Non-equilibrium QCD and Transport @Huizhou

Effects of chiral symmetry restoration on dilepton production in heavy-ion collisions

FUDAN UNIVERSITY

KaiJia Sun
kjsun@fudan.edu.cn
Fudan University
Dec. 9, 2025

In collaboration with Wen-Hao Zhou , Jun Xu, Che Ming Ko

Spontaneous breaking of chiral symmetry & Mass generation (1)

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1,

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinoi

(Received October 27, 1960)

It is suggested that the nucleon mass arises largely as a self-energy of some primary fermion field through the same mechanism as the appearance of energy gap in the theory of superconductivity. The idea can be put into a mathematical formulation utilizing a generalized Hartree-Fock approximation which regards real nucleons as quasi-particle excitations. We consider a simplified model of nonlinear four-fermion interaction which allows a γ_5-gauge group. An interesting consequence of the symmetry is that there arise automatically pseudoscalar zero-mass bound states of nucleon-antinucleon pair which may be regarded as an idealized pion. In addition, massive bound states of nucleon number zero and two are predicted in a simple approximation.
The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

PHYSICAL REVIEW VOLUME 124, NUMBER 1 OCTOBER 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. II*

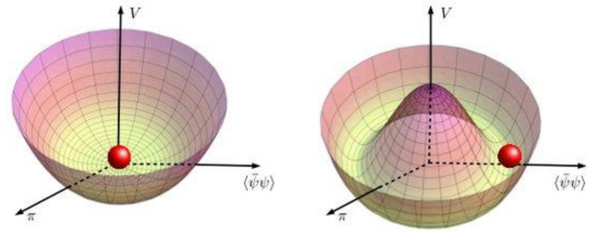
Y. NAMBU AND G. JONA-LASINIO†
Enrico Fermi Institute for Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois

(Received May 10, 1961)

Continuing the program developed in a previous paper, a "superconductive" solution describing the proton-neutron doublet is obtained from a nonlinear spinor field Lagrangian. We find the pions of finite mass as nucleon-antinucleon bound states by introducing a small bare mass into the Lagrangian which otherwise possesses a certain type of the γ_5 invariance. In addition, heavier mesons and two-nucleon bound states are obtained in the same approximation. On the basis of numerical mass relations, it is suggested that the bare nucleon field is similar to the electron-neutrino field, and further speculations are made concerning the complete description of the baryons and leptons.

$$\mathcal{L}_{\rm int} = G\left[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\lambda^a\psi)^2\right] + K\left[\det_f(\bar\psi(1+\gamma_5)\psi) + h.c.\right]$$

Spontaneous symmetry breaking:



$$SU(3)_L \times SU(3)_R \xrightarrow{\text{SSB}} SU(3)_V$$

Order parameter: $\langle\bar uu\rangle = \langle\bar dd\rangle = \langle\bar ss\rangle \neq 0$: Broken

$= 0$: Restored

8 Goldstone bosons: $\pi,\ K,\ \eta_8$

Spontaneous breaking of chiral symmetry & Mass generation (1)

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1,

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinoi

(Received October 27, 1960)

It is suggested that the nucleon mass arises largely as a self-energy of some primary fermion field through the same mechanism as the appearance of energy gap in the theory of superconductivity. The idea can be put into a mathematical formulation utilizing a generalized Hartree-Fock approximation which regards real nucleons as quasi-particle excitations. We consider a simplified model of nonlinear four-fermion interaction which allows a γ_5-gauge group. An interesting consequence of the symmetry is that there arise automatically pseudoscalar zero-mass bound states of nucleon-antinucleon pair which may be regarded as an idealized pion. In addition, massive bound states of nucleon number zero and two are predicted in a simple approximation.
The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

PHYSICAL REVIEW VOLUME 124, NUMBER 1 OCTOBER 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. II*

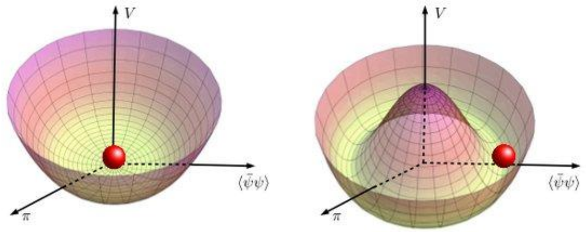
Y. NAMBU AND G. JONA-LASINIO†
Enrico Fermi Institute for Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois

(Received May 10, 1961)

Continuing the program developed in a previous paper, a "superconductive" solution describing the proton-neutron doublet is obtained from a nonlinear spinor field Lagrangian. We find the pions of finite mass as nucleon-antinucleon bound states by introducing a small bare mass into the Lagrangian which otherwise possesses a certain type of the γ_5 invariance. In addition, heavier mesons and two-nucleon bound states are obtained in the same approximation. On the basis of numerical mass relations, it is suggested that the bare nucleon field is similar to the electron-neutrino field, and further speculations are made concerning the complete description of the baryons and leptons.

$$\mathcal{L}_{\rm int} = G\left[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\lambda^a\psi)^2\right] + K\left[\det_f(\bar\psi(1+\gamma_5)\psi) + h.c.\right]$$

Spontaneous symmetry breaking:



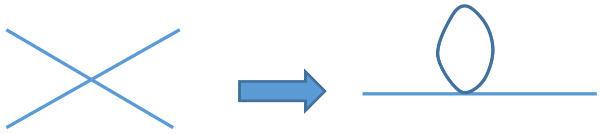
$$SU(3)_L \times SU(3)_R \xrightarrow{\text{SSB}} SU(3)_V$$

Order parameter: $\langle\bar u u\rangle = \langle\bar d d\rangle = \langle\bar s s\rangle \neq 0$: Broken
$= 0$: Restored

8 Goldstone bosons: $\pi,\ K,\ \eta_8$

Mass generation:

In analogy with BCS theory for superconductivity



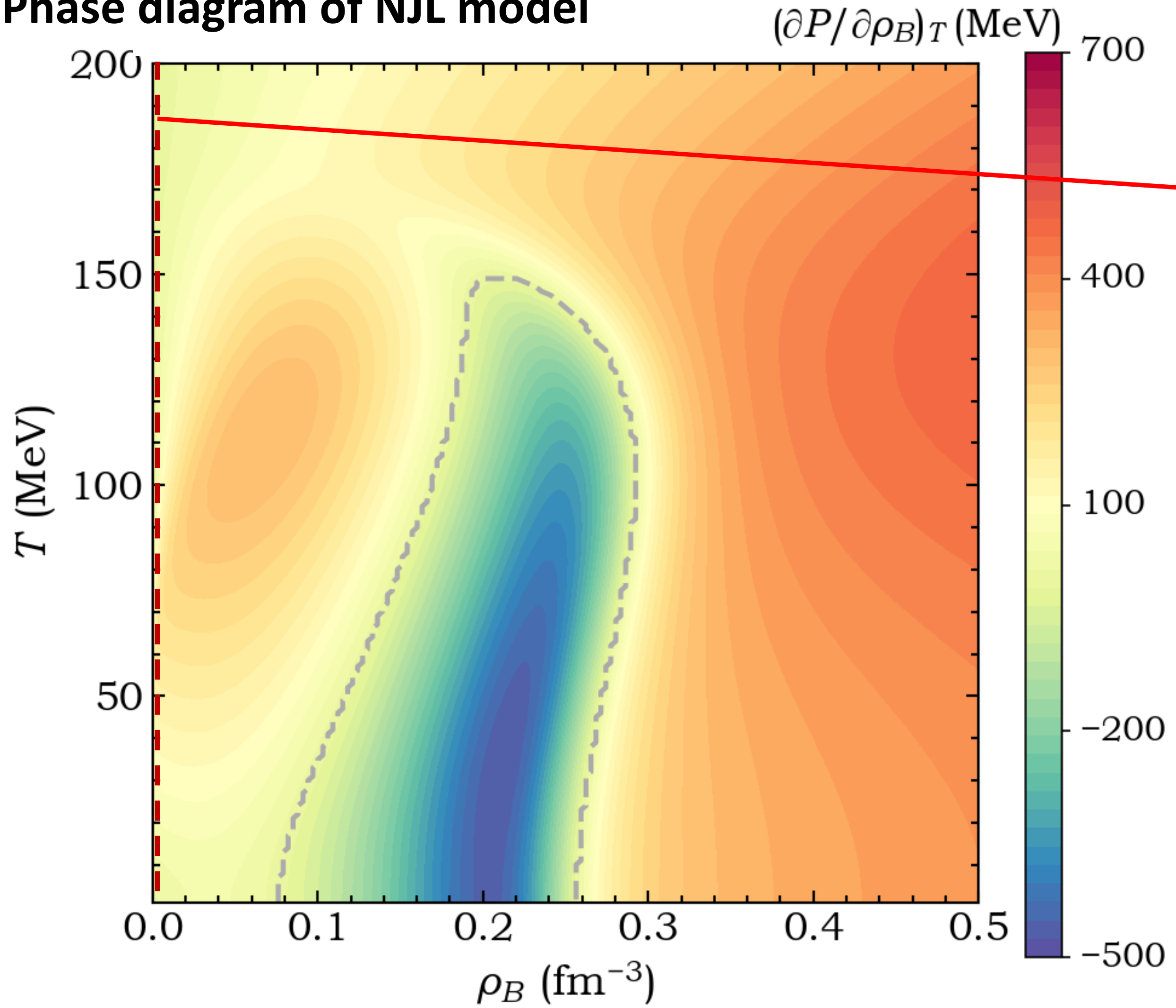
$$M_i = m_i - 4G\,\phi_i + 2K\,\phi_j\,\phi_k$$

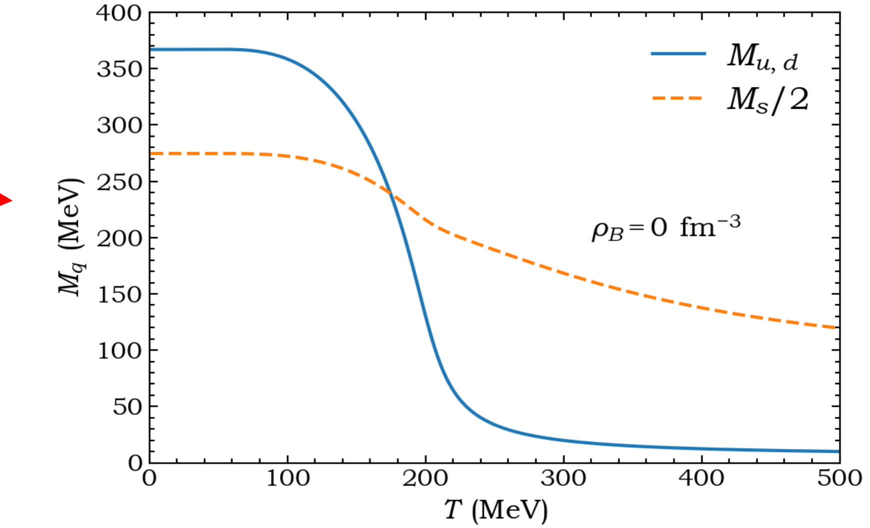
In-medium mass **Current mass** **Condensate**

U. Vogl and W. Weise, PPNP 27, 195 (1991)

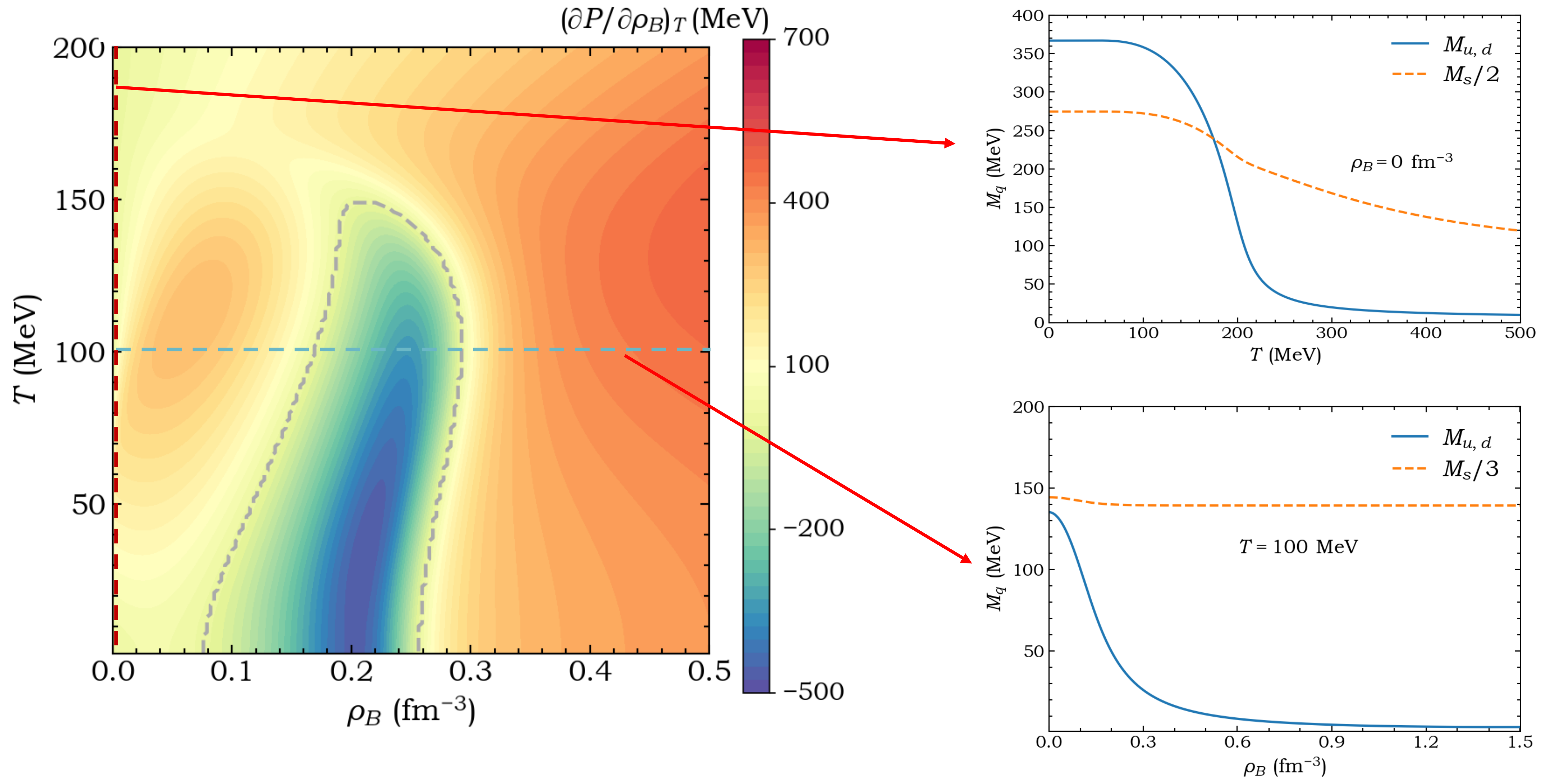
Chiral symmetry restoration (2)

Phase diagram of NJL model





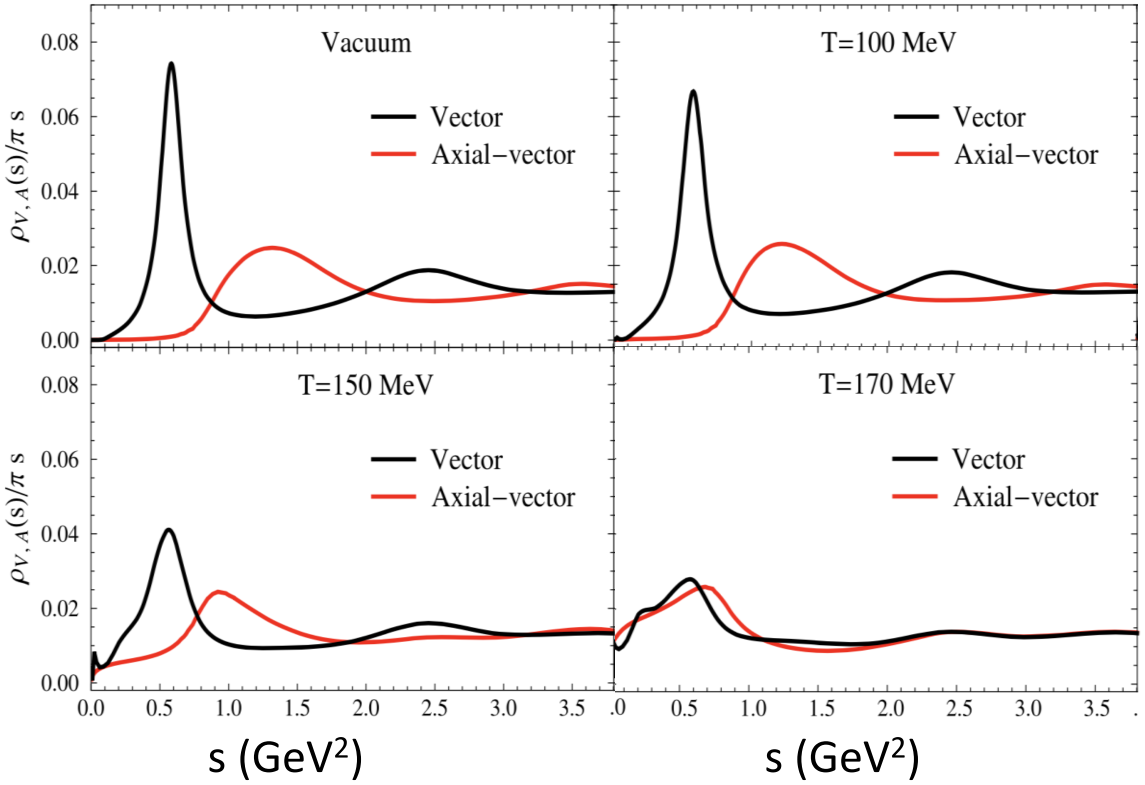
Chiral symmetry restoration (2)



Signals of chiral symmetry restoration? (3)

Degeneracy of chiral partner

$\rho(770)$ and $a_1(1260)$ $\quad m_{a1}^2 \approx m_\rho^2 + 4M_q^2$

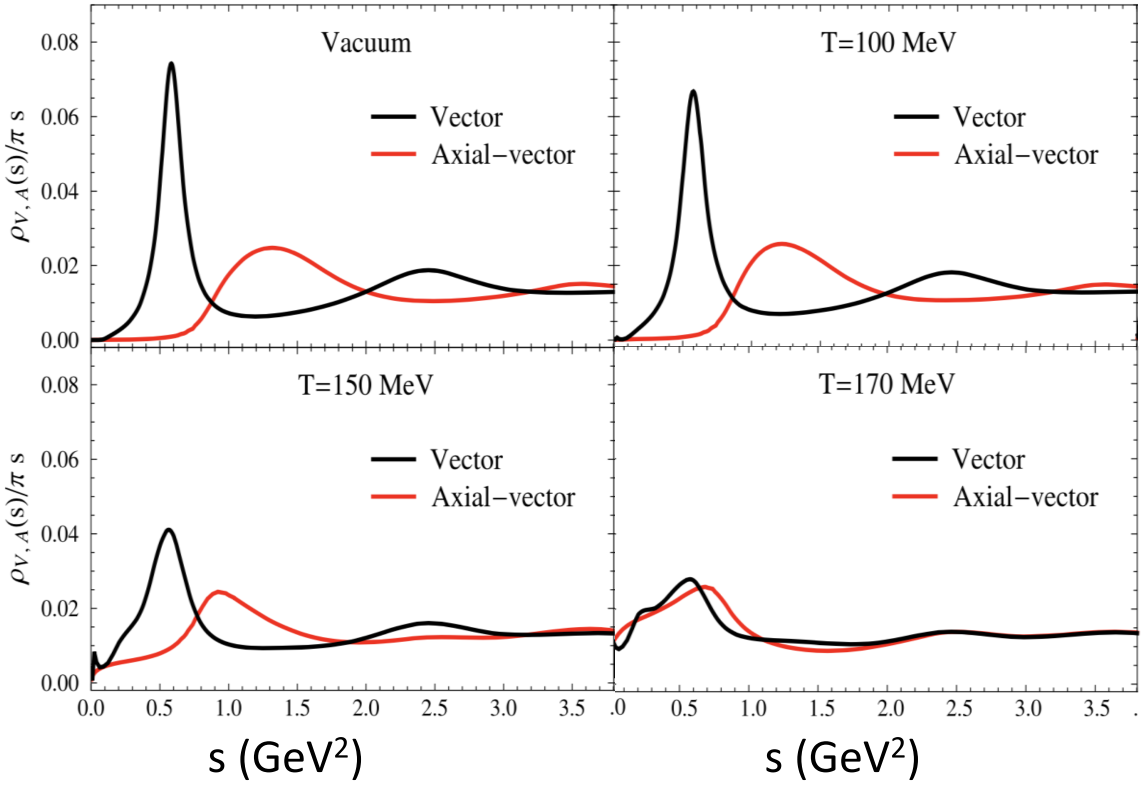


Rapp and Hohler: Phys. Lett. B 731 (2014) 103-109

Signals of chiral symmetry restoration? (3)

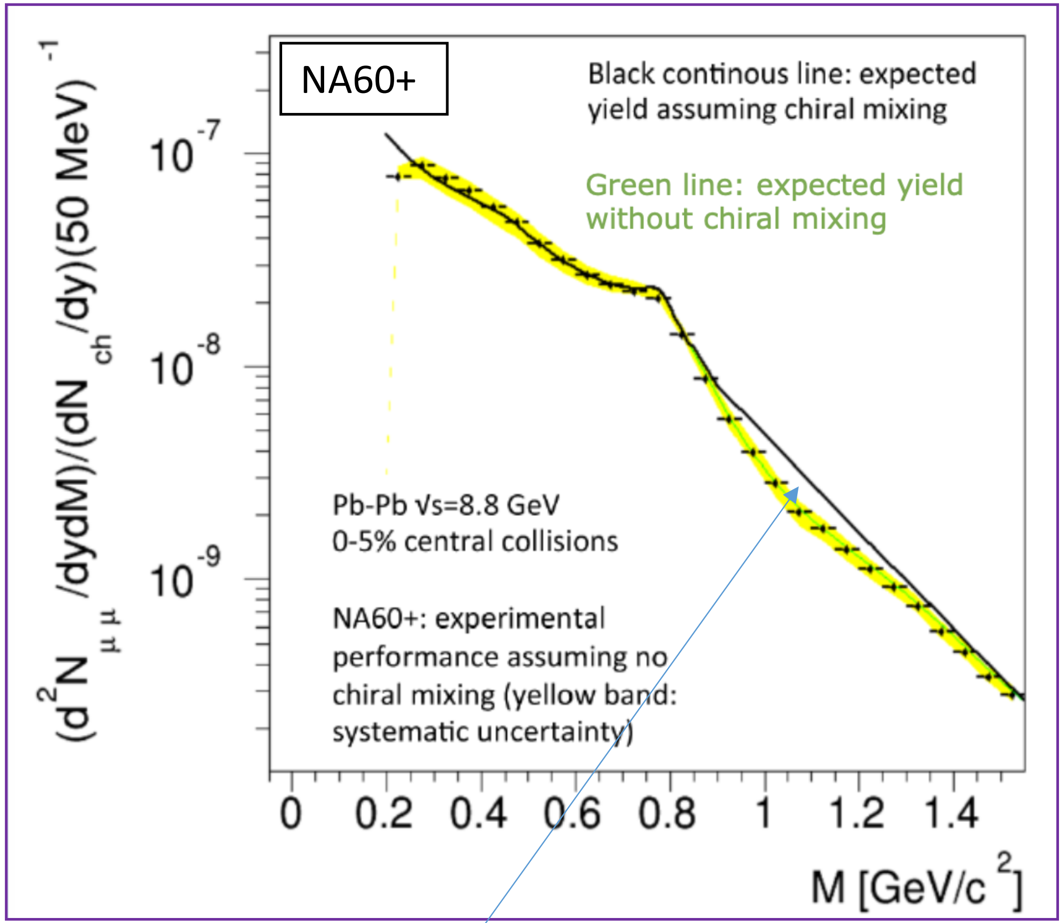
Degeneracy of chiral partner

$\rho(770)$ and $a_1(1260)$ $\quad m_{a1}^2 \approx m_\rho^2 + 4M_q^2$



Rapp and Hohler: Phys. Lett. B 731 (2014) 103-109

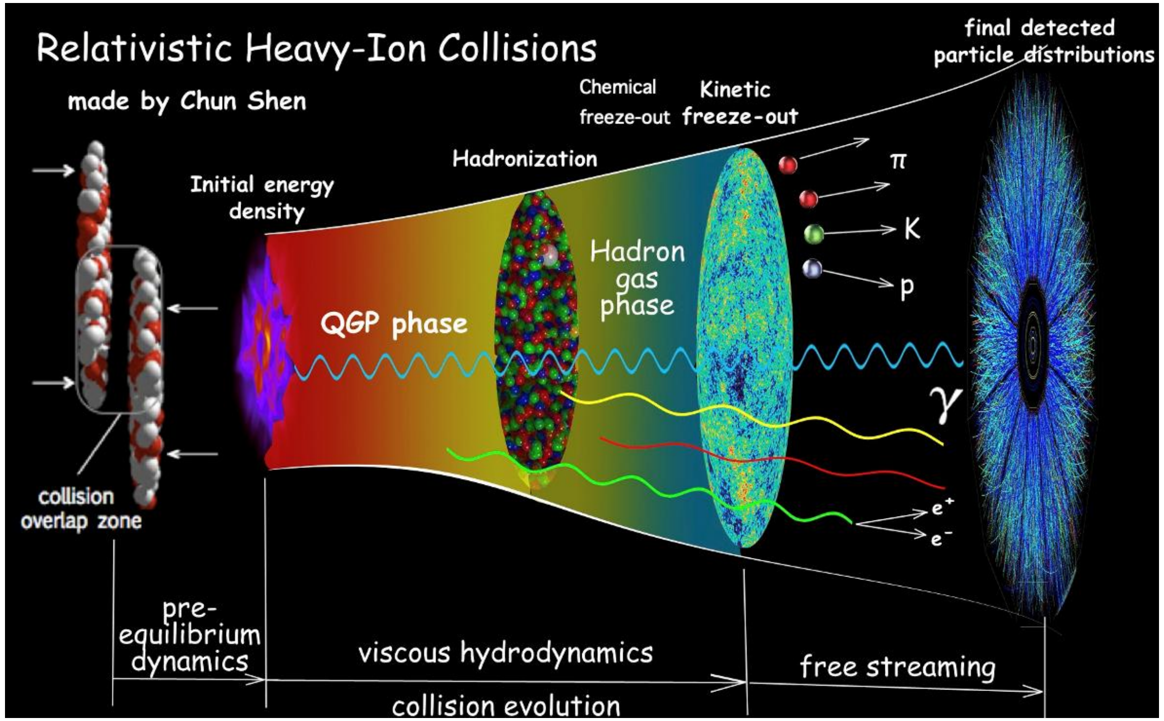
Dilepton spectral function modification

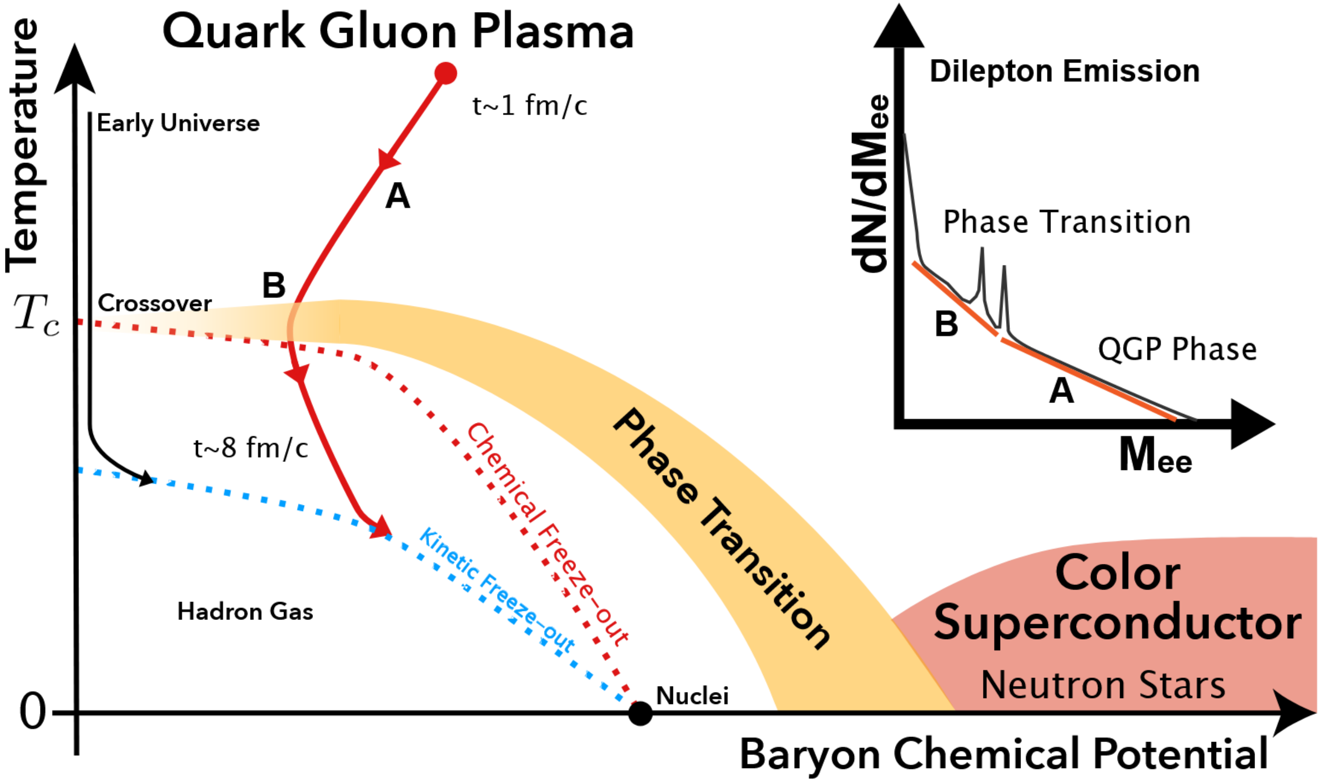


20-30% extra excess

Dilepton probes early dynamics (4)

Dileptons are produced through all stages in heavy-ion collisions





STAR, *Nature Commun. 16 (2025) 1, 9098*

Dilepton production

Slide from Zaochen Ye

Inclusive signals (space-time integral)

Thermal signals:
- QGP radiation
- In-medium ρ decays

+

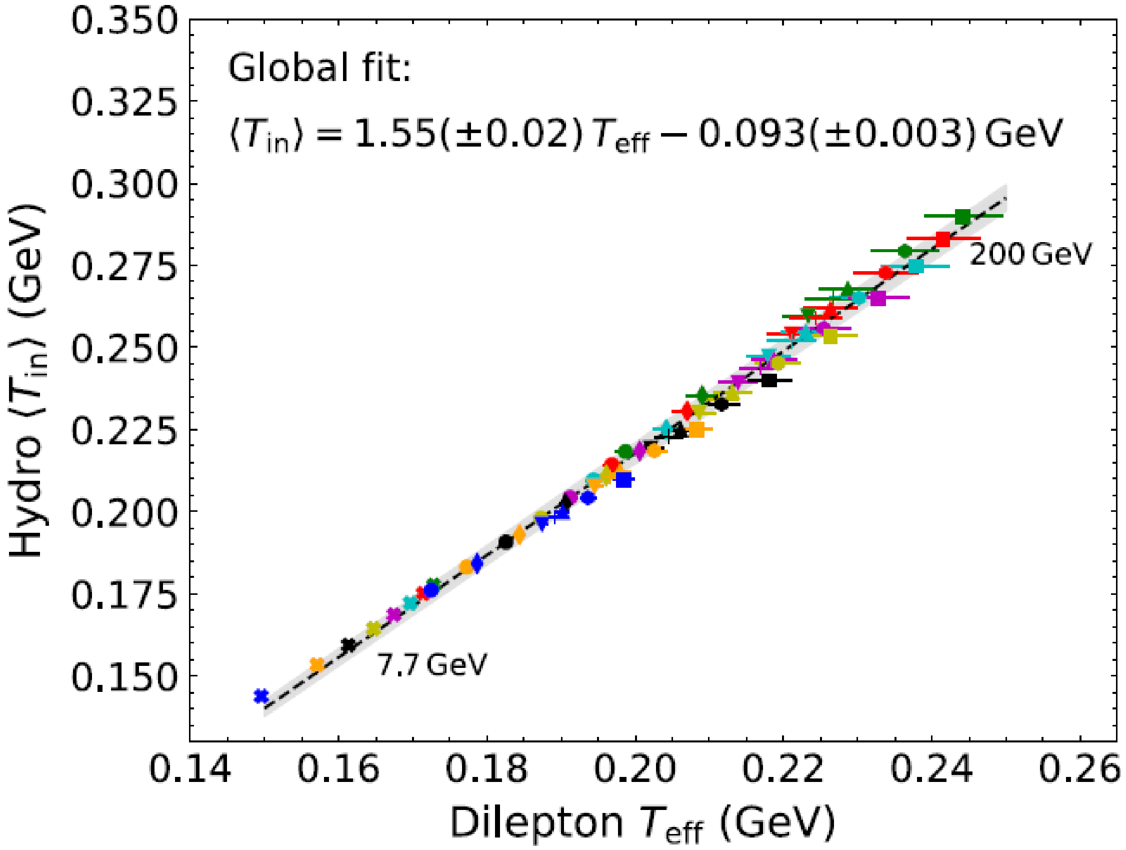
Physical background (Cocktails):
- Drell-Yan
- π^0, η , η' → γe^+e^-
- ω, φ → e^+e^-, ω→ $\pi^0 e^+e^-$, φ → ηe^+e^-
- J/ψ → e^+e^-, $c\bar{c}$ → e^+eX

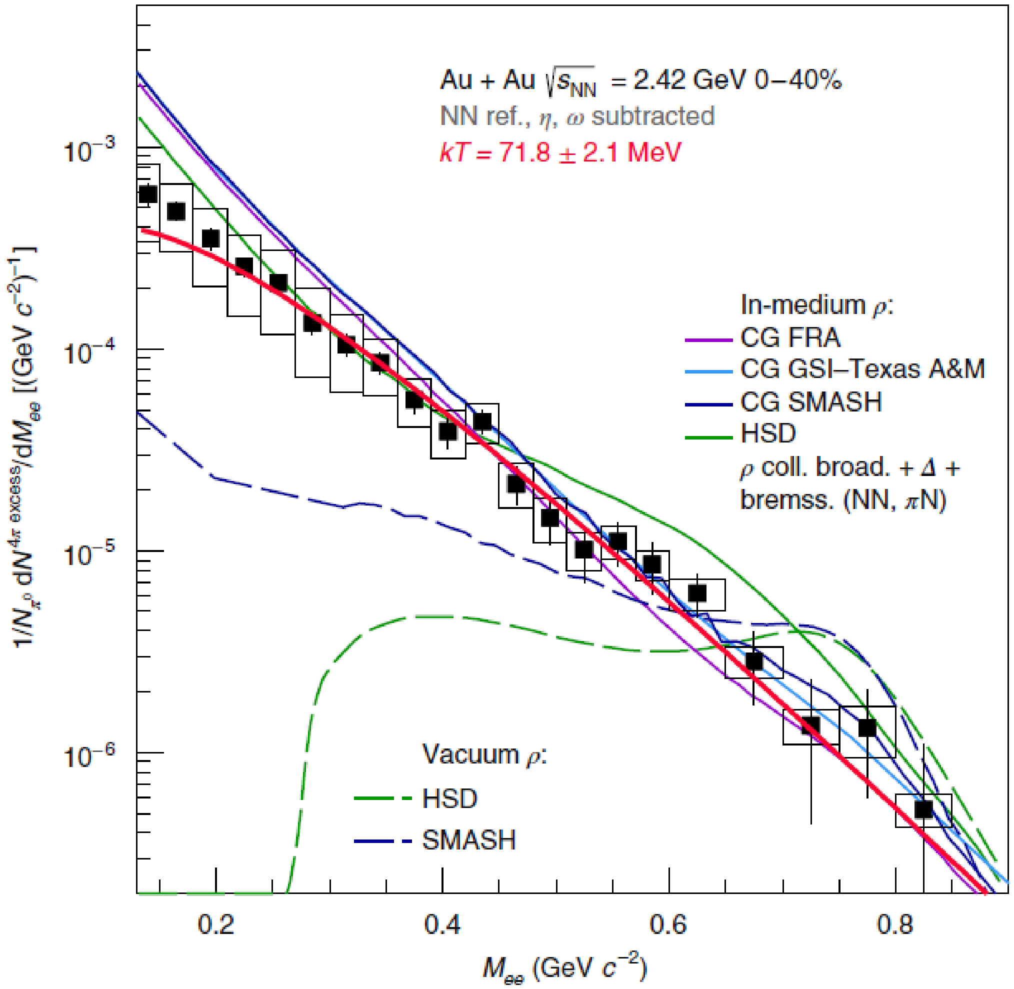
Physical background can be determined using the well-established cocktail simulation techniques

Thermal signals = **Inclusive signals** — **Physical background**

Dilepton production as a thermometer

Initial T



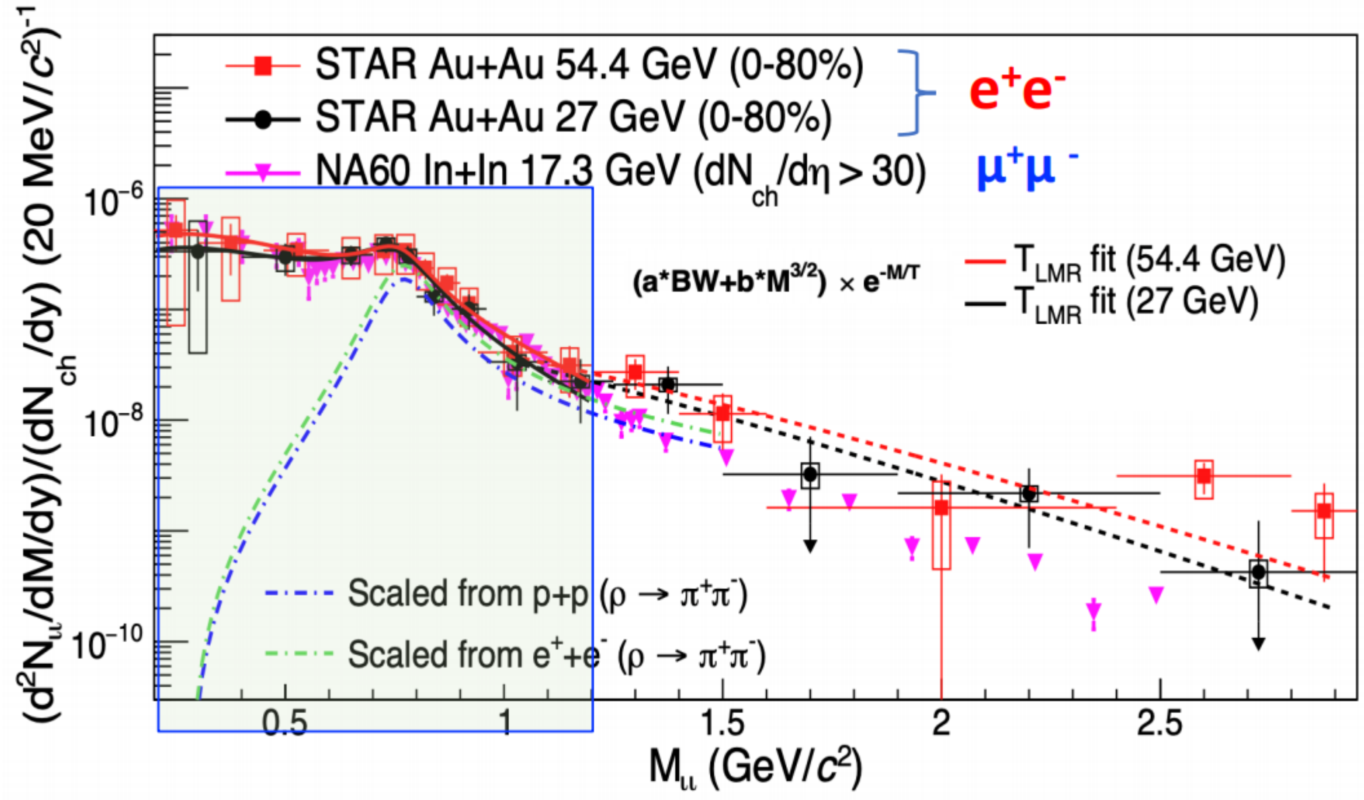


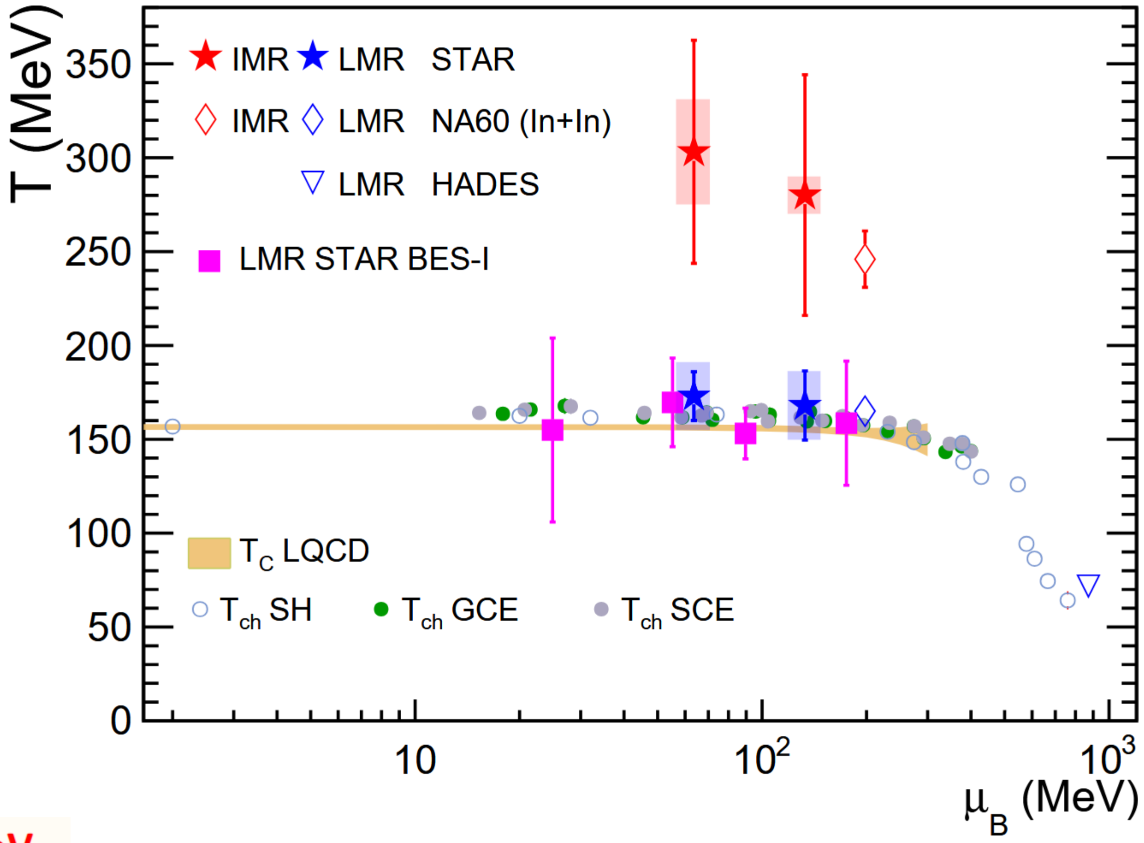
J. Churchill, L. Du, *et al.*, *PRL* **132**, 172301 (2024).

HADES, *Nat. Phys.* **15**, 1040 (2019).

Dilepton production as a thermometer (7)

Slide from Zaochen Ye





- $T^{27GeV}_{LMR} = 167 \pm 21 \pm 18$ (MeV)
- $T^{54.4GeV}_{LMR} = 172 \pm 13 \pm 18$ (MeV)
- $T^{17.3GeV}_{LMR} = 165 \pm 4$ (MeV)

T_{IMR} from STAR: ~300 MeV

T_{IMR} from NA60:

- **246 ± 17 MeV** (1.2<M<2.5 GeV/c²)
- 205 ± 12 MeV (1.2<M<2.0 GeV/c²)

STAR, *Nature Commun. 16 (2025) 1, 9098*

Low Mass Region (LMR): 0.4-1.2 GeV/c^2

Intermediate Mass Region (IMR): 1.0-2.9 GeV/c^2

Dilepton production and QCD phase transitions (8)

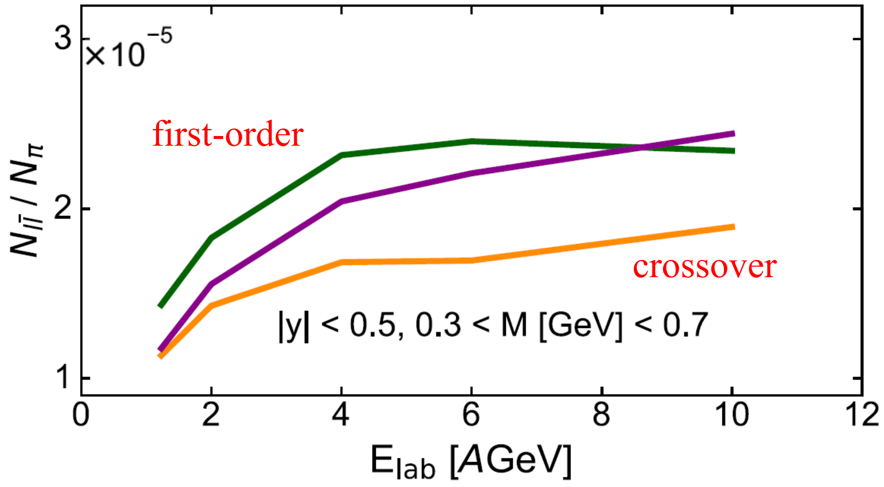
- Dilepton and photon productions are not affected by strong force and emitted continuously at all stages of evolution.

- Hadronic rescatterings tend to wash out the information from the EoS-driven expansion i.e. spinodal decomposition.
  F. Seck *et. al.*, *Phys. Rev. C* **106**, 014904 (2022).

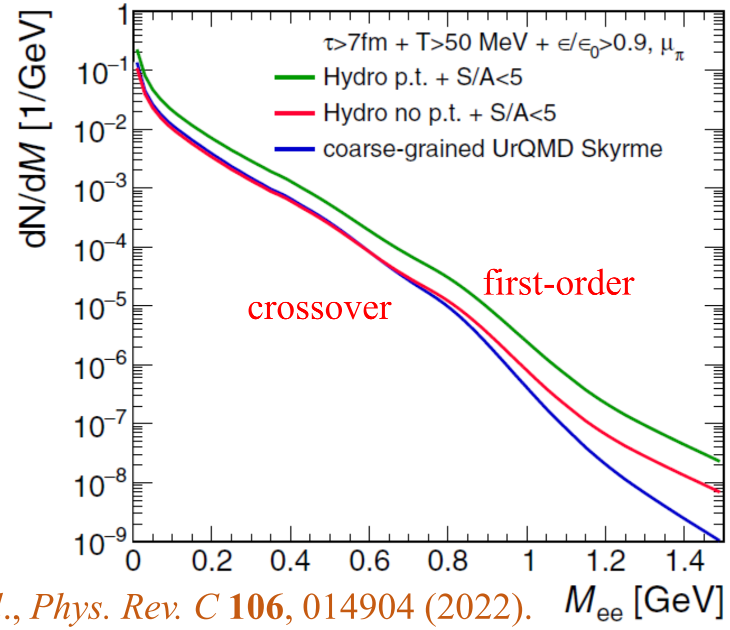
- EM radiation may be sensitive to a local density clumping and a longer lifetime of the fireball when the medium undergoes a first-order phase transition. R. Rapp and H. van Hees, *Phys. Lett. B* **753**, 586 (2016).

- The dilepton production rate is proportional to the square of parton density, more dileptons are produced when the density fluctuation is large.
  F. Li and C. M. Ko, *Phys. Rev. C* **95**, 055203 (2017).

$$\frac{dN_{ll}}{d^4x d^4q} = -\frac{\alpha^2}{3\pi^3}\frac{L(M^2)}{M^2}\,\mathrm{Im}\,\Pi^{\mu}_{\mathrm{em},\mu}(M,q) f^B(q_0;T)$$



O. Savchuk, *et. al.*, *J. Phys. G: Nucl. Part. Phys.* **50**, 125104 (2023).



F. Seck *et. al.*, *Phys. Rev. C* **106**, 014904 (2022).

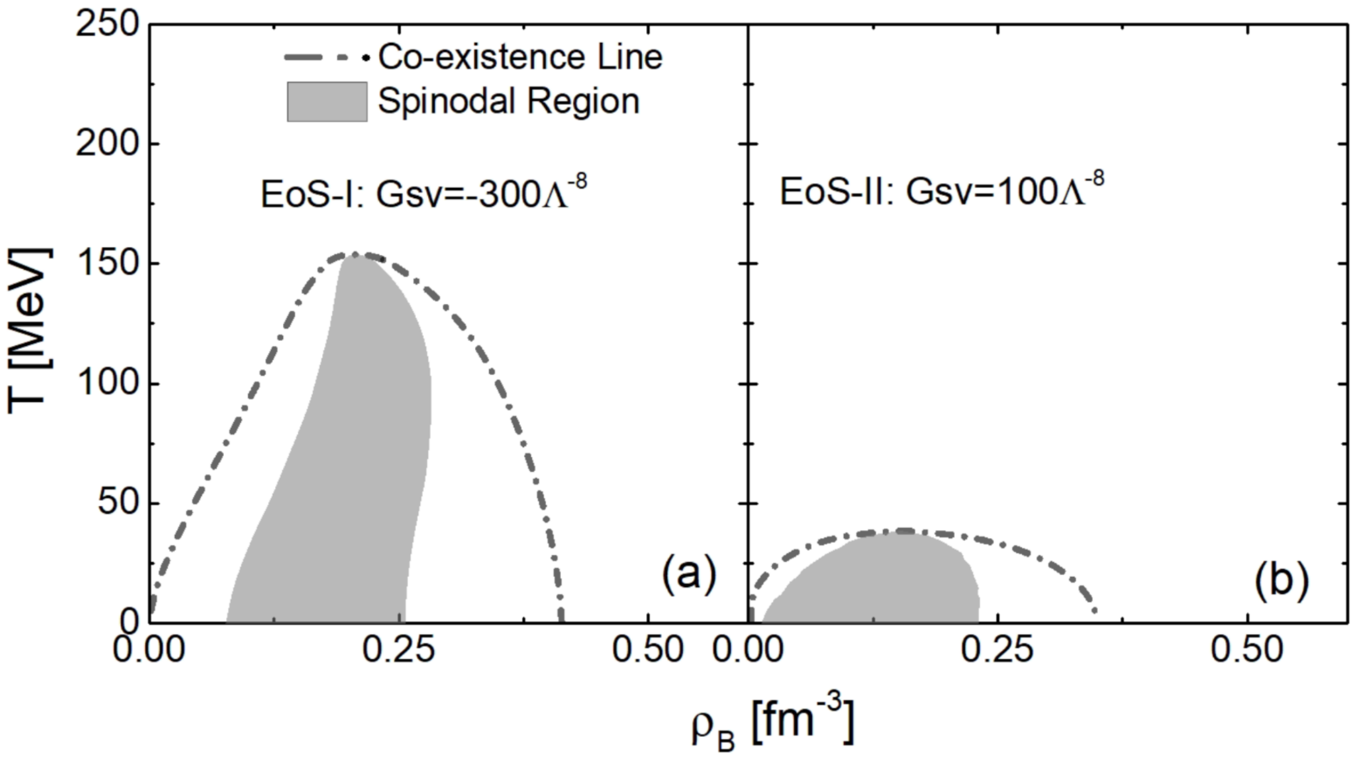
Equation of State (extended NJL model) (9)

The eNJL provides a flexible equation of state (EoS) . The critical temperature can be easily changed by varying the strength of the scalar-vector interaction without affecting the vacuum properties.

Lagrangian density for an extended Nambu-Jona-Lasinio (eNJL) model

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m})\psi + G_S \sum_{a=0}^{8}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] \\
&- K\{[\det[\bar{\psi}(1+\gamma_5)\psi] + \det[\bar{\psi}(1-\gamma_5)\psi]\} \\
&+ G_{SV}\left\{\sum_{a=1}^{3}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2]\right\} \\
&\times \left\{\sum_{a=1}^{3}[(\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\lambda^a\psi)^2]\right\},
\end{aligned}
$$

Λ [MeV]	602.3	$M_{u,d}$ [MeV]	367.7
$G\Lambda^2$	1.835	M_s [MeV]	549.5
$K\Lambda^5$	12.36	$(\langle\bar{u}u\rangle)^{1/3}$ [MeV]	-241.9
$m_{u,d}$ [MeV]	5.5	$(\langle\bar{s}s\rangle)^{1/3}$ [MeV]	-257.7
m_s [MeV]	140.7		



M. Buballa, Phys. Rept. 407, 205 (2005)

K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

Box Simulation

Effective mass:

$$M_u = m_u - 4G_S\phi_u + 2K\phi_d\phi_s - 2G_{SV}(\rho_u+\rho_d)^2(\phi_u+\phi_d),$$
$$M_d = m_d - 4G_S\phi_d + 2K\phi_u\phi_s - 2G_{SV}(\rho_u+\rho_d)^2(\phi_u+\phi_d),$$
$$M_s = m_s - 4G_S\phi_s + 2K\phi_u\phi_d$$

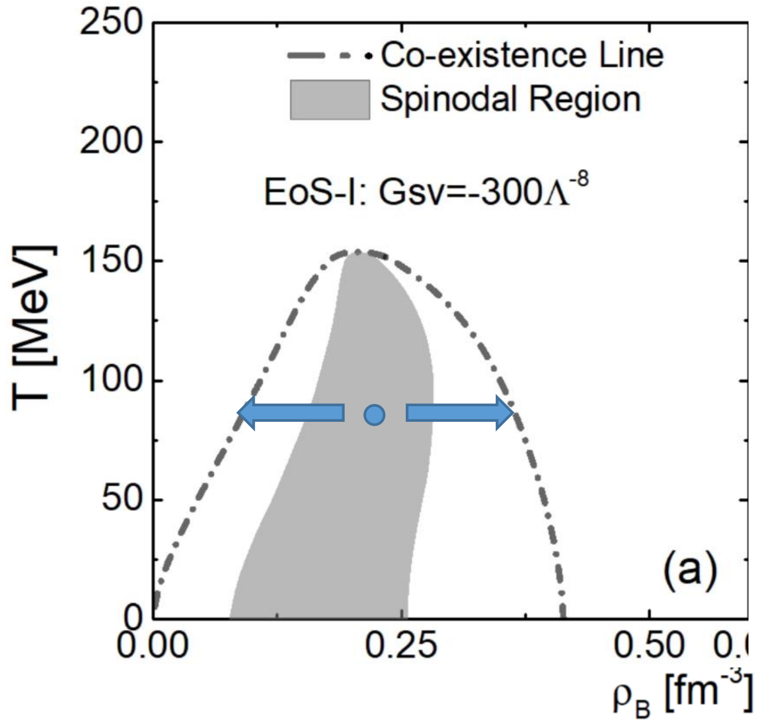
$$\phi_i = -2N_c\int_0^\Lambda \frac{d^3p}{(2\pi\hbar)^3}\frac{M_i}{E_i}(1-f_i-\bar{f}_i)$$

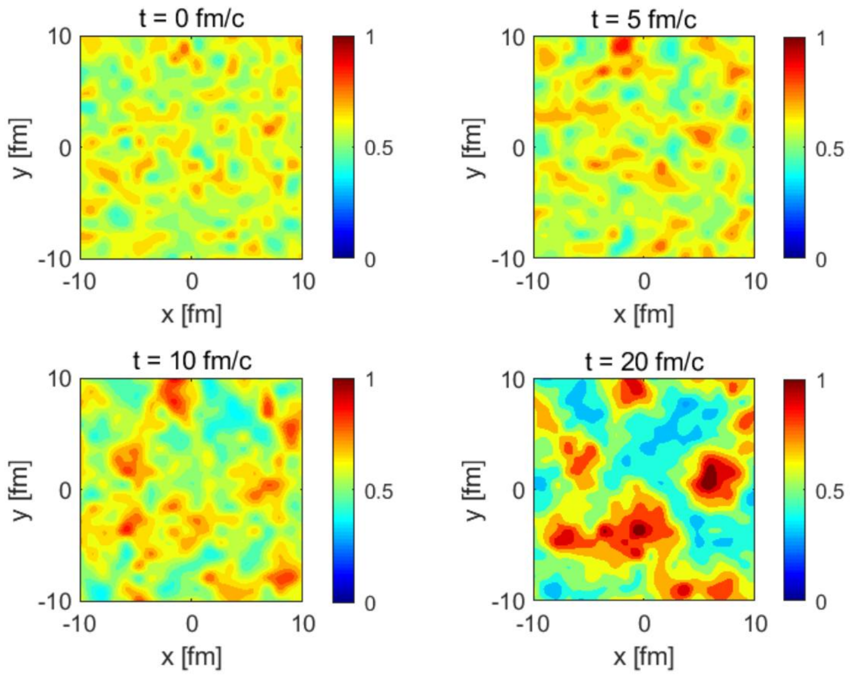
$$\rho_i = 2N_c\int_0^\Lambda \frac{d^3p}{(2\pi\hbar)^3}(f_i-\bar{f}_i)$$

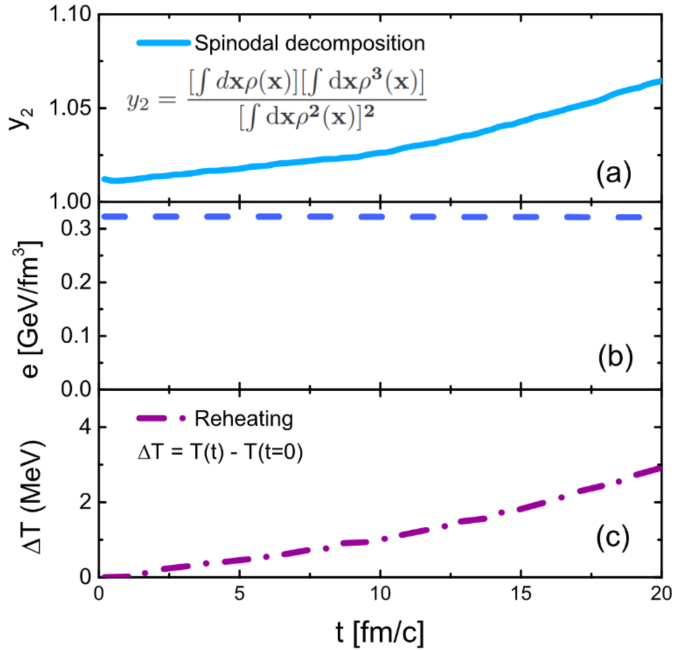
Test –particle method: J. Xu, arXiv:1904.00131 (2019)

$$\frac{d\mathbf{r}}{dt} = \mathbf{v},$$
$$\frac{d\mathbf{p}}{dt} = -\frac{M}{E^*}\boldsymbol{\nabla}_r M \pm \mathbf{E} \pm \mathbf{v}\times\mathbf{B}$$

Strong EM fields







Small irregularities will grow exponentially and soon the evolution becomes 'chaotic'.

For small k, $\Gamma_k = -i\,vk$, $v^2 = \frac{n}{\varepsilon+p}\left(\frac{\partial p}{\partial n}\right)_S$ or $v^2 = \frac{n}{\varepsilon+p}\left(\frac{\partial p}{\partial n}\right)_T < 0$

M. Buballa, Phys. Rept. 407, 205 (2005)
K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

Dilepton production rate and in-medium quark mass

- Cross section for massive quark, anti-quark pair to dilepton

$$\sigma_{q\bar{q}\to e^+e^-}(M) = \frac{4\pi\alpha^2 e_q^2}{3N_c M^2}\left(1+\frac{2M_q^2}{M^2}\right)\left(1-\frac{4M_q^2}{M^2}\right)^{-\frac{1}{2}}$$

- Production rate for dileptons per space-time volume

$$\frac{\mathrm{d}^4N}{\mathrm{d}t\mathrm{d}^3x} = \sum_{q=u,d,s}\int \frac{\mathrm{d}^3\vec{k}_1}{(2\pi)^3}\frac{\mathrm{d}^3\vec{k}_2}{(2\pi)^3} v_{\mathrm{rel}} f_q(\vec{k}_1) f_{\bar{q}}(\vec{k}_2)\sigma_{q\bar{q}\to e^+e^-}(M)$$

⇩

$$\frac{\mathrm{d}^4N}{\mathrm{d}t\mathrm{d}^3x} \propto T\int_{2M_q}^{\infty}\mathrm{d}M\, M^2(M^2-4M_q^2)K_1\left(\frac{M}{T}\right)\sigma(M)$$

⇩ $M_q,\ T \ll M$

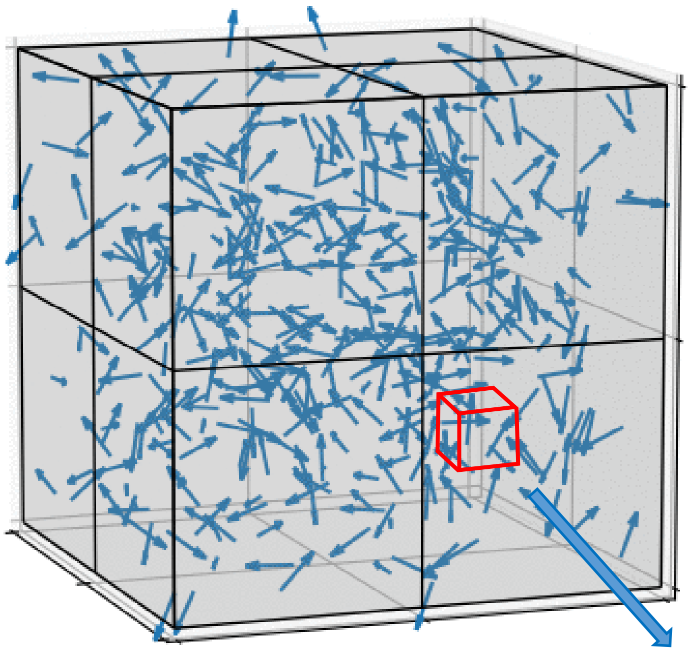
$$\frac{\mathrm{d}^5N}{\mathrm{d}M\mathrm{d}t\mathrm{d}^3x} \propto TM^2\left(1+\frac{2M_q^2}{M^2}\right)\sqrt{1-\frac{4M_q^2}{M^2}}K_1\left(\frac{M}{T}\right)$$

- **By fitting dilepton spectrum, one could extract both the medium temperature and the in-medium quark masses.**

- Dilepton production rate as function of invariant mass

$$\frac{\mathrm{d}^5N}{\mathrm{d}M\mathrm{d}t\mathrm{d}^3x} \propto (MT)^{3/2}e^{-\frac{M}{T}}$$

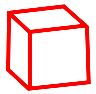
Model validation: box calculation



$$T_{\text{Medium}} = \frac{\sum_n w_n T_n}{\sum_n w_n}$$

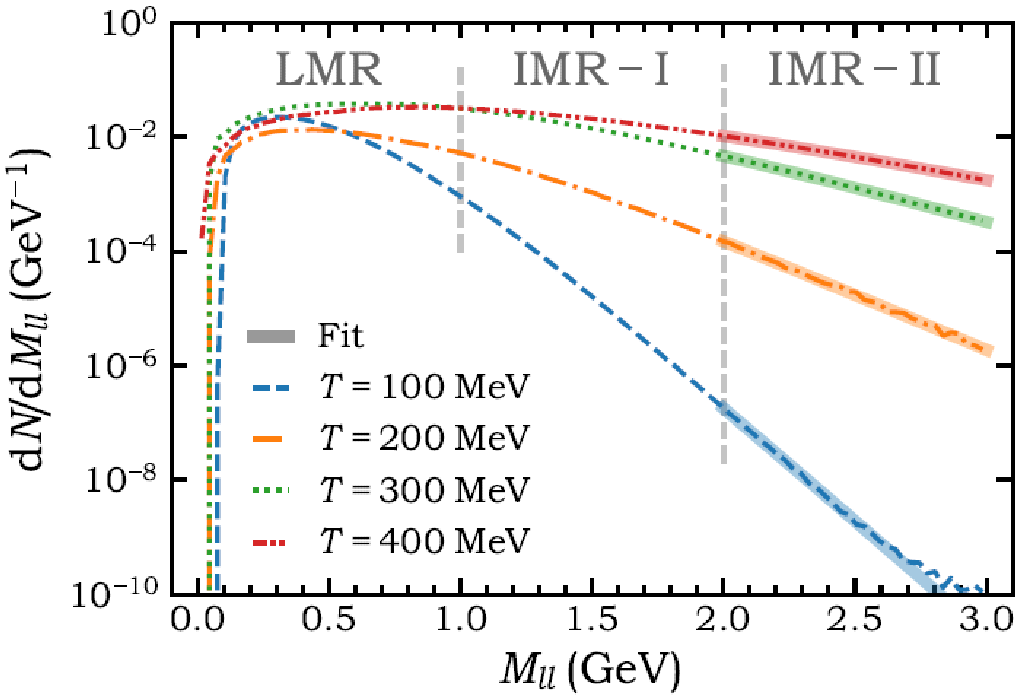
T_n are the local temperature of cells, w_n are the weights, which are the product of density of quark and antiquark.

$$\Delta V = \Delta x \Delta y \Delta z$$



Temperature,
Density,
Energy density,
Effective mass
…

- T_{Medium} is obtained by local cells.
- T_{Dilepton} is obtained by fitting the invariant mass spectrum of dilepton.
- Assuming local thermal equilibrium.



- Invariant mass spectra of dileptons

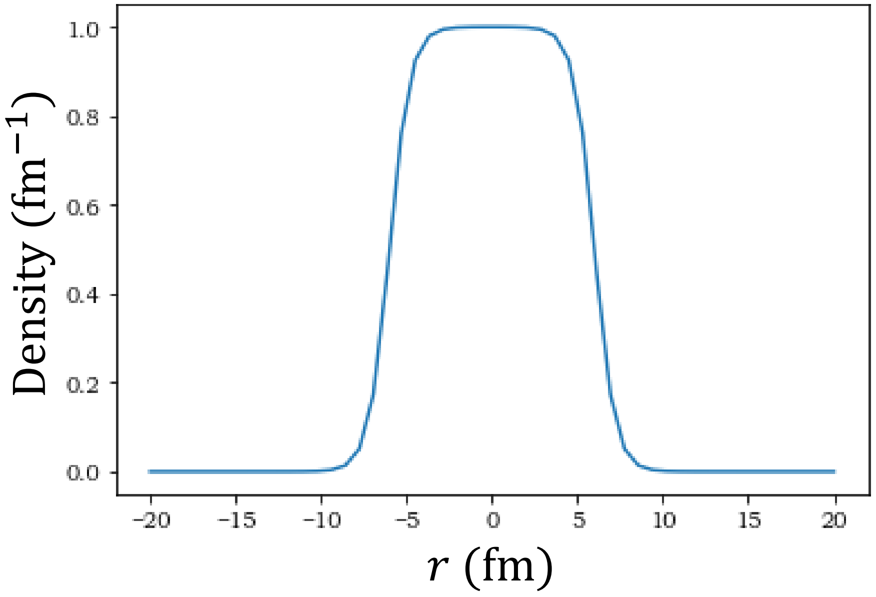
T_{Medium}	100	200	300	400
T_{Dilepton} (IMR)	103	207	317	433
T_{Dilepton} (IMR-I)	103	211	328	455
T_{Dilepton} (IMR-II)	102	201	300	410

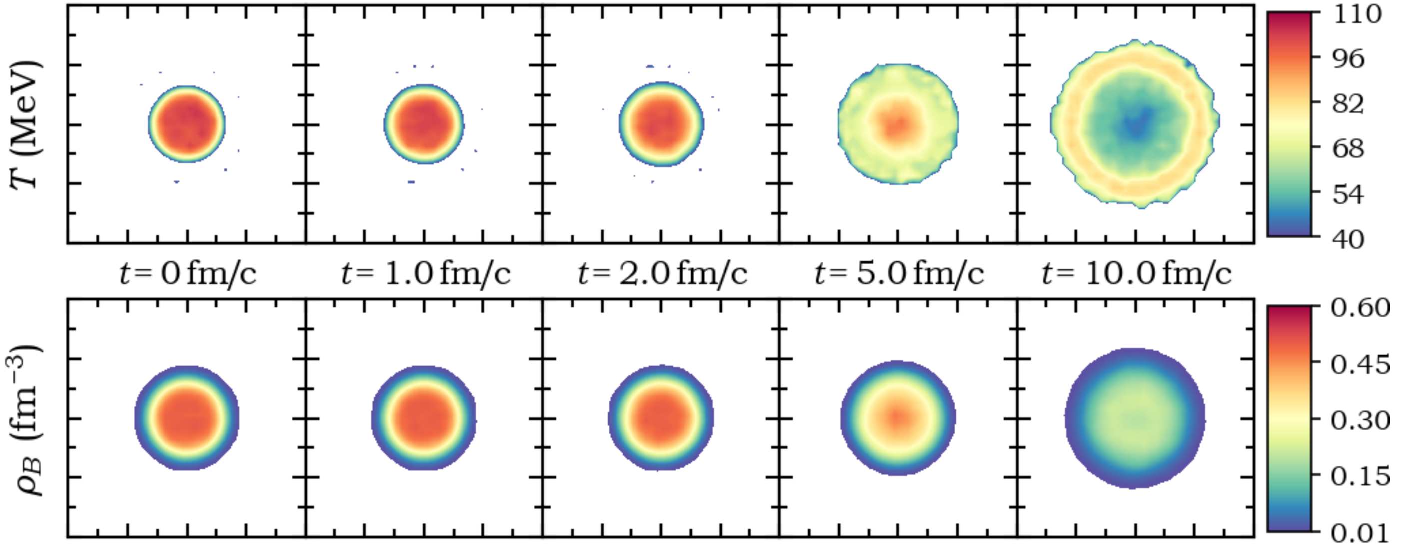
Fireball calculation

- Woods-Saxon type initial profiles

$$\rho_B(r) = \frac{\rho_{\max}}{1+\exp\left((r-R_{\mathrm{WS}})/a_{\mathrm{WS}}\right)}$$

$$T(r) = \frac{T_{\max}}{1+\exp\left((r-R_{\mathrm{WS}})/a_{\mathrm{WS}}\right)}$$

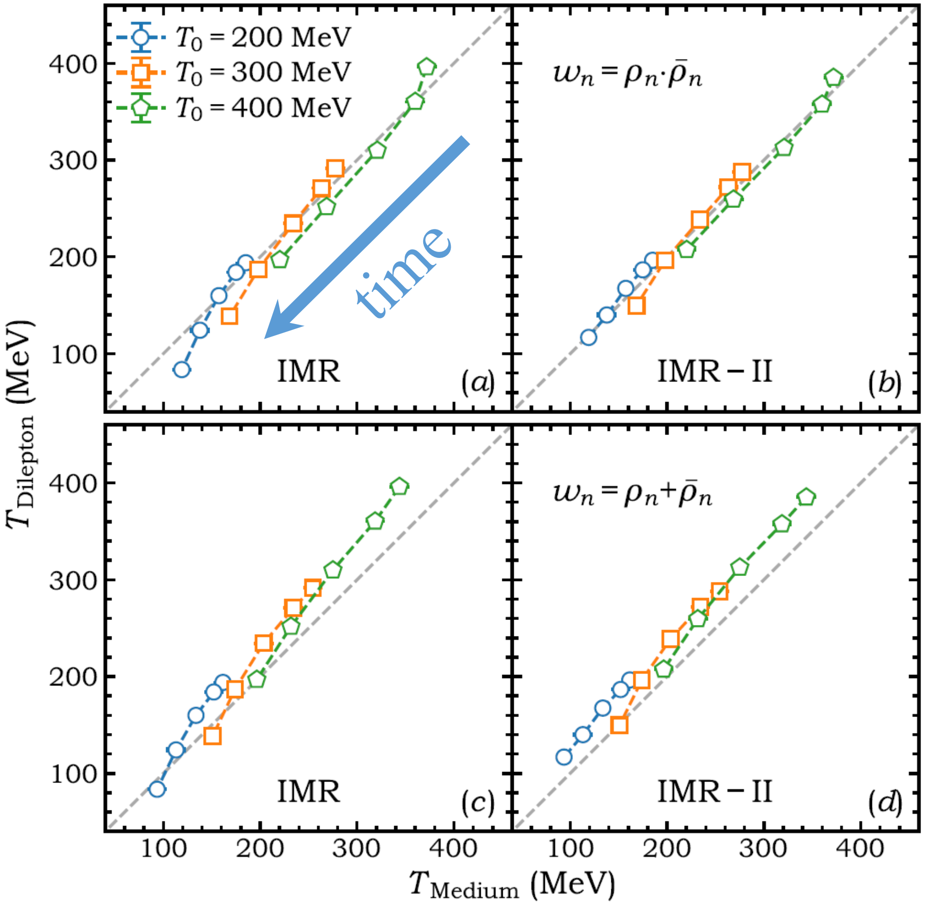




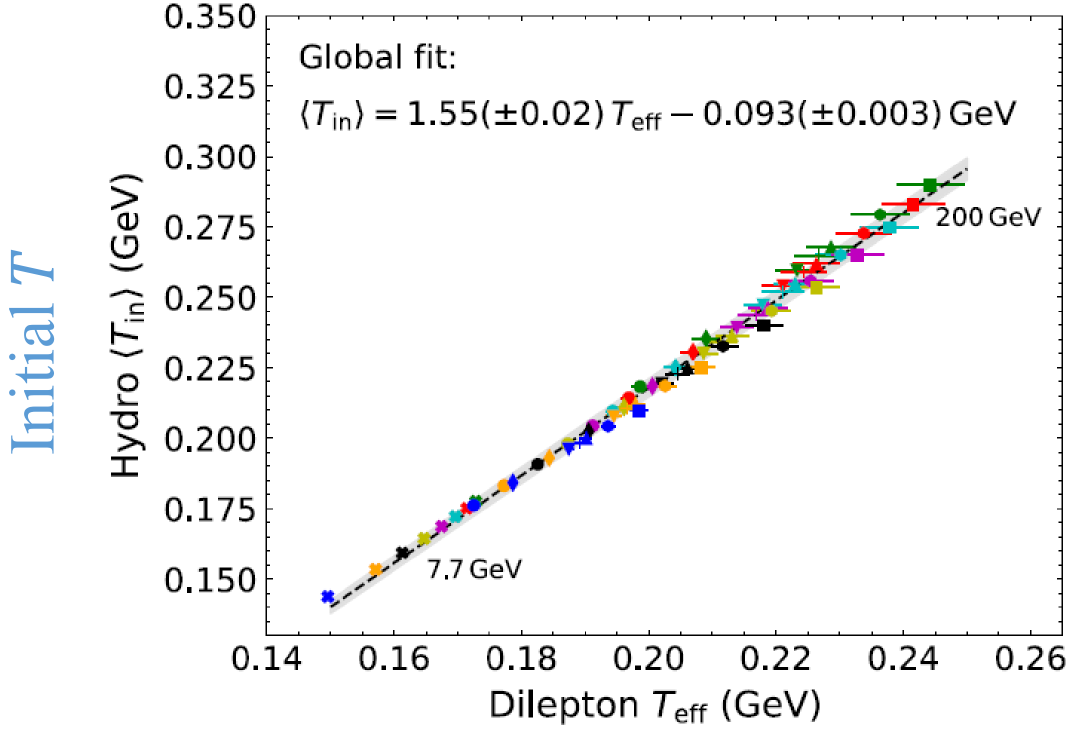
- Contour plots of temperature and net-baryon density

- Fireball becomes bubble structure as time goes on.
- Density gradient-driven expansion.
- $t_{\max} = 10$ fm/c.

Result I: Medium temperature from dilepton production

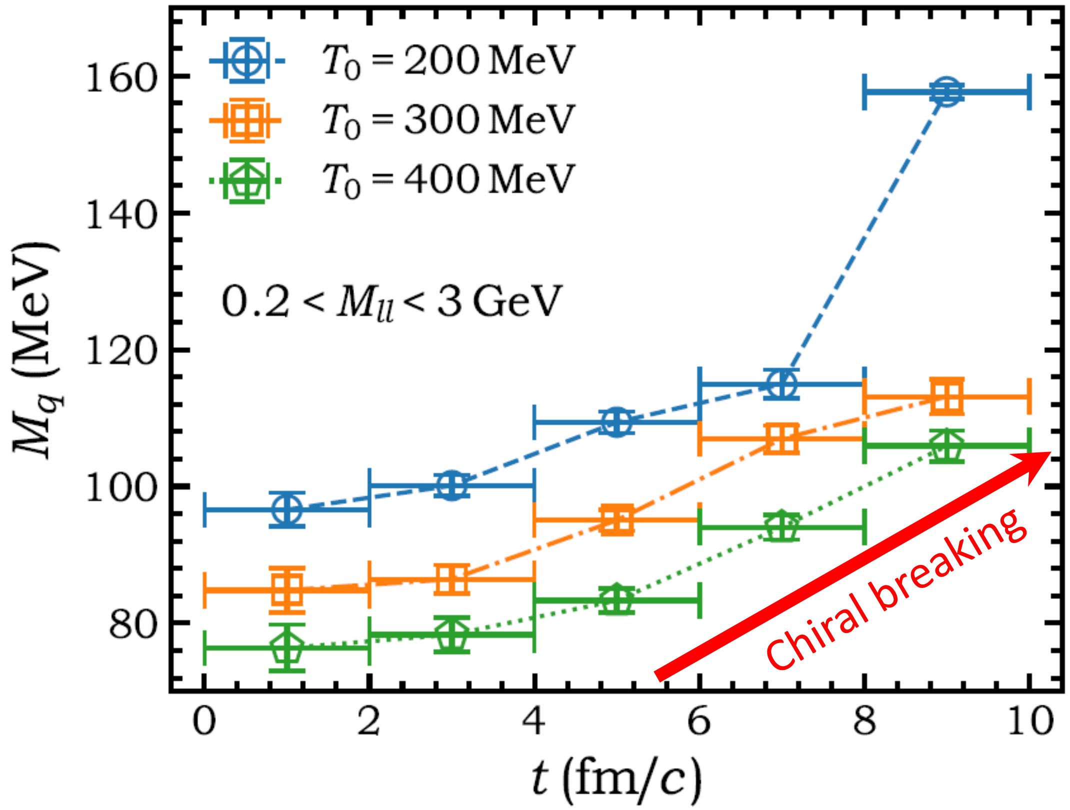


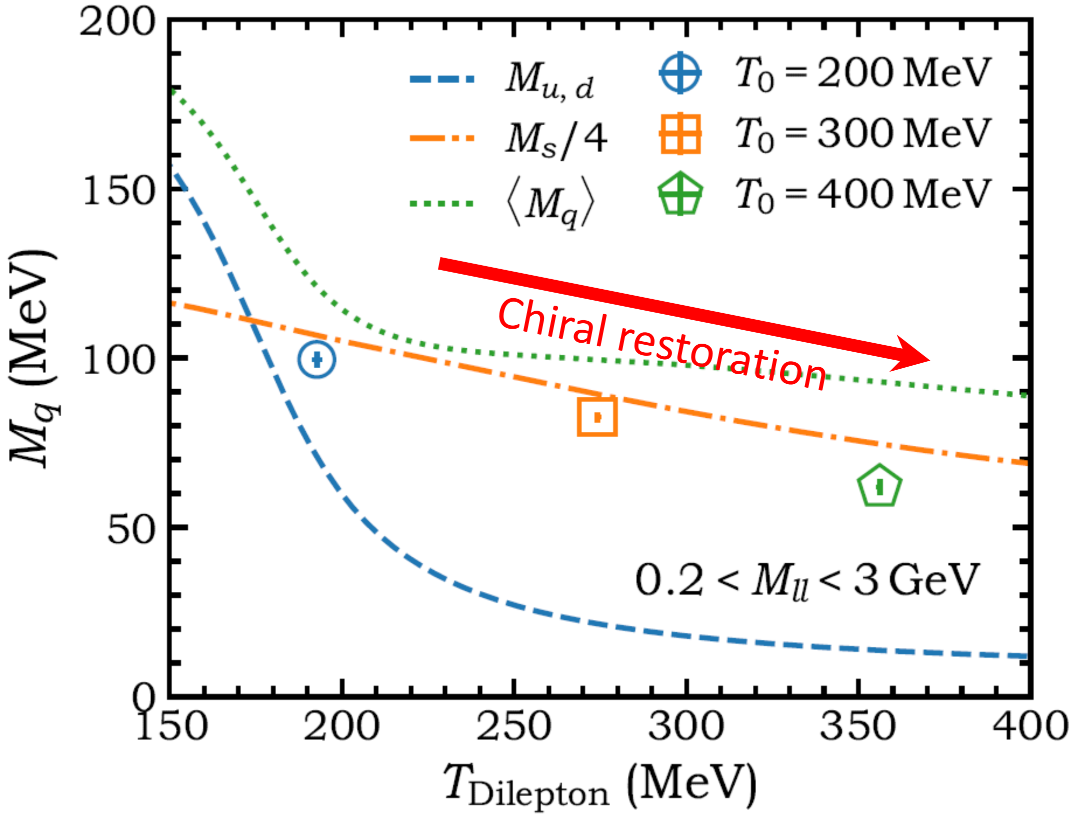
W. H. Zhou, C. M. Ko, and K. J. Sun, *ArXiv* 2412.18895 (2024)



J. Churchill, L. Du, *et al*., *PRL* **132**, 172301 (2024).

Result II: In-medium quark mass from dilepton production





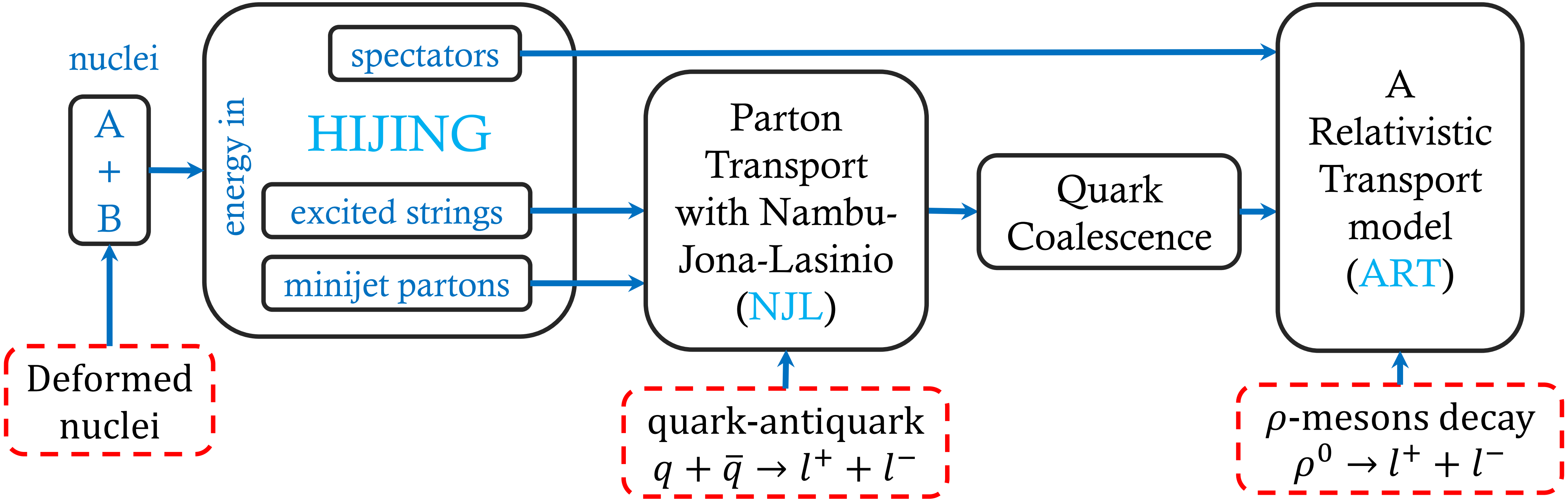
- Effective quark mass as a function of evolution time
- The effective quark mass gradually increases as the system expands due to the spontaneous breaking of chiral symmetry at lower densities and temperatures.

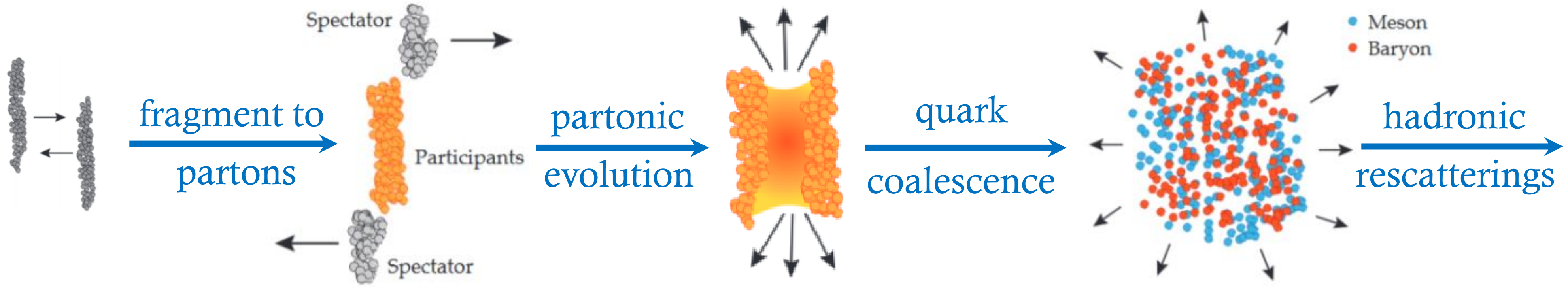
W. H. Zhou, C. M. Ko, and K. J. Sun, *ArXiv* 2412.18895 (2024)

- Temperature and in-medium quark mass extracted from the invariant mass spectra of dileptons
- Results suggest that dilepton production is a sensitive probe to chiral symmetry restoration in heavy-ion collisions.

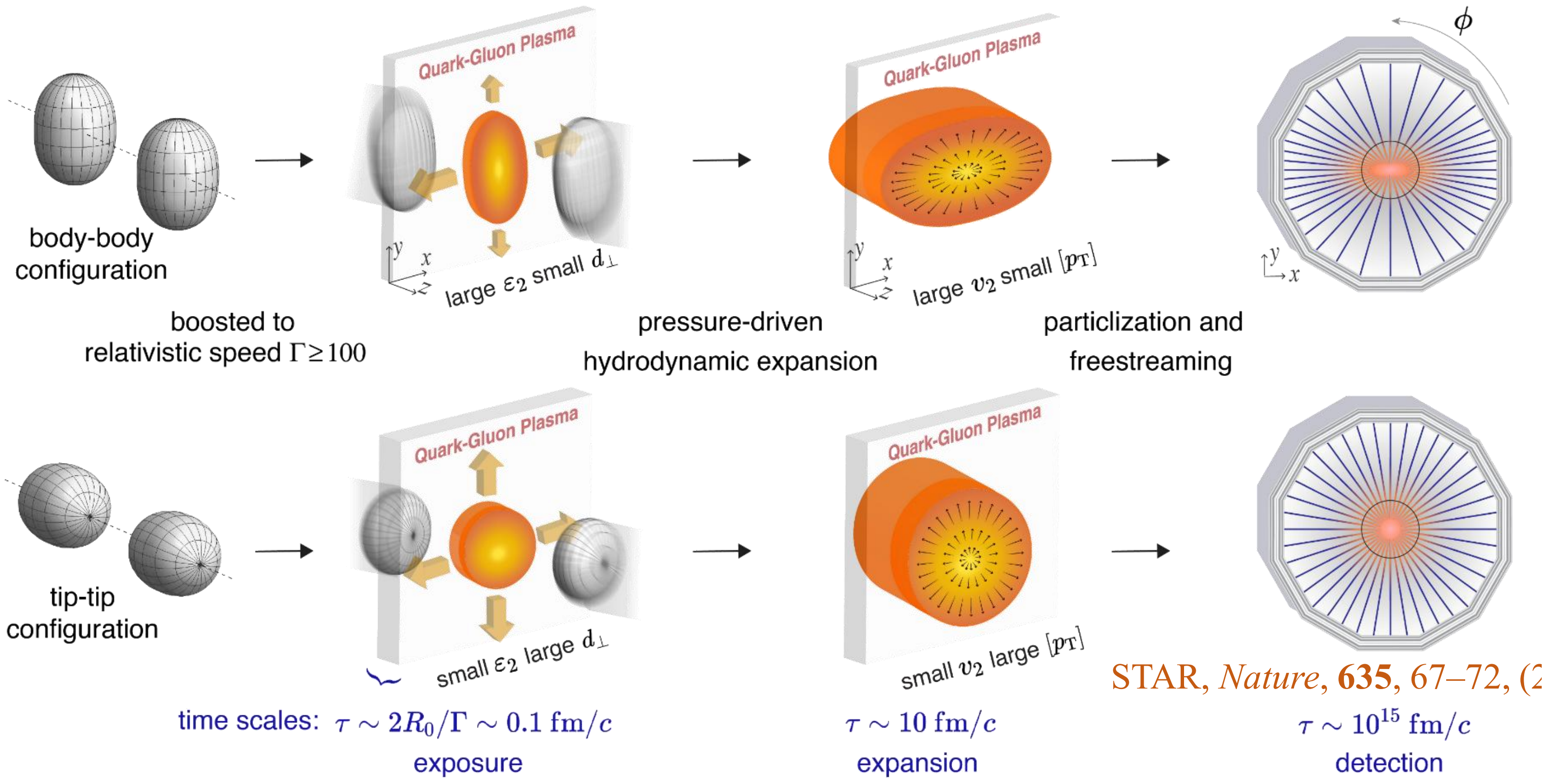
Transport modeling for realistic heavy-ion collisions

W. H. Zhou, L. M. Liu, C. M. Ko, K. J. Sun, and J. Xu, *ArXiv* 2507.18189 (2025)





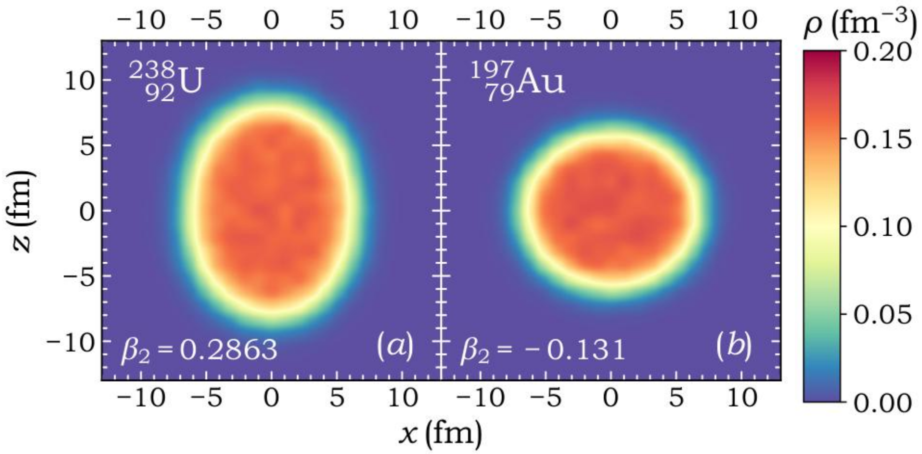
Dilepton production and nuclear shape



STAR, *Nature*, **635**, 67–72, (2

- Woods-Saxon nuclear profiles

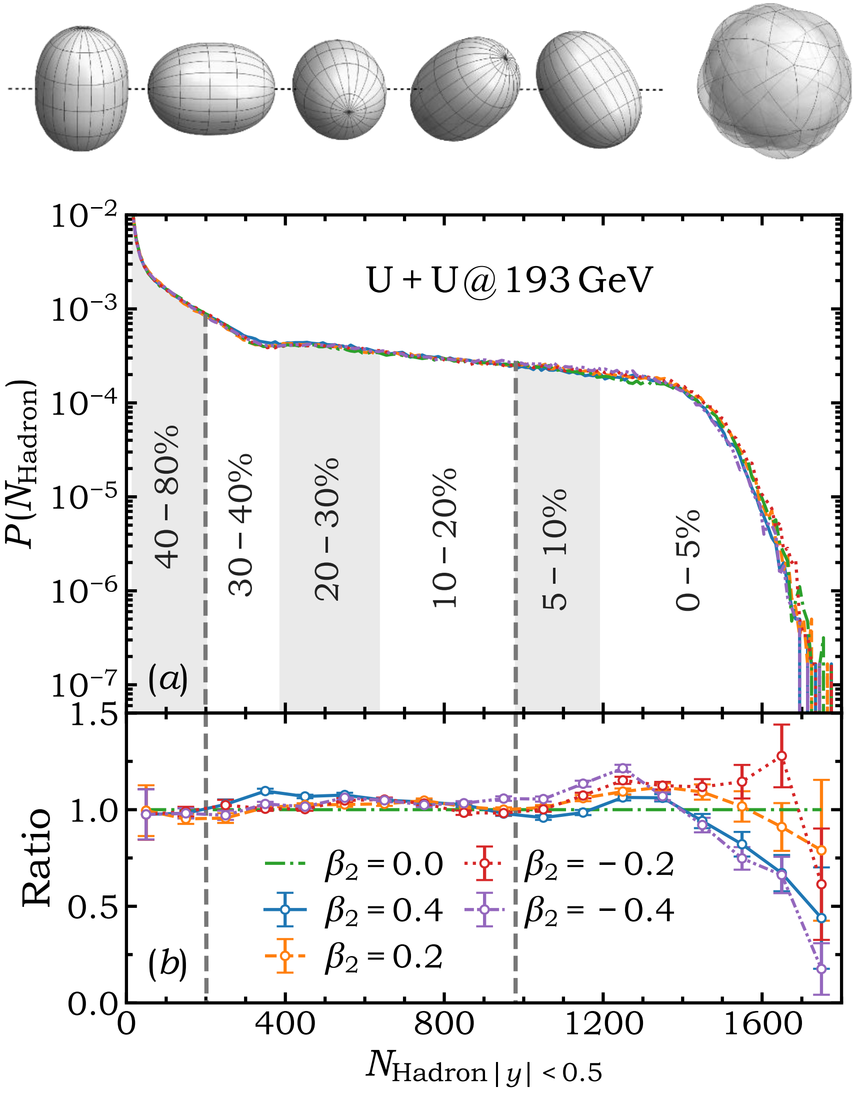
$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+\exp\left[\frac{r-R_{WS}(1+\beta_2 Y_{2,0}(\theta,\phi))}{a_{WS}}\right]}$$

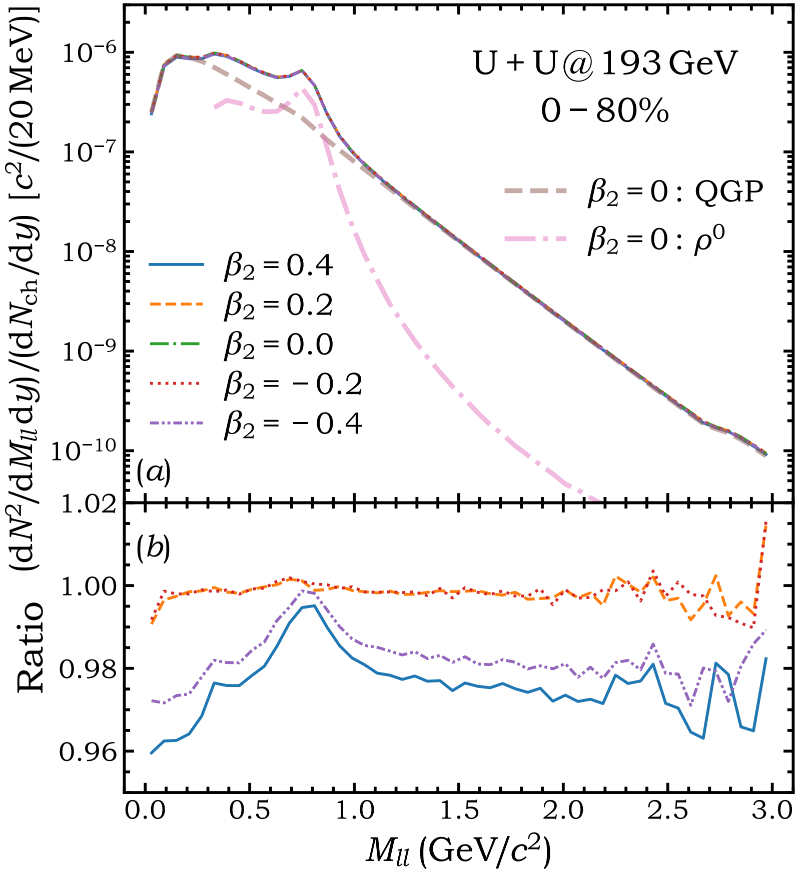


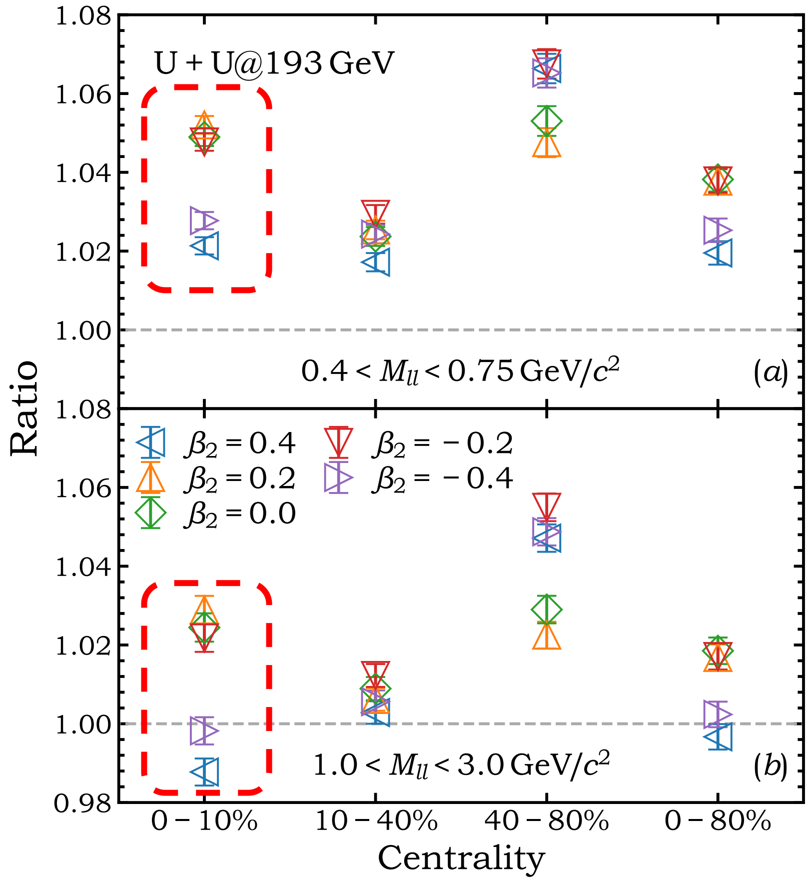
- The spatial anisotropy, caused by the initial deformed nuclear profiles, can be converted into the momentum space anisotropy; STAR, *PRC*, **107**, 024901 (2023).
- Dilepton and photon productions are not affected by strong force and emitted continuously at all stages of evolution. F. Seck, *et al*., *PRC* **106**, 014904 (2022).

J. Jia, *et al*., *PRC* **107**, L021901 (2023); W. Ryssens, *et al*., *PRL* **130**, 212302 (2023); L.-M. Liu, *et al*., *PLB* **838**, 137701 (2023); Z. Wang, *et al*., *PRC* **110**, 034907 (2024); J. Luo, *et al*., *PRC* **108**, 054906 (2023); S. Lin, *et al*., *PRD* **107**, 054004 (2023) ...

Dilepton production in U+U system

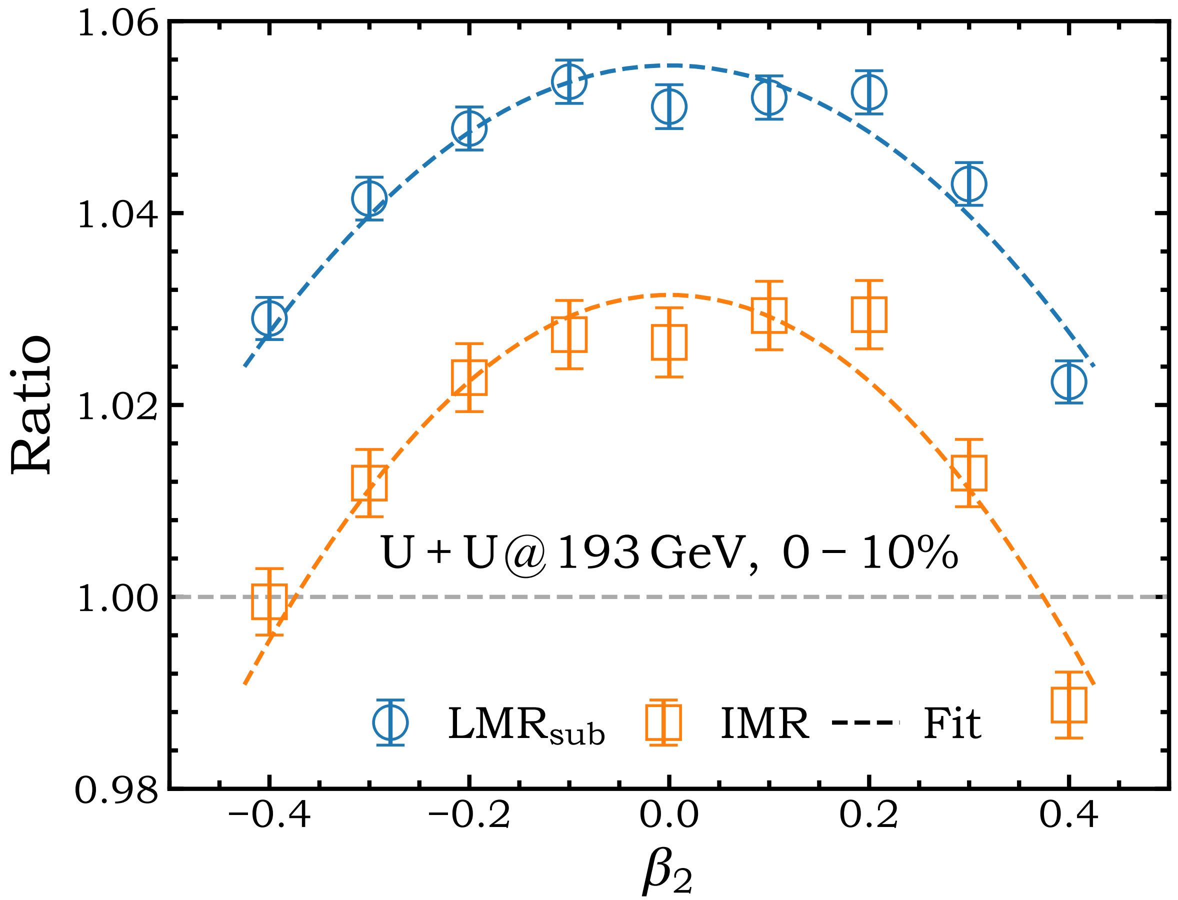






- Larger $|\beta_2|$ has a larger suppression of the multiplicity in central collisions.

Result III: Dilepton probes the shape of U



$$\text{Ratio} = \frac{(\mathrm{d}N_{ll}^{\mathrm{U}}/\mathrm{d}y)/(\mathrm{d}N_{\mathrm{ch}}^{\mathrm{U}}/\mathrm{d}y)}{(\mathrm{d}N_{ll}^{\mathrm{Au}}/\mathrm{d}y)/(\mathrm{d}N_{\mathrm{ch}}^{\mathrm{Au}}/\mathrm{d}y)} = k_2 \cdot \beta_2^2 + k_0$$

Mass region	k_2	k_0
LMR$_{sub}$	-0.174 ± 0.019	1.055 ± 0.002
IMR	-0.225 ± 0.025	1.031 ± 0.002

- The dilepton yields in IMR is more sensitive than in LMR.

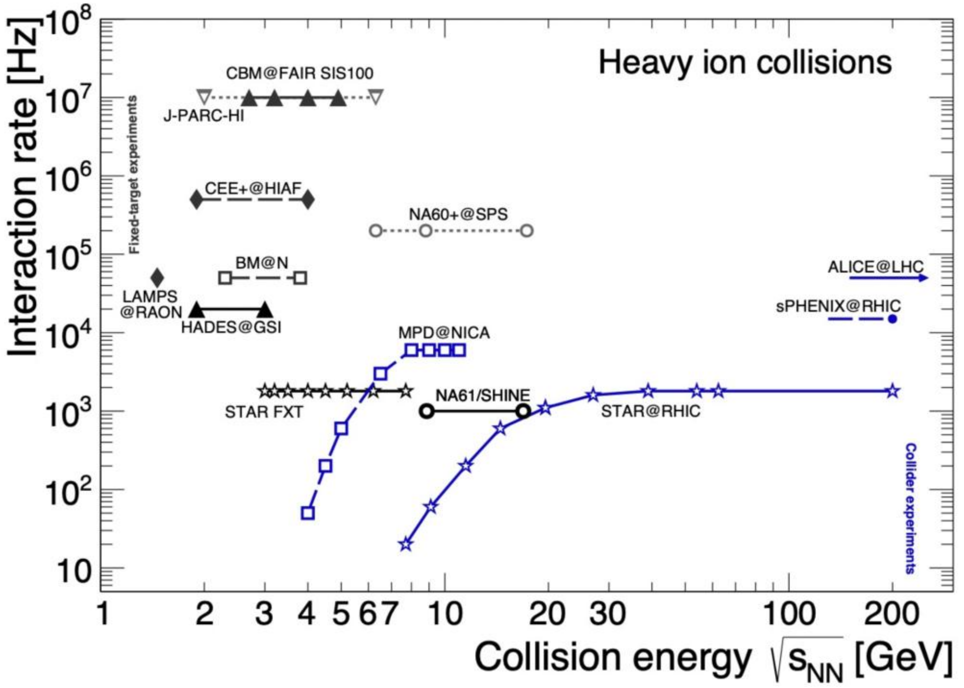
Summary and outlook

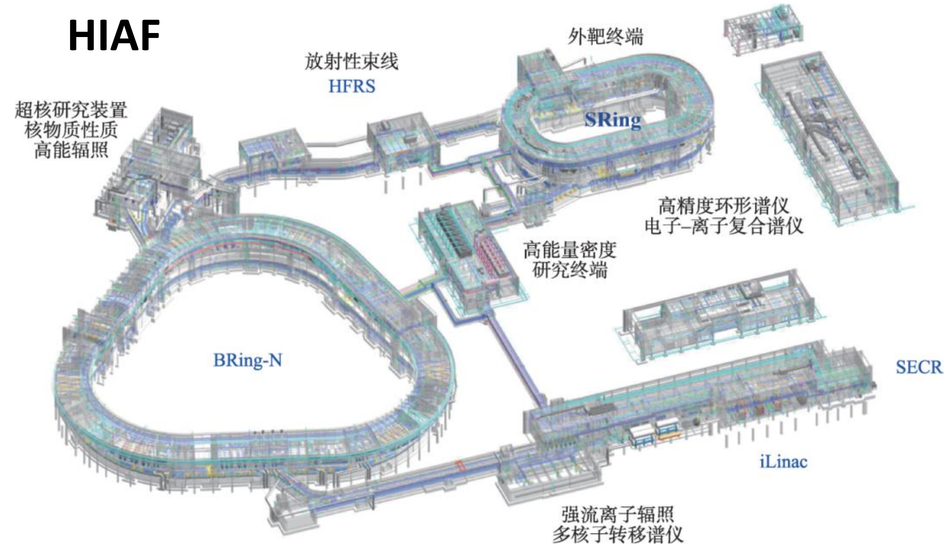
Summary:

- The dilepton production can serve as a thermometer for the produced partonic matter
- The dilepton production provides a feasible tool to probe chiral symmetry restoration
- The dilepton production can also be used to extract deformation of initial colliding nuclei

Outlook:

- Improve calculation on hadronic evolution with chiral symmetry restoration
- Explore the spinodal enhancement effect on dilepton production and the spin polarization of dilepton
- Use dilepton to study chiral symmetry restoration in dense matter at HIAF

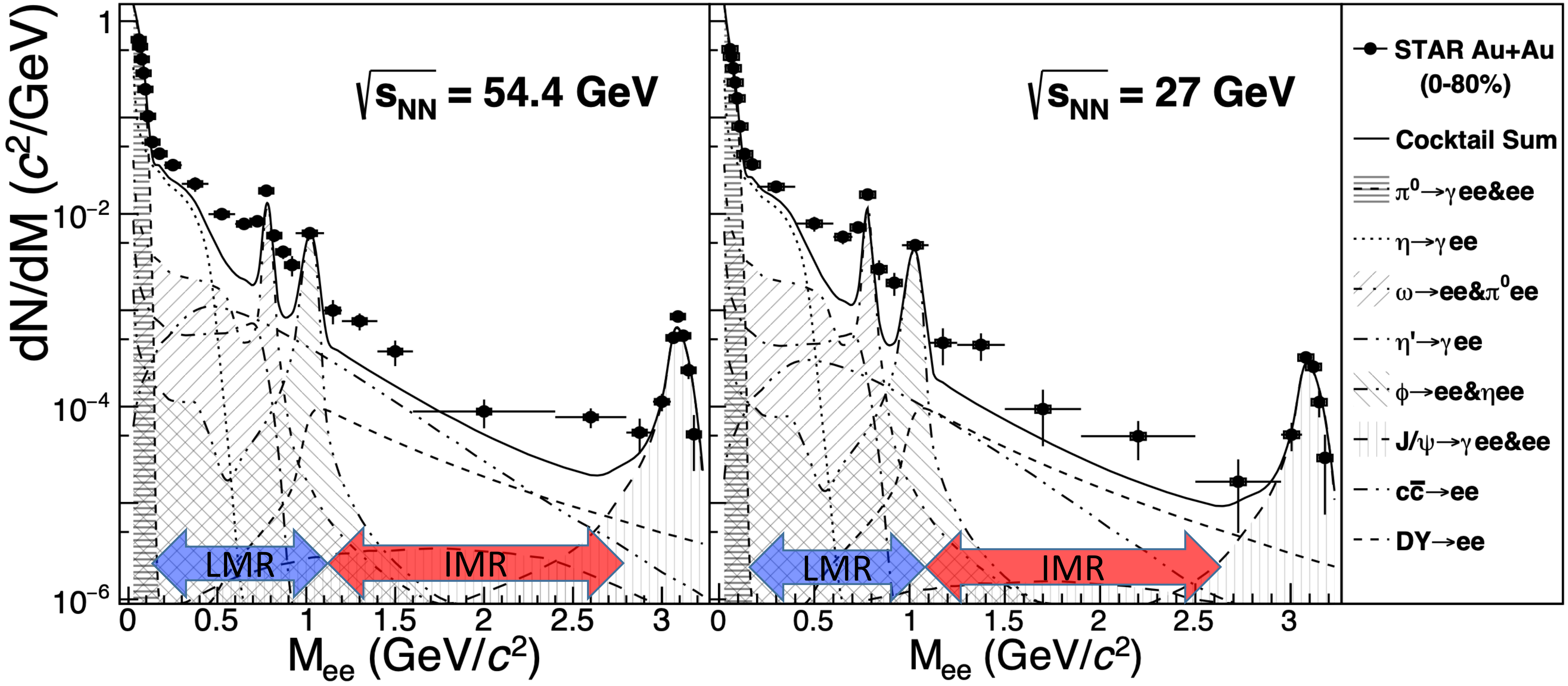




Backup

Examples of Data vs. Physical B.G. (Cocktails)

From ZaochenYe



Clear enhancement compared to cocktail contributions in both low mass region (**LMR**) and intermediate mass region (**IMR**)

Is Chiral Symmetry Partially Restored?

From ZaochenYe

scalar quark condensate

$< q\bar{q} >$ **≠ 0: chiral symmetry breaking;** **= 0: chiral symmetry restored.**

Vector meson (ρ…) → In-medium broaden **ρ** at RHIC, SPS can be well described by models considering **CSR**

Axial-Vector meson (a_1…) — No dilepton decays (chiral partner of ρ)

Direct V vs. A comparison is not available!