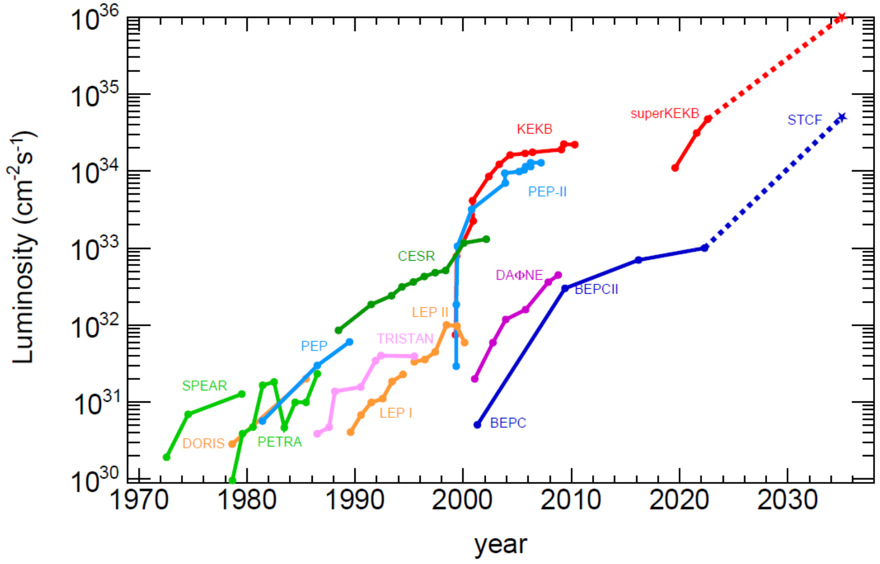
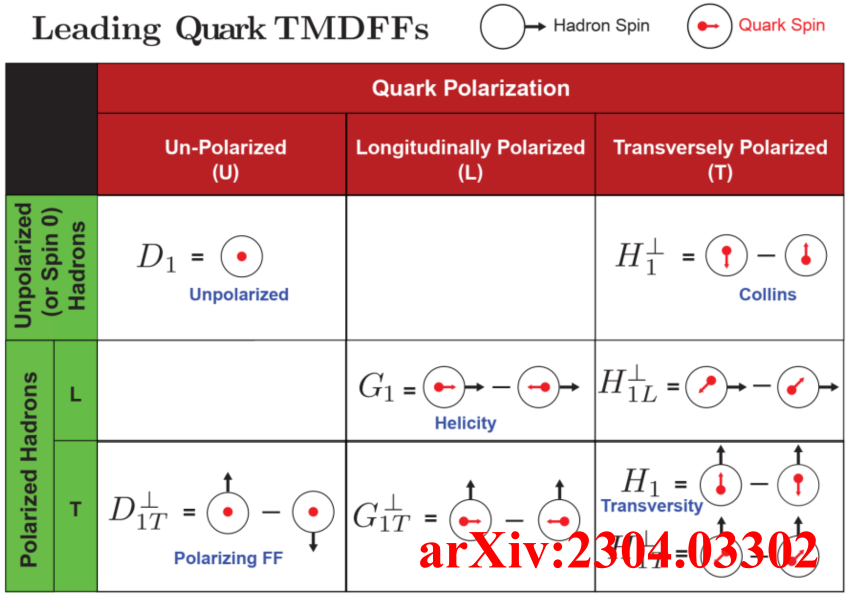
Hadron production at low energy e^+e^-

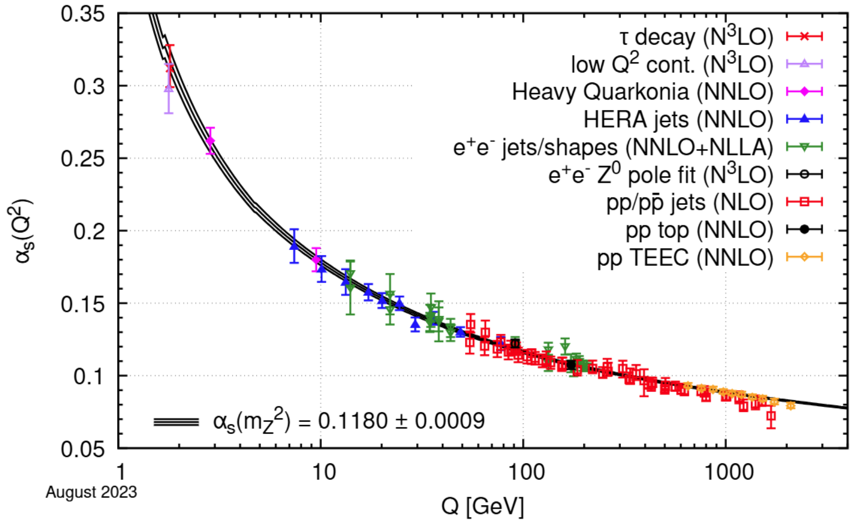
鄢文标(中国科学技术大学)

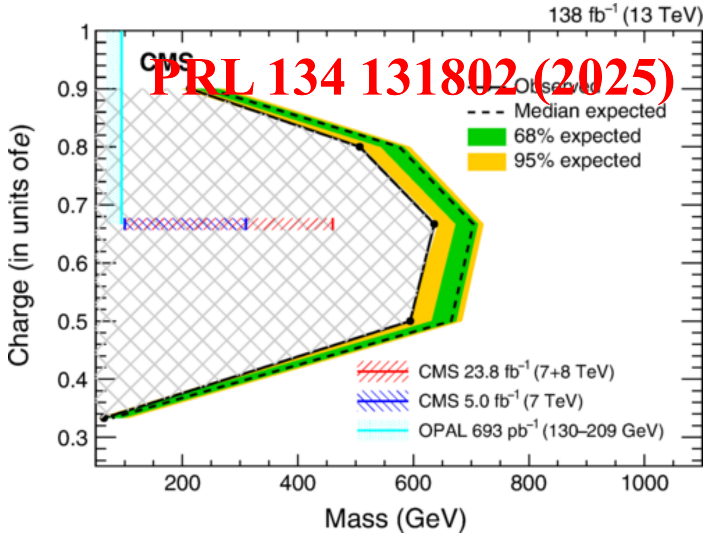




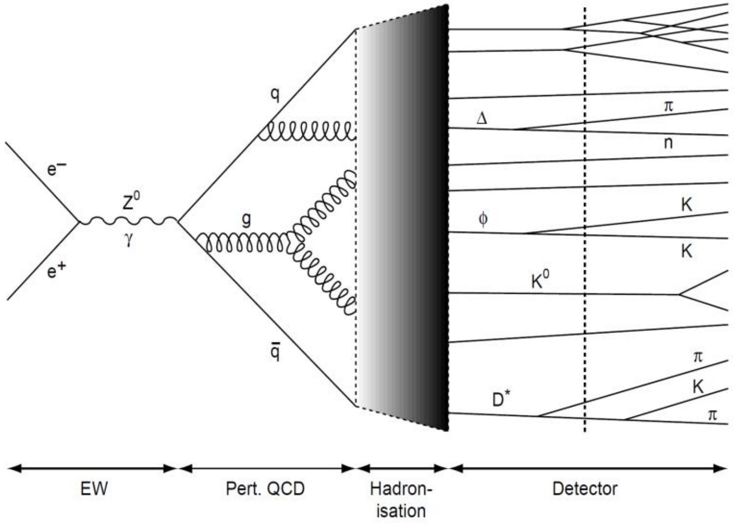
碎裂函数和能量关联研讨会，2025.08.09，兰州

QCD: Asymptotic freedom & Confinement

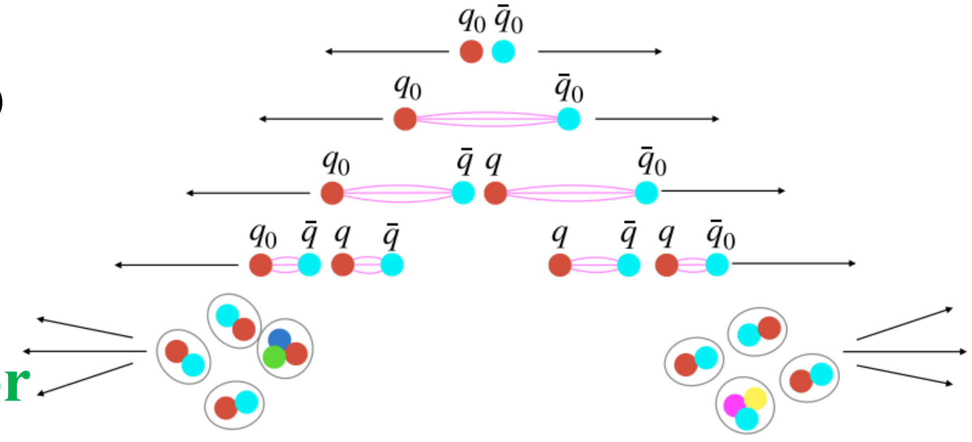




PRL 134 131802 (2025)



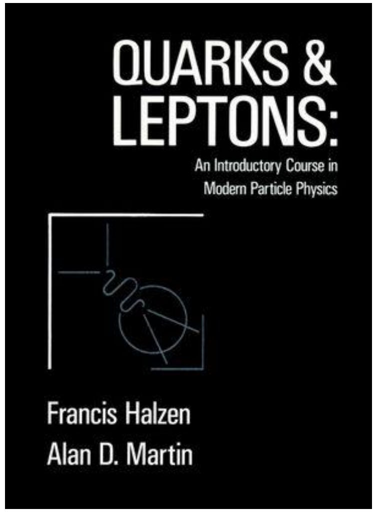
- QCD coupling constant $\alpha_s(Q)$
  - ✓ High Q: asymptotic freedom, perturbative QCD
  - ✓ Low Q: non-perturbative QCD
- Confinement: partons do not exist as free particles, but are always confined in hadron
- Fragmentation function & Energy-energy correlator
  - ✓ Understanding hadronization

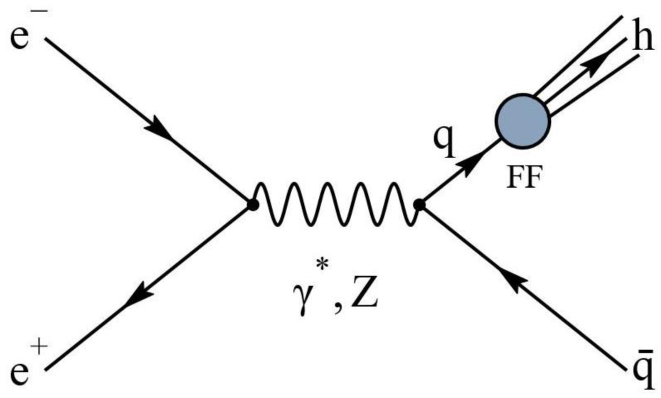


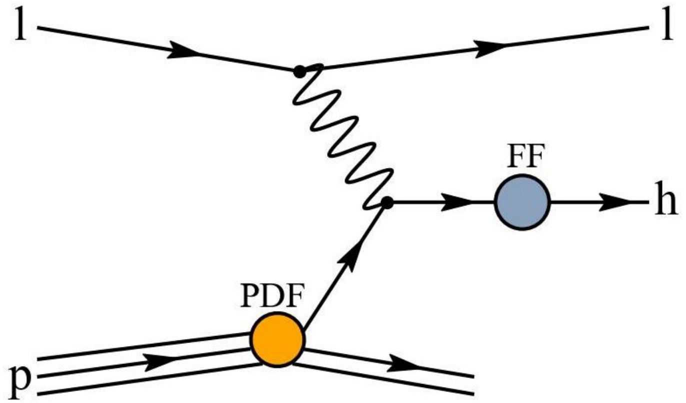
Eur. Phys. J. C85 16(2025)

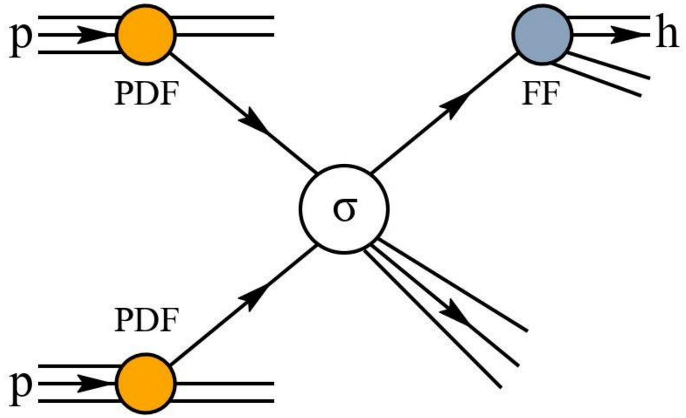
Fragmentation function: integrated $D_1^h(z)$

- Fragmentation function $D_q^h(z)$: probability that hadron h is found in the debris of a parton carrying a fraction z of parton's energy
- Consequence of confinement
- FF: QCD first principle (NOT YET)
  - ✓ FF evolution function: DGLAP
  - ✓ Fitting: parametrization & experimental data
  - ✓ Universality: e^+e^-, DIS, pp, $p\bar{p}$ data
- FFs contribute to virtually all processes









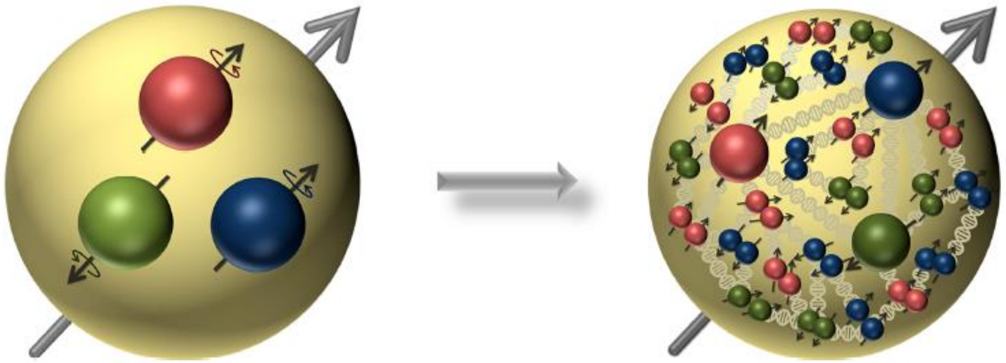
FFs with quark/hadron polarization

Hadron polarization	Quark polarizatiom @ PPNP 91 136 (2016)		
	Unpolarized	Longitudinally	Transversely
Unpolarized	D_1^h		$H_1^{\perp h}$
Longitudinally		G_1^h	$H_{1L}^{\perp h}$
Transversely	$D_{1T}^{\perp h}$	G_{1T}^h	H_1^h $H_{1T}^{\perp h}$

$$D_{hq^\uparrow}(z, P_{h\perp}) = D_1^q(z, P_{h\perp}^2) + H_1^{\perp q}(z, P_{h\perp}^2)\frac{(\hat{\mathbf{k}} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{zM_h}$$

- **Theoretically many more, in particular with polarized hadrons in the final state and transverse momentum dependence (TMD)**

FFs for EIC & EicC



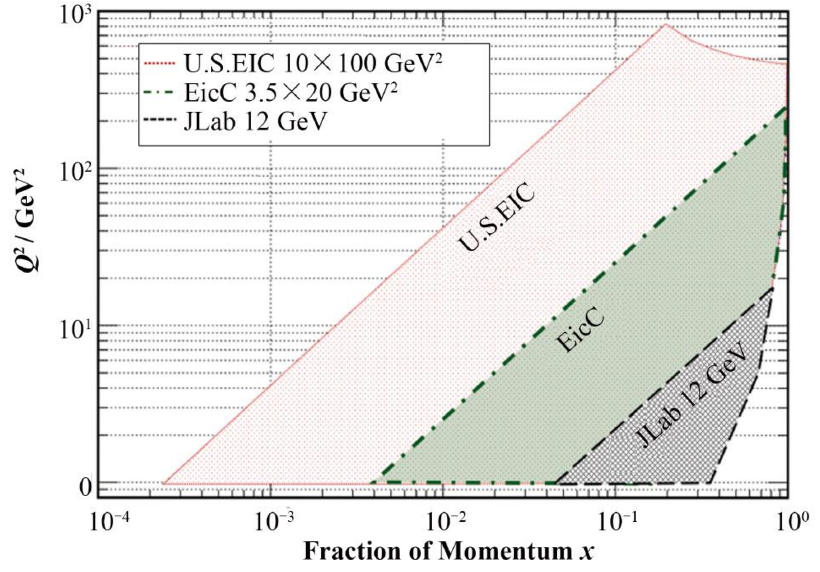
Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

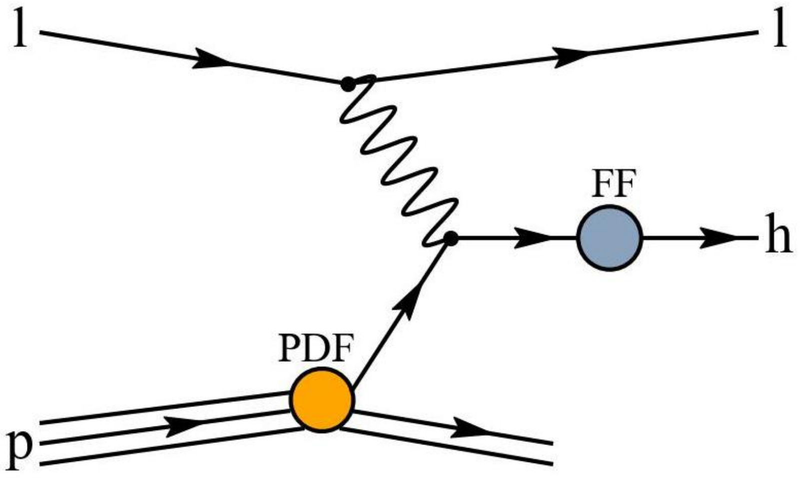
TMD Collaboration

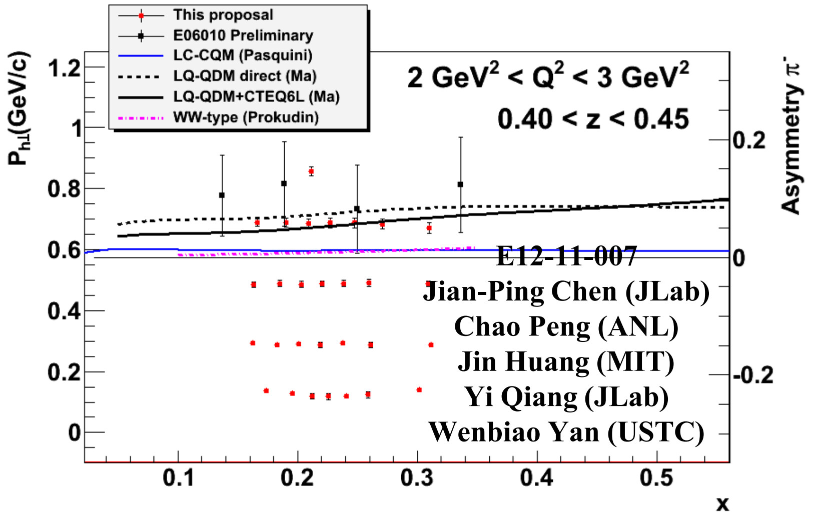
arXiv:2304.03302

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

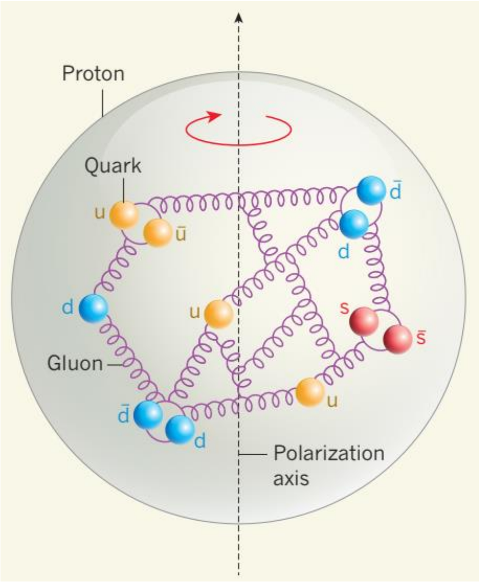


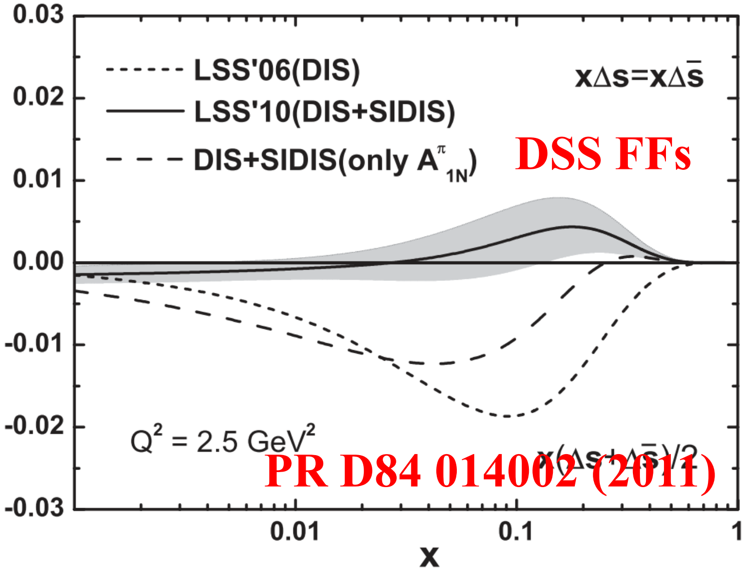


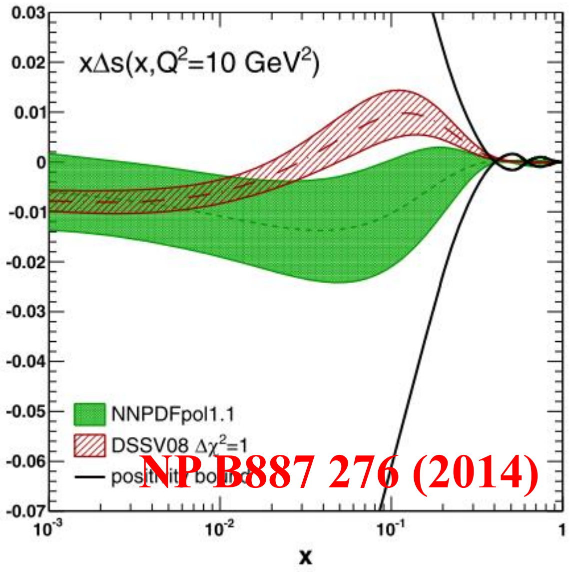


Precise knowledge of FFs will be crucial

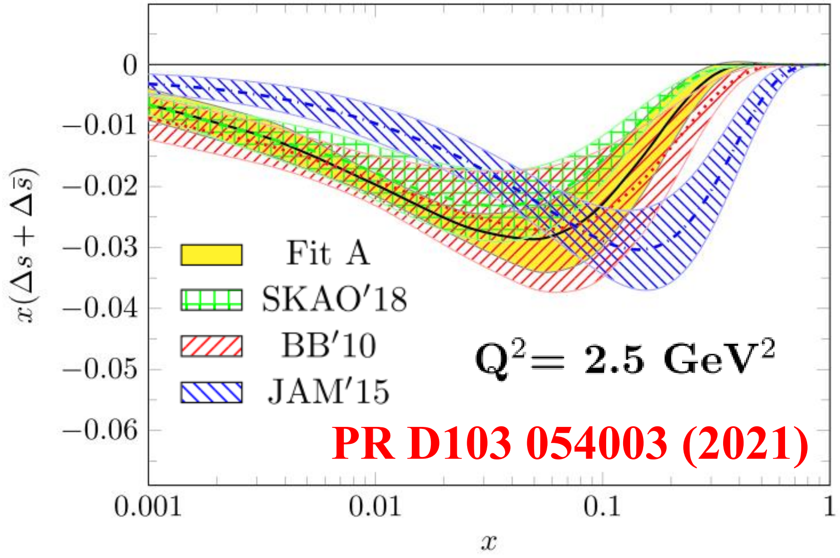
Strange quark polarization puzzle



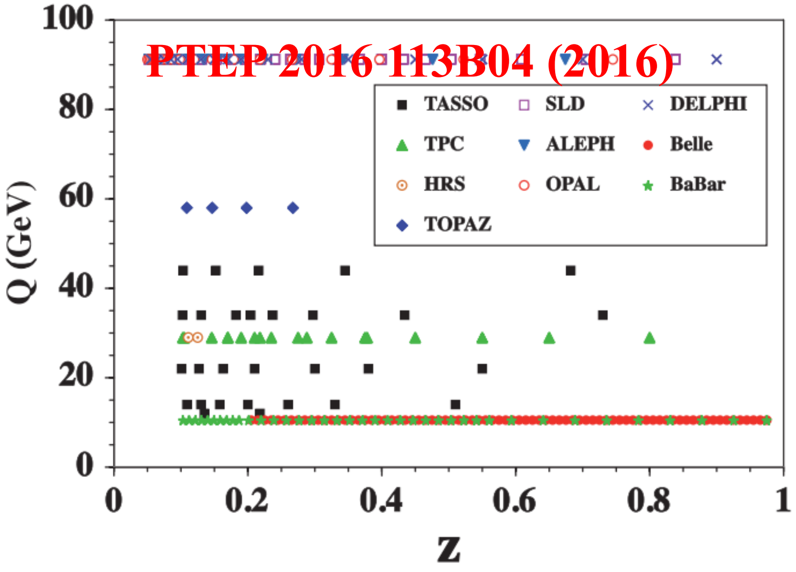




- Strange quark density function: $\Delta s(x)+\Delta\bar{s}(x)$
  - ✓ Inclusive DIS: only proton PDF
    - a. negative for all values of x
  - ✓ Semi-inclusive DIS: proton PDF & kaon FF
    - a. DSS FFs: positive for most of measured x
    - b. NNPDF FF: negative
    - c. JAM FFs: negative
- Reliable FFs knowledge ? Important !!!



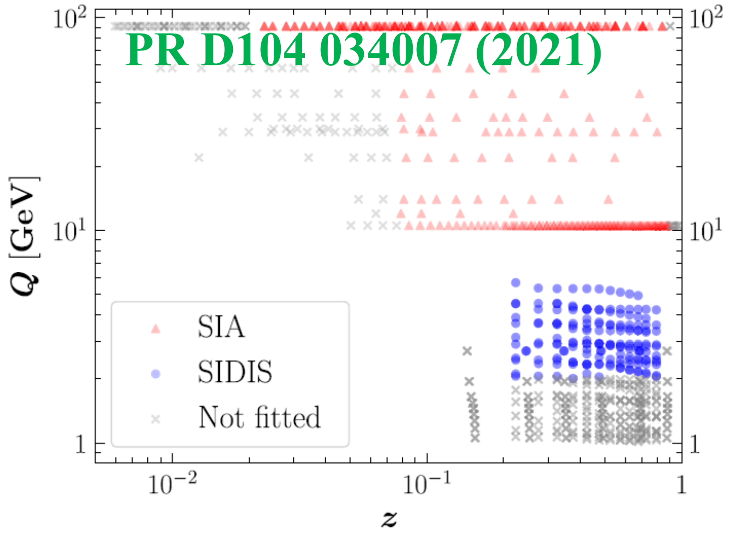
Used data set @ FFs fitting



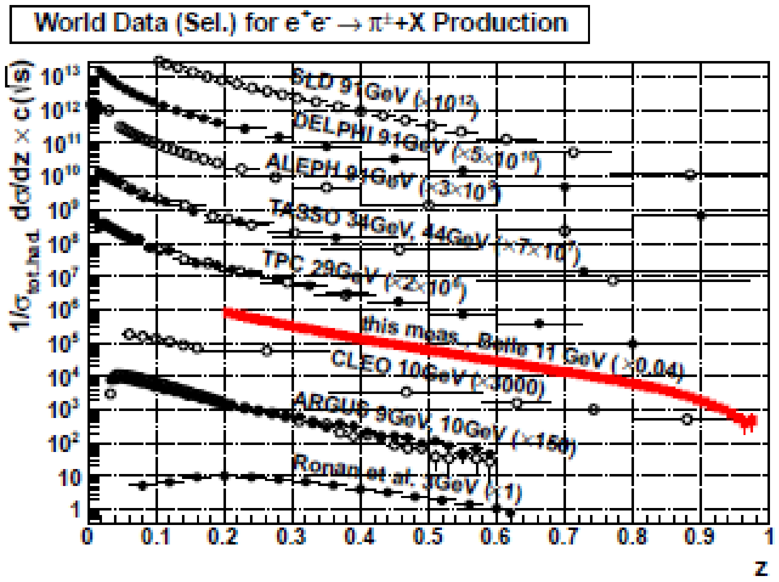
PPNP 91 136 (2016)

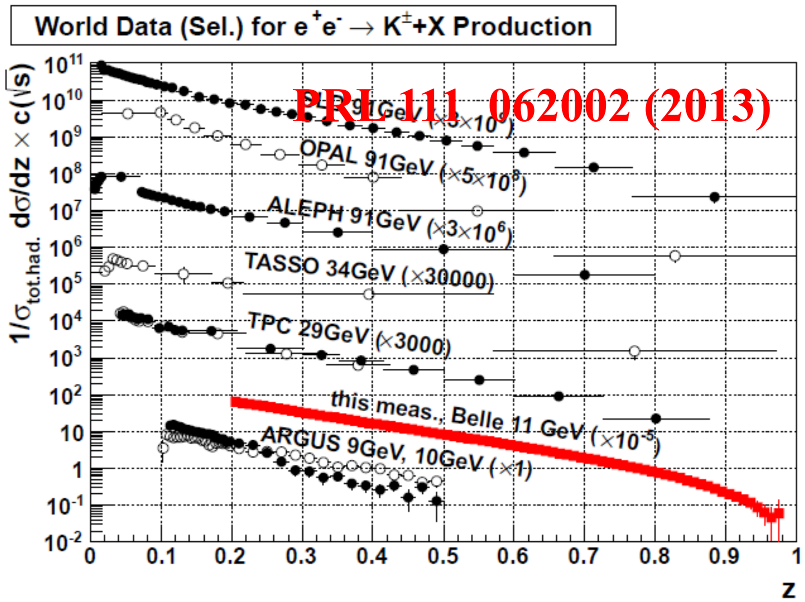
Experiment	Process	$\mathcal{L}[pb^{-1}]$	$Q^2[GeV^2]$	Final states
TPC [288–291]	e^+e^-	140	2[illegible]	$\pi^\pm, K^\pm, p/\bar{p}$
TASSO				
[292–294]	e^+e^-	34	34,44	$\pi^\pm, K^\pm, p/\bar{p}, K^0_S, \Lambda/\bar{\Lambda}$
[295–298]				
SLD [299,300]	e^+e^-	20	M_Z	$\pi^\pm, K^\pm, p, K^0_S, \Lambda/\bar{\Lambda}$
ALEPH [301,302]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K^0_S, \Lambda/\bar{\Lambda}$
DELPHI [303–306]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K^0_S, \Lambda/\bar{\Lambda}$
OPAL [307–310]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K^0_S, \Lambda/\bar{\Lambda}$
H1 [311–313]	$e+p$	500	$27.5(e)+920(p)$	$h^\pm, K^S_0$
ZEUS [314–316]	$e+p$	500	$27.5(e)+920(p)$	$h^\pm$
BELLE [317,318]	e^+e^-	10^6	Near 10.58	$\pi^\pm, K^\pm, p/\bar{p}$
BaBar [319,320]	e^+e^-	$557 \cdot 10^3$	Near 10.58	$\pi^\pm, K^\pm, \eta, p/\bar{p}$
HERMES [321,322]	$e+p(d)$	272 (p) 329 (d)	27.6 fixed target	$\pi^{\pm,0}, K^\pm$
COMPASS [323]	$\mu+p(d)$	775	160 GeV fixed target	$h^\pm$
PHENIX		16×10^{-3}	62.4	
[324–326]	pp	2.5	200	$\pi^{\pm,0}, \eta$
[327–329]		128	510	
STAR				
[330–332]	pp	8	200	$\pi^{0,\pm}, \eta, p/\bar{p}, K^0_S, \Lambda/\bar{\Lambda}$
[333–335]				
ALICE [336]	pp	6×10^{-3}	7×10^3	π^0, η
TOPAZ [337]	e^+e^-	278	52–61.4	$\pi^\pm, K^\pm, K^0_S,$
CDF [338,339]	$p+\bar{p}$	n/a	630 (1800)	$h^\pm, K^0_S \Lambda^0$

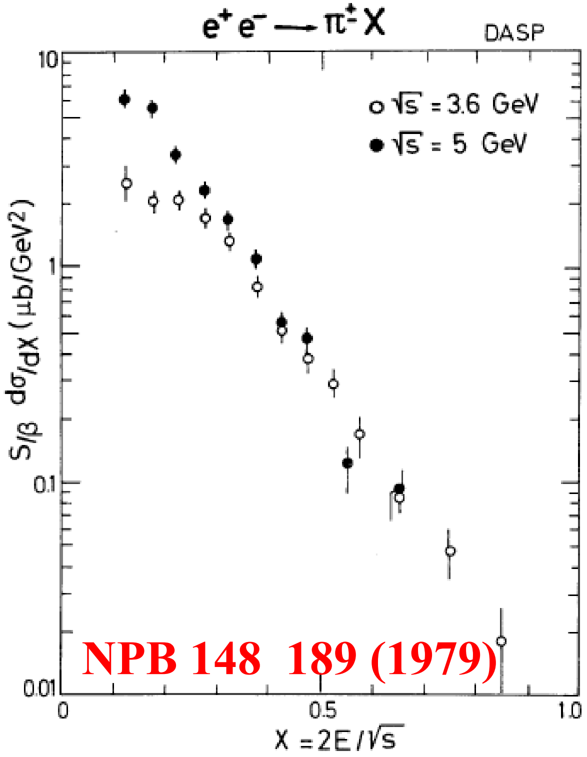
- Updated **HKNS FFs** @ 2016
  - ✓ only e^+e^- ($\sqrt{s} > 10$ GeV) data sets
- **MAPFF1.0 FFs** @ 2021
  - ✓ e^+e^- ($\sqrt{s} > 10$ GeV) and SIDIS data sets
- Data set at $\sqrt{s} < 10$ GeV e^+e^- collision ?



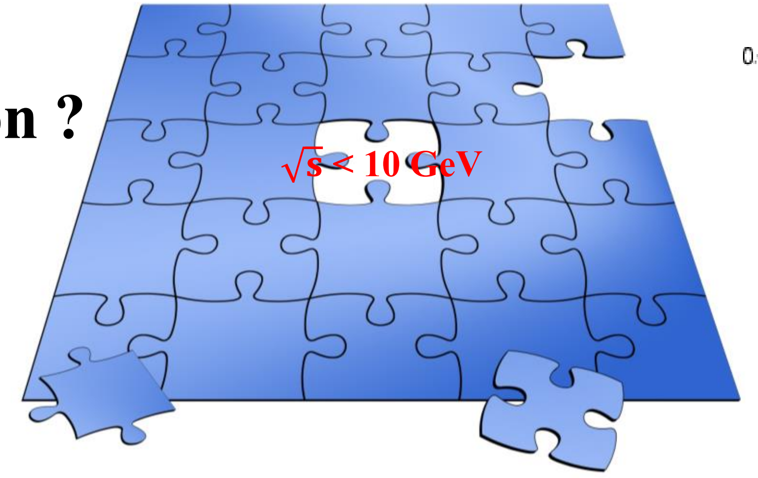
World π & K data on e^+e^-



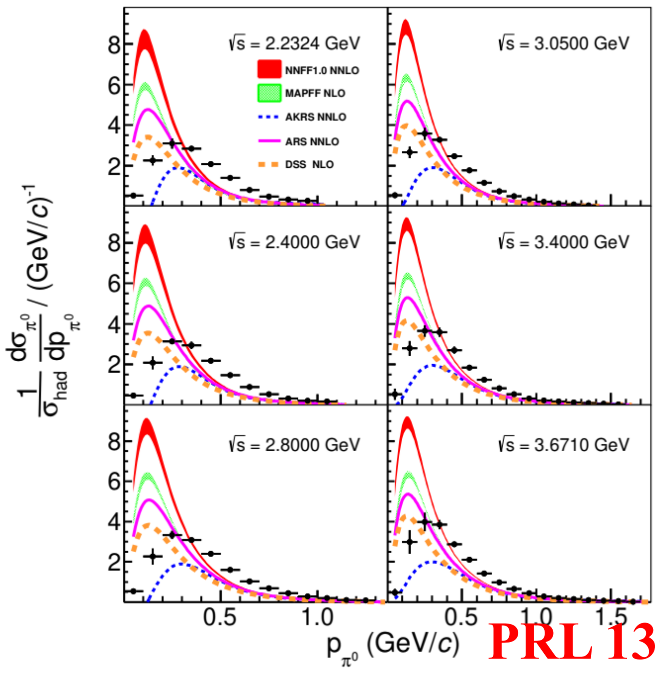


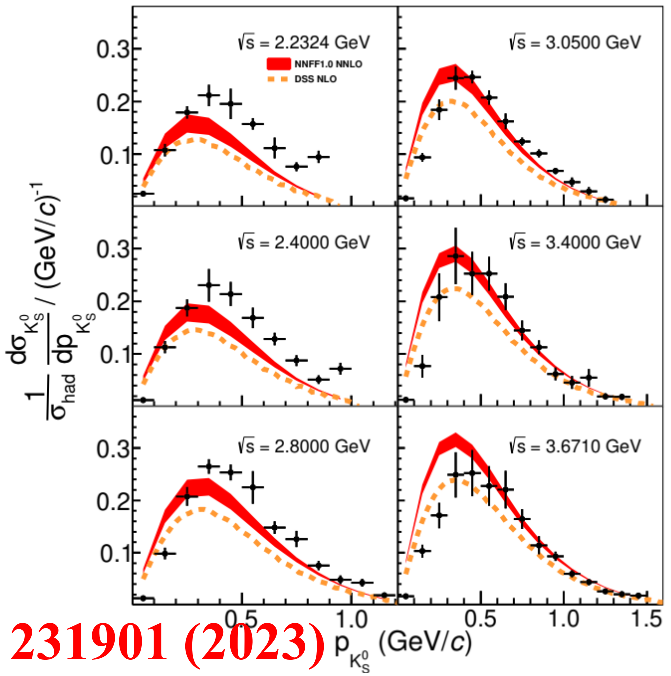


- Data sets at high z ?
- Data sets at $\sqrt{s} < 10$ GeV e^+e^- collision ?
- Charged π data by DASP
  - ✓ stat. uncertainty: 18%
- Chance for BESIII & STCF
  - ✓ Precision

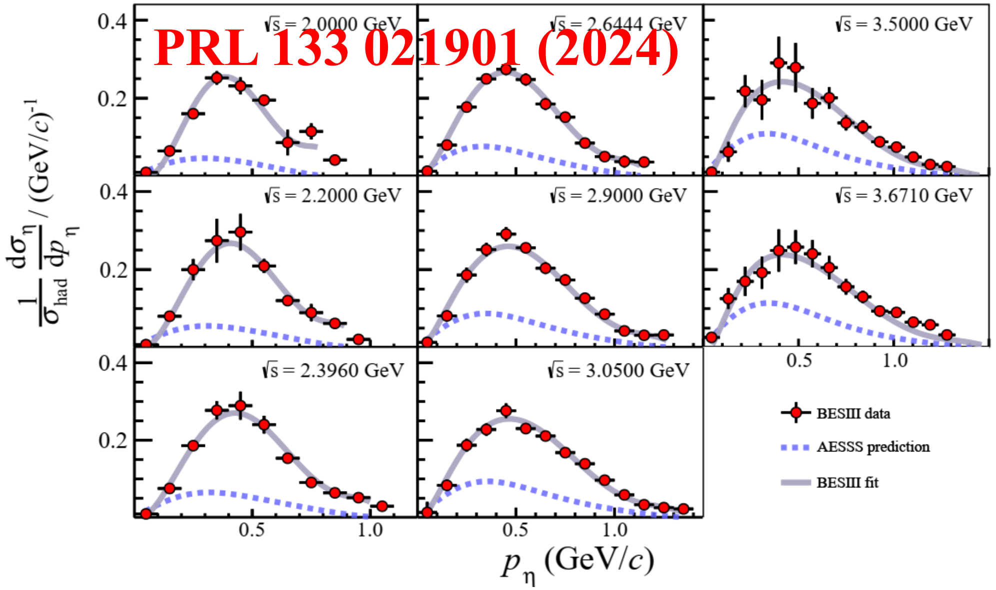


Inclusive $\pi^0/K_S^0/\eta$ vs. what we know

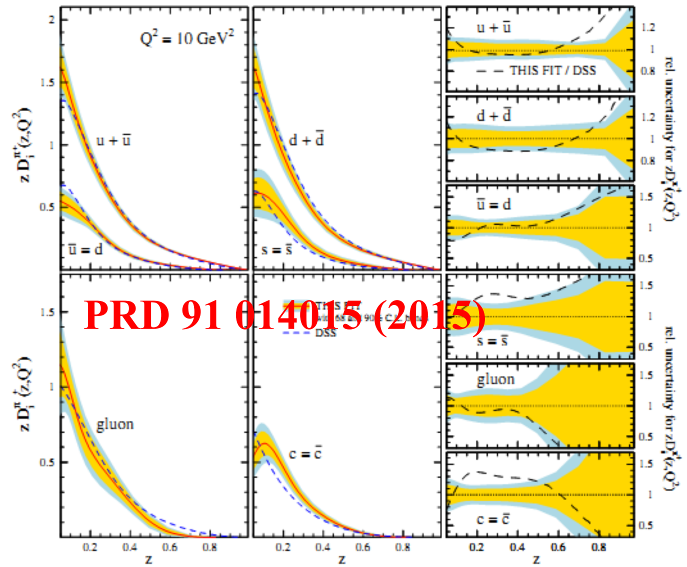




PRL 130 231901 (2023)



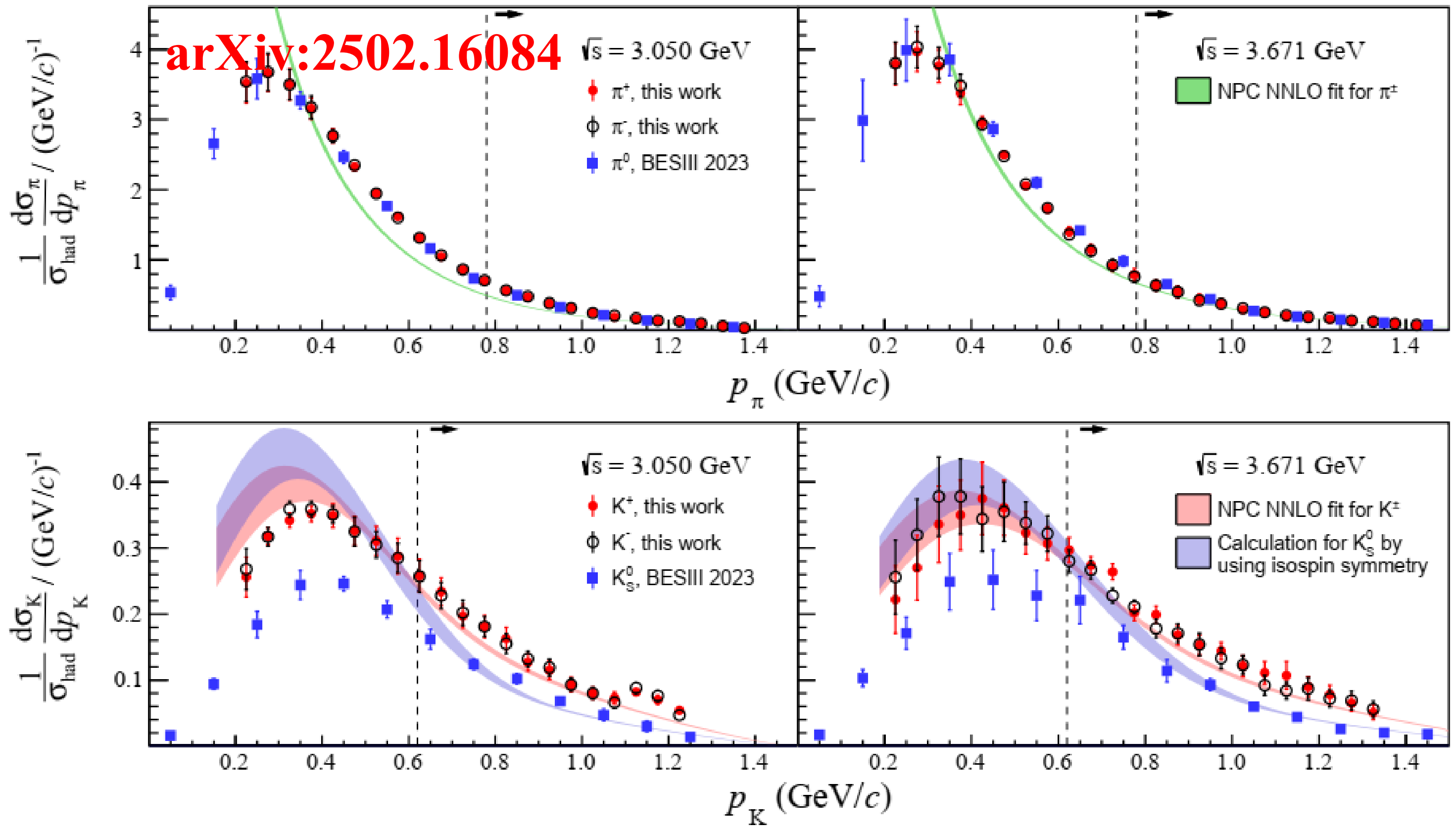
PRL 133 021901 (2024)



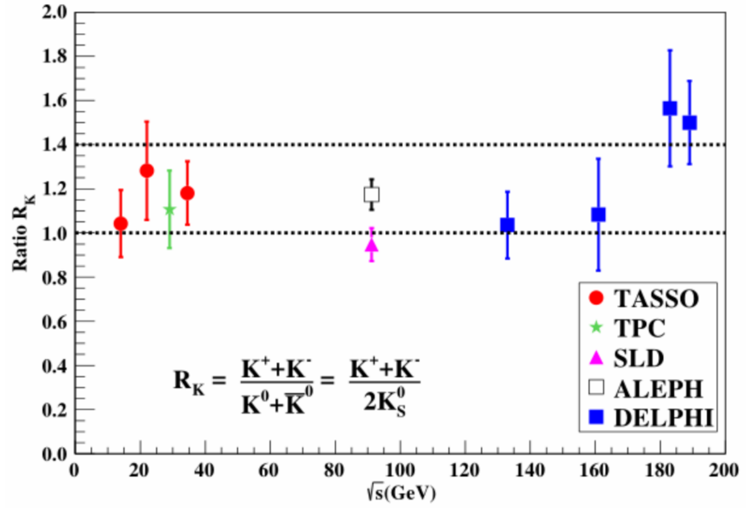
PRD 91 014015 (2015)

- Inclusive π^0 : surprise , disagreement !!!
  - ✓ Best known pion FFs
- Inclusive K_S^0 : not so bad
- Inclusive η : how to understand the discrepancy
  - ✓ NNLO accuracy, hadron mass correction & higher twist

Inclusive $\pi^{\pm}/K^{\pm}$ vs. pQCD



- **Inclusive pion cross section**
  - ✓ $\pi^{+} \approx \pi^{-} \approx \pi^{0}$
  - ✓ **Isospin symmetry**
- **Inclusive kaon cross section**
  - ✓ $K^{+} \approx K^{-} \approx 1.4\ K_S$
  - ✓ **Large isospin violation ?**



$$R_K = \frac{K^+ + K^-}{K^0 + \bar{K}^0} = \frac{K^+ + K^-}{2K_S^0}$$

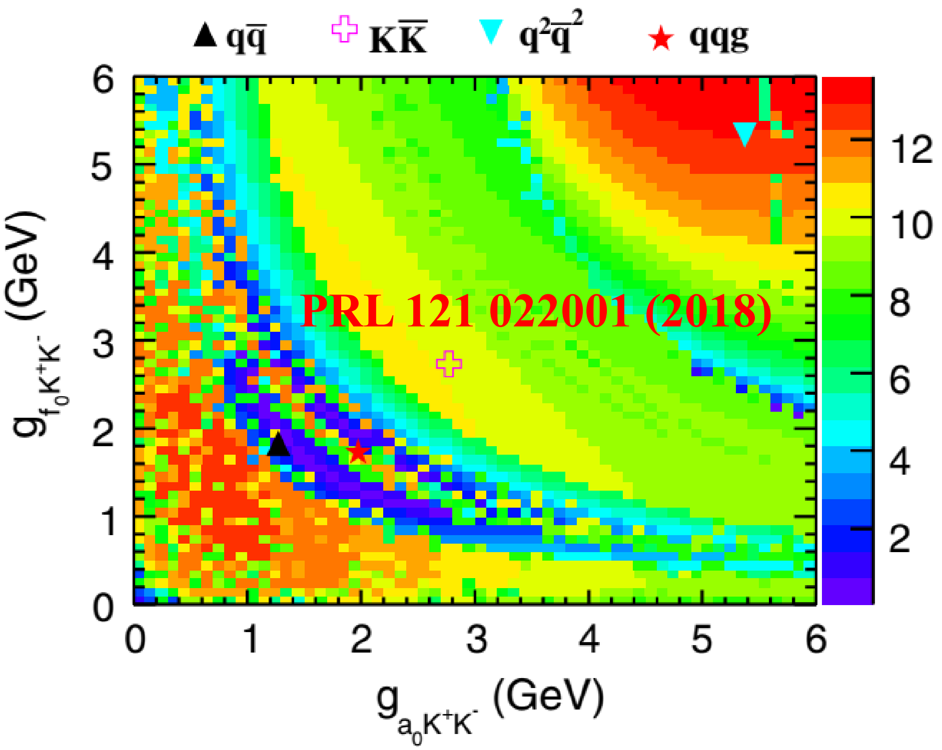
- **NPC NNLO: $\sqrt{s} > 3.0$ GeV & $E_h > 0.8$ GeV @ BESIII**
- **arXiv:2502.17837: Validity of QCD factorization and pQCD calculation at energy scales down to 3 GeV**
- **Test of isospin symmetry with FF fitting**

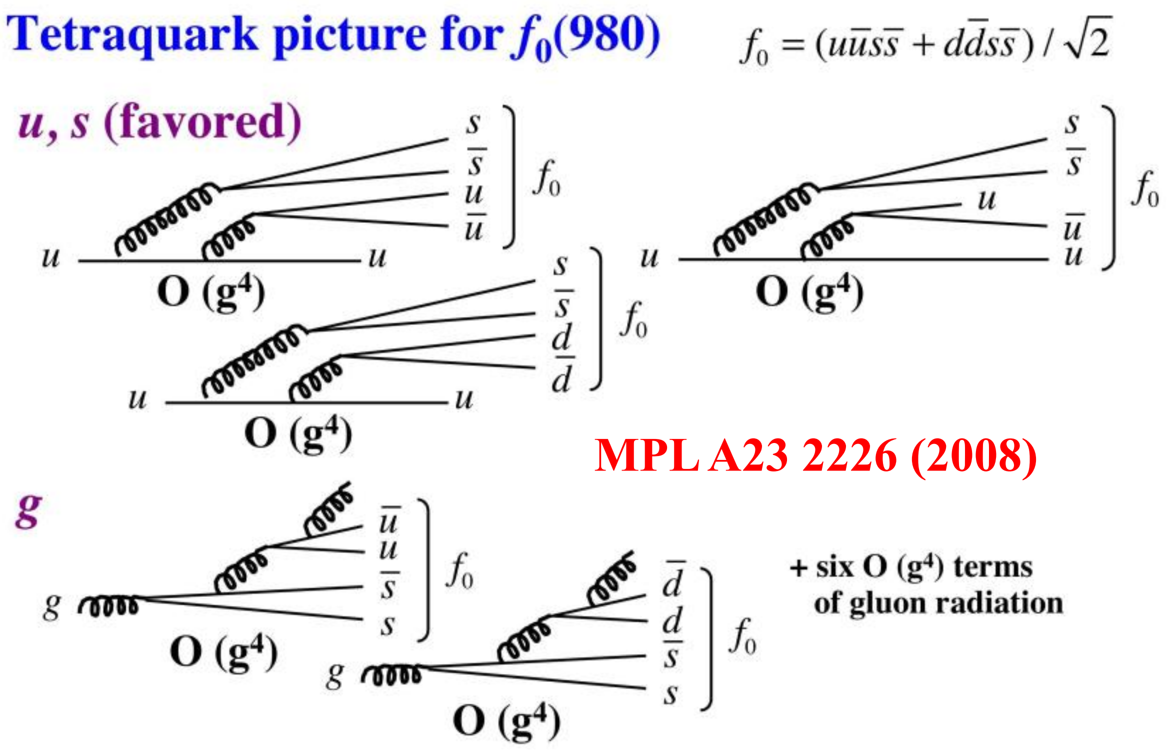
$$D_i^{\pi^0}(z,Q) = \frac{1}{2} D_j^{\pi^{\pm}}(z,Q)$$

$$D_i^{K_S^0}(z,Q) = \frac{1}{2} D_j^{K^{\pm}}(z,Q) \quad \begin{cases} \text{if } i = d(u),\ j = u(d) \\ \text{otherwise } i = j \end{cases}$$

@NPB 803 42 (2008)

Nature of $f_0(980)$





- **Nature of $a_0(980)$ & $f_0(980)$**
  - ✓ **molecule, tetraquark, hybrid ?**
- **$a_0(980)$-$f_0(980)$ mixing @ BESIII**
- **Inclusive $f_0(980)$ production**

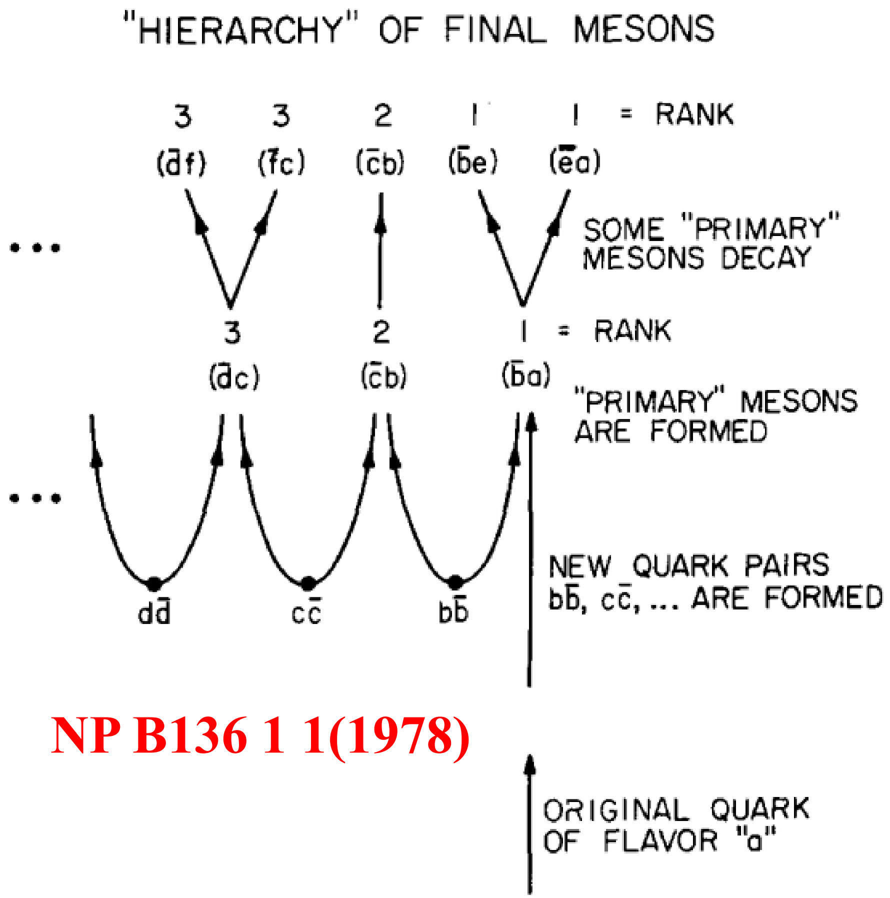
Table 1. Possible $f_0(980)$ configurations and their relations to the second moments and the peak positions for the fragmentation functions of $f_0(980)$.

Type	Configuration	Second moments	Peak positions
Strange $q\bar{q}$	$s\bar{s}$	$M_u < M_s \lesssim M_g$	$z_u^{max} < z_s^{max}$
Tetraquark (or $K\bar{K}$)	$(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})/\sqrt{2}$	$M_u \sim M_s \lesssim M_g$	$z_u^{max} \sim z_s^{max}$

To do list

Theoretically many more, in particular with polarized hadrons in the final state and transverse momentum dependence (TMD)

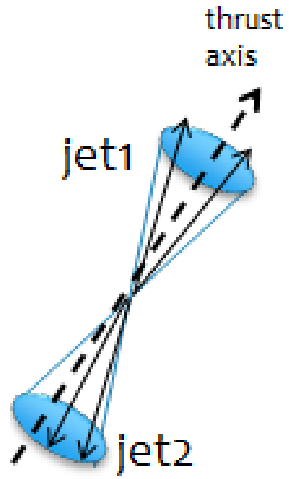
- **TMD FFs:**
  - ✓ reliable thrust axis @ 5.0 GeV
- **Hadrons with spin: spin effect**
  - ✓ hyperon: polarization, weak decay
  - ✓ vector states: spin alignment , strong decay
- **Inclusive di-hadron production**
  - ✓ di-hadron FFs, thrust axis ?
- **Event shape variables**
  - ✓ Thrust & energy-energy correlator
- **Polarization electron beam @ STCF**
  - ✓ MC generator ? what could we do ?
- **More ideas**

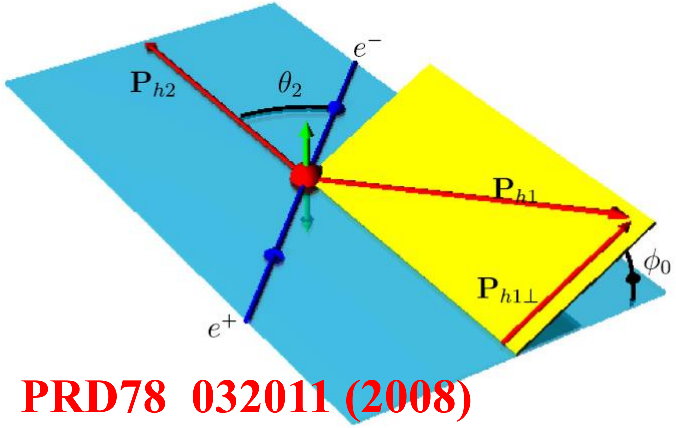


NP B136 1 1(1978)

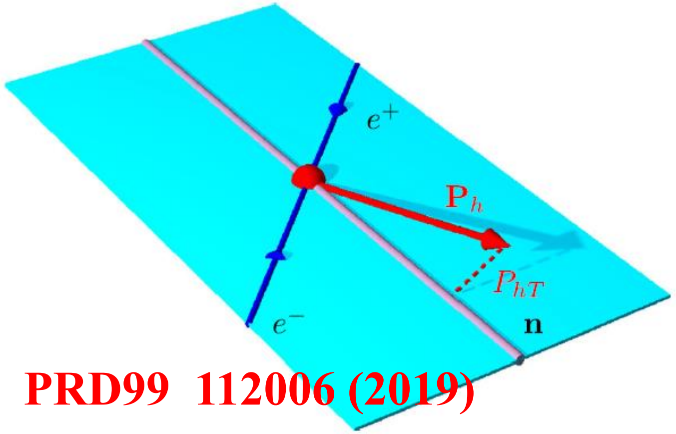
TMD FFs: $D_1^h(z) \Rightarrow D_1^h(z, p_T)$

TMD FF $D_1(z, k_T)$		
$e^+e^- \to h_a h_b X$	$\sum_q e_q^2 D_1^{h_a/q}(z_a, k_{aT}) \otimes D_1^{h_b/\bar{q}}(z_b, k_{bT})$ $+\{q \leftrightarrow \bar{q}\}$	back-to-back production of hadron pair
$e^+e^- \to$ $(h, \text{jet/thrust axis})X$	$\sum_q e_q^2 D_1^{h/q}(z, k_T)$	can access z, k_T

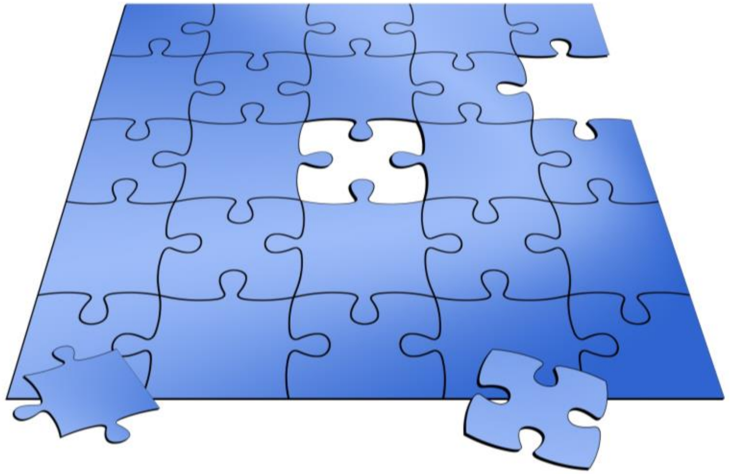




PRD78 032011 (2008)

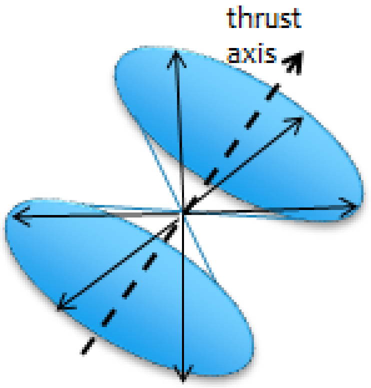


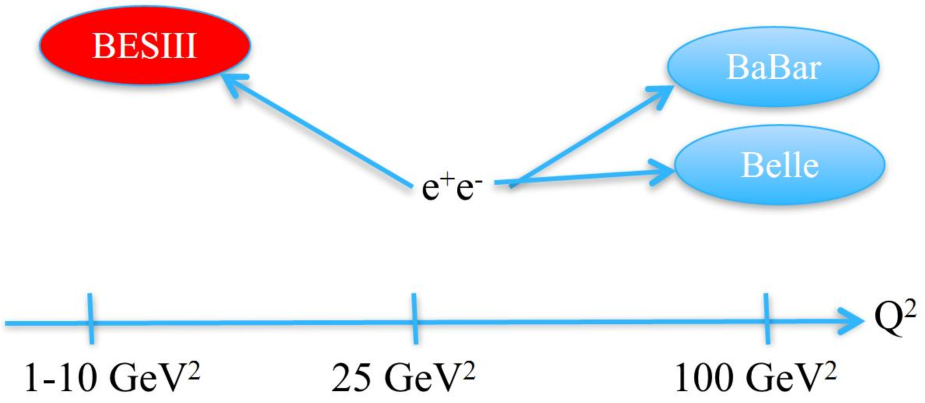
PRD99 112006 (2019)

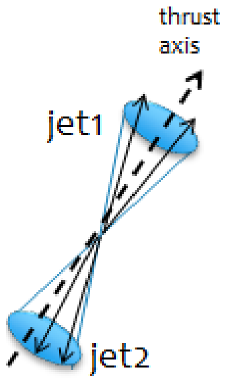


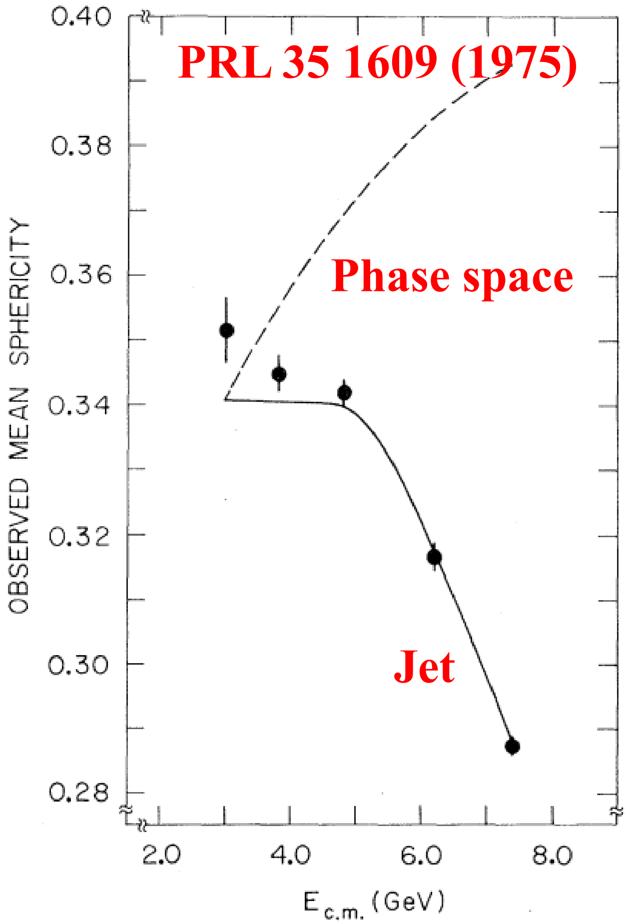
- **Traditional 2-hadron FF**
  - ✓ **Use transverse momentum between two hadron**
  - ✓ **$e^+e^- \to \pi\pi + X$: $\theta_{\pi\pi}$>120°, back-to-back pions**
- **Signal-hadron FF with thrust or jet axis**

Thrust axis: more is different



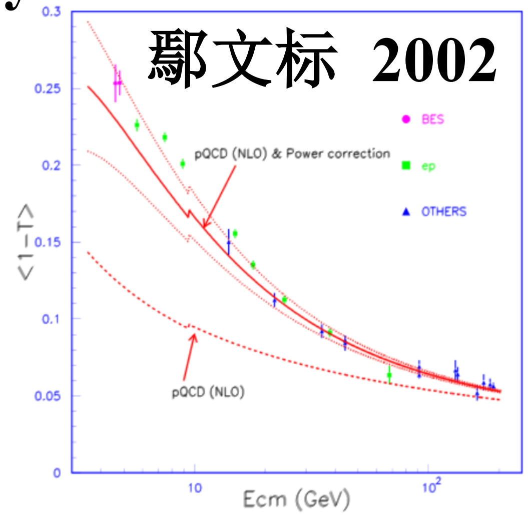






$$T \equiv 1 - \tau = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$

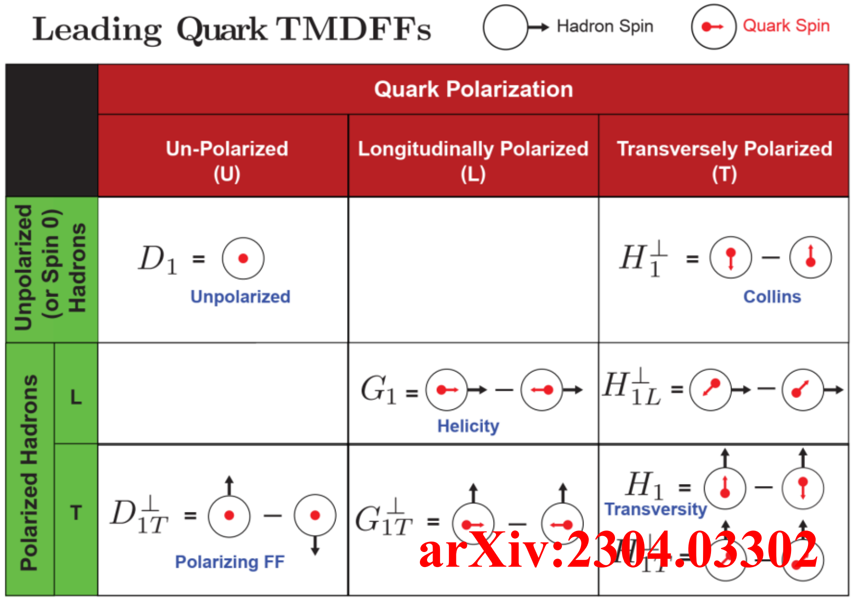
- **Jet structure at BESIII & STCF**
  - ✓ **can reconstruct thrust axis correctly ?**
- **Phase space model vs. Jet model**
  - ✓ **$\sqrt{s} \geq 5$ GeV ? reliable ?**
  - ✓ **evidence for jet structure ?**
- **Advantage of thrust axis**
  - ✓ **Two hadron ∈ same q/q̄ ?**
  - ✓ **back-to-back hadrons**
  - ✓ **di-hadron FFs**

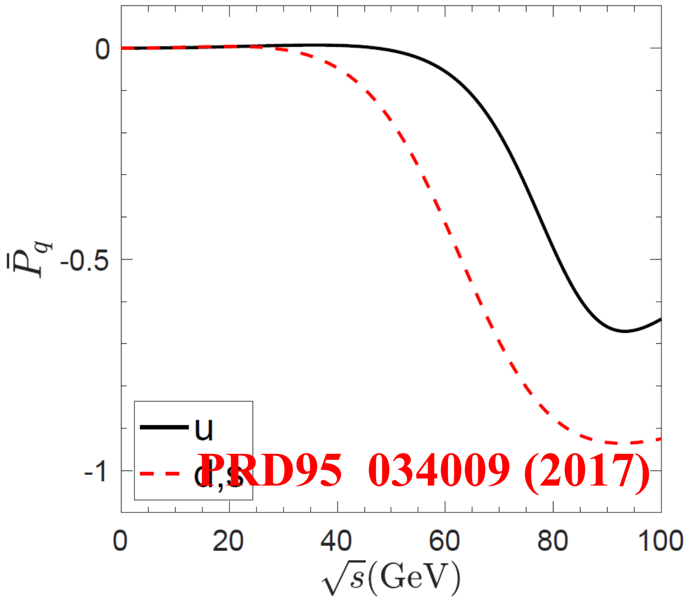


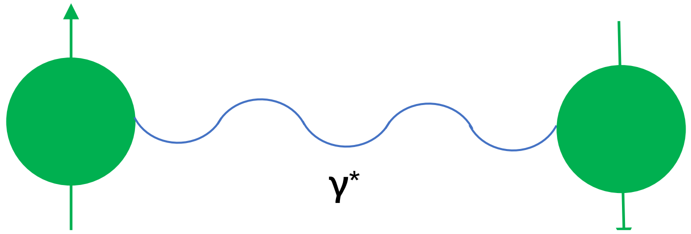
$$M^{\alpha\beta} = \sum_{i=1}^{N_{particle}} p_i^\alpha p_i^\beta \quad (\alpha, \beta = 1, 2, 3)$$

$$\lambda_1 \geq \lambda_2 \geq \lambda_3 \qquad \lambda_1 + \lambda_2 + \lambda_3 = 1$$

Quark polarization





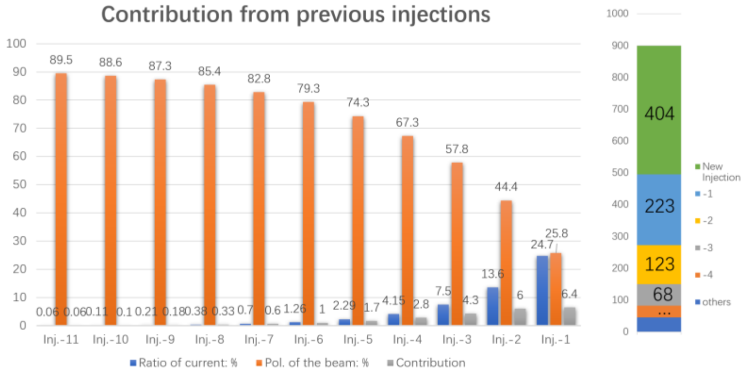


$$W_{\uparrow\downarrow} = \frac{5\sqrt{3}}{16}\frac{e^2\gamma^5\hbar}{m^2c^2\rho^3}\left(1 + \frac{8}{5\sqrt{3}}\right)$$

$$W_{\downarrow\uparrow} = \frac{5\sqrt{3}}{16}\frac{e^2\gamma^5\hbar}{m^2c^2\rho^3}\left(1 - \frac{8}{5\sqrt{3}}\right)$$

- Quark polarization @ $e^+e^- \to \gamma^*/Z \to q\bar{q}$
  - ✓ $q/\bar{q}$: longitudinally polarization
  - ✓ $q/\bar{q}$: transversely polarization suppressed by factor $m_q/\sqrt{s}$
  - ✓ unpolarized $q/\bar{q}$ @ [2, 7] GeV: Yes or No ?
- Sokolv-Ternov effect: transverse polarization of electron/positron beam in storage rings

BEPCII beam polarization



Electron beam
Life time: 2.1 h
Data taking: 1 h
Injection: 0.25 h
P_1=> 23.6%

Positron beam
Life time: 1.8 h
Data taking: 1 h
Injection: 0.25 h
P_2=> 20.1%

P*P = 0.05

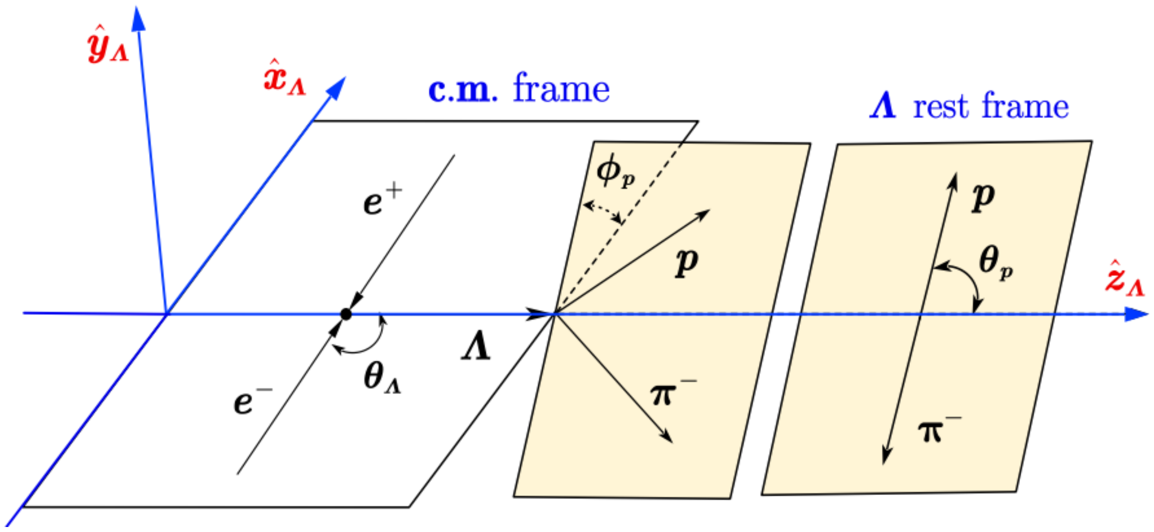
Previous injections contribute to the polarization!

Inclusive Λ production

- **Spin density matrix ρ of spin ½ hyperon**

$$\rho = \frac{1}{2}\boldsymbol{I} + S^i\Sigma^i$$

✓ **S: polarization vector**
✓ **$\Sigma^i = \sigma^i/2$, Pauli matrices**



- **Polarization vector of hyperon with parity violating decay**
- **Daughter proton preferentially decays in direction of Λ's spin (opposite for $\bar{\Lambda}$)**
  ✓ **Decay parameter α**

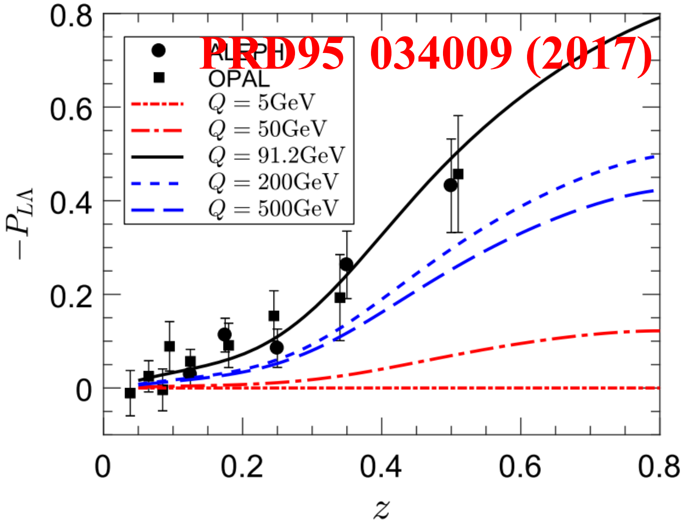
$$\frac{dN}{dcos\theta} = \frac{1}{2}\left(1 + \alpha|\vec{S}|cos\theta\right)$$

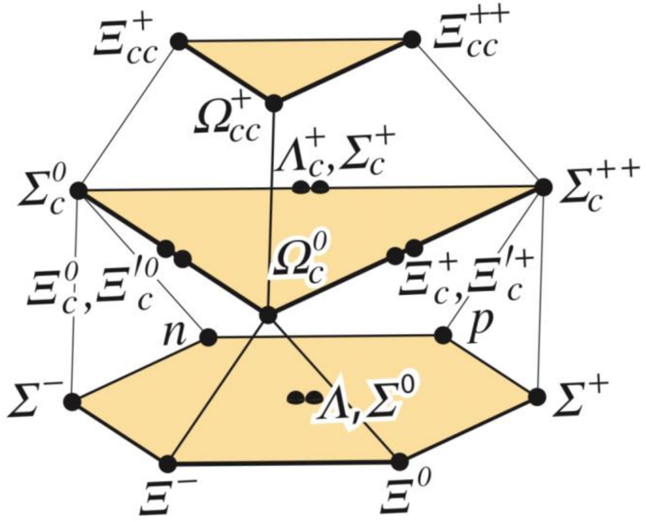
- **Cross section of inclusive Λ**
  ✓ **For Λ FFs**

$$E_p\frac{d\sigma}{d^3p} = \chi\frac{2\alpha^2}{zQ^4}\Big\{[T_0(y)D_1(z) + \lambda_h T_1(y)\Delta D_{1L}(z)] + \frac{4M}{zQ}|\vec{S}_\perp|\sqrt{y(1-y)}[T_2(y)D_T(z)\sin\phi_s - T_3(y)\Delta D_T(z)\cos\phi_s]\Big\}.$$

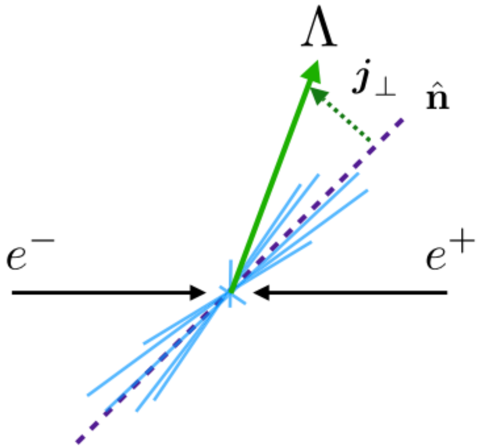
Inclusive Λ production

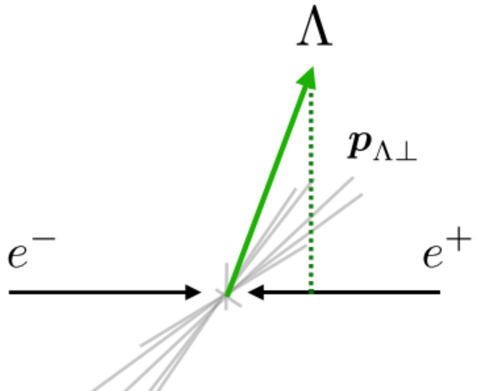
Group	Reaction	$\sqrt{s}$/GeV	Measurements	Result
BES	$e^+ + e^- \to \Lambda + X$	~ 5	$P_T^\Lambda \sim D_T^\Lambda$	Furture
Belle [44]	$e^+ + e^- \to \Lambda/\bar{\Lambda} + X$	10.58	$P_T^\Lambda \sim D_{1T}^{\perp\Lambda}$	~ -0.06
	$e^+ + e^- \to \Lambda/\bar{\Lambda} + h^\pm + X$		$P_T^\Lambda \sim D_1^h D_{1T}^{\perp\Lambda}$	~ 0.1
BelleII [45]	$e^+ + e^- \to \Lambda_c + X$	10.58	Polarization	Furture
	$\Lambda_c \to \Lambda + e^+ + \nu$		Polarization	-0.86 ± 0.04
	$\Lambda_c \to \Lambda + \pi^+$		Polarization	-0.91 ± 0.15
OPAL [46]	$e^+ + e^- \to \Lambda/\bar{\Lambda} + X$	91.2	$P_T^\Lambda \sim D_{1T}^{\perp\Lambda}$	~ 0
	$e^+ + e^- \to \Lambda/\bar{\Lambda} + X$		$P_L^\Lambda \sim G_{1L}^\Lambda$	-0.329
ALEPH [47]	$e^+ + e^- \to \Lambda/\bar{\Lambda} + X$	91.2	$P_T^\Lambda \sim D_{1T}^{\perp\Lambda}$	~ 0
	$e^+ + e^- \to \Lambda/\bar{\Lambda} + X$		$P_L^\Lambda \sim G_{1L}^\Lambda$	-0.32
TASSO [49]	$e^+ + e^- \to \Lambda/\bar{\Lambda} + X$	14, 22, 34	$P_T^\Lambda \sim D_{1T}^{\perp\Lambda}$	~ 0
	$e^+ + e^- \to \Lambda/\bar{\Lambda} + X$		$P_L^\Lambda \sim G_{1L}^\Lambda$	~ 0





- Energy dependence of Λ polarization
- Thrust axis → z axis @ experiment
- Beyond Λ
  - ✓ ground state Σ, Ξ
  - ✓ exotic hyperon Λ(1405)





PL B818 136371 (2021)

Spin alignment of vector meson

$$\begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1,-1} \\ \rho_{01} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

$$\boldsymbol{\rho} = \frac{1}{3}\left(\boldsymbol{I} + \frac{3}{2}S^i\Sigma^i + 3T^{ij}\Sigma^{ij}\right)$$

- T: polarization tensor
- $\Sigma^{ij} = \frac{1}{2}(\Sigma^i\Sigma^j + \Sigma^j\Sigma^i) - \frac{2}{3}\boldsymbol{I}\delta^{ij}$

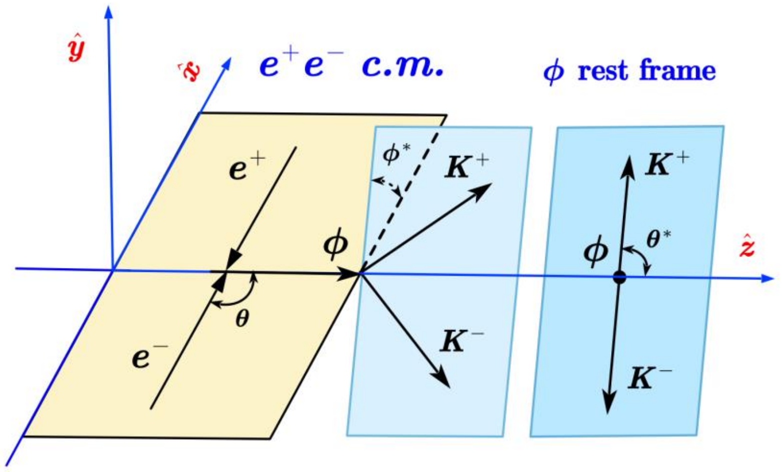
$$\boldsymbol{S} = (S_T^x, S_T^y, S_L),$$

$$\boldsymbol{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3}S_{LL} \end{pmatrix}$$

PRD62 114004 (2000)

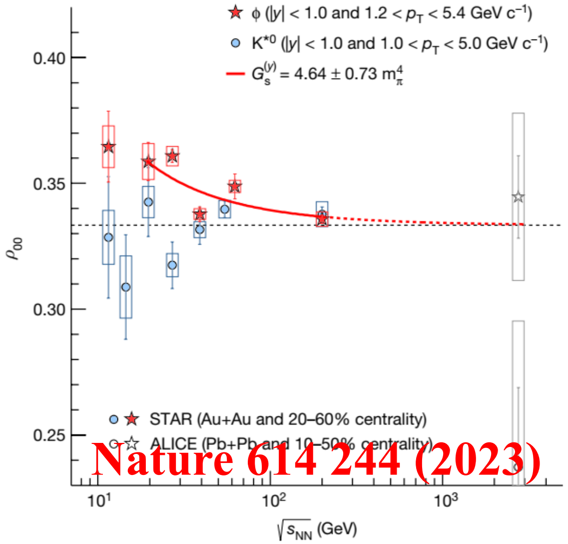
$$\boldsymbol{\rho} = \begin{pmatrix} \frac{1}{3} + \frac{S_{LL}}{3} + \frac{S_L}{2} & \frac{S_{LT}^x - iS_{LT}^y}{2\sqrt{2}} + \frac{S_T^x - iS_T^y}{2\sqrt{2}} & \frac{S_{TT}^{xx} - iS_{TT}^{xy}}{2} \\ \frac{S_{LT}^x + iS_{LT}^y}{2\sqrt{2}} + \frac{S_T^x + iS_T^y}{2\sqrt{2}} & \frac{1}{3} - \frac{2S_{LL}}{3} & \frac{-S_{LT}^x + iS_{LT}^y}{2\sqrt{2}} + \frac{S_T^x - iS_T^y}{2\sqrt{2}} \\ \frac{S_{TT}^{xx} + iS_{TT}^{xy}}{2} & \frac{-S_{LT}^x - iS_{LT}^y}{2\sqrt{2}} + \frac{S_T^x + iS_T^y}{2\sqrt{2}} & \frac{1}{3} + \frac{S_{LL}}{3} - \frac{S_L}{2} \end{pmatrix}$$

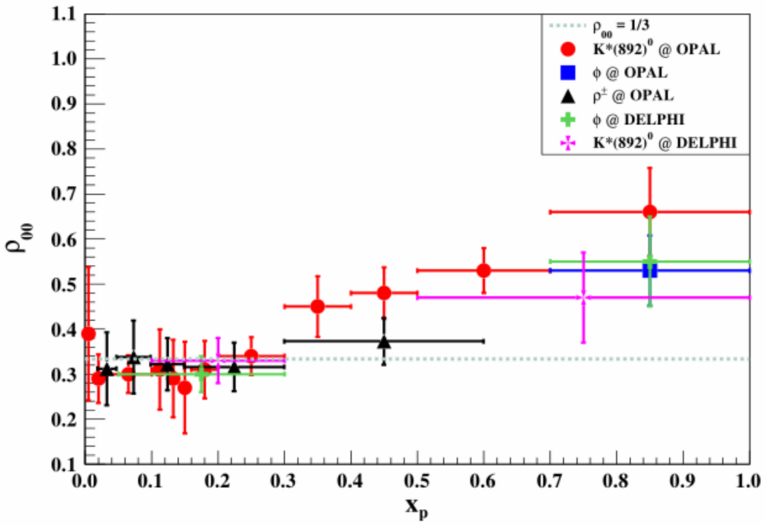
- **Spin density matrix ρ of vector states**
  - ✓ **ρ_{mm} : probability to be |s; s_z = m>**
  - ✓ **$\rho_{00} \neq 1/3$: spin alignment**
- **Angular of decay particle @ rest frame**
  - ✓ **extract some elements, e.g. ρ_{00}**

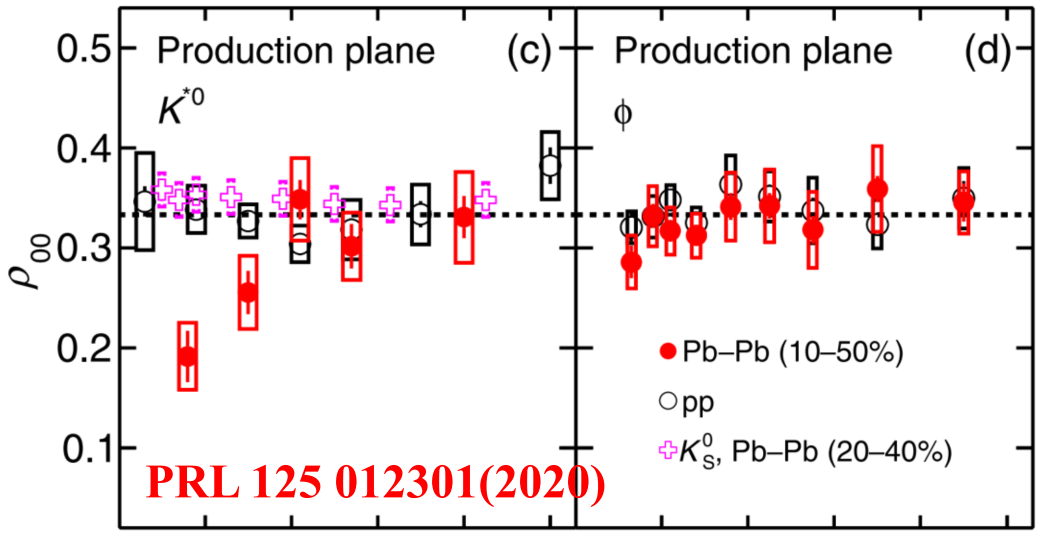


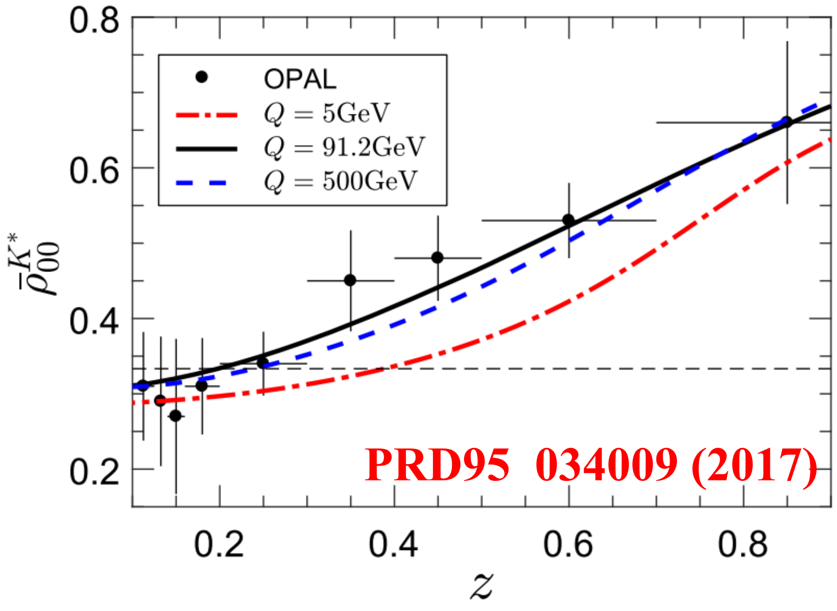
$$\begin{aligned} W(\cos\theta^*, \phi^*) = \; & (3/4\pi)\left[\frac{1}{2}(1-\rho_{00}) + \frac{1}{2}(3\rho_{00}-1)\cos^2\theta^*\right. \\ & -\mathrm{Re}\,\rho_{1-1}\ \sin^2\theta^*\cos 2\phi^* - \frac{1}{\sqrt{2}}\mathrm{Re}\,(\rho_{10}-\rho_{0-1})\sin 2\theta^*\cos\phi^* \\ & \left.+\mathrm{Im}\,\rho_{1-1}\ \sin^2\theta^*\sin 2\phi^* + \frac{1}{\sqrt{2}}\mathrm{Im}\,(\rho_{10}-\rho_{0-1})\sin 2\theta^*\sin\phi^*\right] \end{aligned}$$

Spin alignment of vector meson





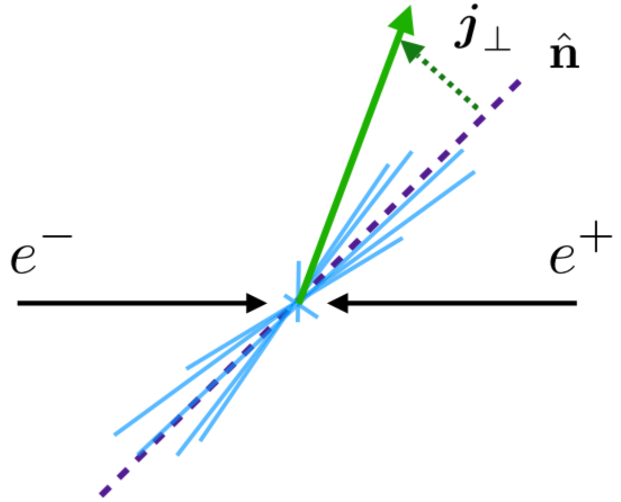




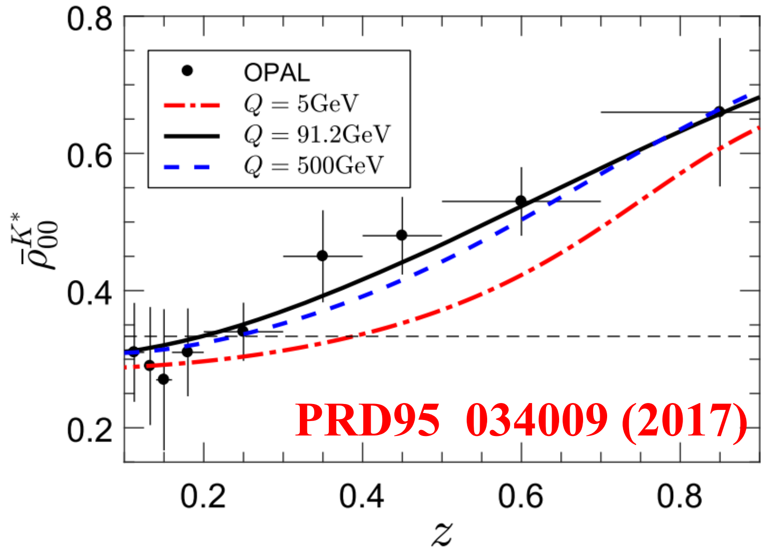
- **Heavy ion collision: QGP & spin-orbital coupling**
  - ✓ **STAR: for phi unexpectedly large than 1/3**
- **e^+e^- collision: fragmentation, Z^0 dominant**
  - ✓ **$x_p < 0.3$, $\rho_{00} \approx 1/3$; $x_p > 0.3$, $\rho_{00} > 1/3$**
- **pp collision: PDF & fragmentation**
- **BESIII: e^+e^- collision, fragmentation, γ^* dominant**

Vector meson: FFs, TMD & spin alignment

$$E\frac{d\sigma}{d^3p}=\frac{2\alpha^2}{zQ^4}\chi\Big\{[T_0(y)D_1(z)+T_0(y)S_{LL}D_{1LL}(z_B)+\lambda_h T_1(y)\Delta D_{1L}(z)]$$
$$+\frac{4M}{zQ}|\vec{S}_\perp|[\tilde{T}_2(y)\sin\varphi_s D_T(z)-\tilde{T}_3(y)\cos\varphi_s\Delta D_T(z)]$$
$$+\frac{4M}{zQ}|\vec{S}_{LT}|[-\tilde{T}_2(y)\cos\varphi_{LT}D_{LT}(z)+\tilde{T}_3(y)\sin\varphi_{LT}\Delta D_{LT}(z)]\Big\}$$



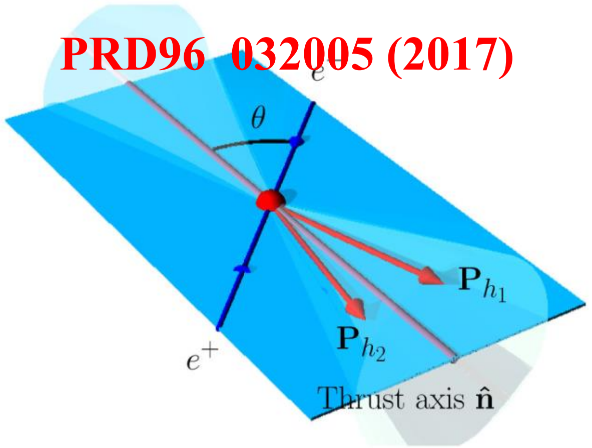
- **Cross section of vector states**
  - ✓ **FFs of vector meson**
  - ✓ **ρ/ω/ϕ/K*(892) FFs: not yet**
- **Using thrust axis**
  - ✓ **Combine TMD & spin alignment**
- **$e^+ e^- \rightarrow$ Vector + π + X**
  - ✓ **Back-to-back vector & π**



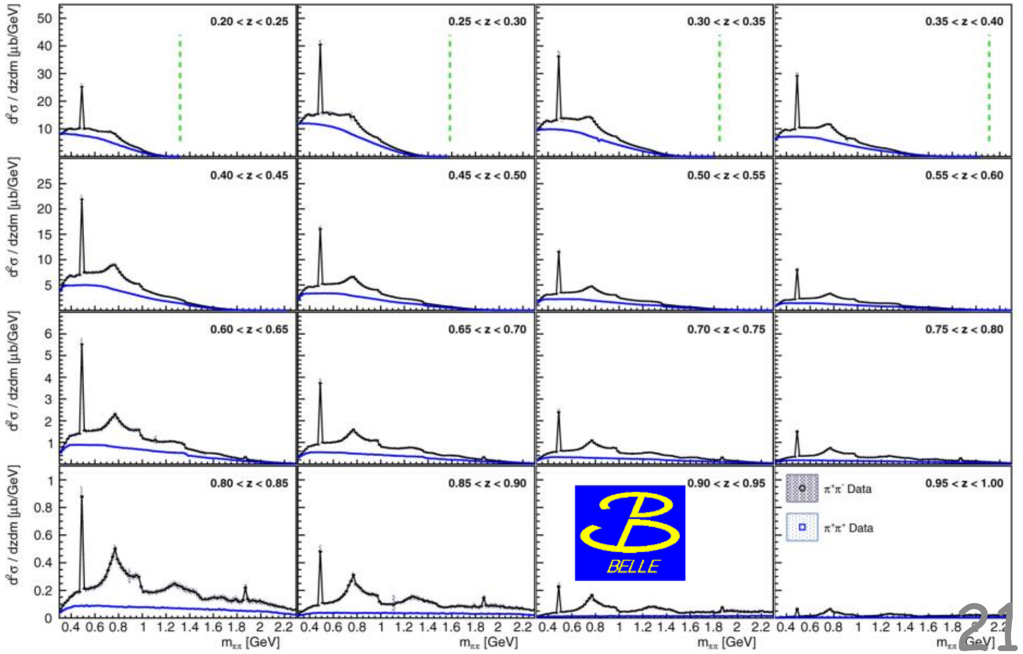
di-hadron FFs

Di-hadron FFs		
$e^+e^- \to (h_1, h_2)X$	$\sum_q e_q^2 D_1^{h_1 h_2/q}(z, M_h)$	for large M_h also $D_1^{h/i}(z)$ contribute
$e^+e^- \to (h_{a1}, h_{a2})\,(h_{b1}, h_{b2})X$	$\sum_q e_q^2 D_1^{h_{a1}h_{a2}/q}(z_a, M_{ha})\, D_1^{h_{b1}h_{b2}/\bar{q}}(z_b, M_{hb}) + \{q \leftrightarrow \bar{q}\}$ $\sum_q e_q^2 H_1^{\sphericalangle\, h_{a1}h_{a2}/q}(z_a, M_{ha})\, H_1^{\sphericalangle\, h_{b1}h_{b2}/\bar{q}}(z_b, M_{hb}) + \{q \leftrightarrow \bar{q}\}$ $\sum_q e_q^2 G_1^{\perp\, h_{a1}h_{a2}/q}(z_a, M_{ha})\, G_1^{\perp\, h_{b1}h_{b2}/\bar{q}}(z_b, M_{hb}) + \{q \leftrightarrow \bar{q}\}$	back-to-back production of di-hadron pair

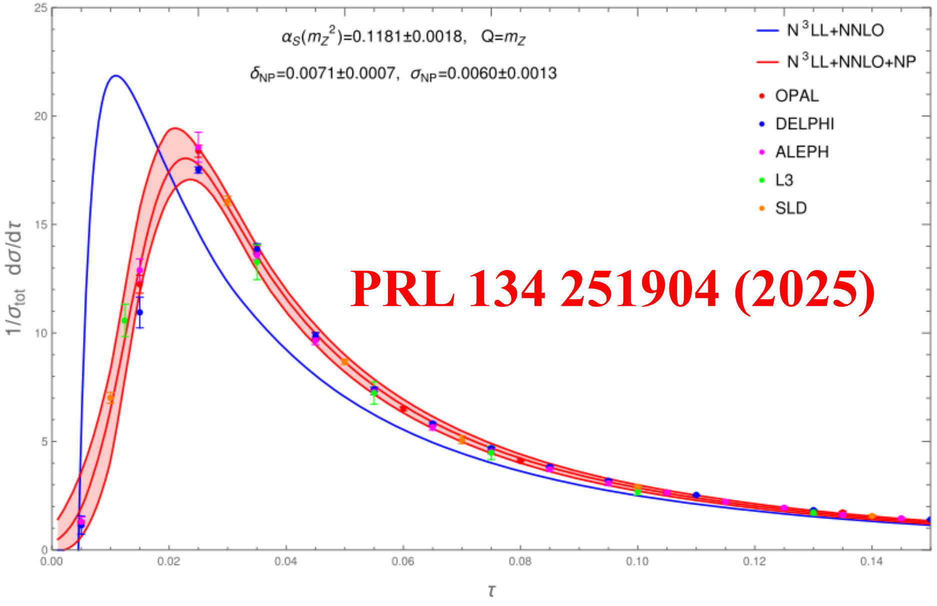
PRD96 032005 (2017)

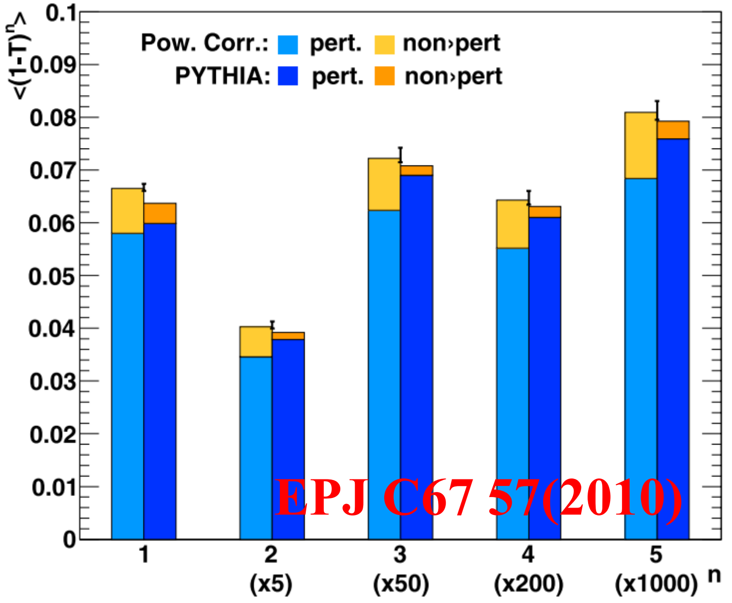


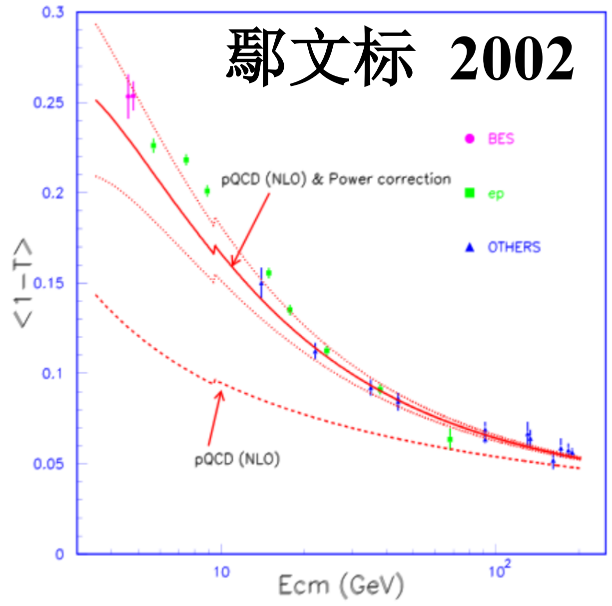
- **Fragmentation of a parton into two hadrons**
  - ✓ **Hadrons emerge from same q or $\bar{q}$**
  - ✓ **Thrust axis or $\theta_{hh} < 60^O$?**
- **Possible results @ BESIII**
  - ✓ **Around 5.0 GeV with thrust axis**
  - ✓ **TMD di-hadron FFs**
  - ✓ **With $\theta_{hh} < 60^O$ method**



Event shape variables: thrust

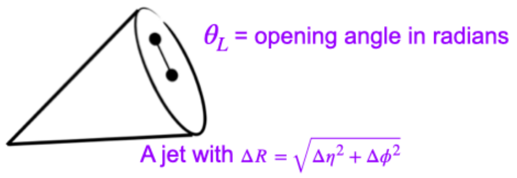






- Event shape explore geometry of hadronic energy-momentum flow
  - ✓ Interplay between perturbative and non- perturbative dynamics
- Thrust: how compatible an event is with two back-to-back objects
- Thrust analysis around 5.0 GeV: important analysis
  - ✓ Shape: experimental data vs. theory prediction
  - ✓ Moment: $\alpha_s(Q)$ running (cross check)

Energy-energy correlator (EEC)

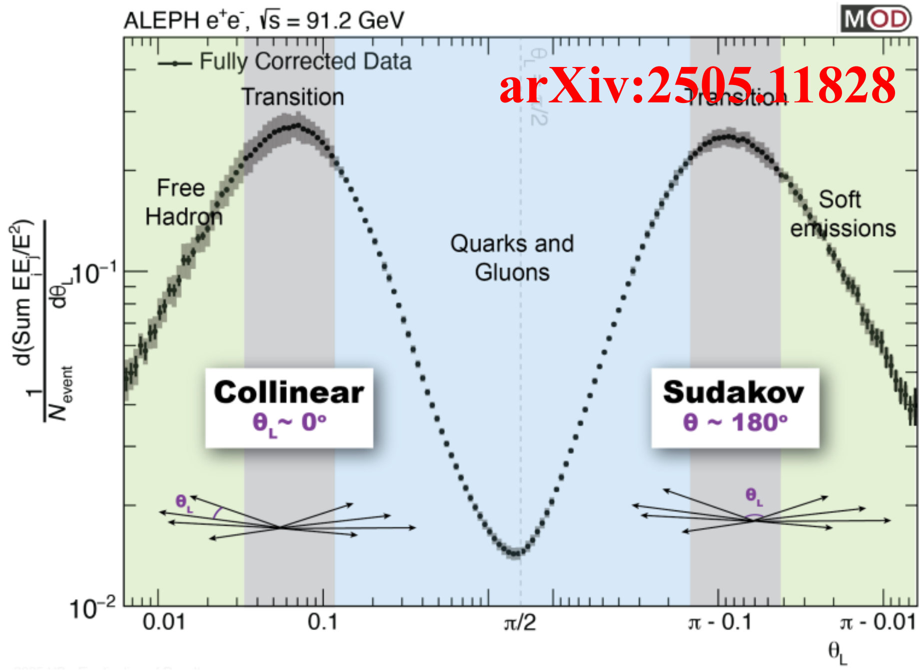


$$\mathrm{E2C}(\theta_\mathrm{L}) = \sum_{\mathrm{i,j}}^{\mathrm{n}} \int d\sigma \frac{E_\mathrm{i} E_\mathrm{j}}{E^2} \delta(\theta_\mathrm{L} - \theta_\mathrm{ij})$$

$$\mathrm{E3C}(\theta_\mathrm{L}) = \sum_{\mathrm{i,j,k}}^{\mathrm{n}} \int d\sigma \frac{E_\mathrm{i} E_\mathrm{j} E_\mathrm{k}}{E^3} \delta(\theta_\mathrm{L} - \max(\theta_\mathrm{ij}, \theta_\mathrm{ik}, \theta_\mathrm{jk}))$$

$$EEC(\theta_L) = \frac{1}{N} \sum_{events}^{N} \frac{1}{\Delta\theta_L} \int_{\theta_L - \Delta\theta_L/2}^{\theta_L - \Delta\theta_L/2} \sum_{ij} \frac{E_i E_j}{E_{beam}^2} \delta(\theta - \theta_{ij}) d\theta$$

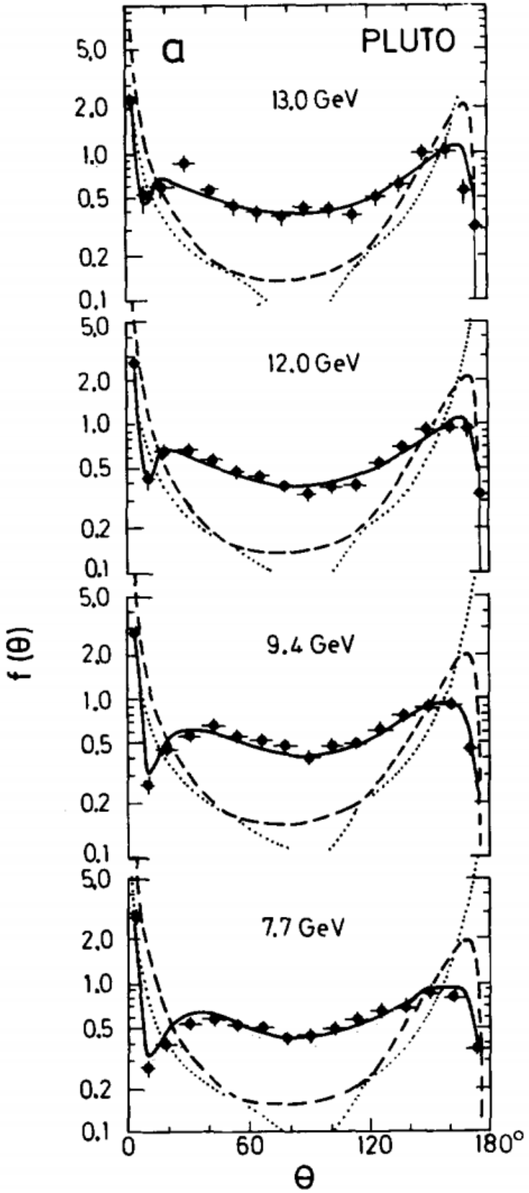
- **EEC describes energy flux in e^+e^- collision**
- **Angular scale resolve and characterize behavior in different QCD regions**
  - ✓ **Small angle: collinear limit of QCD, EEC inside Jets, α_s measurement**
  - ✓ **Large angle: back-to-back, limit, soft recoiling, TMD physics**
  - ✓ **Intermediate angle: fixed-order perturbative expansion**



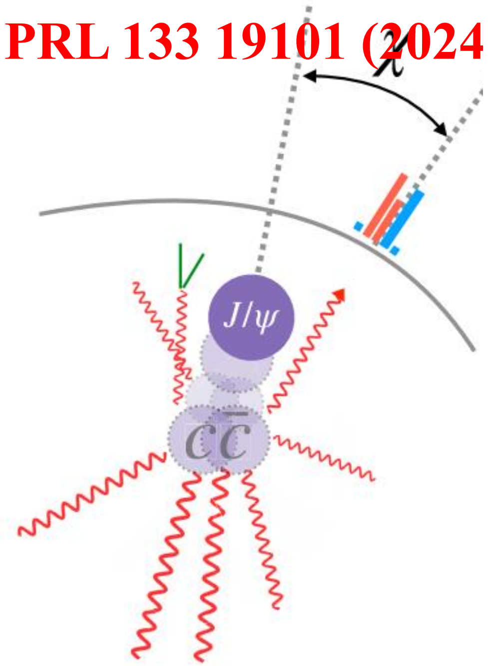
arXiv:2505.11828

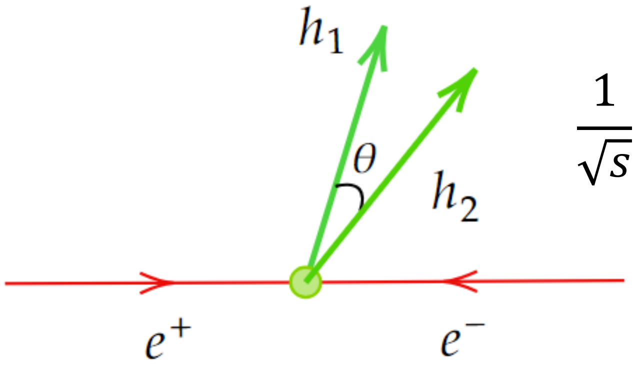
Rich physics @ EEC

Energy-energy correlator (EEC)

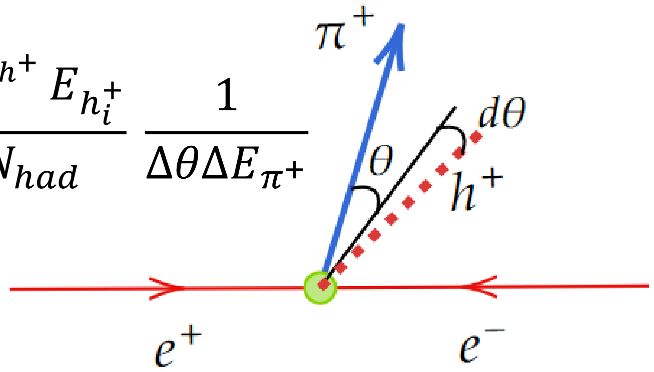


PRL 133 19101 (2024





$$\frac{1}{\sqrt{s}} \frac{\sum_i^{N_{h^+}} E_{h_i^+}}{N_{had}} \frac{1}{\Delta\theta \Delta E_{\pi^+}}$$



$$\frac{1}{\sqrt{s}\cdot\sqrt{s}} \frac{\sum_i^{N_{h_{12}}} E_{h_1} E_{h_2}}{N_{had}} \frac{1}{\Delta\theta}$$

$$\frac{\sum_i^{N_{12}} (z_1 \cdot z_2)}{N_{had}} \frac{1}{\Delta\cos\theta\Delta\phi}$$

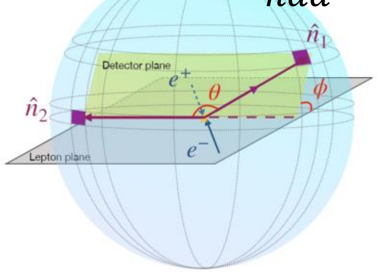


Figure 1: Illustration of EEC for e^+e^- annihilation. Here θ is the angular separation between the two detectors pointing along $\hat{n}_1$ and $\hat{n}_2$ and ϕ is the azimuthal angle between the plane formed by these detectors (referred to as "detector plane", shown as yellow) and the plane generated by the direction $\hat{n}_2$ and the beam (referred to as "lepton plane", shown

- **Energy dependence of EEC**
  - ✓ **Results with $\sqrt{s} < 7$ GeV: not yet**
- **Single inclusive EEC**
- **Azimuthal dependence of energy correlators**
- **Energy correlator around ϕ, mesons ?**
  - ✓ **Strange quark, almost free of resonances decay**

STCF with polarized beam

一个左手极化或右手极化的入射轻子$e^-_{L,R}$与一个无极化的e^+的散射截面分别为：

$$\frac{d\sigma_L}{dt} = \frac{1}{16\pi s^2}\left\{u^2\left|G_{LL}(s)+G_{LL}(t)\right|^2 + t^2\left|G_{LR}(s)\right|^2 + s^2\left|G_{LR}(t)\right|^2\right\},$$

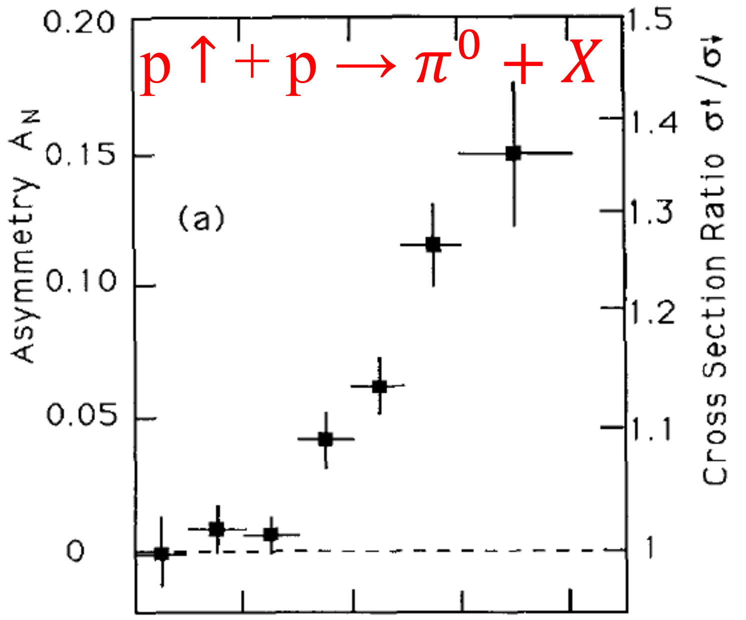
$$\frac{d\sigma_R}{dt} = \frac{1}{16\pi s^2}\left\{u^2\left|G_{RR}(s)+G_{RR}(t)\right|^2 + t^2\left|G_{RL}(s)\right|^2 + s^2\left|G_{RL}(t)\right|^2\right\}.$$

$$A_L = \frac{d\sigma^{\rightarrow} - d\sigma^{\leftarrow}}{d\sigma^{\rightarrow} + d\sigma^{\leftarrow}}$$

- **Longitudinally polarized e⁻ beam @ STCF**
  - ✓ **Effect on fragmentation function ?**
- **Single-spin asymmetry**
  - ✓ **What could we obtain ?**
- **We need theorist's help.**

Large-x_F spin asymmetry in π^0 production by 200-GeV polarized protons*

FNAL E704 Collaboration



Summary

碎裂函数和能量关联研讨会

8-11 August 2025
Asia/Shanghai timezone

- Overview
- Timetable
- Registration
- Participant List

碎裂函数和能量关联研讨会将于2025年08月08日至11日在甘肃省兰州市召开（08月08日报到）。本次研讨会旨在聚焦碎裂函数 (Fragmentation Functions, FFs) 和能量-能量关联 (Energy-Energy Correlations, EEC) 等关键方向，深入梳理当前国内外在该领域的最新研究进展，并就相关前沿问题进行深入交流与探讨。

一个核心目标是展望未来，重点研讨这些物理观测量在未来高能物理实验装置（包括但不限于超级陶粲装置 (STCF)、电子离子对撞机 (EicC) 等）上的研究潜力、挑战与机遇。我们将探讨如何利用这些先进平台深化对强相互作用动力学和夸克胶子等离子体性质的理解。