Preliminary Study of Quantum Entanglement in $e^+e^- \to \tau^+\tau^-$ under $\sqrt{s} = 7$ GeV at STCF

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Outline

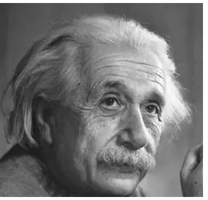
Introduction: Theory

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)



Albert Einstein, Boris Podolsky, and Nathan Rosen questioned whether QM gives a complete description of reality. They introduced a thought experiment (a.k.a EPR paradox)

1935

VOLUME 47, NUMBER 7 PHYSICAL REVIEW LETTERS 17 AUGUST 1981

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France

(Received 30 March 1981)

2022

Alain Aspect`s Bell test experiments (1981-1982)
Used polarized photon pairs to test Bell's inequalities

1981-1982

1964

J.S. Bells`s Theorem Formulated Bell's Inequality: predictions of local hidden variable theories must obey specific statistical limits Provided a testable framework

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received 4 November 1964*)

Today

ATLAS, CMS, BelleII test quantum entanglement in high energy particle pairs ($\bar{t}t$, $\tau^+\tau^-$)

Article | Open access | Published: 18 September 2024

Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration

Nature **633**, 542–547 (2024) | Cite this article

nature
2024

Introduction: Theory

- **Bit**

  In classical computing, a bit can only be in one state, either 0 or 1

- **Qubit**

  Unlike classical bits, qubits can exist in a superposition.

  They can represent both 0 and 1 simultaneously.

  $$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \ (|\alpha|^2 + |\beta|^2 = 1).$$

- **Quantum mixed state**

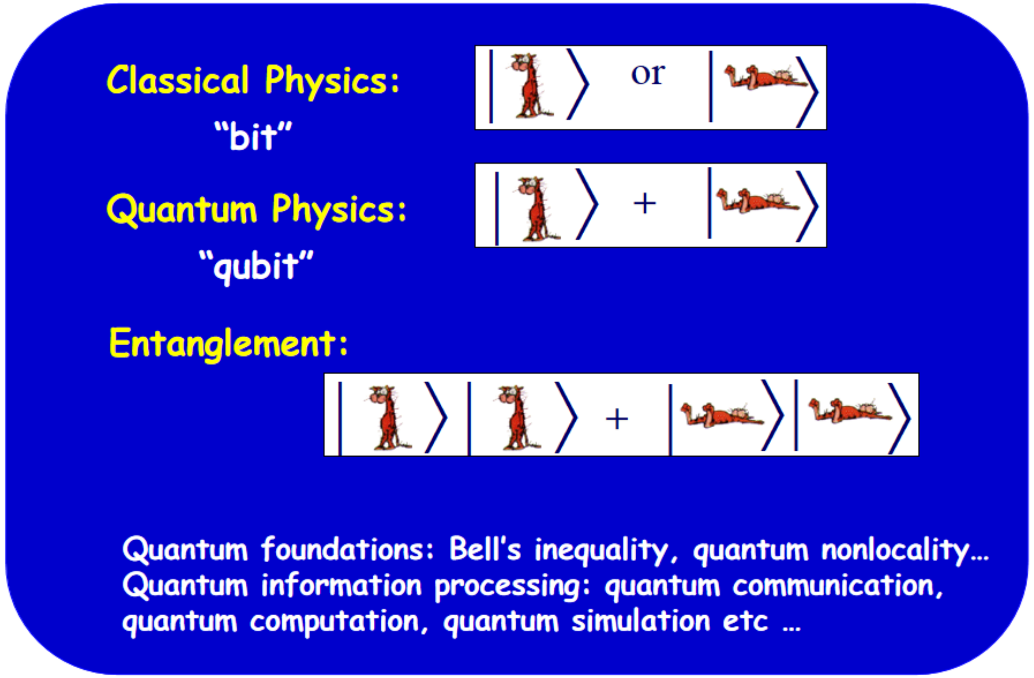
  Quantum mixed state can be described by a density matrix

  $$\rho = \sum p_i |\phi_i\rangle\langle\phi_i|$$

- **General density matrix for a qubit**

  For single bit, $\rho = \frac{I_2 + \sum B_i \sigma^i \otimes I_2}{2}$, where ***B*** is a spin polarization and are Pauli matrices

Quantum Superposition



Introduction: Entanglement observables

- **Spin Density Matrix (SDM) for Two-qubit system ($\tau^+\tau^-$)**
  - Spin quantum state of a τ-pair is described by the spin density matrix

$$\rho = \frac{1}{4}[I\otimes I + \sum_i B_i^+(\sigma_i\otimes I) + \sum_j B_j^-(I\otimes\sigma_j) + \sum_{i,j} C_{ij}(\sigma_i\otimes\sigma_j)]$$

where B_i^+ / B_j^- is spin polarization of τ^+/τ^-, C_{ij} is correlation matrix connecting spin of τ^+/τ^-

- **QE observables based on SDM:**
  - **Concurrence $\mathcal{C}[\rho]$** :

For a bipartite qubit system, an entanglement monotone can be defined as:

no entanglement $0 \leq \mathcal{C}[\rho] = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \leq 1$ maximally entangled

where λ_i are the eigenvalues, in decreasing order, of the matrix

$$R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}, with \tilde{\rho} = (\sigma_2\otimes\sigma_2)\rho^*(\sigma_2\otimes\sigma_2)$$

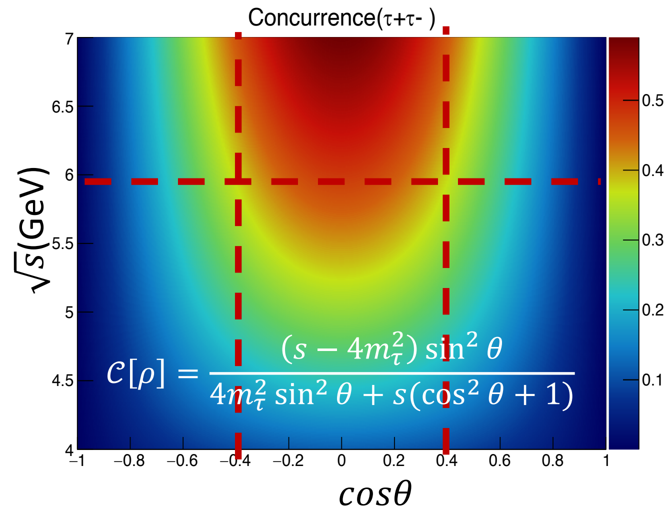
  - **Bell-inequality**: an optimized operator **$m12[C]$**:

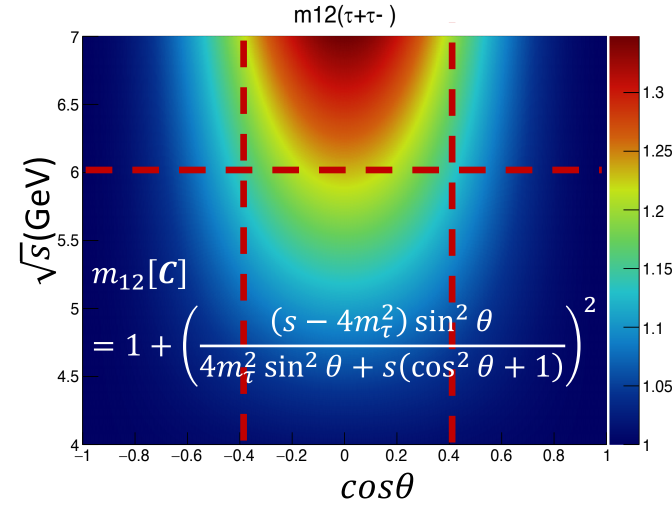
$$m_{12}[C] = m_1 + m_2$$

$m_{12}[C] > 1$ Bell-inequality is violated

where $m_1 \geq m_2 \geq m_3$ are the eigenvalues of the positive semi-definite matrix $M = C^T C$

theoretical signal region





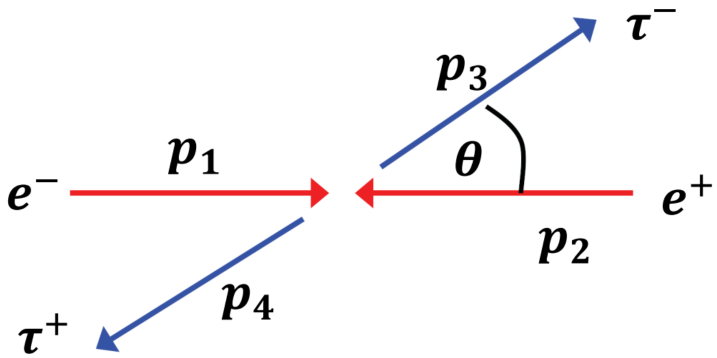
Introduction: Entanglement observables

❑ B, C

- $\rho = \frac{1}{4}[I\otimes I + \sum_i B_i^+ (\sigma_i\otimes I) + \sum_j B_j^- (I\otimes\sigma_j) + \sum_{i,j} C_{ij}(\sigma_i\otimes\sigma_j)]$, where:

$$\boldsymbol{B_i^\pm} = \frac{\mathbf{3}}{\boldsymbol{\kappa_\pm}}\frac{\mathbf{1}}{\boldsymbol{\sigma}}\int \boldsymbol{d\Omega^\pm}\frac{\boldsymbol{d\sigma}}{\boldsymbol{d\Omega^\pm}}(\vec{\boldsymbol{h}}^\pm\cdot\hat{\boldsymbol{e}}_i)$$

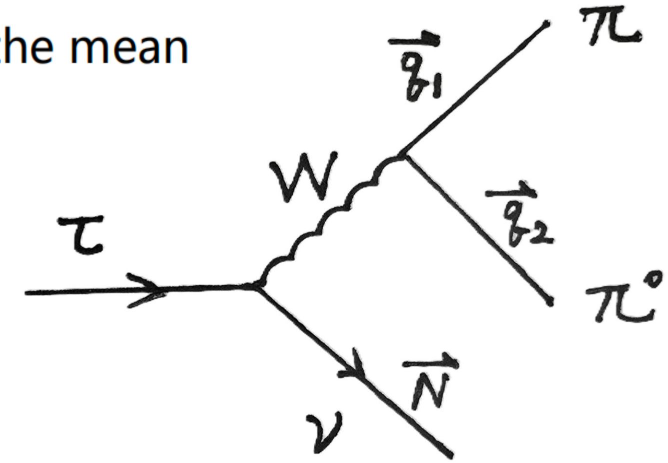
$$\boldsymbol{C_{ij}} = \frac{\mathbf{9}}{\boldsymbol{\kappa_+\kappa_-}}\frac{\mathbf{1}}{\boldsymbol{\sigma}}\int \boldsymbol{d\Omega^+ d\Omega^-}\frac{\boldsymbol{d\sigma}}{\boldsymbol{d\Omega^+ d\Omega^-}}(\vec{\boldsymbol{h}}^+\cdot\hat{\boldsymbol{e}}_i)(\vec{\boldsymbol{h}}^-\cdot\hat{\boldsymbol{e}}_j)$$



- $\kappa_\pm = \pm 1.0$, $\vec{h}^\pm$ are polarimetric vectors for $\tau^\pm$.

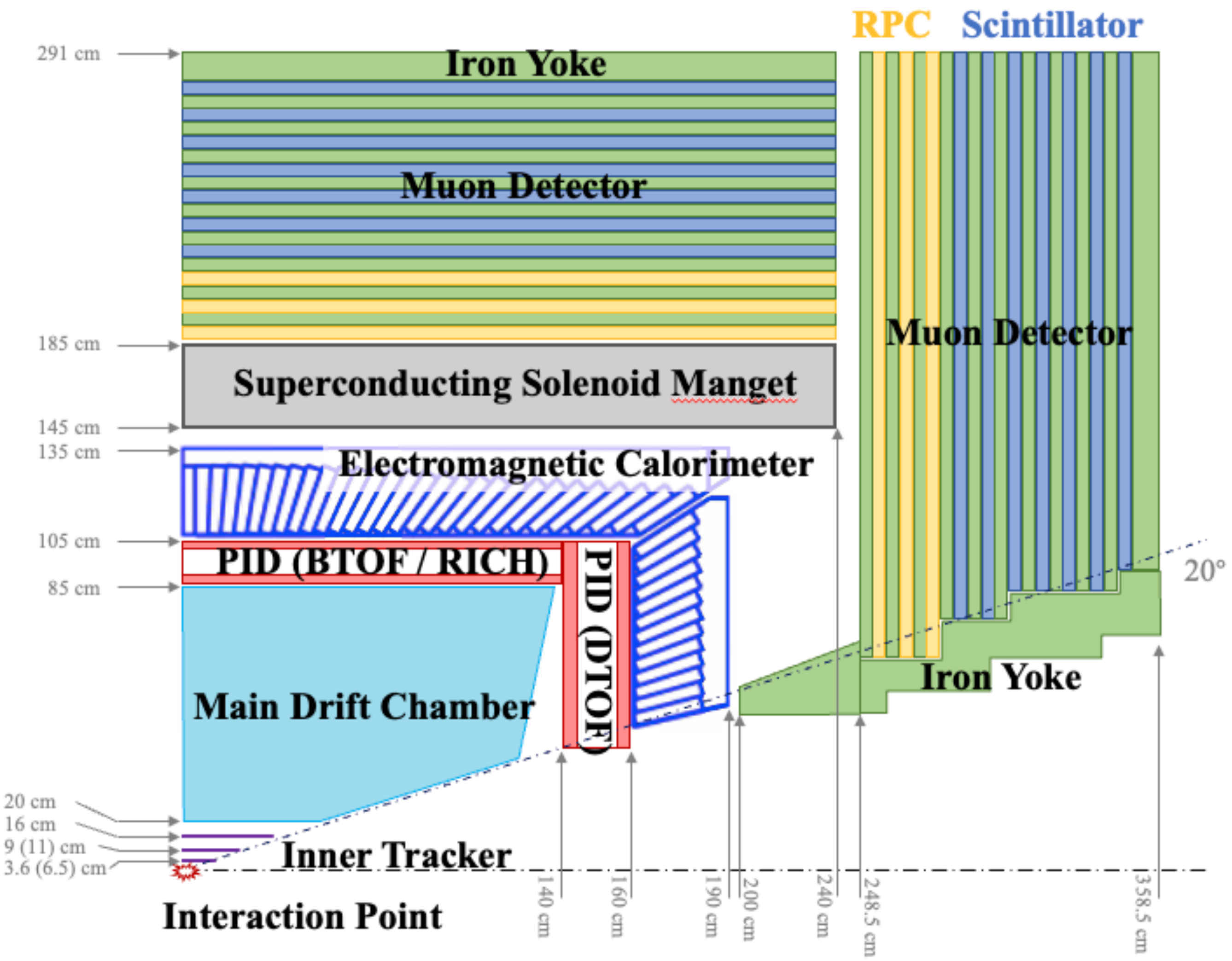
- So the key point is to retrieve $\vec{h}^\pm$, and B, C coefficients correspond to the mean value of $\vec{h}^\pm$ in different directions of $\{\hat{e}_i\}(i = 1,2,3)$, usually $\{\hat{n}, \hat{r}, \hat{k}\}$.

- $\vec{\boldsymbol{h}} \propto -[2(q^\mu N_\mu)\vec{\boldsymbol{q}} - (q^\mu q_\mu)\vec{\boldsymbol{N}}]$



Outline

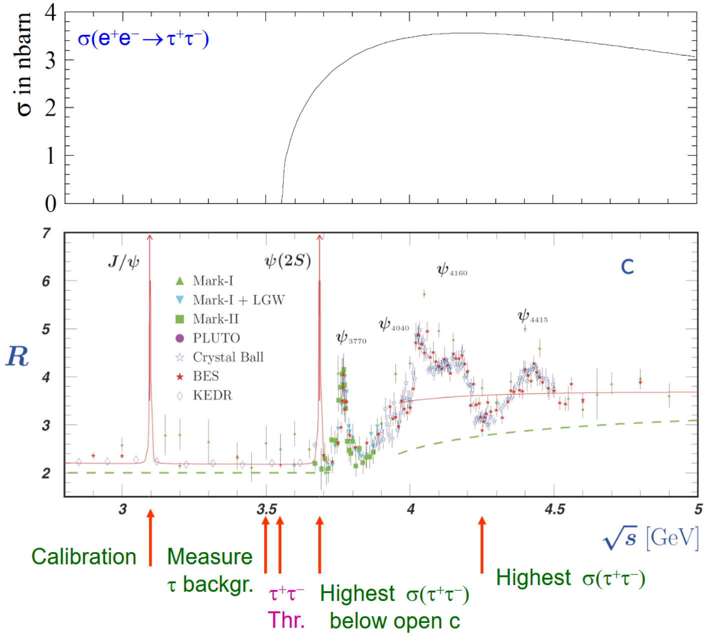
STCF Detector

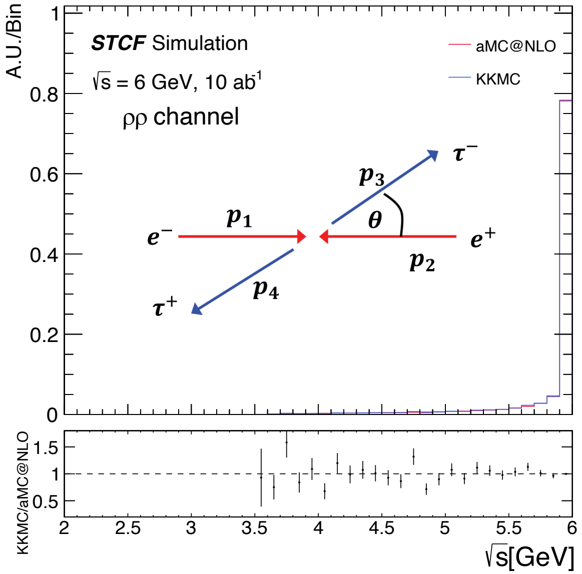


- ITK: Inner Tracker
  - ITKW: cylindrical MPGD
  - ITKM: CMOS M-MAPS
- MDC: Main Drift Chamber, Central tracker
  - π^+/π^- track Reconstruction
- PIDE: Particle Identification-Endcap
  - DTOF: DIRC-like TOF
  - PIDB: Particle Identification-Barrel
  - RICH: Ring Imaging Cherenkov detectors CsI-MPGD
  - BTOF: Barrel-TOF
  - π^+/π^- Identificaton
- ECAL(EMC): Electromagnetic Calorimeter pure CsI + APD
  - π^0 Reconstruction
- MUD: Muon Detector RPC + scintillator

τ-pair Production on STCF

- ❑ τ-pair Production on STCF
  - STCF: 2-7GeV event rate: $3.5\times10^{9}/y(4.26GeV)$
  - E_{cm}=4.26GeV: highest $\sigma(\tau^{+}\tau^{-})$
  - E_{cm}=6.0-7.0GeV: higher significance of Quantum Entanglement
  - Main decay channel: $\tau\rightarrow\pi\nu_{\tau},\tau\rightarrow\rho\nu_{\tau},\tau\rightarrow a_{1}\nu_{\tau},\tau\rightarrow e\nu_{e}\nu_{\tau},\tau\rightarrow\mu\nu_{\mu}\nu_{\tau}$
- ❑ Tau pair production on **Oscar 2.6.2** (Fullsim + digi + reco):
  - ✓ $\sqrt{s}=7GeV,100\ million\ events\sim0.052ab^{-1}$with ISR
  - ✓ Matrix element (LHE) simulated via aMC@NLO
  - ✓ Tau decay simulated via aMC@NLO(taudecay_UFO)
  - ✓ Spin correlations between tau and their decay products are fully considered
- ❑ Background Simulation: Considered different tau decay channel



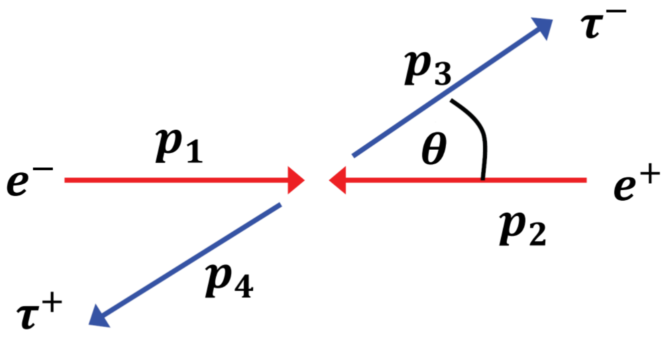


Outline

Reconstruction

- Signal decay: $\tau^{+}\tau^{-} \rightarrow \rho^{+}\rho^{-}\nu\,\bar{\nu}$

  main background: $\tau^{+}\tau^{-} \rightarrow \rho\pi\pi^{0}\pi^{0}, \tau^{+}\tau^{-} \rightarrow \pi\pi\pi^{0}\pi^{0}, \tau^{+}\tau^{-} \rightarrow \rho K^{*}$,etc



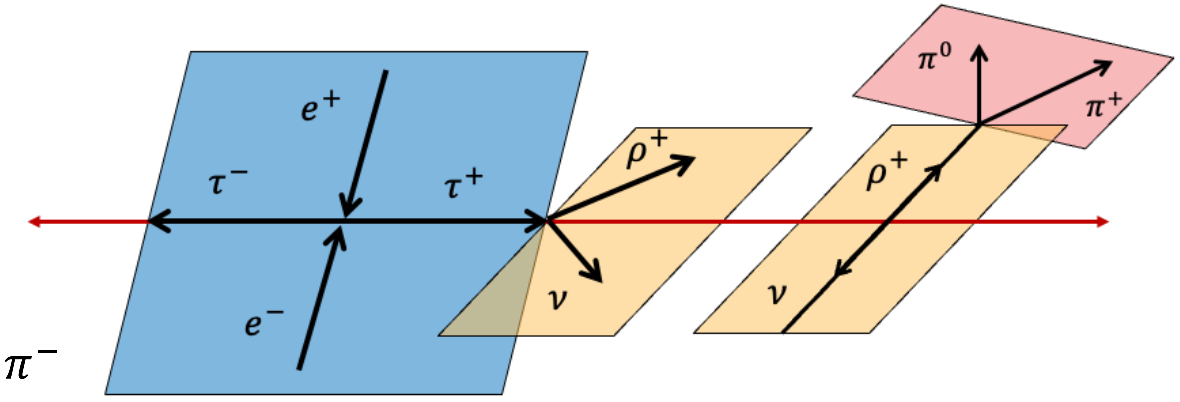
- Selection criteria:

  γ energy $E > 0.05\ GeV$(endcap), $E > 0.025\ GeV$(barrel)

  geometric acceptance: $20^{\circ} < \theta < 160^{\circ}$

- Reconstruction step:

  0. Number of π^{+} = 1, Number of π^{-} = 1
  1. Passed γ-level machine learning selection, Number of γ = 4(separate signal from bkg)
  2. Passed pairing of γ, Passed pairing of π^{0} and π^{+}/ π^{-}
  3. Passed event-level machine learning selection



$e^{+}e^{-}$CM frame τ^{+}rest frame

Step1: π^0 gamma Reconstruction

- Beam background gamma: cut by ECAL(~200ps)
- ISR feature: low energy, relatively forward
- Use BDTG to Select π^0 gamma from other gamma(ISR gamma)
- Input train variable

  E_{seed}:energy deposited in the center crystal of the shower

  E_{total} hits $\cos(\theta)$

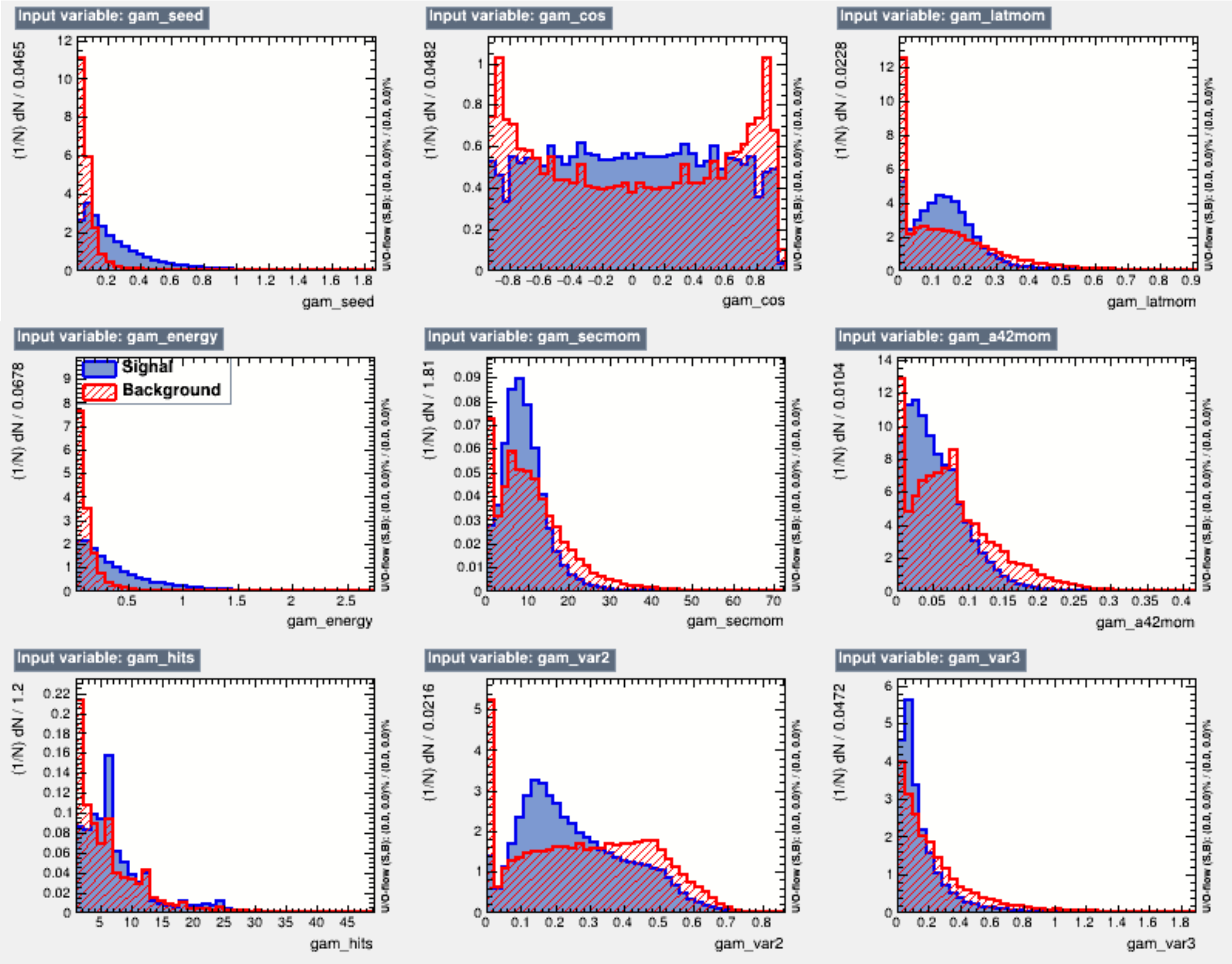
  secondary moment:$\sum E_i r_i^2 / \sum E_i$

  Lateral moment:$\sum_{i=3}^{n} E_i r_i^2 / (E_1 r_0^2 + E_2 r_0^2 + \sum_{i=3}^{n} E_i r_i^2)$

  A_{42}moment: $\sum \frac{E_i}{E_{tot}} f_{4,2}\left(\frac{r_i}{R_0}\right) e^{im\phi}$

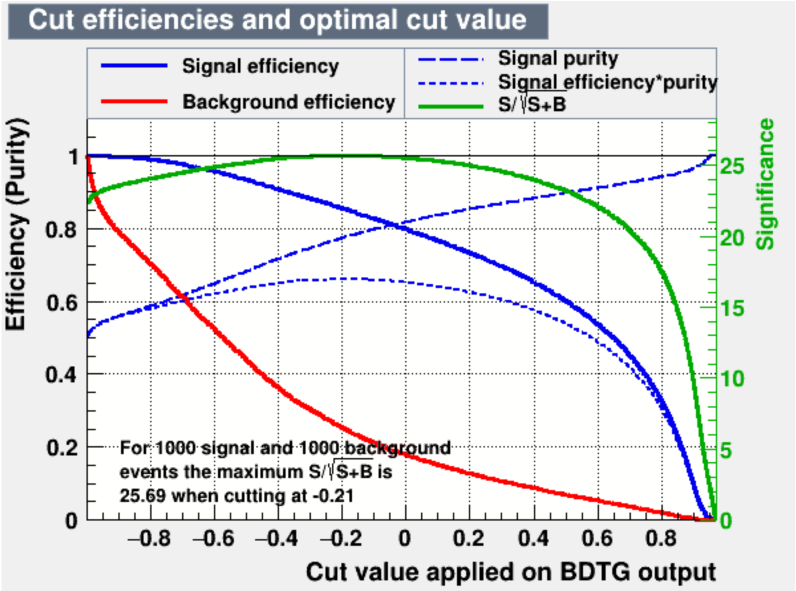
  Variable2= $1 - \frac{E_{seed}}{E_{3x3}}$

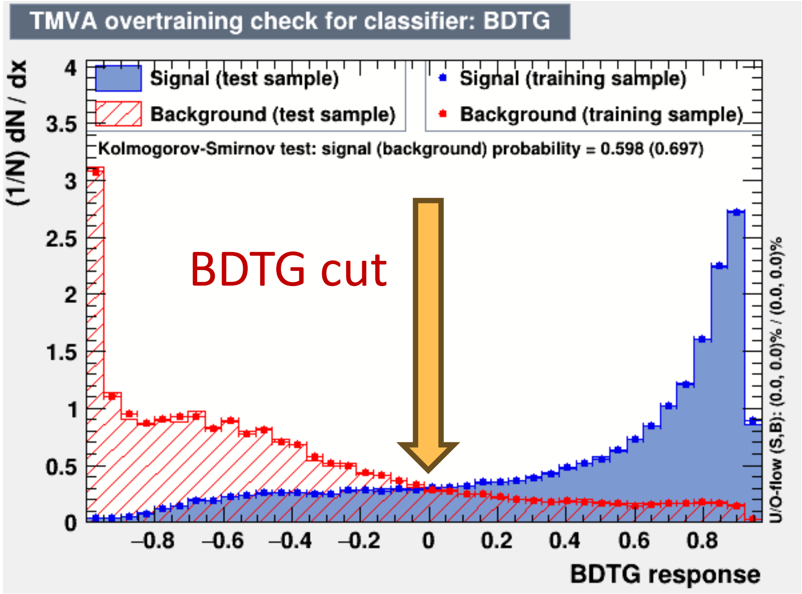
  Variable3= $\begin{cases} 0, & hits = 1 \\ (\frac{E_{tot}}{E_{seed}} - 1)/(hits - 1), & hits > 1 \end{cases}$

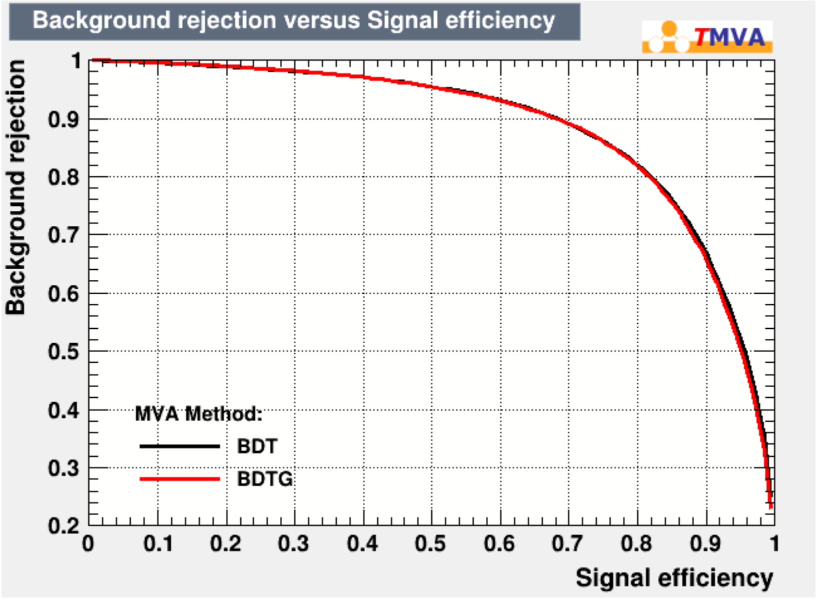


Step1: π^0 gamma Reconstruction

- Main background: ISR gamma
- Set BDTG cut = 0
- retaining about 80% pi0 gamma and 20% other gamma(ISR)
- Efficiency : $\rho\rho$ decay channel(25.2%), all channel(7.3%)





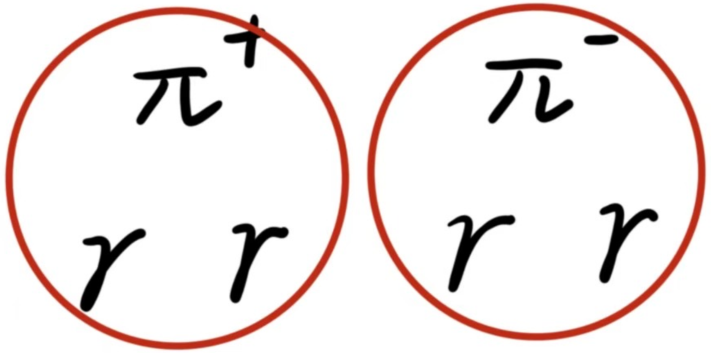


Step2: Event Pairing

- Loop all pairing schemes and select
- Kalman Kinematic Fit (BES)

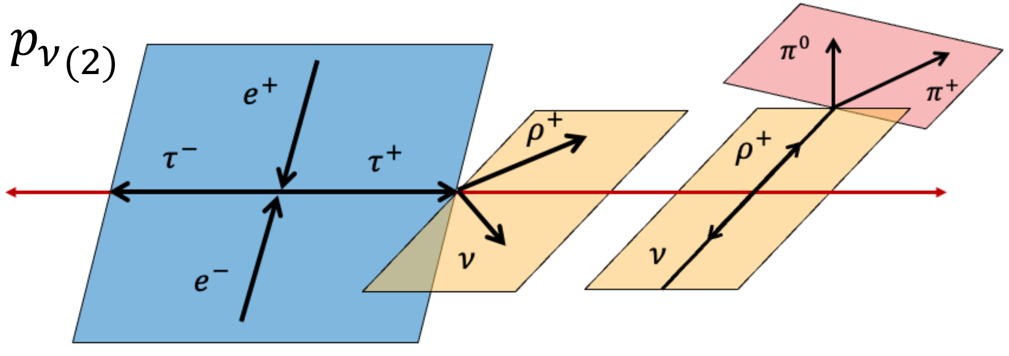
  Pairing gamma

  - Using mass constraints of π^0

  Pairing π^0 with π^+/π^- , Solve the ν energy-momentum $p_{\nu(1)}, p_{\nu(2)}$

  - Add Missing Momentum ν_1, ν_2 (6 unknown quantities)
  - Add Energy-momentum conservation (4 equation)
  - Add Mass constraints of τ (2 equation)

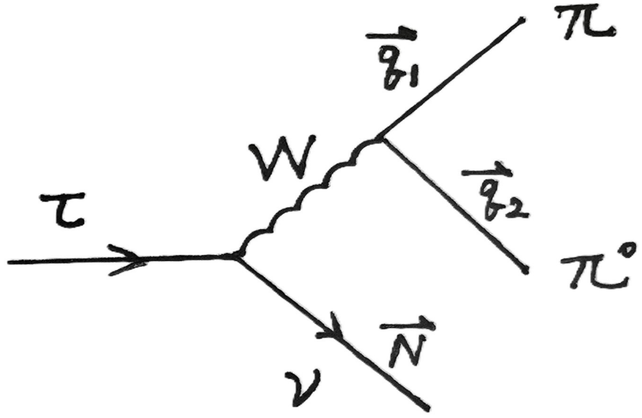


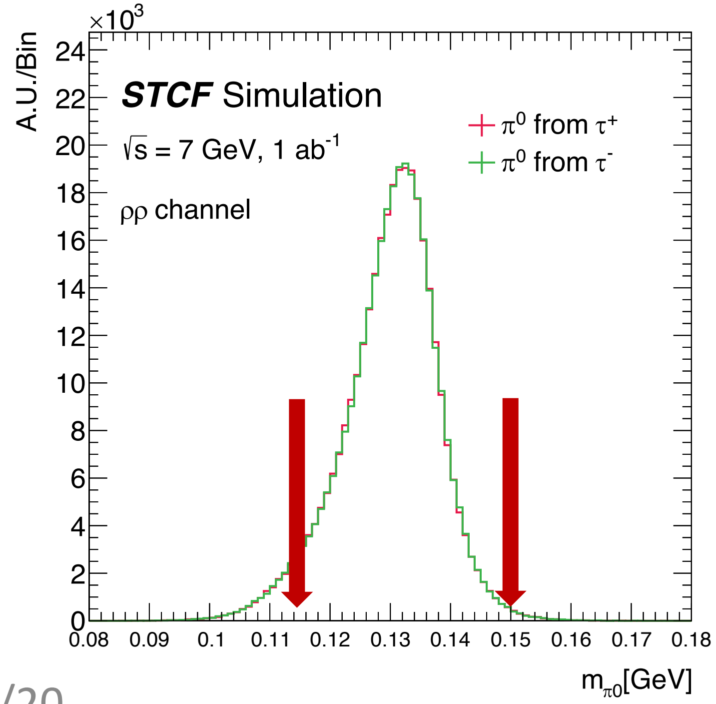
e^+e^-CM frame τ^+rest frame

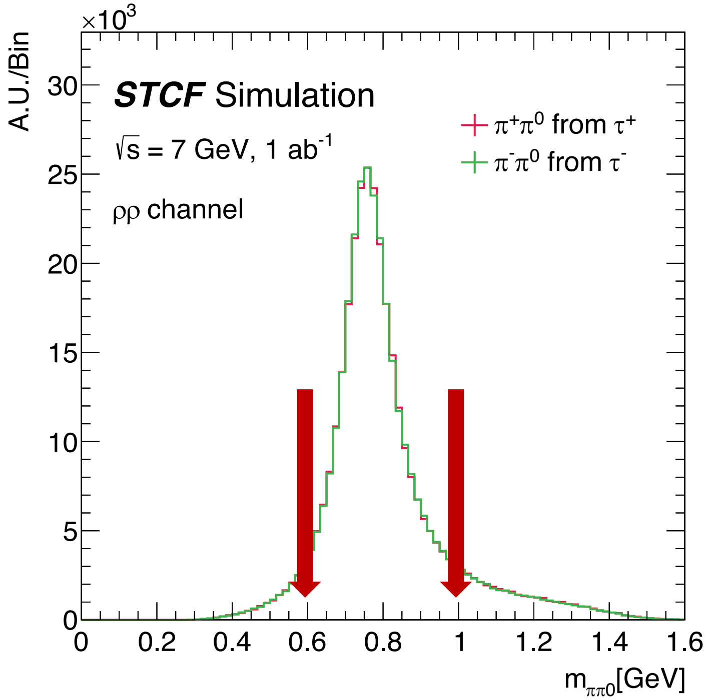
- passed the pairing cut: $\chi^2 < 20$ & $p_\nu = p_{\nu(1)} + p_{\nu(2)} < \frac{E_{total}}{2}$ and select less χ^2
- Efficiency : $\rho\rho$ decay channel(45.9%), all channel(22.4%)
- For $\rho\rho$ decay channel: true pairing: 98.25% , wrong pairing: 0.92% , include ISR: 0.83%

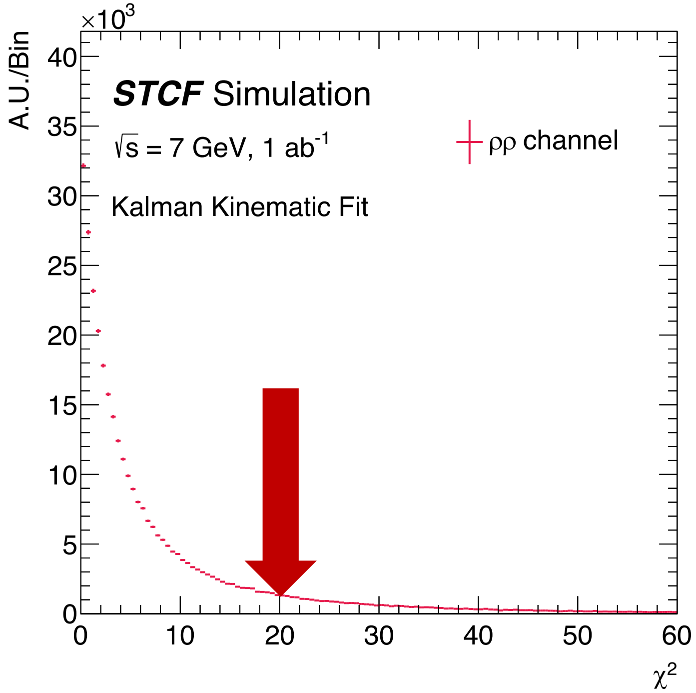
Step2: Event Pairing

- mass constraints of π^0 : $0.115\text{GeV} < m_{\pi 0} < 0.150\text{GeV}$
- mass constraints of $\pi\pi^0$: $0.6\text{GeV} < m_{\pi\pi 0} < 1.0\text{GeV}$
- Kalman Kinematic Fit cut: $\chi^2 < 20$
- Apply this cut at final step to optimize efficiency and reconstruction quality









Step3: $\rho\rho$ event Reconstruction

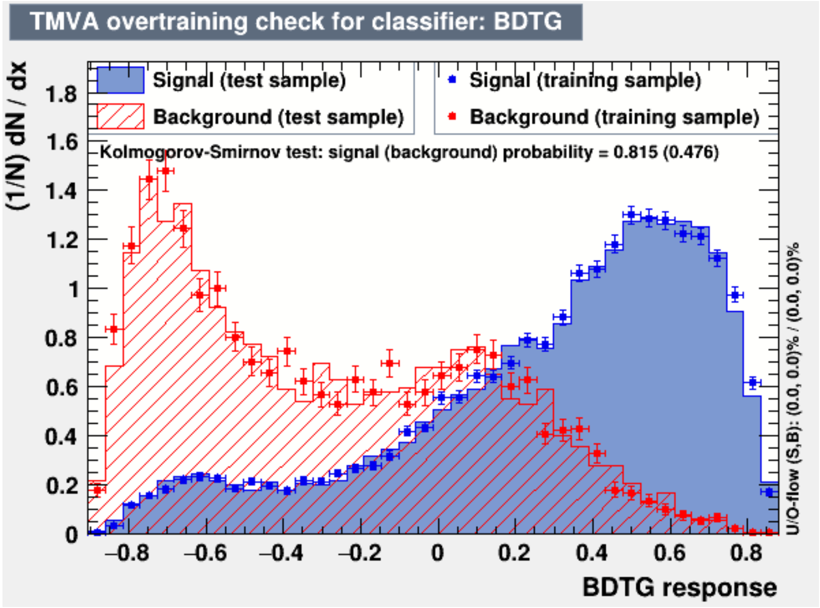
- Main background event: $\rho a(\rho \to \pi\pi^0, a \to \pi\pi^0\pi^0)$,etc
- Use BDTG
- Input train variable

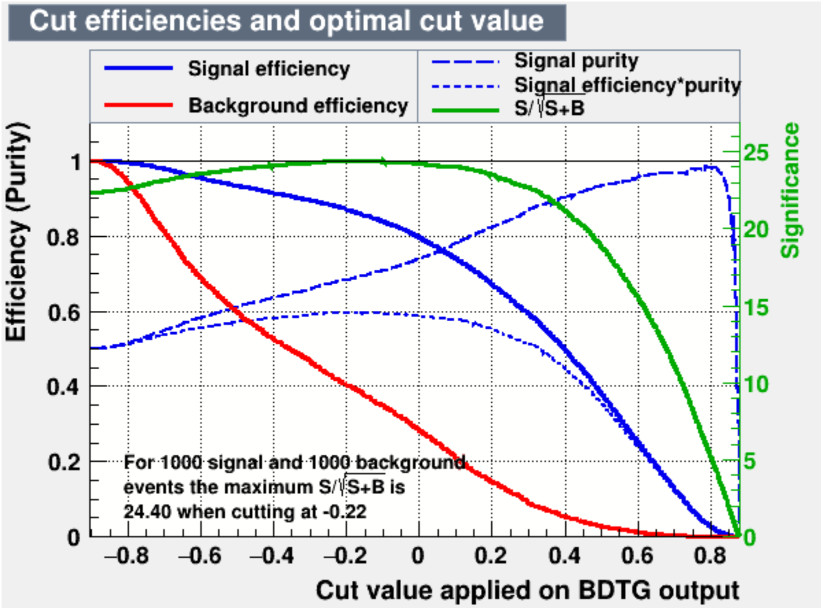
  momentum $p_{\nu(1)}, p_{\nu(2)}, p_{\pi^+}, p_{\pi^-}, 4\ p_\gamma$

  Pairing χ^2, p_ν

  PID: probPi, probMu, probK

  nGamma before gamma-level BDTG cut
- Set BDTG cut = -0.2， retaining about 87% signal event and 40% other event
- Efficiency : $\rho\rho$ decay channel(87.5%), all channel(78.8%)





Reconstruction: cutflow

- $\rho\rho$ decay channel (1000W events test)

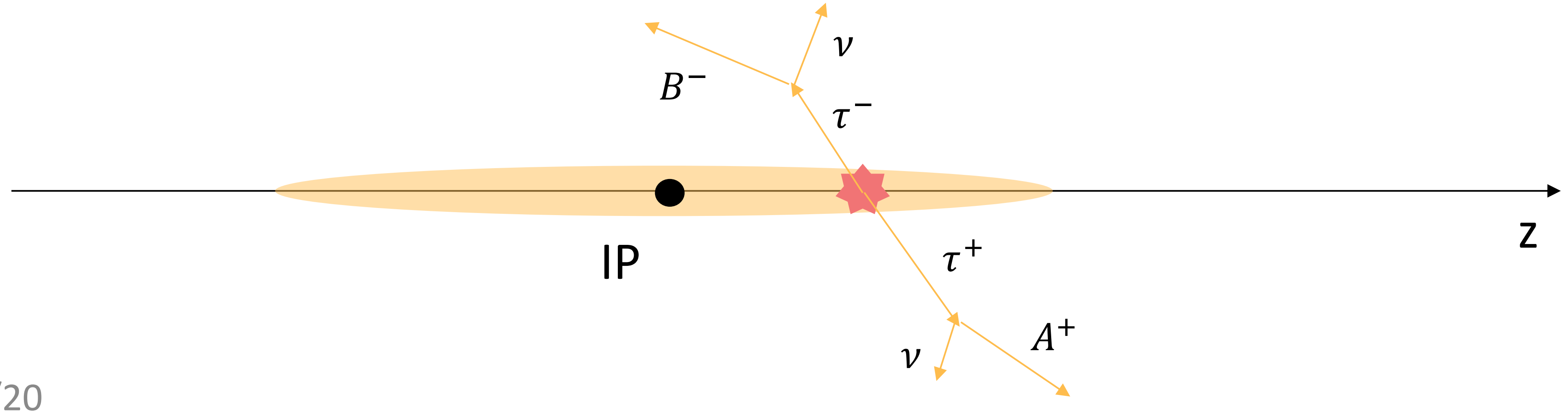
step	Percentage of previous step		Signal purity
	Signal + BKG	Signal	
Total events	-	-	6.9%
Number of charged tracks = 2, total charge = 0	64.8%	78.1%	8.3%
Number of photons = 4	6.4%	21.7%	28.1%
Number of π^+ = 1, Number of π^- = 1	67.4%	79.6%	33.2%
Passed the particle pairing	19.0%	45.2%	78.9%
Passed event-level machine learning selection	75.8%	86.3%	89.9%
Passed the τ momentum reconstruction	95.0%	95.4%	90.3%

Overall efficiency: 0.38% , Signal efficiency: 5.0% , Signal purity: 90.3%

Signal region: true pairing: 88.71% , $\rho\rho$ wrong pairing: 0.83% , include ISR: 0.75% , Background event: 9.71%

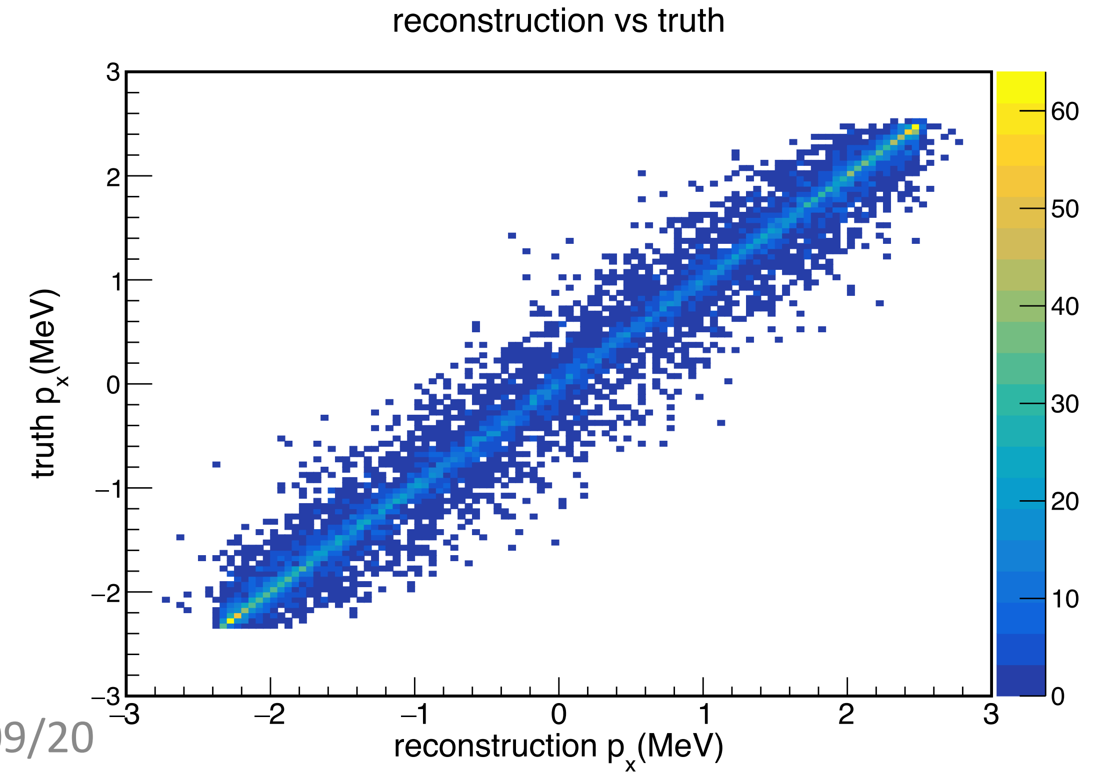
Reconstruction: P_τ

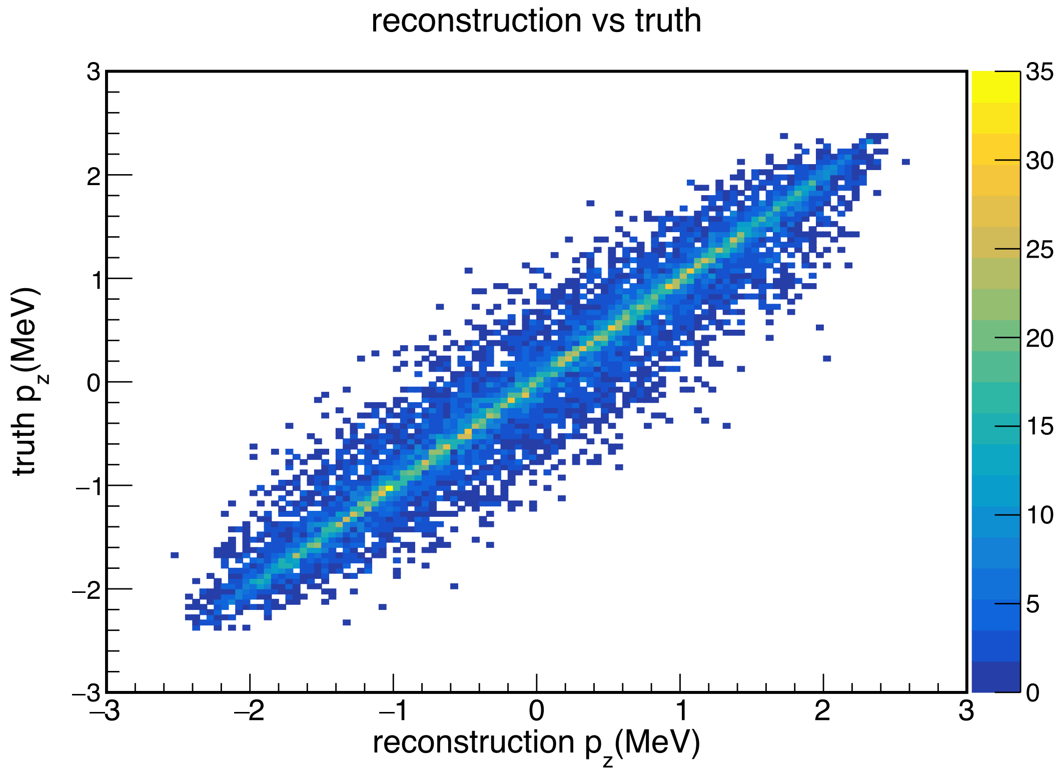
- P_τ reconstruction: solving a set of analytic equations
- Existing problem:
  - Because of two missing ν , the τ flight direction is calculated with a two-fold ambiguity
  - Vertex resolution(σ_Z, σ_T), Beam size(IP $\sigma_Z \sim 0.3mm$), τ pair flight length($\sim 0.2mm$)
- BelleII:
  - huge error method, freely vary the position of the τ decay vertex along π track
  - The reconstructed τ axis and the beam axis intersect



Reconstruction validation: based on truth P_τ

- ❑ P_τ reconstruction: Kalman Kinematic Fit (BES)
- ❑ Event cut:
  - Because of two missing ν , the τ flight direction is calculated with a two-fold ambiguity
  - Select event which two τ flight direction is similar

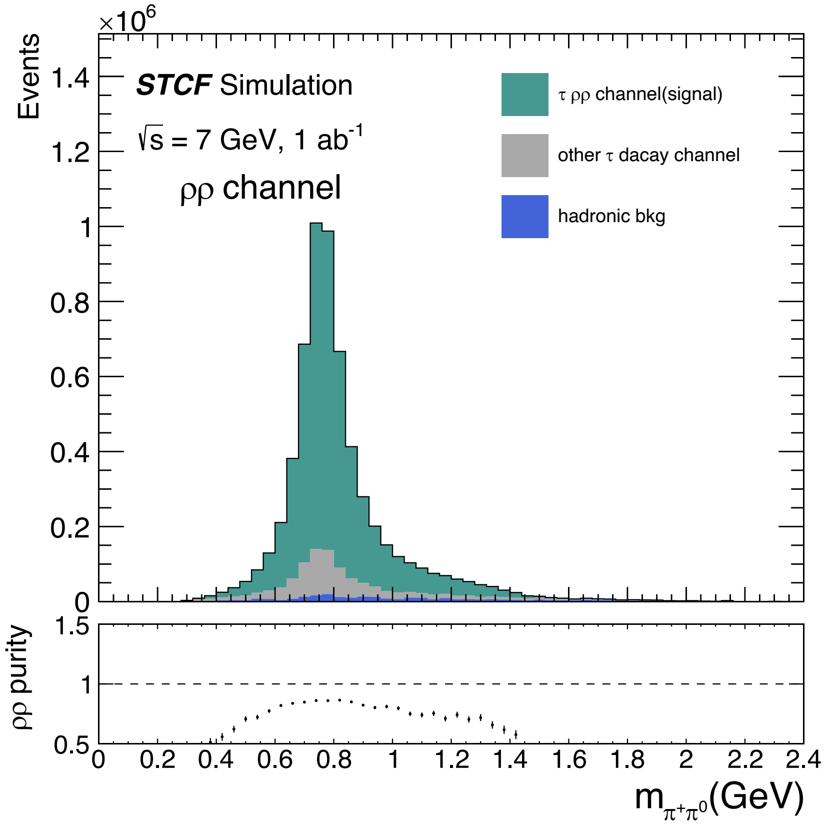


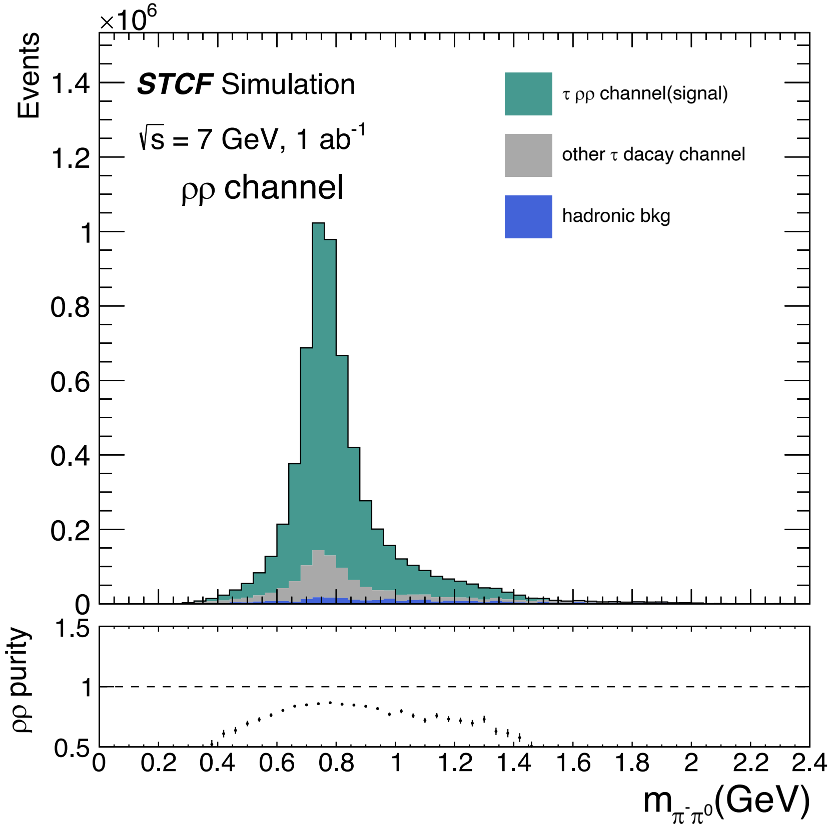


Outline

Physical Background

- Produced other tau decay channel sample (30 million) and hadronic background sample (20 million)--ongoing
- rare tau decay channel, events: $3.7\times10^{8}(1\ ab^{-1})$ select efficiency: 0.0008%
- Hadronic background, events: $6.2\times10^{9}(1\ ab^{-1})$ select efficiency: 0.004%
- Scale all sample to $1\ ab^{-1}$





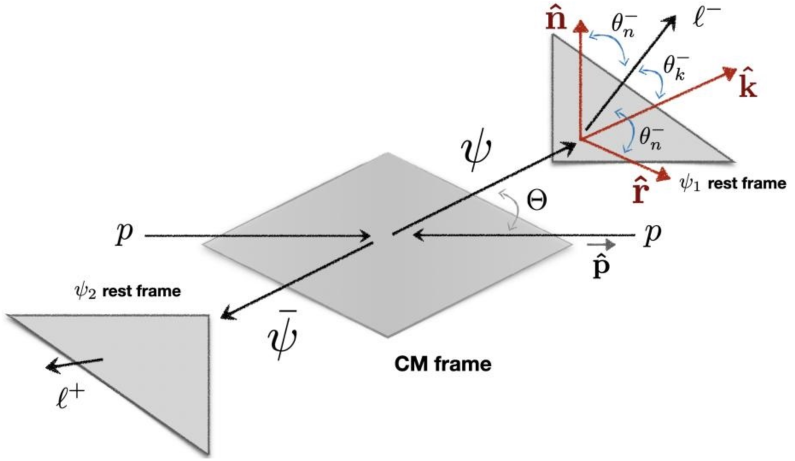
Background and SDM Reconstruction

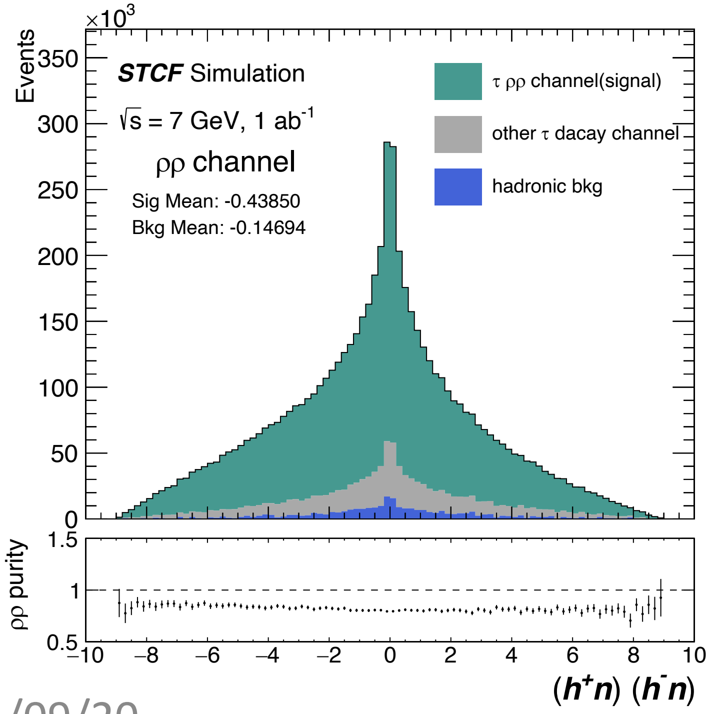
- Spin quantum state of a τ-pair is described by the spin density matrix

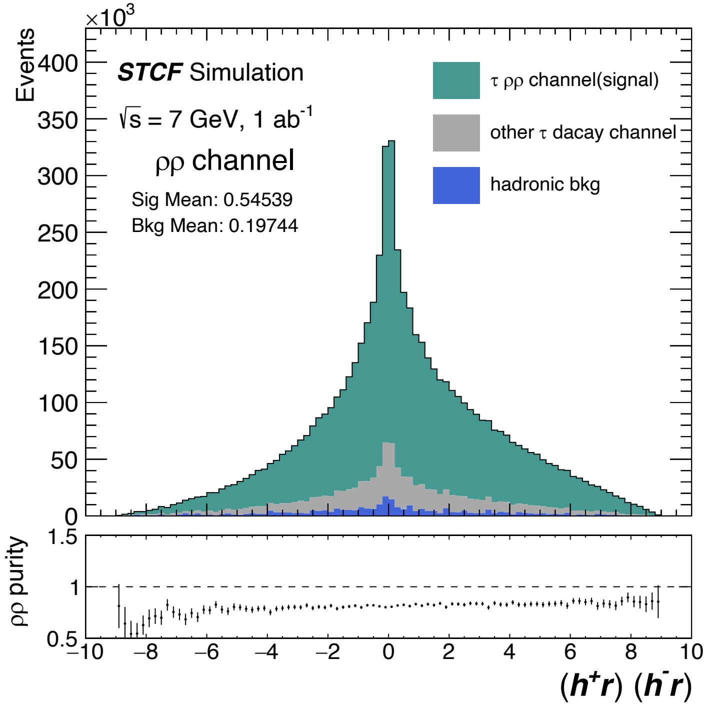
$$\rho = \frac{1}{4}\left[I \otimes I + \sum_i B_i^+ (\sigma_i \otimes I) + \sum_j B_j^- (I \otimes \sigma_j) + \sum_{i,j} C_{ij} (\sigma_i \otimes \sigma_j)\right]$$

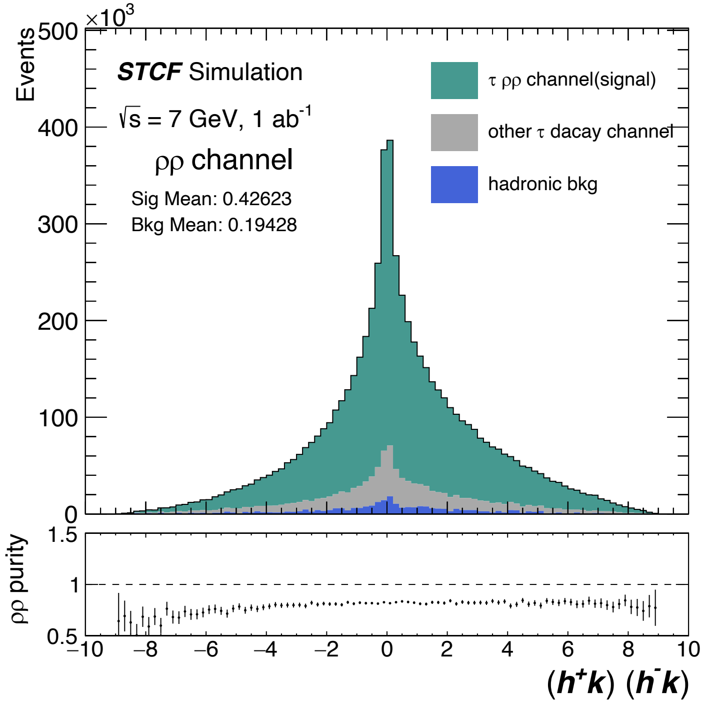
B_i^+ / B_j^- is spin polarization of $\tau^+\tau^-$, C_{ij} is spin correlation matrix

- Using moment method, $\boldsymbol{B}_i^{\pm} = 3\langle \boldsymbol{h}_i^{\pm} \rangle$, $\boldsymbol{C}_{ij}^{\pm} = 9\langle \boldsymbol{h}_i^{+} \boldsymbol{h}_j^{-} \rangle$



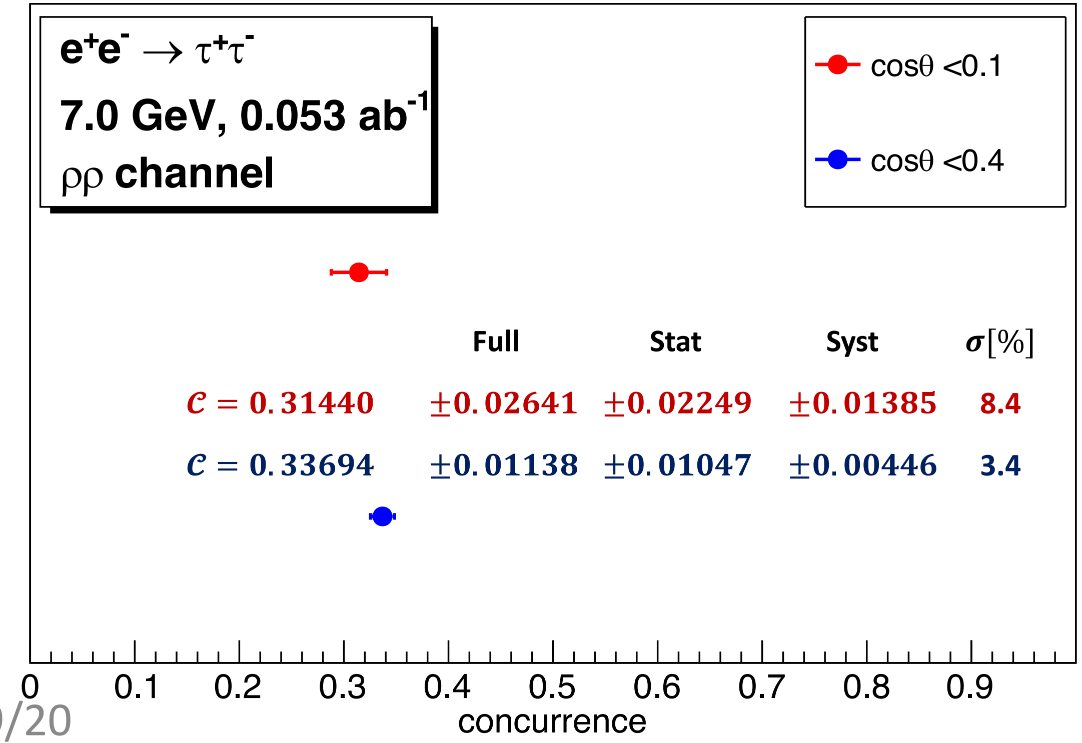


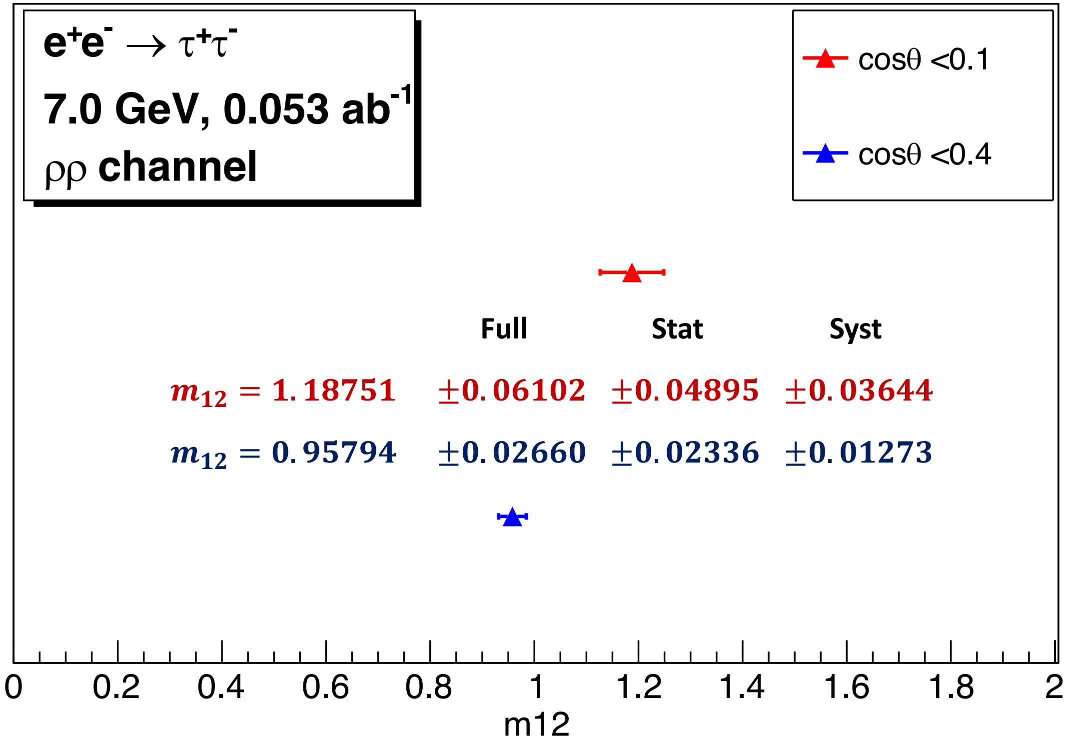




Uncertainties and Systematics

- A sample of 100 million MC events with ISR under **Oscar** full detector simulation
- Only considering $\rho\rho$ channel
- For systematics, we simulate the experimental resolution by randomly varying ("**smearing**") the four-vectors of the charged and neutral particles produced in the τ decays before applying kinematic cut and fit





Results and Prospect

- The prospective value for tau pair concurrence under the STCF luminosity :

$$\boldsymbol{\mathcal{C} = 0.31440 \pm 0.02249[stat.] \pm 0.01385[syst.]}$$

Promising for an witness (>5σ) of the entanglement

- For the **witness $\boldsymbol{m_{12}}$**

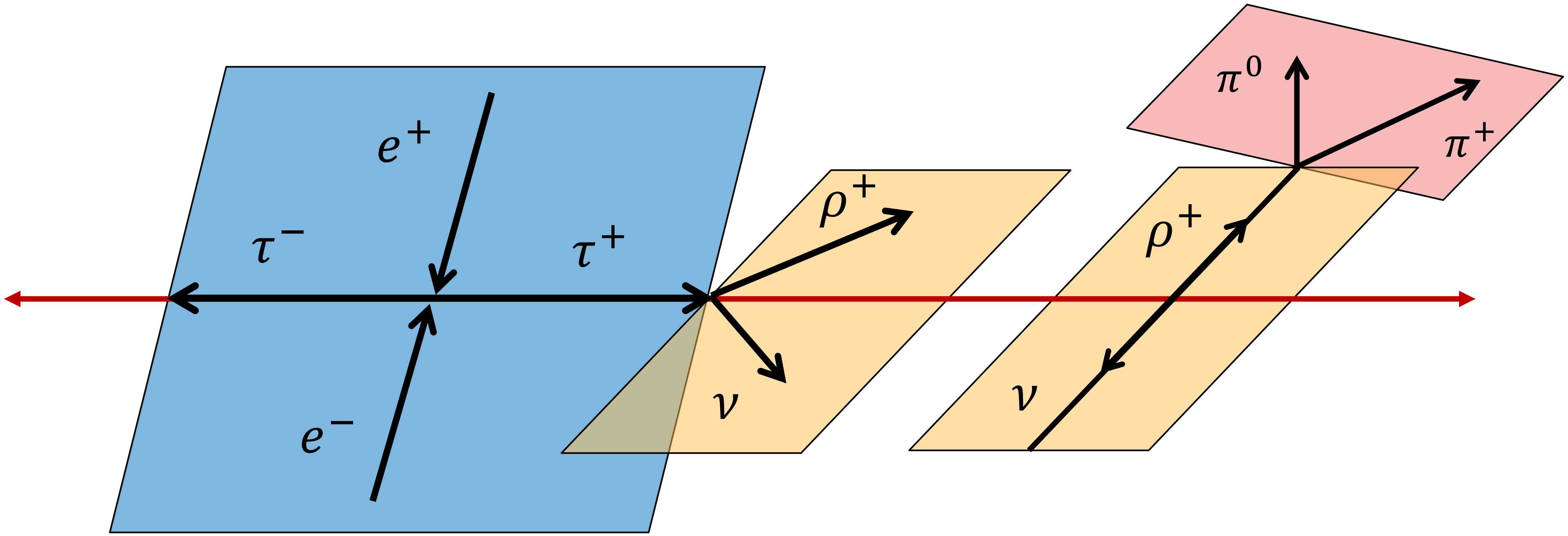
$$\mathbf{m_{12} = 1.18751 \pm 0.04895[\mathit{stat.}] \pm 0.03644[\mathit{syst.}]}$$

Promising for an witness (>3σ) of the violation of Bell's Inequality, to optimize $\mathbf{m_{12}}$ result, we need:

- Vertex fit algorithm for tau pair

BACKUP

Reconstruction

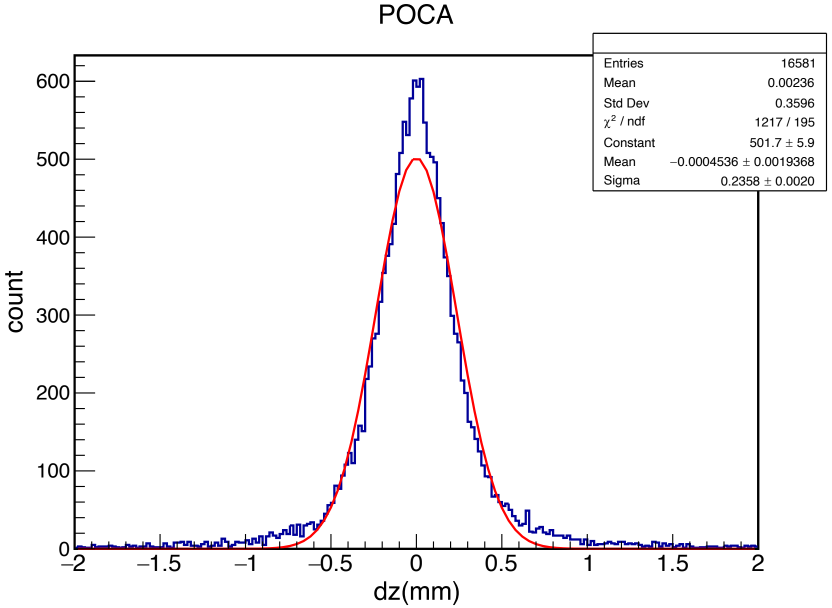


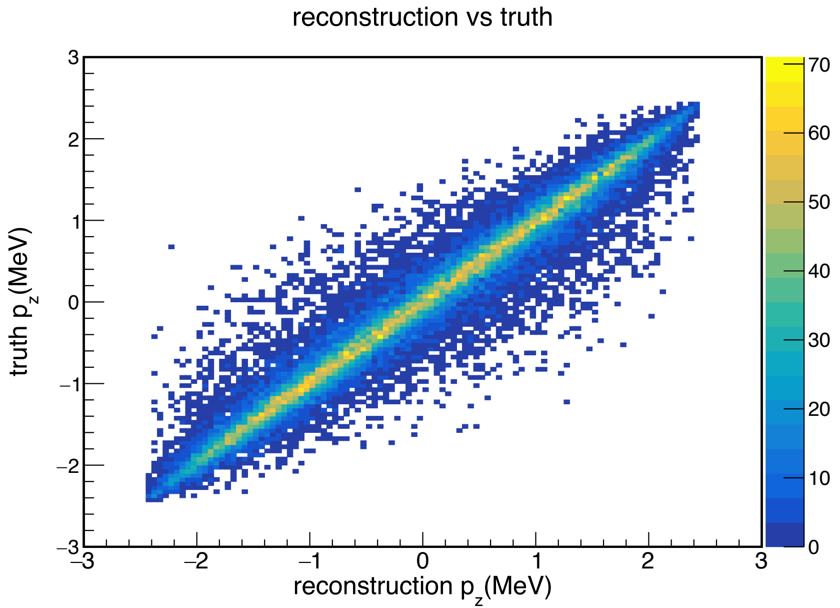
e^+e^-CM frame

τ^+rest frame

Reconstruction: P_τ

- ❑ P_τ reconstruction: solving a set of analytic equations
- ❑ Existing problem:
  - Because of two missing ν , the τ flight direction is calculated with a two-fold ambiguity
  - Vertex resolution/POCA point resolution($\sigma_z \sim 235um, \sigma_T \sim 80um$)
- ❑ Assuming higher resolution($\sigma_z' \sim 80um, \sigma_T' \sim 27um$)
  - Replace reconstruction vertex by smearing vertex
  - We can solve the two-fold ambiguity and reconstruction P_τ





truth $-0.1 < cos\theta < 0.1$

$\omega = \sin(\alpha/2)$	0.1	0.15	0.2	0.25	0.3	0.35	0.4
Events	8491	18639	27019	33057	36837	38816	39573
C11	-0.83305+/-0.03267	-0.83400+/-0.02298	-0.80628+/-0.01904	-0.79281+/-0.01659	-0.75765+/-0.01628	-0.73597+/-0.01594	-0.72234+/-0.01505
C22	0.79958+/-0.03082	0.82032+/-0.02088	0.86102+/-0.01733	0.90361+/-0.01634	0.92631+/-0.01517	0.95014+/-0.01503	0.95993+/-0.01427
C33	0.53357+/-0.03236	0.46969+/-0.02031	0.43612+/-0.01655	0.42276+/-0.01475	0.40934+/-0.01364	0.40405+/-0.01268	0.40145+/-0.01361
m_{12}	1.34316 +/-0.06928	1.37160 +/-0.04541	1.39159 +/-0.04246	1.44545 +/-0.03846	1.43234 +/-0.03698	1.44466 +/-0.03427	1.44351 +/-0.03307

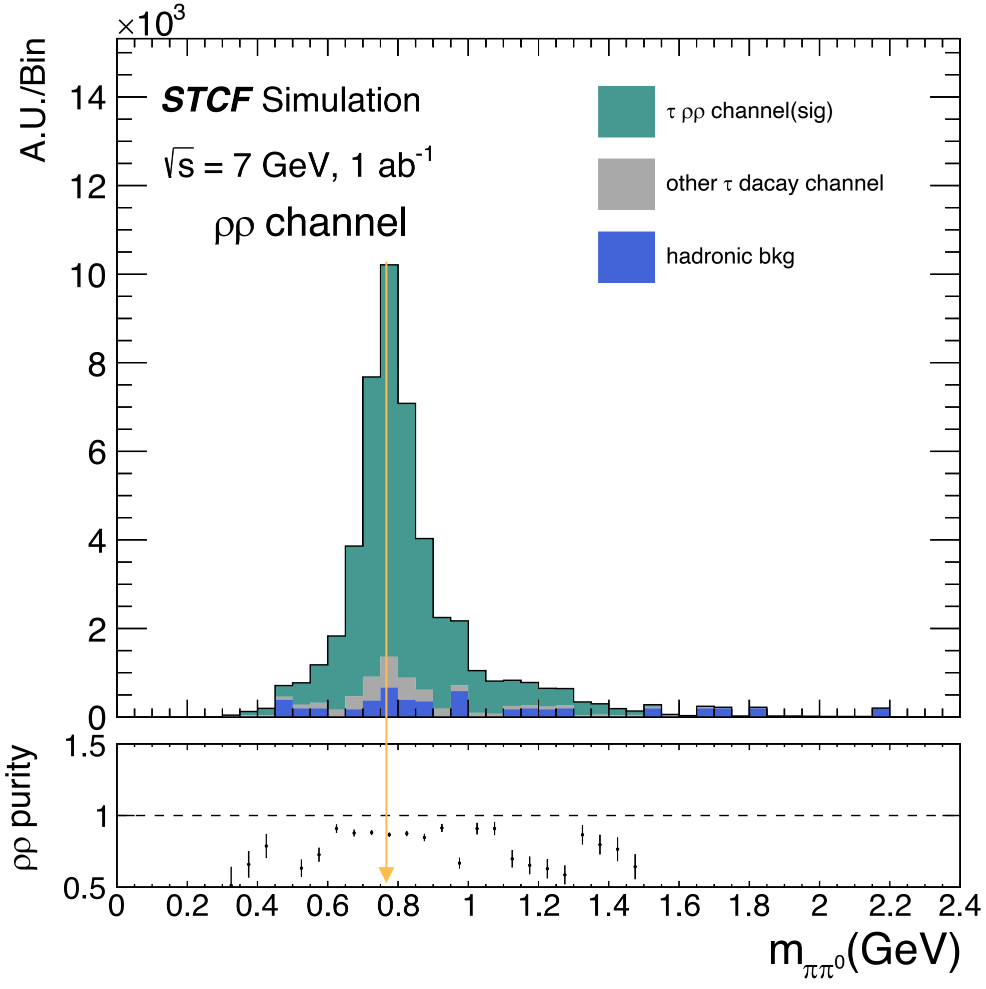
- Event sample: 1亿 （其中$\rho\rho \sim 600W$）
- $\rho\rho$: 34W （purity ~ 90%）
- 用于构建密度矩阵SDM:2W-3W （仅考虑统计误差）

  Concurrence: 0.32162 +/- 0.01778

  m12: 1.14328 +/- 0.04350
- 包含其他衰变道：

  Concurrence: 0.30744 +/- 0.01659

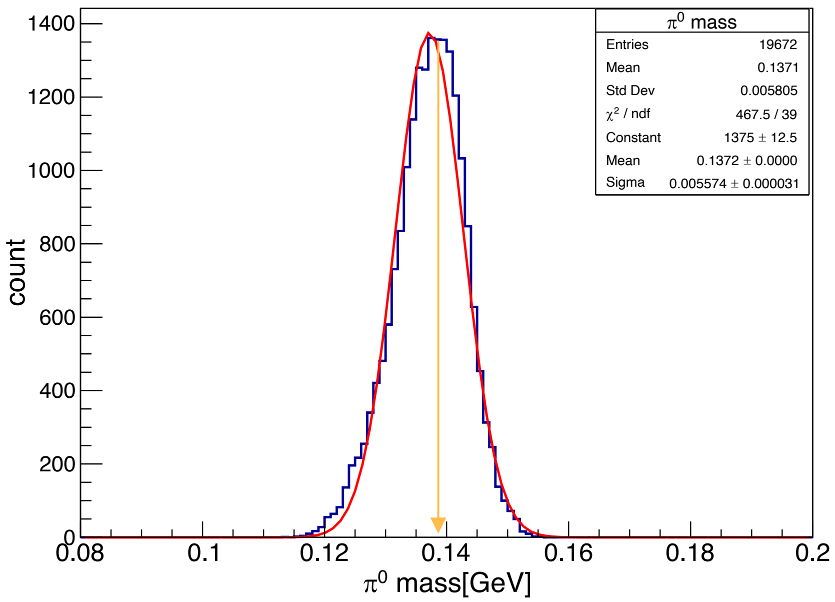
  m12: 1.04791 +/- 0.04141



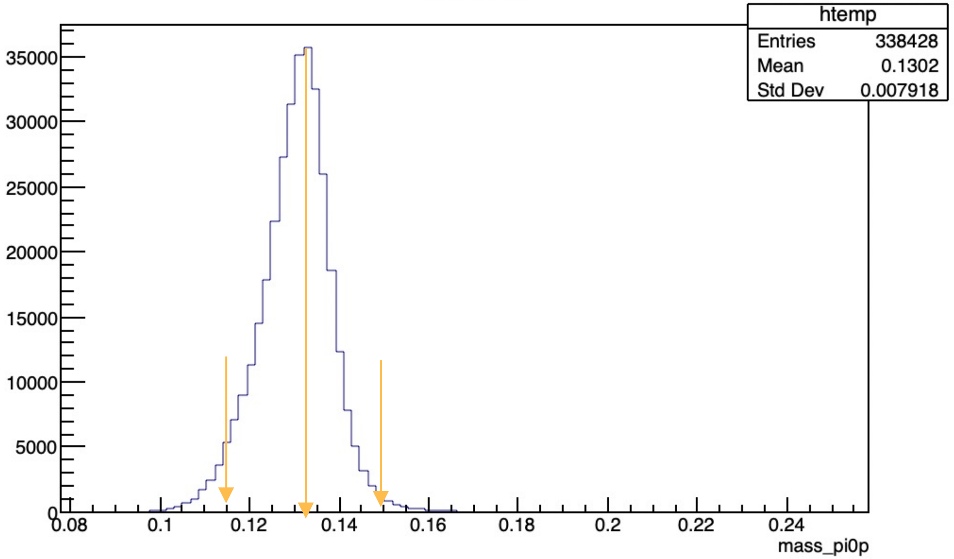
Tau的主要衰变道 1亿

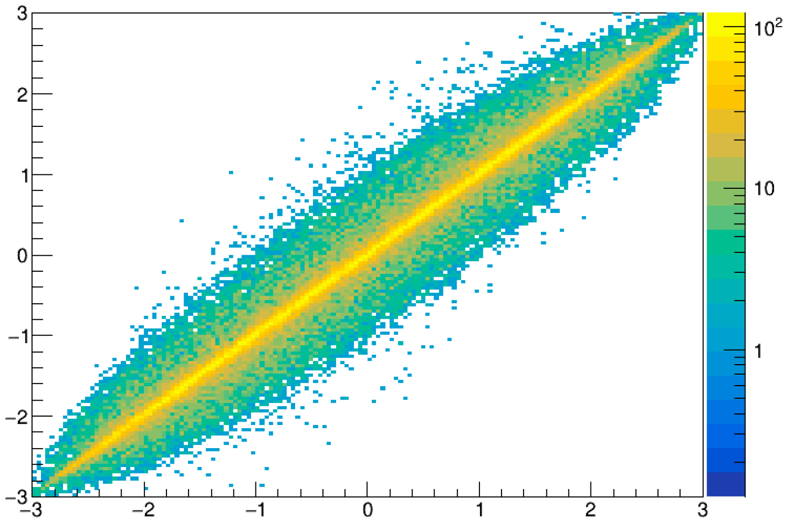
Tau的次要衰变道 3000W

强子本底 2000W

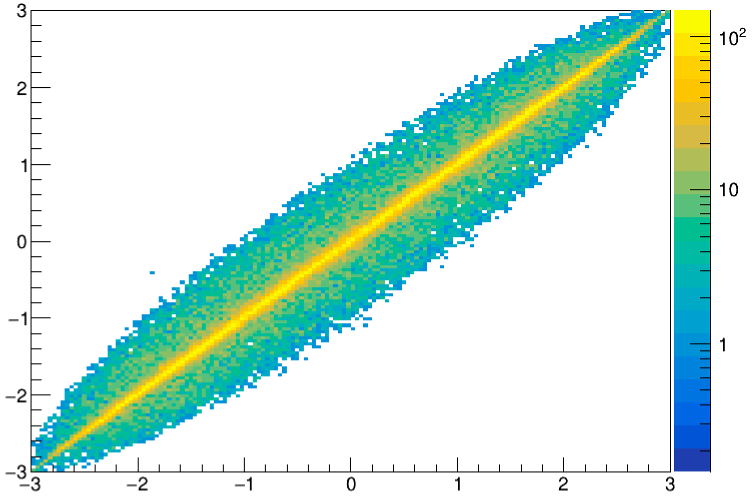


m_{π^0} from τ^+

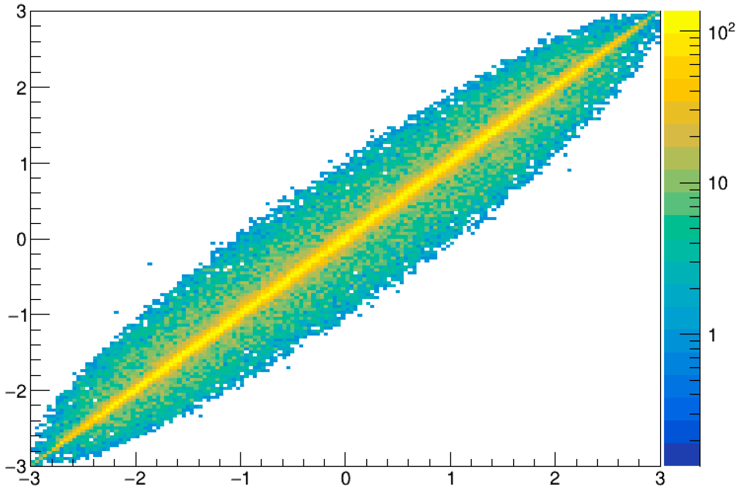




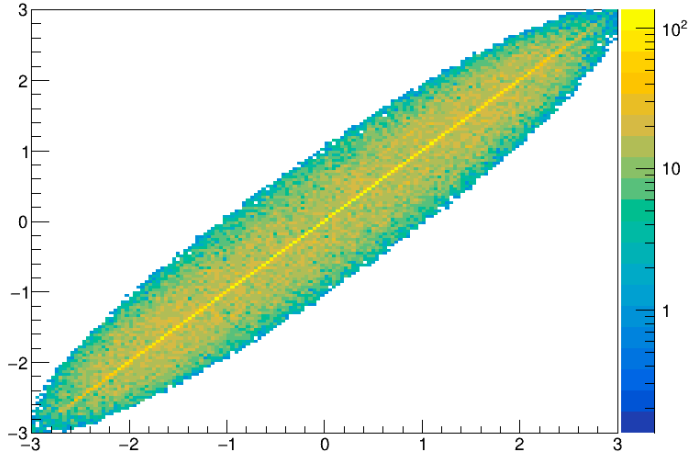
$P_z^{truth}\ vs\ P_z^{fit}$



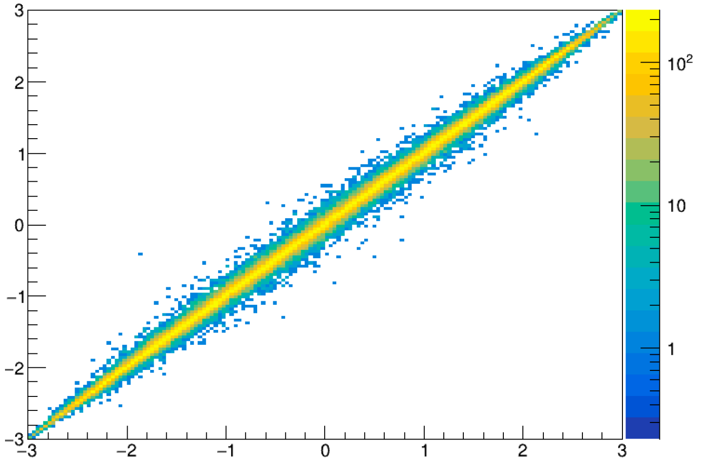
$P_z^{fit}\ vs\ P_z^{calculate1}$

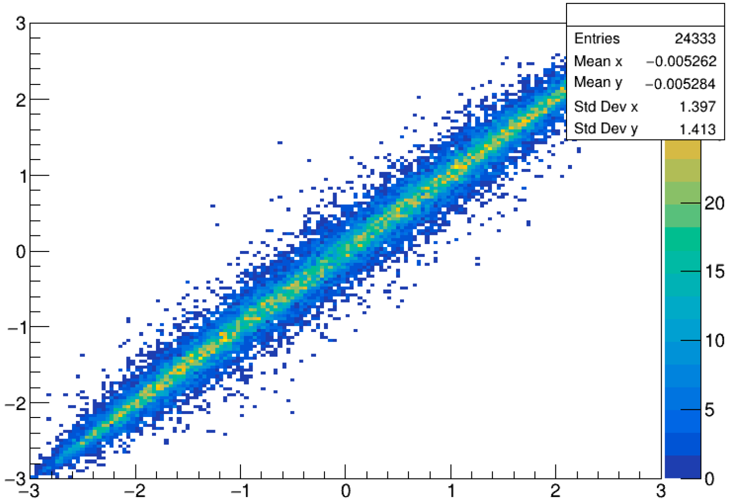


$P_z^{fit}\ vs\ P_z^{calculate2}$



$P_z^{calculate1} vs\ P_z^{calculate2}$





$P_z^{fit}\ vs\ P_z^{calculate}$(选取夹角更小的解)