Maximal Entanglement and Bell Nonlocality at an Electron-Ion Collider

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Wei Qi, Zijing Guo, and BX, • arXiv:2506.12889v1 [hep-ph]



The Einstein-Podolsky-Rosen Paradox

- **The EPR Paper** [Phys. Rev. 47, 777 (1935)]
 - Can Quantum-Mechanical Description of Physical Reality be Considered Complete?
 - Challenge to Copenhagen orthodox interpretation

Quantum Entanglement

- The quintessential phenomenon of QM introduced by Schrödinger in response to the EPR paper.
- Non-local correlations between particles
- Violates local realism assumptions
- Einstein's famous phrase: "God does not play dice"
 - To which Bohr replied: "Einstein, stop telling God what to do"
- The EPR paradox revealed the profound nature of quantum entanglement!



Einstein, Podolsky, and Rosen
"Spooky action at a distance"





Schrödinger, Bohr and Einstein in 1925

ER=EPR Conjecture: Entanglement as Wormhole Geometry

The ER=EPR Conjecture

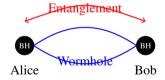
[Maldacena & Susskind, 2013]

Einstein-Rosen Bridge =
Einstein-Podolsky-Rosen Pair
Entanglement ⇔ Wormhole

- EPR correlations create geometric connections
- Wormhole Geometry is holographic manifestation of entanglement
- Non-traversable wormhole no superluminal signaling
- Bridge between QM and GR: unifying general relativity and quantum mechanics into string theory.

Supporting Evidence: Holographic Realization: [Jensen & Karch, 2013]

- EPR pair in AdS_5 space [Xiao, 2008]
- The holographic dual of the EPR pair has two horizons and a string (wormhole) connecting them.



"Entanglement weaves the fabric of spacetime"



Separable vs Entangled States: Two-Qubit Systems

Separable States

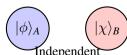
- Can be written as: $|\psi\rangle = |\phi\rangle_A \otimes |\chi\rangle_B$
- No quantum correlations

Examples:

$$|00\rangle = |0\rangle_A \otimes |0\rangle_B, \quad |01\rangle = |0\rangle_A \otimes |1\rangle_B$$

$$|\psi_{\text{sep}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_A \otimes |0\rangle_B$$

$$= \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$



Entangled States

- Cannot be written as product
- Genuine quantum correlations

Bell States (Maximally Entangled):

$$|\Phi^{+}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$|\Phi^{-}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Psi^{+}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Psi^{-}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$





Bell's Theorem

Quantum Indeterminacy

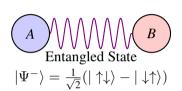
- **Realism:** Quantum indeterminacy reflects our ignorance of hidden variables; outcomes are determined but unknown.
- **Copenhagen:** Indeterminacy is fundamental; outcomes are truly probabilistic until measured.
- Agnosticism: The reality behind quantum events is unknowable; only predictive power of the theory matters.

■ **Bell Nonlocality** [Bell, 1964]

- Bell inequality: It makes an observable difference for Realism vs Copenhagen, and eliminates Agnostic view.
- Decisive evidence supporting QM (Copenhagen).

■ **CHSH Inequality** [Clauser et al., 1969]

- Generalized Bell inequality
- Foundation for quantum information theory



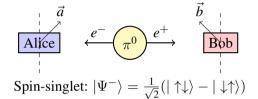


John Stewart Bell



EPRB Experiment: Testing Bell Nonlocality

Einstein-Podolsky-Rosen-Bohm Experiment



Correlation:
$$E(\vec{a}, \vec{b}) = \langle A(\vec{a}) \cdot B(\vec{b}) \rangle$$

Bell CHSH Inequality:

$$\mathbb{B}_{HT} = |E(a,b) - E(a,b') + E(a',b) + E(a',b')| \le 2$$

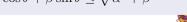
$$\mathbb{B}_{QM} = |\cos \theta_{ab} - \cos \theta_{ab'} + \cos \theta_{a'b} + \cos \theta_{a'b'}| \le 2\sqrt{2}$$

Local Hidden Variable Theory

- Pre-existing density $P(\lambda)$ for λ
- $A(\vec{a}, \lambda) = \pm 1$ predetermined
- $\blacksquare E(\vec{a}, \vec{b}) = \int P(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) d\lambda$
- Local realism: $\mathbb{B}_{HT} \leq 2$

Quantum Mechanics

- No predetermined values
- $E(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} = -\cos \theta_{ab}$
- Nonlocality: $2 < \mathbb{B}_{QM} \le 2\sqrt{2}$ Elementary proof with: $\alpha \cos \theta + \beta \sin \theta < \sqrt{\alpha^2 + \beta^2}$





QM violates Bell inequality \Rightarrow Nature is nonlocal!

Time Reversal Operation and Kramers Degeneracy

Time Reversal for Spin-1/2

$$\mathcal{T}=-i\sigma_{\mathrm{y}}\mathcal{K}$$

$$\mathcal{T}|\uparrow\rangle = |\downarrow\rangle$$
 $\mathcal{T}|\downarrow\rangle = -|\uparrow\rangle$

lacktriangleright K is complex conjugation

$$\tilde{\chi} = \mathcal{T} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -\beta^* \\ \alpha^* \end{pmatrix}$$

- $\langle \tilde{\chi} | \chi \rangle = 0$
- $[\mathcal{T}, H] = 0$ (if time-reversal invariant)
- anti-unitary: $\langle \mathcal{T}\psi | \mathcal{T}\phi \rangle = \langle \psi | \phi \rangle^*$

Kramers Degeneracy

For half-integer spin systems with time-reversal symmetry:

Every energy level is at least doubly degenerate

Proof Sketch:

- Then $H\mathcal{T}|\psi\rangle = E\mathcal{T}|\psi\rangle$
- But $\langle \psi | \mathcal{T} \psi \rangle = 0$ (since $\mathcal{T}^2 = -1$)
- So $|\psi\rangle$ and $\mathcal{T}|\psi\rangle$ are orthogonal
- \blacksquare \Rightarrow At least 2-fold degeneracy





Concurrence: Measuring the Degree of Entanglement (Pure States)

Time Reversal Operation flips spins:

- $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
- [Wootters, 98] flip spins with $\hat{T} = -i\sigma^y \hat{K}$ (Anti-Unitary)
- $|\tilde{\psi}\rangle = \delta^*|00\rangle \gamma^*|01\rangle \beta^*|10\rangle + \alpha^*|11\rangle$, the spin-flipped complex conjugate.
- $C(|\psi\rangle) = 2|\alpha\delta \beta\gamma|$ measures overlap with time-reversed state.
- $\mathcal{C} = 0$: Separable (no entanglement)
- \bullet 0 < \mathcal{C} < 1: Partially entangled
- $\mathbf{C} = 1$: Maximally entangled

C = invariance under time reversal

Separable State

$$\begin{split} |\psi_1\rangle &= |00\rangle \text{ and } \alpha = 1, \beta = \gamma = \delta = 0 \\ \mathcal{C} &= 2|1\cdot 0 - 0\cdot 0| = 0 \end{split}$$

Bell State (Maximally Entangled)

$$\begin{split} |\Phi^{+}\rangle &= \tfrac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\ \mathcal{C} &= 2|\tfrac{1}{2} - 0| = 1 \end{split}$$

Partially Entangled

$$|\psi_2\rangle = \frac{1}{\sqrt{3}}|00\rangle + \sqrt{\frac{2}{3}}|11\rangle$$

$$C = 2|\frac{1}{\sqrt{3}} \cdot \sqrt{\frac{2}{3}}| = \frac{2\sqrt{2}}{3} \approx 0.94$$



Spin Density Matrix for Spin-1/2 Particles

Density Matrix Formalism

For a spin-1/2 particle, the density matrix is:

$$\rho = \frac{\mathbb{1}_2 + \vec{n} \cdot \vec{\sigma}}{2}$$

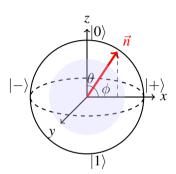
Bloch Vector: $n_i = \langle \sigma_i \rangle = \text{Tr}(\rho \sigma_i)$

- $|\vec{n}| = 1$: Pure state
- $|\vec{n}| < 1$: Mixed state
- $\vec{n} = 0$: Maximally mixed state

For a heavy quark:

- **Production mechanism:** QCD processes determine initial Bloch vectors
- Experimental access: Weak decay measures spin projections $\langle \vec{n} \cdot \vec{\sigma} \rangle$

Bloch Sphere Representation



Geometry encodes quantum information

- $\vec{n} = (0, 0, 1): \rho = |0\rangle\langle 0|$
- $\vec{n} = (1,0,0): \rho = |+\rangle\langle +|$
- $\vec{n} = (0,0,0)$: $\rho = \frac{1}{2} \mathbb{1}_2$ (classical)

Density Matrix and Concurrence for Two-Qubit Systems

Extending to mixed states

Density Matrix Representation:

- Pure state: $\rho = |\psi\rangle\langle\psi|$
- Mixed state: $\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|$
- General form in computational basis:

$$\rho = \begin{pmatrix} \rho_{00,00} & \rho_{00,01} & \rho_{00,10} & \rho_{00,11} \\ \rho_{01,00} & \rho_{01,01} & \rho_{01,10} & \rho_{01,11} \\ \rho_{10,00} & \rho_{10,01} & \rho_{10,10} & \rho_{10,11} \\ \rho_{11,00} & \rho_{11,01} & \rho_{11,10} & \rho_{11,11} \end{pmatrix}$$

Properties:

- Hermitian: $\rho^{\dagger} = \rho$
- Trace $Tr(\rho) = 1$: Non-negative.

Concurrence in general:

[Hill, Wootters, 97; Wootters, 98]

- Define: $\tilde{\rho} = (\sigma_{v} \otimes \sigma_{v}) \rho^{*} (\sigma_{v} \otimes \sigma_{v})$
- Compute: $\mathcal{R} = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$
- Eigenvalues of \mathcal{R} : $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ (descending order)

$$\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

Example - Werner State:
$$\rho_W = p|\Psi^-\rangle\langle\Psi^-| + \frac{1-p}{4}\mathbb{1}_4$$

- p = 1: Pure Bell state
- $\mathcal{C}(\rho_W) = \max\{0, \frac{3p-1}{2}\}\$
- Entangled when p > 1/3



Spin Density Matrix: Physical Interpretation

The most general two-qubit density matrix:

$$\rho = \frac{1}{4} \left(\mathbb{1}_4 + B_i^+ \sigma^i \otimes \mathbb{1}_2 + B_j^- \mathbb{1}_2 \otimes \sigma^j + C_{ij} \sigma^i \otimes \sigma^j \right)$$

Physical Quantities:

$$\blacksquare B_i^+ = \operatorname{Tr} \rho(\sigma_i \otimes \mathbb{1}_2)$$

$$\blacksquare B_i^- = \operatorname{Tr} \rho(\mathbb{1}_2 \otimes \sigma_j)$$

•
$$C_{ij} = \operatorname{Tr} \rho(\sigma_i \otimes \sigma_j)$$

Spin correlation /NB: Not [C]

Special Case:

For Bell states:
$$B_i^+ = B_j^- = 0$$
 (No individual spin polarization)

Dall Ctates & Carrelation Matrices

Bell States & Correlation Matrices:	
State	Correlation Matrix
$ \Psi^{-}\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\rangle - \downarrow\uparrow\rangle)$	$C_{ij} = \operatorname{diag}(-1, -1, -1)$
$ \Psi^{+}\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\rangle + \downarrow\uparrow\rangle)$	$C_{ij} = \operatorname{diag}(1, 1, -1)$
$ \Phi^{+}\rangle = \frac{1}{\sqrt{2}}(\uparrow\uparrow\rangle + \downarrow\downarrow\rangle)$	$C_{ij} = \operatorname{diag}(1, -1, 1)$
$ \Phi^{-}\rangle = \frac{1}{\sqrt{2}}(\uparrow\uparrow\rangle - \downarrow\downarrow\rangle)$	$C_{ij} = \operatorname{diag}(-1, 1, 1)$

For singlet state:

 $C_{ii} = -\delta_{ii}$ means spins are always anti-parallel.

Correlation matrix C_{ii} fully characterizes entanglement structure for Bell states



Entanglement and Bell Nonlocality Conditions

Starting from the spin density matrix:
$$\rho_{\alpha\alpha',\beta\beta'}=rac{1}{4}\left(\mathbb{1}_{\alpha\alpha',\beta\beta'}+C_{ii}\sigma^i_{\alpha\beta}\otimes\sigma^i_{\alpha'\beta'}\right)$$

- Anti-correlated spins: C_{xx} , C_{yy} , C_{zz} < 0
- Def: $D = (C_{xx} + C_{yy} + C_{zz})/3 = \text{tr}C/3$
- D = -1: Perfect anti-correlation

Four eigen values of $\mathcal{R}=\rho$ (since $\tilde{\rho}=\rho$)

$$\lambda_1 = \frac{1}{4}(1 - C_{xx} - C_{yy} - C_{zz}),$$

$$\lambda_2 = \frac{1}{4}(1 + C_{xx} + C_{yy} - C_{zz}),$$

$$\lambda_3 = \frac{1}{4}(1 + C_{xx} - C_{yy} + C_{zz}),$$

$$\lambda_4 = \frac{1}{4}(1 - C_{xx} + C_{yy} + C_{zz}).$$

Entanglement Condition

Concurrence $C[\rho] = \frac{1}{2}(-3D-1) > 0$:

$$D<-rac{1}{3}$$

Bell Nonlocality Condition

For CHSH violation $\mathbb{B} > 2$: [Horodecki, et al, 95]

$$D < -\frac{1}{\sqrt{2}} \approx -0.707$$



Hierarchy: Bell Nonlocality ⊂ Entanglement ⊂ All Quantum States

Top Quark Weak Decay and Spin Transfer

Top Quark Decay: Choose its rest frame

$$t \to W^+ b \to \ell^+ \nu_\ell b, \, \bar{t} \to W^- \bar{b} \to \ell^- \bar{\nu}_\ell \bar{b}$$

Decay Spin Density Matrix:

$$\Gamma_{\pm} = \frac{\mathbb{1}_2 + \kappa_{\pm} \vec{\sigma}_t \cdot \hat{l}_{\pm}}{2}$$

Parity Violating Angular Distribution:

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \kappa_{\pm}\cos\theta$$

- Weak decay (parity violation) provides Spin-momentum correlation
- $\kappa_{\pm} = \pm 1$ ($t\bar{t}$) spin analyzing power
- $\sigma_{l_+l_-} \propto \operatorname{tr}[\Gamma_+ \otimes \Gamma_- \rho] \operatorname{NB} \operatorname{tr}[\sigma^i \sigma^j] = 2\delta^{ij}$

Correlation between di-leptons

$$\frac{d^2\sigma}{\sigma d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2} \left[1 - \hat{l}_+ \cdot C \cdot \hat{l}_- \right]$$

Entanglement Signature

$$\cos \varphi \rangle = -\frac{1}{3}D = -\frac{1}{9}\text{Tr}(C)$$

Experimental Reach:

- Extract D = Tr(C)/3 parameter directly
- Quantum Tomography: all elements of ρ can be measured. [Bernreuther, Heisler, Si, 15; ATLAS, 1612.07004; CMS, 1907.03729]

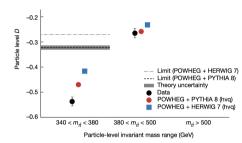
First Observation of Quark Entanglement at the LHC

[ATLAS (Nature 2024):] First observation of entanglement in quarks at the highest-energy. **Entanglement Measure:**

$$D = \operatorname{tr}[C]/3 = -3\langle \cos \phi \rangle$$

where ϕ is the angle between charged leptons in their parent top/antitop rest frames **Key Features:**

- Spin transferred to decay products
- Measured near $t\bar{t}$ threshold
- From atomic physics to high-energy collisions: A new frontier!
- **CMS, STAR, BES-III** more to come.



 $\sqrt{s} = 13 \text{ TeV}, 140 \text{ fb}^{-1} \text{ data } (2015-2018)$

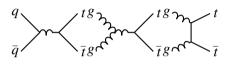
Measured:
$$D < -1/3$$
 (Entanglement criterion) $D = -0.547 \pm 0.002$ (stat.) ± 0.021 (syst.)

- Observed: $> 5\sigma$ from no entanglement
- Yet, Bell Nonlocality: $D < -1/\sqrt{2}$

Theory vs Experiment: Top Quark Entanglement

Quantum State Tomography $\rho_{\alpha\alpha',\beta\beta'}=R_{\alpha\alpha',\beta\beta'}/\text{tr}R$ [Afik, de Nova, 2022]

Top quark pair production



$$R_{lphalpha',etaeta'}=rac{1}{N}\sum \mathcal{M}^*_{t_lphaar{t}_{lpha'}}\mathcal{M}_{t_etaar{t}_{eta'}}$$

- Measured $D \approx -0.54$ near threshold
- Gluon fusion dominance at LHC
- Angular momentum conservation determines spin correlations
- Statistical mixture of $q\bar{q}$ and gg

Near Threshold ($\beta \rightarrow 0$):

- $q\bar{q}$: Separable state (C = 0), since $t\bar{t}$ spin (± 1) is equally mixed along beam.
- gg: Maximally entangled singlet Ψ^-

High Energy ($\beta \rightarrow 1$) with $\theta = \pi/2$:

■ Both channels: Maximally entangled triplet Ψ^+ along \hat{n} with nonzero OAM.

Mixed State at LHC

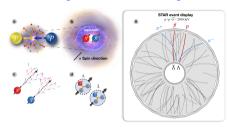
$$\rho = w_{q\bar{q}}\rho_{q\bar{q}} + w_{gg}\rho_{gg}$$



"Observation of Entanglement but not Bell Nonlocality due to Quark channel mixture"

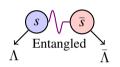
First evidence of spin correlation in $\Lambda \bar{\Lambda}$ hyperon pairs

STAR Collaboration [arXiv:2506.05499] with data from p + p collisions at $\sqrt{s} = 200$ GeV



- Relative polarization (same as *D*): $P_{\Lambda\bar{\Lambda}} = (18 \pm 4)\%$
- Parallel: 1/3; Antiparallel: -1; no spin correlation 0.
- Short-range pairs show maximal entanglement
- Long-range pairs: correlation vanishes (decoherence)
- Evidence for quantum entanglement in QCD vacuum

Entanglement as a Tool



- **QCD Confinement**
- Chiral Symmetry
- **Spin Dynamics**
- Decoherence
- Bell Nonlocality
- Nuclear Medium !?



Λ Hyperon: Nature's Built-in Spin Analyzer

The Hyperon Decays

$$\Lambda \to p + \pi^ \bar{\Lambda} \to \bar{p} + \pi^+$$

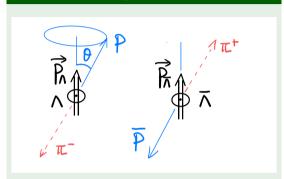
Anisotropic Angular Distribution:

$$\frac{dN_{\Lambda}}{N_{\Lambda}d\cos\theta} = \frac{1}{2} \left(1 + \alpha_{\Lambda} \vec{P}_{\Lambda} \cdot \hat{p}_{p} \right)$$

$$\frac{dN_{\bar{\Lambda}}}{N_{\bar{\Lambda}}d\cos\theta} = \frac{1}{2} \left(1 + \alpha_{\bar{\Lambda}} \vec{P}_{\bar{\Lambda}} \cdot \hat{p}_{\bar{p}} \right)$$

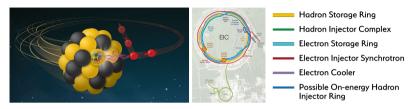
- Asymmetry parameter $\alpha_{\Lambda} \simeq -\alpha_{\bar{\Lambda}} = 0.75$
- Proton predominantly is going off in the direction of the spin of the Lambda.

Self-Analyzing Property



- Weak decay violates parity.
- lacktriangle Proton direction reveals Λ spin direction

EIC Status Update



Aiming to the start of operation in 2031, EIC has reached several milestones:

- Five stages of project Critical Decision approvals:
 - I CD-0 Approve Mission Need

 January 9, 2020: EIC CD-0 and site selection

 Link
 - 2 CD-1 Approve Alternative Selection and Cost Range

 June 29, 2021: EIC CD1 and start of project execution

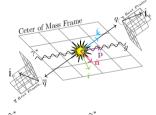
 Link
 - 3 CD-2 Approve Performance Baseline
 - 4 CD-3 Approve Start of Construction
 - 5 CD-4 Approve Start of Operations or Project Completion
- RHIC \rightarrow eRHIC; Energy: 20 \rightarrow 141 GeV; Luminosity: 10^{34} cm⁻²/s; Polarized electron and hadron beams

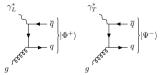


Quark Pair Production in Photon-Gluon Fusion: Longitudinal case

[Qi, Guo, Xiao] • arXiv:2506.12889v1 [hep-ph] **Photon-Gluon Fusion Process** $\gamma_{\lambda=+0}^* + g \rightarrow q + \bar{q}$

$$ho_L = rac{1}{4} \left(\mathbb{1}_4 + C_{ij} \sigma^i \otimes \sigma^j
ight)$$





For $q\bar{q}$ with $\beta \to 0$ and $\theta = \frac{\pi}{2}$

Longitudinal photons contribution:

$$C_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\chi_1 & -\chi_2 \\ 0 & -\chi_2 & \chi_1 \end{pmatrix}$$

with
$$\chi_1$$

$$\chi_1 = \frac{1 - 2z^2 + z^2 \beta^2}{1 - z^2 \beta^2}, \quad \chi_2 = \sqrt{1 - \chi_1^2}.$$

 $lackbox{}{\hspace{0.1cm}}$ ho_L is given by a pure state $= |\Psi\rangle\,\langle\Psi|,$ with

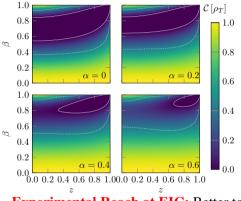
$$|\Psi\rangle = \frac{1}{2}(\sqrt{1+\chi_1},i\sqrt{1-\chi_1},i\sqrt{1-\chi_1},\sqrt{1+\chi_1}).$$

- Near Threshold ($\beta \to 0$): $|\Phi^+\rangle$.
- **High Energy** $(\beta \to 1)$: $|\Phi^+\rangle$.
- $q\bar{q}$ has spin 1 with nonzero OAM and $\mathcal{C}[\rho_L] \equiv 1!$

Always Maximally Entangled! Very Special!

Quark Pair Production in Photon-Gluon Fusion: Transverse case

[Qi, Guo, Xiao] • arXiv:2506.12889v1 [hep-ph] **Transverse photons**: similar to $gg \rightarrow q\bar{q}$ channel.



- Density plots of the concurrence for transverse photon at EIC as functions of β and $z = \cos \theta$ at given $\alpha \equiv Q^2/\hat{s}$.
- Solid lines (boundaries for entanglement $(C[\rho_T] = 0)$) and dashed lines (boundaries for for Bell nonlocality $(\mathcal{N}[\rho_T] = 0)$.
- Near Threshold ($\beta \to 0$): Maximally entangled singlet Ψ^-
- High Energy ($\beta \to 1$) with $\theta = \pi/2$: Maximally entangled triplet Φ^- .

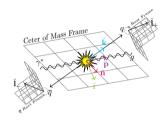
Experimental Reach at EIC: Better to have *LT* separation!

- Low background and Maximal signal at EIC (including ultra-peripheral collisions).
- Possible measurements: $b\bar{b}$ or $c\bar{c}$ or hyperon $\Lambda\bar{\Lambda}$.



Summary and Outlook





- Entanglement and Bell Nonlocality are measurable at high energy collisions.
- EIC offers a unique and clean experimental environment for measuring entanglement and Bell Nonlocality.
- Using entanglement as a tool to probe **nuclear environment** and other QCD effects.
- New opportunities to explore the interplay of quantum information phenomena and high energy and hadronic physics in the years to come.