Probe Non-Perturbative Physics with Energy Correlators

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近代物理所，兰州，8.9.2025

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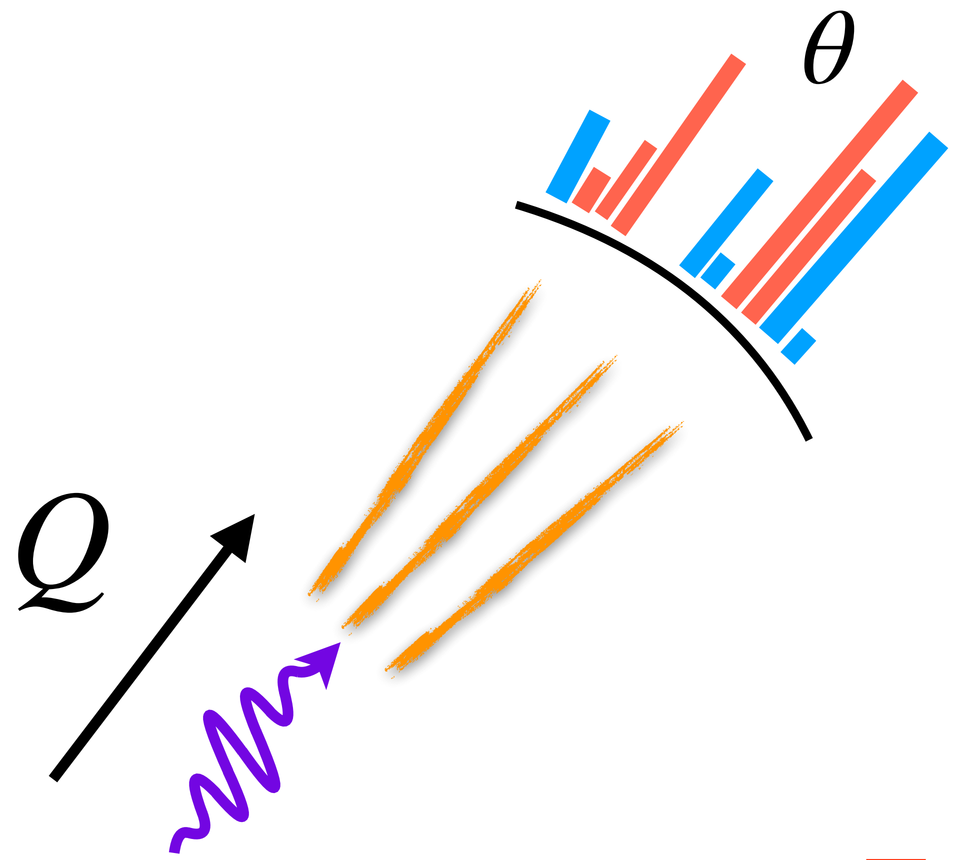
Outline

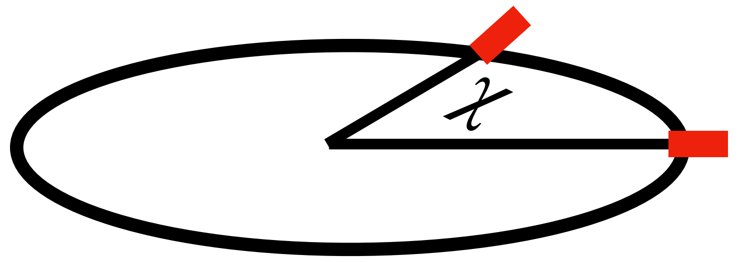
- Non-perturbative properties in the near-side EEC by only dimension and symmetry constraints
  - Warm up: Classic scaling and Dihadron distribution
  - Light-ray OPE, Quantum Scaling and Dihadron Fragmentation
- Conclusion

Energy Correlators

Energy-Energy-Correlator (EEC)

See Hua Xing's talk for the review and reference





$$\text{EEC} = \frac{1}{\sigma}\int d\sigma \sum_{ij} \frac{E_i E_j}{Q^2}\delta(\theta - \theta_{ij})$$

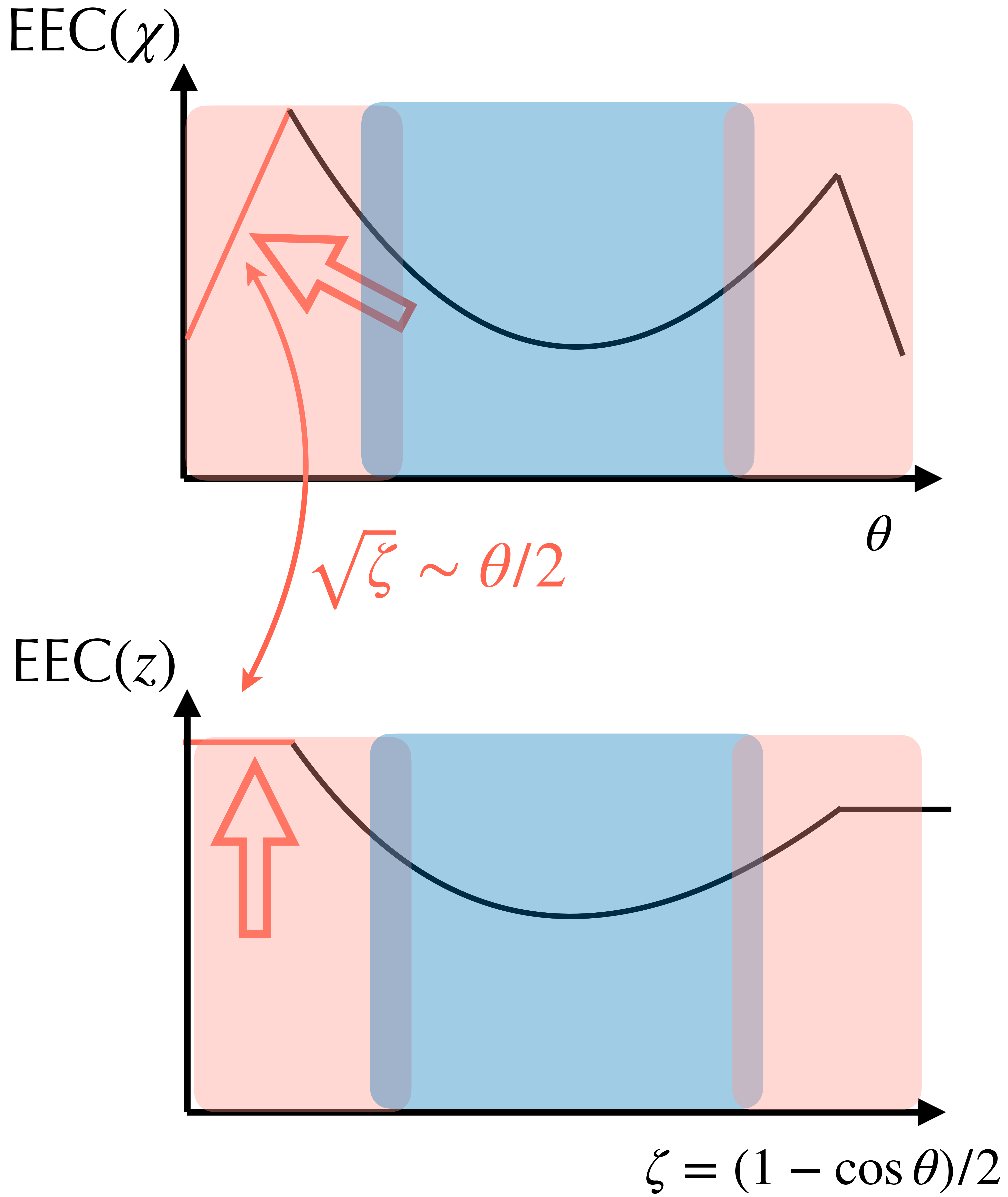
$$= \frac{\langle \mathcal{E}(n_i)\mathcal{E}(n_j)\rangle}{Q^2}$$

Sterman, 1975

Bashman, et al. 1978

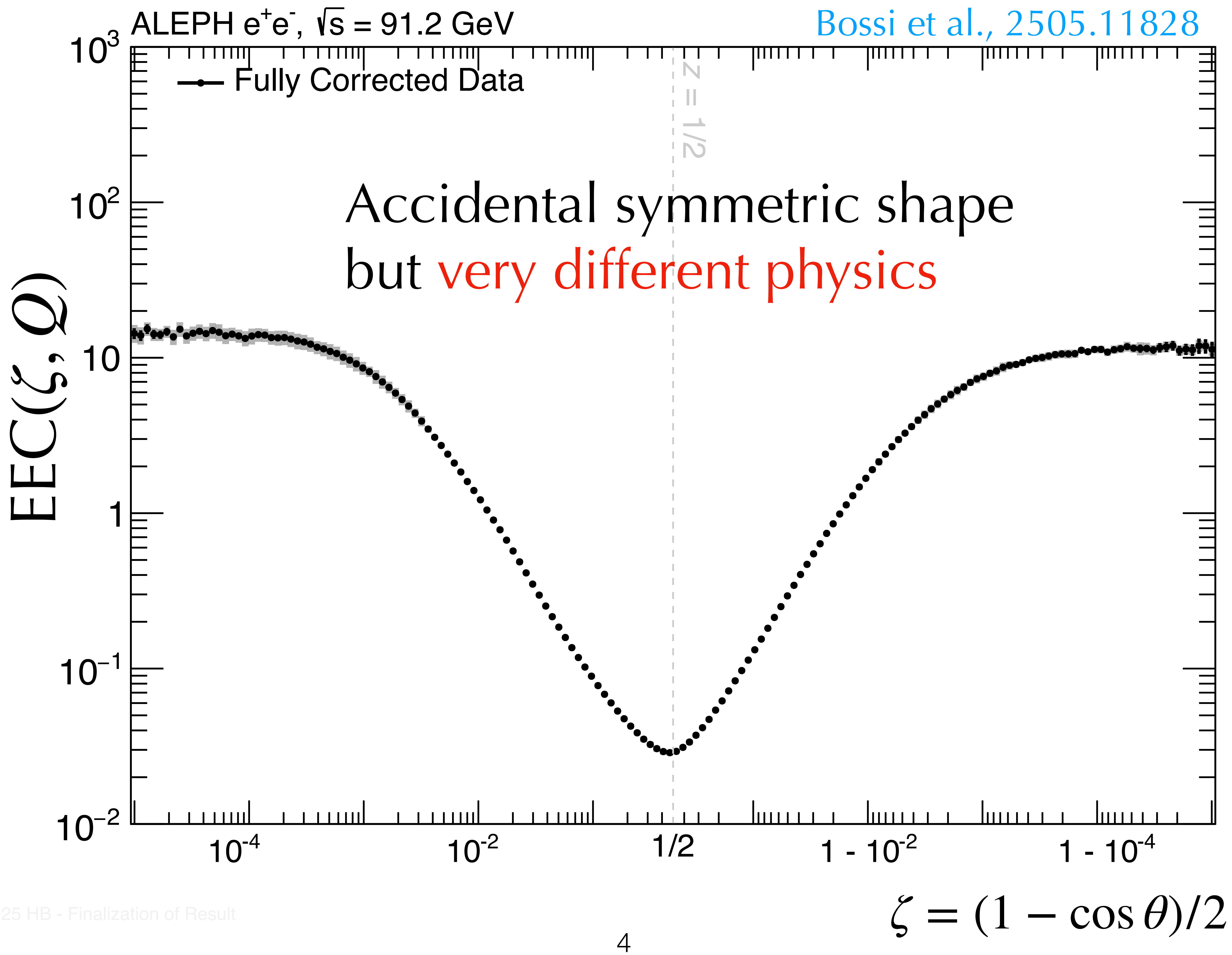
$$\mathcal{E}(n) = \lim_{\bar{n}\cdot x\to\infty} (\bar{n}\cdot x)^{4-2}\int dn \cdot x T_{\mu\nu}\bar{n}^{\mu}\bar{n}^{\nu}$$

Belitsky et al., 1309.1424



Energy Correlators

Real data using track



Energy Correlators

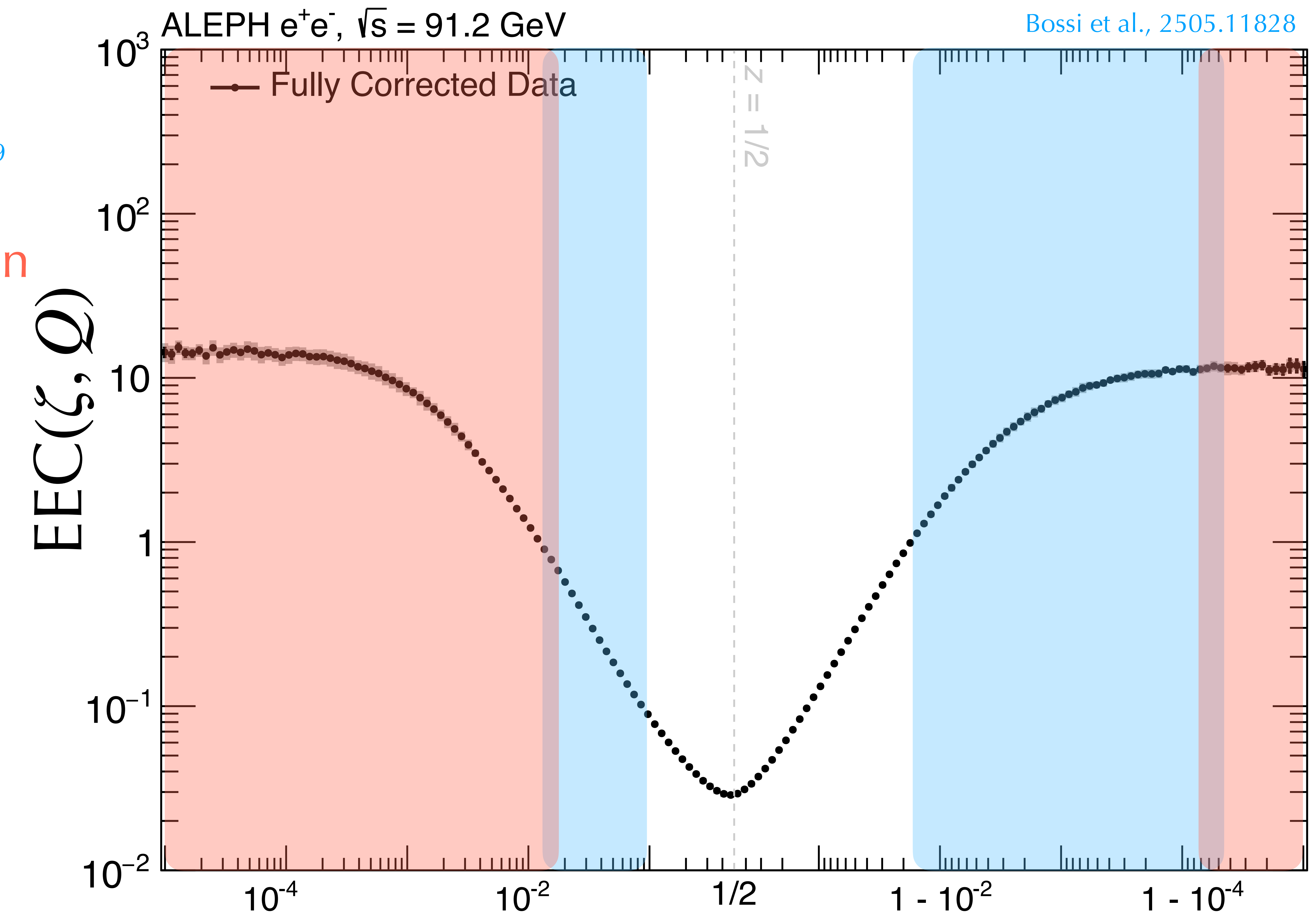
Real data using track

- $\chi \to 0$, collinear
- pQCD $\sim \zeta^{-1+\gamma[3]}$ Dixon, Moult, Zhu, 2019
- Purely Non-Perturbative transition

Collinear region:

More suitable for underlying NP studies

The least understood region



$\zeta = (1 - \cos\theta)/2$

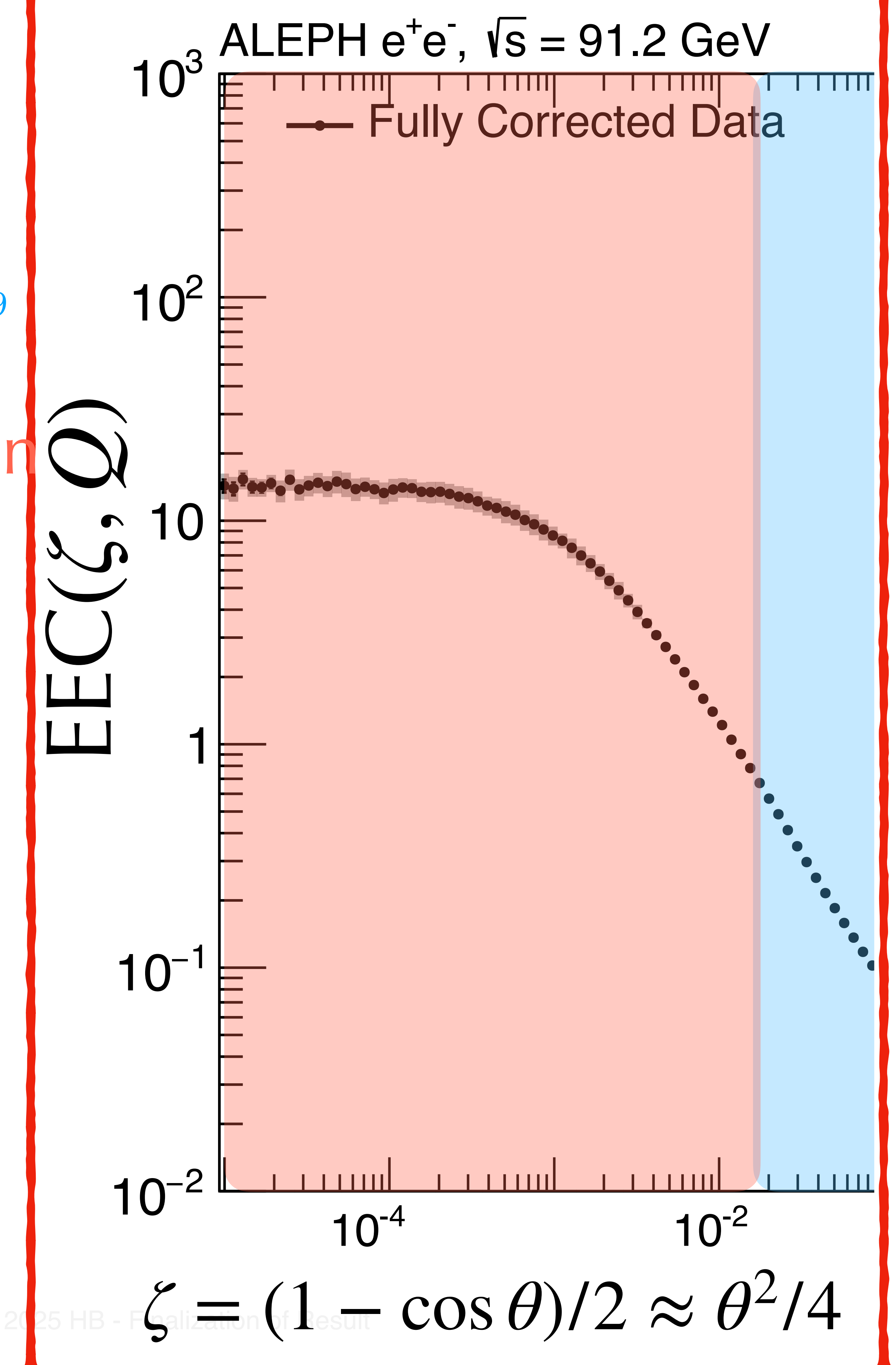
- $\chi \to \pi$, TMD soft $\sim e^{a \ln^2 \frac{1}{\zeta}} e^{-2S_{NP}}$ Moult, Zhu, 2018
- Mostly perturbative
- Suppressed Non-perturbative regime

TMD region:

Conventional approach

Energy Correlators around the Confinement Transition

- $\chi \to 0$, collinear
- pQCD $\sim \zeta^{-1+\gamma[3]}$
  Dixon, Moult, Zhu, 2019
- Purely Non-Perturbative transition



Collinear region:

More suitable for underlying NP studies

The least understood region

CFT

Infinte coupling limit

Hofman, Maldacena, 2008

Large multiplicity expansion

Chicherin, et al., 2023

Holographic model of confinement

Csaki, Ismail, 2024

Csaki, Ferrante, Ismail, 2024

QCD

Free hadron

Komiske, Moult, Thaler, Zhu, PRL 2022

TMD modeling with classic Q^2 scaling

XL, Vogelsang, Yuan, Zhu, PRL 2024

Barata, et al., PRL 2024

Energy Correlators around the Confinement Transition

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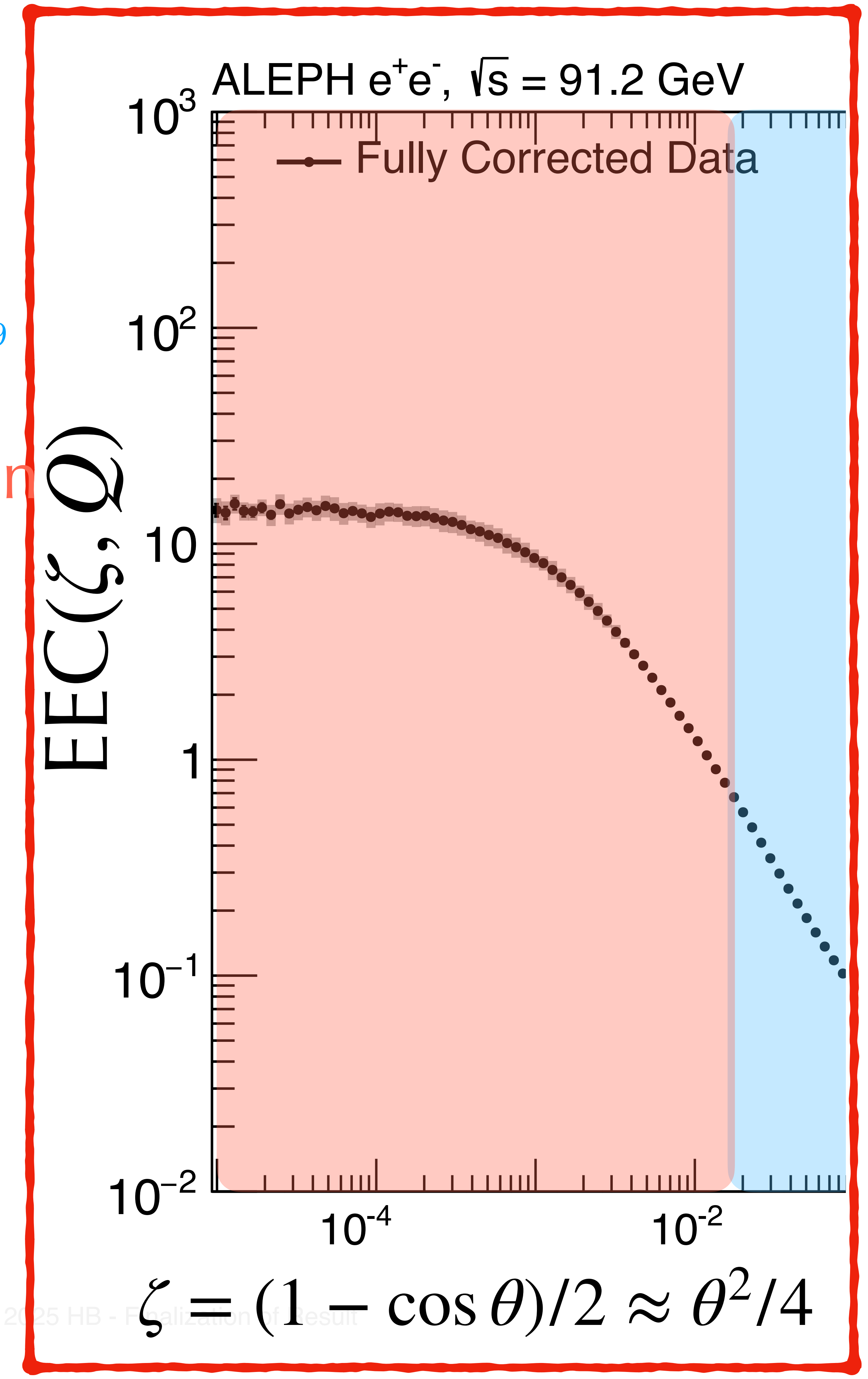


Collinear region:

More suitable for underlying NP studies

The least understood region

ONLY In the last month!

Quantum scaling:

Lee and Stewart 2507.11495

Chang, et al., 2507.15923

Herrmann, et al., 2507.17704

Kang, et al., 2507.17444

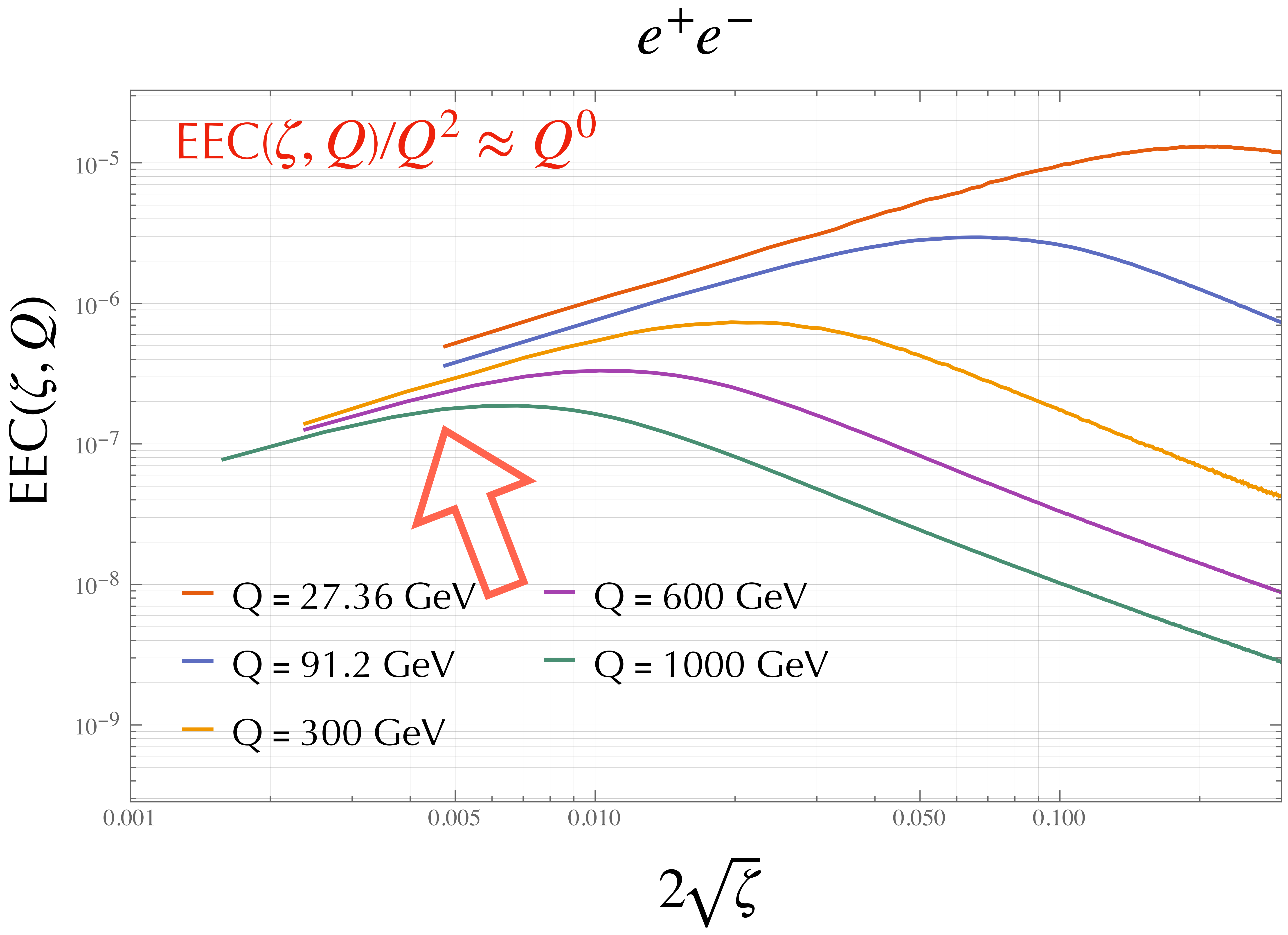
Guo, et al., 2507.15820

EEC spectrum does tell us about the underlying NP physics

Warm up: Classic scaling and Dihadron distribution

Naive dimension analysis

Classic scaling



$\mathrm{EEC}(\zeta) \approx Q^2$ + correction

What does it tell us?

$\mathrm{EEC}(\zeta) = \frac{1}{\sigma}\sum_{ij}\frac{E_iE_j}{Q^2}\sigma_{ij}$ is scaleless

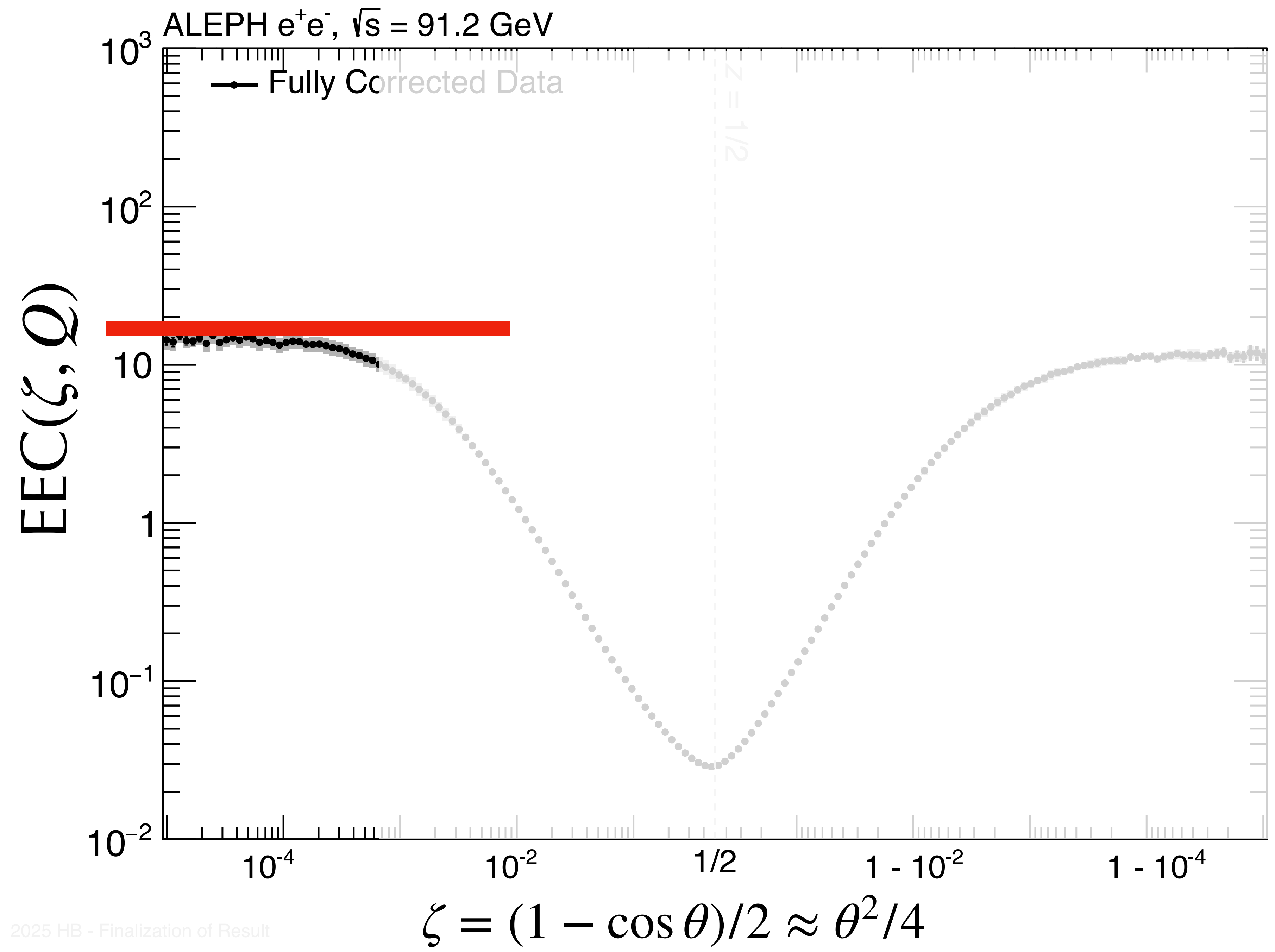
$\Rightarrow$ Additional scale to balance the dimension, different from CFT

Classic scaling

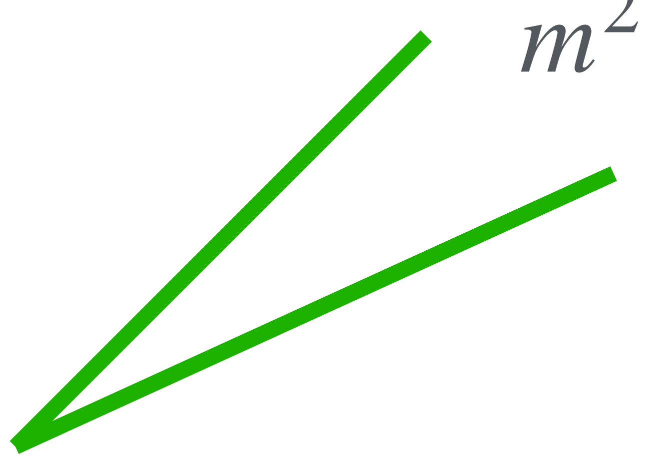
QCD:

$$\text{EEC}(\zeta) \approx \frac{Q^2}{\Lambda^2_{\text{QCD}}} + \text{power correction}$$
$$+ \text{quantum correction}$$

c.f. NP power correction pQCD $\sim \Lambda^a_{\text{QCD}}/Q^a$



$\Lambda^{-2}_{\text{QCD}}$ behavior naturally fits into a non-pert. universal transverse momentum/invariant mass distribution interpretation



$m^2 \sim z_1 z_2 Q^2 \zeta$

e.g.,

$$\frac{d\Sigma}{\sigma dm^2} \sim F_{np}(m) \sim \Lambda^{-2}_{\text{QCD}}$$

$$\Rightarrow \frac{d\Sigma}{\sigma d\zeta} \sim Q^2 F_{np}(Q\sqrt{\zeta}) \sim Q^2 F_{np}(0) + Q^2 (Q\sqrt{\zeta})\, F^{(1)}_{np}(0) + \ldots$$

- Recovers the Q^2 scaling, the $\zeta \to 0$ plateau behavior
- Near-side EEC is only a function of $Q^2\zeta$

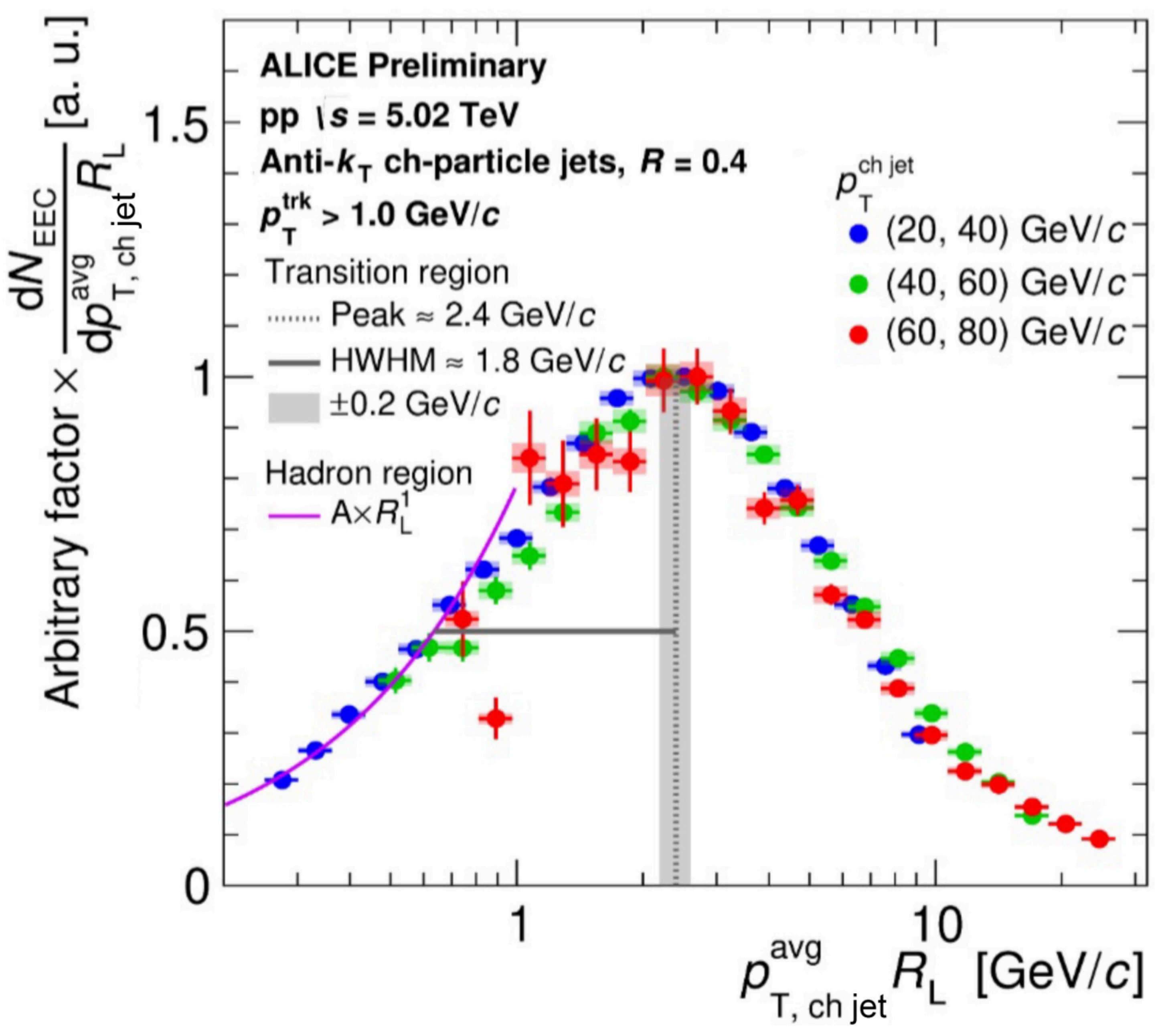
Classic scaling

QCD:

$$\mathrm{EEC}(\zeta) \approx \frac{Q^2}{\Lambda^2_{\mathrm{QCD}}} + \text{power correction}$$

$$+ \text{quantum correction}$$

c.f. NP power correction pQCD $\sim \Lambda^a_{\mathrm{QCD}}/Q^a$



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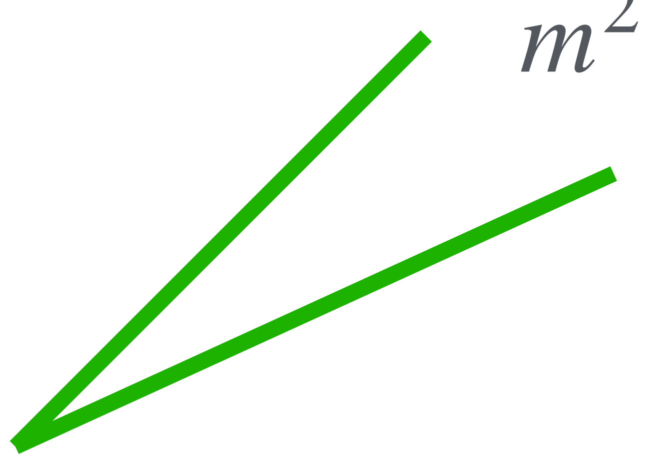


$m^2 \sim z_1 z_2 Q^2 \zeta$

$$\Rightarrow \frac{d\Sigma}{\sigma d\zeta} \sim Q^2 F_{np}(Q\sqrt{\zeta}) \sim Q^2 F_{np}(0) + Q^2 (Q\sqrt{\zeta}) F^{(1)}_{np}(0) + \ldots$$

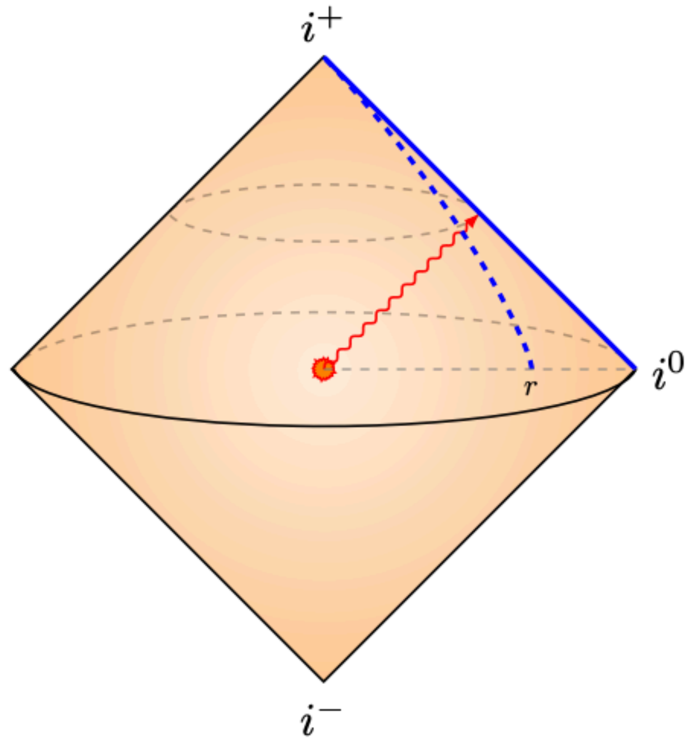
- Recovers the Q^2 scaling, the $\zeta \to 0$ plateau behavior
- Near-side EEC is only a function of $Q^2\zeta$
- **Microscopic structures? Quantum Scaling!**

Light-ray OPE, Quantum scaling and dihadron framentation function

More dimension and symmetry constraints

Quantum scaling by light-ray OPE

Chang, Chen, XL, Simmons-Duffin, Yuan, Zhu, 2507.15923



Scaling under
$n \to \rho n, \bar{n} \to \rho^{-1}\bar{n}$
$\mathbb{O}(\rho n) = \rho^{J_L}\mathbb{O}(n)$

$$\mathbb{O}^{\Delta_L, J_L} \sim \lim_{\bar{n}\cdot x \to \infty} (\bar{n}\cdot x)^{\Delta - J} \int dn \cdot x \, \mathcal{O}^{\Delta, J}_{\mu_1 \ldots \mu_J}(x)\bar{n}^{\mu_1}\ldots\bar{n}^{\mu_J}$$

Light-ray operator

Light transform

Local operator

e.g. $T_{\mu\nu}$
$\bar{\psi}\gamma_{\mu_1} iD_{\mu_2}\ldots iD_{\mu_J}\psi$

Dimension, classic scaling

Spin

1905.01311

$$-\Delta_L = \Delta - 1 - \Delta + J = J - 1$$

$$J_L = -\Delta + J + 1 - J = -\Delta + 1$$

e.g., $\mathcal{E}: \quad -\Delta_L = 2 - 1 = 1\,, \qquad J_L = -4 + 1 = -3$

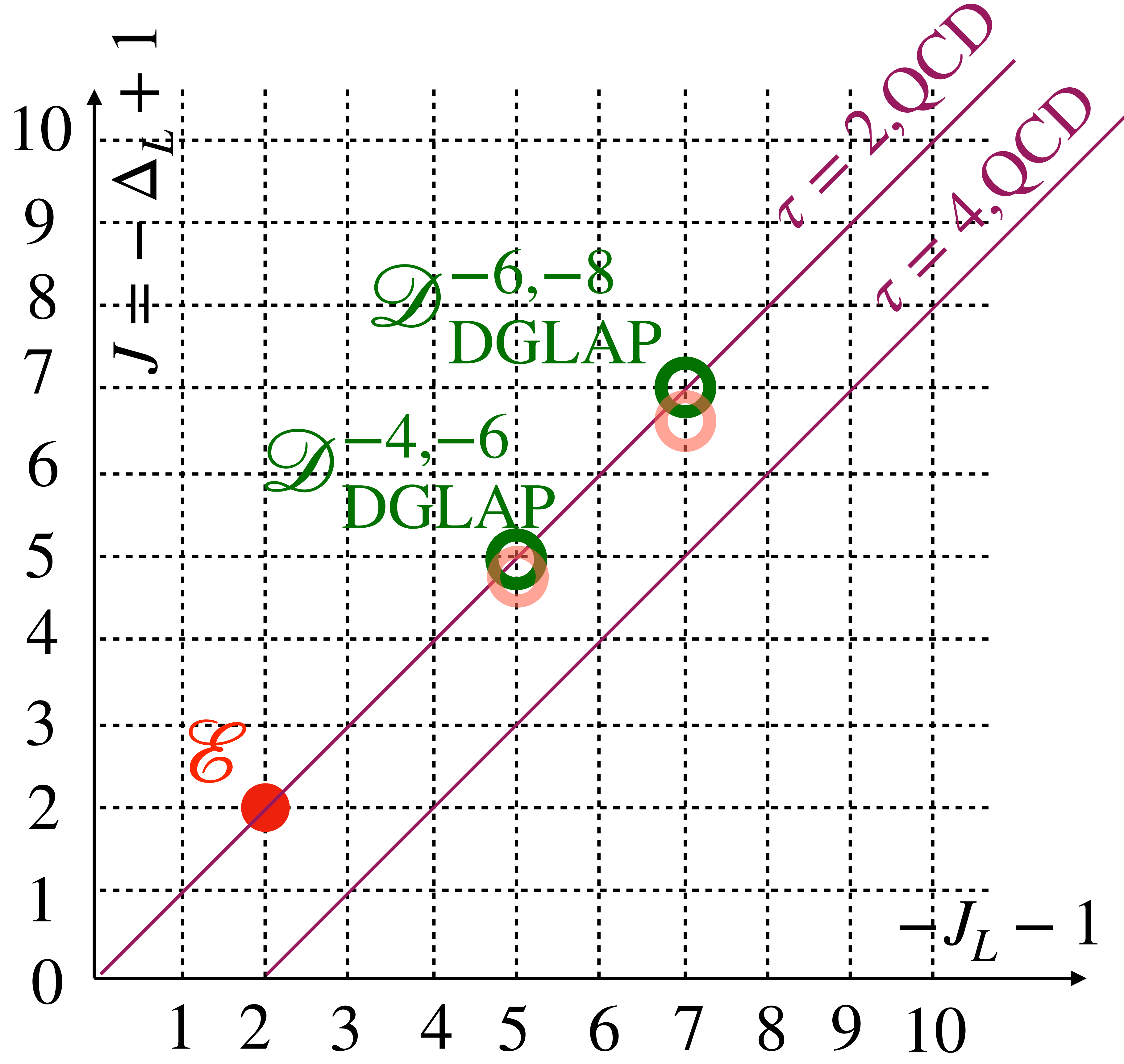
$\mathscr{D}_{\text{DGLAP}}$, Light-ray transformation of the local twist-2 operators:

$$\tau = 2 = \Delta - J = \Delta_L - J_L$$

$$\frac{d\mathscr{D}_{\text{DGLAP}}}{d\ln\mu^2} = \hat{\gamma}(J)\mathscr{D}_{\text{DGLAP}}$$

$\sqrt{\zeta}: \quad -\Delta_L = 0\,, \qquad J_L = 1$

($p_\perp \sim \bar{n}\cdot P\sqrt{\zeta}$ unchanged under boost)



Quantum scaling by light-ray OPE

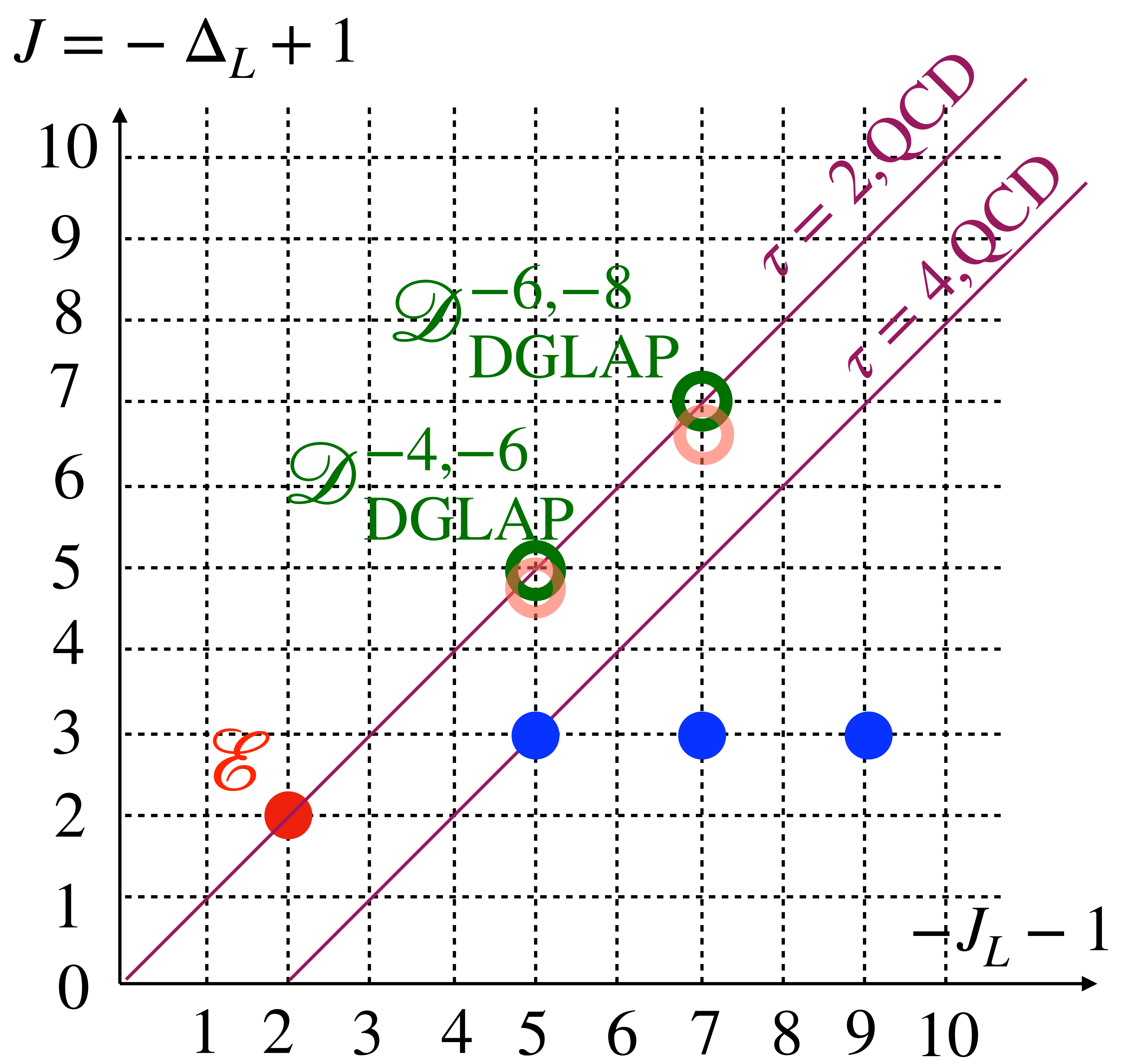
Chang, Chen, XL, Simmons-Duffin, Yuan, Zhu, 2507.15923

$$\mathscr{E}(n_1)\mathscr{E}(n_2) = \sum_{J_L} A_{J_L} \sqrt{\zeta}^{-6-J_L} \mathbb{O}_H^{-2,J_L}(n_2)$$

$-\Delta_{L,\mathscr{E}\mathscr{E}} = 2$

$J_{L,\mathscr{E}\mathscr{E}} = -6$

$-J_L \geq 6$



- Light-ray OPE in the small ζ limit with a hadron EFT ((boost)-Chiral?), in the free hadron regime, assume zero hadron mass
- Equate the Lorentz dimension and Lorentz spin on both sides of the expansion. Keep the dimension unchanged
- Both A_{J_L} and $\mathbb{O}_H$ can be calculated within the hadron EFT

Compare with the previous naive F_{np} expansion, but now we have more information (Lorentz symmetry) on each "$F^{(n)}$"

Quantum scaling by light-ray OPE

Chang, Chen, XL, Simmons-Duffin, Yuan, Zhu, 2507.15923

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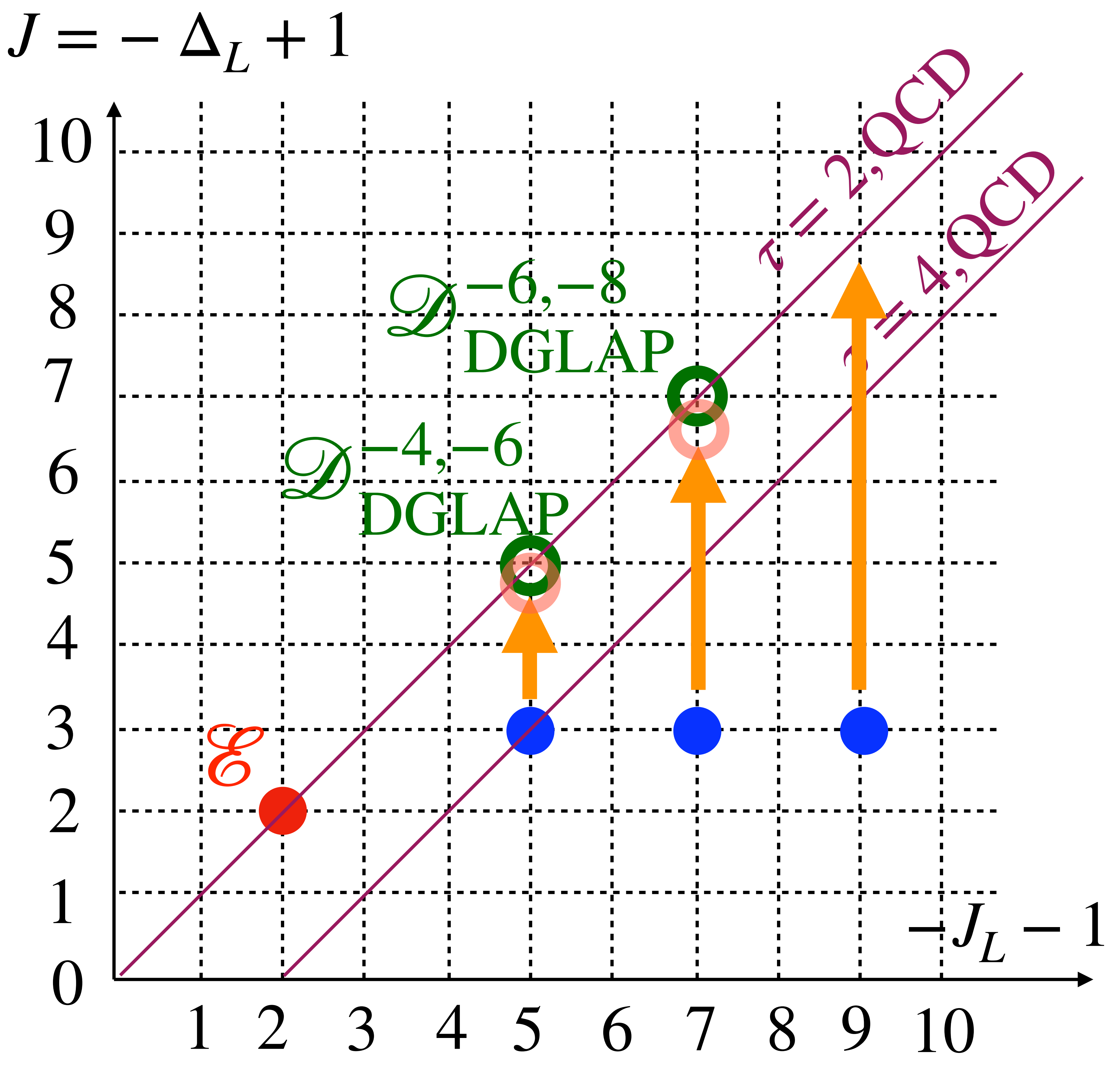
$-\Delta_{L,\mathcal{E}\mathcal{E}} = 2$

$J_{L,\mathcal{E}\mathcal{E}} = -6$

$-J_L \geq 6$

$$\mathbb{O}_H^{-2,J_L} = \sum_{\Delta_L} B_{-2-\Delta_L}(\mu^2, \Lambda^2_{\mathrm{QCD}}) \mathcal{D}_{\Delta_L, J_L}(\mu^2)$$

$$\approx B_{-4-J_L}(\mu^2, \Lambda^2_{\mathrm{QCD}}) \; \mathcal{D}^{J_L+2, J_L}_{\mathrm{DGLAP}}(\mu^2)$$



- Match $\mathbb{O}_H^{-2,J_L}$ (IR) onto QCD light-ray operators (UV), but keep the J_L unchanged
- B is non-perturbative
- Dominant contribution from twist-2

Quantum scaling by light-ray OPE

Chang, Chen, XL, Simmons-Duffin, Yuan, Zhu, 2507.15923

$$\mathscr{E}(n_1)\mathscr{E}(n_2) = \sum_{J_L} A_{J_L} \sqrt{\zeta}^{-6-J_L} \mathbb{O}_H^{-2,J_L}(n_2)$$

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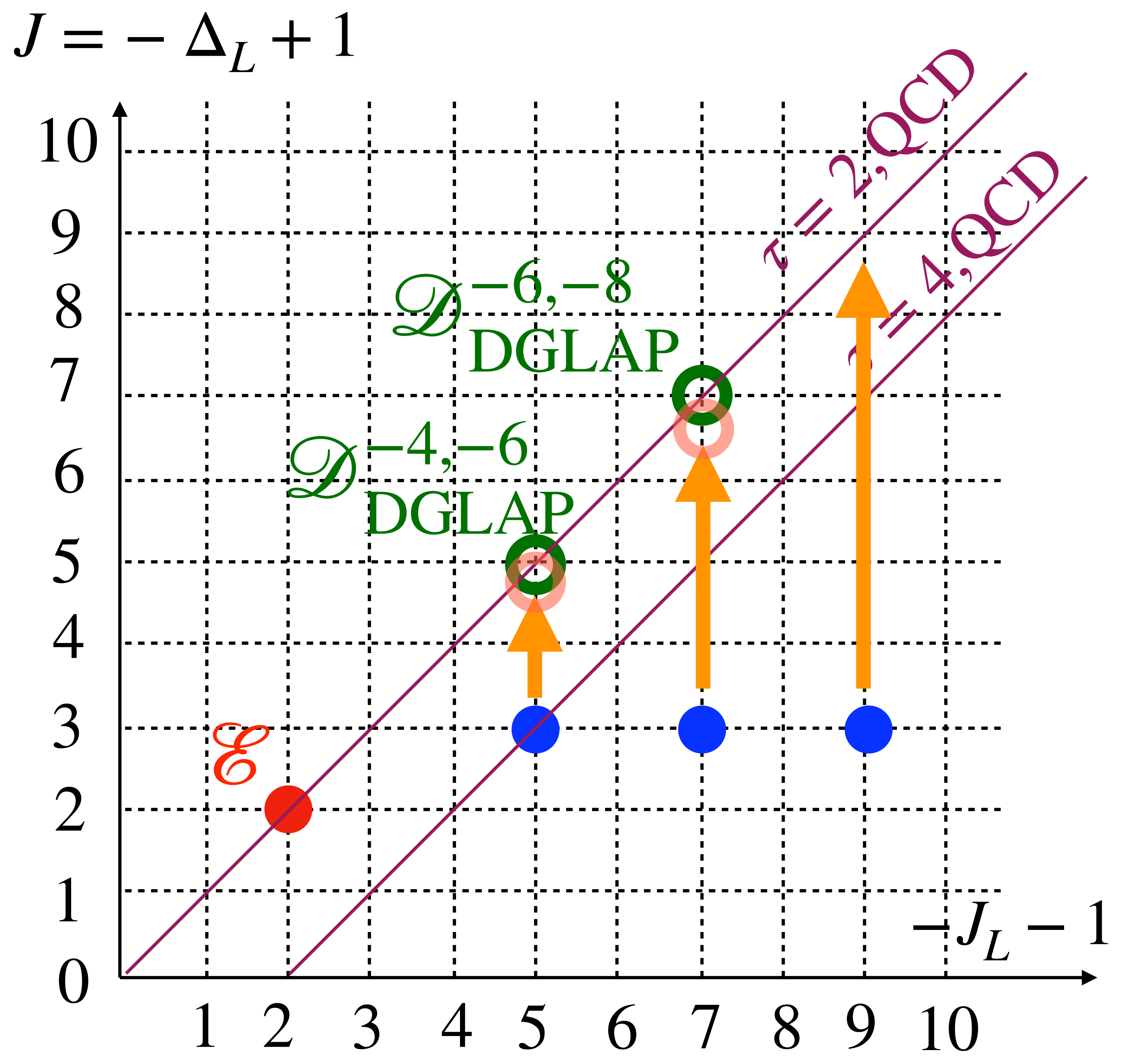


$$\text{EEC}(\zeta, Q) = \frac{\langle \mathscr{E}\mathscr{E} \rangle}{Q^2}$$

$$\approx Q^2 \sum_{-J_L \geq 6} (\sqrt{\zeta} Q)^{-6-J_L} R_{-4-J_L}(\mu^2, \Lambda_{\rm QCD}^2) \; \frac{\langle \mathscr{D}_{\rm DGLAP}^{J_L+2, J_L} \rangle (\mu^2)}{Q^{-(J_L+2)}}$$

$$R_{-4-J_L} = A_{J_L} B_{-4-J_L}$$

Hadron-level interaction

Parton to hadron fragmentation

Quantum scaling by light-ray OPE

Chang, Chen, XL, Simmons-Duffin, Yuan, Zhu, 2507.15923

$$\mathrm{EEC}(\zeta, Q) = \frac{\langle \mathscr{E}\mathscr{E} \rangle}{Q^2}$$

$$\approx Q^2 \sum_{-J_L \geq 6} (\sqrt{\zeta} Q)^{-6-J_L} R_{-4-J_L}(\mu^2, \Lambda^2_{\mathrm{QCD}}) \frac{\langle \mathscr{D}^{J_L+2, J_L}_{\mathrm{DGLAP}} \rangle (\mu^2)}{Q^{-(J_L+2)}}$$

- Q^2 scaling naturally recovers the plateau $(-J_L = 6)$, but also predicts finite ζ terms $(-J_L > 6)$
- Quantum scaling for each term:

$$\frac{d\mathscr{D}_{\mathrm{DGLAP}}}{d\ln\mu^2} = \hat{\gamma}(J)\mathscr{D}_{\mathrm{DGLAP}} = \hat{\gamma}(-J_L - 1)\mathscr{D}_{\mathrm{DGLAP}}$$

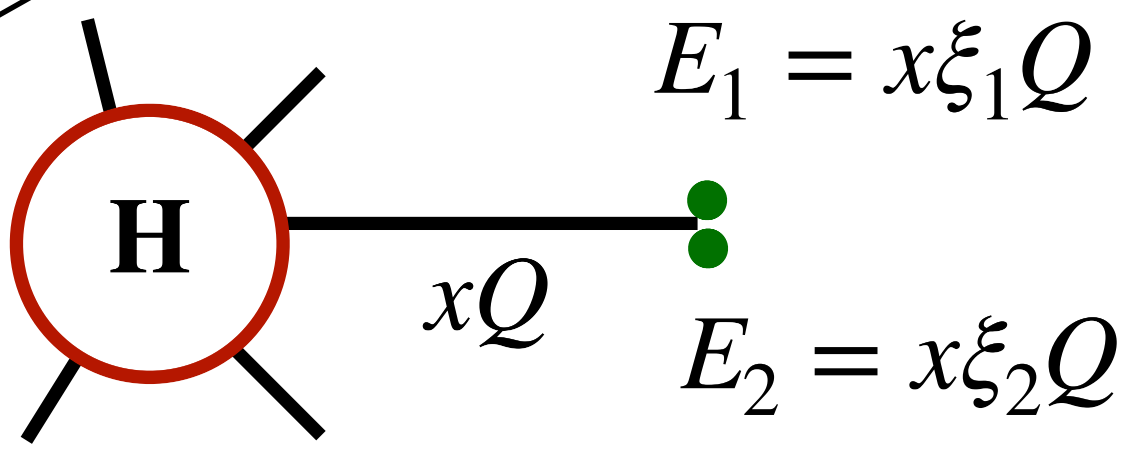
Plateau driven by $\hat{\gamma}(J = 5)$

Connection to Dihadron Fragmentation

Chang, Chen, XL, Simmons-Duffin, Yuan, Zhu, 2507.15923

$$\mathrm{EEC}(\zeta, Q) = \frac{\langle \mathscr{E}\mathscr{E} \rangle}{Q^2}$$

$$\approx Q^2 \sum_{-J_L \geq 6} (\sqrt{\zeta} Q)^{-6-J_L} R_{-4-J_L}(\mu^2, \Lambda^2_{\mathrm{QCD}}) \frac{\langle \mathscr{D}^{J_L+2, J_L}_{\mathrm{DGLAP}} \rangle (\mu^2)}{Q^{-(J_L+2)}}$$



$$\mathrm{EEC}(\zeta, Q) = \int dx H(xQ, \mu^2) \int d\xi_1 d\xi_2 dm^2 [x^2 \xi_1 \xi_2] \, D_{hh}(\xi_1, \xi_2, m) \times \delta\left(\zeta - \frac{m^2}{4x^2 \xi_1 \xi_2 Q^2}\right)$$

$$= 4 \int dx \left[x^4 H(xQ, \mu^2)\right] \int d\xi_1 d\xi_2 \xi_1^2 \xi_2^2 \, D_{hh}(\xi_1, \xi_2, m)$$

$$\sim Q^2 \sum_n (\sqrt{\zeta} Q)^n \int d\xi_1 d\xi_2 \xi_1^{2+n/2} \xi_2^{2+n/2} D^{(n)}_{hh}(\xi_1, \xi_2, 0) \int dx x^{4+n} H(xQ, \mu^2)$$

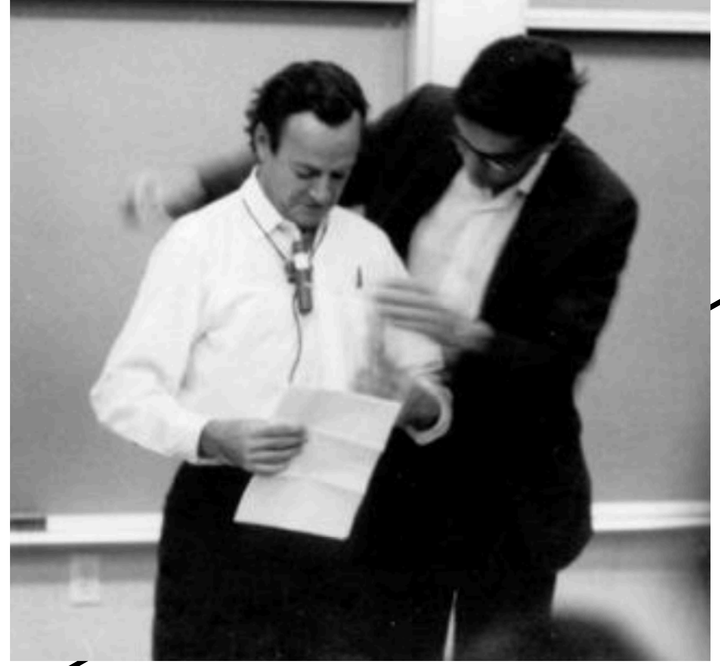
Expanding with $m \lesssim \Lambda_{\mathrm{QCD}}$

Light-ray OPE in the massless free hadron limit

=

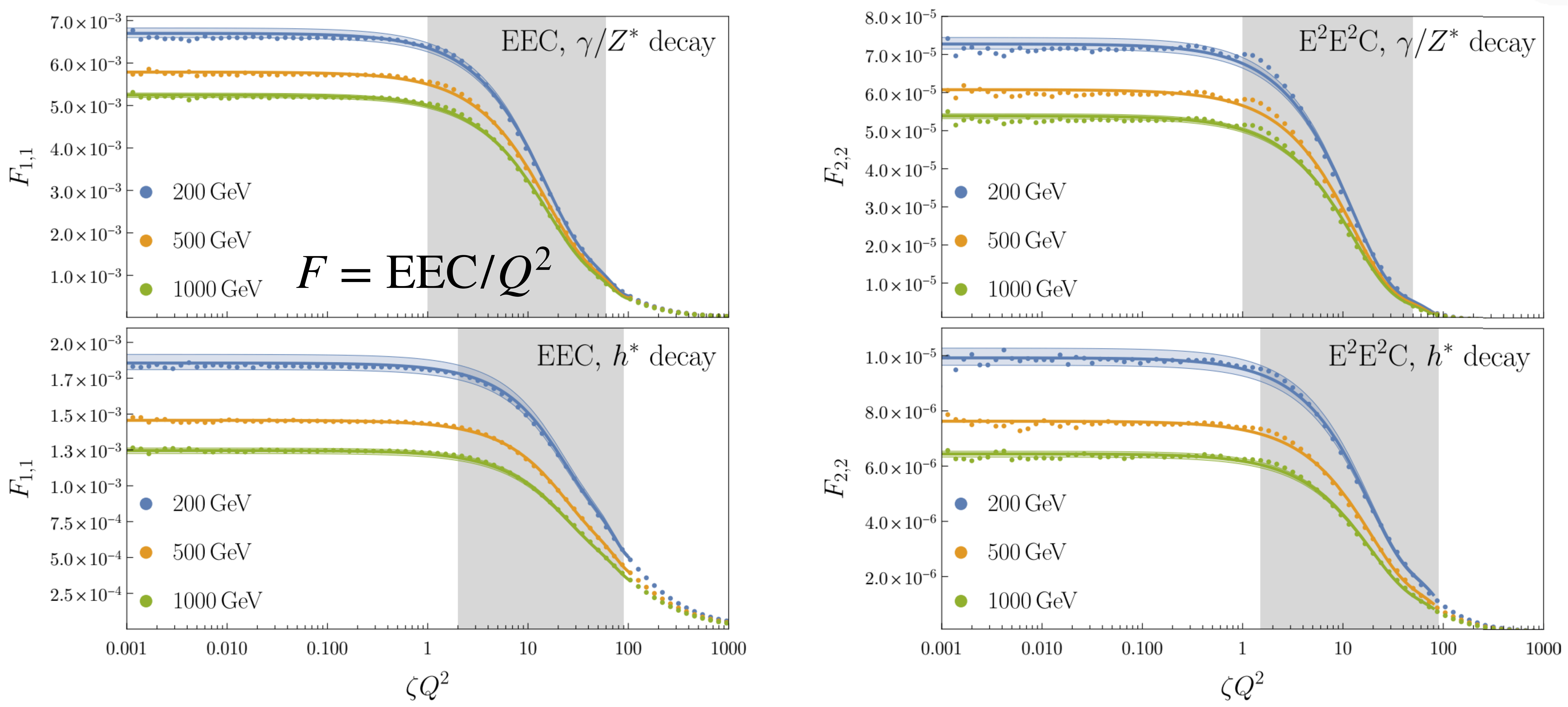
Dihadron fragmentation

Feynman Bjorken



Validation by Pythia (QED off)

Chang, Chen, XL, Simmons-Duffin, Yuan, Zhu, 2507.15923

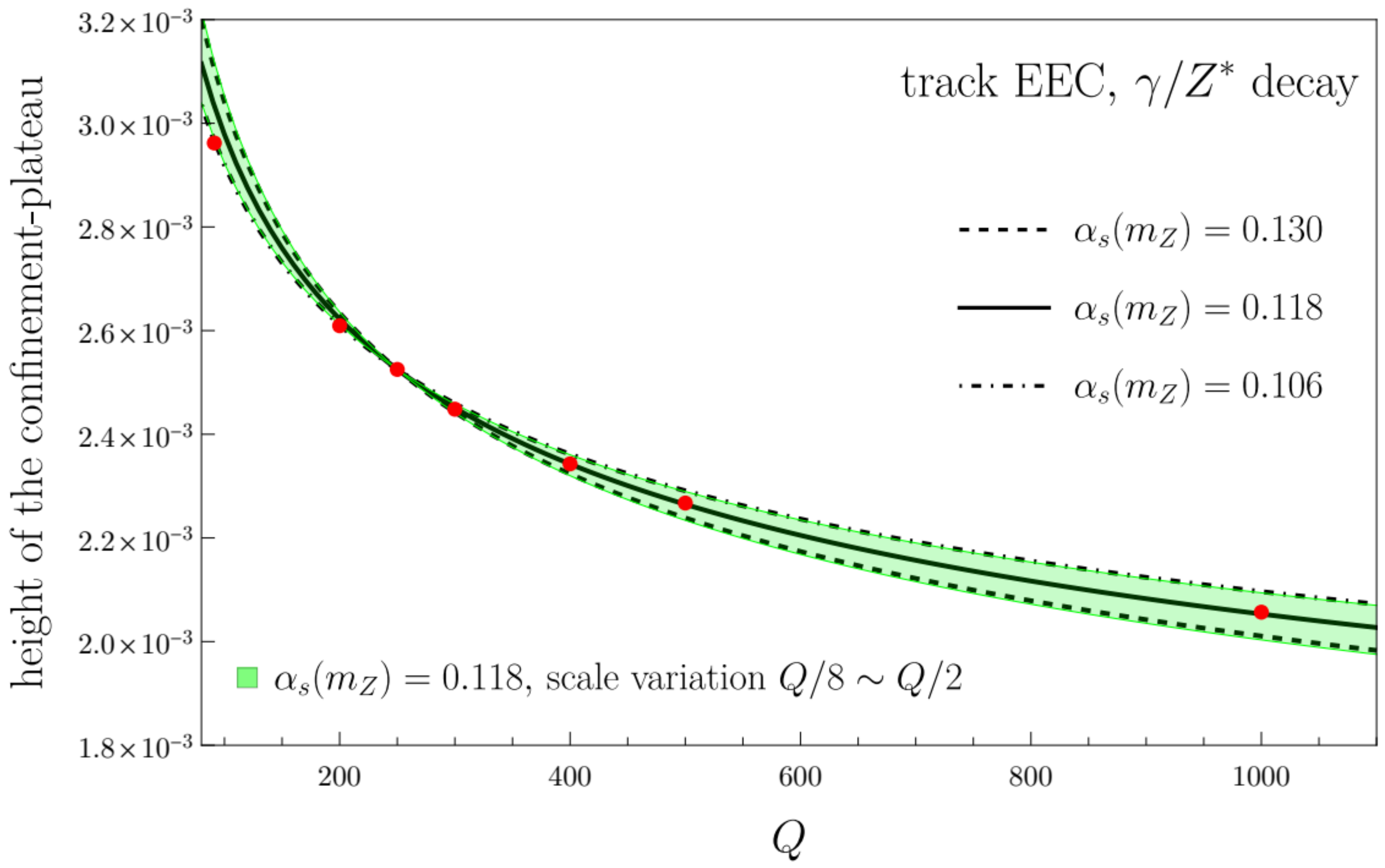


- Fit to 500GeV curve, quantum LL evolution to other Q's
- Excellent agreement!

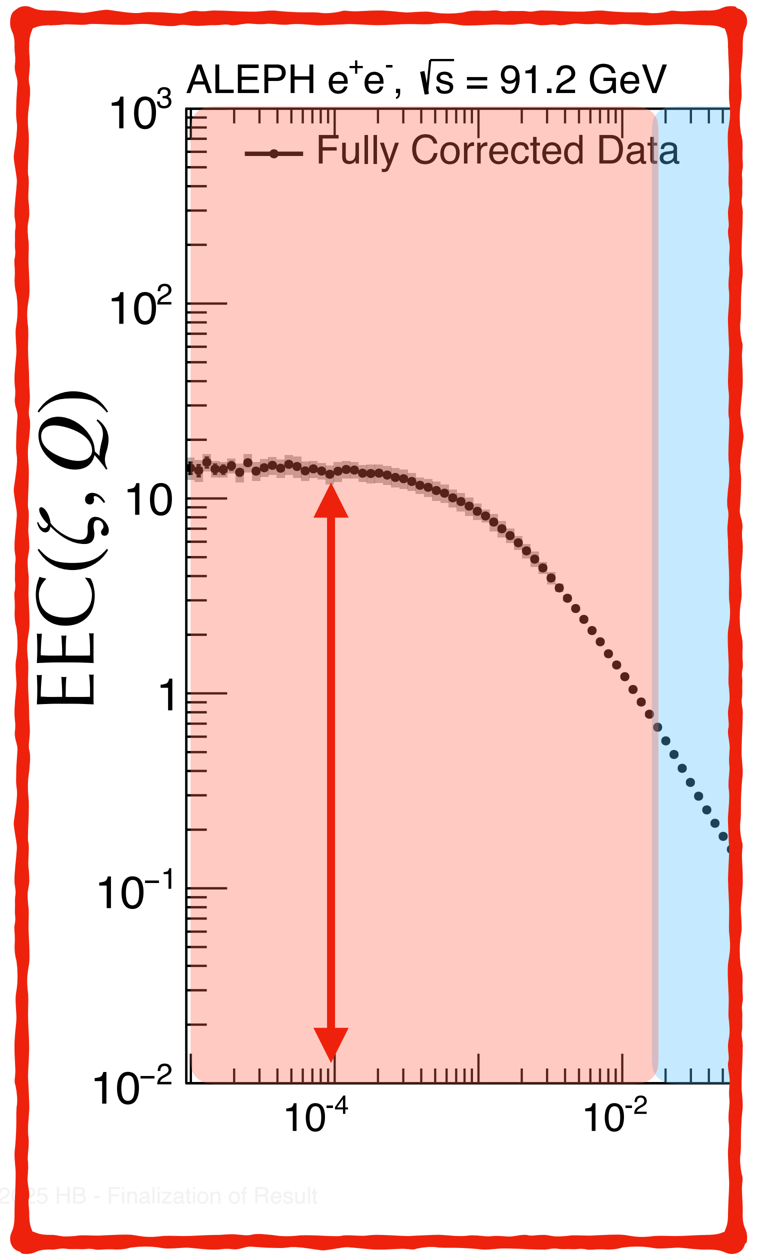
$$\frac{d\mathscr{D}_{\mathrm{DGLAP}}}{d\ln\mu^2} = \hat{\gamma}(J)\mathscr{D}_{\mathrm{DGLAP}} = \hat{\gamma}(-J_L-1)\mathscr{D}_{\mathrm{DGLAP}}$$

Possible α_s fitting

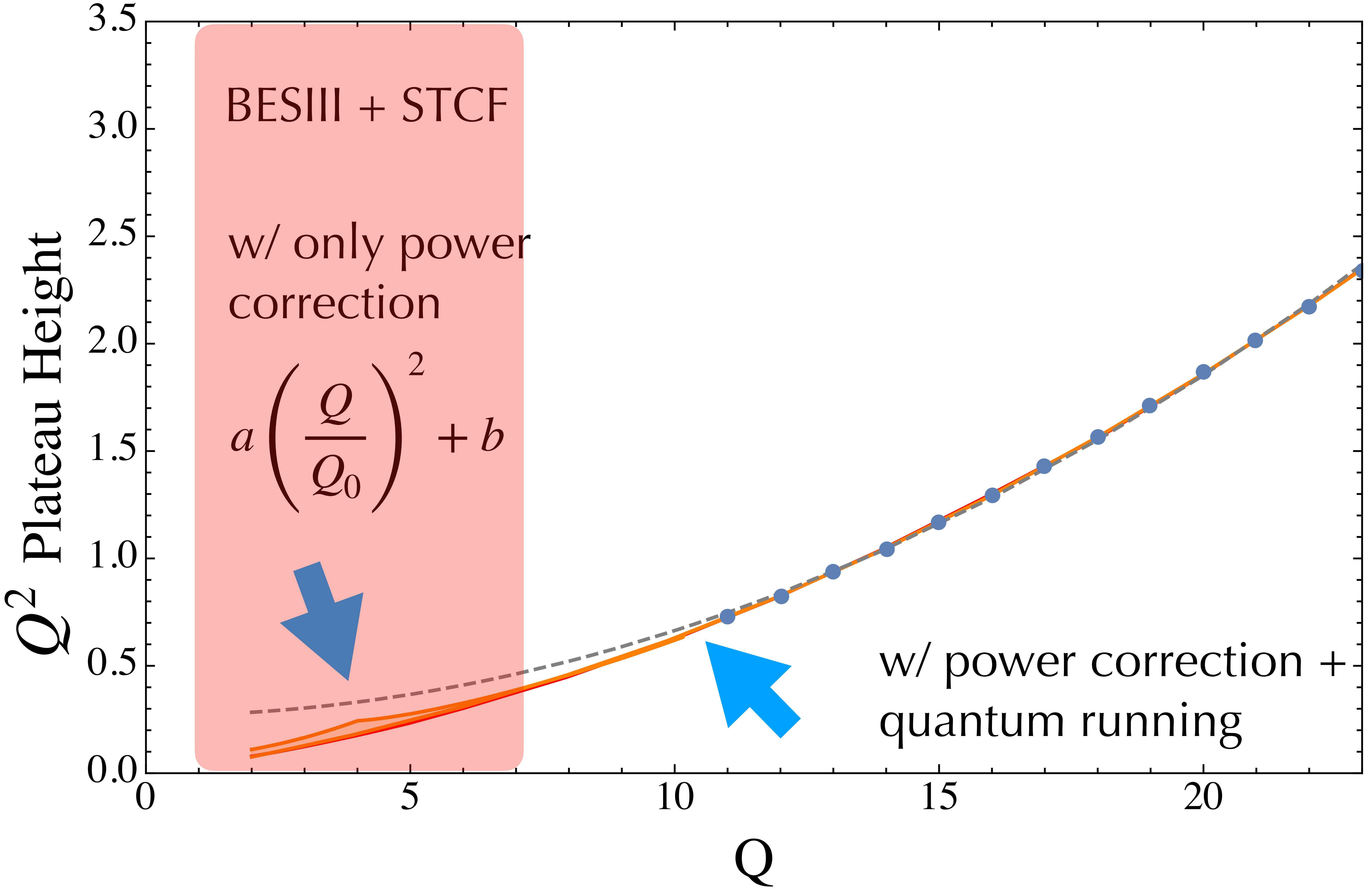
Chang, Chen, XL, Simmons-Duffin, Yuan, Zhu, 2507.15923



- Quantum scaling of the plateau height sensitive to α_s
- Plauteau hight should be experimentally easier to measure precisely (with tracks)
- Higher theory accuracy (NNLL) is ready to achieve



Low energy behavior



Still valid at low energy?

- Quark or hadron d.o.f ?
- Power correction to the di-hadron picture?

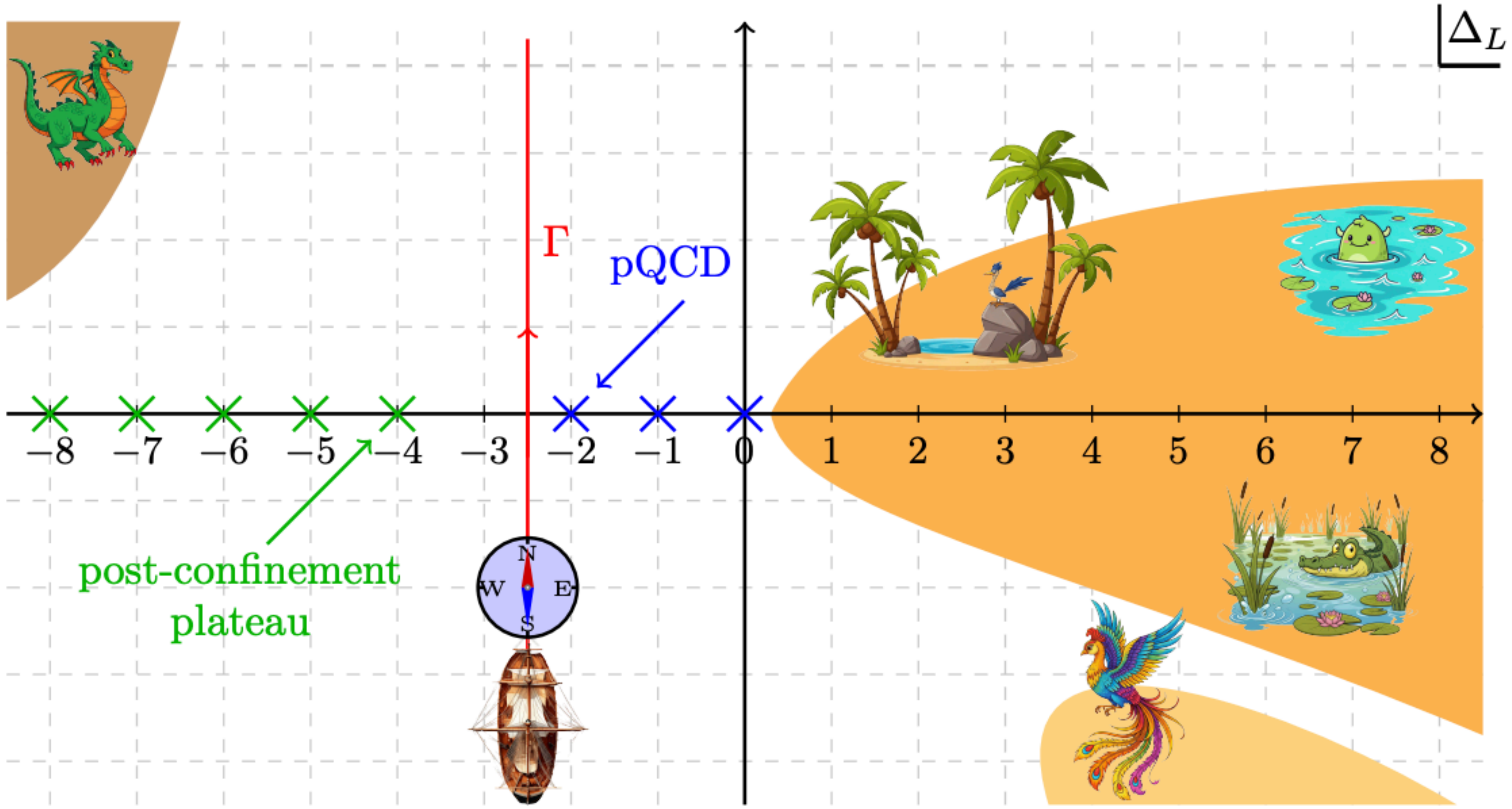
$$a\left(\frac{Q}{Q_0}\right)^2\left(\frac{\alpha_s(Q)}{\alpha_s(Q_0)}\right)^{\gamma(5)} + b\left(\frac{\alpha_s(Q)}{\alpha_s(Q_0)}\right)^{c}$$

Conclusion

- Quantum scaling around transition region
  - Light-ray OPE in the massless free hadron limit = Dihadron fragmentation, new paradigm for hadron structures
  - Hadron level interaction can be naturally encoded in light-ray OPE
  - Possible α_s precision determination in an unexpected regime

Thanks

Back-up



$$\mathrm{EEC}(\zeta, Q) = \sum_i \int_\Gamma \frac{d\Delta_L}{2\pi i} f_{\Delta_L - \tau_i}(\zeta) \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^{\Delta_L+2} \times C_{\Delta_L,i}\left(\mu^2/\Lambda^2_{\mathrm{QCD}}\right) \frac{\langle \mathcal{D}_{\Delta_L,i}(\mu^2)\rangle_Q}{Q^{-\Delta_L}}$$

A conjecture formula for the EEC for generic $\zeta Q^2/\Lambda^2_{\mathrm{QCD}}$ which can reproduce two different expansions in the pre-confinement and post-confinement regimes.