

# Some new observables in QCD spin physics

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## Introduction

## One of the most important discoveries in hadron physics over the past decades is the measurements of large spin asymmetries

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#### Transverse Quark Polarization in Large- $p_T$ Reactions, $e^+e^-$ Jets, and Leptoproduction: A Test of Quantum Chromodynamics

G. L. Kane

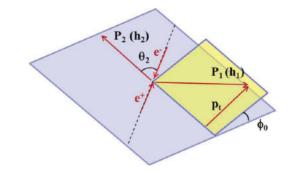
Physics Department, University of Michigan, Ann Arbor, Michigan 48109

and

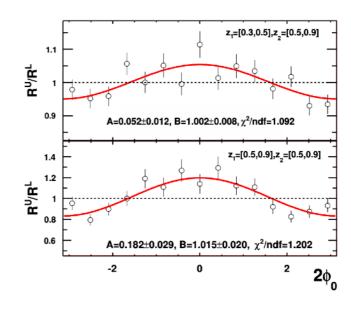
J. Pumplin and W. Repko
Physics Department, Michigan State University, East Lansing, Michigan 48823
(Received 5 July 1978)

quarks. We discuss how to test the predictions. At least for the cases when P is small, tests should be available soon in large- $p_T$  production [where currently  $P(\Lambda) = 25\%$  for  $p_T \gtrsim 2 \text{ GeV}/c$ ], and  $e^+e^-$  reactions. While fragmentation effects could dilute polarizations, they cannot (by parity considerations) induce polarization. Consequent-

ly, observation of significant polarizations in the above reactions would contradict either QCD or its applicability.



#### E.g. Collins asymmetry at BESIII

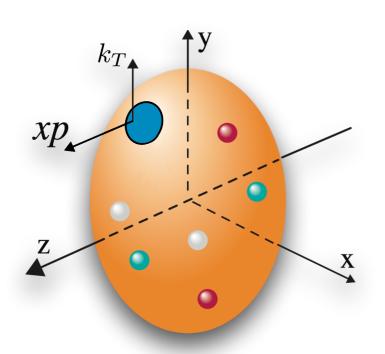


Also see Yixiong Zhou's talk

These experimental measurements can be used to probe the internal structure of hadrons

# 3D imaging of the hadron in the momentum space

- Both longitudinal and transverse motion
- Large Lorentz boost in longitudinal direction ( $P_z$ ), but not in transverse momentum ( $P_x$ ,  $P_y$ ), mapping out the small  $P_T$  is not easy
- Correlation between nucleon spin with parton(quark, gluon) orbital angular momentum



EicC White paper '21

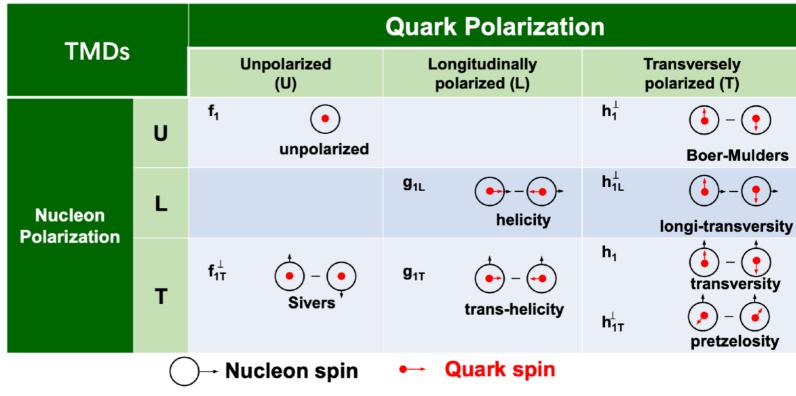
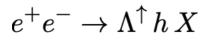


Figure 2.5: The leading-twist quark TMD distributions.

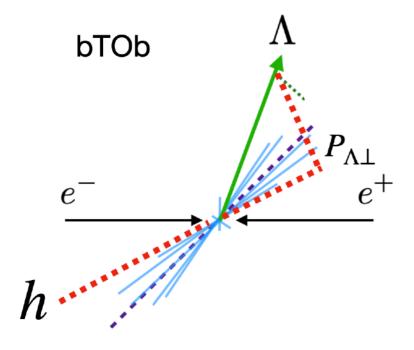
## Transverse Λ polarization in electron positron collisions

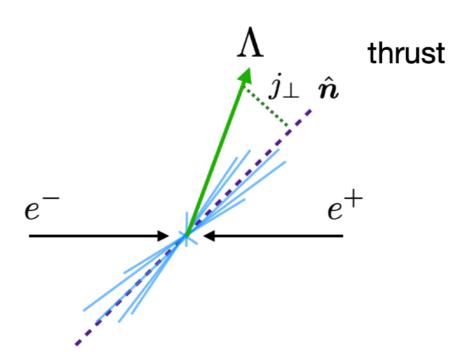
#### The cleanest way to access fragmentation functions

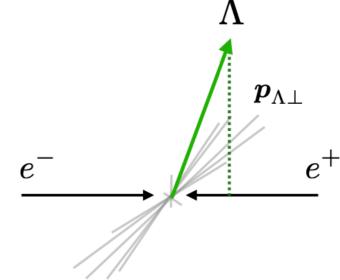


$$e^+e^- \to \Lambda^{\uparrow}(\text{Thrust}) X$$

$$e^+e^- \to \Lambda^{\uparrow} X$$





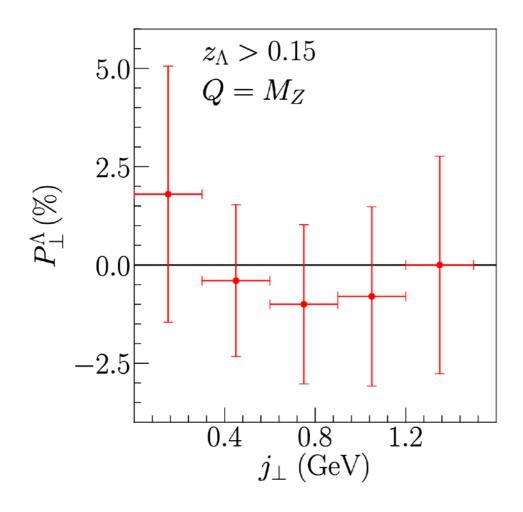


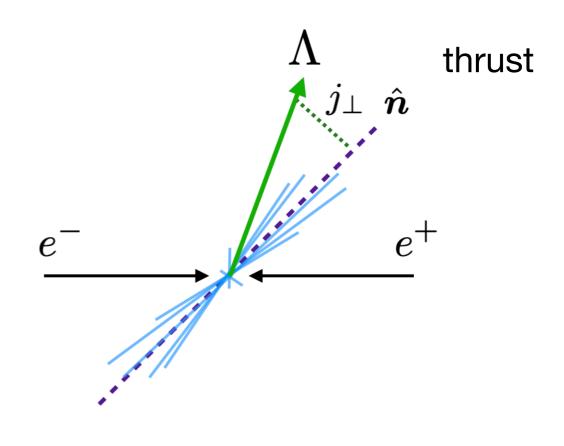
$$T \equiv \max_{\hat{n}} \frac{\sum_{i} |\vec{p_i} \cdot \hat{n}|}{\sum_{i} |\vec{p_i}|}$$

Also see Wenbiao Yan's talk

# Transverse A polarization at the LEP

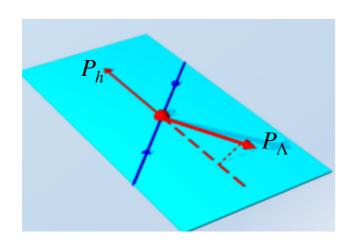


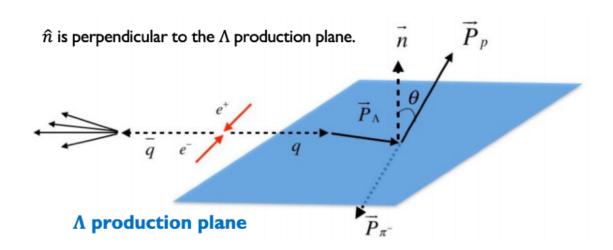




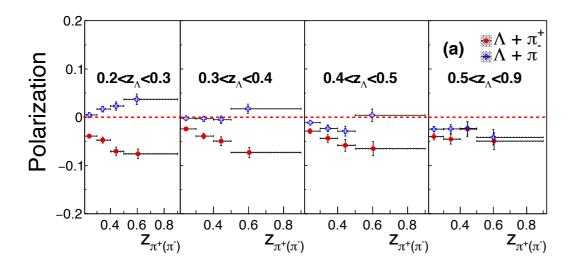
No significant transverse polarization is observed at the LEP

# Transverse A polarization at the Belle

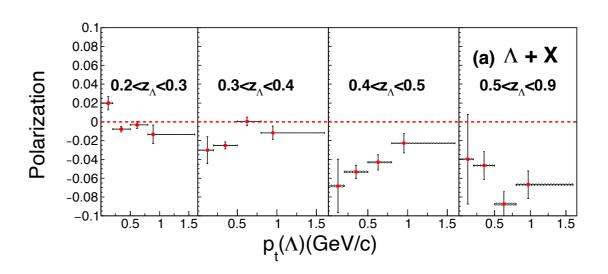




Belle '18 PRL



$$e^+e^- \to \Lambda^{\uparrow} h X$$

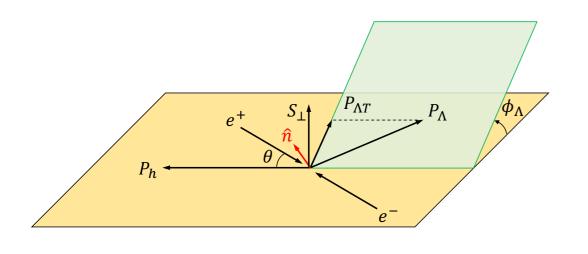


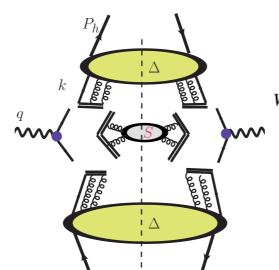
$$e^+e^- \to \Lambda^{\uparrow}(\text{Thrust}) X$$

## Back-to-back Λ+h

$$e^-(\ell) + e^+(\ell') \to \gamma^*(q) \to h(P_h) + \Lambda(P_\Lambda, \mathbf{S}_\perp) + X$$

#### TMD factorization theorem





$$W^{\mu\nu} \stackrel{\text{prelim}}{=} \frac{8\pi^3 z_A z_B}{Q^2} \sum_{f} \operatorname{Tr} k_{A,\gamma}^+ \gamma^- H_f^{\nu}(Q) k_{B,\gamma}^- \gamma^+ \overline{H}_f^{\mu}(Q)$$

$$\times \int \frac{\mathrm{d}^{2-2\epsilon} \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^{2-2\epsilon}} e^{-i\boldsymbol{q}_{h\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \tilde{S}(\boldsymbol{b}_{\mathrm{T}}) \tilde{D}_{1, H_A/f}(z_A, \boldsymbol{b}_{\mathrm{T}}) \tilde{D}_{1, H_B/\bar{f}}(z_B, \boldsymbol{b}_{\mathrm{T}})$$
+ polarized terms.

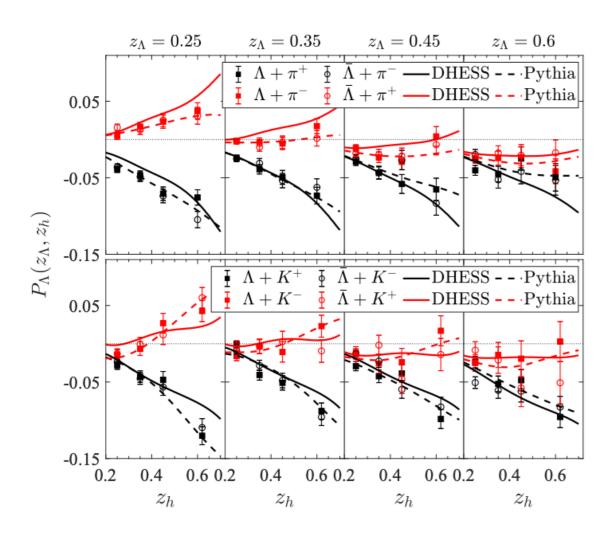
#### Spin-dependent cross section is factorized as:

$$\frac{d\sigma\left(\boldsymbol{S}_{\perp}\right)}{d\mathcal{P}\mathcal{S}d^{2}\boldsymbol{q}_{\perp}} = \sigma_{0}\left\{\mathcal{F}\left[D_{\Lambda/q}D_{h/\bar{q}}\right] + |\boldsymbol{S}_{\perp}|\sin\left(\phi_{S} - \phi_{\Lambda}\right)\frac{1}{z_{\Lambda}M_{\Lambda}}\mathcal{F}\left[\hat{\boldsymbol{P}}_{\Lambda T} \cdot \boldsymbol{p}_{\Lambda \perp}D_{1T,\Lambda/q}^{\perp}D_{h/\bar{q}}\right] + \cdots\right\}$$

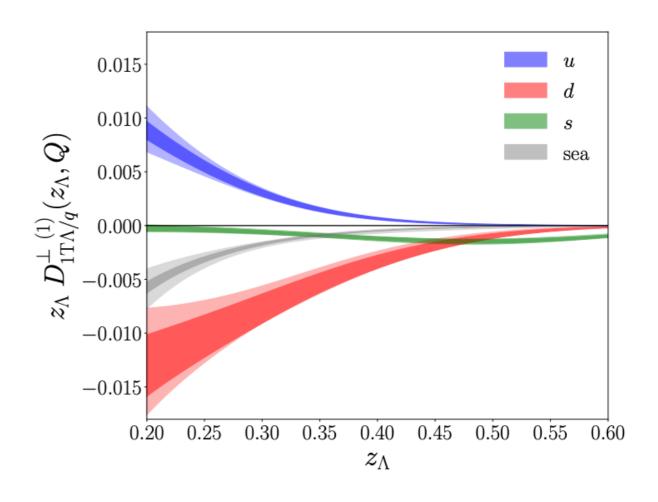
**PFFs: Polarizing Fragmentation Functions** 

## Fitting of PFFs from Λ+h data

Chen, Liang, Pan, Song, Wei '21



Kang, Terry, Vossen, Xu, Zhang '21



See also: D' Alesio, Murgia, Zaccheddu '20 Callos, Kang, Terry '20

... ...

Light bands: the uncertainty from the fit to Belle data

Dark bands: the simultaneous fit of the Belle data and the EIC pseudo-data

## Theory framework on transverse A polarization

$$e^+e^- \to \Lambda^{\uparrow} h X$$

Collins-Soper-Sterman, Ji-Ma-Yuan, Soft-Collinear Effective Theory... ...

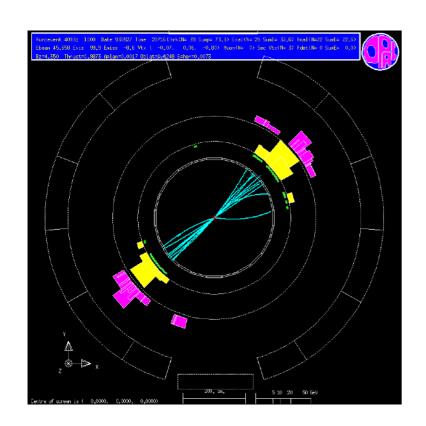
$$e^+e^- \to \Lambda^{\uparrow}(\text{Thrust}) X$$
???

## TMD factorization two scale problem

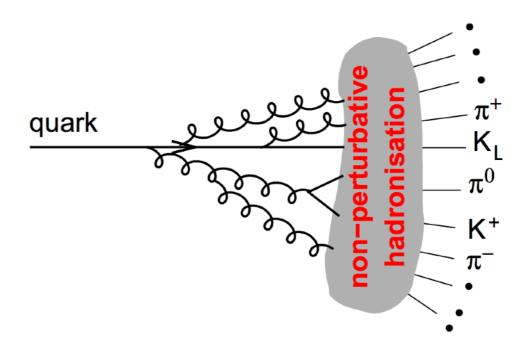
$$\Lambda_{QCD} \lesssim j_{\perp} \ll Q$$

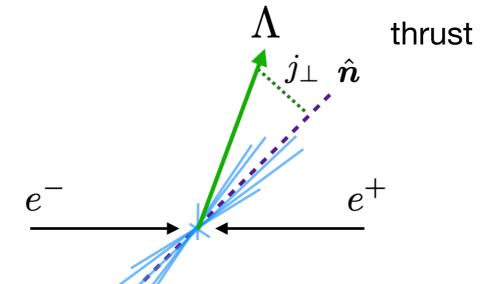
Is it the same (polarizing) fragmentation function in these two measurements ???

# **TMD** factorization for Λ(thrust)



#### Parton fragmentation and hadronization





#### From short to long distances in quantum field theory

$$J(\text{ scale }\mu_2) \sim J(\text{ scale }\mu_1) \exp \left[ \int_{\mu_1}^{\mu_2} \frac{d\mu'}{\mu'} \int dx P(x, \alpha_s(\mu')) \right]$$

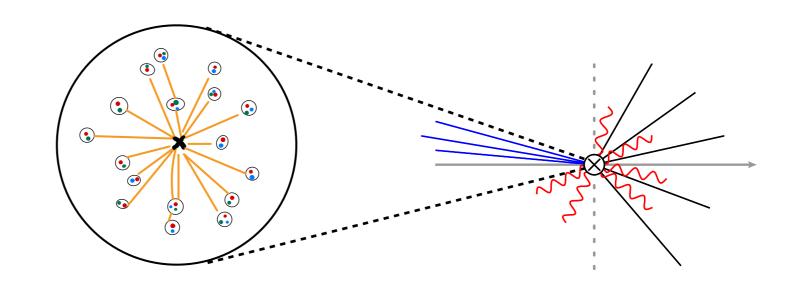
"Jets from Quantum Chromodynamics" Sterman & Weinberg '77

## TMD factorization formula on the jet broadening

(Becher, Rahn, DYS '17 JHEP)

#### **Definition of the broadening:**

$$b_N = \sum_{i \in \text{jets}} \left| \vec{p}_i^{\perp} \right|$$



#### Construction of the theory formalism $b_N \ll Q$

- Two scales in the problem
- Rely on effective field theory: SCET + Jet Effective Theory (Becher, Neubert, Rothen, DYS '16 PRL)

$$\frac{d\sigma}{db_N} = \sum_{f=q,\bar{q},g} \int db_N^s \int d^{d-2}p_N^{\perp} \frac{\mathcal{J}_f\left(b_N - b_N^s, p_N^{\perp}\right)}{\mathcal{J}_f\left(b_N - b_N^s, p_N^{\perp}\right)} \sum_{m=1}^{\infty} \left\langle \mathcal{H}_m^f(\{\underline{n}\}, Q) \otimes \mathcal{S}_m\left(\{\underline{n}\}, b_N^s, -p_N^{\perp}\right) \right\rangle$$

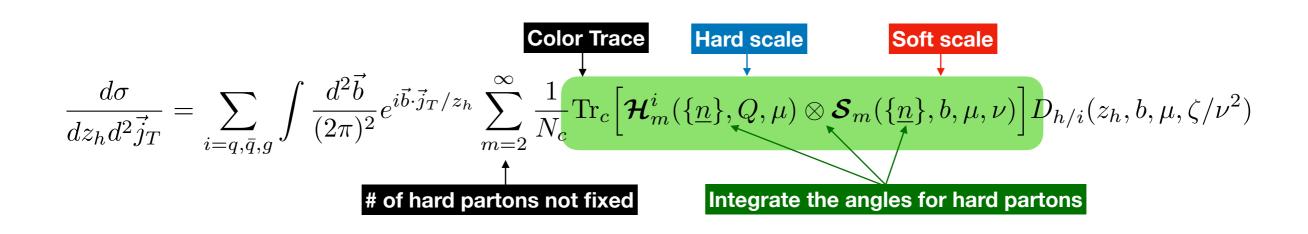
Rapidity divergence cancellation is verified at two-loop order !!!

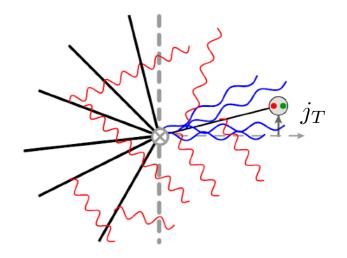
### All-order factorization formula

Kang, DYS, Zhao '20 JHEP

Non-global observables Dasgupta & Salam '01

- Do not exponentiate in a simple manner
- New hard partons acts as new source
- Non-linear evolution, BMS eq @ LL Banfi, Marchesini, Smye '02
- Multi-Wilson-line structure Becher, Neubert, Rothen, DYS '16 PRL
- Super-leading logs resummation at hadron colliders Becher, Neubert, DYS '21 PRL





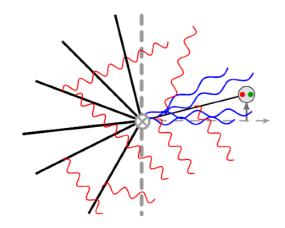
## Factorization on single hadron unpolarized TMDs

Case-I:  $e^-e^+ \to h_1h_2 + X$ 

Global observable, standard TMD factorization

Case-II:  $e^-e^+ \rightarrow h + X$ 

Non-global observable; new TMD factorization



hard:  $p_h \sim Q(1,1,1)$  collinear:  $p_c \sim Q(\lambda^2,1,\lambda)$   $\lambda = j_T/Q \ll 1$ 

soft:  $p_s \sim Q(\lambda, \lambda, \lambda)$ 

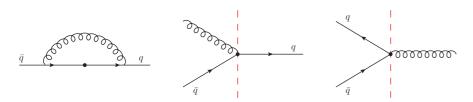
$$\frac{d\sigma}{d^2\boldsymbol{q}_T} \sim H \otimes D_{h_1} \otimes D_{h_2} \otimes S$$

Collins, "Foundations of perturbative QCD"

$$rac{d\sigma}{d^2m{q}_T}\sim D_h\otimes\sum_m\mathcal{H}_m\otimes\mathcal{S}_m$$
Kang, DYS, Zhao '20 JHEP

$$\lambda = j_T/Q \ll 1$$

#### **NLO hard function:**



Divergences are half of the hard function in case-I

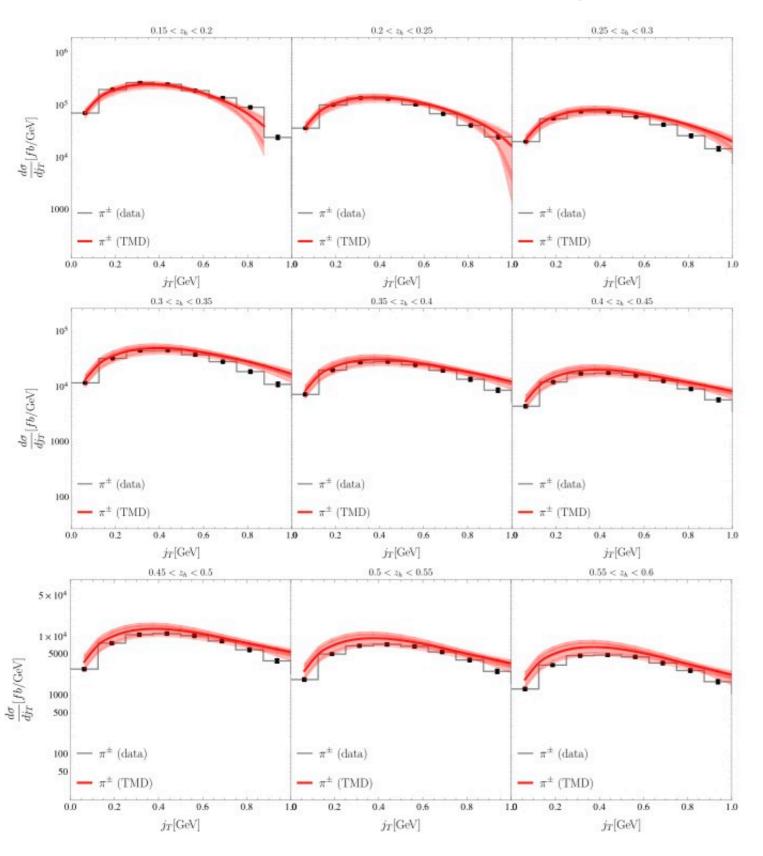
#### **NLO** soft function:

$$\frac{\alpha_s C_F}{2\pi^2} \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \int \frac{dk^+ dk^-}{2} \left(\frac{\mu^2}{\vec{\lambda}_T^2}\right)^{\epsilon} \frac{2n \cdot \bar{n}}{k^+ k^-} \delta^+(k^+ k^- - \vec{\lambda}_T^2) \left|\frac{\nu}{2k_z}\right|^{\eta} \theta \left(1 - \frac{k^+}{k^-}\right) 
= \frac{\alpha_s}{2\pi} C_F \left[\frac{2}{\eta} \left(-\frac{1}{\epsilon} - \ln\left(\frac{\mu^2}{\mu_b^2}\right)\right) + \frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln\left(\frac{\nu^2}{\mu^2}\right)\right]$$

Divergences are half of the soft function in case-I

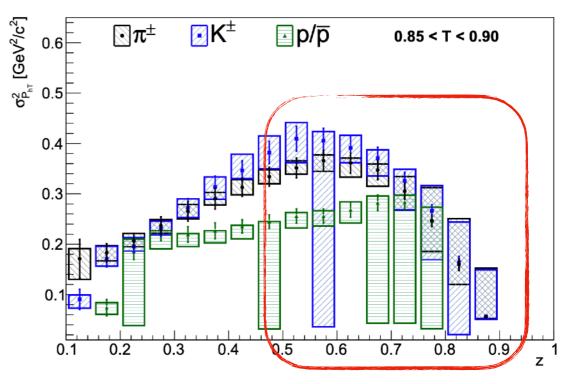
## **Numerical results**

Kang, DYS, Zhao '20 JHEP



- Our TMD resummation formula gives a good description of the shape of  $j_T$  distribution as  $z_h < 0.65$
- As z<sub>h</sub> > 0.65, one needs to also include threshold resummation effects

$$\frac{d\sigma}{dz_h d^2 \vec{j}_T} \propto \frac{1}{\pi \sigma_{j_T}^2} \exp\left(-j_T^2/\sigma_{j_T}^2\right)$$



## Joint threshold and TMD factorization

Kang, DYS, Zhao '20 JHEP

Joint factorization:  $z_h o 1$  &  $j_T \ll Q$ 

Joint TMD and threshold resummation is first developed in Li '98 & Laenen, Sterman, Vogelsang '01

in the threshold region, a new mode: collinear-soft (c-soft) modes contribute

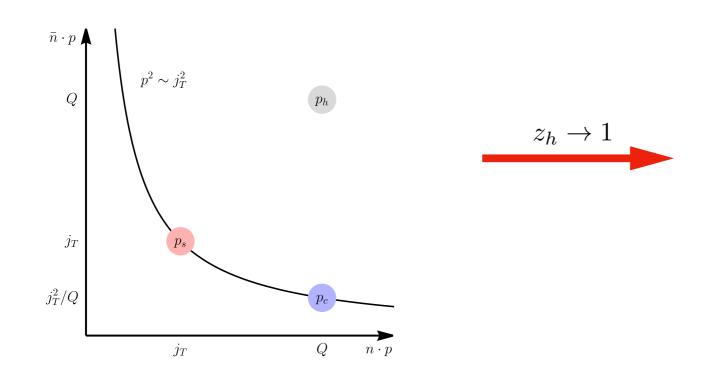
Li, Neill, Zhu '16 & Lustermans, Waalewijn, Zeune '16

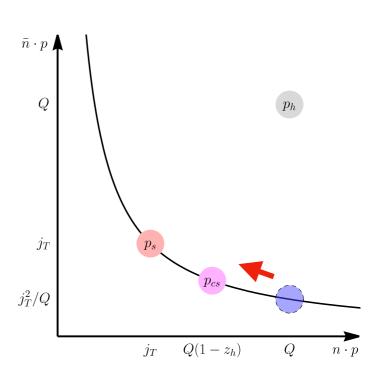
 $p_h \sim Q(1,1,1)$ hard:

collinear:  $p_c \sim Q(\lambda^2, 1, \lambda)$   $z_h \to 1$  c-soft:  $p_{\mathscr{S}} \sim (j_T^2/(Q(1-z_h)), Q(1-z_h), j_T)$ 

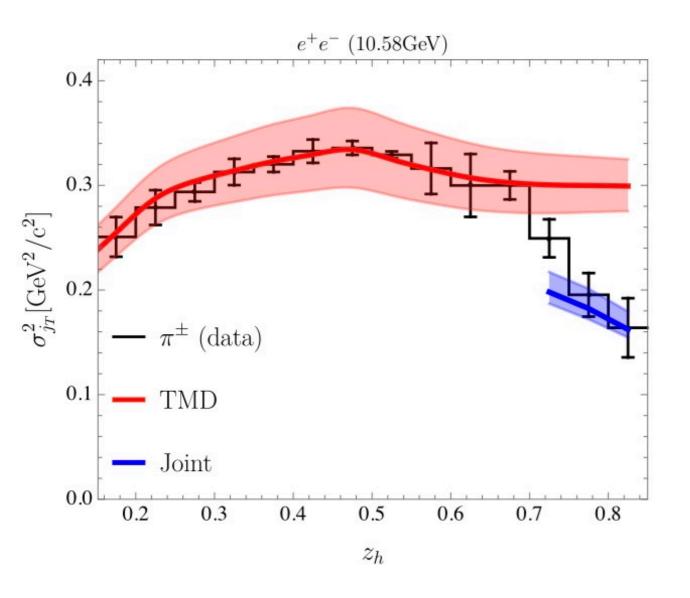
 $p_s \sim Q(\lambda, \lambda, \lambda)$ soft:

#### TMD FFs in the threshold limit





## **Numerical results**

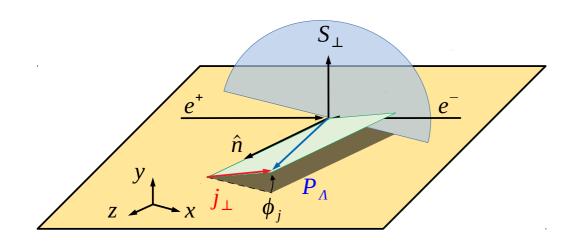


$$\frac{d\sigma}{dz_h d^2 \vec{j}_T} \propto \frac{1}{\pi \sigma_{j_T}^2} \exp\left(-j_T^2/\sigma_{j_T}^2\right)$$

- The Gaussian width of the  $j_T$  distribution given by the TMD formalism freeze to a certain value.
- After including joint threshold and TMD resummation effects, the theoretical predictions are consistent with the data

#### Factorization on transverse polarized Λ hyperon production with the thrust axis

Gamberg, Kang, DYS, Terry, Zhao '21 PLB

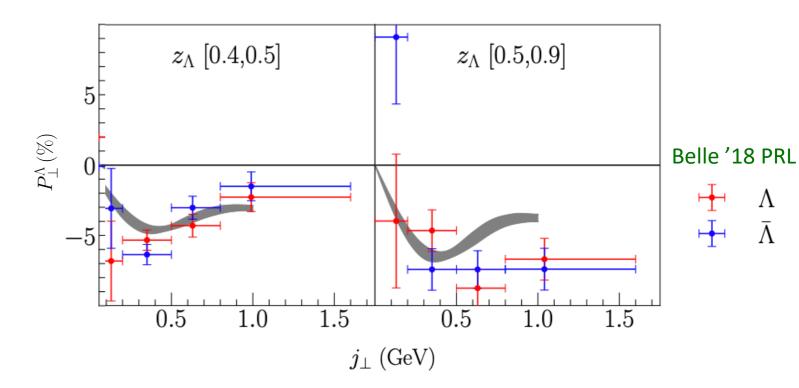


$$P_{\perp}^{\Lambda}(z_{\Lambda},j_{\perp}) = \left. rac{d\Delta\sigma}{dz_{\Lambda}d^2m{j}_{\perp}} 
ight/ rac{d\sigma}{dz_{\Lambda}d^2m{j}_{\perp}}$$

#### Theory results are consistent with Belle data

#### Theory formula including QCD evolution

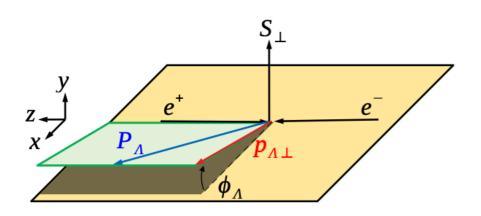
$$\begin{split} \frac{d\Delta\sigma}{dz_{\Lambda}d^{2}j_{\perp}} &= \frac{d\sigma\left(\boldsymbol{S}_{\perp}\right)}{dz_{\Lambda}d^{2}\boldsymbol{j}_{\perp}} - \frac{d\sigma\left(-\boldsymbol{S}_{\perp}\right)}{dz_{\Lambda}d^{2}\boldsymbol{j}_{\perp}} \\ &= \sigma_{0}\sin\left(\phi_{s} - \phi_{j}\right)\sum_{q}e_{q}^{2}\int_{0}^{\infty}\frac{b^{2}db}{2\pi}J_{1}\left(\frac{bj_{\perp}}{z_{\Lambda}}\right) \\ &\times \frac{M_{\Lambda}}{z_{\Lambda}^{2}}D_{1T,\Lambda/q}^{\perp(1)}\left(z_{\Lambda},\mu_{b_{*}}\right)e^{-S_{\mathrm{NP}}^{\perp}\left(b,z_{\Lambda},Q_{0}^{\prime},Q\right)-S_{\mathrm{pert}}\left(\mu_{b_{*}},Q\right)} \\ &\times U_{\mathrm{NG}}\left(\mu_{b_{*}},Q\right) \end{split}$$



This result provides proof of principle that the experimental data can be described using the factorization and resummation formalism that we have introduced.

#### Factorization on transverse polarized Λ hyperon production with the thrust axis

Gamberg, Kang, DYS, Terry, Zhao '21 PLB



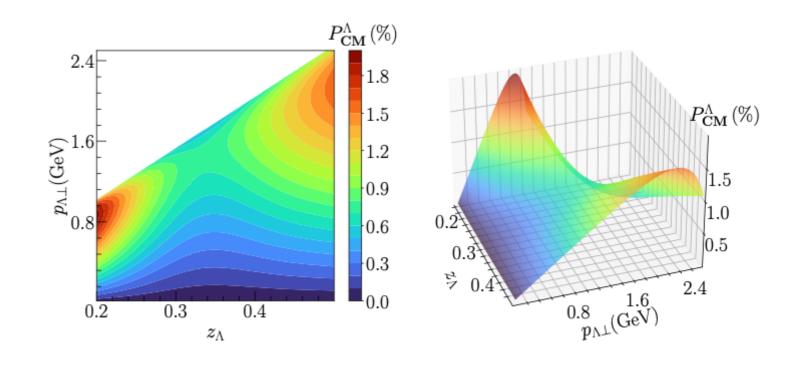
$$P^{\Lambda}_{\mathbf{CM}}(z_{\Lambda},p_{\Lambda\perp}) = \left. rac{d\Delta\sigma}{dz_{\Lambda}\,d^2p_{\Lambda\perp}} 
ight/ rac{d\sigma}{dz_{\Lambda}\,d^2p_{\Lambda\perp}}$$

# Twist-3 theory formula including QCD evolution

$$\begin{split} \frac{d\Delta\sigma}{dz_{\Lambda}\,d^{2}p_{\Lambda\perp}} &= -\sin(\phi_{s}-\phi_{\Lambda})\frac{2N_{c}\alpha_{\mathrm{em}}^{2}}{Q^{4}z_{\Lambda}}\left(\frac{4M_{\Lambda}}{Q}\right)\frac{p_{\Lambda\perp}}{Q} \\ &\times \frac{1}{z_{\Lambda}^{3}}\sum_{q}e_{q}^{2}\frac{D_{T,\Lambda/q}(z_{\Lambda},Q)}{z_{\Lambda}}\,. \end{split}$$

$$egin{aligned} rac{1}{z_\Lambda} D_{T,\Lambda/q}(z_\Lambda,Q) &= -\left(1-z_\Lambda rac{d}{dz_\Lambda}
ight) D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda,Q) \ &-2\int_0^1 deta rac{\Im\left[\hat{D}_{FT}^{qg}(z_\Lambda,Q,eta)
ight]}{(1-eta)^2}\,. \end{aligned}$$

#### **Theory predictions**



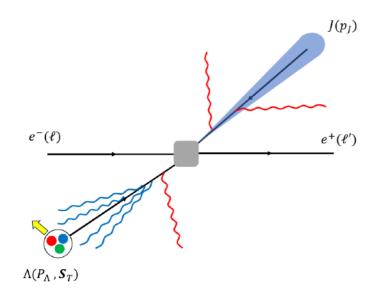
Also see Zhe Zhang's talk

## Transverse Lambda polarization and jet charge

(Gamberg, Kang, DYS, Terry, Zhao in progress)

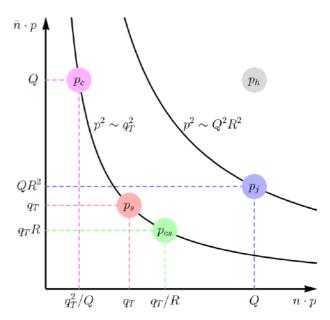
As shown in (Kang, Liu, Mantry, DYS '20 PRL), the jet charge observable is a novel probe of flavor structure for the hadron spin

$$e^-(\ell) + e^+(\ell') \to J(p_J) + \Lambda(P_\Lambda, \mathbf{S}_T) + X$$



#### **Dynamic modes:**

- hard:  $p_h \sim Q(1, 1, 1)$ ,
- soft:  $p_s \sim q_T(1, 1, 1)$ ,
- collinear:  $p_c \sim (q_T^2/Q, Q, q_T)$ ,
- jet:  $p_j \sim Q(1, R^2, R)$ ,
- collinear-soft:  $p_{cs} \sim q_T/R(1, R^2, R)$ ,

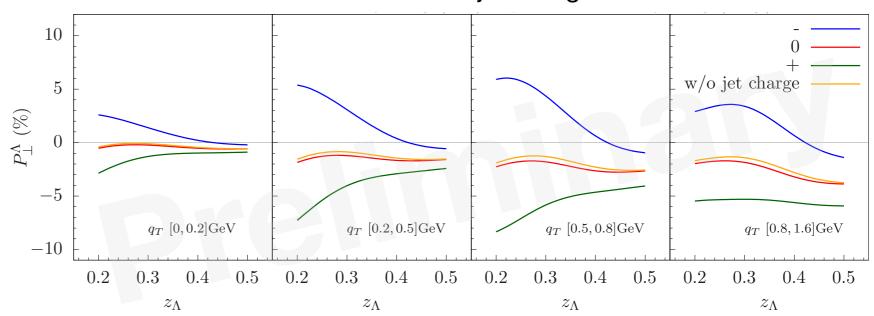


Jet charge definition:  $Q_{\kappa} = \sum_{i} \left(\frac{p_{i,T}}{p_{J}}\right)^{\kappa} Q_{i}$ 

Charge tagged jet function:

$$\mathcal{G}_i(Q_\kappa, p_T R, \mu)$$

#### Polarization w/ jet charge



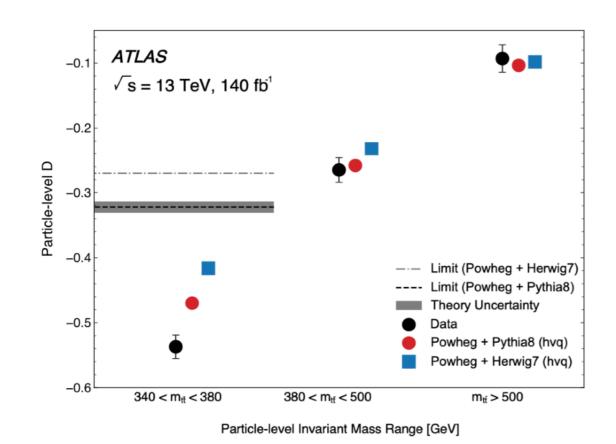
## Quantum information science meets High energy physics

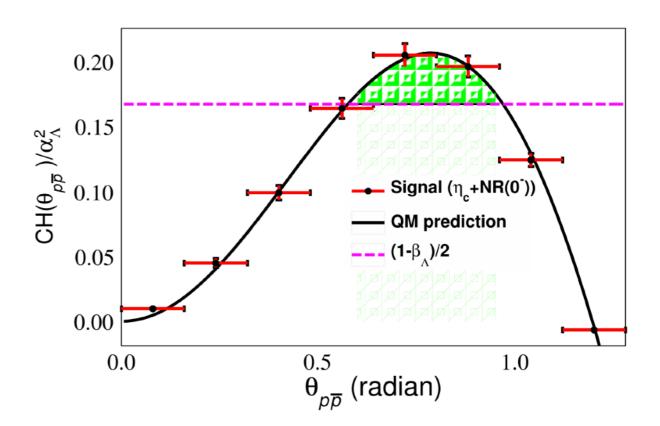
- QIS 4 HEP
  - New experimental probes and fundamental tests
  - Enhanced simulations of quantum systems

"Quantum Information meets High-Energy Physics: Input to the update of the European Strategy for Particle Physics"

A recent review 2504.00086

- HEP 4 QIS
  - A novel testbed for fundamental QIS concepts in extreme regimes



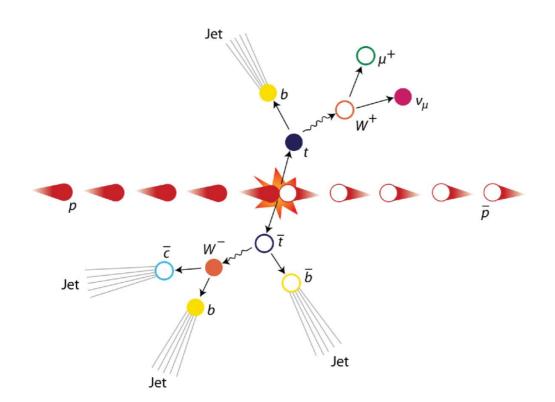


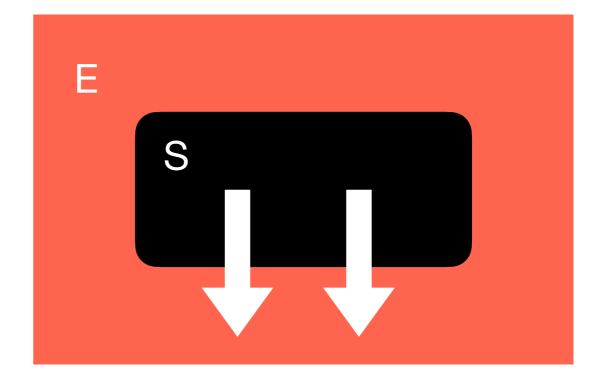
ATLAS Nature 2024

**BESIII Nature Communications 2025** 

## **Decoherence in entangled fermion pairs**

- In the above measurements at the LHC, entangled top quark pairs can not be treated as a closed system
- Top quarks may radiate gluons or photons in the short period of time before decaying, leading to a reduction in quantum spin information, i.e., decoherence.
- Decoherence can be studied by recognizing that realistic quantum systems are always embedded in some environment.
- This interaction with the system results in 'leakage of information' to the environment, decreasing the entanglement between the components of the system.



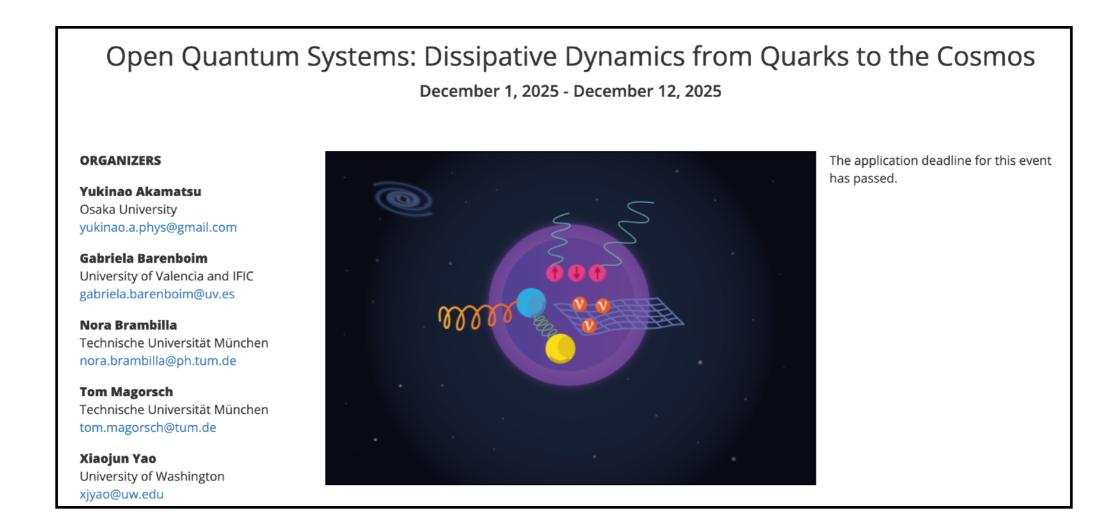


#### Some decoherence effects

- "Infrared quantum information" Soft radiation decrease momentum entanglement (Carney, Chaurette, Neuenfeld, Semenof '17)
- K0 K0bar system (Bertlmann '04) Λ Λbar system (Wu, Qian, Yang, Wang '24)
- Inflation and effective field theory (Burgess, Colas, Holman, Kaplanek '25)
- Black hole horizons decohere superpositions (Biggs, Maldacena '24)

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## Concurrence at LO (closed system)

- We consider  $e^+e^- \rightarrow \tau^+\tau^-$
- The spin state of a τ-lepton pair can be characterized by a two-qubit density operator

$$\hat{\rho} = \frac{1}{4} \left( \hat{I}_2 \otimes \hat{I}_2 + B_i^+ \hat{\sigma}_i \otimes \hat{I}_2 + B_i^- \hat{I}_2 \otimes \hat{\sigma}_i + C_{ij} \hat{\sigma}_i \otimes \hat{\sigma}_j \right)$$

$$\uparrow$$
Spin correlation matrix

At the LO

$$\rho_{\text{LO}} = \frac{1}{4} \left( \hat{I}_2 \otimes \hat{I}_2 + \frac{\sin^2 \theta}{1 + \cos^2 \theta} \hat{\sigma}_1 \otimes \hat{\sigma}_1 + \frac{\sin^2 \theta}{1 + \cos^2 \theta} \hat{\sigma}_2 \otimes \hat{\sigma}_2 - \hat{\sigma}_3 \otimes \hat{\sigma}_3 \right)$$

To probe entanglement, one can calculate the concurrence C

$$\mathcal{C}[\rho_{\rm LO}] = \frac{\sin^2 \theta}{1 + \cos^2 \theta}$$

• Maximum entanglement  $\cos\vartheta = 0$ 

$$\mathcal{C}[\rho_{\mathrm{LO}}] = 1$$
 
$$\frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle)$$

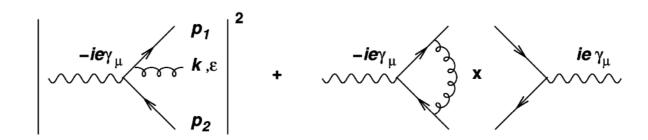
## Quantum maps for open systems

Aoude, Barr, Maltoni, Satrioni '25

The evolution of an open system can represented by a quantum map

$$\mathcal{E}[
ho] = \sum_{j} K_{j} 
ho K_{j}^{\dagger}, \qquad \sum_{j} K_{j}^{\dagger} K_{j} = \mathbb{1},$$

**Kraus operators** 



- The virtual corrections lead to the same final state Hilbert space while the real emission leads to the extra Hilbert space of the environment.
- To obtain the reduced density matrix, we need to trace over the emitted radiation

$$\begin{split} \rho_{\mathrm{LO}+\mathrm{NLO}}^{\mathrm{red}} &= \mathsf{p}_{\mathrm{LO}} \, 1\!\!1 \rho_{\mathrm{LO}} 1\!\!1 + \bar{\mathcal{E}}_{\mathrm{V}}[\rho_{\mathrm{LO}}] + \bar{\mathcal{E}}_{\mathrm{R}}[\rho_{\mathrm{LO}}] \\ \bar{\mathcal{E}}_{\mathrm{V}}[\rho_{\mathrm{LO}}] &= \mathsf{p}_{\mathrm{V}} 1\!\!1 \rho_{\mathrm{LO}} 1\!\!1 \\ \bar{\mathcal{E}}_{\mathrm{R}}[\rho_{\mathrm{LO}}] &= \sum_{j} K_{j} \rho_{\mathrm{LO}} K_{j}^{\dagger} \end{split}$$

Virtual: does not change LO structure (massless)

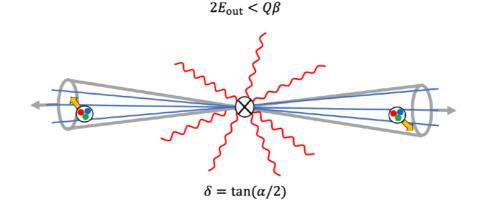
Real: <del>hard</del>, collinear, <del>soft</del>

## Effective field theory for decoherence

J.Y. Gu, S.J. Lin (林士佳), DYS, L.T. Wang, S.X. Yang (杨斯翔) 2508.XXXXX

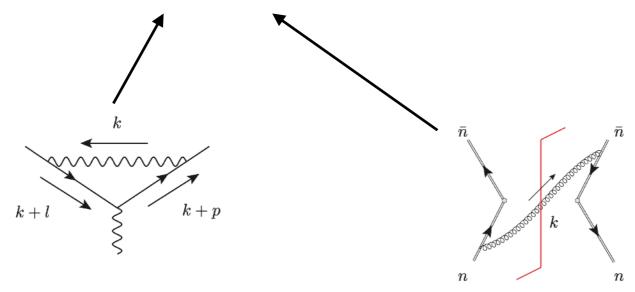
- Radiation should be considered unresolvable if either soft of collinear
- We introduce the energy and angular resolution parameters, which is similar to Sterman-Weinberg cone jet definition (Sterman, Weinberg '77)

Two fermion events:



- We apply soft collinear effective theory (SCET) + Jet Effective Theory (JET) (Becher, Neubert, Rothen, DYS '16 PRL)
- The production matrix

$$\hat{R} = H(Q, \mu) S(Q\beta, \delta, \mu) \hat{J}_f(Q\delta, \lambda, \mu) \hat{R}_{LO} \hat{J}_{\bar{f}}(Q\delta, \lambda, \mu),$$



## Fragmenting density matrix and measurement operator

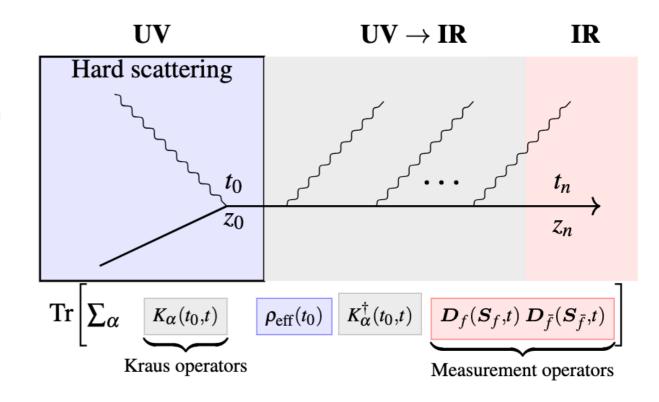
J.Y. Gu, S.J. Lin (林士佳), DYS, L.T. Wang, S.X. Yang (杨斯翔) 2508.XXXXX

#### Fragmenting density matrix

$$\hat{J}_f \!=\! \mathcal{J}_f^U \, \hat{I} \! \otimes \! \hat{I} \! + \! \mathcal{J}_f^L \hat{\sigma}_z \! \otimes \! \hat{\sigma}_z \! + \! \mathcal{J}_f^T (\hat{\sigma}_x \! \otimes \! \hat{\sigma}_x \! + \! \hat{\sigma}_y \! \otimes \! \hat{\sigma}_y)$$

- $J^{U}$ : unpolarized FFs
- J<sup>L</sup>: longitudinal polarized-FFs
- J<sup>T</sup>: transverse polarized-FFs
- The Kraus operators in QED

$$\begin{split} \hat{K}_{(i,j)} &= \hat{K}_{i}^{\ell^{-}} \otimes \hat{K}_{j}^{\ell^{+}} \\ \hat{K}_{0}^{\ell^{-}} &= \hat{K}_{0}^{\ell^{+}} = \sqrt{1 - p^{2}} \, \mathbb{I} \,, \\ \hat{K}_{1}^{\ell^{-}} &= \hat{K}_{1}^{\ell^{+}} = p \, \hat{\sigma}_{3} \,, \quad p = \sqrt{\frac{1}{2} \left[ 1 - \exp\left(-\frac{\alpha}{2\pi}t\right) \right]} \end{split}$$



#### **Concurrence in QED**

$$C(\rho) = \max \left\{ 0, \frac{\sin^2 \theta}{1 + \cos^2 \theta} \left( \frac{Q\delta}{m} \right)^{-\frac{\alpha}{\pi}} \right\}$$

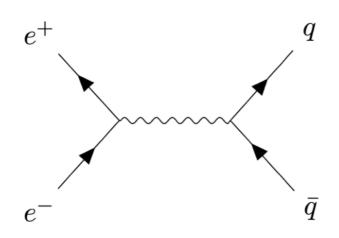
#### Lindblad operators in QED

$$\mathcal{L}_t(\rho_{\text{eff}}) = -\frac{\alpha}{2\pi} \rho_{\text{eff}} + \frac{\alpha}{4\pi} \left[ (\mathbb{I} \otimes \hat{\sigma}_3) \rho_{\text{eff}} (\mathbb{I} \otimes \hat{\sigma}_3) + (\hat{\sigma}_3 \otimes \mathbb{I}) \rho_{\text{eff}} (\hat{\sigma}_3 \otimes \mathbb{I}) \right]$$

#### phase-flip decoherence channe

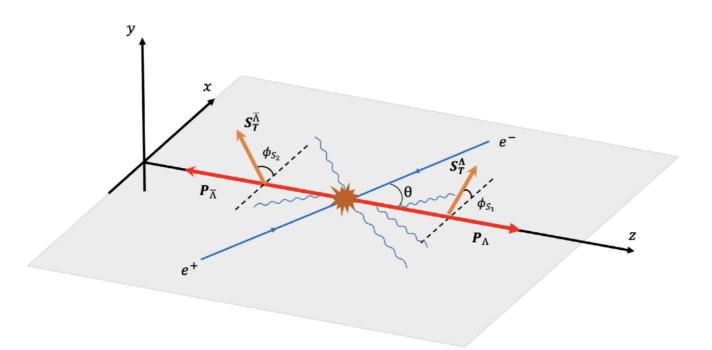
## Spin correlation in A pair production with a thrust cut

S.J. Lin (林士佳), M.J. Liu (刘铭钧), DYS, S.Y. Wei '25



Bell variable Parton-level

$$\mathcal{B}_{+}^{q\bar{q}} = \frac{2\sin^2\theta}{1 + \cos^2\theta}$$



Bell variable Hadron-level

$$\mathcal{B}_+^{\Lambdaar{\Lambda}} = rac{2\,\mathrm{d}\sigma^T}{\mathrm{d}\sigma^U}$$

#### Parton model:

Boer, Jakob, Mulders '97

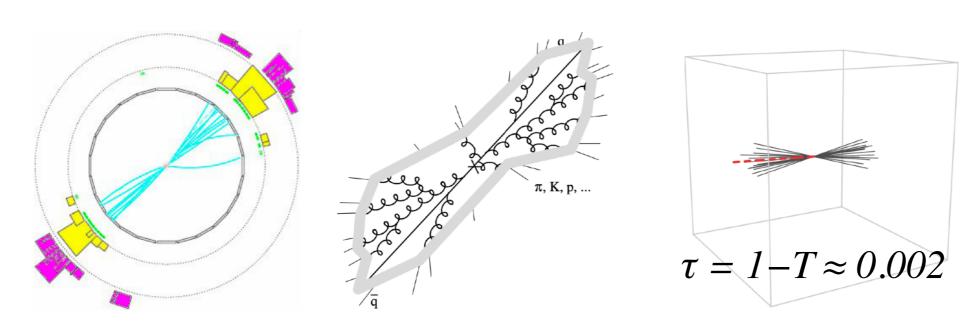
$$\frac{\mathrm{d}\sigma(\boldsymbol{S}^{\Lambda},\boldsymbol{S}^{\bar{\Lambda}})}{\mathrm{d}z_{1}\,\mathrm{d}z_{2}\,\mathrm{d}\Omega} = \sum_{q} e_{q}^{2} \left[ \frac{\mathrm{d}\sigma_{0}^{U}}{\mathrm{d}\Omega} \,\mathcal{D}_{\Lambda/q}^{U}(z_{1},\mu) \,\mathcal{D}_{\bar{\Lambda}/\bar{q}}^{U}(z_{2},\mu) + P_{z}^{\Lambda} P_{z}^{\bar{\Lambda}} \, \frac{\mathrm{d}\sigma_{0}^{L}}{\mathrm{d}\Omega} \,\mathcal{D}_{\Lambda/q}^{L}(z_{1},\mu) \,\mathcal{D}_{\bar{\Lambda}/\bar{q}}^{L}(z_{2},\mu) \right]$$

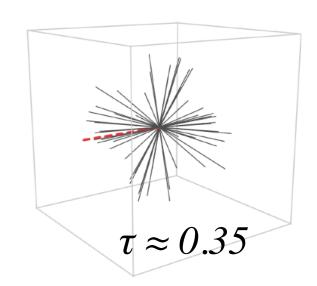
$$+|S_T^{\Lambda}||S_T^{\bar{\Lambda}}|\cos(\phi_{S_1}+\phi_{S_2})\frac{\mathrm{d}\sigma_0^T}{\mathrm{d}\Omega}\,\mathcal{D}_{\Lambda/q}^T(z_1,\mu)\,\mathcal{D}_{\bar{\Lambda}/\bar{q}}^T(z_2,\mu)$$

## Spin correlation in A pair production with a thrust cut

S.J. Lin (林士佳), M.J. Liu (刘铭钧), DYS, S.Y. Wei '25

• We apply the event shape thrust (T) to select two-jet configuration  $T=rac{1}{Q}\max_{ec{n}_T}\sum_i |ec{n}_T\cdotec{p}_i|$ 





The resummation predictions on the polarized cross section

$$\frac{\mathrm{d}\sigma^{\mathcal{P}}(\tau_{\mathrm{cut}})}{\mathrm{d}z_{1}\,\mathrm{d}z_{2}\,\mathrm{d}\Omega} = \int_{0}^{\tau_{\mathrm{cut}}} \mathrm{d}\tau \,\frac{\mathrm{d}\sigma^{\mathcal{P}}}{\mathrm{d}\tau\,\mathrm{d}z_{1}\,\mathrm{d}z_{2}\,\mathrm{d}\Omega}, \qquad \boxed{\mu_{h} = Q, \quad \mu_{J} = Q\sqrt{\tau_{\mathrm{cut}}}, \quad \mu_{s} = Q\tau_{\mathrm{cut}}.}$$

$$= \frac{\mathrm{d}\sigma_{0}^{\mathcal{P}}}{\mathrm{d}\Omega} \exp\left[4C_{F}S(\mu_{h}, \mu_{J}) + 4C_{F}S(\mu_{s}, \mu_{J}) - 2A_{H}(\mu_{h}, \mu_{s}) + 4A_{J}(\mu_{J}, \mu_{s})\right] \left(\frac{Q^{2}}{\mu_{h}^{2}}\right)^{-2C_{F}A_{\mathrm{cusp}}(\mu_{h}, \mu_{J})}$$

$$\times H(Q^{2}, \mu_{h}) \,\widetilde{S}_{T}(\partial_{\eta}, \mu_{s})$$

$$\times \sum_{q} e_{q}^{2} \,\widetilde{\mathcal{G}}_{\Lambda/q}^{\mathcal{P}}\left(z_{1}, \ln \frac{\mu_{s}Q}{\mu_{J}^{2}} + \partial_{\eta}, \mu_{J}\right) \,\widetilde{\mathcal{G}}_{\Lambda/\bar{q}}^{\mathcal{P}}\left(z_{2}, \ln \frac{\mu_{s}Q}{\mu_{J}^{2}} + \partial_{\eta}, \mu_{J}\right) \left(\frac{\tau_{\mathrm{cut}}Q}{\mu_{s}}\right)^{\eta} \frac{e^{-\gamma_{E}\eta}}{\Gamma(1+\eta)} \Big|_{\eta=4C_{E}A_{\mathrm{cusp}}(\mu_{J}, \mu_{s})}.$$

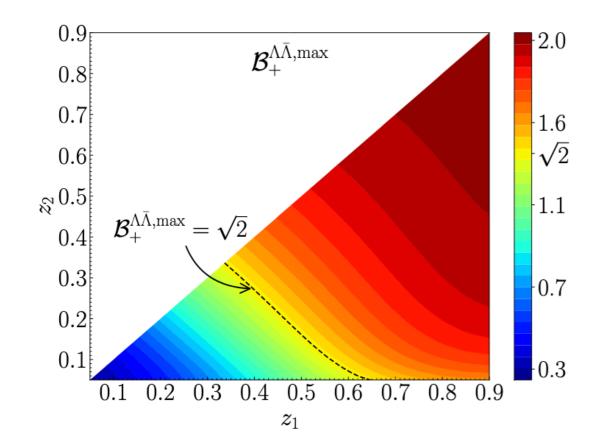
## Bell nonlocality and decoherence

S.J. Lin (林士佳), M.J. Liu (刘铭钧), DYS, S.Y. Wei '25

- For the non-perturbative Λ FFs, we employ the DSV parameterization for the unpolarized Λ FF (de Florian, Stratmann, Vogelsang '97)
- We can utilize theoretical positivity bounds to define their maximal contribution (Soffer '94; Vogelsang '97)

$$|\mathcal{D}^L(z,\mu_0)| \leq \mathcal{D}^U(z,\mu_0), \qquad |\mathcal{D}^T(z,\mu_0)| \leq \frac{1}{2} \left[ \mathcal{D}^U(z,\mu_0) + \mathcal{D}^L(z,\mu_0) \right]$$

ullet We start from the ideal partonic baseline of a maximally entangled  ${\cal B}_+^{qar q}=2$ 



- We observe that under these ideal hadronization assumptions, the Bell variable is suppressed below the partonic maximum of 2
- As expected, this decoherence is reduced at large z, where the hadron carries most of the parent parton's spin information

# Energy correlators in e+e-

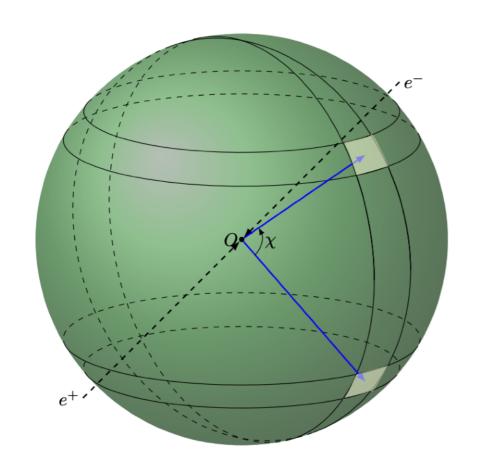
Louis Basham, Brown, Ellis, & Love '78; a recent review Moult, Zhu '25

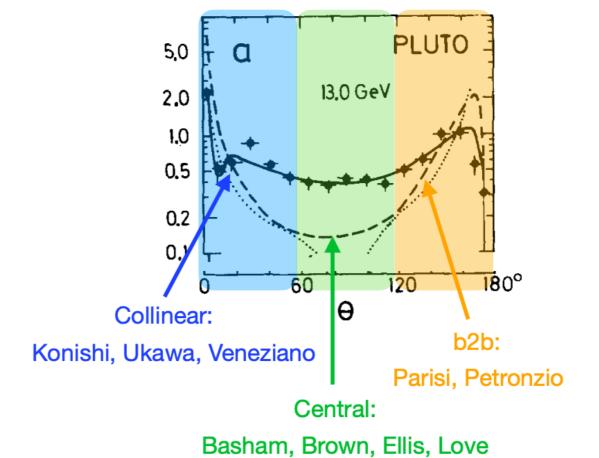
• Correlators of energy flow operators  $\langle \mathcal{OE}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \mathcal{O}^{\dagger} \rangle$  characterize the final state in collider experiments

Energy flow operators: 
$$\mathcal{E}(ec{n}) = \lim_{r o \infty} r^2 \int\limits_0^\infty dt \; n^i T_{0i}(t,rec{n})$$
 Sterman '75

$$EEC_{e^+e^-}(\tau) = \frac{1}{2} \sum_{i,j} \int d\theta_{ij} dz_i dz_j z_i z_j \frac{1}{\sigma} \frac{d\sigma}{d\theta_{ij} dz_i dz_j} \delta\left(\tau - \frac{1 + \cos\theta_{ij}}{2}\right)$$

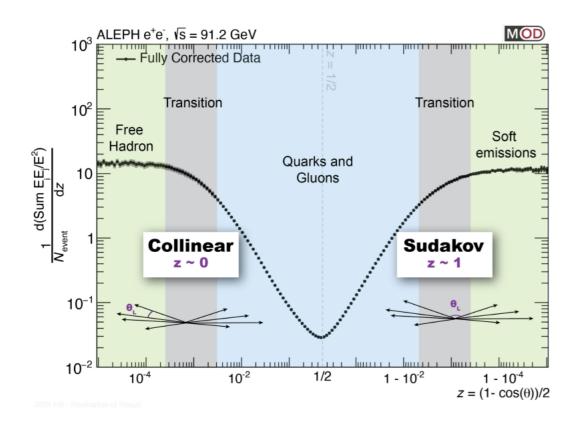
The first measurement at PLUTO 13 GeV

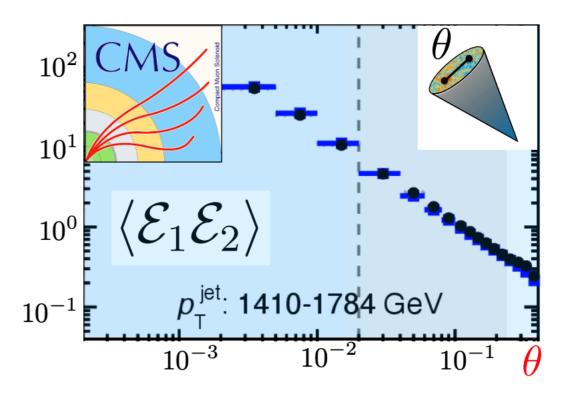




# **Energy correlators**

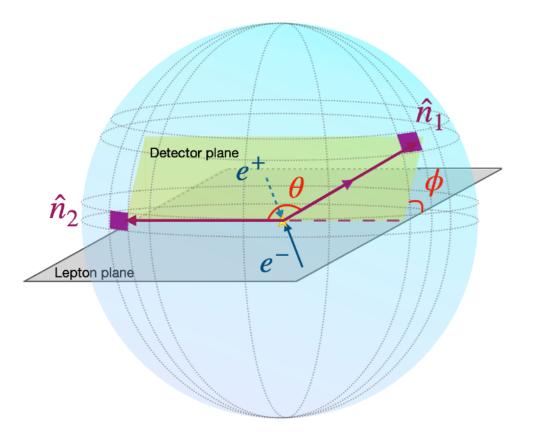
- Energy Correlators are interesting observables for characterizing generic quantum theories (Hofman, Maldacena, 2008)
- Well defined at weak coupling, strong coupling, in a CFT, coupled to gravity
- Simpler in perturbation theory Korchemsky, Sokatchev, Zhiboedov (2013)





# Spin asymmetry of EEC in the large angle limit

- Many spin asymmetries arise from the azimuthal correlations
- Azimuthal angle dependence in the small angle limit Chen, Moult, & Zhu '20; Li, Liu, Yuan, Zhu '23
- We extend the EEC in the back-to-back by considering azimuthal asymmetries associated with the EEC Kang, Lee, DYS, Zhao '23



$$\frac{1}{\sigma_{\text{tot}}} \frac{d\Sigma_{\text{e}^{+}\text{e}^{-}}}{d\tau d\phi} = \frac{\langle \mathcal{OE} \left( \vec{n}_{1} \right) \mathcal{E} \left( \vec{n}_{2} \right) \mathcal{O}^{\dagger} \rangle}{\langle \mathcal{OO}^{\dagger} \rangle}$$

$$EEC_{e^{+}e^{-}}(\tau,\phi) \equiv \frac{d\Sigma_{e^{+}e^{-}}}{d\tau d\phi} = \frac{1}{2} \sum_{1,2} \int d\sigma z_{1} z_{2} \, \delta\left(\tau - \frac{1 + \cos\theta_{12}}{2}\right) \delta(\phi - \phi_{12})$$

## Azimuthal dependent EEC in e<sup>+</sup>e<sup>-</sup>

The standard TMD factorization for the back-to-back di-hadron process

$$\begin{split} \frac{d\sigma}{dz_{i}dz_{j}d^{2}\boldsymbol{q}_{T}} &= \sigma_{0}\,H(Q,\mu)\sum_{q}e_{q}^{2}\int d^{2}\boldsymbol{p}_{1\perp}d^{2}\boldsymbol{p}_{2\perp}d^{2}\boldsymbol{\lambda}_{\perp}\delta^{2}\left(\frac{\boldsymbol{p}_{1\perp}}{z_{1}} + \frac{\boldsymbol{p}_{2\perp}}{z_{2}} - \boldsymbol{\lambda}_{\perp} + \boldsymbol{q}_{T}\right)S(\boldsymbol{\lambda}_{\perp}^{2},\mu,\nu) \\ &\times \left[D_{1,h_{1}/q}^{(u)}(z_{1},\boldsymbol{p}_{1\perp}^{2},\mu,\zeta/\nu^{2})D_{1,h_{2}/\bar{q}}^{(u)}(z_{2},\boldsymbol{p}_{2\perp}^{2},\mu,\zeta/\nu^{2}) + \cos(2\phi_{12})\left(\hat{\boldsymbol{q}}_{T,\alpha}\hat{\boldsymbol{q}}_{T,\beta} - \frac{1}{2}g_{\perp,\alpha\beta}\right)\right. \\ &\times \frac{\boldsymbol{p}_{1\perp}^{\alpha}}{z_{1}M_{1}}H_{1,h_{1}/q}^{\perp(u)}(z_{1},\boldsymbol{p}_{1\perp}^{2},\mu,\zeta/\nu^{2})\frac{\boldsymbol{p}_{2\perp}^{\beta}}{z_{2}M_{2}}H_{1,h_{2}/\bar{q}}^{\perp(u)}(z_{2},\boldsymbol{p}_{2\perp}^{2},\mu,\zeta/\nu^{2})\right]. \end{split}$$

$$z_i = rac{2P_{hi} \cdot q}{Q^2} = rac{2E_i}{Q}$$
 Energy fraction

$$D_{1,h/q}^{(u)}(z,\pmb{p}_{\perp}^2,\mu,\zeta/
u^2)$$
 Unpolarized TMD FF

$$H_{1,h/q}^{\perp(u)}(z,\boldsymbol{p}_{\perp}^2,\mu,\zeta/
u^2)$$
 Collins TMD FF

#### Leading Quark TMDFFs ( )→ Hadron Spin ( •-



	Quark	Spir
\ /		

		Quark Polarization			
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)	
Unpolarized		$D_1$ = $lacktriangle$ Unpolarized		$H_1^{\perp} = \bigcirc - \bigcirc \bigcirc$	
Polarized Hadrons	L		$G_1 = \bigcirc - \bigcirc - \bigcirc \rightarrow$ Helicity	$H_{1L}^{\perp} = \longrightarrow - \longrightarrow$	
	т	$D_{1T}^{\perp} = \bullet - \bullet$ Polarizing FF	$G_{1T}^{\perp} = \stackrel{\uparrow}{\longleftarrow} - \stackrel{\uparrow}{\longleftarrow}$	$H_1 = \begin{array}{c} \uparrow \\ \hline \\ \text{Transversity} \end{array}$ $H_{1T}^{\perp} = \begin{array}{c} \uparrow \\ \hline \\ \end{array}$	

## Azimuthal dependent EEC in e<sup>+</sup>e<sup>-</sup>

The TMD factorization for the azimuthal-dependent EEC in the back-to-back limit

$$\text{EEC}_{e^{+}e^{-}}(\tau,\phi) = \frac{1}{2}\sigma_{0} H(Q,\mu) \sum_{q} e_{q}^{2} \int \frac{b \, db}{2\pi} \left[ J_{0}(b\sqrt{\tau}Q) J_{q}(b,\mu,\zeta) J_{\bar{q}}(b,\mu,\zeta) + \cos(2\phi) \, \frac{b^{2}}{8} J_{2}(b\sqrt{\tau}Q) \, J_{q}^{\perp}(b,\mu,\zeta) J_{\bar{q}}^{\perp}(b,\mu,\zeta) \right]$$

New term: azimuthal asymmetry "Collins-type" EEC jet functions

A similar structure for Winner-take-All jet function was given in W. Lai, X. Liu, M Wang, H. Xing '21 '22

• The unpolarized EEC jet function has a close relation to the unpolarized TMD FFs

$$J_q(b,\mu,\zeta) \equiv \sum_h \int_0^1 dz \, z \, \tilde{D}_{1,h/q}(z,b,\mu,\zeta)$$

Collins-type EEC jet functions are closely connected with the Collins FFs

$$J_q^\perp(b,\mu,\zeta) \equiv \sum_h \int_0^1 dz\, z\, ilde H_{1,h/q}^\perp(z,b,\mu,\zeta)$$

# Collins-type EEC jet function

We introduce Collins-type EEC jet function

$$J_{q}(\boldsymbol{b}, \mu, \zeta) \equiv \sum_{h} \int_{0}^{1} dz z \tilde{D}_{1,h/q}(z, b, \mu_{b_{*}}, \zeta_{i}) e^{-S_{\text{pert}}(\mu, \mu_{b_{*}*}) - S_{\text{NP}}^{D_{1}}(b, Q_{0}, \zeta)} \left(\sqrt{\frac{\zeta}{\zeta_{i}}}\right)^{\kappa(b, \mu_{b_{*}})}$$

$$J_{q}^{\perp}(\boldsymbol{b}, \mu, \zeta) \equiv \sum_{h} \int_{0}^{1} dz z \tilde{H}_{1,h/q}^{\perp}(z, b, \mu_{b_{*}}, \zeta_{i}) e^{-S_{\text{pert}}(\mu, \mu_{b_{*}}) - S_{\text{NP}}^{H_{1}^{\perp}}(b, Q_{0}, \zeta)} \left(\sqrt{\frac{\zeta}{\zeta_{i}}}\right)^{\kappa(b, \mu_{b_{*}})}$$

Collins function in *b*-space

The OPE of the subtracted unpolarized and Collins TMD FFs gives

$$\tilde{D}_{1,h/q}(z,b,\mu,\zeta) = \left[ C_{j\leftarrow q} \otimes D_{1,h/j} \right] (z,b,\mu,\zeta) + \mathcal{O}(b^2 \Lambda_{\text{QCD}}^2), 
\tilde{H}_{1,h/q}^{\perp}(z,b,\mu,\zeta) = \left[ \delta C_{j\leftarrow q}^{\text{Collins}} \otimes \hat{H}_{1,h/j}^{\perp(3)} + A_{j\leftarrow q} \tilde{\otimes} \hat{H}_{F,h/j} \right] (z,b,\mu,\zeta) + \mathcal{O}(b^2 \Lambda_{\text{QCD}}^2),$$

twist-3 FFs (H<sub>F</sub> is ignored)

$$\begin{split} \delta C_{q'\leftarrow q}^{\text{Collins}}\left(z,b,\mu,\zeta\right) = &\delta_{qq'} \left\{ \delta\left(1-z\right) + \frac{\alpha_s}{\pi} \left[ C_F \delta\left(1-z\right) \left( -\frac{L_b^2}{4} + \frac{L_b}{2} \left( \frac{3}{2} + \ln\frac{\mu^2}{\zeta^2} \right) - \frac{\pi^2}{24} \right) \right. \\ &\left. + \left( \ln z - \frac{L_b}{2} \right) \hat{P}_{q\leftarrow q}^c\left(z\right) \right] \right\} + \mathcal{O}(\alpha_s^2) \,, \end{split}$$

## The OPE of the Collins TMD FFs

The OPE of the subtracted unpolarized and Collins TMD FFs gives

$$\tilde{H}_{1,h/q}^{\perp}(z,b,\mu,\zeta) = \left[\delta C_{j\leftarrow q}^{\text{Collins}} \otimes \hat{H}_{1,h/j}^{\perp(3)} + A_{j\leftarrow q} \tilde{\otimes} \hat{H}_{F,h/j}\right](z,b,\mu,\zeta) + \mathcal{O}(b^2 \Lambda_{\text{QCD}}^2)$$

• Standard convolution  $\left[C_{j\leftarrow q}\otimes F_{h/j}\right](z,b,\mu,\zeta)=\int_{z}^{1}rac{dx}{x}C_{j\leftarrow q}\left(rac{z}{x},b,\mu,\zeta
ight)F_{h/j}\left(x,\mu\right)$ 

$$\begin{split} \delta C_{q'\leftarrow q}^{\text{Collins}}\left(z,b,\mu,\zeta\right) = &\delta_{qq'}\bigg\{\delta\left(1-z\right) + \frac{\alpha_s}{\pi}\bigg[C_F\delta\left(1-z\right)\left(-\frac{L_b^2}{4} + \frac{L_b}{2}\left(\frac{3}{2} + \ln\frac{\mu^2}{\zeta^2}\right) - \frac{\pi^2}{24}\right) \\ &+ \left(\ln z - \frac{L_b}{2}\right)\hat{P}_{q\leftarrow q}^c\left(z\right)\bigg]\bigg\} + \mathcal{O}(\alpha_s^2)\,, \end{split}$$

 $\hat{H}_{1,h/j}^{\perp(3)}(z,\mu)$ : twist-3 fragmentation function, related to the first k $_\perp$ -moment of the Collins TMD FF

Double convolution

$$\left[A_{j\leftarrow q}\tilde{\otimes}\hat{H}_{F,h/j}\right](z,b,\mu,\zeta) = \int_{z}^{1} \frac{dx}{x} \int \frac{dz_{1}}{z_{1}^{2}} \operatorname{PV}\left(\frac{1}{\frac{1}{x} - \frac{1}{z_{1}}}\right) A_{j\leftarrow q}\left(\frac{z}{x}, z_{1}, b, \mu, \zeta\right) \hat{H}_{F,h/j}\left(x, z_{1}, \mu\right)$$

starts at the order  $O(\alpha_S)$  and is ignored in our work

## Sum rule

The collinear functions in the OPE matching obey the sum rules

$$\sum_h \int_0^1 dz \, z \, D_{1,h/j} \, (z,\mu) = 1 \,, \quad \text{sum over longitudinal momentum fraction carried by the hadron is 1}$$

$$\sum_h \int_0^1 dz \, \hat{H}_{1,h/q}^{\perp(3)}(z,\mu) = 0 \,. \quad \text{the transverse momentum carried by the final hadron sum to 0 (Schafer-Teryaev sum rule)}$$

• In the OPE region 
$$J_q^{\perp}(b,\mu,\zeta) = \sum_h \int_0^1 dz \, z \, \tilde{H}_{1,h/q}^{\perp}(z,b,\mu,\zeta)$$
  

$$= \sum_h \int_0^1 dz \, \int_z^1 \frac{dx}{x} \, \delta C_{q\leftarrow q}^{\text{Collins}}(\frac{z}{x},b,\mu_{b_*},\zeta) \hat{H}_{1,h/q}^{\perp(3)}(x,\mu_{b_*}) \, e^{-S_{\text{pert}}(\mu,\mu_{b_*})}$$

$$= \int_0^1 d\tau \delta C_{q\leftarrow q}^{\text{Collins}}(\tau,b,\mu_{b_*},\zeta) \left[\sum_h \int_0^1 dx \, \hat{H}_{1,h/q}^{\perp(3)}(x,\mu_{b_*})\right] e^{-S_{\text{pert}}(\mu,\mu_{b_*})}$$

$$= 0,$$

 We find that the Collins-type EEC jet function becomes zero in the OPE region upon neglecting the off-diagonal matching terms.

# Collins-type EEC with subsets of hadrons

• In the small angle limit, the track function formalism was used to study energy correlation between hadrons with specific quantum number  $\langle \mathcal{E}_{\mathbb{S}_1}(\hat{n}_1)\mathcal{E}_{\mathbb{S}_2}(\hat{n}_2)\rangle$  Chang, Procura, Thaler, & Waalewijn '13; Y, Li, Moult, Schrijnder, Waalewijn, H. X. Zhu '21; Jaarsma, Y. Li, Moult, Waalewijn, Z. X. Zhua '22, '23 + H. Chen '22 '23

• In the large angle limit (TMD region), one can also use subset  $\mathbb S$  of hadrons to define the large time time.

jet function

Tacking jet function for the recoil free jets

$$\bar{\mathcal{J}}_q^{(1)} = \mathcal{J}_q^{(1)} + 4C_F \int_0^1 dx \, \frac{1+x^2}{1-x} \ln \frac{x}{1-x} \int_0^1 dz_1 \, T_q(z_1, \mu) 
\times \int_0^1 dz_2 \, T_g(z_2, \mu) \left[\theta(z_1 x - z_2(1-x)) - \theta(x - \frac{1}{2})\right]$$

Chien, Rahn, DYS, Waalewijn & Wu '22 JHEP + Schrignder '21 PLB





$$\sum_{h} \Rightarrow \sum_{h \in \mathbb{S}}$$

E.g. 
$$S =$$
 charged particles

$$S = h$$

hadron full

 $p_{T,I} > 60 \,\text{GeV}, \ |\eta_I| < 2$ 

175

180

R = 0.5

170

 $\Delta \phi [\deg]$ 

----- hadron track

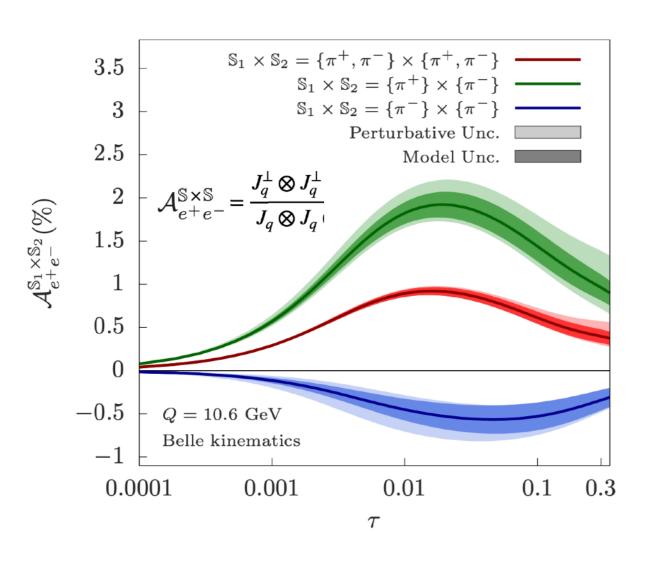
 $d\sigma/d\Delta\phi \, [{
m pb/deg}]$ 

# EEC in e+e-: Collins asymmetry

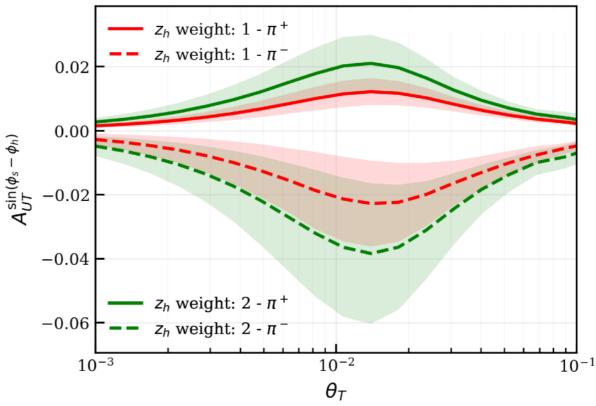
We provide a prediction for Collins asymmetry at Belle kinematics

$$EEC_{e^{+}e^{-}}(\tau,\phi) = \frac{d\Sigma_{e^{+}e^{-}}}{d\tau d\phi} = \frac{1}{2}\sigma_{0} \sum_{q} e_{q}^{2} \int d\mathbf{q}_{T}^{2} \,\delta\left(\tau - \frac{\mathbf{q}_{T}^{2}}{Q^{2}}\right) Z_{uu} \left[1 + \cos(2\phi) \frac{Z_{\text{Collins}}}{Z_{uu}}\right]$$

$$\equiv \frac{1}{2}\sigma_{0} \sum_{q} e_{q}^{2} Z_{uu} \left[1 + \cos(2\phi) A_{e^{+}e^{-}}(\tau Q^{2})\right] ,$$



$$egin{align} Z_{uu} &= \int rac{bdb}{2\pi} J_0(bq_T) J_q(b,\mu,\zeta) J_{ar q}(b,\mu,\zeta) \,, \ Z_{
m Collins} &= \int rac{bdb}{2\pi} rac{b^2}{8} J_2(bq_T) J_q^\perp(b,\mu,\zeta) J_{ar q}^\perp(b,\mu,\zeta) \,. \end{align}$$



# **Summary and Outlook**

- Jets and jet substructures offer new opportunity to understand hadron spin structures
- We develop the factorization framework to study transverse polarization effects for Λ(thrust) production in e<sup>+</sup>e<sup>-</sup> collisions</sup>
  - QCD effective field theory approach, model independent
  - Verify the universality of polarizing fragmentation function
- We apply fragmenting density matrix to investigate decoherence effects arising from soft and collinear radiation in spin entanglement
- Identify a correspondence between open quantum system and renormalization group
- Quantum information theory for hadronization
- We introduce the Collin-type EEC jet function for the first time
- By generalizing the EEC with azimuthal angle dependence, one gets access to spin dependent effects

Thank you