A Recent Study on Hadronic Contributions to Muon g-2

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with Hao-Xiang Pan, Jia-Yu Zhou, S.J.Wang, W. Qin *et.al.*

Based on: arxiv: 2505.00770, PRD111 (2025) 034025, JHEP07 (2023) 037, RPP84 (2021) 076201, JHEP03 (2021) 092, PRD99 (2019) 114015, PRD97 (2018) 036012, PRD95 (2017) 056007, PRD94 (2016) 116061, PRD90 (2014) 036004, PLB736 (2014) 11, PRD88 (2013) 056001, et. al.

The 7th International Workshop on Future Tau-Charm Facilities

Nov. 2025，Huangshan

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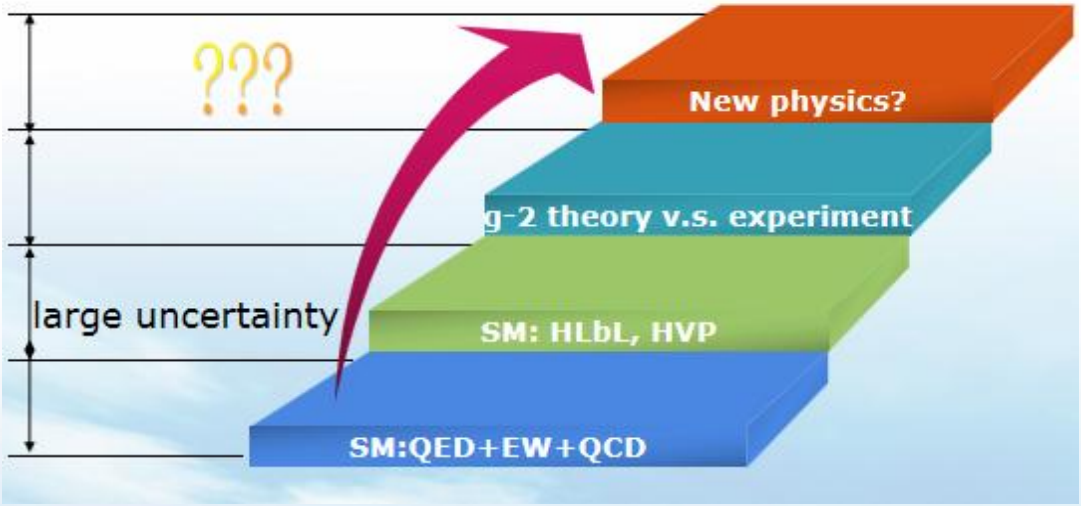
Outlines

1. Introduction
2. HVP
3. Further theory?
4. Summary

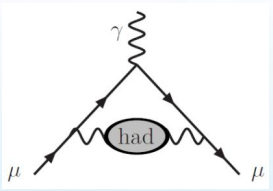
1、Introduction

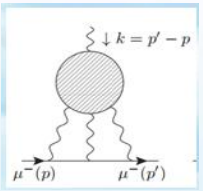


$$a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{QCD}}$$

- HVP, HLbL:

  DD v.s. LQCD?





EXP

arxiv: 2506.03069 [hep-exp]

Phys.Rev.Lett. 131, 161802 (2023)

Phys.Rev.Lett.126, 141801 (2021)

Phys.Rev.D 73, 072003 (2006).

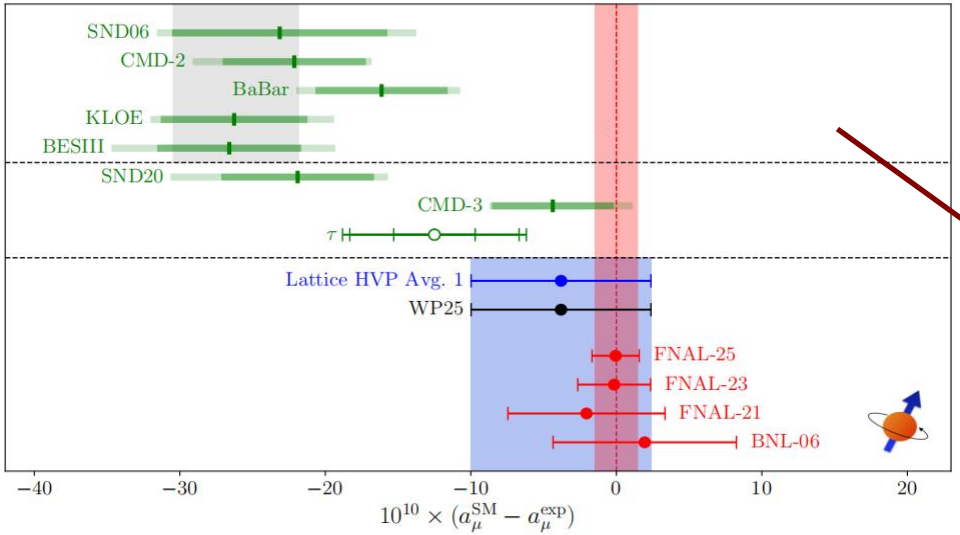
J-PARC, FNAL, CSNS, HIAF?

arxiv: 2505.21476 [hep-ph]

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E989, E821)		Eq. (9.5)	116 592 071.5(14.5)	Refs. [5–8, 10–13]
HVP LO (lattice)	Sec. 3.6.1	Eq. (3.37)	7132(61)	Refs. [14–30]
HVP LO (e^+e^-, τ)	Sec. 2	Table 5	Estimates not provided at this point	
HVP NLO (e^+e^-)	Sec. 2.9	Eq. (2.47)	−99.6(1.3)	Refs. [31, 32]
HVP NNLO (e^+e^-)	Sec. 2.9	Eq. (2.48)	12.4(1)	Ref. [33]
HLbL (phenomenology)	Sec. 5.10	Eq. (5.69)	103.3(8.8)	Refs. [34–57]
HLbL NLO (phenomenology)	Sec. 5.10	Eq. (5.70)	2.6(6)	Ref. [58]
HLbL (lattice)	Sec. 6.2.8	Eq. (6.34)	122.5(9.0)	Refs. [59–63]
HLbL (phenomenology + lattice)	Sec. 9	Eq. (9.2)	112.6(9.6)	Refs. [34–57, 59–63]
QED	Sec. 7.5	Eq. (7.27)	116 584 718.8(2)	Refs. [64–70]
EW	Sec. 8	Eq. (8.12)	154.4(4)	Refs. [51, 71–73]
HVP LO (lattice) + HVP N(N)LO (e^+e^-)	Sec. 9	Eq. (9.1)	7045(61)	Refs. [14–33]
HLbL (phenomenology + lattice + NLO)	Sec. 9	Eq. (9.3)	115.5(9.9)	Refs. [34–63]
Total SM Value	Sec. 9	Eq. (9.4)	116 592 033(62)	Refs. [14–73]
Difference: $\Delta a_\mu \equiv a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$	Sec. 9	Eq. (9.6)	38(63)	

1、Introduction

- WP2025 v.s. WP2020



Contribution	WP25	WP20
HVP LO (lattice)	7132(61)	7116(184)
HVP LO (e^+e^-, τ)	Table 5	6931(40)*
HVP NLO (e^+e^-)	−99.6(1.3)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)	12.4(1)
HLbL (phenomenology)	103.3(8.8)	92(19)
HLbL NLO (phenomenology)	2.6(6)	2(1)
HLbL (lattice)	122.5(9.0)	82(35)
HLbL (phenomenology + lattice)	112.6(9.6)	90(17)
QED	116 584 718.8(2)	116 584 718.931(104)
EW	154.4(4)	153.6(1.0)
HVP (LO + NLO + NNLO)	7045(61)	6845(40)
HLbL (phenomenology + lattice + NLO)	115.5(9.9)	92(18)
Total SM Value	116 592 033(62)	116 591 810(43)

Phys.Rept. 1143 (2025) 1
Phys.Rept.887(2020)1

CERN-TH-2025-101
INT-PUB-25-015
KEK Preprint 2025-22
MITP-25-037
FERMILAB-PUB-25-0344-T
IPARCOS-UCM-25-029
LTH 1403
UWThPh 2025-15
ZU-TH 37/25

The anomalous magnetic moment of the muon in the Standard Model: an update

R. Aliberti [1,2], T. Aoyama [3], E. Balzani [4,5], A. Bashir [6,7], G. Benton [8,9], J. Bijnens [10], V. Biloshytskyi [1,2], T. Blum [11,12], D. Boito [13], M. Bruno [14,15], E. Budassi [16,17], S. Burri [18], L. Cappiello [19], C. M. Carloni Calame [17], M. Cè [14,15], V. Cirigliano [20,21], D. A. Clarke [22], G. Colangelo [†18], L. Cotrozzi [23], M. Cottini [18], I. Danilkin [1,2], M. Davier [†24], M. Della Morte [25], A. Denig [1,2,26], C. DeTar [22], V. Druzhinin [27], G. Eichmann [28], A. X. El-Khadra [†8,9], E. Estrada [29], X. Feng [30,31,32], C. S. Fischer [33,34], R. Frezzotti [35], G. Gagliardi [36], A. Gérardin [37], M. Ghilardi [16,17], D. Giusti [38,39], M. Golterman [40], S. Gonzàlez-Solís [41,42], S. Gottlieb [43], R. Gruber [44], A. Guevara [45], V. Gülpers [46], A. Gurgone [47,48], F. Hagelstein [1,2], M. Hayakawa [49,50], N. Hermansson-Truedsson [10,46], A. Hoecker [51], M. Hoferichter [†18], B.-L. Hoid [1,2], S. Holz [18], R. J. Hudspith [52], F. Ignatov [23], L. Jin [11], N. Kalntis [18], G. Kanwar [46], A. Keshavarzi [53], J. Komijani [44], J. Koponen [1,2], S. Kuberski [54], B. Kubis [55], A. Kupich [27], A. Kupść [56,57], S. Lahert [22], S. Laporta [4,5], C. Lehner [†39], M. Lellmann [1], L. Lellouch [37], T. Leplumey [58,59], J. Leutgeb [60], T. Lin [30], Q. Liu [61], I. Logashenko [27], C. Y. London [13], G. López Castro [29], J. Lüdtke [62], A. Lusiani [48,63], A. Lutz [24], J. Mager [60], B. Malaescu [64], K. Maltman [65,66], M. K. Marinković [44], J. Márquez [29], P. Masjuan [67,68], H. B. Meyer [1,2,26], T. Mibe [†69], N. Miller [26], A. Miramontes [70,71], A. Miranda [67], G. Montagna [16,17], S. E. Müller [72], E. T. Neil [73], A. V. Nesterenko [27], O. Nicrosini [17], M. Nio [50,74], D. Nomura [75], J. Paltrinieri [23], L. Parato [44], J. Parrino [39], V. Pascalutsa [1,2], M. Passera [5,76], S. Peris [67,68], P. Petit Rosàs [23], F. Piccinini [17,77], R. N. Pilato [23], L. Polat [64,24], A. Portelli [46], D. Portillo-Sánchez [29], M. Procura [62], L. Punzi [48,63], K. Raya [7], A. Rebhan [60], C. F. Redmer [1,2], B. L. Roberts [†78], A. Rodríguez-Sánchez [71], P. Roig [29,71], J. Ruiz de Elvira [79], P. Sánchez-Puertas [80], A. Signer [81,59], J. W. Sitison [73], D. Stamen [55], D. Stöckinger [82], H. Stöckinger-Kim [82], P. Stoffer [59,81], Y. Sue [69], P. Tavella [44], T. Teubner [†23], J.-N. Toelstede [59,81], G. Toledo [83], W. J. Torres Bobadilla [23], J. T. Tsang [54], F. P. Ucci [16,17], Y. Ulrich [23], R. S. Van de Water [84], G. Venanzoni [23,48], S. Volkov [85], G. von Hippel [1,2], G. Wang [37], U. Wenger [18], H. Wittig [†1,2,26], A. Wright [23], E. Zaid [23], M. Zanke [55], Z. Zhang [24], M. Zillinger [18], C. Alexandrou [86,87], A. Altherr [44], M. Anderson [1,2], C. Aubin [88], S. Bacchio [87], P. Beltrame [23], A. Beltran [1,2], P. Boyle [89], I. Campos Plasencia [90], I. Caprini [91], B. Chakraborty [92], G. Chanturia [55], A. Crivellin [59], A. Czarnecki [93], L.-Y. Dai [94], T. Dave [23], L. Del Debbio [46], K. Demory [24], D. Djukanovic [26], T. Draper [95], A. Driutti [48], M. Endo [96], F. Erben [54], K. Ferraby [23], J. Finkenrath [54], L. Flower [23], A. Francis [97], E. Gámiz [98], J. Gogniat [18], A. V. Grebe [84], S. Gündogdu [81,59], M. T. Hansen [46], S. Hashimoto [96], H. Hayashii [99], D. W. Hertzog [20], L. A. Heuser [55], L. Hostetler [43], X. T. Hou [100], G. S. Huang [101], T. Iijima [102,49], K. Inami [49], A. Jüttner [54,92], R. Kitano [103], M. Knecht [37], S. Kollatzsch [81,59], A. S. Kronfeld [84], T. Lenz [1,2], G. Levati [18], Q. M. Li [100], Y. P. Liao [100], J. Libby [104], K. F. Liu [95], V. Lubicz [36], M. T. Lynch [8,9], A. T. Lytle [8,9], J. L. Ma [100], K. Miura [96], K. Möhling [82], J. Muskalla [1,2], F. Noël [18], K. Ottnad [1,2], P. Paradisi [4,5], C. T. Peterson [105], A. Pich [71], S. Pitelis [1], S. Plura [1,2], A. Price [106], D. Radic [81,59], A. Radzhabov [27], A. Risch [107], S. Romiti [18], S. Sahoo [108], F. Sannino [25,19], H. Schäfer [55], Y. Schelhaas [1,2], S. I. Serednyakov [27], O. Shekhovtsova [109,110], J. N. Simone [84], S. Simula [111], E. P. Solodov [27], F. M. Stokes [66], M. Vanderhaeghen [1,2], A. Vaquero [112,113], N. Vestergaard [23], W. Wang [1,2], Z. Wąs [114], K. Yamashita [115], Y. B. Yang [116], T. Yoshioka [117], C. Z. Yuan [100], A. S. Zhevlakov [27]

[†]mgm2-sc@lists.physics.illinois.edu

arXiv:2505.21476v2 [hep-ph] 19 Jun 2025

Methods from SM

- Hadronic Part: Methods from SM
  - LQCD
  - Data-driven solutions from experiment
  - Amplitude analysis: model independent

- Only one physical amplitude!
- It should satisfy the fundamental QFT principles
- It should be compatible with the exp results

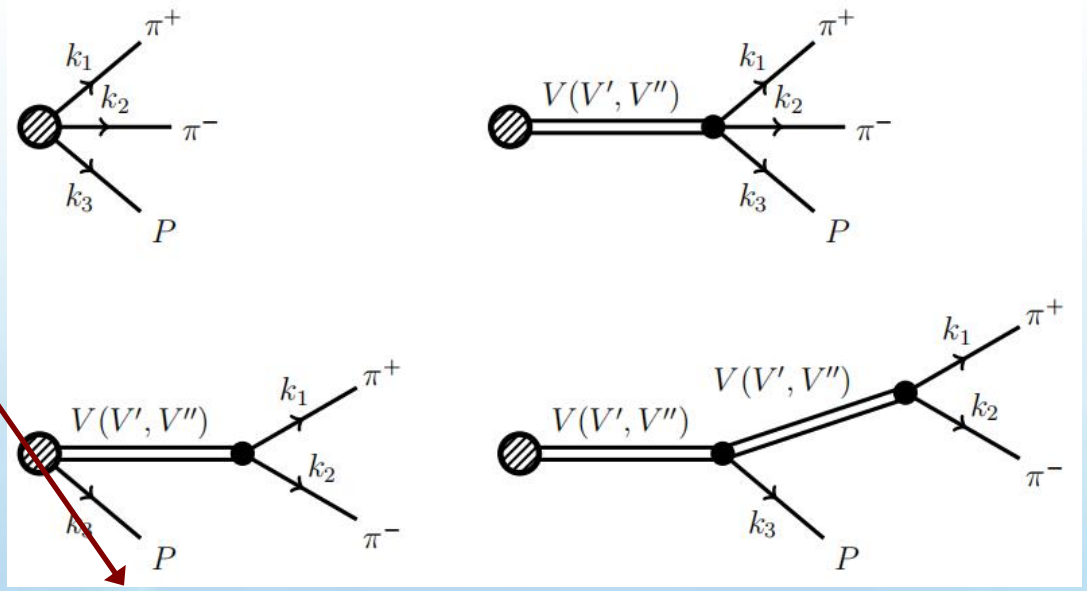
- ChEFT for low energy interaction!
- FSI of light hadrons been taken into account
- Data constraints & constraining data

2、HVP

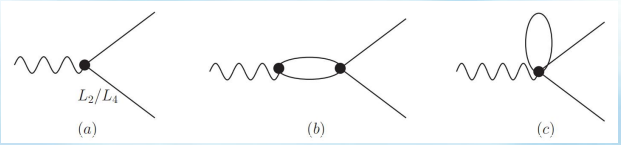
- RChT in the resonance region, excited states?
  - resonances included as new degrees of freedom
  - QCD high energy constraints to reduce LECs
  - 1/Nc expansion

- V', V'' has the same topologies as the ground states

- ππ-KK FSI part by matching with Omens functions and ChPT



Guerrero, et.al., PLB 412 (1997) 382



Wang, Fang, Dai, JHEP07 (2023) 037

$$\frac{1}{M_V^2 - x} \to \frac{1}{M_V^2 - x} + \frac{\beta'_\pi}{M_{V'}^2 - x} + \frac{\beta''_\pi}{M_{V''}^2 - x}$$

Dai, et.al., PRD88 (2013) 056001

Building amplitudes

- We give a combined analysis on several channels:

$$\pi^+\pi^-, K^+K^-, \pi^+\pi^-\pi^0, \pi^+\pi^-\eta \ \ \pi^0\gamma \text{ and } \eta\gamma.$$

  - ρ-ω mixing, origined from Gasser&Leutwyler's

$$F_V^\pi = \left(1 + \frac{F_V G_V}{F^2} Q^2 \left(BW(M_\rho, \Gamma_{\rho,}, Q^2)\right.\right.$$
$$\left. + \beta'_{\pi\pi} BW(M_{\rho'}, \Gamma_{\rho'}, Q^2) + \beta''_{\pi\pi} BW(M_{\rho''}, \Gamma_{\rho''}, Q^2)\right)$$
$$\left(\frac{1}{\sqrt{3}} \sin\theta_V \sin\delta^\rho + \cos\delta\right) \cos\delta$$
$$- \frac{F_V G_V}{F^2} Q^2 \left(BW(M_\omega, \Gamma_{\omega,}, Q^2) + \beta'_{\pi\pi} BW(M_{\omega'}, \Gamma_{\omega'}, Q^2)\right.$$
$$\left. + \beta'''_{\pi\pi} BW(M_{\omega''}, \Gamma_{\omega''}, Q^2)\right) \left(\frac{1}{\sqrt{3}} \sin\theta_V \cos\delta - \sin\delta^\omega\right) \sin\delta^\omega \Bigg)$$
$$\exp\left[\frac{-s}{96\pi^2 F^2}\left(\mathrm{Re}\left[A[m_\pi, M_\rho, Q^2] + \frac{1}{2} A[m_K, M_\rho, Q^2]\right]\right)\right]$$

- Not much freedom for Fit

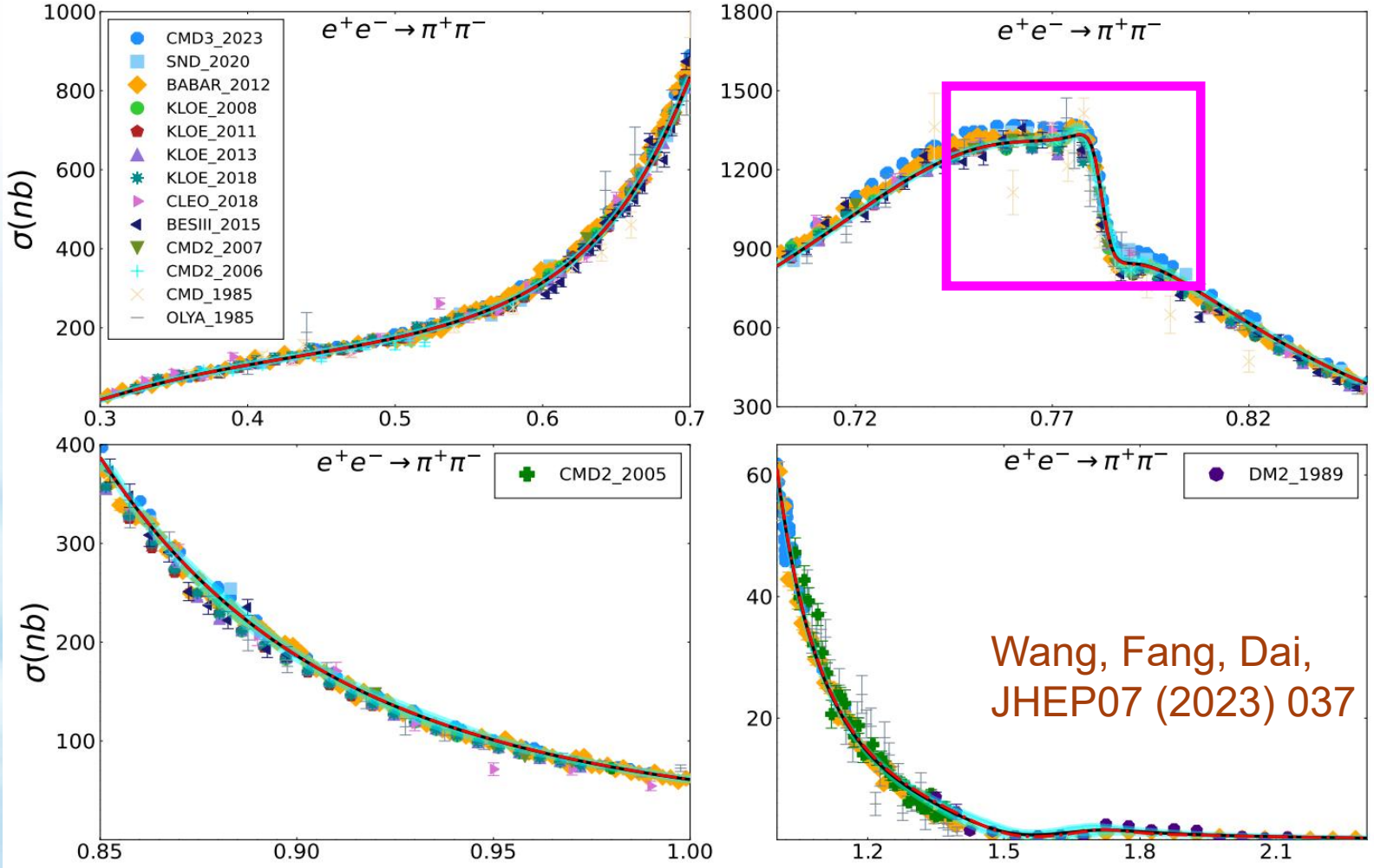
It is 1, from QCD as well as disersion relation constraints

Gasser&Leutwyler, Phys.Rept.87 (1982) 77

Guerrero&Pich, PLB 412 (1997) 382

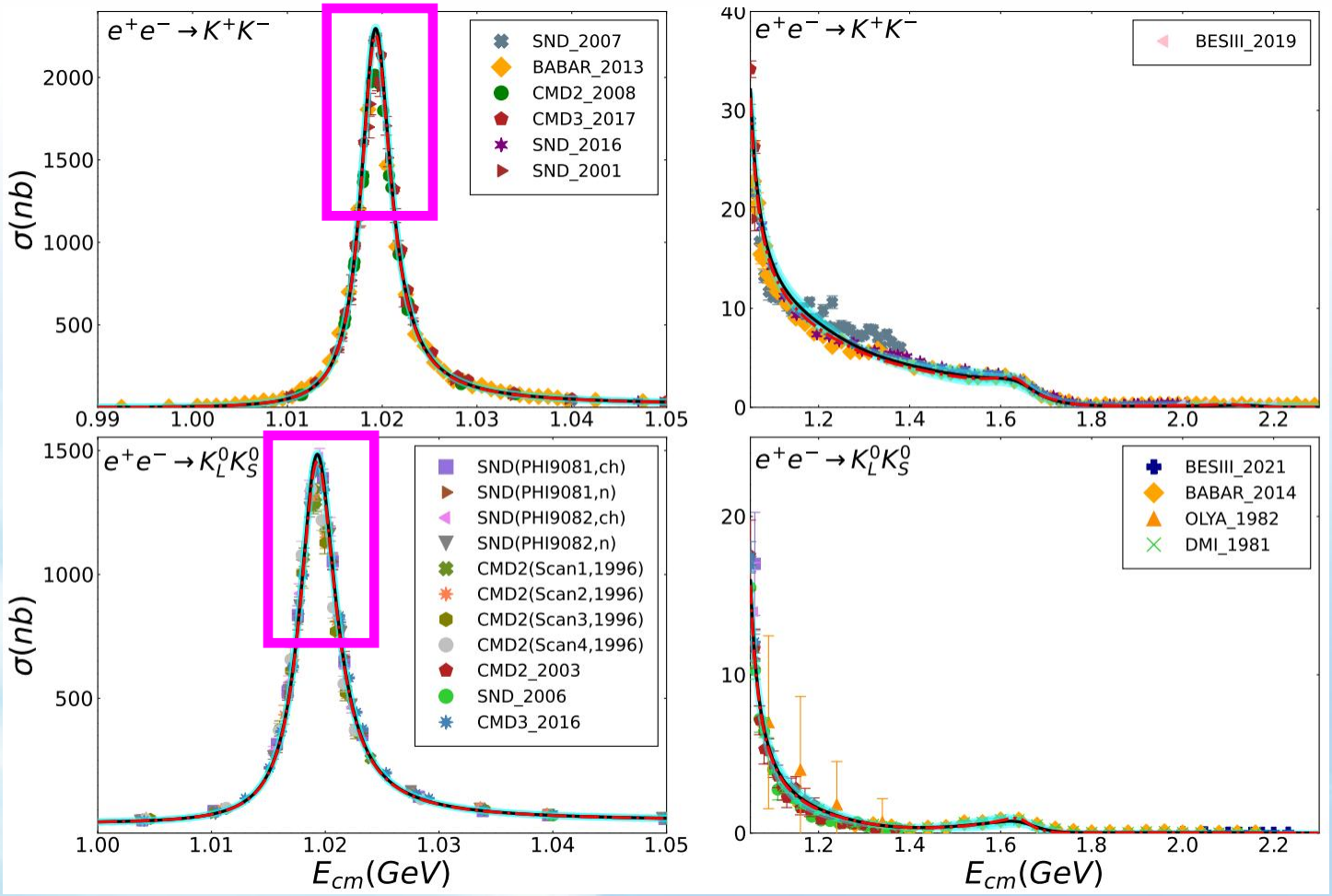
$\pi\pi$

- $\pi\pi$: Now closer to KLOE and BESIII's
- Latest exp: CMD-3, large discrepancy



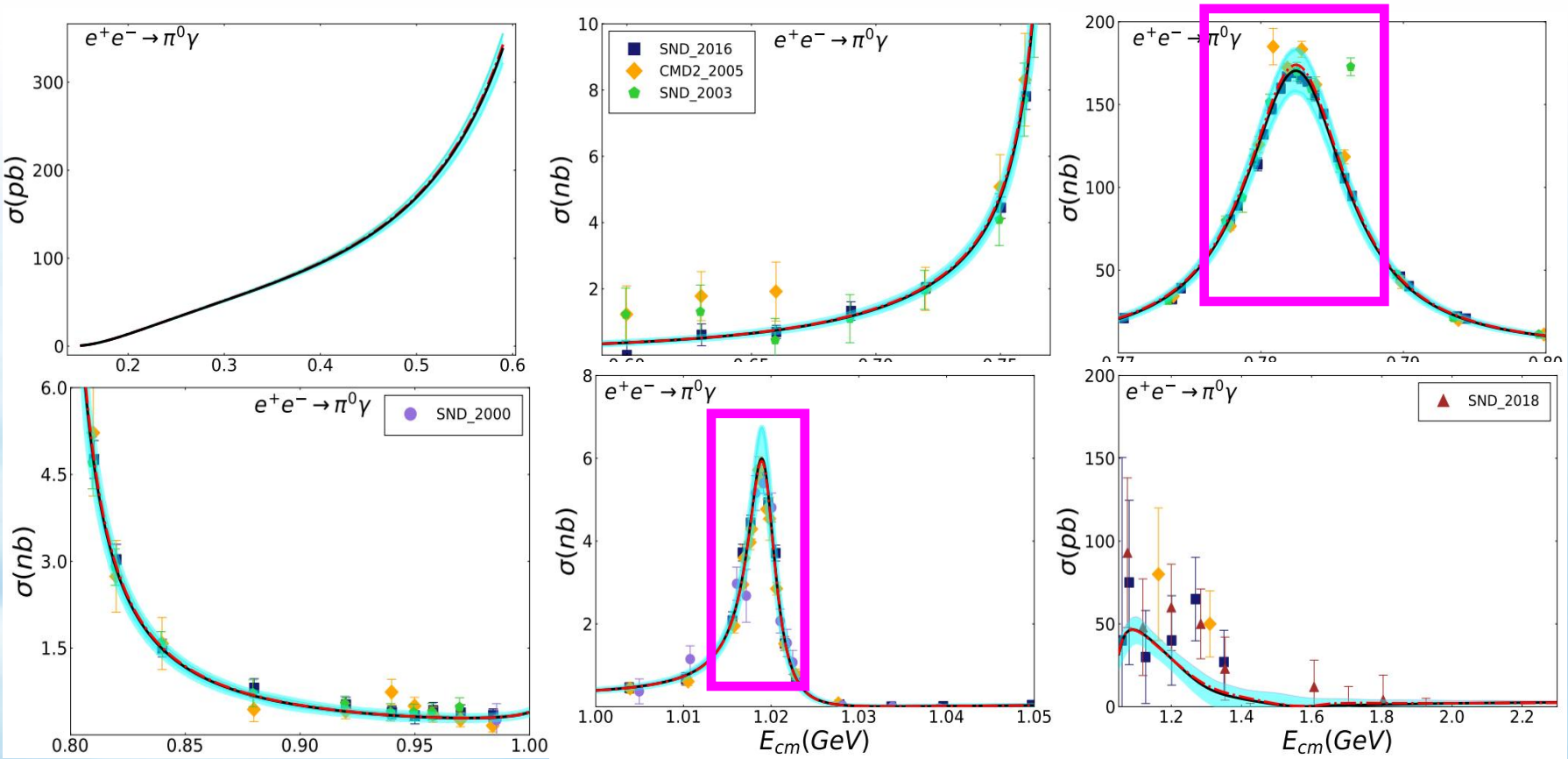
KK

- KK: data in the ϕ 'peak' have large discrepancy
- $K_L K_S$: further direct constraints on $\pi\pi$, KK channels



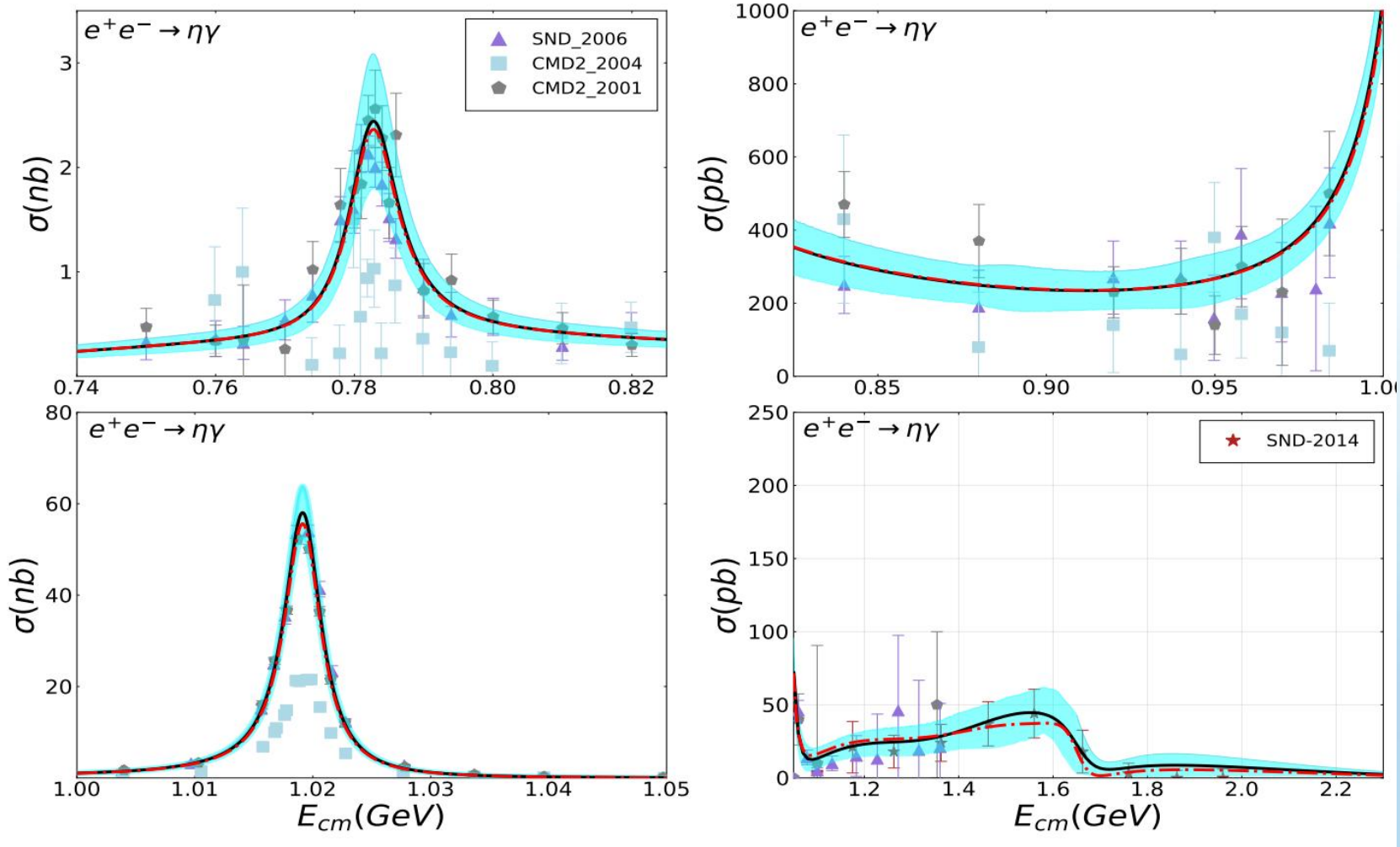
$\pi\gamma$

- $\pi\gamma$: helps to constrain $\pi\pi$, KK channels: ρ, ω, ϕ



ηγ

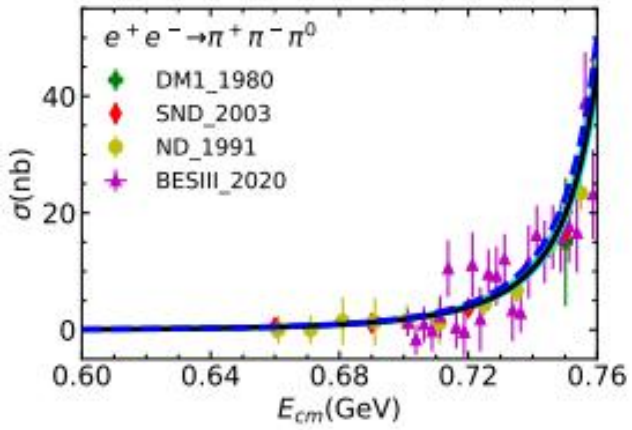
- ηγ: helps to constrain KK, and parameters of ρ, ω, ϕ

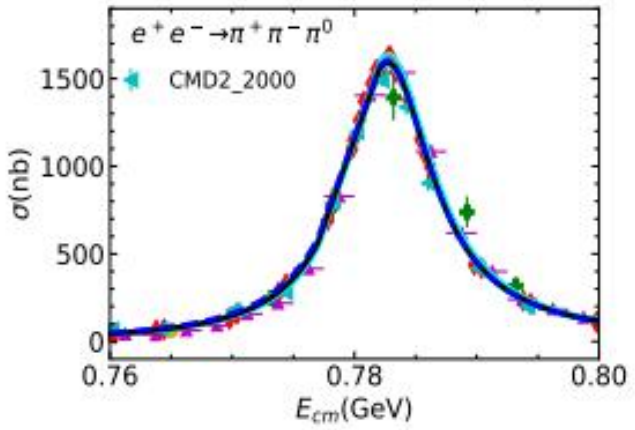


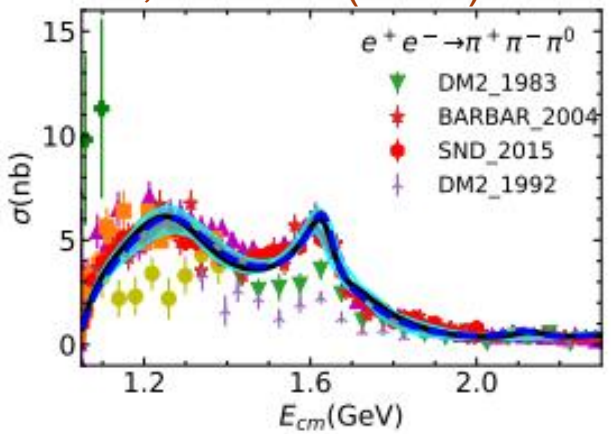
ππ, ππη

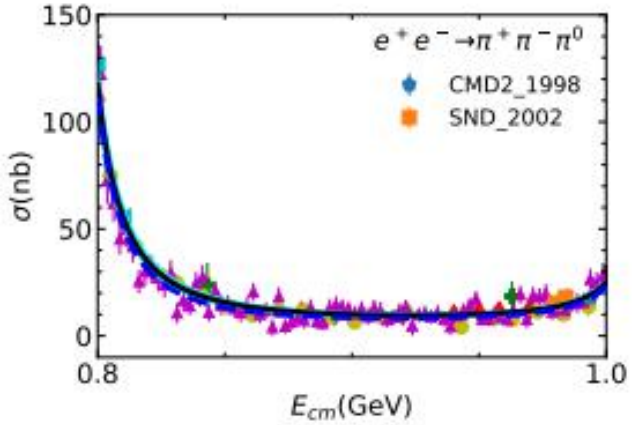
- ππ: needs more precise data in the ω ϕ region
- ππη: check our model

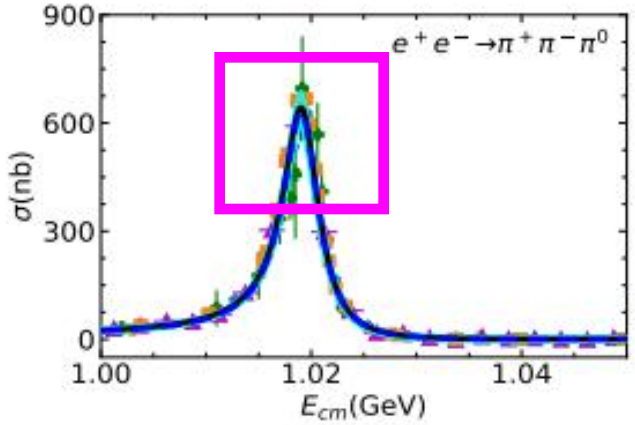
Qin, Dai, Portoles, JHEP03(2021)092

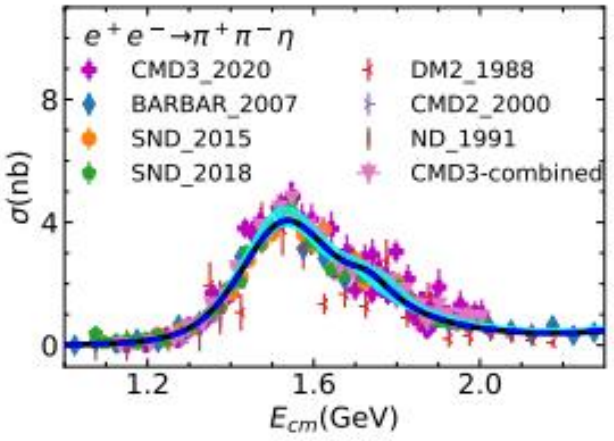






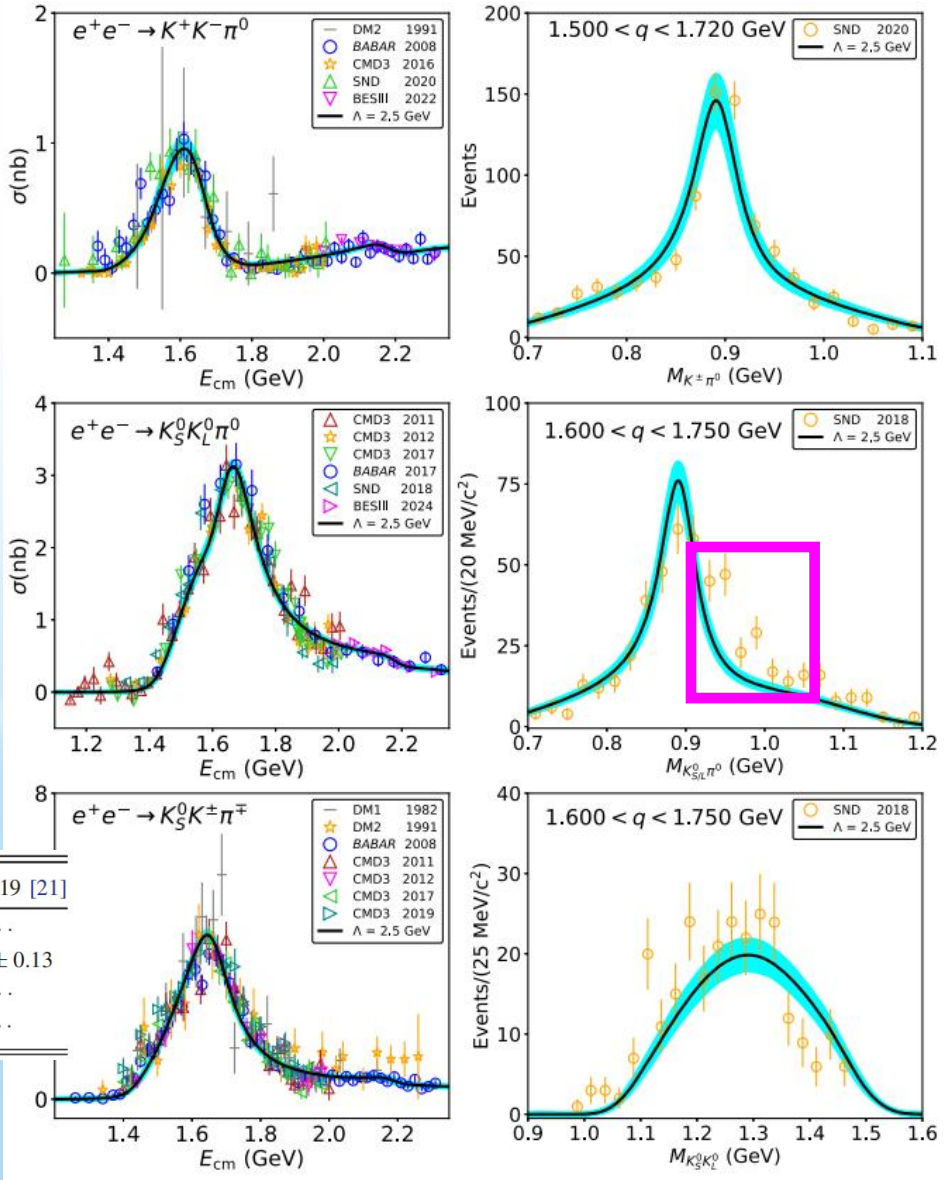






KKπ

- Tensors included
- KKπ: angular distributions are helpful to constrain amplitudes
- Three body rescattering can improve it
- Ours are close to other groups'



$a_\mu^{\mathrm{HVP,LO}} \times 10^{10}$	This analysis	KNT18 [20]	DHMZ19 [21]
$a_\mu^{K\bar{K}\pi}[1.260 \le q \le 1.937\ \mathrm{GeV}]$	2.726 ± 0.063	2.71 ± 0.12	...
$a_\mu^{K\bar{K}\pi}[\mathrm{th} \le q \le 1.8\ \mathrm{GeV}]$	2.407 ± 0.062	2.44 ± 0.11	2.45 ± 0.13
$a_\mu^{K\bar{K}\pi}[\mathrm{th} \le q \le 2\ \mathrm{GeV}]$	2.829 ± 0.064	2.80 ± 0.12	...
$a_\mu^{K\bar{K}\pi}[\mathrm{th} \le q \le 2.3\ \mathrm{GeV}]$	3.178 ± 0.071	...	...

Qin, Qin, Dai, PRD111 (2025) 034025

Contributions to g-2

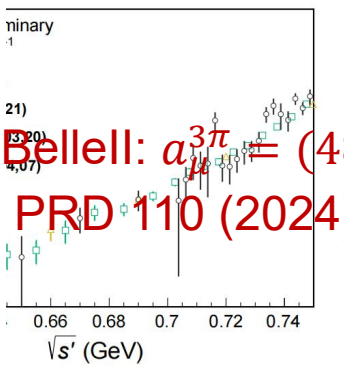
- Ours is close to other data-driven results
  - Ours of $\pi\pi\pi$ is consistent with the recent measurements

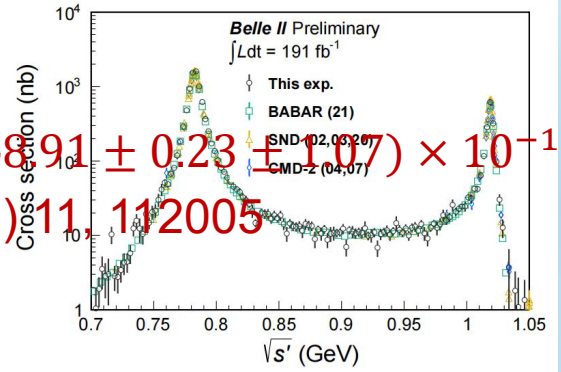
$a_\mu^C \times 10^{-10}$	Ref. [116]	Ref. [29]	Ref. [117]	Ref. [60]	Fit I	Fit II
$a_\mu^{\pi\pi}\|_{\le 0.63\text{GeV}}$	132.8(0.4)(1.0)	—	—	—	132.11±0.63	132.11±0.67
$a_\mu^{\pi\pi}\|_{\le 1\text{GeV}}$	495.0(1.5)(2.1)	—	—	—	498.48±2.34	498.47±2.33
$a_\mu^{\pi\pi}\|_{\le 1.8\text{GeV}}$	—	—	—	507.85±0.83±3.23±0.56	508.89±2.45	508.89±2.45
$a_\mu^{\pi\pi}\|_{\le 2.3\text{GeV}}$	—	—	—	—	509.13±2.48	509.13±2.48
$a_\mu^{KK}\|_{\le 1.1\text{GeV}}$	—	—	—	—	20.73±0.94	20.74±0.88
$a_\mu^{KK}\|_{\le 1.8\text{GeV}}$	—	—	—	23.08±0.20±0.33±0.21	24.35±1.02	24.36±0.97
$a_\mu^{KK}\|_{\le 2.3\text{GeV}}$	—	—	—	—	24.43±1.03	24.44±1.01
$a_\mu^{\pi\pi\pi}\|_{\le 1.8\text{GeV}}$	—	48.55	46.2(8)	46.21±0.40±1.10±0.86	48.55±1.42	48.54±1.39
$a_\mu^{\pi\pi\pi}\|_{\le 2.3\text{GeV}}$	—	—	—	—	48.76±1.45	48.75±1.43
$a_\mu^{\eta\pi\pi}\|_{\le 1.8\text{GeV}}$		1.135	—	1.19±0.02±0.04±0.02	1.28±0.10	1.29±0.09
$a_\mu^{\eta\pi\pi}\|_{\le 2.3\text{GeV}}$	—	—	—	—	1.52±0.12	1.53±0.12
$a_\mu^{\text{HVP.LO}}$	—	—	—	694.0±4.0	699.46±3.41	699.47±3.39
a_μ^{SM}				11659183.1±4.8	11659187.3±3.8	11659187.3±3.9
Δa_μ				26.0 ± 7.9(3.3σ)	21.6 ± 7.4(2.9σ)	21.6 ± 7.4(2.9σ)

Qin, Dai, Portoles, JHEP03(2021)092

$a_\mu^C \times 10^{-10}$	Ref. [15]	Ref. [16]	Ref. [27]	Fit A	Fit
$a_\mu^{\pi\pi}\|\le 0.63\,\text{GeV}$	132.8(0.4)(1.0)		130.10 ±0.67	130.70 ±0.02	129.91
$a_\mu^{\pi\pi}\|\le 1\,\text{GeV}$	495.0(1.5)(2.1)		496.45 ±2.33	496.50±0.93	495.28
$a_\mu^{\pi\pi}\|\le 1.8\,\text{GeV}$	—	507.85±0.83±3.23±0.55	506.69 ±2.45	—	505.64
$a_\mu^{\pi\pi}\|\le 2.3\,\text{GeV}$	—		506.93 ±2.48	—	505.89
$a_\mu^{KK}\|\le 1.1\,\text{GeV}$	—		19.88±0.88	18.92 ±0.41	19.42
$a_\mu^{KK}\|\le 1.8\,\text{GeV}$	—	23.08±0.20±0.33±0.21	23.44 ±0.97	—	22.99
$a_\mu^{KK}\|\le 2.3\,\text{GeV}$	—		23.52 ±1.01	—	23.03
$a_\mu^{K_L^0K_S^0}\|\le 1.1\,\text{GeV}$				11.79±0.27	12.02
$a_\mu^{K_L^0K_S^0}\|\le 1.8\,\text{GeV}$	—	12.82±0.06±0.18±0.15		—	12.67±0.24
$a_\mu^{K_L^0K_S^0}\|\le 2.3\,\text{GeV}$				—	12.67±0.24
$a_\mu^{\pi^0\gamma}\|\le 0.63\,\text{GeV}$				0.15±0.01	0.15±0.01
$a_\mu^{\pi^0\gamma}\|\le 1\,\text{GeV}$	—			4.66±0.04	4.55±0.08
$a_\mu^{\pi^0\gamma}\|\le 1.8\,\text{GeV}$	—	4.41±0.06±0.04±0.07		—	4.61±0.09
$a_\mu^{\pi^0\gamma}\|\le 2.3\,\text{GeV}$	—			—	4.61±0.10
$a_\mu^{\eta\gamma}\|\le 1\,\text{GeV}$	—			0.20±0.02	0.19±0.02
$a_\mu^{\eta\gamma}\|\le 1.8\,\text{GeV}$	—	0.65±0.02±0.01±0.01		—	0.66±0.06
$a_\mu^{\eta\gamma}\|\le 2.3\,\text{GeV}$	—			—	0.66±0.06
$a_\mu^{\pi\pi\pi}\|\le 1.8\,\text{GeV}$	—	46.21±0.40±1.10±0.86	48.46±1.42		
$a_\mu^{\pi\pi\pi}\|\le 2.3\,\text{GeV}$	—		48.68±1.45		
$a_\mu^{\pi\pi\eta}\|\le 1.8\,\text{GeV}$	—	1.19±0.02±0.04±0.02			
$a_\mu^{\pi\pi\eta}\|\le 2.3\,\text{GeV}$	—		1.49±0.12		
$a_\mu^{HVP.LO}$	—	694.0±4.0	695.54±3.35	—	694.10±3.14
a_μ^{SM}	—	11659183.1±4.8	11659183.4±3.7	—	11659182.0±3.5
Δa_μ	—	26.0±7.9(3.3σ)	22.7±5.5(4.1σ)	—	24.1±5.4(4.5σ)

Wang, Fang, Dai, JHEP07 (2023) 037





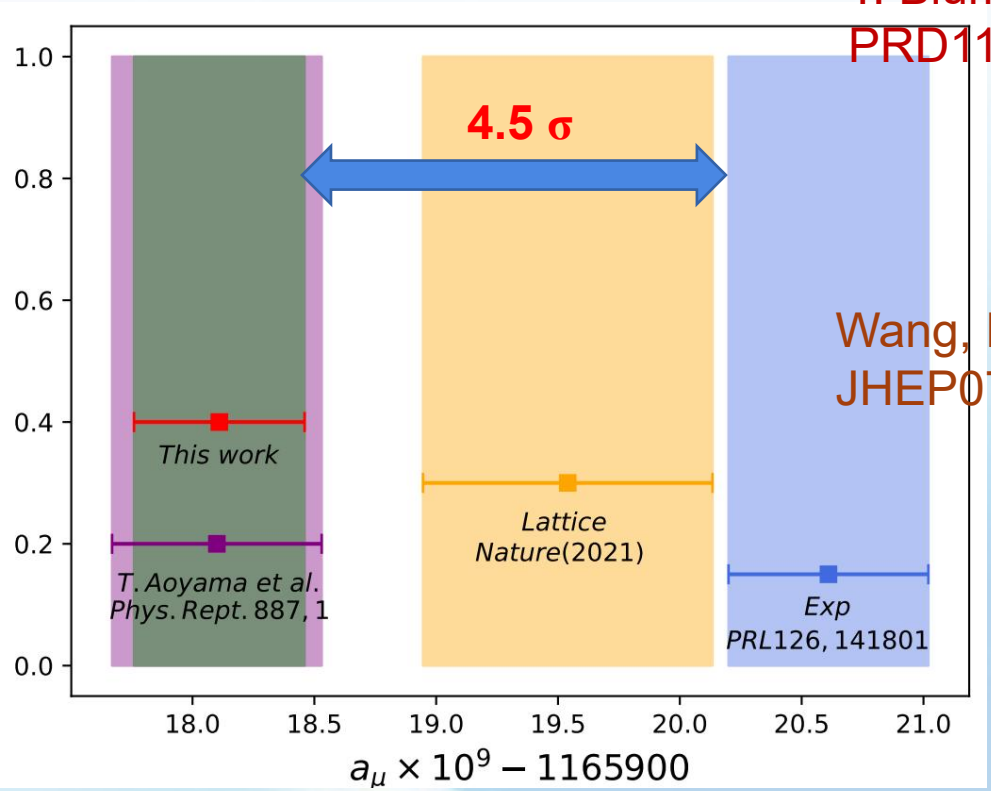
BelleII: $a_\mu^{3\pi} = (48.91 \pm 0.23 \pm 1.07) \times 10^{-10}$

PRD 110 (2024)11, 112005

Discrepancy

- Ours: a_μ=11659181.1 ±3.5 × 10^{-11}
- Obvious discrepancy from LQCD or latest exp.
  - 3.9σ If HLBL part repleaced with latest LQCD's

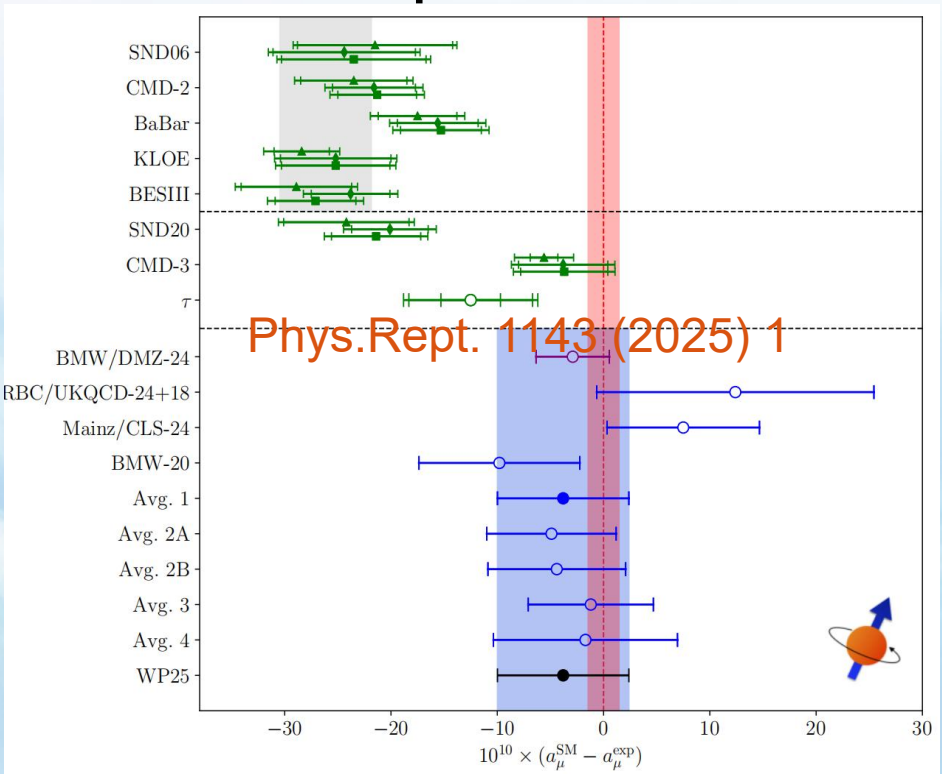
T. Blum, et.al.,
PRD111 (2025)014501

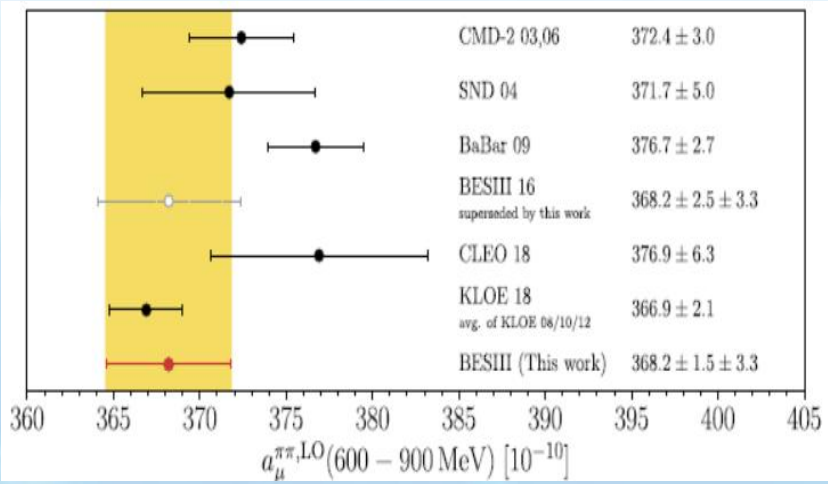


Wang, Fang, Dai,
JHEP07 (2023) 037

Future experiment?

- CMD-3's (2023) is close to the LQCD result
- ISR, FSR?
- Scalar part?
- Future experiments? BESIII, KLOE, BELLEII,STCF?





Guangshun Huang, talk at HNU

3、Further theory?

- Check the adaptability of our model

Energy region	CMD-2	BaBar	KLOE	BESIII	SND20	CMD-3
0.63-0.958(Gev)	361.5	-	-	-	-	-
0.3-1.8(Ge)	-	513.54	-	-	-	-
0.35-0.85(Gev^2)	-	-	377.5	-	-	-
0.6-0.9(Gev)	-	-	-	36	-	-
0.525-0.883(Gev)	-	-	-	-	409.79	-
0.6-0.88(Gev)	-	-	-	-	-	379.3
Rcht1	360.04	512.43	377.60	368.24	408.33	377.62

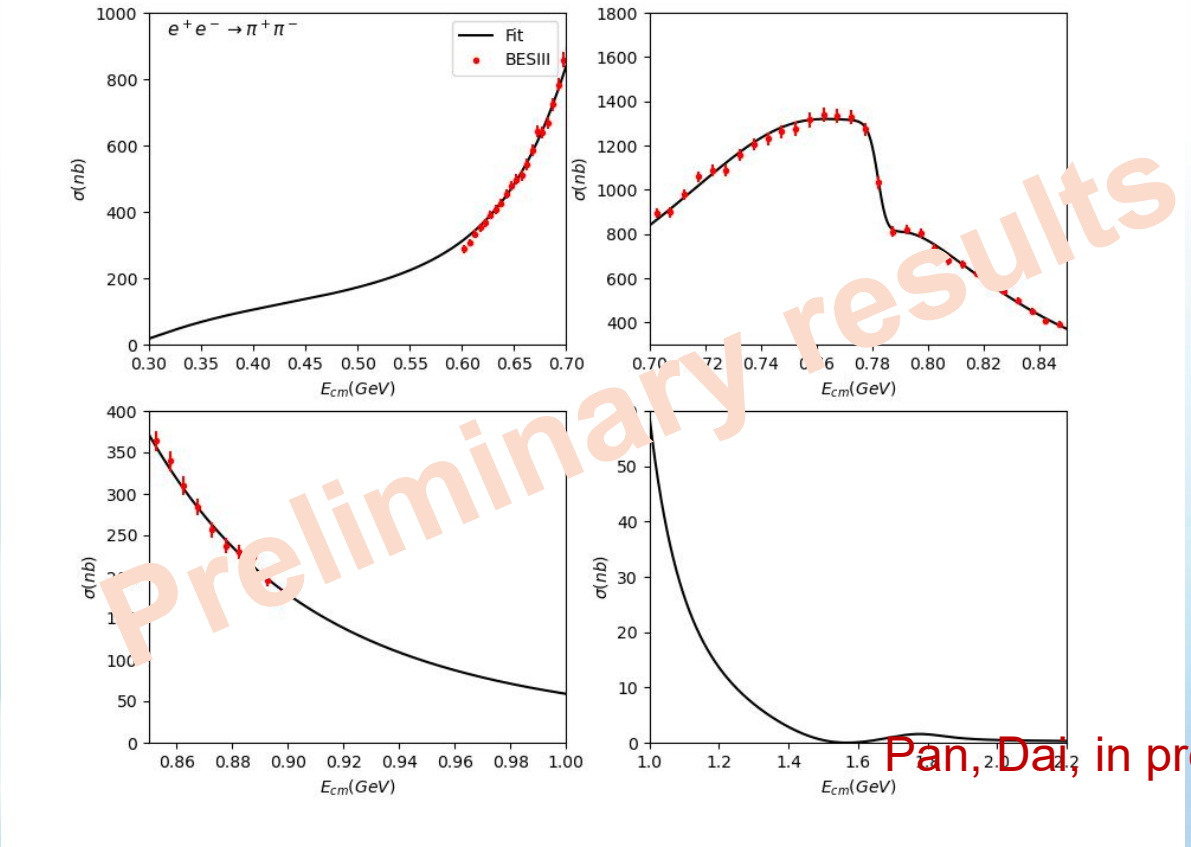
Pan, Dai, in preparation

Preliminary results

- Ours is consistent with each dataset

BESIII

- Fit to BESIII data

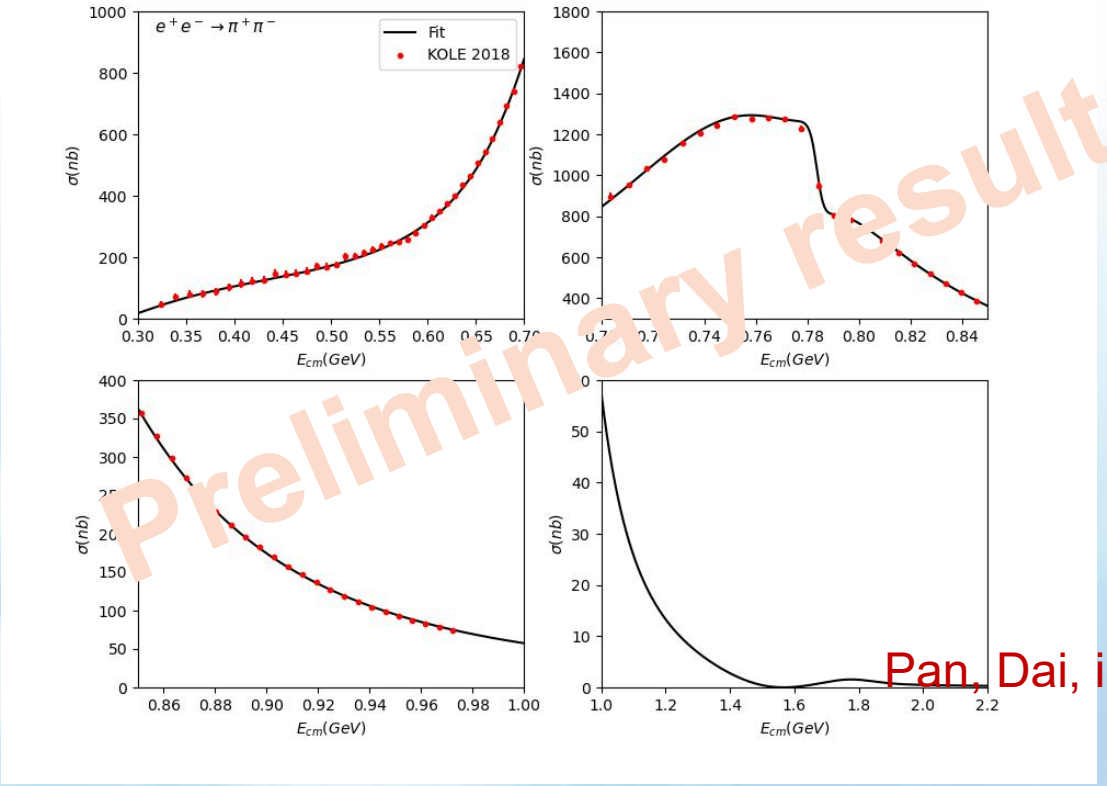


Pan, Dai, in preparation

- Measurements on 0.3-0.6, 0.9-2.3GeV can be helpful

KLOE

- Fit to KLOE data

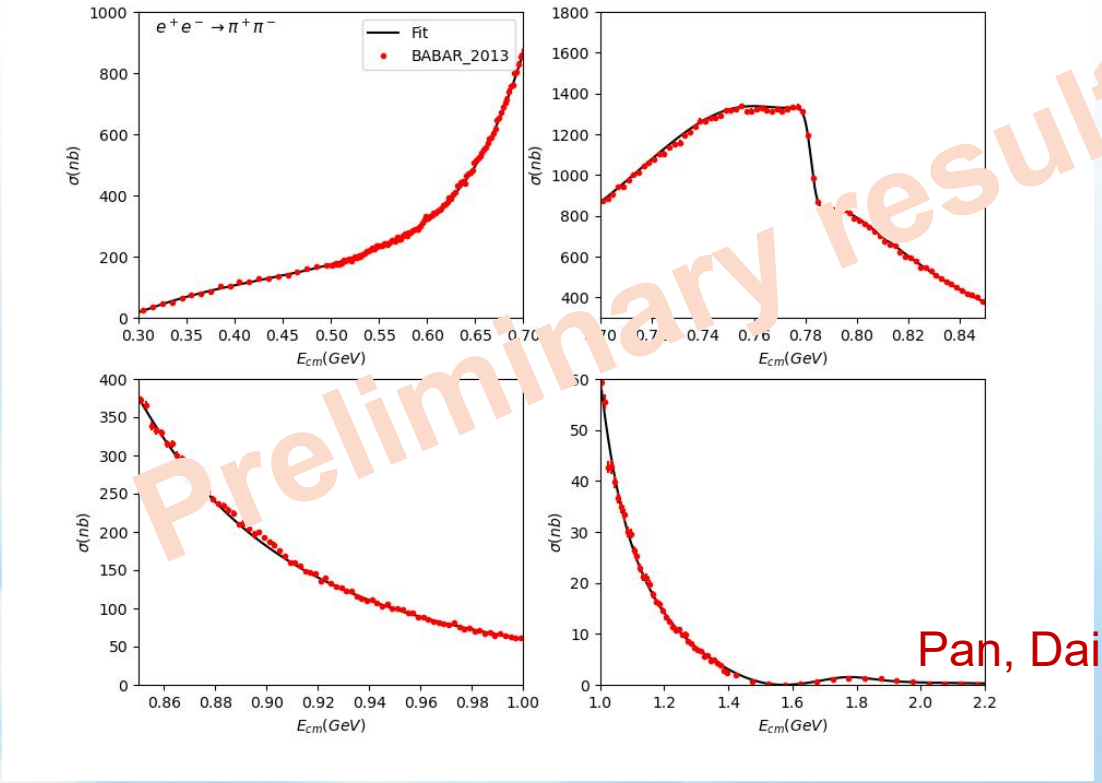


Pan, Dai, in preparation

- Measurements on 0.98-2.3 GeV can be helpful

BABAR

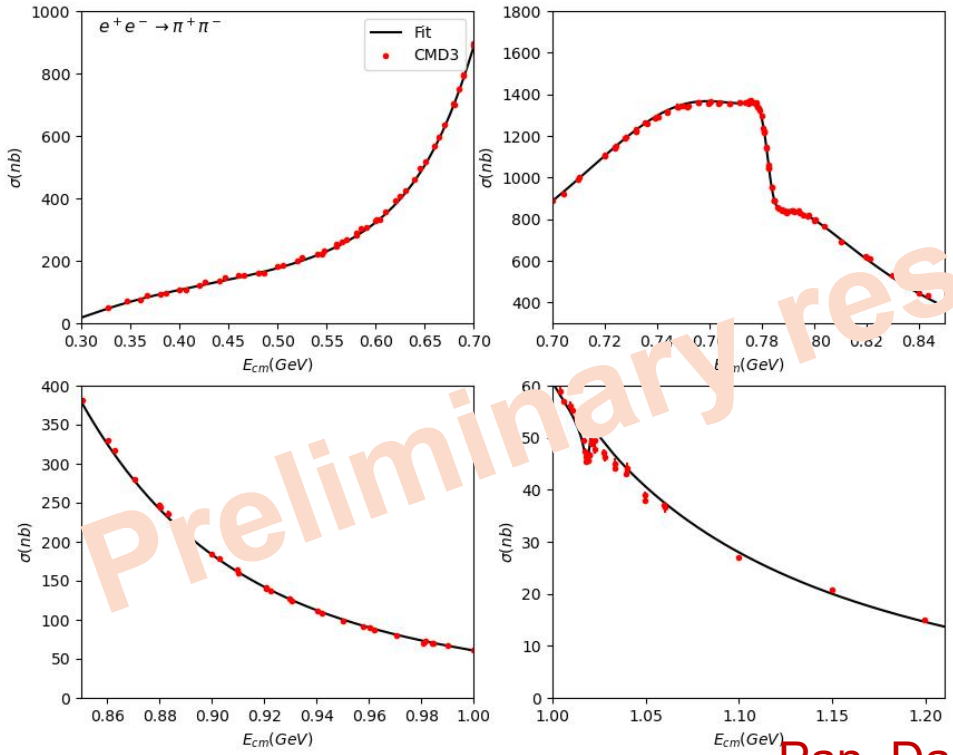
- Fit to BABAR data



Pan, Dai, in preparation

CMD-3

- Fit to CMD-3data



Preliminary results

Pan, Dai, in preparation

- Suggested $\rho - \phi$ mixing
  - Ours: $\Gamma_{\phi\to\pi\pi} = 4.061 \times 10^{-7} GeV$
  - PDG: $4.04 \pm 0.81 \times 10^{-7} GeV$

HVP from each dataset

- Individual analysis to each dataset (0.3-1.8GeV)

Experiment	Percentage covered	CHKLS	DHMZ	KNTW	Rcht1	chi/dof
SND06	93.80%	508.2(7.3)	505.3(6.7)	506.2(6.8)	504.63	1.51
CMD-2	8[illegible]%	506.2(5.0)	508.1(3.9)	508.4(3.7)	505.95	1.25
BaBar	99.90%	[illegible]	514.1(3.8)	514.4(3.8)	512.98	1.75
KLOE	97.20%	501.3(2.6, [illegible]	[illegible]	504.5(5.1)	500.60	1.50
BESIII	72.80%	500.8(5.2)	505.9(3.7)	5[illegible]	502.29	0.87
SND20	80.30%	505.5(5.9)	509.6(3.6)	508.3(4.2)	50[illegible]1	1.36
CMD-3	98.90%	524.1(1.3)	525.9(4.2)	526.0(4.1)	521.60	3.11

Preliminary results

Pan, Dai, in preparation

- Our model describe all datasets well
- A bit smaller result for CMD-3 data

Global analysis

- With or without CMD-3data

Rcht1	without CMD3	Global
aμ^HVP	506.33	518.04
chi/dof	2.59	5.79

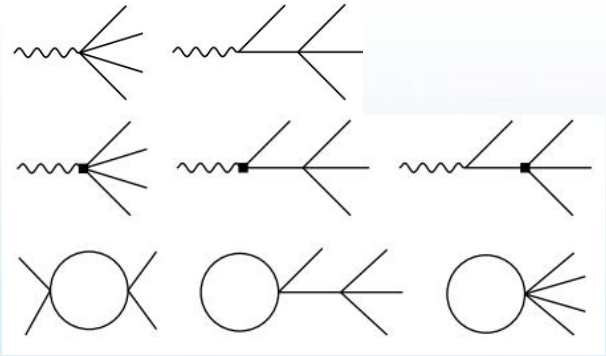
Preliminary results

Four body final states?

- ChPT up to O(p^4), for $\gamma^* \to 4\pi$

$$\mathcal{L}_{(2)}^{\mathrm{ChPT}} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+ \rangle$$

$$\begin{aligned}\mathcal{L}_{(4)}^{\mathrm{ChPT}} &= L_1\langle u_\mu u^\mu\rangle\langle u_\nu u^\nu\rangle + L_2\langle u_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle \\ &+ L_3\langle u_\mu u^\mu u_\nu u^\nu\rangle + L_4\langle u_\mu u^\mu\rangle\langle \chi_+\rangle + L_5\langle u_\mu u^\mu \chi_+\rangle \\ &+ L_6\langle \chi_+\rangle^2 + L_7\langle \chi_-\rangle^2 + \frac{L_8}{2}\langle \chi_+^2 + \chi_-^2\rangle \\ &- iL_9\langle f_+^{\mu\nu} u_\mu u_\nu\rangle + \frac{L_{10}}{4}\langle f_+^2 - f_-^2\rangle \\ &+ H_1\langle f_{\mu\nu}^R f_R^{\mu\nu} + f_{\mu\nu}^L f_L^{\mu\nu}\rangle + H_2\langle \chi\chi^\dagger\rangle.\end{aligned}$$

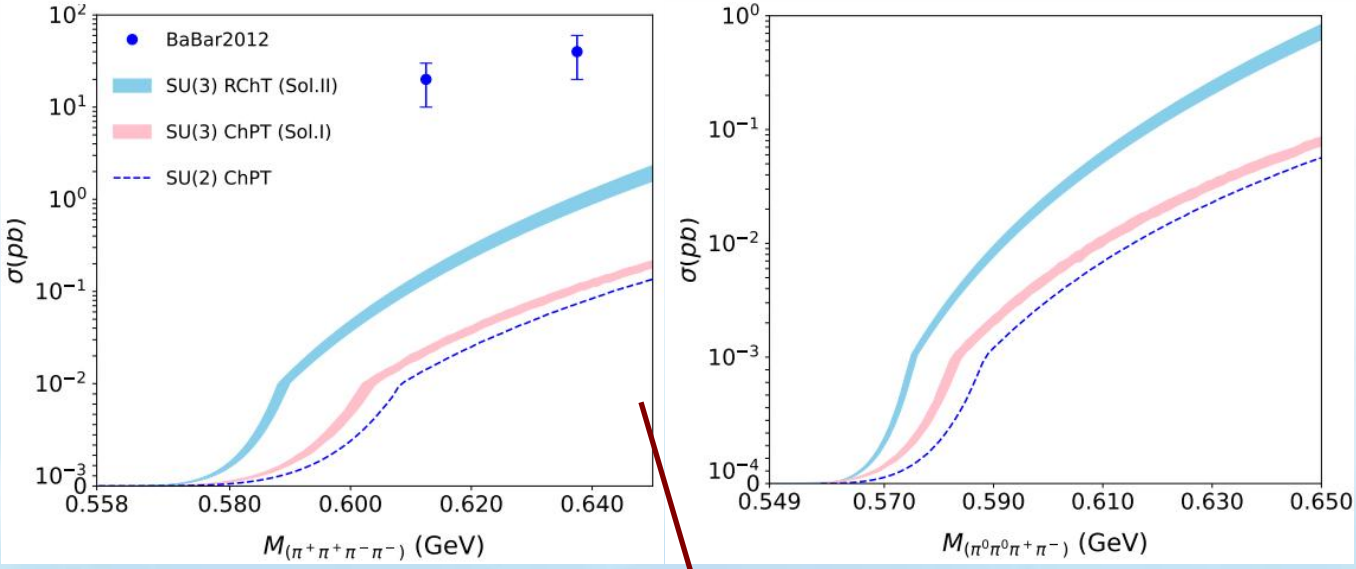


J. Bijnens, G. Ecker, ARNPS 64, 149 (2014)

- LECs well fixed, good prediction
- Only in the low energy region!

$\pi\pi\pi\pi$

- ChPT's 2-3 orders smaller than the data
- RChT's 1-2 orders smaller than the data

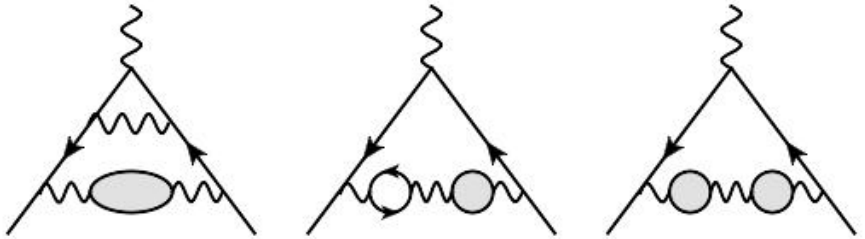


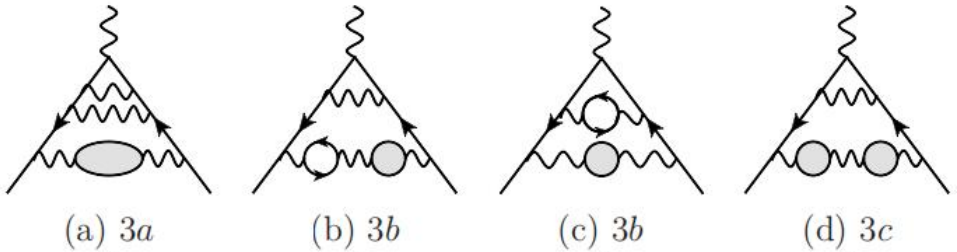
Zhou, Pan, Dai, arxiv: 2505.00770 [hep-ph]

- FSI effects
- Resonance contribution
- Furture measurements

HVP: NLO, NNLO?

- More channels (also high energy ones) to give a complete estimation?





$\times 10^{-12}$	$\pi\pi$	KK	$\pi\pi\pi$	$\pi\pi\eta$	Sum(4)	Kurz et al. (PLB 734,144)
2a	-1369±8	-79.8±2.8	-145±3	-5.93±0.46	-1600±9	-2090
2b	776±5	37.6±1.3	74.7±1.8	2.37±0.18	891±5	1068
2c			22.4±0.2		22.4±0.2	35
$a_\mu^{\rm NLO}$					-687±10	-987±9
3a	45.4±0.3	3.11±0.11	5.20±0.12	0.267±0.021	54.0±0.3	80
3b	-24.8±0.2	-1.62±0.06	-2.78±0.06	-0.131±0.010	-29.3±0.2	-41
3bLBL	58.0±0.3	3.47±0.12	6.19±0.14	0.268±0.021	67.9±0.4	91
3c			-2.34±0.02		-2.34±0.02	-6
3d			0.0249±0.0004		0.0249±0.0004	0.05
$a_\mu^{\rm NNLO}$					90.3±0.5	124±1

Kurz, et.al.
PLB 734 (2014) 144

Refine our results by considering other channels of three, four body final states .

4、Summary

FSI

Amplitude analsysis connects QFT principles and Exp. ChEFT+FSI are good tools for AA.

HVP

Ours has a significant discrepancy with the latest FNAL's. Processes of multi-body channels needs to be studied both from theory and exp. sides.

Test

Our model is applied to test each dataset. Our HVP from CMD-3 dataset is a bit smaller than others. The discrepancy between CMD-3 and other exp. groups is still unknown

Next?

Further study of multi-body final states processes--to refine the NLO HVP results; Discrepancy between LQCD v.s. data driven+ChEFT+FSI? τ-ee, Isospin breaking? NNLO ISR?

Thank You For your patience !

Backup

- Estimations from different experiments

Experiment	Percentage covered			
e^+e^-		CHKLS	DHMZ	KNTW
SND06	93.8%	508.2(7.3)	505.3(6.7)	506.2(6.8)
CMD-2	88.6%	506.2(5.0)	508.1(3.9)	508.4(3.7)
BaBar	99.9%	512.2(3.7)	514.1(3.8)	514.4(3.8)
KLOE	97.2%	501.3(2.6)	504.5(5.2)	504.5(5.1)
BESIII	72.8%	500.8(5.2)	505.9(3.7)	502.6(3.8)
SND20	80.3%	505.5(5.9)	509.6(3.6)	508.3(4.2)
CMD-3	98.9%	524.1(1.3)	525.9(4.2)	526.0(4.1)
τ		DHLMZ-23+LMR-24+WP25		
Belle+CLEO+ALEPH+OPAL	100%	517.2(5.8)		

Phys.Rept. 1143 (2025) 1

- The total IB effects are compatiable with LQCD's

	SD window		int window		LD window		full HVP	
	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$
$\pi^0\gamma$	0.16(0)	–	1.52(2)	–	2.70(4)	–	4.38(6)	–
$\eta\gamma$	0.05(0)	–	0.34(1)	–	0.31(1)	–	0.70(2)	–
$\omega(\to\pi^0\gamma)\pi^0$	0.15(0)	–	0.54(1)	–	0.19(0)	–	0.88(2)	–
FSR (2π)	0.12(0)	–	1.17(1)	–	3.13(3)	–	4.42(4)	–
FSR (3π)	0.03(0)	–	0.20(0)	–	0.28(1)	–	0.51(1)	–
FSR (K^+K^-)	0.07(0)	–	0.39(2)	–	0.29(2)	–	0.75(4)	–
ρ–ω mixing (2π)	–	0.06(1)	–	0.86(6)	–	2.87(12)	–	3.79(19)
ρ–ω mixing (3π)	–	−0.13(3)	–	−1.03(27)	–	−1.52(40)	–	−2.68(70)
pion mass (2π)	0.04(8)	–	−0.09(56)	–	−7.62(63)	–	−7.67(94)	–
kaon mass (K^+K^-)	−0.29(1)	0.44(2)	−1.71(9)	2.63(14)	−1.24(6)	1.91(10)	−3.24(17)	4.98(26)
kaon mass ($\bar{K}^0K^0$)	0.00(0)	−0.41(2)	−0.01(0)	−2.44(12)	−0.01(0)	−1.78(9)	−0.02(0)	−4.62(23)
sum	0.33(8)	−0.04(4)	2.34(57)	0.02(33)	−1.97(63)	1.48(44)	0.71(95)	1.47(80)
RBC/UKQCD18	–	–	0.0(2)	0.1(3)	–	–	−1.0(6.6)	10.6(8.0)
BMW20	–	–	−0.09(6)	0.52(4)	–	–	−1.5(6)	1.9(1.2)
JLM21	–	–	–	–	–	–	–	3.32(89)

From Bai-Long Hoid's talk at STCF2025