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Two-meson and meson-axion production from tau lepton

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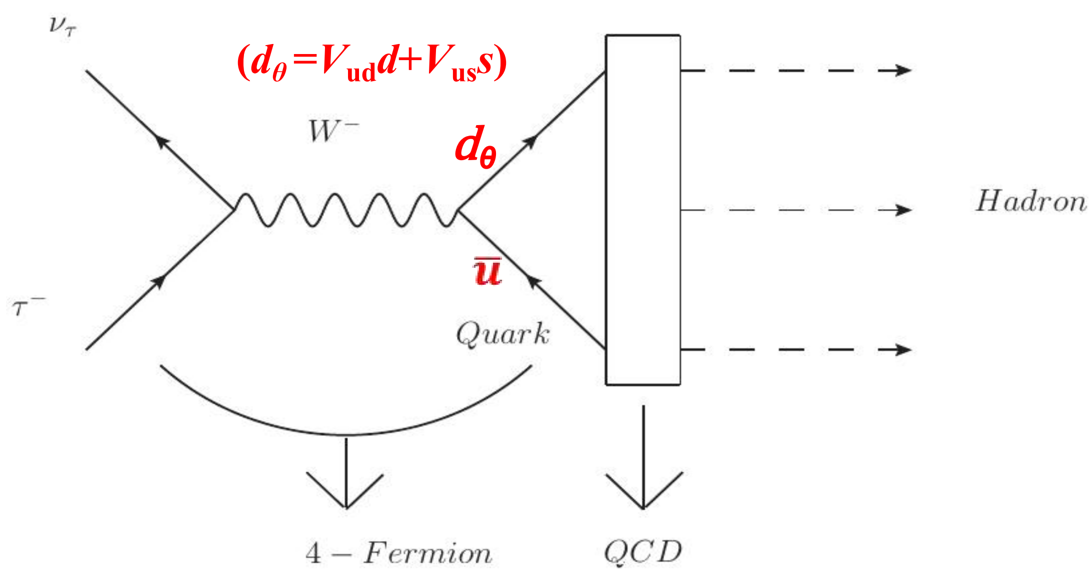
Hadronic tau decays: a unique feature for tau lepton



Hadronic V-A currents

$$\mathbf{H}_\mu = \langle H^- | \bar{u}\gamma_\mu (1-\gamma_5) d_\theta \, e^{iL_{QCD}} | 0 \rangle$$

Chiral EFT is the low energy realization of QCD:

$$e^{i\mathcal{Z}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}G_\mu \; e^{i\int d^4x \mathcal{L}_{QCD}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}u \; e^{i\int d^4x \mathcal{L}_{EFT}(v_\mu, a_\mu, s, p)}$$

$$\mathcal{L}^{QCD} = \mathcal{L}_0^{QCD} + \bar{q}\gamma^\mu (v_\mu + a_\mu \gamma_5) q - \bar{q}(s - i\gamma_5 p) q$$

v_μ, a_μ, s, p are the external source fields .

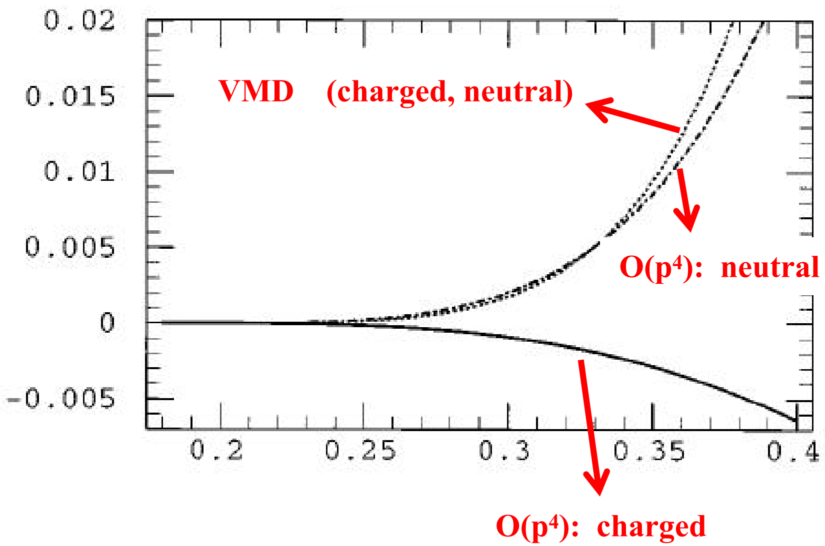
Chiral symmetry is RELEVANT to tau decays

Example: $\tau \to \nu_\tau \pi\pi\pi$ transition amplitudes in the low energy region
VMD models do not automatically respect chiral symmetry.

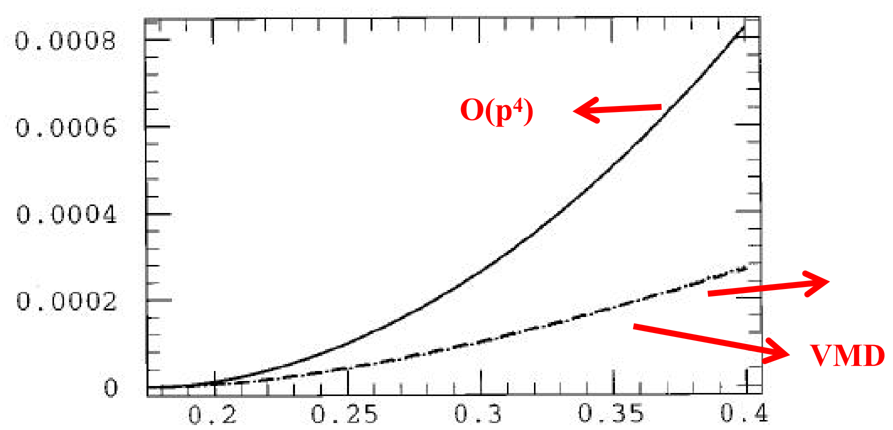
$$J_\alpha = -i\frac{2\sqrt{2}}{3f_\pi}\mathrm{BW}_a(Q^2)(B_\rho(s_2)V_{1\alpha} + B_\rho(s_1)V_{2\alpha})$$

[Kuhn,Santamaria, ZPC'90]

W_D structure function



W_{SA} structure function (neutral channel)



[Colangelo, et al., PRD'96]

- **Resonance chiral theory** **implements the constraint of chiral symmetry from the very beginning in the construction of the Lagrangians.**

Resonance chiral theory (RχT)

Chiral group: $G = SU(3)_L \times SU(3)_R$, $H = SU(3)_V$, $u(\phi) = G/H$

Resonances : $R \stackrel{G}{\Longrightarrow} h\,R\,h^\dagger, \quad h \in H$

pNGB and external sources : $X = u_\mu,\ \chi_\pm,\ f_\pm^{\mu\nu},\ h_{\mu\nu}$

Operators	P	C	h.c.	chiral order
u	$u^\dagger$	u^T	$u^\dagger$	1
Γ_μ	Γ^μ	$-{\Gamma_\mu}^T$	$-\Gamma_\mu$	p
u_μ	$-u^\mu$	u_μ^T	u_μ	p
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$	$\pm\chi_\pm$	p^2
$f_{\mu\nu\,\pm}$	$\pm f_\pm^{\mu\nu}$	$\mp f_{\mu\nu\,\pm}^T$	$f_{\mu\nu\,\pm}$	p^2
$h_{\mu\nu}$	$-h^{\mu\nu}$	$h_{\mu\nu}^T$	$h_{\mu\nu}$	p^2

Operators	P	C	h.c.
$V_{\mu\nu}$	$V^{\mu\nu}$	$-V_{\mu\nu}^T$	$V_{\mu\nu}$
$A_{\mu\nu}$	$-A^{\mu\nu}$	$A_{\mu\nu}^T$	$A_{\mu\nu}$
S	S	S^T	S
P	$-P$	P^T	P

Minimal RχT Lagrangian [Ecker, *et al.*, '89]

$$
\begin{aligned}
\mathcal{L}_{2V} &= \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle\,,\\
\mathcal{L}_{2A} &= \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle\,,\\
\mathcal{L}_{2S} &= c_d\langle\, S u_\mu u^\mu\,\rangle + c_m\langle\, S\chi_+\,\rangle\,,\\
\mathcal{L}_{2P} &= id_m\langle\, P\chi_-\,\rangle.
\end{aligned}
$$

Operators beyond minimal
[Cirigliano, *et al.*, '04]:

$$\mathcal{L}_{VAP} = \lambda_1^{VA}\langle [V^{\mu\nu}, A_{\mu\nu}]\chi_-\rangle + \ldots\,,$$

[Ruiz-Femenia, Pich and Portolés, '03]

$$
\begin{aligned}
\mathcal{L}_{VVP} &= d_1\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \ldots\,,\\
\mathcal{L}_{VJP} &= \frac{c_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \ldots\,,
\end{aligned}
$$

$$d_4\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\}u_\alpha\rangle$$

QCD dynamics in RχT

- Low energy QCD: implemented from the construction of RχT
- Intermediate energy: explicit resonance states
- High energy information: to match the same physical objects in RχT and QCD, $\langle\, J(x_n) \cdot\cdot J(0)\,\rangle^{\mathrm{R}\chi\mathrm{T}} = \langle\, J(x_n) \cdot\cdot J(0)\,\rangle^{\mathrm{QCD}}$.

  For example: $\pi\pi$ vector form factor

$$
\begin{aligned}
\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{R}\chi\mathrm{T}} &= 1 + \frac{F_V G_V}{F^2}\frac{q^2}{M_V^2 - q^2}\,, \\
\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{QCD}} &\to 0, \qquad \mathrm{for}\ q^2 \to \infty
\end{aligned}
$$

  This leads to

$$
[\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{R}\chi\mathrm{T}} = [\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{QCD}} \Longrightarrow F_V G_V = F^2
$$

Quick glance at axion/axion-like particle (ALP)

$$\mathcal{L}_{\rm QCD} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \theta\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- **Strong CP problem: $\theta < 10^{-10}$, constrained by neutron EDM.**
- **Peccei-Quinn mechanism: θ promoted as a dynamical field, i.e., axion, corresponding to a pseudo-Nambu-Goldstone boson from the spontaneous breaking of the $U(1)_{PQ}$ symmetry** [Peccei,Quinn,PRL'77] [Weinberg,PRL'78] [Wilzeck,PRL'78]
- **Axion: intensively and broadly studied in particle physics**
- **Model-independent axion interaction:**

  $\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$, **inevitably introducing the axion-hadron interactions !**

- **Axion-like particle (ALP): non-vanishing bare mass $m_{a,0}$**

$$\frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- **Previous studies of ALP in tau decays focused on the axion-lepton interacion.** **Model-independent ALP interaction has not been studied in the contex of tau decays. Our work fills this gap.**

Unified study of $\tau \to P_1 P_2 (P+axion)\, \nu_\tau$

[Hao, Duan, ZHG, 2507.00383, to appear in FOP]

$$\langle P_1 P_2 | \bar{D}\gamma^\mu u | 0\rangle = \left[(p_2 - p_1)^\mu - \frac{\Delta_{P_2 P_1}}{s} q^\mu\right] F_+^{P_1 P_2}(s) + \frac{\Delta_{Du}}{s} q^\mu \widehat{F}_0^{P_1 P_2}(s)$$

(D=d or s) (vector FF) (scalar FF)

$$\Delta_{P_2 P_1} = m_{P_2}^2 - m_{P_1}^2\,, \qquad \Delta_{Du} = B_0(m_D - m_u)\,, \qquad q_\mu = (p_1 + p_2)_\mu, \qquad s = q^2.$$

- **Invariant-mass distribution of P_1P_2**

$$\frac{d\Gamma_{\tau\to P_1 P_2 \nu_\tau}}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{48\pi^3 s} S_{\rm EW}\, |V_{uD}|^2 \left(1 - \frac{s}{M_\tau^2}\right)\left\{\left(1 + \frac{2s}{M_\tau^2}\right) q_{P_1P_2}^3(s) \left|F_+^{P_1P_2}(s)\right|^2 + \frac{3\Delta_{Du}^2}{4s} q_{P_1P_2}(s) \left|\widehat{F}_0^{P_1P_2}(s)\right|^2\right\}$$

- **Forward-Backward (FB) asymmetry distribution**

$$A_{FB}(s) = \frac{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha} - \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha}}{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha} + \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha}} = \frac{\Delta_{Du}\, q_{P_1P_2}(s)\, \Re\left[F_+^{P_1P_2}(s)\, \widehat{F}_0^{P_1P_2*}(s)\right]}{\frac{2\sqrt{s}}{3}\left(1 + \frac{2s}{M_\tau^2}\right) q_{P_1P_2}^2(s) \left|F_+^{P_1P_2}(s)\right|^2 + \frac{\Delta_{Du}^2}{2\sqrt{s}} \left|\widehat{F}_0^{P_1P_2}(s)\right|^2}$$

α: **angle between the momenta of P_1 and τ in the P_1P_2 rest frame**

$\tau \to \pi^- \pi^0 \nu_\tau$: $\Delta_{PP'} \to 0$ (Isospin breaking term), scalar F_0 negligible, vector F_+ dominant!

$\tau \to K\pi\nu_\tau$: $\Delta_{PP'} \neq 0$, both scalar F_0 and vector F_+ are relevant!

Calculation of P_1P_2 / $P\,a$ form factors

Generic effective axion Lagrangian for light-flavor quarks

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

(1) Remove the $aG\tilde{G}$ term via the quark axial transformation: $q \to e^{i\frac{a}{2f_a}\gamma_5 Q_a} q$

(2) Alternatively, one can also explicitly keep the $aG\tilde{G}$ term and match it to Chiral EFT

<u>Reminiscent</u>: QCD $U(1)_A$ anomaly that is caused by topological charge density

$\omega(x) = \alpha_s G_{\mu\nu}\tilde{G}^{\mu\nu}/(8\pi)$ **, is responsible for the massive singlet η_0.**

Axion could be similarly included as the η_0 mass via the U(3) χPT:

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle + \frac{F^2}{12}M_0^2 X^2$$

$$U = u^2 = e^{i\frac{\sqrt{2}\Phi}{F}}\,, \qquad \chi = 2B(s+ip)\,, \qquad \chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

$$u_\mu = i u^\dagger D_\mu U u^\dagger\,, \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \qquad X = \log(\det U) - i\frac{a}{f_a}$$

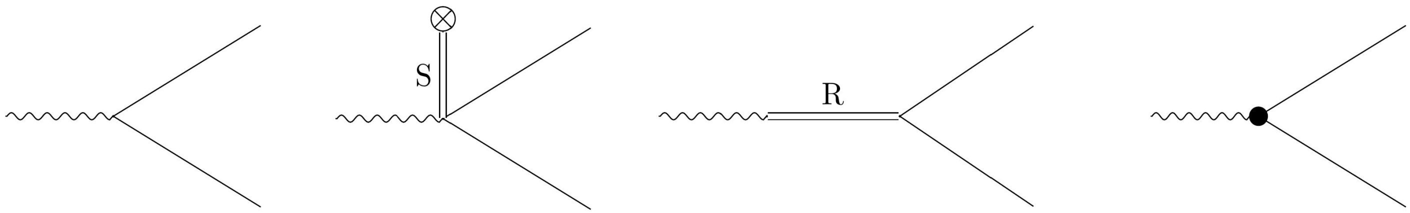
- **Q_a is not needed in U(3) χPT.**
- **δ expansion scheme: $\delta \sim O(p^2) \sim O(m_q) \sim O(1/N_c)$; $M_0^2 \sim O(1/N_c)$.**
- **Axion interactions enter via the axion-meson mixing terms at LO.**

Calculation of P_1P_2 / $P\,a$ form factors

Relevant Resonance Chiral Lagrangians

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{12}M_0^2X^2 \qquad \mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu}u^\mu u^\nu\rangle\,,$$

$$\mathcal{L}_\Lambda^{\rm NLO,U(3)} = -\frac{F^2\,\Lambda_1}{12}D^\mu X D_\mu X - \frac{F^2\,\Lambda_2}{12}X\langle\chi_-\rangle \qquad \mathcal{L}_S = c_d\langle S u_\mu u^\mu\rangle + c_m\langle S\chi_+\rangle\,,$$



(a) (b) (c) (d)

(a) (b)

- **Mixing relations of π⁰-η-η'-a (axion) at NLO**

[Gao,ZHG,Oller,Zhou, JHEP'23]
[Gao,Hao,ZHG,Oller,Zhou, EPJC'25]

$$\begin{pmatrix}\hat{\pi}^0\\ \hat{\eta}\\ \hat{\eta}'\\ \hat{a}\end{pmatrix} = \begin{pmatrix} 1+z_{11} & c_\theta(-v_{12}+z_{12})+s_\theta(-v_{13}+z_{13}) & -s_\theta(-v_{12}+z_{12})+c_\theta(-v_{13}+z_{13}) & -v_{14}+z_{14}\\ v_{12}+z_{21} & c_\theta(1+z_{22})+s_\theta(z_{23}-v_{23}) & -s_\theta(1+z_{22})+c_\theta(z_{23}-v_{23}) & -v_{24}+z_{24}\\ v_{13}+z_{31} & c_\theta(z_{32}+v_{23})+s_\theta(1+z_{33}) & -s_\theta(z_{32}+v_{23})+c_\theta(1+z_{33}) & -v_{34}+z_{34}\\ v_{41}+z_{41} & c_\theta(v_{42}+z_{42})+s_\theta(v_{43}+z_{43}) & -s_\theta(v_{42}+z_{42})+c_\theta(v_{43}+z_{43}) & 1+v_{44}+z_{44}\end{pmatrix}\begin{pmatrix}\pi^0\\ \eta_8\\ \eta_0\\ a\end{pmatrix}$$

v_{ij}: LO terms; z_{ij}: NLO terms (L_5, L_8, Λ_1, Λ_2) or [(L_5, L_8)~ (c_dc_m, c_mc_m)/M^2_S , Λ_1, Λ_2]

- **Scalar form factor** $\langle P_1 P_2|(m_D - m_u)\bar{D}u|0\rangle$

$$\Delta_{du} = B_0(m_d - m_u) = \Delta_{du}^{\rm Phy}\left\{1 + \frac{m_K^2}{F^2}\left[\frac{16c_m(c_d - c_m)}{M_S^2} + \frac{16c'_m(c'_d - c'_m)}{M_{S'}^2}\right]\right\}$$

$$\Delta_{du}^{\rm Phy} = m_{K^0}^2 - m_{K^+}^2 - (m_{\pi^0}^2 - m_{\pi^+}^2)$$

⇒

$$(m_d - m_u)\langle\pi^- P|\bar{d}u|0\rangle = \Delta_{du}^{\rm Phy} F_0^{\pi^- P}(s) \qquad \widehat{F}_0^{\pi^- P}(s) = \frac{\Delta_{du}^{\rm Phy}}{\Delta_{du}} F_0^{\pi^- P}(s)$$

⇒

$$\langle\pi^- P|\bar{d}\gamma^\mu u|0\rangle = \left[(p_P - p_\pi)^\mu - \frac{\Delta_{P\pi}}{s}q^\mu\right]F_+^{\pi^- P}(s) + \frac{\Delta_{du}^{\rm Phy}}{s}q^\mu F_0^{\pi^- P}(s)$$

- **Finiteness of the above expression at *s* = 0 requires**

$$F_+^{\pi^-(\eta,\eta',a)}(0) = \frac{\Delta_{du}^{\rm Phy}}{\Delta_{(\eta,\eta',a)\pi}} F_0^{\pi^-(\eta,\eta',a)}(0)$$

✓ **Non-trivial confirmation is verified in chiral EFT!**

Some explicit expressions for Form Factors

- **VFF-ππ**

$$F_{+}^{\pi^-\pi^0}(s) = -\frac{\sqrt{2}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s)$$

$$G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) = \frac{G_V F_V s + F^2(M_\rho^2 - s)}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} - \frac{G'_V F'_V s}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} - \frac{G''_V F''_V s}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)}\,,$$

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F_\pi^2}\left[\sigma_{\pi\pi}^3(s) + \frac{1}{2}\sigma_{KK}^3(s)\right]\,,\quad \Gamma_{\rho',\rho''}(s) = \Gamma_{\rho',\rho''}\frac{s}{M_{\rho',\rho''}^2}\frac{\sigma_{\pi\pi}^3(s)}{\sigma_{\pi\pi}^3(M_{\rho',\rho''}^2)}\,,\quad \sigma_{P_1P_2}(s) = \frac{2q_{P_1P_2}(s)}{\sqrt{s}}\theta[s-(m_{P_1}+m_{P_2})^2]$$

- **VFF-πη/πη'/πa**

$$F_{+}^{\pi^-\eta}(s) = -\frac{\sqrt{2}v_{12}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{12} - v_{13}y_{23}^{(0)}\right)\,,$$

$$F_{+}^{\pi^-\eta'}(s) = -\frac{\sqrt{2}v_{13}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{13} + v_{12}y_{23}^{(0)}\right)\,,$$

$$F_{+}^{\pi^-a}(s) = -\frac{\sqrt{2}v_{41}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{14} + v_{12}\,y_{24}^{(0)} + v_{13}\,y_{34}^{(0)}\right)$$

- **SFF-πη/πη'/πa**

$$F_0^{\pi^-\eta}(s) = \sqrt{\frac{2}{3}}(c_\theta - \sqrt{2}s_\theta) + \frac{1}{\sqrt{3}}(\Lambda_1 - 2\Lambda_2)s_\theta - \frac{1}{\sqrt{3}}y_{23}(2c_\theta + \sqrt{2}s_\theta) + 4\sqrt{\frac{2}{3}}\frac{c_\theta - \sqrt{2}s_\theta}{F^2}\Bigg\{ \left[\frac{c_m(c_m - c_d)2m_\pi^2 + c_m c_d(s + m_\pi^2 - m_\eta^2)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} - \frac{2c_m(c_m - c_d)(2m_K^2 - m_\pi^2)}{M_S^2}\right] + \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \to c'_{m,d}, M_{a'_0}, \Gamma_{a'_0}, M_{S'}\right]\Bigg\},$$

$$F_0^{\pi^-\eta}(s) = \sqrt{\frac{2}{3}}(c_\theta - \sqrt{2}s_\theta) + \frac{1}{\sqrt{3}}(\Lambda_1 - 2\Lambda_2)s_\theta - \frac{1}{\sqrt{3}}y_{23}(2c_\theta + \sqrt{2}s_\theta) + 4\sqrt{\frac{2}{3}}\frac{c_\theta - \sqrt{2}s_\theta}{F^2}\Bigg\{ \left[\frac{c_m(c_m - c_d)2m_\pi^2 + c_m c_d(s + m_\pi^2 - m_\eta^2)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} - \frac{2c_m(c_m - c_d)(2m_K^2 - m_\pi^2)}{M_S^2}\right] + \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \to c'_{m,d}, M_{a'_0}, \Gamma_{a'_0}, M_{S'}\right]\Bigg\},$$

$$F_0^{\pi^-a}(s) = \frac{(\sqrt{2}c_\theta - 2s_\theta)}{\sqrt{3}}v_{24}^{(0)} + \frac{(2c_\theta + \sqrt{2}s_\theta)}{\sqrt{3}}v_{34}^{(0)} - \frac{2}{\sqrt{3}}\left(s_\theta\, v_{24}^{(0)} - c_\theta\, v_{34}^{(0)} + \frac{F}{\sqrt{6}f_a}\right)(\Lambda_2 - \frac{1}{2}\Lambda_1) + \frac{(\sqrt{2}c_\theta - 2s_\theta)}{\sqrt{3}}y_{24}^{(0)} + \frac{(2c_\theta + \sqrt{2}s_\theta)}{\sqrt{3}}y_{34}^{(0)} + \frac{4(\sqrt{2}c_\theta - 2s_\theta)v_{24}^{(0)} + 4(2c_\theta + \sqrt{2}s_\theta)v_{34}^{(0)}}{\sqrt{3}F^2}\Bigg\{ \left[\frac{2c_m^2\, m_\pi^2 + c_m c_d\left(s - m_a^2 - m_\pi^2\right)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} + \frac{2c_m(c_d - c_m)\left(2m_K^2 - m_\pi^2\right)}{M_S^2}\right] + \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \to c'_{m,d}, M_{a'_0}, \Gamma_{a'_0}, M_{S'}\right]\Bigg\}.$$

Fits to experimental spectra and BRs

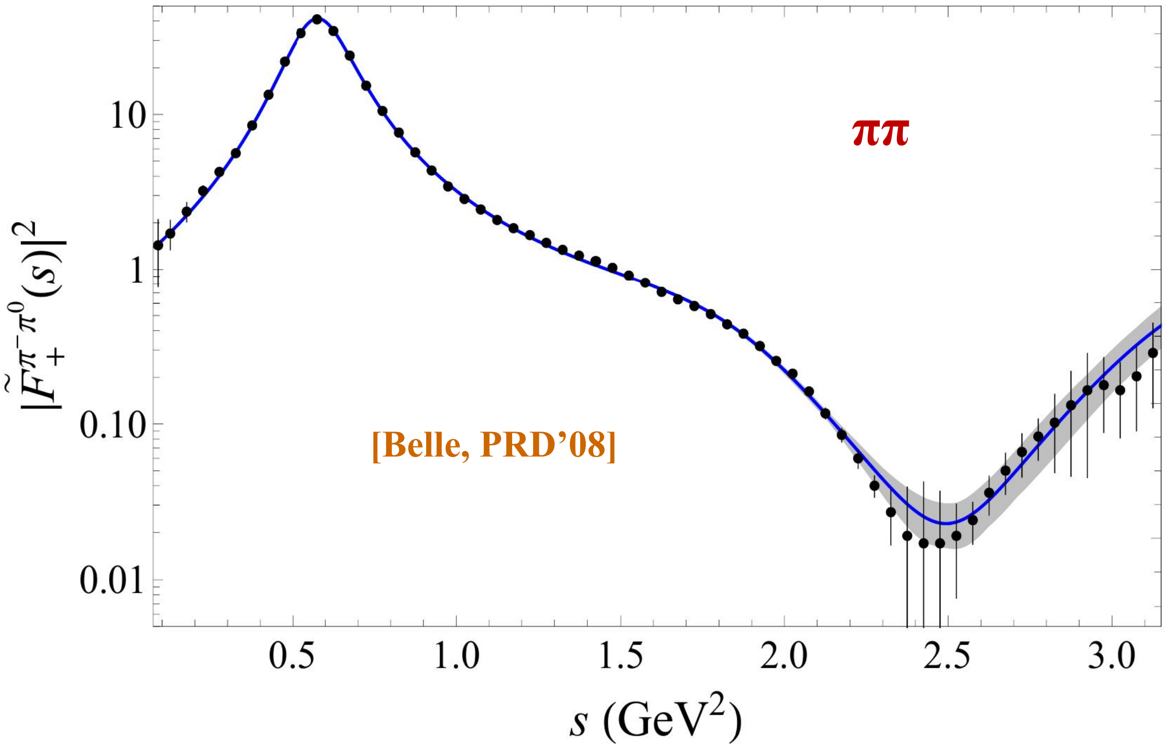
$G_V\, F_V(\mathrm{GeV}^2) \times 10^3$	$10.26^{+0.01}_{-0.01}$	$G'_V\, F'_V(\mathrm{GeV}^2) \times 10^3$	$0.64^{+0.03}_{-0.02}$
$G''_V\, F''_V(\mathrm{GeV}^2) \times 10^3$	$-0.94^{+0.05}_{-0.05}$	$M_\rho(\mathrm{GeV})$	$0.7738^{+0.0003}_{-0.0003}$
$M_{\rho'}(\mathrm{GeV})$	$1.409^{+0.004}_{-0.004}$	$\Gamma_{\rho'}(\mathrm{GeV})$	$0.338^{+0.012}_{-0.010}$
$M_{\rho''}(\mathrm{GeV})$	$1.842^{+0.012}_{-0.013}$	$\Gamma_{\rho''}(\mathrm{GeV})$	$0.268^{+0.025}_{-0.026}$
$c'_m(\mathrm{GeV})$	$0.053^{+0.007}_{-0.009}$	$M_{K^*}(\mathrm{GeV})$	$0.8956^{+0.0002}_{-0.0002}$
$\Gamma_{K^*}(\mathrm{GeV})$	$0.0477^{+0.0005}_{-0.0005}$	$M_{K^{*\prime}}(\mathrm{GeV})$	$1.339^{+0.009}_{-0.009}$
$\overline{B}_{K_S\pi^-} \times 10^3$	$3.98^{+0.04}_{-0.04}$	$\overline{B}_{K^-\eta} \times 10^4$	$1.34^{+0.04}_{-0.04}$
$\chi^2/\mathrm{d.o.f}$	$271.5/(182-14) = 1.61$		

$$c_m c_d + c'_m c'_d = \frac{F^2}{4}$$

c_m= 27 MeV, c_d=15 MeV

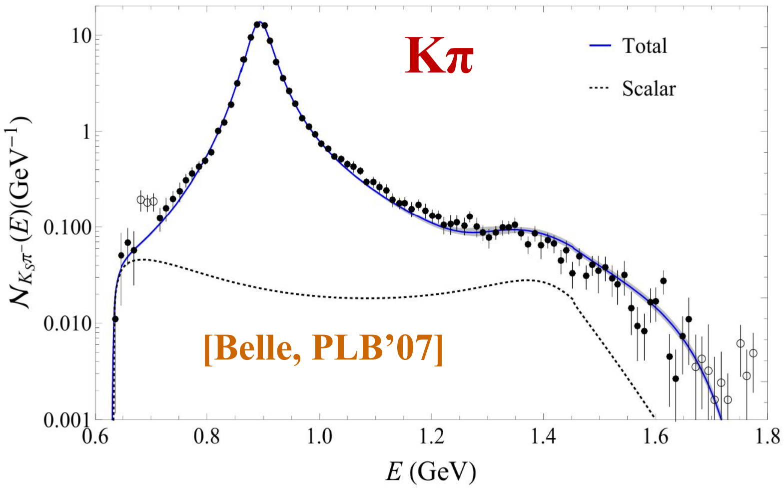
[ZHG,Oller, PRD'11]

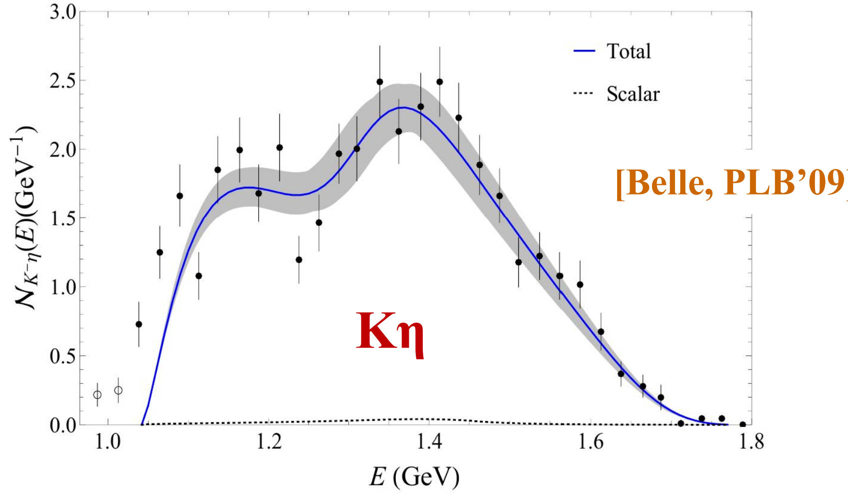
Parameters for a_0(980)/a_0(1450)/K_0*(700)//K_0*(1430) are fixed to their pole positions .



- **Crucial inputs to address muon g-2**
- **Most precise spectra is from Belle;**

  but most precise BR is from ALEPH: 25.47(13)%
- **Coherent precise measurements of both spectra and BR from one Exp would be invaluable!**

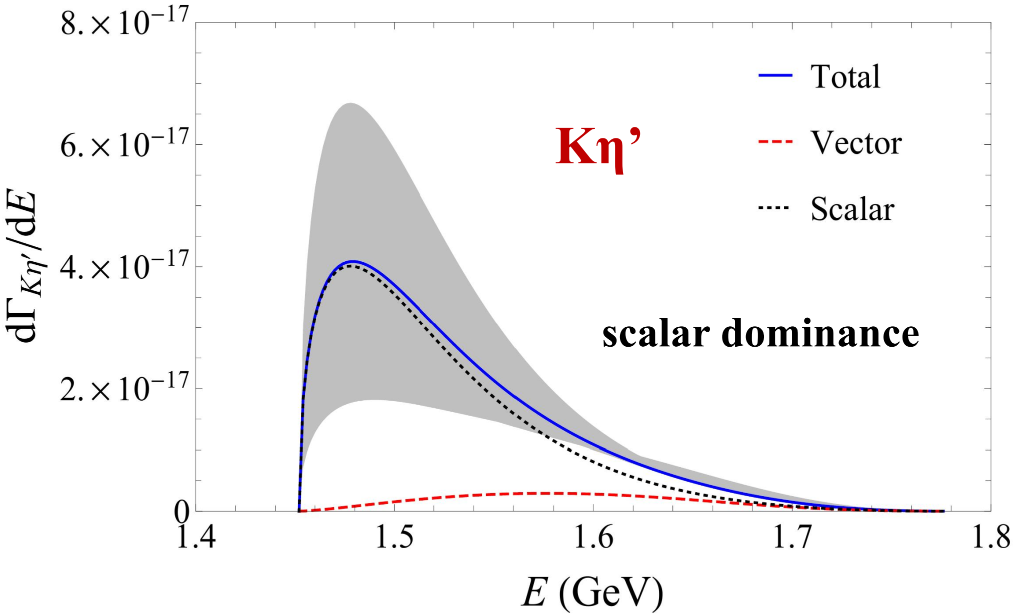




F_+^{KP}: K*, K*(1410), K*(1680)

F_0^{KP}: κ, $K^*_0(1430)$

prediction to Kη’



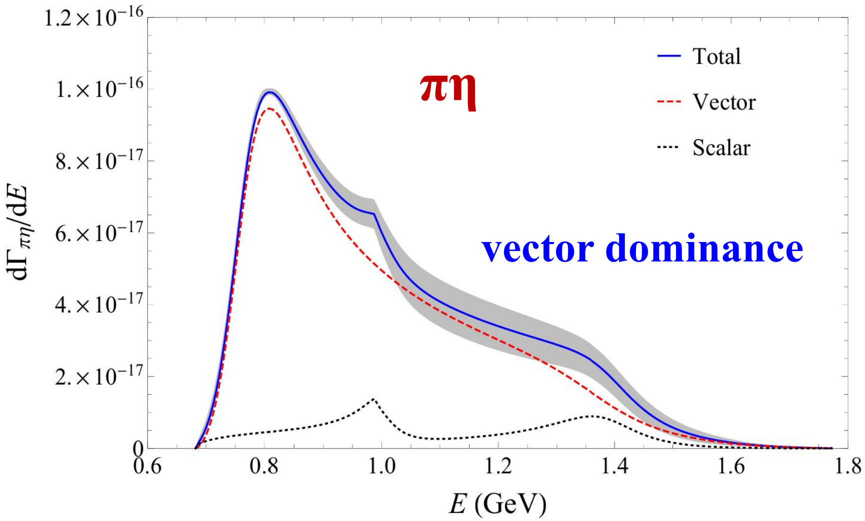
$BR(K^- \eta')^{Theo} = (2.0 \pm 1.0)\times 10^{-6}$

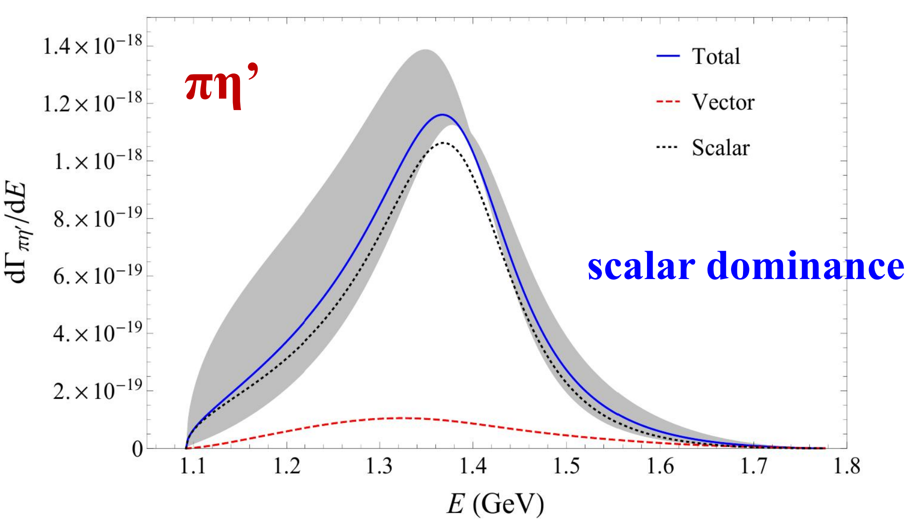
$BR(K^- \eta')^{Exp,\ BaBar} < 2.4\times 10^{-6}$

Predictions to $\tau \to \pi^- \eta^{(\prime)} \nu_\tau$ (Cabibbo allowed): second-class currents

Processes driven by second-class currents are suppressed by isospin breaking:

$$\langle \pi^- P | \bar{d}\gamma^\mu u | 0 \rangle = \left[(p_P - p_\pi)^\mu - \frac{\Delta_{P\pi}}{s} q^\mu \right] F_+^{\pi^- P}(s) + \frac{\Delta_{du}^{\rm Phy}}{s} q^\mu F_0^{\pi^- P}(s)$$



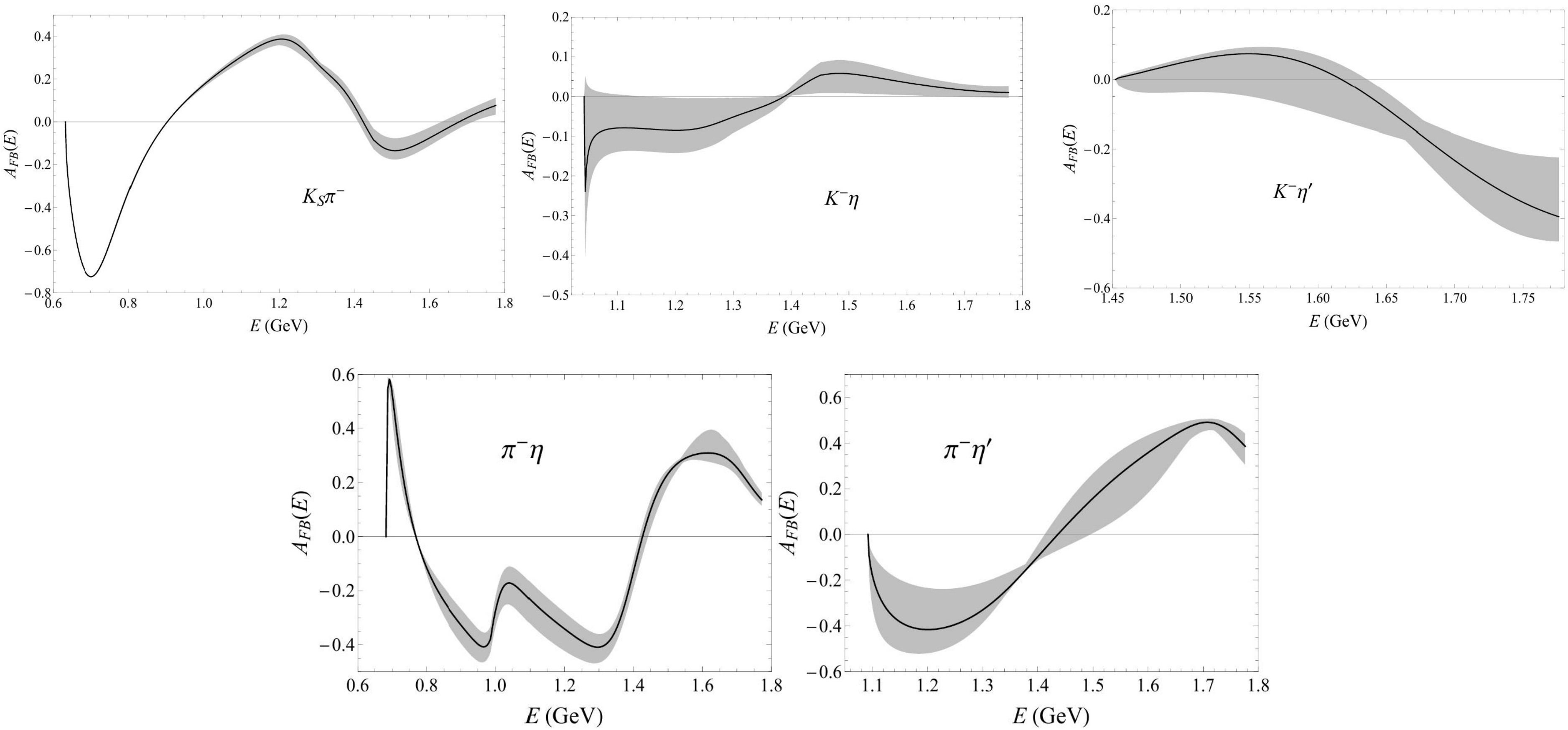


Channel	Total	Vector	Scalar	Exp Limits
$\tau^- \to \pi^- \eta \nu_\tau$ $(\times 10^5)$	$1.63^{+0.14}_{-0.14}$	$1.43^{+0.18}_{-0.21}$	$0.20^{+0.07}_{-0.04}$	<9.9 (BaBar) [69] <7.3 (Belle) [70]
$\tau^- \to \pi^- \eta' \nu_\tau$ $(\times 10^7)$	$1.17^{+0.36}_{-0.07}$	$0.14^{+0.09}_{-0.08}$	$1.03^{+0.44}_{-0.16}$	<40 (BaBar) [71]

[Hao, Duan, ZHG, 2507.00383, to appear in FOP]

Predictions to Forward-Backward asymmetries

$$\frac{\Delta_{Du}\, q_{P_1P_2}(s)\, \Re\left[F_+^{P_1P_2}(s)\, \widehat{F}_0^{P_1P_2*}(s)\right]}{\frac{2\sqrt{s}}{3}\left(1+\frac{2s}{M_\tau^2}\right) q_{P_1P_2}^2(s)\left|F_+^{P_1P_2}(s)\right|^2 + \frac{\Delta_{Du}^2}{2\sqrt{s}}\left|\widehat{F}_0^{P_1P_2}(s)\right|^2}$$

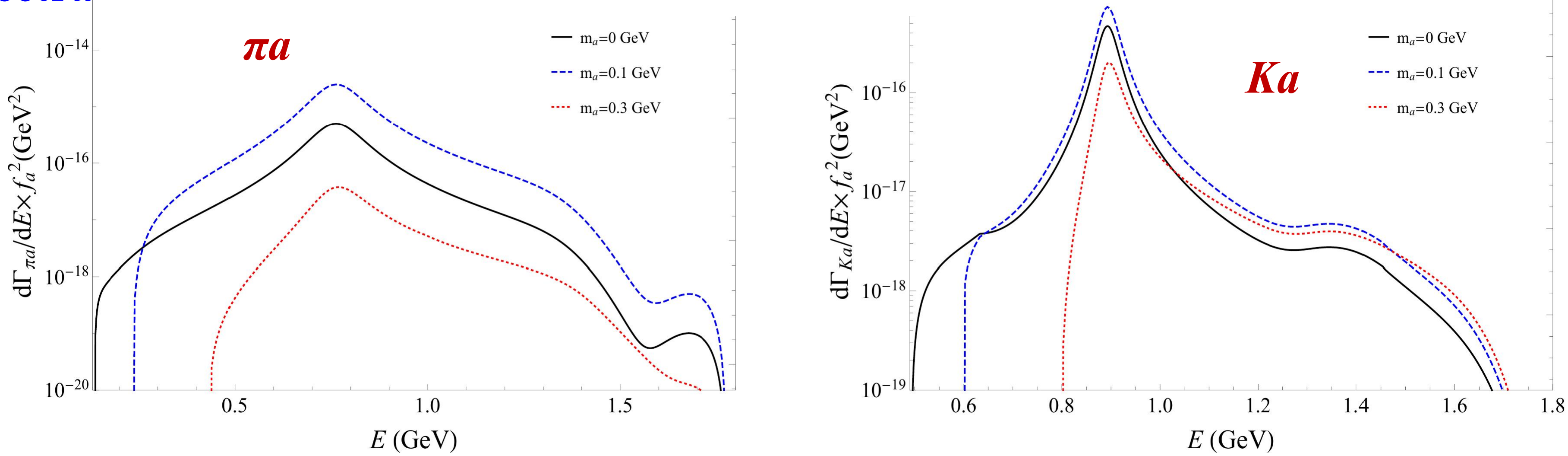


- **Measurement on A_{FB} can determine the curcial inputs for CPV**

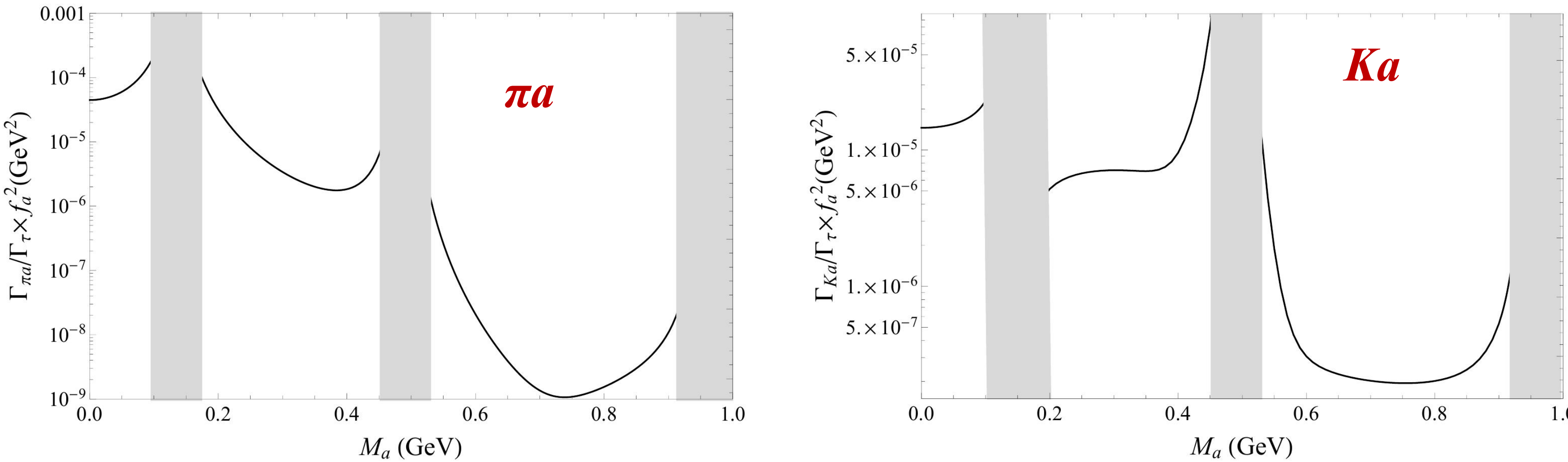
Predictions to ALP-meson production in tau decays

Spectra

vector dominance



BRs



- **Hadronic resonances enhance the decay widths involving ALPs by around one order of magnitudes, with respect to the results from leading-order χPT.**

Summary

Two-boson tau decays are interesting in many aspects:

- $\tau^- \to \nu_\tau \pi^0 \pi^-$ [crucial inputs for muon g-2];
- $\tau^- \to \nu_\tau (K\pi, K\eta, K\eta')^-$ [V_{us}, strange hadron resonances, possible CPV];
- Second-class currents: $\tau^- \to \nu_\tau \pi^- \eta/\eta'$;
- Forward-Backward asymmetries in the τ decays;
- Significant enhancement of axion production with hadronic resonances is observed in $\tau^- \to \nu_\tau\, a\, K^-/\pi^-$.

Thanks very much for your listening！