Revealing strong α-clustering in hot and dense nuclear matter via intermediate-energy heavy-ion collisions

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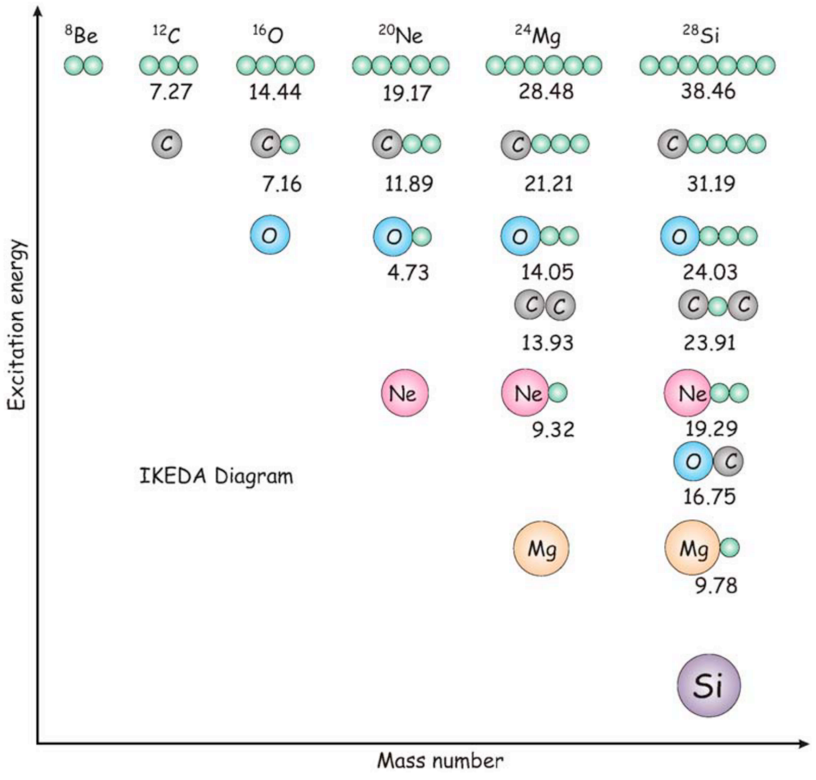
R. Wang, Y.-G. Ma, L-W. Chen, C. M. Ko, K.-J. Sun & **ZZ**, PRC108, L031601(2023)
R. Wang, **ZZ**, S. Burrell, M. Colonna, & E. G. Lanza, arXiv: 2506.16437
R. Wang, **ZZ**, Y.-G. Ma, L.-W. Chen, C. M. Ko & K.-J. Sun, arXiv: 2507.16613

Outline

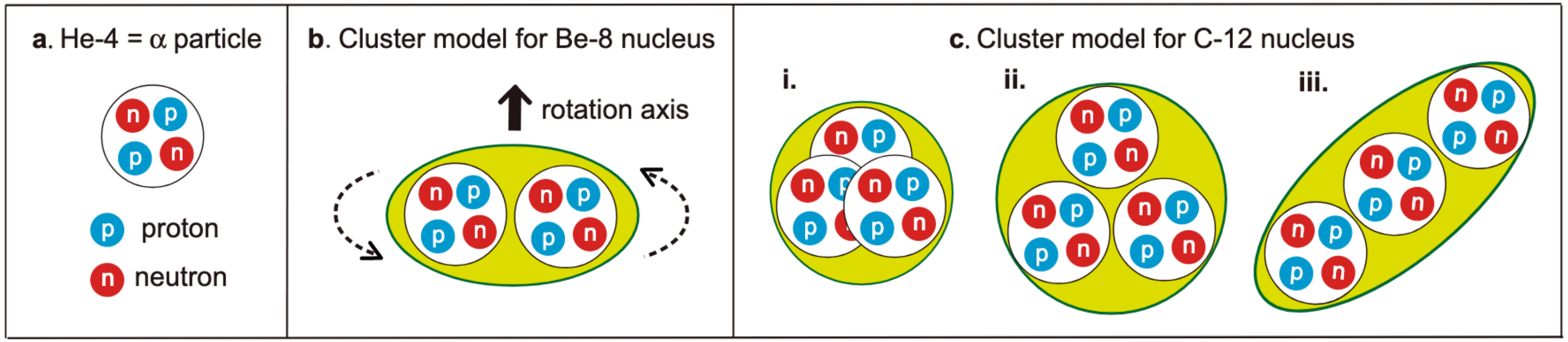
- Introduction
- Kinetic approach of light-nuclei production in intermediate-energy heavy-ion collisions
- Bayesian inference of α-cluster Mott effect in warm and dense nuclear matter
- Summary

Alpha clusters in light nuclei



Oertzen et al. Phys. Rept. 432, 43 (2006)

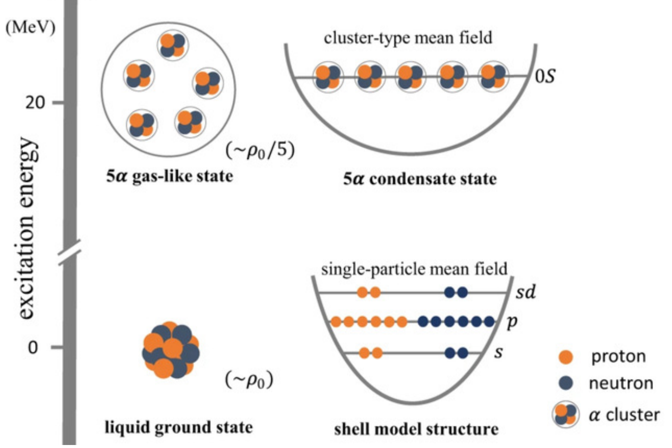
马余刚, 核技术,2023,46(08):8-29



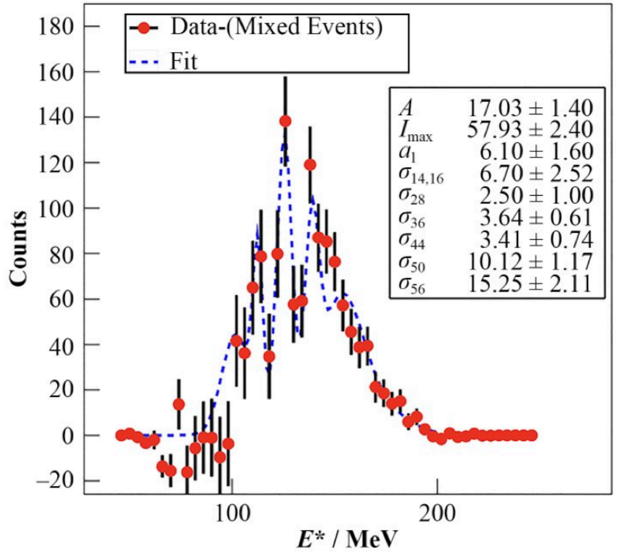
T. Otsuka et al., Nat. Commun. 13, 2234 (2022)

Ne-20

Si-28



B. Zhou et al. Nat. Commun. 14, 8206 (2023)



X. G. Cao et al., PRC 99, 014606 (2019)

Alpha clusters at surface of heavy nuclei

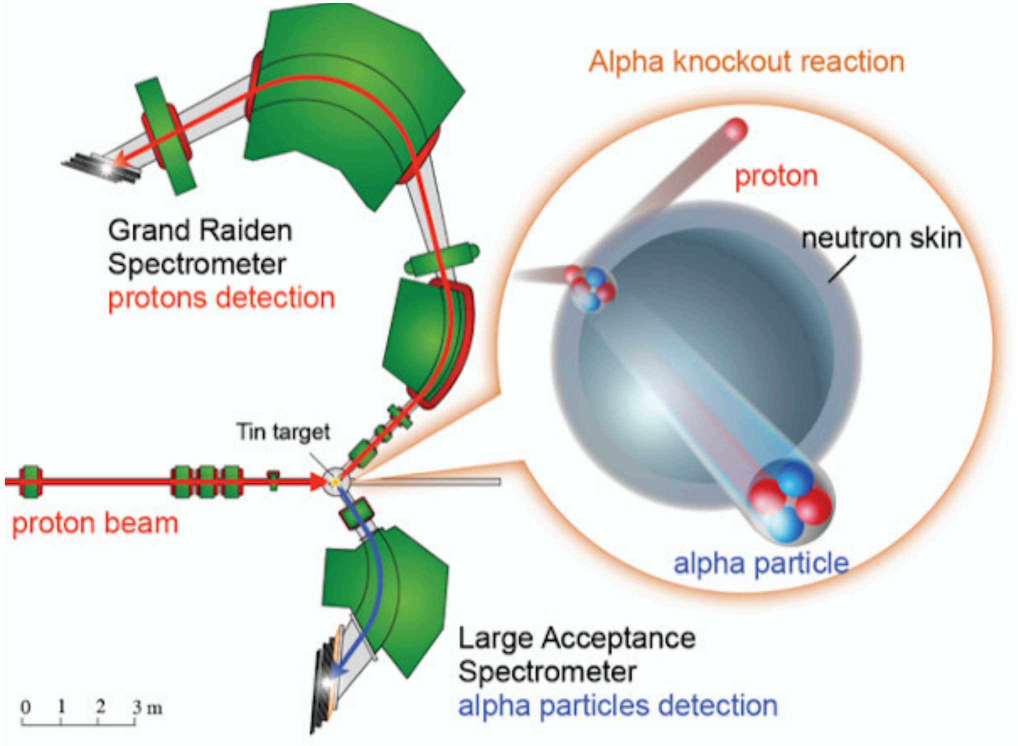
RESEARCH

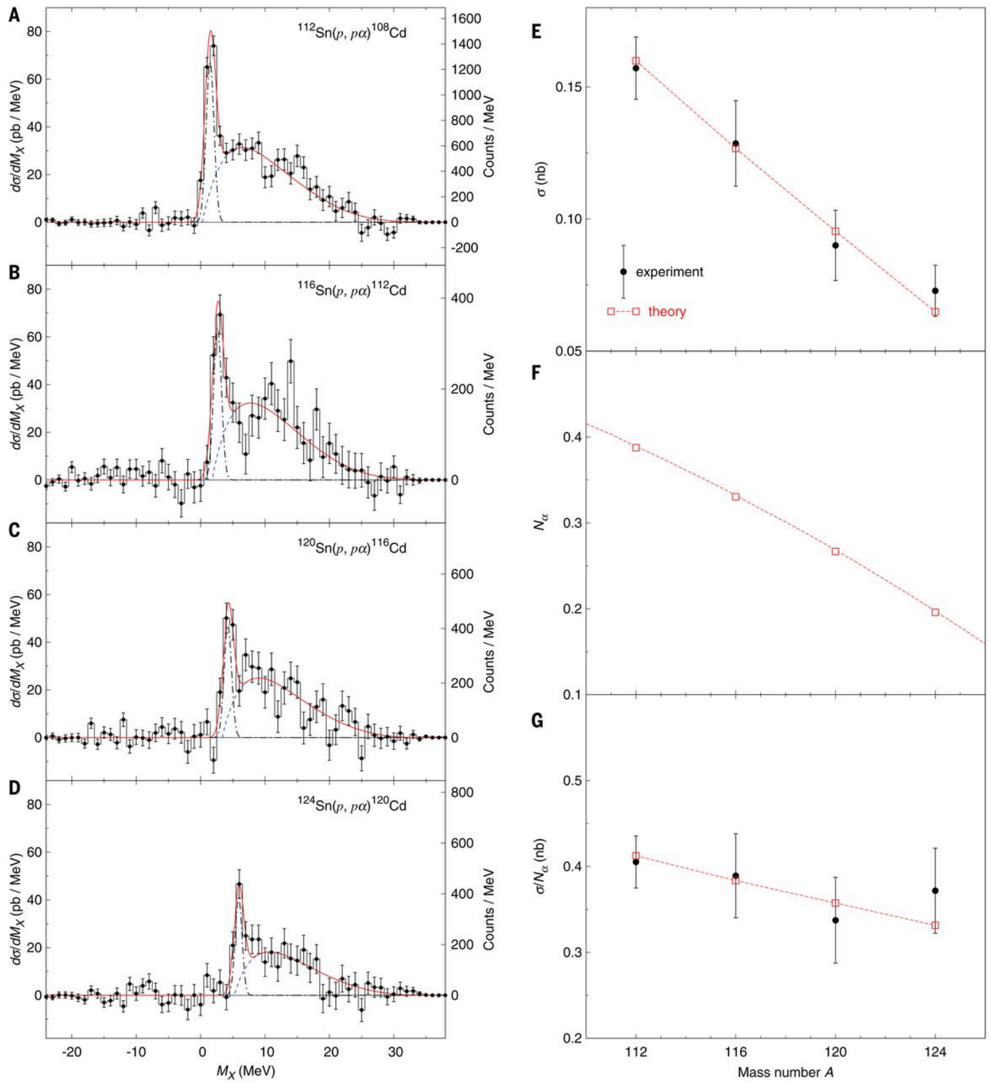
REPORT

NUCLEAR PHYSICS

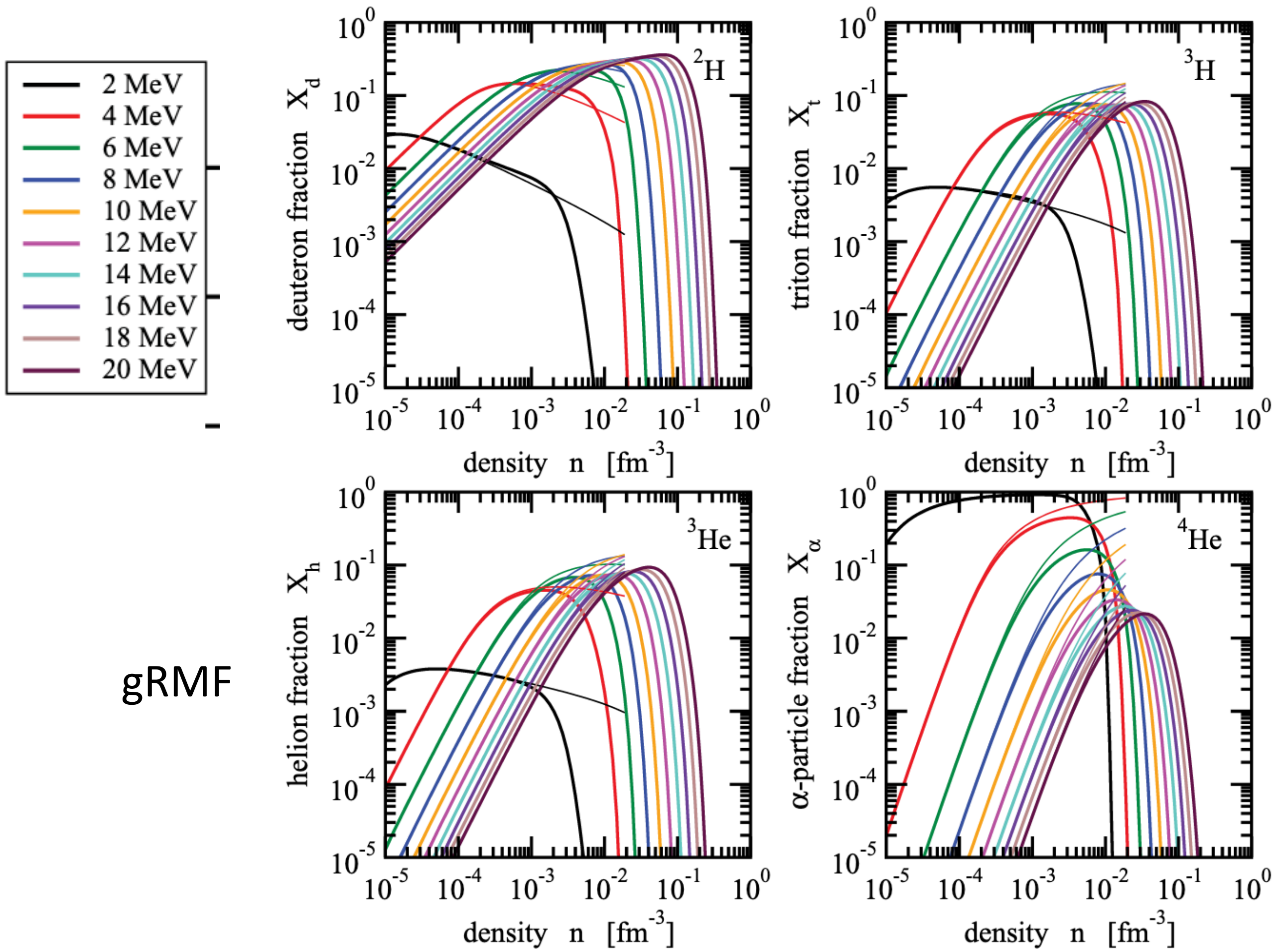
Formation of α clusters in dilute neutron-rich matter

Junki Tanaka[1,2,3]*, Zaihong Yang[3,4]*, Stefan Typel[1,2], Satoshi Adachi[4], Shiwei Bai[5], Patrik van Beek[1], Didier Beaumel[6], Yuki Fujikawa[7], Jiaxing Han[5], Sebastian Heil[1], Siwei Huang[5], Azusa Inoue[4], Ying Jiang[5], Marco Knösel[1], Nobuyuki Kobayashi[4], Yuki Kubota[3], Wei Liu[5], Jianling Lou[5], Yukie Maeda[8], Yohei Matsuda[9], Kenjiro Miki[10], Shoken Nakamura[4], Kazuyuki Ogata[4,11], Valerii Panin[3], Heiko Scheit[1], Fabia Schindler[1], Philipp Schrock[12], Dmytro Symochko[1], Atsushi Tamii[4], Tomohiro Uesaka[3], Vadim Wagner[1], Kazuki Yoshida[13], Juzo Zenihiro[3,7], Thomas Aumann[1,2,14]

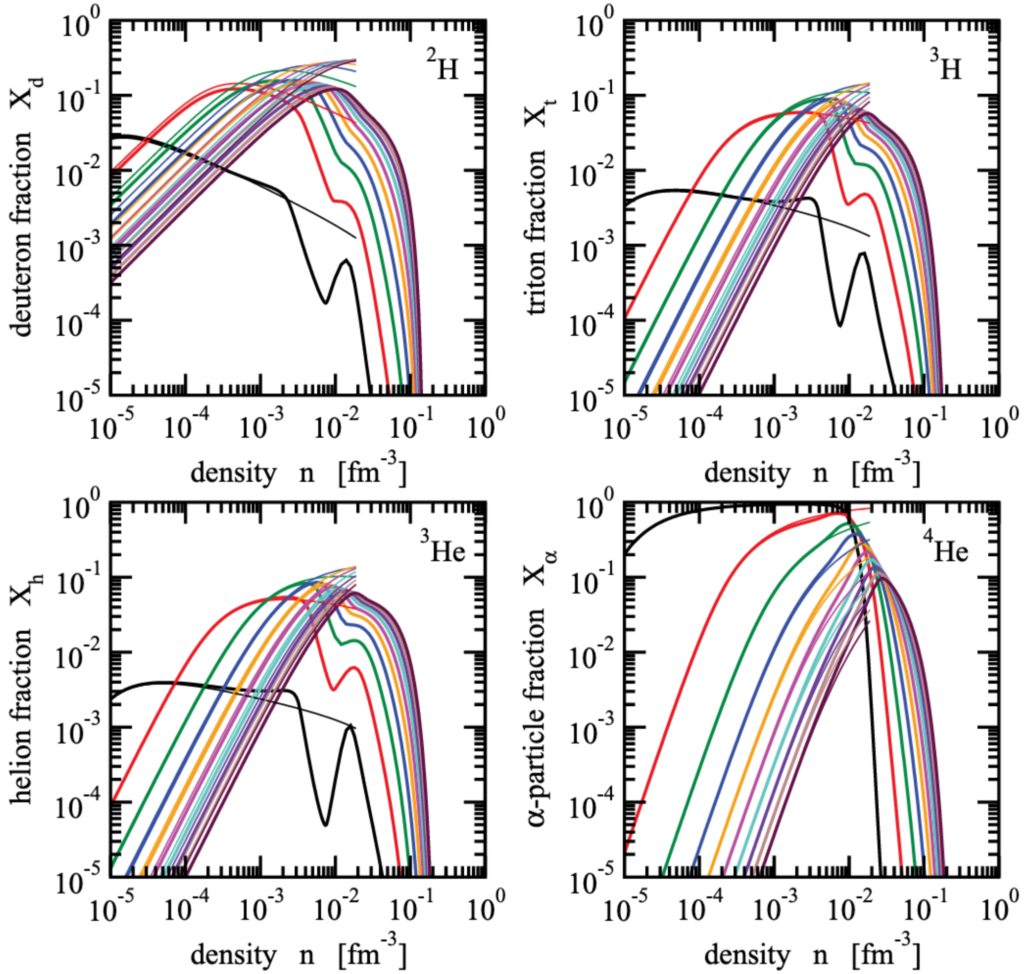




Alpha clusters in dilute nuclear matter



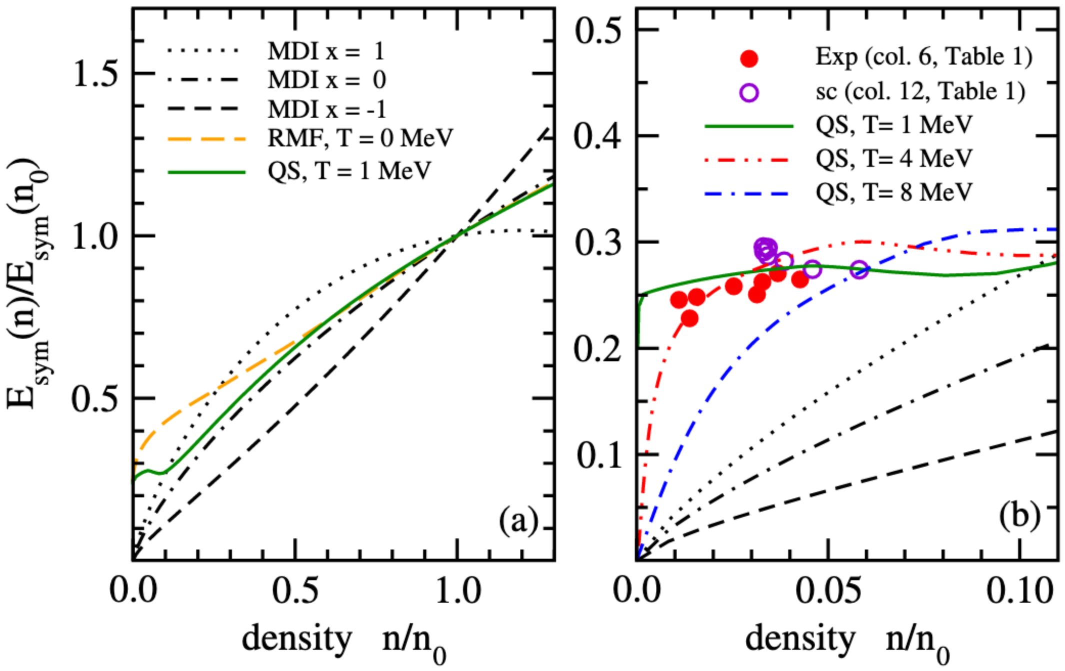
S. Typel et al. PRC 81, 015803



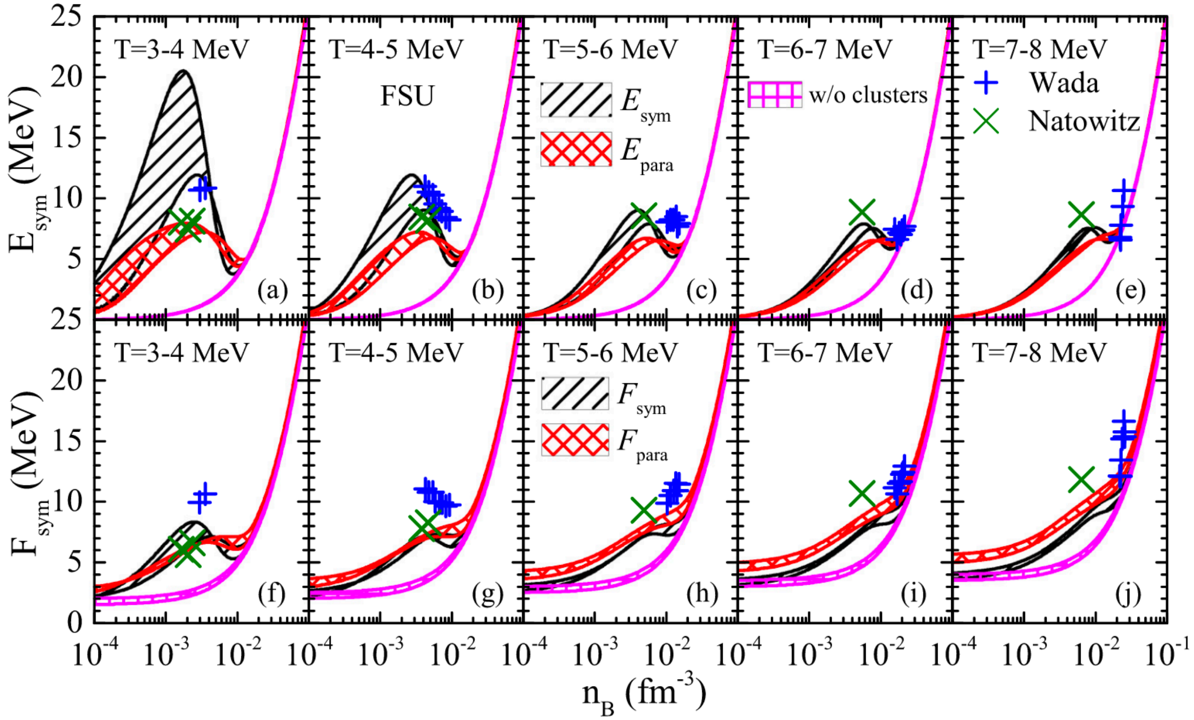
quantum statistical formulation

- ◆ Large cluster fractions in dilute nuclear matter: how do they affect the EOS?
- ◆ Why nuclear matter become uniform at high densities?

Clustering effects on symmetry energy



J. B. Natowitz et al., PRL 104, 202501 (2010)

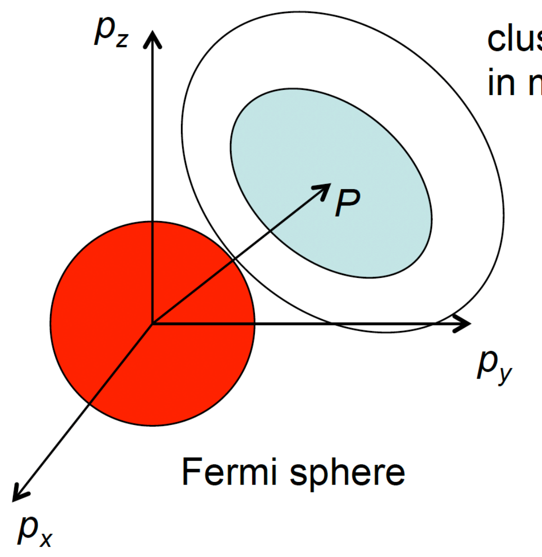


Z. W. Zhang and L. W. Chen, PRC95, 064330 (2017)

◆Strong clustering correlations at low densities significantly increase nuclear symmetry energy.

Pauli blocking and Mott effect

Taken from Röpke's talk



cluster wave function (deuteron, alpha,…) in momentum space

P - center of mass momentum

The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum *P*

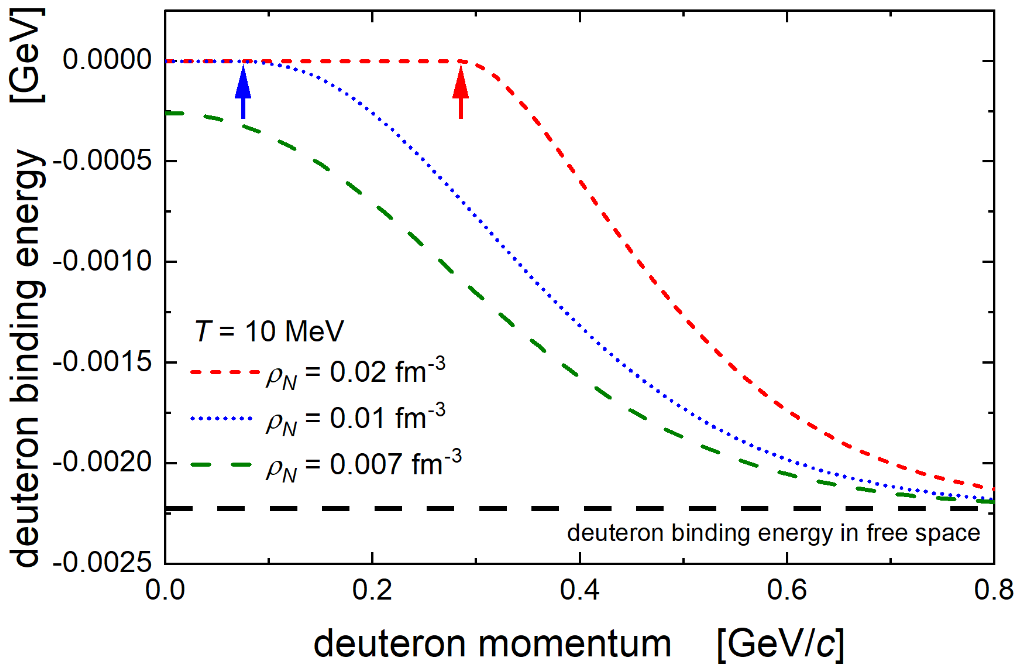
momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

In-medium Schrodinger equation

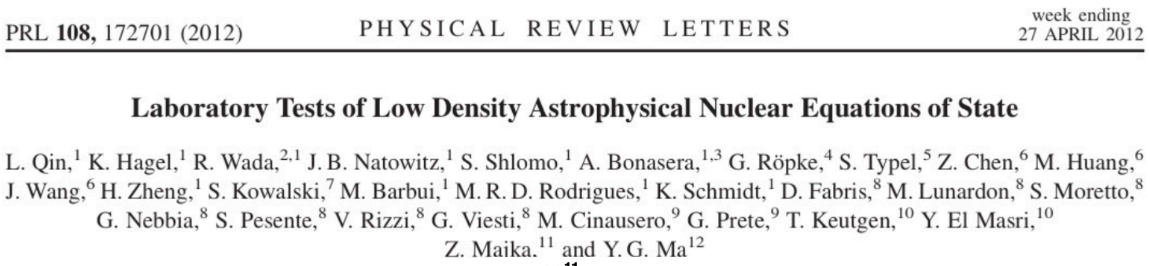
$$[\epsilon_n(\vec{p}_1) + \epsilon_p(\vec{p}_2) - E_d(\vec{P})]\langle\vec{q}|\phi_d\rangle + [1 - f_n(\vec{p}_1) - f_p(\vec{p}_2)]\int \frac{d\vec{q}'}{(2\pi\hbar)^3}\langle\vec{q}|\hat{V}|\vec{q}'\rangle\langle\vec{q}'|\phi_d\rangle = 0$$

G. Röpke, L. Münchow, and H. Schulz, NPA 379, 536 (1982).
G. Röpke, M. Schmidt, L. Münchow, and H. Schulz， NPA 399, 587 (1983)



- **Mott effect**: a light nucleus will dissolve if the phase-space density of its surrounding nucleons is too large.
- α-particle fraction at high densities reduces due to the Mott effect.
- How about hot and dense nuclear matter?

Medium effects of light clusters in Fermi-energy HICs


PRL **108,** 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

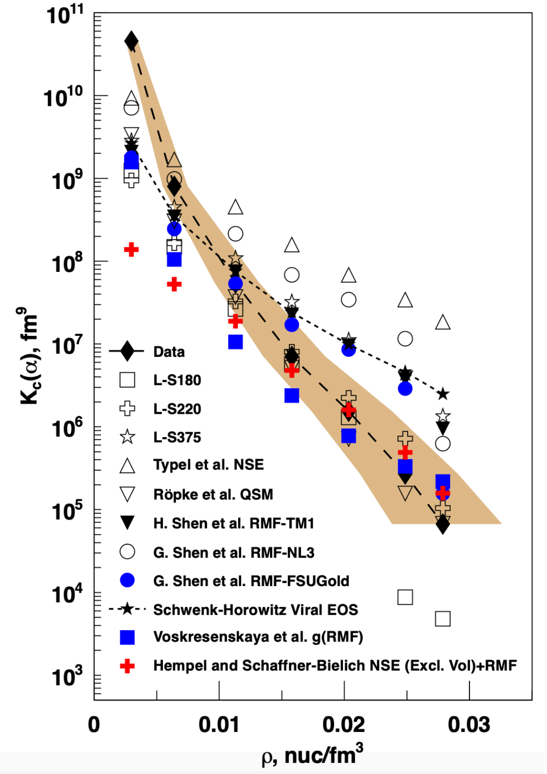
L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Maika,[11] and Y. G. Ma[12]

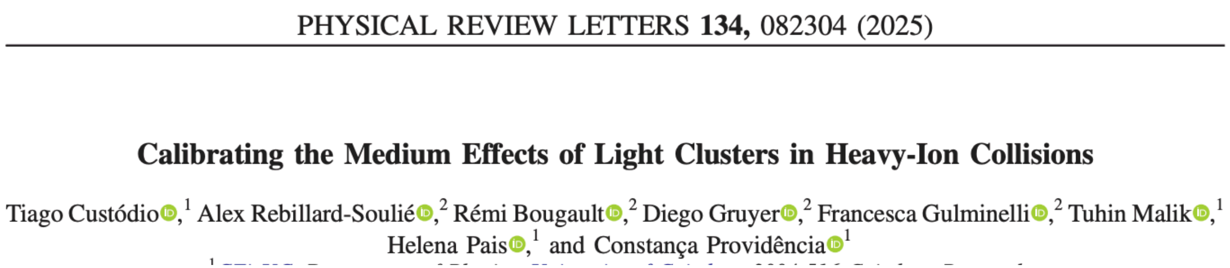
$$K_c(A,Z) = \frac{\rho(A,Z)}{\rho_p^Z \rho_n^{(A-Z)}}$$

◆Ek = 47 A MeV

◆Light clusters freeze-out at low densities.

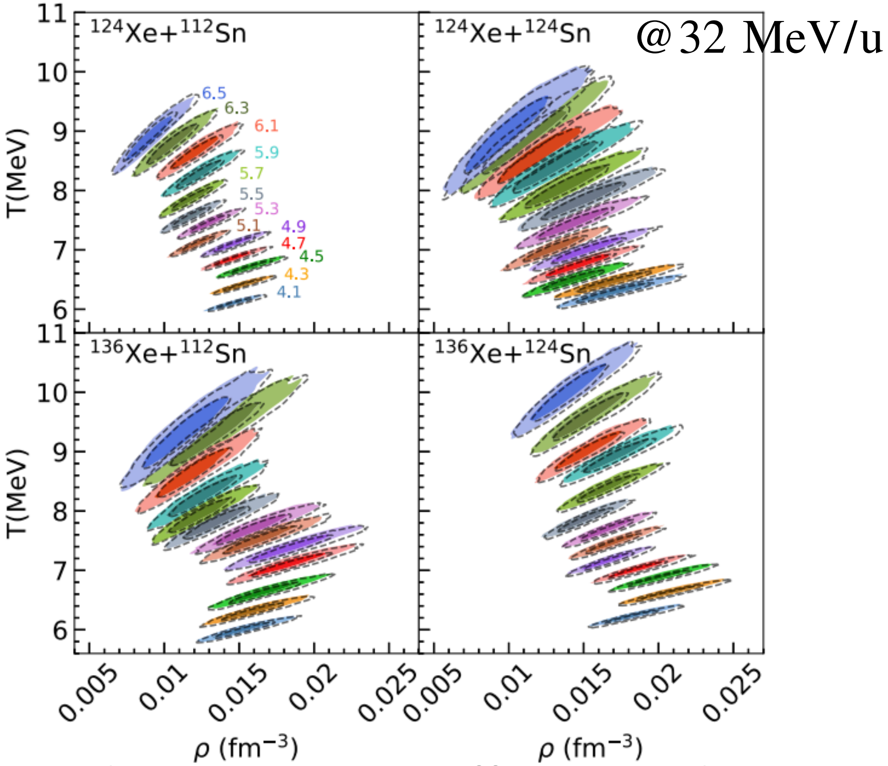
◆temperature 5-11 MeV




PHYSICAL REVIEW LETTERS **134,** 082304 (2025)

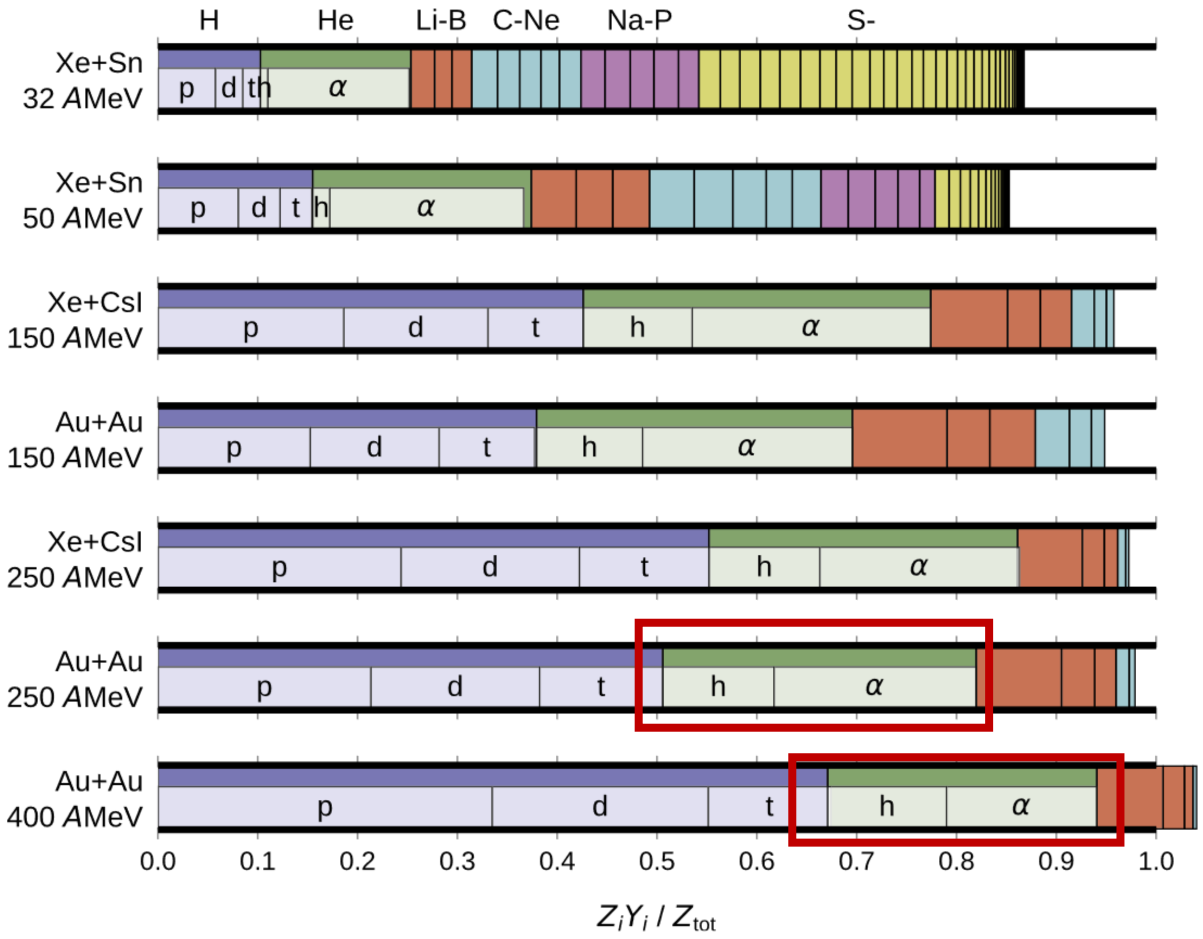
Calibrating the Medium Effects of Light Clusters in Heavy-Ion Collisions

Tiago Custódio,[1] Alex Rebillard-Soulié,[2] Rémi Bougault,[2] Diego Gruyer,[2] Francesca Gulminelli,[2] Tuhin Malik,[1] Helena Pais,[1] and Constança Providência[1]



✦Fermi energy HICs offer a unique experimental avenue for probing mediums effects on clusters in warm and dilute nuclear matter. L. Qin et al., PRL 108, 172701 (2012); K. Hagel et al., PRL 108, 062702 (2012); H. Pais et al., PRL 125, 012701 (2020)

Light nuclei in intermediate-energy HICs

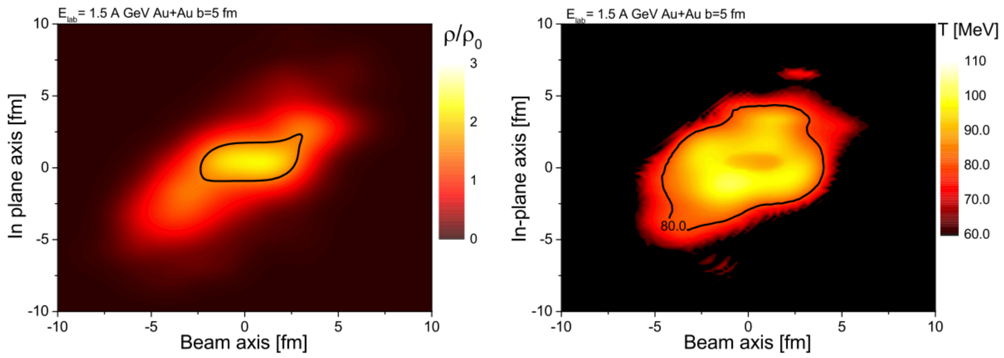


A. Ono, Progress in Particle and Nuclear Physics 105 (2019) 139–179

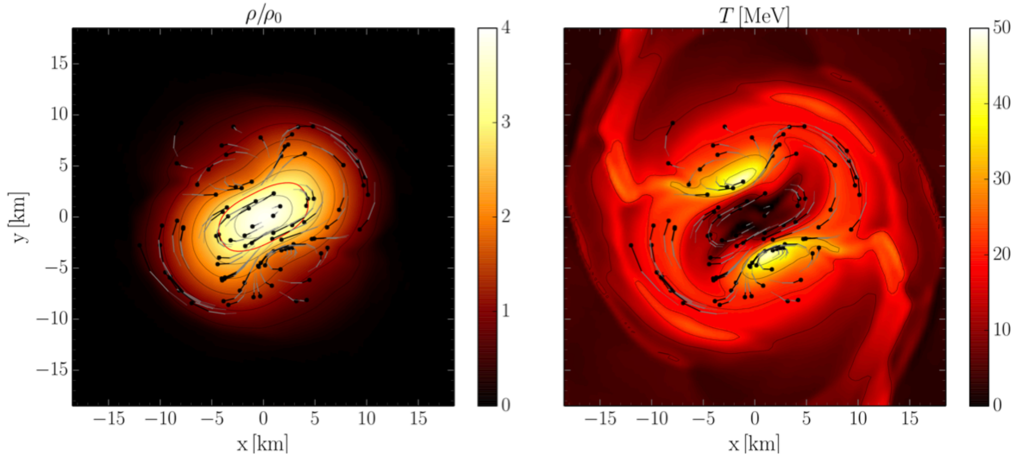
FOPI Collaboration, Nuclear Physics A 848 (2010) 366–427

✦ Large α yield challenges transport models for HICs

Microscopic collision: Heavy-ion collisions



Macroscopic collision: Neutron star mergers



✦Hot and dense nuclear matter forms in intermediate-energy HICs

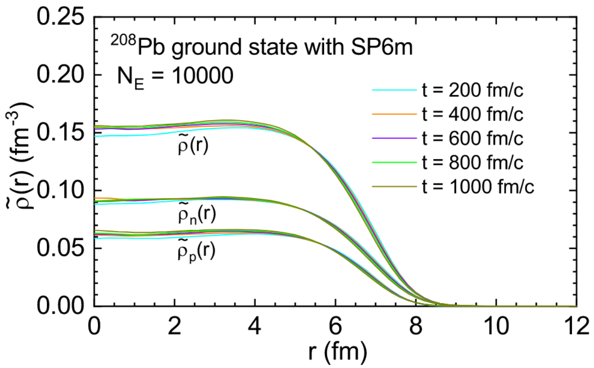
Kinetic approach of light-nuclei production in intermediate-energy heavy-ion collisions

R. Wang, Y.-G. Ma, L-W. Chen, C. M. Ko, K.-J. Sun & **ZZ**, PRC108, L031601(2023)

Lattice Boltzmann-Uehling-Uhlenbeck (LBUU) model

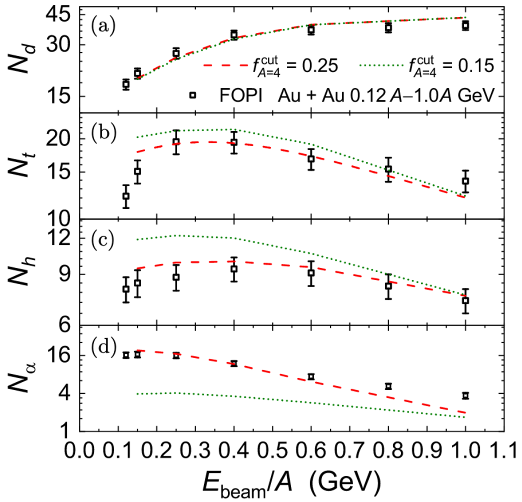
Lattice Hamiltonian Vlasov

R. Wang, L.-W. Chen & **ZZ**, PRC 2019



+ Light clusters, Δ, π

R. Wang, Y.-G. Ma, L.-W. Chen et al., PRC 2023

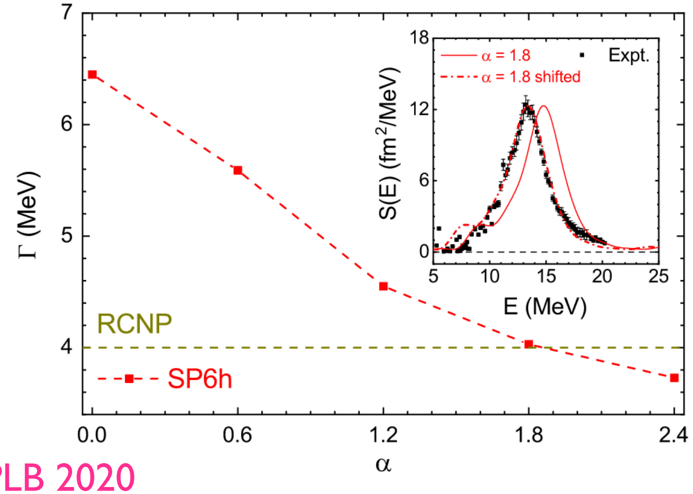


+ Λ, K, Σ . . .

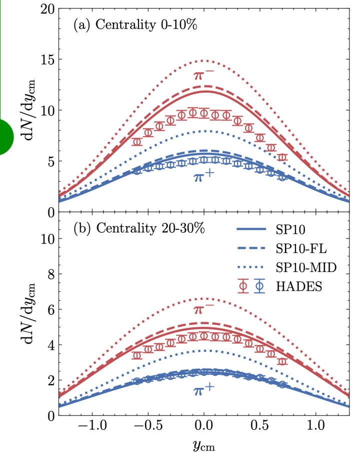
(In progress)

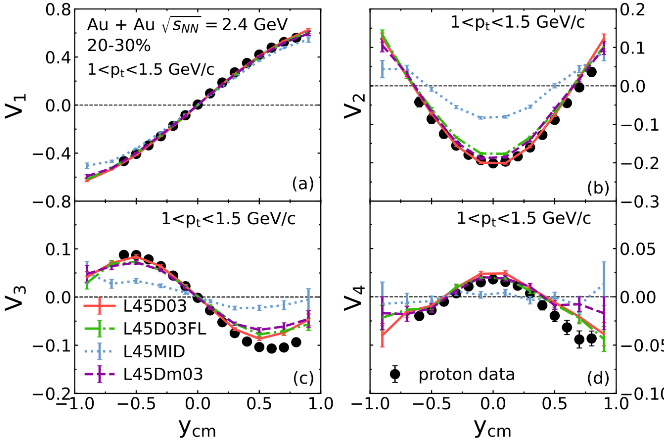
+ Stochastic method for collisions (LBUU)

R. Wang, **ZZ**, L.-W. Chen et al. PLB 2020



+ Δ^*, N^* (for HADES-Energy HICs)





L. Li, S.-P. Wang, **ZZ** et al., arXiv: 2509.21099

L. Li, S.-P. Wang, R. Wang et al. arXiv: 2511.20387

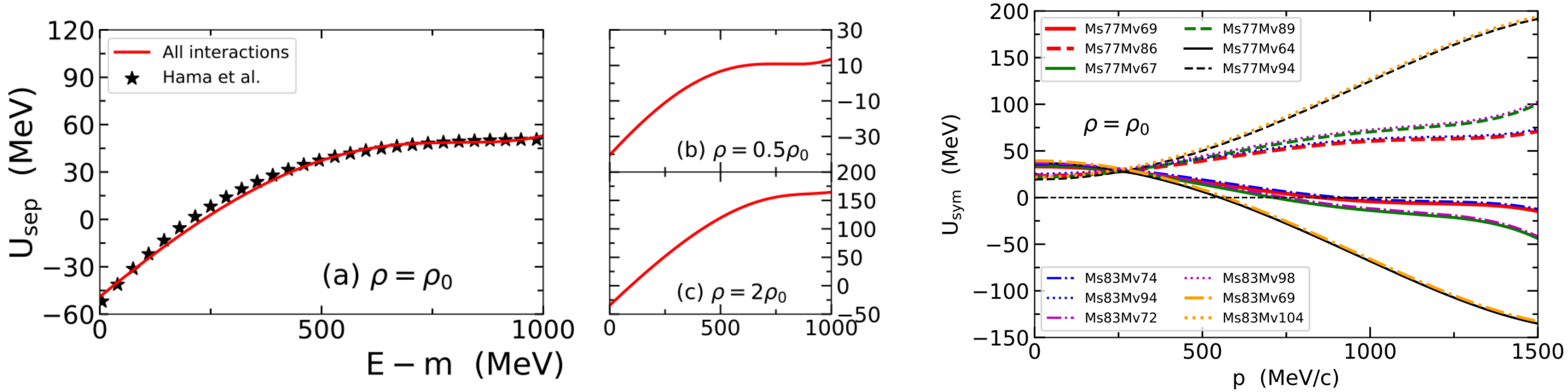
LBUU model

✦**GPU parallel**: High accuracy with large number of test particles.

✦N5LO Skyrme pseudo-potential:

- Reproduce empirical optical potential (SNM)
- More flexible isopin- & momentume-dependece of mean-field (n-p effective mass splitting?)

R. Wang et al., PRC 98, 054618 (2018)
S.-P. Wang et al. PRC 109, 554623 (2024); 111, 054605 (2025)



✦ Successful applications across nuclear structure and intermediate-energy HICs

- Nuclear giant resonance (~20 MeV) — PRC 99, 044609 (2019); Front. Phys. 8:330 (2020); PRC 104, 044603 (2021); PRC 112 (2025) 4, 044614
- In-medium NN cross sections — PLB 807, 135532 (2020); PRC 108, 064603 (2023)
- Light nuclei production (0.15-1 GeV/u) — PRC108, L031601 (2023); arXiv:2507.16613
- Pion production (1.23 GeV/u) — arXiv: 2509.21099
- Flow observables (1.23 GeV/u) — arXiv: 2511.20387

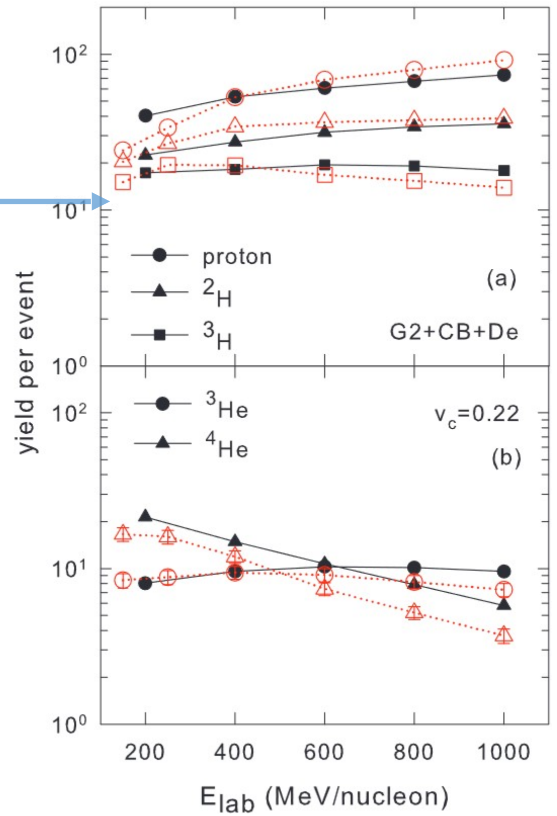
Theoretical approaches for light nuclei production

- Minimum spanning tree and variants

  e.g., FRIGA, A. Le Fèvre, et. al, Physical Review C 100, 034904 (2019)

- Coalescence (+ statistical decay)

  e.g., A. S. Botvina, et. al, Physical Review C 103, 064602 (2021)

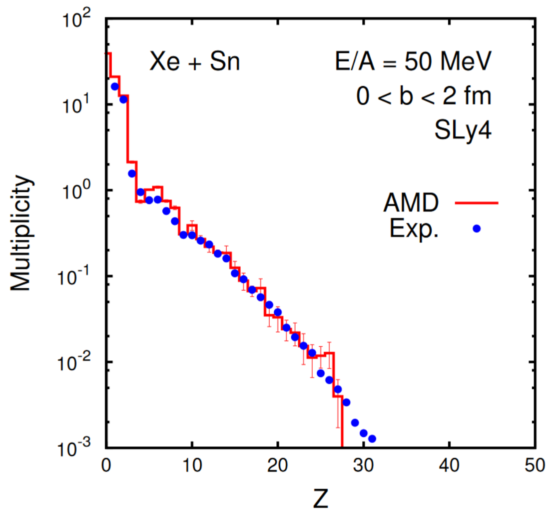


- Treating light nuclei dynamically
  - Anti-symmetrized molecular dynamics (AMD)

    All light nuclei (up to A = 4) have been included. With inter-cluster correlations, achieves a very good description of the measured multiplicity for low incident energies

    A. Ono, Journal of Physics: Conference Series 420 (2013) 012103

AMD with dynamical light nuclei and inter-cluster correlation



  - LQMD H.-G. Chen and Z.-Q. Feng Phys.Rev.C 109 (2024) 2, L021602
  - Kinetic approach/Boltzmann–Uehling–Uhlenbeck equation

    Light nuclei with A = 2 (d) and A = 3 (t, h) have been included.

    P. Danielewicz and G. F. Bertsch, Nuclear Physics A533, (1991) 712-748

    P. Danielewicz and Q. Pan, Physical Review C 46, 2002-2011 (1992)

    K.J. Sun, R. Wang, C.M. Ko, Y-G. Ma & C. Shen. Nature Commun. 15. 1074 (2024)

See also SMASH, PHQMD

Light nuclei in kinetic approach

Describe production and dissociation of light nuclei through **many-body scatterings**:

For example

α loss term

$$K_\alpha^> f_\alpha = \frac{\mathcal{S}_{5'} f_\alpha}{2E_\alpha} \int \prod_{i=1'}^{5'} \frac{\mathrm{d}\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{\mathrm{d}\vec{p}_N}{(2\pi\hbar)^3 2E_N} \overline{\left|\mathcal{M}_{N\alpha\to NNNNN}\right|^2} g_N f_N \prod_{i=1'}^{5'} (1 \pm f_i)(2\pi)^4 \delta^4\left(\sum_{i=1'}^{5'} p_i - p_N - p_\alpha\right)$$

$$+\frac{\mathcal{S}_{3'} f_\alpha}{2E_\alpha} \int \prod_{i=1'}^{3'} \frac{\mathrm{d}\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{\mathrm{d}\vec{p}_N}{(2\pi\hbar)^3 2E_N} \overline{\left|\mathcal{M}_{N\alpha\to NNt}\right|^2} g_N f_N \prod_{i=1'}^{3'} (1 \pm f_i)(2\pi)^4 \delta^4\left(\sum_{i=1'}^{3'} p_i - p_N - p_\alpha\right) + t \to h$$

$$+\frac{\mathcal{S}_{2'} f_\alpha}{2E_\alpha} \int \prod_{i=1'}^{2'} \frac{\mathrm{d}\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{\mathrm{d}\vec{p}_N}{(2\pi\hbar)^3 2E_N} \overline{\left|\mathcal{M}_{N\alpha\to dt}\right|^2} g_N f_N \prod_{i=1'}^{2'} (1 \pm f_i)(2\pi)^4 \delta^4\left(\sum_{i=1'}^{2'} p_i - p_N - p_\alpha\right) + t \to h$$

+ elastic part.

- A = 2 $NNN \leftrightarrow Nd$
- A = 3 $NNNN \leftrightarrow Nt(h)$
- A = 4 $NNNNN \leftrightarrow N\alpha$, $NNt(h) \leftrightarrow N\alpha$, $dt(h) \leftrightarrow N\alpha$

See also pBUU, AMD, SMASH, PHQMD, and LQMD

- Many body **transition amplitudes** e.g., $\overline{\left|\mathcal{M}_{Npn\leftrightarrow Nd}\right|^2}$
- The medium effect of light nuclei – **Mott effect**

R. Wang et al., Phys.Rev.C 108 (2023) 3, L031601

Impulse approximation

$$\overline{\left|\mathscr{M}_{Nd\to NNN}\right|^2} \approx F(\sqrt{s}) \sum_{\text{spectator nucleons}} \boxed{\left|\left\langle \vec{k} \mid \phi_d \right\rangle\right|^2} \overline{\left|\mathscr{M}_{NN\to NN}\right|^2}$$

Internal wave function of light nuclei

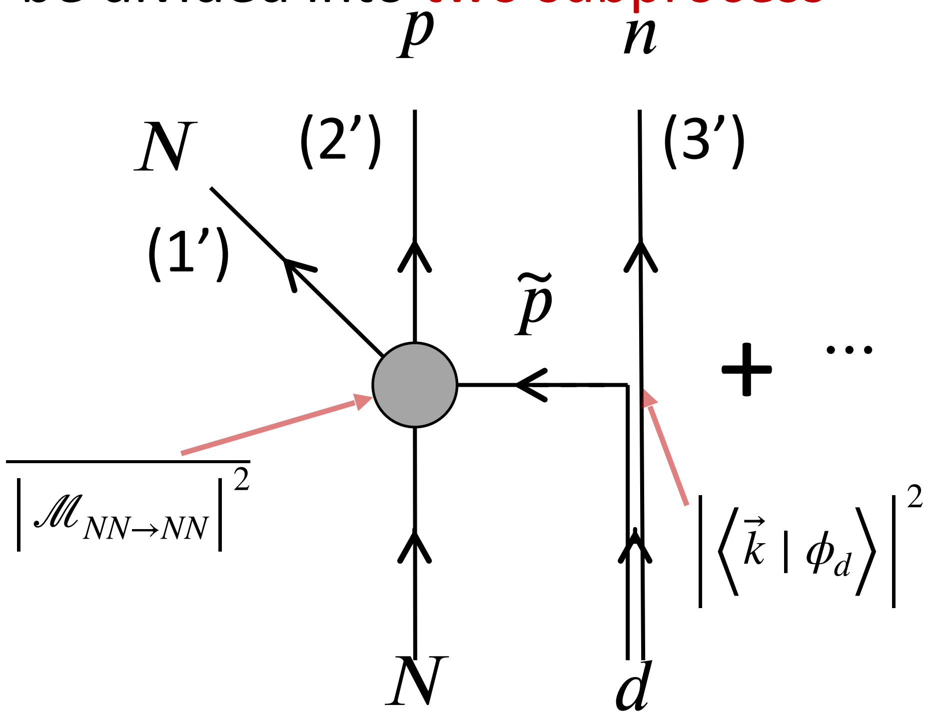
$\vec{k}$ are the standard Jacobian relative momenta

Under IA, $\sigma_{Nd}^{total} \sim \sigma_{Np}^{total} + \sigma_{Nn}^{total}$

$F(\sqrt{s})$ is a factor to 1) account for the failure of IA, 2) exclude the elastic part of N-d from the total amplitude.

They should be determined by comparing with experimental N-d N-t N-α in-elastic cross sections.

In transport approach, under IA the scattering $NNN \leftrightarrow Nd$ can be divided into two subprocess



IA could ensure the **detailed balance condition** of many-body scatterings like $NNN \leftrightarrow Nd$

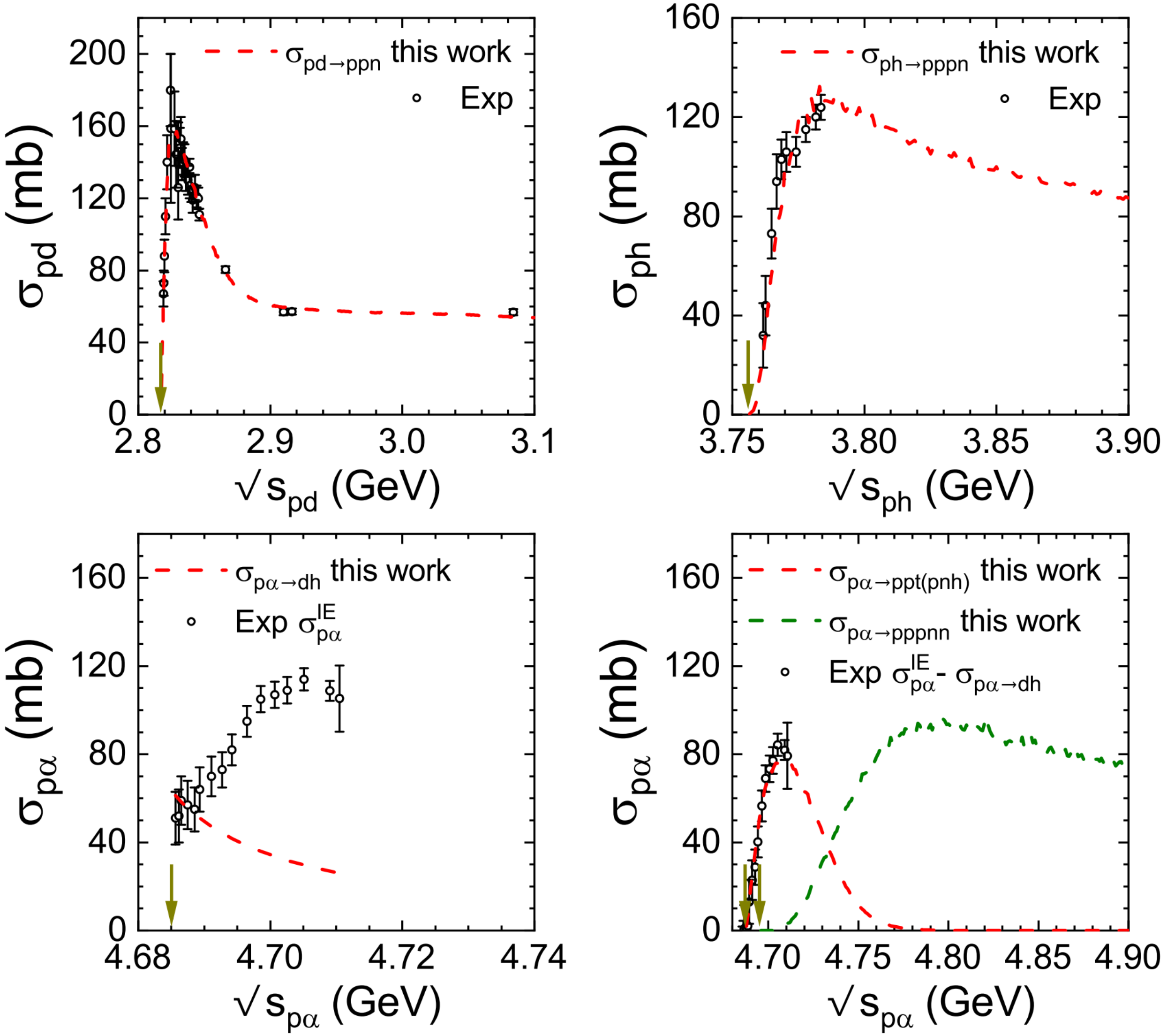
Cross sections

$$\overline{\left|\mathscr{M}_{N\alpha\to NNNNN}\right|^2} \approx \boxed{F(\sqrt{s})} \sum_{\substack{\text{spectator}\\ \text{nucleons}}} \left|\left\langle \vec{k}\vec{k}_\lambda\vec{k}_\mu \mid \phi_\alpha \right\rangle\right|^2 \overline{\left|M_{NN\to NN}\right|^2}$$

- A natural feature of F is it approaches to 1 as $\sqrt{s}$ increases. (For large incident energy, IA becomes very good, and the reaction is dominated by in-elastic channels)

- Different parameterizations of $F\left(\sqrt{s}\right)$ for different many-body scattering channels are adopted to properly reproduce the experimental N-d, N-h and N-α inelastic cross sections.

- $N\alpha \leftrightarrow NNt(h)$ and $N\alpha \leftrightarrow dt(h)$ should be included to reproduce the experimental N-α inelastic cross sections at small $\sqrt{s}$. Assumption has to be made of the branching ratios of $N\alpha$ scattering



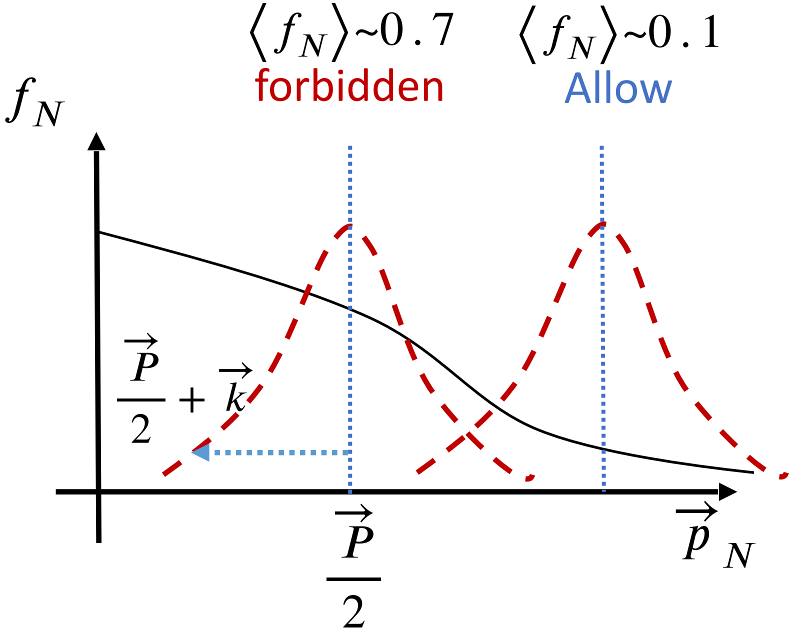
R. Wang et al., Phys.Rev.C 108 (2023) 3, L031601

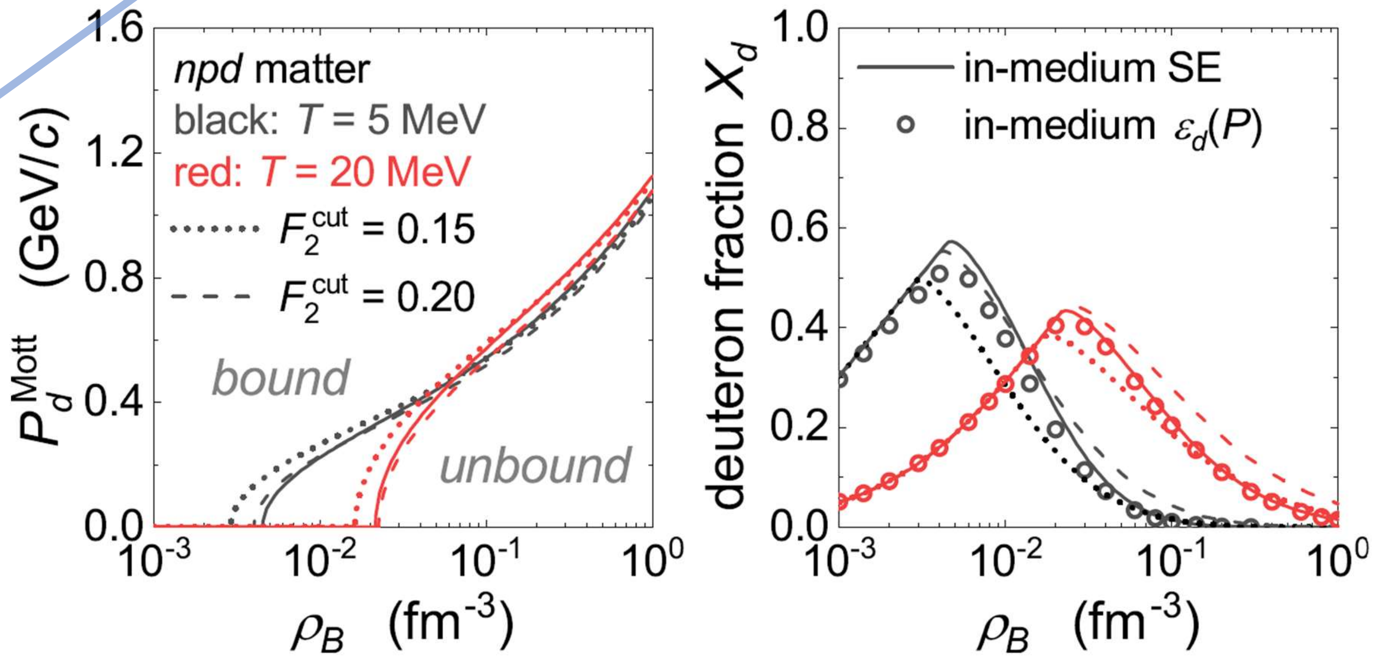
Mott effect in kinetic approach

The Mott effect can be effectively introduced into the kinetic approach through a phase-space cut-off.

Momentum distribution of the constituent nucleon, related to the light-nuclei internal wave function

$$\langle f_N\rangle_i(\vec{P}) \equiv \int f_N^{\mathrm{tot}}\left(\frac{\vec{P}}{A_i}+\vec{p}\right)\left|\phi_i(\vec{p})\right|^2 \mathrm{d}\vec{p} \leqslant f^{\mathrm{cut}}_{A=A_i}$$





$f^{cut}_{A=2}$, $f^{cut}_{A=3}$, and $f^{cut}_{A=4}$ can be treated as surrogates of the strength of light-nuclei Mott effect

R. Wang, **ZZ**, S. Burrello, et al., arXiv:2506.16437

Light-nuclei yields

Central Au+Au collision at 0.4*A* GeV

Multiplicities and charge balance for Au + Au at $E/A = 0.40$ GeV and $b_0 < 0.15$.

$Z=1$	106.0 ± 4.0	p	52.9 ± 2.7	$Z=2$	21.3 ± 1.9	^{3}He	9.4 ± 0.9
		d	34.2 ± 2.1			^{4}He	11.9 ± 1.1
		t	19.4 ± 1.8				
π^+	0.95 ± 0.08	π^-	2.80 ± 0.14				

Li	3.5 ± 0.4	Be	0.84 ± 0.09
B	0.27 ± 0.03	C	0.096 ± 0.01

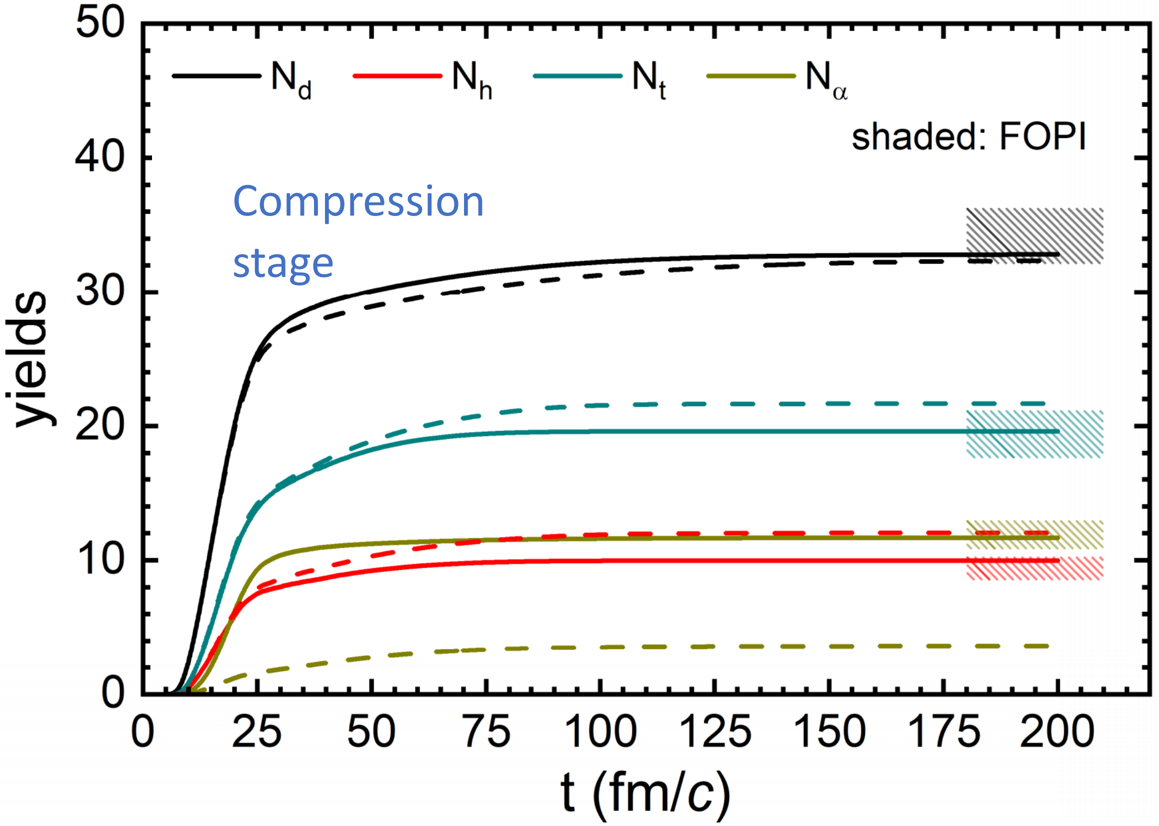
Charge balance LCP: 148.6 or 94.1%
Charge balance LCP + pions: 146.8 or 92.9%
Charge in HC (Z3–Z6): 15.77 ± 1.6 or $(10.0 \pm 1.0)\%$
Total balance: 102.9%

FOPI data at 0.4*A* GeV

To reproduce the measured light-nuclei yields (Note N_α is larger than N_h), one needs to use a larger $f^{cut}_{A=4}$ compared with $f^{cut}_{A=3}$, which means the Mott effect of α-particle should be weaker than that of triton/helium3.

A smaller $f^{cut}_{A=4}$ leads to a significant decrease of N_α.

Time evolution of light-nuclei yields



R. Wang et al., Phys.Rev.C 108 (2023) 3, L031601

Bayesian inference of α-cluster Mott effect in warm and dense nuclear matter

R. Wang, **ZZ**, S. Burrell, M. Colonna, & E. G. Lanza, arXiv: 2506.16437
R. Wang, **ZZ**, Y.-G. Ma, L.-W. Chen, C. M. Ko & K.-J. Sun, arXiv: 2507.16613

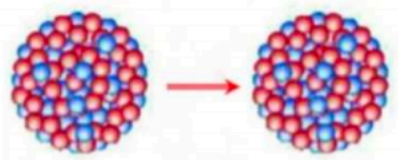
HICs versus nuclear matter

Kinetic equation of the occupation number f_τ

$$(\partial_t+\vec{\nabla}_p\epsilon_\tau\cdot\vec{\nabla}_r-\vec{\nabla}_r\epsilon_\tau\cdot\vec{\nabla}_p)f_\tau=\mathcal{K}_\tau^<[f_n,f_p,f_d,\cdots](1\pm f_\tau)-\mathcal{K}_\tau^>[f_n,f_p,f_d,\cdots]f_\tau,\quad \tau=n,p,d,t,h,\alpha$$

Heavy-ion collisions

By initializing f_τ of each particle species (initially only nucleons) and let them evolve according to the kinetic equation



Parameters characterizing the strength of in-medium effects can be empirically determined by comparison with experimental data, such as light-nuclei yields.

Nuclear matter

By imposing equilibrium conditions on the collision integral→ f_τ^{eq} →Particle fractions (Standard thermal model + in-medium effects on light clusters)

Infer light-cluster compositions of nuclear matter at given conditions(densities and temperatures where light clusters freeze out in HICs)

Collision integral $\longrightarrow$ equilibrium

Taking the example of $Nnp \leftrightarrow Nd$, at equilibrium

$$f_\tau(\vec{p}_1) f_n(\vec{p}_2) f_p(\vec{p}_3) [1 - f_\tau(\vec{p}_4)] [1 + f_d(\vec{p}_5)] = f_\tau(\vec{p}_4) f_d(\vec{p}_5) [1 - f_\tau(\vec{p}_1)] [1 - f_n(\vec{p}_2)] [1 - f_p(\vec{p}_3)]$$

Pauli blocking   Bose enhancement

For light clusters $\dfrac{f_d}{1 + f_d} \sim e^{-\beta \epsilon_d}$ $\Longrightarrow$ Bose(Fermi) distribution

The in-medium effects enter the light-cluster distribution function by means of the Mott momentum P_ν^{Mott}

$$\boxed{f_\nu(\vec{P}) = \frac{H\left(|\vec{P}| - P_\nu^{\mathrm{Mott}}\right)}{\exp\{\beta[\epsilon_\nu(\vec{P}) - \mu_\nu]\} \pm 1}}$$

Heaviside step function

Clustered Nuclear matter

In nuclear matter, for given $\beta(T)$, ρ_n^{tot}, and ρ_p^{tot}

- Light clusters f_ν

$$f_\nu^{\mathrm{eq}}(\vec{P}) = \frac{H\left(|\vec{P}| - P_\nu^{\mathrm{Mott}}\right)}{\exp\{\beta[\epsilon_\nu(\vec{P}) - \mu_\nu]\} \pm 1}$$

- Nucleons f_τ

$$f_\tau^{\mathrm{eq}}(\vec{p}) = \frac{1 - f_\tau^{\mathrm{clu}}(\vec{p})}{\exp\{\beta[\epsilon_\tau(\vec{p}) - \mu_\tau]\} + 1}$$

- Obtain Mott momenta P_ν^{Mott} from the criterion

$$f_\tau^{\mathrm{eq}}(\vec{p}) + f_\tau^{\mathrm{clu}}(\vec{p})$$

$$\langle f_\tau \rangle_\nu(\vec{P}) \equiv \int \tilde{f}_\tau\left(\frac{\vec{P}}{A} + \vec{q}\right)|\phi_\nu(\vec{q})|^2 \mathrm{d}\vec{q} \leqslant F_A^{\mathrm{cut}}$$

Equilibrium condition

$$\mu_\nu = N\mu_n + Z\mu_p$$

The bound-state contribution to nucleon occupation

$$f_n^{\mathrm{clu}}(\vec{p}) = \sum_\nu^{\mathrm{clu}} N \int \frac{\mathrm{d}\vec{P}}{(2\pi\hbar)^3} f_\nu^{\mathrm{eq}}(\vec{P}) |\phi_{\nu,\vec{P}}(\vec{p})|^2$$

$$f_p^{\mathrm{clu}}(\vec{p}) = \sum_\nu^{\mathrm{clu}} Z \int \frac{\mathrm{d}\vec{P}}{(2\pi\hbar)^3} f_\nu^{\mathrm{eq}}(\vec{P}) |\phi_{\nu,\vec{P}}(\vec{p})|^2$$

Binding energies

Probability distributions (internal wave functions)

As an approximation, their free-space values are adopted

Obtain μ_n, μ_p, P_d^{Mott}, P_t^{Mott}, P_h^{Mott}, P_α^{Mott}, and thus f_ν and f_τ

R. Wang, **ZZ**, S. Burrello, et al., arXiv:2506.16437

Bayesian inference

Data: d, t, h, α yields in Au+Au central collisions at energies of 0.25 - 0.6*A* GeV.

- Influence of spinodal decomposition (larger clusters) in the low-energy region.
- Pion catalysis reactions, i.e., $\pi NN \leftrightarrow \pi d$, may contribute to the light nuclei yields at $E_{beam} > 0.6A$ GeV

surmise 0.2.1

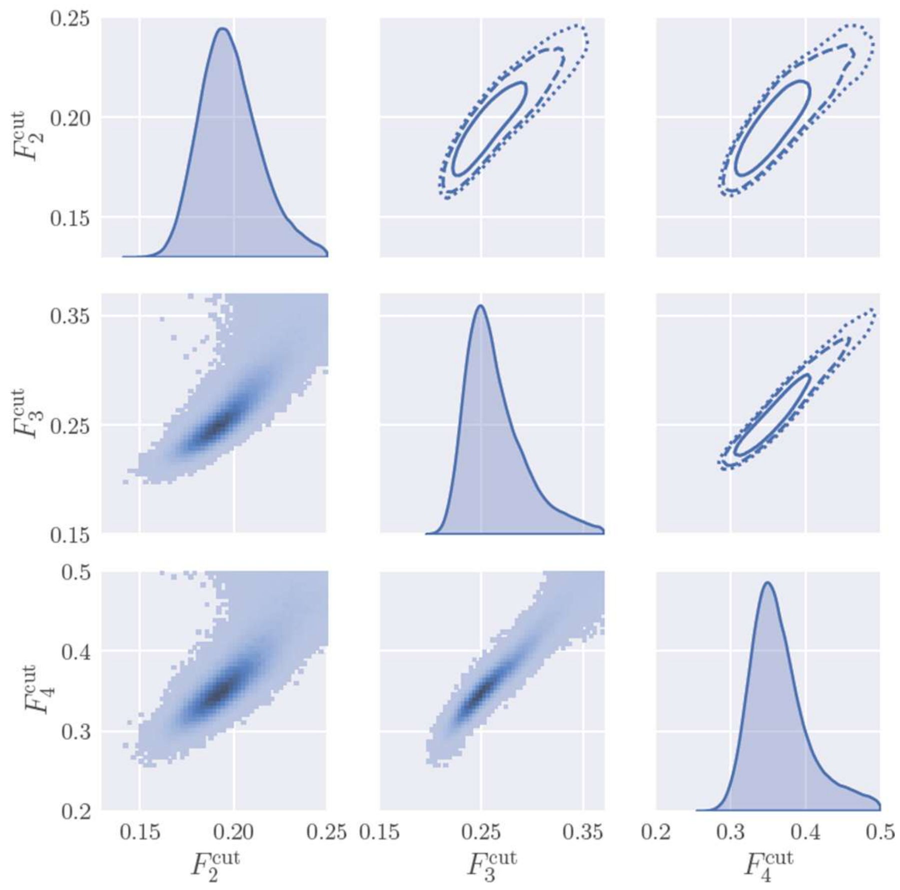
```
pip install surmise
```

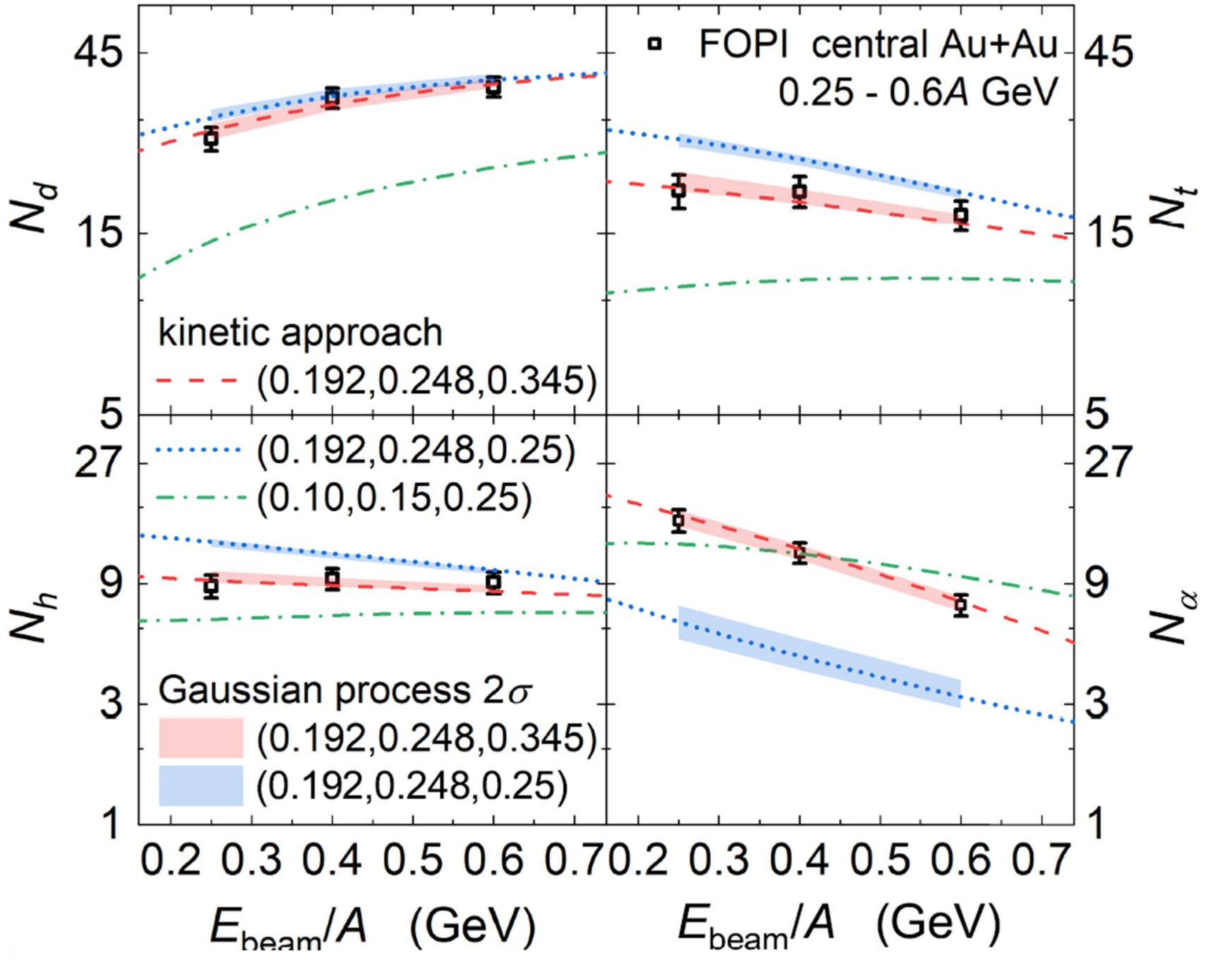
- 45 parameter sets from **Latin Hypercube Sampling**
- **Kinetic approach** calculation
- **Emulator**: Principal component analysis + Gaussian process
- Metropolis-Hastings (MH) algorithm for **MCMC**

BAND

Bayesian Analysis of Nuclear Dynamics

Light-nuclei yields





90% confidence level: $F_2^{cut} = 0.192^{+0.023}_{-0.013}$, $F_3^{cut} = 0.248^{+0.046}_{-0.023}$, and $F_4^{cut} = 0.345^{+0.073}_{-0.030}$.

R. Wang, Z. Zhang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, in preparation

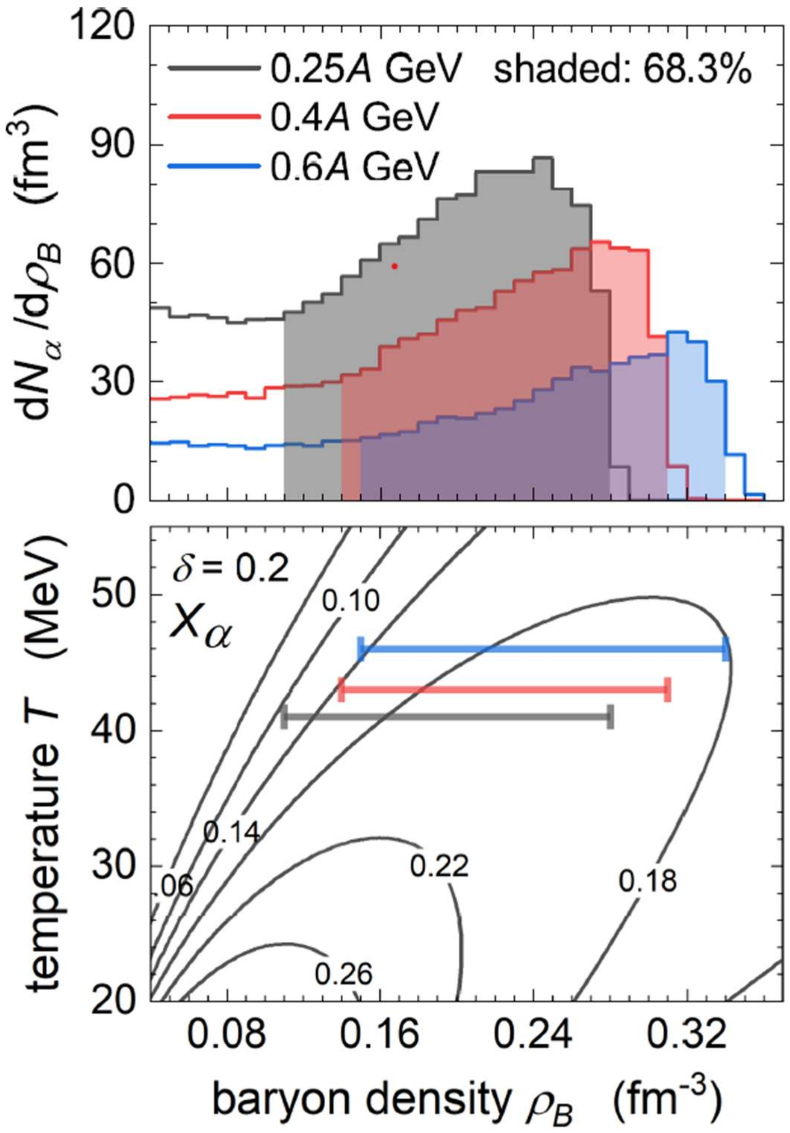
α-particle fraction

- According to the present kinetic approach, in intermediate-energy heavy-ion collisions, light nuclei are mainly formatted and freeze-out chemically at high densities ($NNNNN \leftrightarrow N\alpha$), especially for α-particles.

Contradict!

- It is generally thought that the dense nuclear matter can be regarded almost as a uniform nucleon liquid.

α-particle fraction ~ 0.2 for in nuclear matter at around $\rho = 0.266\ \mathrm{fm}^{-3}$ with T = 19.5 MeV, $\rho = 0.306\ \mathrm{fm}^{-3}$ with T = 33.9 MeV, and $\rho = 0.340\ \mathrm{fm}^{-3}$ with T = 51.1 MeV.



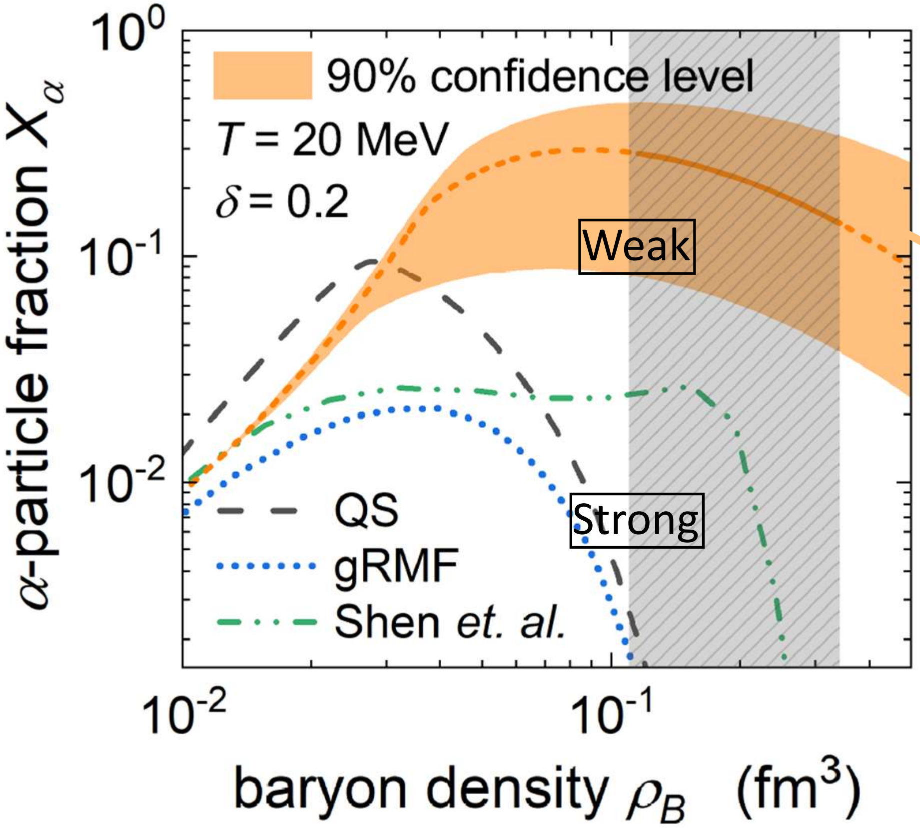
R. Wang, **ZZ**, Y.-G. Ma, et al., arXiv:2507.16613

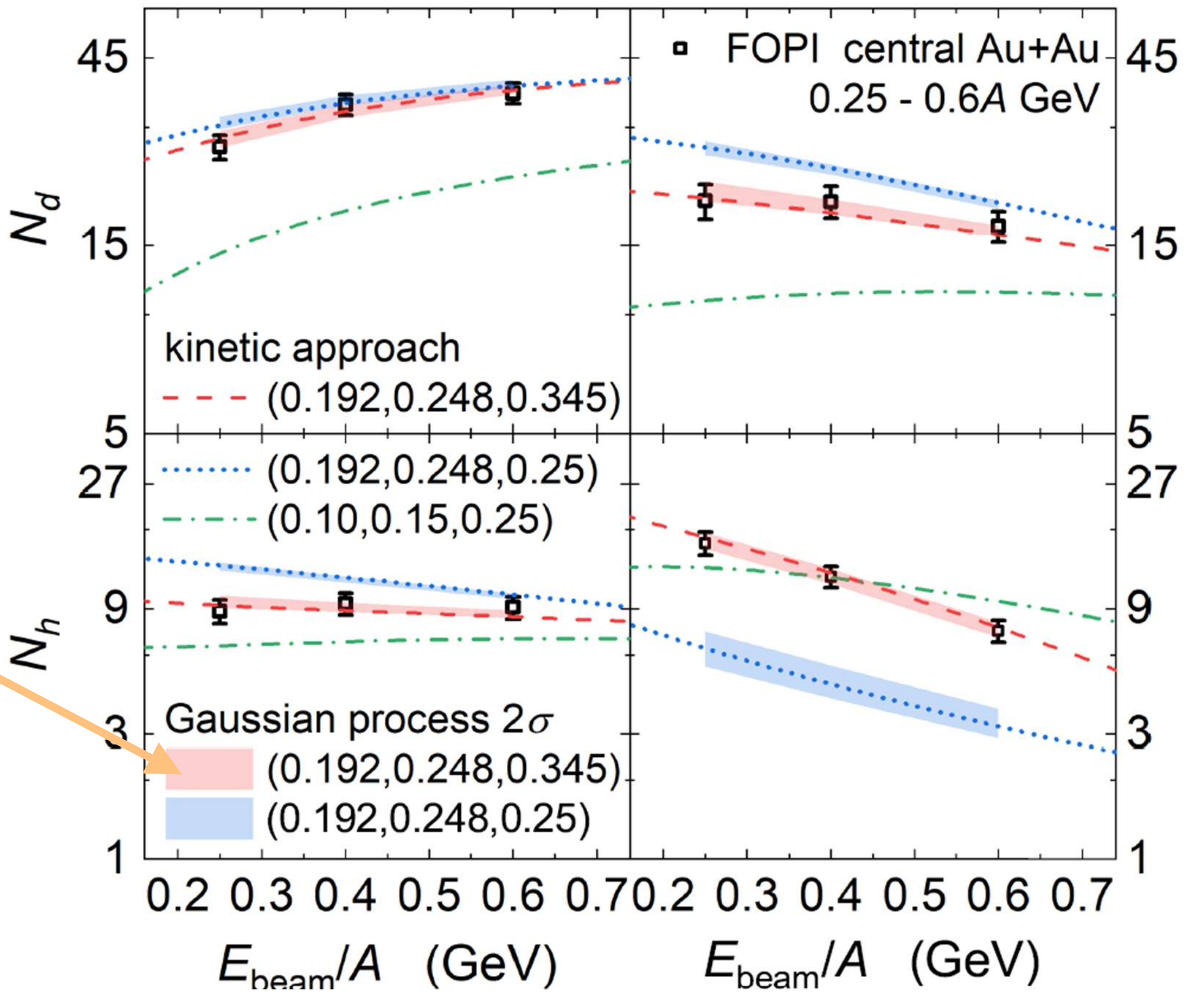
α-particle fraction

Comparison with other approaches

- QS & gRMF - Mott momentum from in-medium Schrodinger equation in low density regions
- Shen et. al - exclusion of α-particle in coordinate space

T = 20 MeV





R. Wang, **ZZ**, Y.-G. Ma, et al., arXiv:2507.16613

Summary

- The FOPI data on light-nuclei yields can be reasonably reproduced within the present kinetic approach.
- Large yield of α-particle is a consequence of its weaker Mott effect.
- FOPI data of light-nuclei yields indicate an unexpectedly **high α-particle fraction** in **warm and dense nuclear matter.**

Acknowledgements

- Rui Wang, Stefano Burrello, Maria Colonna, Edoardo G. Lanza (INFN)
- Lie-Wen Chen (SJTU)
- Yu-Gang Ma, Kai-Jia Sun (FDU)
- Che Ming Ko (TAMU)

Thanks for your attention

Light nuclei in kinetic approach

Kinetic equations are derived based on the closed time-path Green's function formulism

For example, in the deuteron case, the two-body Green's function G_2 satisfies an equation

$$G_2 = \mathcal{G}_2 + \frac{1}{4}\mathcal{G}_2 v G_2$$

The light nuclei are realized as poles of the many-body Green's function.
In the vicinity of the pole, we have

$$i\left\langle x \left| G_2^<(P,\Omega,R,T) \right| x' \right\rangle \sim \langle x | \phi(P,R,T) | \rangle \langle \phi(P,R,T) x' \rangle f_2(P,R,T) 2\pi\delta[\Omega - E(P,R,T)]$$

$$i\left\langle x \left| G_2^>(P,\Omega,R,T) \right| x' \right\rangle \sim \langle x | \phi(P,R,T) | \rangle \langle \phi(P,R,T) x' \rangle \left[1 + f_2(P,R,T)\right] 2\pi\delta[\Omega - E(P,R,T)]$$

P. Danielewicz and G. F. Bertsch, Nuclear Physics A533, 712-748 (1991)

Finally leads to equations of the occupation number f_τ of light nuclei

$$\left(\partial_t + \vec{\nabla}_p \epsilon_\tau \cdot \vec{\nabla}_r - \vec{\nabla}_r \epsilon_\tau \cdot \vec{\nabla}_p\right) f_\tau = \mathcal{K}_\tau^< \left[f_n, f_p, f_d, \cdots\right] \left(1 \pm f_\tau\right) - \mathcal{K}_\tau^> \left[f_n, f_p, f_d, \cdots\right] f_\tau, \quad \tau = n, p, d, t, h, \alpha$$

stochastic approach

$$\mathscr{C}_{22} = \frac{1}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{v}\int\frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'}f_1'f_2'$$
$$\times\left|\mathscr{M}_{1'2'\to 12}\right|^2(2\pi)^4\delta^{(4)}\left(p_1'+p_2'-p_1-p_2\right)-\frac{1}{2E_1}$$
$$\times\int\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{v}\int\frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'}f_1f_2$$
$$\times\left|\mathscr{M}_{12\to 1'2'}\right|^2(2\pi)^4\delta^{(4)}\left(p_1+p_2-p_1'-p_2'\right).$$

$$f_i = \frac{\Delta N_i}{\frac{1}{(2\pi)^3}\Delta^3x\Delta^3p_i},\quad i=1,2$$

$$\sigma_{22} = \frac{1}{2s}\frac{1}{v}\int\frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'}\left|\mathscr{M}_{12\to 1'2'}\right|^2$$
$$\times(2\pi)^4\delta^{(4)}\left(p_1+p_2-p_1'-p_2'\right),$$

$$\frac{\Delta N_{\mathrm{coll}}^{2\to 2}}{\Delta t\frac{1}{(2\pi)^3}\Delta^3x\Delta^3p_1} = \frac{1}{2E_1}\frac{\Delta^3p_2}{(2\pi)^3 2E_2}f_1f_2$$
$$\times\frac{1}{v}\int\frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'}\left|\mathscr{M}_{12\to 1'2'}\right|^2$$
$$\times(2\pi)^4\delta^{(4)}\left(p_1+p_2-p_1'-p_2'\right)$$

$$P_{22} = \frac{\Delta N_{\mathrm{coll}}^2}{\Delta N_1\Delta N_2} = v_{\mathrm{rel}}\sigma_{22}\frac{\Delta t}{\Delta^3x}$$

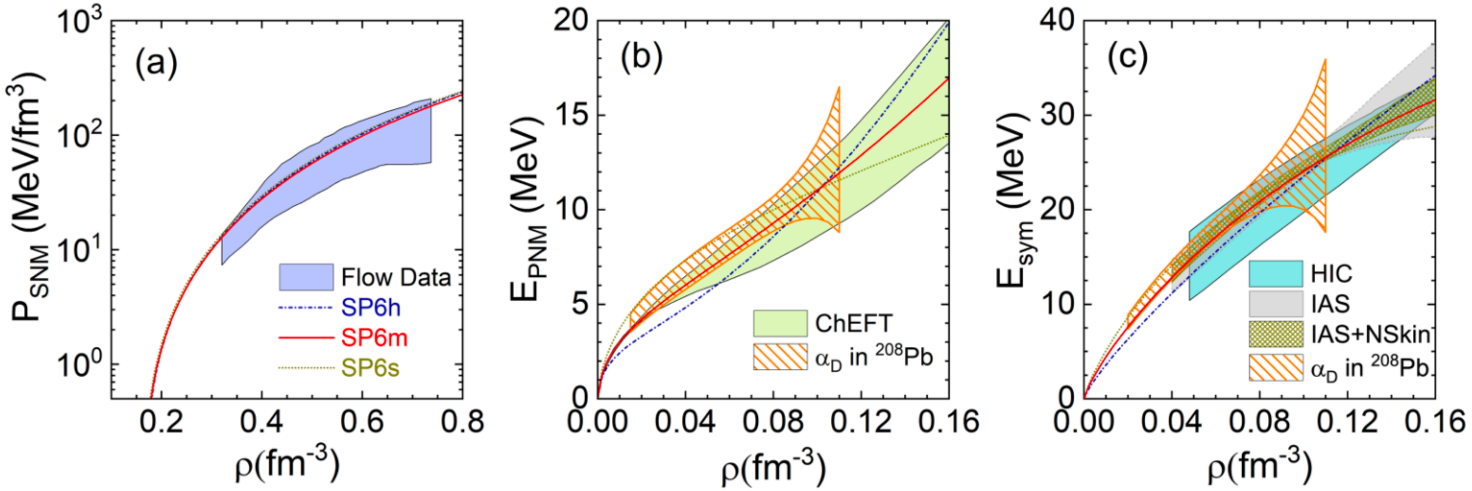
Skyrme pseudo potential

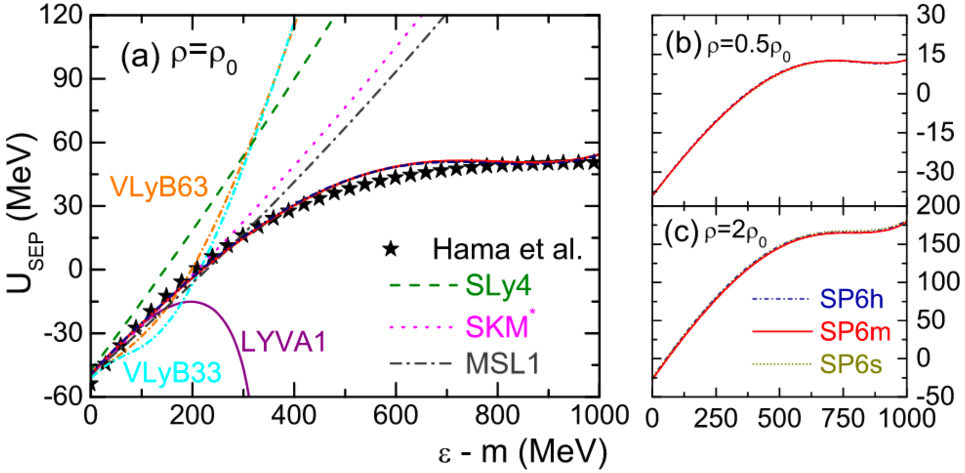
PHYSICAL REVIEW C **98**, 054618 (2018)

Extended Skyrme interactions for transport model simulations of heavy-ion collisions

Rui Wang, Lie-Wen Chen,* and Ying Zhou

SP6m





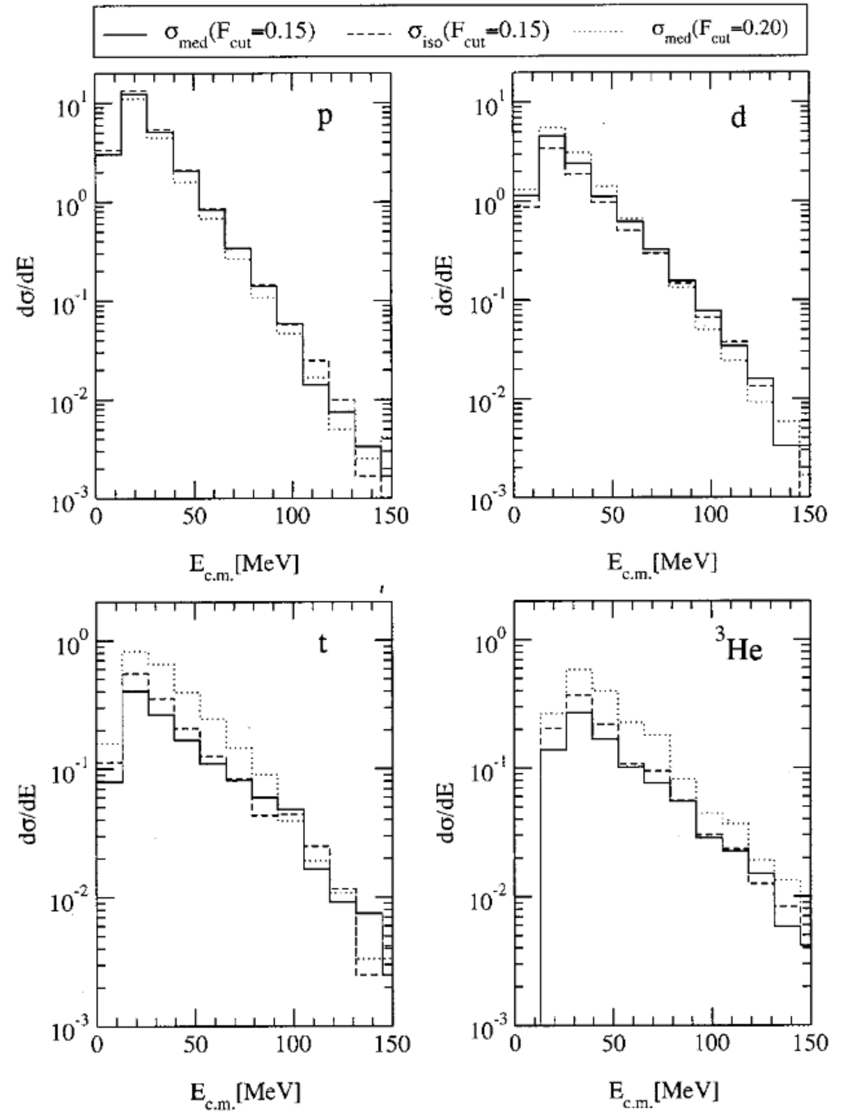
Light nuclei in kinetic approach

- Production of deuterons and pions in a transport model of energetic heavy-ion reactions
  P. Danielewicz and G. F. Bertsch, NPA533, (1991) 712-748
  Establish the framework, A = 2 light nucleus was introduced.

- Blast of light fragments from central heavy-ion collisions
  P. Danielewicz and Q. Pan, PRC46, 2002-2011 (1992)
  A = 3 light nuclei were further introduced.

- Medium corrections in the formation of light charged particles in heavy ion reactions
  C. Kuhrts, M. Beyer, P. Danielewicz, and G. Ropke, PRC63, 034605 (2001)
  Medium corrections, especially the Mott effect, were examined for light nuclei with A <= 3.

D. D. S. Coupland, W. G. Lynch, M. B. Tsang, P. Danielewicz and Y. Zhang, PRC 84, 054603 (2011)
Influence of light nuclei on isospin transport ratios was studied



C. Kuhrts, et. al., PRC63, 034605 (2001)

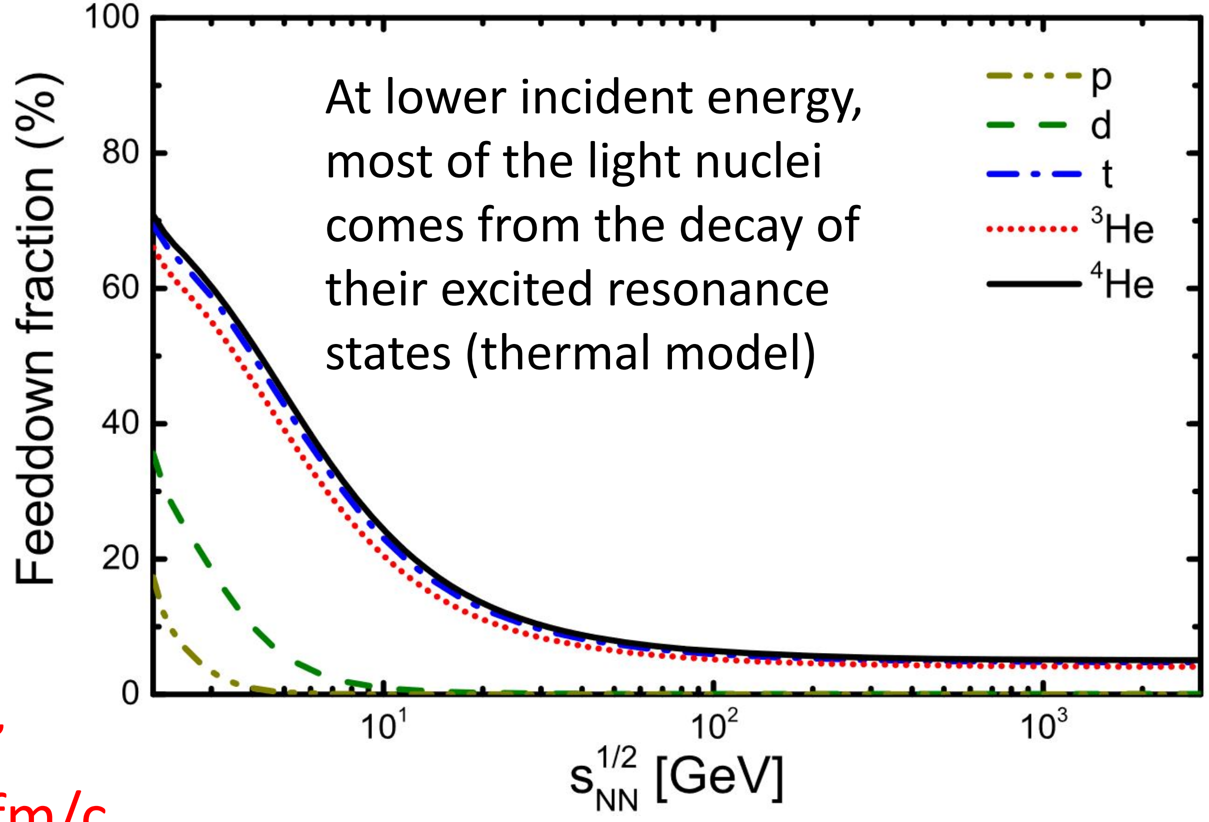
Feeddown effect from unstable nuclei

$A=4$	E_x (MeV)	J^π	Decay channels
^{4}H	g.s.	2^-	n(100%)
	0.31	1^-	n(100%)
	2.08	0^-	n(100%)
	2.83	1^-	n(100%)
^{4}He	g.s.	0^+	stable
	20.21	0^+	p(100%)
	21.01	0^-	n(23.8%), p(76.2%)
	21.84	2^-	n(37.3%), p(62.7%)
	23.33	2^-	n(47.3%), p(52.7%)
	23.64	1^-	n(44.5%), p(55.5%)
	24.25	1^-	n(47.0%), p(50.5%), d(2.5%)
	25.28	0^-	n(48.3%), p(51.7%)
	25.95	1^-	n(48.5%), p(51.5%)
	27.42	2^+	n(3%), p(3%), d(94%)
	28.31	1^+	n(47%), p(48%), d(5%)
	28.37	1^-	n(2%), p(2%), d(96%)
	28.39	2^-	n(0.25%), p(0.25%), d(99.5%)
	28.64	0^-	d(100%)
	28.67	2^+	d(100%)
	29.89	2^+	n(0.4%), p(0.4%), d(99.2%)
^{4}Li	g.s.	2^-	p(100%)
	0.32	1^-	p(100%)
	2.08	0^-	p(100%)
	2.85	1^-	p(100%)

$A=5$	E_x (MeV)	J^π	Decay channels
^{5}H	g.s.	$\frac{1}{2}^+$	2n(100%)
^{5}He	g.s.	$\frac{3}{2}^-$	n(100%)
	1.27	$\frac{1}{2}^-$	n(100%)
	16.84	$\frac{3}{2}^+$	n(60%), d(40%)
^{5}Li	g.s.	$\frac{3}{2}^-$	p(100%)
	1.49	$\frac{1}{2}^-$	p(100%)
	16.87	$\frac{3}{2}^+$	p(70%), n(30%)

Apart from the ground state nuclei, their excited resonance states can also be produced.

Lifetime ~ 20 – 200 fm/c



V. Vovchenko, et al., PLB 809 (2020) 135746

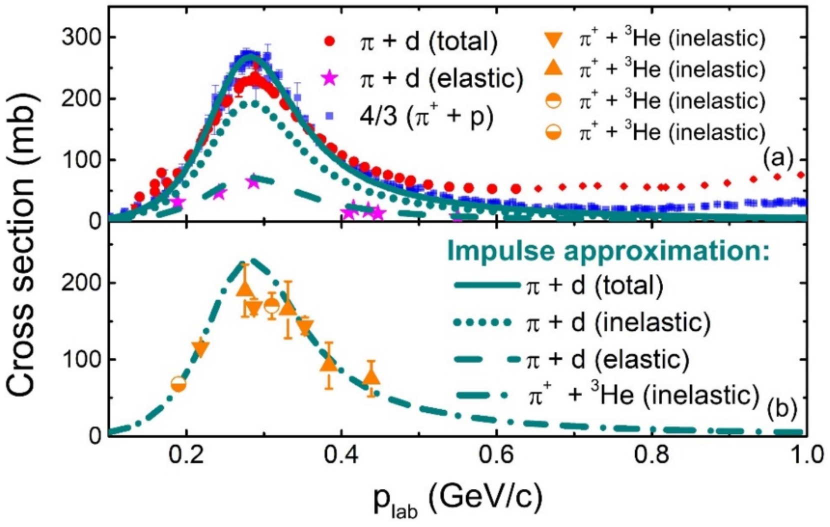
Impulse approximation

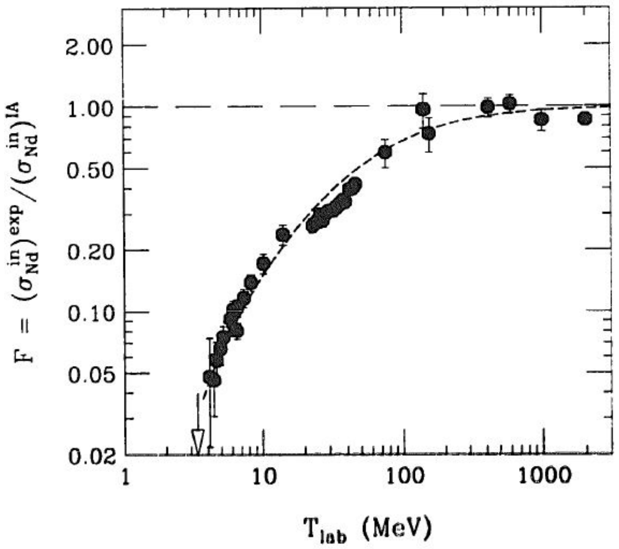
Only consider the momentum-space, or one can replace it with $\left\langle \vec{r}_N \vec{p}_N \middle| \phi_{LN} \right\rangle$ to including the internal degrees of freedom of light nuclei

One can evaluate the scattering probabilities of many-body scatterings based on the impluse approximation.

$$\underbrace{\overline{|M^{\pi pn\to\pi d}|^2} = \overline{|M^{\pi d\to\pi pn}|^2}}_{\text{Detailed balance}} \approx \underbrace{F\left(2m_d|\langle\vec{p}_p|\phi_d\rangle|^2\overline{|M^{\pi n\to\pi n}|^2} + 2m_d|\langle\vec{p}_n|\phi_d\rangle|^2\overline{|M^{\pi p\to\pi p}|^2}\right)}_{\text{Impulse approximation (IA)}}$$

F is a factor to exclude the elastic part from the total amplitude. It can be a constant (for pion catalysis) or a parameterization (for nucleon catalysis)





P. Danielewicz and G. F. Bertsch, NPA533, 712-748 (1991)

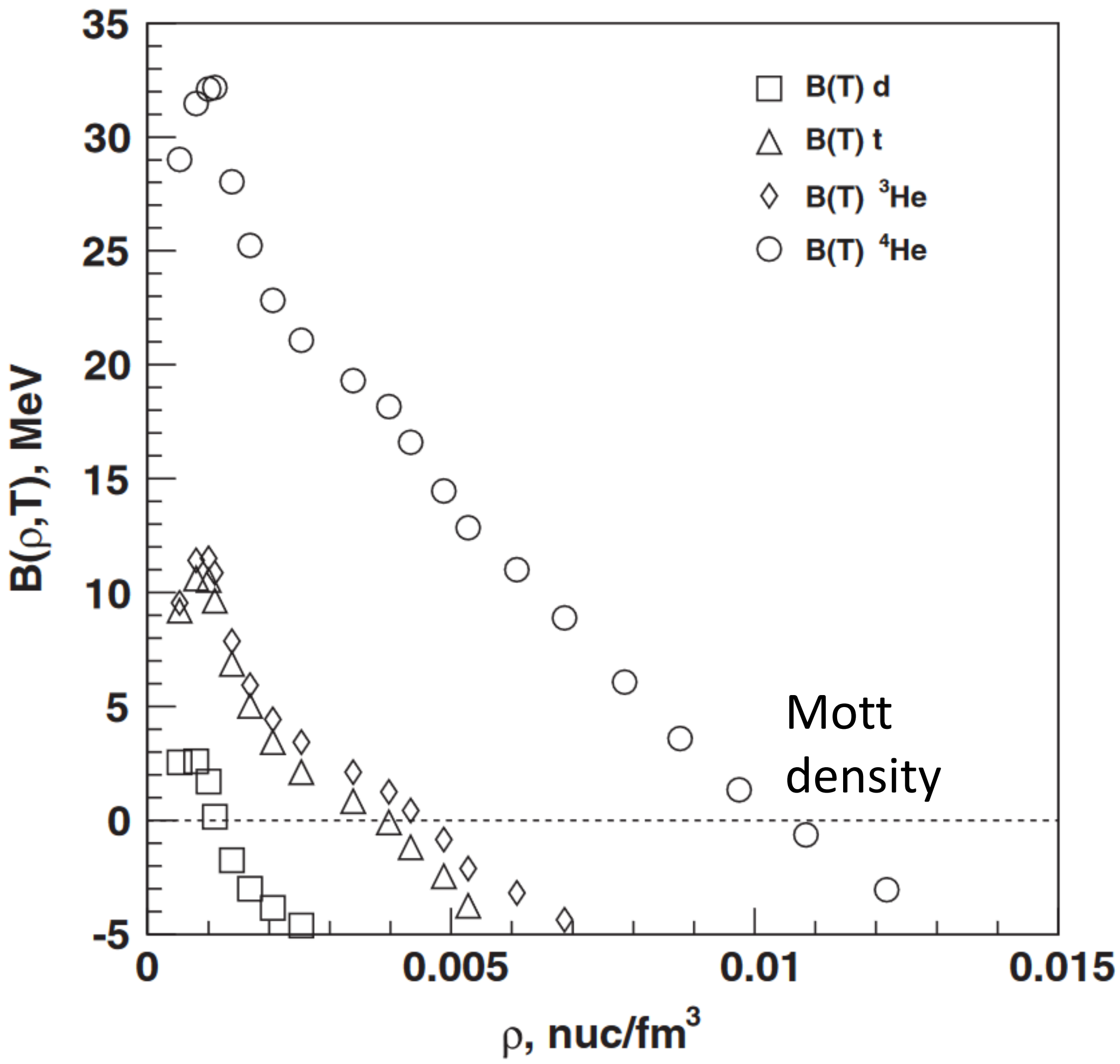
Mott point

A light nucleus can not bind if the surrounding nucleon phase space is too dense

Experimental deduction from light nuclei yields of HICs at NIMROD TAMU.

Based on the assumption that there exists a thermalized intermediate source during the reaction, the binding energies are obtained by a thermal fit of light nuclei yields.

$$\rho(A,Z) = \frac{N(A,Z)}{V} = \frac{A^{3/2}\lambda_T^{3(A-1)}\omega(A,Z)}{(2s_p+1)^Z(2s_n+1)^{A-Z}} \times \rho_p^Z \rho_n^{A-Z} \exp\left(\frac{B(A,Z)}{T}\right)$$



K. Hagel et. al., Physical Review Letter 108, 062702 (2012)

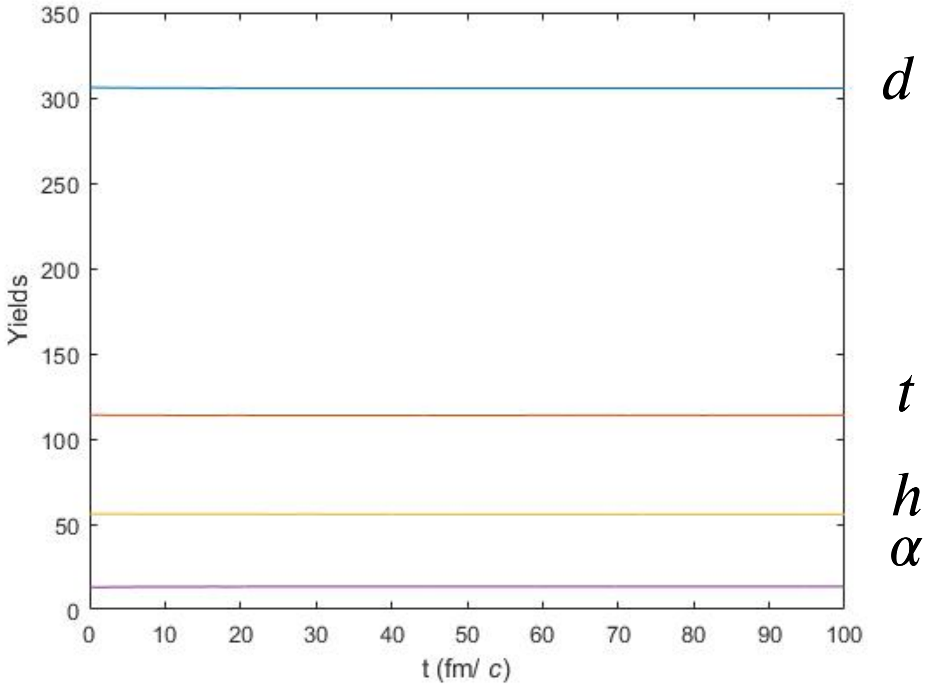
Test in box

T = 53 MeV, L = 20 fm

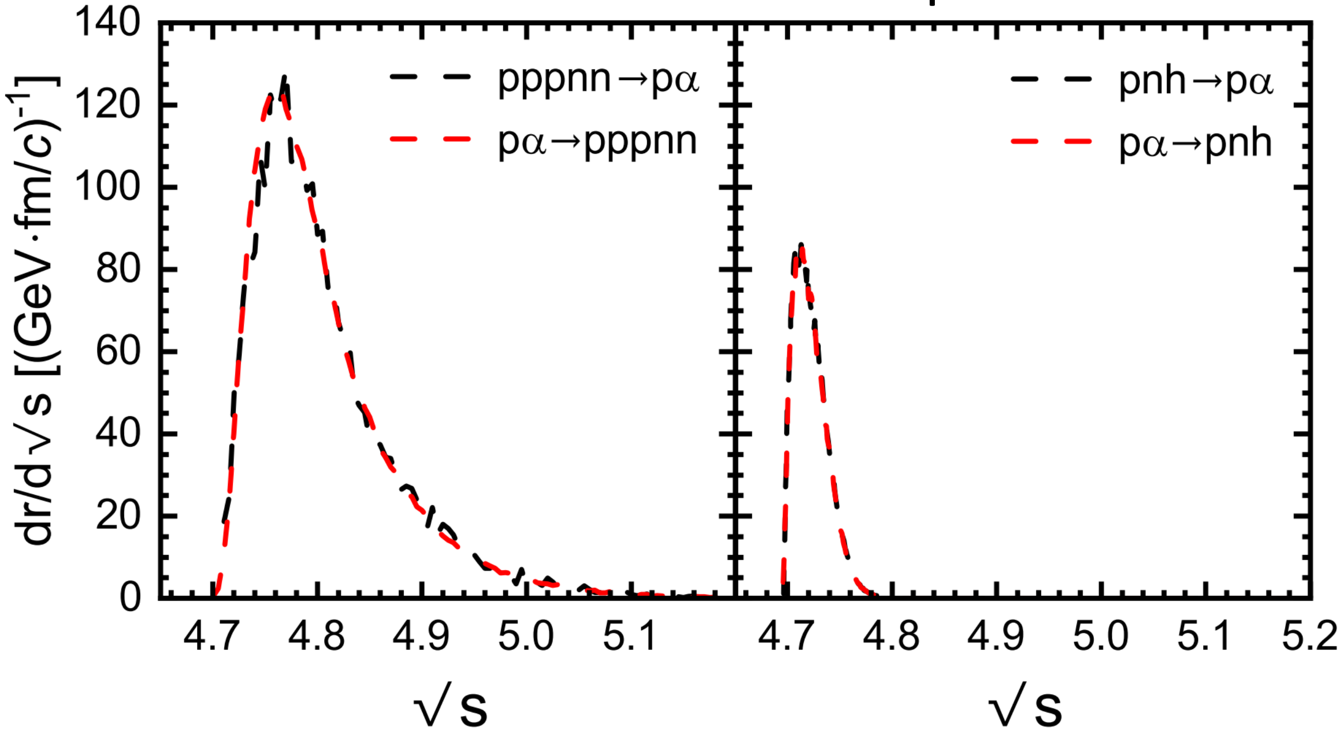
Neglecting quantum statistics, Mott effect, and binding energy of light nuclei

Initially, the system is in thermal equilibrium (Boltzmann distribution) with $N_p = 246$, $N_n = 502$, $N_d = 306$, $N_h = 56$, $N_t = 114$ and $N_\alpha = 13$

Time evolution of light nuclei yields



Collision rate dependence on $\sqrt{s}$ of two channels related to α-particle



Cross section and branching ratio

For light nuclei with A>2, they can be prodcued from different channels, thus the branching ratio of these channels should be consdiered (for example $\sigma_{\pi NNN\to\pi t}:\sigma_{\pi Nd\to\pi t}$). However, currently these quantities are not available experimentally. It seems there are two way to deal with it.

- Estimate them by the overlap of there internal wave function (e. g. $\langle NNN|t\rangle:\langle Nd|t\rangle$), as what we have done in our previous work (arXiv:2106.12742).
- Combine the contributions from different channels effectively into one single channel. This was adopted in Pawel's early work (Physical Review C 46, 2002-2011, 1992). However, one should test whether this simplification affect the final result.

Clustered Nuclear matter

Taking the example of $Nnp \leftrightarrow Nd$

$$f_\tau(\vec{p}_1)f_n(\vec{p}_2)f_p(\vec{p}_3)[1-f_\tau(\vec{p}_4)][1+f_d(\vec{p}_5)] = f_\tau(\vec{p}_4)f_d(\vec{p}_5)[1-f_\tau(\vec{p}_1)][1-f_n(\vec{p}_2)][1-f_p(\vec{p}_3)]$$

- Scenario 1: treat light clusters as completely independent degrees of freedom

$$\frac{f_\tau}{1-f_\tau} \sim e^{-\beta\epsilon_\tau} \quad \Longrightarrow \quad f_\tau^{\rm eq}(\vec{p}) = \frac{1}{\exp\{\beta[\epsilon_\tau(\vec{p})-\mu_\tau]\}+1}$$

$$\langle f_\tau\rangle_\nu(\vec{P}) = \int \tilde{f}_\tau\left(\frac{\vec{P}}{A}+\vec{q}\right)|\phi_\nu(\vec{q})|^2 {\rm d}\vec{q} \leqslant F_A^{\rm cut}$$

- Scenario 2: constituent nucleons bound in light clusters should also contribute to the one-body nucleon occupation (a quasi free nucleon can "feel" the Pauli blocking from both other quasi-free nucleons and constituent nucleons bound in light clusters)

The Pauli blocking factor in the collision integral is modified correspondingly

$$1-f_\tau(\vec{p}) \rightarrow 1-f_\tau(\vec{p})-f_\tau^{\rm clu}(\vec{p})$$

$f_\tau^{\rm clu}$ represents the bound-state contribution to nucleon occupation

$$\frac{f_\tau}{1-f_\tau-f_\tau^{\rm clu}} \sim e^{-\beta\epsilon_\tau} \quad \Longrightarrow \quad \boxed{f_\tau^{\rm eq}(\vec{p}) = \frac{1-f_\tau^{\rm clu}(\vec{p})}{\exp\{\beta[\epsilon_\tau(\vec{p})-\mu_\tau]\}+1}}$$

$$f_n^{\rm clu}(\vec{p}) = \sum_\nu^{\rm clu} N \int \frac{{\rm d}\vec{P}}{(2\pi\hbar)^3} f_\nu^{\rm eq}(\vec{P})|\phi_{\nu,\vec{P}}(\vec{p})|^2$$

$$f_p^{\rm clu}(\vec{p}) = \sum_\nu^{\rm clu} Z \int \frac{{\rm d}\vec{P}}{(2\pi\hbar)^3} f_\nu^{\rm eq}(\vec{P})|\phi_{\nu,\vec{P}}(\vec{p})|^2$$

$$f_\tau^{\rm eq}(\vec{p}) + f_\tau^{\rm clu}(\vec{p})$$

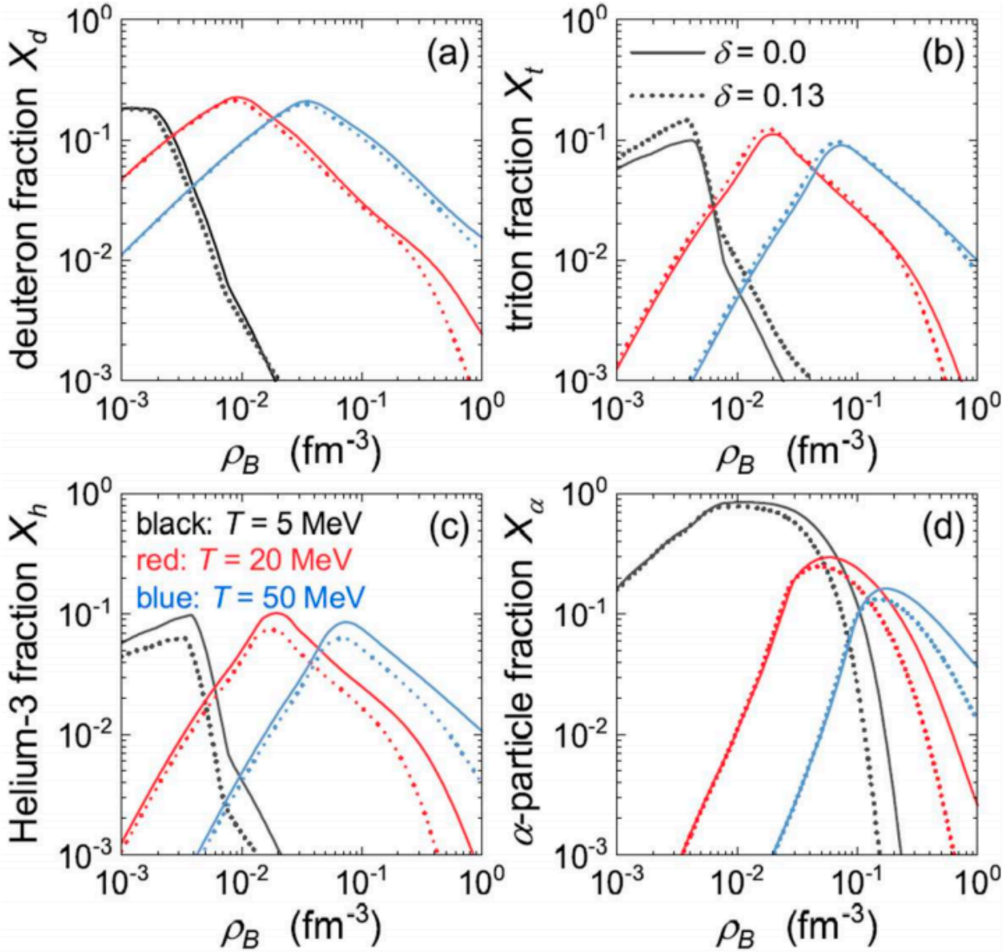
$$\langle f_N\rangle_\nu(\vec{P}) = \int \tilde{f}_N\left(\frac{\vec{P}}{A}+\vec{q}\right)|\phi_\nu(\vec{q})|^2 {\rm d}\vec{q} \leqslant F_A^{\rm cut}$$

Clustered Nuclear matter

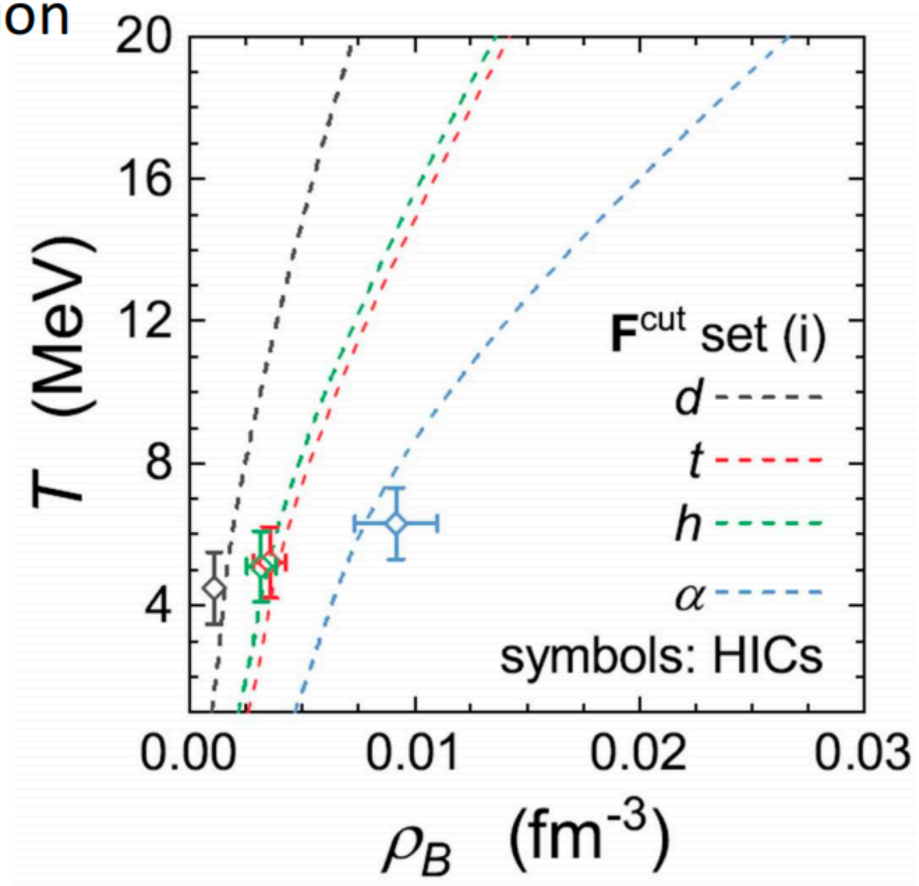
$F_2^{\text{cut}} = 0.1, F_3^{\text{cut}} = 0.15, F_4^{\text{cut}} = 0.25$

Employing standard Skyrme interaction to calculate $\epsilon_\tau(\vec{p})$ and $\epsilon_\upsilon(\vec{P})$

Light-cluster fractions



Mott densities: comparison with experimental deduction

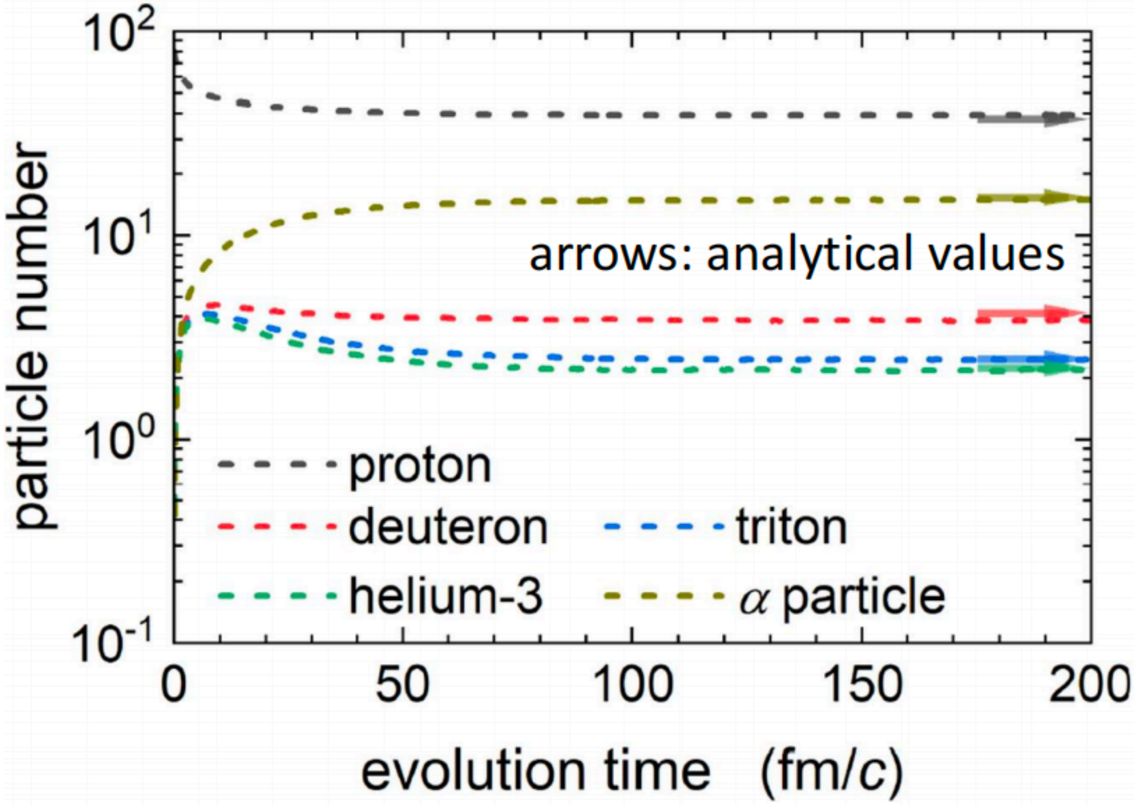


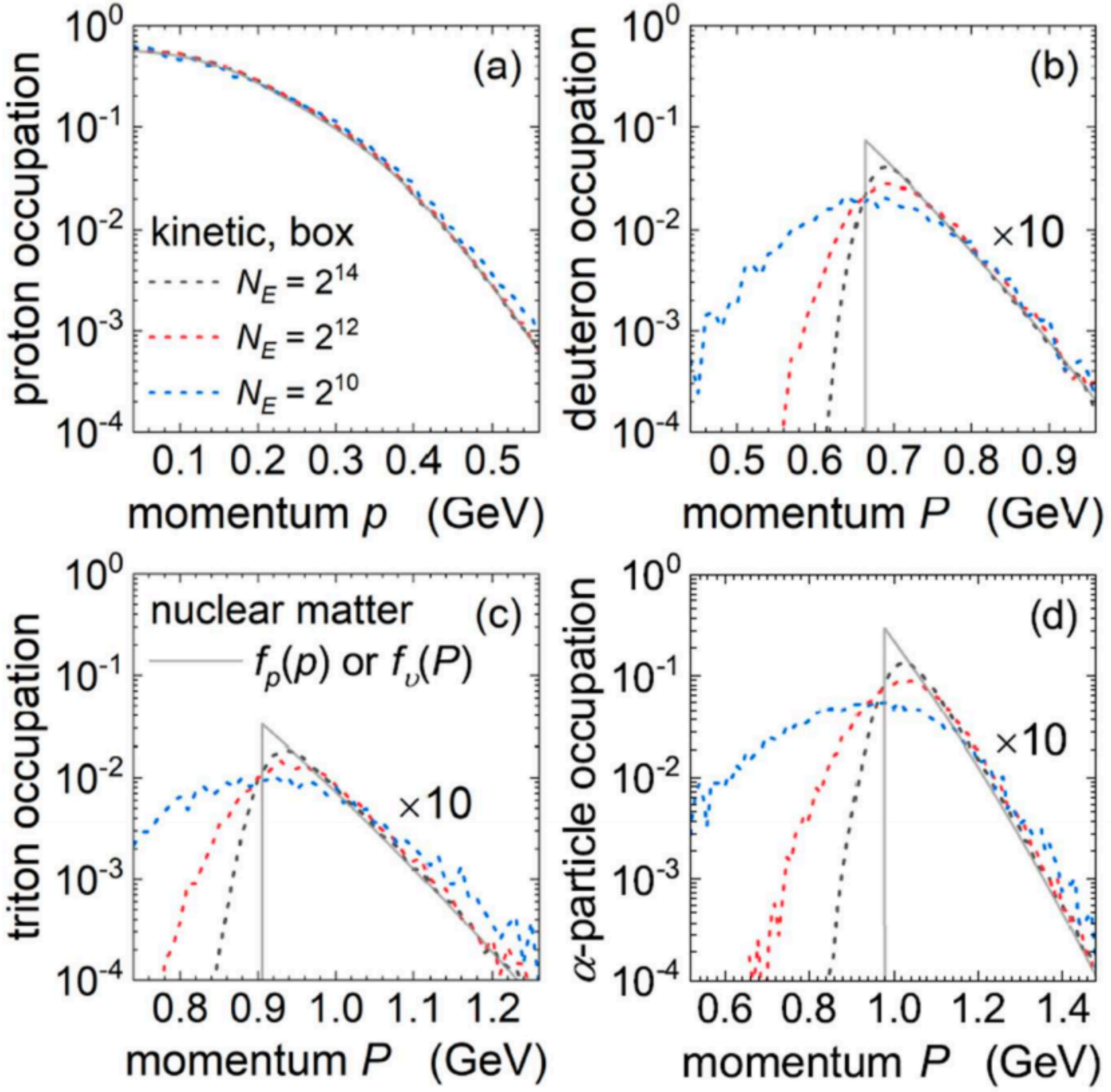
R. Wang, Z. Zhang, S. Burrello, et. al., arXiv:2506.16437 [nucl-th]

Numerical calculation of nuclear matter

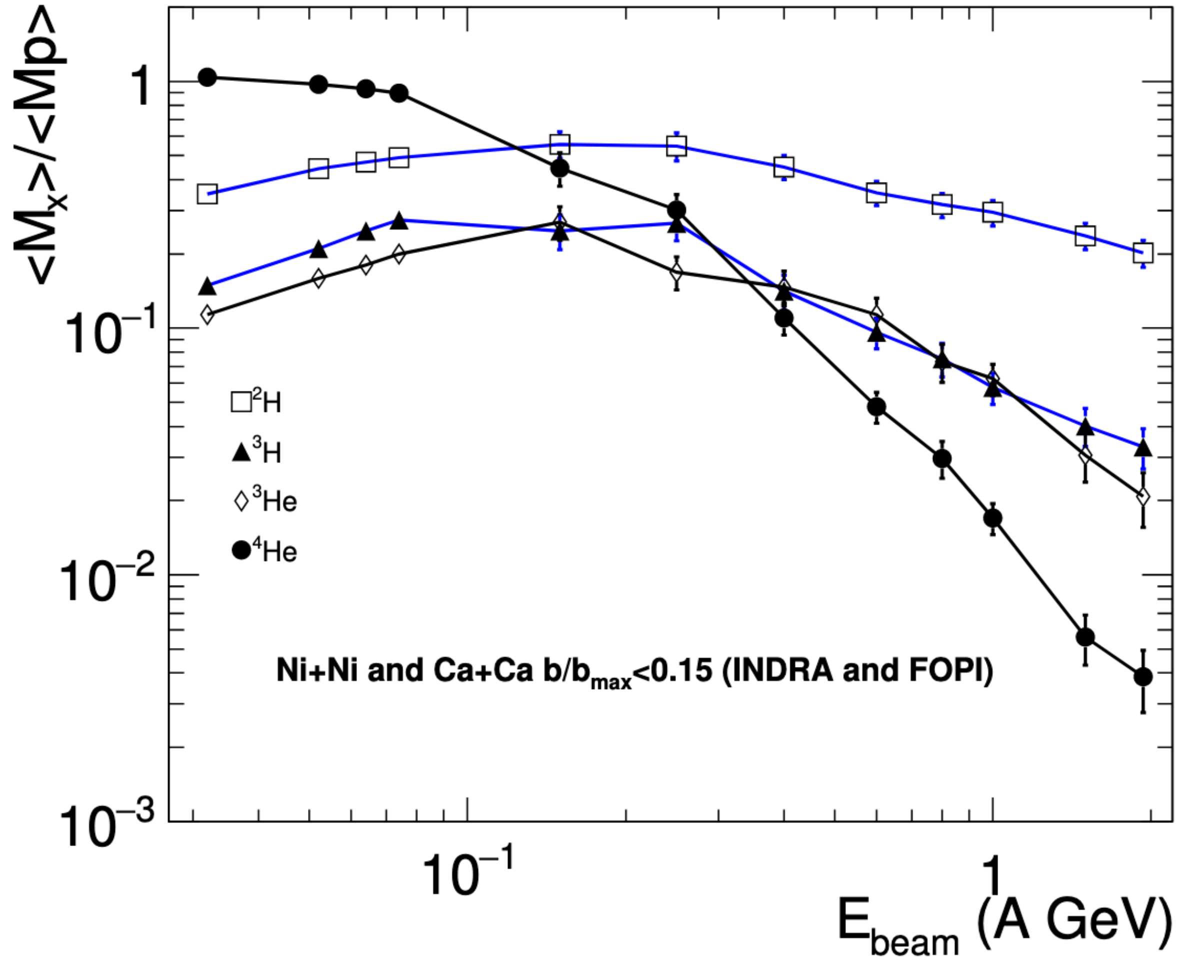
Validation: box calculation

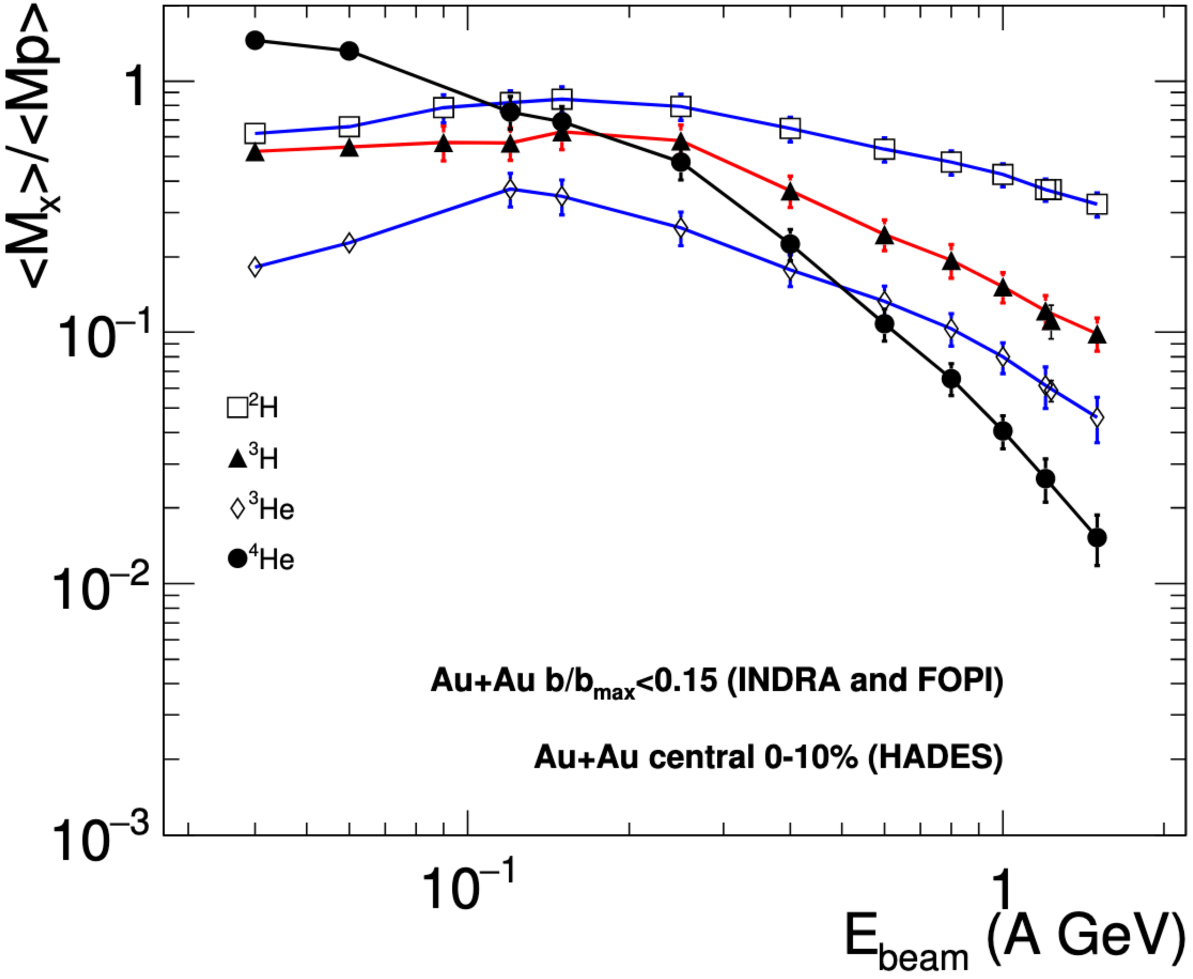
As the number of ensembles N_E (or number of test particles) used in the kinetic approach increases, the resulting light-cluster occupations approach their analytical counterparts





R. Wang, Z. Zhang, Y.-G. Ma, *et al.*, arXiv:2507.16613 [nucl-th]





R. Bougault, Symmetry 13, 1406 (2021).