Twist-4 FFs in Electron Positron Annihilation

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Yantai University

2025.08.10

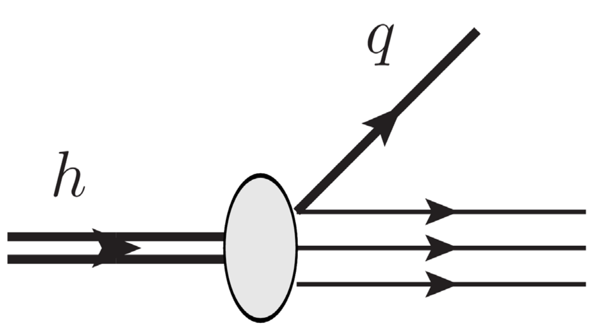
FFs and EEC @ Lanzhou

Outlines

1. Fragmentation functions (FFs)
2. Higher twist fragmentation functions (HTFFs)
3. HTFFs in annihilation
4. Calculations
5. Summary (to be continued…)

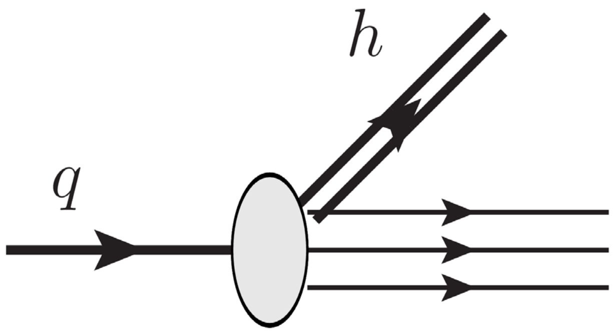
1. Fragmentation functions

- $f_1^{q/h}(x)$: Probability density



hadron / quark	U	L	T
U	f_1 density		$f_{1T}^{\perp}$ Sivers
L		g_{1L}	g_{1T}
T	$h_1^{\perp}$ Boer-Mulders	$h_{1L}^{\perp}$	h_{1T} Transversity $h_{1T}^{\perp}$

- $D_1^{h/q}(z)$: Probability density

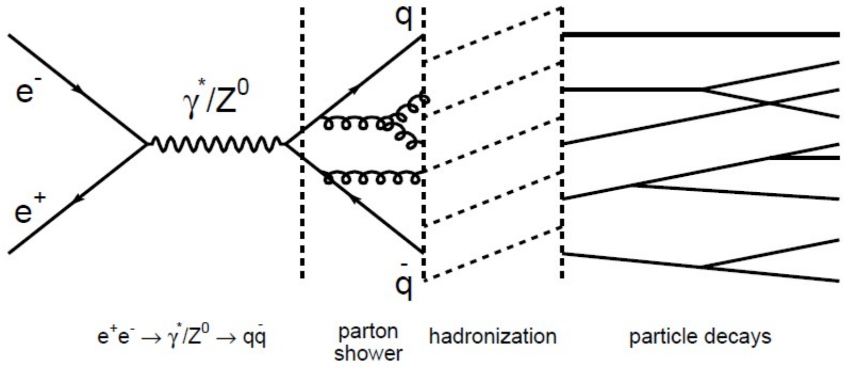


hadron / quark	U	L	T
U	D_1 density		$D_{1T}^{\perp}$
L		G_{1L} longitudinal	G_{1T}
T	$H_1^{\perp}$ Collins	$H_{1L}^{\perp}$	H_{1T} transverse $H_{1T}^{\perp}$

1. Fragmentation functions

- Hadronization

D_1: probability density

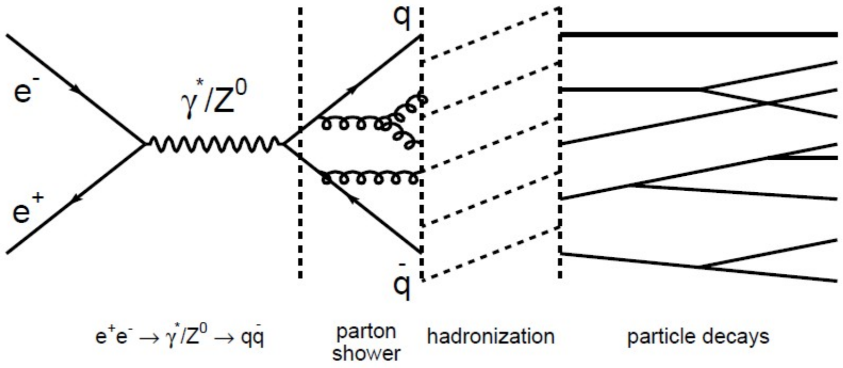


- ➢ LUND Model
- ➢ Webber Model
- ➢ QCM (Shandong)
- ➢ ……

1. Fragmentation functions

- Hadronization

D_1: probability density



- ➢ LUND Model
- ➢ Webber Model
- ➢ QCM (Shandong)
- ➢ ……

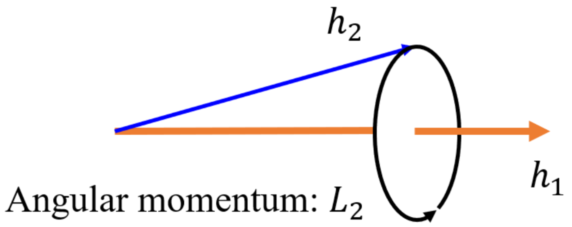
- How quark polarization transfers to hadrons ?

$D_{1T}^{\perp}$: unpolarized quark → transversely polarized hadron;

G_{1L}: longitudinally polarized quark → longitudinally polarized hadron;

$H_1^{\perp}$: transversely polarized quark → unpolarized hadron.

Orbital angular momentum?



1. Fragmentation functions

- Quark polarizations → jet handedness

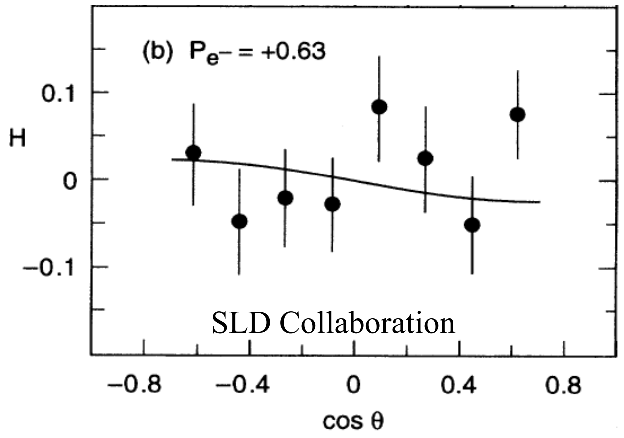
$$H = \frac{N_R(X>0) - N_L(X<0)}{N} = \alpha P \qquad \alpha = \frac{N_R^+ - N_L^+}{n^+} = \frac{N_L^- - N_R^-}{n^-}$$

M. Stratmann and W. Vogelsang, Phys. Lett. B 295 (1992) 277-282;

A.V. Efremov, L. Mankiewicz and N. A. Tornqvist, Phys. Lett. B 284 (1992) 394-400;

M. G. Ryskin, Phys. Lett. B 319 (1993) 346-347;

SLD Collaboration, Phys. Rev. Lett. 74. 1512.



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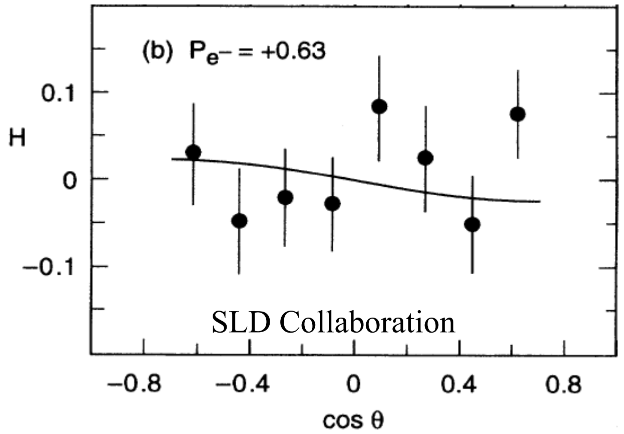

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DiFFs

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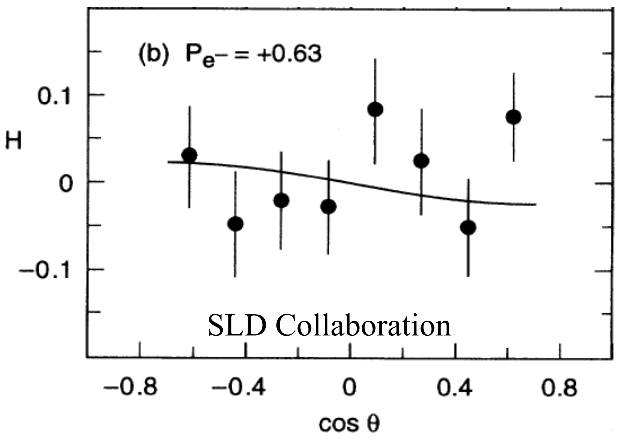


DiFFs

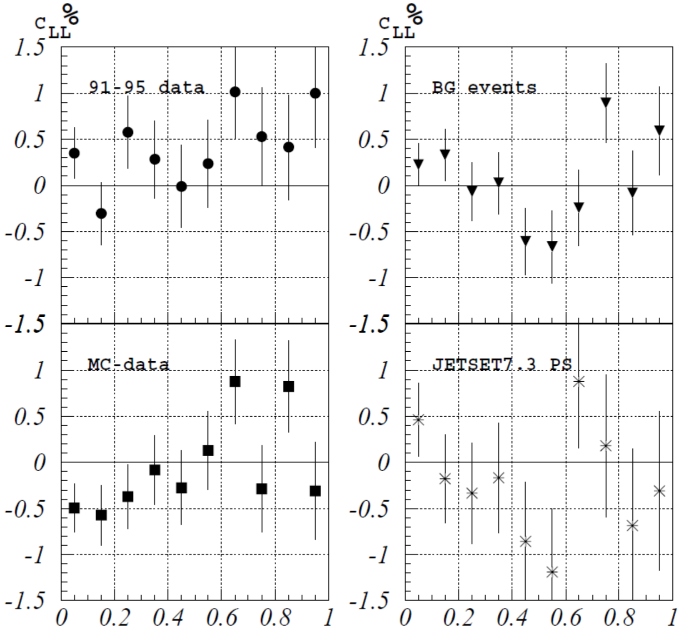
- Handedness correlations

Favoured Q: $C^{q,\bar{q}} = -H^q H^{\bar{q}}$ $\qquad C_Q^{q,\bar{q}} = -\left(\alpha_Q^q\right)^2$

Favoured Q + Disfavoured q:

$$C_h^{q,\bar{q}} = \frac{1}{4}\left[(\alpha_h+\beta_h)^2 P_Q^q P_Q^{\bar{q}} + (\alpha_h-\beta_h)^2 P_q^q P_q^{\bar{q}} + (\alpha_h^2-\beta_h^2)\left(P_Q^q P_q^{\bar{q}} + P_q^q P_Q^{\bar{q}}\right)\right]$$

A.V. Efremov and L.G. Tkatchev, Acta Physica Polonica B, Vol. 29 (1998), No. 5
W. Yang, Nucl. Phys. A 1022 (2022) 122440



2. Higher twist fragmentation functions

- Leading twist FFs

quark \ hadron	U	L	T
U	D_1 density		$D_{1T}^{\perp}$ −
L		G_{1L} − longitudinal	G_{1T} −
T	$H_1^{\perp}$ − Collins	$H_{1L}^{\perp}$ −	H_{1T} − transverse $H_{1T}^{\perp}$ −

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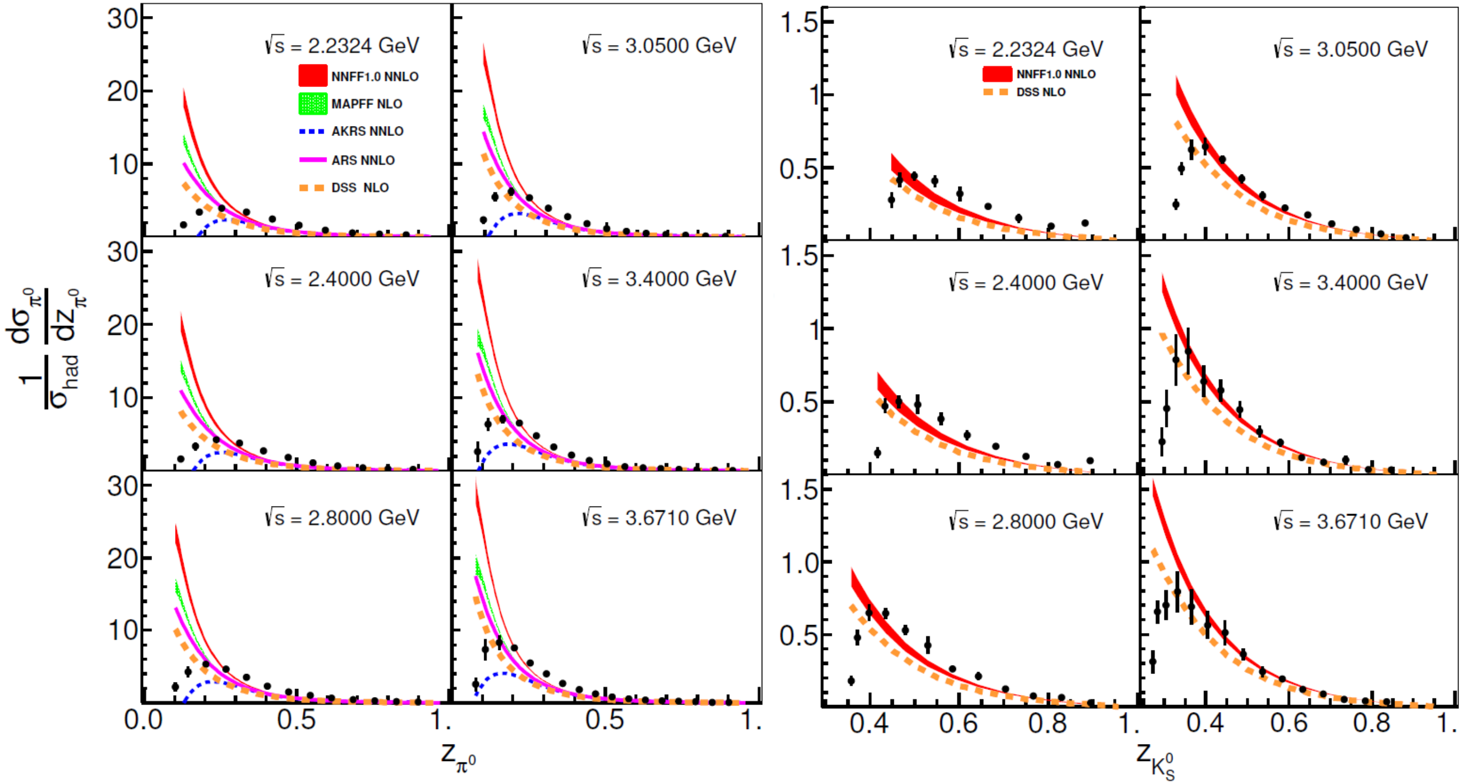

➢ Fitting experimental data, Leading twist FFs + higher twist FFs; $D_{t2} + D_{t4} = C$

2. Higher twist fragmentation functions

- Leading twist FFs

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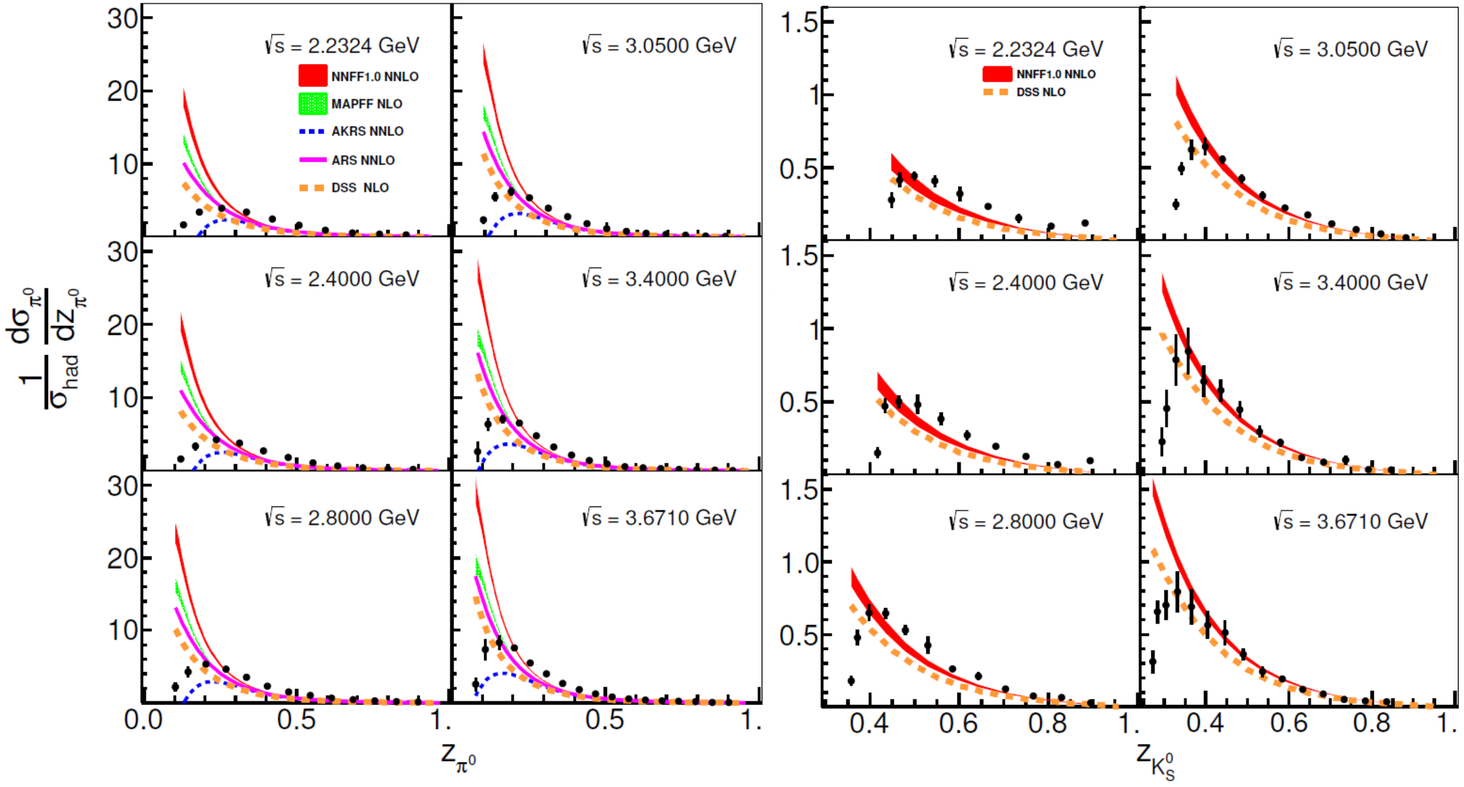


- ➢ Fitting experimental data, Leading twist FFs + higher twist FFs; $D_{t2} + D_{t4} = C$
- ➢ Universality of higher twist FFs, $D_{t2}^{e^+e^-} = D_{t2}^{SIDIS}$, $D_{t4}^{e^+e^-} \xrightarrow{?} D_{t4}^{SIDIS}$. **Factorization ?**

3. HTFFs in annihilation

- $e^+ + e^- \to h + X$

The differential cross section,

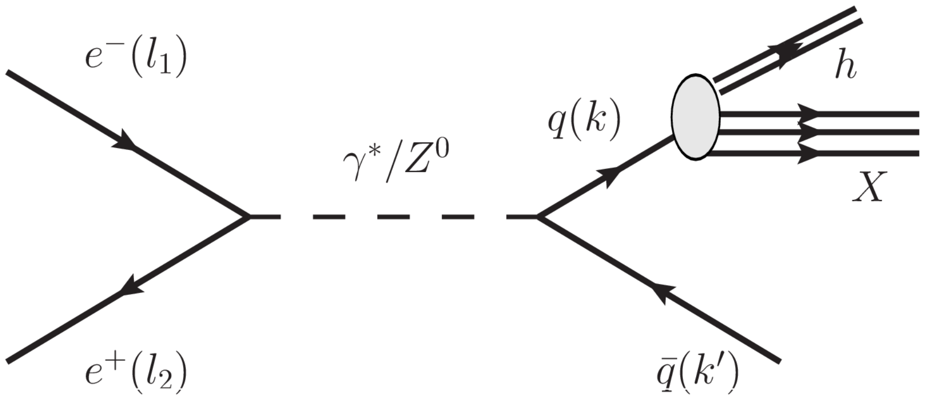
$$\frac{d\sigma}{dzd\cos\theta} = \frac{\pi\beta z\alpha^2}{2Q^4} L_{\mu\nu} W^{\mu\nu}$$

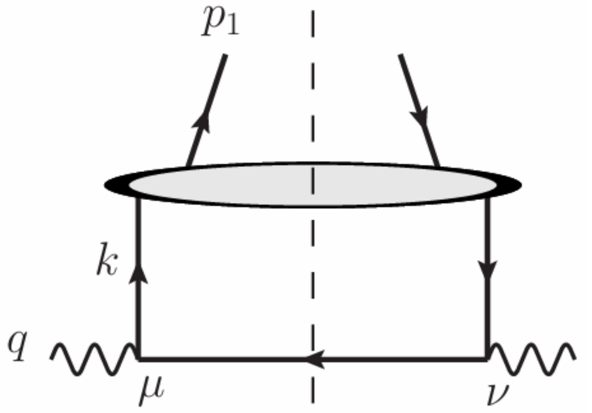
The leptonic tensor,

$$L_{\mu\nu}(l_1, l_2) = 2[l_{1\mu} l_{2\nu} + l_{1\nu} l_{2\mu} - g_{\mu\nu}(l_1 \cdot l_2)] + 2i\lambda_e \epsilon_{\mu\nu\rho\sigma} l_1^\rho l_2^\sigma$$

The hadronic tensor,

$$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Pi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(k,q)\right]$$



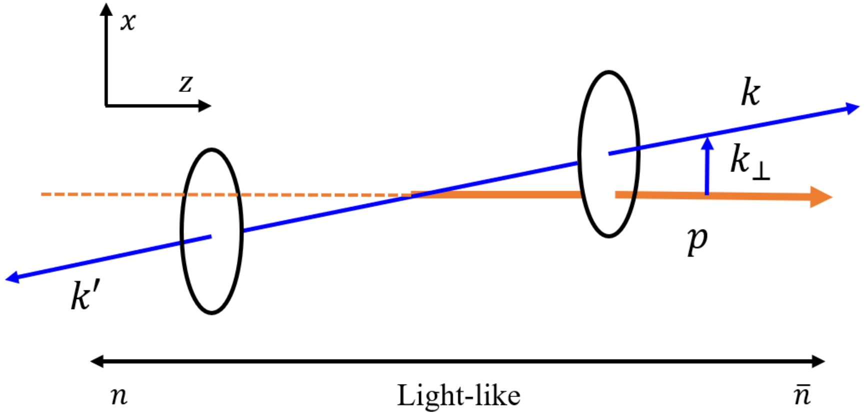


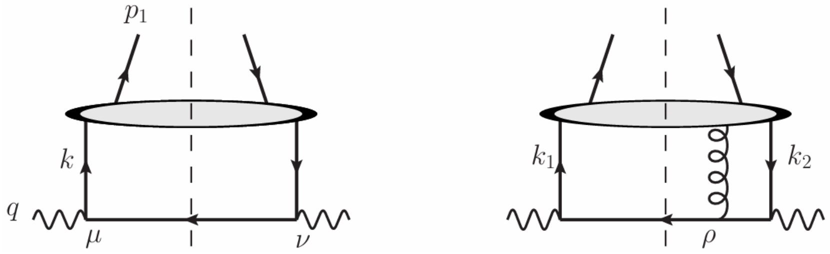
Handbag diagram

S. Y. Wei, Y. k. Song, and Z. t. Liang, Phys. Rev. D 89, 014024 (2014).
S. Y. Wei, K. b. Chen, Y. k. Song, and Z. t. Liang, Phys. Rev. D 91, 034015 (2015).

3. HTFFs in annihilation

- $e^+ + e^- \to h + X$





$$\hat{\Pi}^{(0)} \sim \langle 0|\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)|0\rangle \to \langle 0|\psi(0)\mathcal{L}(0,\infty)|hX\rangle\langle hX|\bar{\psi}(\xi)|0\rangle,$$

$$\hat{\Pi}_\rho^{(1)} \sim \langle 0|\psi(0)A_\rho(y)|hX\rangle\langle hX|\bar{\psi}(\xi)|0\rangle$$

$$\to \langle 0|\psi(0)iD_{\perp\rho}|hX\rangle\langle hX|\bar{\psi}(\xi)|0\rangle + \langle 0|\psi(0)A^+|hX\rangle\langle hX|\bar{\psi}(\xi)|0\rangle.$$

- The produced hadron moves along the z-direction.
- $p^\mu \to (p\cdot n)\,\bar{n}^\mu$.

$$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Pi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(k,q)\right]$$

1. light-cone: $\hat{H}_{\mu\nu}^{(0)}(k,q) \to \gamma\cdot n,$

   $\hat{\Pi}^{(0)} \to \gamma\cdot\bar{n},$ $\quad\gamma\cdot n$

   leading twist, higher twist

2. transverse: $\hat{H}_{\mu\nu}^{(0)}(k,q) \to k_{\perp\alpha} = \omega_\alpha^\beta k_\beta,$

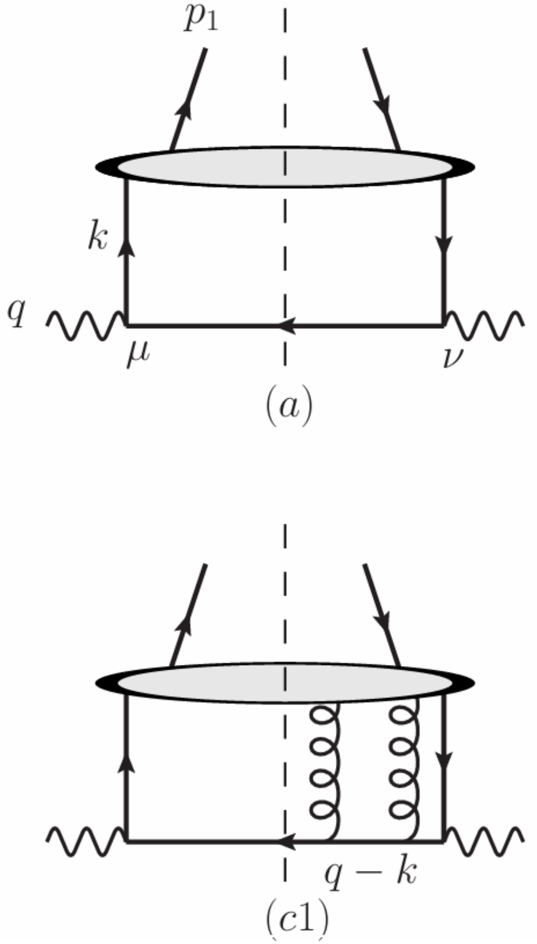
$$A_\alpha = \omega_\alpha^\beta A_\beta + A\cdot n\,\bar{n}_\alpha,$$

$$\omega_\alpha^\beta k_\beta + \omega_\alpha^\beta A_\beta \to i\partial_{\perp\alpha} + A_{\perp\alpha} \to iD_{\perp\alpha},$$

$$A\cdot n\,\bar{n}_\alpha + (A\cdot n\,\bar{n}_\alpha)^2 + \cdots \to \mathcal{L}(\xi,\infty),$$

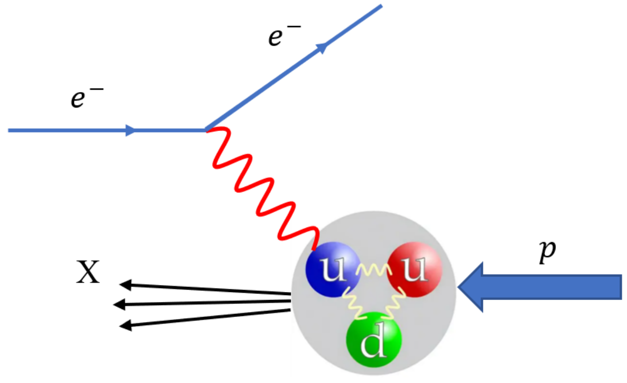
3. HTFFs in annihilation

- $e^+ + e^- \rightarrow h + X$



4. Calculations

- Deeply inelastic scattering (DIS)



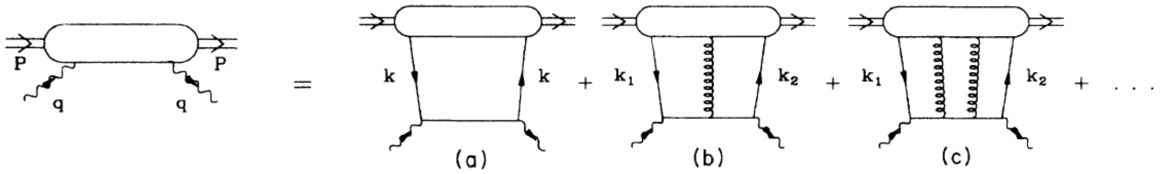
$$e^-(l,\lambda_e) + N(P,S) \to e^-(l') + X,$$

1. Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) \to \sum_n C_{12}^n(x)\,\mathcal{O}_n(0), \qquad x \to 0.$$

- K. G. Wilson, Phys. Rev. 179 (1969) 1499–1512 .
- R. A. Brandt, G. Preparata, Nucl. Phys. B 27 (1971) 541–567.
- N. H. Christ, B. Hasslacher, A. H. Mueller, Phys. Rev. D 6 (1972) 3543.

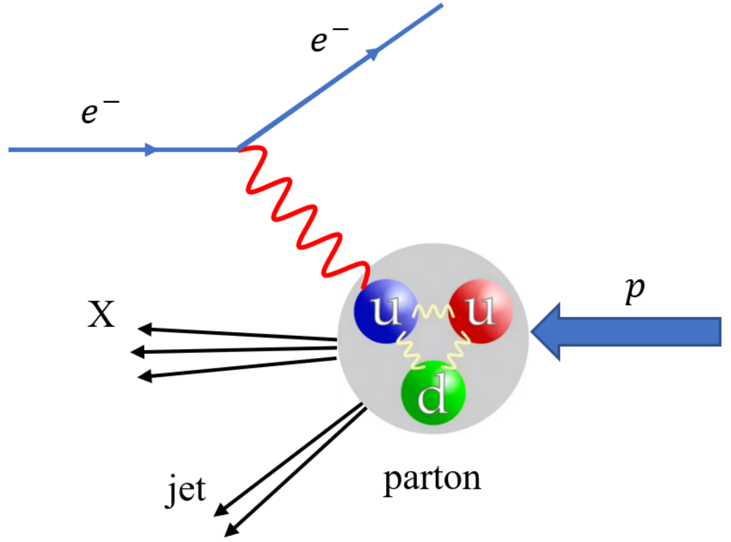
2. Feynman diagram approach (Collinear Expansion)

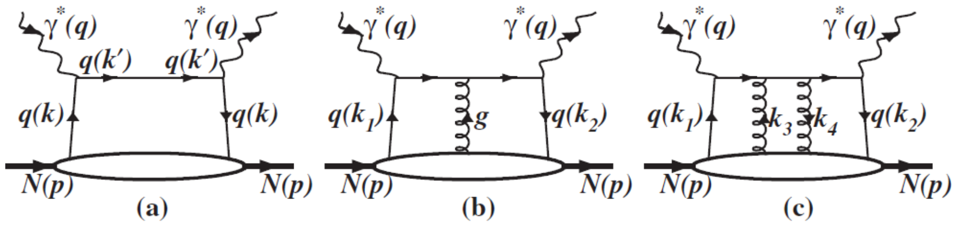


- R. K. Ellis, W. Furmanski and R. Petronzio, Nucl. Phys. B 207, 1 (1982); Nucl. Phys. B 212, 29 (1983).
- J. W. Qiu and G. F. Sterman, Nucl. Phys. B 353, 105 (1991); Nucl. Phys. B 353, 137 (1991).
- J. W. Qiu, Phys. Rev. D 42, 30 (1990).

4. Calculations

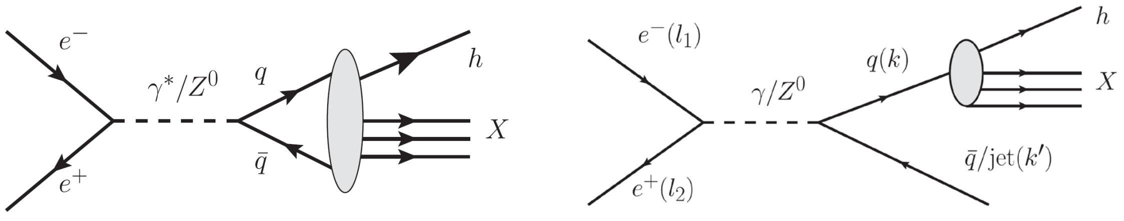
- Semi-inclusive DIS

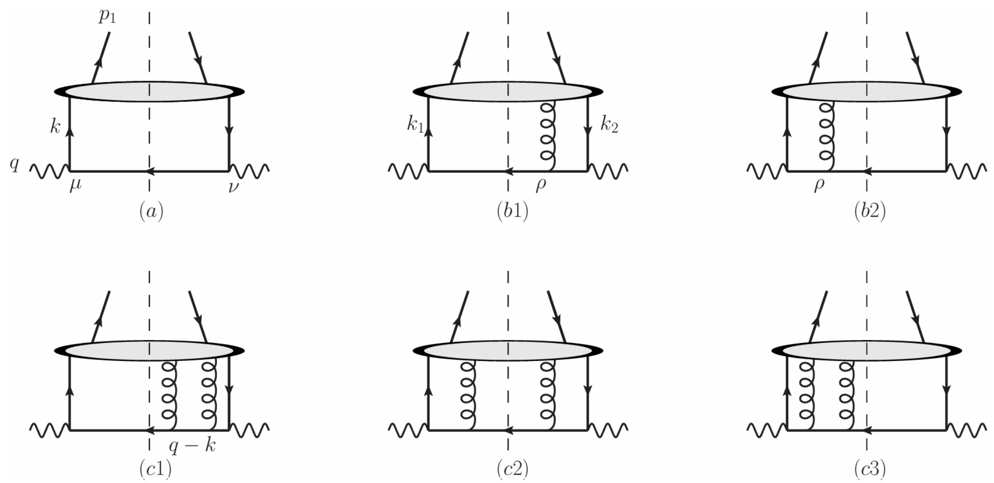




Z. T. Liang and X. N. Wang, Phys. Rev. D 75, 094002 (2007)

- Electron positron annihilation





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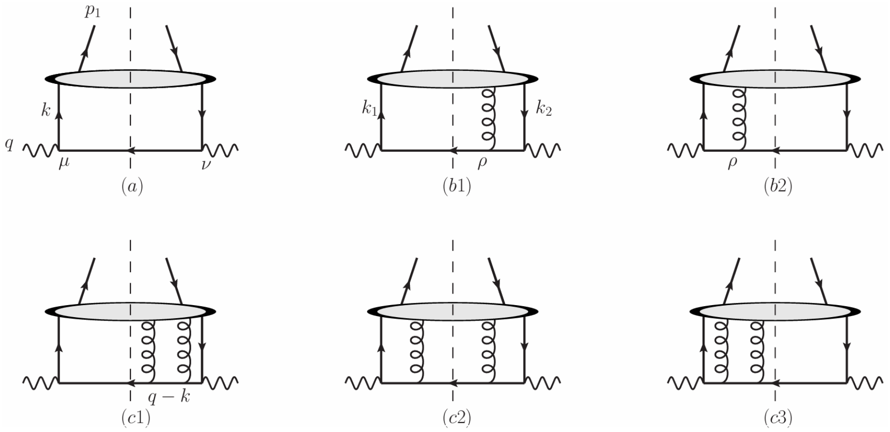
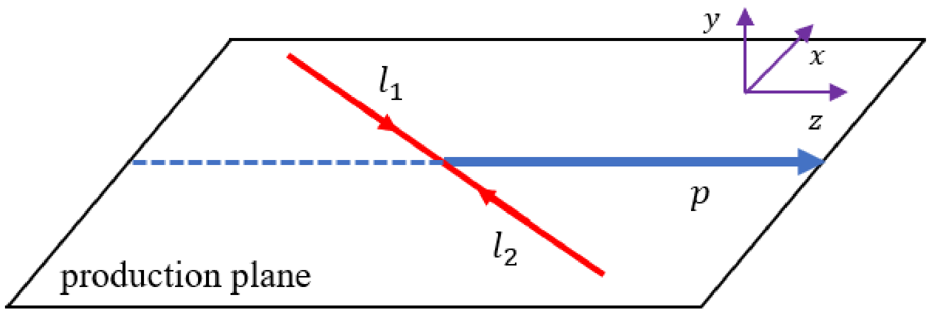
4. Calculations

- Calculations in the parton model





- Light-cone coordinates

$$\bar{n} = \left(1,0,\vec{0}_\perp\right), \qquad n^\mu = \left(0,1,\vec{0}_\perp\right)$$

- Parton model

$$\frac{d\sigma}{dz d\cos\theta} = \frac{\pi\beta z\alpha^2}{2Q^4} L_{\mu\nu} W^{\mu\nu}$$

$$W_{\mu\nu} = \sum_{j,c} W_{\mu\nu}^{(j,c)} = W_{\mu\nu}^{(0)} + W_{\mu\nu}^{(1,L)} + W_{\mu\nu}^{(1,R)} + \ldots,$$

$$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Pi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(k,q)\right]$$

$$\hat{\Pi}^{(0)}(k,p,S) = \frac{1}{2\pi}\int d^4\xi e^{-ik\xi}\sum_X \langle 0|\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)|0\rangle,$$

$$\hat{H}_{\mu\nu}^{(0)}(k,q) = (2\pi)\Gamma_\mu^q(\not{q}-\not{k})\Gamma_\nu^q \delta^+\left[(q-k)^2\right].$$

$$W_{\mu\nu}^{(1,c)} = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Pi}_\rho^{(1,c)}\hat{H}_{\mu\nu}^{(1,c)\rho}\right],$$

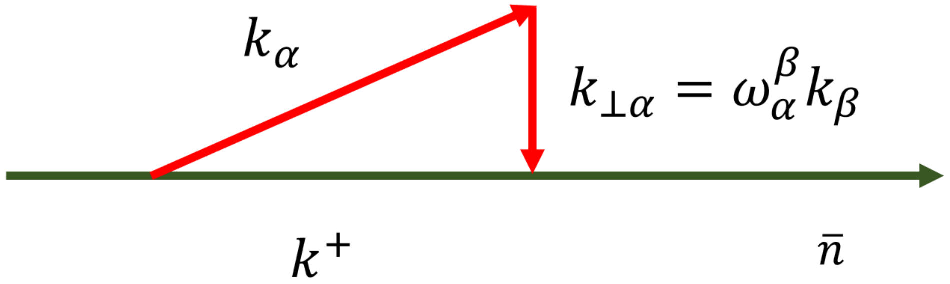
$$W_{\mu\nu}^{(2,c)} = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Pi}_{\rho\sigma}^{(2,c)}\hat{H}_{\mu\nu}^{(2,c)\rho\sigma}\right].$$

4. Calculations

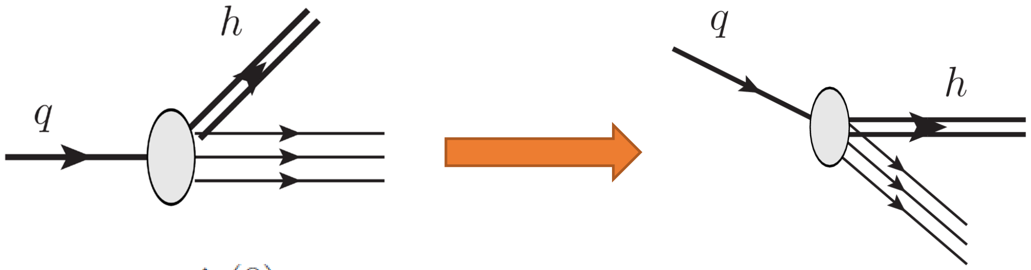
- Decompose the gluon field

$$A_\alpha(y) = A^+(y)\bar{n}_\alpha + \omega_\alpha^{\ \beta} A_\beta(y).$$

Projection operator: $\omega_\alpha^{\ \beta} \equiv g_\alpha^{\ \beta} - \bar{n}_\alpha n^\beta$



- Expansion of the hard part



$$\hat{H}^{(0)}_{\mu\nu} = \hat{H}^{(0)}_{\mu\nu}(z) + \omega_\alpha^{\ \beta} k_\beta \frac{\partial \hat{H}^{(0)}_{\mu\nu}(z)}{\partial k_\alpha} + \omega_\alpha^{\ \beta} k_\beta \omega_\rho^{\ \sigma} k_\sigma \frac{1}{2} \frac{\partial^2 \hat{H}^{(0)}_{\mu\nu}(z)}{\partial k_\alpha \partial k_\rho} + \cdots,$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu} = \hat{H}^{(1,L)\rho}_{\mu\nu}(z_1, z_2) + \omega_\alpha^{\ \beta} k_{1\beta} \frac{\partial \hat{H}^{(1,L)\rho}_{\mu\nu}(z_1, z_2)}{\partial k_{1\alpha}} + \omega_\alpha^{\ \beta} k_{2\beta} \frac{\partial \hat{H}^{(1,L)\rho}_{\mu\nu}(z_1, z_2)}{\partial k_{2\alpha}} + \cdots,$$

…

4. Calculations

- Apply the Ward identity

$\hat{H}_{\mu\nu}^{(0)}$:

$$\hat{H}_{\mu\nu}^{(0)}(k,q) = (2\pi)\Gamma_\mu^q(\not{q}-\not{k})\Gamma_\nu^q\delta^+\left[(q-k)^2\right]$$

$$\frac{\partial \hat{H}_{\mu\nu}^{(0)}(z)}{\partial k_\alpha} = -\hat{H}_{\mu\nu}^{(1,L)\alpha}(z,z) - \hat{H}_{\mu\nu}^{(1,R)\alpha}(z,z),$$

$\hat{H}_{\mu\nu}^{(1,L)\rho}$:

$$\hat{H}_{\mu\nu}^{(1,L)\rho} =(2\pi)\Gamma_\mu^q(\not{q}-\not{k}_1)\gamma^\rho\frac{\not{k}_2-\not{q}}{(k_2-q)^2-i\epsilon}\Gamma_\nu^q\delta^+\left[(q-k_1)^2\right]$$

$$p_\alpha\hat{H}_{\mu\nu}^{(1,L)\alpha}(z_1,z_2) = \frac{z_1 z_2}{z_1-z_2+i\epsilon}H_{\mu\nu}^{(0)}(z_1).$$

⋮ ⋮

4. Calculations

- Add all the terms with the same hard part together

$$W_{\mu\nu} = W_{\mu\nu}^{(0)} + W_{\mu\nu}^{(1,c)} + W_{\mu\nu}^{(2,c)} + \cdots$$

$$\sim Tr\left[\hat{H}_{\mu\nu}^{(0)}(k)\hat{\Pi}^{(0)}\right] + Tr\left[\hat{H}_{\mu\nu}^{(1,c)}(k_1,k_2)\hat{\Pi}^{1,c}\right] + \cdots$$

$$= Tr\left[\hat{H}_{\mu\nu}^{(0)}\hat{\Pi}^{(0)} + \frac{\partial \hat{H}_{\mu\nu}^{(0)}}{\partial k_\alpha}k_{\perp\alpha}\hat{\Pi}^{(0)} + \frac{1}{2}\frac{\partial^2 \hat{H}_{\mu\nu}^{(0)}}{\partial k_\alpha \partial k_\beta}k_{\perp\alpha}k_{\perp\beta}\hat{\Pi}^{(0)}\right] + \cdots$$

$$+Tr\left[\hat{H}_{\mu\nu}^{(1,c)}\hat{\Pi}^{(1,c)} + \frac{\partial \hat{H}_{\mu\nu}^{(1,c)}}{\partial k_{1\alpha}}k_{\perp 1\alpha}\hat{\Pi}^{(1,c)} + \frac{\partial^2 \hat{H}_{\mu\nu}^{1,c}}{\partial k_{2\beta}}k_{\perp 2\beta}\hat{\Pi}^{(1,c)}\right] + \cdots$$

$$\sim Tr\left[\left(\hat{\boldsymbol{H}}_{\boldsymbol{\mu\nu}}^{(\boldsymbol{0})}\hat{\boldsymbol{\Pi}}^{(\boldsymbol{0})}\right) + \hat{H}_{\mu\nu}^{(1,c)}\hat{\Pi}^{(1)}(i\partial_{\perp\alpha}) + \frac{1}{2}\frac{\partial^2 \hat{H}_{\mu\nu}^{(0)}}{\partial k_\alpha \partial k_\beta}k_{\perp\alpha}k_{\perp\beta}\hat{\Pi}^{(0)}\right] + \cdots$$

$$+Tr\left[\left(\hat{\boldsymbol{H}}_{\boldsymbol{\mu\nu}}^{(\boldsymbol{0})}\hat{\boldsymbol{\Pi}}^{(\boldsymbol{1,c})}(\boldsymbol{g}\boldsymbol{A}_{\boldsymbol{\alpha}}^{+})\right) + \hat{H}_{\mu\nu}^{(1,c)}\hat{\Pi}^{(1,c)}(gA_{\perp\alpha}) + \frac{\partial \hat{H}_{\mu\nu}^{(1,c)}}{\partial k_{1\alpha}}k_{\perp 1\alpha}\hat{\Pi}^{(1,c)} + \frac{\partial^2 \hat{H}_{\mu\nu}^{1,c}}{\partial k_{2\beta}}k_{\perp 2\beta}\hat{\Pi}^{(1,c)}\right] + \cdots$$

$$\frac{\partial \hat{H}_{\mu\nu}^{(0)}(z)}{\partial k_\alpha} = -\hat{H}_{\mu\nu}^{(1,L)\alpha}(z,z) - \hat{H}_{\mu\nu}^{(1,R)\alpha}(z,z),$$

$$p_\alpha \hat{H}_{\mu\nu}^{(1,L)\alpha}(z_1,z_2) = \frac{z_1 z_2}{z_1 - z_2 + i\epsilon}H_{\mu\nu}^{(0)}(z_1).$$

$$k_{\perp\alpha}\hat{\Pi}^{(0)} \to \hat{\Pi}^{(1)}(i\partial_{\perp\alpha})$$

$$\hat{\Pi}^{(1,c)} = \hat{\boldsymbol{\Pi}}^{(\boldsymbol{1,c})}(\boldsymbol{g}\boldsymbol{A}_{\boldsymbol{\alpha}}^{+}) + \hat{\Pi}^{(1,c)}(gA_{\perp\alpha})$$

4. Calculations

- Add all the terms with the same hard part together

$$W_{\mu\nu} = W_{\mu\nu}^{(0)} + W_{\mu\nu}^{(1,c)} + W_{\mu\nu}^{(2,c)} + \cdots$$

$$\sim Tr\left[\left(\boldsymbol{\widehat{H}_{\mu\nu}^{(0)}\widehat{\Pi}^{(0)}}\right) + \widehat{H}_{\mu\nu}^{(1,c)}\widehat{\Pi}^{(1)}(i\partial_{\perp\alpha}) + \frac{1}{2}\frac{\partial^2\widehat{H}_{\mu\nu}^{(0)}}{\partial k_\alpha \partial k_\beta} k_{\perp\alpha}k_{\perp\beta}\widehat{\Pi}^{(0)}\right] + \cdots$$

$$+Tr\left[\left(\boldsymbol{\widehat{H}_{\mu\nu}^{(0)}\widehat{\Pi}^{(1,c)}(gA_\alpha^+)}\right) + \widehat{H}_{\mu\nu}^{(1,c)}\widehat{\Pi}^{(1,c)}(gA_{\perp\alpha}) + \frac{\partial\widehat{H}_{\mu\nu}^{(1,c)}}{\partial k_{1\alpha}} k_{\perp 1\alpha}\widehat{\Pi}^{(1,c)} + \frac{\partial^2\widehat{H}_{\mu\nu}^{1,c}}{\partial k_{2\beta}} k_{\perp 2\beta}\widehat{\Pi}^{(1,c)}\right] + \cdots$$

$$\widehat{\Pi}^{(0)} = p^+ \int \frac{d\xi^-}{2\pi} e^{-k^+\xi^-} \sum_X \langle 0|\boldsymbol{\mathcal{L}}^\dagger(\boldsymbol{0},\infty)\psi(0)|hX\rangle \langle hX|\bar{\psi}(\xi^-)\boldsymbol{\mathcal{L}}(\boldsymbol{\xi}^-,\infty)|0\rangle,$$

$$\widehat{\Pi}_\alpha^{(1)} = p^+ \int \frac{d\xi^-}{2\pi} e^{-k^+\xi^-} \sum_X \langle 0|\boldsymbol{\mathcal{L}}^\dagger(\boldsymbol{0},\infty)D_{\perp\alpha}(0)\psi(0)|hX\rangle \langle hX|\bar{\psi}(\xi^-)\boldsymbol{\mathcal{L}}(\boldsymbol{\xi}^-,\infty)|0\rangle.$$

Gauge link: $\mathcal{L}(\xi^-,\infty) = \mathcal{P}\, e^{ig\int_{\xi^-}^{\infty} d\eta^-\, A^+(\eta^-)}$, and covariant derivative: $D_{\perp\alpha} = \partial_{\perp\alpha} + igA_{\perp\alpha}$.

4. Calculations

- Hadronic tensors

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\hat{\Xi}^{(0)}],$$

$$\tilde{W}^{(1,L)}_{\mu\nu} = -\frac{1}{4(p\cdot q)}\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\hat{\Xi}^{(1)}_{\rho}],$$

$$\tilde{W}^{(2,M)}_{\mu\nu} = \frac{1}{4(p\cdot q)^2}\mathrm{Tr}[\hat{h}^{(2)\rho\sigma}_{\mu\nu}\hat{\Xi}^{(2,M)}_{\rho\sigma}],$$

$$\tilde{W}^{(2,L)}_{\mu\nu} = \frac{1}{4(p\cdot q)^2}\mathrm{Tr}[\hat{N}^{(2)\rho\sigma}_{\mu\nu}\hat{\Xi}^{(2)}_{\rho\sigma} + \hat{h}^{(1)\rho}_{\mu\nu}\hat{\Xi}^{(2\prime)}_{\rho}],$$

- Hard parts

$$\hat{h}^{(0)}_{\mu\nu} = \frac{1}{p^+}\gamma_\mu \not{n}\gamma_\nu,$$

$$\hat{h}^{(1)\rho}_{\mu\nu} = \gamma_\mu \not{n}\gamma^\rho \not{\bar{n}}\gamma_\nu,$$

$$\hat{N}^{(2)\rho\sigma}_{\mu\nu} = q^-\gamma_\mu\gamma^\rho\not{n}\gamma^\sigma\gamma_\nu,$$

$$\hat{h}^{(2)\rho\sigma}_{\mu\nu} = \frac{p^+}{2}\gamma_\mu\not{\bar{n}}\gamma^\rho\not{n}\gamma^\sigma\not{\bar{n}}\gamma_\nu.$$

S. y. Wei, Y. k. Song and Z. t. Liang, Phys. Rev. D 89, no. 1, 014024 (2014)

- Gauge invariant correlators

$$\hat{\Xi}^{(0)} = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ip^+\xi^-/z}\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle \times \langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle,$$

$$\hat{\Xi}^{(1)}_{\rho} = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ip^+\xi^-/z}\langle 0|\mathcal{L}^\dagger(0,\infty)D_\rho(0)\psi(0)|hX\rangle \times \langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle,$$

$$\hat{\Xi}^{(2M)}_{\rho\sigma} = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ip^+\xi^-/z}\langle 0|\mathcal{L}^\dagger(0,\infty)D_\rho(0)\psi(0)|hX\rangle \times \langle hX|\bar{\psi}(\xi^-)D_\sigma(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle,$$

$$\hat{\Xi}^{(2\prime)}_{\rho} = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ip^+\xi^-/z} p^\sigma\langle 0|\mathcal{L}^\dagger(0,\infty)D_\rho(0)D_\sigma(0) \times \psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle,$$

$$\hat{\Xi}^{(2)}_{\rho\sigma} = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ip^+\xi^-/z}\langle 0|\mathcal{L}^\dagger(\eta^-,\infty)D_\rho(\eta^-)D_\sigma(\eta^-) \times \mathcal{L}^\dagger(0,\eta^-)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle.$$

4. Collinear expansion

- Deeply inelastic scattering

1. inclusive

$$e^- + N \to e^- + X$$

2. semi-inclusive

➢ Current region jet

$$e^- + N \to e^- + jet + X$$

➢ Target region hadron

$$e^- + N \to e^- + h + X$$

- Electron positron annihilation

1. inclusive

$$e^- + e^+ \to h + X$$

2. semi-inclusive

➢ Back to back hadron

$$e^- + e^+ \to h + jet + X$$

➢ Back to back dihadron

$$e^- + e^+ \to h_1 h_2 + jet + X$$

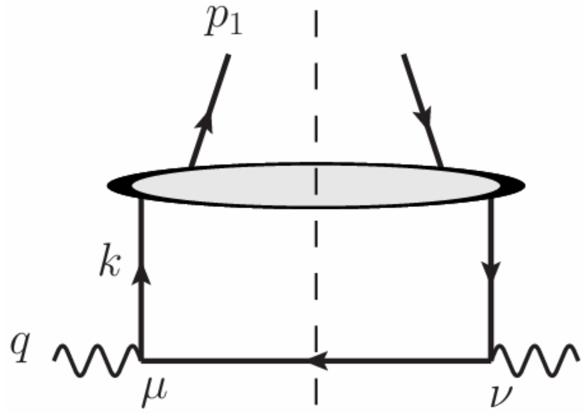
4. Calculations

- 0-gluon scattering amplitude

1. Hadronic tensor

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\hat{\Xi}^{(0)}], \qquad \hat{h}^{(0)}_{\mu\nu} = \frac{1}{p^+}\gamma_\mu \not{n} \gamma_\nu,$$

$$\hat{\Xi}^{(0)} = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ip^+\xi^-/z} \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$



2. Calculate traces and decomposing the correlator

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\not{n}] = -\frac{4}{p^+}g_{\perp\mu\nu}, \qquad z\Xi^{(0)}_\alpha = p^+\bar{n}_\alpha D_1 - M\tilde{S}_{T\alpha}D_T + \frac{M^2}{p^+}n_\alpha D_3,$$

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\gamma^5\not{n}] = \frac{4}{p^+}i\varepsilon_{\perp\mu\nu}, \qquad z\tilde{\Xi}^{(0)}_\alpha = -p^+\bar{n}_\alpha\lambda_h G_{1L} + MS_{T\alpha}G_T - \frac{M^2}{p^+}n_\alpha\lambda_h G_{3L}.$$

3. Leading twist result

$$z\tilde{W}_{t2\mu\nu} = -2g_{\perp\mu\nu}D_1 - 2i\lambda_h\varepsilon_{\perp\mu\nu}G_{1L}.$$

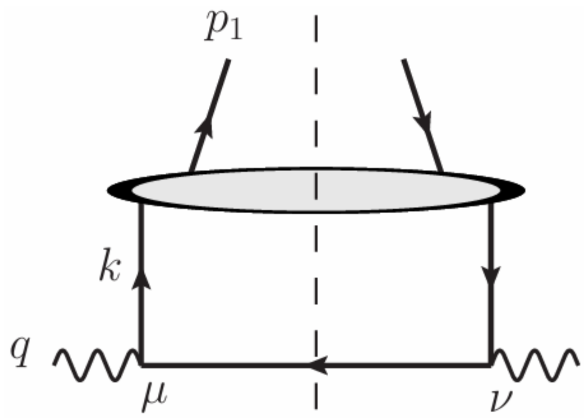
4. Calculations

- 0-gluon scattering amplitude

1. Hadronic tensor

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\hat{\Xi}^{(0)}], \qquad \hat{h}^{(0)}_{\mu\nu} = \frac{1}{p^+}\gamma_\mu \not{n} \gamma_\nu,$$

$$\hat{\Xi}^{(0)} = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ip^+\xi^-/z} \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$



2. Calculate traces and decomposing the correlator

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\not{n}] = -\frac{4}{p^+} g_{\perp\mu\nu}, \qquad z\Xi^{(0)}_\alpha = p^+\bar{n}_\alpha D_1 - M\tilde{S}_{T\alpha}D_T + \frac{M^2}{p^+}n_\alpha D_3,$$

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\gamma^5\not{n}] = \frac{4}{p^+} i\varepsilon_{\perp\mu\nu}, \qquad z\tilde{\Xi}^{(0)}_\alpha = -p^+\bar{n}_\alpha\lambda_h G_{1L} + MS_{T\alpha}G_T - \frac{M^2}{p^+}n_\alpha\lambda_h G_{3L}.$$

	U	L
T-2	D_1	G_{1L}

3. Leading twist result

$$z\tilde{W}_{t2\mu\nu} = -2g_{\perp\mu\nu}D_1 - 2i\lambda_h\varepsilon_{\perp\mu\nu}G_{1L}.$$

Leading twist

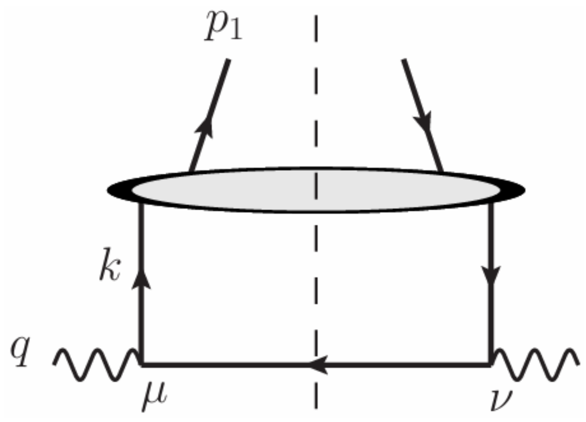
4. Calculations

- 0-gluon scattering amplitude

1. Hadronic tensor

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\hat{\Xi}^{(0)}], \qquad \hat{h}^{(0)}_{\mu\nu} = \frac{1}{p^+}\gamma_\mu \not{n}\gamma_\nu,$$

$$\hat{\Xi}^{(0)} = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ip^+\xi^-/z}\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$



2. Calculate traces and decomposing the correlator

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\not{n}] = -\frac{4}{p^+}g_{\perp\mu\nu}, \qquad z\Xi^{(0)}_\alpha = p^+\bar{n}_\alpha D_1 - M\tilde{S}_{T\alpha}D_T + \frac{M^2}{p^+}n_\alpha D_3,$$

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\gamma^5\not{n}] = \frac{4}{p^+}i\varepsilon_{\perp\mu\nu}, \qquad z\tilde{\Xi}^{(0)}_\alpha = -p^+\bar{n}_\alpha\lambda_h G_{1L} + MS_{T\alpha}G_T - \frac{M^2}{p^+}n_\alpha\lambda_h G_{3L}.$$

Leading twist   Twist-3

	U	L
T-2	D_1	G_{1L}
T-3	D_T	G_T

3. Leading twist result

$$z\tilde{W}_{t2\mu\nu} = -2g_{\perp\mu\nu}D_1 - 2i\lambda_h\varepsilon_{\perp\mu\nu}G_{1L}.$$

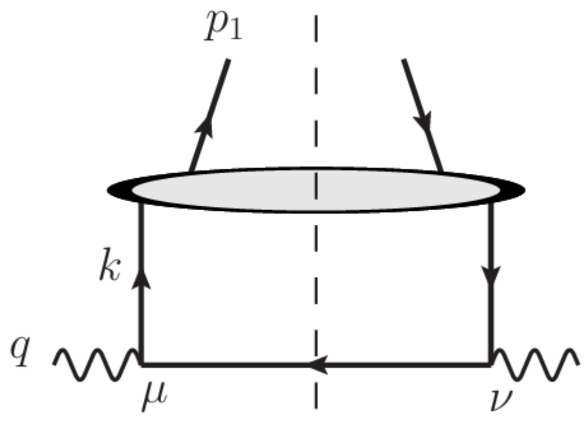
4. Calculations

- 0-gluon scattering amplitude

1. Hadronic tensor

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\hat{\Xi}^{(0)}], \qquad \hat{h}^{(0)}_{\mu\nu} = \frac{1}{p^+}\gamma_\mu \not{n}\gamma_\nu,$$

$$\hat{\Xi}^{(0)} = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ip^+\xi^-/z}\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$



2. Calculate traces and decomposing the correlator

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\not{n}] = -\frac{4}{p^+}g_{\perp\mu\nu}, \qquad z\Xi^{(0)}_\alpha = p^+\bar{n}_\alpha D_1 - M\tilde{S}_{T\alpha}D_T + \frac{M^2}{p^+}n_\alpha D_3,$$

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\gamma^5\not{n}] = \frac{4}{p^+}i\varepsilon_{\perp\mu\nu}, \qquad z\tilde{\Xi}^{(0)}_\alpha = -p^+\bar{n}_\alpha\lambda_h G_{1L} + MS_{T\alpha}G_T - \frac{M^2}{p^+}n_\alpha\lambda_h G_{3L}.$$

Leading twist — Twist-3 — Twist-4

	U	L
T-2	D_1	G_{1L}
T-3	D_T	G_T
T-4	D_3	G_{3L}

3. Leading twist result

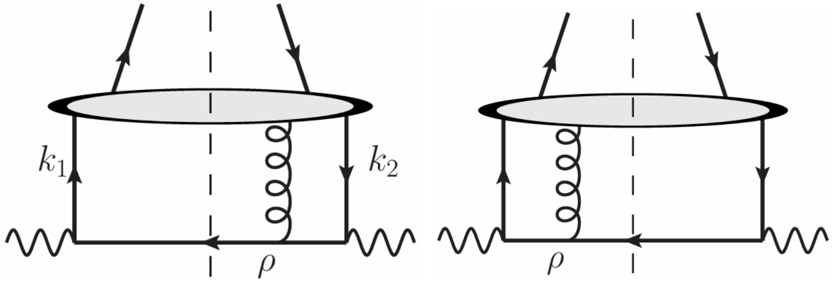
$$z\tilde{W}_{t2\mu\nu} = -2g_{\perp\mu\nu}D_1 - 2i\lambda_h\varepsilon_{\perp\mu\nu}G_{1L}.$$

4. Calculations

- 1-gluon scattering amplitude

$$z\Xi^{(1)}_{\rho\alpha} = -p^+\bar{n}_\alpha M\tilde{S}_{T\rho}D_{dT} + M^2 g_{\perp\rho\alpha}D_{3d} + i\lambda_h M^2\varepsilon_{\perp\rho\alpha}D_{3dL},$$

$$z\tilde{\Xi}^{(1)}_{\rho\alpha} = -ip^+\bar{n}_\alpha M S_{T\rho}G_{dT} + iM^2\varepsilon_{\perp\rho\alpha}G_{3d} + \lambda_h M^2 g_{\perp\rho\alpha}G_{3dL}.$$



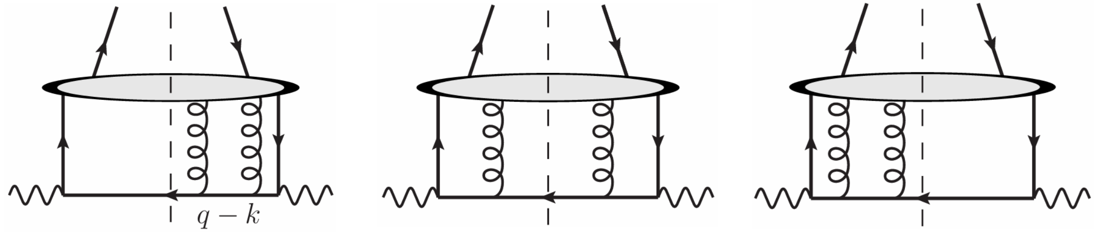
$$z\tilde{W}^{(1)L}_{t3\mu\nu} = -\frac{2M}{q^-}\left[\varepsilon^S_{\perp\mu}\bar{n}_\nu D_{dT} + \varepsilon^S_{\perp\mu}\bar{n}_\nu G_{dT}\right]$$

$$z\tilde{W}^{(1)L}_{t4\mu\nu} = -\frac{4M^2}{p\cdot q}n_\mu n_\nu\left(D_{3d} - G_{3d}\right)$$

- 2-gluon scattering amplitude

$$z\Xi^{(2)}_{\rho\sigma\alpha} = p^+\bar{n}_\alpha\left[M^2 g_{\perp\rho\sigma}D_{3dd} - i\lambda_h M^2\varepsilon_{\perp\rho\sigma}D_{3ddL}\right],$$

$$z\tilde{\Xi}^{(2)}_{\rho\sigma\alpha} = p^+\bar{n}_\alpha\left[iM^2\varepsilon_{\perp\rho\sigma}G_{3dd} - \lambda_h M^2 g_{\perp\rho\sigma}G_{3ddL}\right].$$



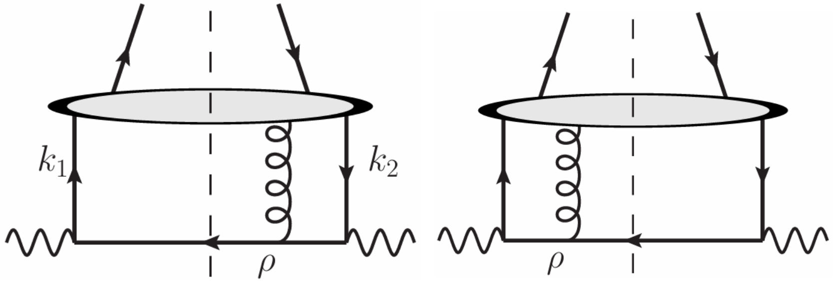
$$z\tilde{W}^{(2)L}_{t4\mu\nu} = \frac{2M^2}{p\cdot q}g_{\perp\mu\nu}\left(D_{3dd} - G_{3dd}\right),$$

$$z\tilde{W}^{(2)M}_{t4\mu\nu} = -\frac{4M^2}{(p\cdot q)^2}p_\mu p_\nu\left(D^M_{3dd} + G^M_{3dd}\right)$$

4. Calculations

- 1-gluon scattering amplitude

$$z\Xi^{(1)}_{\rho\alpha} = -p^+\bar{n}_\alpha M\tilde{S}_{T\rho}D_{dT} + M^2 g_{\perp\rho\alpha}D_{3d} + i\lambda_h M^2\varepsilon_{\perp\rho\alpha}D_{3dL},$$
$$z\tilde{\Xi}^{(1)}_{\rho\alpha} = -ip^+\bar{n}_\alpha M S_{T\rho}G_{dT} + iM^2\varepsilon_{\perp\rho\alpha}G_{3d} + \lambda_h M^2 g_{\perp\rho\alpha}G_{3dL}.$$

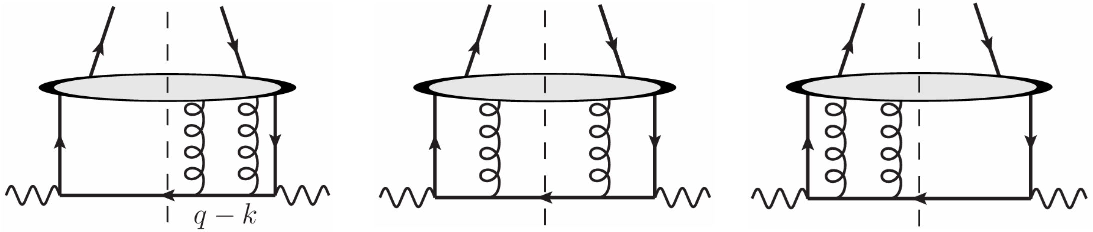


$$z\tilde{W}^{(1)L}_{t3\mu\nu} = -\frac{2M}{q^-}\left[\varepsilon^S_{\perp\mu}\bar{n}_\nu D_{dT} + \varepsilon^S_{\perp\mu}\bar{n}_\nu G_{dT}\right]$$

$$z\tilde{W}^{(1)L}_{t4\mu\nu} = -\frac{4M^2}{p\cdot q}n_\mu n_\nu\left(D_{3d} - G_{3d}\right)$$

- 2-gluon scattering amplitude

$$z\Xi^{(2)}_{\rho\sigma\alpha} = p^+\bar{n}_\alpha\left[M^2 g_{\perp\rho\sigma}D_{3dd} - i\lambda_h M^2\varepsilon_{\perp\rho\sigma}D_{3ddL}\right],$$
$$z\tilde{\Xi}^{(2)}_{\rho\sigma\alpha} = p^+\bar{n}_\alpha\left[iM^2\varepsilon_{\perp\rho\sigma}G_{3dd} - \lambda_h M^2 g_{\perp\rho\sigma}G_{3ddL}\right].$$



$$z\tilde{W}^{(2)L}_{t4\mu\nu} = \frac{2M^2}{p\cdot q}g_{\perp\mu\nu}\left(D_{3dd} - G_{3dd}\right),$$

$$z\tilde{W}^{(2)M}_{t4\mu\nu} = -\frac{4M^2}{(p\cdot q)^2}p_\mu p_\nu\left(D^M_{3dd} + G^M_{3dd}\right)$$

4. Calculations

- Hadronic tensor up to twist-4

$$
\begin{aligned}
zW_{\mu\nu} = &- 2g_{\perp\mu\nu}D_1 - 2i\lambda_h\varepsilon_{\perp\mu\nu}G_{1L} \\
&- \frac{2M}{p\cdot q}\left[\varepsilon^S_{T\{\mu}\bar{q}_{\nu\}}D_T + i\varepsilon^S_{T[\mu}\bar{q}_{\nu]}G_T\right] \\
&+ \frac{4M^2}{p\cdot q}\bar{q}_\mu\bar{q}_\nu D_3 + \frac{4M^2}{p\cdot q}g_{\mu\nu}D_{-3dd} \\
&+ \frac{M^2}{(p\cdot q)}\left\{g_{\perp\mu\nu}D_{4q} + i\lambda_h\varepsilon_{\perp\mu\nu}G_{4qL}\right\}.
\end{aligned}
$$

→ Leading twist

4. Calculations

- Hadronic tensor up to twist-4

$$
\begin{aligned}
zW_{\mu\nu} = & -2g_{\perp\mu\nu}D_1 - 2i\lambda_h\varepsilon_{\perp\mu\nu}G_{1L} \\
& -\frac{2M}{p\cdot q}\left[\varepsilon^S_{T\{\mu}\bar{q}_{\nu\}}D_T + i\varepsilon^S_{T[\mu}\bar{q}_{\nu]}G_T\right] \\
& +\frac{4M^2}{p\cdot q}\bar{q}_\mu\bar{q}_\nu D_3 + \frac{4M^2}{p\cdot q}g_{\mu\nu}D_{-3dd} \\
& +\frac{M^2}{(p\cdot q)}\left\{g_{\perp\mu\nu}D_{4q} + i\lambda_h\varepsilon_{\perp\mu\nu}G_{4qL}\right\}.
\end{aligned}
$$

→ Leading twist

→ Twist-3

4. Calculations

- Hadronic tensor up to twist-4

$$
\begin{aligned}
zW_{\mu\nu} = & -2g_{\perp\mu\nu}D_1 - 2i\lambda_h\varepsilon_{\perp\mu\nu}G_{1L} \\
& - \frac{2M}{p\cdot q}\left[\varepsilon^S_{T\{\mu}\bar{q}_{\nu\}}D_T + i\varepsilon^S_{T[\mu}\bar{q}_{\nu]}G_T\right] \\
& + \frac{4M^2}{p\cdot q}\bar{q}_\mu\bar{q}_\nu D_3 + \frac{4M^2}{p\cdot q}g_{\mu\nu}D_{-3dd} \\
& + \frac{M^2}{(p\cdot q)}\left\{g_{\perp\mu\nu}D_{4q} + i\lambda_h\varepsilon_{\perp\mu\nu}G_{4qL}\right\}.
\end{aligned}
$$

→ Leading twist

→ Twist-3

→ Twist-4, Multi-gluon scattering

4. Calculations

- Hadronic tensor up to twist-4

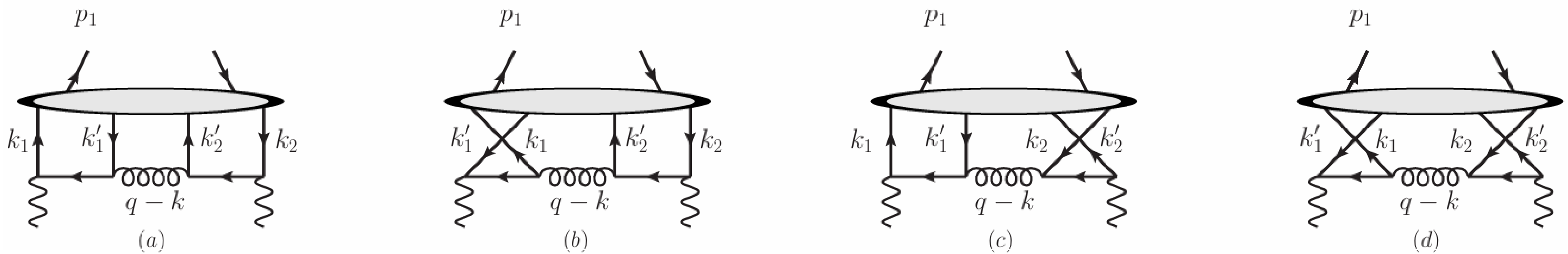
$$
\begin{aligned}
zW_{\mu\nu} = & -2g_{\perp\mu\nu}D_1 - 2i\lambda_h\varepsilon_{\perp\mu\nu}G_{1L} \\
& - \frac{2M}{p\cdot q}\left[\varepsilon^S_{T\{\mu}\bar{q}_{\nu\}}D_T + i\varepsilon^S_{T[\mu}\bar{q}_{\nu]}G_T\right] \\
& + \frac{4M^2}{p\cdot q}\bar{q}_\mu\bar{q}_\nu D_3 + \frac{4M^2}{p\cdot q}g_{\mu\nu}D_{-3dd} \\
& + \frac{M^2}{(p\cdot q)}\left\{g_{\perp\mu\nu}D_{4q} + i\lambda_h\varepsilon_{\perp\mu\nu}G_{4qL}\right\}.
\end{aligned}
$$

→ Leading twist

→ Twist-3

→ Twist-4, Multi-gluon scattering

→ Twist-4, Multi-quark scattering



4. Calculations

- Cross section up to twist-4

$$\frac{d\sigma}{dz\,d\cos\theta} = \sum_a e_a^2 \frac{\pi\beta z\alpha^2}{2Q^4}\Big\{(1+\cos^2\theta)\Big[D_1 - \frac{4\kappa_M^2}{z}\mathrm{Re}(D_{3dd}-G_{3dd}) - \frac{\kappa_M^2}{z}D_{4q}\Big]$$
$$+\beta^2\frac{8\kappa_M^2}{z^2}\sin^2\theta D_3 - |S_T|\beta\frac{\kappa_M}{z}\sin 2\theta\sin\phi_S D_T$$
$$+2\lambda_e\Big[S_L\cos\theta\Big(G_{1L}-\frac{\kappa_M^2}{z}G_{4qL}\Big) - |S_T|\beta\frac{\kappa_M}{z}\sin\theta\cos\phi_S G_T\Big]\Big\},$$

where $\kappa_M \equiv M/Q$ is a typical suppression factor of high twist contributions.

$$F_{U,U}^1 = \sum_a e_a^2\Big[D_1 - \frac{4\kappa_M^2}{z}\mathrm{Re}(D_{3dd}-G_{3dd}) - \frac{\kappa_M^2}{z}D_{4q}\Big],$$

$$F_{U,U}^2 = \sum_a e_a^2\beta^2\frac{8\kappa_M^2}{z^2}D_3,$$

…

…

5. Summary

- Fragmentation functions are important in describing hadronization process;
- Higher twist fragmentation function deserve more attention;
- ……

5. Summary

- Fragmentation functions are important in describing hadronization process;
- Higher twist fragmentation function deserve more attention;

Thank you!

Backup

OPE

$$\bar q\gamma^\mu q(x)\,\bar q\gamma^\nu q(0) = \bar q(x)\gamma^\mu q(x)\bar q(0)\gamma^\nu q(0) + \bar q(x)\gamma^\mu q(x)\,\bar q(0)\gamma^\nu q(0) + \cdots \quad (1)$$

利用

$$\frac{1}{(i\partial+q)^2} = \frac{-1}{Q^2-2iq\cdot\partial+\partial^2} = -\frac{1}{Q^2}\sum_{n=0}^{\infty}\left(\frac{2iq\cdot\partial-\partial^2}{Q^2}\right)^n \quad (2)$$

$$\frac{1}{2}(\gamma^\mu\gamma^\alpha\gamma^\nu+\gamma^\nu\gamma^\alpha\gamma^\mu) = g^{\mu\alpha}\gamma^\nu+\gamma^\mu g^{\alpha\nu}-g^{\mu\nu}\gamma^\alpha \quad (3)$$

有

$$\int d^4x\, e^{iqx}\,\bar q(x)\gamma^\mu q(x)\,\bar q(0)\gamma^\nu q(0) = \bar q\gamma^\mu\frac{i(i\not\partial+\not q)}{(i\partial+q)^2}\gamma^\nu q(0)$$

$$= -i\bar q(2\gamma^\mu(i\partial^\nu)-g^{\mu\nu}\not q)\frac{1}{Q^2}\sum_{n=0}^{\infty}\left(\frac{2iq\cdot\partial}{Q^2}\right)^n q \quad (4)$$

$$= -i\bar q\,2\gamma^\mu(i\partial^\nu)(i\partial)^n q\sum_{n=0}^{\infty}\frac{(2q)^n}{Q^{2n+2}} \quad \text{(两项和)}$$

⇓

$$i\int d^4x\, e^{iq\cdot x}J^\mu(x)J^\nu(0) = \sum_f Q_f^2\Big[4\sum_{n=2}^{\infty}\frac{(2q^{\mu_1})\cdots(2q^{\mu_{n-2}})}{(Q^2)^{n-1}}O_f^{(n)\mu\nu\mu_1\cdots\mu_{n-2}}$$

$$-g^{\mu\nu}\sum_{n=2}^{\infty}\frac{(2q^{\mu_1})\cdots(2q^{\mu_n})}{(Q^2)^n}O_f^{(n)\mu_1\cdots\mu_n}\Big]+\cdots \quad (5)$$

$$\langle J^\mu(x)J^\nu(0)\rangle = \sum_i C_i O_i \quad (6)$$

$$\langle J^\mu(x)J^\nu(0)\rangle \to 6;\quad O_i\to d;\quad C_i\to 6-d \quad (7)$$

$$\int d^4x\, e^{iqx}\langle J^\mu(x)J^\nu(0)\rangle = \sum_i\int d^4x\, e^{iqx}C_i O_i = \sum_i\tilde C_i O_i \quad (8)$$

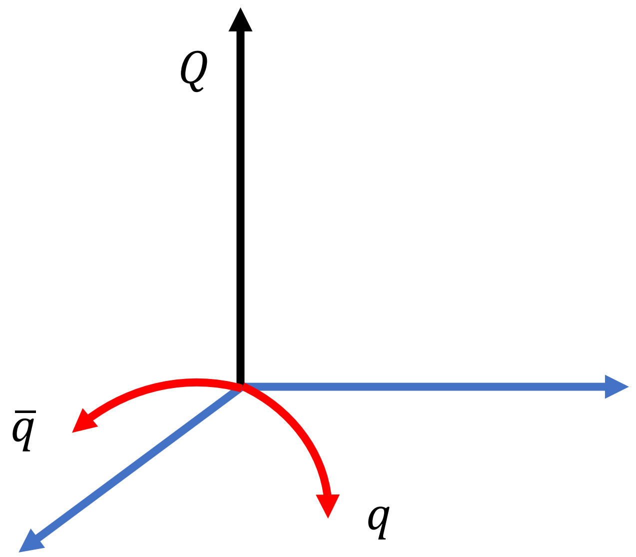
$$\tilde C_i \sim 2-d \sim \left(\frac{1}{Q}\right)^{d-2}.$$

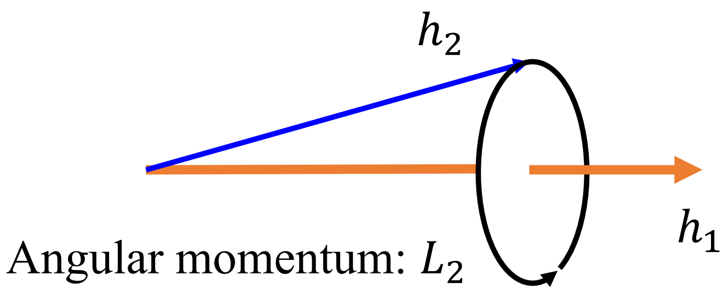
考虑自旋. $\sim\left(\frac{1}{Q}\right)^{d-2-s} = \left(\frac{1}{Q}\right)^{t-2}$ (9)

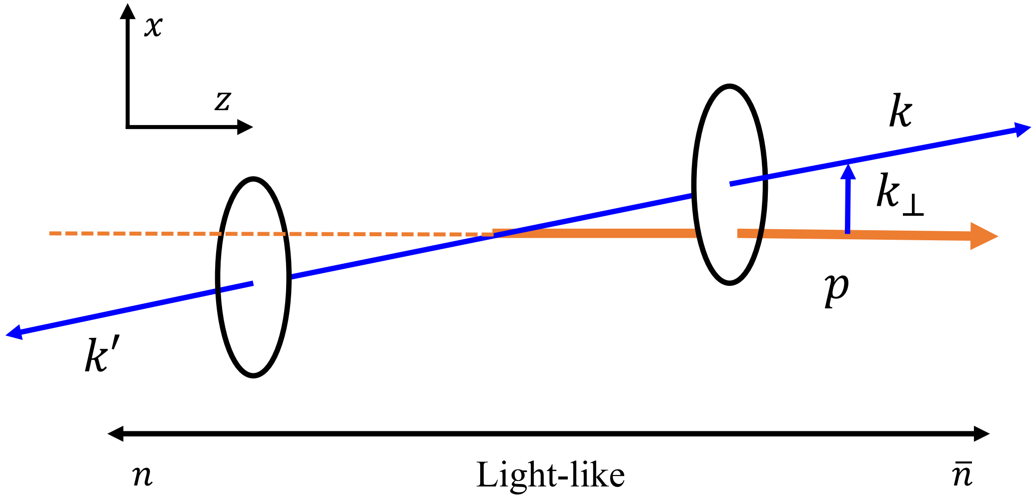
t : twist.

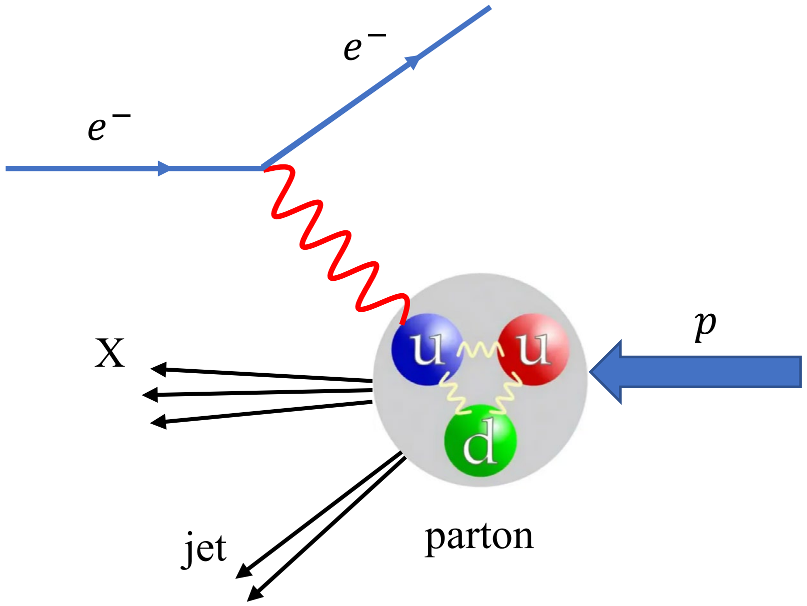
Backup

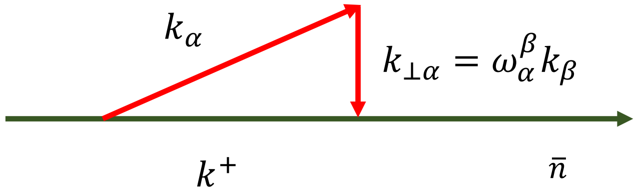
The origin of the handedness in the jet fragmentation.

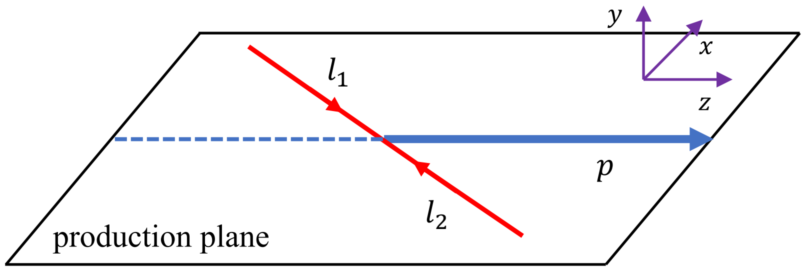



h_2
h_1
Angular momentum: L_2


x
z
k
$k_\perp$
p
k'
n
Light-like
$\bar{n}$


e^-
e^-
X
u
u
d
p
jet
parton


k_α
$k_{\perp\alpha} = \omega_\alpha^\beta k_\beta$
k^+
$\bar{n}$



y
x
z
l_1
p
l_2
production plane