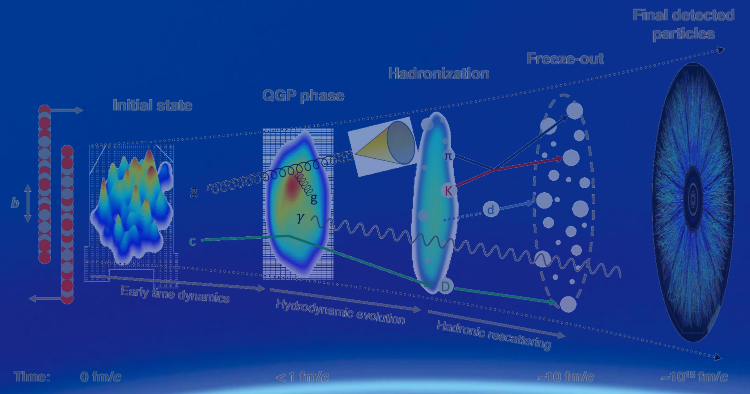
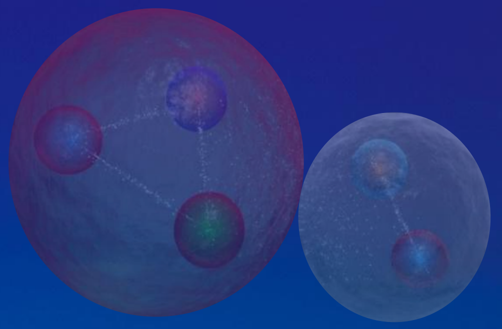
Non-equilibrium QCD and Transport Dec.09-12, 2025 Huizhou Guangdong Prov.

Isospin-dependent cross sections & equation of states at high densities



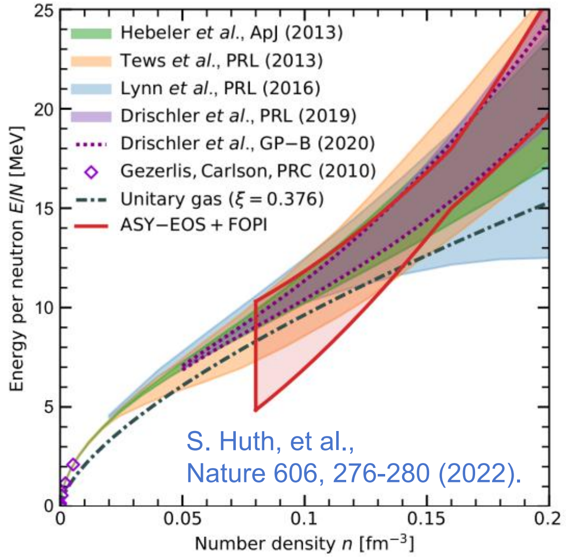
Qingfeng Li
Huzhou Uni.

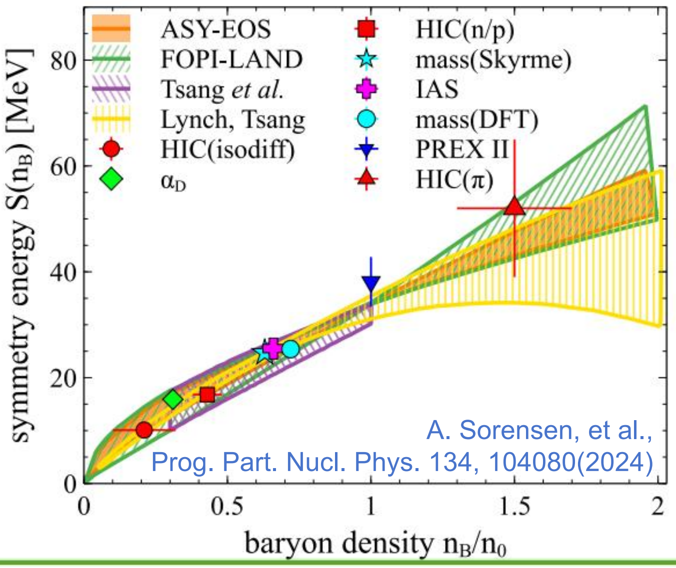


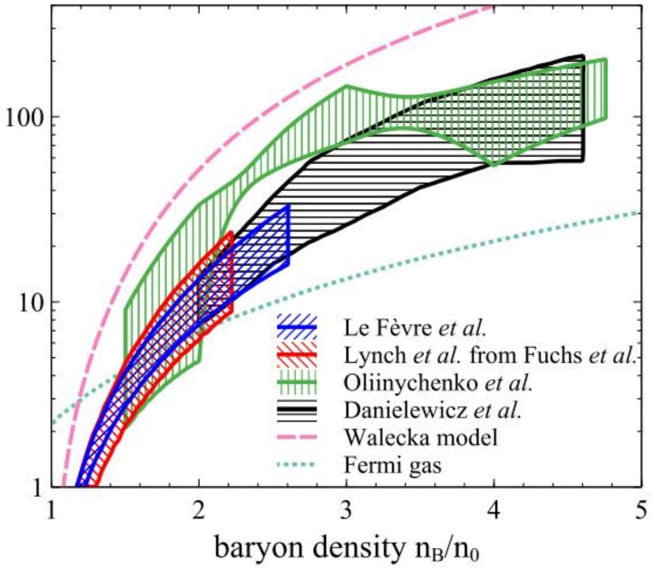
Collaborators: Manzi Nan, Yangyang Liu, Pengcheng Li, Yongjia Wang
Zhuxia Li, Yingxun Zhang, Zhigang Xiao, Gaochan Yong, Paolo Russotto

CONTENTS

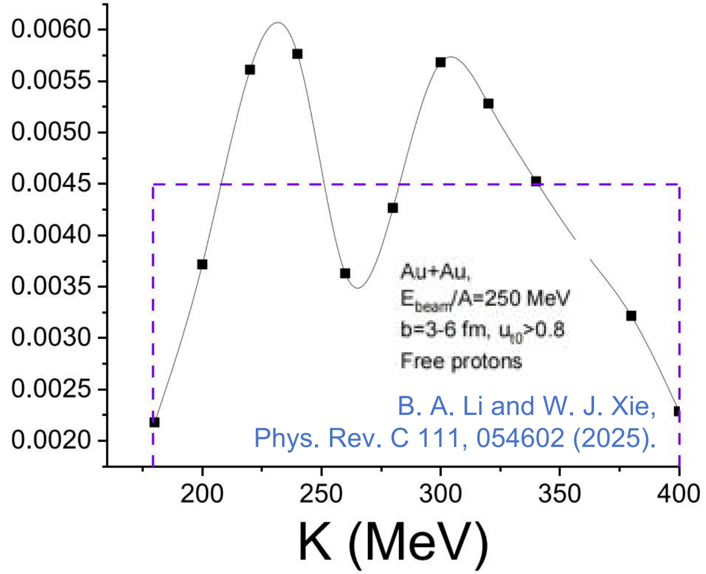
Ⅰ、Current status of EoS constraints (现状)

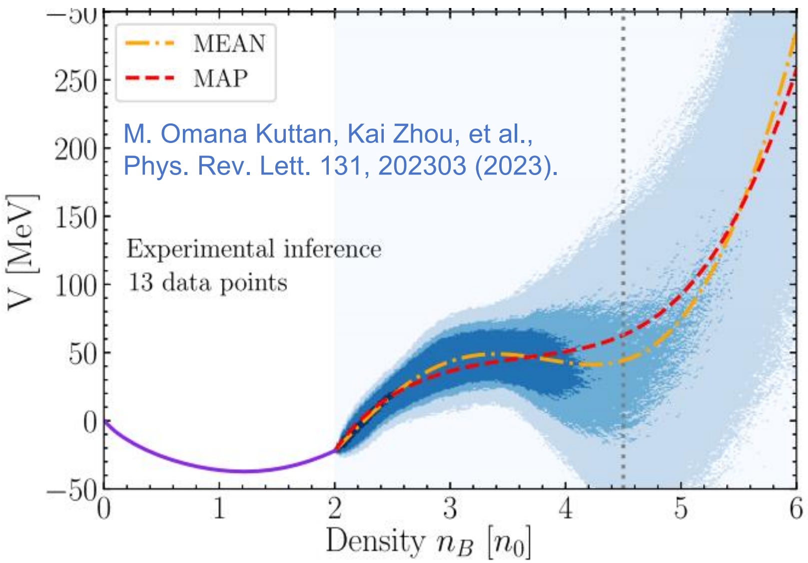


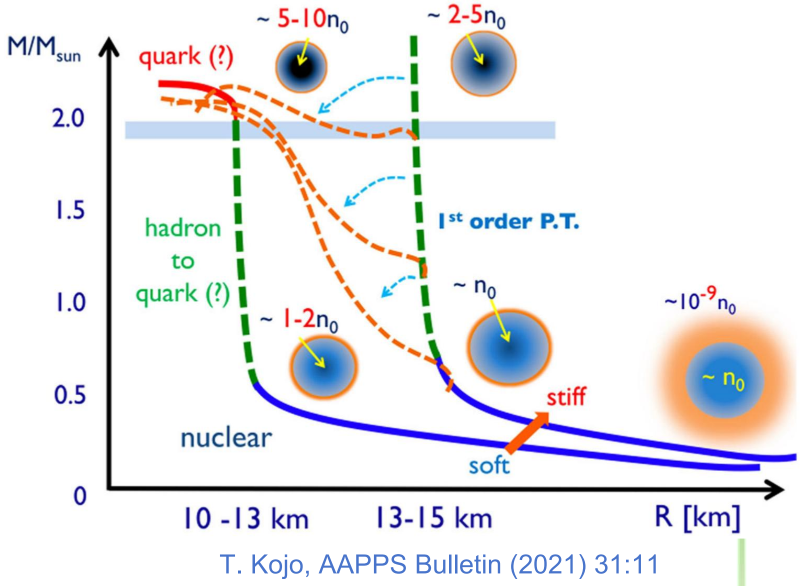




Larger uncertainties esp. at higher densities !!!





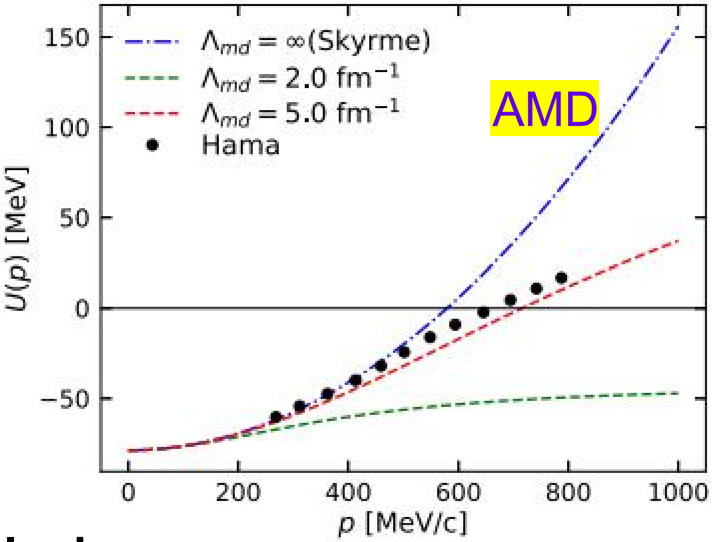


EoS vs. density: soft → stiff

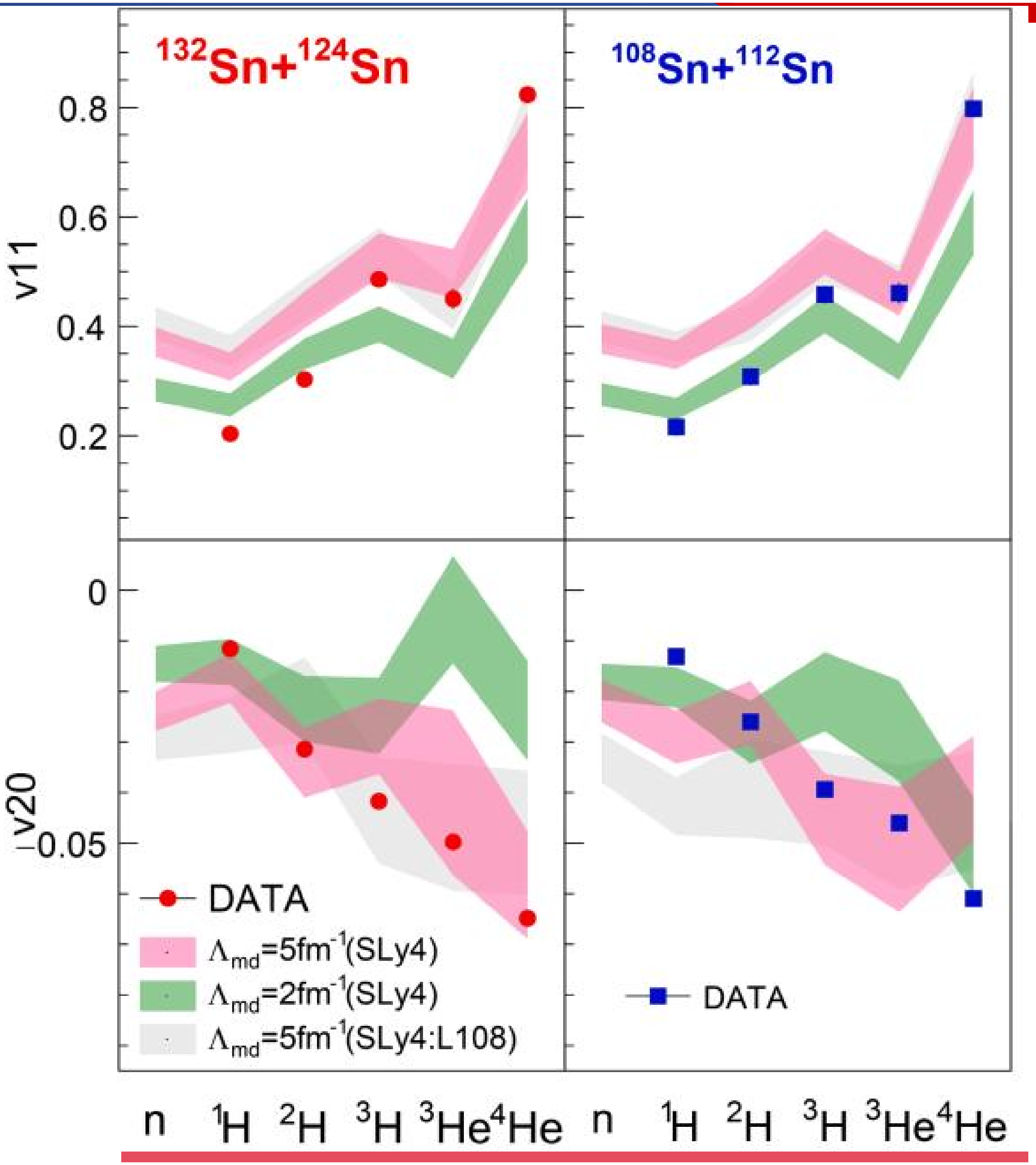
Ⅰ、status of SπRIT

Phys Lett B 813 (2021) 136016

Directed and elliptic flow observations in Sn + Sn collisions with radioactive beams at 270 MeV/u Phys. Lett. B 871 (2025) 139970



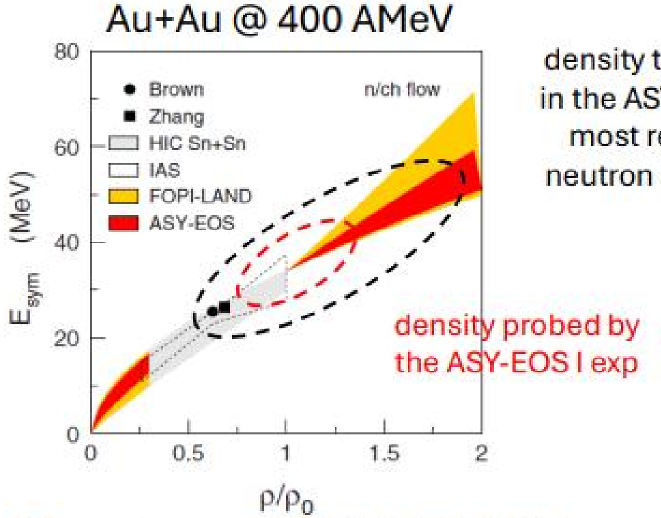
Main conclusions:
The results reveal that both v1 and v2 values are enhanced when the mean field incorporates stronger momentum dependence. In addition, v2 exhibits larger sensitivity to symmetry energy than v1 but the overall sensitivity to symmetry energy is low.
In an attempt to describe the data, different parameter sets have been tried. Most describe either the hydrogen flow or helium flow reasonably well but none describes the flow of all particles simultaneously.



Ⅰ、status of ASY-EOS I & II

Y2005:1st talk for GSI, since then, successful collaboration over 20 years!
Y2009: PPNP62, 425; Y2011:PLB697,471; Y2016:PRC94, 034608…

S122 (ASY-EOS II) : advancing Symmetry Energy studies towards high densities



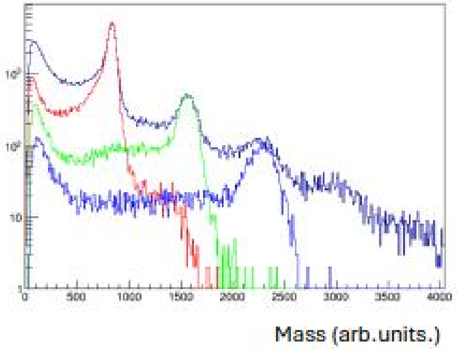
P. Russotto et al., PRC 94, 034608 (2016)

density to be probed in the ASY-EOS II exp, most relevant for neutron star physics

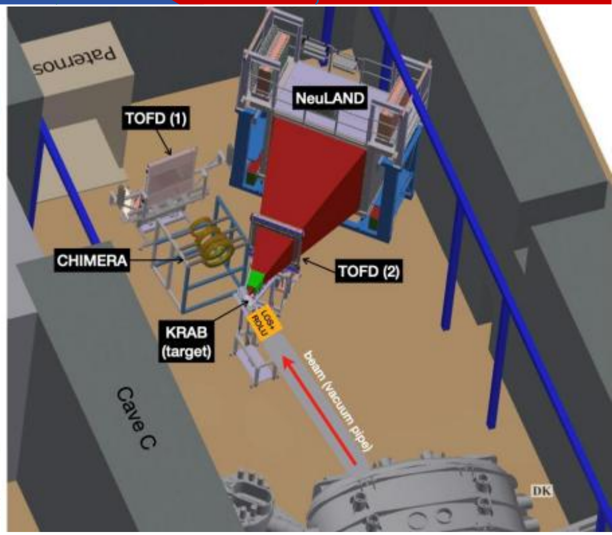
Energy [AMeV]
280
400
600
1000

improved resolution

KRAB: new detector for reaction plane determination and on-beam centrality selection

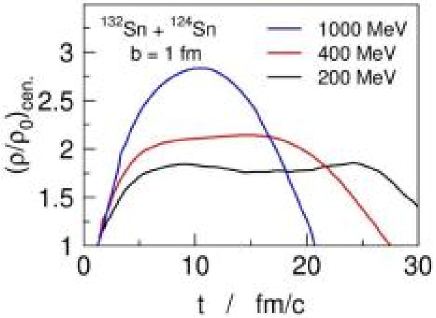


NeuLAND: capability to resolve p,d,t

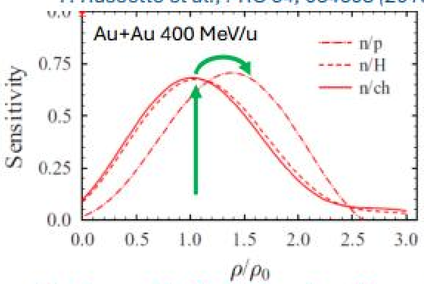


and higher densities

Li, Bao-An, NPA 708, 365 (2002)

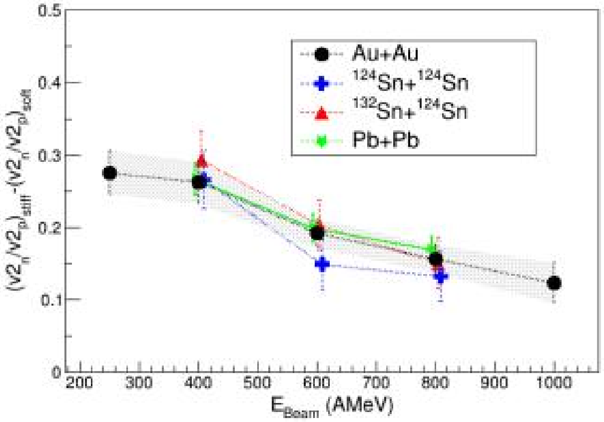


P. Russotto et al., PRC 94, 034608 (2016)

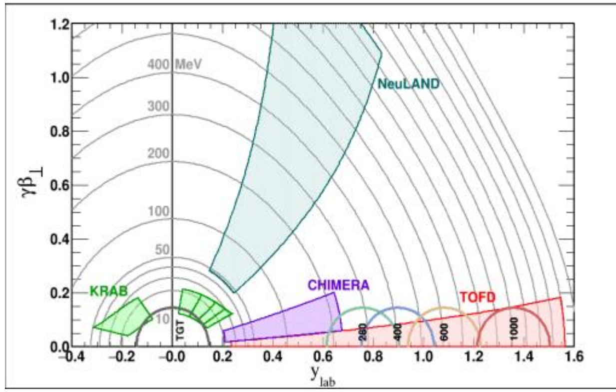


n/p observable → higher densities probed

Higher incident energies → higher densities reached (probed?)



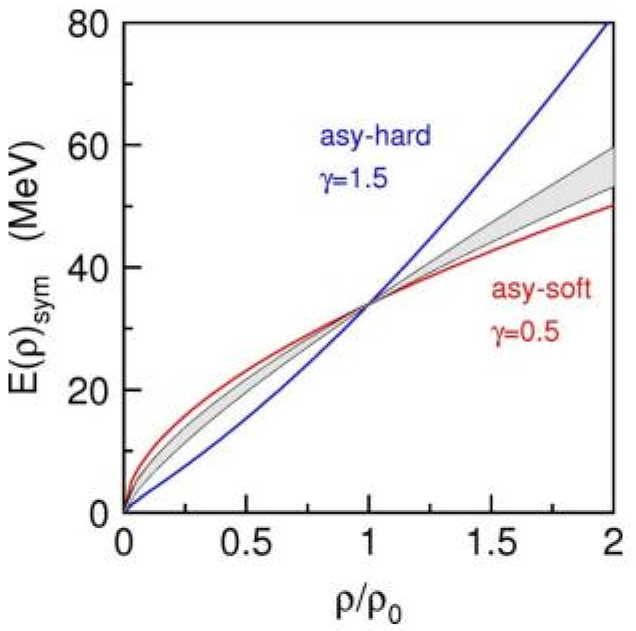
UrQMD simulations: good sensitivity to EOS parametrization up to 1 GeV/A



Ⅰ、status of our previous and current contributions

n/p flows: symmetry energy at high-densities

ASY-EOS



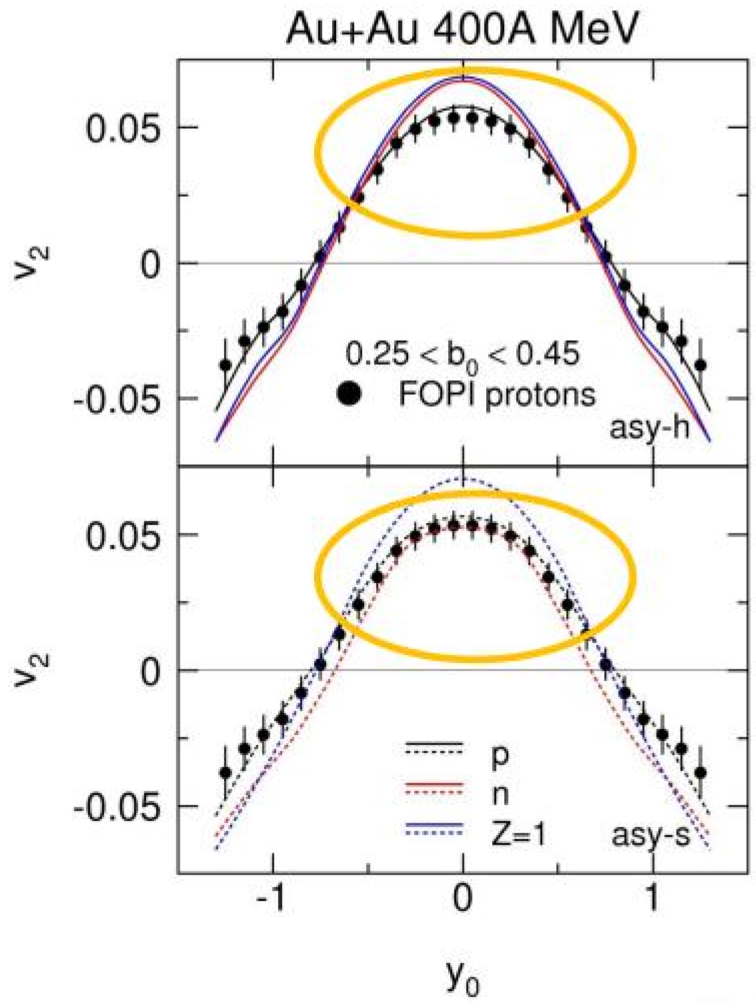
Parametrization for SE used in UrQMD* model:

$$E_{sym} = E_{sym}^{pot} + E_{sym}^{kin} = 22\text{MeV}\cdot(\rho/\rho_0)^{\gamma} + 12\text{MeV}\cdot(\rho/\rho_0)^{2/3}$$

Investigation of the symmetry energy in the high-density regime: the ASY-EOS II experiment at GSI

P.Russotto[1] et (many) al.*

on behalf of R3B and CHIRONE Collaborations



UrQMD*: Qingfeng Li
Data. W. Reisdorf et al.

Ad.1 （插播1）

Bayesian inferences on covariant density functionals from multimessenger astrophysical data: The influences of parametrizations of density dependent couplings

Submitted to PLB➔the isovector channel requires further refinement.

Guo-Jun Wei[c], Armen Sedrakian[d,e], Yong-Jia Wang[b], Qing-Feng Li[b,f], Fu-Hu Liu[a]

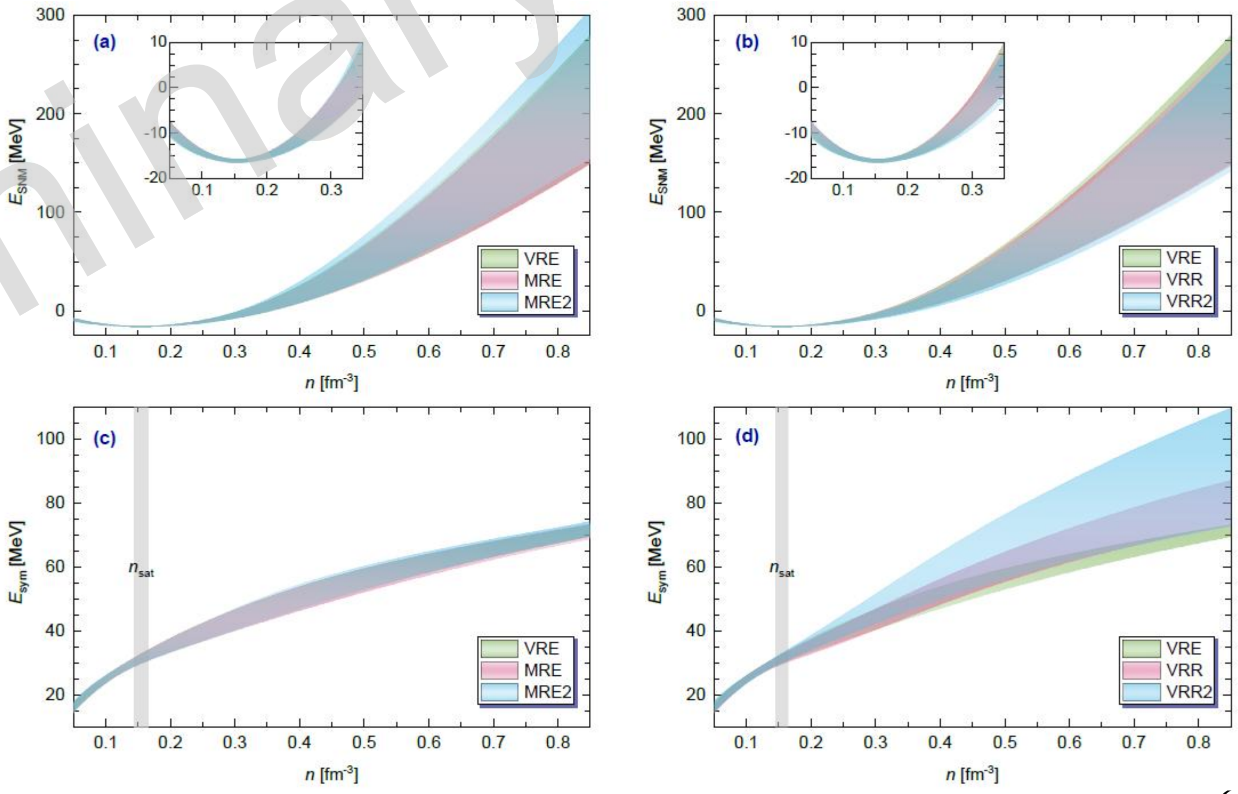
Keywords: Equation of state, Compact stars, Covariant density functional, Bayesian inference

$$n_s \equiv \langle \bar{\psi}\psi \rangle = \sum_{n=n,p} \frac{1}{\pi^2} \int_0^{k_{\mathrm{Fn}}} \frac{m^*_{\mathrm{D,n}}}{\sqrt{k^2 + m^{*2}_{\mathrm{D,n}}}} k^2 dk,$$

$$n_v \equiv \langle \psi^\dagger \psi \rangle = \sum_{n=n,p} \frac{1}{\pi^2} \int_0^{k_{\mathrm{Fn}}} k^2 dk = \sum_{n=n,p} \frac{1}{3\pi^2} k^3_{\mathrm{Fn}},$$

$$g_m(n) = g_m(n_{\mathrm{ref}}) f_m(r)$$

Model	Type	f_σ, f_ω	f_ρ	No.
VRE	V	R, (9) and (10)	E	7
MRE	M	R, (9) and (10)	E	7
MRE2	M	R, (9)	E	8
VRR	V	R, (9) and (10)	R, (11)	7
VRR2	V	R, (9) and (10)	R, (9)	8



Ad.2 （插播2）

Constraining the Phase-Transition EoS using the Energy Dependence of Directed Flow

Zhi-Min Wu[1,2], Gao-Chan Yong[1,2,3,*] and Qingfeng Li[4,2]

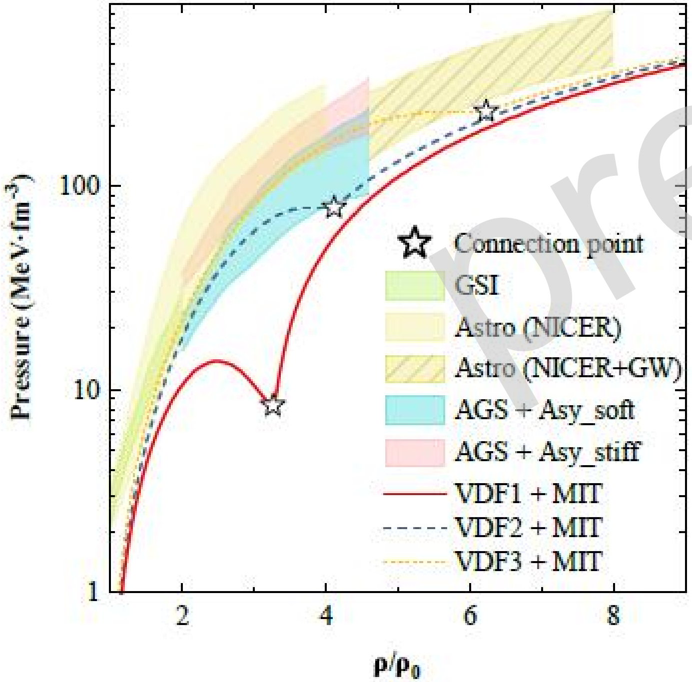
Submitted to PRD→the "safe" density domain before P.T..

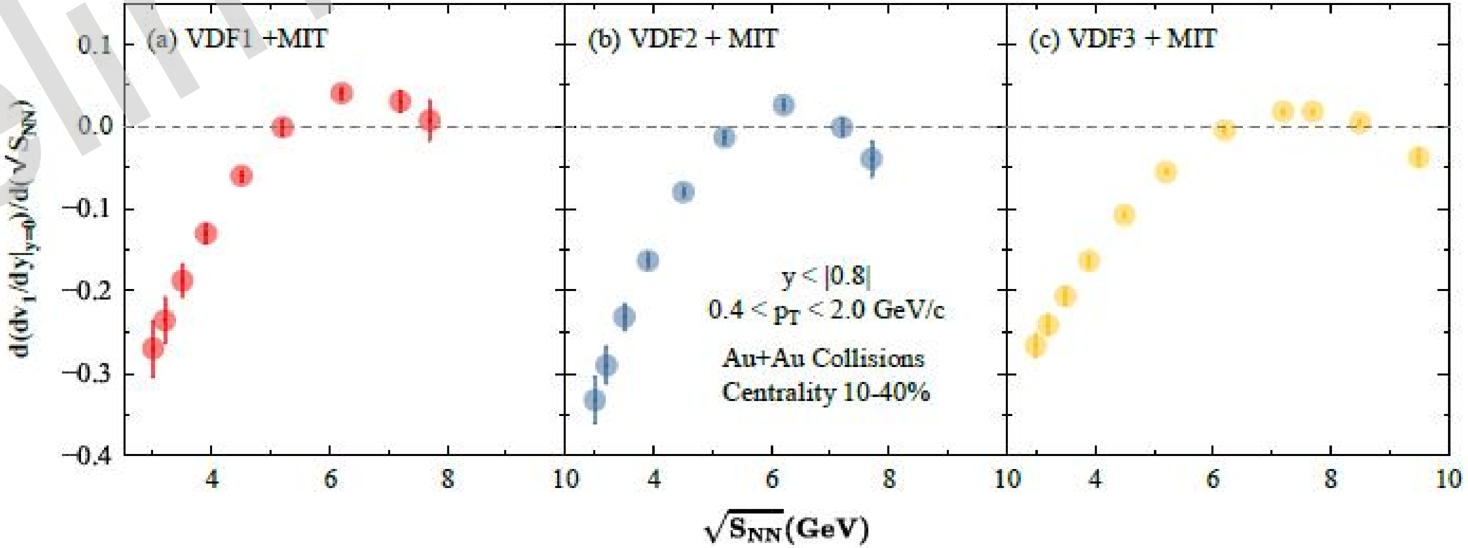
relativistic Vector Density Functional (VDF)

$$U(\rho) = \sum_{i=1}^{N} \widetilde{C}_i (\rho/\rho_0)^{b_i - 1},$$

EoS	$T_c^{(N)}$ [MeV]	$\rho_c^{(Q)}$ $[\rho_0]$	$T_c^{(Q)}$ [MeV]	η_L $[\rho_0]$	η_R $[\rho_0]$	K_0 [MeV]
VDF1	18	3.0	100	2.50	3.315	261
VDF2	18	4.0	50	3.85	4.124	279
VDF3	22	6.0	50	5.80	6.177	356

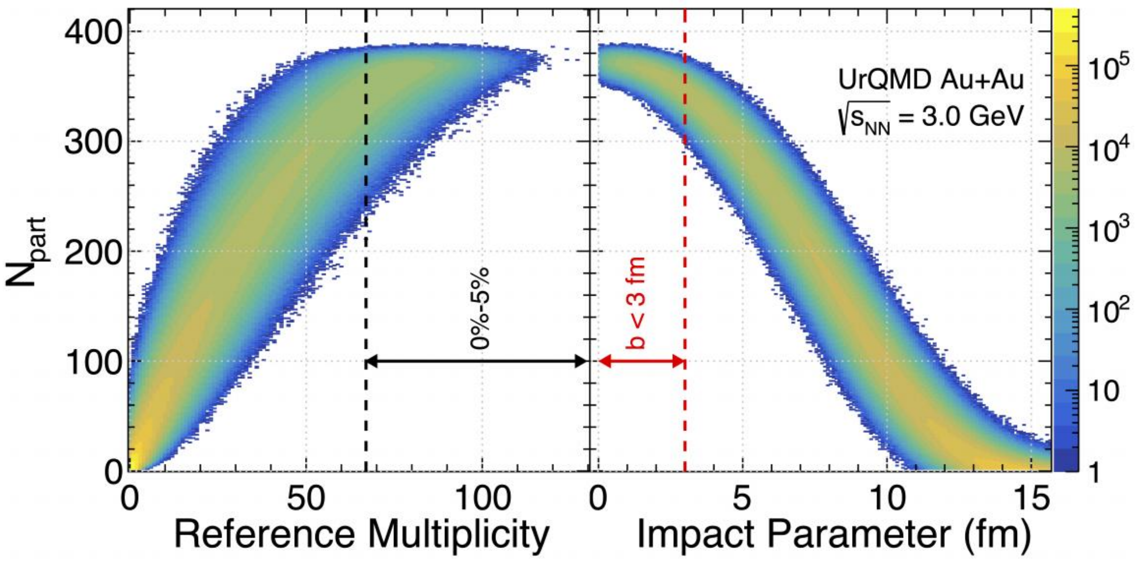
We propose a hybrid equation of state (VDF+MIT EoS) to describe the hadron-quark phase transition in dense nuclear matter. By coupling this EoS with the AMPT-HC transport model and comparing to recent experimental data on proton and Λ directed flow v_1, we constrain the transition to likely occur near $5\rho_0$–$6\rho_0$, ruling out transitions below $3\rho_0$. Furthermore, we introduce the energy derivative of the mid-rapidity v_1 slope, $d(dv_1/dy)/d(\sqrt{s_{NN}})$, as a weakly model-dependent observable. Its zero crossing provides a direct signature of the phase transition critical point, offering a new tool for mapping the QCD phase diagram in future experiments.

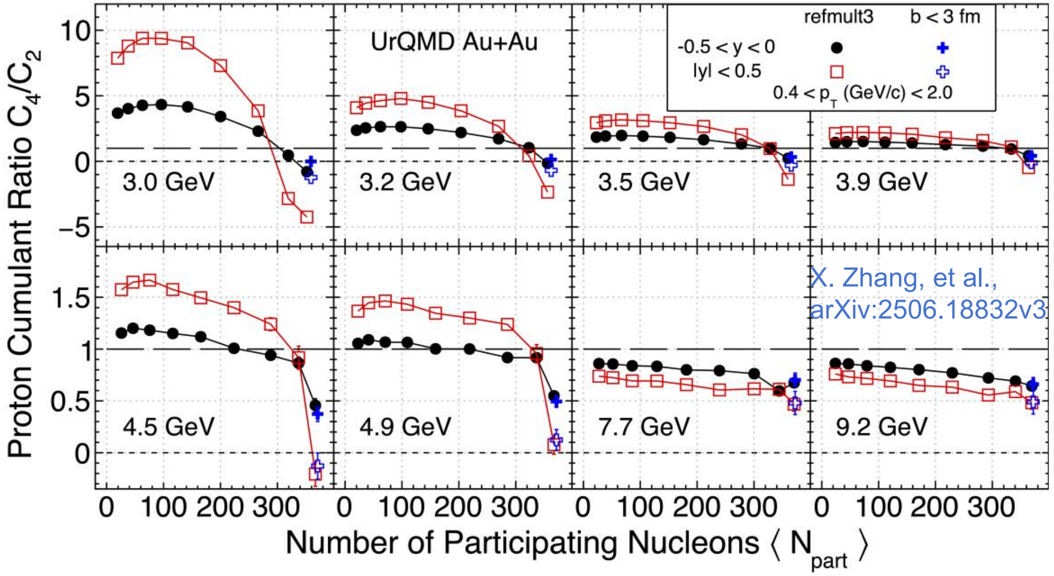


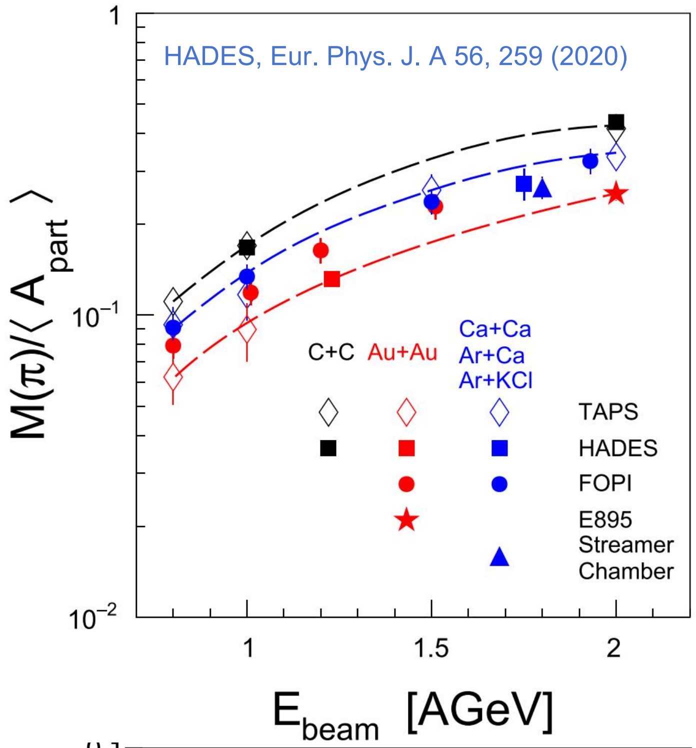


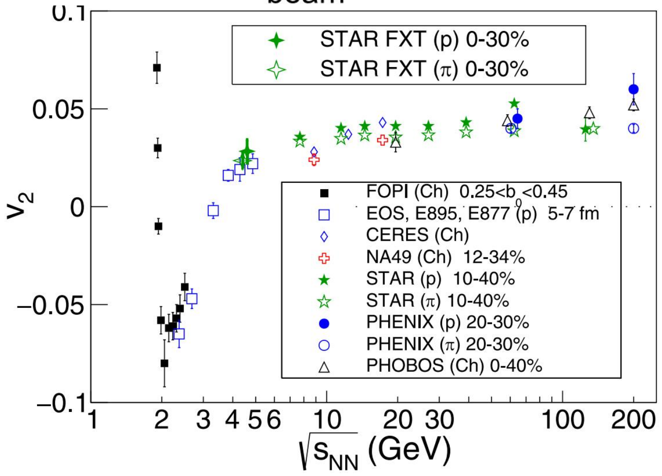
Ⅰ、Challenges and Opportunities (机会 and/or 挑战)

① 理论与实验的差异（apple to apple?）

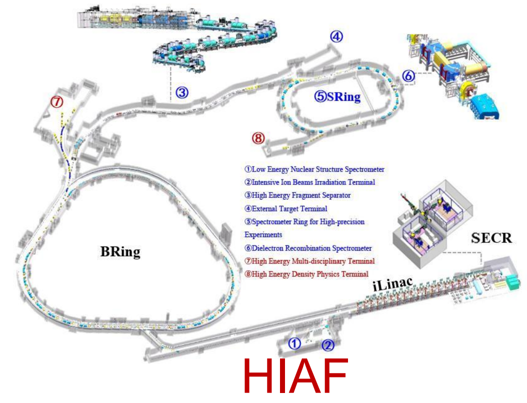


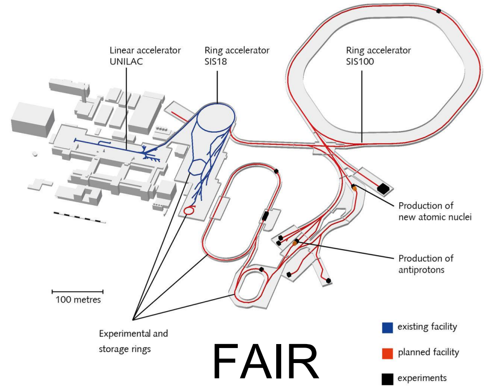


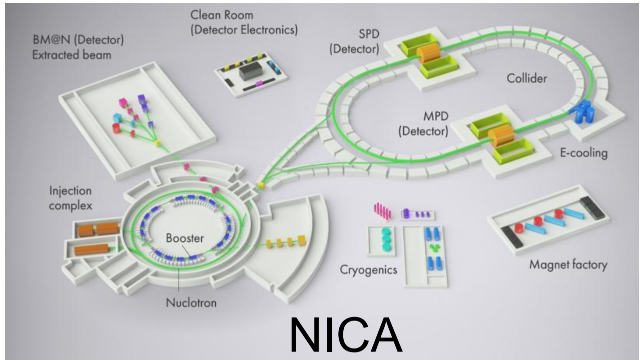




STAR, Phys. Rev. C 103, 034908 (2021)

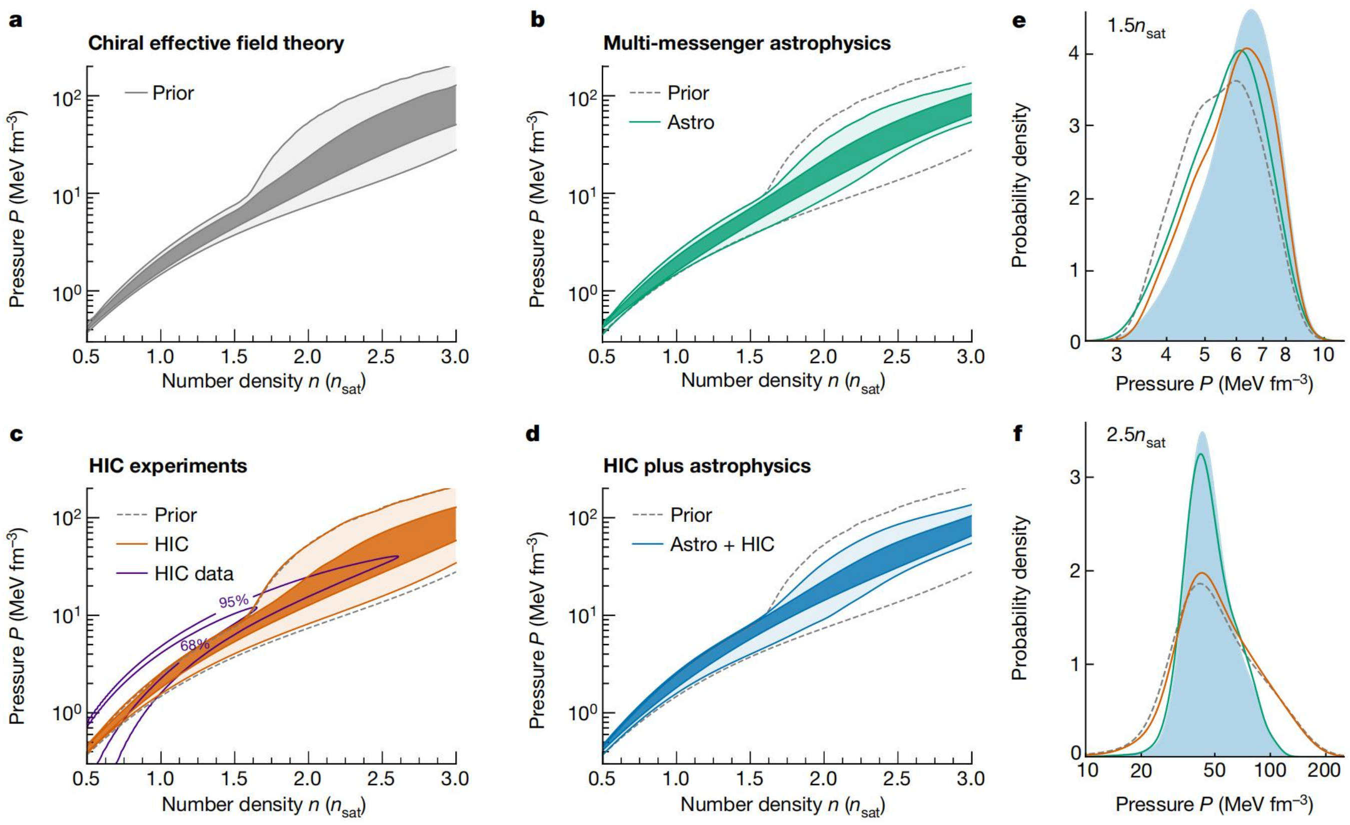


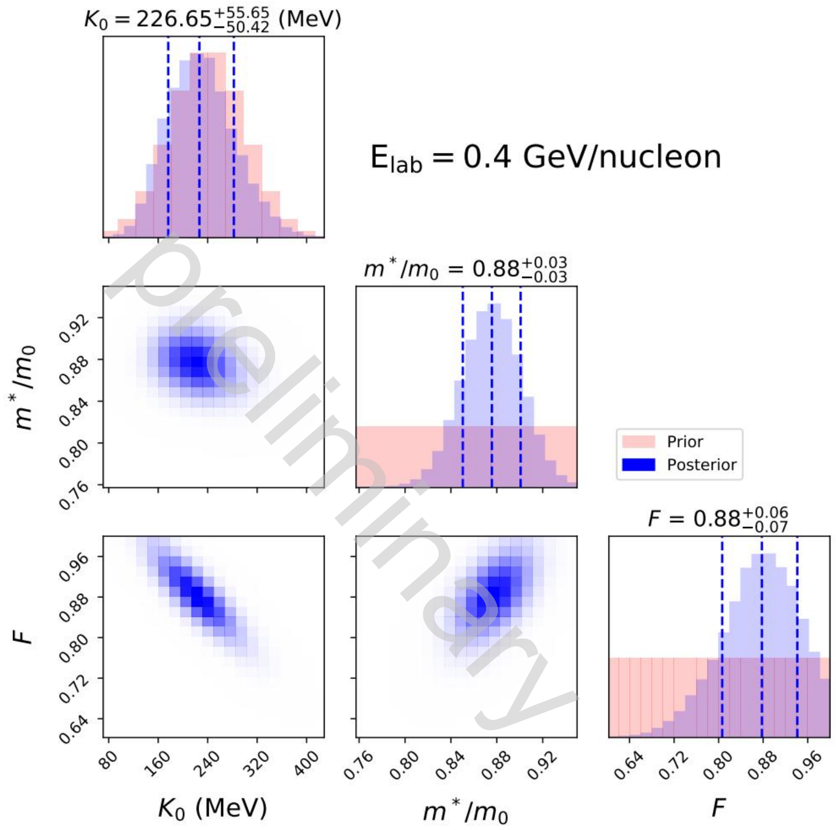




Ⅰ、Challenges and Opportunities

② 不确定性来源分析:方法上——贝叶斯分析+AI





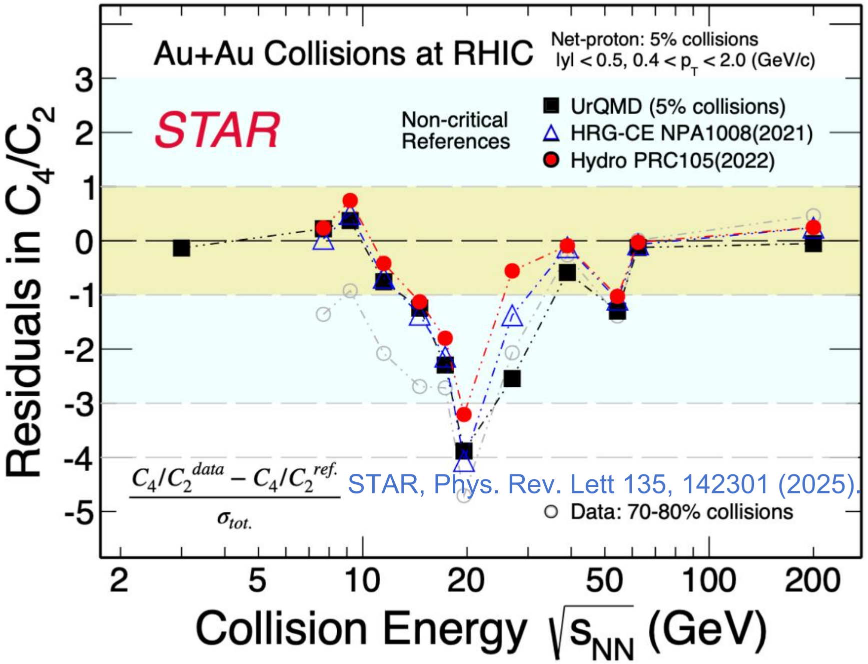
Joint analyses can shed light on the properties of neutron-rich supranuclear matter over the density range probed in neutron stars.

S. Huth, P. T. H. Pang, I. Tews, T. Dietrich, et al. Nature 606, 276-280 (2022).
G. J. Wei, M. Z. Nan, P. C. Li, Y. J. Wang, **Q. F. Li***, G. C. Yong and F. H. Liu, arXiv:2509.03406.

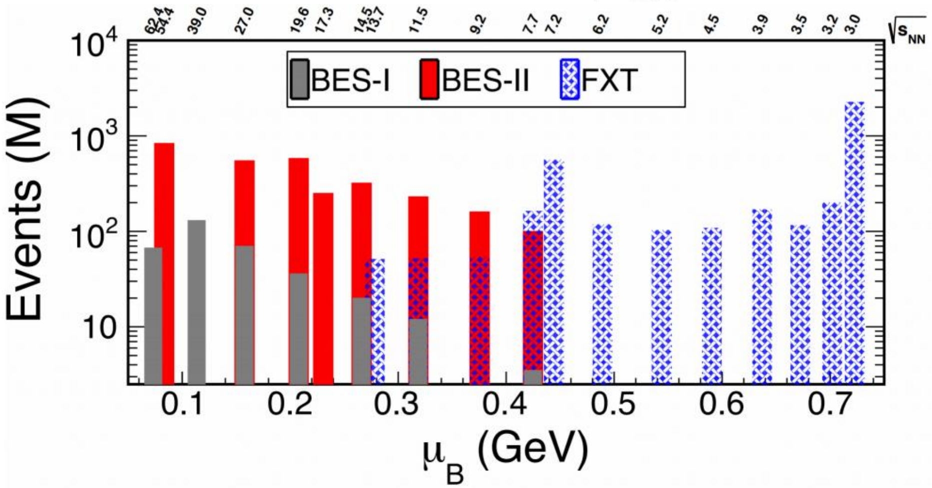
Ⅰ、Challenges and Opportunities

② 不确定性还来自…

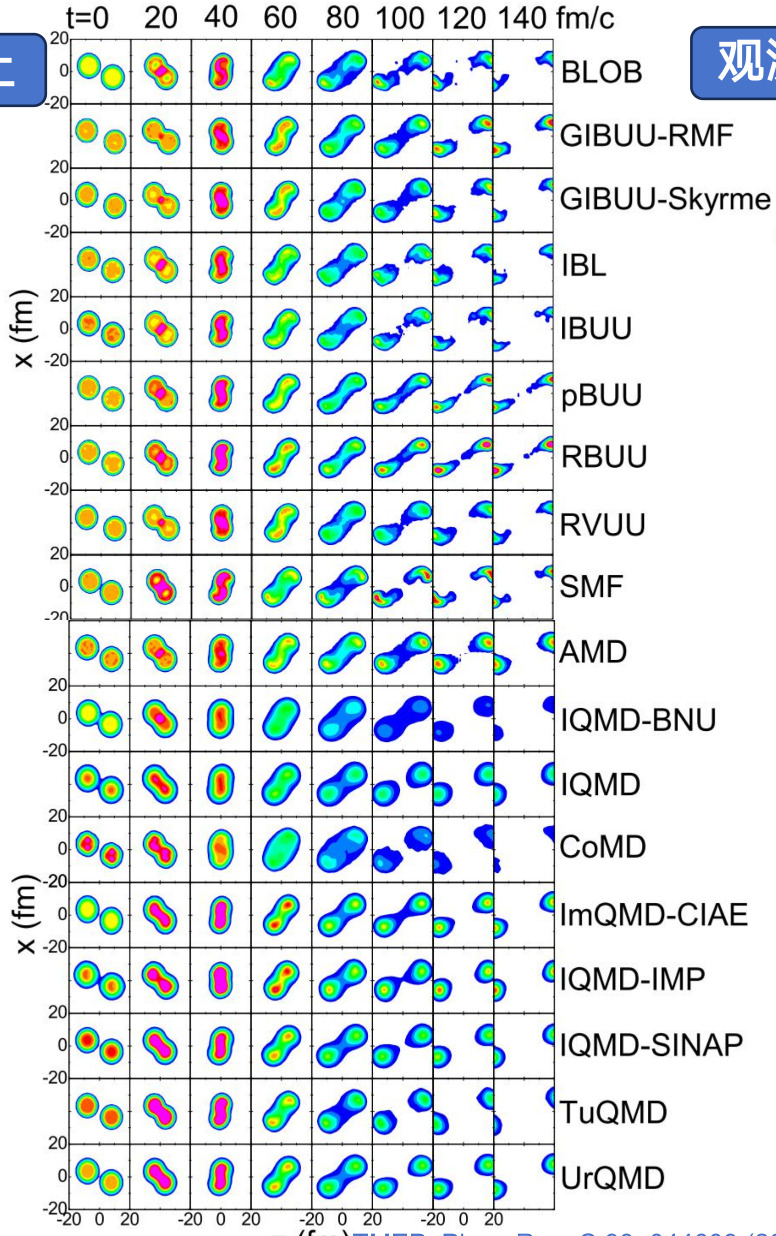
实验上



STAR, Phys. Rev. Lett 135, 142301 (2025).

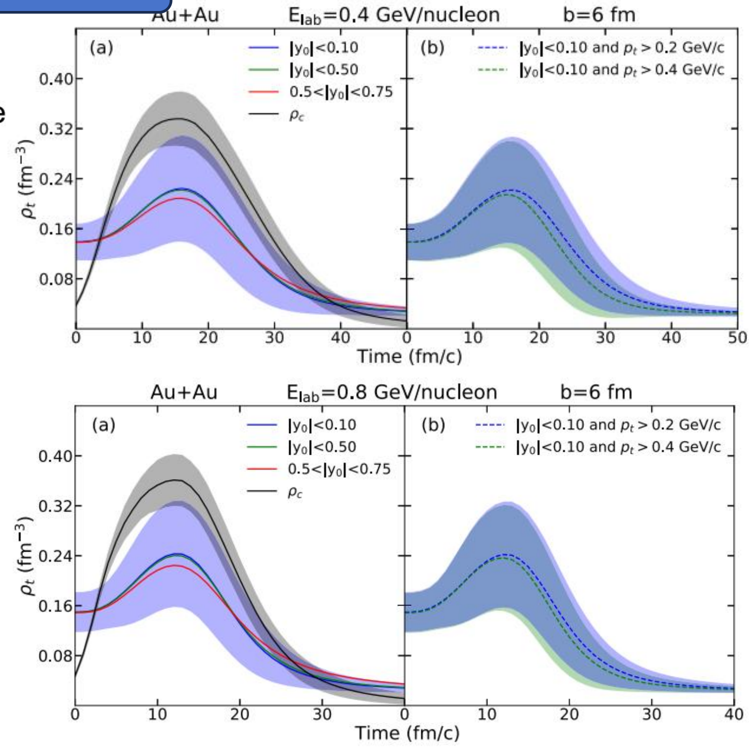


模型上



TMEP, Phys. Rev. C 93, 044609 (2016); PPNP 125, 103962(2022).

观测量上

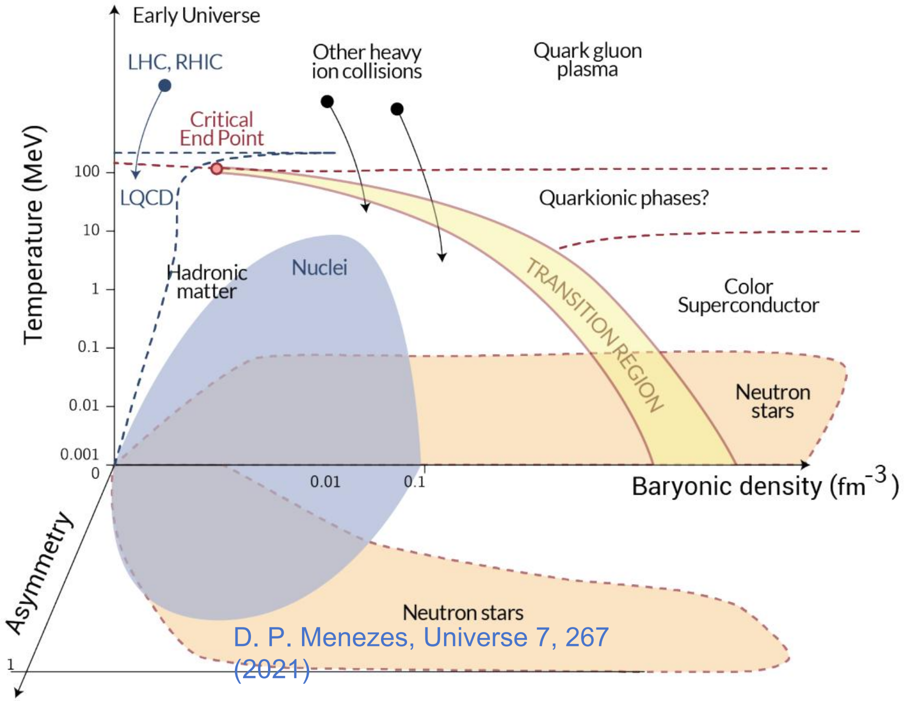


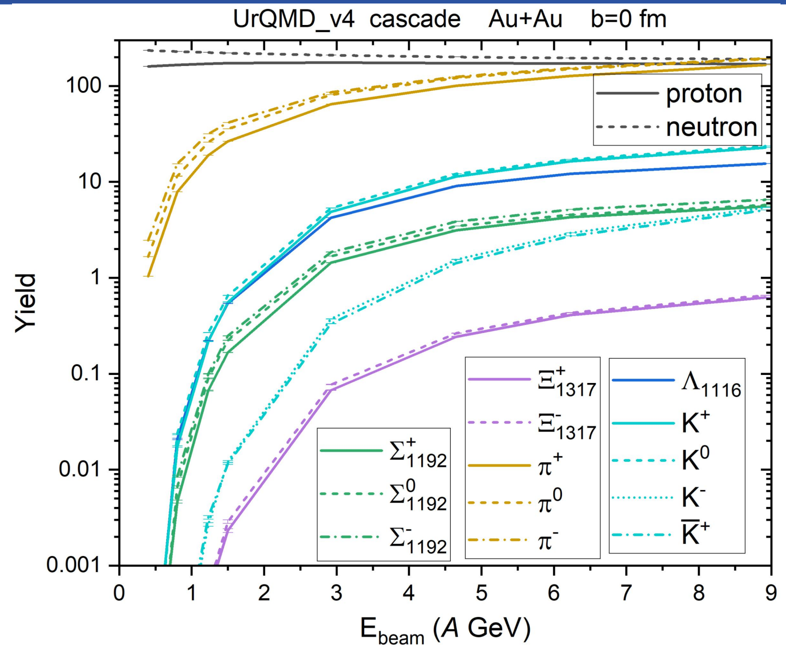
Y. J. Wang, et al., Phys. Rev C 110, 044606 (2024)

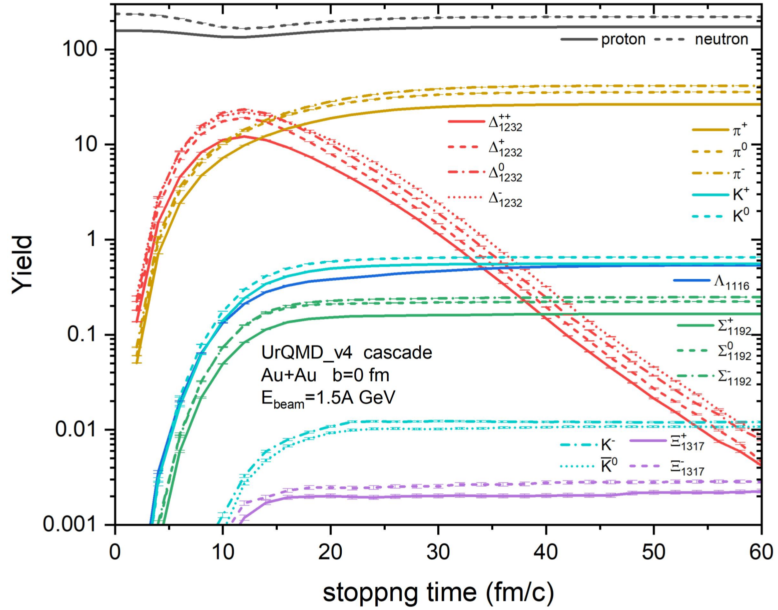
Continue to work on data, model, method, and observables !

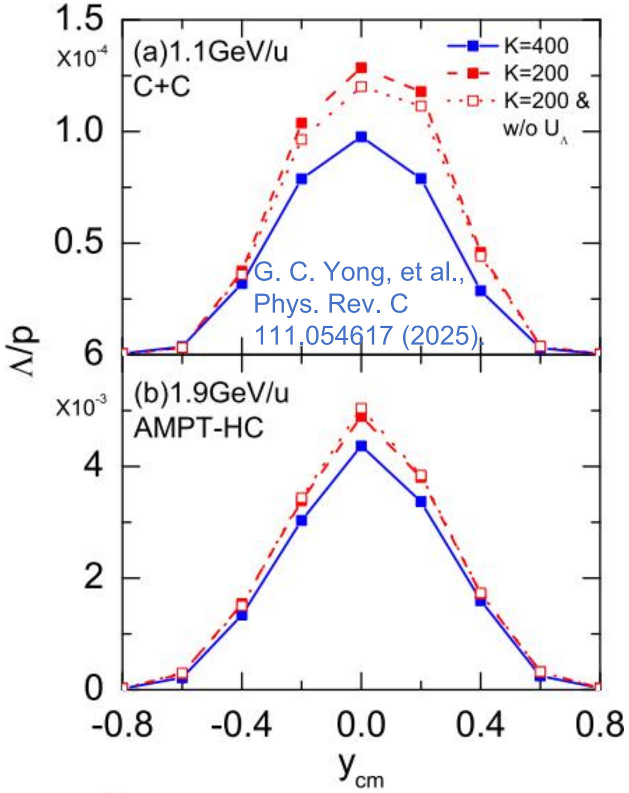
Ⅰ、Challenges and Opportunities

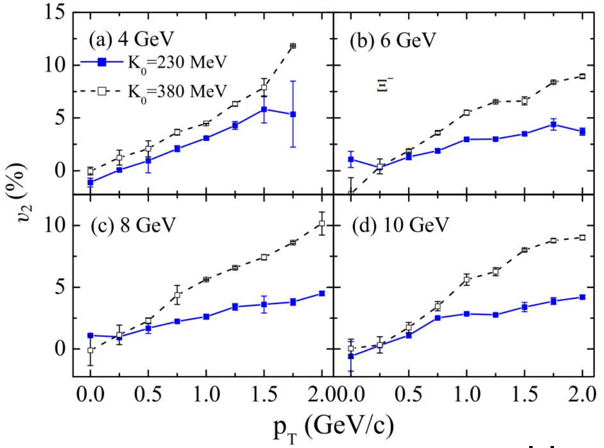
③ 复杂的核环境和丰富的粒子种类











Y. Gao, et al., Phys. Rev. C 112.024909 (2025).

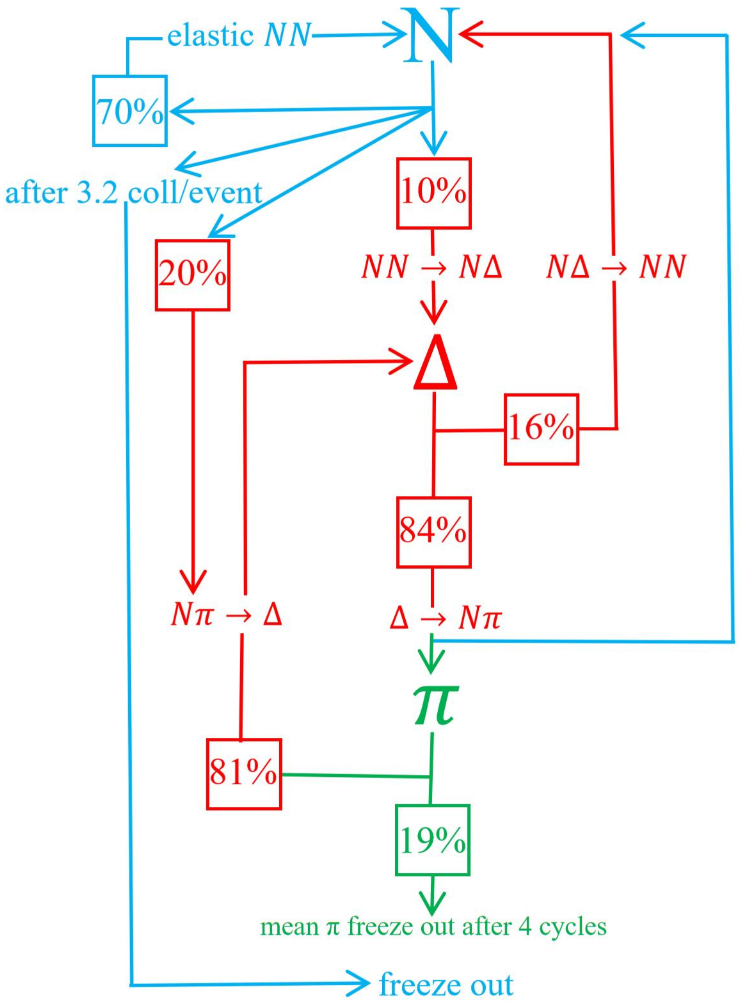
density, temperature, asymmetry, ...

N、*Δ*、*p*iπ、*K*、*Λ*、*Σ*、*Ξ*、…

Ⅱ 、Two-body cross sections (非平衡动力学的特点)

Example: the N-Δ-π cycle in the IQMD model

Au+Au@1A GeV $b \leq 5$ fm



S. A. Bass, et al., Prog. Part. Nucl. Phys. 41, 255-369 (1998)

Au+Au Collisions @ 1 GeV/nucleon:

- relevant hadronic degrees of freedom: nucleon, Δ(1232) & pion

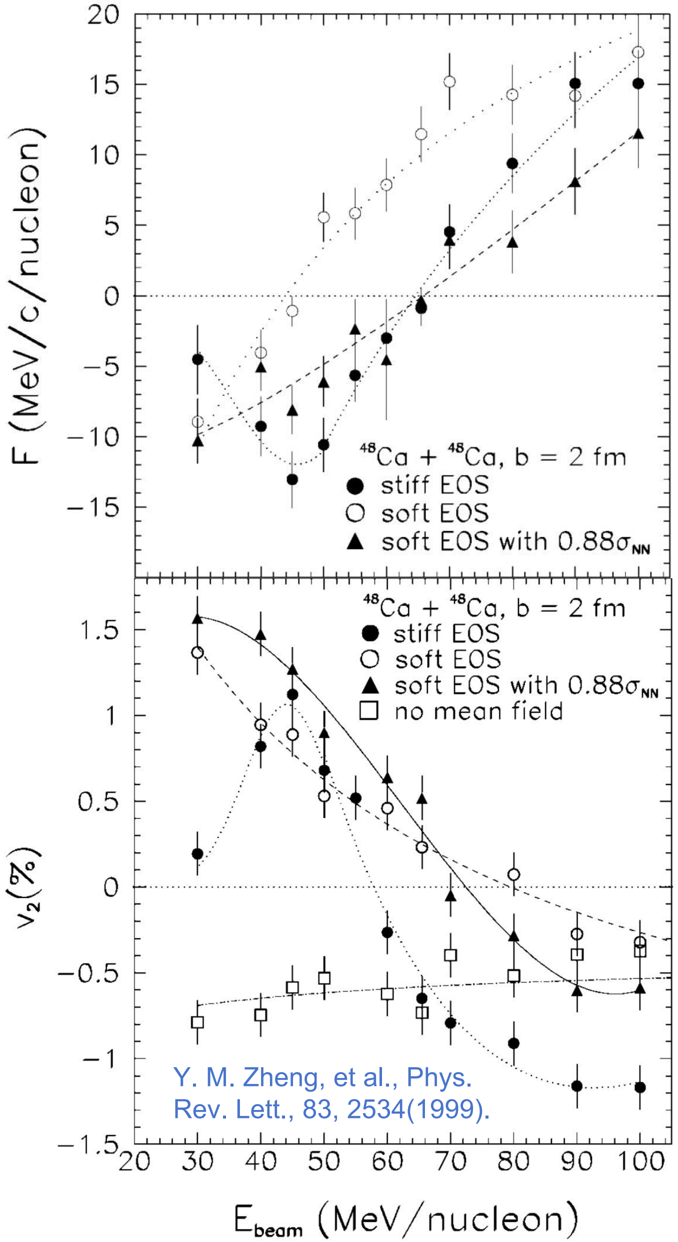
Particle Production/Absorption:

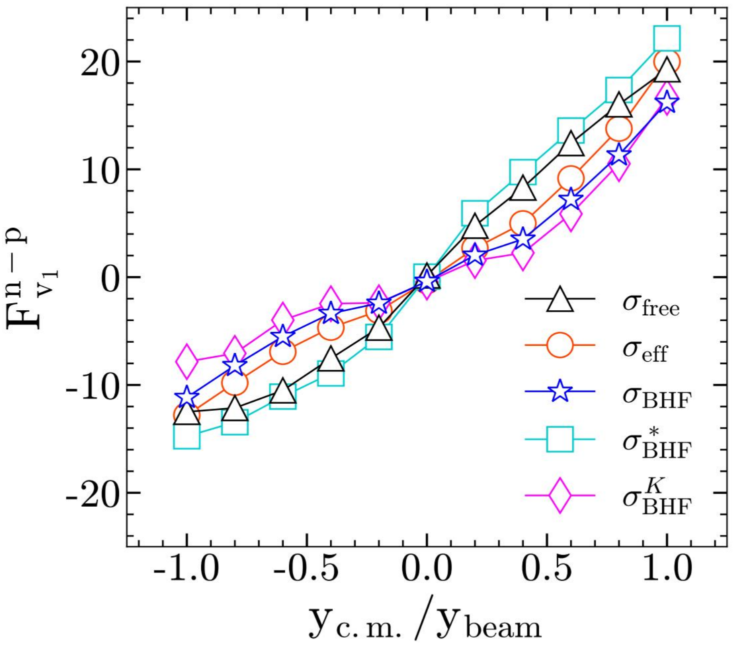
- pion production via Δ(1232)→N+π
- pion absorption requires two steps: π+N→Δ(1232) & Δ(1232)+N→N+N

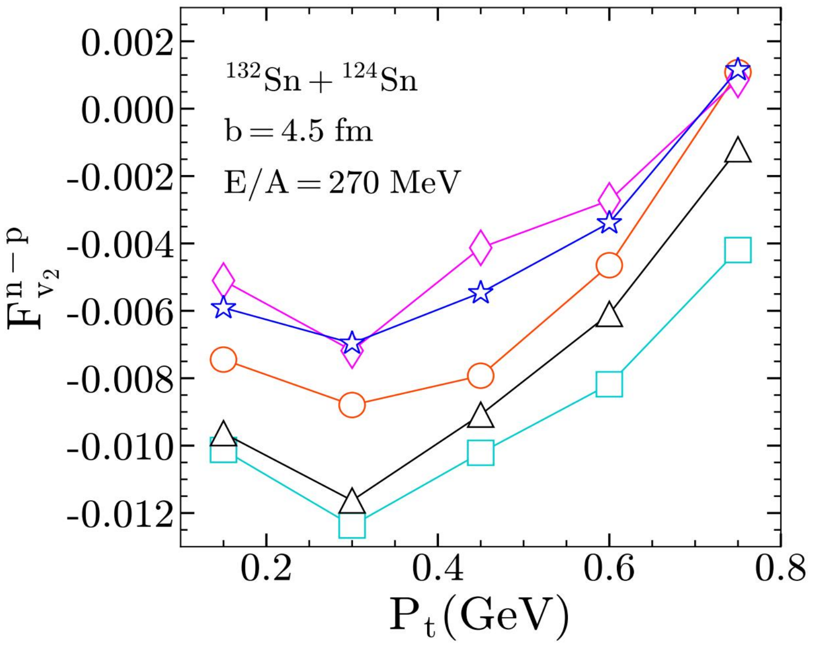
- Δ(1232) plays a crucial role in the particle production dynamics
- the average pion goes through approx. 4 Δ-cycles before freeze-out !

Process ID#	Description
1	$NN \to N\Delta$
2	$NN \to NN^*$
3	$NN \to N\Delta^*$
4	$NN \to \Delta\Delta$
5	$NN \to \Delta N^*$
6	$NN \to \Delta\Delta^*$
7	$NN \to N^*N^*, N^*\Delta^*, \Delta^*\Delta^*$
8	$N\Delta \to \Delta\Delta$
10	$MB \to B'$
11	$MM \to M'$
13	BB (but not pp or pn) elastic scattering
14	inelastic scattering (no string excitation)
15	$BB \to$ 2 strings
17	pn elastic
19	pp elastic
20	decay
22	$B\bar{B}$ elastic
23	$B\bar{B}$ annihilation $\to$ 1 string
24	$B\bar{B}$ diffractive $\to$ 2 strings
26	MB elastic scattering
27	$MB, MM \to$ 1 string
28	$MB, MM \to$ 2 strings
30	$N\Delta \to NN$
31	$\Delta\Delta \to \Delta N$
32	$\Delta\Delta \to NN$
35	$N\Delta$ inelastic
36	Danielewicz forward delay ($MB \to B'$)
37	Danielewicz forward delay ($MM \to M'$)
38	MM elastic scattering
39	$B\bar{B}$ inelastic scattering (no annihilation)
80	Periodic wall: Particle crosses the box wall
81	Solid wall: Particle reflects off the box wall
91	Fluidization
96	Particlization

Ⅱ 、Two-body cross sections & EoS

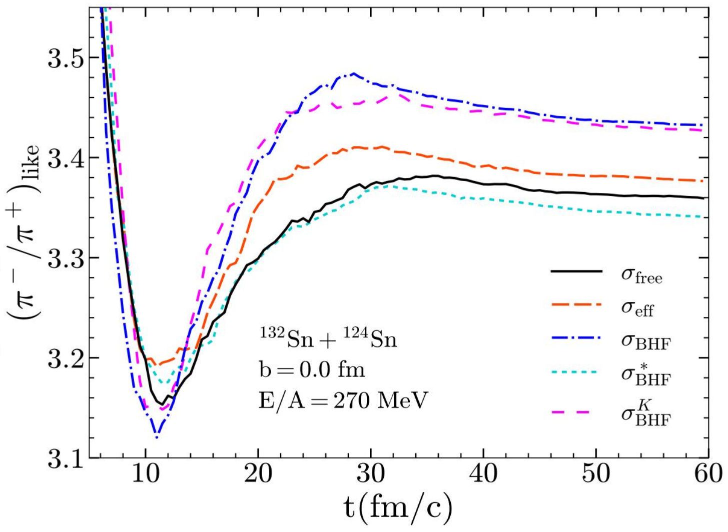






$$\sigma(\rho,\beta,k,K) = \frac{M^{*2}}{16\pi^2\hbar^4}\sum_{S,J}\sum_{L',L}\left[1-(-1)^{S+L+T}\right]^2 \times \frac{2J+1}{4\pi}\left|G^{SJ}_{L'L}(\rho,\beta,k,K)\right|^2 , \quad (3)$$

where $G^{SJ}_{L'L} = \langle k, L'SJ|G(e_2)|k, LSJ\rangle$ is the matrix ele-



S. C. Han, et al., arXiv:2507.23476v1

Ⅱ、Two-body cross sections (eg1: NN elastic)

- In-medium correction factors applied to vacuum cross sections

G. D.~Westfall, et al., PRL 71, 1986-1989 (1993); J. Y. Liu, et al., PRL 86, 975 (2001).

$$\sigma_{NN}^{*} = F * \sigma_{NN}^{free} = (1 + a\frac{\rho}{\rho_0})\sigma_{NN}^{free}$$

- Employing effective nucleon masses in nuclear medium

B. A. Li, et al., PRC 72, 064611 (2005), W. M. Guo, et al., PRC 111, 024612 (2025).

$$F = \sigma_{NN}^{*}/\sigma_{NN}^{free} = (m_{NN}^{*}/m_{NN})^2$$

- Phenomenological formula

$$\sigma_{nn} = (13.73 - 15.04\beta^{-1} + 8.76\beta^{-2} + 68.67\beta^{4}) \times \frac{1.0 + 7.772E_{lab}^{0.06}\rho^{1.48}}{1.0 + 18.01\rho^{1.46}},$$

$$\sigma_{np} = (-70.67 - 18.18\beta^{-1} + 25.26\beta^{-2} + 113.85\beta) \times \frac{1.0 + 20.88E_{lab}^{0.04}\rho^{2.02}}{1.0 + 35.86\rho^{1.90}},$$

X. Z. Cai, et al., PRC58, 572-575 (1998).

- Giant Dipole Resonance

R. Wang, et al., PLB 807, 135532(2020); Y. D. Song, et al., PRC 108, 064603(2023)

- Brueckner theories

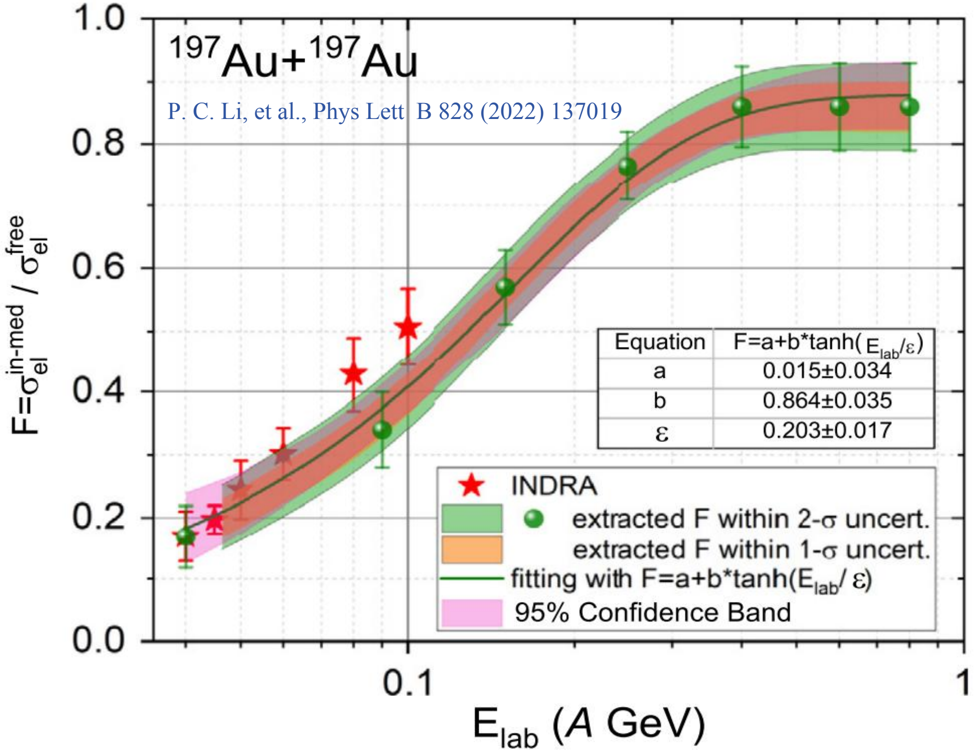
G. Q. Li, et al., PRC 48, 1702 (1993), F. Sammarruca et al., PRC 73, 014001(2006).

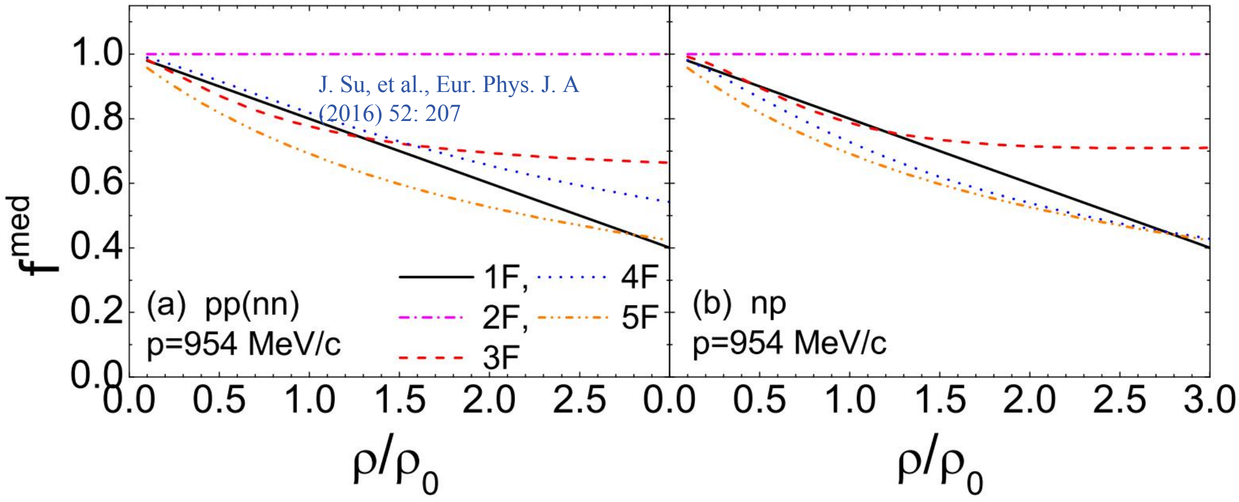
- The closed time path Green's function approach

G. J. Mao, et al., PRC 49, 3137 (1994), Q. F. Li, et al., PRC 69, 017601(2004).

- Relativistic ***ab initio*** calculations

T. Wang, et al., arXiv:2503.18103v1



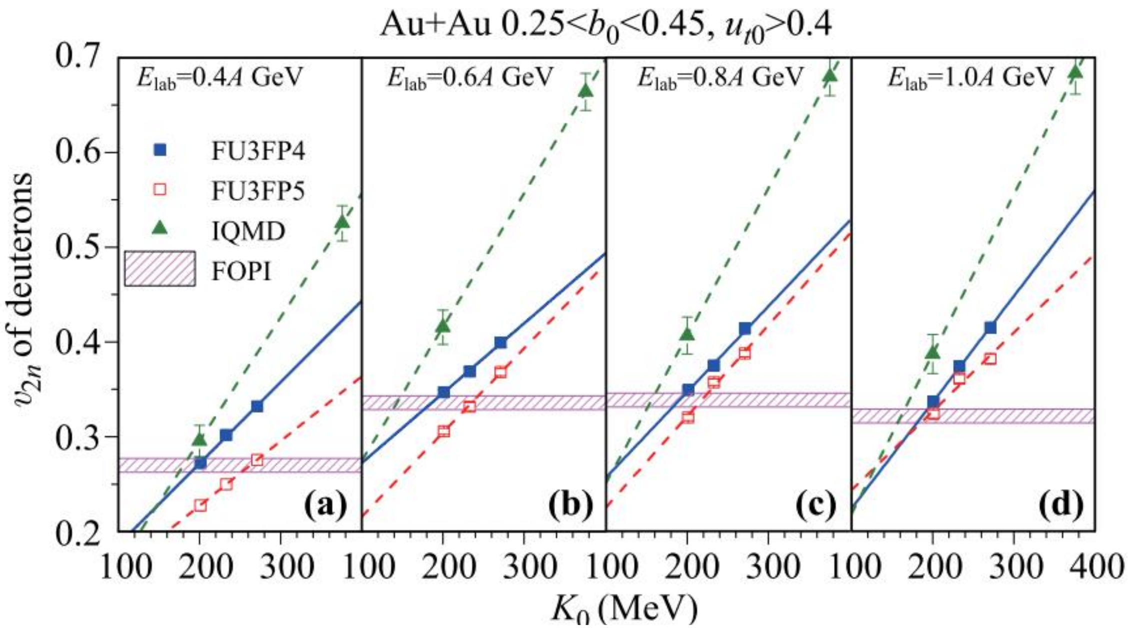


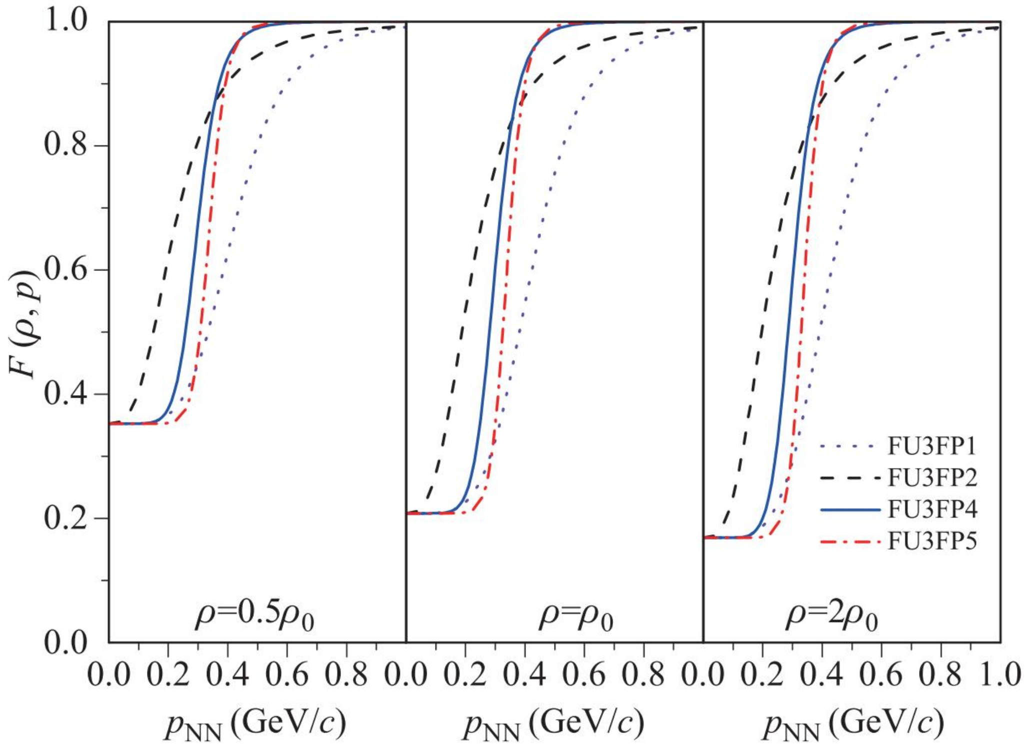
Two-body cross sections (eg1: NN elastic)

$$\sigma^*_{\text{tot}} = \sigma_{\text{in}} + \sigma^*_{\text{el}} = \sigma_{\text{in}} + F(\rho,p)\sigma_{\text{el}},$$

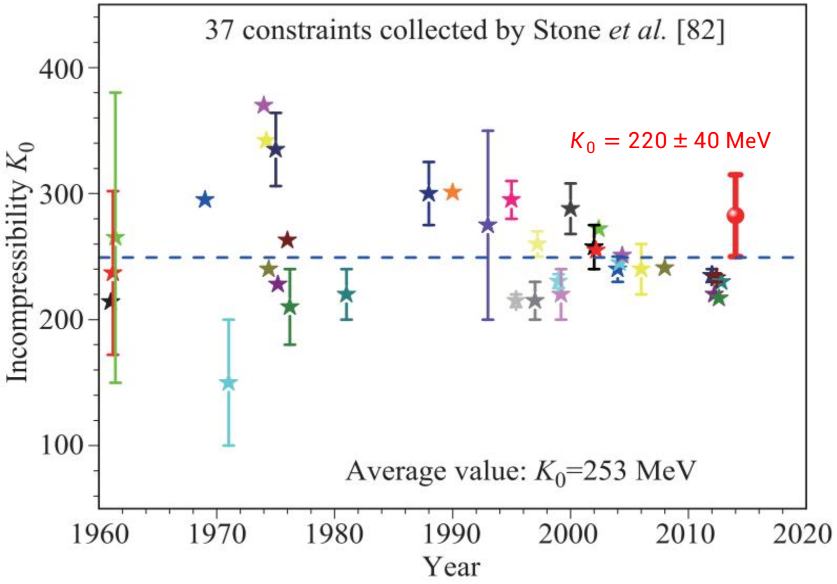
$$F(\rho,p) = \begin{cases} f_0 & p_{NN} > 1\ \text{GeV}/c, \\ \frac{F_\rho - f_0}{1+(p_{NN}/p_0)^\kappa} + f_0 & p_{NN} \leqslant 1\ \text{GeV}/c, \end{cases}$$

$$F_\rho = \lambda + (1-\lambda)\exp\left[-\frac{\rho}{\zeta\rho_0}\right]$$





散射过程的处理对状态方程信息抽取的重要性！



Q. F. Li, et. al., J. Phys. G 32, 407 (2006). Q. F. Li, *et al.*, Phys. Rev. C **83**, 044617 (2011), Y. J. Wang, Q. F. Li, Front. Phys. **15**, 44302 (2020).

Ⅱ、Two-body cross sections (eg2: NN inelastic)

- Phenomenological in-medium cross section

$$\sigma^{*}_{NN\to N\Delta} = R * \sigma^{free}_{NN\to N\Delta}$$

T. Song, et al., PRC 91, 014901 (2015), Z. Z. Zhang, et al., PRC 103, 034615 (2021), W. M. Guo, et al., PRC 111, 024612 (2025).

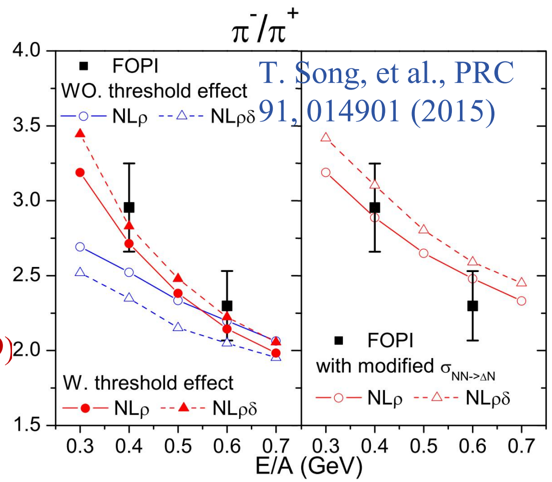
- The closed time path Green's function approach

Q. F. Li, et al., PLB 773 (2017) 557–562. Q. F. Li, et al., Sci. China-Phys. Mech. Astron. 62, 972011 (2019)

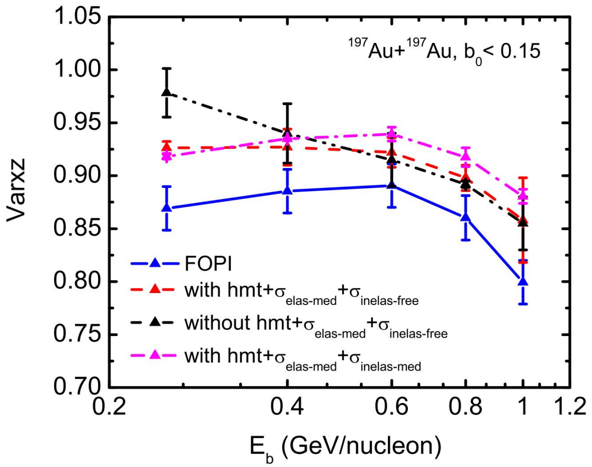
- One-boson exchange model

Y. Cui, et al., PRC 98, 054605 (2018), CPC 43, 024105 (2019), CPC 44, 024106 (2020).

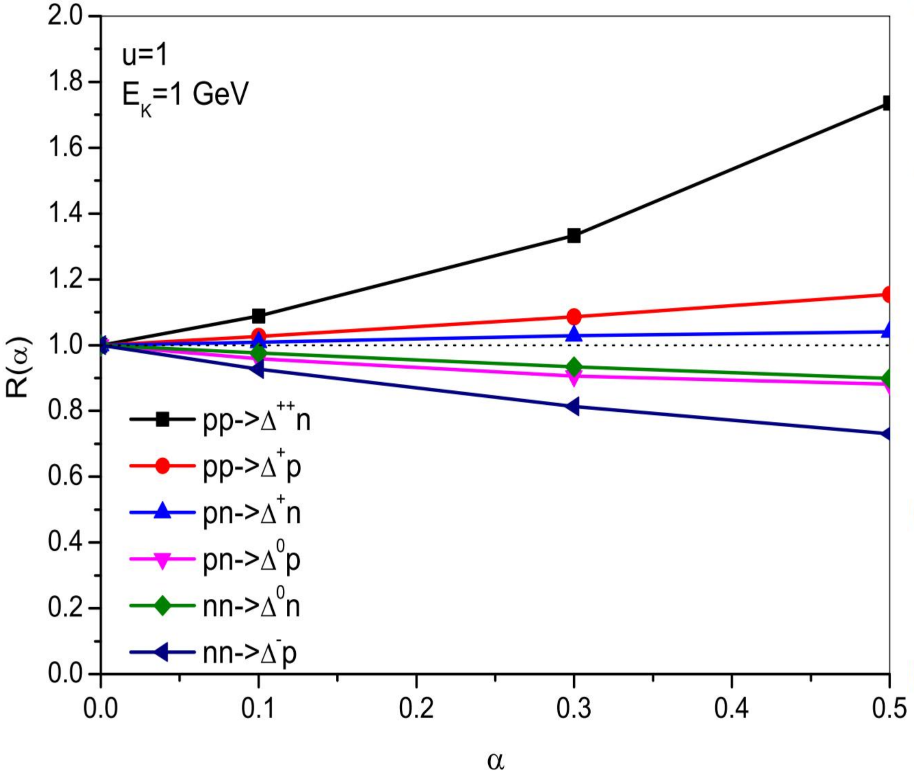
- …

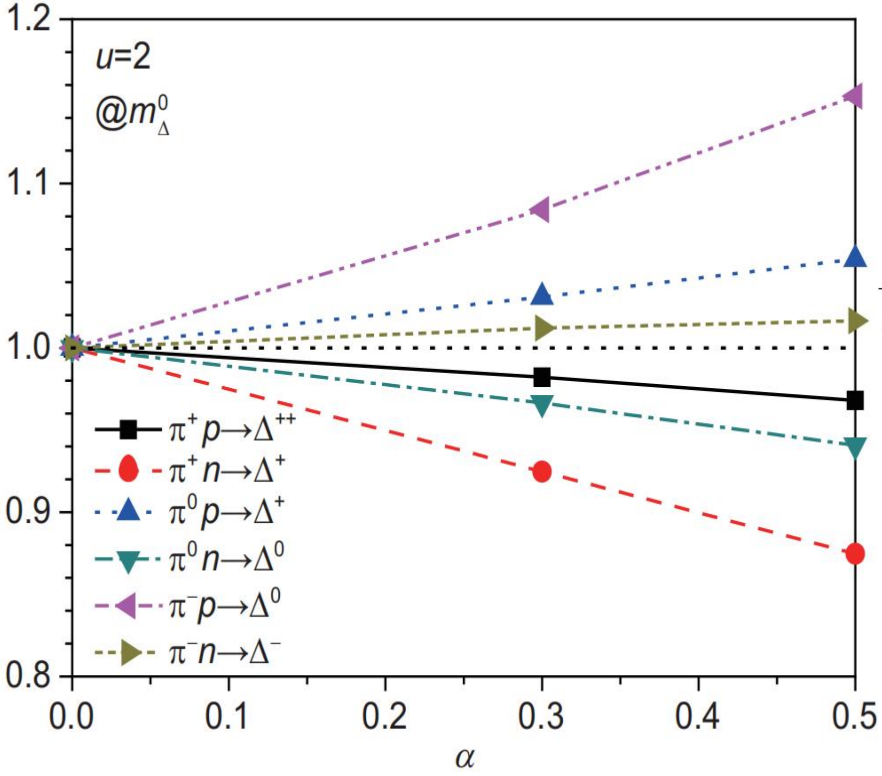


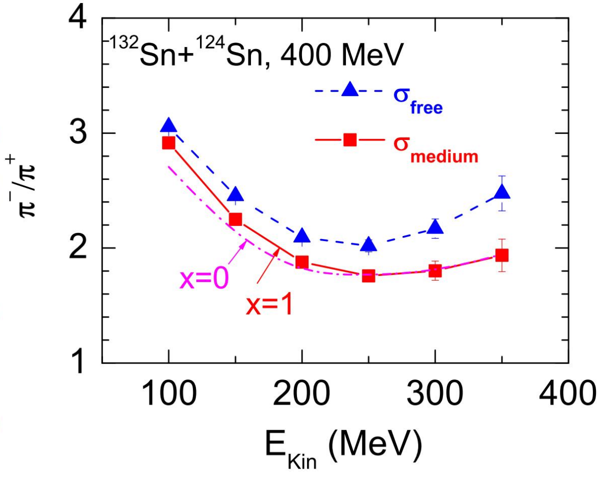
T. Song, et al., PRC 91, 014901 (2015)



W. M. Guo, et al., PRC 111, 024612 (2025).







Z. Z. Zhang, et al., PRC 103, 034615 (2021)

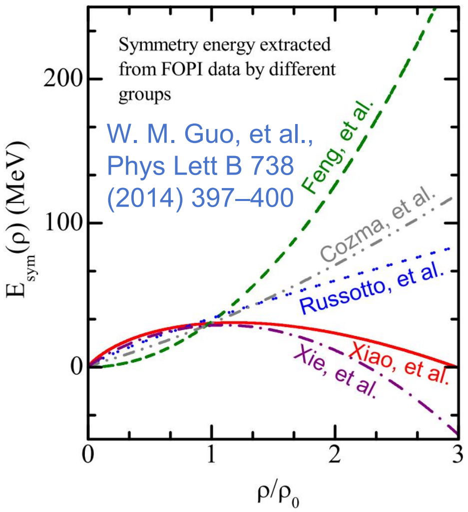
Ⅱ、Two-body cross section for EoS (eg2: NN inelastic)

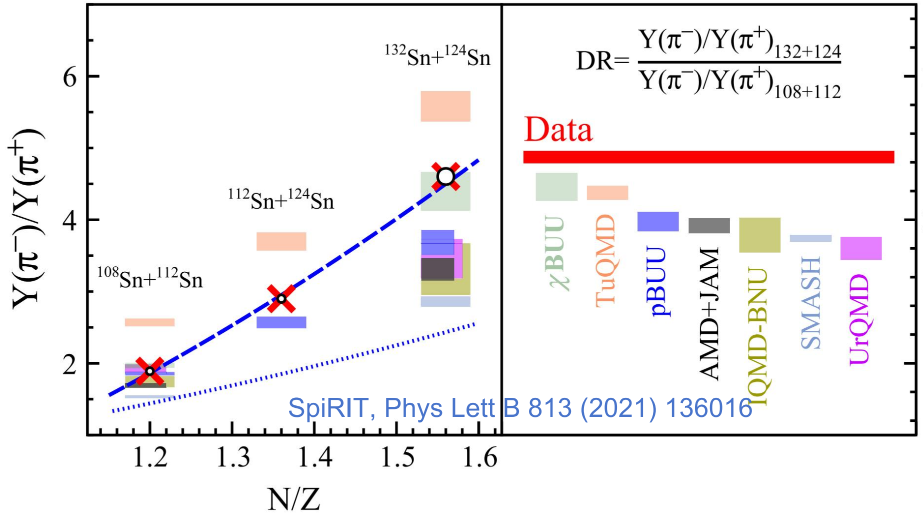
Probing the equation of state with pions
Q Li, Z Li, S Soff, M Bleicher, H Stöcker
Journal of Physics G: Nuclear and Particle Physics, 2005 · iopscience.iop.org

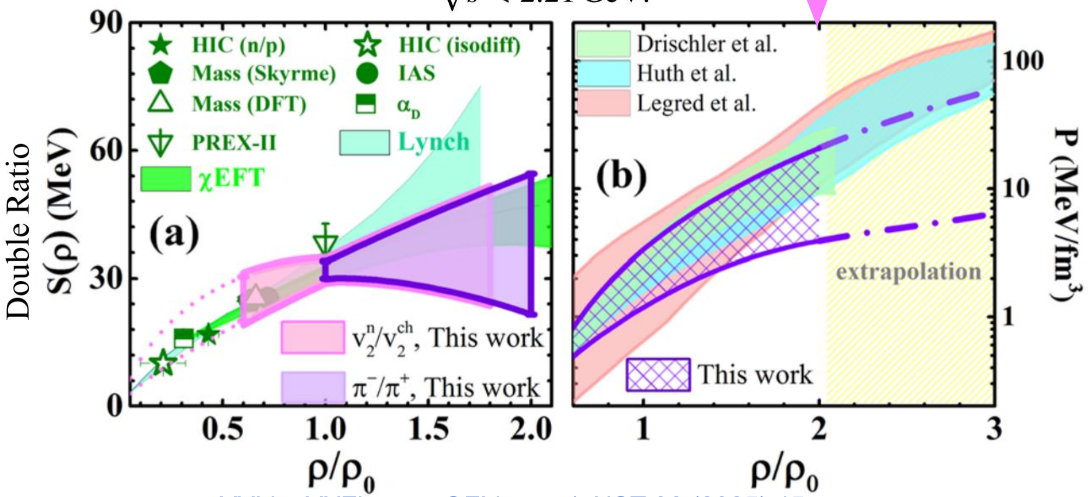
引用：120

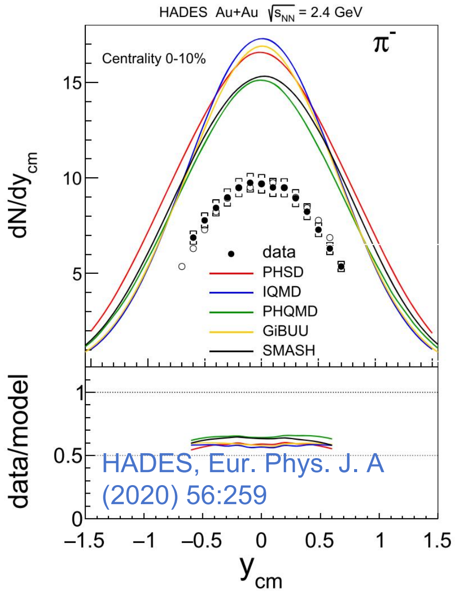
$$\sigma_{NN\to N\Delta}(\sqrt{s}) = A_1 + \frac{4A_2 \times e^{-(\sqrt{s}-A_3)/A_4}}{(1+e^{-(\sqrt{s}-A_3)/A_4})^2},$$

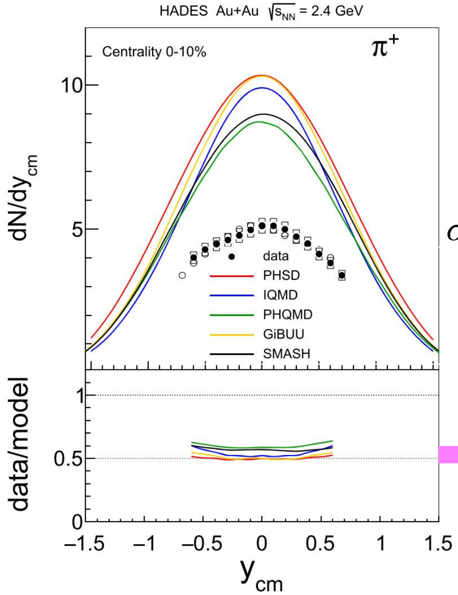
$$\sqrt{s} < 2.21\ \text{GeV}.$$





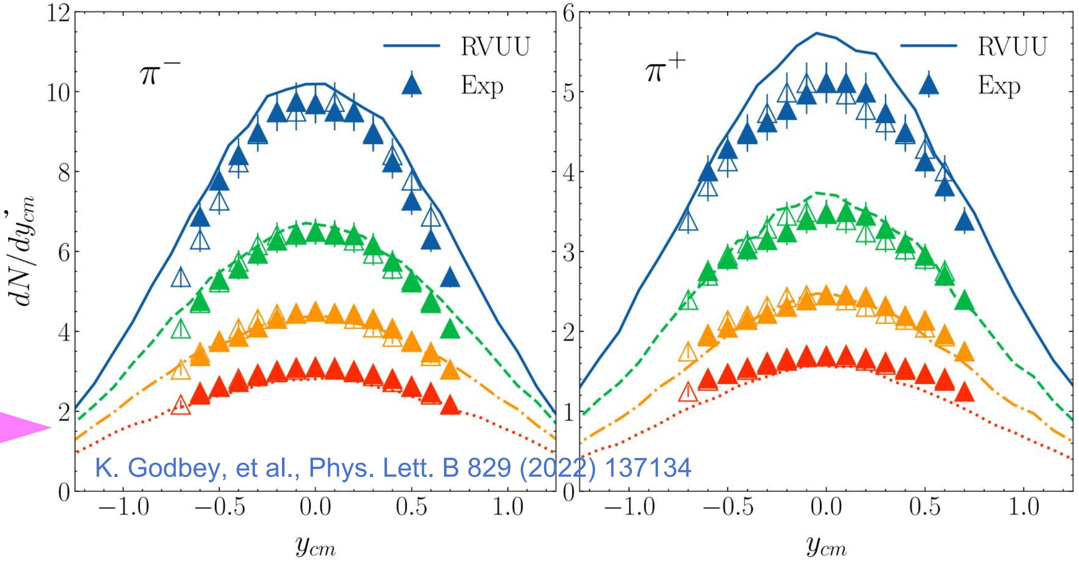




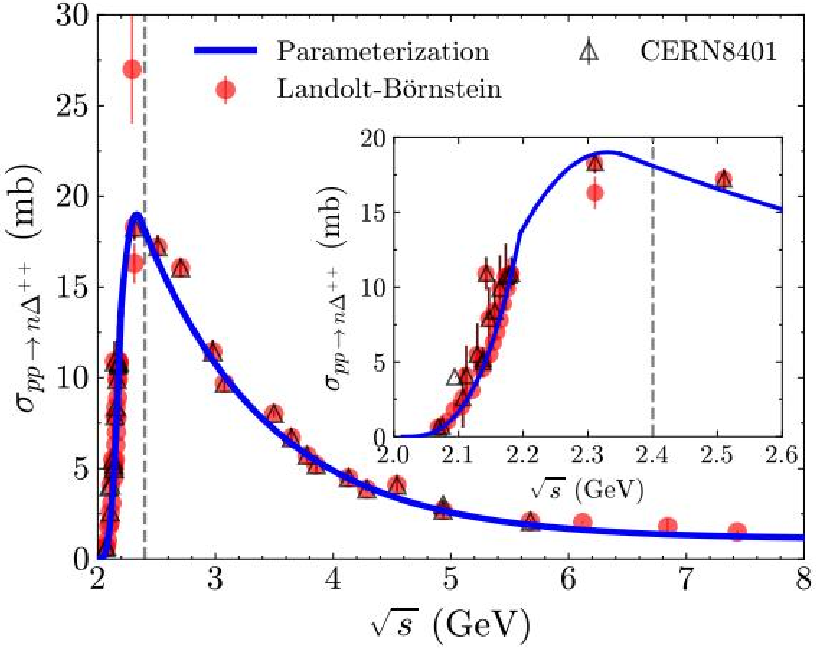


$$\sigma^*_{NN\to N\Delta}(\rho) = \sigma_{NN\to N\Delta}\, e^{-\alpha^{\pm}(\rho/\rho_0)^{3/2}},$$

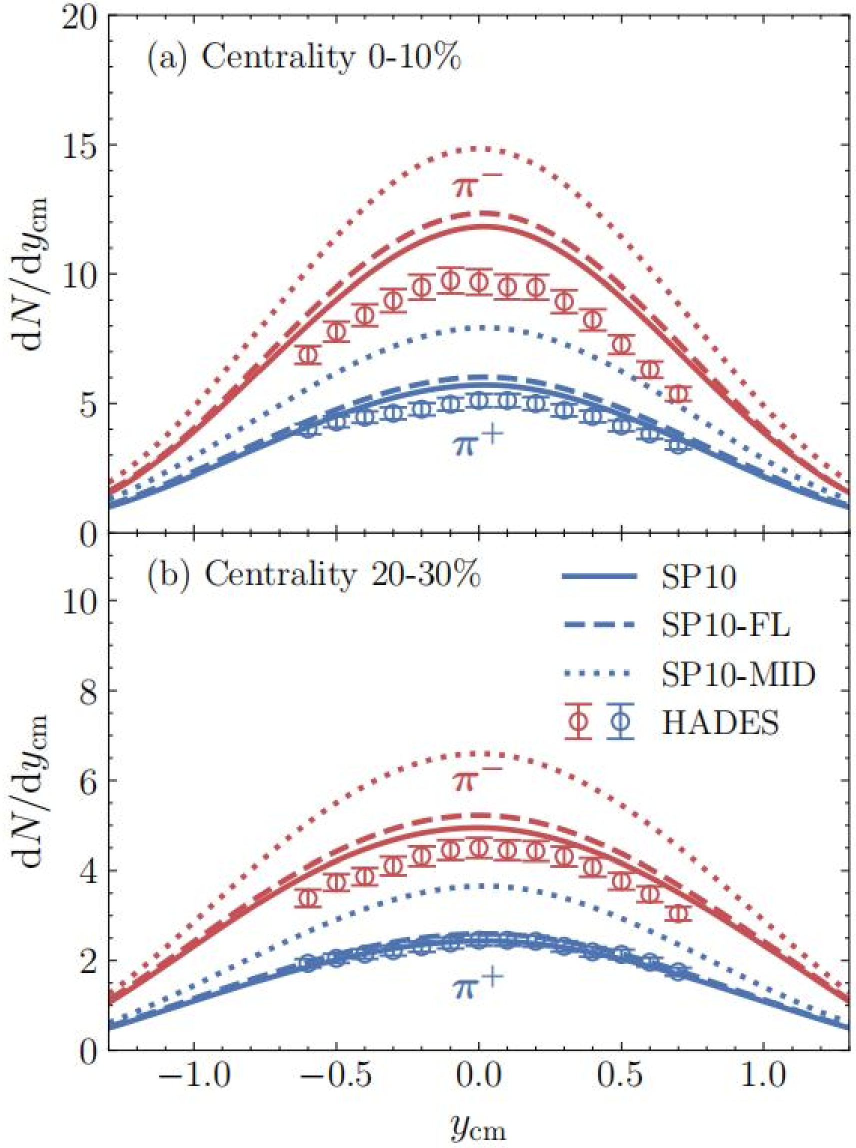
	$M(\pi^-)$	$M(\pi^+)$
HADES	$11.1 \pm 0.6 \pm 0.6$	$6.0 \pm 0.3 \pm 0.3$
$\alpha^+ = \alpha^- = 0$	17.2	8.7
$\alpha^+ = \alpha^- = 0.6$	11.8	5.4
$\alpha^+ = 0.39,\ \alpha^- = 0.7$	11.3	6.2



Ⅱ、Two-body cross section for EoS (eg2: NN inelastic)



	SP10	SP10-FL	SP10-MID
a_0 (MeV)	-64.97	-64.44	-52.84
a_2 (MeV fm^2)	7.104	6.829	0
a_4 (MeV fm^4)	-0.1628	-0.1719	0
a_6 (MeV fm^6)	1.731×10^{-3}	1.908×10^{-3}	0
a_8 (MeV fm^8)	-8.614×10^{-6}	-9.672×10^{-6}	0
a_{10} (MeV fm^{10})	1.621×10^{-8}	1.827×10^{-8}	0
b_0 (MeV)	-34.70	-34.70	-32.43
b_2 (MeV fm^2)	-3.323	-3.323	0
b_4 (MeV fm^4)	2.733×10^{-2}	2.733×10^{-2}	0
b_6 (MeV fm^6)	-8.992×10^{-5}	-8.992×10^{-5}	0
b_8 (MeV fm^8)	1.5849×10^{-7}	1.5849×10^{-7}	0
b_{10} (MeV fm^{10})	-1.738×10^{-10}	-1.738×10^{-10}	0



X. Li, et al., arXiv:2509.21099v1: Lattice Boltzmann Uehling-Uhlenbeck (LBUU) transport model

Ⅲ 、Self-consistent RBUU transport theory

50年前~

Annals of Physics
Volume 83, Issue 2, April 1974, Pages 491-529

A theory of highly condensed matter ☆

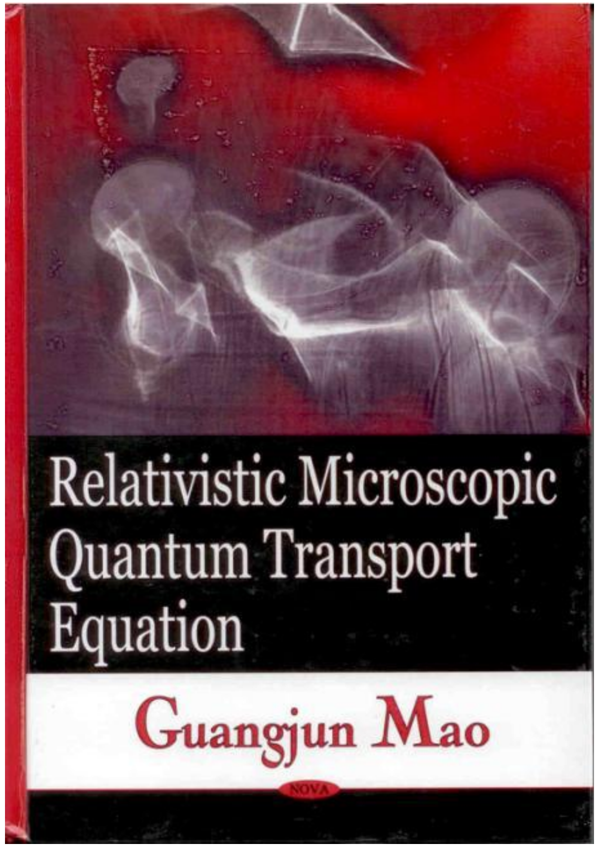
J.D Walecka

40年前~

Physics Reports
Volume 118, Issues 1–2, February 1985, Pages 1-131

Equilibrium and nonequilibrium formalisms made unified 闭合时间回路格林函数方法

Kuang-chao Chou, Zhao-bin Su *, Bai-lin Hao, Lu Yu



Publisher: Nova Science Pub Inc; UK ed. edition (April 8, 2005)

ISBN-10: 159454395X

ISBN-13: 978-1594543951

20年前~

BUU equation

$$\left(\partial_t + \frac{\mathbf{p}}{m}\cdot\nabla_{\mathbf{R}} - \nabla_{\mathbf{R}}U(R,t)\cdot\nabla_{\mathbf{p}}\right)f(\mathbf{p},\mathbf{R},t) \equiv I(f)$$

相对论化

sc-RBUU equation

30年前~

$$\left\{\left[\partial^{\mu}_{X} - \sum\nolimits^{\mu\nu}_{HF}(X,p,t)\partial^{p}_{\nu} - \partial^{\nu}_{p}\,\mathrm{Re}\sum\nolimits^{\mu}_{F}(X,p,t)\partial^{X}_{\nu}\right]\frac{p_{\mu}}{M^{*}_{\mathrm{Re}}} + \partial^{X}_{\nu}\left[\sum\nolimits_{H}(X) + \mathrm{Re}\sum\nolimits_{F}(X,p,t)\right]\partial^{\nu}_{p} - \partial^{\nu}_{p}\,\mathrm{Re}\sum\nolimits_{F}(X,p,t)\partial^{X}_{\nu}\right\}\times\frac{M}{E(\vec{p})}f(X,\vec{p})$$

$$= \frac{1}{2}\int\frac{d\vec{p}_2}{(2\pi)^3}\frac{d\vec{p}_3}{(2\pi)^3}\frac{d\vec{p}_4}{(2\pi)^3}(2\pi)^4\delta(\vec{p}+\vec{p}_2-\vec{p}_3-\vec{p}_4)\delta(E(\vec{p})+E(p_2)-E(p_3)-E(p_4))$$

$$\times W_{el,in}(\vec{p},p_2,p_3,p_4)\left(F^{2}_{el,in} - F^{1}_{el,in}\right)$$

Ⅲ 、Self-consistent RBUU transport theory

The collision term can be expressed as:

$$C(x,p)=\frac{1}{2}\int\frac{d^3p_2}{(2\pi)^3}\int\frac{d^3p_3}{(2\pi)^3}\int\frac{d^3p_4}{(2\pi)^3}(2\pi)^4\delta^{(4)}\left(p+p_2-p_3-p_4\right)W\left(p,p_2,p_3,p_4\right)\left(F_2-F_1\right),$$

$$W\left(p,p_2,p_3,p_4\right)=G_1\left(p,p_2,p_3,p_4\right)+G_2\left(p,p_2,p_3,p_4\right)+p_3\leftrightarrow p_4,$$

$$\begin{aligned}F_1&=f(\mathbf{x},\mathbf{p},\tau)f(\mathbf{x},\mathbf{p}_2,\tau)[1-f_\Delta(\mathbf{x},\mathbf{p}_3,\tau)][1-f\left(\mathbf{x},\mathbf{p}_4,\tau\right)],\\F_2&=[1-f(\mathbf{x},\mathbf{p},\tau)][1-f\left(\mathbf{x},\mathbf{p}_2,\tau\right)]f_\Delta\left(\mathbf{x},\mathbf{p}_3,\tau\right)f\left(\mathbf{x},\mathbf{p}_4,\tau\right),\end{aligned}$$

The relationship between the scattering cross section and the collision term is:

自洽化

$$\int\frac{d^3p_3}{(2\pi)^3}\int\frac{d^3p_4}{(2\pi)^3}(2\pi)^4\delta^{(4)}\left(p+p_2-p_3-p_4\right)W\left(p,p_2,p_3,p_4\right)=\int v\sigma(s,t)d\Omega.$$

Therefore the collision term can be expressed as:

$$C(x,p)=\frac{1}{2}\int\frac{d^3p_2}{(2\pi)^3}v\sigma(s,t)\left(F_2-F_1\right)d\Omega.$$

理论需要长期主义，期间填满
理解深化、实验验证、框架更新

G. J. Mao, Z. X. Li, Y. Z. Zhuo, Y. L. Han and Z. Q. Yu, Phys. Rev. C 49, 3137-3146 (1994).

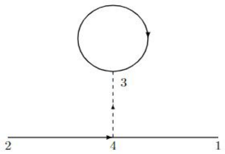
Ⅳ、NN elastic cross section (density-dep.)

L_I is the interaction Lagrangian density of nucleons coupled to σ, ω, ρ, and π mesons and reads as

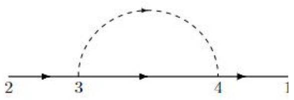
$$L_I = g_\sigma \bar{\Psi}\Psi\boldsymbol{\sigma} - g_\omega \bar{\Psi}\gamma_\mu \Psi\omega^\mu + g_\pi \bar{\Psi}\gamma_\mu\gamma_5 \boldsymbol{\tau}\cdot\Psi\partial^\mu\boldsymbol{\pi} - \frac{1}{2}g_\rho \bar{\Psi}\gamma_\mu \boldsymbol{\tau}\cdot\Psi\boldsymbol{\rho}^\mu,$$

- σ^*_{np} depends on the baryon density weakly, while $\sigma^*_{pp(nn)}$ depends on density significantly.
- The ρ meson field plays a dominant role in the isospin dependence of the NN elastic cross sections.

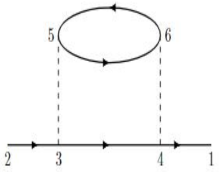
Feynman diagram



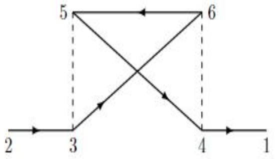
Hartree term



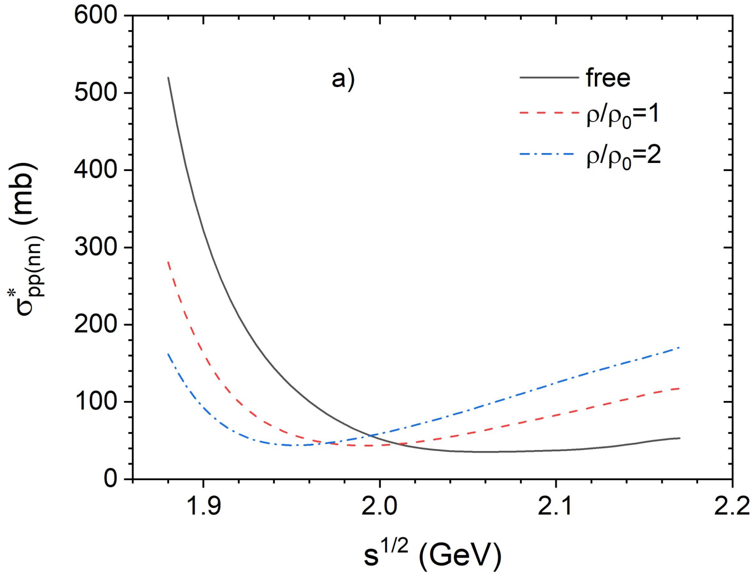
Fock term

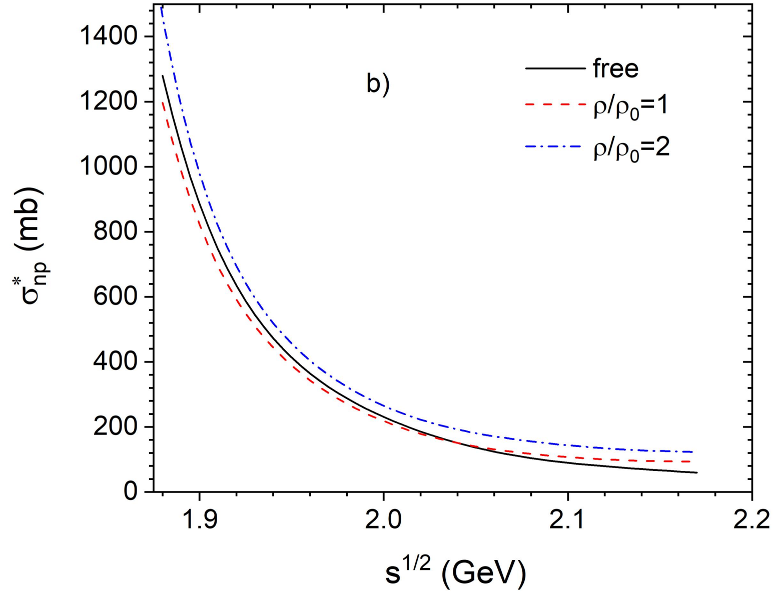


Born direct term



Born exchanged term





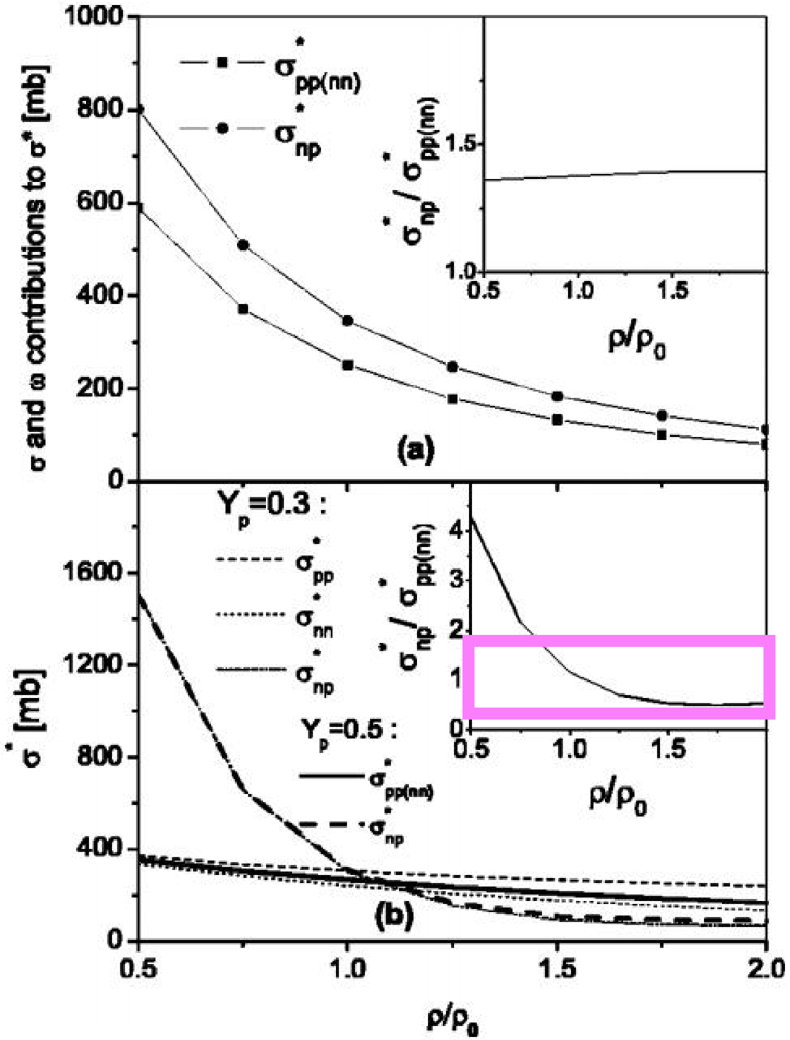
Q. F. Li, Z. X. Li, G. J. Mao, Phys. Rev. C 62, 014606(2000).

Ⅳ 、NN elastic cross section (isospin-dep.)

The scalar-isovector δ meson is further introduced into the effective Lagrangian to calculate the in-medium NN elastic cross section

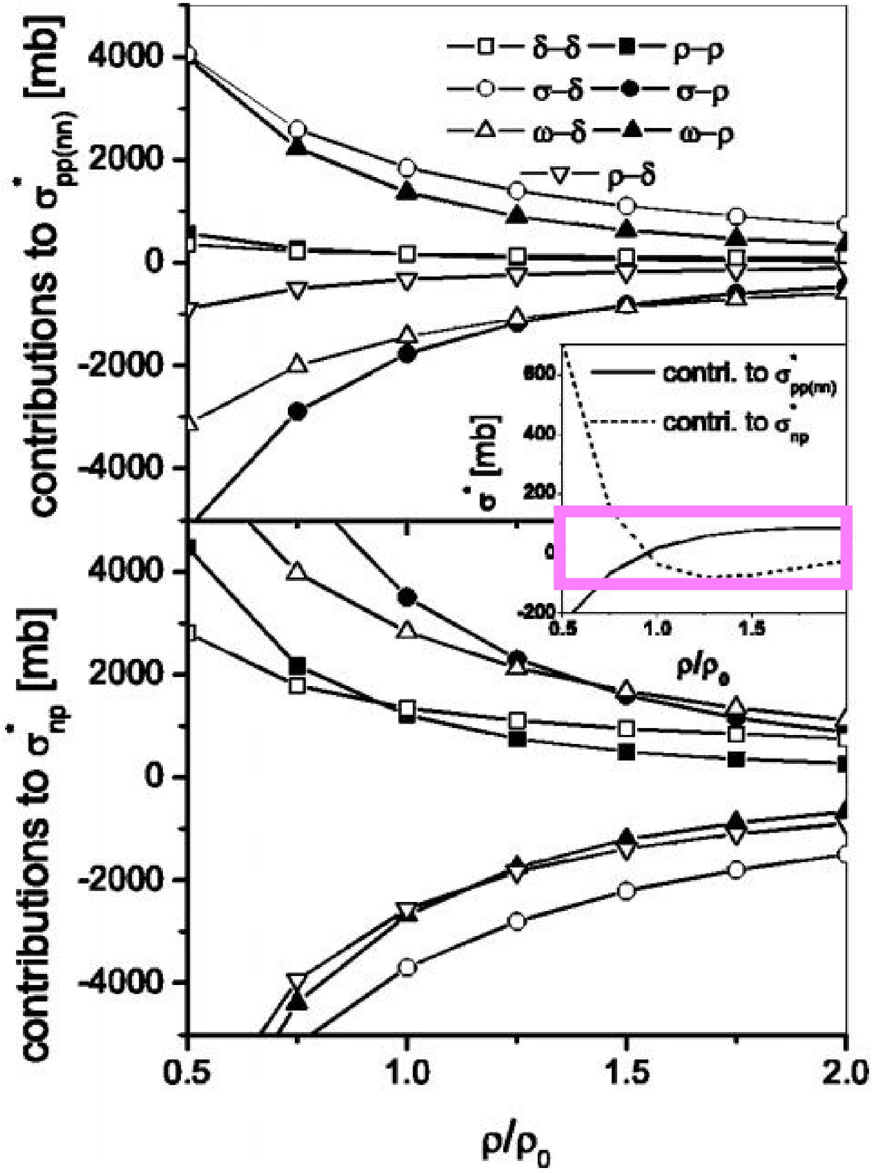
$$L=\overline{\Psi}[i\gamma_\mu\partial^\mu - M_N]\Psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}F_{\mu\nu}\cdot F^{\mu\nu} + \frac{1}{2}\partial_\mu\boldsymbol{\delta}\partial^\mu\boldsymbol{\delta}$$
$$-\frac{1}{4}L_{\mu\nu}\cdot L^{\mu\nu} - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{2}m_\delta^2\boldsymbol{\delta}^2 + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu$$
$$+g_\sigma\overline{\Psi}\Psi\sigma - g_\omega\overline{\Psi}\gamma_\mu\Psi\omega^\mu + g_\delta\overline{\Psi}\boldsymbol{\tau}\cdot\Psi\boldsymbol{\delta}$$
$$-\frac{1}{2}g_\rho\overline{\Psi}\gamma_\mu\boldsymbol{\tau}\cdot\Psi\boldsymbol{\rho}^\mu,$$

The density dependence of σ^*_{np} and $\sigma^*_{pp(nn)}$ is very different: at low densities, the σ^*_{np} is about three to four times larger than $\sigma^*_{pp(nn)}$; while at dense nuclear matter the isospin effect on in-medium NN cross section largely washes out.

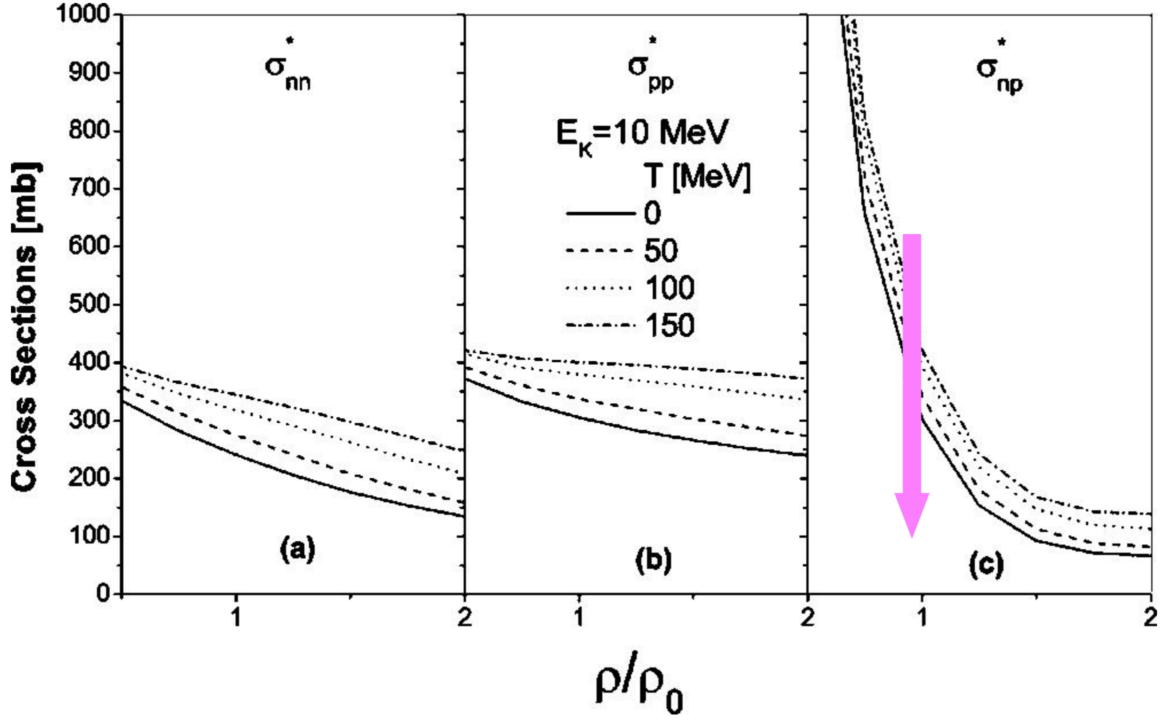


Q. F. Li, Z. X. Li, E. G. Zhao, Phys. Rev. C 69, 017601 (2004).

Ⅳ、NN elastic cross section (density-, isospin-, temp-dep.)



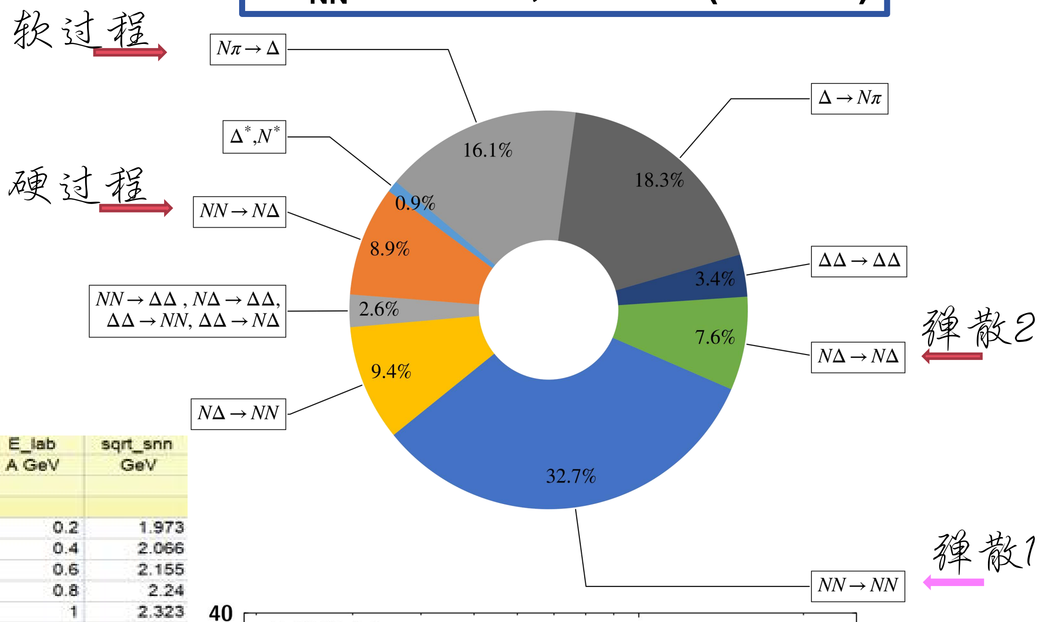
Q. F. Li, Z. X. Li, E. G. Zhao, Phys. Rev. C 69, 017601 (2004).



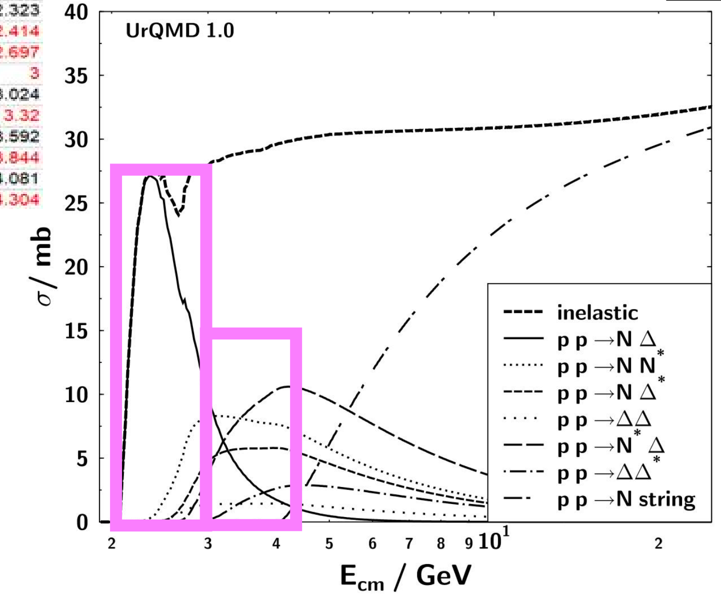
- The isospin effect on the density dependence of the in-medium NN elastic cross section is dominantly contributed by the isovector δ and ρ mesons.
- The temperature effect of nuclear medium on σ^*_{np} and $\sigma^*_{pp(nn)}$ is weaker compared with the density effect.

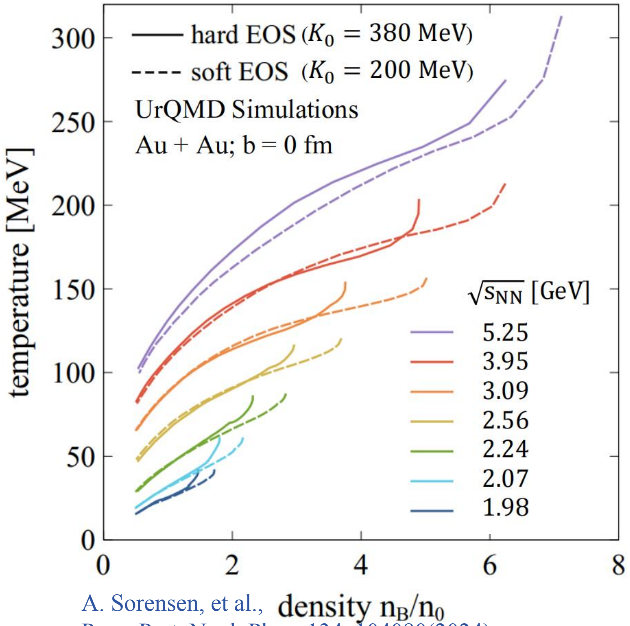
Ⅴ 、Δ- related cross sections （进入多共振态“混合”模式的必由之路）

√s_{NN}=2.6 GeV, Au+Au (b=0 fm)

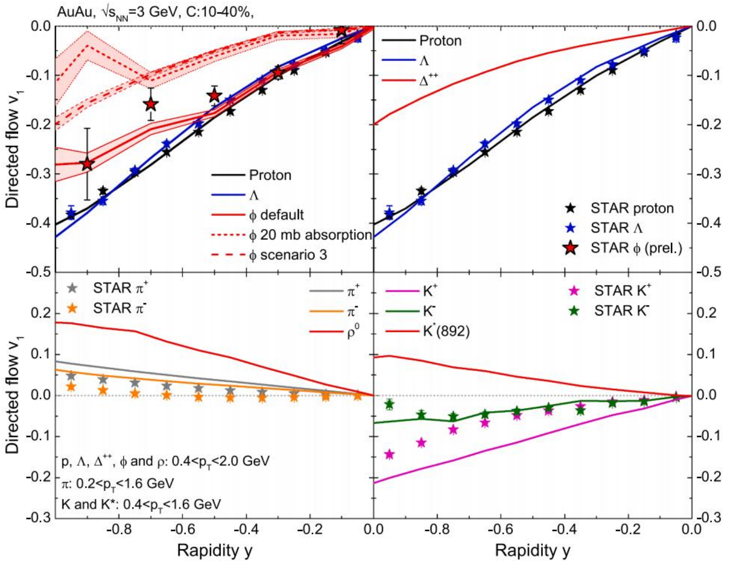


E_lab A GeV	sqrt_snn GeV
0.2	1.973
0.4	2.066
0.6	2.155
0.8	2.24
1	2.323
1.23	2.414
2	2.697
2.921	3
3	3.024
4	3.32
5	3.592
6	3.844
7	4.081
8	4.304





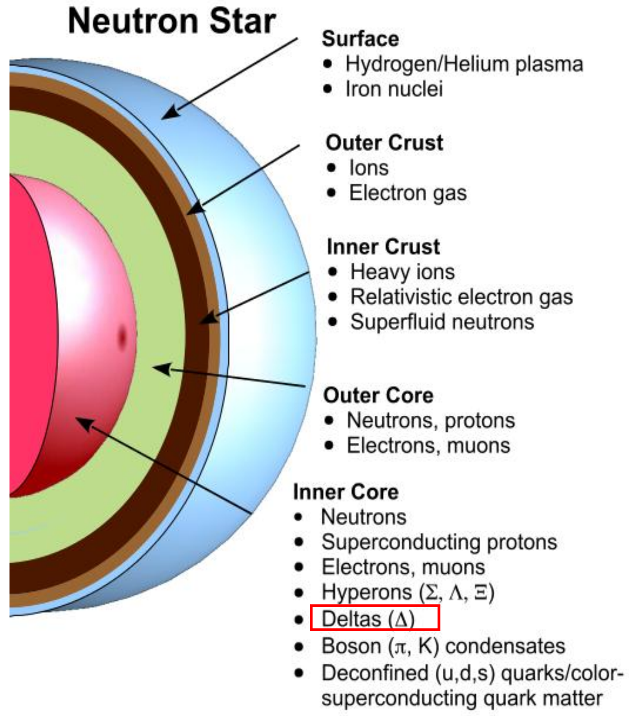
A. Sorensen, et al., Prog. Part. Nucl. Phys. 134, 104080(2024).



Jan Steinheimer, et al., Phys. Lett. B 869 (2025) 139870

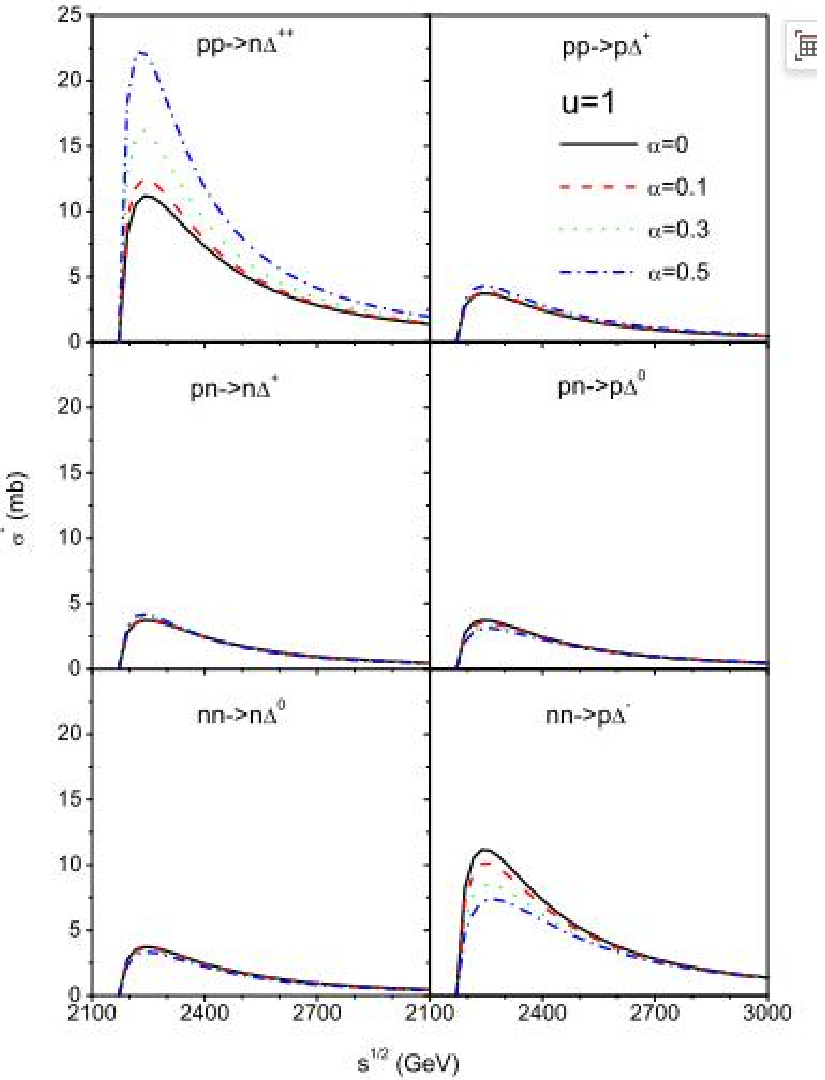
neutron star

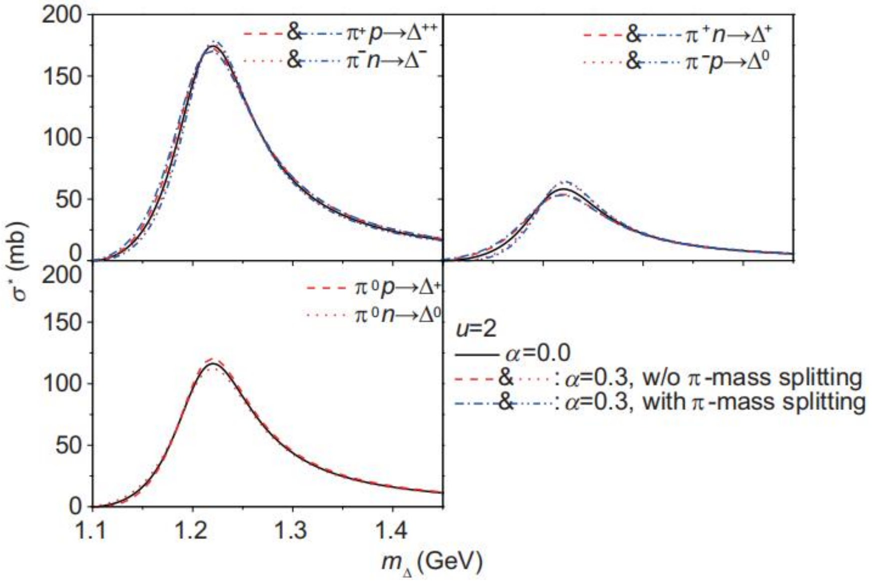
R. B. Jacobsen, V. Dexheimer and R. Luciano Sonego Farias, Astron. Nachr. 344, e230038 (2023).

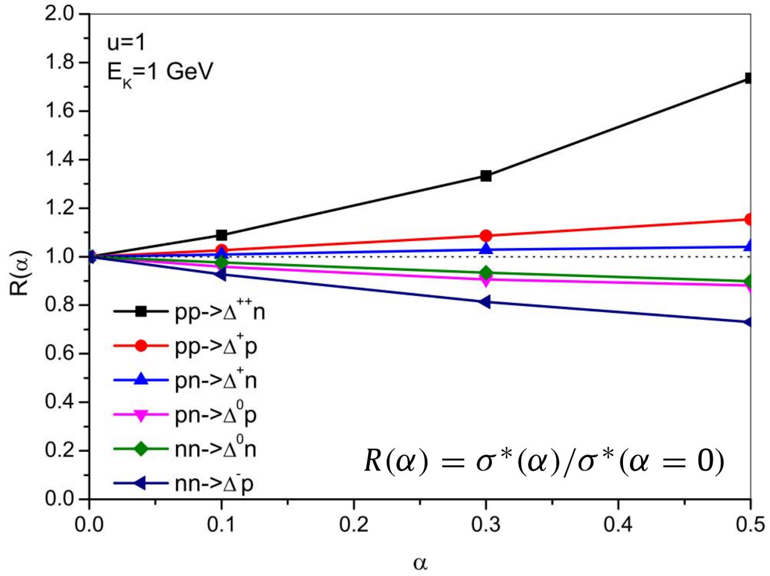


同：较大的密度范围
异：温度、非平衡、引力、更大的同位旋不对称度、物质种类…

Ⅴ、hard- and soft-Δ production cross sections







- Similar to the NN elastic ones, the reductions of NN inelastic cross sections is also isospin dependent and the effect is largest and of opposite sign for the Δ^{++} and Δ^{-} states.
- The mass-splitting effect on $\sigma^*_{\pi N\to\Delta}$ which is mainly from the mass splittings of nucleons and Δ baryons,
- The largest mass-splitting influence is reflected in the production of Δ^0 and Δ^+ isobars.

Q. F. Li, Z. X. Li, Phys. Lett. B 773, 557-562 (2017) & Sci. China-Phys. Mech. Astron. 62, 972011 (2019).

V 、 hard-Δ production cross section

➢ Temperature dependence (σ, ω, ρ, δ)

M Nan, P Li, G Wei, X Xiang, W Zuo, Q Li arXiv:2510.09337

$$\mathcal{L}_F = \bar{\Psi}\left[i\gamma_\mu\partial^\mu - m_N\right]\Psi + \bar{\Psi}_{\Delta\nu}\left[i\gamma_\mu\partial^\mu - m_\Delta\right]\Psi_\Delta^\nu + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\delta}\partial^\mu\vec{\delta} - \frac{1}{4}F_{\mu\nu}\cdot F^{\mu\nu} - \frac{1}{4}\vec{L}_{\mu\nu}\cdot\vec{L}^{\mu\nu} - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{2}m_\delta^2\vec{\delta}^2 + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - U_{NL}(\sigma,\omega^\mu,\vec{\delta},\vec{\rho}^\mu),$$

$$\mathcal{L}_I = g_{NN}^\sigma\bar{\Psi}\Psi\sigma + g_{NN}^\delta\bar{\Psi}\vec{\tau}\cdot\Psi\vec{\delta} - g_{NN}^\omega\bar{\Psi}\gamma_\mu\Psi\omega^\mu - g_{NN}^\rho\bar{\Psi}\gamma_\mu\vec{\tau}\cdot\Psi\vec{\rho}^\mu + g_{\Delta\Delta}^\sigma\bar{\Psi}_\Delta\Psi_\Delta\sigma + g_{\Delta\Delta}^\delta\bar{\Psi}_\Delta\vec{\tau}\cdot\Psi_\Delta\vec{\delta} - g_{\Delta\Delta}^\omega\bar{\Psi}_\Delta\gamma_\mu\Psi_\Delta\omega^\mu - g_{\Delta\Delta}^\rho\bar{\Psi}_\Delta\gamma_\mu\vec{\tau}\cdot\Psi_\Delta\vec{\rho}^\mu,$$

$$m_{\Delta^{++}/\Delta^-}^* = m_\Delta - g_{\Delta\Delta}^\sigma\sigma \mp g_{\Delta\Delta}^\delta\delta,$$
$$m_{\Delta^+/\Delta^0}^* = m_\Delta - g_{\Delta\Delta}^\sigma\sigma \mp \tfrac{1}{3}g_{\Delta\Delta}^\delta\delta.$$

$$m_{p/n}^* = m_N - g_{NN}^\sigma\sigma \mp g_{NN}^\delta\delta$$

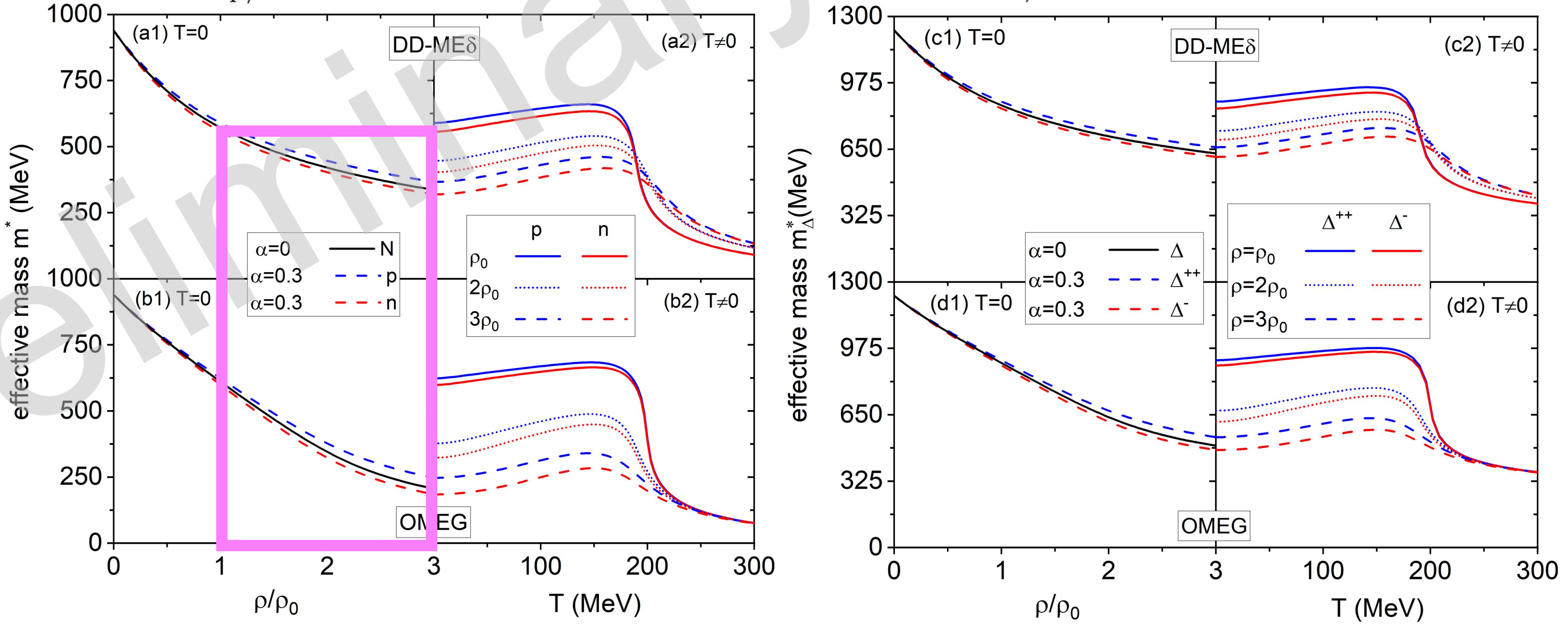
- Density-dependent coupling constants DD-MEδ

X. Roca-Maza, X. Vinas, M. Centelles, P. Ring and P. Schuck, Phys. Rev. C 84, 054309 (2011)

- Nonlinear-dependent coupling constants OMEG

N. Zabari, S. Kubis, and W. W´ojcik, Phys. Rev. C 99, 035209 (2019)
&
Phys. Rev. C 100, 015808 (2019)

$\Lambda_{\sigma\delta} = 50, \quad \Lambda_{\omega\rho} = 173.77$

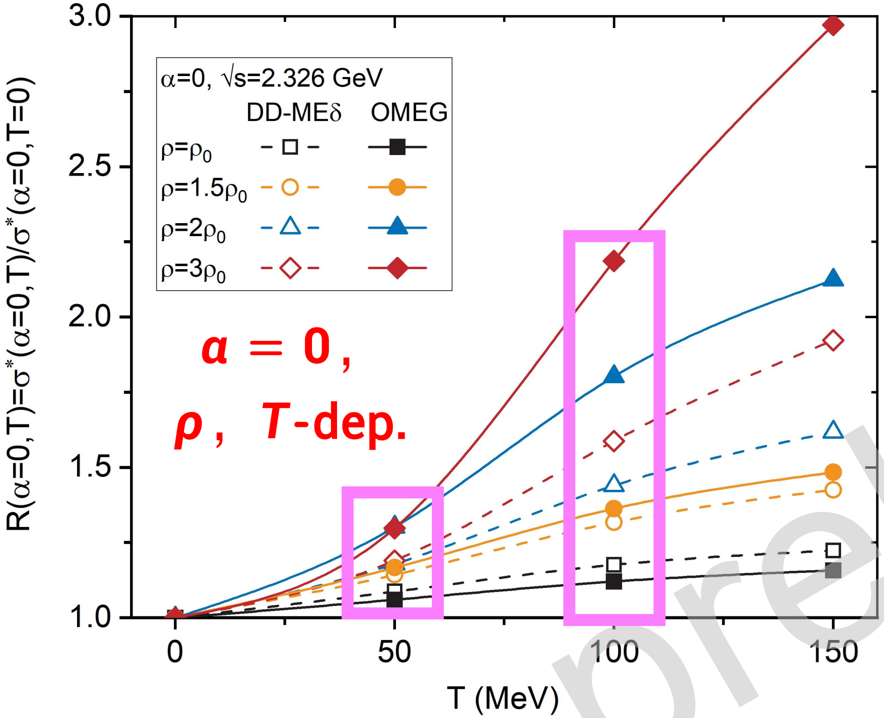


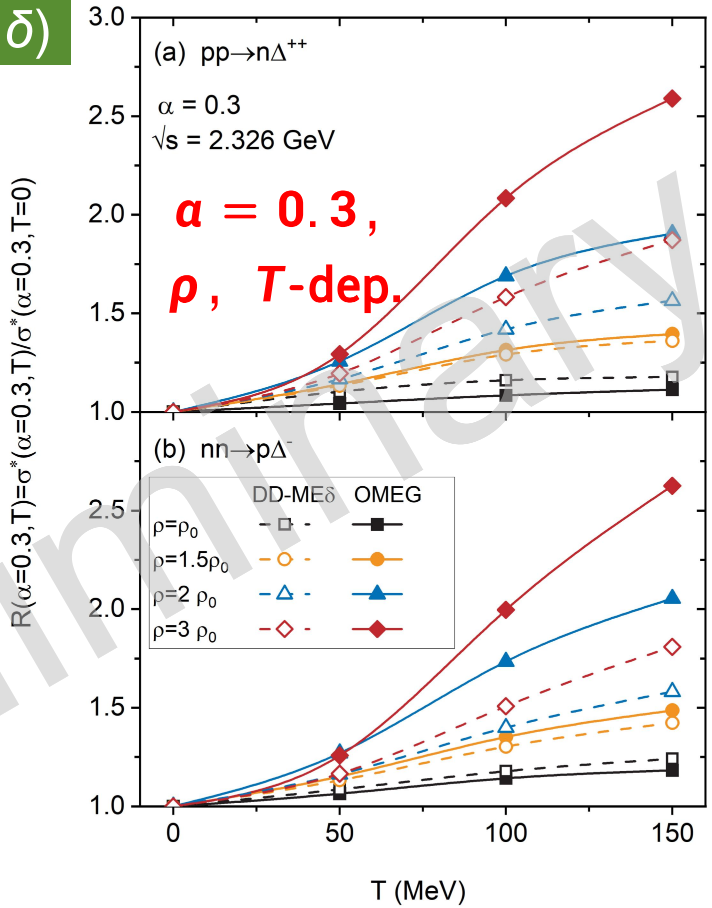
➢ Both coupling parameter sets yield similar temperature-dependent baryon effective mass and the mass splitting.

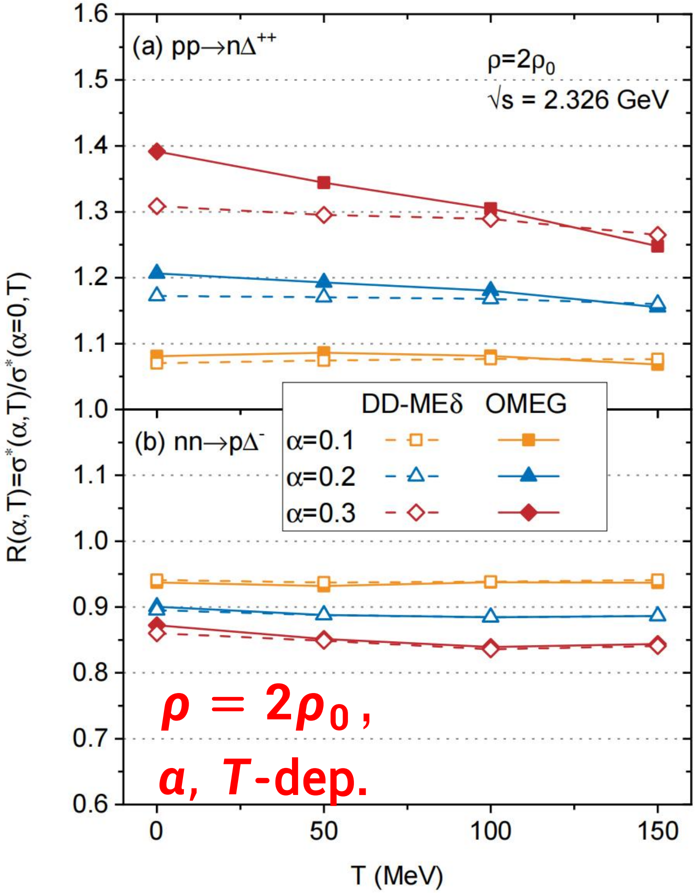
➢ The OMEG set exhibits a stronger density dependence, especially at high densities ($\rho \gtrsim 1.5\rho_0$).

Ⅴ 、 hard-Δ production cross section

➢ Temperature dependence (σ, ω, ρ, δ)



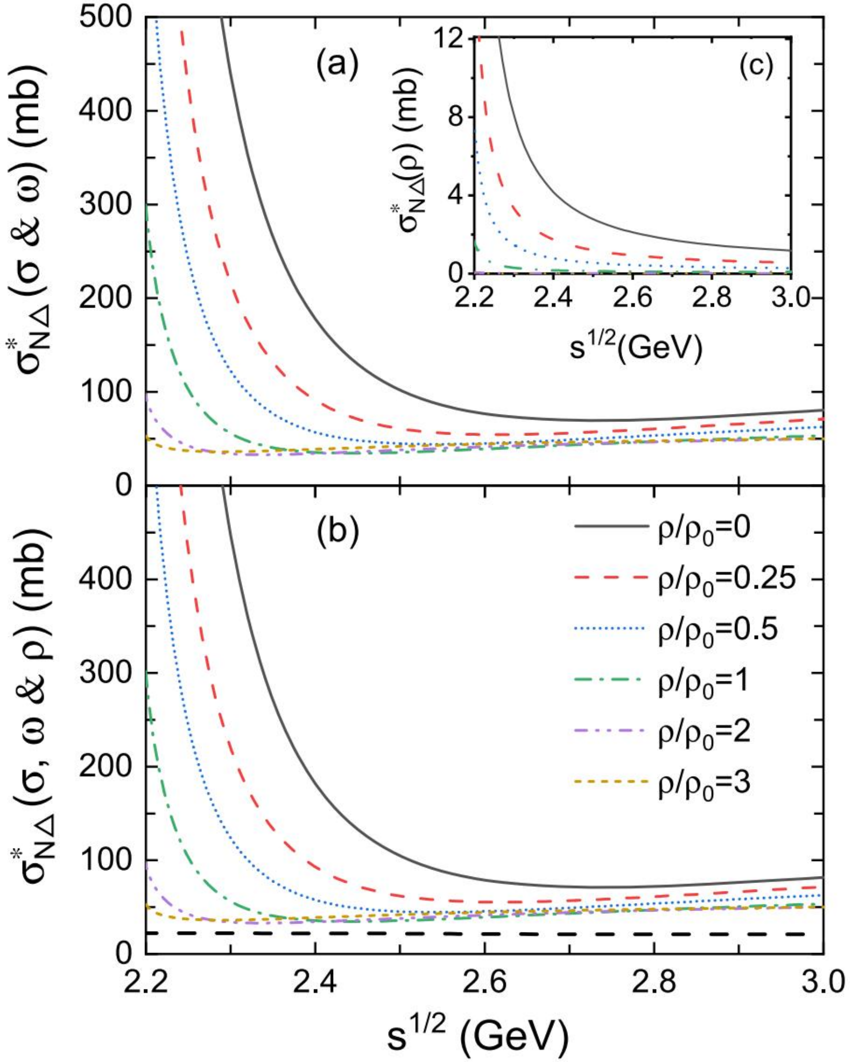




- ➢ As the temperature T increases, the Δ stiff production cross sections increase significantly.
- ➢ The ratio $R(\alpha, T)$ for both parameter sets shows similar T dependences in lower density ($\rho \leq 1.5\rho_0$), while at higher density, the model dependence becomes more obvious.
- ➢ The finite-temperature enhancement is more pronounced in isospin-symmetric matter.
- ➢ The temperature effect on the isospin dependence of the cross section is complex due to the model dependence.

Ⅴ、NΔ elastic cross section (density-dep.)

➢ Density dependence (σ, ω, ρ)



$$L_I = g^{\sigma}_{NN}\bar{\Psi}\Psi\sigma - g^{\omega}_{NN}\bar{\Psi}\gamma_\mu\Psi\omega^\mu - g^{\rho}_{NN}\bar{\Psi}\gamma_\mu\boldsymbol{\tau}\cdot\Psi\boldsymbol{\rho}^\mu$$
$$+ g^{\sigma}_{\Delta\Delta}\bar{\Psi}_\Delta\Psi_\Delta\sigma - g^{\omega}_{\Delta\Delta}\bar{\Psi}_\Delta\gamma_\mu\Psi_\Delta\omega^\mu$$
$$- g^{\rho}_{\Delta\Delta}\bar{\Psi}_\Delta\gamma_\mu\boldsymbol{\tau}\cdot\Psi_\Delta\boldsymbol{\rho}^\mu.$$

Also, in the coupling 'constants':

$$g^i_{NN}(\rho) = g_i(\rho_{sat})f_i(\xi), \quad i = \sigma, \omega, \quad f_i(\xi) = a_i\frac{1 + b_i(\xi + d_i)^2}{1 + c_i(\xi + d_i)^2}, \quad \xi = \frac{\rho}{\rho_{sat}}.$$

$$g^{\rho}_{NN}(\rho) = g_\rho(\rho_{sat})e^{-a_\rho(\xi - 1)}. \qquad \frac{g^{\sigma}_{\Delta\Delta}}{g^{\sigma}_{NN}} = 1.0, \quad \frac{g^{\omega}_{\Delta\Delta}}{g^{\omega}_{NN}} = 0.8, \quad \frac{g^{\rho}_{\Delta\Delta}}{g^{\rho}_{NN}} = 0.7;$$

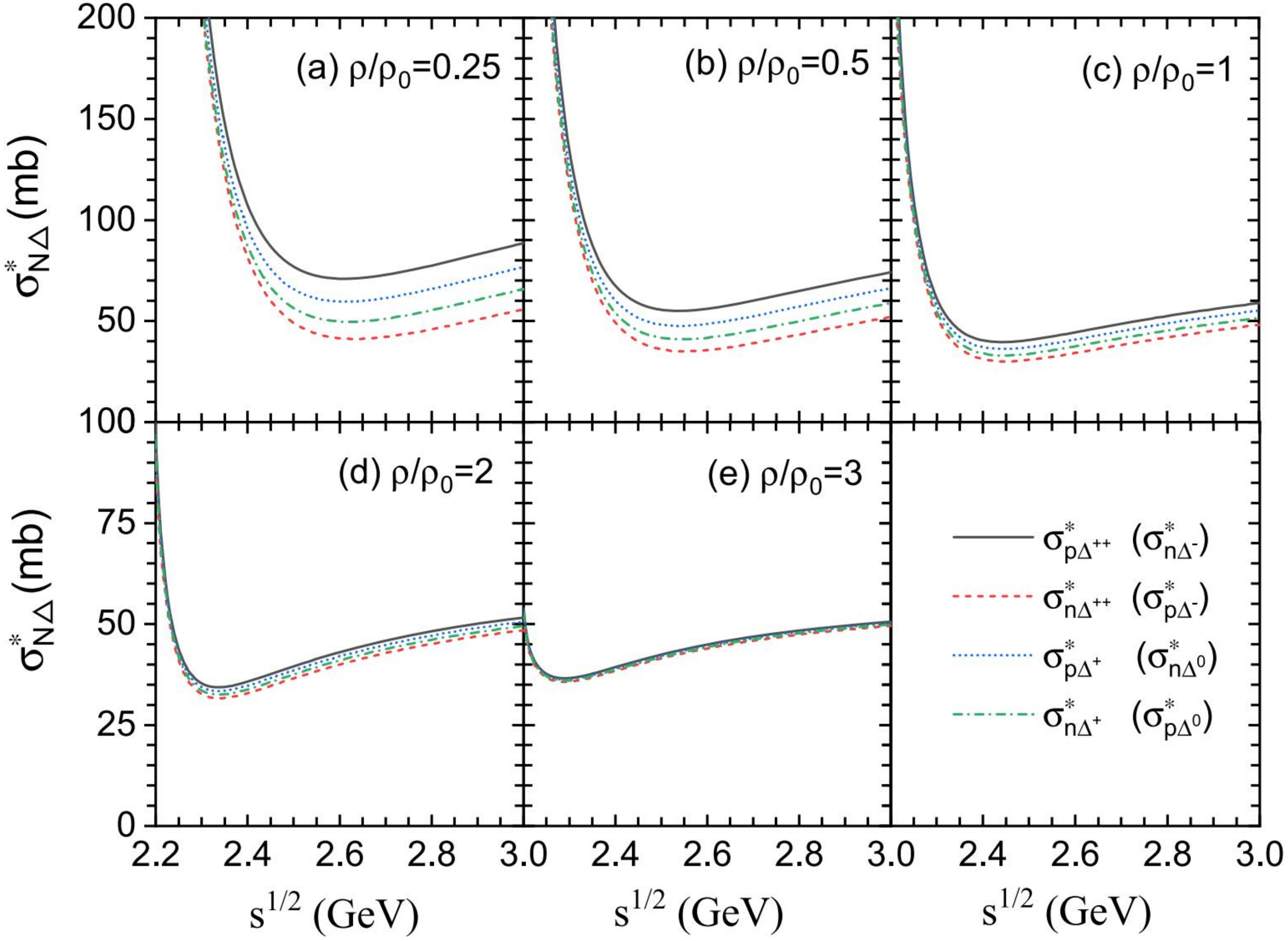
A. R. Raduta, Phys. Lett. B 814, 136070 (2021).
G. A. Lalazissis, T. Niksic, D. Vretenar, and P. Ring, Phys. Rev. C 71, 024312 (2005).

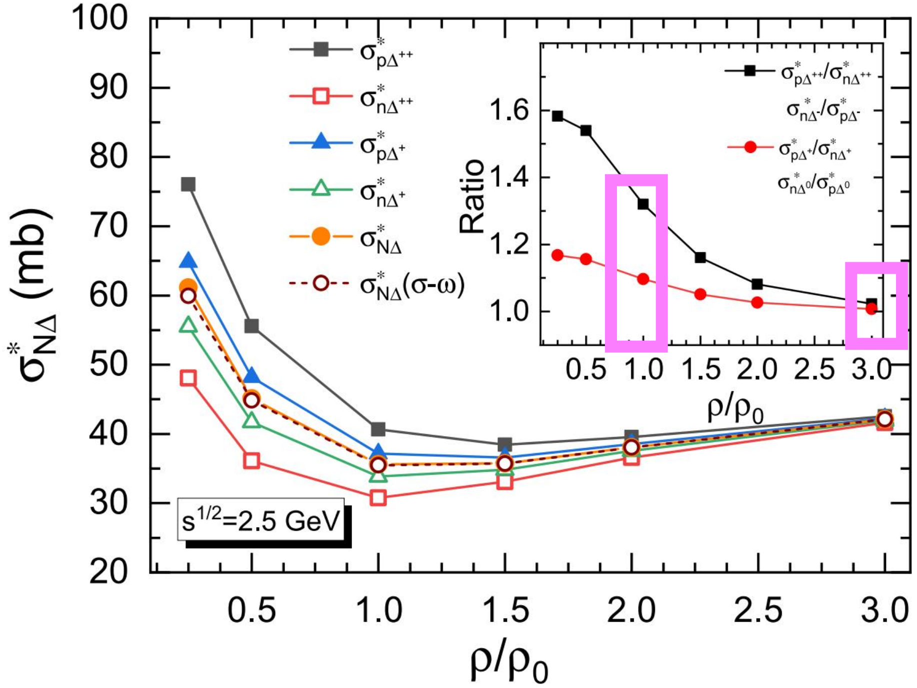
➢ An overall suppression of $N\Delta \to N\Delta$ cross section, especially at low energies.
➢ Due to the density-dependent coupling constants adopted in this work, the decrease in cross section with density is faster.
➢ Compared to σ and ω, the contribution of ρ meson exchange is weak especially at high energies.

M. Z. Nan, P. C. Li, Y. J. Wang, Q. F. Li, W. Zuo, Eur. Phys. J. A 60, 131 (2024).

Ⅴ、NΔ elastic cross section (density-dep.)

➢ Density dependence (σ, ω, ρ)





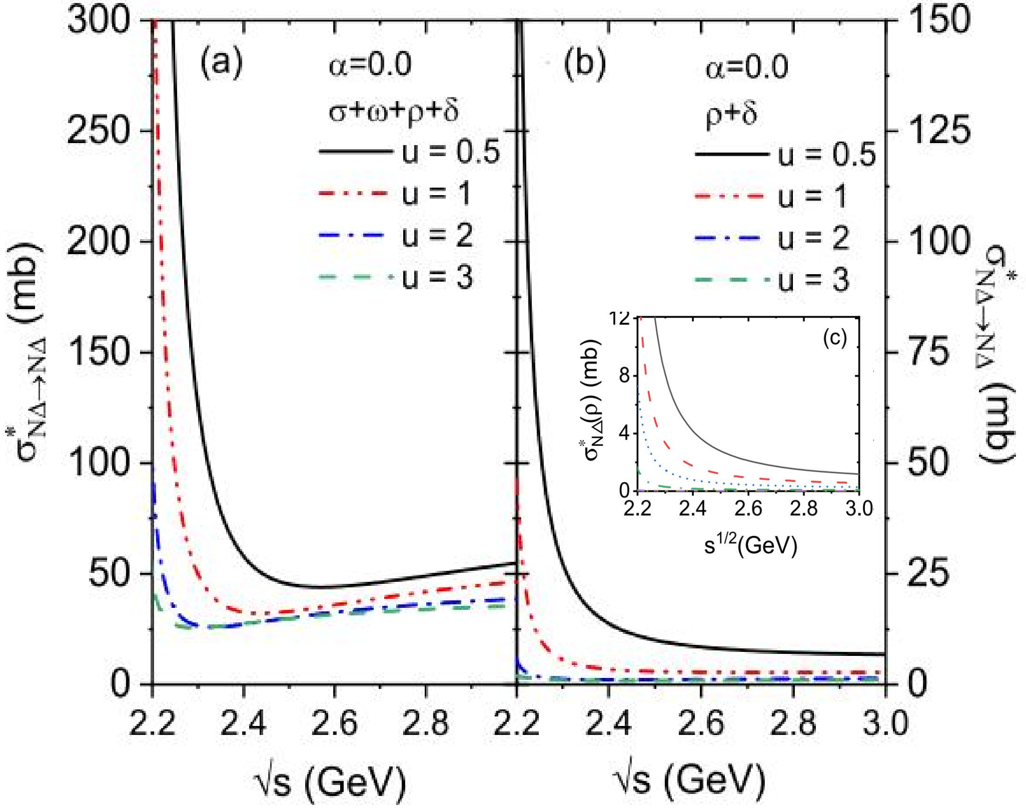
- The inclusion of ρ meson exchange leads to obvious isospin effect especially at low densities.
- With increasing density, the isospin effect is fading away…
- When the density reaches 3ρ0, the isospin effect almost disappears.

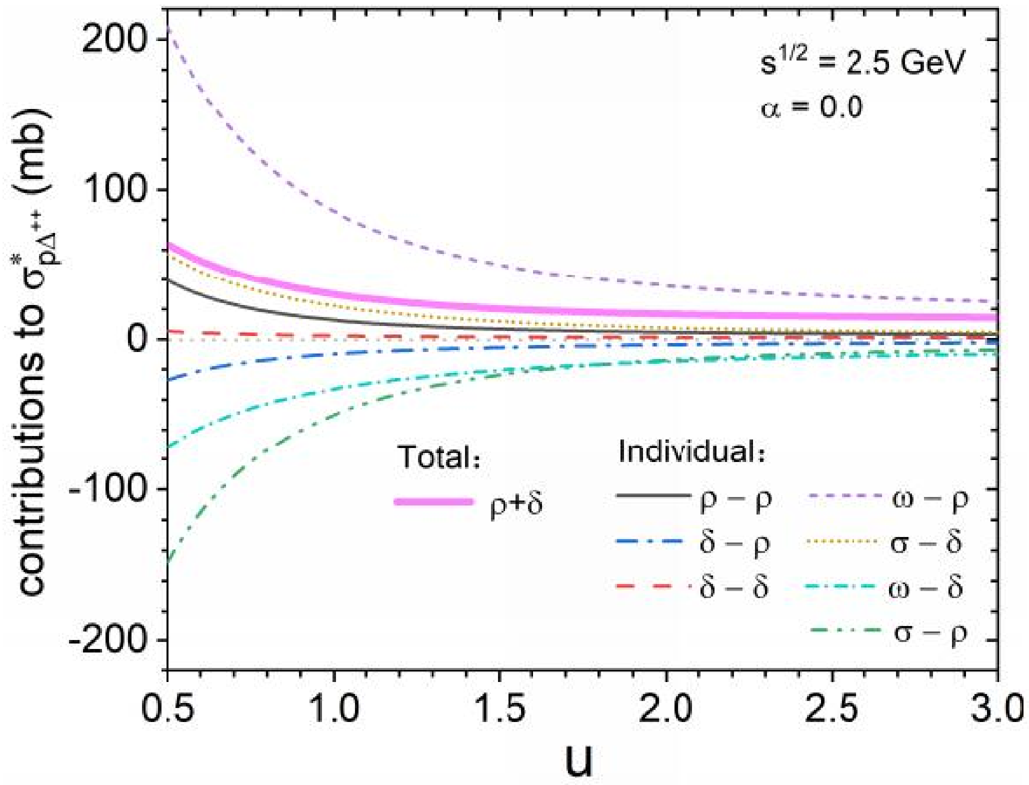
M. Z. Nan, P. C. Li, Y. J. Wang, Q. F. Li, W. Zuo, Eur. Phys. J. A 60, 131 (2024).

Ⅴ、NΔ elastic cross section (isospin-dep.)

➢ Density dependence (σ, ω, ρ, δ)

M. Z. Nan, P. C. Li, Q. F. Li, W. Zuo, Chin. Phys. C 49, 094112 (2025)



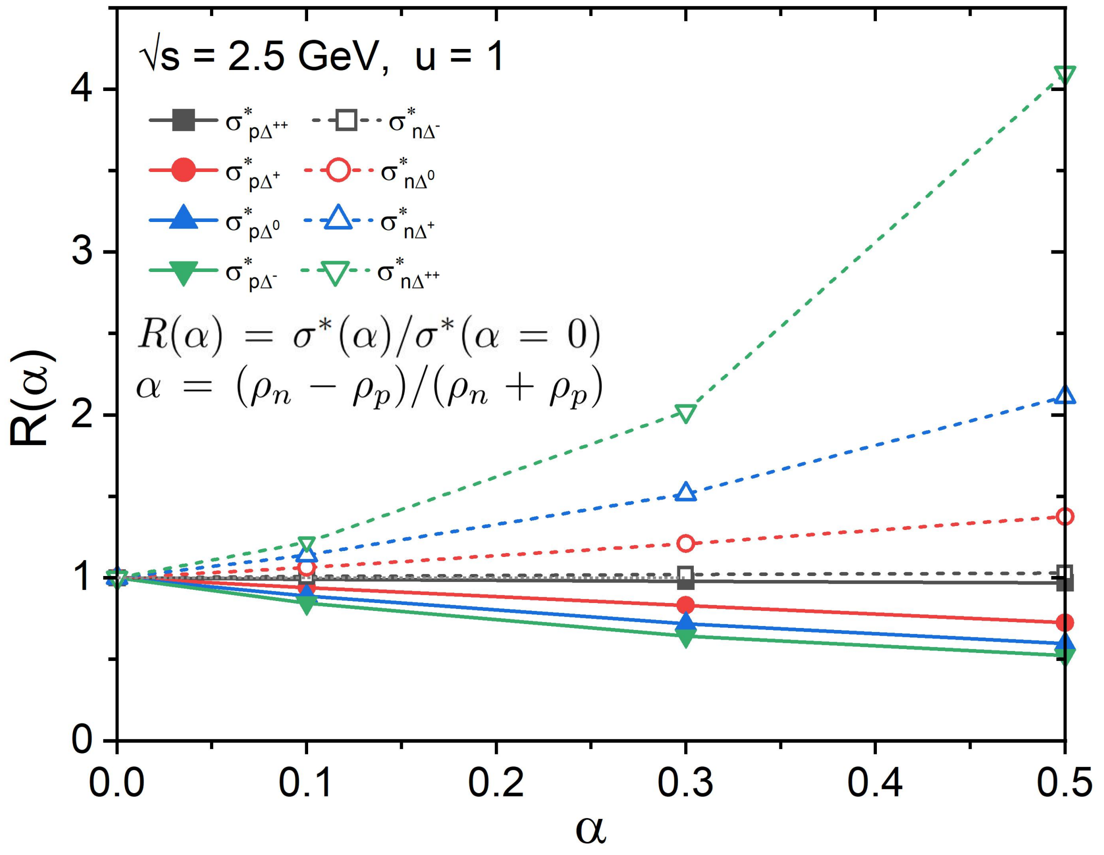


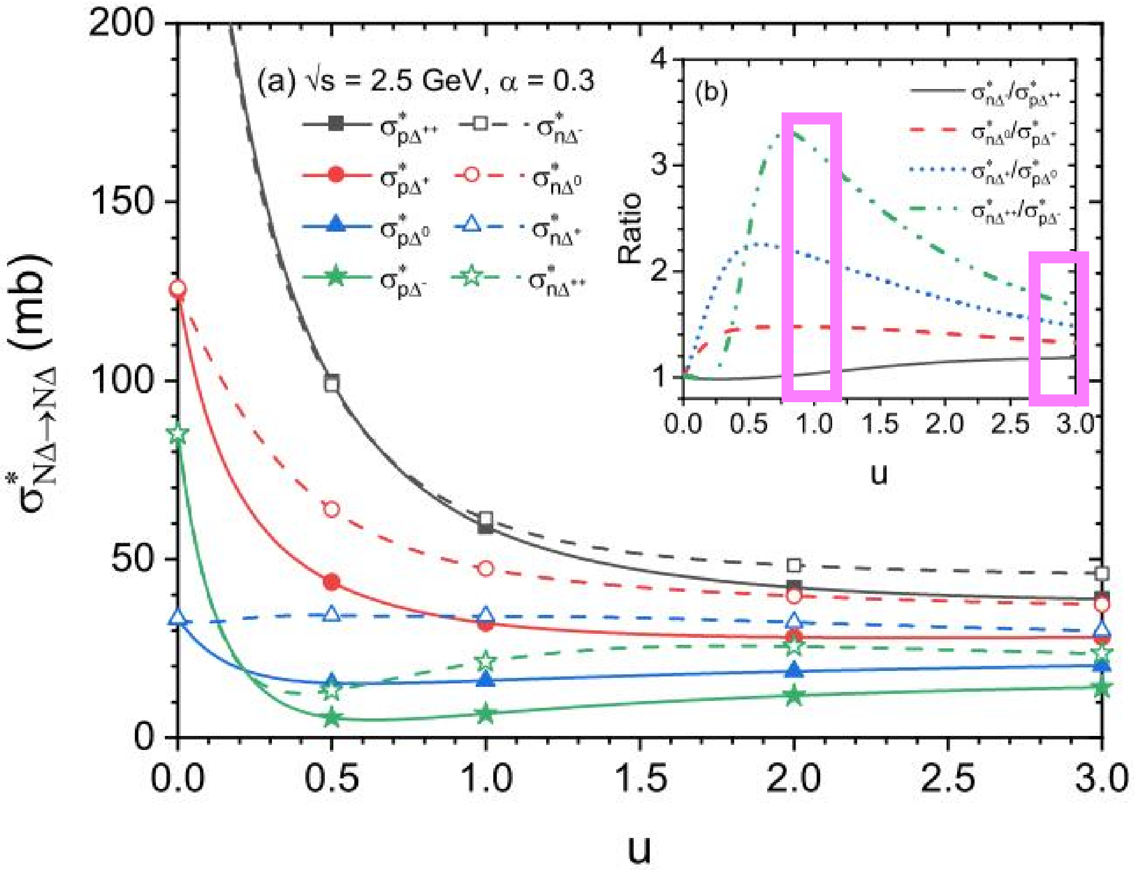
- $\sigma^*_{N\Delta}$ decreases with increasing density, indicating a visible density dependent suppression of nuclear medium.
- The ρ and δ meson related-terms have a larger contribution than that of ρ meson field.
- The contribution of each meson exchange term decreases with increasing reduced density, the baryon-baryon-meson coupling constants and the effective masses of nucleons and Δ particles.
- Obvious cancellation effect, but the net-contribution of ρ and δ related exchange terms to the $\sigma^*_{p\Delta^{++}}$ is larger than 0.

Ⅴ、NΔ elastic cross section (isospin-dep.)

➢ Density dependence (σ, ω, ρ, δ)

M. Z. Nan, P. C. Li, Q. F. Li, W. Zuo, Chin. Phys. C 49, 094112 (2025)



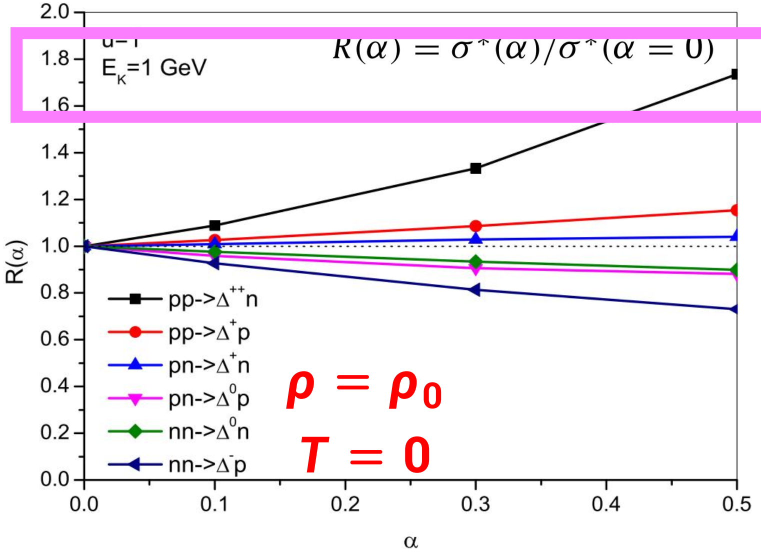


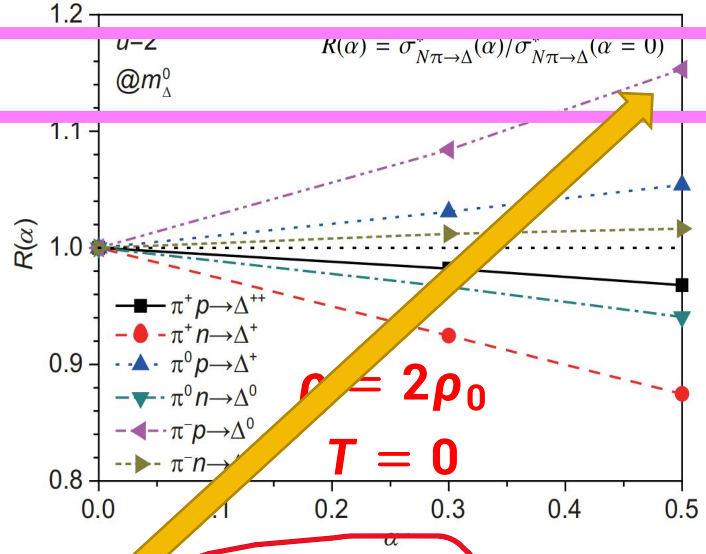
- $R(\alpha)$ for pΔ channels is decreased, while that for nΔ channels is increased as α increases from 0.0 to 0.5, since the contribution of δ meson exchange to the effective masses of protons, neutrons and Δ-isobars have opposite signs.
- The isospin effect, which introduced by isovector ρ and δ meson fields, in NΔ → NΔ channel should not be negligible even at such a high energy and density.

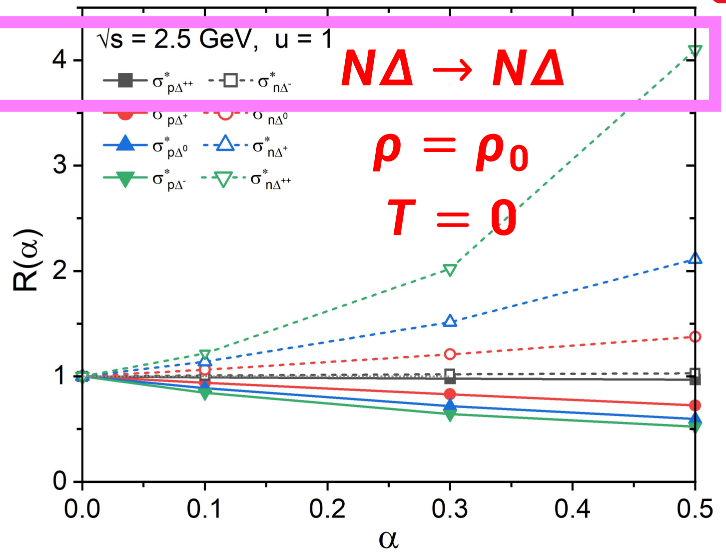
V 、NN inelastic cross section

➤ Isospin dependence (σ, ω, ρ, δ)

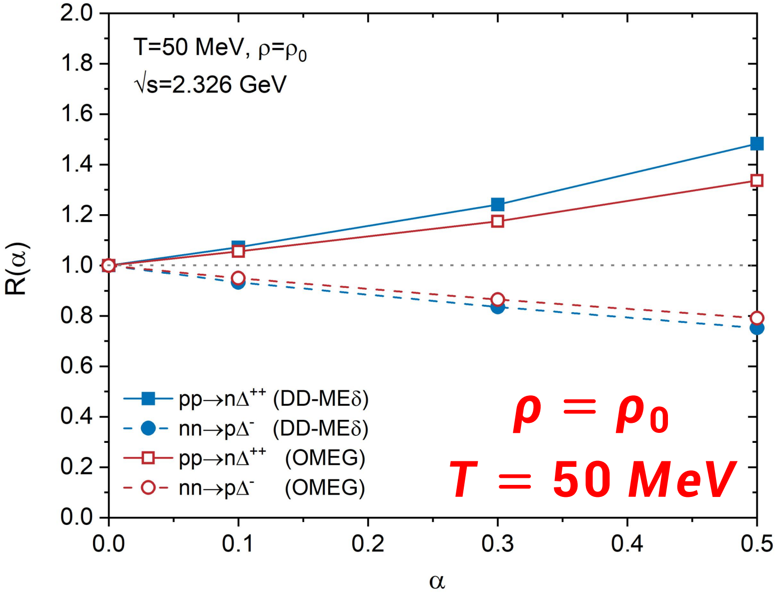
Amplifier







Adjuster!



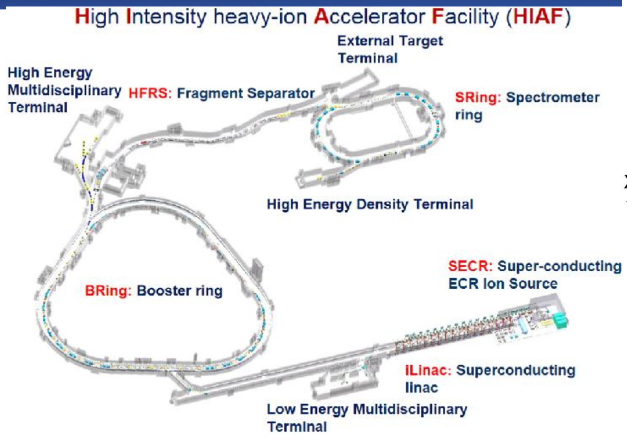
The largest splitting:	$pp \to n\Delta^{++}$	$\pi^- p \to \Delta^0$	$n\Delta^{++} \to n\Delta^{++}$
	...	...	...
	$nn \to p\Delta^-$	$\pi^+ n \to \Delta^+$	$p\Delta^- \to p\Delta^-$
The smallest splitting:	$pn \to n\Delta^+$	$\pi^- n \to \Delta^-$	$p\Delta^{++} \to p\Delta^{++}$
	...	...	...
	$nn \to n\Delta^0$	$\pi^+ p \to \Delta^{++}$	$n\Delta^- \to n\Delta^-$

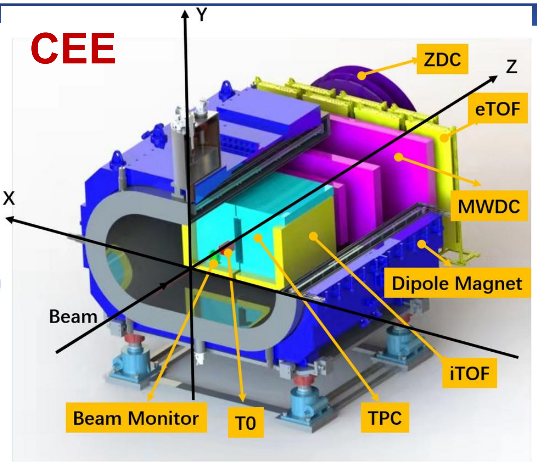
VI、Summary and Outlook

- ✓ The non-equilibrium dynamic theory and model are more critical to understanding the higher dense nuclear matter;
- ✓ The self-consistent RBUU transport theory is further developed as requested;
- ✓ The **energy-, density-, isospin-, and temperature**-dependent NN→NN、NN→NΔ、Nπ → Δ、NΔ → NΔ cross sections are analyzed within sc-RBUU framework, and partly introduced into the UrQMD model;
- ✓ The influence of the in-medium effects of cross-sections on EoS sensitive observables plays important roles and should be carefully considered. Further investigations related to its transport theory, model simulation, and sensitive observables are urgently required.

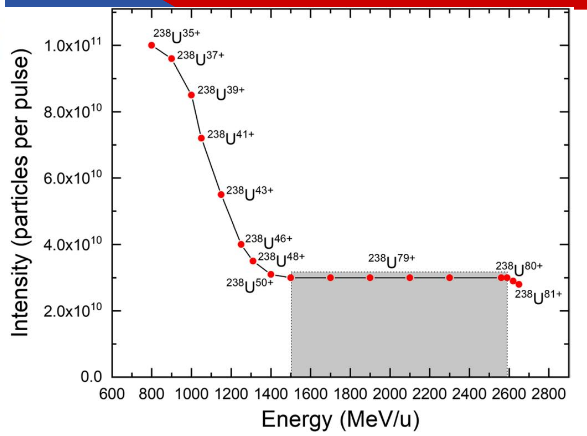
➢ **What shall we do for the running HIAF?➔One stronger transport model, fully taking the above concerns into account!**

VI、What are we supposed to do for EOS? **Organized & Targeted pre-research is very important !!!**





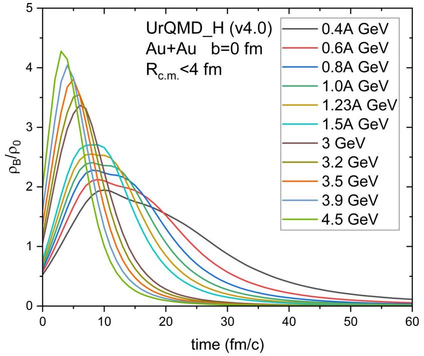
Ion species	Energy (GeV/u)	Intensity (ppp)
p	9.3	2.0×10^{12}
$^{18}O^{6+}$	2.6	6.0×10^{11}
$^{78}Kr^{19+}$	1.7	3.0×10^{11}
$^{209}Bi^{31+}$	0.85	1.2×10^{11}
$^{238}U^{35+}$	0.835	1.0×10^{11}

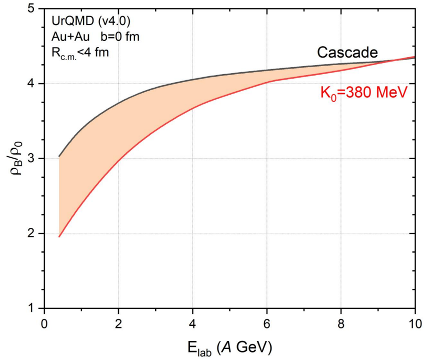


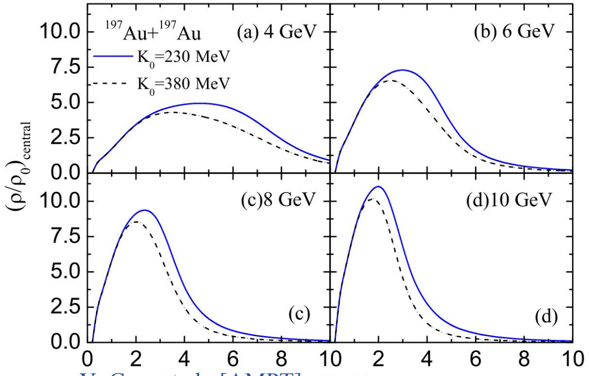
X. H. Zhou, J. C. Yang, [HIAF project Team], AAPPS Bull. 32, 35 (2022).

- More than SπRIT+ASY-EOS➔
  - C+C/Au, UU/AuAu…
  - $E_{lab}=0.2A\sim4A$ GeV
  - $1\sim4\rho_0$
- By measuring **"CES"** (BES' younger brother)➔
  - π, K
  - Neutron, proton, clusters
  - Λ、Σ, Hypernuclei
- To see see ("Two CC")➔
  - Collective flow (sensitive to the EoS and cross-section)
  - Correlation (sensitive to the EoS)

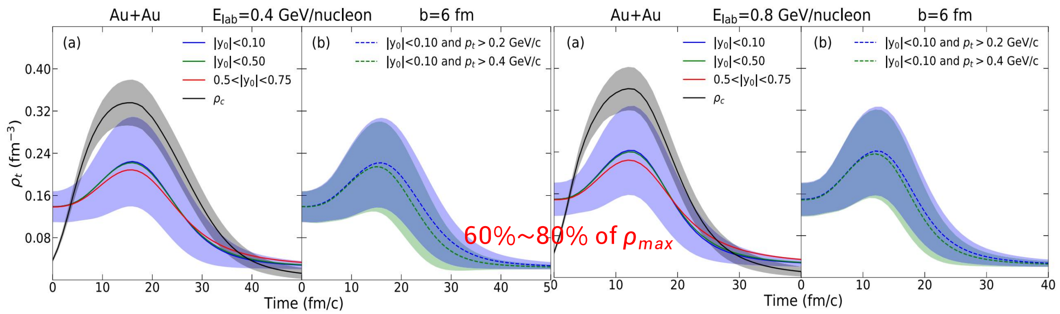
E_lab	sqrt(s)
0.4	2.066
0.8	2.241
1	2.323
1.2	2.402
1.23	2.414
1.5	2.517
2	2.697
2.921	3
3	3.024
4	3.32
4.654	3.5
5	3.592
6	3.844
6.232	3.9
7	4.081
8	4.304
8.918	4.5
9	4.517
10	4.72

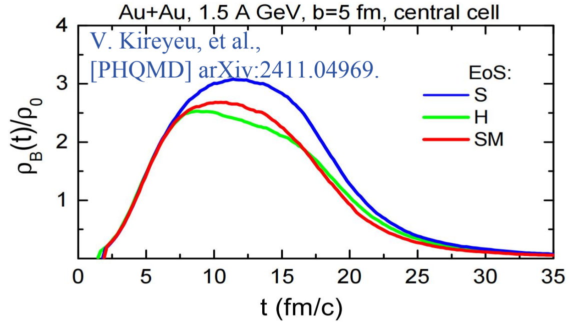






Y. Gao, et al., [AMPT] PRC 112, 024909 (2025)





V. Kireyeu, et al., [PHQMD] arXiv:2411.04969.

致谢：国基金重点项目：12335008；科技部重点研发项目：2023YFA1606400

Thanks for your attention!

感谢您的聆听！

李庆峰
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湖州师范学院

超重：点穴式进攻
状态方程：长征