Global Analyses of Fragmentation Functions from NPC

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on behalf of
the Non-Perturbative Physics Collaboration

碎裂函数与能量关联研讨会, 兰州
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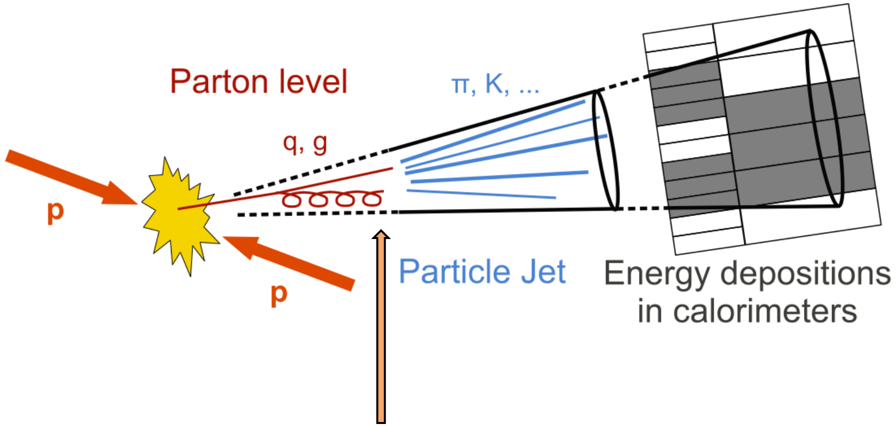
Fragmentation functions (FFs) as extension of the parton model

Bjorken, et al., ‘71
FFs proposed

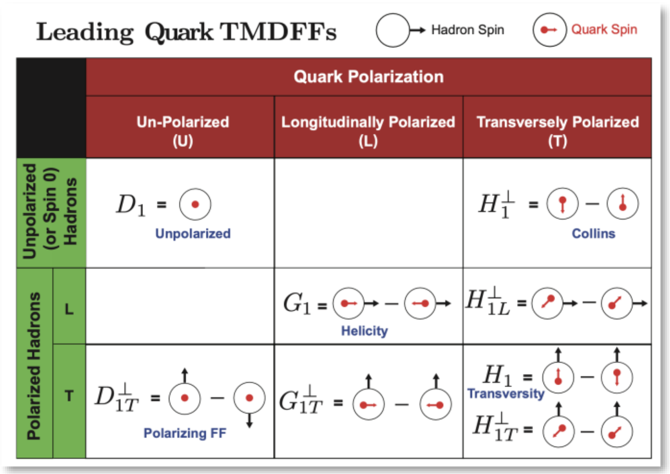
1969 **1971** **1977**

Feynman, ‘69
Parton model

Field, Feynman, ‘77
First determination



Collinear FFs: $$D_{h\leftarrow i}(z \equiv p_h^+/p_i^+)$$



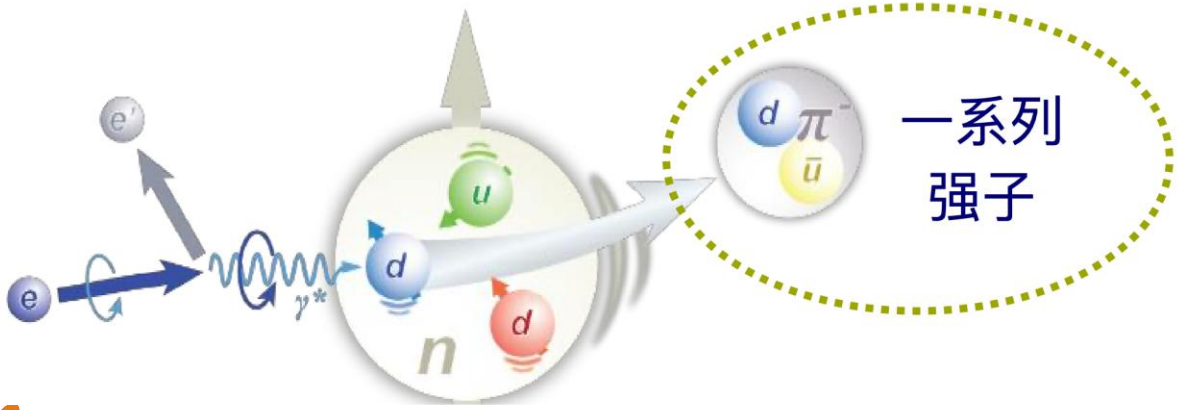
[2304.03302]

$D_{h\leftarrow i}(z)$: number density of hadron h carrying a fraction of momentum z of parton i

Why FFs: key ingredients of QCD factorization framework

- 1969 — Feynman, '69: Parton model
- 1971 — Bjorken et al., '71: FFs proposed
- 1977 — Field, Feynman, '77: First determination
- 1982 — Collins, Soper, '82: Definition in QCD
- 1989 — Collins et al., '89: Factorization thm.

➢ Semi-Inclusive DIS (SIDIS) : e + N -> e + **h** + X:

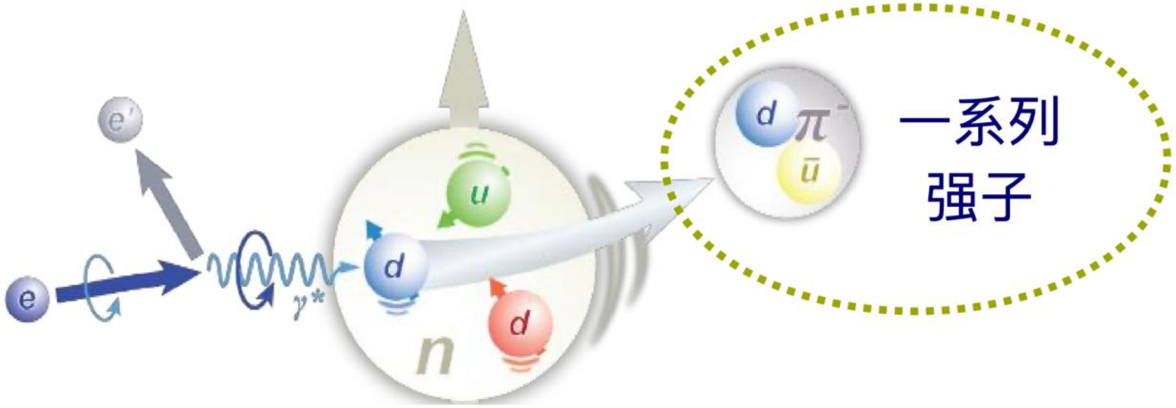


$$\frac{d^3\sigma_h}{dx\,dy\,dz_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x,y,z) \otimes D_{h/j}(z_h) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)$$

PDF FF

Why FFs: phenomenologial application

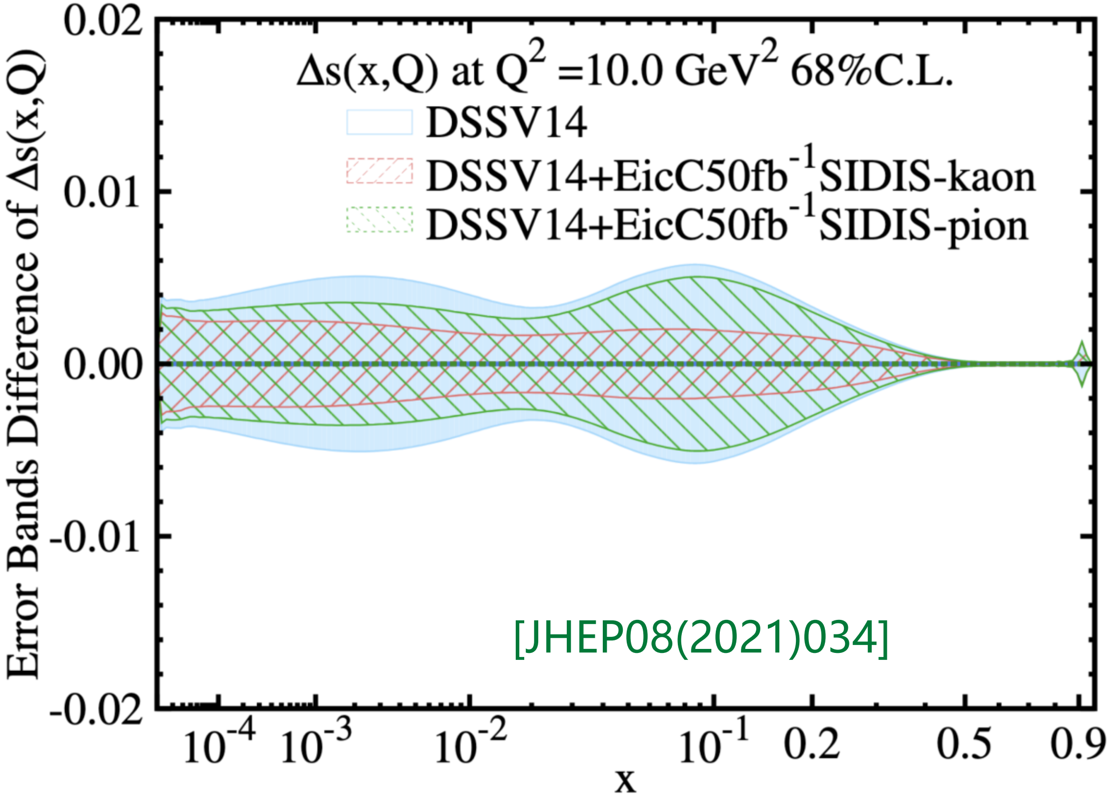
Identified hadron helps discriminate the initial parton



FFs are key inputs of pPDFs determination

$$g_1(x, Q^2, z) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x, Q^2) D^{q\to h}(Q^2, z) + \Delta\bar{q}(x, Q^2) D^{\bar{q}\to h}(Q^2, z)\right]$$

pPDF FF

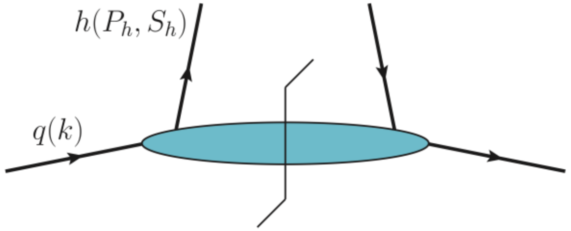


[JHEP08(2021)034]

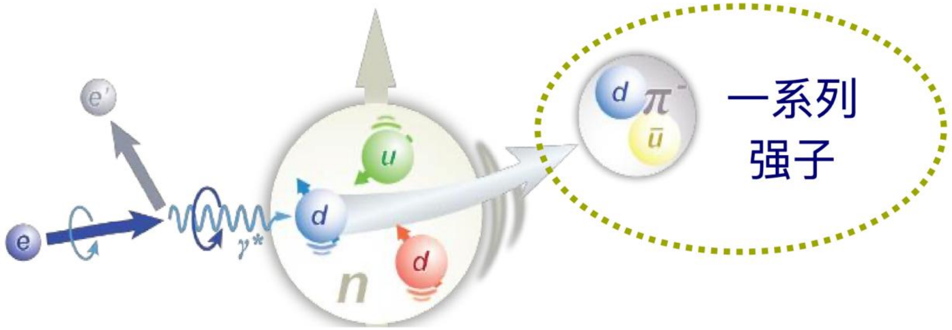
Extraction of FFs

❖ Field theory definiton of the collinear (integrated) quark FFs [Collins, Soper '82]

$$D_{h/q}(z) = \frac{z}{4}\sum_X \int \frac{d\xi^+}{2\pi} e^{iP_h^- \xi^+/z} \mathrm{Tr}\Big[\langle 0 | \mathcal{W}(\infty^+, \xi^+)\ \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X\rangle \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T)\ \mathcal{W}(0^+, \infty^+) | 0\rangle\ \gamma^-\Big]$$



❖ Global analyses of **ee(SIA), ep(SIDIS) and pp** data based on factorization formula.



$$\frac{d^3\sigma_h}{dx\,dy\,dz_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x, y, z) \otimes D_{h/j}(z_h) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

measurement　　input　　pQCD　　to be determined. Universal !

NPC collaboration gathering on July 19th 2025

in neighborhood of Huizhou city (host of EICc)

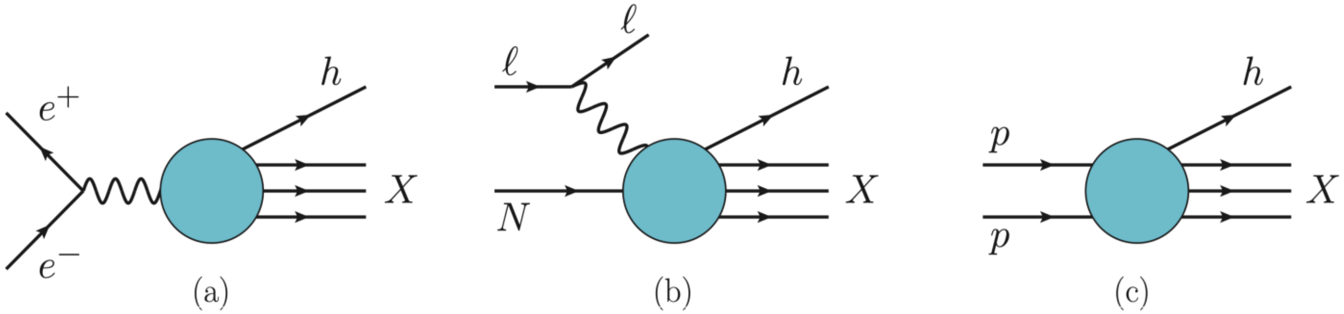
Jun Gao, ChongYang Liu, Meng Yang Li, **XiaoMin Shen**, **HongXi Xing**, **YuXiang Zhao**, Bin Zhou, YiYu Zhou
Shanghai JiaoTong Univ., South China Normal Univ., Institute of Modern Physics, CAS

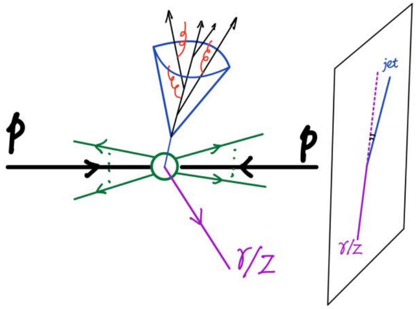
NPC=Non-Perturbative Physics Collaboration

Slide from Jun Gao

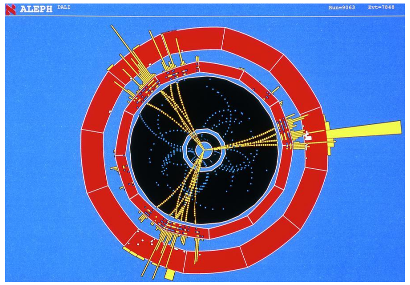
Automated calculation tool: FMNLO

- a hybrid scheme of phase-space slicing method and local subtraction
- **automation** of fragmentation calculations for arbitrary hard processes up to NLO, interfaced to MG5_aMC@NLO





Hadron in jet (pp)



Hadron in jet (ee)

- partonic cross sections saved as interpolation tables for **fast** convolution with FFs
- code publicly available [Liu, **XS**, Zhou, Gao, 2305.14620 (JHEP)]

NPC analyses of FFs to light charged hadrons $(\pi^{\pm}, K^{\pm}, p/\bar{p})$

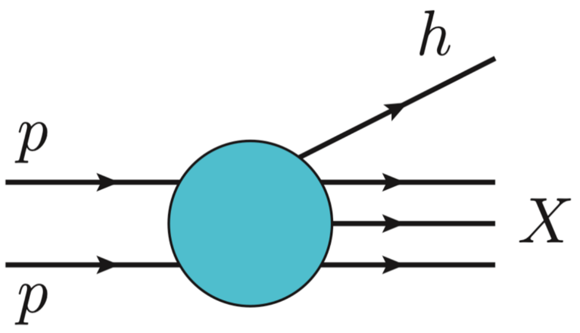
Gao, Liu, **XS**, Xing, Zhao, ***PRL 132, 261903*, '24**
Gao, Liu, **XS**, Xing, Zhao***, PRD 110, 114019*, '24** (Editors' suggestion)

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	**NPC**
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓
pp hadron in jet	✗	✗	✗	✗	✗	✓
FFs	$\pi^{\pm}$,$K^{\pm}$,p	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p,$h^{\pm}$ η	$\pi^{\pm}$	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO

Only some of the recent global analyses are shown here.

New opportunities in probing FFs at the LHC

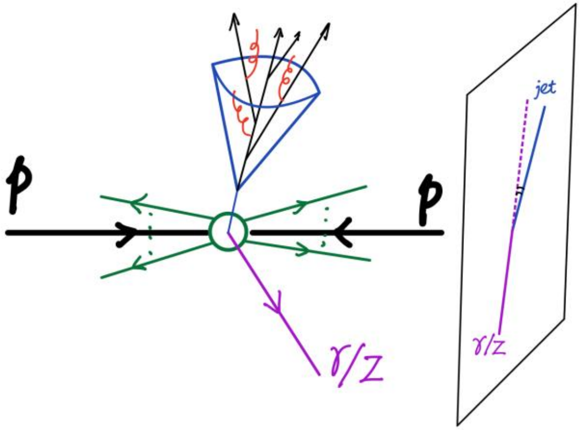
- Inclusive hadron production (STAR, ALICE)



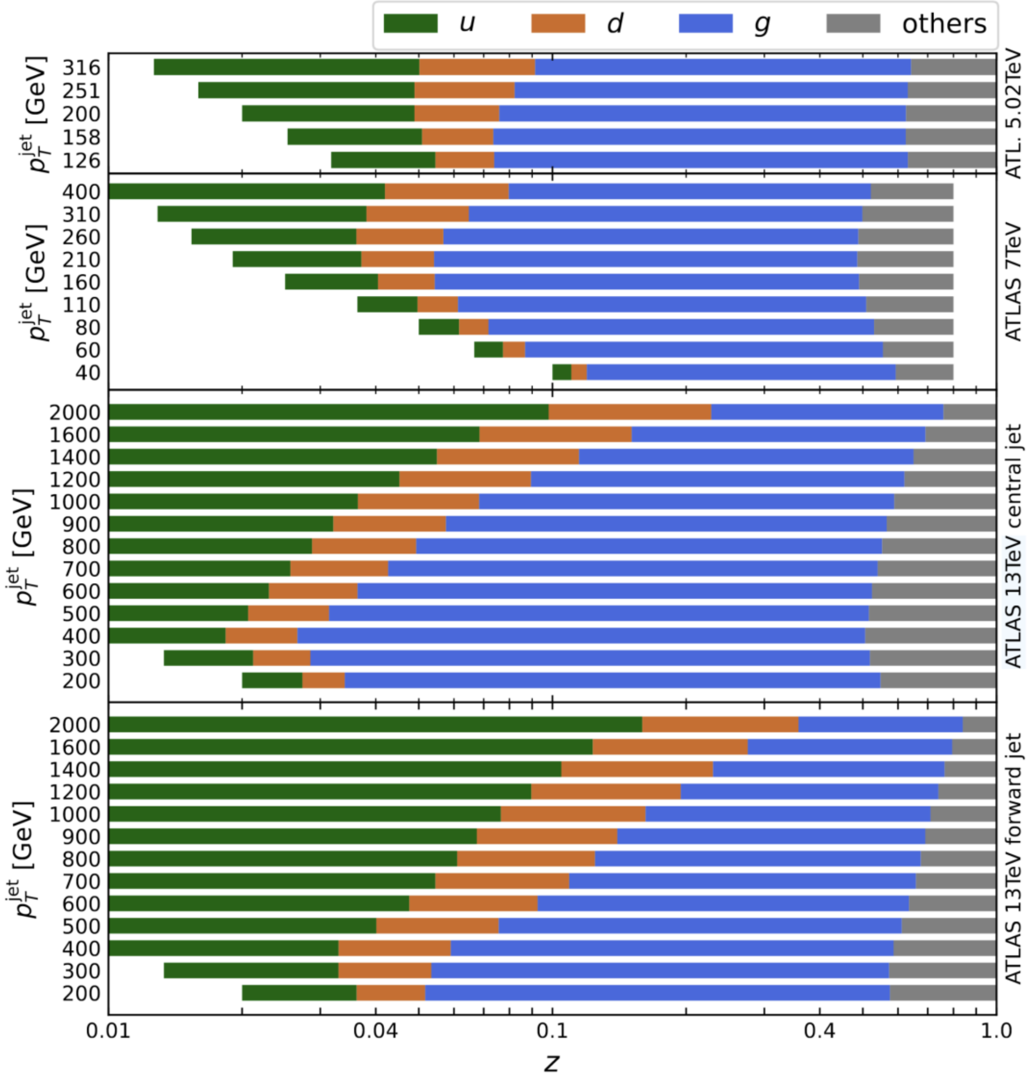
sensitive to gluon FFs.
However,

$$\sigma^h = \mathrm{PDF} \otimes \mathrm{PDF} \otimes \hat{\sigma}_i \otimes D_{h \leftarrow i}(z)$$

- Hadron in jet (ATLAS, CMS, LHCb)



$$\frac{p_{T,h}}{p_{T,j}} \xrightarrow{\text{LO}} z$$



Hadron-inside-jet measurements offer direct probe of z dependence

Simultaneous fit of $\pi^{\pm}, K^{\pm}, p/\bar{p}$ FFs

Experiments	N_{pt}	χ^2	χ^2/N_{pt}
ATLAS jets †	446	350.8	0.79
ATLAS $Z/\gamma+$jet †	15	31.8	2.12
CMS $Z/\gamma+$jet †	15	17.3	1.15
LHCb $Z+$jet	20	30.6	1.53
ALICE inc. hadron	147	150.6	1.02
STAR inc. hadron	60	42.2	0.70
pp sum	703	623.3	0.89
TASSO	8	7.0	0.88
TPC	12	11.6	0.97
OPAL	20	16.3	0.81
OPAL (202 GeV) †	17	24.2	1.42
ALEPH	42	31.4	0.75
DELPHI	78	36.4	0.47
DELPHI (189 GeV)	9	15.3	1.70
SLD	198	211.6	1.07
SIA sum	384	353.8	0.92
H1 †	16	12.5	0.78
H1 (asy.) †	14	12.2	0.87
ZEUS †	32	65.5	2.05
COMPASS (06l)	124	107.3	0.87
COMPASS (16p)	97	56.8	0.59
SIDIS sum	283	254.4	0.90
Global total	1370	1231.5	0.90

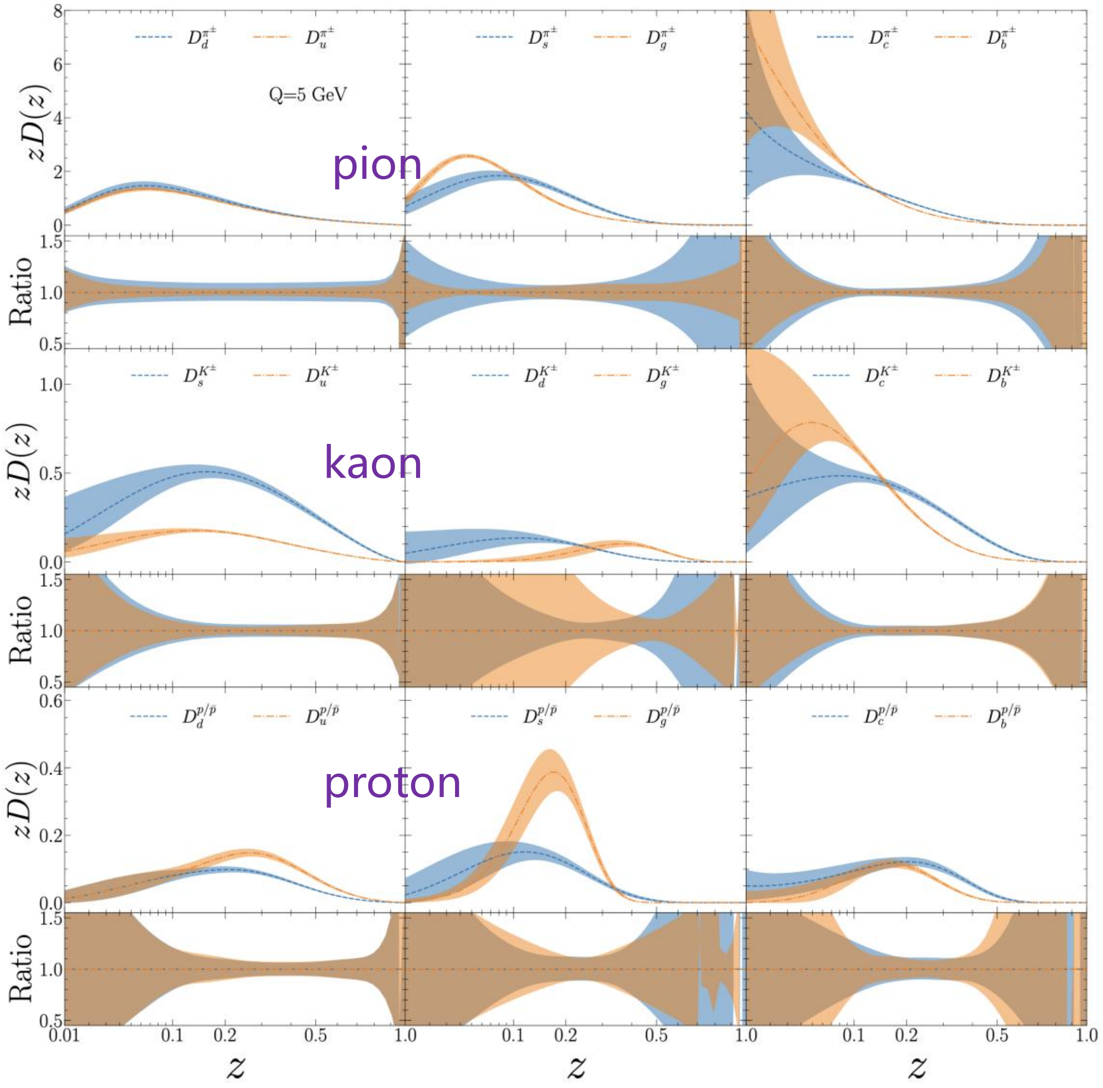
$$zD_i^h(z, Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^{m} a_{i,n}^h (\sqrt{z})^n\right)$$

parton-to-π^+	favored	α	β	a_0	a_1	a_2	d.o.f.
u	Y						5
$\bar{d} \simeq u$	Y	-	-		-	-	1
$\bar{u} = d$	N					x	4
$s = \bar{s} \simeq \bar{u}$	N	-				x	3
$c = \bar{c}$	N					x	4
$b = b$	N					x	4
g	N		F				4

parton-to-K^+	favored	α	β	a_0	a_1	a_2	d.o.f.
u	Y					x	4
$\bar{s} \simeq u$	Y	-	-		-	x	1
$\bar{u}=d=\bar{d}=s$	N					x	4
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F			x	3

parton-to-p	favored	α	β	a_0	a_1	a_2	d.o.f.
$u = 2d$	Y					x	4
$\bar{u} = \bar{d} = s = \bar{s}$	N				x	x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F			x	3

The NPC23 FF sets



Gao, Liu, **XS**, Xing, Zhao, PRL '24,
Gao, Liu, **XS**, Xing, Zhao, PRD '24(Editors' suggestion)

- High precision determination of FFs for charged hadrons.
- Resulting FFs are publicly available:

LHAPDF 6.5.5

Main page	PDF sets	Class hierarchy	Examples	More...

2070000	NPC23_PIp_nlo	(tarball)	(info file)	127
2070200	NPC23_KAp_nlo	(tarball)	(info file)	127
2070400	NPC23_PRp_nlo	(tarball)	(info file)	127
2070600	NPC23_PIm_nlo	(tarball)	(info file)	127
2070800	NPC23_KAm_nlo	(tarball)	(info file)	127
2071000	NPC23_PRm_nlo	(tarball)	(info file)	127
2071200	NPC23_PIsum_nlo	(tarball)	(info file)	127
2071400	NPC23_KAsum_nlo	(tarball)	(info file)	127
2071600	NPC23_PRsum_nlo	(tarball)	(info file)	127
2071800	NPC23_CHHAp_nlo	(tarball)	(info file)	127
2072000	NPC23_CHHAm_nlo	(tarball)	(info file)	127
2072200	NPC23_CHHAsum_nlo	(tarball)	(info file)	127

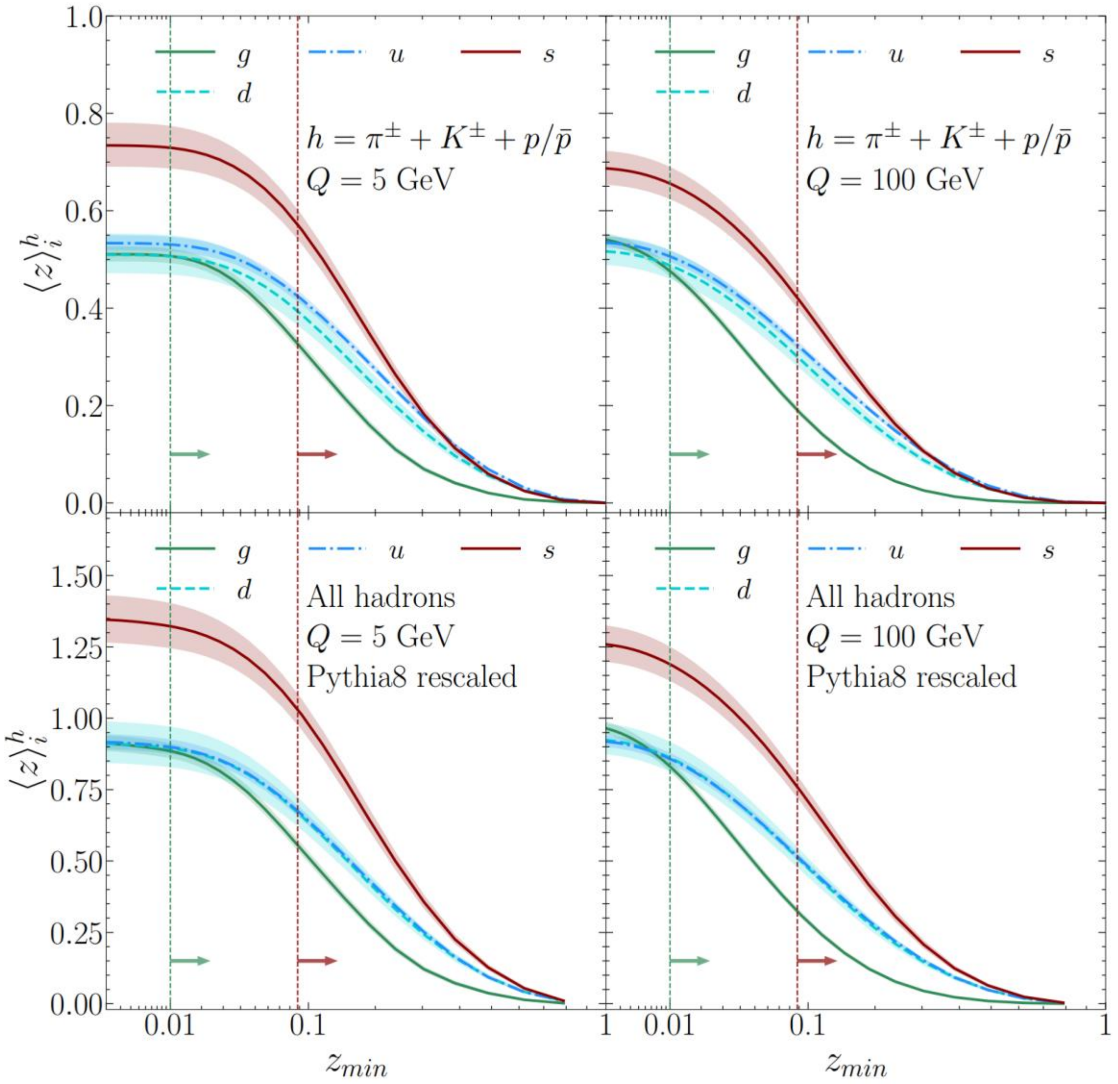
Application: momentum sum rule

Gao, Liu, **XS**, Xing, Zhao, PRL '24

sum rule $$\sum_h \int_0^1 \left[z D_{h\leftarrow i}(z) \right] \mathrm{d}z = 1$$

$$\lim_{z_{\min}\to 0}\left(\sum_h \int_{z_{\min}}^1 \left[z D_{h\leftarrow i}(z) \right] \mathrm{d}z \right)$$

$\langle z\rangle_i^h$	$g(z>0.01)$	$u(z>0.01)$	$d(z>0.01)$	$s(z>0.088)$
π^+	$0.200^{+0.008}_{-0.008}$	$0.262^{+0.017}_{-0.016}$	$0.128^{+0.020}_{-0.019}$	$0.161^{+0.013}_{-0.013}$
K^+	$0.018^{+0.004}_{-0.003}$	$0.058^{+0.005}_{-0.004}$	$0.019^{+0.004}_{-0.004}$	$0.015^{+0.002}_{-0.002}$
p	$0.035^{+0.006}_{-0.005}$	$0.044^{+0.004}_{-0.004}$	$0.022^{+0.002}_{-0.002}$	$0.015^{+0.002}_{-0.002}$
π^-	$0.200^{+0.008}_{-0.008}$	$0.128^{+0.020}_{-0.019}$	$0.299^{+0.054}_{-0.049}$	$0.161^{+0.013}_{-0.013}$
K^-	$0.018^{+0.004}_{-0.003}$	$0.019^{+0.004}_{-0.004}$	$0.019^{+0.004}_{-0.004}$	$0.205^{+0.014}_{-0.013}$
$\bar{p}$	$0.035^{+0.006}_{-0.005}$	$0.019^{+0.003}_{-0.003}$	$0.019^{+0.003}_{-0.003}$	$0.015^{+0.002}_{-0.002}$
Sum	$0.507^{+0.014}_{-0.013}$	$0.531^{+0.015}_{-0.013}$	$0.506^{+0.042}_{-0.037}$	$0.572^{+0.029}_{-0.028}$



NPC analyses of FFs to light neutral hadrons (K^0, η, Λ)

Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, '25

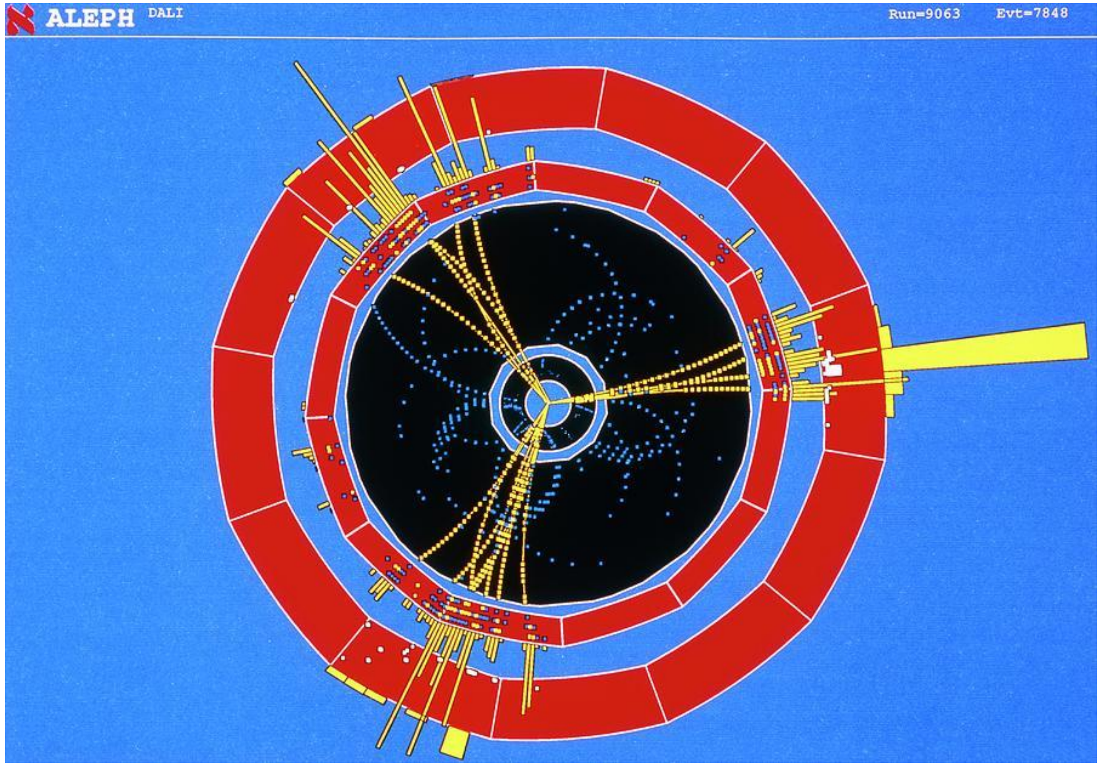
collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓
pp hadron in jet	✗	✗	✗	✗	✗	✓
FFs	$\pi^\pm$,$K^\pm$,p	$\pi^\pm$,$K^\pm$	$\pi^\pm$,$K^\pm$,p,$h^\pm$ η	$\pi^\pm$	$\pi^\pm$,$K^\pm$	$\pi^\pm$,$K^\pm$,p,$h^\pm$ K^0,η,Λ
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO

Only some of the recent global analyses are shown here.

Data included in global analyses for the first time

collaboration	year	$\sqrt{s}$ [GeV]	observable	$N_{\rm pt}(K_0^S)$	$N_{\rm pt}(\Lambda)$	$N_{\rm pt}(\eta)$	$N_{\rm pt}(\pi^0)$
ZEUS [85]	2012	318	$\frac{1}{N_{\rm DIS}}\frac{dN_h}{dz_p}$	5, 5 & 2	5, 3 & 1	–	–
CMS [125]	2011	900	$\frac{dN_h}{dp_T}\Big/\frac{dN_{K_S^0}}{dp_T}$	–	4	–	–
STAR [126]	2010	200	$\frac{Ed^3\sigma_h}{d^3\vec{p}}$	–	–	–	8
PHENIX [127]	2011	200	$\frac{Ed^3\sigma_h}{d^3\vec{p}}\Big/\frac{Ed^3\sigma_{\pi^0}}{d^3\vec{p}}$	–	–	14	–
PHENIX [128]	2007	200	$\frac{Ed^3\sigma_h}{d^3\vec{p}}$	–	–	–	17
PHENIX [129]	2016	510	$\frac{Ed^3\sigma_h}{d^3\vec{p}}$	–	–	–	22
ALICE [130]	2017	2760	$\frac{Ed^3\sigma_h}{d^3\vec{p}}\Big/\frac{Ed^3\sigma_{\pi^0}}{d^3\vec{p}}$	–	–	6	–
ALICE [130]	2017	2760	$\frac{Ed^3\sigma_h}{d^3\vec{p}}$	–	–	–	16
ALICE [131]	2012	7000	$\frac{Ed^3\sigma_h}{d^3\vec{p}}\Big/\frac{Ed^3\sigma_{\pi^0}}{d^3\vec{p}}$	–	–	4	–
ALICE [131]	2012	7000	$\frac{Ed^3\sigma_h}{d^3\vec{p}}$	–	–	–	13
ALICE [132]	2018	8000	$\frac{Ed^3\sigma_h}{d^3\vec{p}}\Big/\frac{Ed^3\sigma_{\pi^0}}{d^3\vec{p}}$	–	–	13	–
ALICE [132]	2018	8000	$\frac{Ed^3\sigma_h}{d^3\vec{p}}$	–	–	–	24
ALICE [133]	2021	13000 & 7000	$\frac{dN_h^{13\ \rm TeV}}{dp_T}\Big/\frac{dN_h^{7\ \rm TeV}}{dp_T}$	10	7	–	–
ALICE [133]	2021	13000	$\frac{dN_h}{dp_T}\Big/\frac{dN_{\pi^\pm}}{dp_T}$	15	–	–	–
ALICE [134]	2024	13000	$\frac{Ed^3\sigma_h}{d^3\vec{p}}\Big/\frac{Ed^3\sigma_{\pi^0}}{d^3\vec{p}}$	–	–	14	–

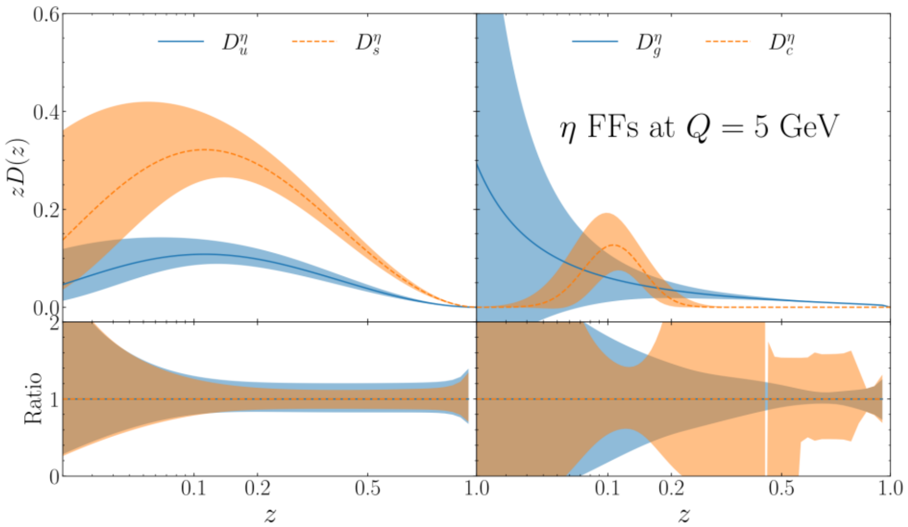
- SIDIS data from ZUES
- pp data from LHC
- hadron-in-jet data (ALEPH three-jet events)

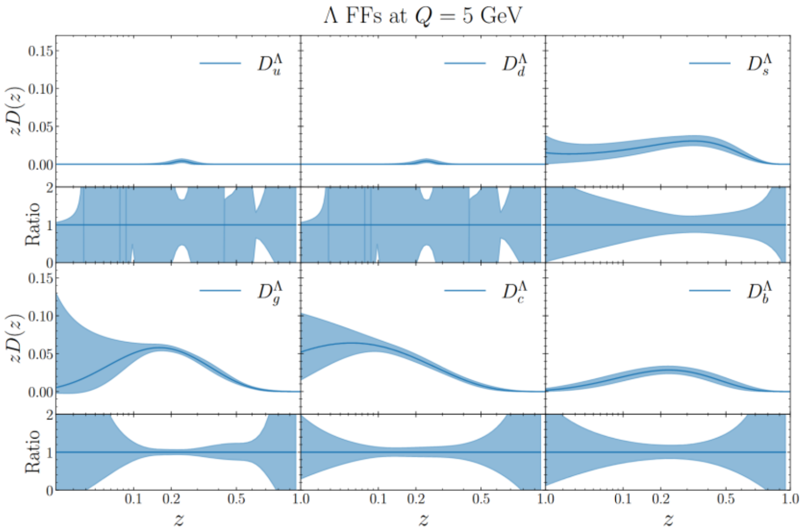


The NPC23 FF sets

$$zD_i^h(z, Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^{m} a_{i,n}^h z^{n/2}\right)$$

parton-to-K_S^0	favored	a_0	α	β	a_1
$u = \overline{u}$	✗	✓	✓	✓	✓
$d = \overline{d}$	✓	✓	$= \alpha_u$	✓	✓
$s = \overline{s}$	✓	✓	$= \alpha_u$	$= \beta_d$	$= a_{1,d}$
$c = \overline{c}$	✗	✓	✓	✓	✓
$b = \overline{b}$	✗	✓	✓	✓	✓
g	✗	✓	✓	✓	✓
parton-to-Λ	**favored**	**a_0**	**α**	**β**	**a_1**
$u = \overline{u} = d = \overline{d}$	✓	✓	✓	✓	✓
$s = \overline{s}$	✓	✓	✓	✓	✓
$c = \overline{c}$	✗	✓	✓	✓	✗
$b = \overline{b}$	✗	✓	✓	✓	✗
g	✗	✓	✓	✓	✓
parton-to-η	**favored**	**a_0**	**α**	**β**	**a_1**
$u = \overline{u} = d = \overline{d}$	✓	✓	✓	✓	✓
$s = \overline{s}$	✓	✓	$= \alpha_u$	$= \beta_u$	$= a_{1,u}$
$c = \overline{c} = b = \overline{b}$	✗	✓	✓	✓	✗
g	✗	✓	✓	✓	✓





What can we learn from neutral + charged hadron FFs

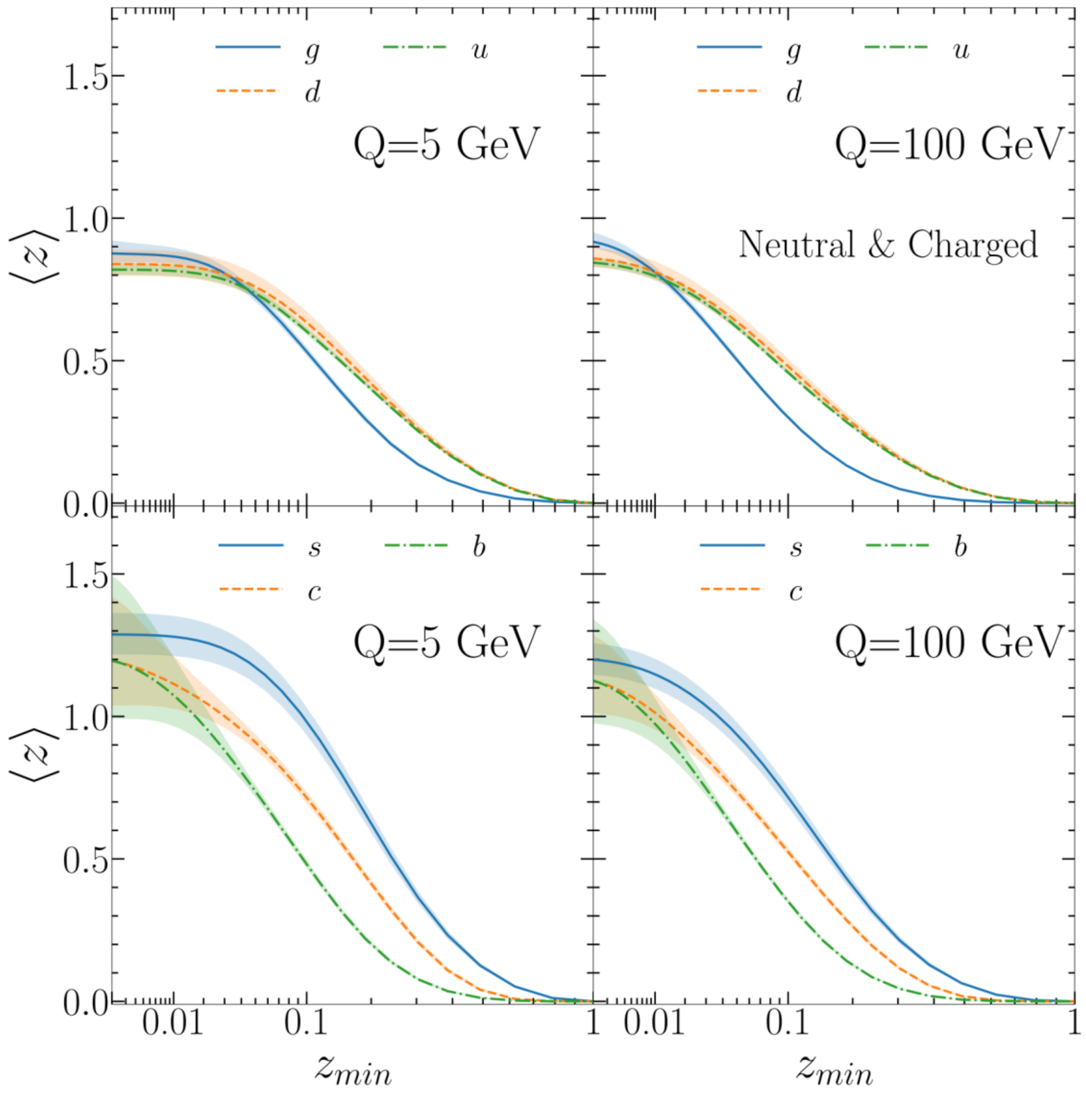
Momentum sum rule revisited

Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, '25

sum rule $$\sum_h \int_0^1 \left[z D_{h\leftarrow i}(z) \right] \mathrm{d}z = 1$$

$$\lim_{z_{\min}\to 0}\left(\sum_h \int_{z_{\min}}^1 \left[z D_{h\leftarrow i}(z) \right] \mathrm{d}z \right)$$

hadron	g ($z_{\min}=0.05$)	d ($z_{\min}=0.05$)	u ($z_{\min}=0.05$)	s ($z_{\min}=0.05$)	c ($z_{\min}=0.05$)	b ($z_{\min}=0.05$)
$K_S^0 + K_L^0$	$0.0922^{+0.0051}_{-0.0040}$	$0.0940^{+0.0278}_{-0.0223}$	$0.0155^{+0.0046}_{-0.0047}$	$0.1486^{+0.0280}_{-0.0243}$	$0.1829^{+0.0112}_{-0.0105}$	$0.0685^{+0.0092}_{-0.0082}$
$\Lambda + \bar{\Lambda}$	$0.0385^{+0.0049}_{-0.0037}$	$0.0006^{+0.0006}_{-0.0004}$	$0.0006^{+0.0006}_{-0.0004}$	$0.0296^{+0.0070}_{-0.0059}$	$0.0321^{+0.0038}_{-0.0034}$	$0.0228^{+0.0052}_{-0.0044}$
η	$0.0192^{+0.0083}_{-0.0077}$	$0.0420^{+0.0092}_{-0.0068}$	$0.0420^{+0.0092}_{-0.0068}$	$0.1247^{+0.0189}_{-0.0174}$	$0.0119^{+0.0071}_{-0.0056}$	$0.0119^{+0.0071}_{-0.0056}$
π^0	$0.1577^{+0.0072}_{-0.0072}$	$0.1829^{+0.0076}_{-0.0065}$	$0.1829^{+0.0076}_{-0.0065}$	$0.1954^{+0.0160}_{-0.0156}$	$0.1674^{+0.0076}_{-0.0076}$	$0.1636^{+0.0060}_{-0.0057}$
sum	$0.3076^{+0.0131}_{-0.0119}$	$0.3194^{+0.0303}_{-0.0242}$	$0.2409^{+0.0128}_{-0.0105}$	$0.4984^{+0.0380}_{-0.0342}$	$0.3944^{+0.0157}_{-0.0146}$	$0.2668^{+0.0141}_{-0.0123}$

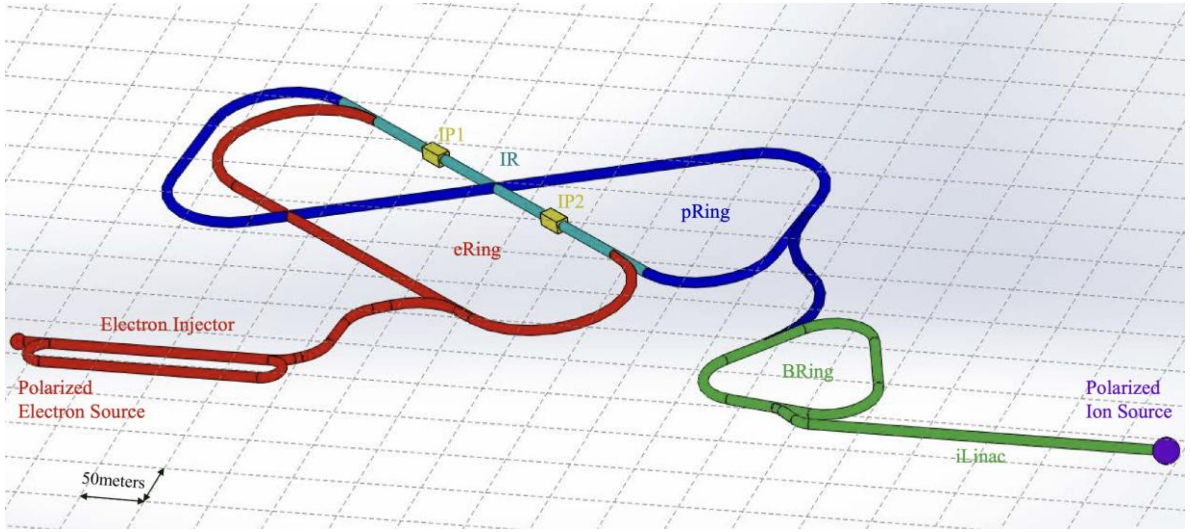


NPC analyses of FFs at NNLO + constraints on PDFs

[Gao, **XS**, Xing, Zhao, Zhou, ***PRL 135, 041902***, 2025]

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓	✗
pp hadron in jet	✗	✗	✗	✗	✗	✓	✗
FFs	$\pi^{\pm}$,$K^{\pm}$,p	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p,$h^{\pm}$ η	$\pi^{\pm}$	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p,$h^{\pm}$ K^0,η,Λ	$\pi^{\pm}$,$K^{\pm}$
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO	NNLO

The need for high-precision FFs extractions



- The Electron-Ion Collider (EIC)
  - start operation in the early 2030s
  - unprecedented access to structure of the proton
  - **FFs** as keys ingredients of SIDIS at the EIC

- Efforts from China
  - ep collisions: EicC
  - ee collisions: BESIII measurements

  See Linqin Huang & Wenbiao Yan's talk

High-precision (NNLO, N3LO) FFs as key output & input

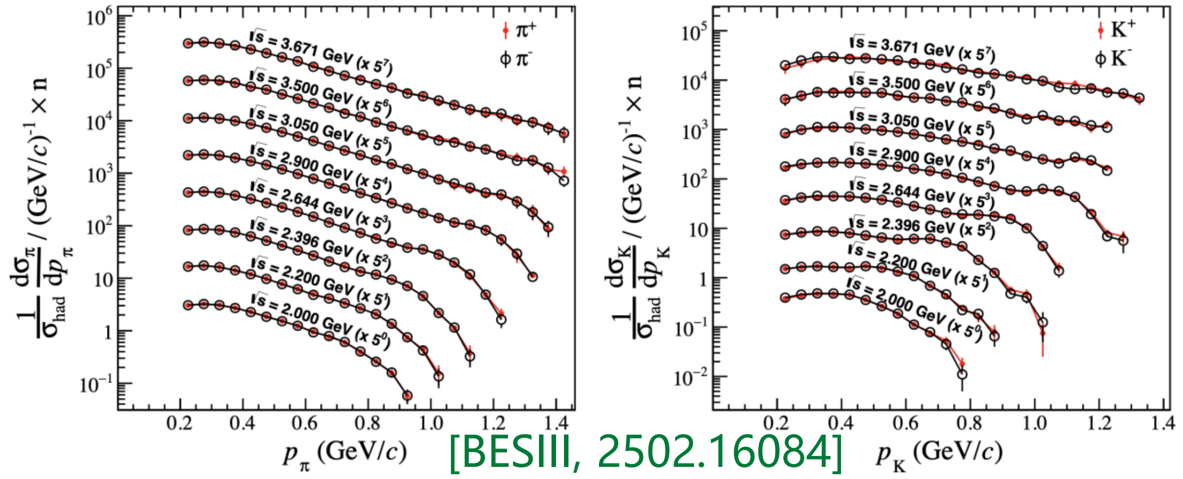
First global analysis of FFs at NNLO

[Gao, **XS**, Xing, Zhao, Zhou, ***PRL 135, 041902***, 2025]

❖ Data

- $\pi^{\pm}, K^{\pm}$: SIA(ee) and SIDIS(lepton+N) data with $Q > 3\ \&\ 2$ GeV
- recent BESIII measurement included

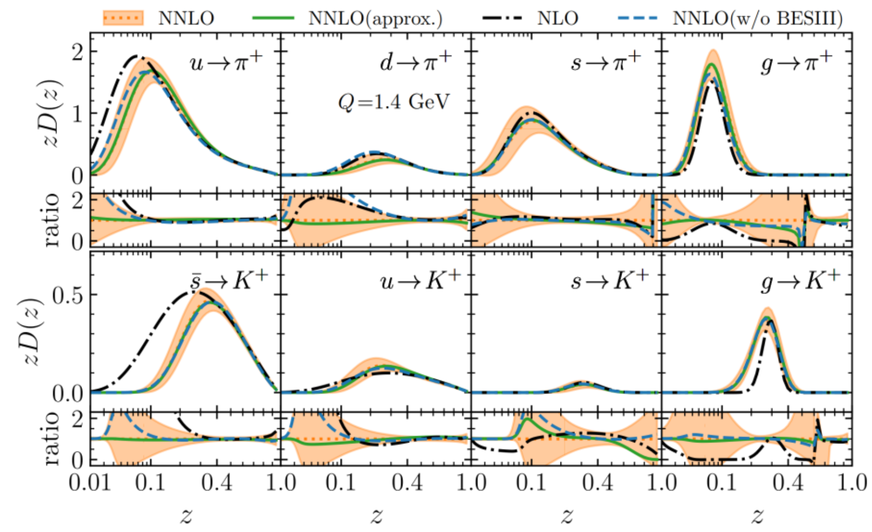
  See Wenbiao Yan & Linqin Huang's talk



[BESIII, 2502.16084]

❖ First global analysis of FFs at NNLO

- SIDIS: coefficient functions at NNLO from [Bonino+, '24] and [Goyal+, '24]
- SIA: NNLO
- 3-loop DGLAP evolution [Mitov, Moch, Vogt, Almasy], [Chen, Yang, Zhu, Zhu, '20]
- include scale variations into the covarinace matrix



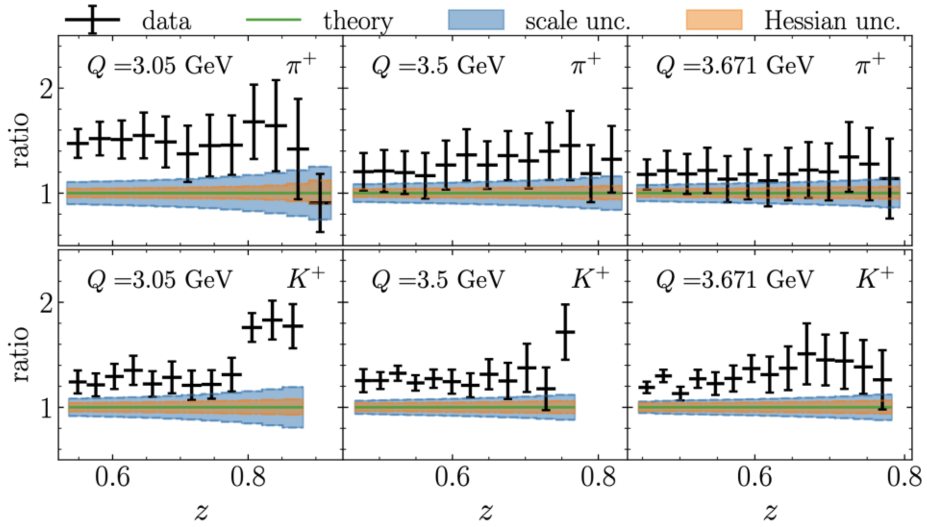
LHAgrids of our FFs available in the LHAPDF repository

Test on universality of FFs at low Q

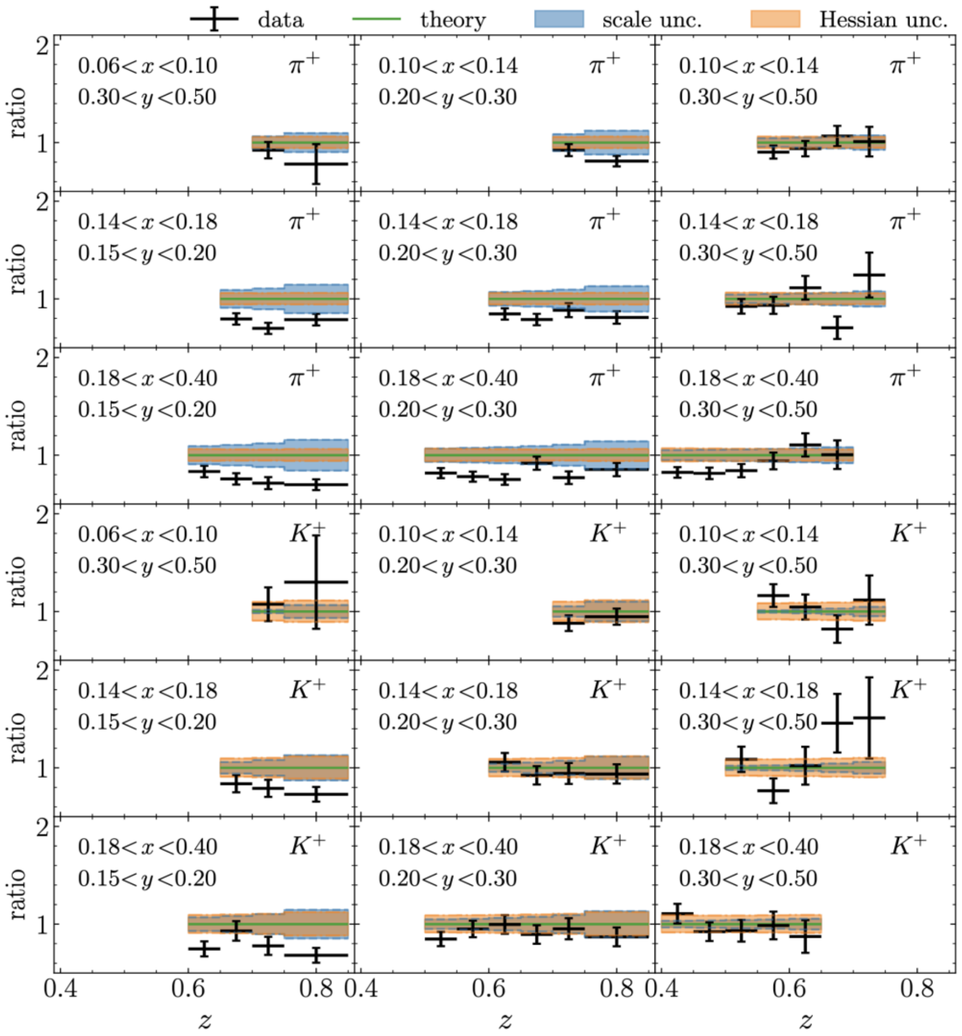
Quality of NNLO fit of SIA and SIDIS data

	BESIII		COMPASS		B-factories		HE-SIA		global		
$E_{h,\min}$[GeV]	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	χ^2	$\chi^2/N_{\rm pt}$
0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77

Theory v.s. data for BESIII (SIA)



Theory v.s. data for COMPASS06(SIDIS)



Satisfactory agreements found for both SIA and SIDIS data with Q~3 GeV

Application: constraining proton PDFs at NNLO

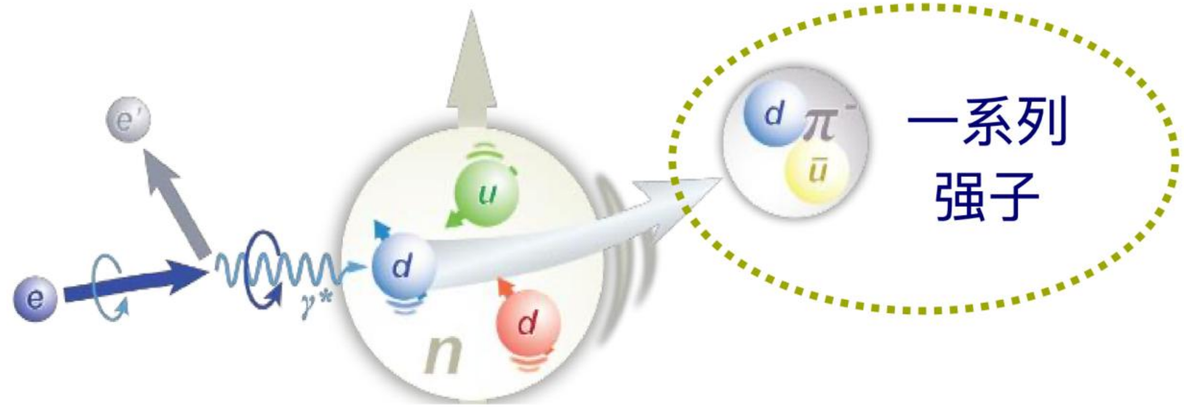
SIDIS measurement depends on both PDFs and FFs

$$\frac{\mathrm{d}^3\sigma_h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x,y,z) \otimes D_{h/j}(z_h) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

Unpol. PDF FF

$$g_1(x,Q^2,z) = \frac{1}{2}\sum_q e_q^2 [\Delta q(x,Q^2) D^{q\to h}(Q^2,z) + \Delta\overline{q}(x,Q^2) D^{\overline{q}\to h}(Q^2,z)]$$

Pol.PDF FF



LO cross section difference of $K^{\pm}$ with iso-scalar target

$$\frac{\mathrm{d}\sigma^{K^+}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} - \frac{\mathrm{d}\sigma^{K^-}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z}$$

$$\sim 2\Big(u_v(x) + d_v(x)\Big)\Big(D_u^{K^+}(z) - D_{\bar{u}}^{K^+}(z)\Big)$$

$$+\Big(s(x) - \bar{s}(x)\Big)\Big(D_s^{K^+}(z) - D_{\bar{s}}^{K^+}(z)\Big) + \cdots$$

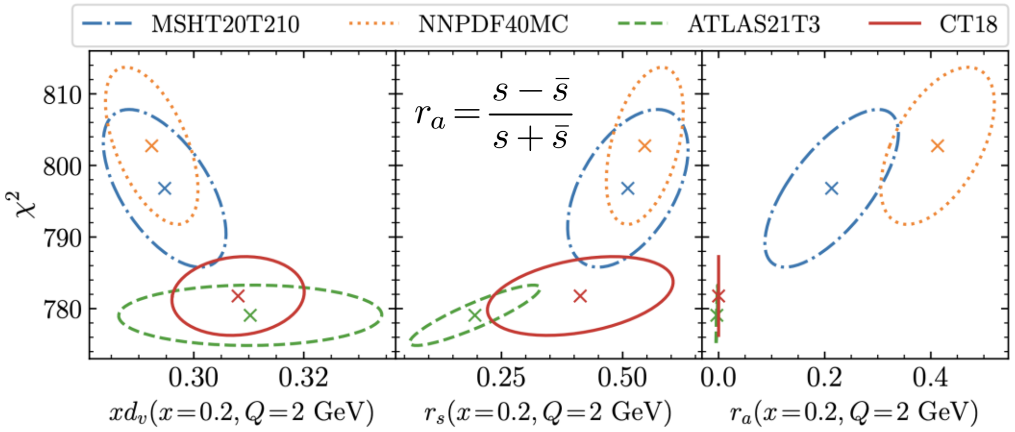
PDF FF

sensitive to strangeness asymmetry

$$r_a = \frac{s - \bar{s}}{s + \bar{s}}$$

Application: constraining proton PDFs at NNLO

- Correlation between chi2 and PDFs

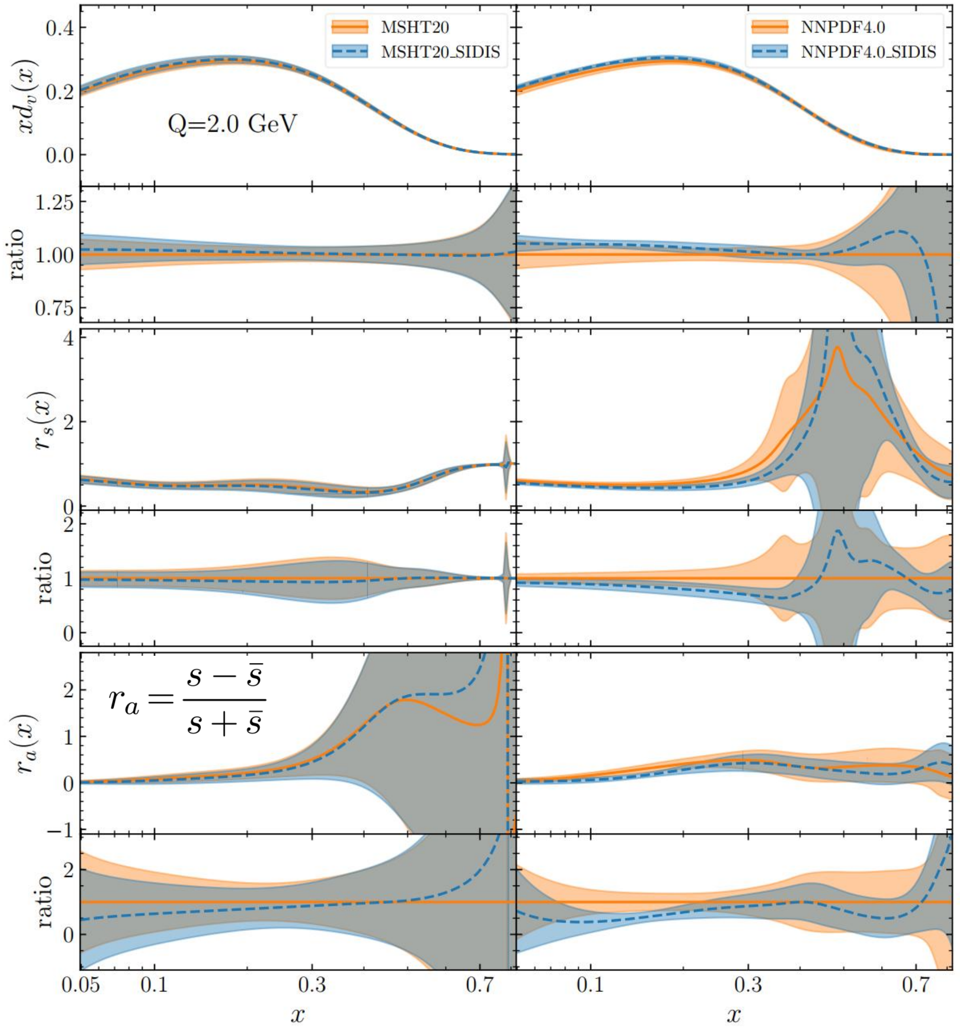


- Modified PDFs with the inclusion of SIDIS data

  - Reweighting of the NNPDF4.0 PDF set
  - Profiling of the MSHT20 PDF set

  [ref: Phys.Rept.742(2018)1]

PDF variables before/after reweighting/profiling



[Gao, **XS**, Xing, Zhao, Zhou, ***PRL 135, 041902***, 2025]

Summary

- FFs are essential non-perturbative inputs for precision calculations of hadron production cross sections from first principle of QCD.
- NPC23 fit framework: developed based on FMNLO [Liu, **XS**, Zhou, Gao, 2305.14620 (JHEP)]
- NPC23 FF sets: the most precise and complete FF sets to date

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓	✗
pp hadron in jet	✗	✗	✗	✗	✗	✓	✗
FFs	$\pi^{\pm}$,$K^{\pm}$,p	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p,$h^{\pm}$ η	$\pi^{\pm}$	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p,$h^{\pm}$ K^0,η,Λ	$\pi^{\pm}$,$K^{\pm}$
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO	NNLO

Gao, Liu, **XS**, Xing, Zhao, *PRL 132, 261903*, 2024
Gao, Liu, **XS**, Xing, Zhao, *PRD 110, 114019*, (Editors' suggestion), 2024
Gao, **XS**, Xing, Zhao, Zhou, *PRL 135, 041902*, 2025
Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, 2503.21311

Thank you for your attention!

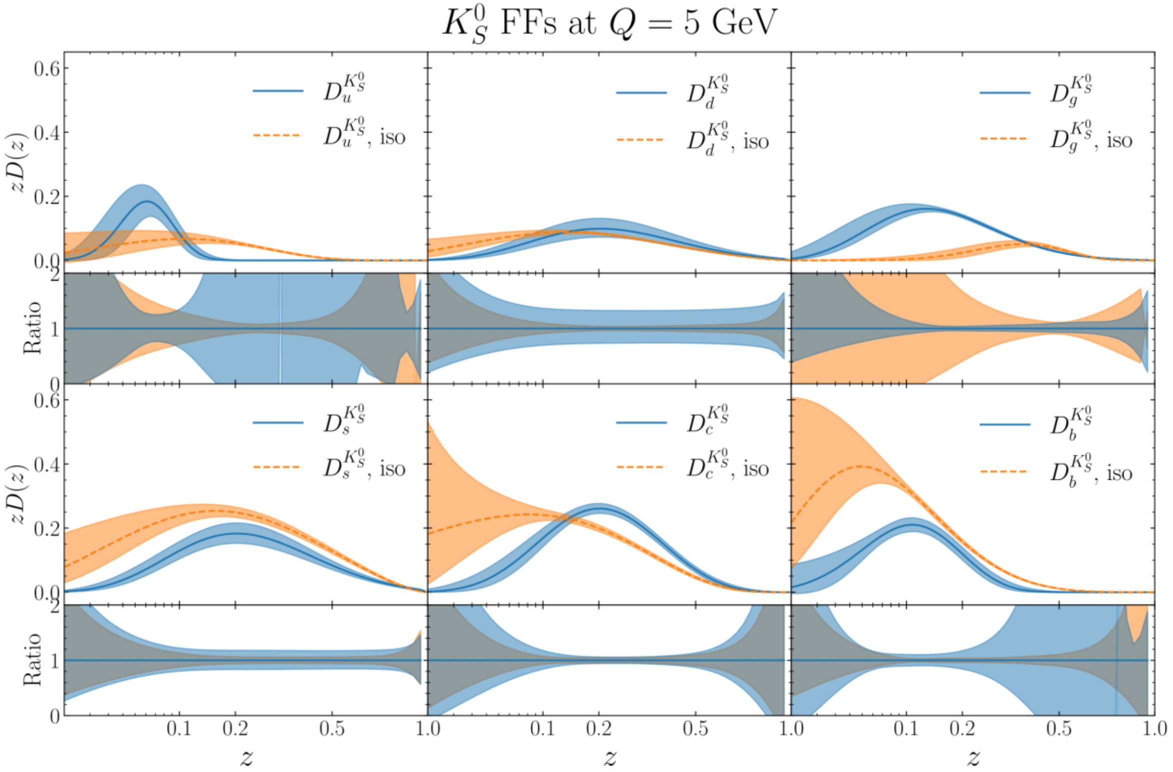
Backup slides

What can we learn from neutral + charged hadron FFs

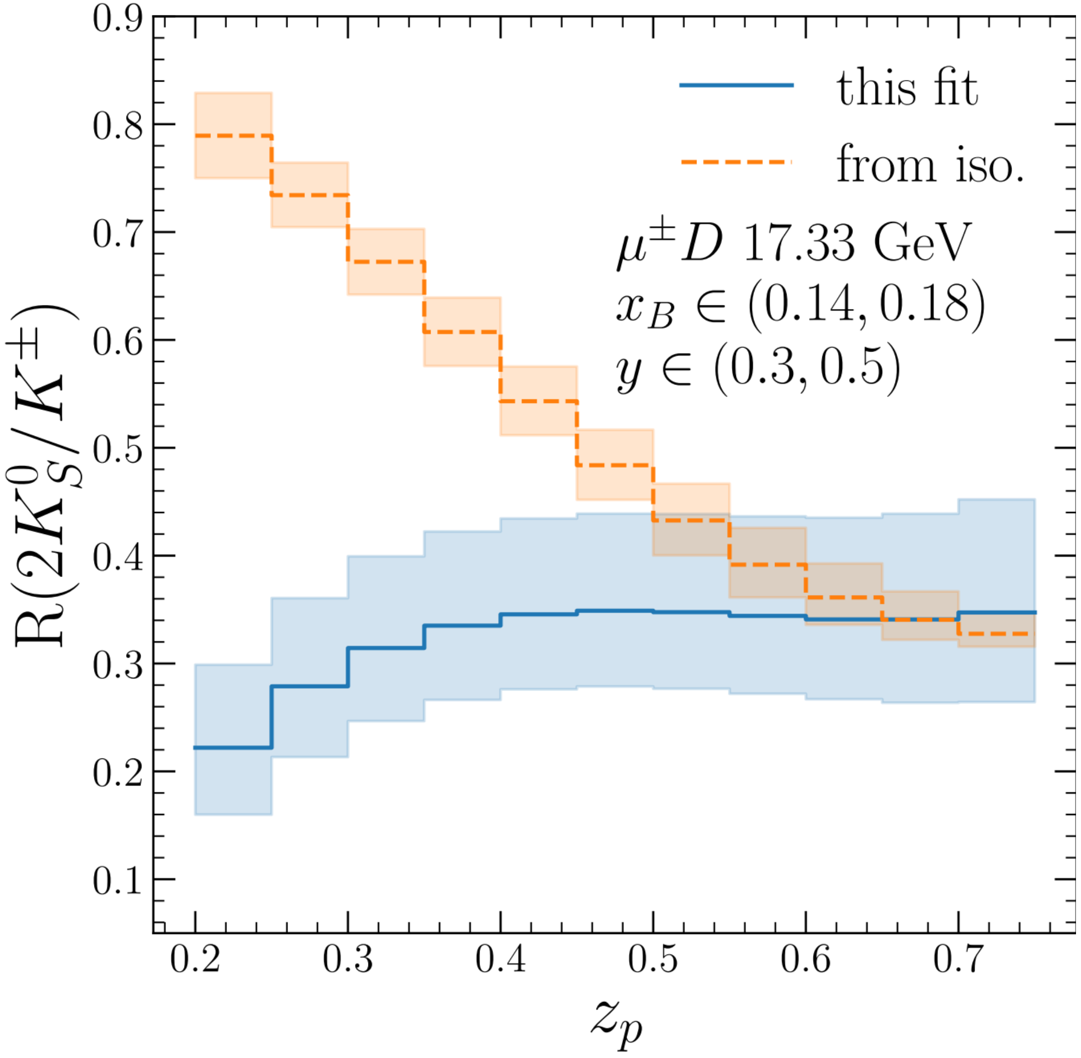
Test the isospin symmetry

$$D_q^{K_S^0,\mathrm{iso}} = \frac{1}{2}\left(D_{q'}^{K^+} + D_{q'}^{K^-}\right),\ q(q') = u(d) \text{ or } d(u)$$

$$D_i^{K_S^0,\mathrm{iso}} = \frac{1}{2}\left(D_i^{K^+} + D_i^{K^-}\right),\ i = g, s, c, b$$



Predictions of K_S^0 production at COMAPSS.



Fit quality

collaboration	year	$\sqrt{s}$ [GeV]	χ^2	N_{pt}	χ^2/N_{pt}
Belle [100]	2024	10.58	14.9	35	0.43
TASSO [95]	1985	14	5.12	9	0.57
TASSO [95]	1985	22	5.54	6	0.92
TASSO [95]	1985	34	14.86	13	1.14
TASSO [96]	1990	14.8	10.89	9	1.21
TASSO [96]	1990	21.5	3.40	6	0.57
TASSO [96]	1990	34.5	19.54	13	1.50
TASSO [96]	1990	35	13.52	13	1.04
TASSO [96]	1990	42.6	32.01	13	2.46
TPC [93]	1984	29	3.02	8	0.38
MARK II [94]	1985	29	11.06	17	0.65
HRS [97]	1987	29	36.33	12	3.03
CELLO [98]	1990	35	3.48	9	0.39
TOPAZ [99]	1995	58	0.18	4	0.05
OPAL [87]	1991	91.2	8.03	7	1.15
OPAL [88]	1995	91.2	12.97	16	0.81
OPAL [89]	2000	91.2	8.98	16	0.56
ALEPH [90]	1998	91.2	7.22	16	0.45
ALEPH [101]	2000	91.2	13.24	14	0.95
ALEPH jet 1 [101]	2000	91.2	14.85	12	1.24
ALEPH jet 2 [101]	2000	91.2	9.34	13	0.72
ALEPH jet 3 [101]	2000	91.2	9.28	11	0.84
DELPHI [91]	1995	91.2	8.12	13	0.62
SLD [92]	1999	91.2	7.61	9	0.85
SLD c-tagged [92]	1999	91.2	23.23	9	2.58
SLD b-tagged [92]	1999	91.2	11.27	9	1.30
SIA sum			308.43	312	0.99
ZEUS $Q^2 \in (160, 640)$ GeV^2 [85]	2012	318	4.38	5	0.88
ZEUS $Q^2 \in (640, 2560)$ GeV^2 [85]	2012	318	2.97	5	0.59
ZEUS $Q^2 \in (2560, 10240)$ GeV^2 [85]	2012	318	3.00	2	1.50
SIDIS sum			10.35	12	0.86
ALICE $\left(N_{K_S^0}^{13\text{ TeV}}/N_{K_S^0}^{7\text{ TeV}}\right)$ [133]	2021	13000 & 7000	3.19	10	0.32
ALICE $\left(N_{K_S^0}/N_{\pi^\pm}\right)$ [133]	2021	13000	5.96	15	0.40
pp sum			9.15	25	0.37
total sum			**327.93**	**349**	**0.94**

K_S^0

collaboration	year	$\sqrt{s}$ [GeV]	χ^2	N_{pt}	χ^2/N_{pt}
ARGUS [106]	1990	9.46	5.94	6	0.99
HRS [107]	1988	29	18.07	13	1.39
JADE [110]	1985	34.4	2.29	2	1.14
JADE [112]	1990	35	3.29	3	1.09
CELLO [113]	1990	35	3.47	5	0.69
L3 [123]	1992	91.2	5.83	4	1.46
L3 [124]	1994	91.2	10.46	10	1.05
ALEPH [117]	1992	91.2	1.48	8	0.18
ALEPH [101]	2000	91.2	18.39	18	1.02
ALEPH jet 1 [101]	2000	91.2	11.26	7	1.61
ALEPH jet 2 [101]	2000	91.2	1.95	6	0.33
ALEPH jet 3 [101]	2000	91.2	10.49	4	2.62
ALEPH [120]	2002	91.2	17.18	5	3.44
OPAL [116]	1998	91.2	7.12	11	0.65
SIA sum			117.20	102	1.15
PHENIX [127]	2011	200	7.61	14	0.51
ALICE [130]	2017	2760	5.37	6	0.90
ALICE [131]	2012	7000	1.26	4	0.32
ALICE [132]	2018	8000	12.64	13	0.97
ALICE [134]	2024	13000	7.11	14	0.51
pp sum			33.99	51	0.67
total sum			**151.21**	**153**	**0.99**

η

collaboration	year	$\sqrt{s}$ [GeV]	χ^2	N_{pt}	χ^2/N_{pt}
TASSO [95]	1985	14	1.25	3	0.42
TASSO [95]	1985	22	2.81	4	0.70
TASSO [95]	1985	34	4.37	7	0.62
SLAC [108]	1985	29	22.46	15	1.50
HRS [97]	1987	29	6.05	8	0.76
TASSO [109]	1981	33	4.48	5	0.90
TASSO [111]	1989	34.8	16.23	10	1.62
TASSO [111]	1989	42.1	1.49	5	0.30
CELLO [98]	1990	35	3.43	7	0.49
DELPHI [121]	1993	91.2	10.49	7	1.50
ALEPH [118]	1994	91.2	6.36	14	0.45
ALEPH [90]	1998	91.2	3.60	16	0.22
ALEPH jet 1 [101]	2000	91.2	16.58	13	1.28
ALEPH jet 2 [101]	2000	91.2	3.53	12	0.29
ALEPH jet 3 [101]	2000	91.2	3.38	9	0.38
OPAL [115]	1997	91.2	6.23	12	0.52
SLD [92]	1999	91.2	4.19	9	0.47
SLD c-tagged [92]	1999	91.2	11.41	5	2.28
SLD b-tagged [92]	1999	91.2	1.17	5	0.23
SIA sum			129.52	166	0.78
ZEUS $Q^2 \in (160, 640)$ GeV^2 [85]	2012	318	13.30	5	2.66
ZEUS $Q^2 \in (640, 2560)$ GeV^2 [85]	2012	318	1.13	3	0.38
ZEUS $Q^2 \in (2560, 10240)$ GeV^2 [85]	2012	318	0.004	1	0.004
SIDIS sum			14.43	9	1.60
CMS $\left(N_\Lambda/N_{K_S^0}\right)$ [125]	2011	900	2.97	4	0.74
ALICE $\left(N_\Lambda^{13\text{ TeV}}/N_\Lambda^{7\text{ TeV}}\right)$ [133]	2021	13000 & 7000	1.71	7	0.24
pp sum			4.68	11	0.43
total sum			**148.63**	**186**	**0.80**

Λ

NPC NNLO FFs analysis: the framework

[Gao, **XS**, Xing, Zhao, Zhou, 2502.17837]

- Joint determination of FFs to charged pion, kaon at NNLO in QCD
- Parameterization at $Q_0 = 1.4$ GeV for light flavors, and threshold for heavy quarks.

$$zD_i^h(z, Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^{m} a_{i,n}^h z^{n/2}\right)$$

- Charge conjugation symmetry

$$D_{u\to\pi^+}(z, Q) = D_{\bar{u}\to\pi^-}(z, Q)$$

- Isospin symmetry

$$D_{u\to\pi^+}(z, Q_0) = D_{\bar{d}\to\pi^+}(z, Q_0)$$

flavor	favored	a_0	α	β	a_1	a_2
$u=\bar{d}$	✓	✓	✓	✓	✓	✓
$d=\bar{u}$	✗	✓	✓	✓	✓	✓
$s=\bar{s}$	✗	✓	$=\alpha_d$	✓	✓	✓
$c=\bar{c}$	✗	✓	✓	✓	✓	✓
$b=\bar{b}$	✗	✓	✓	✓	✓	✓
g	✗	✓	✓	✓	✓	✗

flavor	favored	a_0	α	β	a_1	a_2
u	✓	✓	✓	✓	✓	✓
$\bar{s}$	✓	✓	$=\alpha_u$	$=\beta_u$	✓	✓
$s=\bar{u}=d=\bar{d}$	✗	✓	✓	✓	✓	✗
$c=\bar{c}$	✗	✓	✓	✓	✓	✓
$b=\bar{b}$	✗	✓	✓	✓	✓	✓
g	✗	✓	✓	✓	✓	✗