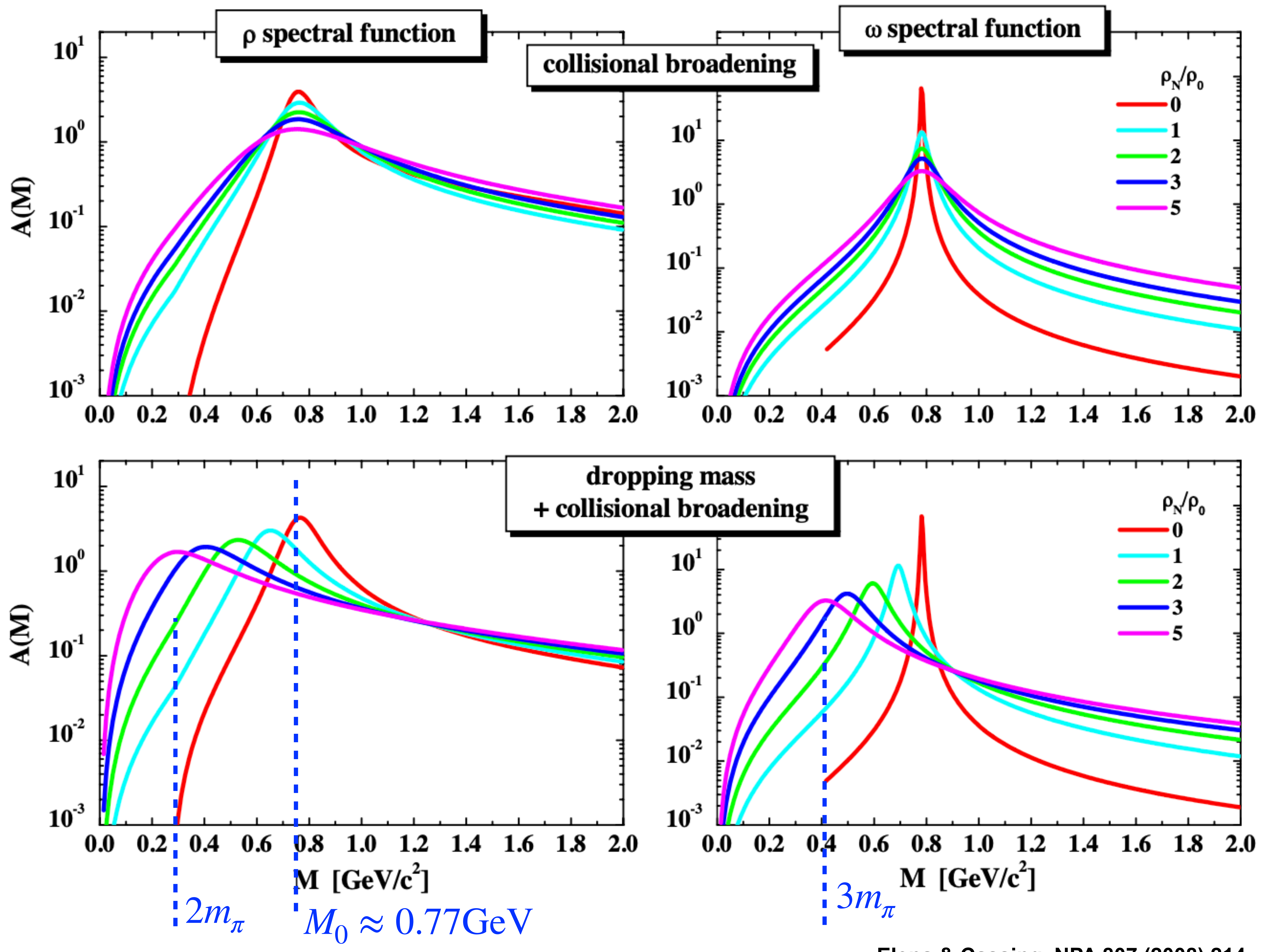
$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma_i^{ret} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma_{(i)}^{ret}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

W. Cassing, Eur. Phys. J. ST 168 (2009) 3

Equations of motion (independent on Γ) ➔ BUU limit

Off-shell vs. on-shell transport dynamics



Elena & Cassing, NPA 807 (2008) 214

Spectral function (Breit-Wigner):

$$A_V(M,\rho_N) = C_1 \cdot \frac{2}{\pi} \frac{M^2 \Gamma_V^*(M,\rho_N)}{(M^2 - M_0^{*2}(\rho_N))^2 + (M\Gamma_V^*(M,\rho_N))^2} \,.$$

Vacuum + collisional broadening:

$$\Gamma_V^*(M,|\vec{p}|,\rho_N) = \Gamma_V(M) + \Gamma_{coll}(M,|\vec{p}|,\rho_N).$$

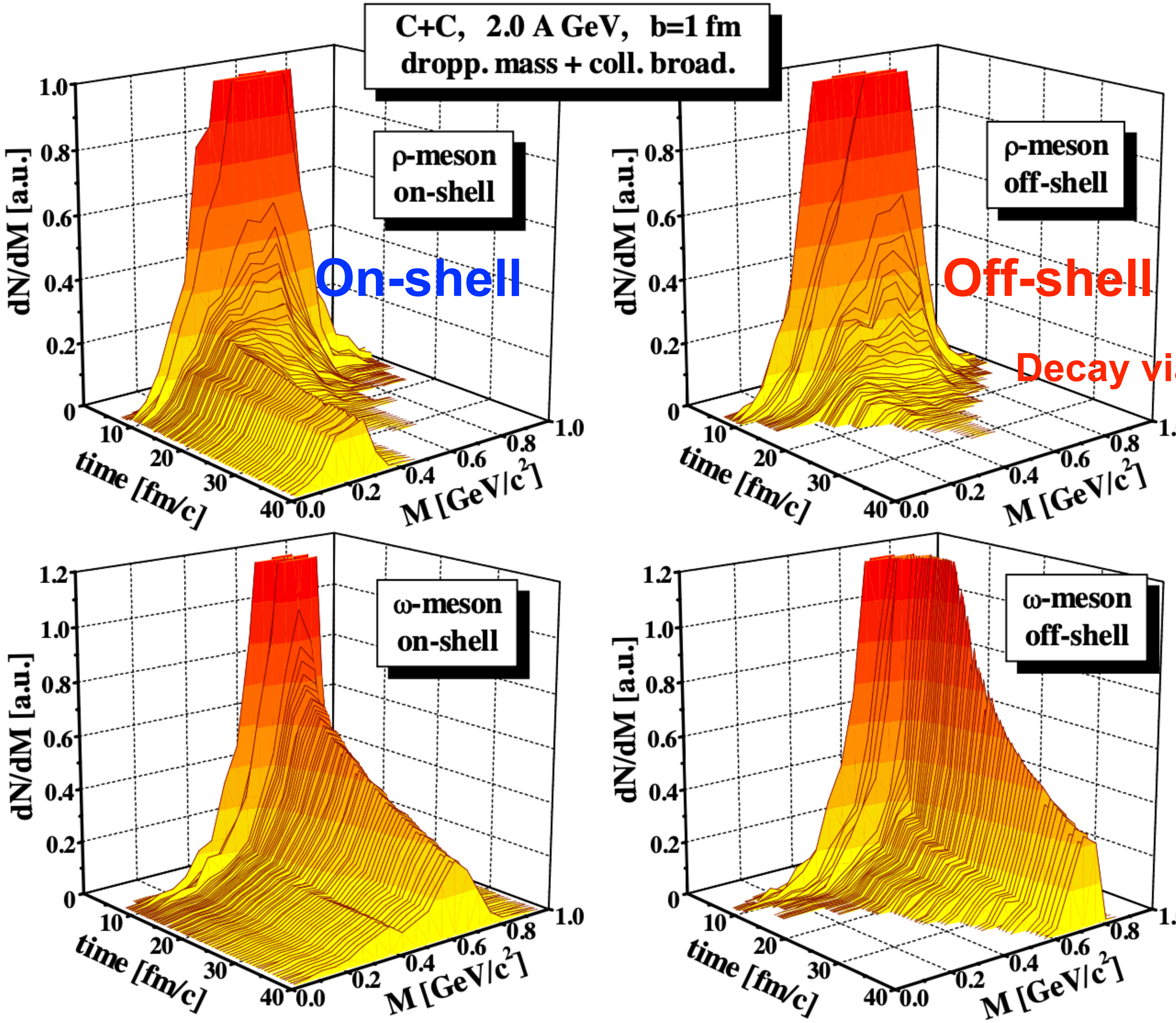
$$\Gamma_\rho(M) \simeq \Gamma_{\rho\to\pi\pi}(M) = \Gamma_0 \left(\frac{M_0}{M}\right)^2 \left(\frac{q}{q_0}\right)^3 F(M)$$

$$\Gamma_{coll}(M,|\vec{p}|,\rho_N) = \gamma\ \rho_N < v\ \sigma_{VN}^{tot} > \approx\ \alpha_{coll}\ \frac{\rho_N}{\rho_0}.$$

Mass shifts (dropping mass):

$$M_0^*(\rho_N) = \frac{M_0}{(1+\alpha\rho_N/\rho_0)}$$

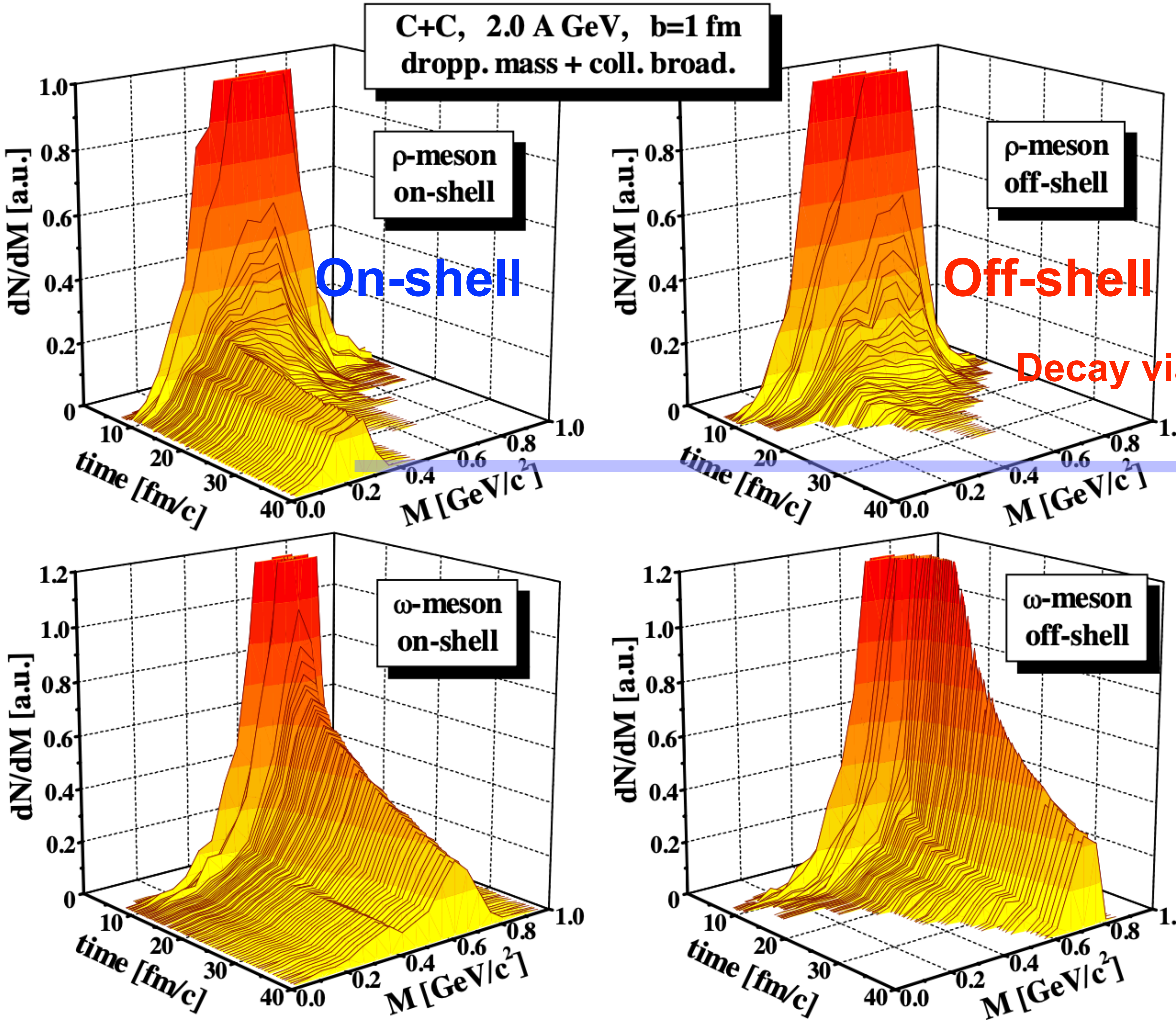
Off-shell vs. on-shell transport dynamics

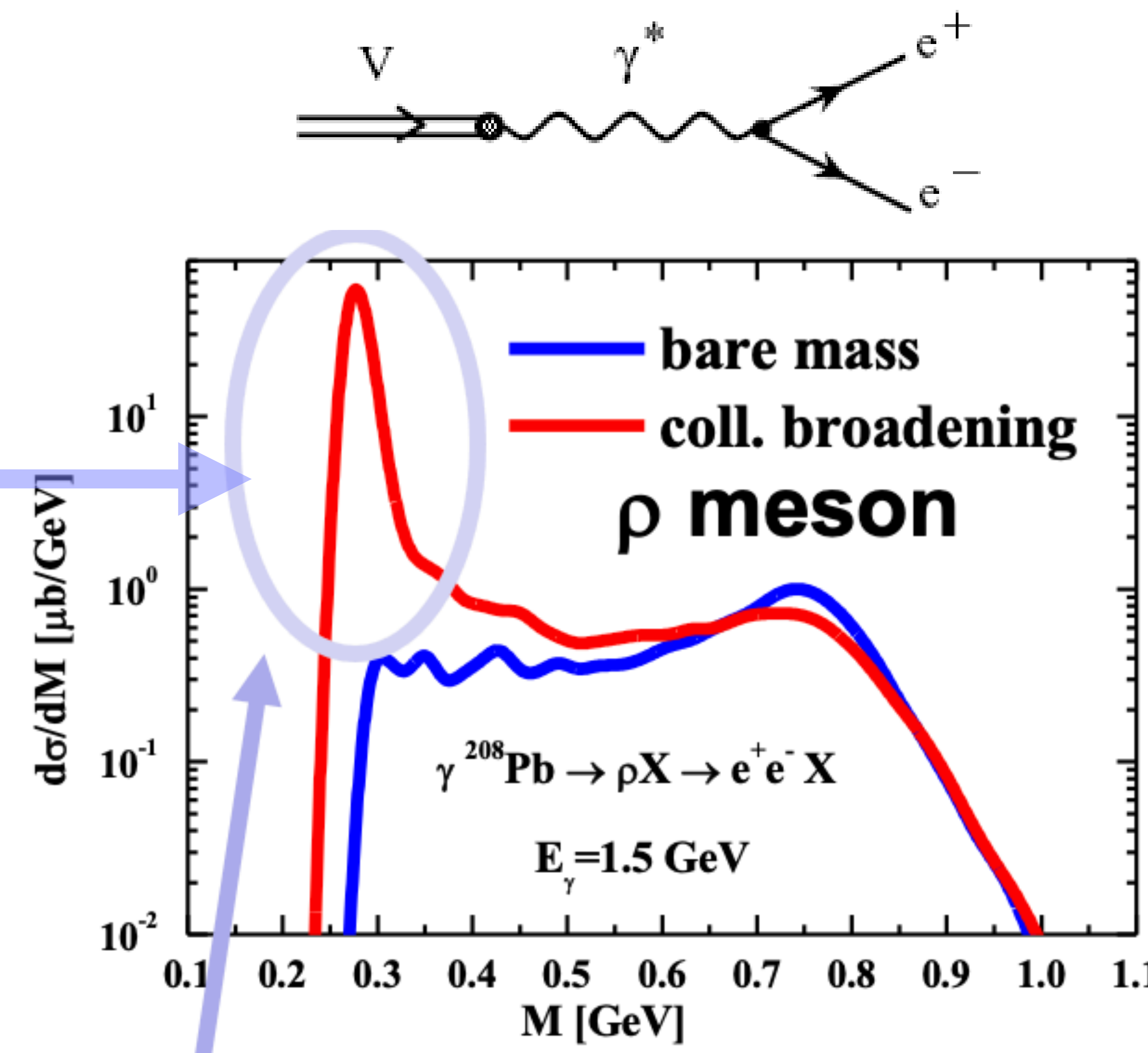


Decay via: $\rho N \to \pi N, \ldots$

The off-shell spectral function **becomes on-shell**
in the vacuum **dynamically** by propagation!

Off-shell vs. on-shell transport dynamics





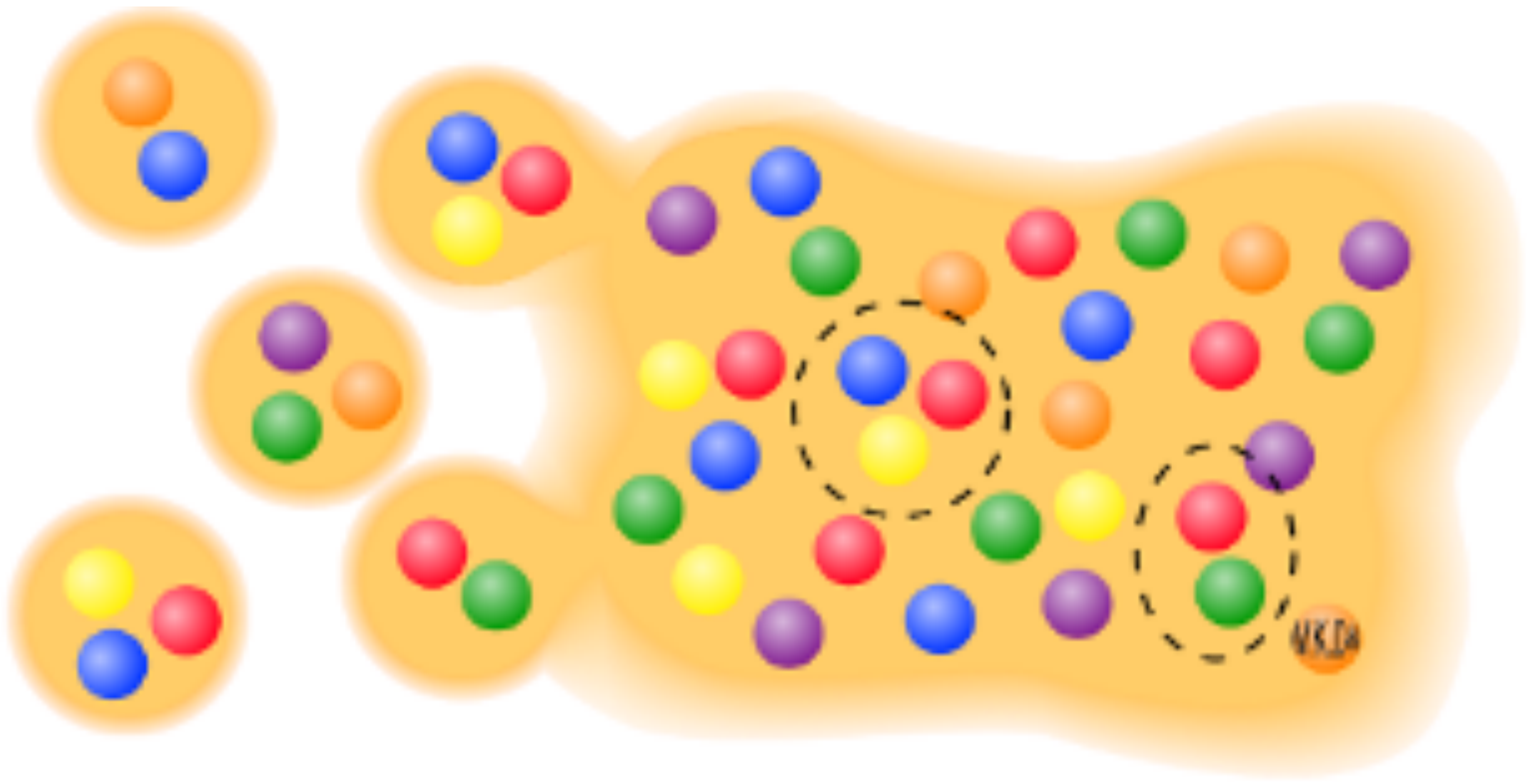
GiBUU: M. Effenberger et al, PRC60 (1999) 027601

low mass ρ and ω mesons live forever (and shine "fake" dileptons)

The off-shell spectral function becomes on-shell in the vacuum dynamically by propagation!

PHSD:

II. Modeling of sQGP for high energy nuclear collisions in microscopic transport theory

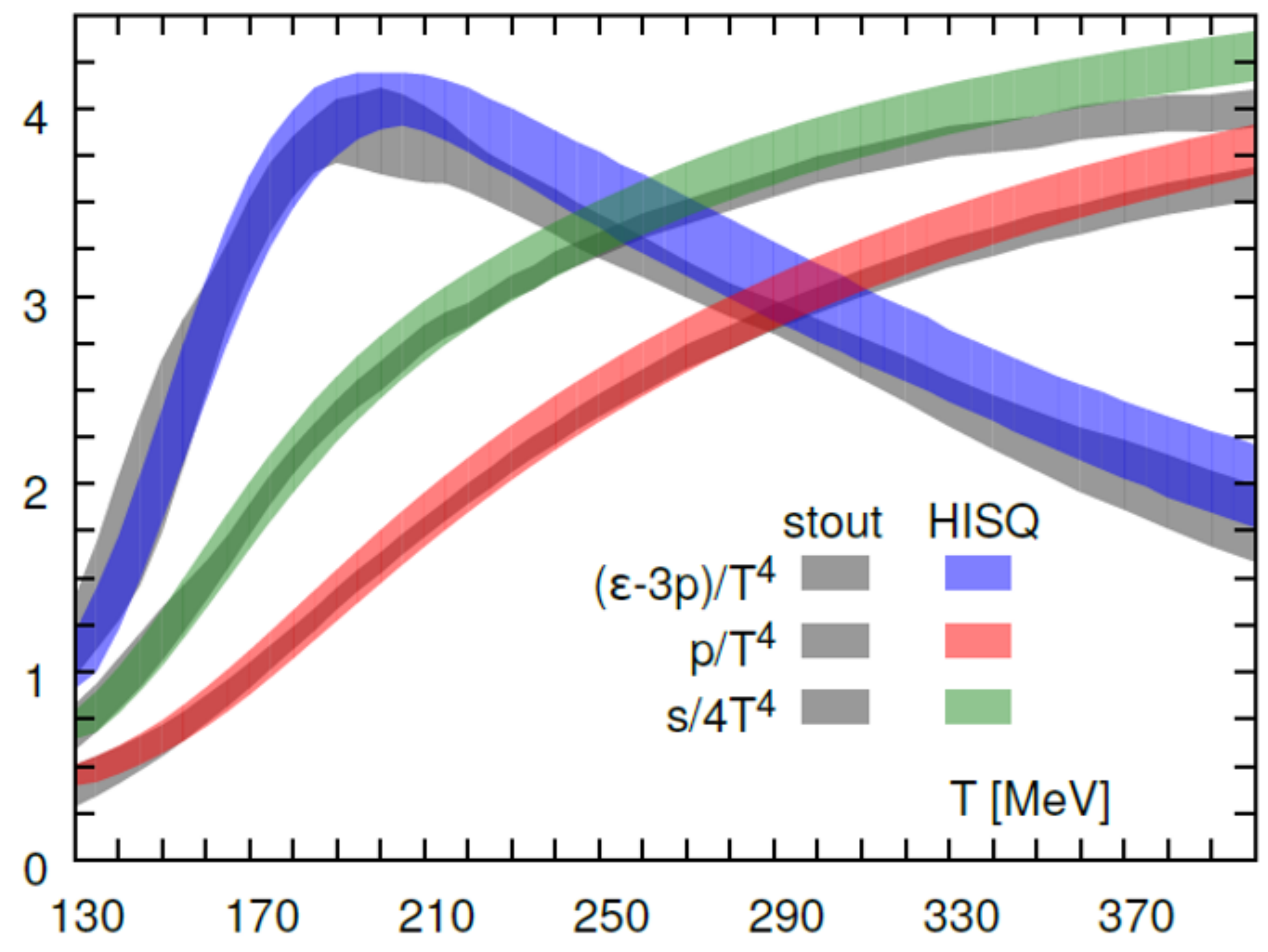


Description of the partonic and hadronic phase

- How to model a QGP phase in line with lQCD data?
- How to solve the hadronization problem?

pQCD based models: AMPT, BAMPS…

Macroscopic (EoS) —> microscopic parton properties (mass, interactions,…)



DQPM – effective model for the description of non-perturbative (strongly interacting) QCD based on lQCD EoS

Strongly coupling QGP

—> a strongly interacting quasiparticle system

Dynamical QuasiParticle Model (DQPM)

Theoretical basis: resummed single-particle Green's functions ➔ quark (gluon) propagator

gluon propagator: $\Delta^{-1} = P^2 - \Pi$ & quark propagator $S_q^{-1} = P^2 - \Sigma_q$

gluon self-energy: $\Pi = M_g^2 - i2\gamma_g\omega$ & quark self-energy: $\Sigma_q = M_q^2 - i2\gamma_q\omega$

➡ **Modeling of the quark/gluon masses and widths (ansatz inspired by HTL calculations)**

Masses:

$$M^2_{q(\bar{q})}(T,\mu_B) = \frac{N_c^2-1}{8N_c} g^2(T,\mu_B)\left(T^2 + \frac{\mu_q^2}{\pi^2}\right)$$

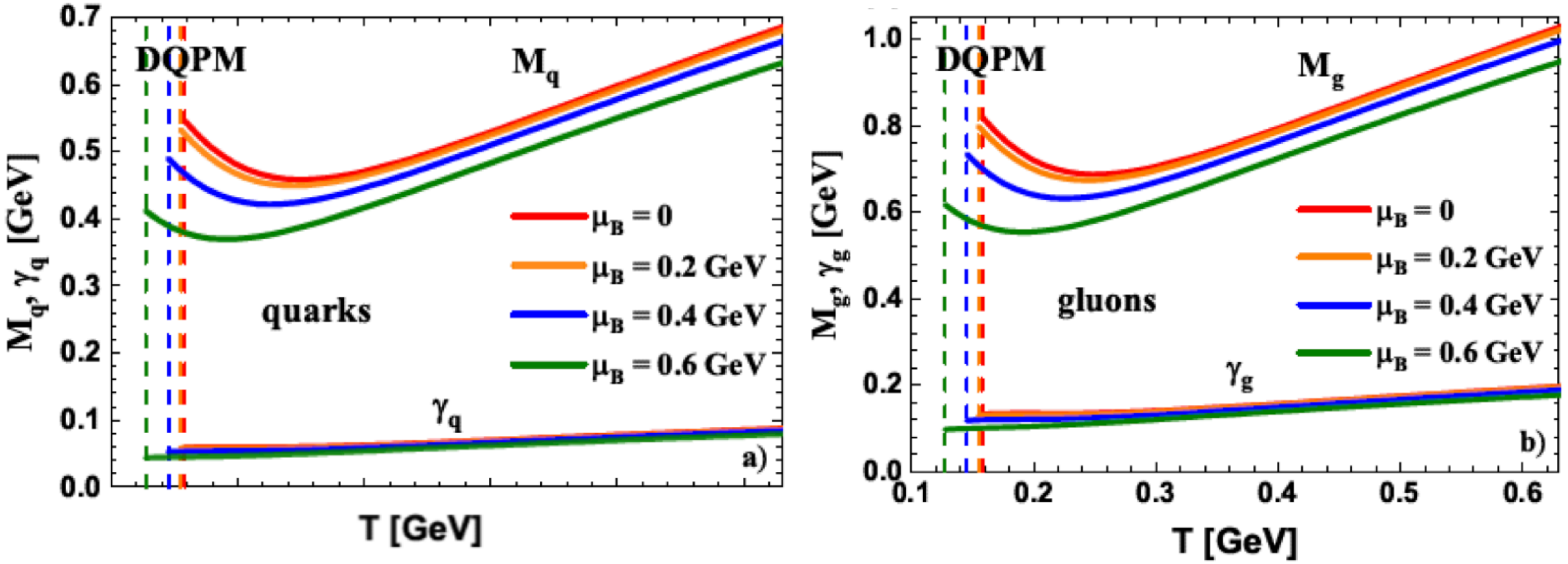
$$M^2_g(T,\mu_B) = \frac{g^2(T,\mu_B)}{6}\left(\left(N_c + \frac{1}{2}N_f\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right)$$

Widths:

$$\gamma_{q(\bar{q})}(T,\mu_B) = \frac{1}{3}\frac{N_c^2-1}{2N_c}\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c}{g^2(T,\mu_B)}+1\right)$$

$$\gamma_g(T,\mu_B) = \frac{1}{3}N_c\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c}{g^2(T,\mu_B)}+1\right)$$

➔ **(T, μ_B) dependent coupling constant can be determined from lattice results**



H. Berrehrah et al, PRC 93 (2016) 044914, Int.J.Mod.Phys. E25 (2016) 1642003

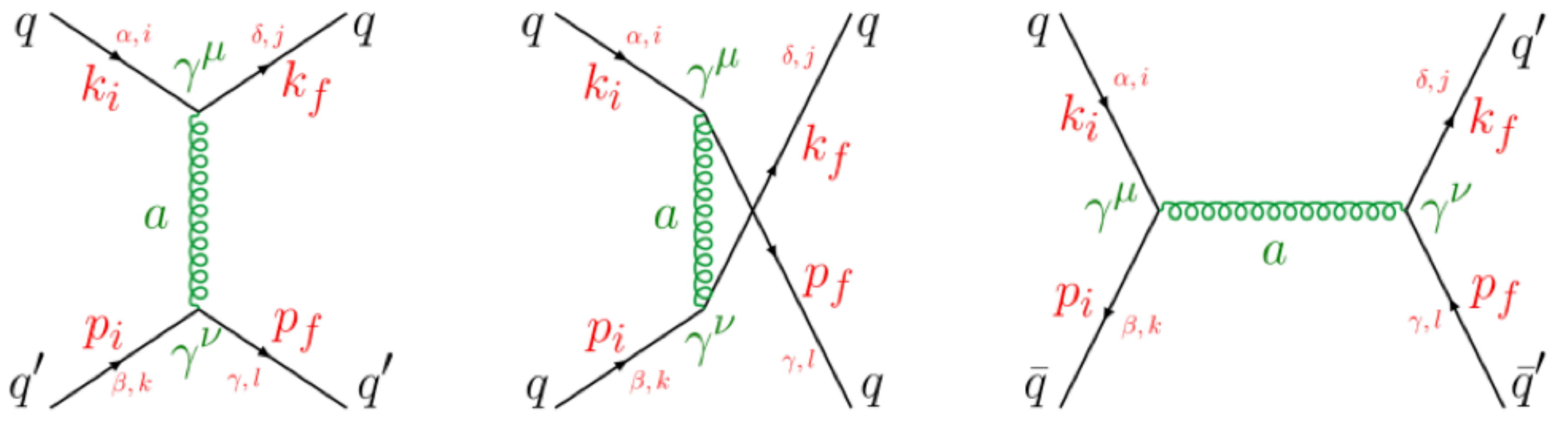
➡ **Machine learning method (Olga; Fupeng): without the above ansatz !**

Partonic interactions: matrix elements

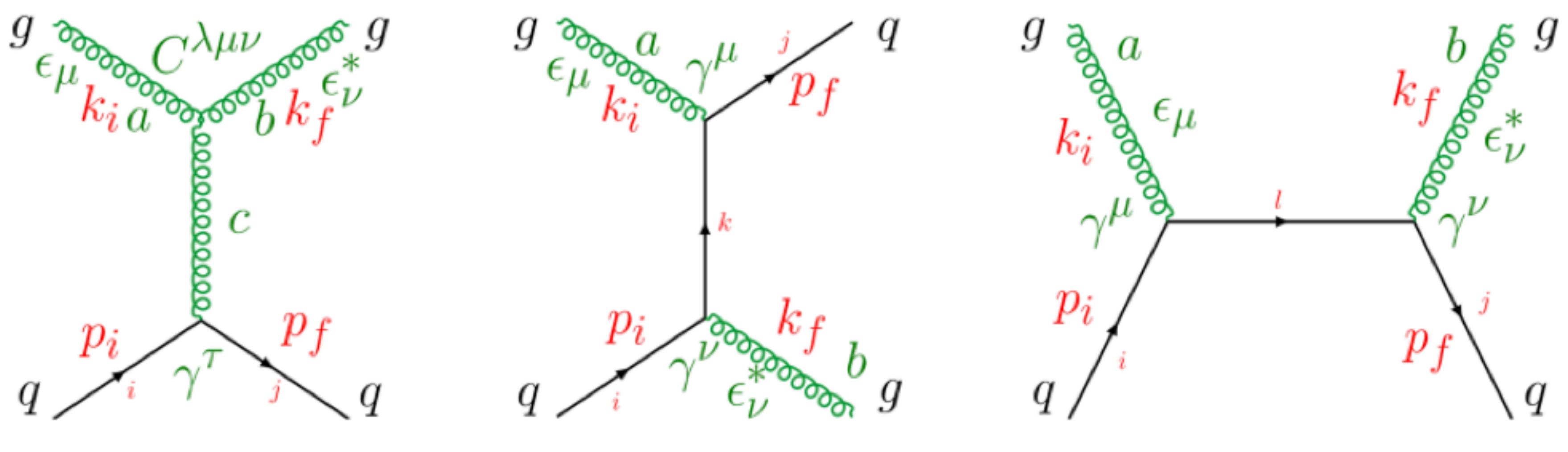
So far, only leading order diagrams included in PHSD

H. Berrehrah et al, PRC 93 (2016) 044914.
P. Moreau et al., PRC100 (2019) 014911.

$qq' \to qq'$



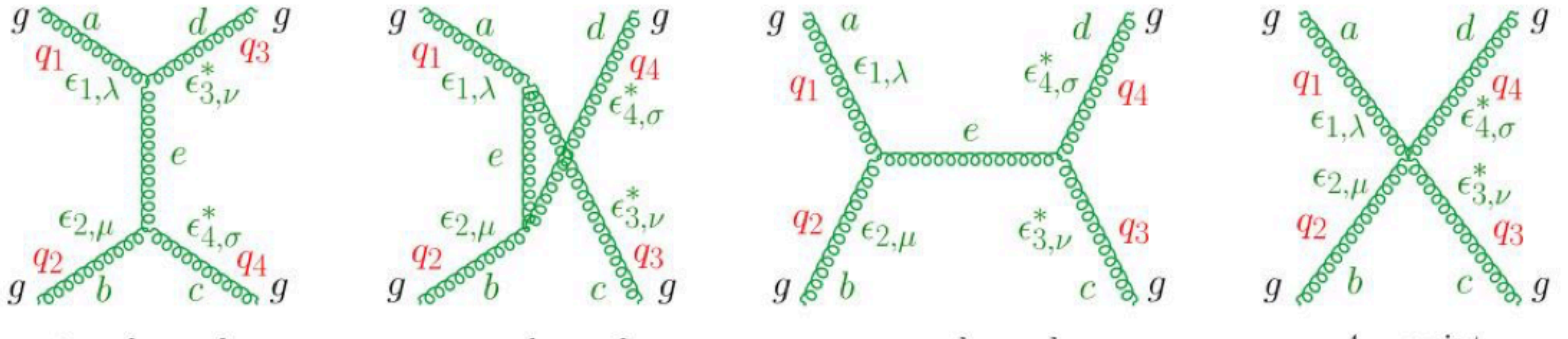
$gq \to gq$



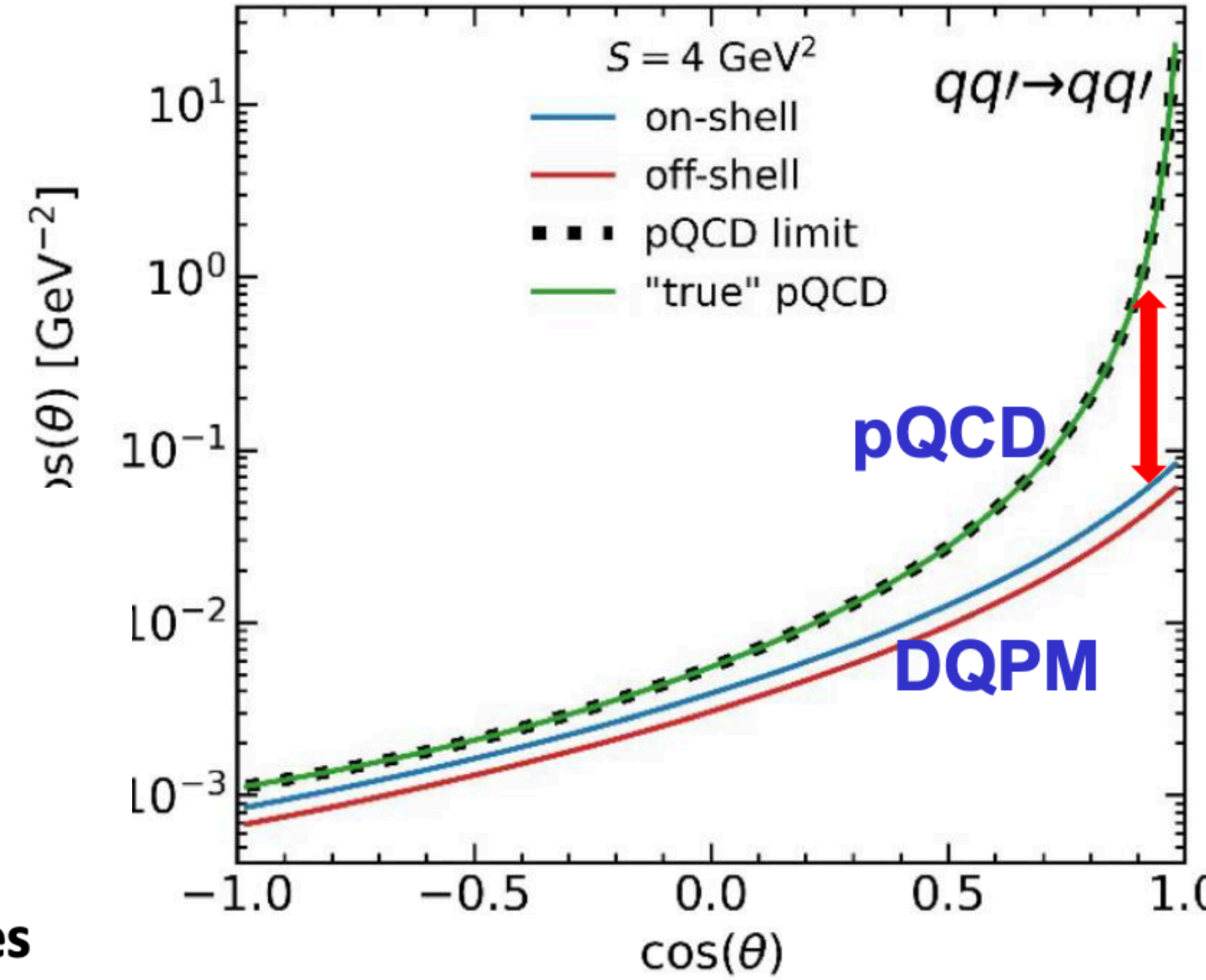
$$= -i\delta_{ab}\frac{g^{\mu\nu} - q^\mu q^\nu / M_g^2}{q^2 - M_g^2 + 2i\gamma_g q_0}$$

$$= i\delta_{ij}\frac{\not{q} + M_q}{q^2 - M_q^2 + 2i\gamma_q q_0}$$

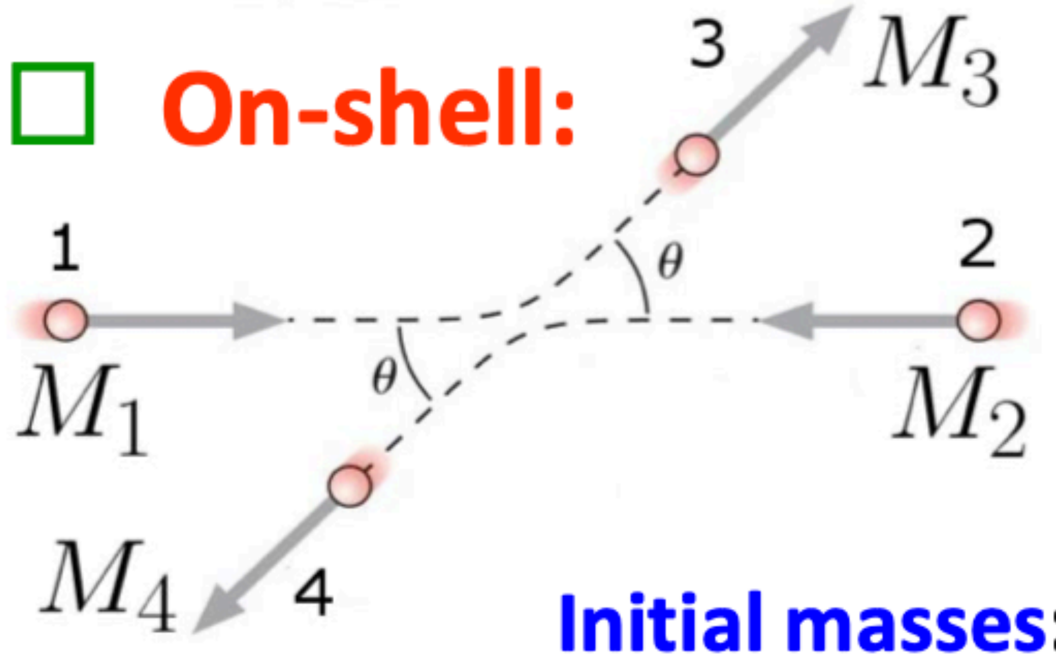
$gg \to gg$



$g \to q\bar{q}$
$q\bar{q} \to g$

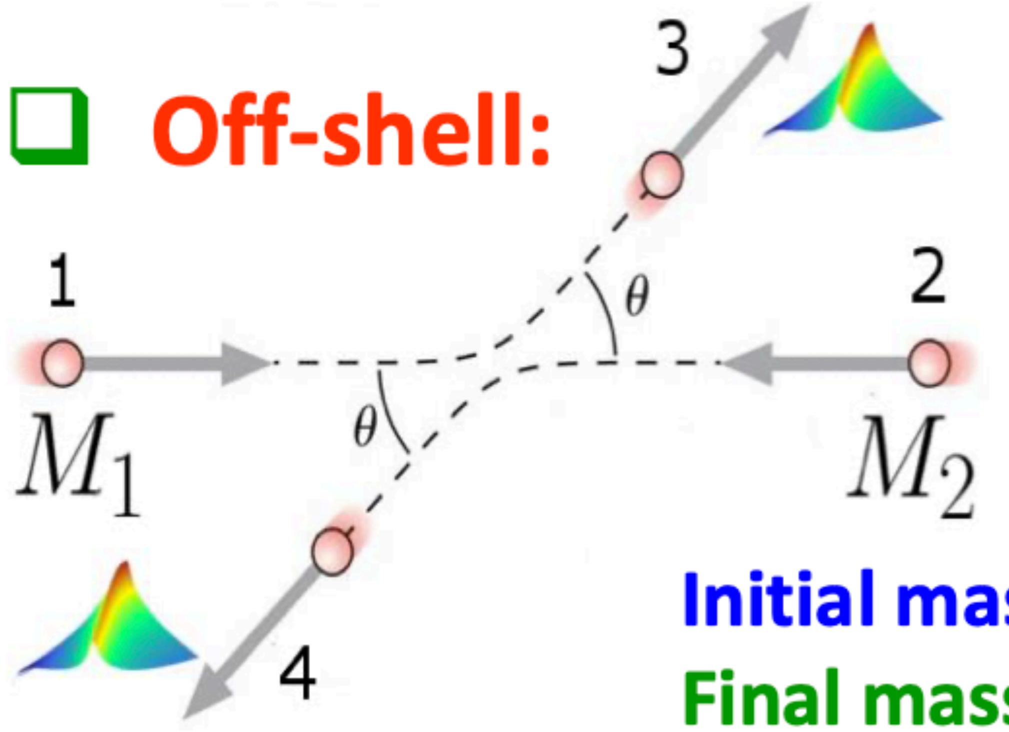


☐ **On-shell:**



Initial masses: pole masses
Final masses: pole masses

❑ **Off-shell:**



Initial masses: pole masses
Final masses: integration over spectral functions

Off-shell parton hadronization

The hadronization occurs by quark-antiquark or 3 quark/3 antiquark recombination which is described by covariant transition rates —> energy-momentum conservation.

For meson:

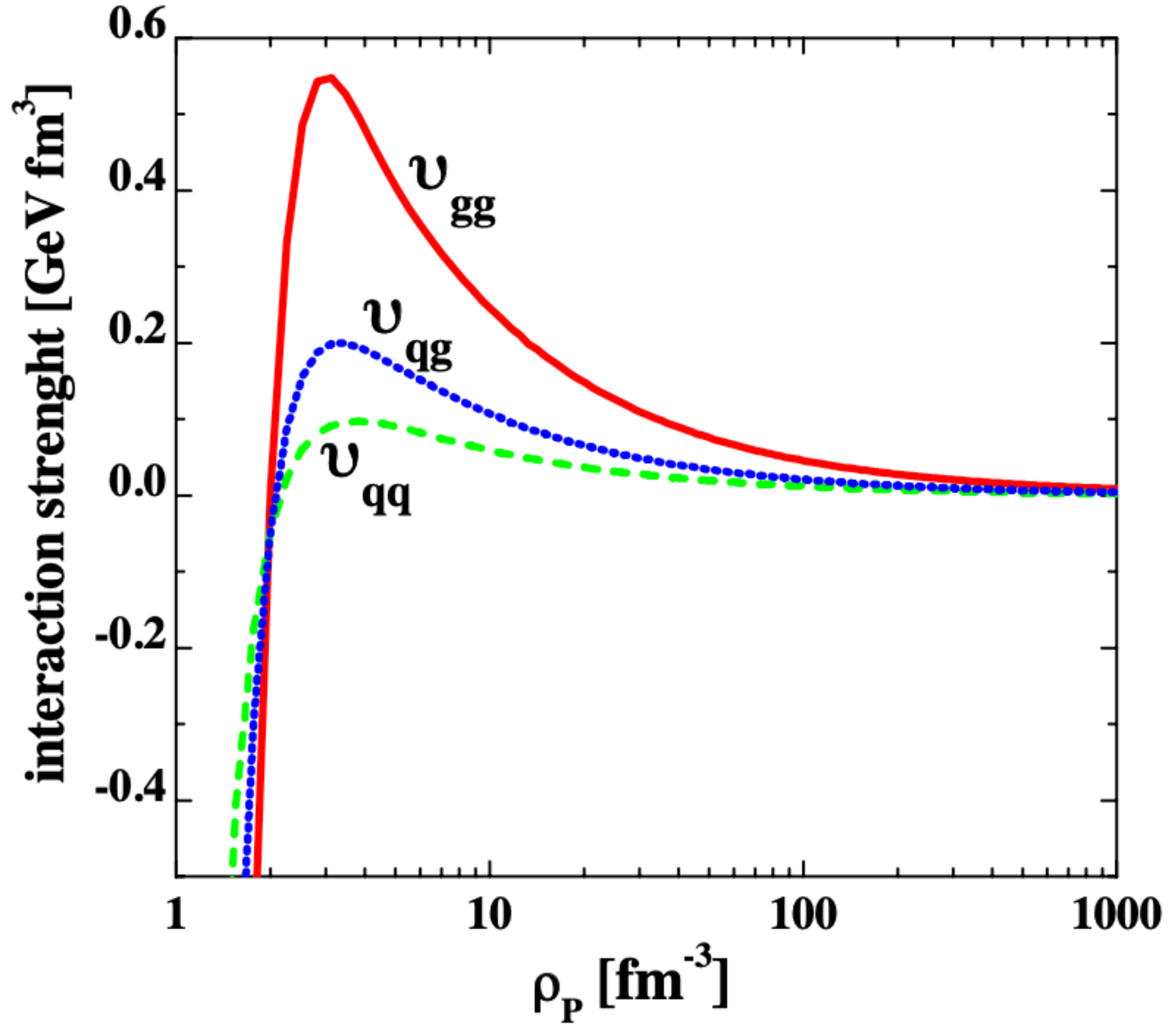
W. Cassing and E. Bratkovskaya, *PRC* 78 (2008) 034919.

$$\frac{dN_m(x,p)}{d^4x d^4p} = Tr_q Tr_{\bar{q}}\ \delta^4(p - p_q - p_{\bar{q}})\ \delta^4\left(\frac{x_q + x_{\bar{q}}}{2} - x\right)$$
$$\times \omega_q\ \rho_q(p_q)\ \omega_{\bar{q}}\ \rho_{\bar{q}}(p_{\bar{q}})\ |v_{q\bar{q}}|^2\ W_m(x_q - x_{\bar{q}}, (p_q - p_{\bar{q}})/2)$$
$$\times N_q(x_q, p_q)\ N_{\bar{q}}(x_{\bar{q}}, p_{\bar{q}})\ \delta(\text{flavor, color}).$$

$$Tr_j = \sum_j \int d^4x_j \int \frac{d^4p_j}{(2\pi)^4}$$ **Sum over spin, flavor, color**

N_q **: phase-space density**

$v_{q\bar{q}}$ **: effective quark-antiquark interaction from the DQPM**



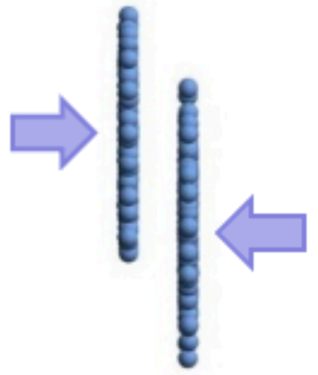
$$W_m(\xi, p_\xi) = \exp\left(\frac{\xi^2}{2b^2}\right)\ \exp\left(2b^2(p_\xi^2 - (M_q - M_{\bar{q}})^2/4)\right)$$

phase-space distribution of the formed "pre-hadron" (later decay to ground states)

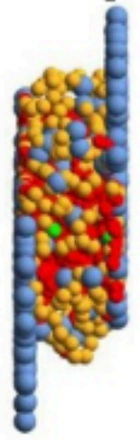
Parton-Hadron-String-Dynamics (PHSD)

PHSD: based on the solution of generalized off-shell transport equations derived from Kadanoff-Baym many-body theory. Website: http://theory.gsi.de/~ebratkov/phsd-project/PHSD/index5.html

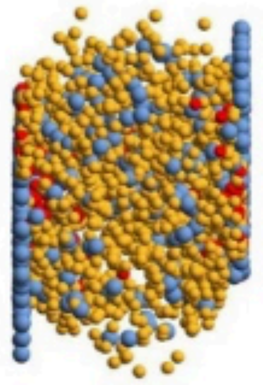
Initial A+A collision



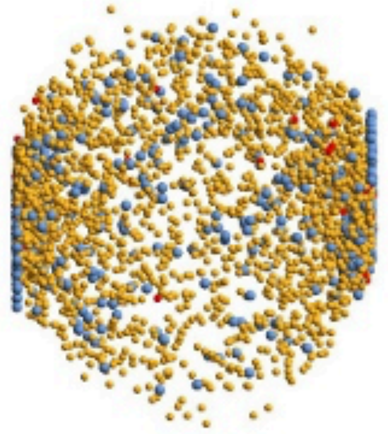
Partonic phase



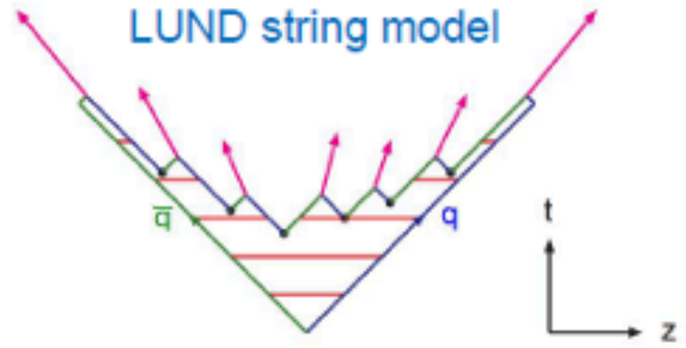
Hadronization



Hadronic phase

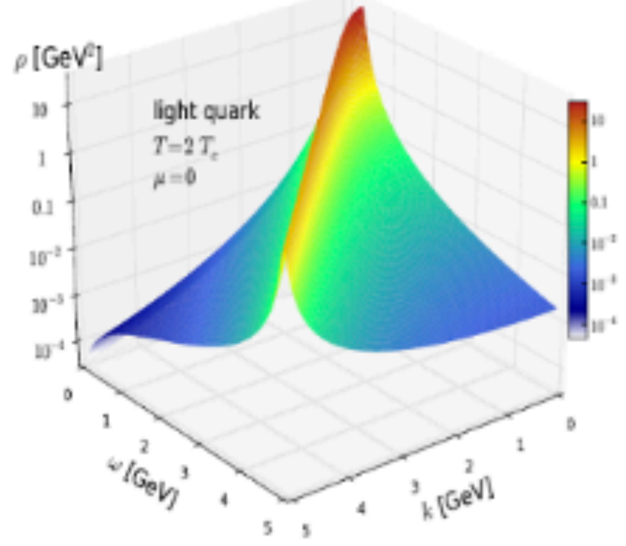


- ❑ **Initial A+A collisions :**
  N+N → string formation → decay to pre-hadrons + leading hadrons



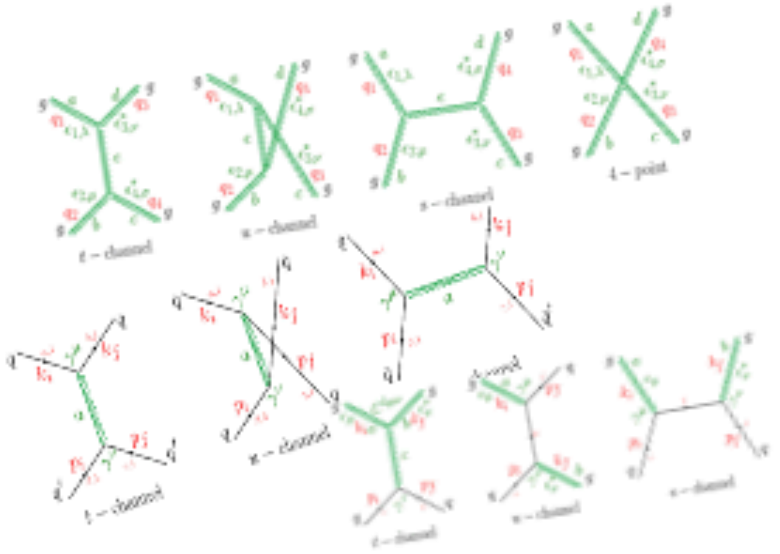
- ❑ **Formation of QGP stage if local $\varepsilon > \varepsilon_{critical}$:**
  dissolution of pre-hadrons → partons

- ❑ **Partonic phase - QGP:**
  QGP is described by the Dynamical QuasiParticle Model (DQPM) matched to reproduce lattice QCD EoS for finite T and μ_B (crossover)

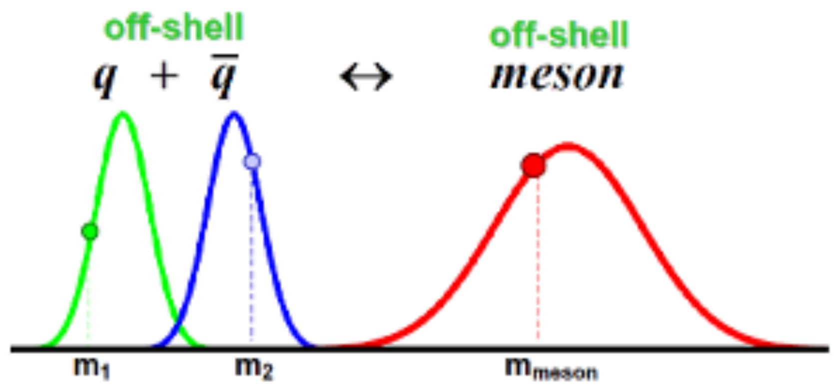


- Degrees-of-freedom: strongly interacting quasiparticles: massive quarks and gluons (g, q, q_{bar}) with sizeable collisional widths in a self-generated mean-field potential

- Interactions: (quasi-)elastic and inelastic collisions of partons



- ❑ **Hadronization to colorless off-shell mesons and baryons:**
  Strict 4-momentum and quantum number conservation



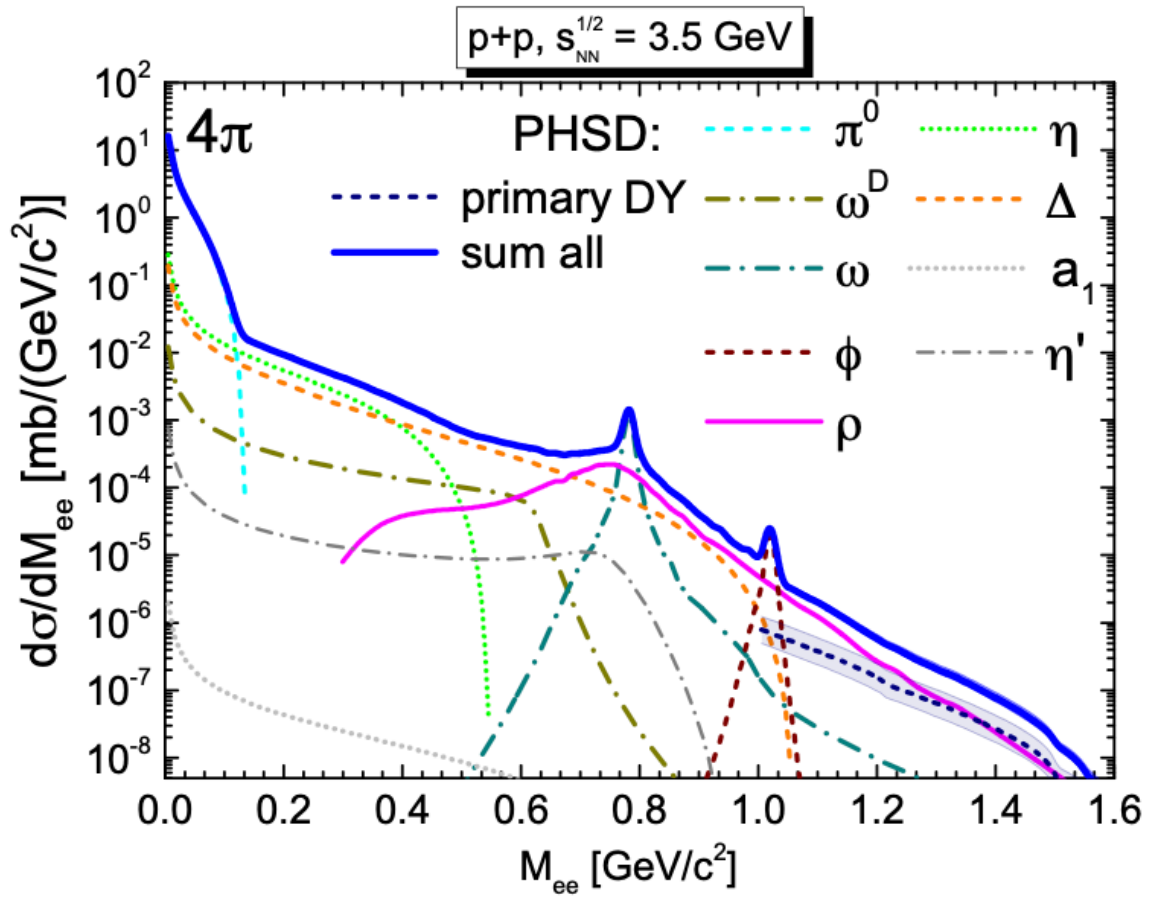
- ❑ **Hadronic phase: hadron-hadron interactions – off-shell HSD**
  including $n \leftrightarrow m$ selected reactions (for strangeness, anti-baryons, deuteron production)

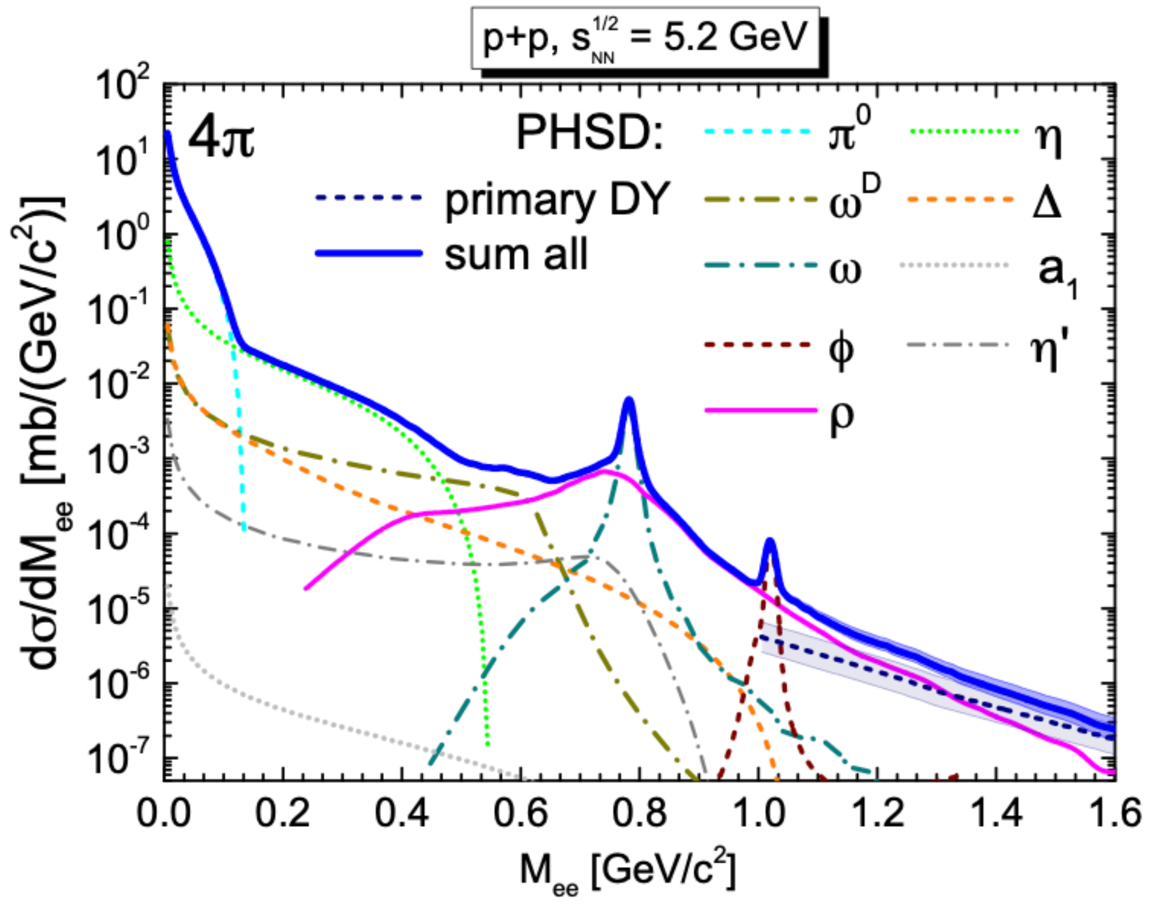
Highlights

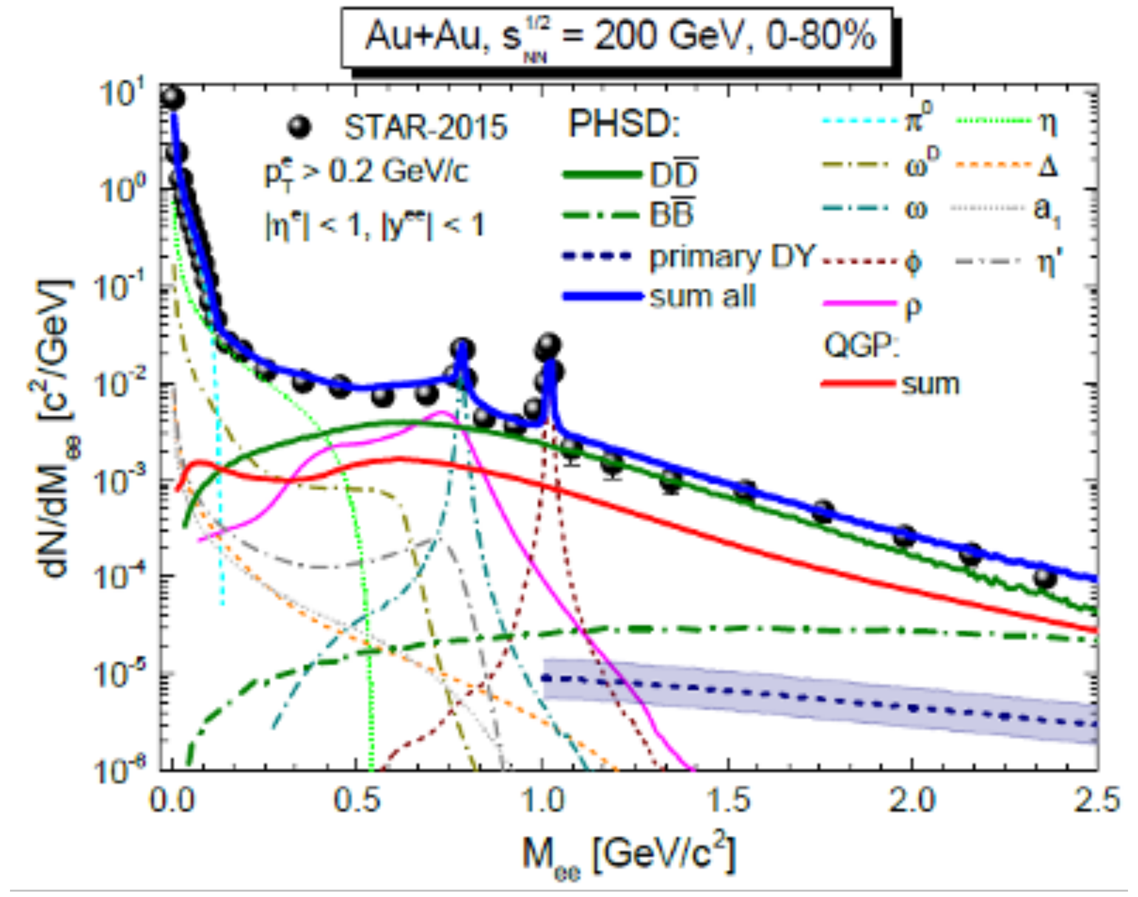
❑ **PHSD provides a good description of bulk observables (y-, p_T-distributions, flow coefficients v_n, …) from SIS to LHC**

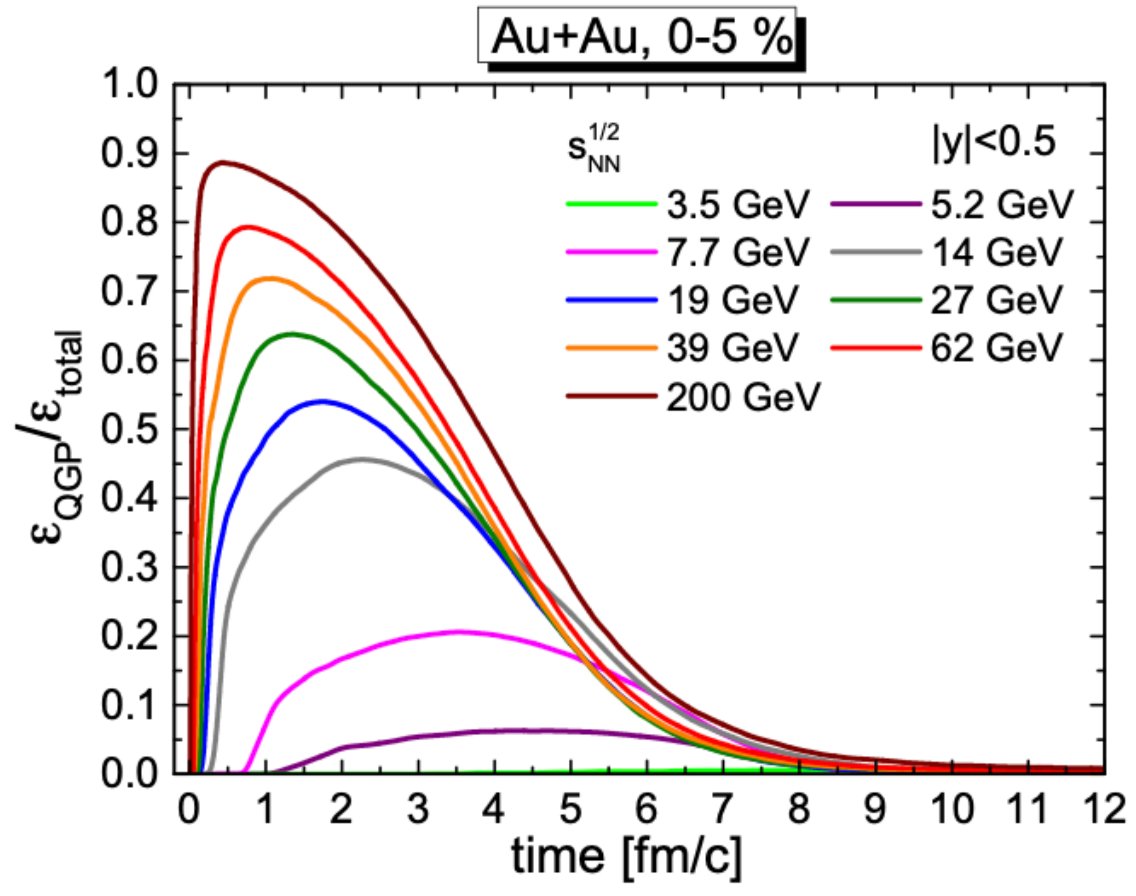
W. Cassing, E. Bratkovskaya, Phys.Rept. 308 (1999) 65; E. Bratkovskaya, J. Aichelin, M. Thomere, S. Vogel, M. Bleicher, PRC 87 (2013) 064907;T. Song et al., Phys. Rev. C97 (2018), 064907; PRC 103, 044901 (2021); A. Palmese et al., PRC94 (2016) 044912;…

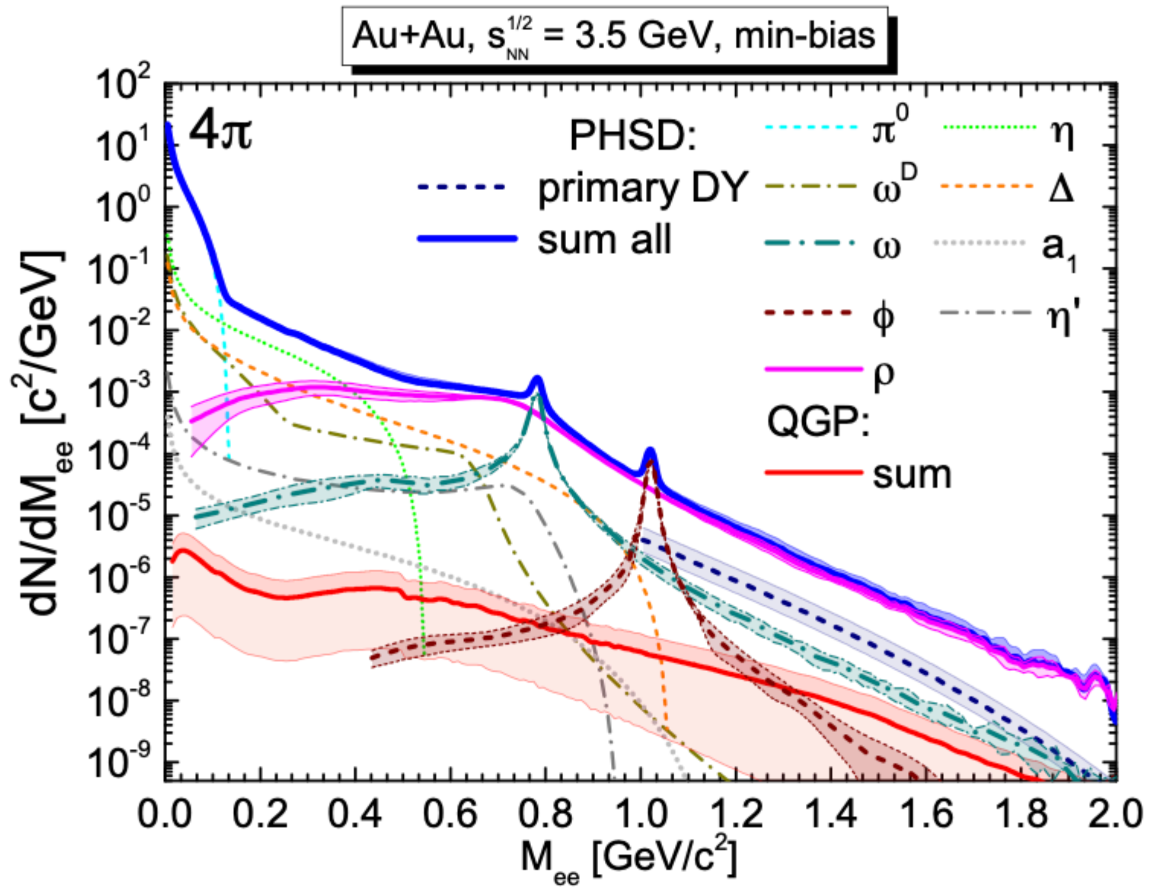
Recent: EM probe —> dilepton spectra including a collisional broadening of the vector meson spectral functions + primary DY + QGP + correlated charm

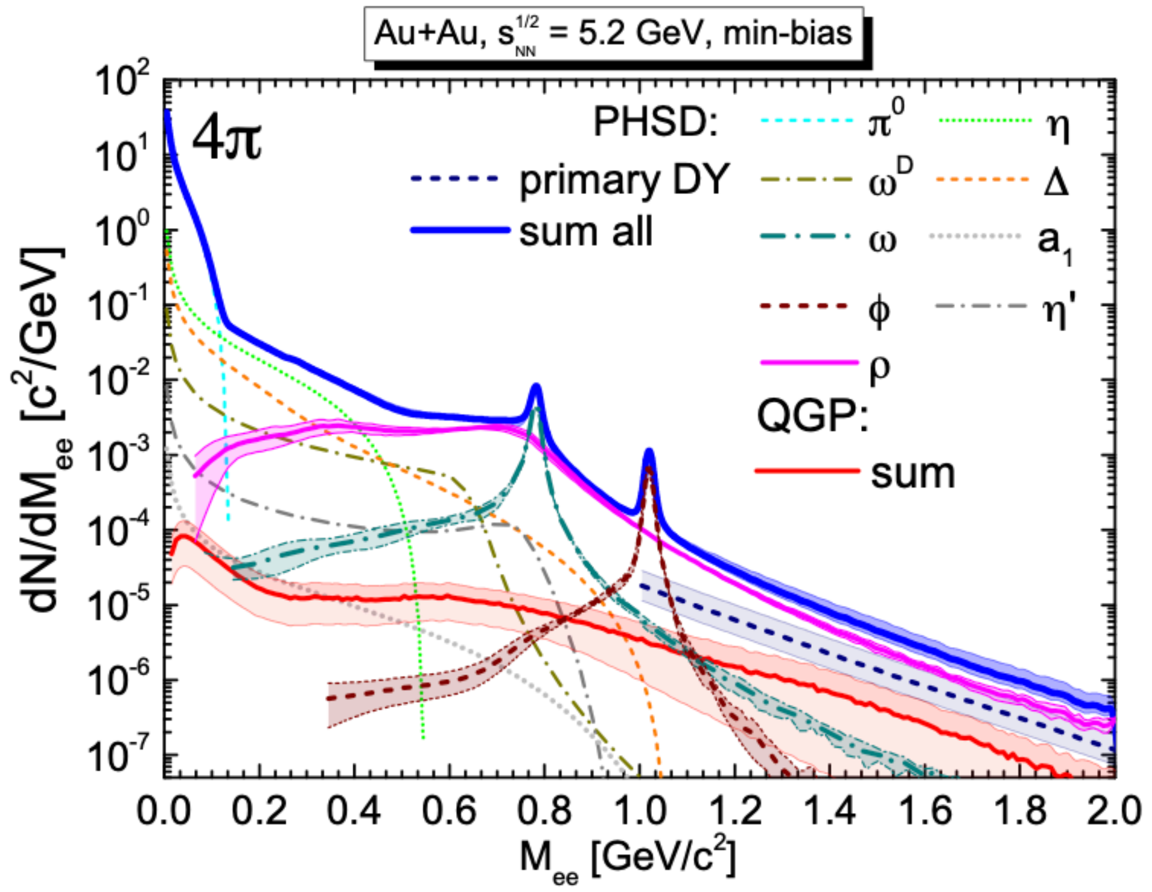










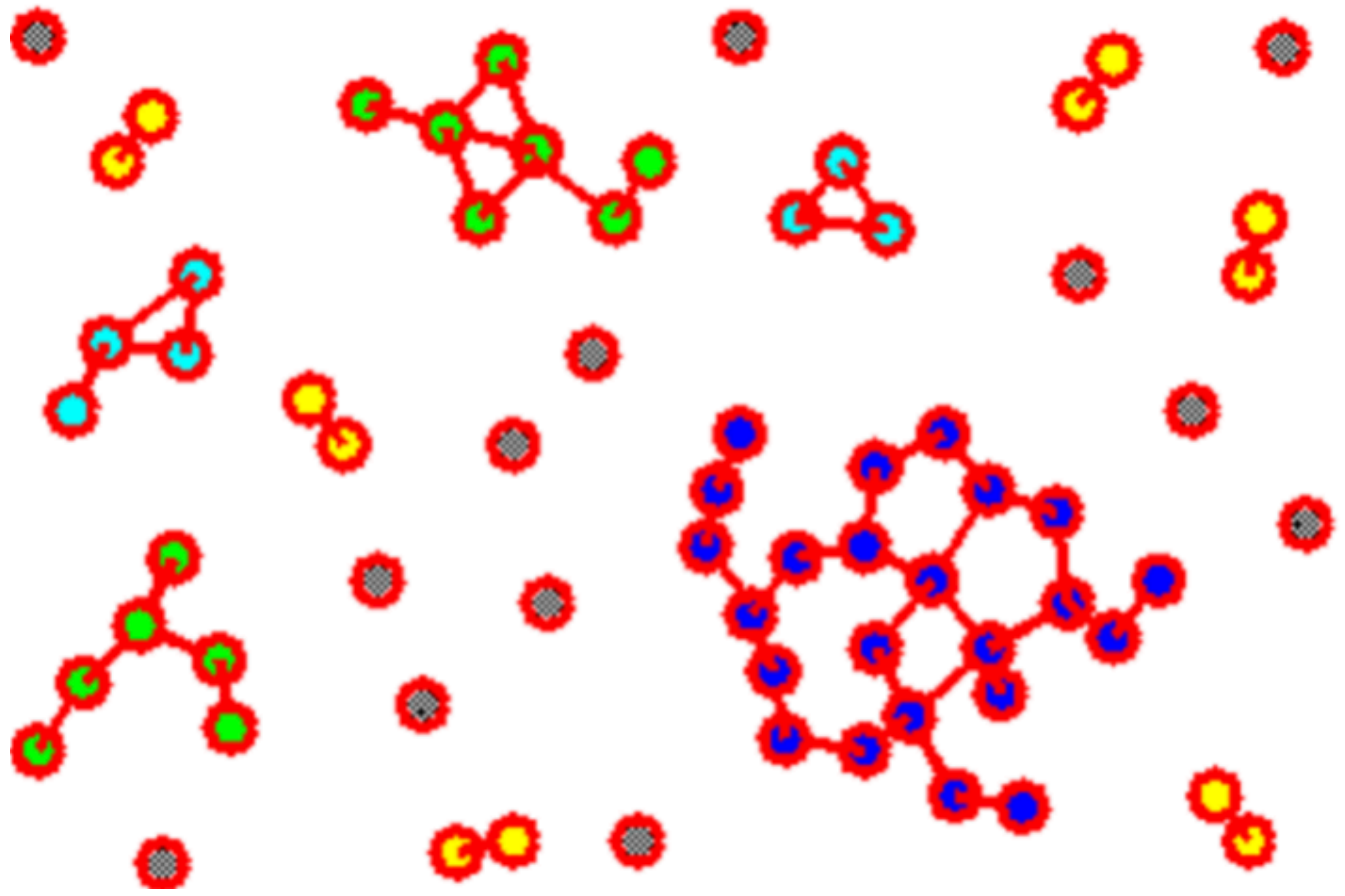


Clearly see the contributions from the different parts!

Adrian W. R. Jorge, T. Song, Q. Zhou, E. Bratkovskaya, *PRC* 111 (2025) 6, 064904.

PHQMD:

I. QMD for nucleons evolutions



BUU vs. QMD

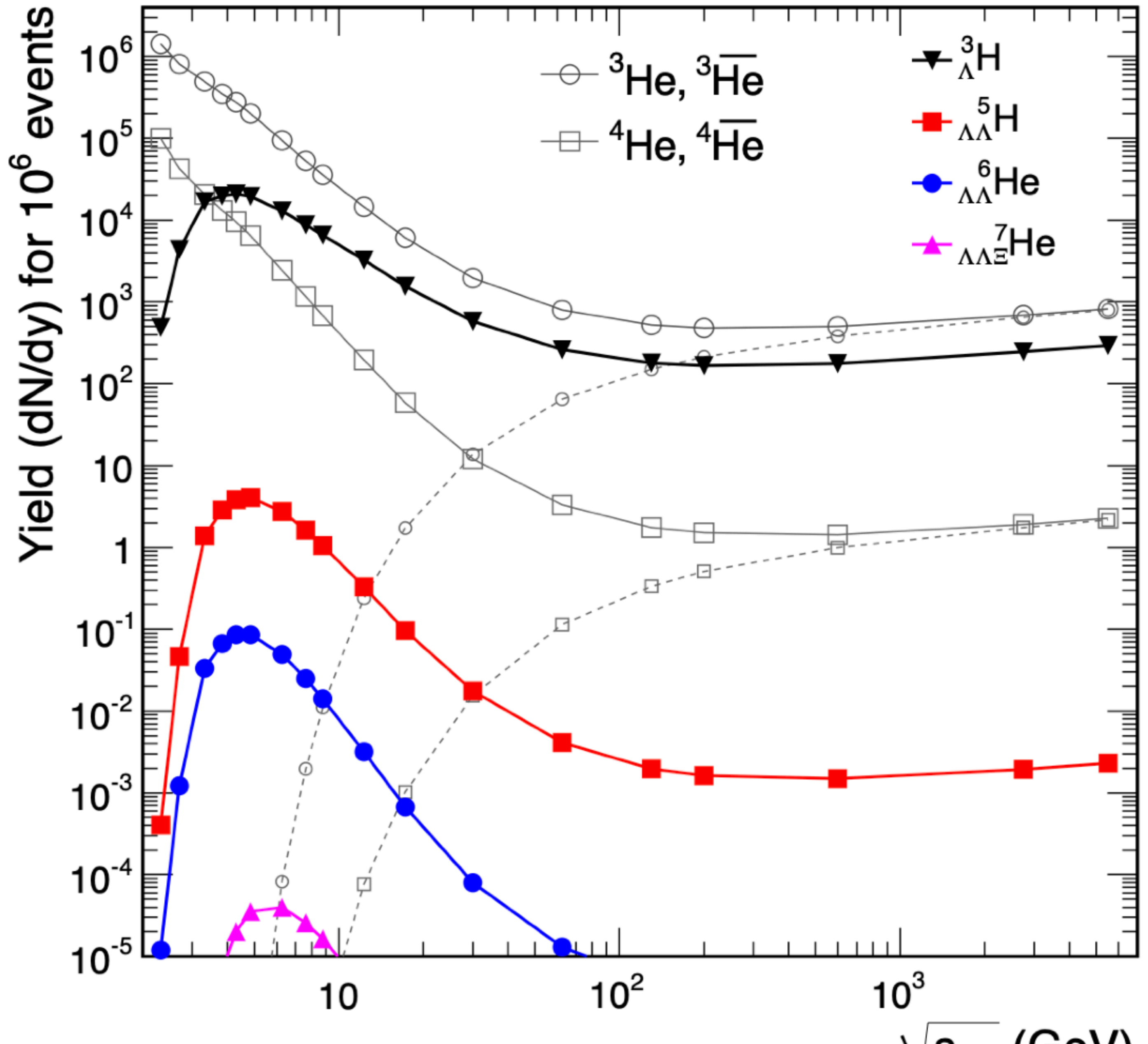
BUU (Boltzmann–Uehling–Uhlenbeck) vs. QMD (Quantum Molecular Dynamics) **Jun Xu's talk**

TMEP collaboration, PPNP, 125 (2022)103962. ….

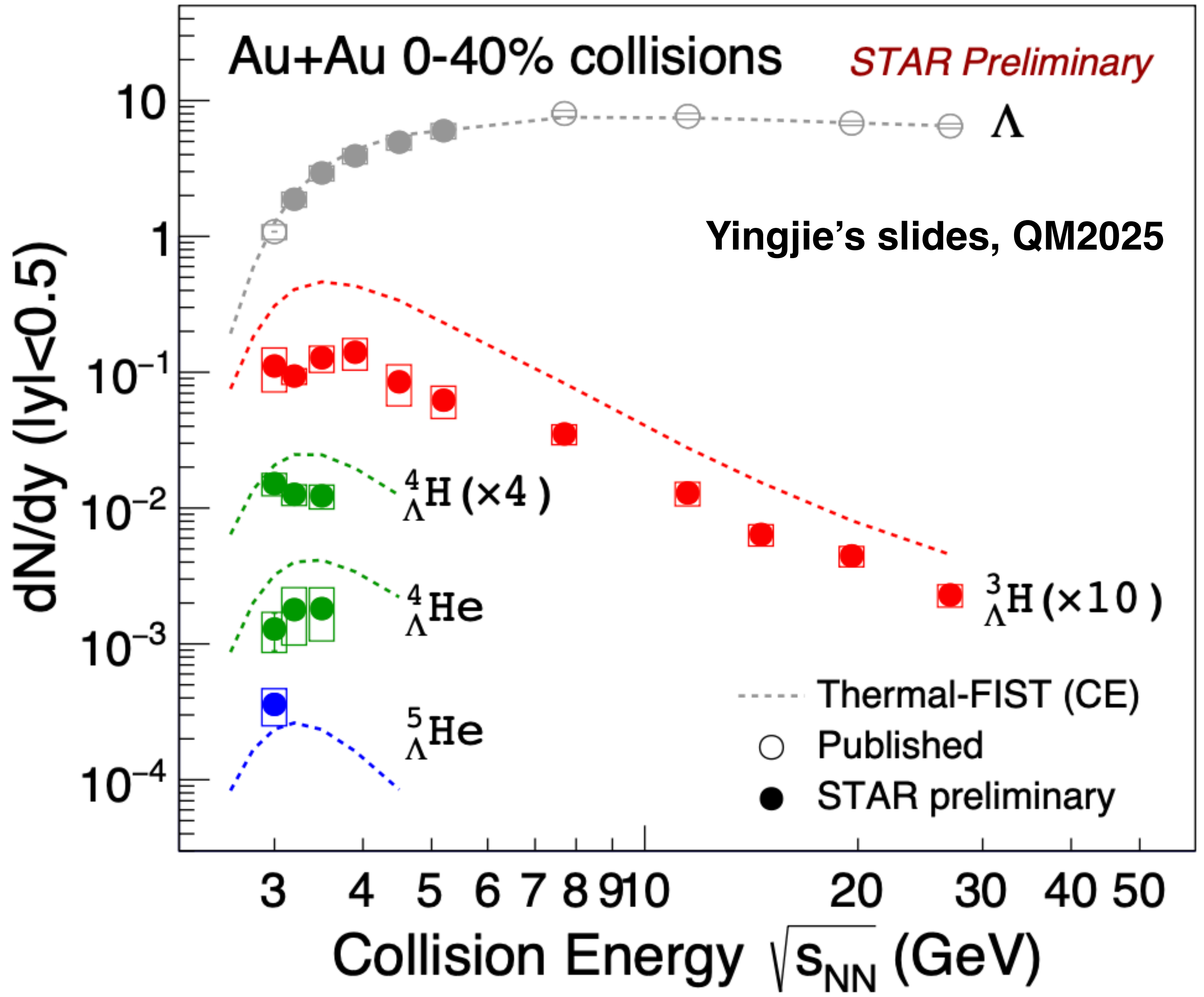
Why QMD?

- ❑ **"Bulk" observables for hadrons are rather similar in QMD and MF!**
- ❑ **Cluster formation is sensitive to nucleon dynamics:**

- **QMD – allows to keep over time NN correlations by potential interaction**
- **MF – correlations are smeared out**



A. Andronic, et al. Phys.Lett.B 697 (2011) 203-207



The production of nuclear cluster are largely enhanced in heavy ion collisions at RHIC BESII, CBM, HIAF, NICA etc.

Probe the EoS, clusterization mechanism, Y–N, Y–Y interaction,…

BUU vs. QMD

BUU (Boltzmann–Uehling–Uhlenbeck) vs. QMD (Quantum Molecular Dynamics) Jun Xu's talk

TMEP collaboration, PPNP, 125 (2022)103962. ….

Why QMD?

- ❑"Bulk" observables for hadrons are rather similar in QMD and MF!
- ❑Cluster formation is sensitive to nucleon dynamics:
  - QMD – allows to keep over time NN correlations by potential interaction
  - MF – correlations are smeared out

QMD (AMD, IQMD, ImQMD, LQMD, JAM, ….) is a n-body model but is limited to energies < 1.5 AGeV —> describes fragments at SIS energies

UrQMD is a n-body model but makes clusterization via coalescence and a statistical fragmentation model

PHQMD is a new n-body model not limited to low beam energies —> includes the formation of a quark-gluon plasma at higher energies, at low energies similar to QMD

PHQMD

PHQMD

E. Bratkovskaya & J. Aichelin

PHQMD: a **unified n-body microscopic transport approach** for the description of heavy-ion collisions and **dynamical cluster formation** from low to ultra-relativistic energies

Realization: combined model **PHQMD = (PHSD & QMD) & (MST/SACA)**

J. Aichelin et al., PRC 101 (2020) 044905;
S. Gläßel et al., PRC 105 (2022) 1;
G. Coci et al., PRC 108 (2023) 1, 014902

Parton-Hadron-Quantum-Molecular Dynamics

↓

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

J.Aichelin, Phys. Rept. 202 (1991)

↓

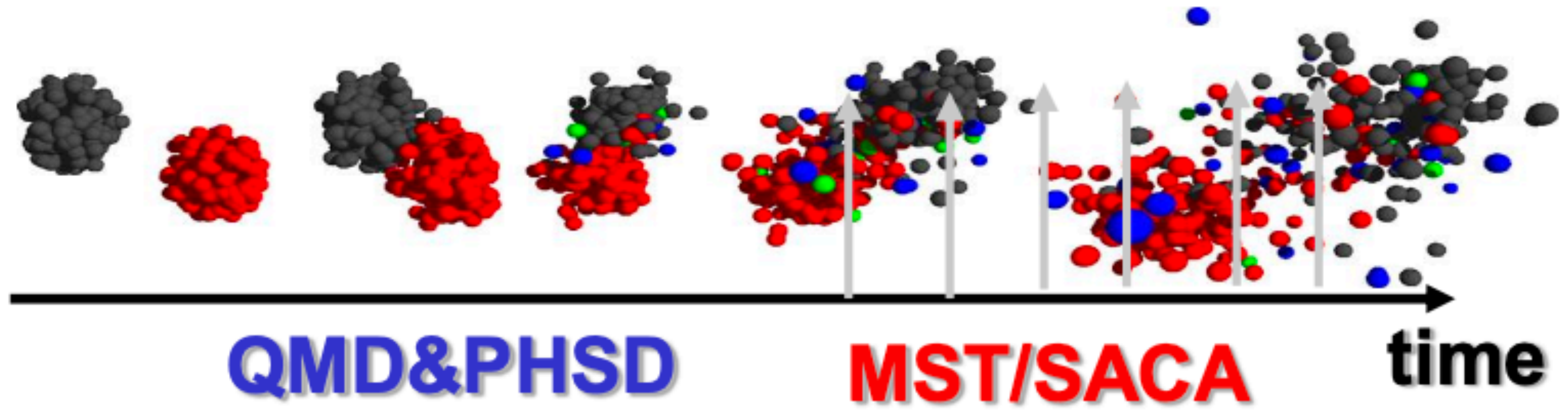
Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215

↓

Cluster recognition:
SACA (Simulated Annealing Clusterization Algorithm)
or MST (Minimum Spanning Tree)

R. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266



PHQMD: EoM

Jun Xu's talk

Generalized Ritz variational principle:

$$\delta \int_{t_1}^{t_2} dt \langle \psi(t) | i \frac{d}{dt} - H | \psi(t) \rangle = 0.$$

Many-body Hamiltonian:

$$H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j \neq i} V_{i,j})$$

Many-body wave function:

$$\psi(t) = \prod_{i=1}^{N} \psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t)$$

Ansatz: trial wave function for one particle "i" : Gaussian with width L centered at "r_{i0}", "p_{i0}"

$$\psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = C \, e^{-\frac{1}{4L}\left(\mathbf{r}_i - \mathbf{r}_{i0}(t) - \frac{\mathbf{p}_{i0}(t)}{m} t\right)^2} \cdot e^{i \mathbf{p}_{i0}(t)(\mathbf{r}_i - \mathbf{r}_{i0}(t))} \cdot e^{-i \frac{\mathbf{p}_{i0}^2(t)}{2m} t}$$

Equations-of-motion (EoM) in coordinate and momentum space:

$$\dot{r}_{i0} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial \langle H \rangle}{\partial r_{i0}}$$

PHQMD: Potential

2-body potential:

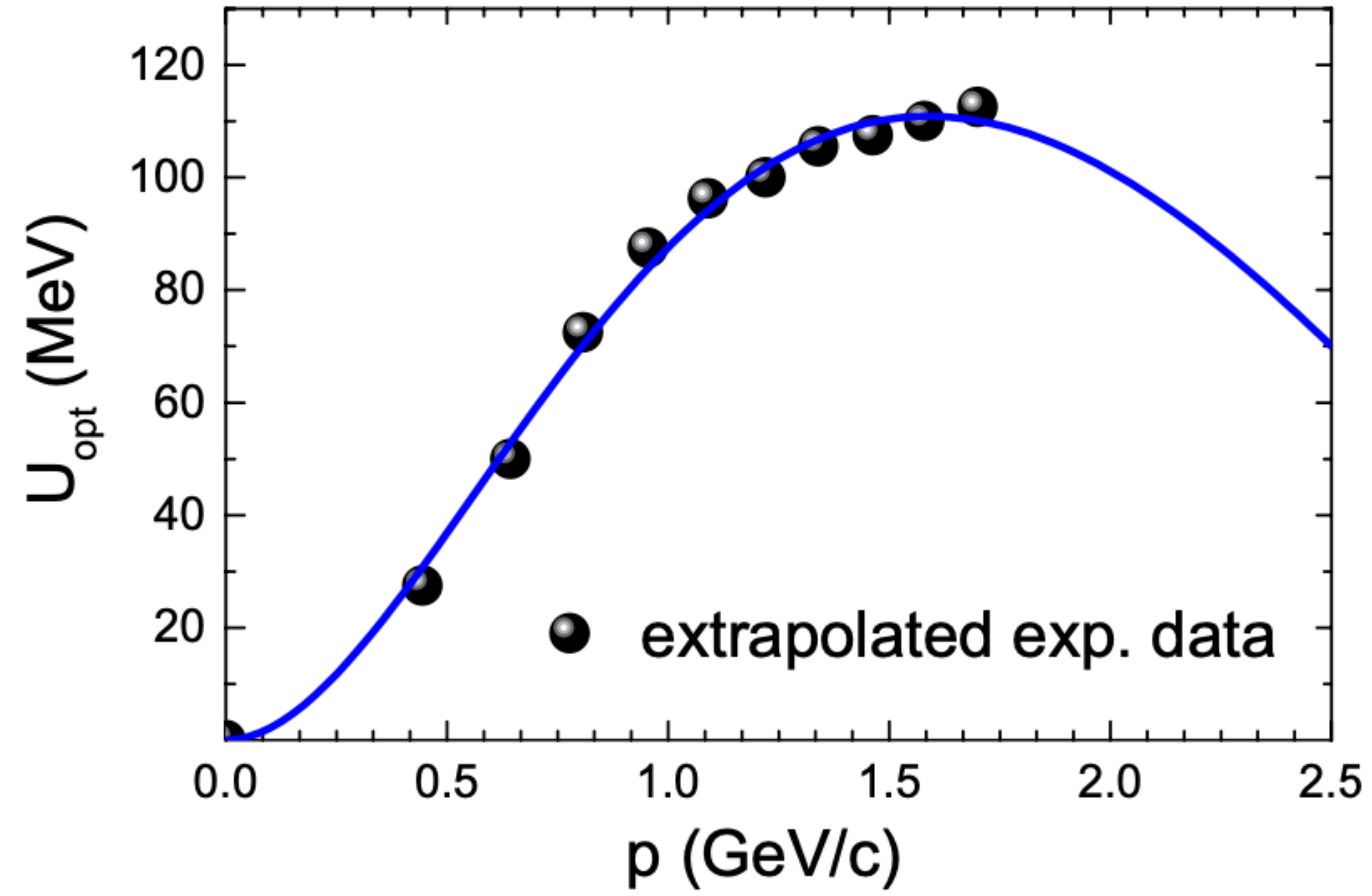
$$V_{ij} = V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, \mathbf{p}_{i0}, \mathbf{p}_{j0}, t)$$
$$= V_{\text{Skyrme loc}} + V_{\text{mom}} + V_{\text{Coul}}$$
$$= \frac{1}{2} t_1 \delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{\gamma+1} t_2 \delta(\mathbf{r}_i - \mathbf{r}_j)\, \rho^{\gamma-1}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, t)$$

Skyrme

$$+ V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{p}_{i0}, \mathbf{p}_{j0}) + \frac{1}{2} \frac{Z_i Z_j e^2}{|\mathbf{r}_i - \mathbf{r}_j|},$$

momentum dependent

Coulomb



In infinite matter a potential corresponds to the EoS:

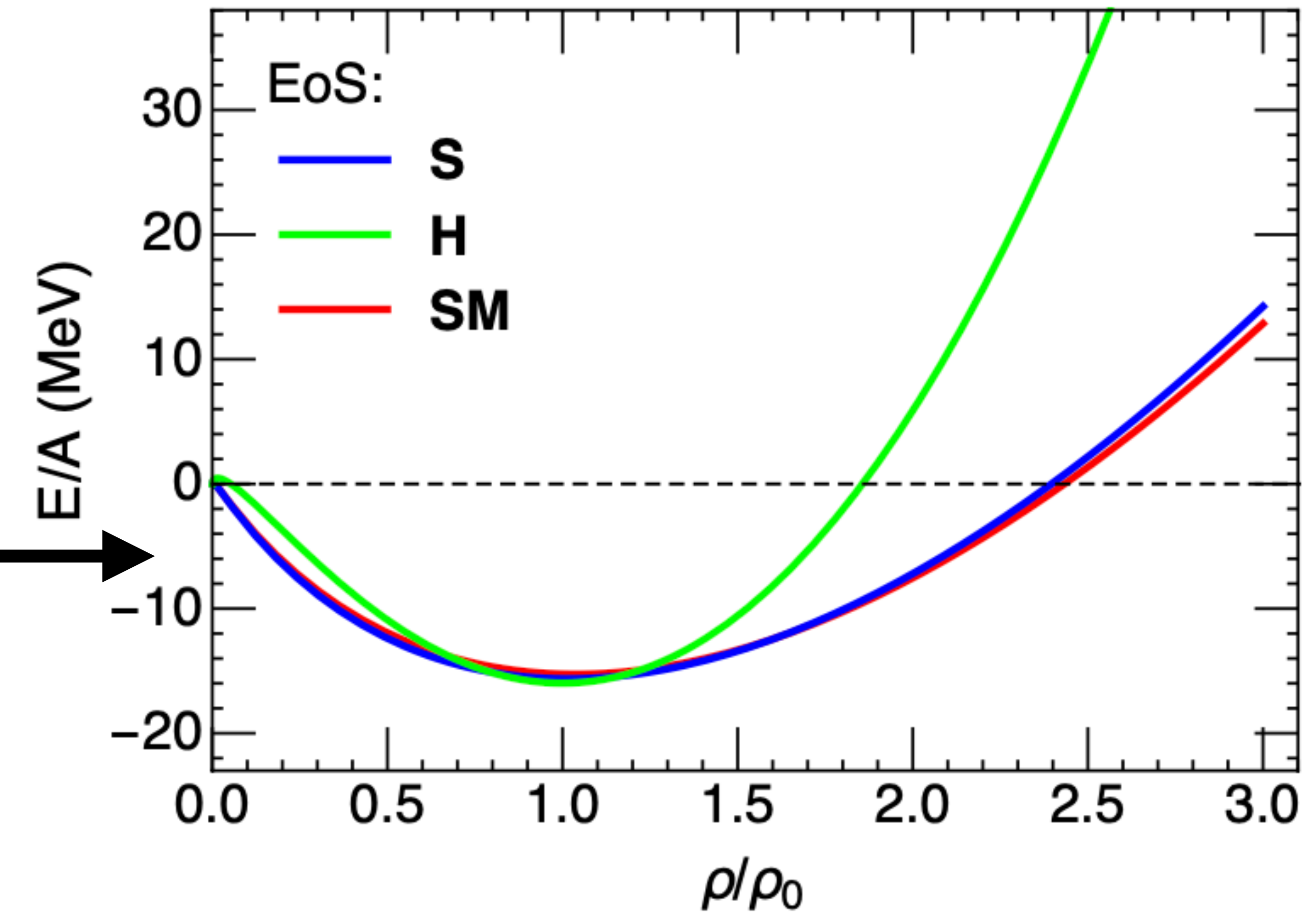
$$E/A(\rho) = \frac{3}{5} E_F + V_{Skyrme\ stat}(\rho) + V_{mom}(\rho)$$

$$V_{Skyrme\ stat} = \alpha \frac{\rho}{\rho_0} + \beta \left(\frac{\rho}{\rho_0}\right)^{\gamma}; \qquad V_{mom} = (a\Delta p + b\Delta p^2)\ exp(-c\sqrt{\Delta p})\ \frac{\rho}{\rho_0}$$

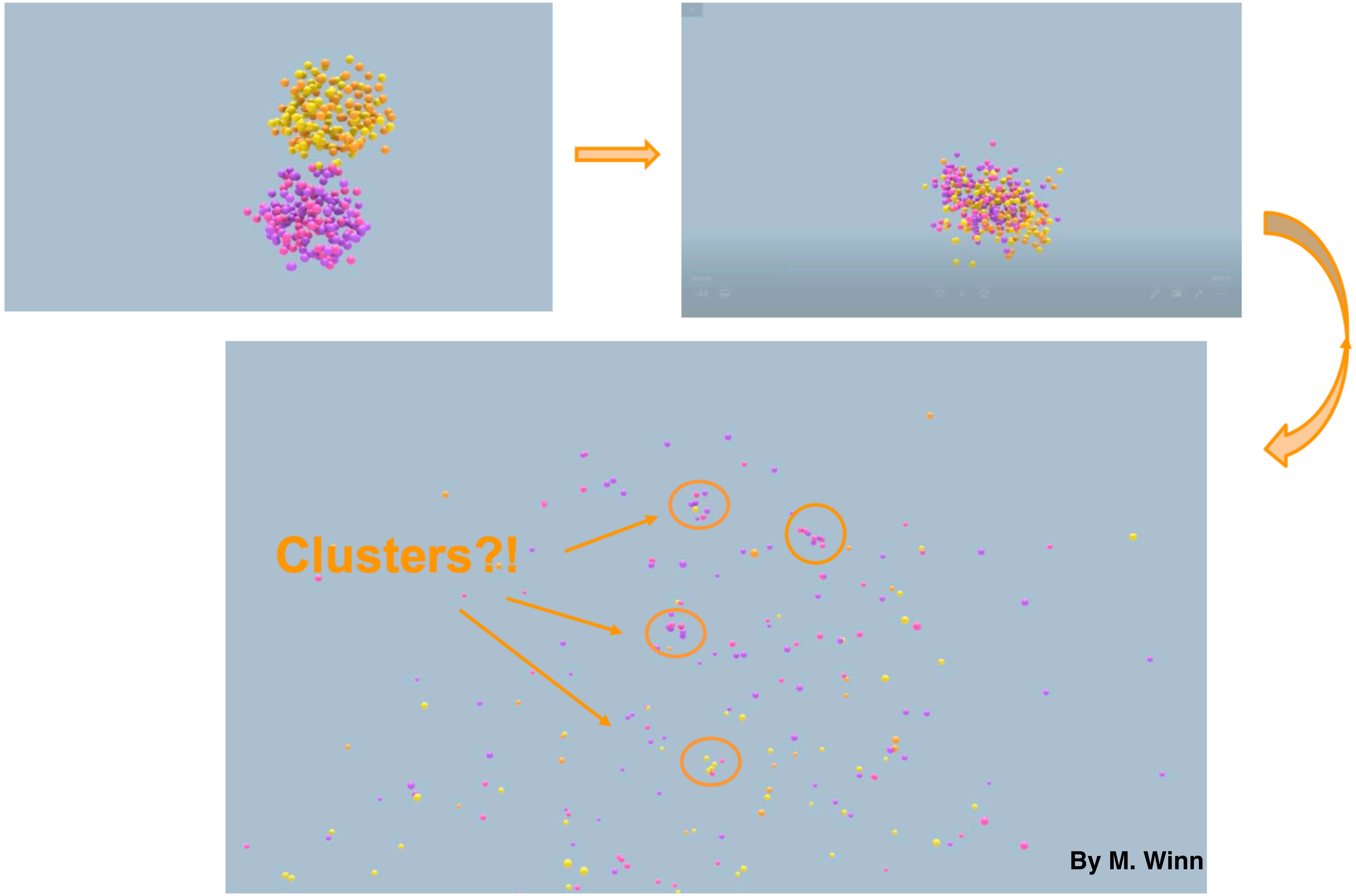
Compression modulus K of nuclear matter:

$$K = -V\frac{dP}{dV} = 9\rho^2 \frac{\partial^2(E/A(\rho))}{(\partial\rho)^2}|_{\rho=\rho_0}.$$

E.o.S.	$\alpha[MeV]$	$\beta[MeV]$	γ	K [MeV]
S	-383.5	329.5	1.15	200
H	-125.3	71.0	2.0	380
SM	-478.87	413.76	1.10	200
	a $[MeV^{-1}]$	b$[MeV^{-2}]$	c$[MeV^{-1}]$	
	236.326	-20.73	0.901	



PHQMD: Cluster formation



PHQMD: MST

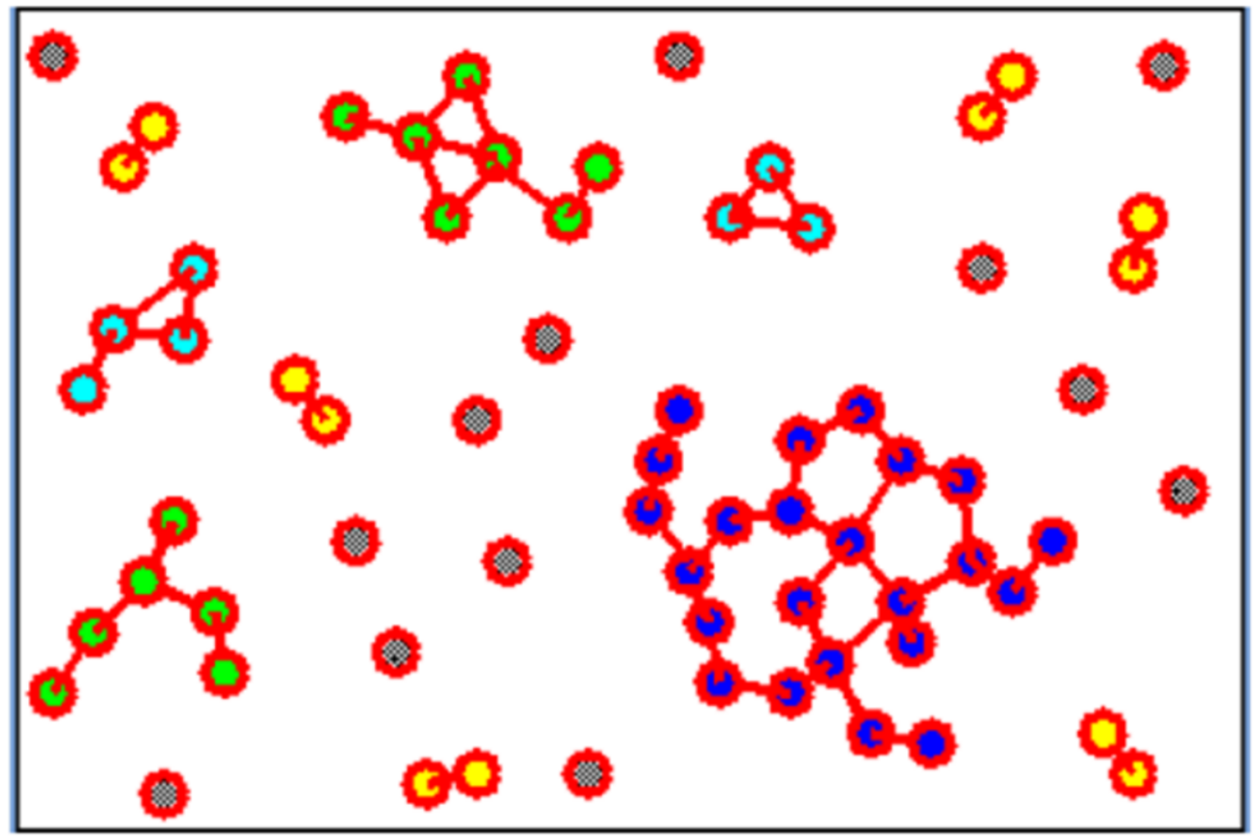
R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

The Minimum Spanning Tree (MST) is a cluster recognition method applicable for the (asymptotic) final states where coordinate space correlations may only survive for bound states.

1. Two particles are 'bound' if their distance in the cluster rest frame fulfills:

$$|\vec{r_i} - \vec{r_j}| \leq 4 \text{ fm}$$

2. Particle is bound to a cluster if it binds with at least one particle of the cluster



Advanced MST

G. Coci et al., Phys.Rev.C 108 (2023) 014902

3. Negative binding energy for identified clusters
4. Stabilization procedure – recombine the final "lost" nucleons back into clusters if they left the cluster without rescattering

PHQMD: Kinetic

1. hadronic inelastic reactions $NN \leftrightarrow d\pi$, $\pi NN \leftrightarrow d\pi$, $NNN \leftrightarrow dN$

2. hadronic elastic $\pi + d$, $N + d$ reactions

SMASH: D. Oliinychenko et al., PRC 99 (2019) 044907;
AMPT: R. Wang et al. PRC 108 (2023) 3, L031601;
PHQMD: G. Coci et al., PRC 108 (2023) 014902.

$$P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3E_4E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

Energy and momentum of final particles

2,3-body phase space integrals [Byckling, Kajantie]

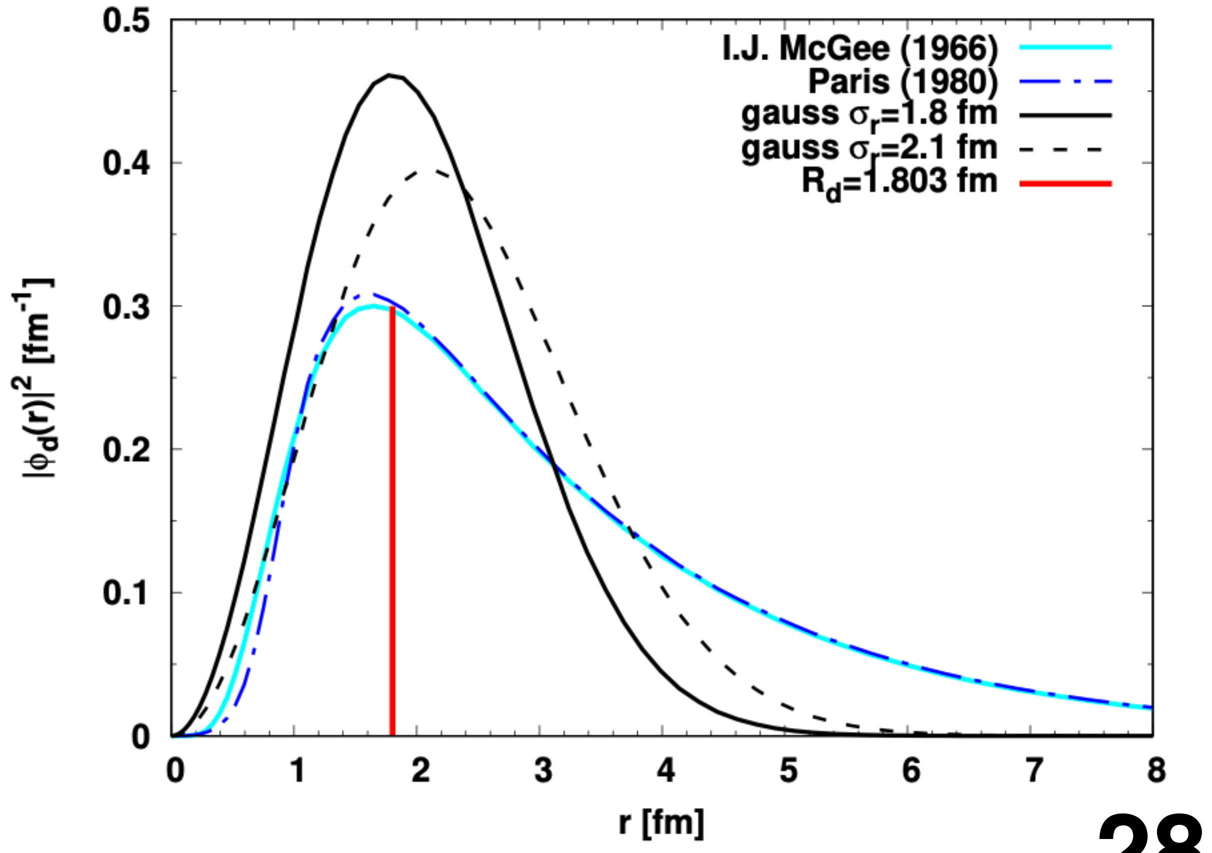
$$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$$
$$\pi^- + p + p \leftrightarrow \pi^0 + d$$
$$\pi^+ + n + n \leftrightarrow \pi^0 + d$$
$$\pi^0 + p + p \leftrightarrow \pi^+ + d$$
$$\pi^0 + n + n \leftrightarrow \pi^- + d$$

+ inclusion of all possible isospin channels (enhance d production)
+ accounting of quantum properties of d (suppress d production) :

1) the finite-size of d in coordinate space (d is not a point-like particle)
   assume that a d can not be formed in a high density region

   Excluded-Volume Condition: $|\vec{r}(i)^* - \vec{r}(d)^*| < R_d$

2) the momentum correlations of p and n inside d
   project the relative momentum of $p + n$ pair on d wave-function in momentum space $|\phi_d(p)|^2$ which lead to a strong reduction of d production.



PHQMD: Coalescence (optional)

Clusters formation at a freeze-out time by coalescence radii in coordinate and momentum space

$$N_A = g_A \int \prod_{i=1}^{A} d^3 r_i \, d^3 p_i \; f_i(\mathbf{r}_i, \mathbf{p}_i) \; W_A\big(\{\mathbf{r}_i, \mathbf{p}_i\}\big)$$

Coalescence probability

Box coalescence (parameters from UrQMD): $\Delta \mathrm{p} <$ **0.285 GeV and** $\Delta \mathrm{r} <$ **3.575 fm.**

P. Hillmann, K. Ka¨fer, J. Steinheimer, V. Vovchenko and M. Bleicher, J. Phys. G 49, no.5, 055107 (2022)

Other approach—>
Gaussian-form of the Wigner density： $W_2^{1S}(\mathbf{r}, \mathbf{q}) = 8e^{-\frac{\mathbf{r}^2}{\sigma^2} - \mathbf{q}^2\sigma^2}.$

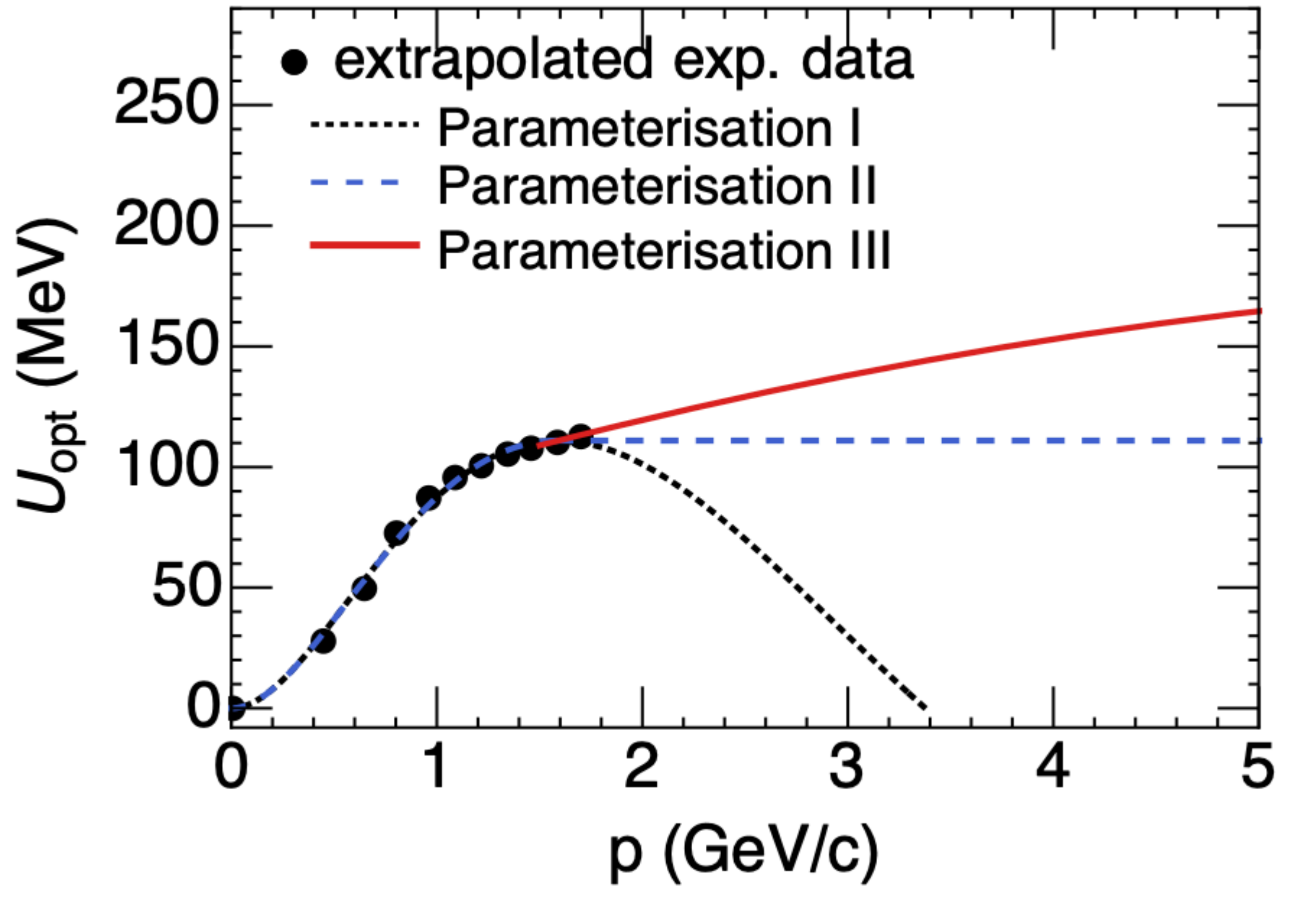
R. Scheibl and U. Heinz, Phys. Rev. C 59, 1585 (1999)
L. Zhu, C. Ko, and X. Yin, Phys. Rev. C 92, 064911 (2015)
K. Sun and C. Ko, Phys. Rev. C 103, 064909 (2021)
D. Liu, et al. Phys. Lett. B 855, 138855.
Q. Lin, et al. arXiv:2503.01128.
R. Wang, et al. Phys.Rev.C 112 (2025) 3, 034908 (2024)

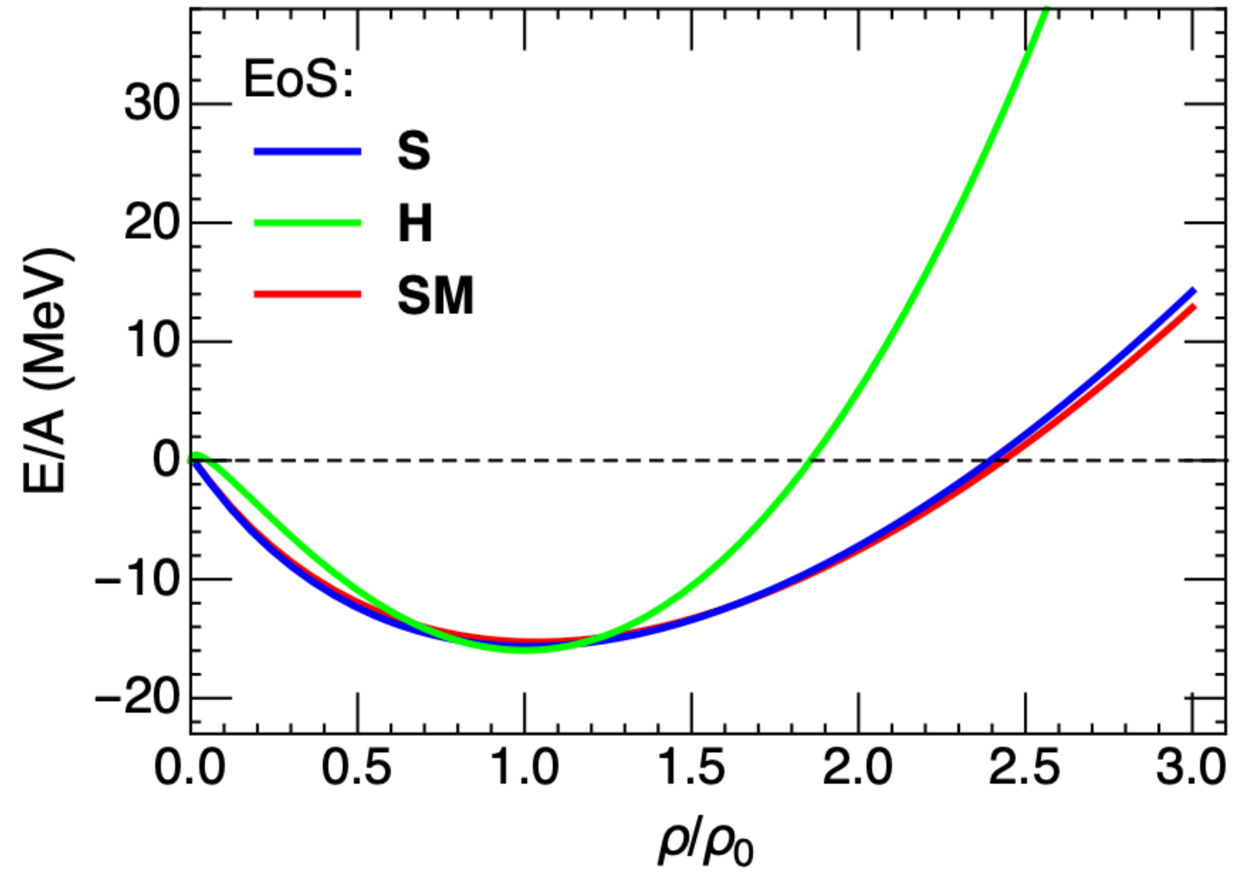
Highlights

❑ **PHQMD provides a good description of cluster and hypernuclei observables (y-, p_T-distributions, flow coefficients v_n, …)**

S. Gläßel et al, *Phys.Rev.C* 105 (2022) 1, 014908. G. Coci et al., PRC 108 (2023) 014902. V. Kireyeu et al., arXiv:2411.04969; …

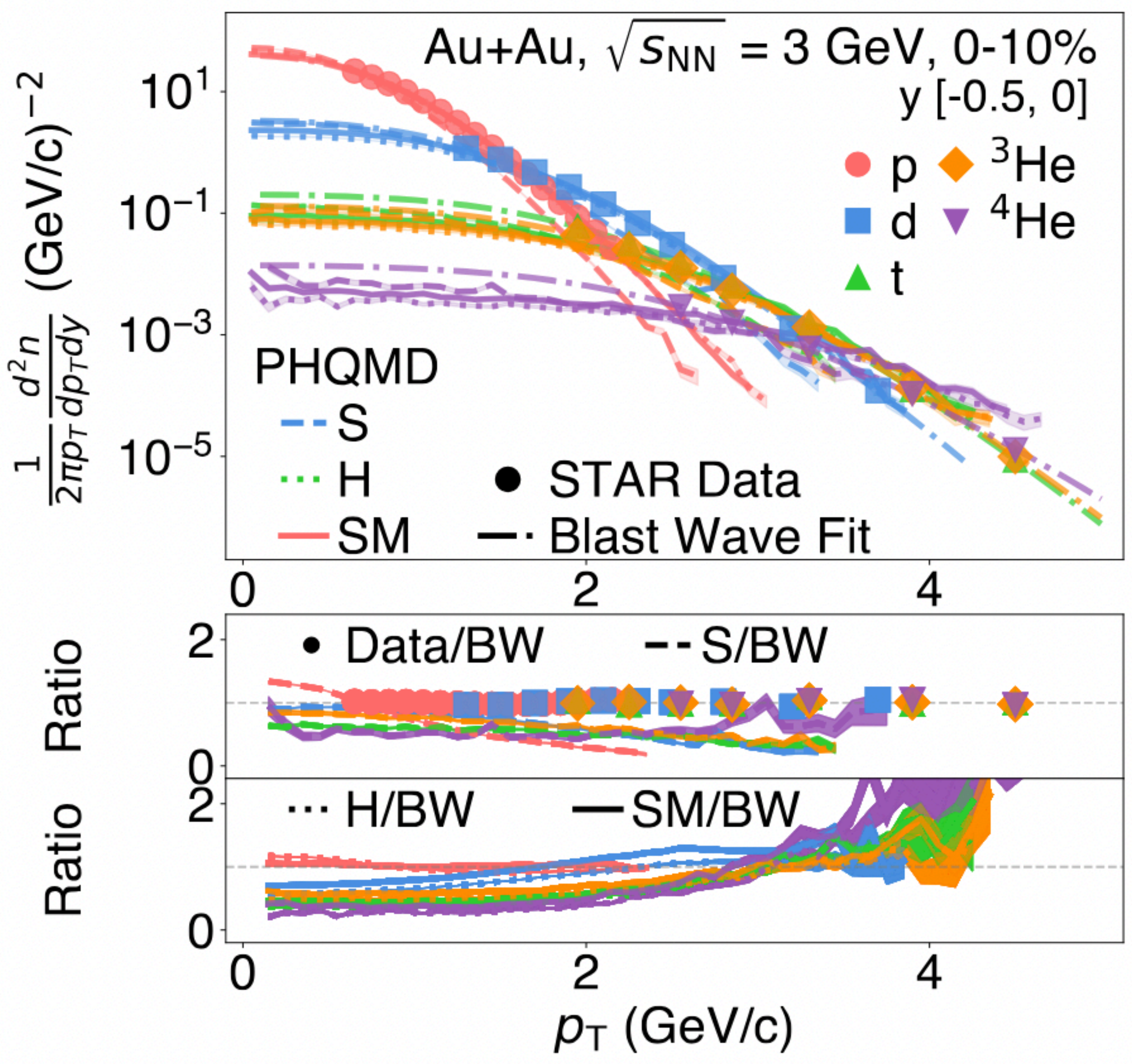
Recent: probe EoS with clusters and hypernuclei —> support SM, need more data at higher energy (4-7GeV) to probe the mom-dependent potential.

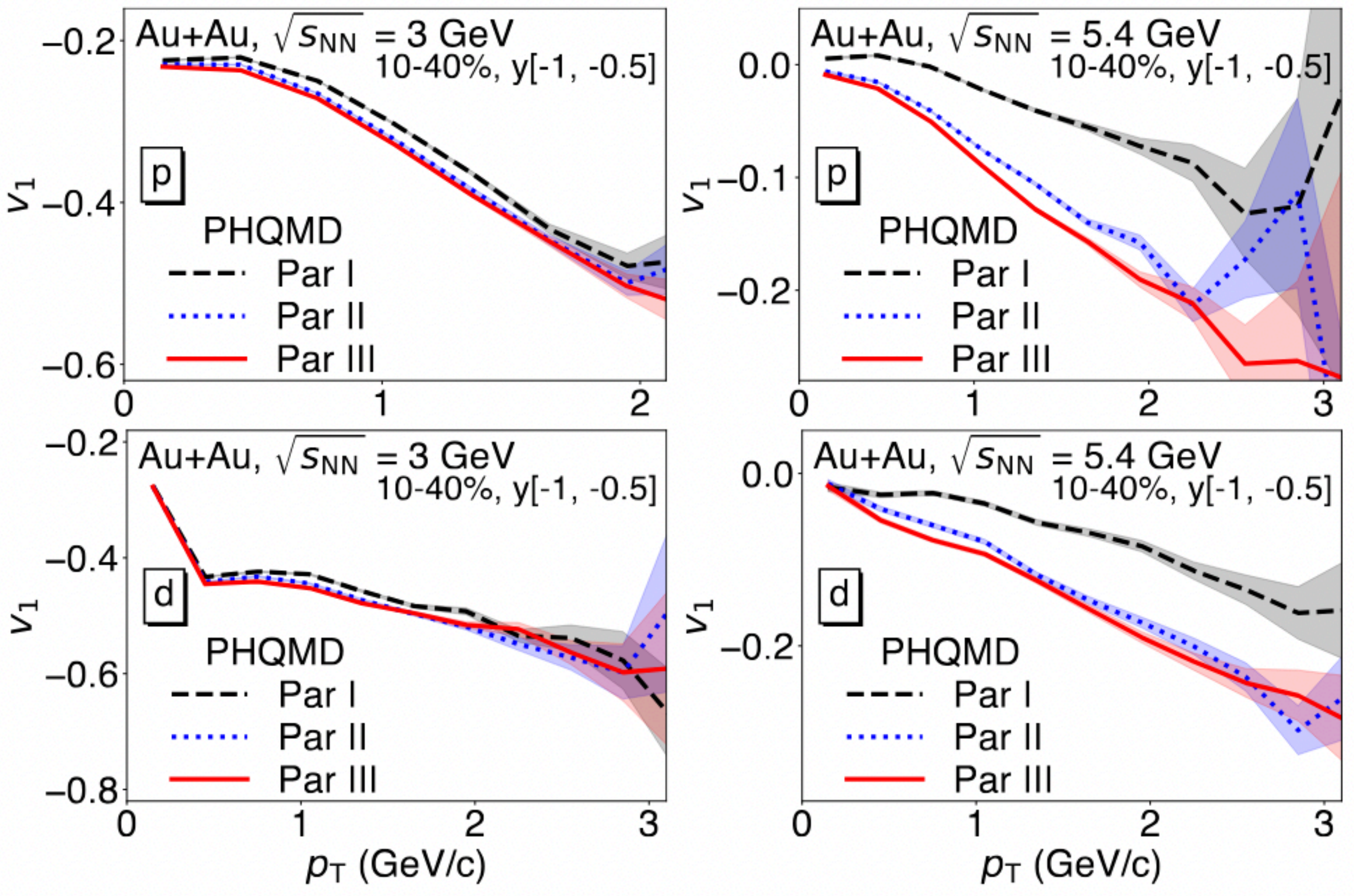




Probing of EoS with clusters and hypernuclei arXiv: 2507.14255.

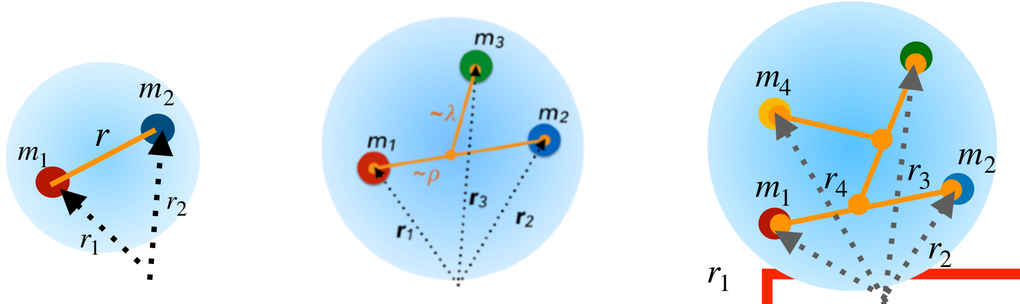
Yingjie Zhou[1], Iouri Vassiliev[1], Yue-Hang Leung[2], Norbert Herrmann[2,1], Susanne Gläßel[3], Viktar Kireyeu[4], Michael Winn[5], Jörg Aichelin[5,6], Christoph Blume[2,1,7], Elena Bratkovskaya[1,7,8], Vadim Voronyuk[4], Nu Xu[9,10], Jiaxing Zhao[7,8]





Highlights

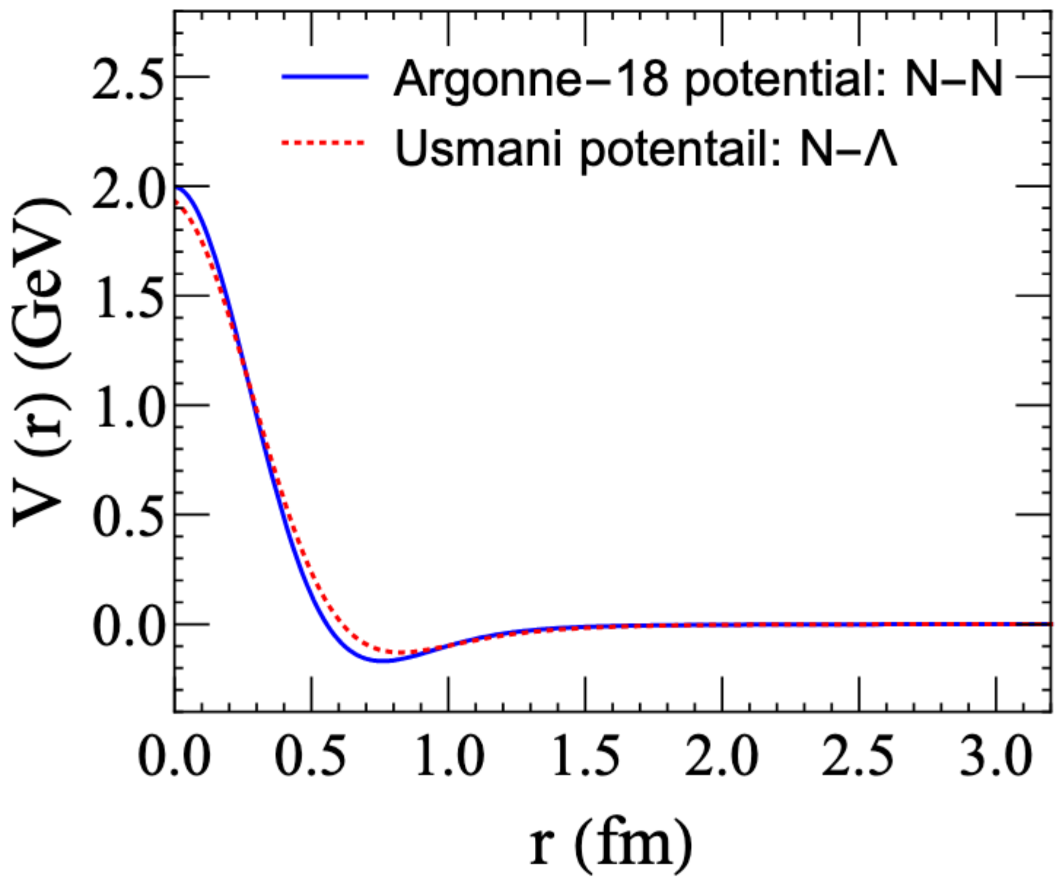
Recent: More realistic Wigner functions from N-body Schrödinger equations—>

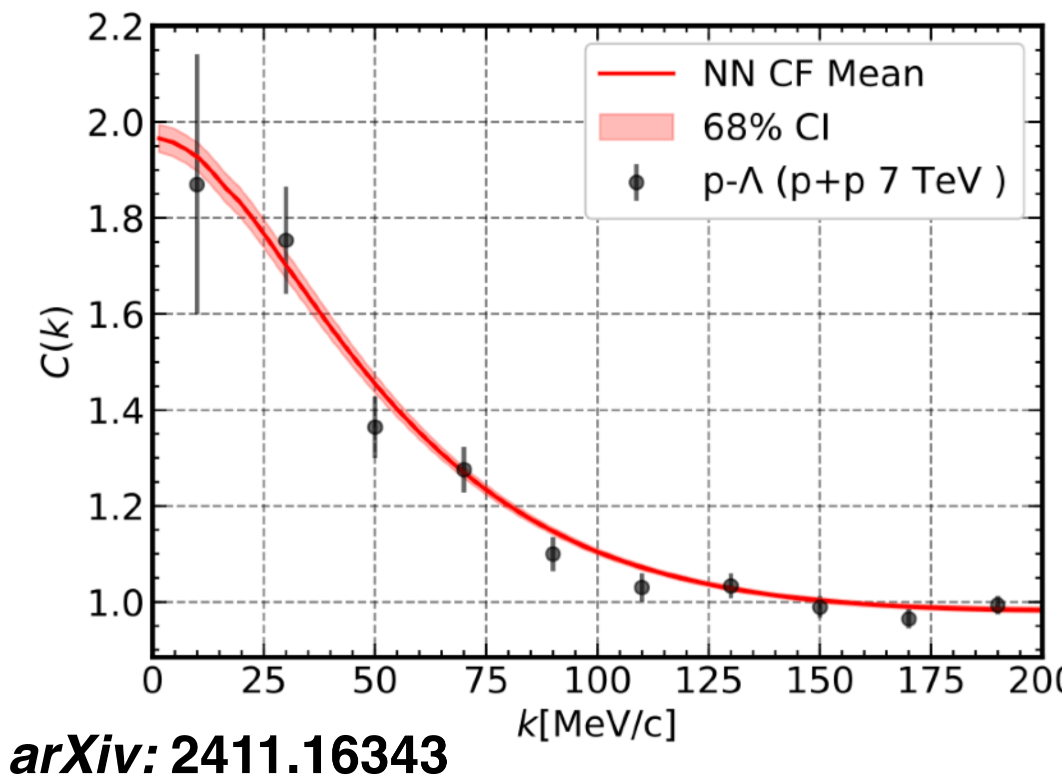


$$\left[\frac{1}{2\mu}\left(\frac{1}{\rho^{3N-4}}\frac{d}{d\rho}\rho^{3N-4}\frac{d}{d\rho}-\frac{K(K+3N-5)}{\rho^2}\right)+E_r\right]R_\kappa=\sum_{\kappa'}V_{\kappa\kappa'}R_{\kappa'}$$

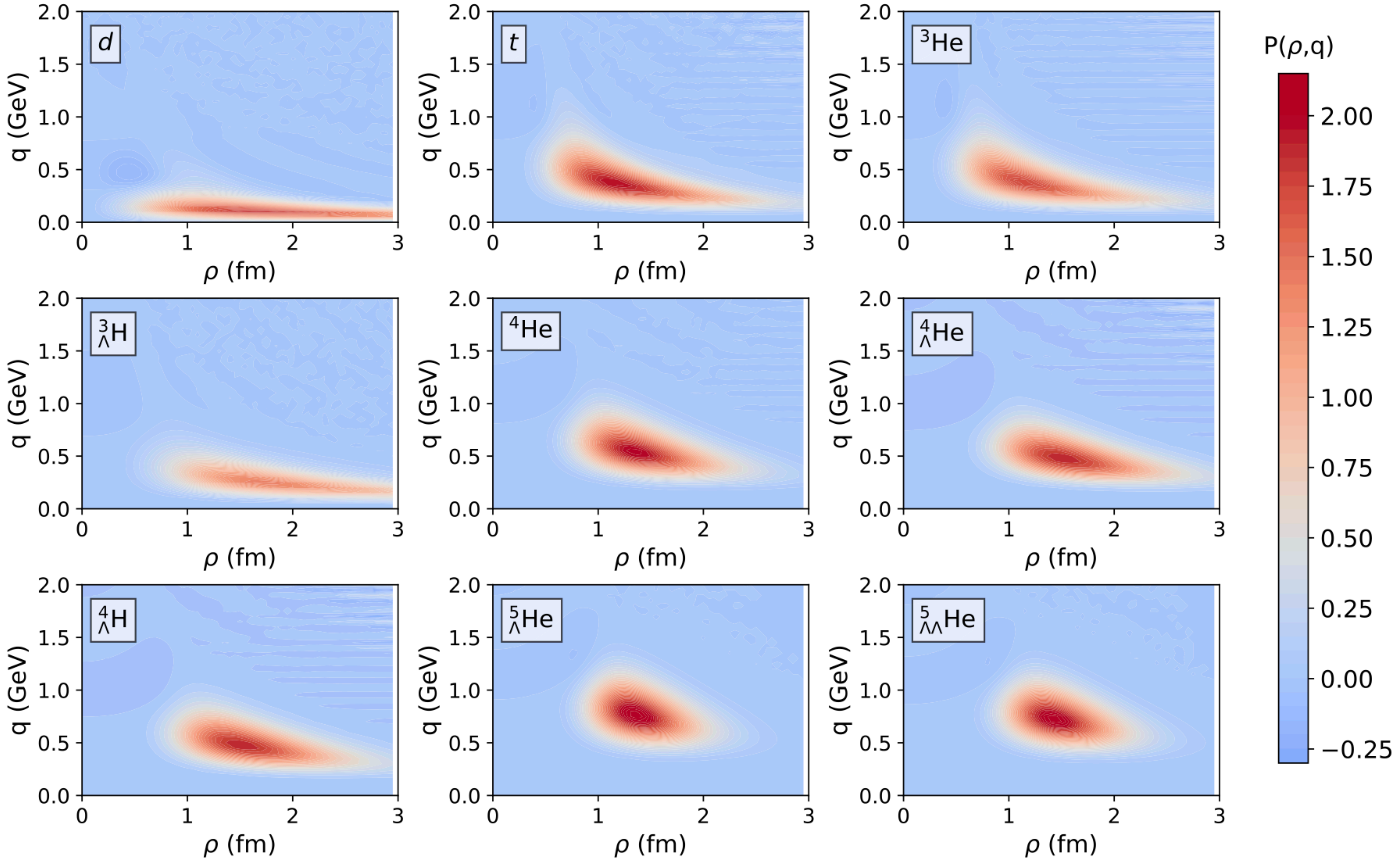
J. Zhao, J. Aichelin, E. Bratkovskaya,
PRC 112 (2025) 6, 064902.

$$W_N(\mathbf{r}_1,\ldots,\mathbf{r}_N;\mathbf{p}_1,\ldots,\mathbf{p}_N)=\int d^3\mathbf{s}_1\ldots d^3\mathbf{s}_N e^{-i\sum_{k=1}^{N}\mathbf{p}_k\cdot\mathbf{s}_k}\Psi^*\left(\mathbf{r}_1+\frac{\mathbf{s}_1}{2},\ldots,\mathbf{r}_N+\frac{\mathbf{s}_N}{2}\right)\Psi\left(\mathbf{r}_1-\frac{\mathbf{s}_1}{2},\ldots,\mathbf{r}_N-\frac{\mathbf{s}_N}{2}\right)$$





L. Wang, J. Zhao *arXiv:* 2411.16343

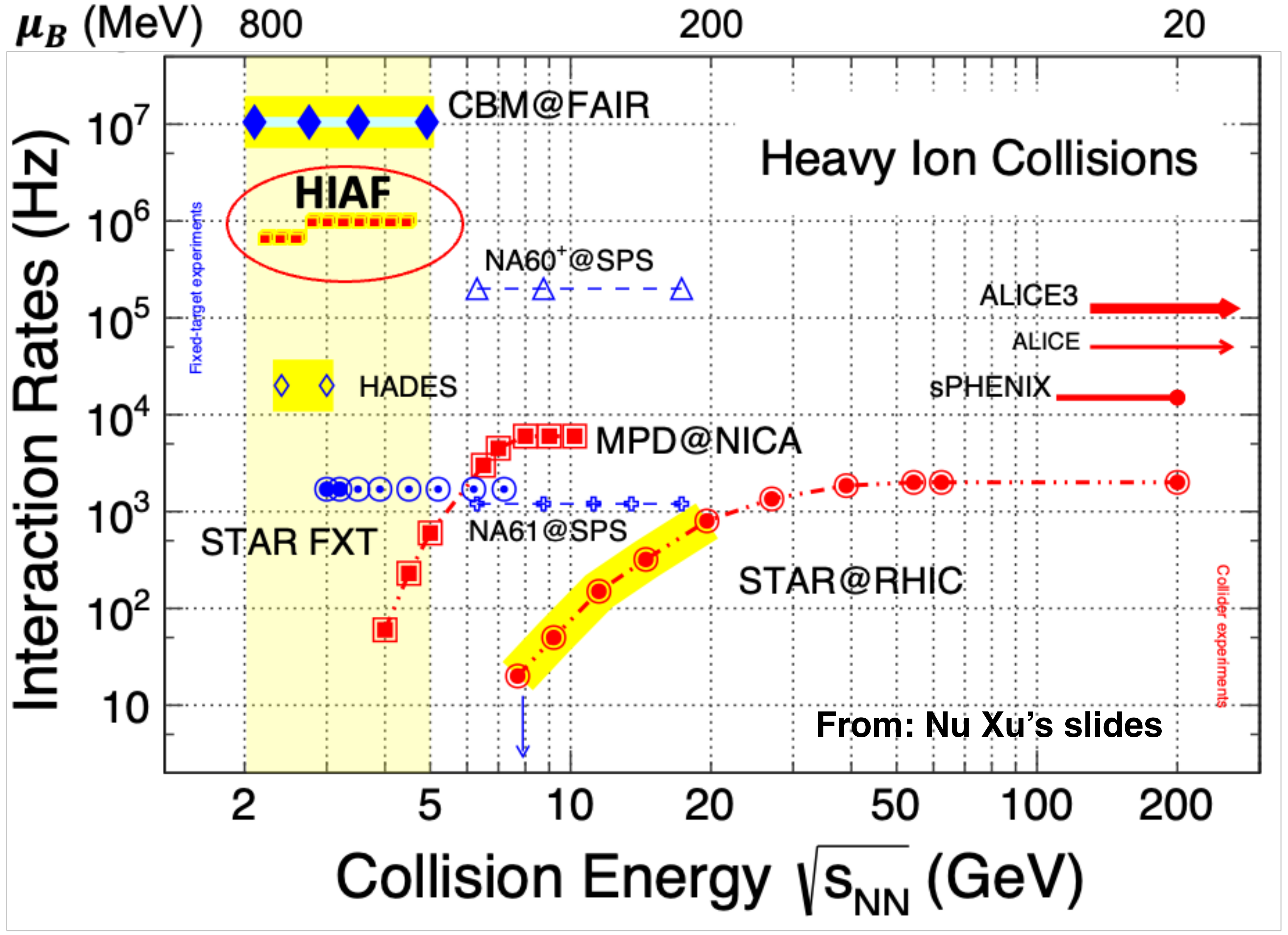


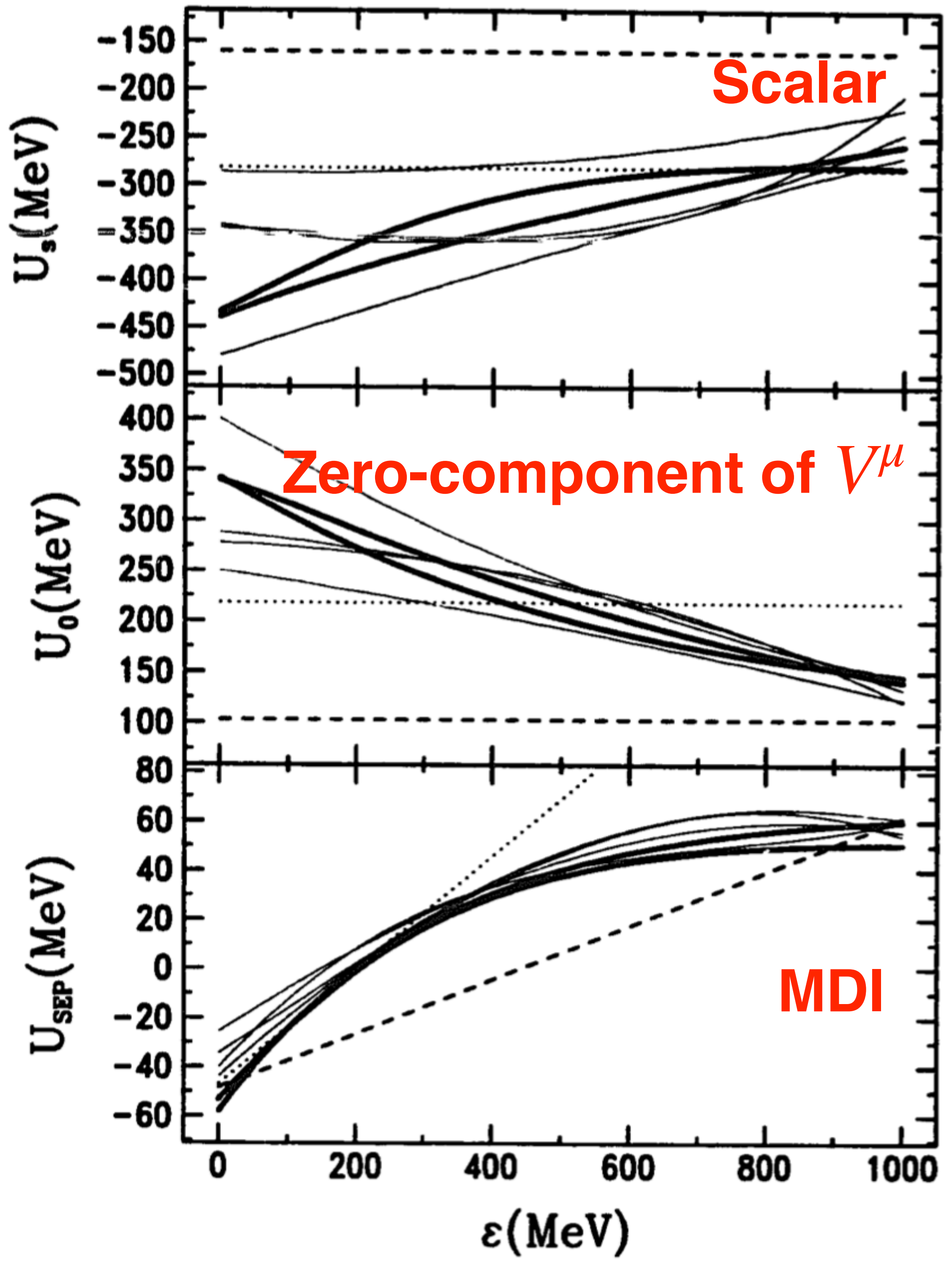
PHQMD:

II. Relativistic QMD for FAIR, NICA, HIAF energies

Relativistic version

- ➡ Relativistic effects play a crucial role—> a fully covariant N-body QMD approach is needed for probing the EoS, cluster formation,…
- ➡ N-N interaction in the medium includes a scalar potential U_s and a vector potential V^μ





PHYSICAL REVIEW C VOLUME 47, NUMBER 1 JANUARY 1993

Global Dirac phenomenology for proton-nucleus elastic scattering

E. D. Cooper, S. Hama, and B. C. Clark
Department of Physics, The Ohio State University, Columbus, Ohio 43210

R. L. Mercer
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

(Received 31 August 1992)

Energy-dependent global Dirac optical model potentials are found by fitting proton elastic scattering data in the energy range 20–1040 MeV for ^{12}C, ^{16}O, ^{40}Ca, ^{90}Zr, and ^{208}Pb. Three different energy- and atomic-mass-number-dependent global Dirac optical potentials are also obtained. A number of characteristic features of the potentials are discussed. In addition, the mean free path, the effective mass m_e^*, the Dirac mass M^*, and the relativistic energy shift E^* are calculated.

$$\{\boldsymbol{\alpha}\cdot\boldsymbol{p}+\beta(m+U_s^{\rm tot}(r;\varepsilon))+U_0^{\rm tot}(r;\varepsilon)\}\Psi(r;\varepsilon)=E\Psi(r;\varepsilon)$$

$$\left\{\frac{p^2}{2m}+U_{\rm SEP}(r;\varepsilon)+U_{\rm s.o.}(r;\varepsilon)\frac{\boldsymbol{\sigma}\cdot\boldsymbol{L}}{r}+C(r;\varepsilon)\right\}\Psi_>=\frac{p_\infty^2}{2m}\Psi_>\,.$$

$$U_{\rm SEP}=U_s^{\rm tot}+U_0^{\rm tot}+\frac{1}{2m}(U_s^{\rm tot^2}-U_0^{\rm tot^2})+\frac{U_0^{\rm tot}}{m}\varepsilon\,.$$

Schrödinger equivalent potential (SEP)

Relativistic version

LORENTZ-COVARIANT DESCRIPTION OF INTERMEDIATE ENERGY HEAVY-ION REACTIONS IN RELATIVISTIC QUANTUM MOLECULAR DYNAMICS*

Tomoyuki MARUYAMA, S.W. HUANG, N. OHTSUKA[1], Guoqiang LI[2] and Amand FAESSLER
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Received 1 February 1991
(Revised 3 May 1991)

RQMD

Poincaré Invariant Hamiltonian Dynamics: Modelling Multi-hadronic Interactions in a Phase Space Approach

HEINZ SORGE, HORST STÖCKER, AND WALTER GREINER

Institut für Theoretische Physik,
Johann Wolfgang Goethe-Universität
Frankfurt/Main, Germany

Received January 9, 1989

Momentum-dependent potential and collective flows within the relativistic quantum molecular dynamics approach based on relativistic mean-field theory

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JAM.RMF

Relativistic version

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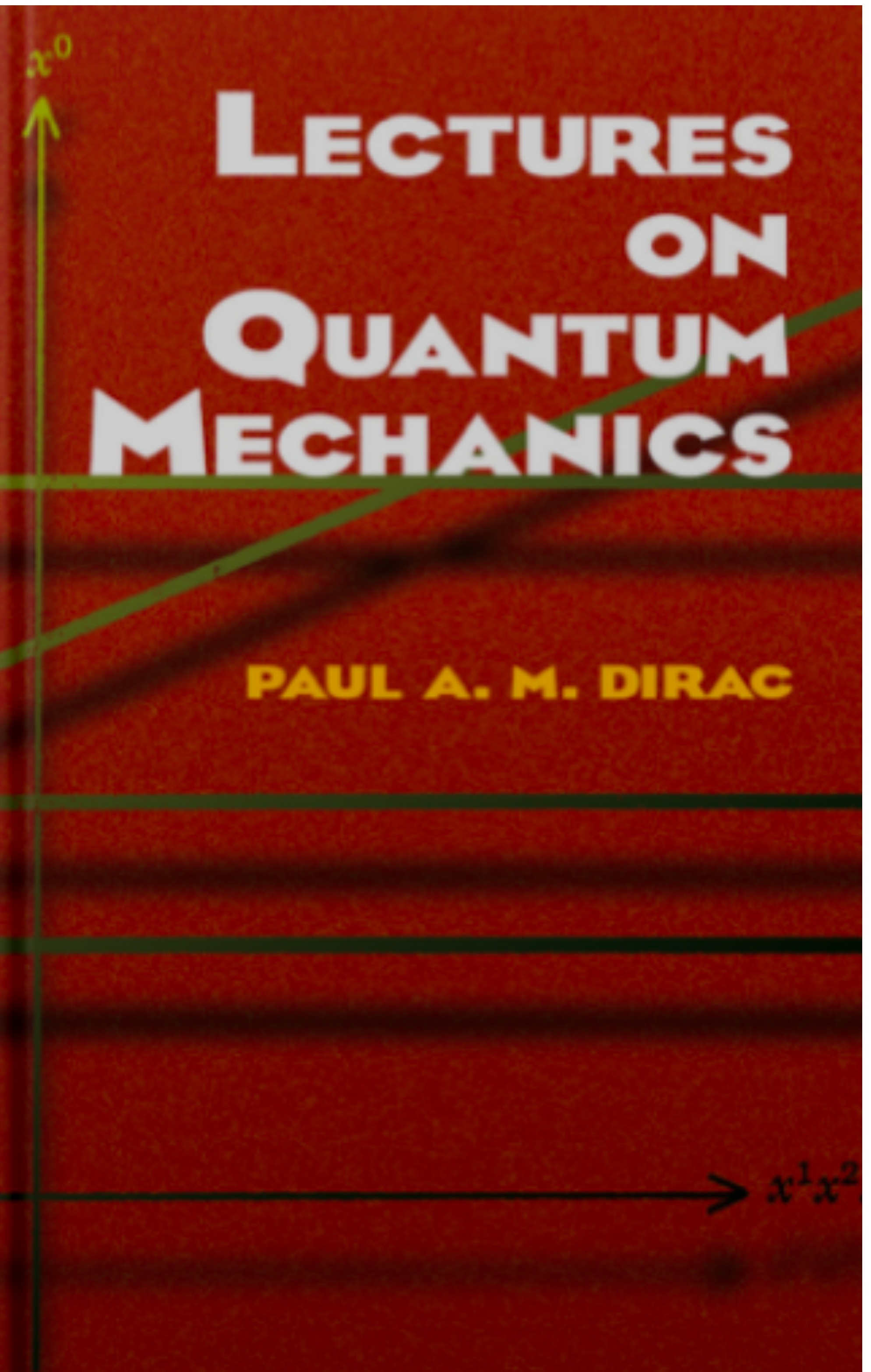

Dirac constraint dynamics: For N-body system,

8N dof.

N first-class constraints (on-shell conditions), $H_i = p_{i,\mu}p_i^\mu - m_i$,

7N dof.

Additional N second-class constraints (the last constraint serve to pick a clock for all particles)

6N+1 dof. **Solvable!**

RQMD, UrQMD only with scalar potential (Skyrme-type potentials)

JAM.RMF with both the scalar and vector potential (RMF)

Relativistic QMD

On-shell conditions

$$H_1 = p^*_{1,\mu} p_1^{*\mu} - m_1^2 + \Phi_1,$$
$$H_2 = p^*_{2,\mu} p_2^{*\mu} - m_2^2 + \Phi_2.$$

$$p^*_{i,\mu} \equiv p_{i,\mu} - A_{i,\mu}$$

$$\Phi_1 = \Phi_2 = \Phi(\sqrt{-q_T^2})$$

$$q_T^\mu = \left(g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} \right) q_\nu = q^\mu - \frac{q_\nu P^\nu}{P^2} P^\mu$$

Second-class constraints

$$\chi_1 = \frac{1}{2}(q_1^\mu - q_2^\mu) U_\mu = 0, \quad \chi_2 = \frac{1}{2}(q_1^\mu + q_2^\mu) U_\mu - \tau = 0,$$

Poincaré invariant (world lines condition)

Equations of motion(Poisson brackets)

$$\frac{dq_i^\mu}{d\tau} = \{q_i^\mu, H\} = \lambda_1 \{q_i^\mu, H_1\} + \lambda_2 \{q_i^\mu, H_2\},$$
$$\frac{dp_i^\mu}{d\tau} = \{p_i^\mu, H\} = \lambda_1 \{p_i^\mu, H_1\} + \lambda_2 \{p_i^\mu, H_2\}.$$

$$\frac{dq_i^\mu}{d\tau} = \sum_{k=1}^{2} \lambda_k \left(2 p_k^{*\nu} \frac{\partial p^*_{k,\nu}}{\partial p_{i,\mu}} \right),$$
$$\frac{dp_i^\mu}{d\tau} = -\sum_{k=1}^{2} \lambda_k \left(2 p_k^{*\nu} \frac{\partial p^*_{k,\nu}}{\partial q_{i,\mu}} + \frac{\partial \Phi_k}{\partial q_{i,\mu}} \right).$$

Only scalar potentail

RQMD/UrQMD

$$\frac{dq_i^\mu}{d\tau} = \frac{p_i^\mu}{p_i^\mu U_\mu},$$
$$\frac{dp_i^\mu}{d\tau} = -\sum_{k=1}^{2} \frac{1}{2 p_k^\mu U_\mu} \frac{\partial \Phi_k}{\partial q_{i,\mu}},$$

Neglects the momentum-dependent derivatives & CoM frame second-class constraints

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$$\frac{d\mathbf{q}_i}{d\tau} = \frac{\mathbf{p}_i^*}{p_i^{*0}} + \sum_{k=1}^{2} \left[\frac{\mathbf{p}_k^*}{p_k^{*0}} \frac{\partial V_k}{\partial \mathbf{p}_i} + \frac{m_k^*}{p_k^{*0}} \frac{\partial m_k^*}{\partial \mathbf{p}_i} \right],$$
$$\frac{d\mathbf{p}_i}{d\tau} = -\sum_{k=1}^{2} \left[\frac{\mathbf{p}_i^*}{p_i^{*0}} \frac{\partial V_{k,v}}{\partial \mathbf{q}_i} + \frac{m_k^*}{p_k^{*0}} \frac{\partial m_k^*}{\partial \mathbf{q}_i} \right].$$

J. Zhao, J. Aichelin, E. Bratkovskaya, arXiv: 2511.13975

Relativistic QMD

Formulation of fully covariant Quantum-Molecular Dynamics for an N-body system with scalar and vector potentials

Jiaxing Zhao,[a,b] Joerg Aichelin,[c,d] Elena Bratkovskaya[e,a,b]

[a] *Helmholtz Research Academy Hessen for FAIR (HFHF), GSI Helmholtz Center for Heavy Ion Research. Campus Frankfurt, 60438 Frankfurt, Germany*

$$\frac{dq_i^\mu}{d\tau} = \sum_{k=1}^{2} \lambda_k \left(2p_k^{*\nu} \frac{\partial p^*_{k,\nu}}{\partial p_{i,\mu}} \right),$$

$$\frac{dp_i^\mu}{d\tau} = -\sum_{k=1}^{2} \lambda_k \left(2p_k^{*\nu} \frac{\partial p^*_{k,\nu}}{\partial q_{i,\mu}} + \frac{\partial \Phi_k}{\partial q_{i,\mu}} \right).$$

- Non-relativistic limit is fulfilled.
- 1-body relativistic case is the same as the traditional way.
- The invarinance of different choices of second-class constraints are proved.
- The frame independence of the evolution is checked.
- Distinct dynamical roles of scalar and vector potentials in 2 and 4-body case.
- A generalization for N-body EoM and solution algorithm are given.

PHRQMD is coming soon !

Relativistic QMD

Density-dependent vector and scalar potentials:

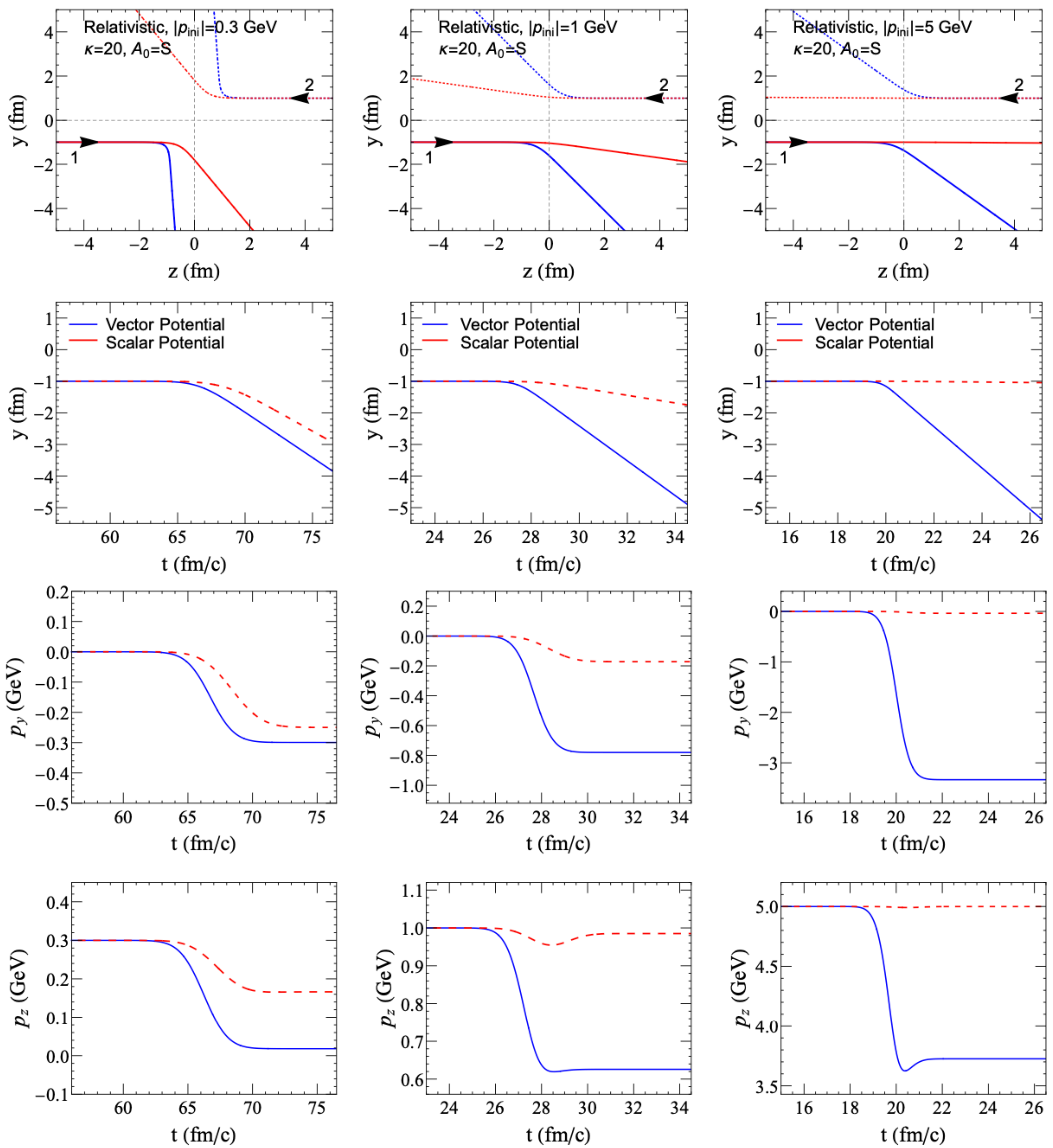
$$A_i^\mu = \sum_{j\neq i}^{N} p_j^{*\mu} \rho_{ij}(q_T), \quad S_i = \sum_{j\neq i}^{N} m_i \rho_{ij}(q_T),$$

Test for 2-body case:

$$A_1^\mu = p_2^{*\mu} \rho_{12}(q_T),$$
$$A_2^\mu = p_1^{*\mu} \rho_{12}(q_T),$$
$$\Phi_1 = \Phi_2 = 2\mu S = 2\mu m \rho_{12}(q_T),$$
$$\rho_{12}(q_T) = \frac{\kappa}{(2\pi L)^{3/2}} \exp(q_T^2/(2L)),$$

Due to the "Lorentz force" in vector potential, trajectories change more than that with scalar potential —> a larger v1.

Trajectories with Vector or Scalar potential; w/o collisions



Summary

Based on the Kadanoff-Baym equation - the PHSD was developed to describe the strongly-interaction hadronic and partonic matter created in p+A and heavy-ion collisions from SIS to LHC energies.

PHQMD is a combinaton of the PHSD and QMD. It includes correlations and was develped to describe nuclear cluster and hypernuclei productions in heavy ion collisions from SIS to RHIC energies.

➡ Cluster formation mechanism (excited states, internal structure,…).

➡ Relativistic QMD framework with both scalar and vector potentials for upcoming HIAF, NICA, FAIR energies.

➡ Spin-QMD for spin evolutions and polarizations.

Thank you !

Advantages of Kadanoff-Baym dynamics vs Boltzmann

Kadanoff-Baym equations:

- propagate two-point Green functions $G^{<}(x,p) \rightarrow A(x,p)N(x,p)$ in 8 dimensions
- $G^{<}$ carries information not only on the occupation number N_{XP}, but also on the particle properties, interactions and correlations via spectral function A_{XP}
  ➔ off-shell approach

Boltzmann equations:

- propagate phase space distribution function $f(\vec{r}, \vec{p}, t)$ in 6+1 dimensions
- works well for small coupling
  = weakly interacting system,
  ➔ on-shell approach

Kadanoff-Baym eqs. can be solved exactly for model cases such as Φ^4 – theory

S. Juchem, W. Cassing, and C. Greiner, Phys. Rev. D 69 (2004) 025006; Nucl. Phys. A 743 (2004) 92

Dyson-Schwinger equation

- **Dyson-Schwinger equation** (follows from Schrödinger eq.):

$$G(x,y) = G_0(x,y) + G_0(x,y)\Sigma(x,y)G(x,y)$$

Dyson-Schwinger equation on the closed-time-path reads in matrix form:

$$\begin{pmatrix} G^c(x,y) & G^<(x,y) \\ G^>(x,y) & G^a(x,y) \end{pmatrix} = \begin{pmatrix} G_0^c(x,y) & G_0^<(x,y) \\ G_0^>(x,y) & G_0^a(x,y) \end{pmatrix} +$$

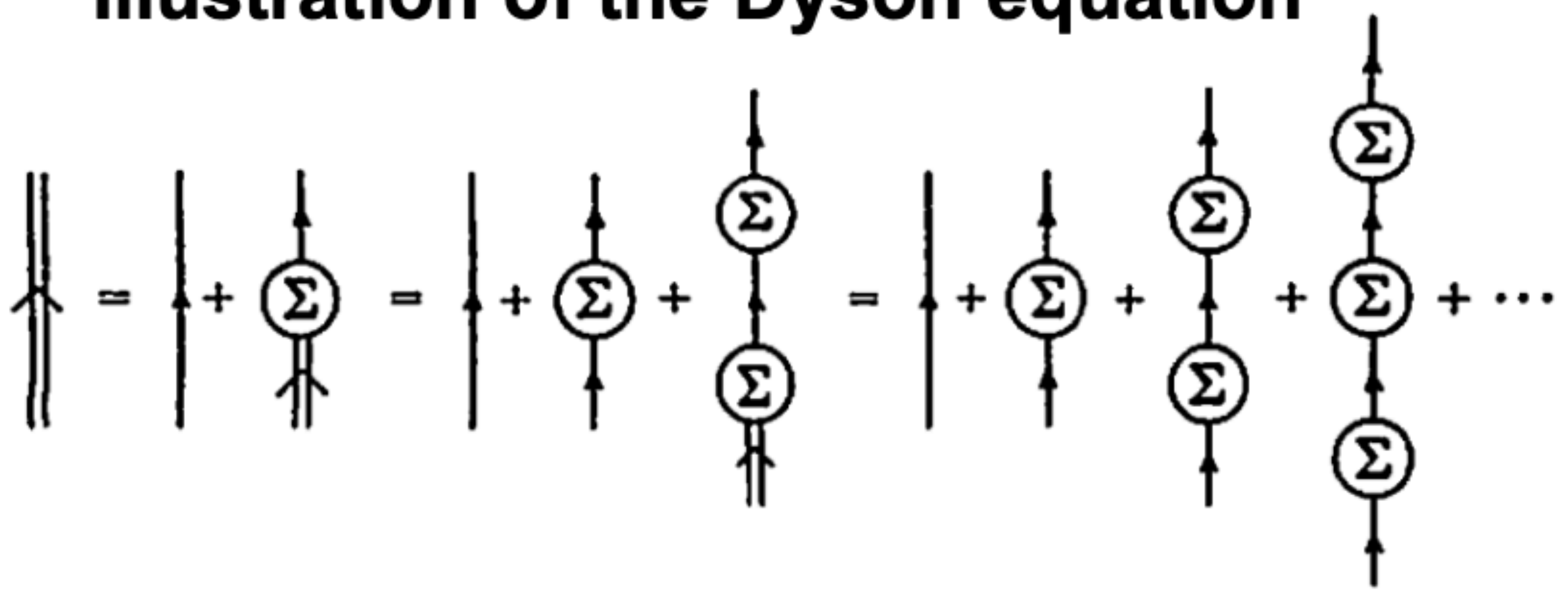
$$\begin{pmatrix} G_0^c(x,x') & G_0^<(x,x') \\ G_0^>(x,x') & G_0^a(x,x') \end{pmatrix} \odot \begin{pmatrix} \Sigma^c(x',y') & -\Sigma^<(x',y') \\ -\Sigma^>(x',y') & \Sigma^a(x',y') \end{pmatrix} \odot \begin{pmatrix} G^c(y',y) & G^<(y',y) \\ G^>(y',y) & G^a(y',y) \end{pmatrix}$$

Free propagator for Bose case:

$$\hat{G}_{0x}^{-1} = -(\partial_\mu^x \partial_x^\mu + m^2)$$

$$\hat{G}_{0x}^{-1} G_0^{R/A}(x,y) = \delta(x-y)$$

Illustration of the Dyson equation



From Kadanoff-Baym equation to transport equation

➤ separate all retarded and advanced quantities – Geen functions and self- energies – into real and imaginary parts:

$$S_{XP}^{ret,adv} = Re S_{XP}^{ret} \mp \frac{i}{2} A_{XP}, \qquad \Sigma_{XP}^{ret,adv} = Re \Sigma_{XP}^{ret} \mp \frac{i}{2} \Gamma_{XP}$$

The imaginary part of the retarded propagator is given by the normalized spectral function A_{XP}:

$$A_{XP} = i\left[S_{XP}^{ret} - S_{XP}^{adv}\right] = -2\, Im\, S_{XP}^{ret}$$

$$\int \frac{dP_0^2}{4\pi} A_{XP} = 1$$

The imaginary part of the selfenergy corresponds to the width Γ_{XP}; then from Dyson-Schwinger equation:

$$Re S_{XP}^{ret} = \frac{P^2 - M_0^2 - Re\Sigma_{XP}^{ret}}{\Gamma_{XP}} A_{XP}$$

algebraic solution

The spectral function A_{XP} in first order gradient expansion (for bosons) :

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma_{XP}^{ret})^2 + \Gamma_{XP}^2/4}$$

The real part of the retarded propagator in first order gradient expansion :

$$Re S_{XP}^{ret} = \frac{P^2 - M_0^2 - Re\Sigma_{XP}^{ret}}{(P^2 - M_0^2 - Re\Sigma_{XP}^{ret})^2 + \Gamma_{XP}^2/4}$$

A_{XP} and $Re\Sigma_{XP}^{ret}$ in first order gradient expansion depend ONLY on Σ_{XP}^{ret} !

Dynamical transport model: collision terms

❑ BUU eq. for different particles of type *i=1,…n*

Hadronic transport models: BUU, IQMD, UrQMD, GiBUU, HSD, JAM, SMASH, ...

$$Df_i \equiv \frac{d}{dt} f_i = I_{coll} \left[f_1, f_2, ..., f_n \right]$$

Drift term=Vlasov eq. collision term

$$i: \quad Baryons: \; p, n, \Delta(1232), N(1440), N(1535), ..., \Lambda, \Sigma, \Sigma^*, \Xi, \Omega; \; \Lambda_C$$

$$Mesons: \; \pi, \eta, K, \bar{K}, \rho, \omega, K^*, \eta', \phi, a_1, ..., D, \bar{D}, J/\Psi, \Psi', ...$$

➔ coupled set of BUU equations for different particles of type *i=1,…n*

$$\begin{cases} Df_N = I_{coll} \left[f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ... \right] \\ Df_\Delta = I_{coll} \left[f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ... \right] \\ ... \\ Df_\pi = I_{coll} \left[f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ... \right] \\ ... \end{cases}$$

Elementary hadronic interactions

Consider all possible interactions – eleastic and inelastic collisions - for the sytem of *(N,R,m)*, where *N*-nucleons, *R*- resonances, *m*-mesons, and resonance decays

Low energy collisions:

- **binary 2←→2 and 2←→3(4) reactions**
- **1←→2 : formation and decay of baryonic and mesonic resonances**

BB ←→ B´B´

BB ←→ B´B´m

mB ←→ m´B´

mB ←→ B´

mm ←→ m´m´

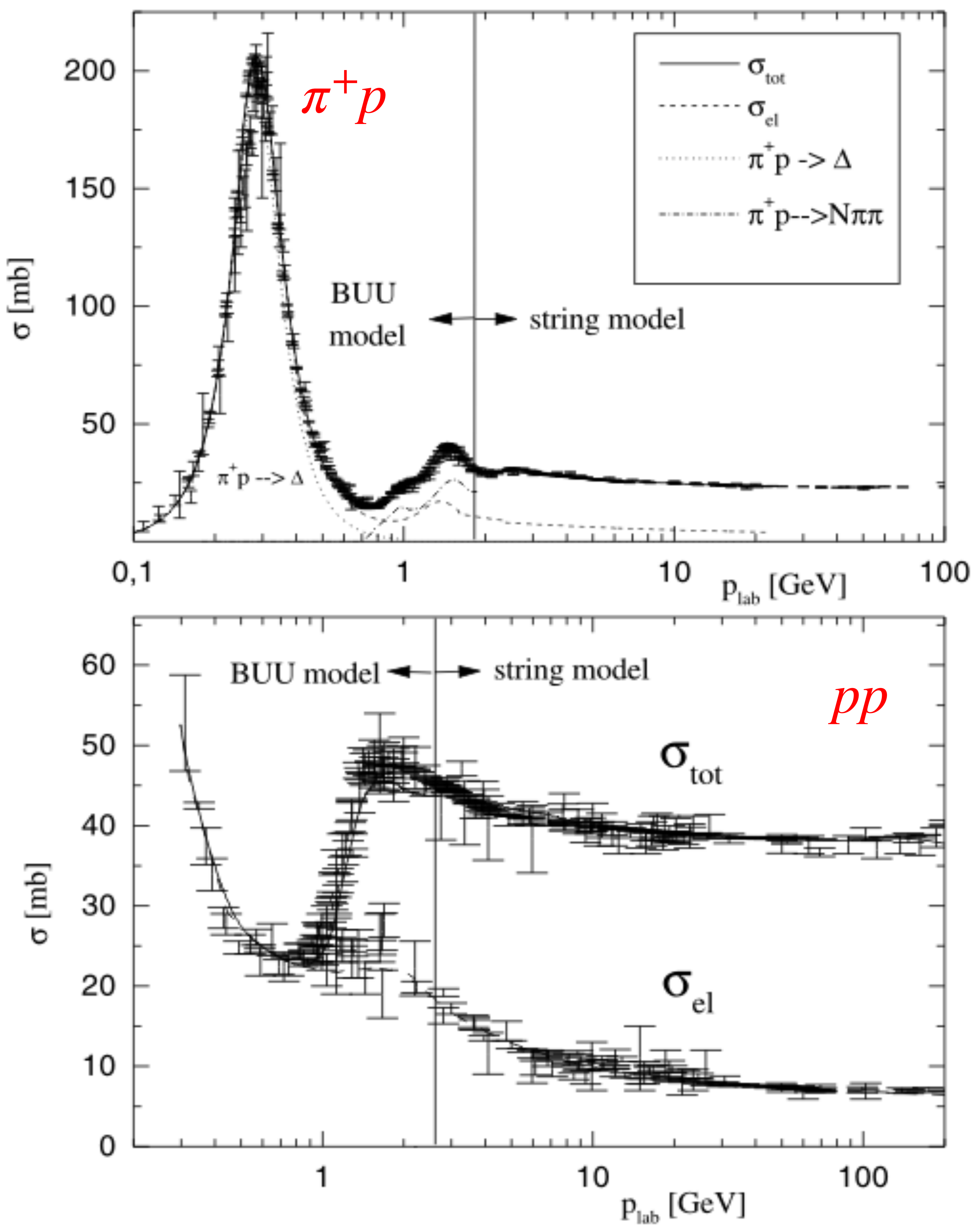
***mm ←→ m´* . . .**

Baryons:

B = p, n, Δ(1232), N(1440), N(1535), ...

Mesons:

$M = \pi, \eta, \rho, \omega, \phi$, ...



Collision term in off-shell transport approach

Collision term for reaction 1+2->3+4:

(Generalized off-shell collision integral for *n* ←→ *m* reactions:) W. Cassing, NPA 700 (2002) 618

$$I_{coll}(X,\vec{P},M^2) = Tr_2 Tr_3 Tr_4 A(X,\vec{P},M^2) A(X,\vec{P}_2,M_2^2) A(X,\vec{P}_3,M_3^2) A(X,\vec{P}_4,M_4^2)$$

spectral functions

$$|G((\vec{P},M^2)+(\vec{P}_2,M_2^2) \to (\vec{P}_3,M_3^2)+(\vec{P}_4,M_4^2))|^2_{\mathcal{A},\mathcal{S}} \; \delta^{(4)}(P+P_2-P_3-P_4)$$

$$[N_{X\vec{P}_3M_3^2} N_{X\vec{P}_4M_4^2} \bar{f}_{X\vec{P}M^2} \bar{f}_{X\vec{P}_2M_2^2} - N_{X\vec{P}M^2} N_{X\vec{P}_2M_2^2} \bar{f}_{X\vec{P}_3M_3^2} \bar{f}_{X\vec{P}_4M_4^2}]$$

Gain term Loss term

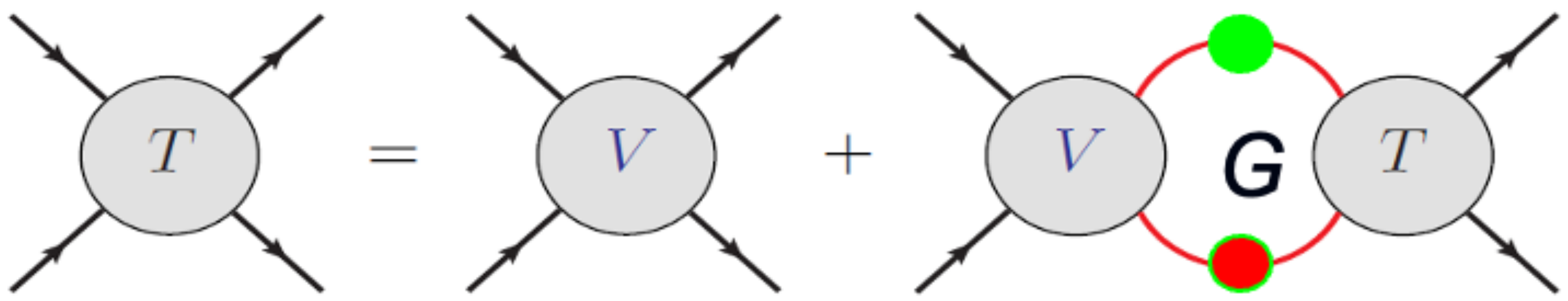
$$\bar{f}_{X\vec{P}M^2} = 1 + \eta N_{X\vec{P}M^2}$$ $\eta = 1$ for bosons, -1 for fermions

The trace over particles 2,3,4 reads explicitly

fermions: $Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dM_2^2}{2\sqrt{\vec{P}_2^2 + M_2^2}}$

bosons: $Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dP_{0,2}^2}{2}$

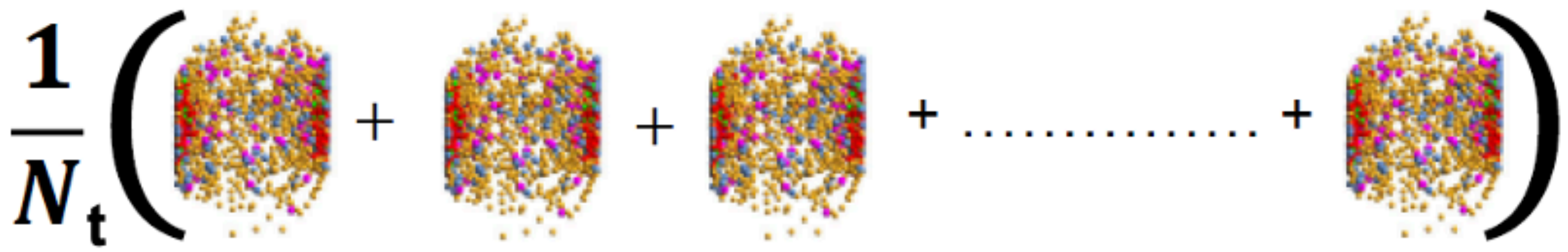
The off-shell transport approach and the particle spectral functions are fully determined once the in-medium transition amplitudes G are known in their off-shell dependence!



Numerical solution of BUU EoM

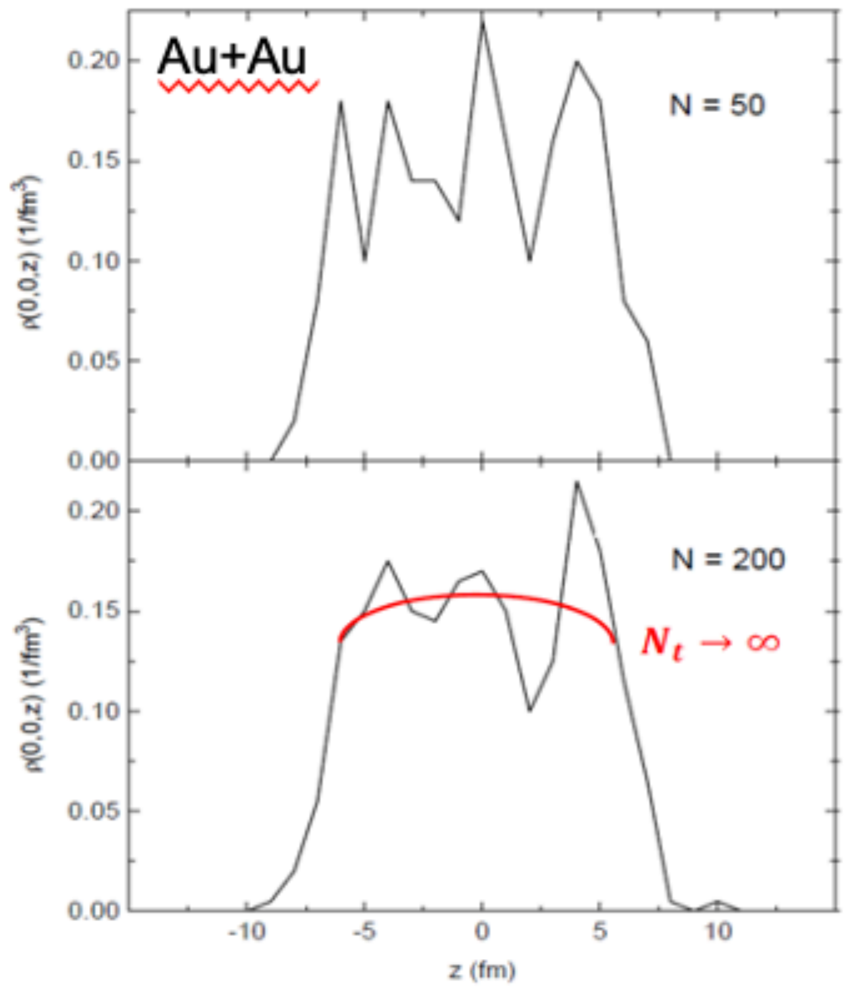
Testparticle method or **method of parallel ensembles :**
the 1-body phase space distribution function is described as a sum of N point-like particles (δ –functions).
In the limit of large number of parallel ensembles $N_t \to \infty$

$$f(\vec{r},\vec{p},t)=\frac{1}{N_t}\sum_{i=1}^{N\cdot N_t}\delta(\vec{r}-\vec{r}_i(t))\delta(\vec{p}-\vec{p}_i(t))$$ **is a solution of Vlasov EoM**



1 + 2 + 3 + ... + N

- **Testparticle method provides a smooth density distribution for calculation of mean-field potential for particle propagation.**
  - No exchange of particles between the parallel ensembles, particles collide only inside one ensemble



➔ **Propagation of test-particles**
in time following 'classical' EoM:

$$\dot{\vec{r}}_i = \frac{d\vec{r}_i}{dt} = \frac{\vec{p}_i}{m_i}$$

$$\dot{\vec{p}}_i = \frac{d\vec{p}_i}{dt} = -\vec{\nabla}_{\vec{r}_i} U(\vec{r}_i, t)$$