Fragmentation in Deep Inelastic Scattering

碎裂函数和能量关联研讨会
August 8th-11th, 2025 @ Lanzhou, Gansu, China

Tianbo Liu (刘天博)

Key Laboratory of Particle Physics and Particle Irradiation (MOE)
Institute of Frontier and Interdisciplinary Science, Shandong University
Southern Center for Nuclear-Science Theory, IMP, CAS

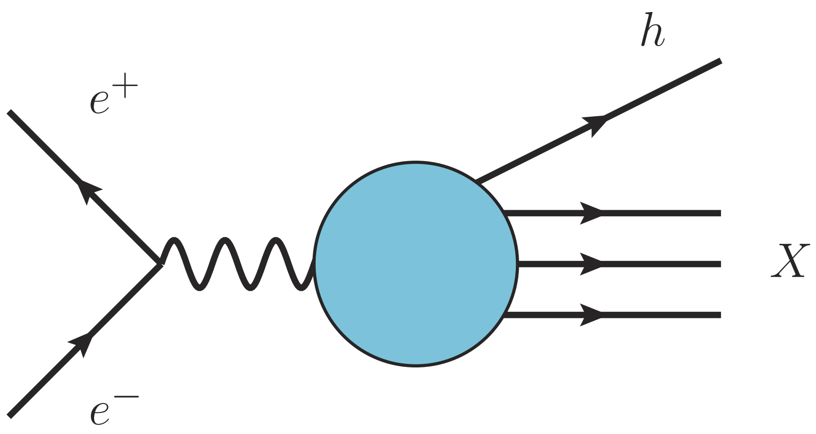
In collaboration with: Hongxin Dong, Zuo-tang Liang, Peng Sun, Ke Yang, Chunhua Zeng, Xiaoyan Zhao, Yuxiang Zhao, Ya-jin Zhou and EicC working groups

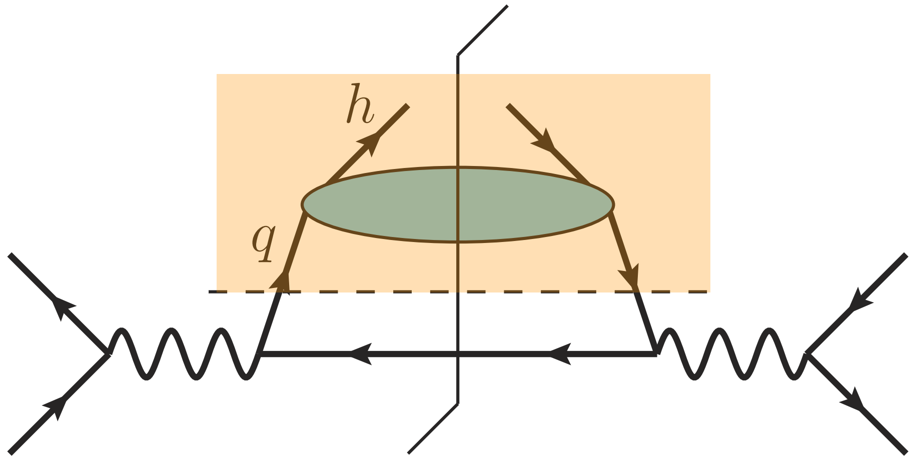
SHANDONG UNIVERSITY

SCNT Southern Center for Nuclear-Science Theory

Semi-inclusive e^+e^- Annihilation

Factorization: $e^+e^- \to hX$





- large momentum scale $Q \gg \Lambda_{\text{QCD}}$ as a short-distance probe
- approximate the cross section: $\sigma \sim H(Q) \otimes D_q^h(z, \mu)$
- overall corrections suppressed by $1/Q^n$
- the emergence of color neutral hadrons from quarks/gluons
- predictive power relies on

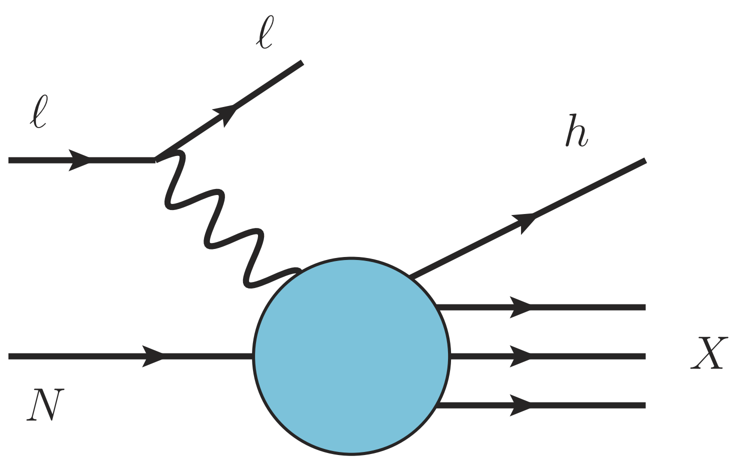
  — the precision of the probe

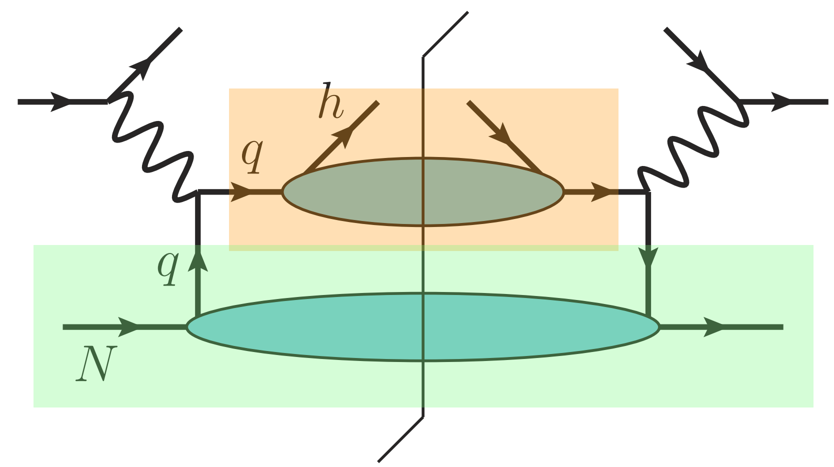
  — the universality of $D_q^h(z, \mu)$

No initial hadron effects
but not enough for flavor separation

Complementary Processes

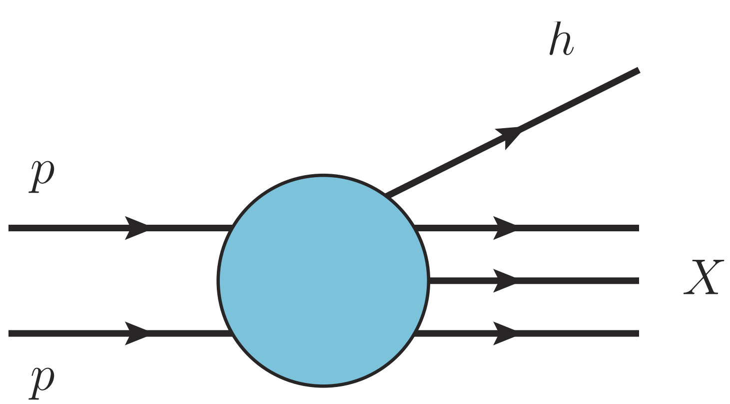
Semi-inclusive deep inelastic scattering: $\ell N \to \ell h X$

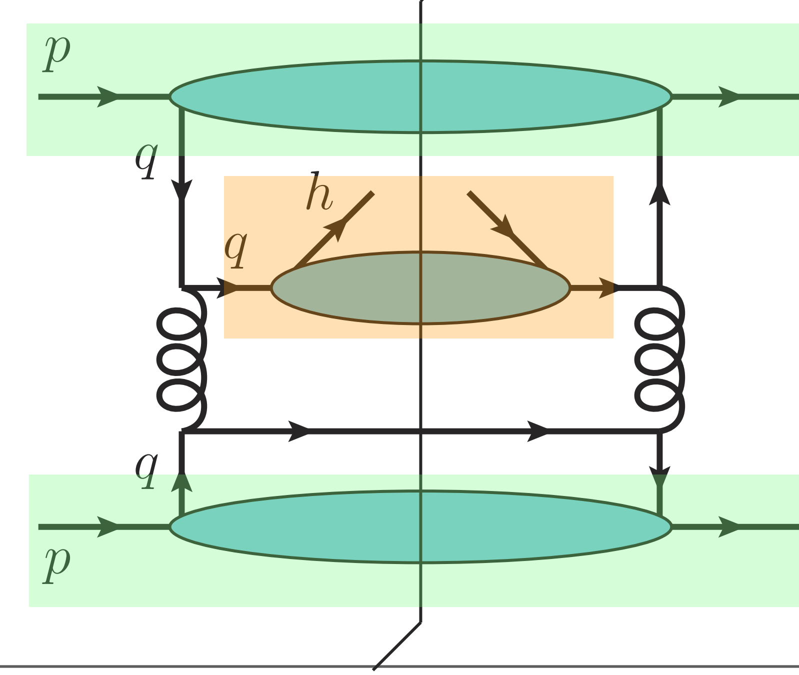




$$\sigma \sim H(Q) \otimes f(x, \mu) \otimes D(z, \mu)$$

Inclusive hadron production in hadron-hadron collision: $pp \to hX$



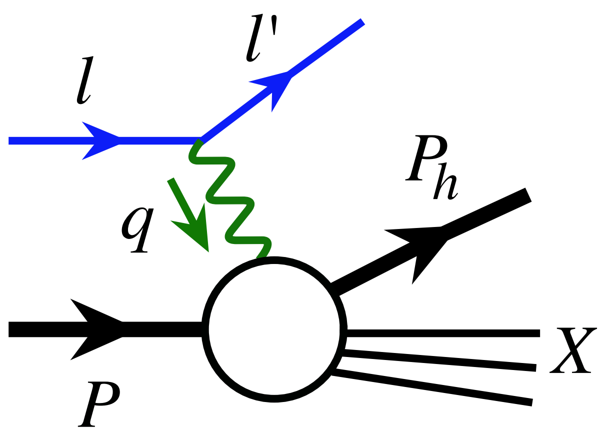


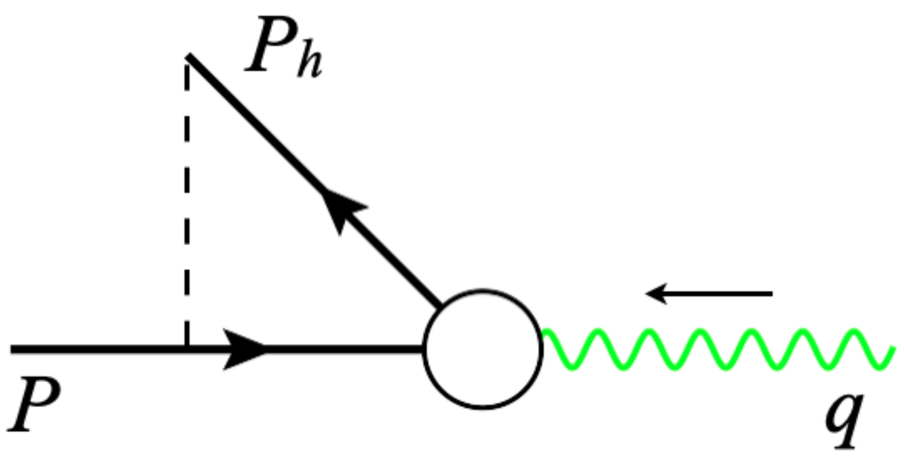
$$\sigma \sim H(Q) \otimes f(x_1, \mu) \otimes f(x_2, \mu) \otimes D(z, \mu)$$

Semi-inclusive Deep Inelastic Scattering

Semi-inclusive DIS: a final state hadron (P_h) is identified

- enable us to explore the emergence of color neutral hadrons from colored quarks/gluons
- flavor dependence by selecting different types of observed hadrons: pions, kaons, …
- a large momentum transfer Q provides a short-distance probe
- an additional and adjustable momentum scale P_{hT}
- multidimensional imaging of the nucleon





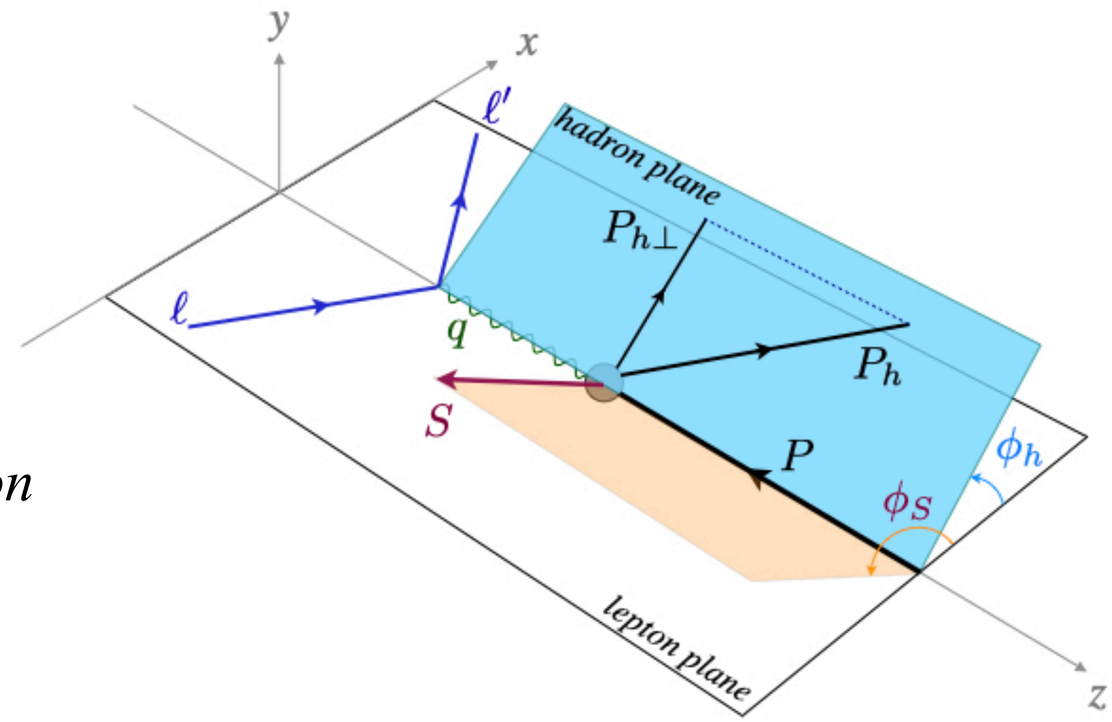
Structure Functions of SIDIS

SIDIS differential cross section

in terms of 18 structure functions

$$F_{AB,C}(x_B, z, P_{hT}^2, Q^2)$$

A: lepton polarization
B: nucleon polarization
C: virtual photon polarization



$$\frac{d\sigma}{dx_B\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\phi_S} = \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)$$

$$\times\Big\{F_{UU,T}+\epsilon F_{UU,L}+\sqrt{2\epsilon(1+\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h+\epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h+\lambda_e\sqrt{2\epsilon(1-\epsilon)}F_{LU}^{\sin\phi_h}\sin\phi_h$$

$$+S_L\left[\sqrt{2\epsilon(1+\epsilon)}F_{UL}^{\sin\phi_h}\sin\phi_h+\epsilon F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\right]+\lambda_e S_L\left[\sqrt{1-\epsilon^2}F_{LL}+\sqrt{2\epsilon(1-\epsilon)}F_{LL}^{\cos\phi_h}\cos\phi_h\right]$$

$$+S_T\Big[\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\sin(\phi_h-\phi_S)+\epsilon F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)$$

$$+\epsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\quad+\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}\sin\phi_S+\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S)\Big]$$

$$+\lambda_e S_T\Big[\sqrt{1-\epsilon^2}F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)$$

$$+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos\phi_S}\cos\phi_S+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big]\Big\}$$

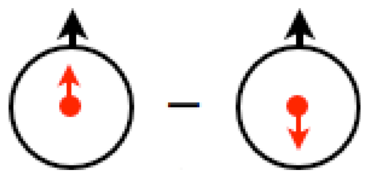
Hadron h is unpolarized or its spin is not measured.

TMD Parton Distribution Functions

		Quark Polarization		
		U	L	T
Nucleon Polarization	U	f_1 unpolarized		$h_1^{\perp}$ Boer-Mulders
	L		g_{1L} helicity	$h_{1L}^{\perp}$ longi-transversity (worm-gear)
	T	$f_{1T}^{\perp}$ Sivers	g_{1T} trans-helicity (worm-gear)	h_1 transversity $h_{1T}^{\perp}$ pretzelosity

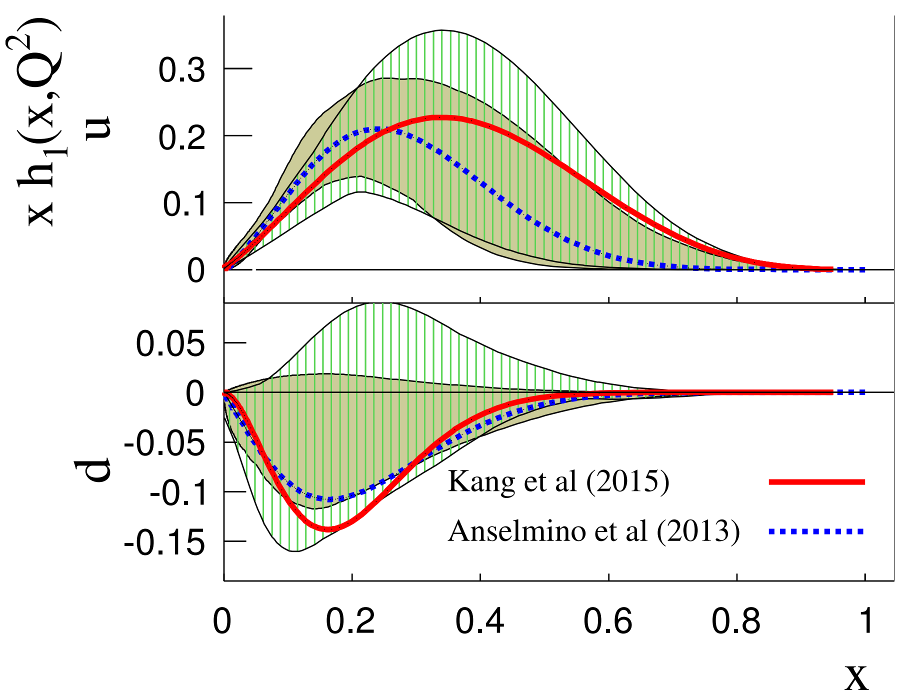
Transversity Distribution

Transversity distribution

h_1  (Collinear & TMD)

A transverse counter part to the longitudinal spin structure: helicity g_{1L}, but NOT the same.

Phenomenological extractions



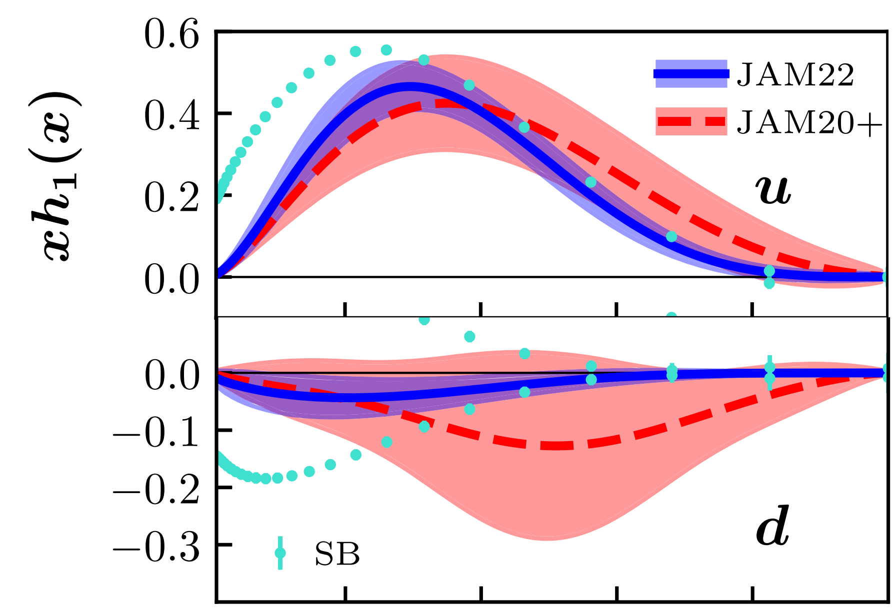
Z.-B. Kang, A. Prokudin, P. Sun, F. Yuan, PRD 93, 014009 (2016).

Chiral-odd:

No mixing with gluons
Valence dominant
Couple to another chiral-odd function.

Effect in SIDIS:
transverse single spin asymmetry (Collins asymmetry)

$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1(x,k_T^2) \otimes H_1^\perp(z,p_T^2)$$

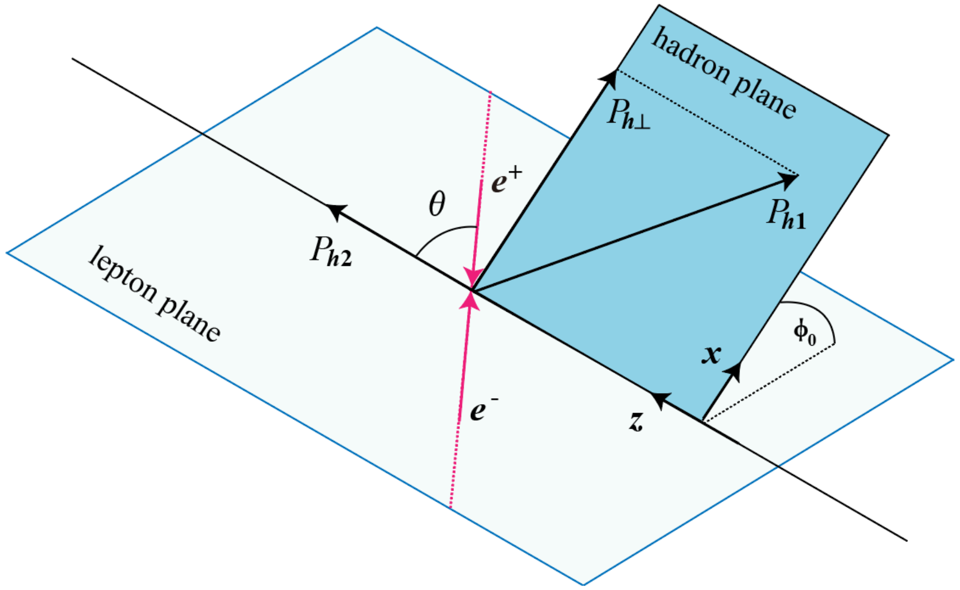


JAM Collaboration, PRD 104, 034014 (2022).

Assuming vanishing transverse polarization of sea quarks!

Complementary Process

Semi-inclusive e^+e^- annihilation: $e^+e^- \to h_1 h_2 X$



$$\frac{d^5\sigma}{dz_1 dz_2 d^2\boldsymbol{P}_{h\perp} d\cos\theta} = \frac{3\pi\alpha^2}{2Q^2} z_1^2 z_2^2 \left[\left(1+\cos^2\theta\right) F_{UU}^{h_1h_2} + \sin^2\theta\cos\left(2\phi_0\right) F_{\text{Collins}}^{h_1h_2}\right]$$

In TMD region: h_1 and h_2 are near back-to-back, $P_{hT} \ll Q$ $\qquad F_{\text{Collins}}^{h_1h_2} \sim H_1^{\perp h_1} \otimes H_1^{\perp h_2}$

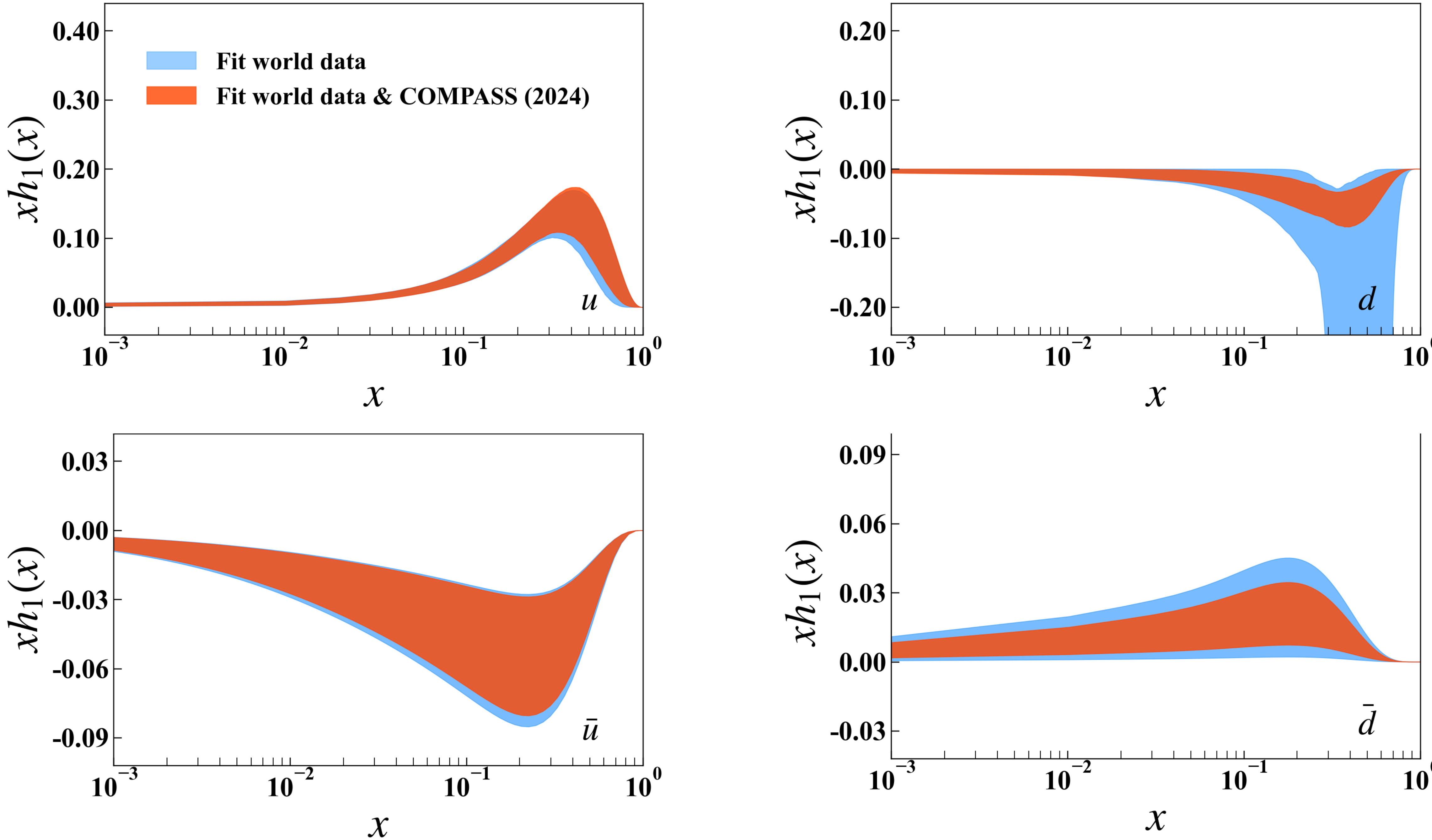
Experimental measurements:

Belle: $\sqrt{s} = 10.58\,\text{GeV}$ Phys. Rev. D 78 (2008) 032011; 86 (2012) 039905(E).

BaBar: $\sqrt{s} = 10.6$ GeV Phys. Rev. D 90 (2014) 052003; Phys. Rev. D 92 (2015) 111101.

BESIII: $\sqrt{s} = 3.68$ GeV Phys. Rev. Lett. 116 (2016) 042001.

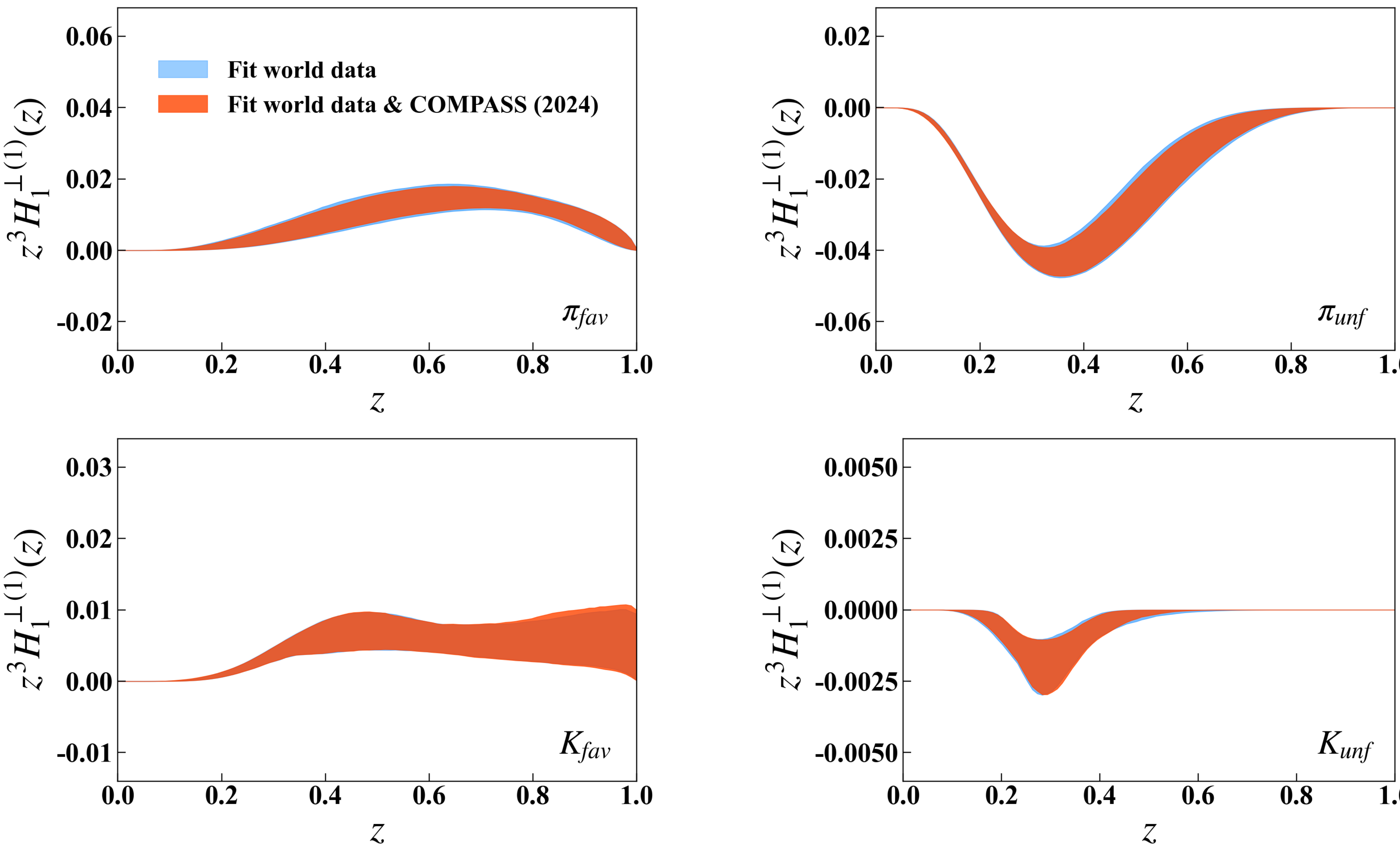
Transversity Distributions



Reconfirmed by new COMPASS data.
Significant improvement on d and $\bar{d}$ distributions.

C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

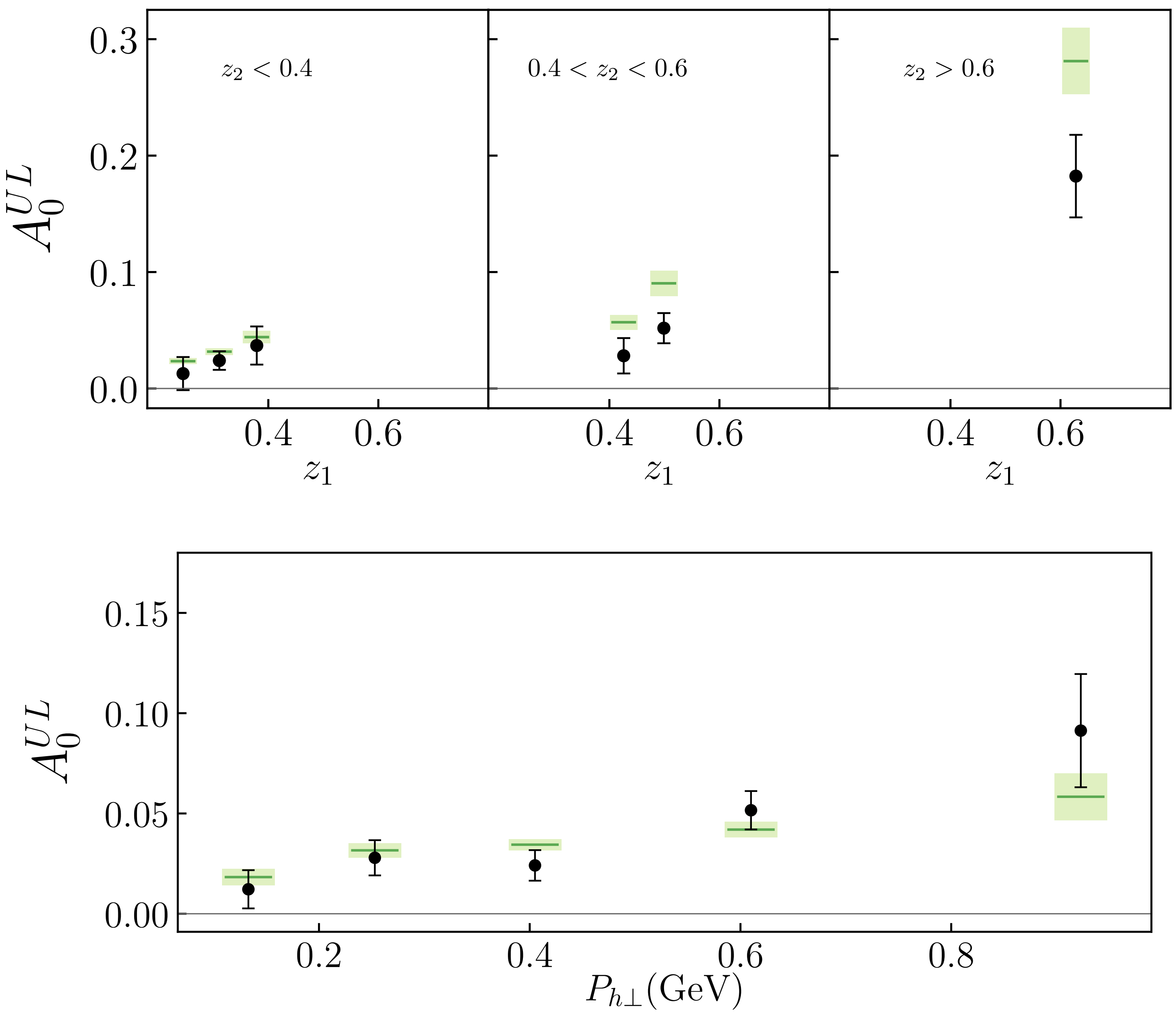
Collins Fragmentation Functions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

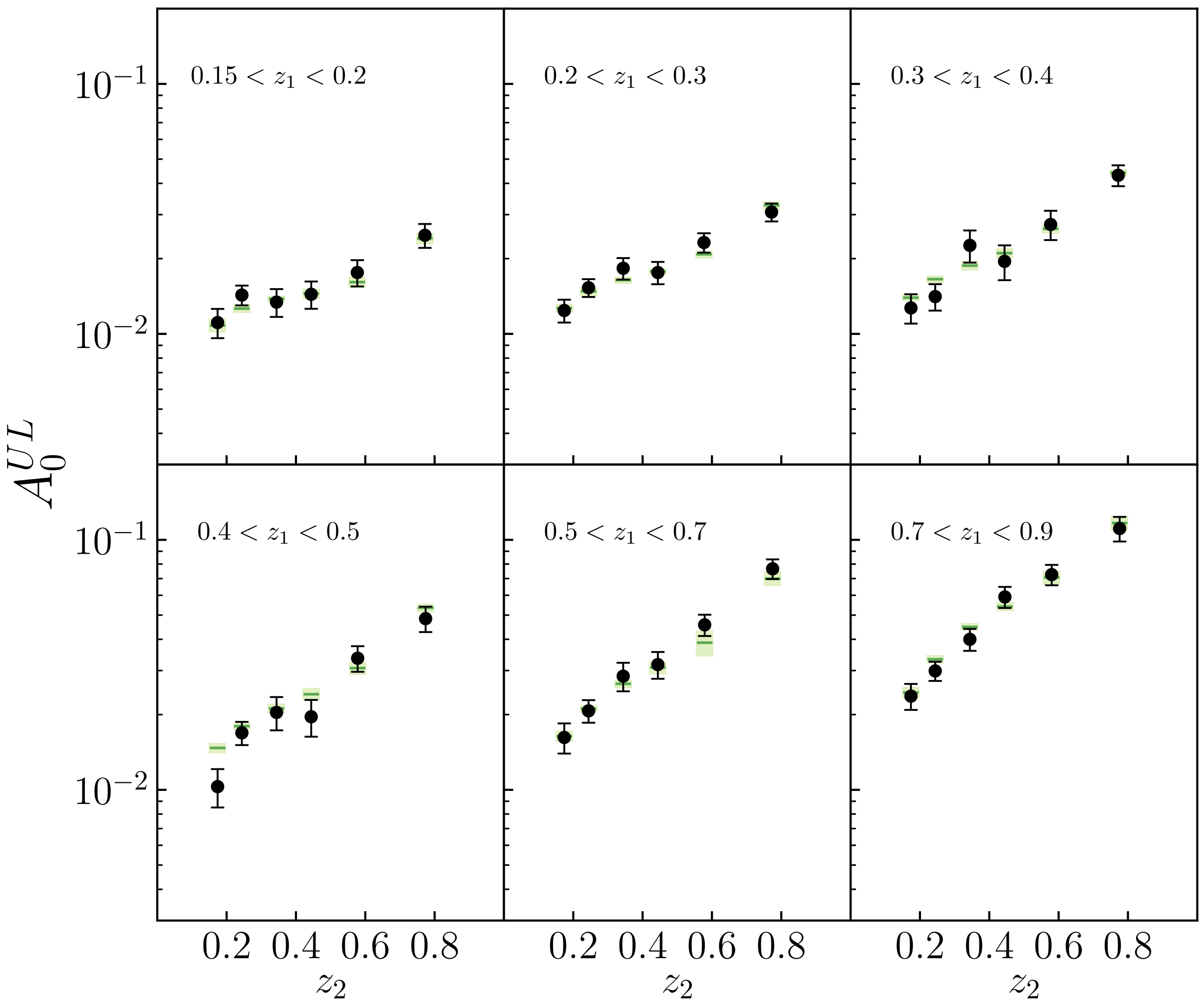
Comparison with BESIII

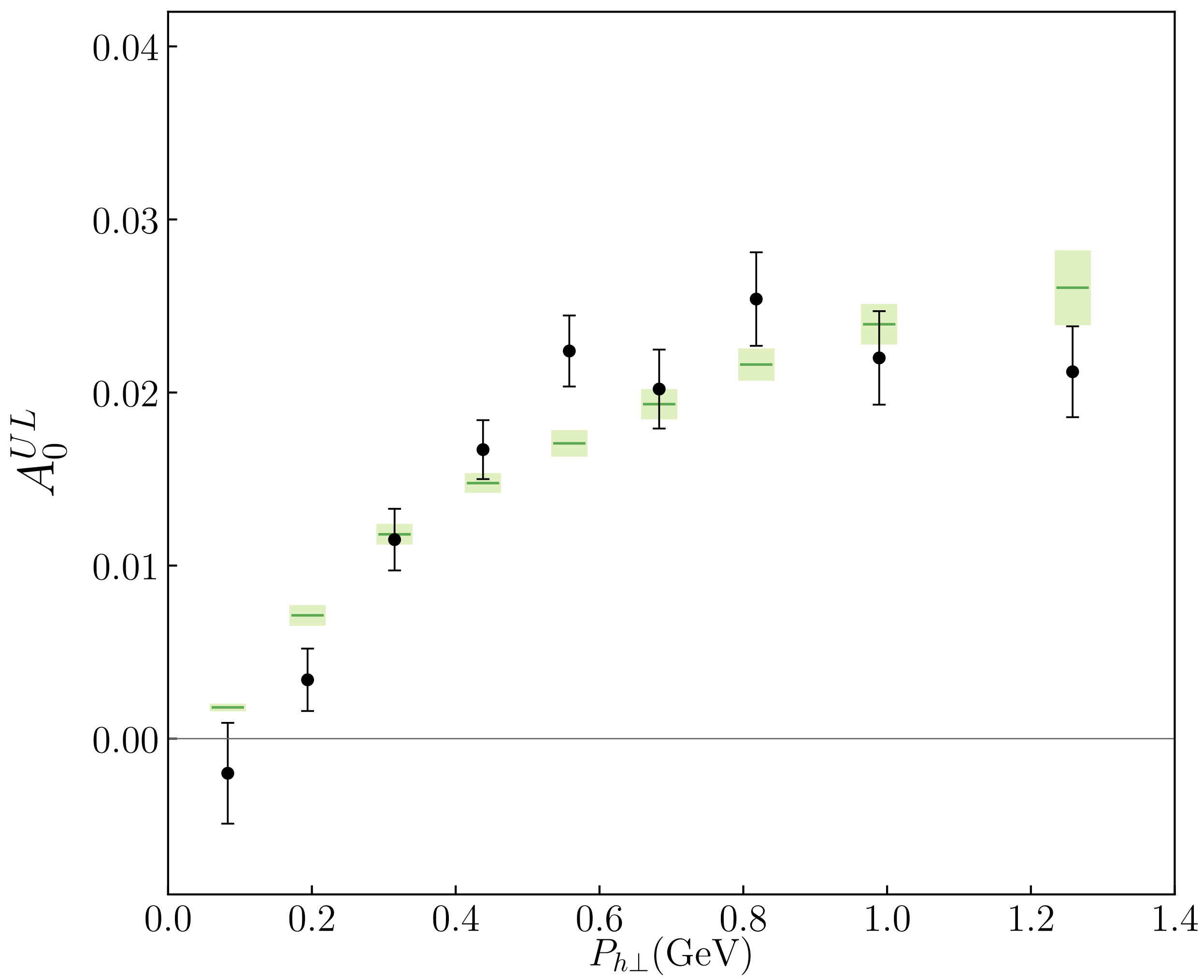


C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

Comparison with BaBar

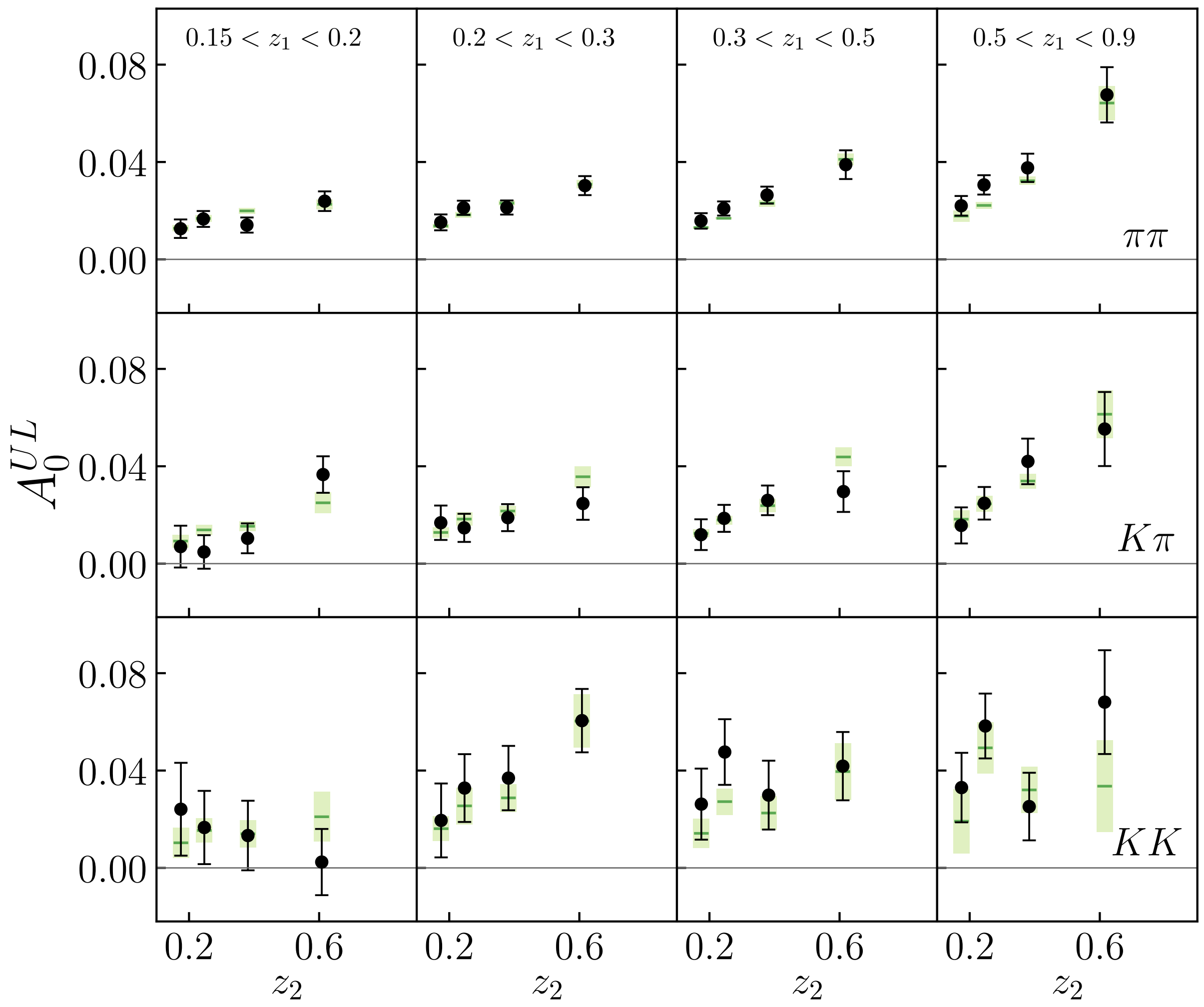




C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

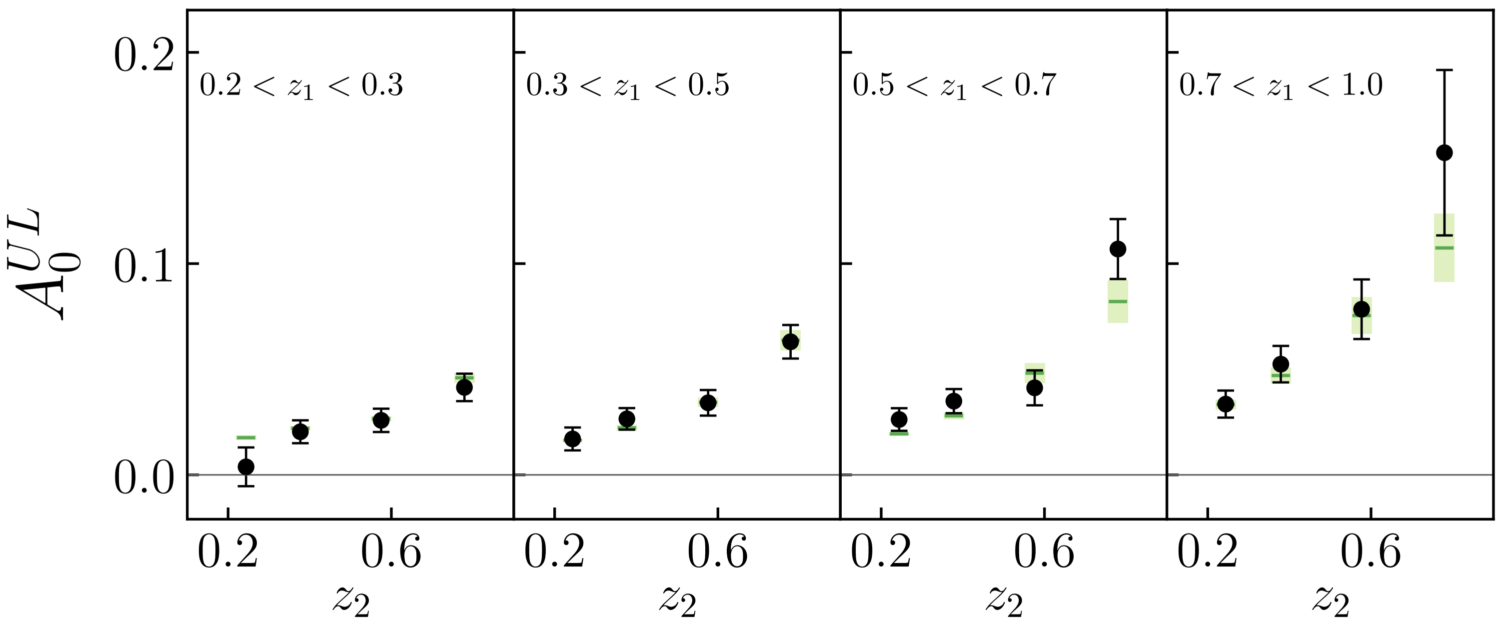
Comparison with BaBar



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

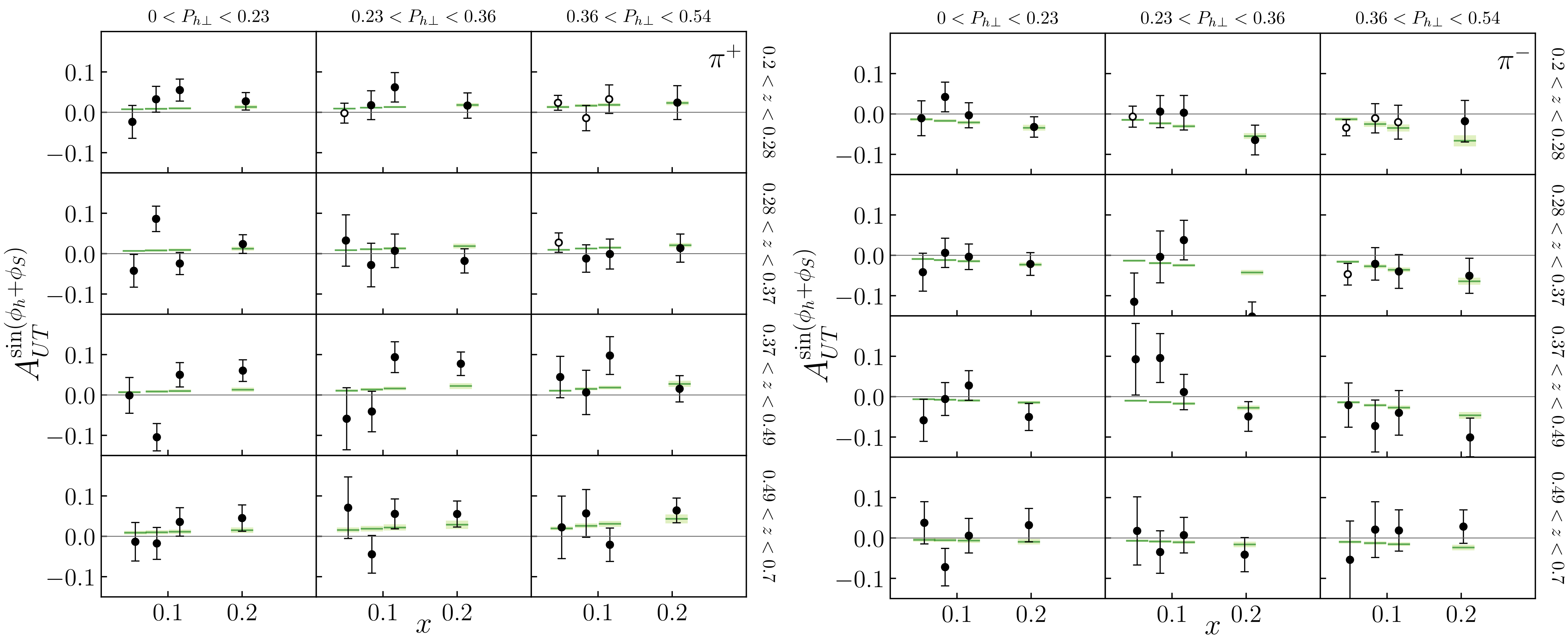
Comparison with Belle



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

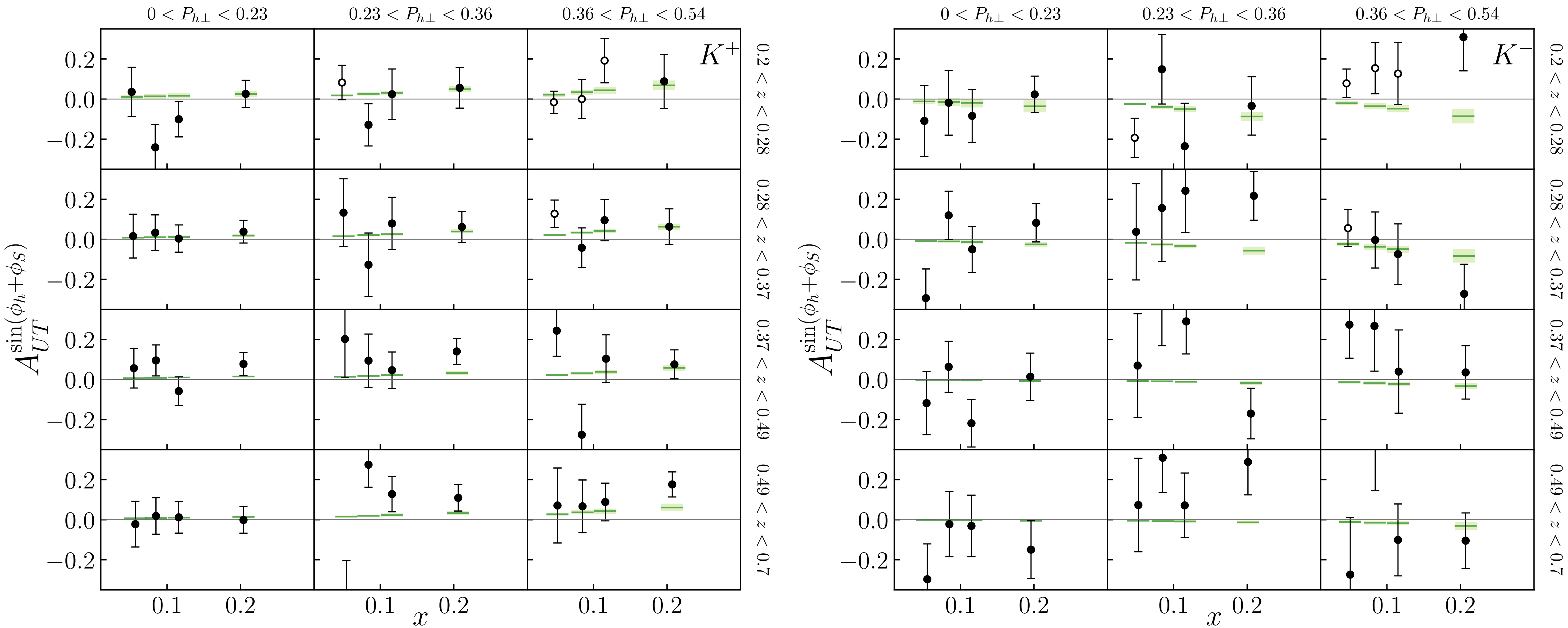
Comparison with HERMES



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

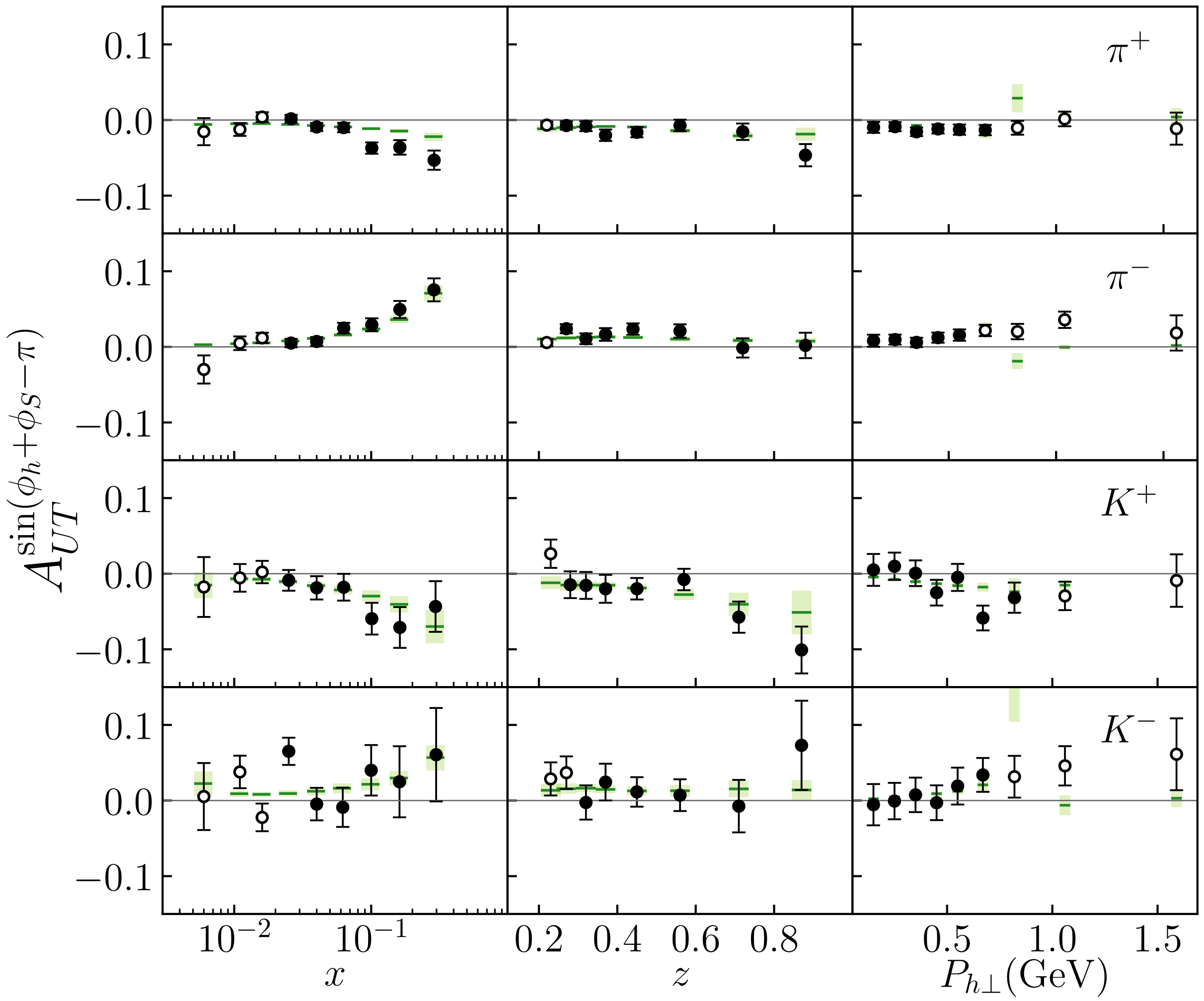
Comparison with HERMES

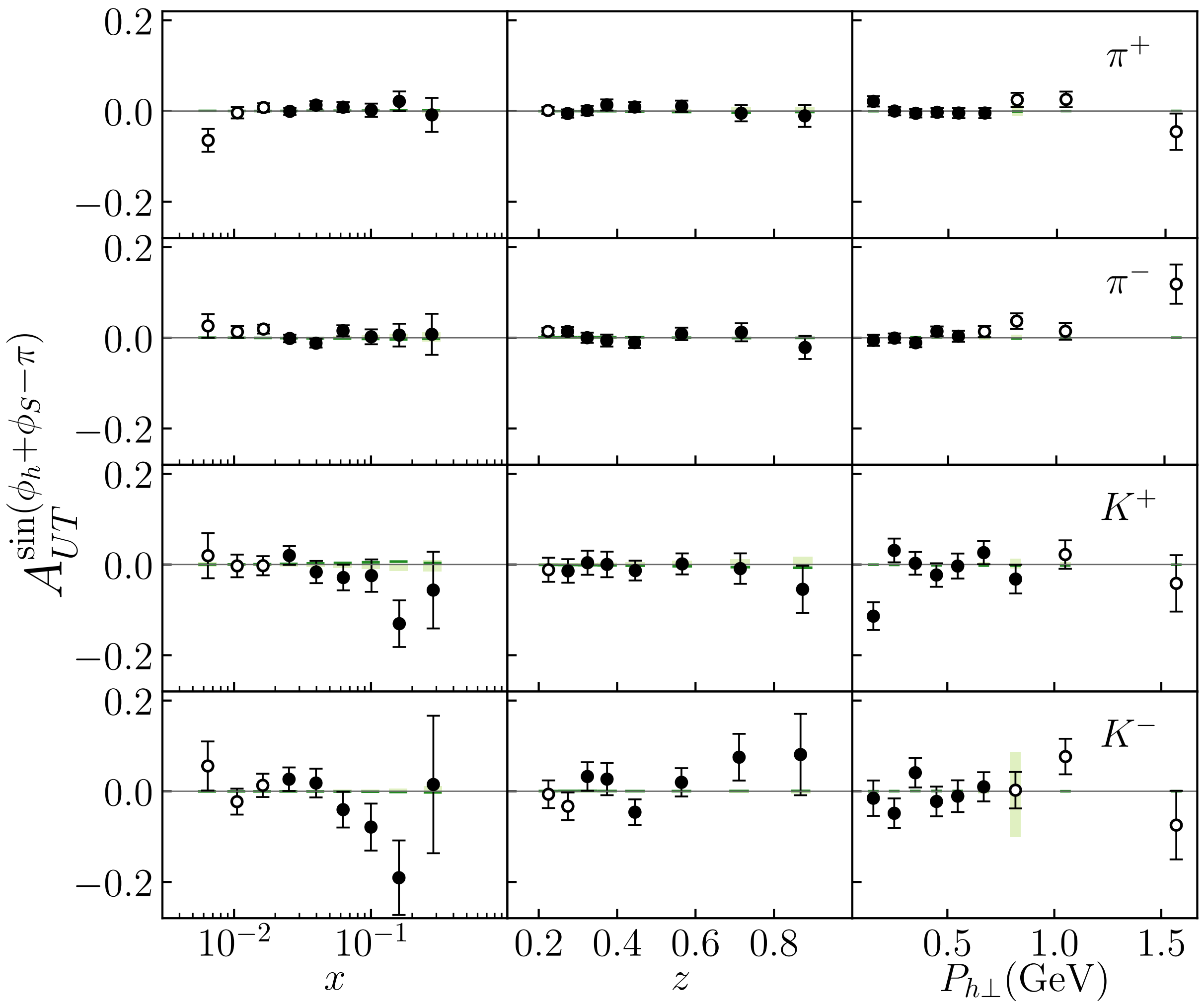


C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

Comparison with COMPASS

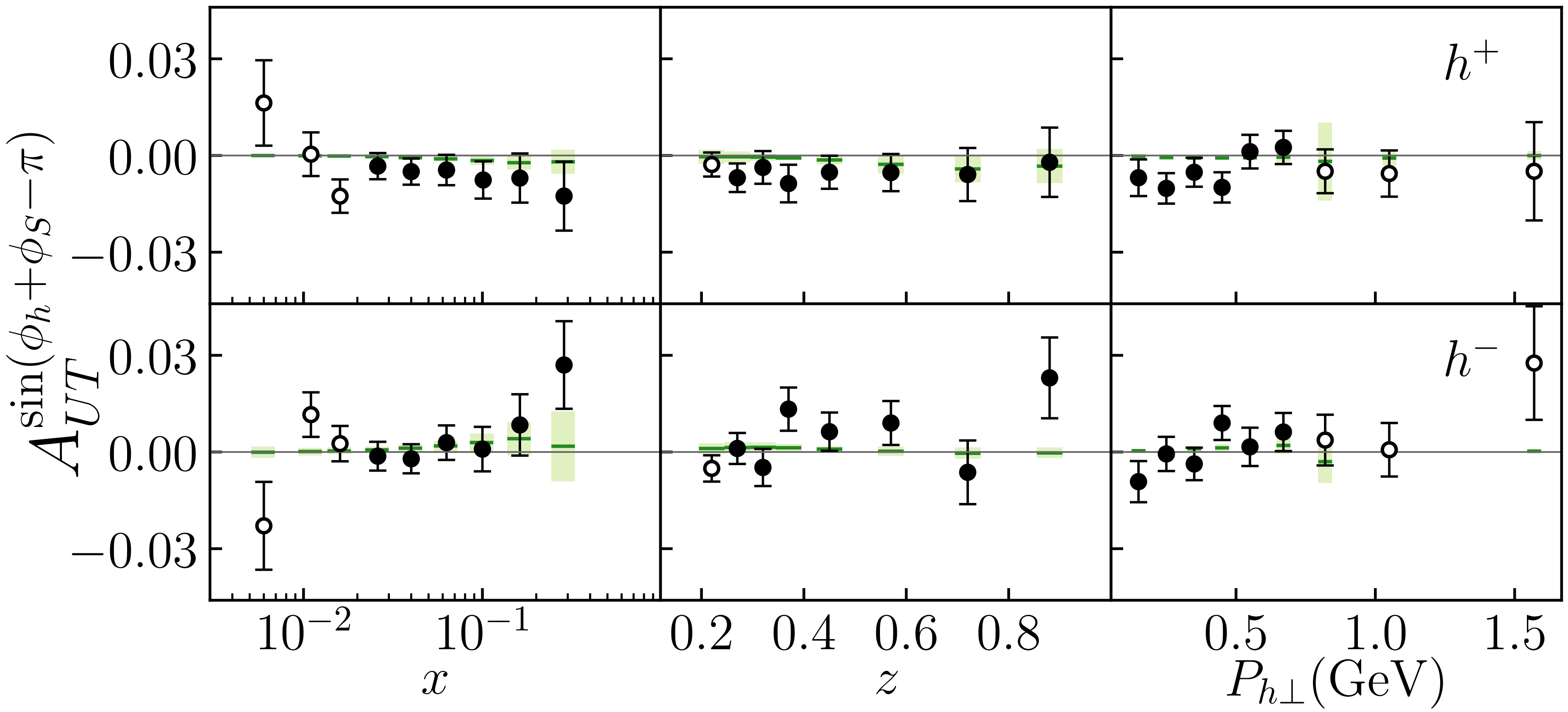




C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

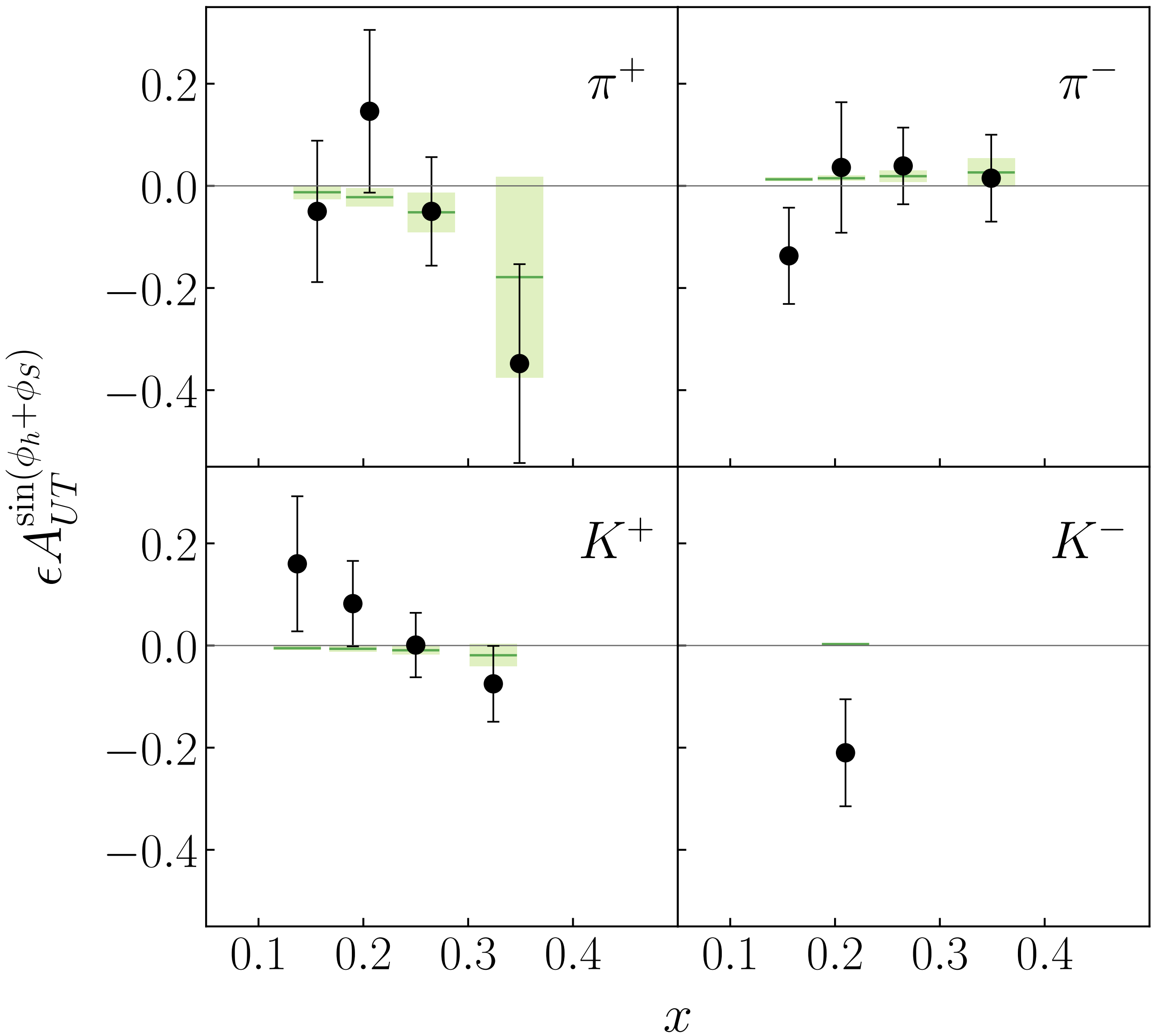
Comparison with COMPASS



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

Comparison with JLab HallA



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Imposing a Bound?

Question:

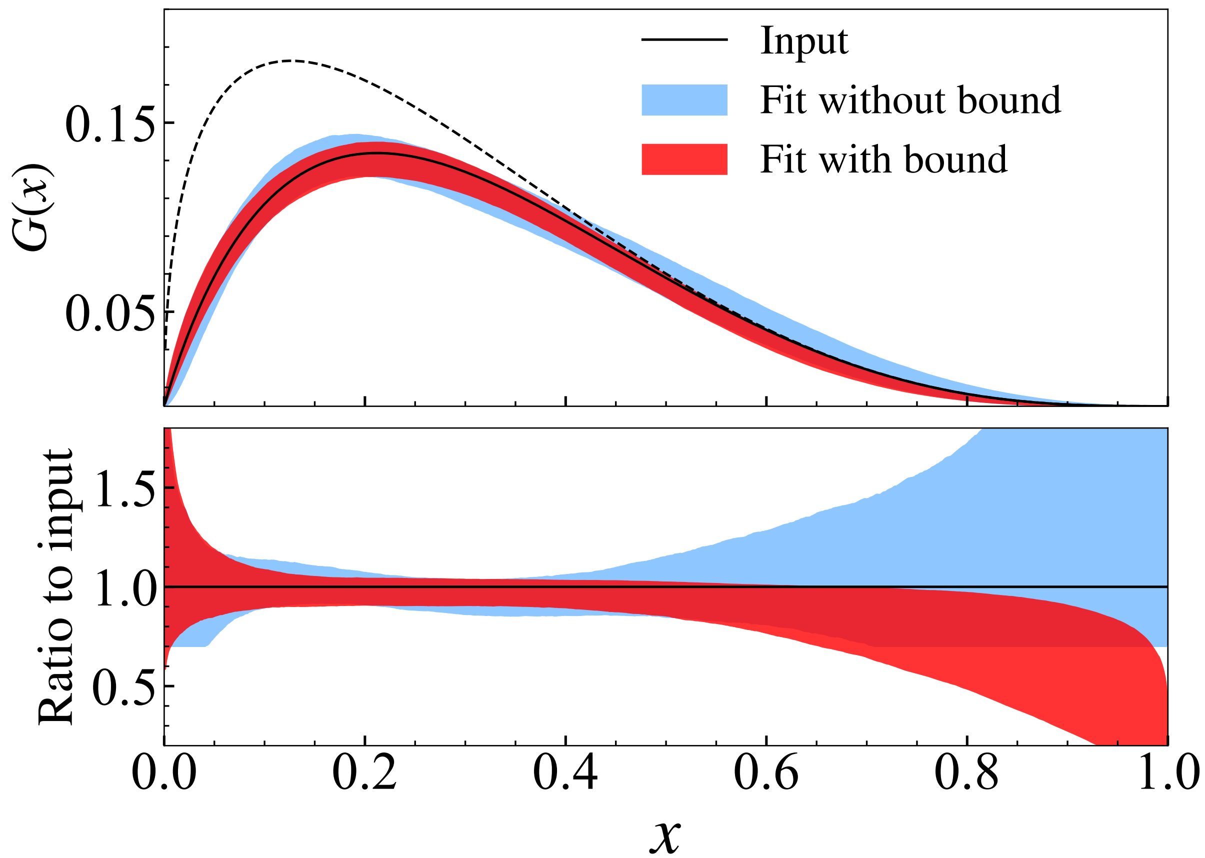
Whether we should impose the Soffer's bound $|h_1(x)| \leq [f_1(x) + g_1(x)]/2$ in the fit?
Similar situations in many other quantities.

A toy model test:

Input some "pol" function $G(x)$
and "unpol" function $F(x)$
Generate asymmetry "data"

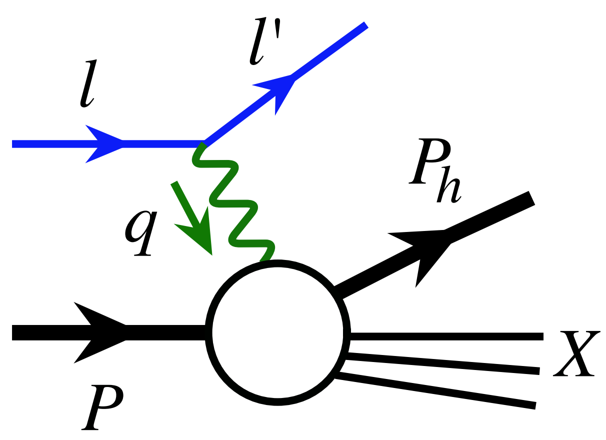
Fit the "data" with or without
imposing the bound $G(x) \leq F(x)$

Imposing a bound in the fit may bias the fit result!



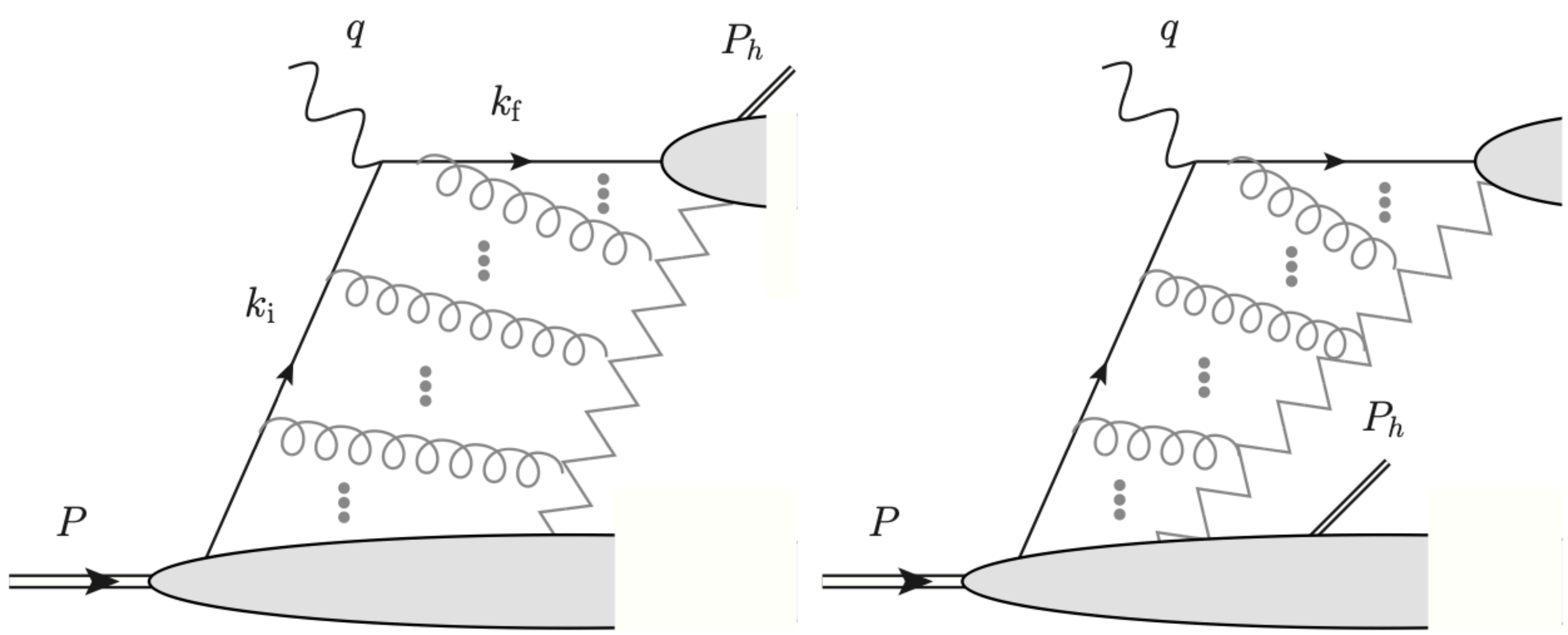
C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

How is the Hadron Produced?



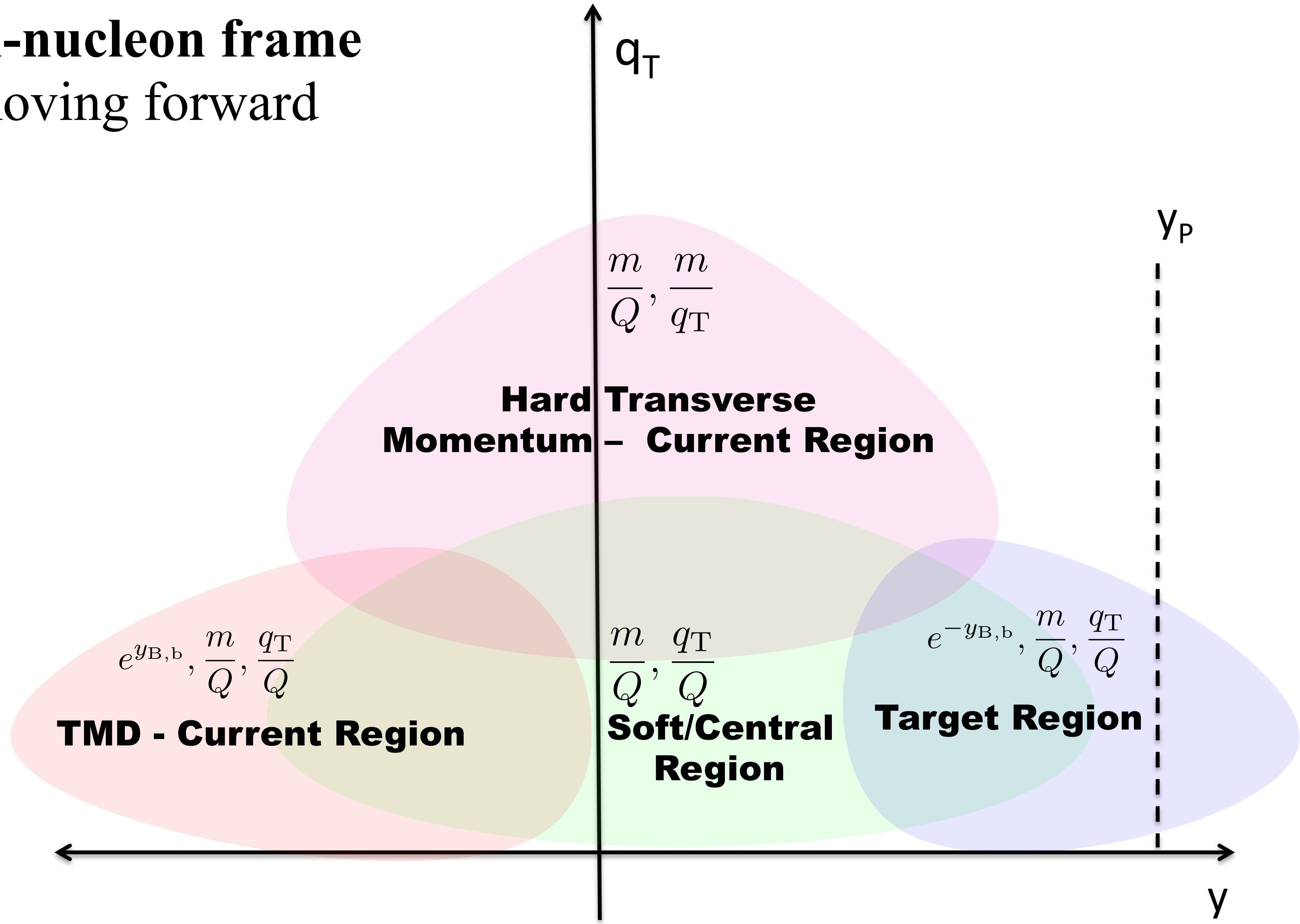
What we know:
A hadron, e.g. h, is observed in the final state, with momentum P_h (and polarization S_h)

What we do not know:
Whether it was produced from the struck quark/parton, the remnant of the target, or …?



SIDIS Kinematic Regions

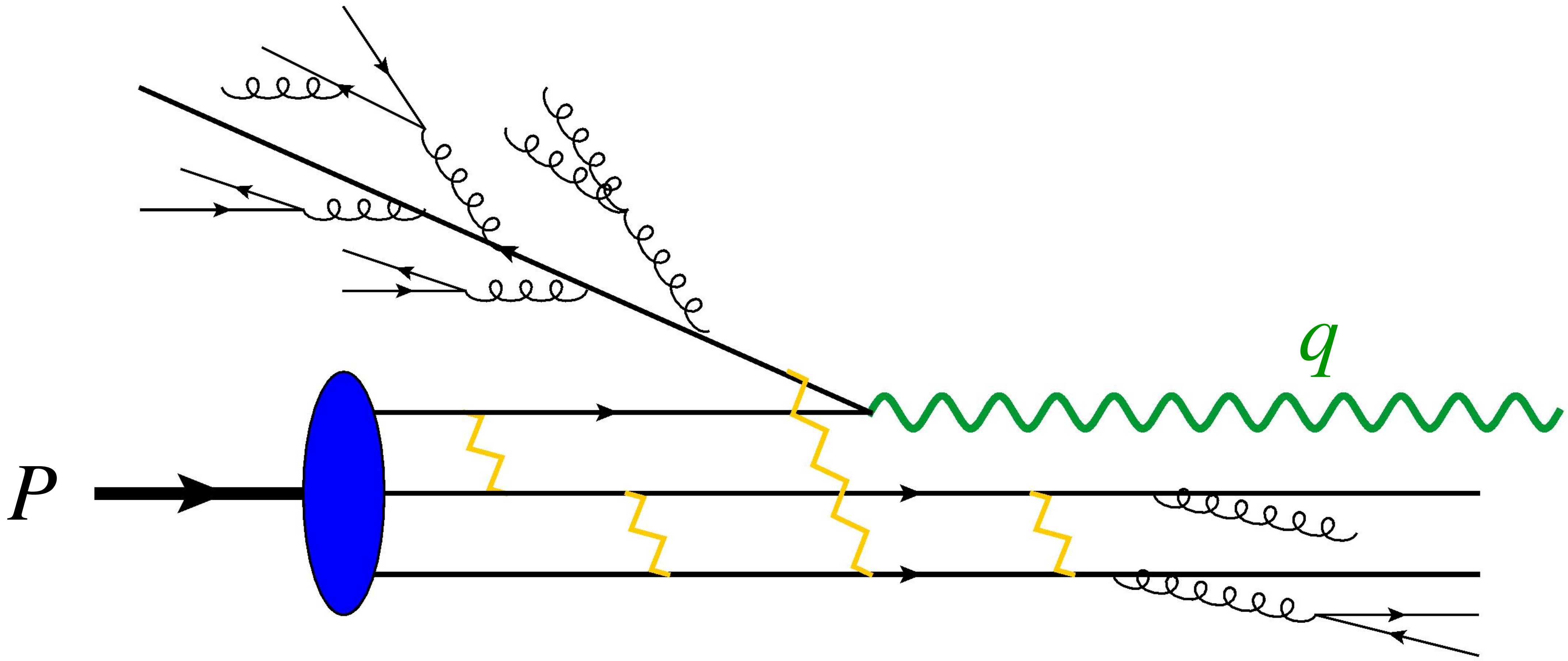
In photon-nucleon frame
nucleon moving forward



[Figure from JHEP10(2019)122]

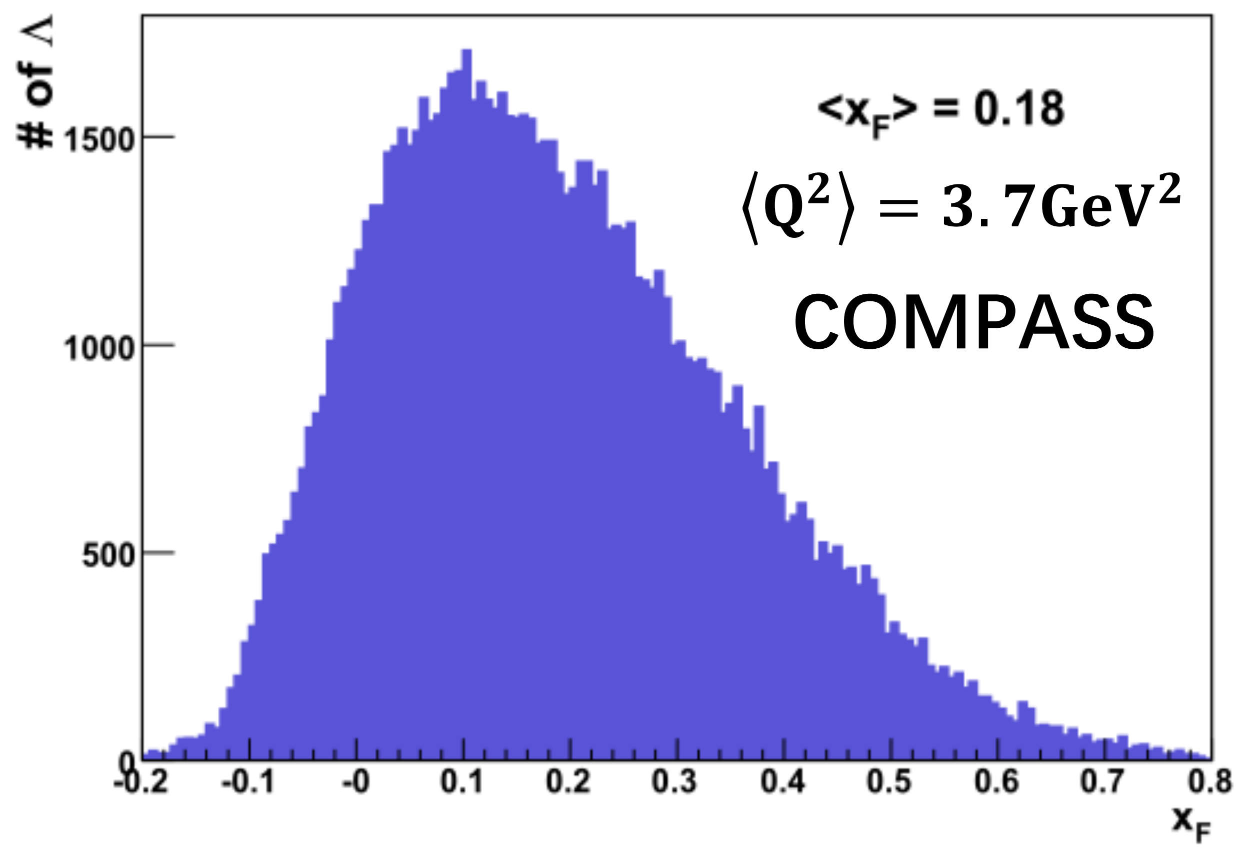
This is an ideal picture we imagined.

SIDIS Kinematic Regions



At high energy limit: enough phase space for shower, to match the quantum number of hadrons
Energy is mostly transferred to the struck quark, produced hadrons dominated by current fragmentation

Events Distribution



$$x_F = \frac{P_{hL}}{W}$$

In photon-nucleon frame, with nucleon moving backward.

No clear separation between current region and target region!

Some Attempt

"R" criteria:

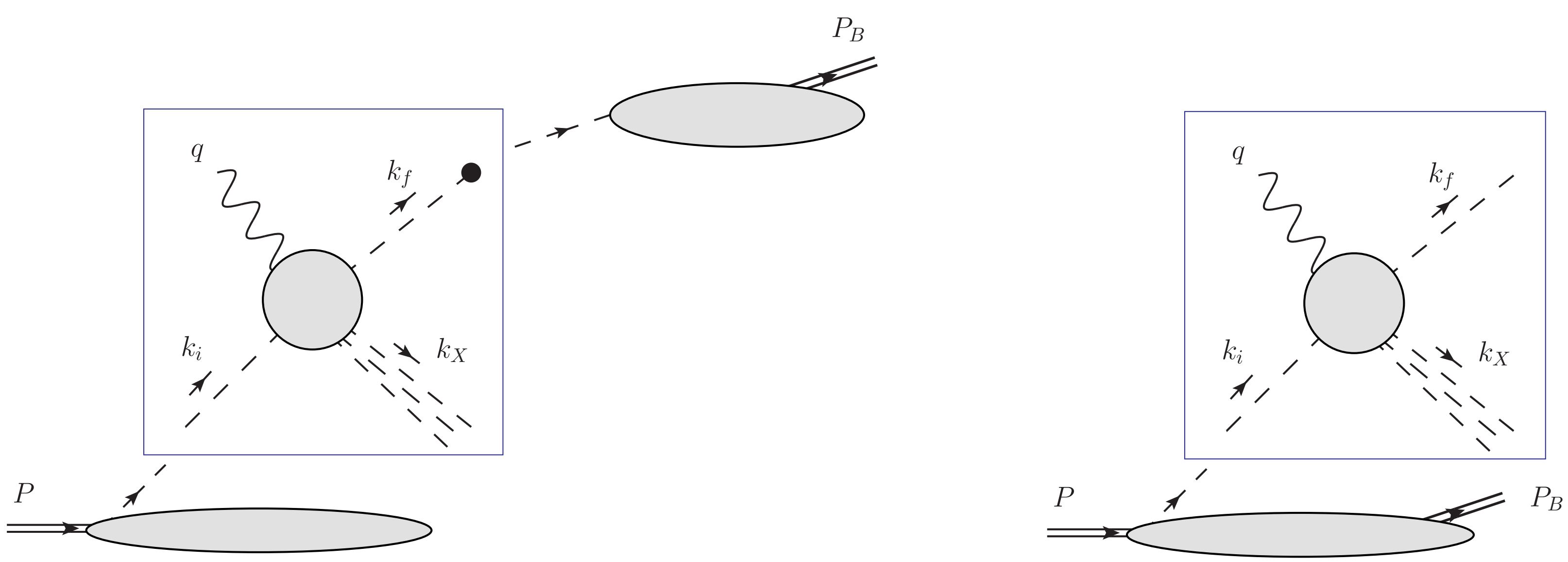
General hardness ratio: $$R_0 \equiv \max\left(\frac{k_i^2}{Q^2}, \frac{k_f^2}{Q^2}\right)$$

Collinearity ratio: $$R_1 \equiv \left|\frac{P_h \cdot k_f}{P_h \cdot k_i}\right|$$

R_1 small for current region
R_1^{-1} small for target region

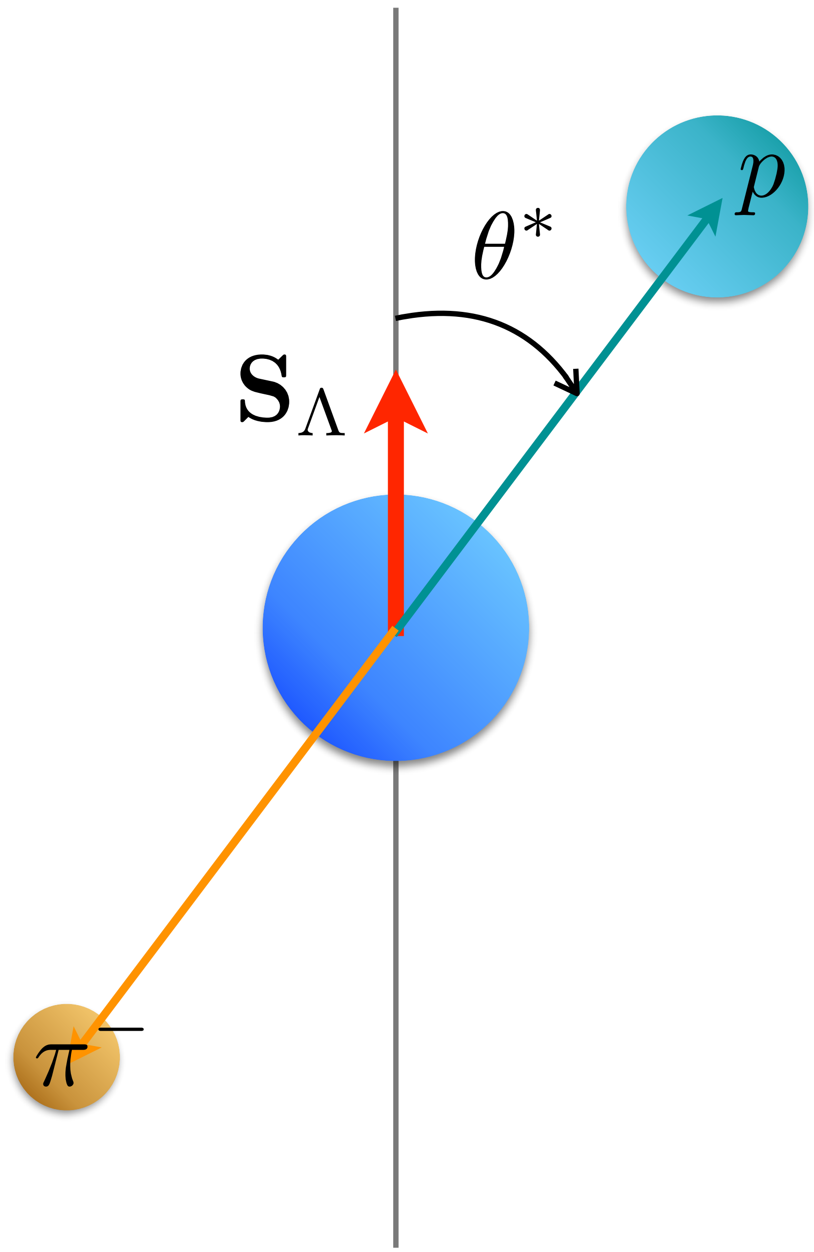
Transverse hardness ratio: $$R_2 \equiv \left|\frac{(k_f - q)^2}{Q^2}\right|$$

R_2 small for TMD region



M. Boglione, A. Dotson, L. Gamberg, S. Gordon, J. O. Gonzalez-Hernandez, A. Prokudin, T.C. Rogers, N. Sato, JHEP 10 (2019) 122.

Analyze Λ Polarization



Decay channel: $\Lambda \to p\pi^-$

branch ratio: $64.1 \pm 0.5\%$

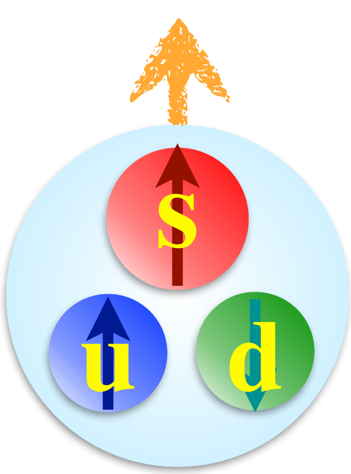
parity violating weak decay

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta^*} \propto \mathcal{A}\,(1 + \alpha_\Lambda P_\Lambda \cos\theta^*)$$

decay parameter: $\alpha_\Lambda = 0.748 \pm 0.007$

[Current PDG value]

Allow to measure the spin (polarization) of the produced Λ



valence component: $|uds\rangle$

spin dominated by s quark

Sensitive to nucleon strange sea and its polarization via SIDIS

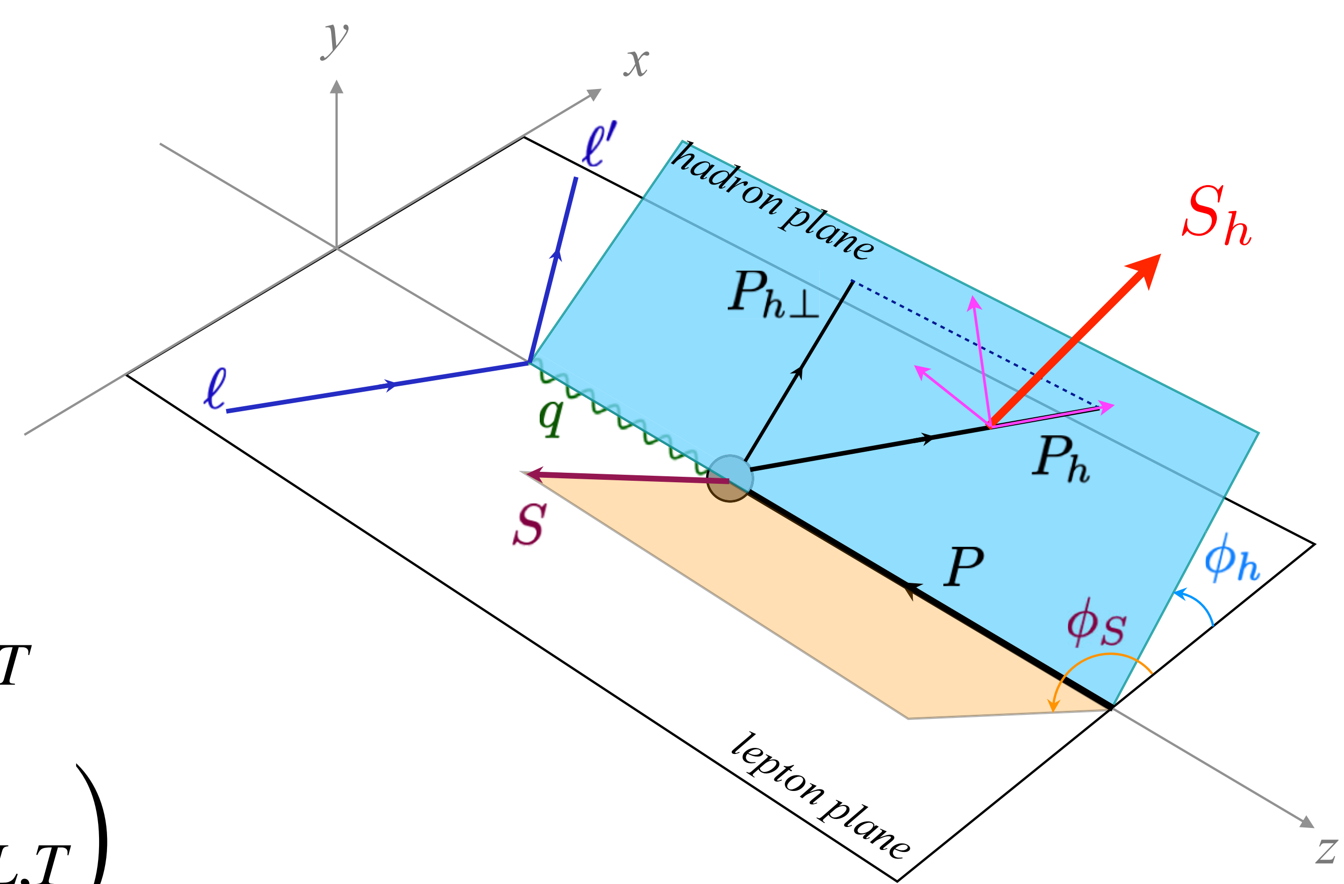
SIDIS with Polarized Λ

$$\frac{d\sigma}{dxdydzd\phi_h d\phi_s dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

$$\times\Big[\mathscr{F}_{U,U} + S_{hL}\mathscr{F}_{U,L} + \left|S_{hT}\right|\mathscr{F}_{U,T}$$

$$+S_L\left(\mathscr{F}_{L,U} + S_{hL}\mathscr{F}_{L,L} + \left|S_{hT}\right|\mathscr{F}_{L,T}\right)$$

$$+\left|S_T\right|\left(\mathscr{F}_{T,U} + S_{h_L}\mathscr{F}_{T,L} + \left|S_{hT}\right|\mathscr{F}_{T,T}\right)\Big]$$



$\mathscr{F}_{A,B}$ — A: target nucleon polarization; B: produced hadron (Λ) polarization

TMD Fragmentation Functions

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = Unpolarized		$H_1^{\perp}$ = − Collins
Polarized Hadrons	L		G_1 = − Helicity	$H_{1L}^{\perp}$ = − Worm-gear
Polarized Hadrons	T	$D_{1T}^{\perp}$ = − Sivers-type	$G_{1T}^{\perp}$ = − Worm-gear	H_1 = − Transversity $H_{1T}^{\perp}$ = − Pretzelosity

Hadron Spin Quark Spin [Figure from TMD Handbook]

Longitudinal Spin Transfer

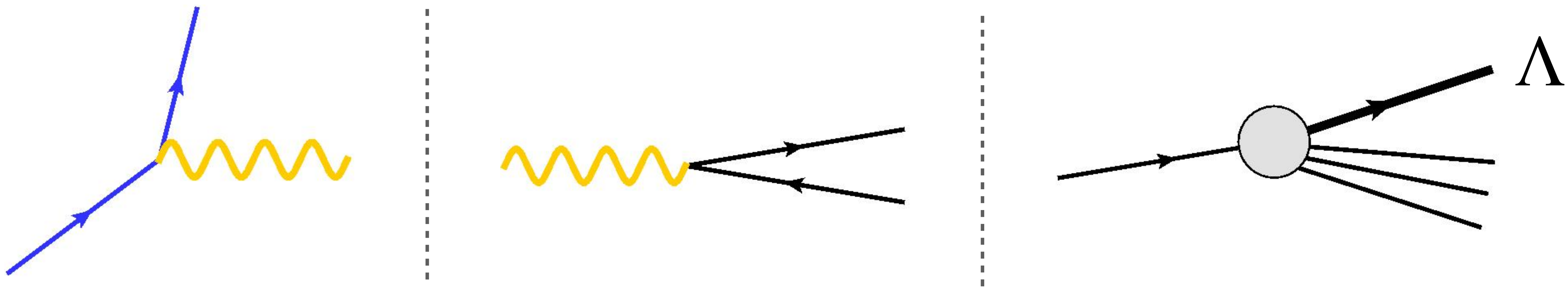
Longitudinal polarized lepton beam on unpolarized nucleon target

$$D_{LL}^{\Lambda} = \frac{G_{U,L}\left(x, Q^2, z\right)}{F_{U,U}\left(x, Q^2, z\right)}$$

$$G_{U,L} \sim f_1(x) \otimes G_1(z)$$

$$F_{U,L} \sim f_1(x) \otimes D_1(z)$$

Illustration of D_{LL}^{Λ} assuming current fragmentation:



emission of polarized photon

preference to pick quark with certain helicity

transfer to Λ polarization

Target vs. Current on D_{LL}^{Λ}

Questions:

Whether TF can compete with CF contributions?

What is the expected effect by TF on D_{LL} measurement?

Quark densities in the nucleon:

$u, d > \bar{u}, \bar{d} > s, \bar{s}$

Current fragmentation:

favored channels: u, d, s

In the target nucleon remnant:

- *ud, us, ds* pairs have better chance to produce Λ than *u, d, s*
- considering quark densities in the nucleon, *ud* pair is expected the dominant channel

X. Zhao, Z.-t. Liang, TL, Y.-j. Zhou, Phys. Rev. Lett. 134 (2025) 231901.

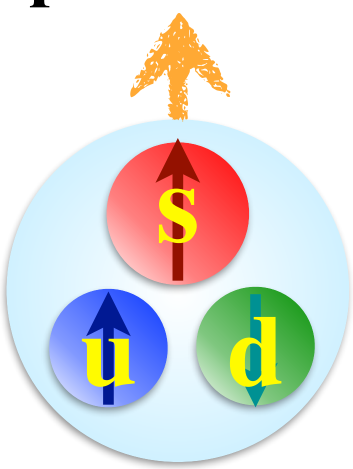
Target vs. Current on D_{LL}^{Λ}

Questions:

Whether TF can compete with CF contributions?

What is the expected effect by TF on D_{LL} measurement?

Spin-flavor wave function



$$\Lambda^{\uparrow} = \frac{1}{\sqrt{3}}(ud)_{0,0}s^{\uparrow} + \frac{1}{\sqrt{12}}(us)_{0,0}d^{\uparrow} - \frac{1}{\sqrt{12}}(ds)_{0,0}u^{\uparrow}$$
$$+\frac{1}{2}\left(\sqrt{\frac{2}{3}}(us)_{1,1}d^{\downarrow} - \sqrt{\frac{1}{3}}(us)_{1,0}d^{\uparrow}\right) - \frac{1}{2}\left(\sqrt{\frac{2}{3}}(ds)_{1,1}u^{\downarrow} - \sqrt{\frac{1}{3}}(ds)_{1,0}u^{\uparrow}\right)$$

In the target nucleon remnant:

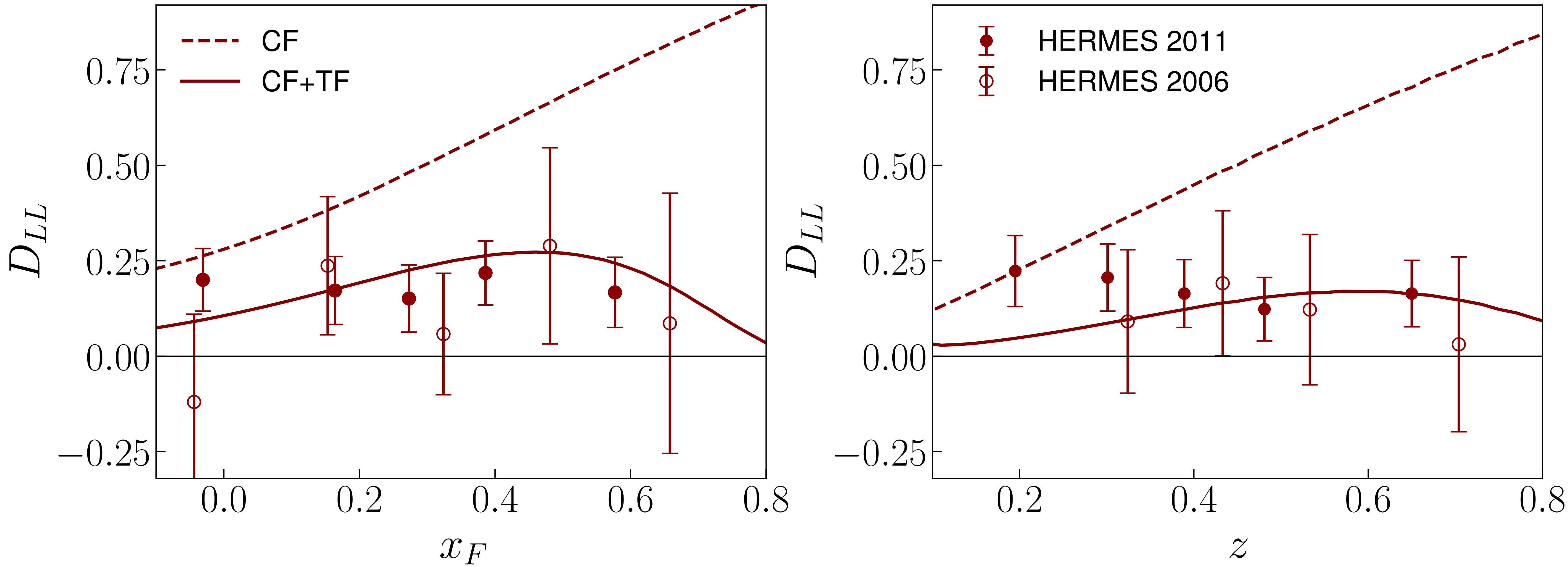
- the target remnant is not directly polarized by the virtual photon
- but can indirectly learn the polarization as constrained by the nucleon wave function
- *ud* pair in a isoscalar state is more likely to produce Λ and will be unpolarized

Expect: the target fragmentation will dilute/suppress the overall measured D_{LL}

X. Zhao, Z.-t. Liang, TL, Y.-j. Zhou, Phys. Rev. Lett. 134 (2025) 231901.

Spin Transfer Estimation

Compare with HERMES data



HERMES Collaboration, Phys. Rev. D 74 (2006) 072004; J. Phys. Conf. Ser. 295 (2011) 012114.

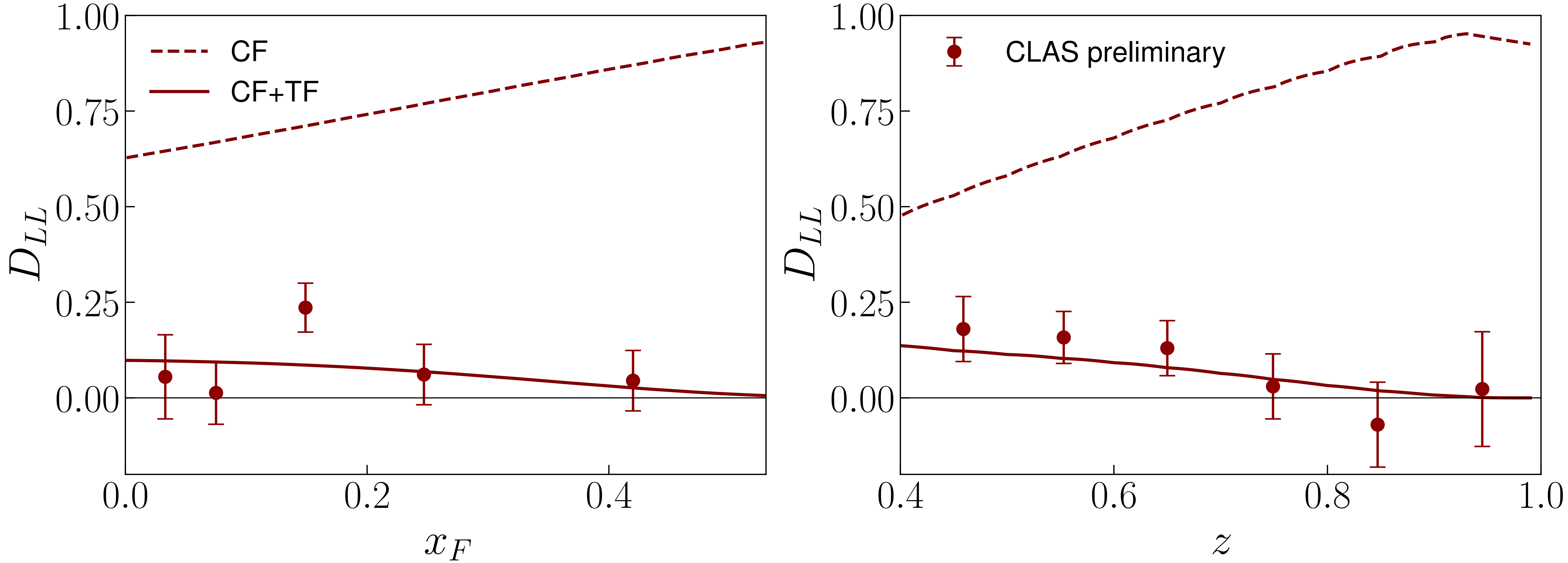
Longitudinal spin transfer D_{LL} to Λ is significantly suppressed by target fragmentation contributions, even at large x_F or z.

Including only the leading TF channel can describe the data well.

X. Zhao, Z.-t. Liang, TL, Y.-j. Zhou, Phys. Rev. Lett. 134 (2025) 231901.

Spin Transfer Estimation

Compare with JLab-CLAS data



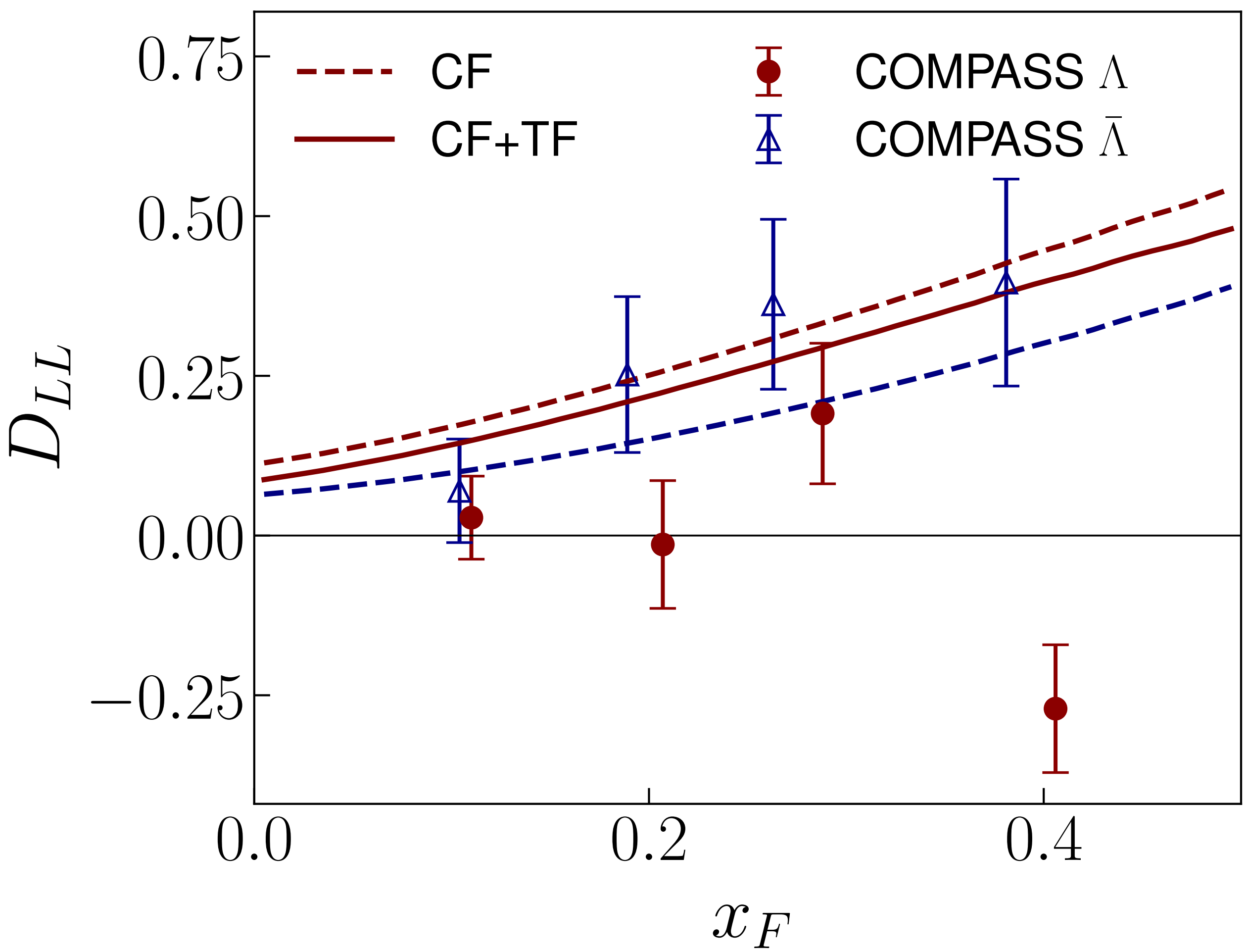
CLAS Collaboration, JPS Conf. Proc. 37 (2022) 020304.

At lower energies, the suppression effect becomes more significant.
Including only the leading TF channel can describe the data well.

X. Zhao, Z.-t. Liang, TL, Y.-j. Zhou, Phys. Rev. Lett. 134 (2025) 231901.

Spin Transfer Estimation

Compare with COMPASS data



COMPASS Collaboration, Eur. Phys. J. C 64 (2009) 171.

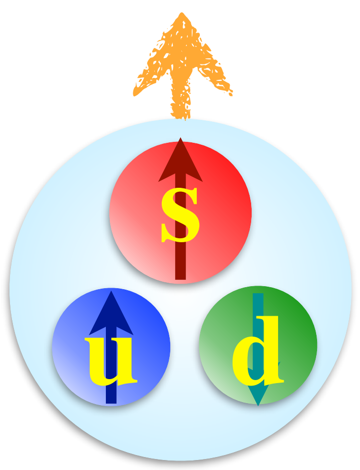
- The suppression effect reduces at higher energies, as expected.
- Including the leading TF channel alleviates the tension, but still deviates from data.
- Need detailed analysis including more channels.

- For $\bar{\Lambda}$ production, target fragmentation cannot compete with current fragmentation.
- Only current fragmentation can describe the data.

X. Zhao, Z.-t. Liang, TL, Y.-j. Zhou, Phys. Rev. Lett. 134 (2025) 231901.

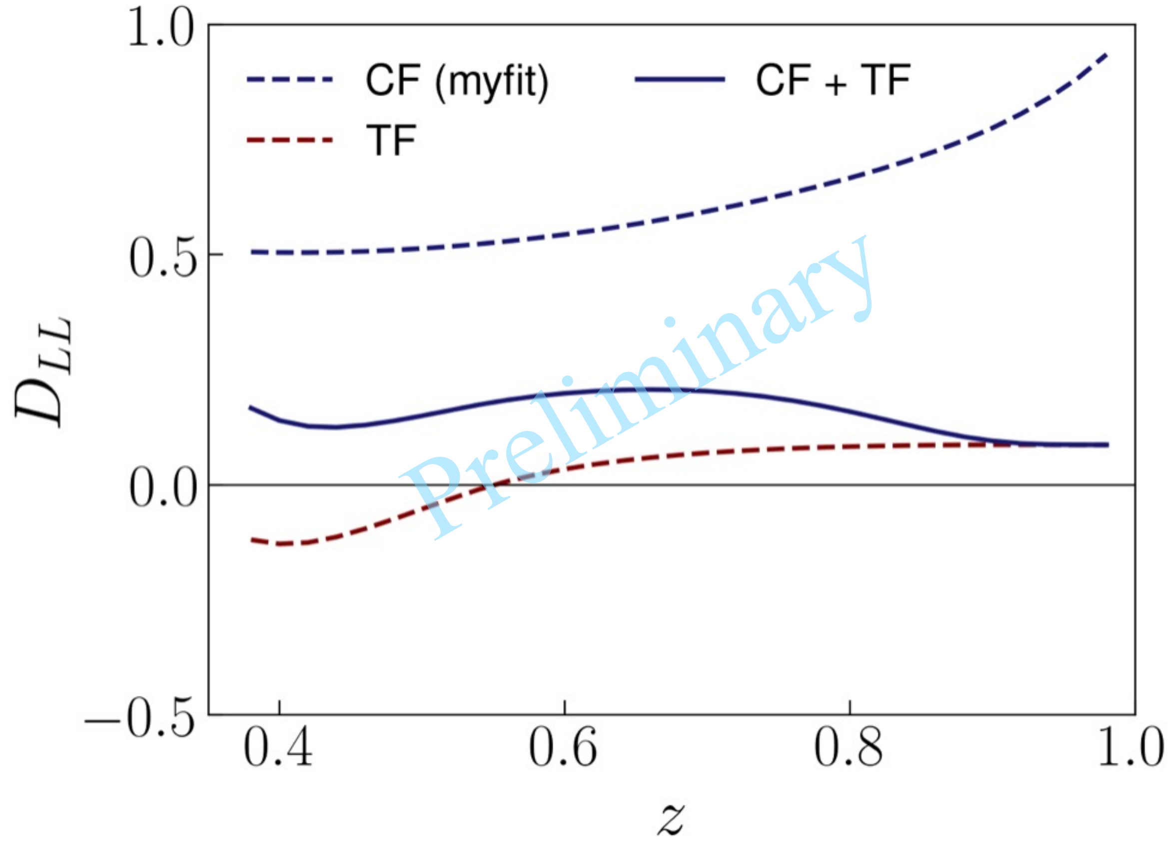
Spin Transfer to Σ Hyperon

Spin-flavor wave function



$$\Sigma^{0\uparrow} = \frac{1}{3}(ud)_{1,0}s^\uparrow - \frac{1}{6}(us)_{1,0}d^\uparrow - \frac{1}{6}(ds)_{1,0}u^\uparrow - \frac{\sqrt{2}}{3}(ud)_{1,1}s^\downarrow + \frac{1}{3\sqrt{2}}(us)_{1,1}d^\downarrow$$
$$+\frac{1}{3\sqrt{2}}(ds)_{1,1}u^\downarrow - \frac{1}{2}(us)_{0,0}d^\uparrow - \frac{1}{2}(ds)_{0,0}u^\uparrow$$

Longitudinal spin transfer



X. Zhao, TL, Y.-j. Zhou, 2025

Summary

- Deep inelastic scattering is a powerful tool to study parton distribution and fragmentation functions.
- At existing fixed-target SIDIS experiments, one cannot clearly separate the current and target fragmentation regions of Λ production events.
- Spin is a sensitive observable to deepen our understanding of hadronization mechanism.
- Target fragmentation plays an important role in understanding spin-related observables, such as the spin-transfer that has been measured by HERMES and COMPASS, JLab-CLAS.
- Global analysis of fragmentation functions including SIDIS data should carefully consider the target fragmentation effects.
- Electron-positron collision, free from target fragmentation, has unique advantage in measuring current fragmentation functions, although not enough for flavor separation.
- More opportunities at future facilities STCF, EicC, HIAF and other facilities worldwide.

Thank you!

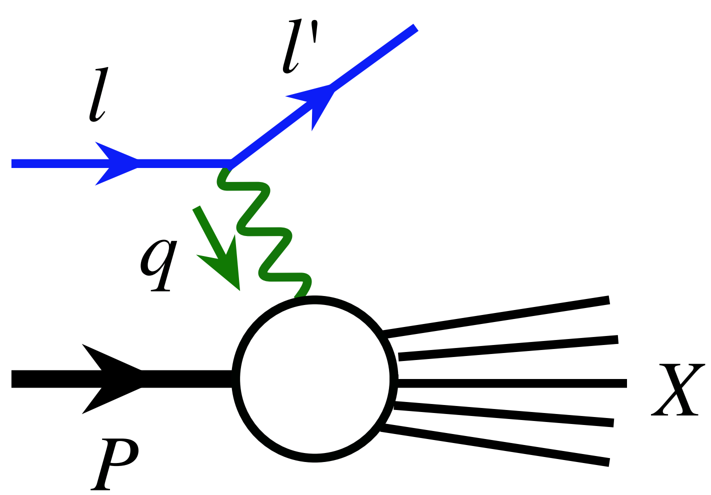
Lepton-Hadron Deep Inelastic Scattering

Inclusive DIS at a large momentum transfer

$Q \gg \Lambda_{\text{QCD}}$

Modern "Rutherford" experiment.

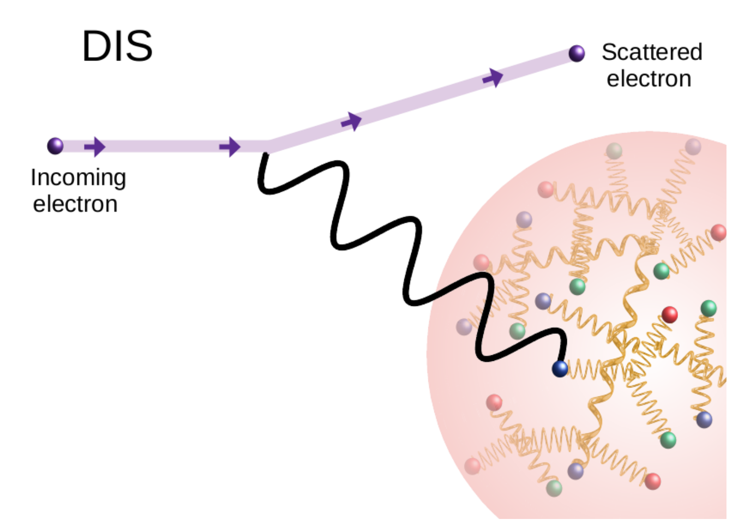
- dominated by the scattering of the lepton off an active quark/parton
- not sensitive to the dynamics at a hadronic scale ~ 1/fm
- collinear factorization: $\sigma \propto H(Q) \otimes f(x, \mu)$
- overall corrections suppressed by $1/Q^n$



- indirectly "see" quarks, gluons and their dynamics
- predictive power relies on

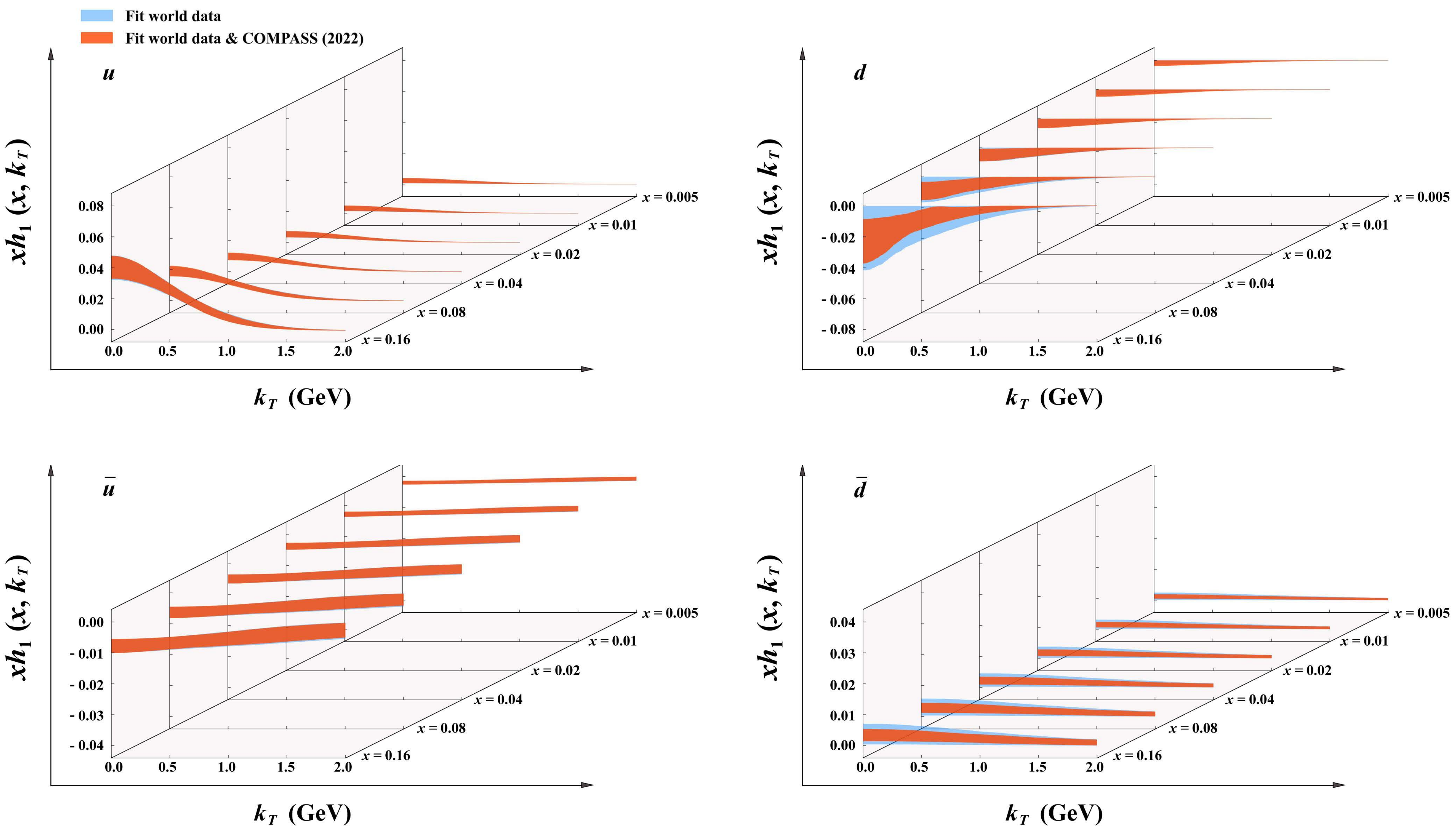
— precision of the probe

— universality of $f(x, \mu)$



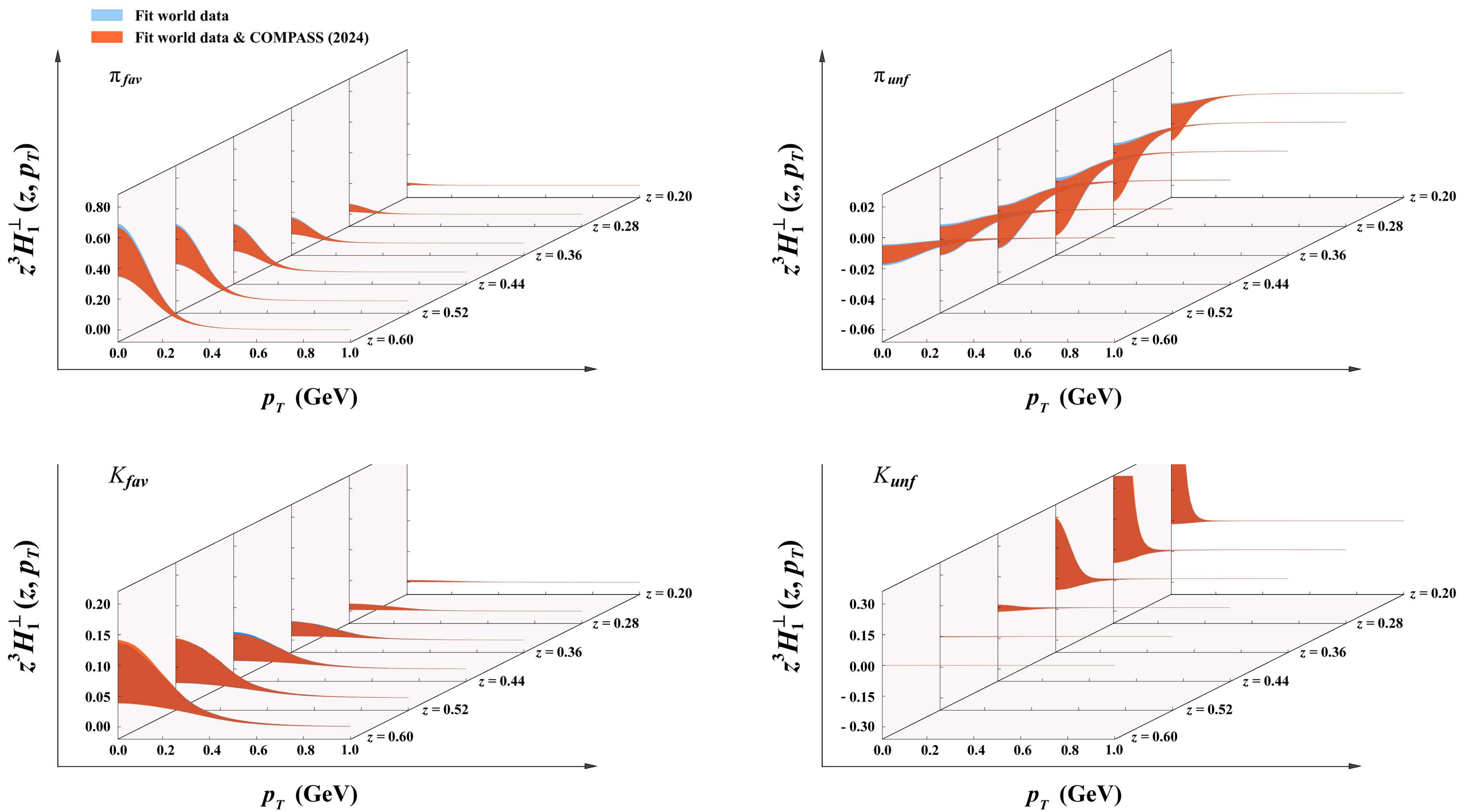
[Figure from DESY-21-099]

Transversity Distributions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Collins Fragmentation Functions



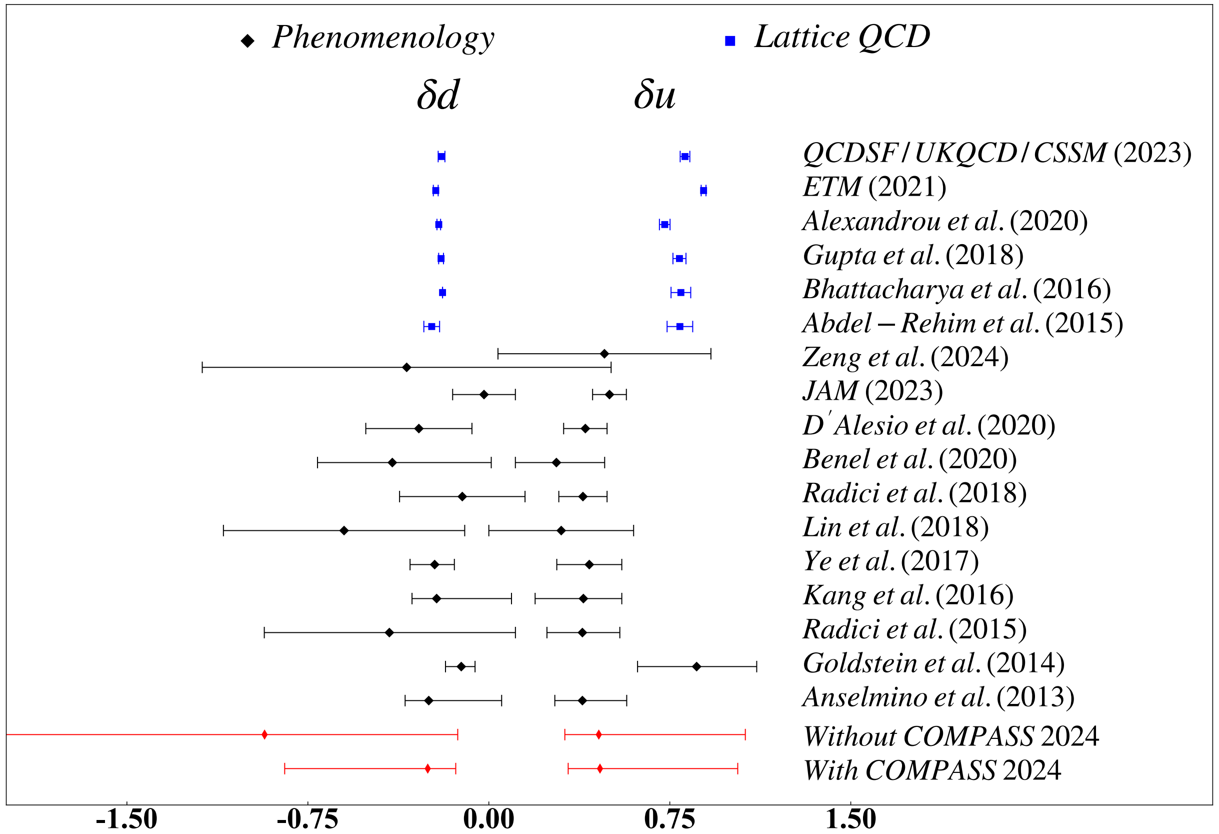
C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

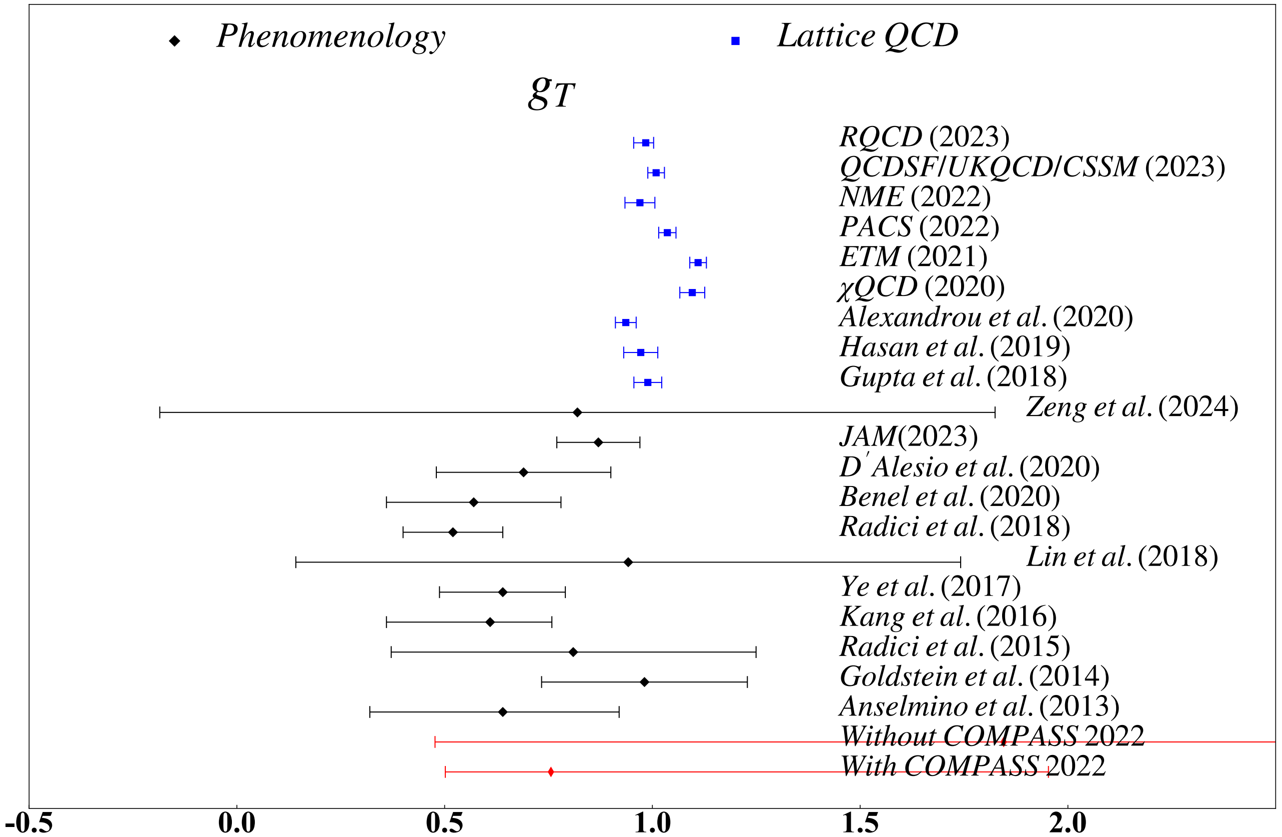
Tensor Charge

Tensor charge

$$\langle P,S|\bar{\psi}^q i\sigma^{\mu\nu}\gamma_5\psi^q|P,S\rangle = g_T^q \bar{u}(P,S) i\sigma^{\mu\nu}\gamma_5 u(P,S) \qquad g_T^q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)]\,dx$$

- A fundamental QCD quantity: matrix element of local operators.
- Moment of the transversity distribution: valence quark dominant.
- Calculable in lattice QCD.





Larger uncertainties when including anti-quarks (less biased)

Compatible with lattice QCD calculations

C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Kinematic Analysis

Basis tensors:

$$h_U^{S\mu\nu} = \left\{ \tilde{g}^{\mu\nu}, \tilde{P}^\mu \tilde{P}^\nu, \tilde{P}^{\{\mu} \tilde{P}_h^{\nu\}}, \tilde{P}_h^\mu \tilde{P}_h^\nu, \epsilon^{\{\mu q P P_h} \tilde{P}^{\nu\}}, \epsilon^{\{\mu q P P_h} \tilde{P}_h^{\nu\}} \right\}$$

$$h_U^{A\mu\nu} = \left\{ \tilde{P}^{[\mu} \tilde{P}_h^{\nu]}, \epsilon^{\mu\nu q P}, \epsilon^{\mu\nu q P_h} \right\}$$

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

$$\tilde{P}^\mu = P^\mu - \frac{P \cdot q}{q^2} q^\mu$$

Polarization dependent scalars and pseudoscalars:

$$\left\{ \epsilon^{P P_h q S}, P_h \cdot S, q \cdot S \right\} \qquad \left\{ \epsilon^{P P_h q S_h}, P \cdot S_h, q \cdot S_h \right\}$$

$$\Big\{ S \cdot S_h, \left(P \cdot S_h\right)\left(P_h \cdot S\right), \left(P_h \cdot S\right)\left(q \cdot S_h\right), \left(P \cdot S_h\right)(q \cdot S), \left(q \cdot S_h\right)(q \cdot S),$$

$$\epsilon^{P P_h S S_h}, \epsilon^{P q S S_h}, \epsilon^{P_h q S S_h}, \epsilon^{P P_h q S_h}(S \cdot q) \Big\}$$

Combination of basis tensors and spin-dependent scalars/pseudoscalars gives the complete set.

K.-b. Chen, W.-h. Yang, S.-y. Wei, Z.-t. Liang, Phys. Rev. D 94 (2016) 034003;
J. Zhao, Z. Zhang, TL, Z.-t. Liang, Y.-j. Zhou, Phys. Rev. D 109 (2024) 074017.

SIDIS Cross Section with Polarized Λ

Longitudinally polarized Λ

unpolarized nucleon:

$$\mathscr{F}_{U,L} = \sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{U,L}^{\sin\phi_h} + \varepsilon\sin 2\phi_h F_{U,L}^{\sin 2\phi_h} + \lambda_e\left(\sqrt{1-\varepsilon^2}G_{U,L} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h G_{U,L}^{\cos\phi_h}\right)$$ 4

longitudinally polarized nucleon:

$$\mathscr{F}_{L,L} = F_{L,L}^{T} + \varepsilon F_{L,L}^{L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{L,L}^{\cos\phi_h} + \varepsilon\cos 2\phi_h F_{L,L}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h G_{L,L}^{\sin\phi_h}$$ 5

transversally polarized nucleon:

$$\begin{aligned}\mathscr{F}_{T,L} = {} & \cos(\phi_h-\phi_T)\left(F_{T,L}^{T\cos(\phi_h-\phi_T)} + \varepsilon F_{T,L}^{L\cos(\phi_h-\phi_T)}\right)\\ & + \sqrt{2\varepsilon(1+\varepsilon)}\left(\cos\phi_T F_{T,L}^{\cos\phi_T} + \cos(2\phi_h-\phi_T) F_{T,L}^{\cos(2\phi_h-\phi_T)}\right)\\ & + \varepsilon\left(\cos(\phi_h+\phi_T) F_{T,L}^{\cos(\phi_h+\phi_T)} + \cos(3\phi_h-\phi_T) F_{T,L}^{\cos(3\phi_h-\phi_T)}\right)\\ & + \lambda_e\left[\sqrt{1-\varepsilon^2}\sin(\phi_h-\phi_T) G_{T,L}^{\sin(\phi_h-\phi_T)}\right.\\ & \left. + \sqrt{2\varepsilon(1-\varepsilon)}\left(\sin\phi_T G_{T,L}^{\sin\phi_T} + \sin(2\phi_h-\phi_T) G_{T,L}^{\sin(2\phi_h-\phi_T)}\right)\right]\end{aligned}$$ 9

J. Zhao, Z. Zhang, TL, Z.-t. Liang, Y.-j. Zhou, Phys. Rev. D 109 (2024) 074017

SIDIS Cross Section with Polarized Λ

Transversally polarized Λ

unpolarized nucleon:

$$\begin{aligned}\mathscr{F}_{U,T} &= \sin\phi_{hT}\left(F_{U,T}^{T\sin\phi_{hT}} + \varepsilon F_{U,T}^{L\sin\phi_{hT}}\right)\\ &+\sqrt{2\varepsilon(1+\varepsilon)}\left(\sin\left(\phi_h-\phi_{hT}\right)F_{U,T}^{\sin(\phi_h-\phi_hT)} + \sin\left(\phi_h+\phi_{hT}\right)F_{U,T}^{\sin(\phi_h+\phi_{hT})}\right)\\ &+\varepsilon\left(\sin\left(2\phi_h-\phi_{hT}\right)F_{U,T}^{\sin(2\phi_h-\phi_{hT})} + \sin\left(2\phi_h+\phi_{hT}\right)F_{U,T}^{\sin(2\phi_h+\phi_{hT})}\right)\\ &+\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos\phi_{hT}G_{U,T}^{\cos\phi_{hT}}\\ &+\sqrt{2\varepsilon(1-\varepsilon)}\left(\cos\left(\phi_h-\phi_{hT}\right)G_{U,T}^{\cos(\phi_h-\phi_{hT})} + \cos\left(\phi_h+\phi_{hT}\right)G_{U,T}^{\cos(\phi_h+\phi_{hT})}\right)\Big]\end{aligned}$$

9

longitudinally polarized nucleon:

$$\begin{aligned}\mathscr{F}_{L,T} &= \cos\phi_{hT}\left(F_{L,T}^{T\cos\phi_{hT}} + \varepsilon F_{L,T}^{L\cos\phi_{hT}}\right)\\ &+\sqrt{2\varepsilon(1+\varepsilon)}\left(\cos\left(\phi_h-\phi_{hT}\right)F_{L,T}^{\cos(\phi_h-\phi_hT)} + \cos\left(\phi_h+\phi_{hT}\right)F_{L,T}^{\cos(\phi_h+\phi_{hT})}\right)\\ &+\varepsilon\left(\cos\left(2\phi_h-\phi_{hT}\right)F_{L,T}^{\cos(2\phi_h-\phi_hT)} + \cos\left(2\phi_h+\phi_{hT}\right)F_{L,T}^{\cos(2\phi_h+\phi_{hT})}\right)\\ &+\lambda_e\Big[\sqrt{1-\varepsilon^2}\sin\phi_{hT}G_{L,T}^{\sin\phi_hT}\\ &+\sqrt{2\varepsilon(1-\varepsilon)}\left(\sin\left(\phi_h-\phi_{hT}\right)G_{L,T}^{\sin(\phi_h-\phi_hT)} + \sin\left(\phi_h+\phi_{hT}\right)G_{L,T}^{\sin(\phi_h+\phi_hT)}\right)\Big]\end{aligned}$$

9

J. Zhao, Z. Zhang, TL, Z.-t. Liang, Y.-j. Zhou, Phys. Rev. D 109 (2024) 074017

SIDIS Cross Section with Polarized Λ

Transversally polarized Λ

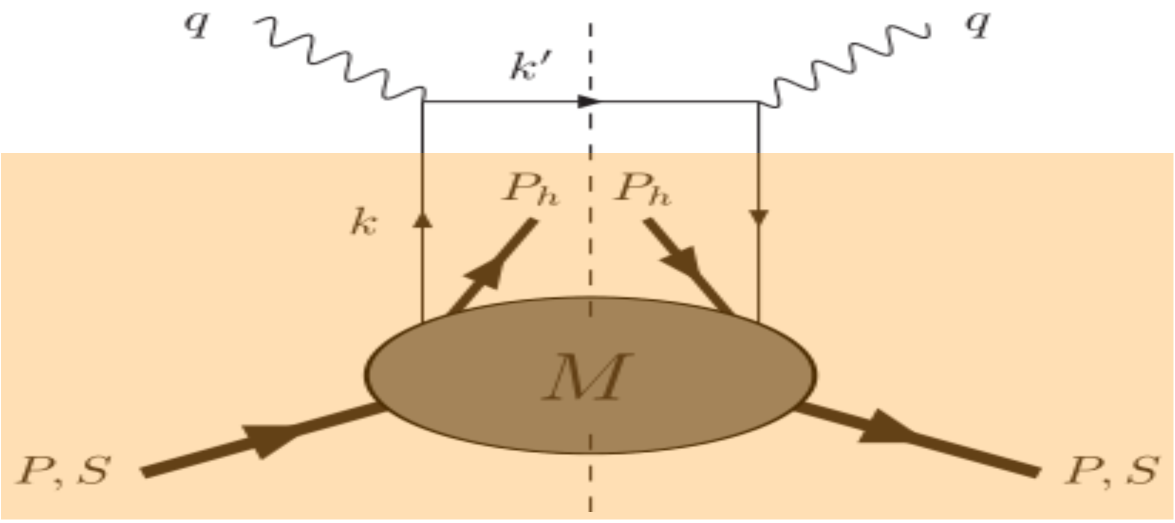
transversally polarized nucleon:

$$
\begin{aligned}
\mathscr{F}_{T,T} = {} & \cos\left(\phi_h - \phi_{hT} - \phi_T\right)\left(F_{T,T}^{T\cos(\phi_h - \phi_{hT} - \phi_T)} + \varepsilon F_{T,T}^{L\cos(\phi_h - \phi_{hT} - \phi_T)}\right) \\
& + \cos\left(\phi_h + \phi_{hT} - \phi_T\right)\left(F_{T,T}^{T\cos(\phi_h + \phi_{hT} - \phi_T)} + \varepsilon F_{T,T}^{L\cos(\phi_h + \phi_{hT} - \phi_T)}\right) \\
& + \sqrt{2\varepsilon(1+\varepsilon)}\left(\cos\left(\phi_{hT} - \phi_T\right) F_{T,T}^{\cos(\phi_{hT} - \phi_T)} + \cos\left(\phi_{hT} + \phi_T\right) F_{T,T}^{\cos(\phi_{hT} + \phi_T)}\right. \\
& \left. + \cos\left(2\phi_h - \phi_{hT} - \phi_T\right) F_{T,T}^{\cos(2\phi_h - \phi_{hT} - \phi_T)} + \cos\left(2\phi_h + \phi_{hT} - \phi_T\right) F_{T,T}^{\cos(2\phi_h + \phi_{hT} - \phi_T)}\right) \\
& + \varepsilon\left(\cos\left(\phi_h - \phi_{hT} + \phi_T\right) F_{T,T}^{\cos(\phi_h - \phi_h + \phi_T)} + \cos\left(\phi_h + \phi_{hT} + \phi_T\right) F_{T,T}^{\cos(\phi_h + \phi_{hT} + \phi_T)}\right. \\
& \left. + \cos\left(3\phi_h - \phi_{hT} - \phi_T\right) F_{T,T}^{\cos(3\phi_h - \phi_{hT} - \phi_T)} + \cos\left(3\phi_h + \phi_{hT} - \phi_T\right) F_{T,T}^{\cos(3\phi_h + \phi_{hT} - \phi_T)}\right) \\
& + \lambda_e \left[\sqrt{2\varepsilon(1-\varepsilon)}\left(\sin\left(\phi_{hT} - \phi_T\right) G_{T,T}^{\sin(\phi_h - \phi_T)} + \sin\left(\phi_{hT} + \phi_T\right) G_{T,T}^{\sin(\phi_{hT} + \phi_T)}\right.\right. \\
& \left. + \sin\left(2\phi_h - \phi_{hT} - \phi_T\right) G_{T,T}^{\sin(2\phi_h - \phi_{hT} - \phi_T)} + \sin\left(2\phi_h + \phi_{hT} - \phi_T\right) G_{T,T}^{\sin(2\phi_h + \phi_{hT} - \phi_T)}\right) \\
& \left. + \sqrt{1-\varepsilon^2}\left(\sin\left(\phi_h - \phi_{hT} - \phi_T\right) G_{T,T}^{\sin(\phi_h - \phi_{hT} - \phi_T)} + \sin\left(\phi_h + \phi_{hT} - \phi_T\right) G_{T,T}^{\sin(\phi_h + \phi_{hT} - \phi_T)}\right)\right]
\end{aligned}
\tag{18}
$$

J. Zhao, Z. Zhang, TL, Z.-t. Liang, Y.-j. Zhou, Phys. Rev. D 109 (2024) 074017

Fracture Functions

Correlation function



$$\mathscr{M}_{ij}\left(k;P,S;P_\Lambda,S_\Lambda\right) = \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{ik\xi} \langle P,S\,|\,\bar{\Psi}_j(0)\,\big|\,P_\Lambda,S_\Lambda;X\rangle\,\langle P_\Lambda,S_\Lambda;X\big|\,\Psi_i(\xi)\,|\,P,S\rangle$$

decomposition with Dirac matrices:

$$\mathscr{M} = \frac{1}{2}\left(\mathscr{S}I + \mathscr{V}_\mu\gamma^\mu + \mathscr{A}_\mu\gamma^\mu\gamma_5 + i\mathscr{P}\gamma_5 + i\mathscr{T}_{\mu\nu}\sigma^{\mu\nu}\gamma_5\right)$$

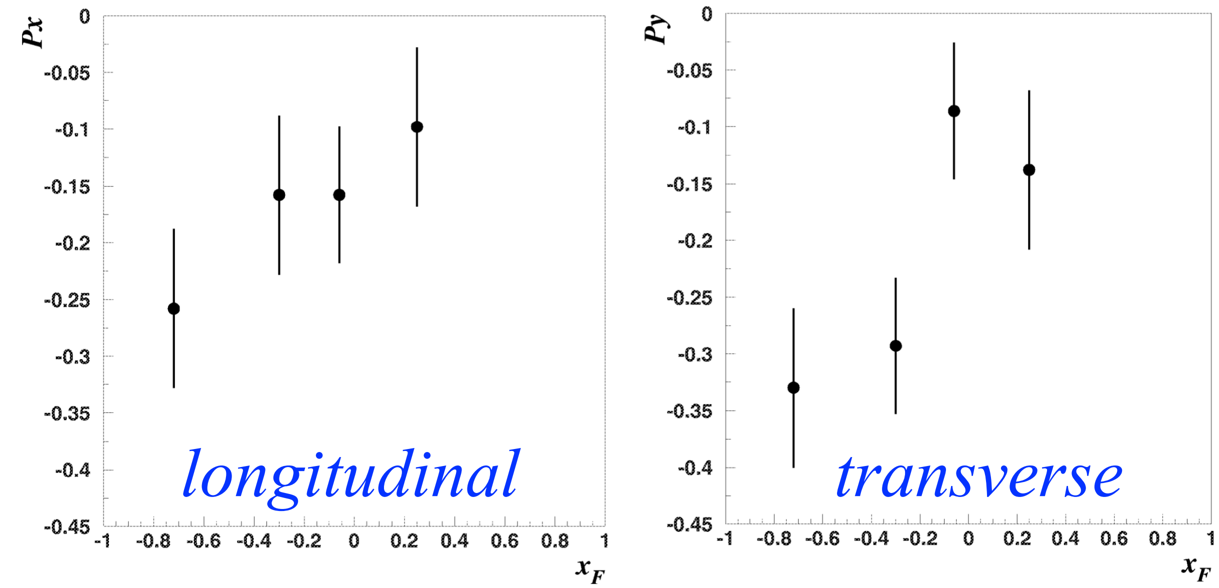
leading-twist terms can be projected by: $\gamma^+,\quad \gamma^+\gamma_5,\quad i\sigma^{i+}\gamma_5$

M. Anselmino, V. Barone, A. Kotzinian, Phys. Lett. B 699 (2011) 108.

Λ Polarization Lepton-Hadron Scattering

Neutrino scattering experiments

G.T. Jones *et al.* Birmingham-Bonn-CERN-London-Munich-Oxford, Z. Phys. C 28 (1985) 23.
WA59 Collaboration, Z. Phys. C 53 (1992) 207.
E632 Phys. Rev. D 50 (1994) 6691.
NOMAD Collaboration, Nucl. Phys. B 588 (2000) 3.
NOMAD Collaboration, Nucl. Phys. B 605 (2001) 3.



NOMAD Collaboration, Nucl. Phys. B 588 (2000) 3.

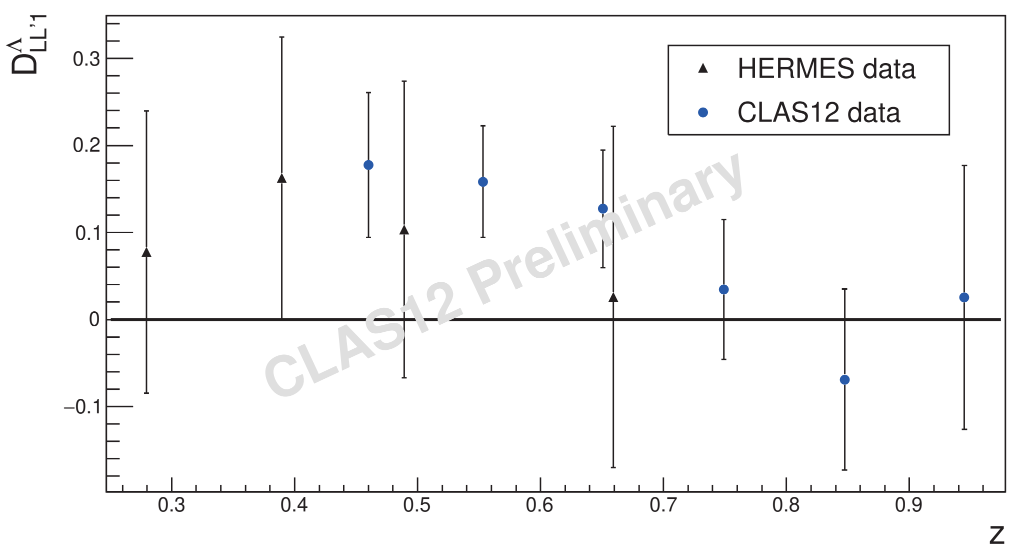
TABLE VI. Λ Polarization. The x axis is along the W boson direction in the Λ rest frame. The other two axes are perpendicular to the W, with the y axis being transverse to the production plane, and the z axis inside the production plane.

Reaction [Ref.]	P_x	P_y	P_z
ν-Ne [This expt.]	-0.38 ± 0.16	-0.04 ± 0.17	-0.17 ± 0.18
$\bar{\nu}$-Ne [15]	-0.56 ± 0.13	-0.02 ± 0.13	0.08 ± 0.13
ν-p [26]	-0.10 ± 0.14	-0.02 ± 0.16	0.12 ± 0.15
$\bar{\nu}$-p [26]	-0.24 ± 0.17	-0.05 ± 0.16	-0.20 ± 0.17
ν-Ne, $x_F < 0$ [This expt.]	-0.43 ± 0.20	-0.06 ± 0.19	-0.45 ± 0.19
$\bar{\nu}$-Ne, $x_F < 0$ [15]	-0.63 ± 0.13	-0.02 ± 0.14	0.12 ± 0.14
ν-p, $x_F < 0$ [26]	-0.29 ± 0.18	-0.09 ± 0.19	0.19 ± 0.18
$\bar{\nu}$-p, $x_F < 0$ [26]	-0.38 ± 0.18	0.02 ± 0.18	-0.17 ± 0.18
K^0 control sample [This expt.]	-0.02 ± 0.08	0.10 ± 0.08	-0.10 ± 0.08

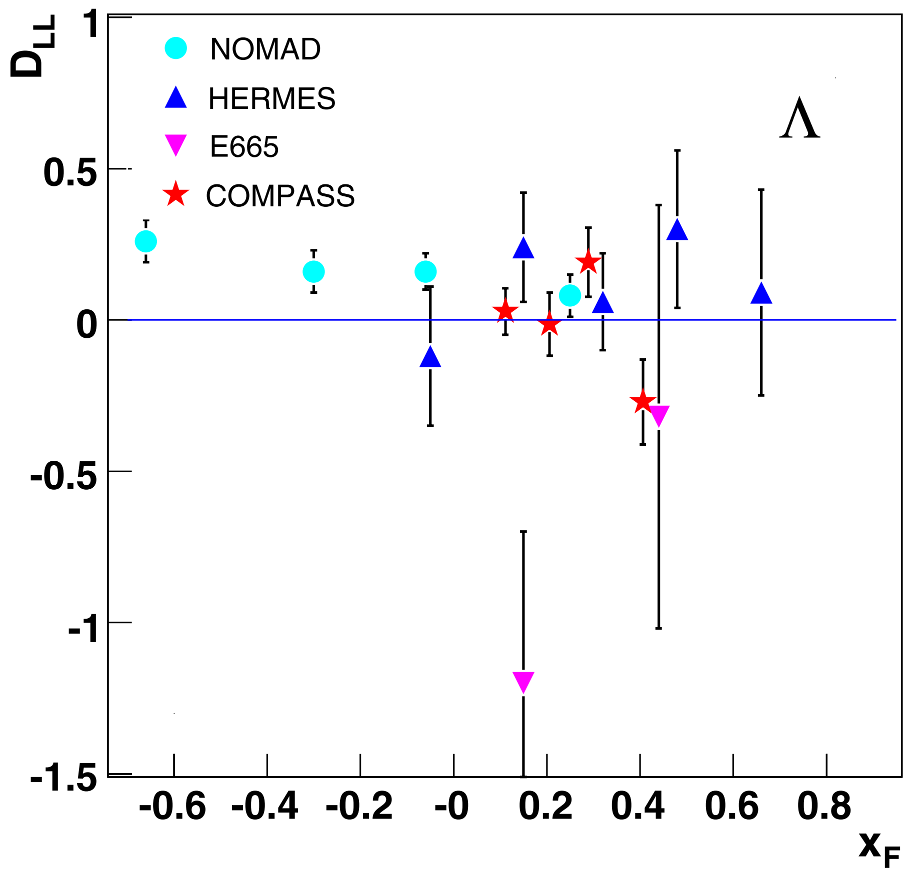
E632 Phys. Rev. D 50 (1994) 6691.

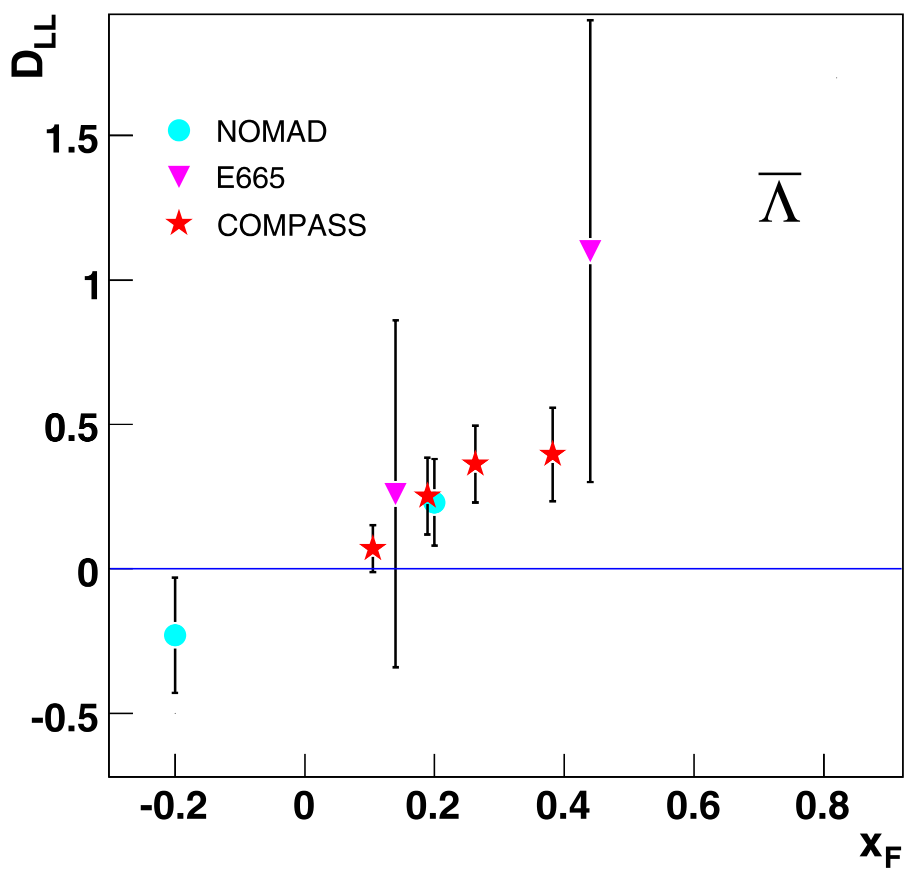
Spin Transfer with Polarized e/μ Beam

Longitudinal polarized e/μ beam on unpolarized target



E665 Collaboration, Eur. Phys. J. C 17 (2000) 263.
HERMES Collaboration, Phys. Rev. D 74 (2006) 072004.
COMPASS Collaboration, Eur. Phys. J. C 64 (2009) 171.
CLAS Collaboration, JPS conf. Proc. 37 (2022) 020304.





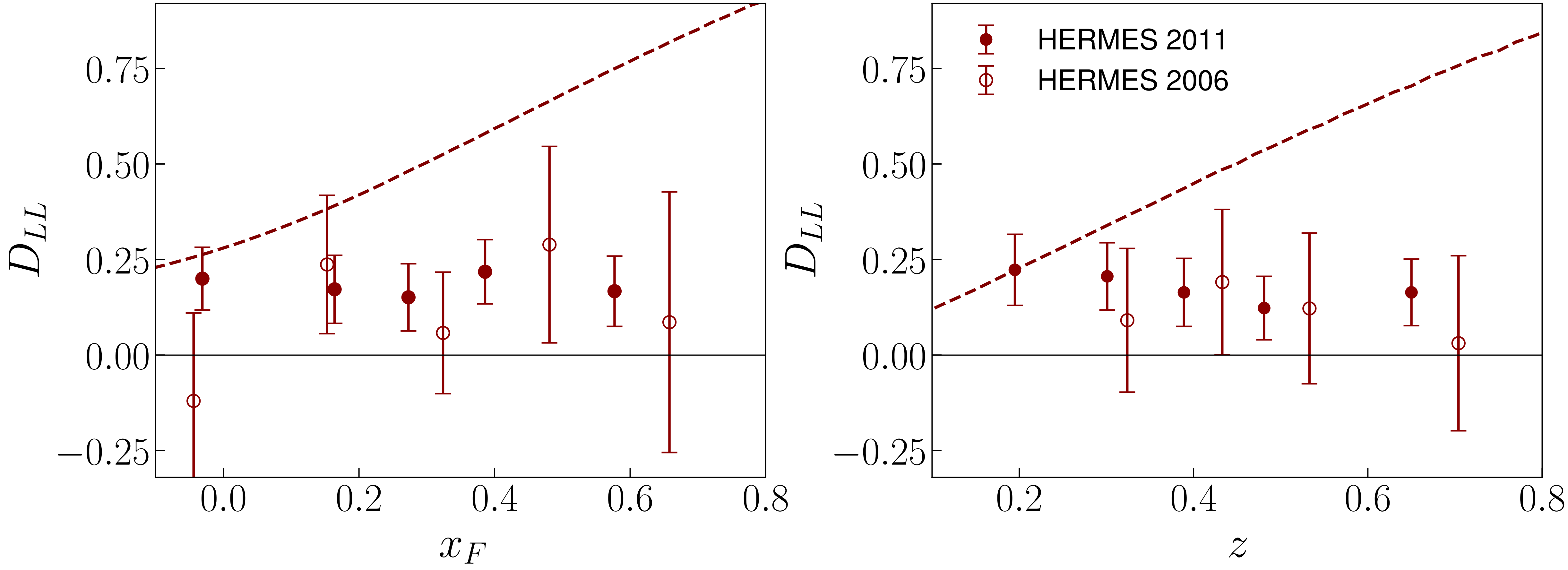
Longitudinal Spin Transfer

Longitudinal polarized lepton beam on unpolarized nucleon target

$$D_{LL}^{\Lambda} = \frac{G_{U,L}\left(x, Q^2, z\right)}{F_{U,U}\left(x, Q^2, z\right)}$$

$$G_{U,L} \sim f_1(x) \otimes G_1(z)$$

$$F_{U,L} \sim f_1(x) \otimes D_1(z)$$



Theory curves based on JR14 PDF parametrization and DSV FF

Tension between data and theory with current fragmentation mechanism

Structure Functions in Target Region

$$F_{U,U}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} M_{UU}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$F_{L,L}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} M_{LL}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$F_{T,T}^{\cos(\phi_h-\phi_T)} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\left[M_{TT}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right) + \frac{P_{h\perp}^2}{m_h^2} M_{TT}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)\right]$$

$$F_{T,T}^{T\cos(2\phi_h-\phi_T-\phi_{hT})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}^2}{m_h^2} M_{TT}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$F_{T,U}^{T\sin(\phi_h-\phi_T)} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{TU}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$F_{U,T}^{T\sin(\phi_h-\phi_{hT})} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{UT}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$F_{L,T}^{\cos(\phi_h-\phi_T)} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{LT}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$F_{T,L}^{\cos(\phi_h-\phi_{hT})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{TL}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$G_{L,U}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} \Delta M_{LU}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$G_{U,L}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} \Delta M_{UL}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$G_{T,U}^{\cos(\phi_h-\phi_T)} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{TU}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$G_{U,T}^{\cos(\phi_h-\phi_{hT})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{UT}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$G_{L,T}^{T\sin(\phi_h-\phi_h T)} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{LT}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$G_{T,L}^{T\sin(\phi_h-\phi_T)} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{TL}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$G_{T,T}^{\sin(\phi_T-\phi_h T)} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\left[\Delta M_{TT}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right) + \frac{P_{h\perp}^2}{m_h^2} \Delta M_{TT}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)\right]$$

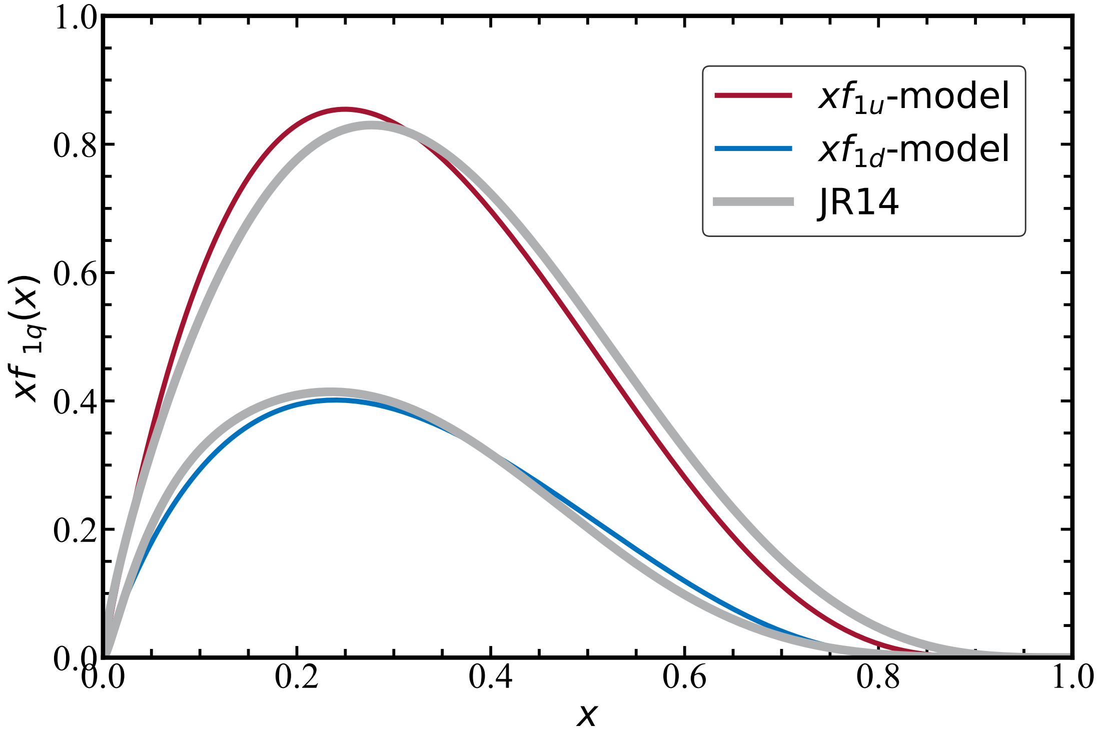
$$G_{T,T}^{\sin(2\phi_h-\phi_T-\phi_{h\tau})} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}^2}{m_h^2} \Delta M_{TT}^h\left(x, \zeta, \mathbf{P}_{h\perp}^2\right)$$

$$\zeta = \frac{P_h^+}{P^+}$$

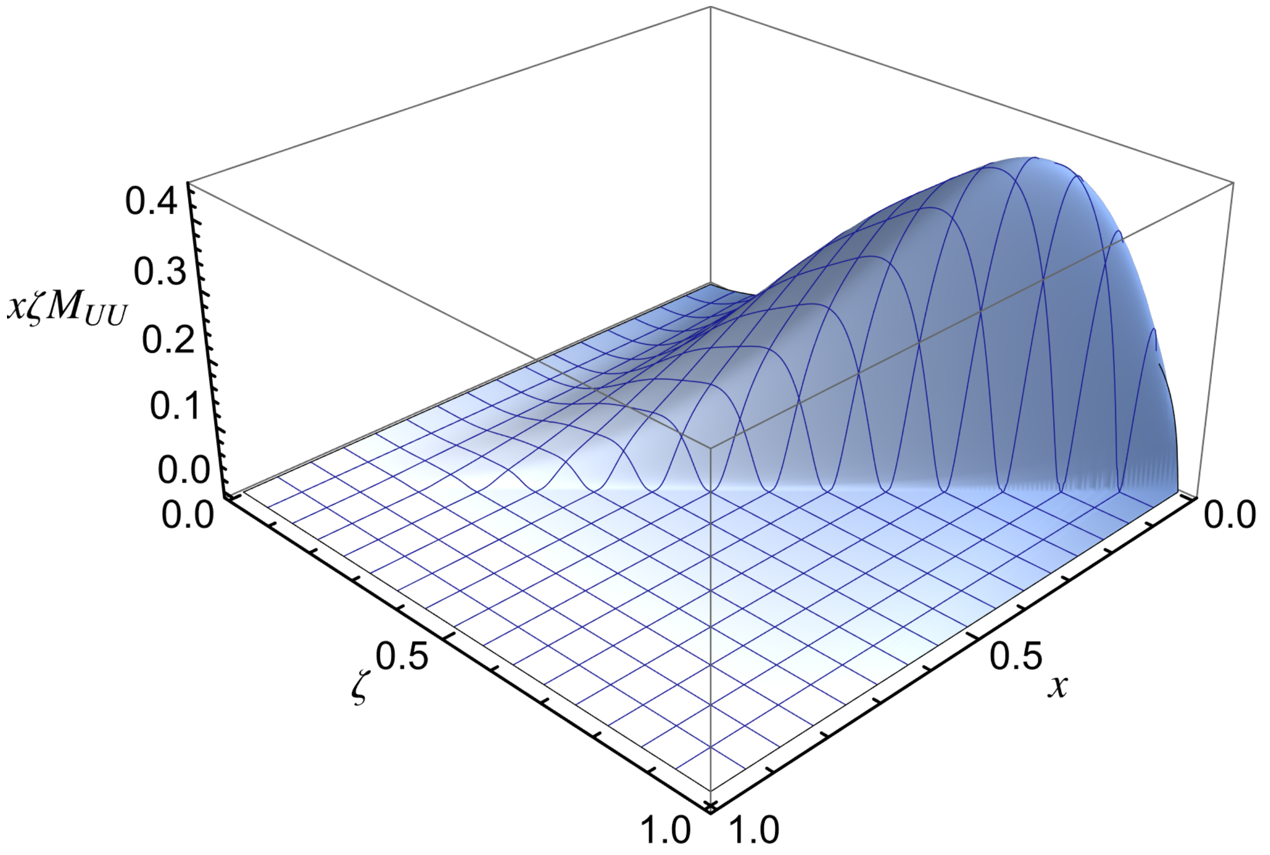
Modeling the Fracture Function

A diquark model estimation of the fraction function

Effective vertex (baryon-quark-diquark): $g_s \bar{\Psi}\phi\psi + g_a \bar{\Psi}\gamma^\mu\gamma_5 A_\mu \psi + \text{ h.c.}$



model parameters tuned to match proton unpolarized PDF



model result of fracture function

X. Zhao, Z.-t. Liang, TL, Y.-j. Zhou, Phys. Rev. Lett. 134 (2025) 231901.