Workshop on Non-equilibrium QCD and Transport

2025. 12. 9-12 Huizhou, Guangdong, China

Transport theory and models for low-intermediate energy HICs

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Outline

1. Transport theory and models for low-intermediate energy HICs

2. Refinements on QMD-like models

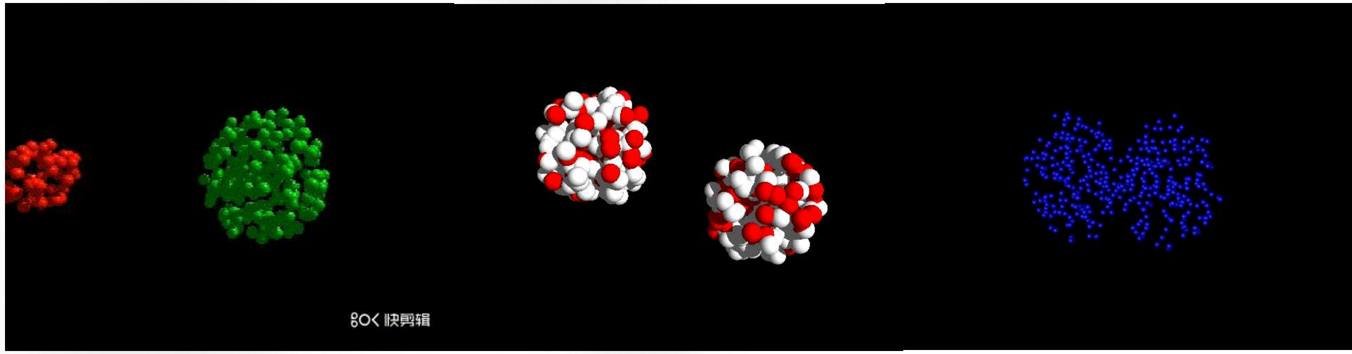
3. Effective mass splitting and SRC

4. Outlooks

Transport phenomena

In daily life

In HICs



nucleons, baryons, mesons, …
Microscopic, N-body system,
time-dependent

Transport theory

DeepSeek: Transport theory is a macroscopic and microscopic framework that studies the processes of transfer or "transport" of physical quantities (such as mass, momentum, energy, and electric charge) through a medium **under non-equilibrium** driving forces.

ChatGPT: Transport theory describes how particles, energy, or physical quantities **move and evolve through a medium under the combined effects of drift, diffusion, and interactions. How particle distributions and macroscopic fluxes change in space, time, and energy due to collisions, external forces, and medium properties.**

19th： Origin of transport theory

- James Clerk Maxwell (1860): velocity distribution of gas moleculars from statistical view point, **Maxwell velocity distribution**

- Ludwig Boltzmann (1872): Blotzmann equation and H-theorem, which marks the official birth of transport theory。

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla_r f + \boldsymbol{F} \cdot \nabla_p f = \int d^3 p_1 d\Omega \, v_{rel} \sigma [f'(\boldsymbol{p}) f'_1(\boldsymbol{p_1}) - f(\boldsymbol{p}) f_1(\boldsymbol{p_1})]$$

$$H(t) = \int f(r,p,t) \ln f(r,p,t) d^3 r d^3 p, \quad \frac{\partial H}{\partial t} \leq 0$$

The Boltzmann equation describes the time evolution of the single-particle distribution function in dilute gases and serves as the foundation for all subsequent transport theories.

- **Quantum extension： Uehling-Uehlenbeck factor, Phys.Rev.43.552**

APRIL 1, 1933 PHYSICAL REVIEW VOLUME 43

Transport Phenomena in Einstein-Bose and Fermi-Dirac Gases. I

E. A. Uehling and G. E. Uhlenbeck, *University of Michigan*

(Received February 13, 1933)

The theory of gases in nonstationary states, given by Lorentz and Enskog, is generalized for the quantum statistics to give the hydrodynamical equations and the distribution function in first and second approximation, and formal expressions for the viscosity and heat conductivity coefficients. These results are valid for all statistics and for all degrees of degeneration. Two essential points contribute to this generality: (a) Exact expressions independent of statistics and degeneration can be given for the coordinate and time derivatives of the coefficient A of the equilibrium distribution function in terms of the pressure and temperature gradients and time derivatives, though a closed expression for this coefficient as a function of v and T is known only in limiting cases; (b) The function W of the general equation of state for ideal gases in all statistics

$$pv=(RT/M)W(v^{\frac{2}{3}}T)$$

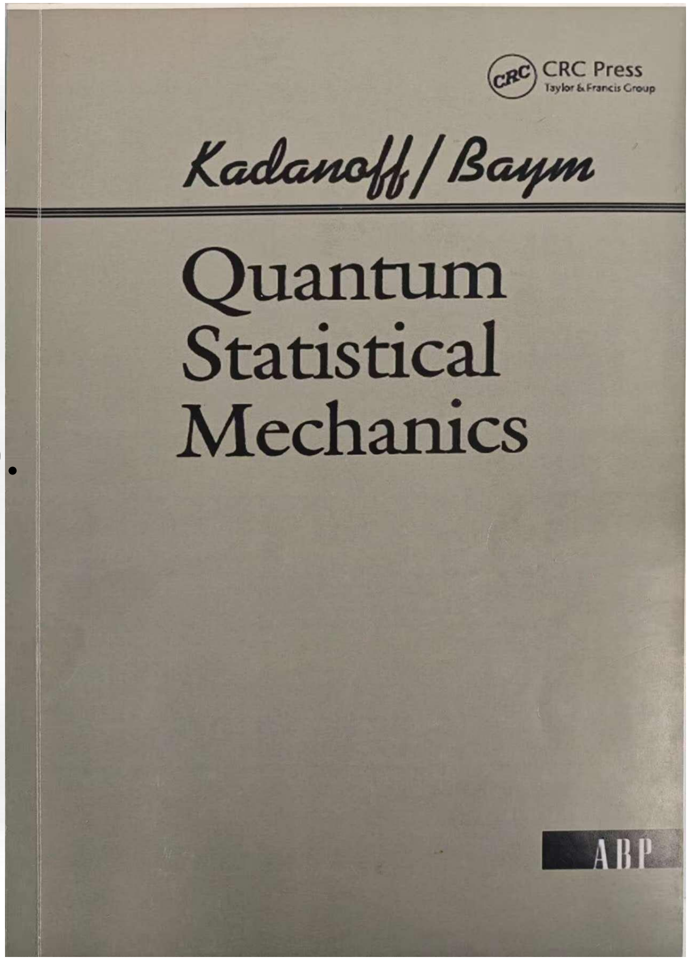
is adiabatically invariant. Numerical values of the viscosity and heat conductivity coefficients, which should come out of the formal theory on the introduction of special assumptions about the molecular forces, have not yet been obtained. It is our hope that these results, when found, may furnish an experimental test of the existence of Einstein-Bose statistics in real gases, as is required by theory.

Mid-20th century： quantum and non-equilibrium statistics

- Bogoliubov (1940s–1950s) systematically derived the Boltzmann equation from microscopic many-body quantum mechanics using the **BBGKY** hierarchy (Bogoliubov–Born–Green–Kirkwood–Yvon hierarchy).

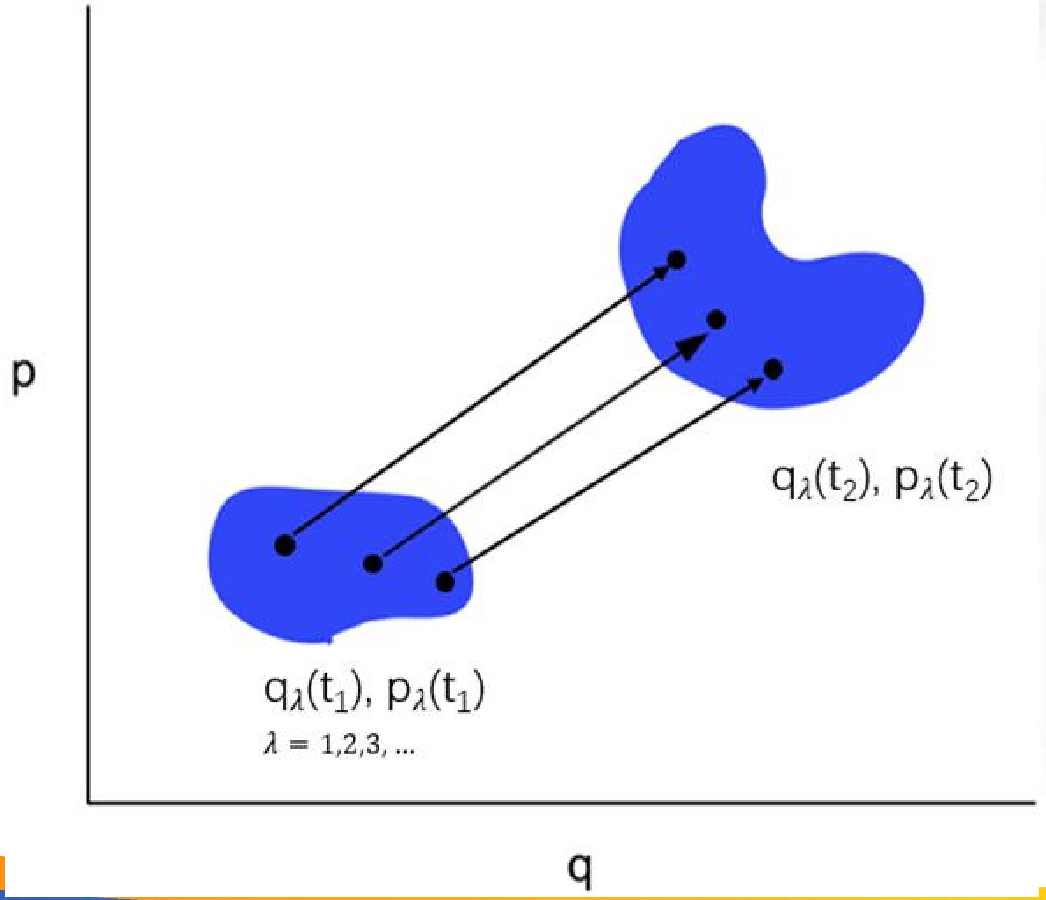
molecular chaos hypothesis (Stosszahlansatz), which assumes that two-particle correlations decay rapidly after collisions.

- Kadanoff and Baym (1962) established the framework of quantum transport theory based on the non-equilibrium Green’s function (NEGF) formalism,



1、 Classical

BBGKY hierarchy– Bogolyubov (46年), Brown(49年), Green(49年), Kirwood(46年), Yvon(37年)



N-body distribution function, $f_N(r_1,p_1, \ldots, r_N, p_N)$

Liouville Equation for N-body system,

$$\frac{\partial f_N}{\partial t} = \{H_N, f_N\} \qquad \{A, B\} = \sum_k \left[\frac{\partial A}{\partial \vec{r}_k} \cdot \frac{\partial B}{\partial \vec{p}_k} - \frac{\partial A}{\partial \vec{p}_k} \cdot \frac{\partial B}{\partial \vec{r}_k}\right]$$

Hamiltonian

$$H_N = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{1\le i<j\le N} V_{ij} + \sum_{1\le i<j<k\le N} V_{ijk} + \cdots$$

V_{ij} 两体相互作用势能

$$H_N = \sum_{i=1}^{N} \left[\frac{p_i^2}{2m} + \phi(r_i)\right] + \sum_{1\le i\le j\le N} \tilde{V}_{ij}$$

注意： $\phi(r)$ single-particle potential, $\tilde{V}_{ij}$ *two-body residue interaction*

Curse of dimensionality：griding phase space， 100 mesh points on each dimension, ＝10^{12A}。A=200, number of mesh points＝10^{2400}

化繁为简、以简驭繁

Simplify complexity, master complexity with simplicity.

l −body distribution function

$$f_l = \frac{1}{V^{l-N}} \int f_N d^6\xi_{l+1} \ldots d^6\xi_N$$

$$\frac{\partial f_l}{\partial t} = \{H_l, f_l\} + \frac{N-l}{V} \int \sum_{i=1}^{l} (\nabla_{r_i} V_{i,l+1}) \cdot (\nabla_{p_i} f_{l+1}) \ldots d^6\xi_{l+1}$$

BBGKY hierarchy

1^{st} BBGKY hierarchy equation

$$\frac{\partial f_1(r,p,t)}{\partial t} = \{H_1, f_1\} + \frac{N-1}{V}\int \sum_{i=1}^{1} (\nabla_r V(r_1, r_2)) \cdot \nabla_p f_2(r_1, p_1; r_2, p_2) \ldots d^6\xi_2$$

H_1:s ingle-particle Hamiltonian

$V(r_1,r_2)$ two-body residue int.

f_2: two-body phase space distributions

$$\frac{\partial}{\partial t} f_1(r,p,t) + \frac{\vec{p}}{m} \cdot \nabla_r f_1(r,p,t) + \nabla\phi(r) \cdot \nabla_p f(r,p,t) = \left(\frac{\partial f_1}{\partial t}\right)_{coll}$$

$$\left(\frac{\partial f_1}{\partial t}\right)_{coll} = \frac{N-1}{V}\int \sum_{i=1}^{1} (\nabla_r V(r, r_2)) \cdot \nabla_p f_2(r, p; r_2, p_2) \ldots d^6\xi_2$$

- **Dilute gas**

2、 Quantum-Nonrelativistic

Schrodinger Eq. for N-body system

$\psi(x_1, \cdots, x_n)$ N-body wave function

$$i\hbar\frac{\partial\psi(x_1,\ldots,x_n)}{\partial t} = \left(-\sum_{i=1}^{N}\frac{\hbar^2}{2m}\nabla_i^2 + V(x_1,\ldots,x_n)\right)\psi(x_1,\ldots,x_n)$$

N-body density matrix

$$\hat{\rho}^{(n)} = |\psi\rangle\langle\psi| \qquad i\hbar\partial_t\hat{\rho}^{(n)} = [H, \hat{\rho}^{(n)}]$$

If we only consider two-body interaction,

$$H=\sum_i T_i+\frac{1}{2}\sum_{i,j(i\neq j)} V_{ij}$$

Define Liuviole operatore, $L_i=[T_i,]$ and $L_{ij}=[\frac{1}{2}V_{ij},]$,

and reduced density matrix,

$$\rho^{(k)}=\frac{N!}{(N-k)!}tr_{(k+1,\cdots,N)}\rho^{(N)}$$

$$\boxed{\frac{\partial}{\partial t}\rho^{(k)}=-i\frac{N!}{(N-k)!}tr_{(k+1,\cdots,N)}\left(\sum_{i=1}^{k}L_i+\sum_{i,j(i\neq j)}^{k}L_{ij}+(n-k)\sum_{i=1}^{k}L_{i,k+1}\right)\rho^{(N)}}$$

$$\frac{\partial}{\partial t}\rho^{(1)}(t) = -\mathrm{i}\left[L_1\rho^{(1)}(t) + \mathrm{tr}_{(2)}\, L_{12}\rho^{(2)}(t)\right],$$
$$\frac{\partial}{\partial t}\rho^{(2)}(t) = -\mathrm{i}\left[\left(L_1 + L_2 + L_{12}\right)\rho^{(2)}(t) + \mathrm{tr}_{(3)}\left(L_{13} + L_{23}\right)\rho^{(3)}(t)\right]$$

In case of

$$\rho^{(2)} = \rho_1^{(1)}\rho_2^{(2)} - \rho_2^{(1)}\rho_1^{(2)} = A_{12}\rho_1^{(1)}\,\rho_2^{(2)}$$

We can get the TDHF eq. without L_{12}

$$\frac{\partial}{\partial t}\rho_1^{(1)}(t) =- i[\frac{p^2}{2m} + U(\rho^{(1)}),\, \rho_1^{(1)}]$$

- 能量适用范围，库伦位垒上几个MeV

单体平均场势为

$$U(\rho^{(1)}) = tr_2 A_{12} V_{12} \rho_2^{(1)}$$

Considering the two-body correlations,

$$\rho^{(2)} = A_{12}\rho_1^{(1)}\rho_2^{(2)} + \sigma_{12}$$

$$\rho^{(2)} = (1 - C_{12})A_{12}\rho_1^{(1)}\rho_2^{(2)}$$

The Hamiltonian can be written as single-body Hamiltoian+residual interaction

$$H = H_0 + v_{12}$$

随机核子-核子散射+涨落项

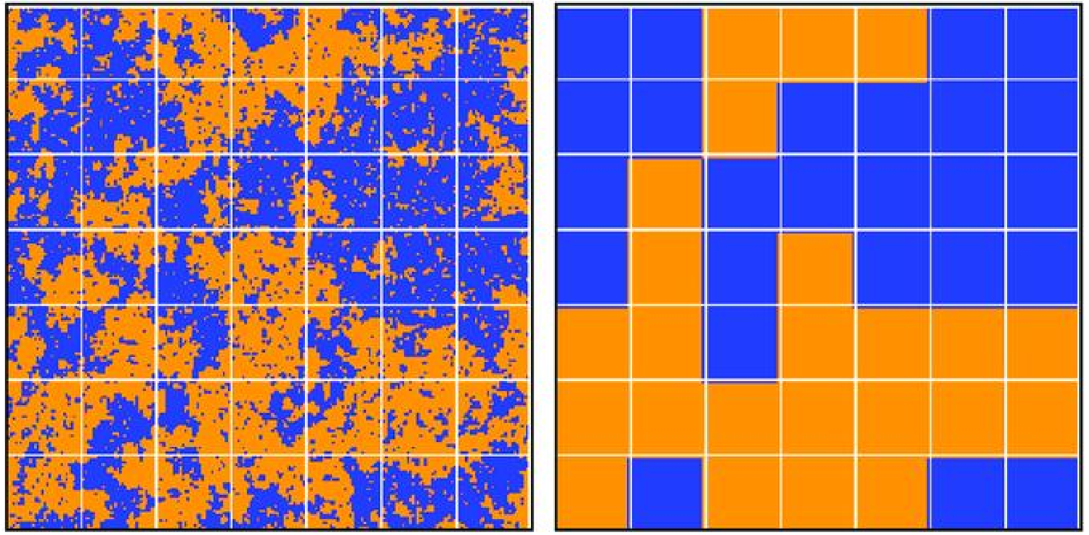
$$i\hbar\frac{\partial}{\partial t}\hat{\rho} = \left[H_0[\hat{\rho}],\ \hat{\rho}\right] + \boxed{K\left[\hat{\rho}, v_{12}\right] + \delta K\left[v_{12}, \hat{\sigma}_{12}\right]}$$

- 能量适用范围，10A-500A MeV

TDHF+ Initial fluctuation
Lacroix, Ayik, Yilmaz, PRC(2012)
Lacroix et al., EPJA52(2016)

SMF, BLOB models: Napolitani, Colonna, PLB726(2013)
M. Colonna, PPNP113 (2020) 103775

Coarse-graining



- Projector operator

$$C_\mu = d_\mu^{-1} \sum_{m,m' \in \mu} |mm\rangle\langle m'm'|$$

Wigner transformation

$$f(\mathbf{r}, \mathbf{p}) = \int e^{-(i/\hbar)\mathbf{p}\cdot\mathbf{s}} \langle \mathbf{r} + \tfrac{1}{2}\mathbf{s} | \hat{\rho} | \mathbf{r} - \tfrac{1}{2}\mathbf{s} \rangle d\mathbf{s},$$

PHYSICAL REVIEW C VOLUME 36, NUMBER 2 AUGUST 1987

Kinetic equation of nuclear gas

Bo-jun Yang and Shu-guang Yao
Department of Technical Physics and Radioelectronics, Peking University, Beijing, China
(Received 27 January 1987)

The kinetic equation on nucl
proved Boltzmann-Uehling-Uhl
body effects.

第 13 卷 第 7 期 高能物理与核物理 Vol. 13, No. 7
1989 年 7 月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS July, 1989

非相对论 BUU 方程*

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卓益忠 (中国原子能研究院,北京)

第 14 卷 第 6 期 高能物理与核物理 Vol. 14, No. 6
1990 年 6 月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS June, 1990

费米能区重离子碰撞动力学*
(I) 动力学的能量关系

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摘 要

本文基于包括了平均场效应,两体碰撞和泡利阻塞效应的 Boltzmann-Uehling-Uhlenbeck理论,系统研究了$^{20}Ne+^{20}Ne$碰撞系统在轰击能量为5—150MeV/u能区内中心碰撞时,反应机制从全融合、非全融合向碎裂变迁的过程. 讨论了平均场、两体碰撞和泡利阻塞效应随轰击能量的变化及它们对反应机制的影响.

Transport models

PHYSICS REPORTS (Review Section of Physics Letters) 160, No. 4 (1988) 189–233. North-Holland, Amsterdam

A GUIDE TO MICROSCOPIC MODELS FOR INTERMEDIATE ENERGY HEAVY ION COLLISIONS

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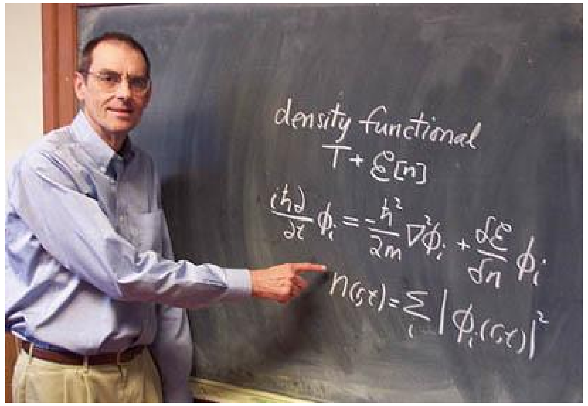
Contents:

Received September 1987



PHYSICS REPORTS (Review Section of Physics Letters) 202, Nos. 5 & 6 (1991) 233–360. North-Holland

"QUANTUM" MOLECULAR DYNAMICS—A DYNAMICAL MICROSCOPIC *n*-BODY APPROACH TO INVESTIGATE FRAGMENT FORMATION AND THE NUCLEAR EQUATION OF STATE IN HEAVY ION COLLISIONS

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Editor: G.E. Brown Received June 1990

A, BUU type: *f(r,p,t)* one body phase space density

$$\frac{\partial f}{\partial t}+v\cdot\nabla_{\mathbf{r}}f-\nabla_{\mathbf{r}}U\cdot\nabla_{\mathbf{p}}f=-\frac{1}{(2\pi)^6}\int d^3p_2d^3p_{2'}d\Omega\frac{d\sigma}{d\Omega}v_{12}\times\{[ff_2(1-f_{1'})(1-f_{2'})]-[f_{1'}f_{2'}(1-f)(1-f_2)]\times(2\pi)^3\delta^3(\mathbf{p}+\mathbf{p}_2-\mathbf{p}_{1'}-\mathbf{p}_{2'})\}$$

Mean field

Two-body collision: occurs between test part.

$$\boxed{f(\mathbf{r},\mathbf{p})\cong\frac{1}{\tilde{N}}\sum_{i=1}^{N\tilde{N}}\delta(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{p}-\mathbf{p}_i)}$$

Solved with test particle methods, CYWang, PRC25,1460(1982).

$$d\vec{r}_i/dt=\nabla_{\vec{p}_i}H;\quad d\vec{p}_i/dt=-\nabla_{\vec{r}_i}H.$$

1, Relativistic kinematic codes: IBUU04, LHV, ……

2, Nonrelativistic codes: SMF, BUU-VM, ……

3, Covariant codes: pBUU, RBUU,RVUU, GiBUU

4, with fluctuation: SMF, BLOB, IBL……

$$\left(\frac{\partial}{\partial t}+\frac{\boldsymbol{p}}{m}\cdot\boldsymbol{\nabla}_r-\boldsymbol{\nabla}_r U\cdot\boldsymbol{\nabla}_p\right) f(\boldsymbol{r},\boldsymbol{p};t) = I_{coll}(\boldsymbol{r},\boldsymbol{p};t)+\delta I_{coll}\,.$$

5, with cluster formation: pBUU, LBUU……

P.Danielewicz, G.F. Bertsch, Nucl.Phys.A533, 712(1991)
Rui Wang, et al., arxiv.org/pdf/2507.16613

Describe deuteron with the help of two-particle green function

$$iG_2(x_1,x_2,t,x_1',x_2',t')=\langle T\{\psi(x_1,t)\psi(x_2,t)\psi^\dagger(x_2',t')\psi^\dagger(x_1',t')\}\rangle\,.$$

$$\Downarrow$$

$$\frac{\partial f_2}{\partial T}+\frac{\partial E}{\partial \boldsymbol{P}}\cdot\frac{\partial f_2}{\partial \boldsymbol{R}}-\frac{\partial E}{\partial \boldsymbol{R}}\cdot\frac{\partial f_2}{\partial \boldsymbol{P}}+\mathscr{D}'f_2=\mathscr{K}^<(1+f_2)-\mathscr{K}^>f_2\,.$$

B, QMD type: solve N-body equation of motion

$$\phi_i(\mathbf{r}_i,t)=\frac{1}{(2\pi\sigma_r^2)^{3/4}}\exp[-\frac{(\mathbf{r}-\mathbf{r}_i)^2}{4\sigma_r^2}+\frac{i\mathbf{p}_i\cdot\mathbf{r}}{\hbar}]$$

$$\psi(\boldsymbol{r}_1,\cdots,\boldsymbol{r}_N)=\phi_{k_1}(\boldsymbol{r}_1)\phi_{k_2}(\boldsymbol{r}_2)\cdots\phi_{k_N}(\boldsymbol{r}_N),$$

$$f_N(\boldsymbol{r}_1,\cdots,\boldsymbol{r}_N;\boldsymbol{p}_1,\cdots,\boldsymbol{p}_N)=\prod_{i=1}^{N}f(\boldsymbol{r}_i,\boldsymbol{p}_i)$$

Two body collision: occurs between nucleons

$$\left(\frac{\partial}{\partial t}+\sum_i\frac{\boldsymbol{p}_i}{m}\cdot\nabla_{\boldsymbol{r}_i}\right)f_N(\boldsymbol{r}_1,\cdots,\boldsymbol{r}_N,\boldsymbol{p}_1,\cdots,\boldsymbol{p}_N;t)=\int\Pi_i\mathrm{d}^3p_i\mathrm{d}^3Q_i\mathrm{d}^3q_i\mathrm{e}^{\mathrm{i}\boldsymbol{r}_i\cdot\boldsymbol{p}_i}$$
$$\cdot f_0^{(n)}(Q_1,\ldots,Q_N,q_1,\ldots,q_N;t)$$
$$\cdot[I_1(T)+I_2(T)+I_3(T)], \qquad (6)$$

$$\dot{\mathbf{r}}_i=\frac{\mathbf{p}_i}{m}+\nabla_{\mathbf{p}_i}\sum_j\langle V_{ij}\rangle=\nabla_{\mathbf{p}_i}\sum_j\langle H\rangle$$
$$\dot{\mathbf{p}}_i=-\nabla_{\mathbf{r}_i}\sum_j\langle V_{ij}\rangle=-\nabla_{\mathbf{r}_i}\sum_j\langle H\rangle$$

nucleon

Rearrange whole nucleon-> large fluctuation

QMD, IQMD, UrQMD, ImQMD, LQMD, EQMD, AMD, FMD, ……

C. Antisymmetric molecular dynamics models

- 能量适用范围， 10A-300A MeV

N-体波函数

$$|\Phi\rangle = \frac{1}{\sqrt{N!}} det\left[\varphi_i\left(j\right)\right] \tag{1}$$

or

$$|\Phi\rangle = \mathcal{A}\left[\varphi_1\left(1\right)\varphi_2\left(2\right)\cdots\varphi_N\left(N\right)\right] \tag{2}$$

where

$$\varphi_i = \phi_{\mathbf{Z}_i}\chi_{\alpha_i} = \phi_{\mathbf{Z}_i}\tau_i s_i.$$

Akira Ono, PTP87(1992)1185

$$\langle \mathbf{r} \mid \phi_{\mathbf{Z}_i}\rangle = (\frac{2\nu}{\pi})^{3/4}\exp[-\nu(\mathbf{r}-\frac{\mathbf{Z}_i}{\sqrt{\nu}})^2]$$

波包中心的演化方程

$$i\hbar\sum_{j=1}^{N}\sum_{\tau=1}^{3} C_{i\sigma,j\tau}\dot{Z}_{j\tau} = \frac{\partial}{\partial Z_{i\sigma}^*}\mathcal{H}$$

Stochastic equation of motion for the wave packet centroids Z:

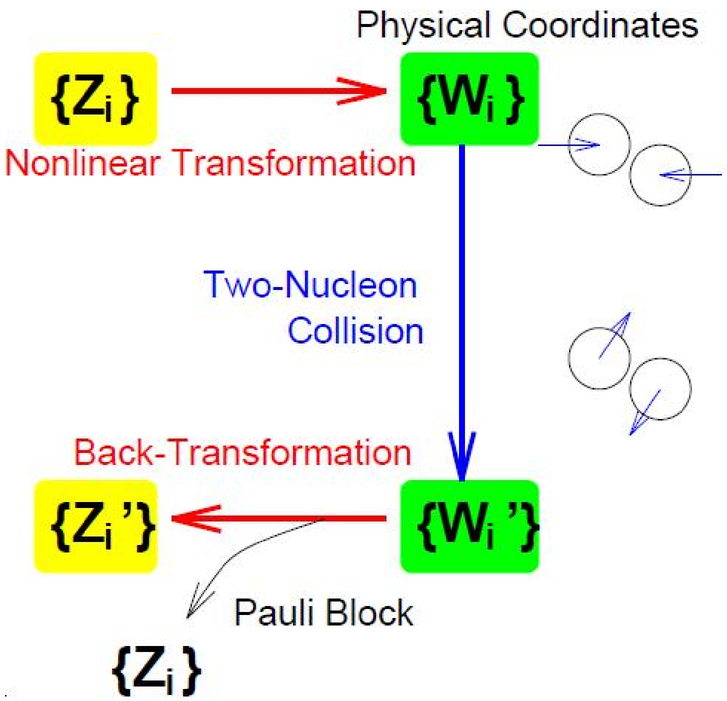
$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\mathrm{PB}} + \quad \text{stochastic NN collisions}$$

Two-Nucleon Collisions

Physical coordinates are used in two-nucleon collisions. PTP87 (1992) 1185

$$\mathbf{W}_k = \sum_j \left(\sqrt{Q(Z)}\right)_{kj} \mathbf{Z}_j = \sqrt{\nu}\mathbf{R}_k + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{P}_k, \qquad f_{\mathrm{W}}(\mathbf{r},\mathbf{p}) = 8\sum_{k=1}^{A} e^{-2\nu(\mathbf{r}-\mathbf{R}_k)^2-(\mathbf{p}-\mathbf{P}_k)^2/2\hbar^2\nu}$$

Collision between 1 and 2 : $W = (\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3, \ldots, \mathbf{W}_A) \Rightarrow W' = (\mathbf{W}'_1, \mathbf{W}'_2, \mathbf{W}_3, \ldots, \mathbf{W}_A)$

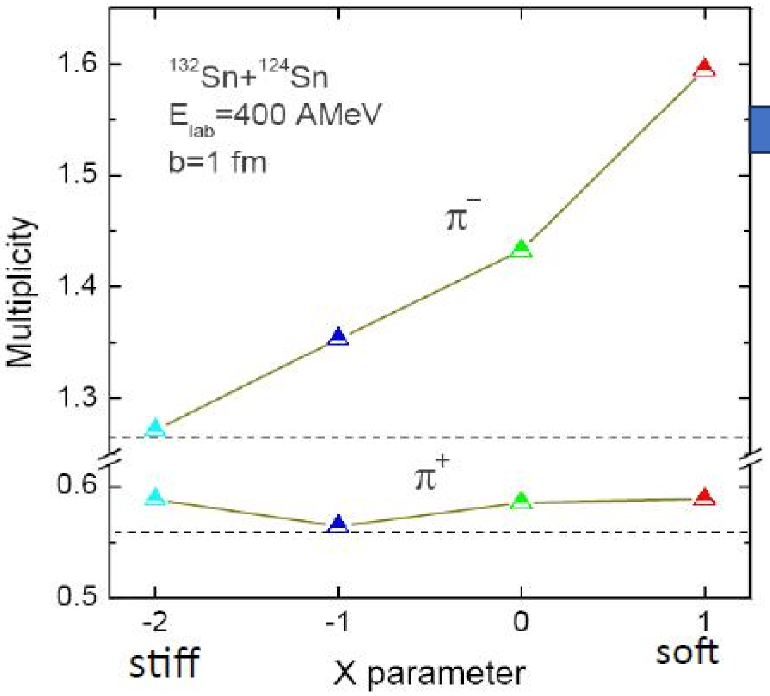


$$\mathbf{P}_1 = \tfrac{1}{2}\mathbf{p}_{\mathrm{tot}} + \mathbf{p}_{\mathrm{rel}}$$
$$\mathbf{P}_2 = \tfrac{1}{2}\mathbf{p}_{\mathrm{tot}} - \mathbf{p}_{\mathrm{rel}}$$
$$\Downarrow\ d\sigma_{NN}/d\Omega$$
$$\mathbf{P}'_1 = \tfrac{1}{2}\mathbf{p}_{\mathrm{tot}} + p'_{\mathrm{rel}}\hat{\mathbf{n}}$$
$$\mathbf{P}'_2 = \tfrac{1}{2}\mathbf{p}_{\mathrm{tot}} - p'_{\mathrm{rel}}\hat{\mathbf{n}}$$

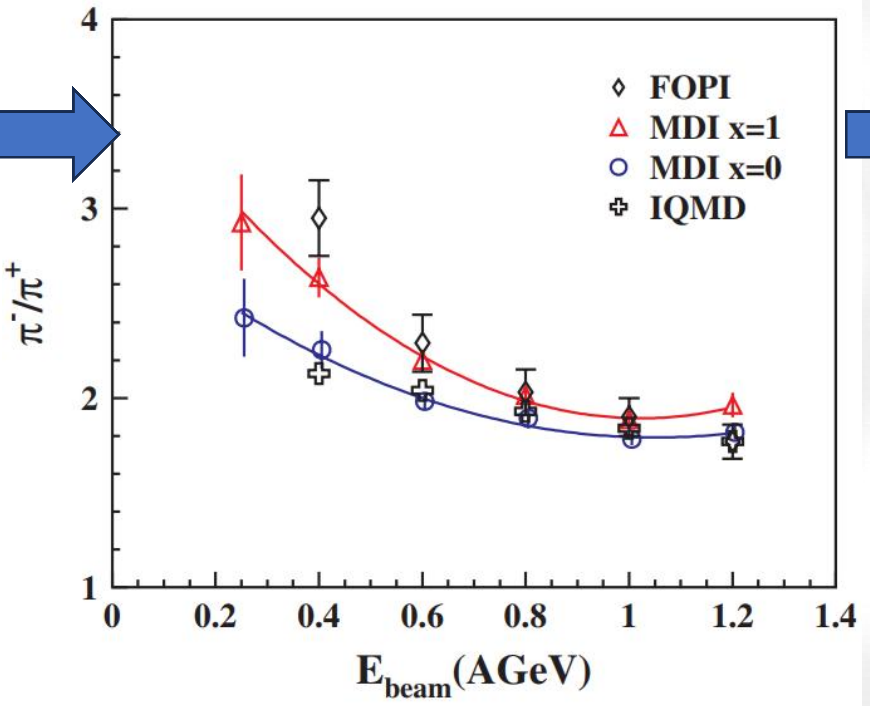
Challenges on low-intermediate energy transport models

• Debates on the constraints of symmetry energy from transport models

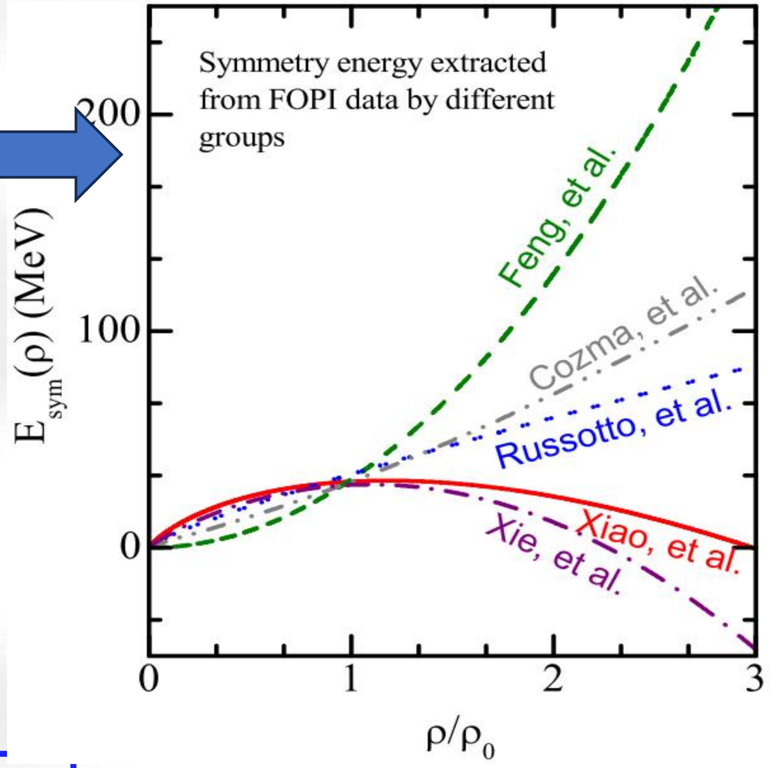
B. A. Li, PRL 88, 192701 (2002)



Z.G.Xiao, PRL102, 062502(2009)



W.M.Guo, GCYong et al, PLB738,397(2014)



π^-/π^+

1, Why there is strong model dependence?

2, Which density region is probed by pi-/pi+?

L = 5 – 144 MeV

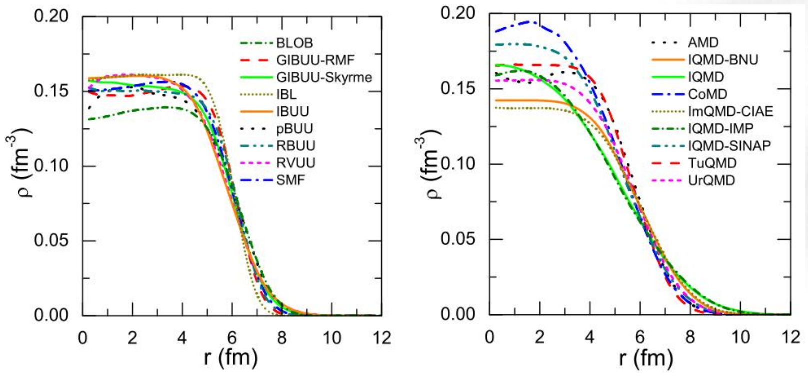

International Workshop on Simulations of
Low and Intermediate Energy
Heavy Ion Collisions
(Transport 2014)
Jan 8-12, 2014, Shanghai, China

• Transport model evaluation project-benchmark the collision and MF

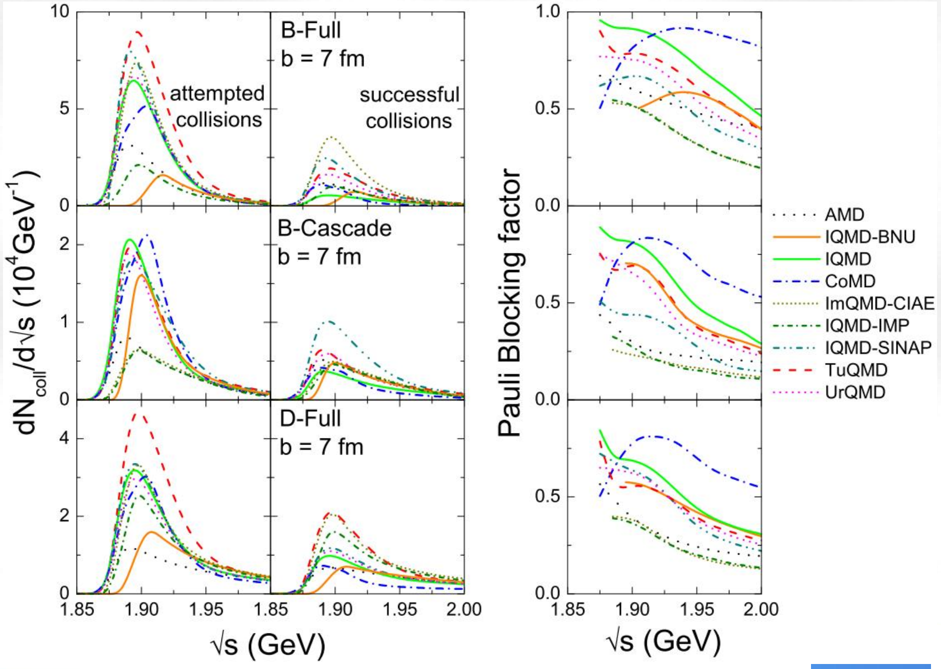
PHYSICAL REVIEW C **93**, 044609 (2016)

Understanding transport simulations of heavy-ion collisions at 100*A* and 400*A* MeV: Comparison of heavy-ion transport codes under controlled conditions

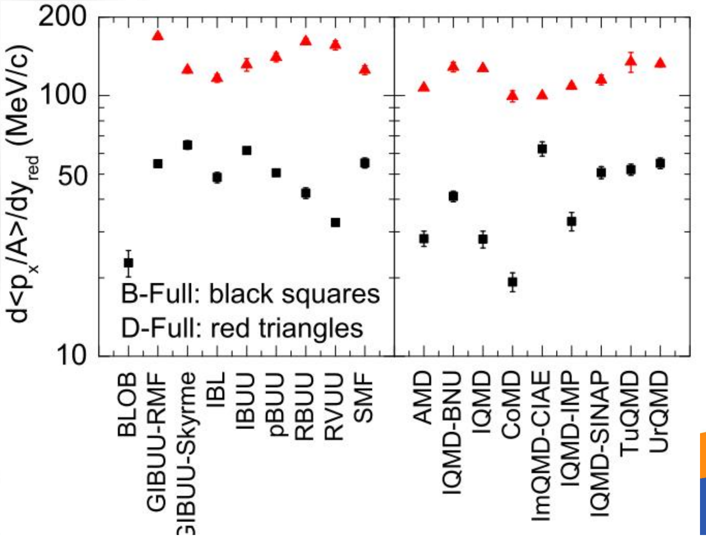
Jun Xu,[1,*] Lie-Wen Chen,[2,†] ManYee Betty Tsang,[3,‡] Hermann Wolter,[4,§] Ying-Xun Zhang,[5,‖] Joerg Aichelin,[6] Maria Colonna,[7] Dan Cozma,[8] Pawel Danielewicz,[3] Zhao-Qing Feng,[9] Arnaud Le Fèvre,[10] Theodoros Gaitanos,[11] Christoph Hartnack,[6] Kyungil Kim,[12] Youngman Kim,[12] Che-Ming Ko,[13] Bao-An Li,[14] Qing-Feng Li,[15] Zhu-Xia Li,[5] Paolo Napolitani,[16] Akira Ono,[17] Massimo Papa,[18] Taesoo Song,[19] Jun Su,[20] Jun-Long Tian,[21] Ning Wang,[22] Yong-Jia Wang,[15] Janus Weil,[19] Wen-Jie Xie,[23] Feng-Shou Zhang,[24] and Guo-Qiang Zhang[1]



1) Initialization ?



2) Collisions and Pauli blocking



3) Flow: BUU is great than QMD，400AMeV, 15% difference

PHYSICAL REVIEW C **97**, 034625 (2018)

Comparison of heavy-ion transport simulations: Collision integral in a box

Ying-Xun Zhang,[1,2,*] Yong-Jia Wang,[3,†] Maria Colonna,[4,‡] Pawel Danielewicz,[5,§] Akira Ono,[6,∥] Manyee Betty Ts
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Che-Ming Ko,[14] Bao-An Li,[15] Qing-Feng Li,[3,11] Zhu-Xia Li,[1] Swagata Mallik,[16] Yasushi Nara,[17] Tatsuhiko Oga
Akira Ohnishi,[19] Dmytro Oliinychenko,[20] Massimo Papa,[4] Hannah Petersen,[20,21,22] Jun Su,[23] Taesoo Song,[20,21] Janu

PHYSICAL REVIEW C **104**, 024603 (2021)

Comparison of heavy-ion transport simulations: Mean-field dynamics in a box

Maria Colonna,[1,*] Ying-Xun Zhang,[2,3,†] Yong-Jia Wang,[4,‡] Dan Cozma,[5] Pawel Danielewicz,[6,§] Che Ming Ko,[7] Akira Ono,[8,∥]
Manyee Betty Tsang,[6,¶] Rui Wang,[9,10] Hermann Wolter,[11,#] Jun Xu,[12,9,**] Zhen Zhang,[13] Lie-Wen Chen,[14] Hui-Gan Cheng,[15]
Hannah Elfner,[16,17,18] Zhao-Qing Feng,[15] Myungkuk Kim,[19] Youngman Kim,[20] Sangyong Jeon,[21] Chang-Hwan Lee,[22]
Bao-An Li,[23] Qing-Feng Li,[4,24] Zhu-Xia Li,[2] Swagata Mallik,[25] Dmytro Oliinychenko,[26,27] Jun Su,[13] Taesoo Song,[16,28]

PHYSICAL REVIEW C **100**, 044617 (2019)

Comparison of heavy-ion transport simulatio
Collision integral with pions and Δ resonances in

Akira Ono,[1,*] Jun Xu,[2,3,†] Maria Colonna,[4] Pawel Danielewicz,[5] Che Ming Ko,[6] Many
Hermann Wolter,[8] Ying-Xun Zhang,[9,10] Lie-Wen Chen,[11] Dan Cozma,[12] Hannah Elf
Natsumi Ikeno,[17,18] Bao-An Li,[19] Swagata Mallik,[20] Yasushi Nara,[21] Tatsuhiko C
Dmytro Oliinychenko,[24] Jun Su,[25] Taesoo Song,[13] Feng-Shou Zhang,[26,27]
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[2]*Shanghai Advanced Research Institute, Chinese Academy of Sciences, Shangh*
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Review

Transport model comparison studies of intermediate-energy heavy-ion collisions

Hermann Wolter [1,*], Maria Colonna [2], Dan Cozma [3], Pawel Danielewicz [4,5], Che Ming Ko [6], Rohit Kumar [4], Akira Ono [7], ManYee Betty Tsang [4,5], Jun Xu [8,9], Ying-Xun Zhang [10,11], Elena Bratkovskaya [12,13], Zhao-Qing Feng [14], Theodoros Gaitanos [15], Arnaud Le Fèvre [12], Natsumi Ikeno [16], Youngman Kim [17], Swagata Mallik [18], Paolo Napolitani [19], Dmytro Oliinychenko [20], Tatsuhiko Ogawa [21], Massimo Papa [2], Jun Su [22], Rui Wang [9,23], Yong-Jia Wang [24], Janus Weil [25], Feng-Shou Zhang [26,27], Guo-Qiang Zhang [9], Zhen Zhang [22], Joerg Aichelin [28], Wolfgang Cassing [25], Lie-Wen Chen [29], Hui-Gan Cheng [14],

1, give a benchmark on how to treat the attempted collision√
2, treat the mean field with lattice method
3, Pauli blocking should be improved

PHYSICAL REVIEW C **109**, 044609 (2024)

Comparing pion production in transport simulations of heavy-ion collisions at 270*A* MeV under controlled conditions

Jun Xu,[1,*] Hermann Wolter,[2,†] Maria Colonna,[3,‡] Mircea Dan Cozma,[4,§] Pawel Danielewicz,[5,6,∥] Che Ming Ko,[7,¶] Akira Ono,[8,#] ManYee Betty Tsang,[5,6,**] Ying-Xun Zhang,[9,10,††] Hui-Gan Cheng,[11] Natsumi Ikeno,[12,13] Rohit Kumar,[5] Jun Su,[14] Hua Zheng,[15] Zhen Zhang,[14] Lie-Wen Chen,[16] Zhao-Qing Feng,[11] Christoph Hartnack,[17] Arnaud Le Fèvre,[18] Bao-An Li,[19] Yasushi Nara,[20] Akira Ohnishi,[21,‡‡] and Feng-Shou Zhang[22,23]
(TMEP Collaboration)
[1]*School of Physics Science and Engineering, Tongji University, Shanghai 200092, China*
[2]*Faculty of Physics, University of Munich, D-85748 Garching, Germany*

Table 1
List of transport models that participated in the TMEP code comparisons discussed in this paper. The columns give the information on the name of the code, the main correspondents of the code, the energy range intended for the code, the treatment of effects of relativity (see Section 2.1), and the comparisons in which the code participated. The different comparisons are listed in the last column in the table by a numbers n, which refer to the subsections 3.n, where they are described in detail: n = 1 for Au+Au collisions around 1 AGeV, n = 2 for Au+Au collision at 100 and 400 AMeV, n = 3 for box-Vlasov, n = 4 for box-cascade with only nucleons, n = 5 for box-cascade with pion and Δ resonance production, and n = 6 for the prediction of pion ratios for Sn+Sn collisions.

BUU Type	Code Correspondents	Energy Range [A GeV]	Relativity	Comparisons
BLOB	P. Napolitani, M. Colonna	0.01–0.5	non-rel	2
BUU-VM	S. Mallik	0.02–1	rel	3,4,5
DJBUU	Y. Kim, S. Jeon, M. Kim, C.-H. Lee, K. Kim	0.05–2	cov	3
GiBUU	J. Weil, T. Gaitanos, K. Gallmeister, U. Mosel	0.05–40	rel/cov	1,2,3,4
IBL	W.J. Xie, F.S. Zhang	0.05–2	rel	2
IBUU	J. Xu, L.W. Chen, B.A. Li	0.05–2	rel	2,3,4,5
LBUU(LHV)	R. Wang, Z. Zhang, L.-W. Chen	0.01–1.5	rel	3
pBUU	P. Danielewicz	0.01–12	rel	1,2,3,4,5,6
PHSD	E. Bratkovskaya, W. Cassing	0.1–200	rel/cov	1,6
RBUU	T. Gaitanos	0.05–2	cov	1,2
RVUU	Z. Zhang, C.M. Ko, T. Song	0.05–2	cov	1,2,3,4,5
SMASH	D. Oliinychenko, H. Elfner, A. Sorensen	0.5–200	cov	3,4,5,6
SMF	M. Colonna, P. Napolitani	0.01–0.5	non-rel	2,3,4
χBUU	Z. Zhang, C.M. Ko	0.01–0.5	non-rel	6
QMD Type	Code Corespondents	Energy Range [AGeV]	Relativity	Comparisons
AMD	A. Ono	0.01–0.3	non-rel	2
AMD+JAM	N. Ikeno, A. Ono	0.01–0.3	non-rel+rel	6
BQMD/IQMD	A. Le Fèvre, J. Aichelin, C. Hartnack, R. Kumar	0.05–2	rel	1,2,6
CoMD	M. Papa	0.01–0.3	non-rel	2,4
ImQMD	Y.X. Zhang, N. Wang, Z.X. Li	0.02–0.4	rel	2,3,4
IQMD-BNU	J. Su, F.S. Zhang	0.05–2	rel	2,3,4,5,6
IQMD-SINAP	G.Q. Zhang	0.05–2	rel	2
JAM	A. Ono, N. Ikeno, Y. Nara, A. Ohnishi	1–158	rel	4,5
JQMD 2.0	T. Ogawa, K. Niita, S. Hashimoto, T. Sato	0.01–3	rel	4,5
LQMD(IQMD-IMP)	Z.Q. Feng, H.G. Cheng	0.01–10	rel	2,3,4,5
TuQMD/dcQMD	D. Cozma	0.1–2	rel	1,2,3,4,5,6
UrQMD	Y. J. Wang, Q. F. Li, Y. X. Zhang	0.05–200	rel	1,2,3,4,6

Identifying a problem is only the first step; addressing it is what drives scientific progress

2 Refinements on QMD-like models

Initialization, Mean field, Collisions, Pauli blocking

ImQMD model

2003
ImQMD
EDF

2005
ImQMD05
S(ρ), MDI

S(ρ),
Rn/p, Ri(y)
PLB, PRL

2009--
TMEP collaboration
Benchmark
MF, Collision
4 PRC, PPNP…

2012-2022
ImQMD
-
Skyrme/MSL,
-Lattice (suggested)
PRC, PL

2023
ImQMD23
-Extended Skyrme
-Consistence
-New PB
PRC ,

Refinements in ImQMD

- **Nucleonic potential energy are derived from ‘full’ Skyrme energy density functional**

$$U_N = \int (u_{loc} + u_{md}) d^3 r$$

$$u_{\mathrm{loc}} = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\eta+1}\frac{\rho^{\eta+1}}{\rho_0^{\eta}} + \frac{g_{\mathrm{sur}}}{2\rho_0}(\nabla\rho)^2 + \frac{g_{\mathrm{sur,iso}}}{\rho_0}[\nabla(\rho_n - \rho_p)]^2 + A_{\mathrm{sym}}\frac{\rho^2}{\rho_0}\delta^2 + B_{\mathrm{sym}}\frac{\rho^{\eta+1}}{\rho_0^{\eta}}\delta^2$$

$$u_{md} = C_0 \sum_{ij} \int d^3p d^3p' f_i(\mathbf{r},\mathbf{p}) f_j(\mathbf{r},\mathbf{p}')(\mathbf{p}-\mathbf{p}')^2 + D_0 \sum_{ij\in n} \int d^3p d^3p' f_i(\mathbf{r},\mathbf{p}) f_j(\mathbf{r},\mathbf{p}')(\mathbf{p}-\mathbf{p}')^2 + D_0 \sum_{ij\in p} \int d^3p d^3p' f_i(\mathbf{r},\mathbf{p}) f_j(\mathbf{r},\mathbf{p}')(\mathbf{p}-\mathbf{p}')^2.$$

- **‘Consistent’ treating the initialization and propagation.**

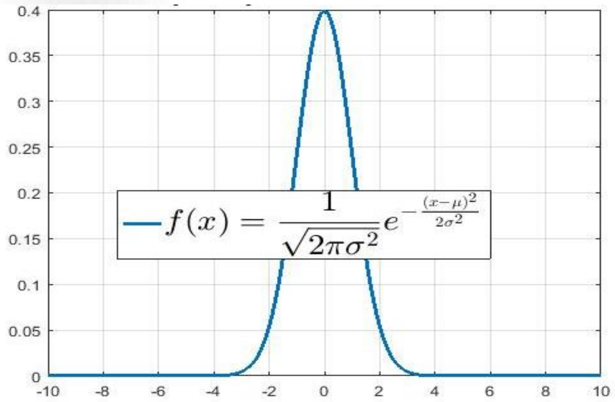
R and BE are calculated under the same Hamiltonian

restricted density variational method (RDV)

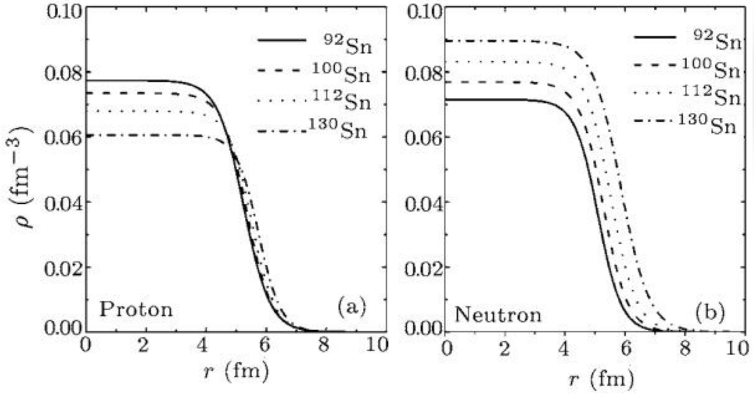
Junping Yang, Yingxun Zhang*, et al., PRC104, 024605 (2021)

$$E=\int \mathcal{H}dr=\int \frac{\hbar^2}{2m}[\tau_n(\mathbf{r})+\tau_p(\mathbf{r})]+\mathcal{H}_{sky}+\mathcal{H}_{\mathrm{coul}}dr.$$

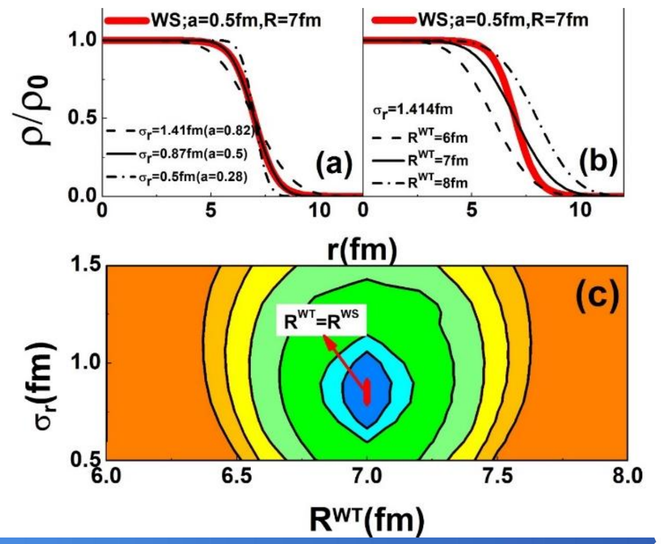
$$\rho_i(\boldsymbol{r})=\rho_{0i}\left[1+\exp\left(\frac{r-R_{0i}}{a_i}\right)\right]^{-1},\quad i=\{n,p\}$$



Liu Min et al 2006 Chin. Phys. Lett. **23** 804



- $\mathbf{R=R^{WT}}$
- $\boldsymbol{\sigma_r\approx 1.71a}$

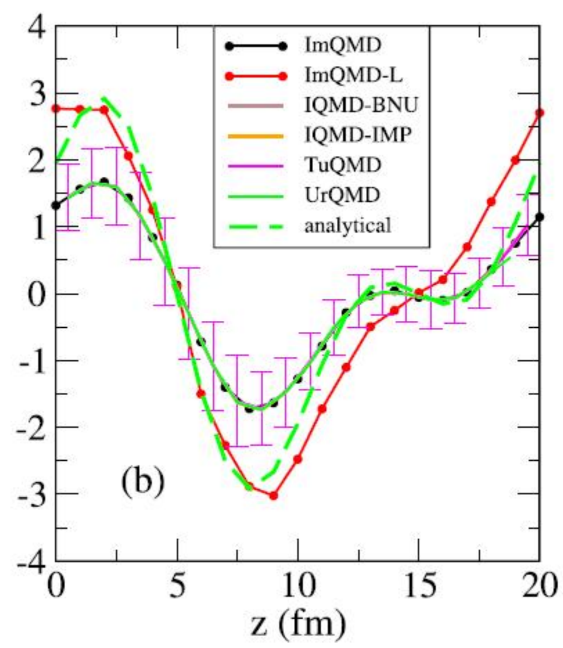


- **'Accurate' treating the nonlinear density dependent interaction term**

$$U = \int u(r) d^3r = \frac{\alpha}{2} \sum_i \left< \frac{\rho}{\rho_0} \right>_i + \frac{\beta}{\gamma+1} \sum_i \left< \frac{\rho^\gamma}{\rho_0^\gamma} \right>_i$$

Deviation: ~30 %

$$\frac{\beta}{\gamma+1} \sum_i \left< \frac{\rho^\gamma}{\rho_0^\gamma} \right>_i \approx \frac{\beta}{\gamma+1} \sum_i \left< \frac{\rho}{\rho_0} \right>_i^\gamma + O(\rho)$$



ImQMD-L

Junping Yang, Yingxun Zhang*, et al., PRC104, 024605 (2021)

Deviation: < 5 %

$$\frac{\beta}{\gamma+1} \sum_i \left< \frac{\rho^\gamma}{\rho_0^\gamma} \right>_i$$

$$\dot{\vec{p}}_i = -\frac{\partial U_3}{\partial R_i} = -\frac{\beta}{\gamma+1} \frac{\partial}{\partial R_i} \sum_m \left< \frac{\rho}{\rho_0} \right>_m^\gamma$$

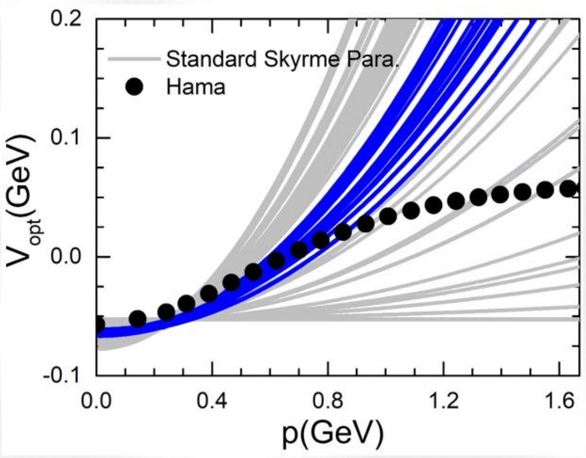
- **'Extended' Skyrme MDI, width of WF is considered into u_{md}**

$$g(\mathbf{p}-\mathbf{p}') = a_0\delta(\boldsymbol{r}-\boldsymbol{r}')(\boldsymbol{p}-\boldsymbol{p}')^2$$

Inspired by N3LO of pseudo-Skyrme potential, Extended Skyrme-type of MDI

$$g(\mathbf{p}-\mathbf{p}') = \delta(\boldsymbol{r}-\boldsymbol{r}')\sum_{l=0}^{N} b_l(\boldsymbol{p}-\boldsymbol{p}')^{2l}$$

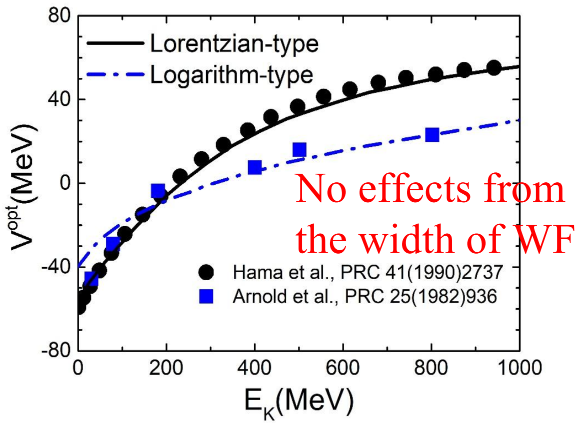
Junping Yang, Xiang Chen, … **YXZ,** PRC109, 054624(2024).



- **Δp^2 type**

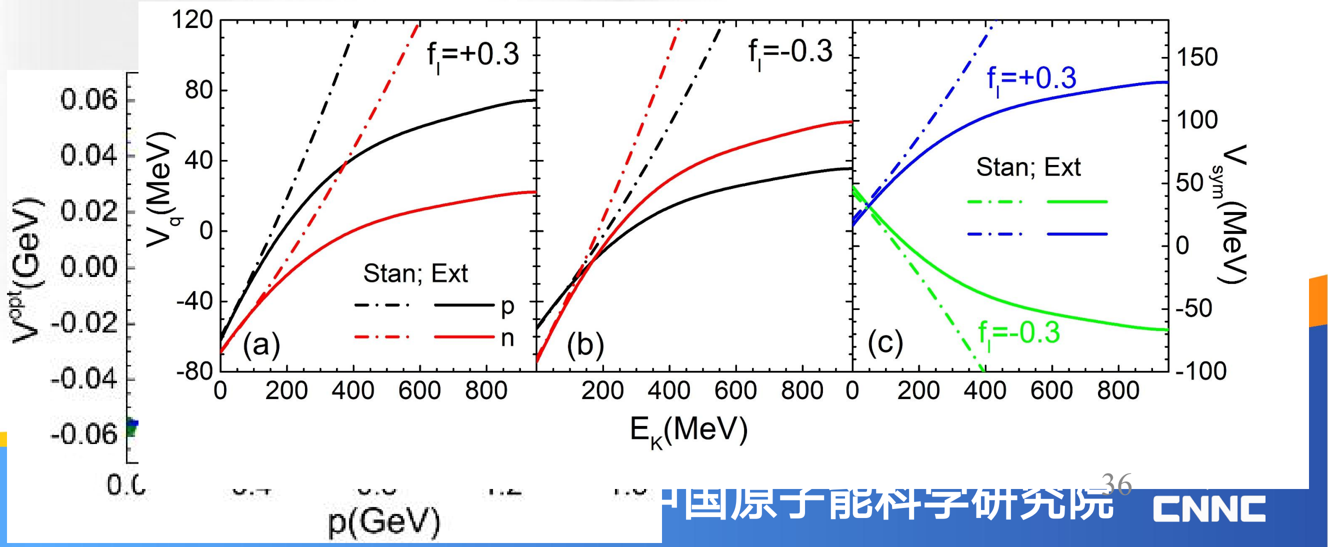
- **Logarithm-type:**

$$g(\mathbf{p}-\mathbf{p}') = 1.57\ln^2(1+t_4(\Delta\boldsymbol{p})^2)$$



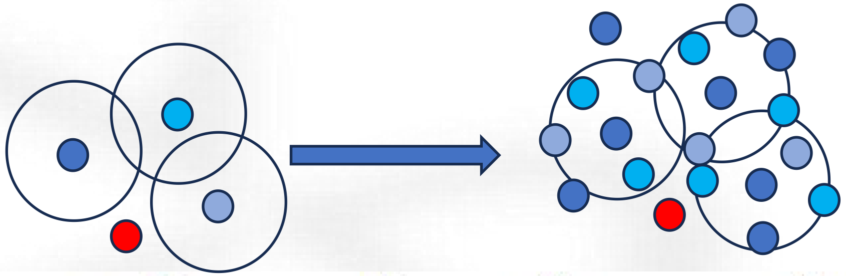
- **Lorentzian-type:**

$$g(\mathbf{p}-\mathbf{p}') = \sum_{k=1,2}\frac{C_{ex}^{(k)}}{1+[(\boldsymbol{p}-\boldsymbol{p}')/u_k]^2}$$



- **A new Pauli blocking occupation probability algorithms**

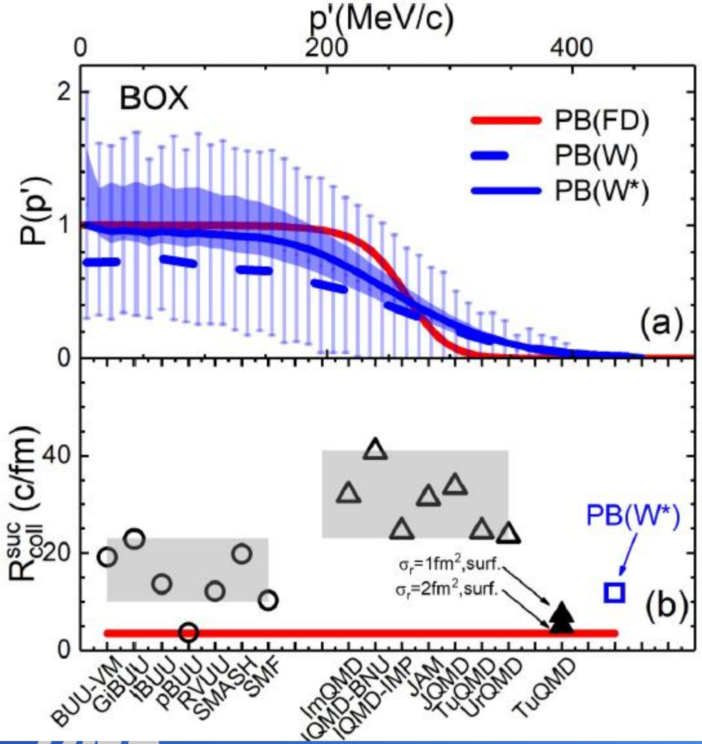
$$P_{block} = 1 - (1 - P_i)(1 - P_j)$$

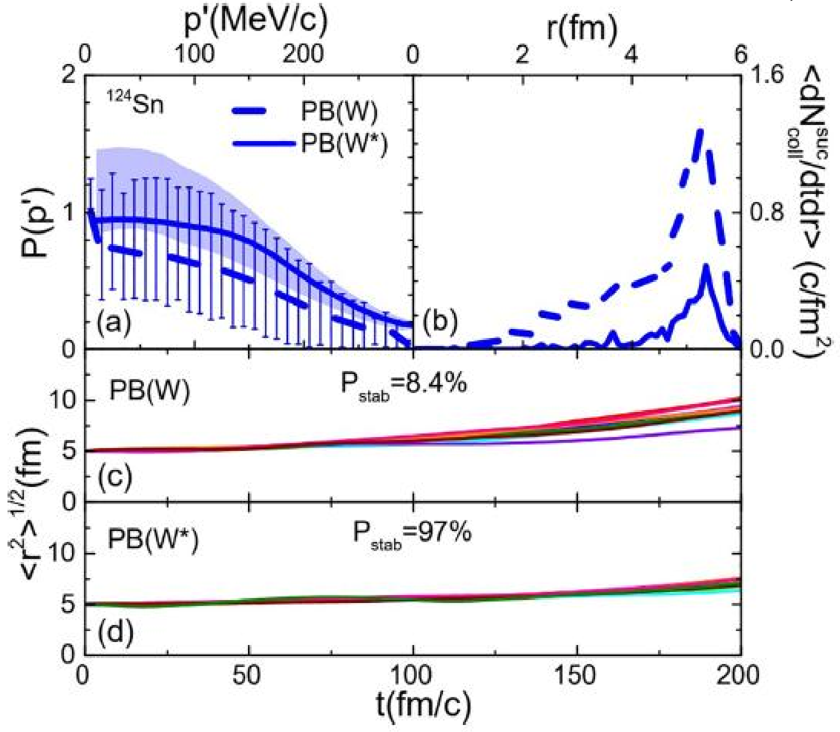


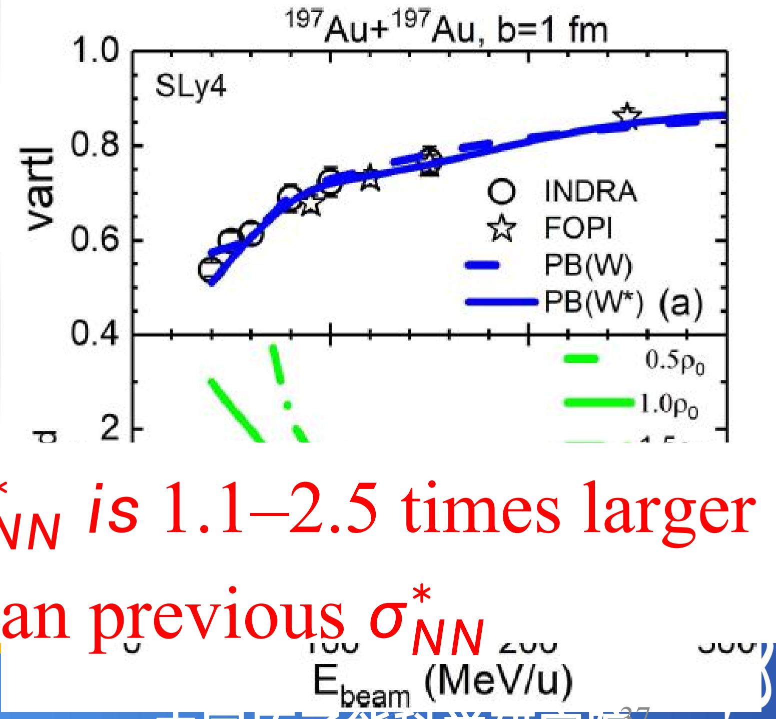
$$P(\mathbf{r}_i, \mathbf{p}_i') = \frac{1}{4/h^3} \sum_{j=1(j\neq i)}^{N} \tilde{f}_j(\mathbf{r}_i, \mathbf{p}_i').$$

$$\tilde{f}_j(\mathbf{r}, \mathbf{p}) = \frac{1}{M} \sum_{\lambda=1}^{M} \frac{1}{(\pi\hbar)^3} \exp\left[-\frac{(\mathbf{r}-\mathbf{r}_j^{(\lambda)})^2}{2\sigma_r^2} - \frac{(\mathbf{p}-\mathbf{p}_j^{(\lambda)})^2}{2\sigma_p^2}\right]$$

X.Chen, J.P. Yang, et al., Phys.Rev.C-Letter 109, L021604 (2024)



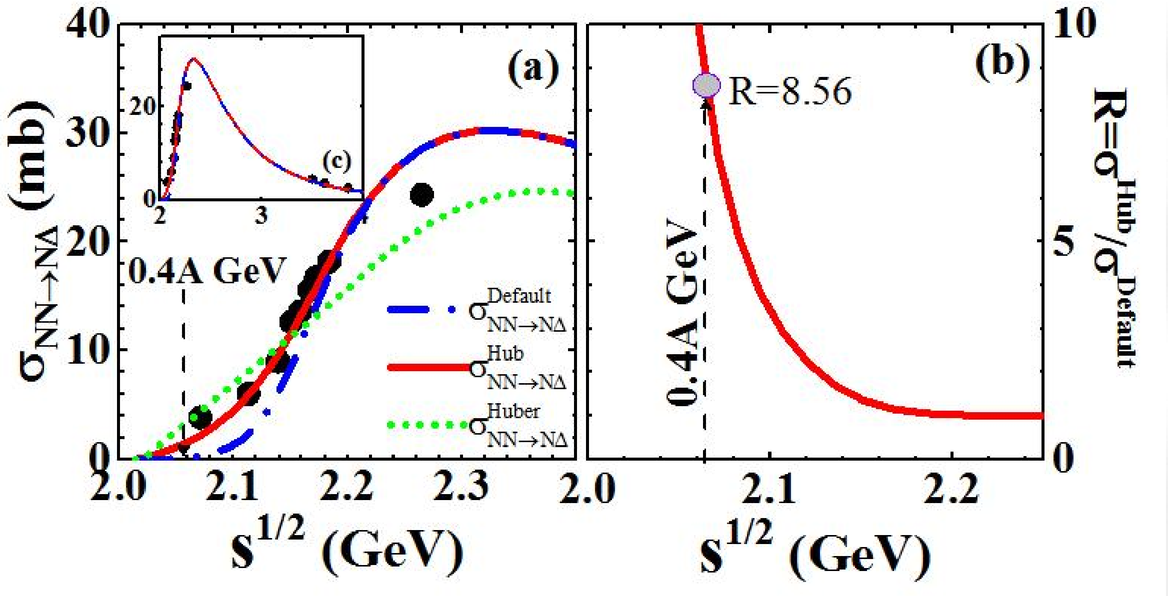




σ_{NN}^{*} *is* 1.1–2.5 times larger than previous σ_{NN}^{*}

Some refinements in UrQMD

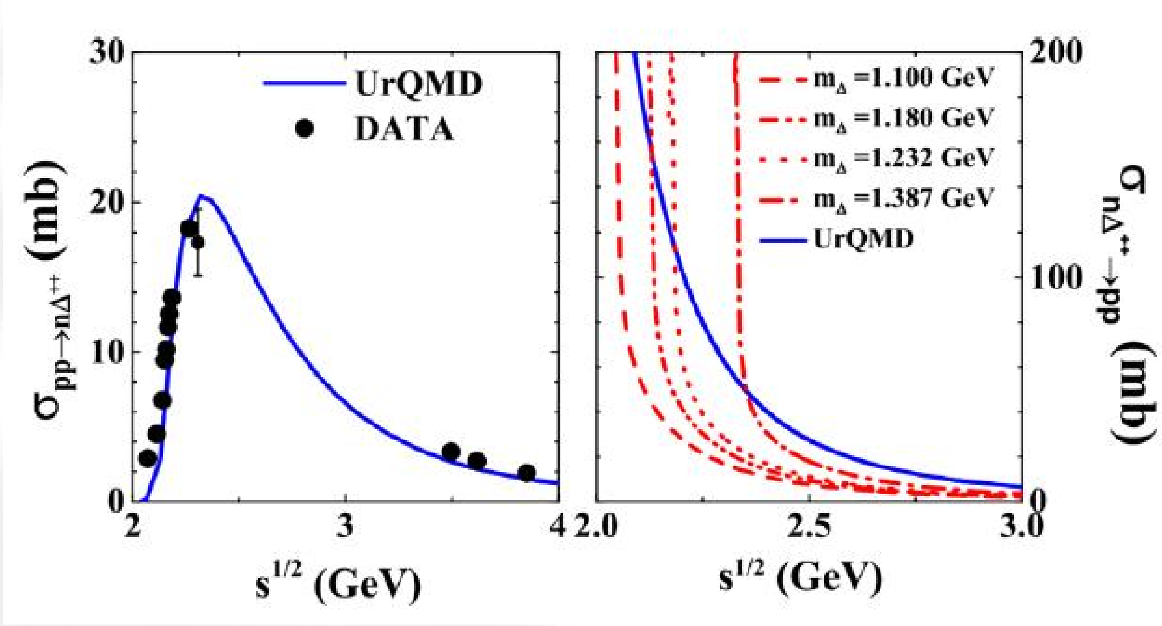
- $NN \rightarrow N\Delta$ cross sections



$$\sigma_{NN\rightarrow N\Delta}(\sqrt{s}) = A_1 + \frac{4A_2 * e^{-(\sqrt{s}-A_3)/A_4}}{(1+e^{-(\sqrt{s}-A_3)/A_4})^2},$$

$$\sqrt{s} < 2.21\text{GeV}$$

- $N\Delta \rightarrow NN$ cross sections



1, Y. Cui, Y. X. Zhang, and Z. X. Li, Chin. Phys. C **44**, 024106 (2020).
2, YY. Liu, et al. Phys.Rev.C 2021

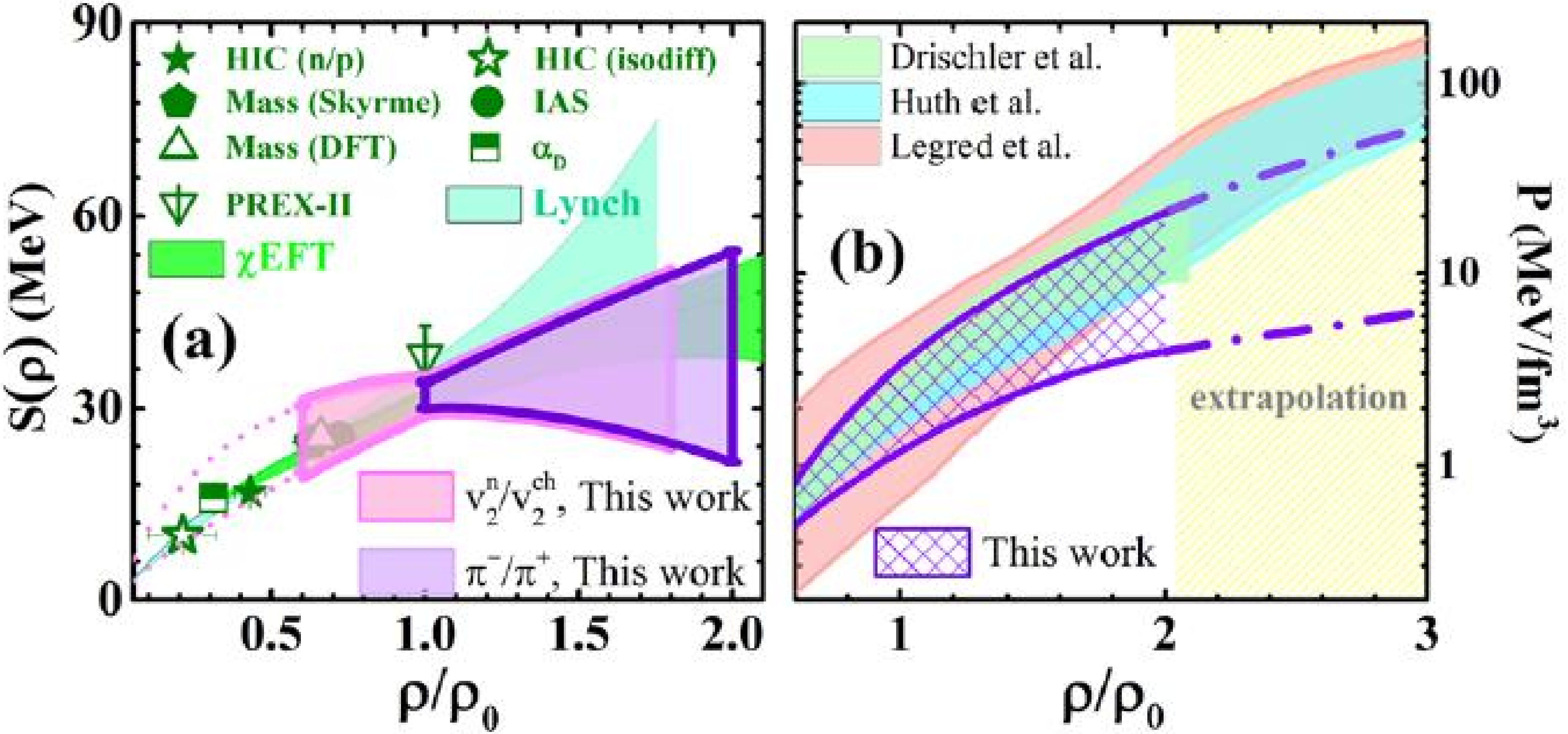
- *MDI* fitting with Hama's optical potential data

Y.Y. Liu, et al., NST2025

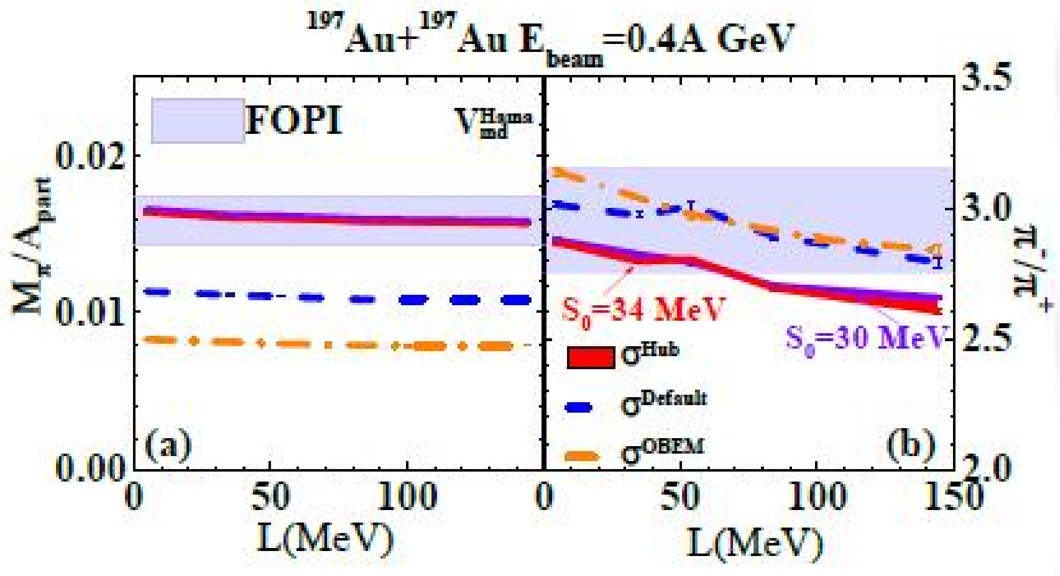
Table 4 Status of transport models for describing 15 experimental observables from the published papers.

observable	experimental data	IBUU	IBL	LQMD	pBUU	RVUU	χBUU	TuQMD	UrQMD-HZU	UrQMD-CIAE
$v_1^n(p_t/A)$	ASY-EOS[11]	-	-	-	-	-	-	+	+	+
$v_2^n(p_t/A)$	ASY-EOS[11]	-	-	-	-	-	-	-	+	+
$v_1^{ch}(p_t/A)$	ASY-EOS[11]	-	-	-	-	-	-	-	+	+
$v_2^{ch}(p_t/A)$	ASY-EOS[11]	-	-	-	-	-	-	-	+	+
v_2^n/v_2^{ch}	ASY-EOS[11]	-	-	-	-	-	-	+	+	+
$v_2^n(p_t/A)$	FOPI[8]	-	-	-	-	-	-	-	+	+
$v_2^H(p_t/A)$	FOPI[8]	-	-	-	-	-	-	-	+	+
$v_2^n(y_0)$	FOPI[8]	+[58]	-	-	-	-	-	-	+	+
$v_2^p(y_0)$	FOPI[8]	+[58]	-	-	-	-	-	-	+	+
v_2^n/v_2^H	FOPI[8]	-	-	-	-	-	-	+	+	+
v_2^n/v_2^p	FOPI[8]	+[58]	-	-	-	-	-	+	+	+
$v_2^n - v_2^H$	FOPI[8]	-	-	-	-	-	-	-	+	+
$v_2^n - v_2^p$	FOPI[8]	-	-	-	-	-	-	-	+	+
$M(\pi)$	FOPI[54]	+ [15,58]	+[17]	+[16]	+[18]	+[19,59]	+[60]	+[20]	-	+[21]
π^-/π^+	FOPI[54]	+ [15,58]	+[17]	+[16]	+[18]	+[19,59]	+[60]	+[20]	-	+[21]

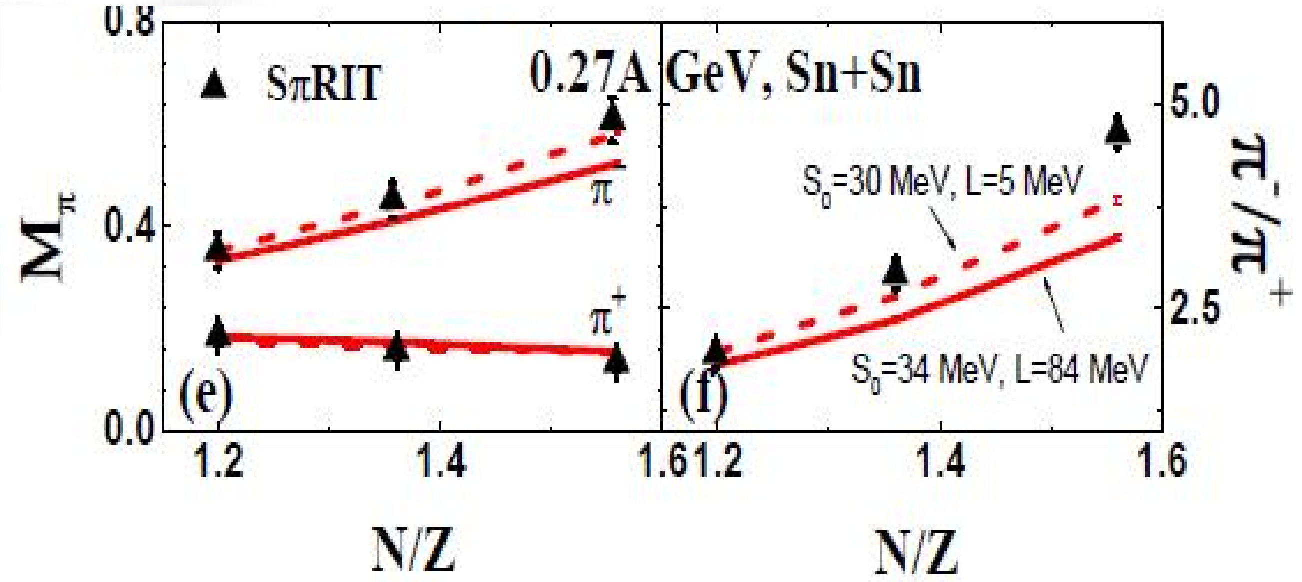
更新后的UrQMD利用了ASY-EOS, FOPI的数据给出的对称能约束



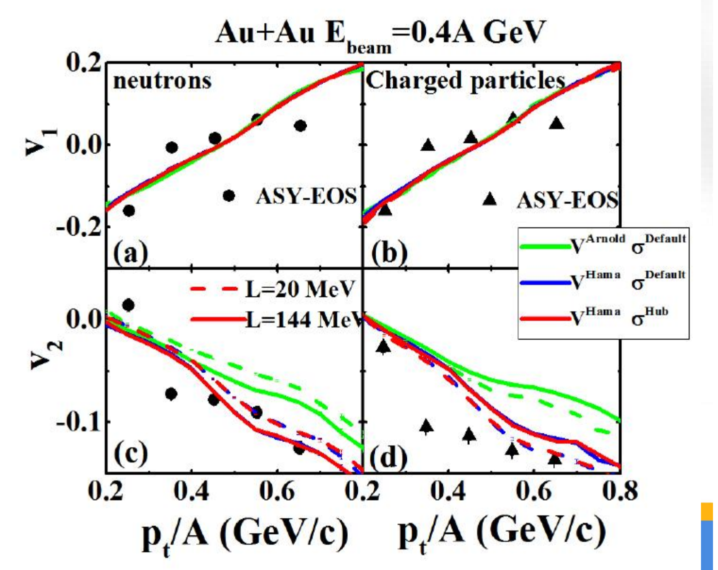
Y.Y. Liu, et al.,



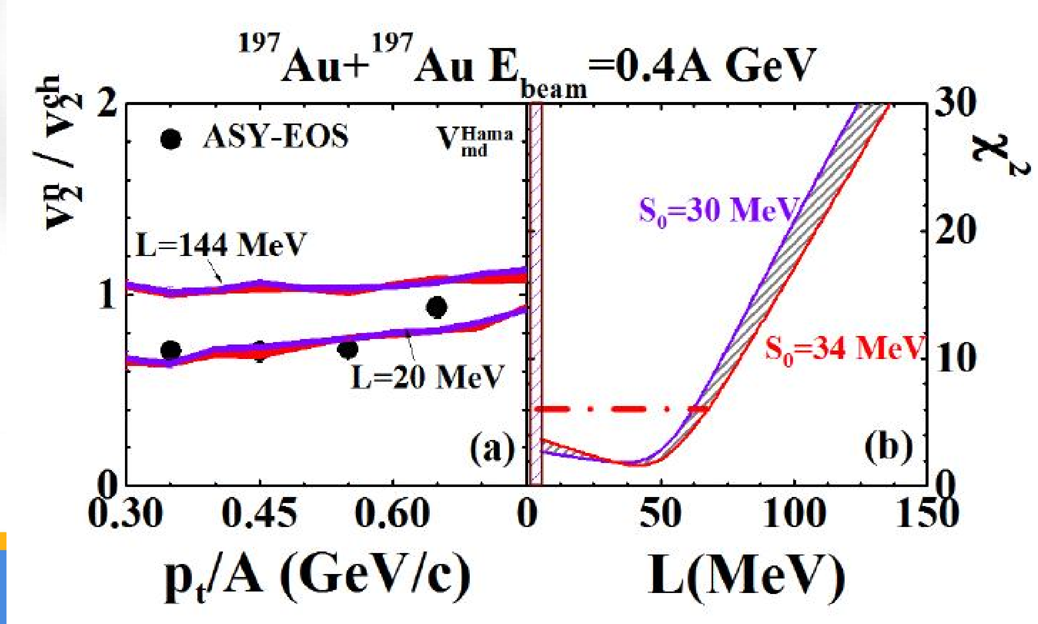
^{197}Au+^{197}Au E_{beam}=0.4A GeV
FOPI
V_{md}^{Hama}
M_π/A_{part}
π^-/π^+
S_0=34 MeV
S_0=30 MeV
σ^{Hub}
$\sigma^{Default}$
σ^{OBEM}
(a)
(b)
L(MeV)
L(MeV)



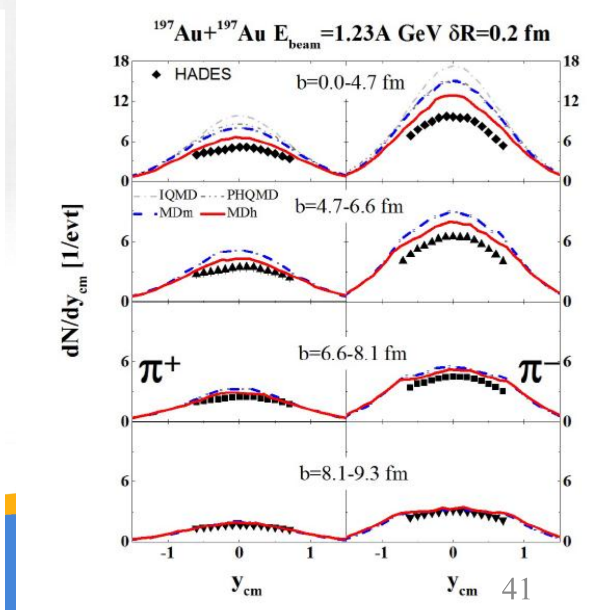
SπRIT
0.27A GeV, Sn+Sn
M_π
π^-/π^+
π^-
π^+
S_0=30 MeV, L=5 MeV
S_0=34 MeV, L=84 MeV
(e)
(f)
N/Z
N/Z



Au+Au E_{beam}=0.4A GeV
neutrons
Charged particles
ASY-EOS
ASY-EOS
v_1
v_2
L=20 MeV
L=144 MeV
V^{Arnold} $\sigma^{Default}$
V^{Hama} $\sigma^{Default}$
V^{Hama} σ^{Hub}
(a)
(b)
(c)
(d)
p_t/A (GeV/c)
p_t/A (GeV/c)



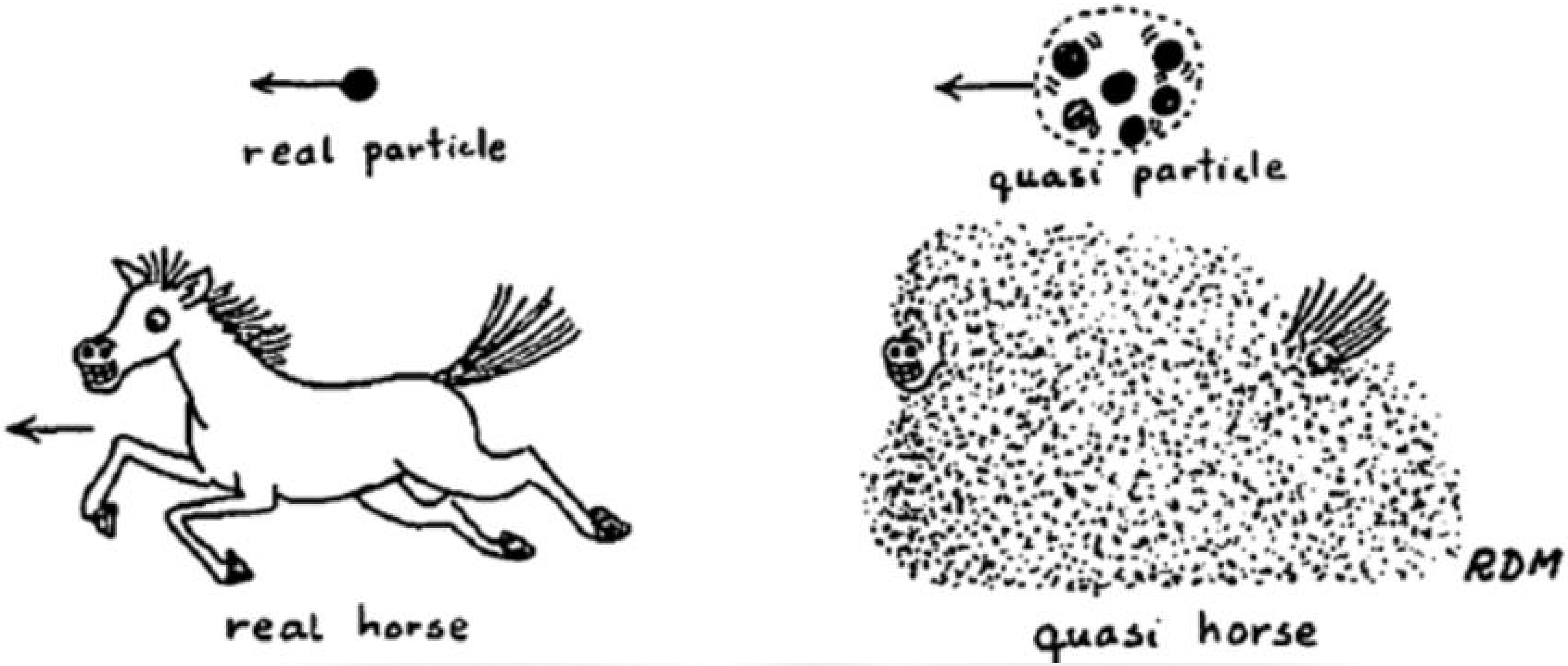
^{197}Au+^{197}Au E_{beam}=0.4A GeV
ASY-EOS
V_{md}^{Hama}
v_2^n/v_2^{ch}
χ^2
L=144 MeV
L=20 MeV
S_0=30 MeV
S_0=34 MeV
(a)
(b)
p_t/A (GeV/c)
L(MeV)



^{197}Au+^{197}Au E_{beam}=1.23A GeV δR=0.2 fm
HADES
b=0.0-4.7 fm
IQMD
PHQMD
MDm
MDh
b=4.7-6.6 fm
b=6.6-8.1 fm
b=8.1-9.3 fm
π^+
π^-
dN/dy_{cm} [1/evt]
y_{cm}
y_{cm}

3，Effective mass splitting and SRC

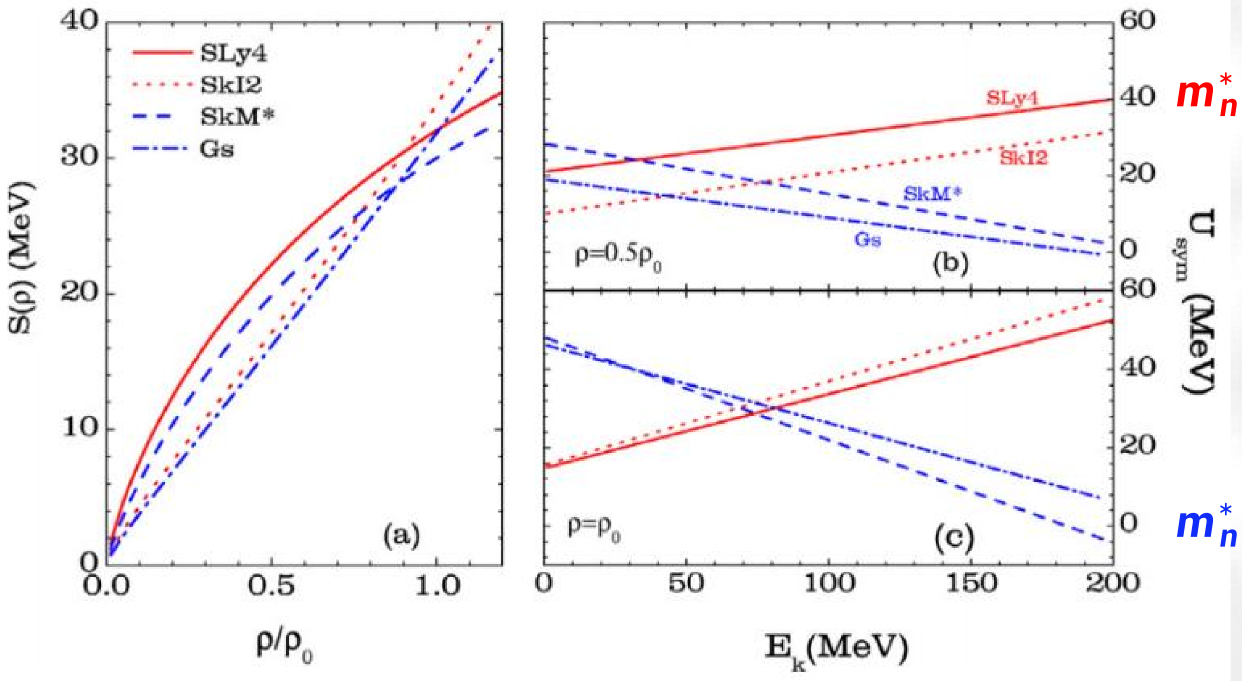
$$\frac{p^2}{2m} + U(r,p) = \frac{p^2}{2m^*} + U_0(r)$$



K-effective mass

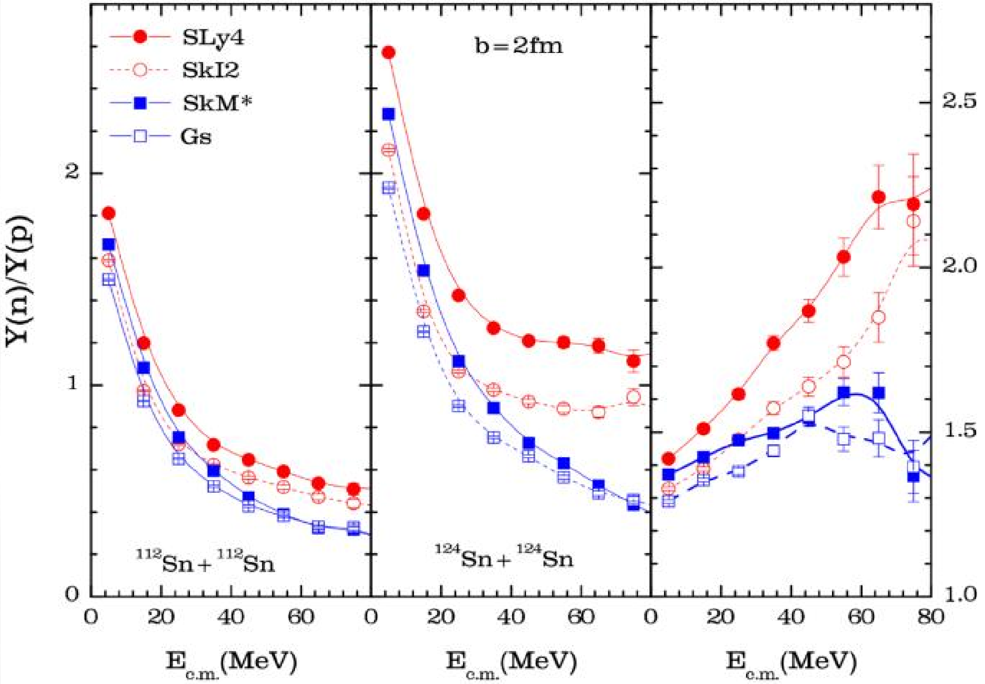
$$\frac{m}{m_q^*} = 1 + \frac{m}{p}\frac{\partial V_q}{\partial p}, \quad q = n, p.$$

Zhang Y X et al., Phys. Lett. B ***732*** *(2014) 186*



$m_n^* < m_p^*$

$m_n^* > m_p^*$



$$U_{sym}(\rho, p) = \frac{V_n - V_p}{2\delta}$$

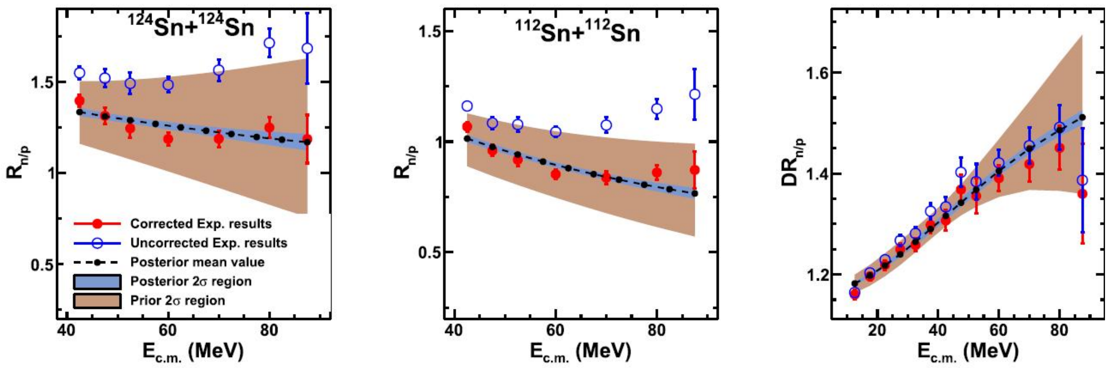
- Same symmetry energy, but different symmetry potential! Thus, different n/p ratios

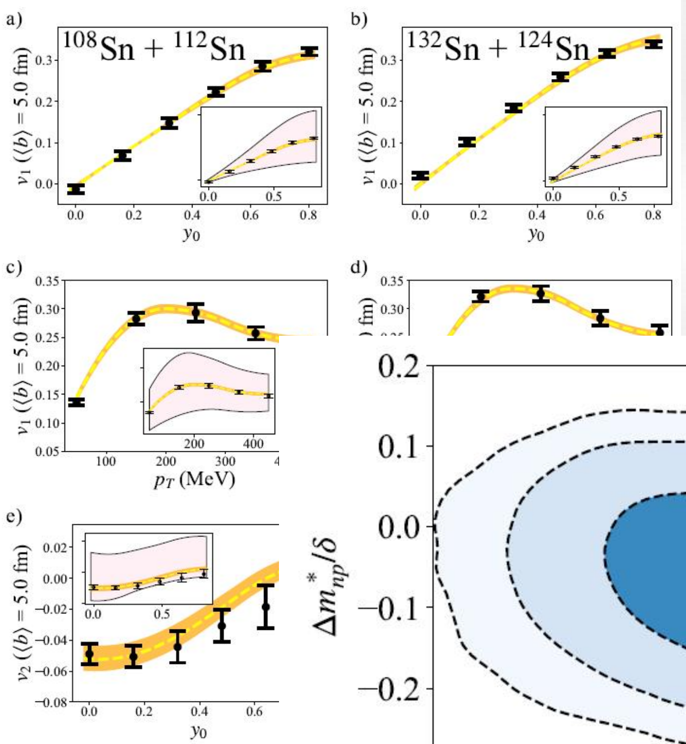
- **Effective mass splitting**

$$\Delta m^*_{np} \approx -\left(\frac{m^*}{m}\right)^2 4m\delta \frac{\partial V_{sym}}{\partial p^2}$$

P. Moreforce, C.Tsang, Y. Zhang, et al., PLB799(2019)135045

C.Y. Tsang, et al., PLB853, 138661(2024).





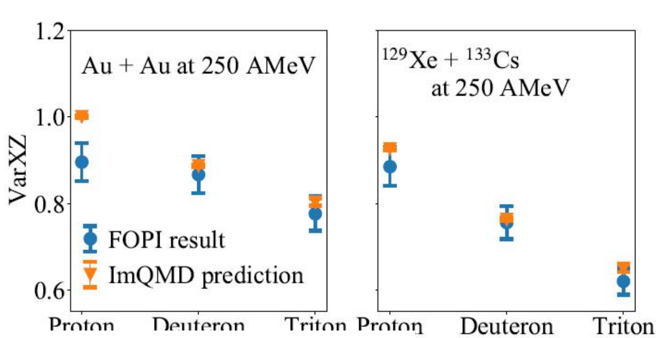
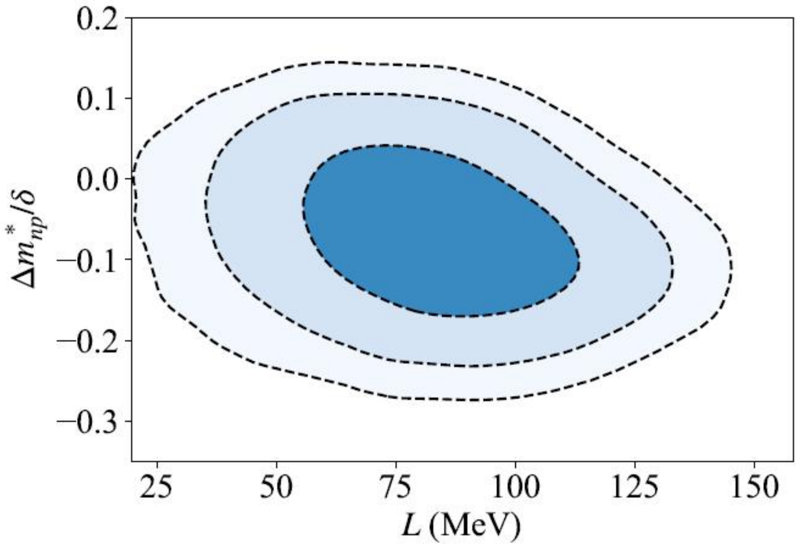


Table 2
Mean μ and sigma σ value of the posterior distribution for S_0, L and f_I.

	Only $DR_{n/p}$	Only $SR_{n/p}$	$DR_{n/p}$ and $SR_{n/p}$
S_0	$\mu = 29.9$ $\sigma = 2.5$	$\mu = 28.7$ $\sigma = 2.8$	$\mu = 28.8$ $\sigma = 1.9$
L	– –	$\mu = 45.1$ $\sigma = 12.9$	$\mu = 49.6$ $\sigma = 13.7$
f_I	$\mu = 0.037$ $\sigma = 0.164$	$\mu = 0.025$ $\sigma = 0.076$	$\mu = 0.063$ $\sigma = 0.057$



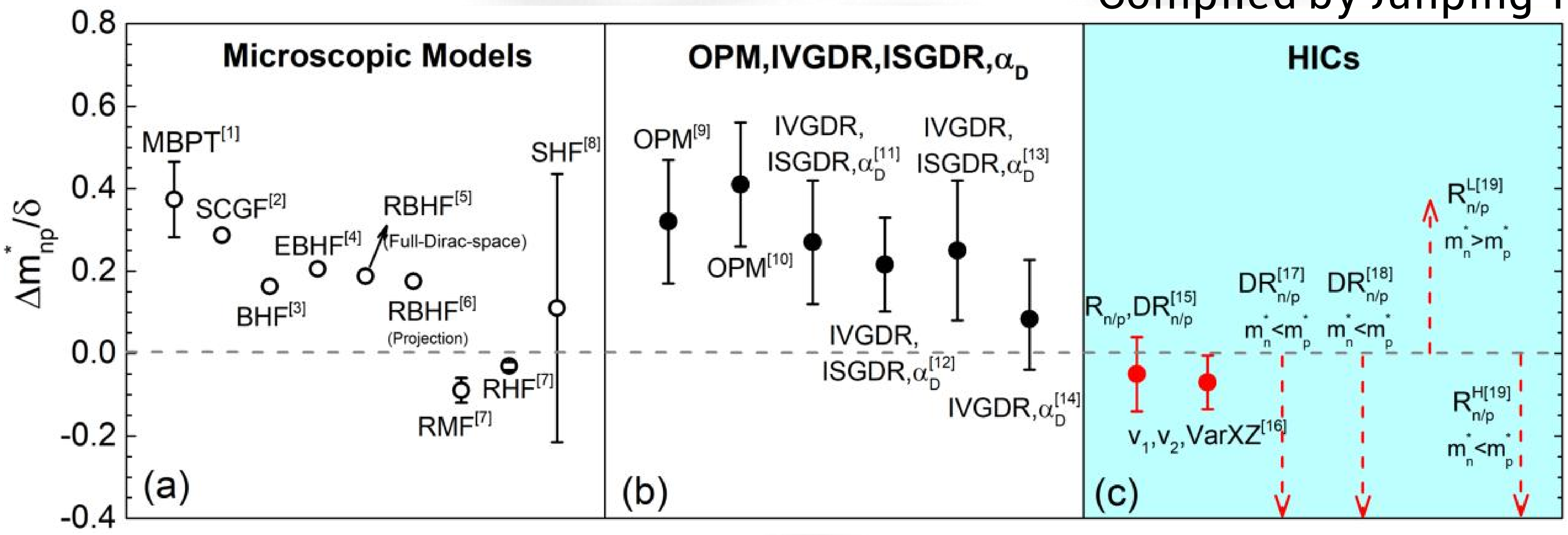
For Standard Skyrme interaction,

$$f_I = \frac{m}{m_s^*} - \frac{m}{m_v^*} = \frac{1}{2\delta}\left(\frac{m}{m_n^*} - \frac{m}{m_v^*}\right) = 2D_0 m\rho$$

Tension on the constraints of neutron-proton effective mass splitting

1) Microscopic model: MBPT, SCGF, BHF, EBHF, HVH, RBHF, ($m_n^* > m_p^*$)
2) Optical potential ($m_n^* > m_p^*$)
3) RBHF ($m_n^* > m_p^*$ and $m_n^* < m_p^*$, depends on the energy)
4) HICs ($m_n^* < m_p^*$, **with larger uncertianty**) ***at*** ???

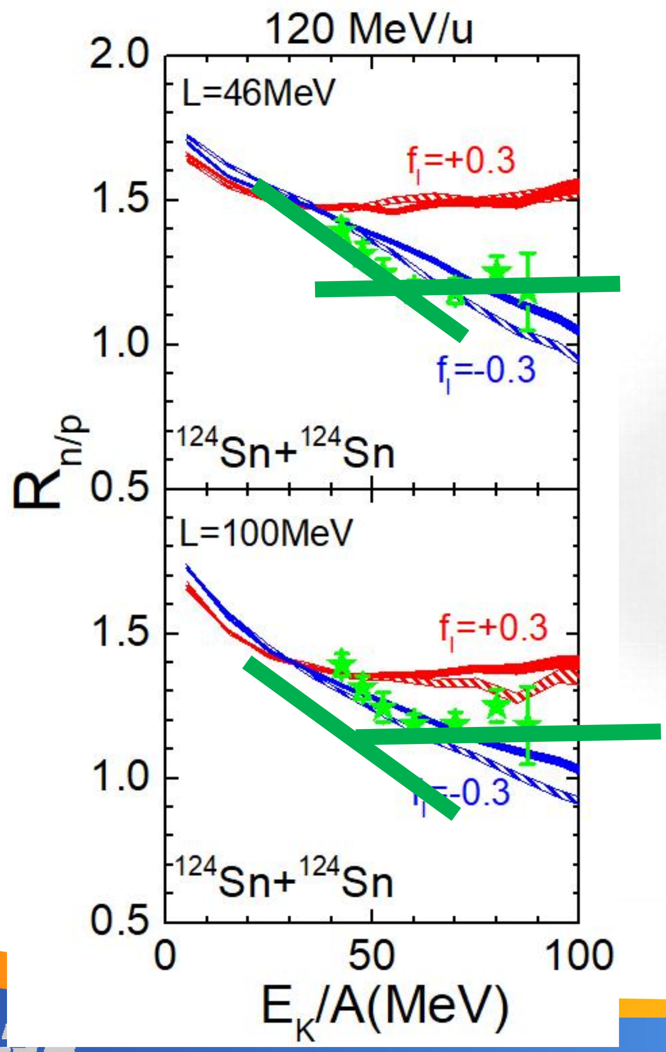
Compiled by Junping Yang



- We need further understanding the effective mass splitting constraints in HICs, based on a refined transport models

A fresh look at the energy spectral of R(n/p)

Junping Yang, Xiang Chen, Ying Cui, Yangyang Liu, Zhuxia Li, Yingxun Zhang, PRC109, 054624(2024).



$$R_{n/p} = \frac{\mathrm{Y(n)}}{\mathrm{Y(p)}} \propto \exp\left(\frac{\mu_n - \mu_p}{T}\right) = \exp\left(\frac{2V_{asy}\delta}{T}\right) \quad (1)$$

$$\Delta m^*_{np} \approx -(\frac{m^*}{m})^2 4m\delta \frac{\partial V_{asy}}{\partial p^2} \quad (2)$$

$$R_{n/p} \propto \exp\left[\frac{2(V^0_{asy} + \frac{\partial V_{asy}}{\partial p^2}p^2 + \cdots)\delta}{T}\right]$$

$$\approx \exp\left[\frac{2V^0_{asy}\delta}{T}\right]\exp\left[-\frac{(\frac{m}{m^*})^2\Delta m^*_{np}}{T}E'_k\right]. \quad (3)$$

$$\boxed{S_{n/p} = \frac{\partial lnR_{n/p}}{\partial E_k/A} = -\frac{\lambda}{T}(\frac{m}{m^*})^2\Delta m^*_{np}}$$

$$E_k^{'} = \lambda E_k/A$$

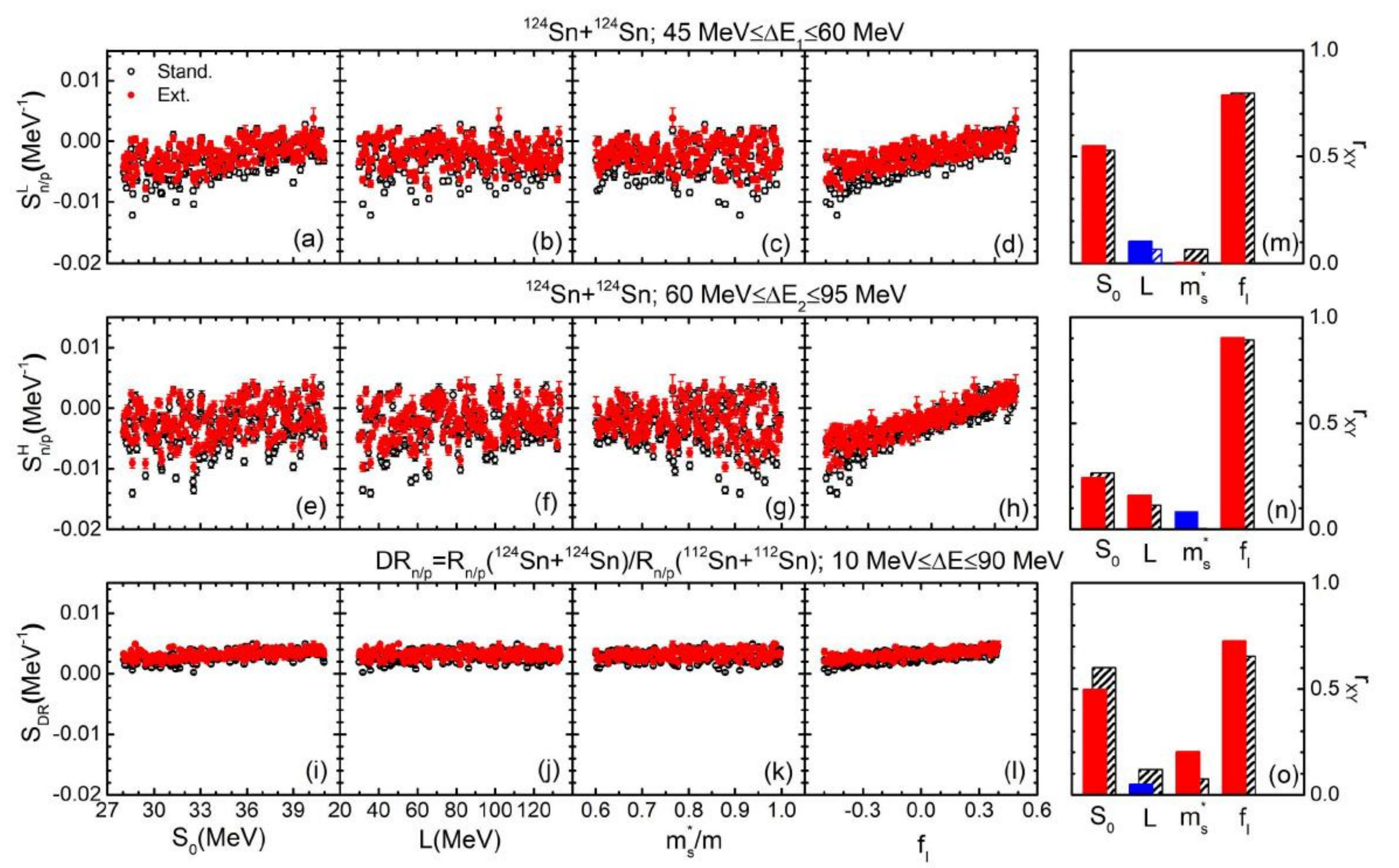
✓ **$S_{n/p}$ is directly related to the Δm^*_{np}！！！**

DNP

中国原子能科学研究院 CNNC

Correlation coefficients between Sn/p and NMPs, in ImQMD model

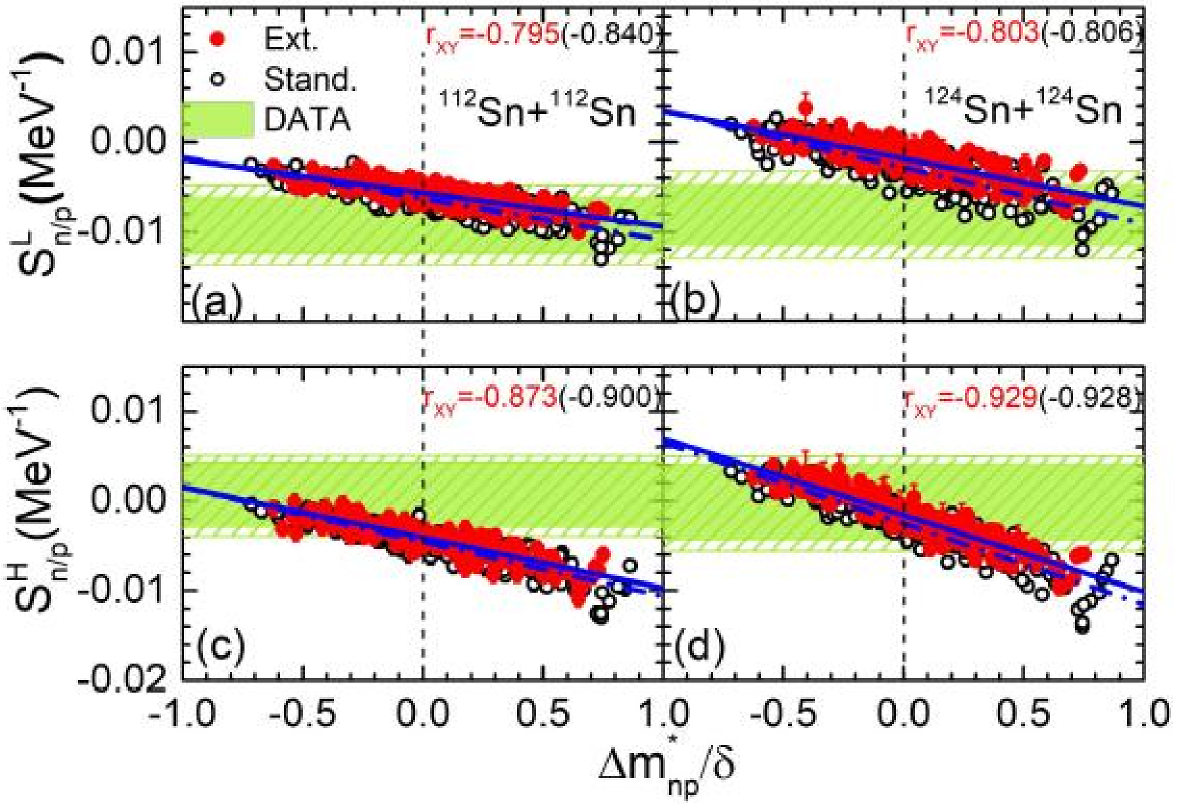
JP Yang, MQSun, Y Cui, ZX Li, K Zhao, Yingxun Zhang, arXiv:2506.17973.

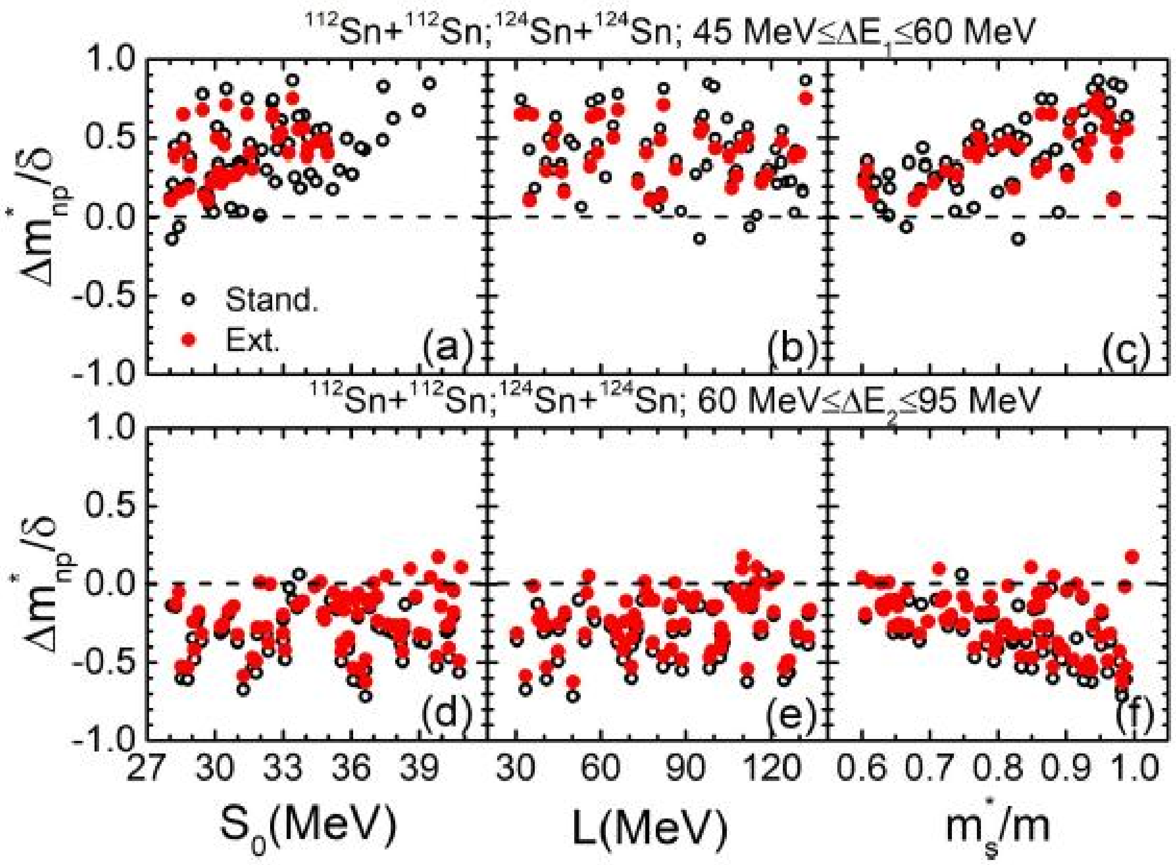


Understanding Sn/p from transport model

Sn/p is strongly correlated wit h effective mass splitting

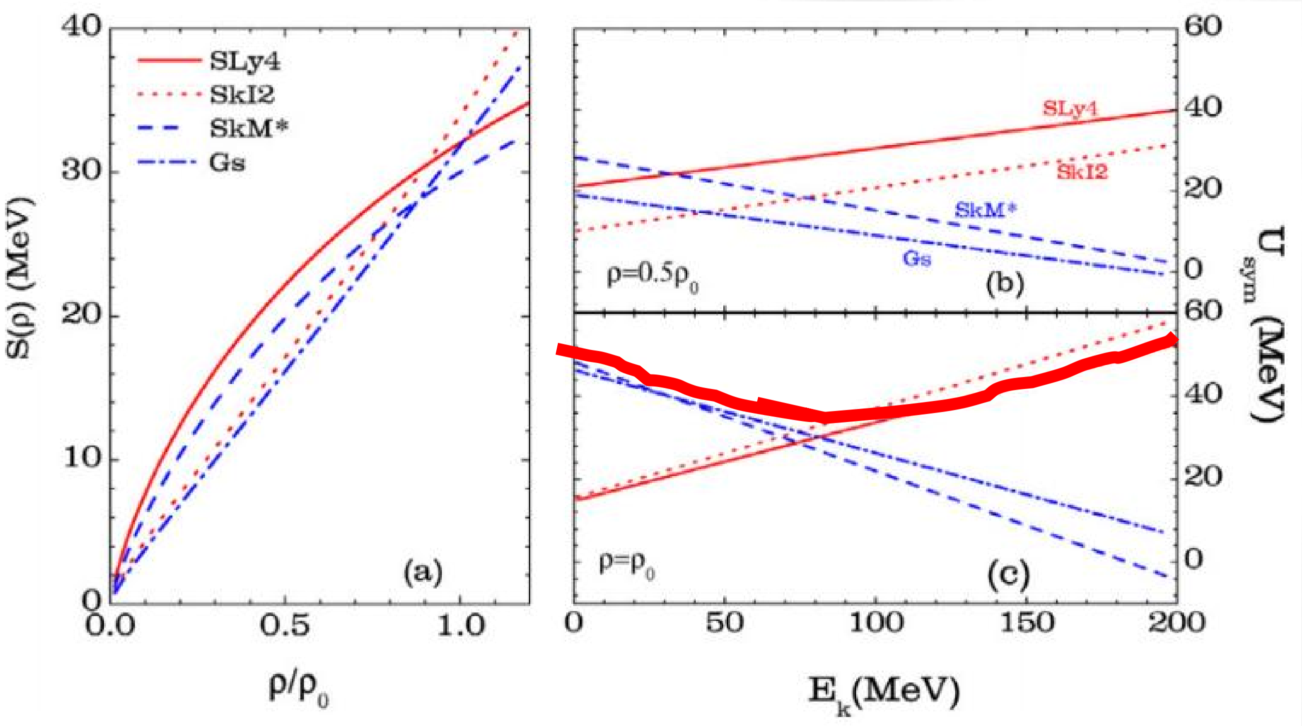
Constraints on effective mass splitting

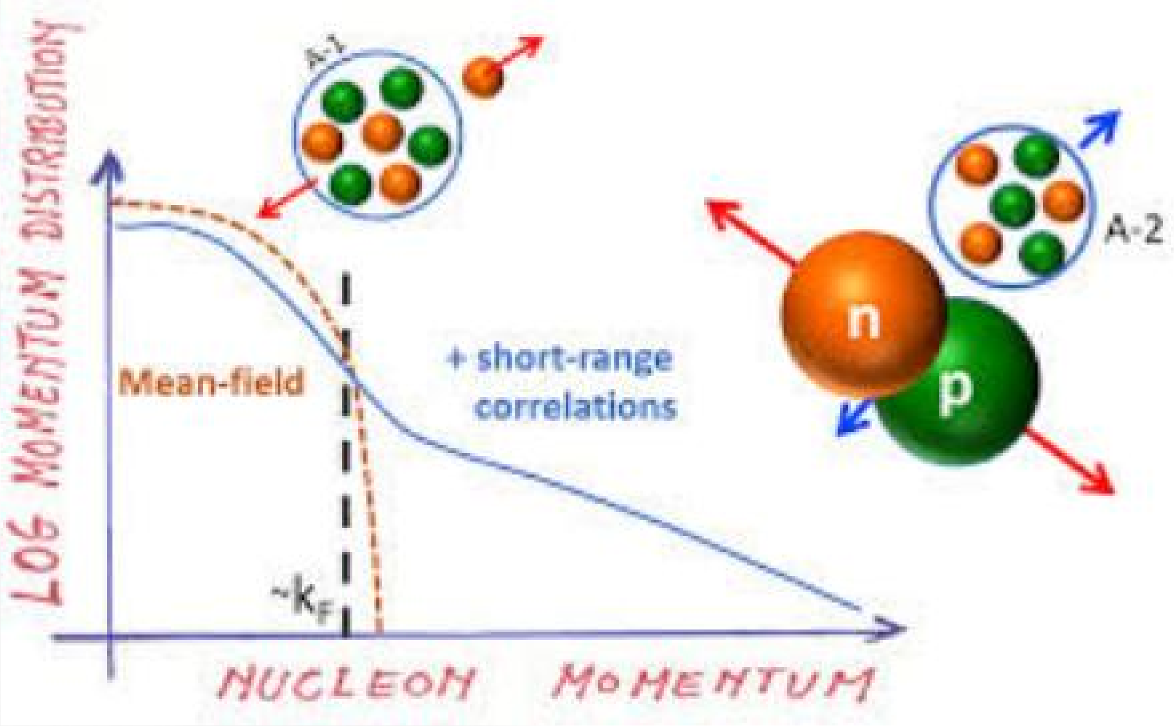




Junping Yang, MQSun, Y Cui, ZX Li, K Zhao, Yingxun Zhang,
Phys.Rev.C 112, 044604(2025)

Possible reasons ?





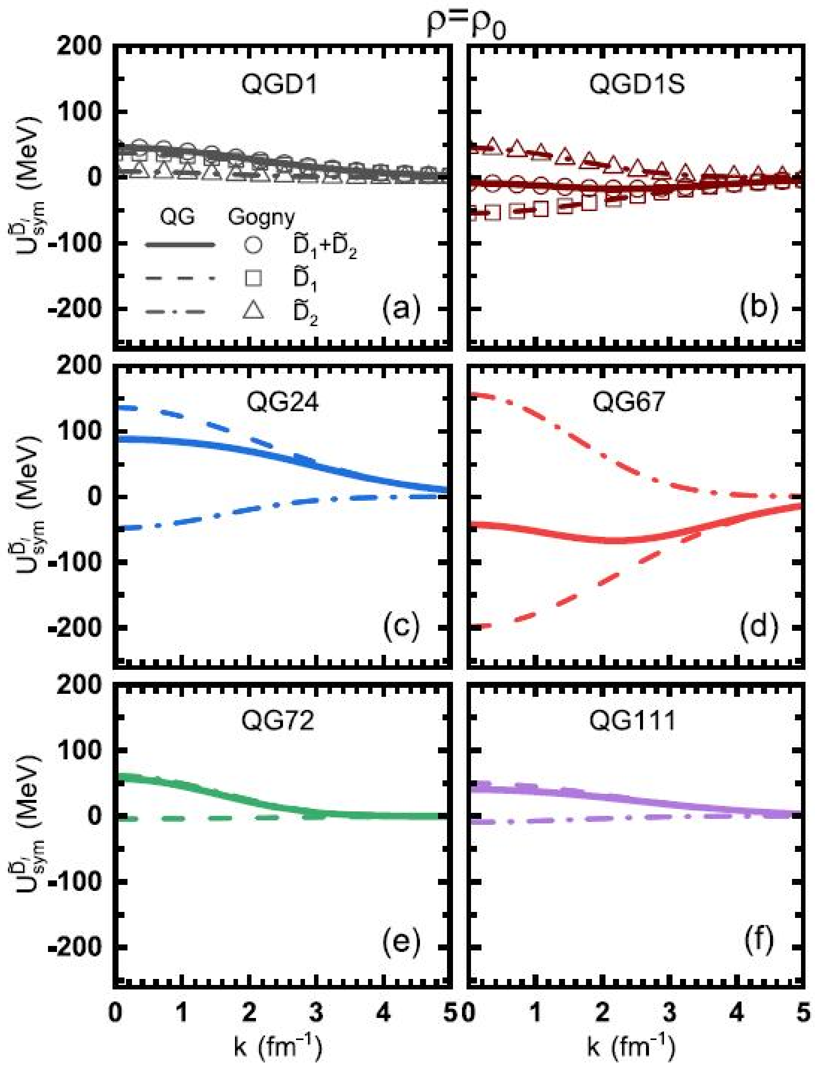
Non-monotonic momentum dependent symmetry potential

Meiqi Sun, Dandan Niu, Junping Yang, Ying Cui, Zhuxia Li, Qiang Zhao , Kai Zhao and Yingxun Zhang, Phys.Rev.C **111**, 064607 (2025)

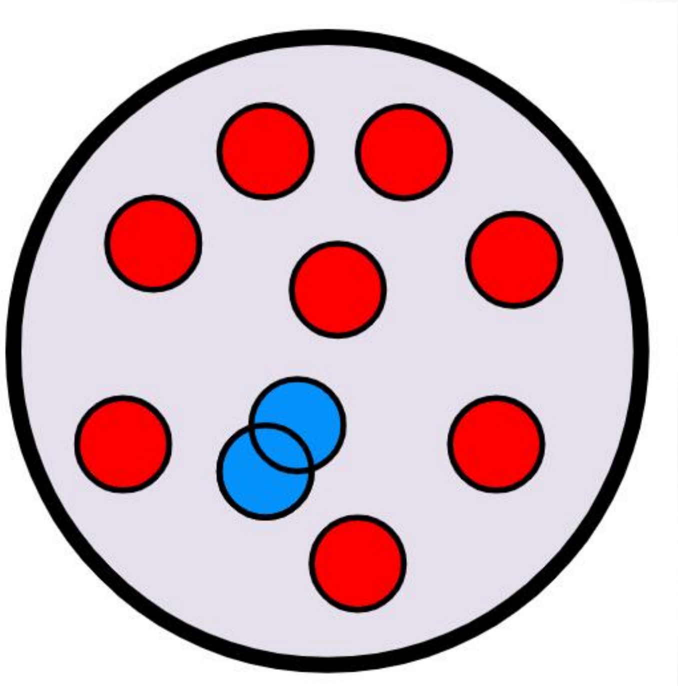
$$\hat{v}_{ij} = \sum_{l=1}^{2}\Big[(\tilde{A}_l + \tilde{B}_l\hat{P}_\tau)e^{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\mu_l^2}}$$

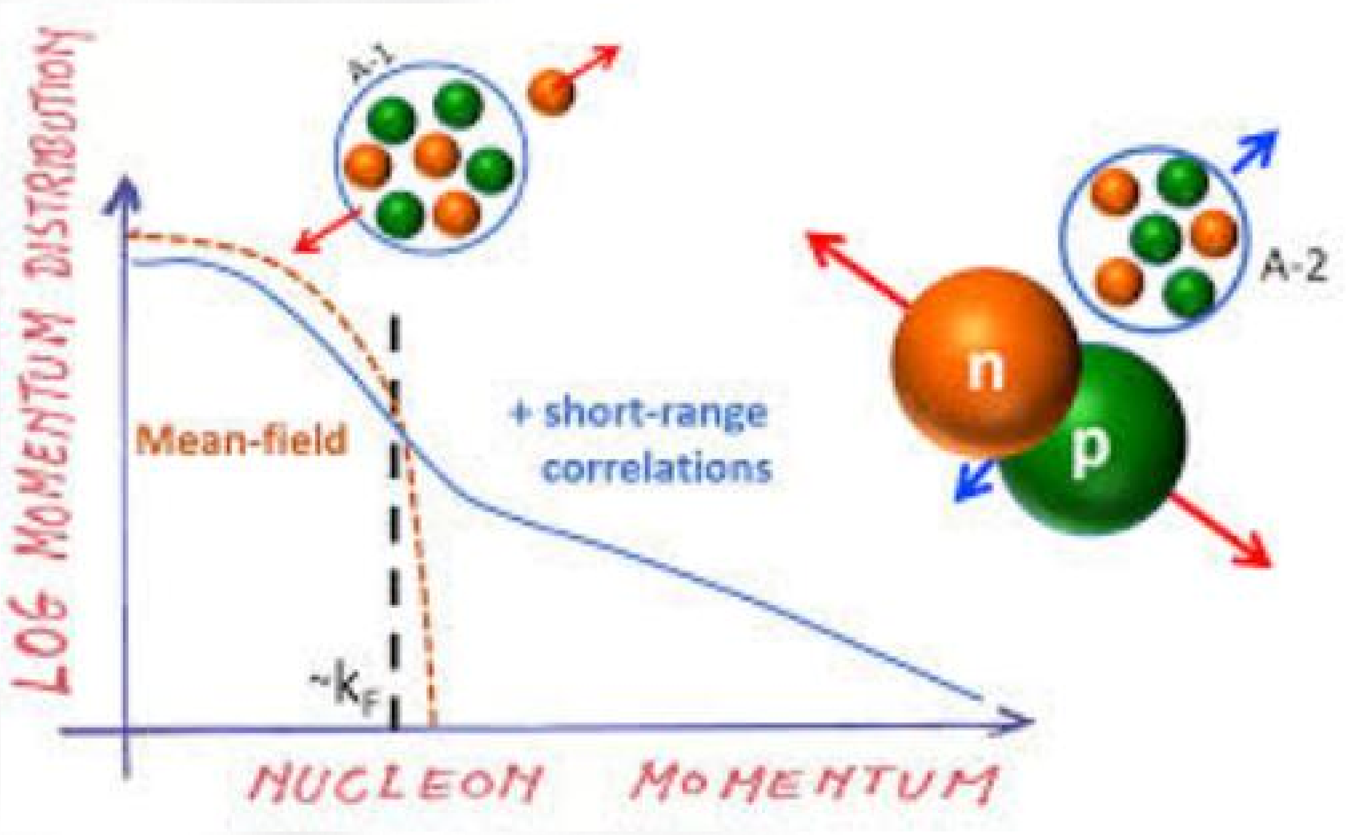
$$+ (\tilde{C}_l + \tilde{D}_l\hat{P}_\tau)e^{-\frac{\mu_l^2(\mathbf{k}_i-\mathbf{k}_j)^2}{4}} e^{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\mu_l^2}}\Big]$$

$$+ (\beta_0 + \beta_1\hat{P}_\tau)\rho^\sigma\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)\delta(\mathbf{r}_1-\mathbf{r}_2).$$



Short-Range correlation





J. M. Cavedon et al. Phys. Rev. Lett. (1982), L. Lapikas, Nucl. Phys. A. 553, 297 (1993); J. Kelly, Adv. Nucl. Phys. 23, 75 (1996); R. Subedi et al. *Science*, 320 , 1476 (2008)

Description SRC in initial state

$$\psi(\mathbf{r}_1, \ldots, \mathbf{r}_A) = F(\mathbf{r}_1, \ldots, \mathbf{r}_A)\phi_0(\mathbf{r}_1, \ldots, \mathbf{r}_A)$$

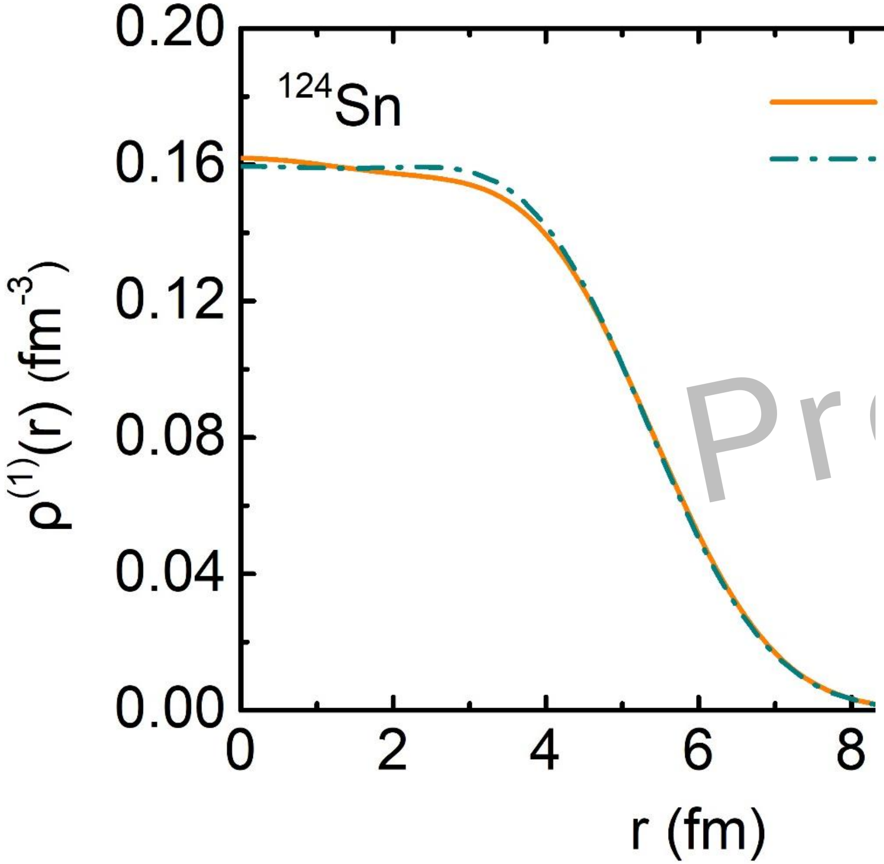
Exact WF

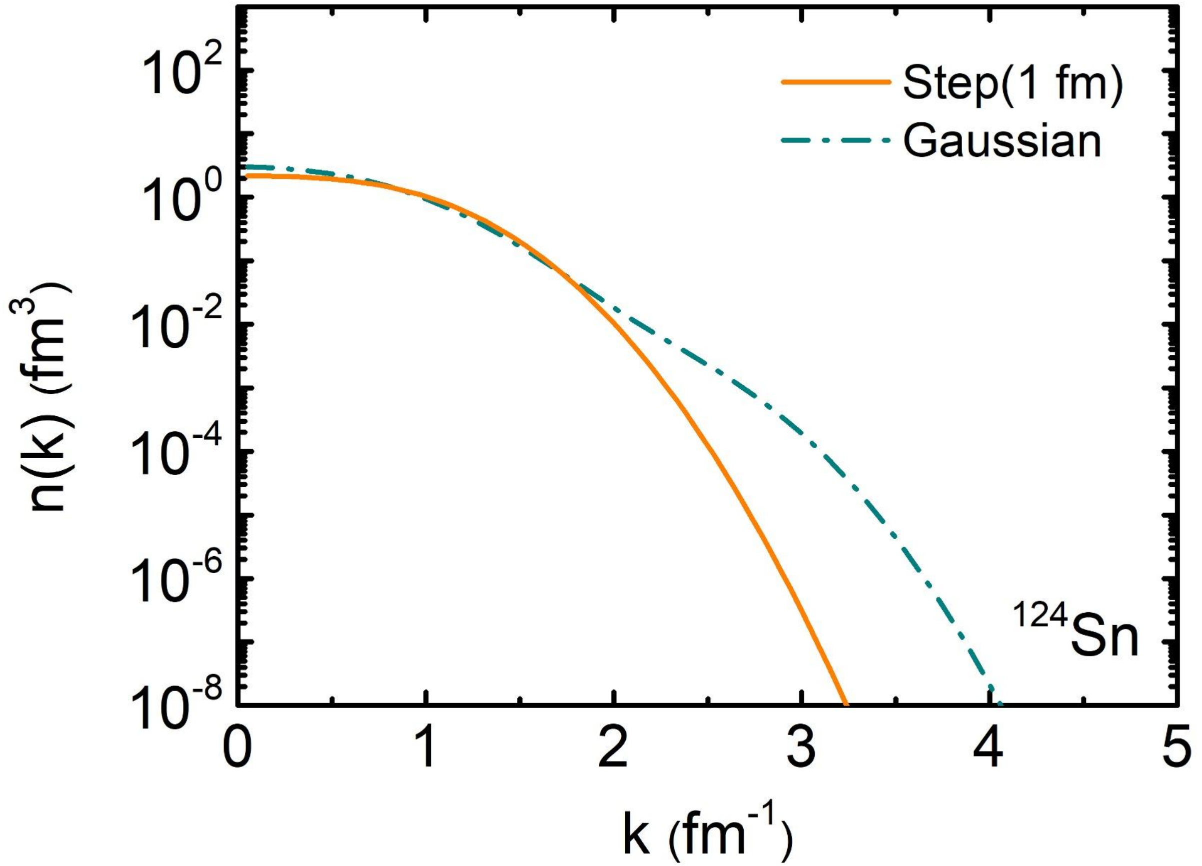
Mean field WF

$$\rho^{(1)}(\mathbf{r}_1) = A \int |\psi(\mathbf{r}_1, \ldots, \mathbf{r}_A)|^2 d^3\mathbf{r}_2 \cdots d^3\mathbf{r}_A$$

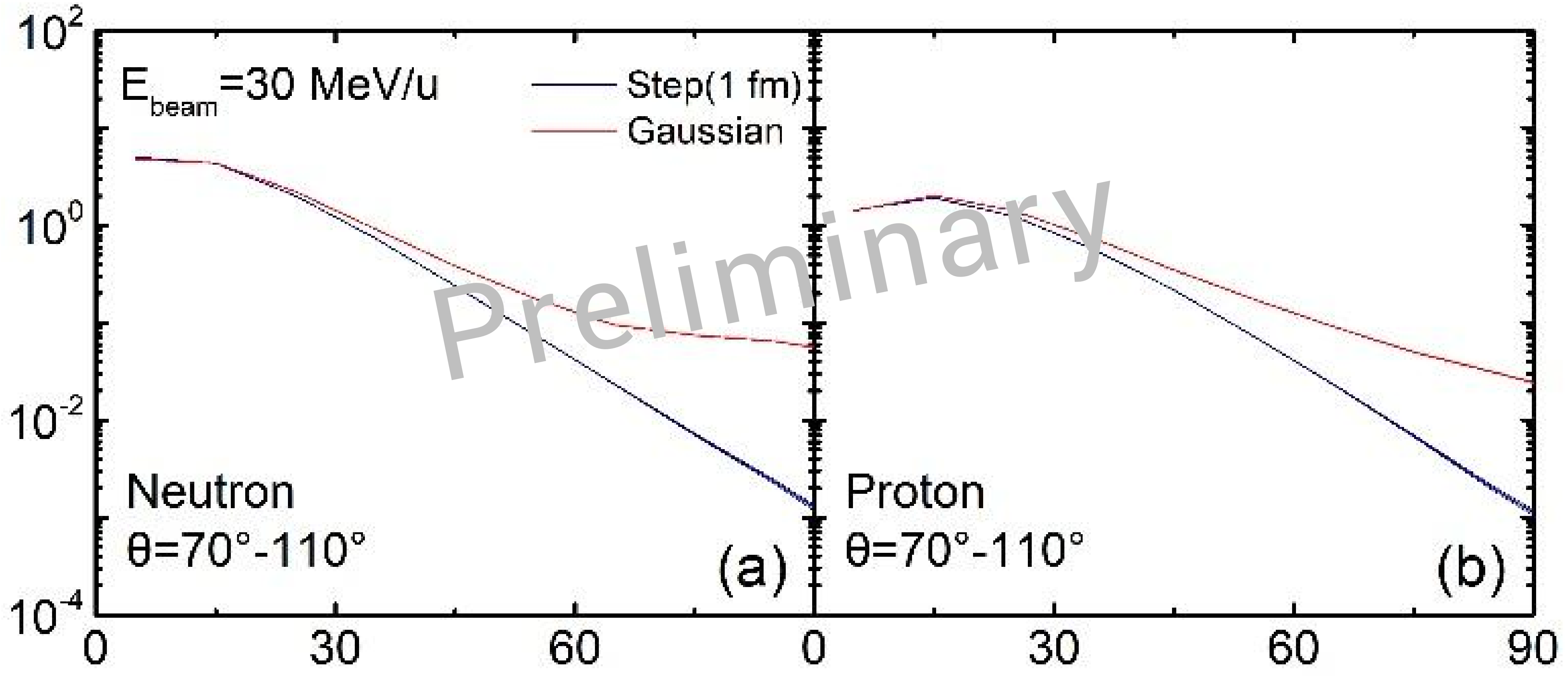
$$\rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2) \approx \frac{A-1}{2A} \rho^{(1)}(\mathbf{r}_1)\rho^{(1)}(\mathbf{r}_2)[1 - C(\mathbf{r}_1, \mathbf{r}_2)]$$

Luqi Li, Yingxun Zhang, in preparation





Luqi Li, Yingxun Zhang, in preparation



4， Outlook

1、Enhance the reliability of transport models

2、Including correlation in transport equation and models

$f(1,2,t) = g(1,2)\, f(1)\, f(2)$ $\quad$ $g(1,2)$ is two-body correlative function

$$\frac{\partial f_1(\mathbf{x}_1)}{\partial t} + \frac{\mathbf{p}_1}{m}\cdot\frac{\partial f_1(\mathbf{x}_1)}{\partial \mathbf{q}_1} - \nabla U_1\cdot\frac{\partial f_1(\mathbf{x}_1)}{\partial \mathbf{p}_1} = \frac{\pi}{\hbar(2\pi)^9}\int d\mathbf{p}_2 d\mathbf{p}_1' d\mathbf{p}_2' \,|\langle \mathbf{p}_1\mathbf{p}_2 | V_{12} | \mathbf{p}_1'\mathbf{p}_2'\rangle|^2$$
$$\times\{f_1(\mathbf{x}_1')f_1(\mathbf{x}_2')[1-f_1(\mathbf{x}_1)][1-f_1(\mathbf{x}_2)]$$
$$-f_1(\mathbf{x}_1)f_1(\mathbf{x}_2)[1-f_1(\mathbf{x}_1')][1-f_1(\mathbf{x}_2')]\}\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1'-\mathbf{p}_2')\,.$$

$$\frac{\partial f_1(\mathbf{x}_1)}{\partial t} + \frac{\mathbf{p}_1}{m}\cdot\frac{\partial f_1(\mathbf{x}_1)}{\partial \mathbf{q}_1} + \mathcal{F}\cdot\frac{\partial f_1(\mathbf{x}_1)}{\partial \mathbf{p}_1} = \frac{\pi}{(2\pi)^3\hbar}\int d\mathbf{k}\left(e^{(\hbar\mathbf{k}/2)\cdot\partial/\partial\mathbf{p}_1} - e^{-(\hbar\mathbf{k}/2)\cdot\partial/\partial\mathbf{p}_1}\right)$$
$$\times\int d\mathbf{p}_2\,\delta\left[\mathbf{k}\cdot\left[\frac{\mathbf{p}_1}{m}-\frac{\mathbf{p}_2}{m}\right]\right]\frac{\tilde{V}_{12}^2(\mathbf{k})}{|1+(1/\hbar)\tilde{V}_{23}\Psi|^2}$$
$$\times[f_1^+(\mathbf{x}_1)f_1^-(\mathbf{x}_2) - f_1^-(\mathbf{x}_1)f_1^+(\mathbf{x}_2)]\,.$$

Bo-jun Yang, et al., PRC36, 1987

Collaborators:

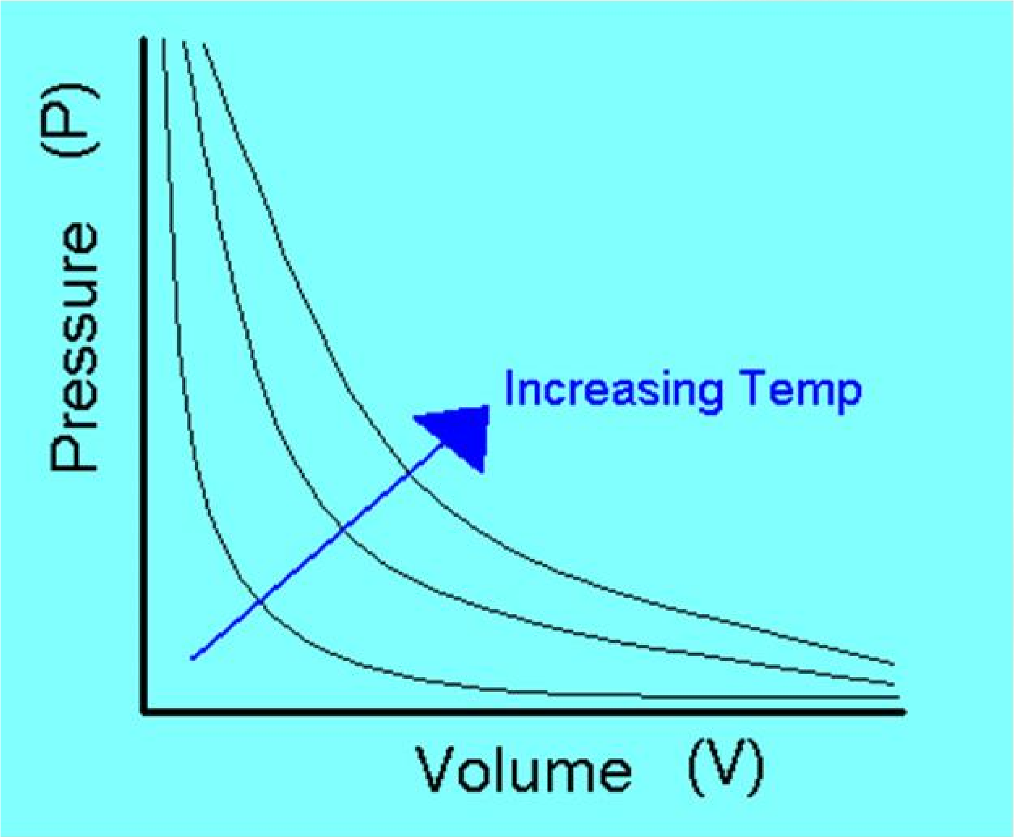
- Meiqi Sun, Junping Yang, Xiang Chen, Ying Cui, Yangyang Liu, Li Li, Kai Zhao, Zhuxia Li, (CIAE)
- Yongjia Wang, Qingfeng Li (HZU)
- TMEP collaborations
- Jun Xu, Zhaoqing Feng

Thanks for your attention!

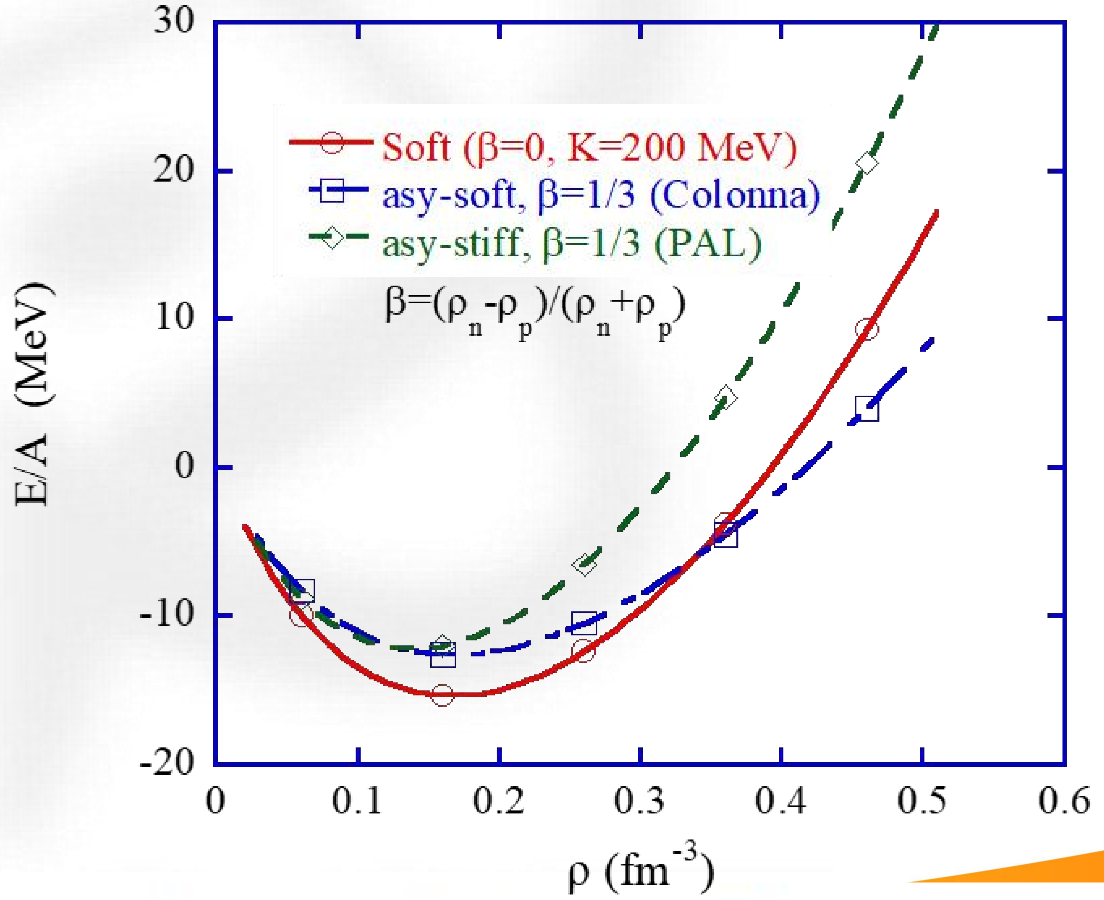
三，模型的一些应用-状态方程和有效质量劈裂的再研究

•输运模型研究的物理内容之一：理解致密核物质状态方程

气体状态方程



核物质状态方程



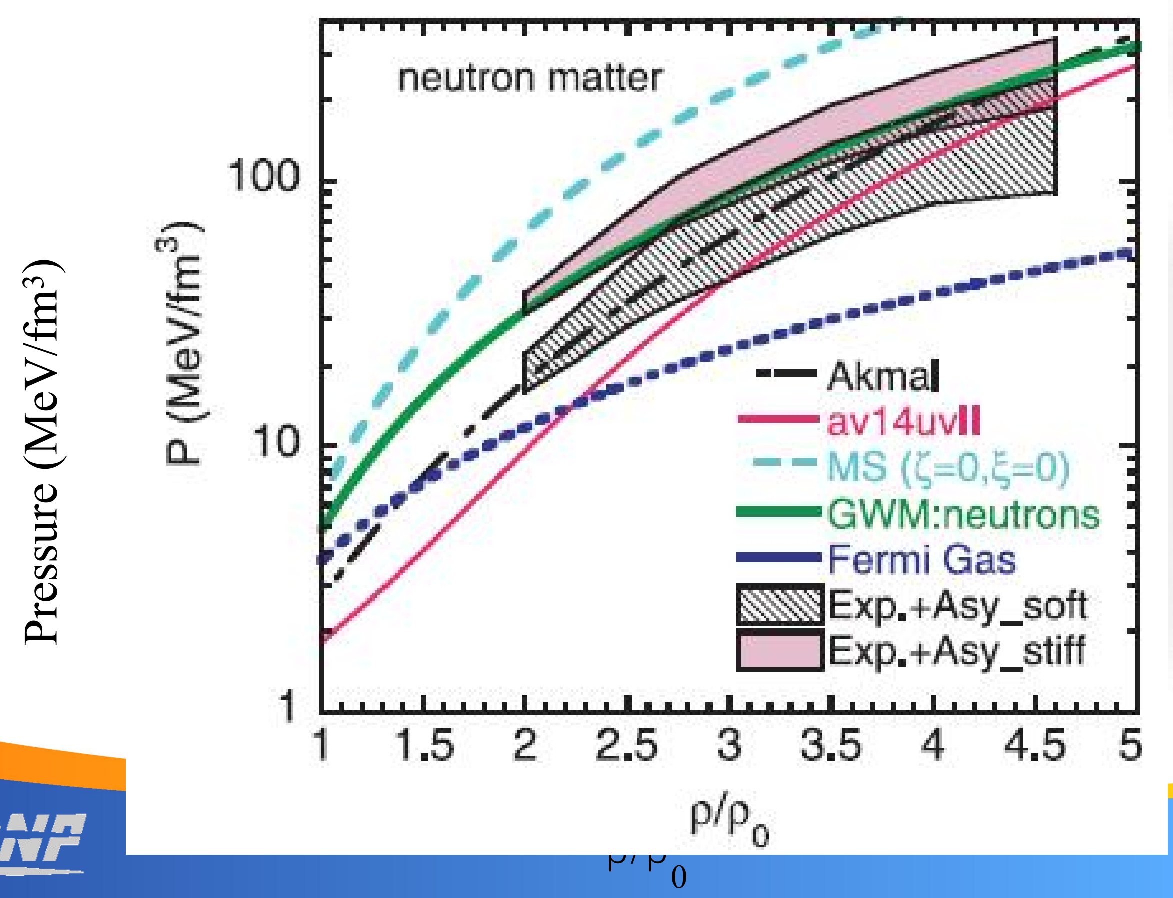
$$E(\rho,\delta)=E(\rho,\delta=0)+E_{sym}$$
$$=E(\rho,\delta=0)+S(\rho)\delta^2+\cdots.$$

同位旋非对称核物质状态方程

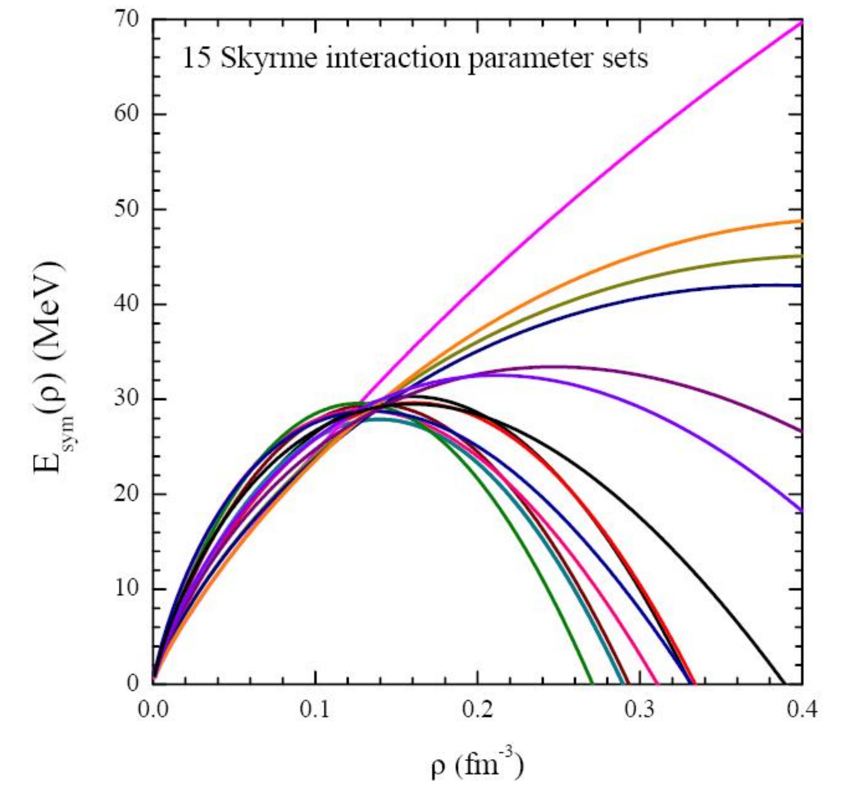
$$E/A(\rho,\delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho)\ ;\quad \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$$

$S(\rho)$: density dependence of symmetry energy

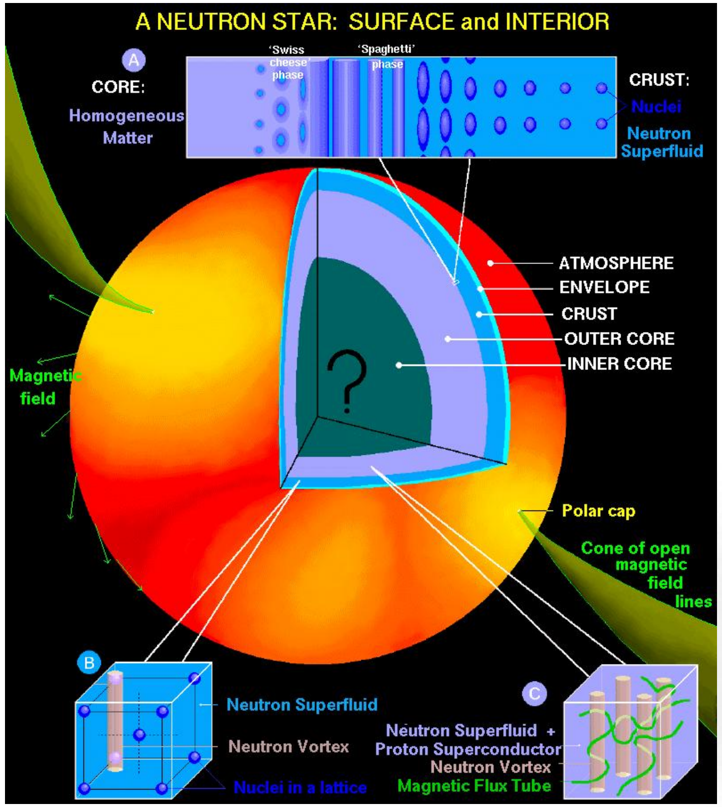
Danielewicz, Lacey, Lynch, Science 298,1592 (2002)



Symmetry energy: Esym=$S(\rho)\delta^2$



- Neutron star

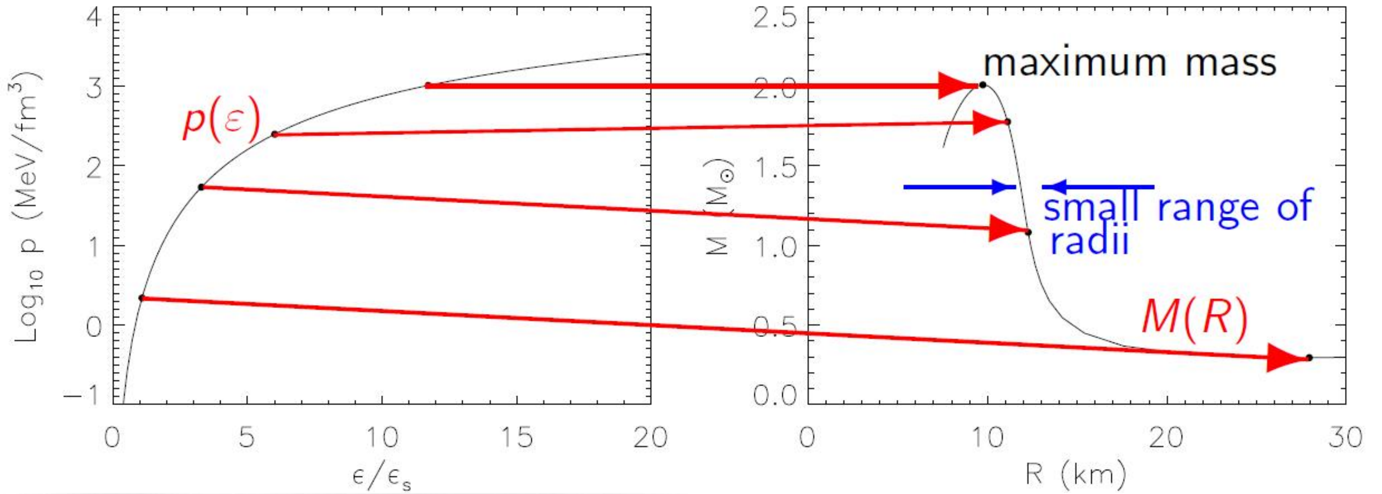


- *Mass=?*
- *Radius= ?*
- ......

Tolman-Oppenheimer-Volkov equations

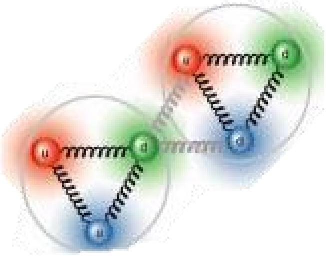
$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2+4\pi pr^3)(\varepsilon+p)}{r(r-2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$

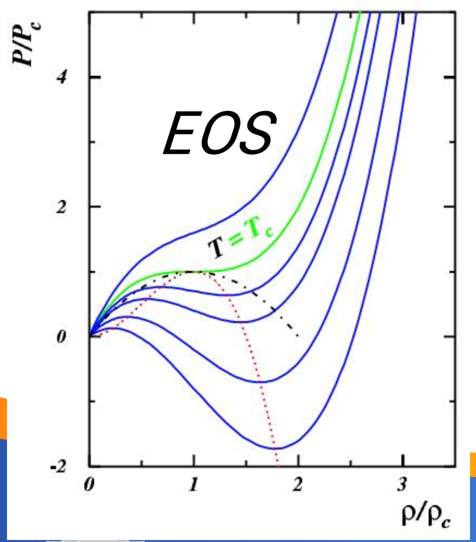


- Mass-Radius relationship, tilde deformability, ...
- EOS at T=0 MeV!

EOS is not a direct input for the transport models



Effective interaction v_{ij}



QMD type model:
Hamiltonian

$$H = \langle \Phi | \sum_i T_i + \sum_{i<j} v_{ij} | \Phi \rangle$$

$$V_i = \sum_j \langle ij | v_{ij} | ij - ji \rangle - V_R$$

Single particle potential:
BUU type

Vlasov

$$\dot{\boldsymbol{r}}_i = \frac{\partial H}{\partial \boldsymbol{p}_i}$$

$$\dot{\boldsymbol{p}}_i = -\frac{\partial H}{\partial \boldsymbol{r}_i}$$

N. propagation

$$\dot{\boldsymbol{r}}_i = \frac{\partial V}{\partial \boldsymbol{p}_i}$$

$$\dot{\boldsymbol{p}}_i = -\frac{\partial V}{\partial \boldsymbol{r}_i}$$

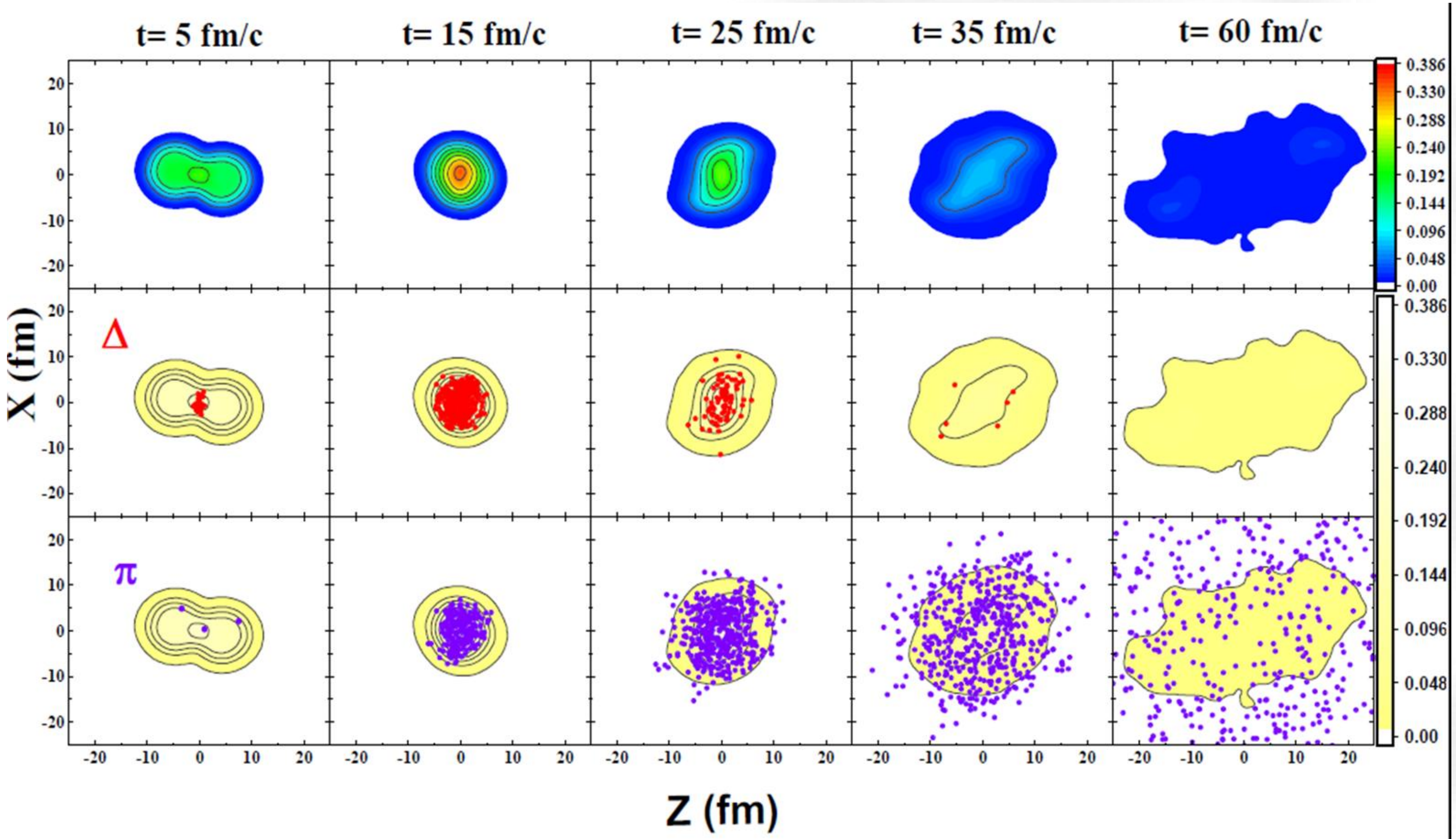
TP propagation

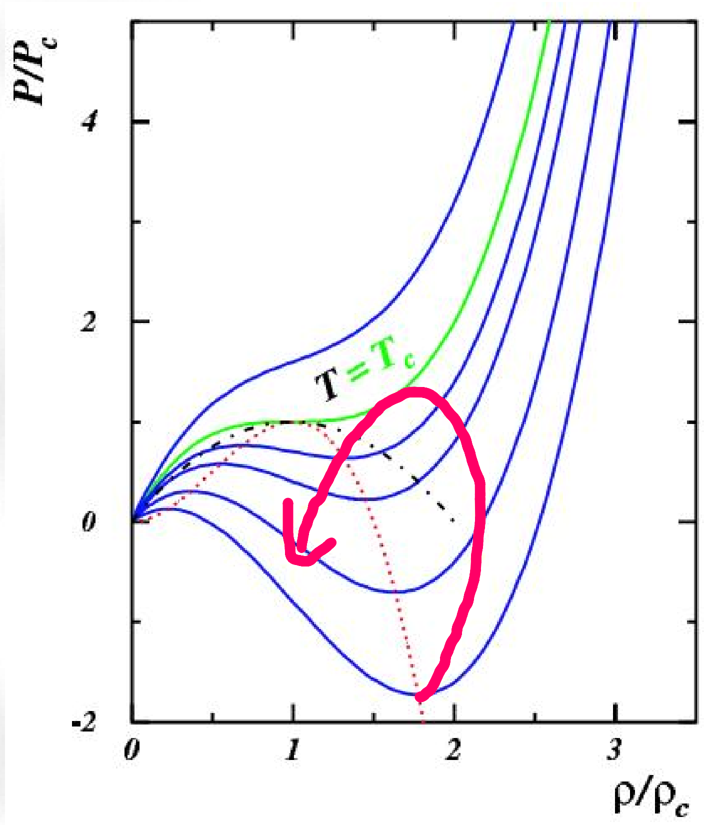
Collisions

- $\sigma_{NN}, \frac{d\sigma}{d\Omega}$
- Pauli blocking

- Heavy ion collisions

YYLiu, YJWang, YCui, ZXLi, QFLi, YXZhang., PRC103, 014616(2021)





- Pi-/pi+, n/p ratios, isospin diffusion, v_2n/v2_p, …. sensitive to the stiffness of symmetry energy
- EOS at $T \neq 0$ MeV

PHYSICAL REVIEW C **85**, 024305 (2012)

Single-nucleon potential decomposition of the nuclear symmetry energy

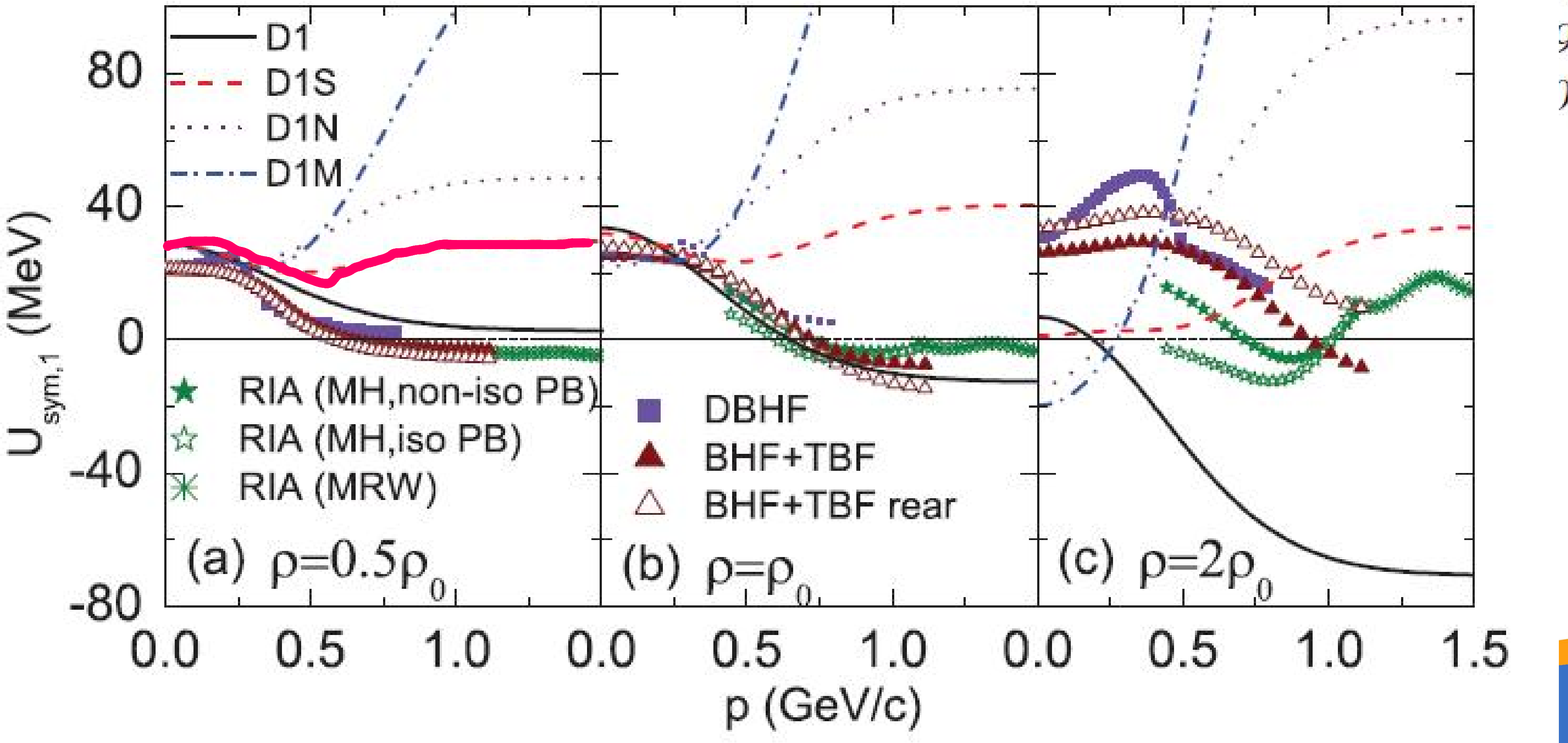
Rong Chen,[1] Bao-Jun Cai,[1] Lie-Wen Chen,[1,2,*] Bao-An Li,[3] Xiao-Hua Li,[1,4] and Chang Xu[5]

[1]*Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China*

[2]*Center of* … *0000, China*

[3]*Departme* … *9-3011, USA*

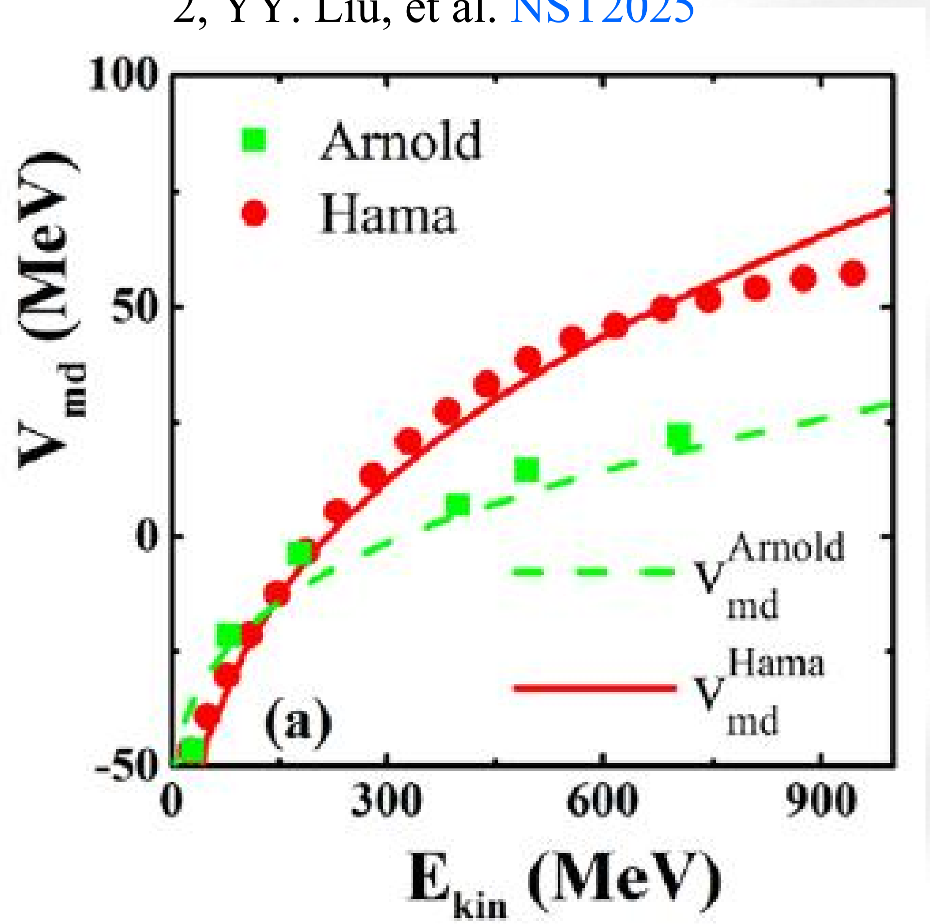
[4]*School* … *01, China*

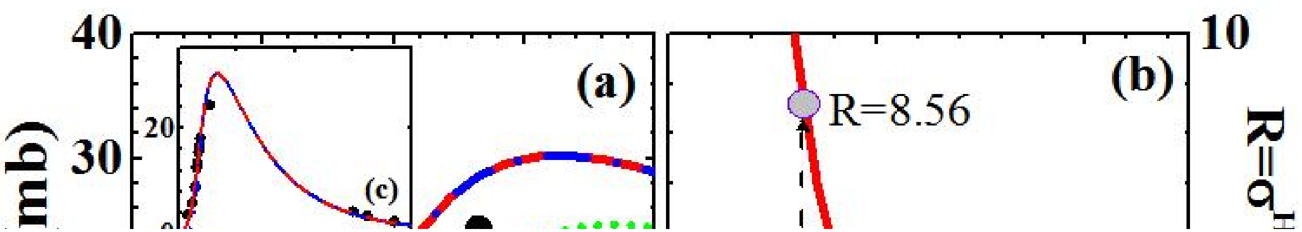


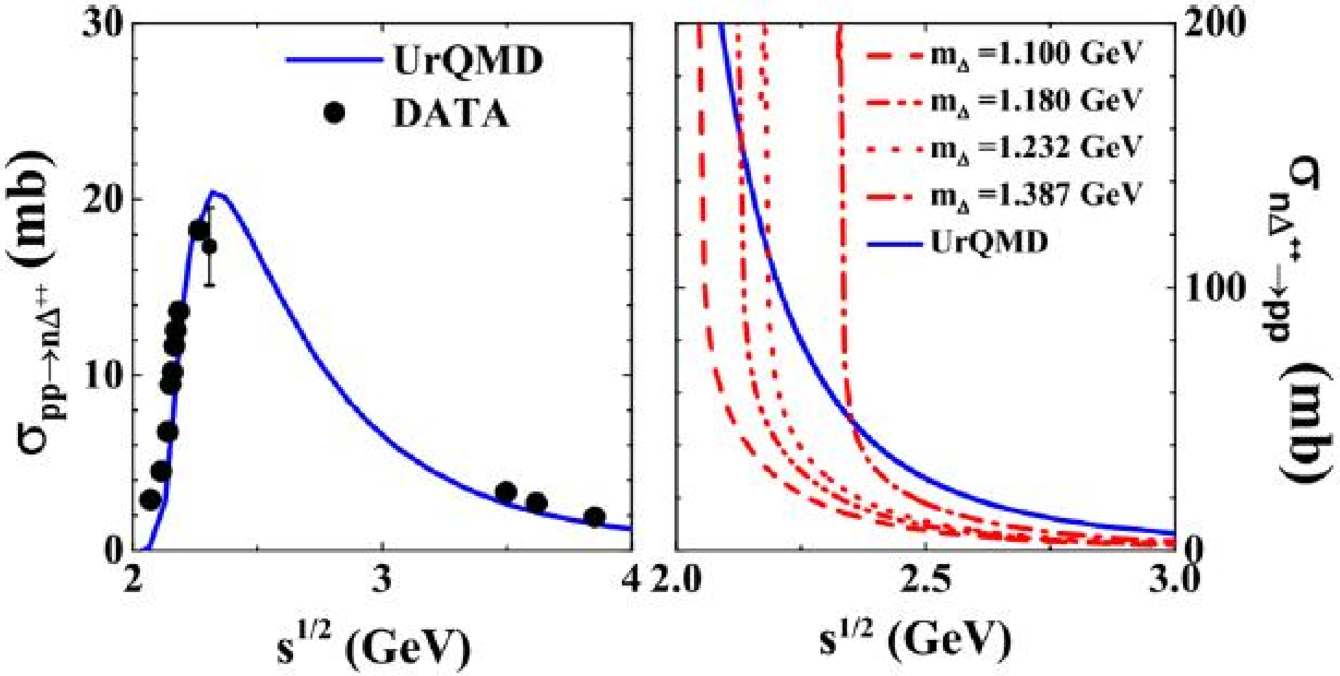
学研究院 CNNC

Some refinements in UrQMD

2, YY. Liu, et al. NST2025







更新原有UrQMD中的动量相关势

1, Y. Cui, Y. X. Zhang, and Z. X. Li, Chin. Phys. C **44**, 024106 (2020).
2, YY. Liu, et al. Phys.Rev.C 2021

- **Skryme energy density functional is used, and all the parameters are as same as we used in the nuclear structure studies**

E. Chabanat, et al., Nucl. Phys. A 627, 710–746 (1997)

$$V(\vec{r}_1,\vec{r}_2) = t_0(1+x_0P_\sigma)\delta(\vec{r}) + \frac{1}{2}t_1(1+x_1P_\sigma)[P'^2\delta(\vec{\mathbf{r}})+\delta(\vec{\mathbf{r}})P^2] + t_2(1+x_2P_\sigma)P'^2\cdot\delta(\vec{\mathbf{r}})P^2 + \frac{1}{6}(1+x_3P_\sigma)[\rho(R)]^\sigma\delta(\vec{\mathbf{r}})$$

$$\vec{r} = \vec{r}_1 - \vec{r}_2, \qquad R = \frac{1}{2}(\vec{\mathbf{r}}_1+\vec{\mathbf{r}}_2)$$

$$P = \frac{1}{2\mathrm{i}}(\nabla_1 - \nabla_2),$$

P' cc of P acting on the left

Y. Zhang, M. Tsang, Z. Li et al.,. Phys. Lett. B 732, 186–190 (2014)

$$u_{\mathrm{loc}} = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\eta+1}\frac{\rho^{\eta+1}}{\rho_0^\eta} + \frac{g_{sur}}{2\rho_0}(\nabla\rho)^2 + \frac{g_{sur,iso}}{\rho_0}[\nabla(\rho_n-\rho_p)]^2 + A_{\mathrm{sym}}\frac{\rho^2}{\rho_0}\delta^2 + B_{\mathrm{sym}}\frac{\rho^{\eta+1}}{\rho_0^\eta}\delta^2.$$

$$u_{md} = C_0\sum_{ij}\int \mathrm{d}^3p\mathrm{d}^3p' f_i(\mathbf{r},\mathbf{p})f_j(\mathbf{r},\mathbf{p}')(\mathbf{p}-\mathbf{p}')^2 + D_0\sum_{ij\in n}\int \mathrm{d}^3p\mathrm{d}^3p' f_i(\mathbf{r},\mathbf{p})f_j(\mathbf{r},\mathbf{p}')(\mathbf{p}-\mathbf{p}')^2 + D_0\sum_{ij\in p}\int \mathrm{d}^3p\mathrm{d}^3p' f_i(\mathbf{r},\mathbf{p})f_j(\mathbf{r},\mathbf{p}')(\mathbf{p}-\mathbf{p}')^2$$

重离子碰撞输运模型中，经常计算的是集体流、粒子能谱等各种物理量随时间的演化，这就意味着我们希望研究 $< O_H(t) >$

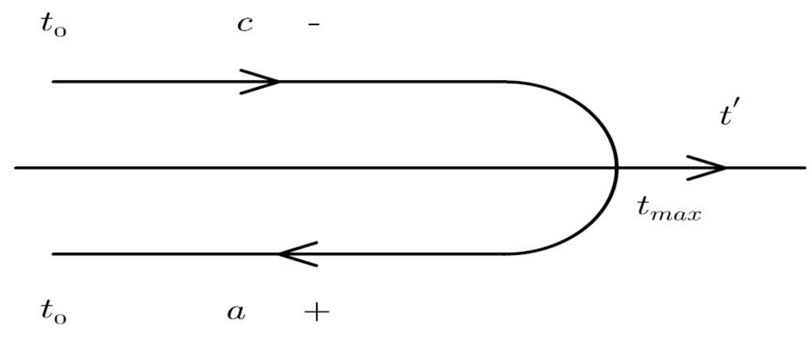
在海森堡表项中

$$\hat{O}_H(t) = \hat{U}(t_0, t)\hat{O}_I(t)\hat{U}(t, t_0)$$

$$\hat{U}(t, t_0) = \mathrm{T}^c\left[exp\left(-i\int_{t_0}^{t} dt' H_I(t')\right)\right]$$

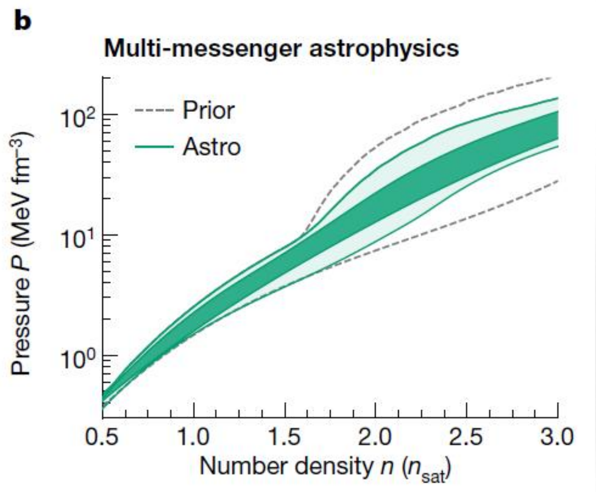
$$\hat{U}(t_0, t) = \mathrm{T}^a\left[exp\left(-i\int_{t}^{t_0} dt' H_I(t')\right)\right]$$

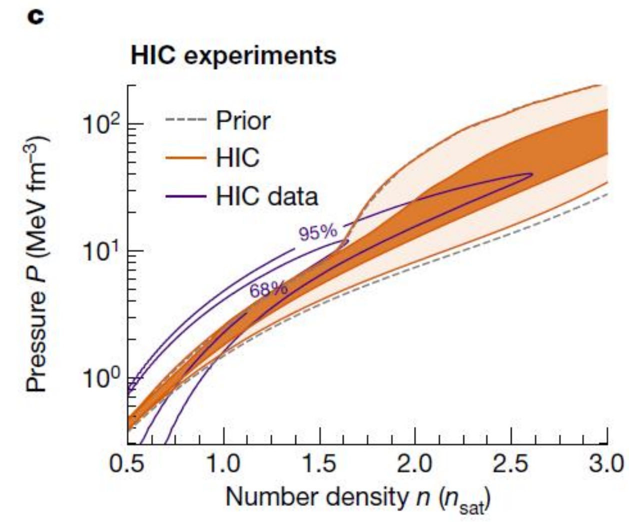
G.J. Mao, RELATIVISTIC MICROSCOPIC QUANTUM TRANSPORT EQUATION,



- **Discrepancy on the constraints of symmetry**

Huth, et al., Nature606, 276(2022)





- **Density dependence of the symmetry energy also depends on the effective mass**

$$S(\rho) = \frac{\hbar^2}{6m}\left(\frac{3\pi^2\rho}{2}\right)^{2/3} + A_{\rm sym}\frac{\rho}{\rho_0} + B_{\rm sym}\left(\frac{\rho}{\rho_0}\right)^{\eta} + C_{\rm sym}(m_s^*, m_v^*)\left(\frac{\rho}{\rho_0}\right)^{5/3}.$$

Boltzmann 方程

$$\frac{\partial f_2}{\partial t} = \{H_2, f_2\}$$

$$= \{(\nabla_{r_1} V(r_1, r_2)) \cdot \nabla_{p_1} + (\nabla_{r_2} V(r_1, r_2)) \cdot \nabla_{p_2} - \frac{p_1}{m} \cdot \nabla_{r_1} - \frac{p_2}{m} \cdot \nabla_{r_2}\} f_2(r_1, p_1; r_2, p_2, t) = 0$$

- 实际上的主要应用是求各种观测量的期望值或相关函数。物理上的观测量一般是一体或者二体算符，为了求它们的期望值，只需要约化的一体或者二体概率密度
- **量子拓展：Uehling-Uehlenbeck 因子（Phys.Rev.43.552）**

假设 $f_2(r_1,p_1;r_2,p_2,t)=f(r_1,p_1,t)f(r_2,p_2,t)$

即两个粒子于t时刻，其一在(r_1,p_1)处，另一必在在(r_2,p_2)处的几率等于它们分别在（r_1,p_1）和（r_2,p_2）处几率的乘积

$$\left(\frac{\partial f_1}{\partial t}\right)_{coll}=\frac{N-1}{V}\int\sum_{i=1}^{1}(\nabla_r V(r,r_2))\cdot\nabla_p f_2(r,p;r_2,p_2)\ldots d^6\xi_2$$

$$\left(\frac{\partial f_1}{\partial t}\right)_{coll}=-[\nabla_r\bar{\phi}(r,t)]\cdot[\nabla_p f_1(r,p,t)]$$

$$\bar{\boldsymbol{\phi}}(\boldsymbol{r},\boldsymbol{t})=\frac{\boldsymbol{N}}{\boldsymbol{V}}\int \boldsymbol{V}(\boldsymbol{r},\boldsymbol{r}_2)\boldsymbol{f}_1(\boldsymbol{r}_2,\boldsymbol{p}_2,\boldsymbol{t})d\boldsymbol{r}_2 d\boldsymbol{p}_2$$

Vlasov 方程

$$\frac{\partial}{\partial t}f_1(r,p,t)+\frac{\vec{p}}{m}\cdot\nabla_r f_1(r,p,t)+\nabla[\phi(r)+\bar{\boldsymbol{\phi}}(\boldsymbol{r})]\cdot\nabla_p f(r,p,t)=0$$

第二级的BBGKY链方程

$$\frac{\partial f_2}{\partial t} = \{H_2, f_2\} + \frac{N-2}{V} \int \sum_{i=1}^{2} (\nabla_{r_i} V(r_i, r_3)) \cdot \nabla_{p_i} f_3 \ldots d^6\xi_3$$

$$H_2 = \sum_{i=1}^{2} [\frac{p_i^2}{2m} + \phi(r_i)] + V_{12}$$

假设三次碰撞不重要 $f_3 = 0$

- 假设碰撞过程时间$\tau_c \ll$二次碰撞间隔，即碰撞为瞬时发生。**稀薄气体，短程力作用**
- 粒子相互作用范围内，外场$\phi(r)$变化很小，视为常数，则$\frac{\partial f_2}{\partial t} = 0$

PHYSICAL REVIEW C VOLUME 36, NUMBER 2 AUGUST 1987

Kinetic equation of nuclear gas

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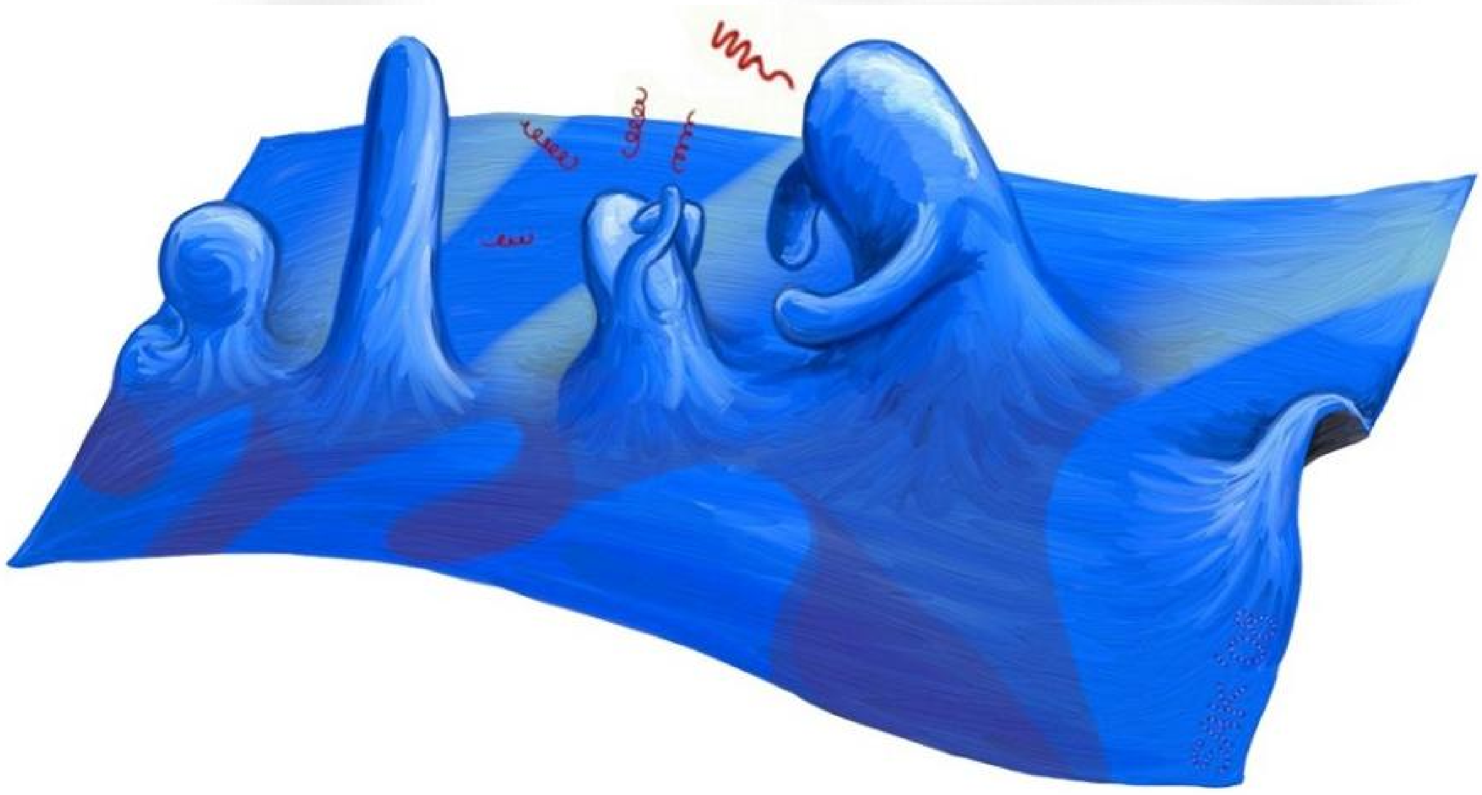
The kinetic equation on nuclear gas is derived by means of the Bogoliubov approach. It is an improved Boltzmann-Uehling-Uhlenbeck equation including correctional binary collisions with many-body effects.

Bogoliubov假设：在分子关联时间（correlation time）之后，系统中粒子间的高阶关联逐渐减弱，系统的多体分布函数可以用低阶分布函数的乘积近似表示。

$$f(1,2,t) \to f(1)f(2), \quad t \gg \tau_c$$

3、Quantum Relativistic



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相对论 Boltzmann–Uehling–Uhlenbeck 方程

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PHYSICAL REVIEW C VOLUME 44, NUMBER 2 AUGUST 1991

Transition from binary processes to multifragmentation in quantum molecular dynamics for intermediate energy heavy ion collisions

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(Received 19 July 1990)

We study the transition from fusion-fission phenomena at about 20 MeV/nucleon multifragmentation at 100–200 MeV/nucleon in the reaction $^{16}O+^{80}Br$ employing the quantum molecular dynamics model. The time evolution of the density and mass distribution, the charged-particle multiplicity, and spectra as well as angular distributions of light particles are investigated. The results exhibit the transition of the disassembly mechanism, but no sharp change is found. The results are in good agreement with recently measured 4π data.

以Fermion的场算符推导格林函数

$$\boxed{iG_F(1,2) \equiv \langle T[\Psi_H(1)\bar{\Psi}_H(2)]\rangle,}$$

Ψ为Fermion的场算符，1，2代表时空点x1,x2

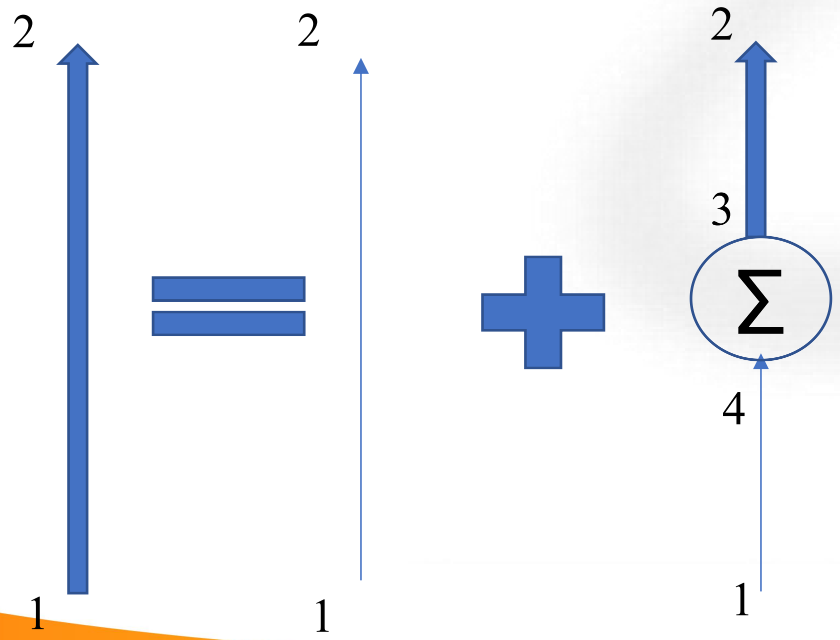
$$iG_F(1,2) = \begin{pmatrix} iG_F^{--}(1,2) & iG_F^{-+}(1,2) \\ iG_F^{+-}(1,2) & iG_F^{++}(1,2) \end{pmatrix}$$

$$iG_F^{--}(1,2) = \langle T^c\Psi_H(1)\bar{\Psi}_H(2)\rangle,$$
$$iG_F^{++}(1,2) = \langle T^a\Psi_H(1)\bar{\Psi}_H(2)\rangle,$$
$$iG_F^{+-}(1,2) = \langle \Psi_H(1)\bar{\Psi}_H(2)\rangle,$$
$$iG_F^{-+}(1,2) = -\langle \bar{\Psi}_H(2)\Psi_H(1)\rangle,$$

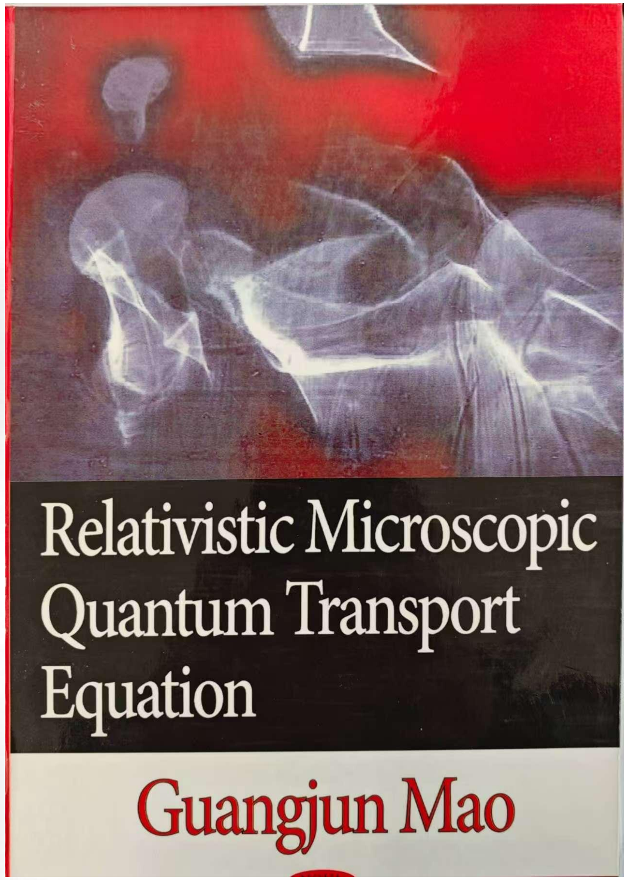
Single-particle Green function

$$iG(1,2)\delta_{\tau_1\tau_2'} = \langle T[exp(-i\int dx H_I(x))\psi_I(1)\bar{\psi}_I(2)]\rangle,$$

$$iG(1,2) = iG^0(1,2) + \int dx_3 \int dx_4 G^0(1,4)\Sigma(4,3)iG(3,2),$$



闭合时间回路



$$\begin{aligned}
\langle \hat{O}_H(t)\rangle &= \langle \hat{U}(t_0,t)\hat{O}_I(t)\hat{U}(t,t_0)\rangle \\
&= \langle \mathrm{T}^a \left[exp\left(-i\int_t^{t_0} dt' H_I(t')\right)\right]\hat{O}_I(t)\mathrm{T}^c\left[exp\left(-i\int_{t_0}^{t} dt' H_I(t')\right)\right]\rangle \\
&= \langle \mathrm{T}\left[exp\left(-i\int_{t_0}^{t_0} dt' H_I(t')\right)\hat{O}_I(t)\right]\rangle.
\end{aligned}$$

Introducing a differential operator

$$\hat{G}_{01}^{-1} = i\gamma \cdot \partial_1 - M_N \tag{3.17}$$

and operating it on the both sides of Eq. (3.5), with the help of the Dirac equation one has

$$\hat{G}_{01}^{-1}\, iG(1,2) = i\delta(1,2) + \int dx_3 \Sigma(1,3) iG(3,2), \tag{3.18}$$

$$\delta(1,2) = \begin{pmatrix} \delta^4(x_1 - x_2) & 0 \\ 0 & -\delta^4(x_1 - x_2) \end{pmatrix}$$

注意：1，2代表x1,x2

$$\begin{aligned} [i\gamma \cdot \partial_1 \ &- \ M_N - \Sigma_H(1)] iG^{-+}(1,2) \\ &= \int_{t_0}^{t'} dx_3 [\mathrm{Re}\Sigma_F^{--}(1,3)] iG^{-+}(3,2) + \int_{t'}^{t_0} dx_3 \Sigma_F^{-+}(1,3) i[\mathrm{Re}G^{++}(3,2)] \\ &+ \int_{t_0}^{t_1} dx_3 [\Sigma_{Born}^{+-}(1,3) - \Sigma_{Born}^{-+}(1,3)] iG^{-+}(3,2) \\ &- \int_{t_0}^{t_2} dx_3 \Sigma_{Born}^{-+}(1,3) [iG^{+-}(3,2) - iG^{-+}(3,2)]. \end{aligned} \tag{3.20}$$

$X = \frac{x_1+x_2}{2}$, $y = x_1 - x_2$, 对G做Wigner变换$\int e^{iPy}dy$,

$$\{\frac{i}{2}\gamma\cdot\partial_X + \gamma\cdot P - M_N - \Sigma_{HF}(X,P) + \frac{i}{2}\partial_\nu^X\Sigma_{HF}(X,P)\partial_P^\nu - \frac{i}{2}\partial_P^\nu\Sigma_{HF}(X,P)\partial_\nu^X\}iG^{-+}(X,P) = F_C(X,P),$$

Canonical variable X,P to kinetic variable x,p

$$\begin{aligned}\{[\partial_{\mathrm{x}}^\mu &- \Sigma_{HF}^{\mu\nu}(x,p)\partial_\nu^{\mathrm{p}} - \partial_{\mathrm{p}}^\nu\Sigma_{HF}^\mu(x,p)\partial_\nu^{\mathrm{x}}]p_\mu \\ &+ m^*[\partial_\nu^{\mathrm{x}}\Sigma_{HF}^{\mathrm{S}}(x,p)\partial_{\mathrm{p}}^\nu - \partial_{\mathrm{p}}^\nu\Sigma_{HF}^{\mathrm{S}}(x,p)\partial_\nu^{\mathrm{x}}]\}\frac{f(\mathbf{x},\mathbf{p},t)}{E^*} \\ &= C_N(x,p).\end{aligned}$$

RBUU方程

$$\frac{1}{4}\mathrm{tr}[iG^{-+}(x,p)] = -\frac{\pi m^*}{E^*}Z_F\delta(p_0 - E^*(p))f(\mathbf{x},\mathbf{p},t),$$

$$p^\mu(x,p) = P^\mu - \Sigma_{HF}^\mu(x,p),$$

$$m^*(x,p) = M_N + \Sigma_{HF}^S(x,p).$$

More general

$$G^{<}(1,2) := G^{-+} = i\langle\bar{\psi}(1)\ \psi(2)\rangle \xrightarrow{\text{Wigner transf}} iA(r,p)F(x,p)$$

$$G^{>}(1,2) := G^{+-} = i\langle\psi(1)\ \bar{\psi}(2)\rangle \xrightarrow{\text{Wigner transf}} iA(r,p)(1-F(x,p))$$

A(r,p) spectral function

$$\boxed{A(x,p) \propto \frac{2\Gamma(x,p)}{(p*^2 - m*^2) + \Gamma^2(x,p)}} \Rightarrow \boxed{\propto \delta(p*^2 - m*^2)\ \Theta(p*^0)}$$

"off-shell transport" (particles with widths) (Buss et al., Phys. Rep. 512, 1 (2012)

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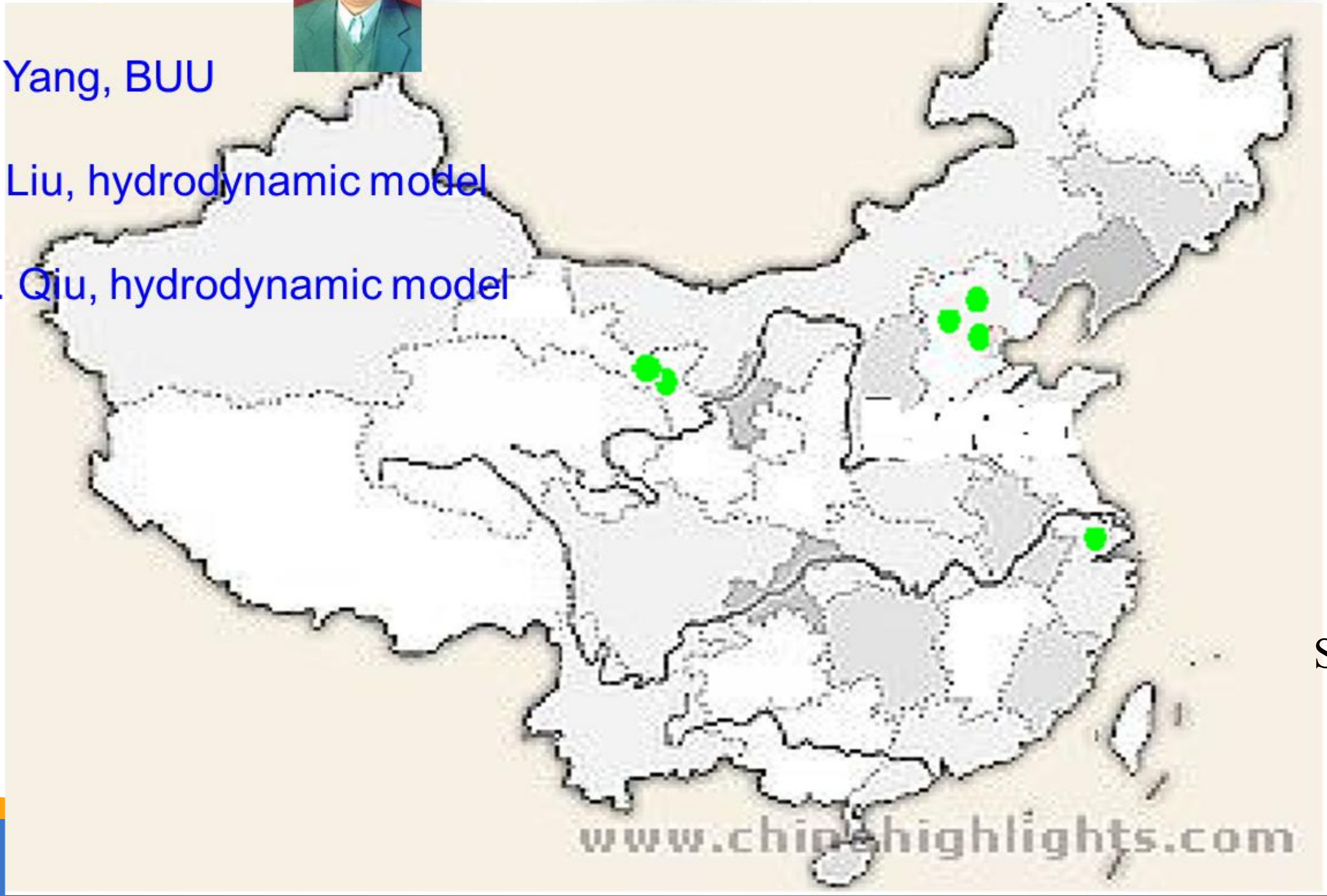

Slides from Fengshou Zhang