Phenomenological study on semi-leptonic inclusive decay of charm meson

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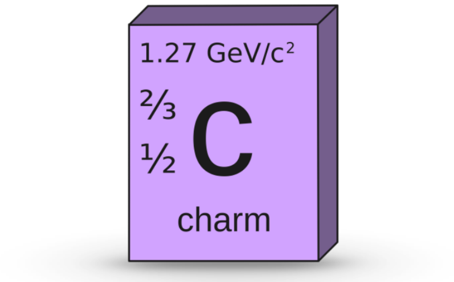

The 7th International Workshop on FTCF- HUANG SHAN

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    - **Determine the HQET basic parameters**
    - **Determine the CKM matrix elements**

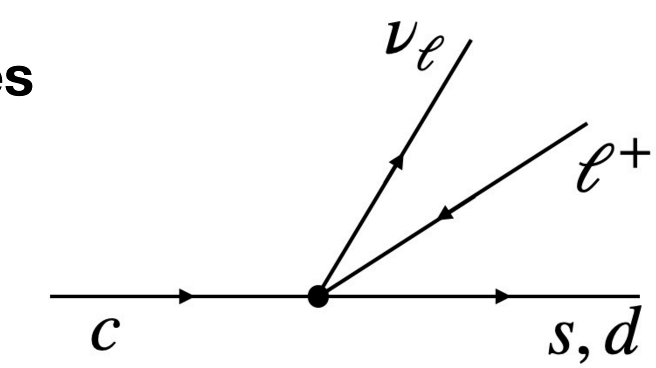
4. **Summary**

1. Why we study semi-leptonic inclusive D meson decays

What and Why?

- **Experimental detection of partial final state particles**

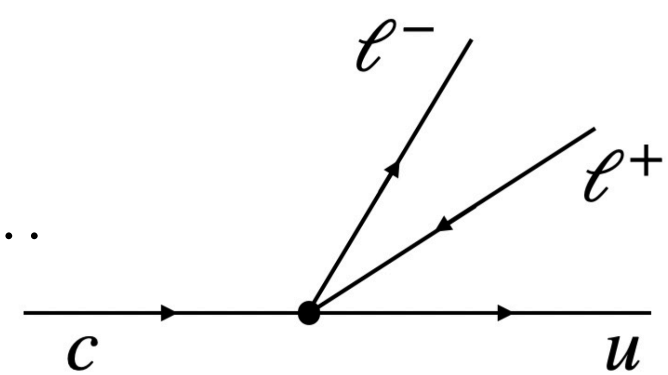
➡ $D \to e^+ X$ ($D \to e^+ \nu_e X$, only e^+ is detected)



- **Sum of a group of exclusive channels**

➡ $D^0 \to e^+ X_s = D \to e^+ \nu_e K^-, e^+ \nu_e K^- \pi^0, e^+ \nu_e \bar{K}^0 \pi^-, \ldots$

➡ $D^0 \to e^+ X_d = D \to e^+ \nu_e \pi^-, e^+ \nu_e \pi^- \pi^0, e^+ \nu_e \pi^- \pi^+ \pi^-, \ldots$



- **Compared to exclusive decays: Better theoretical control**

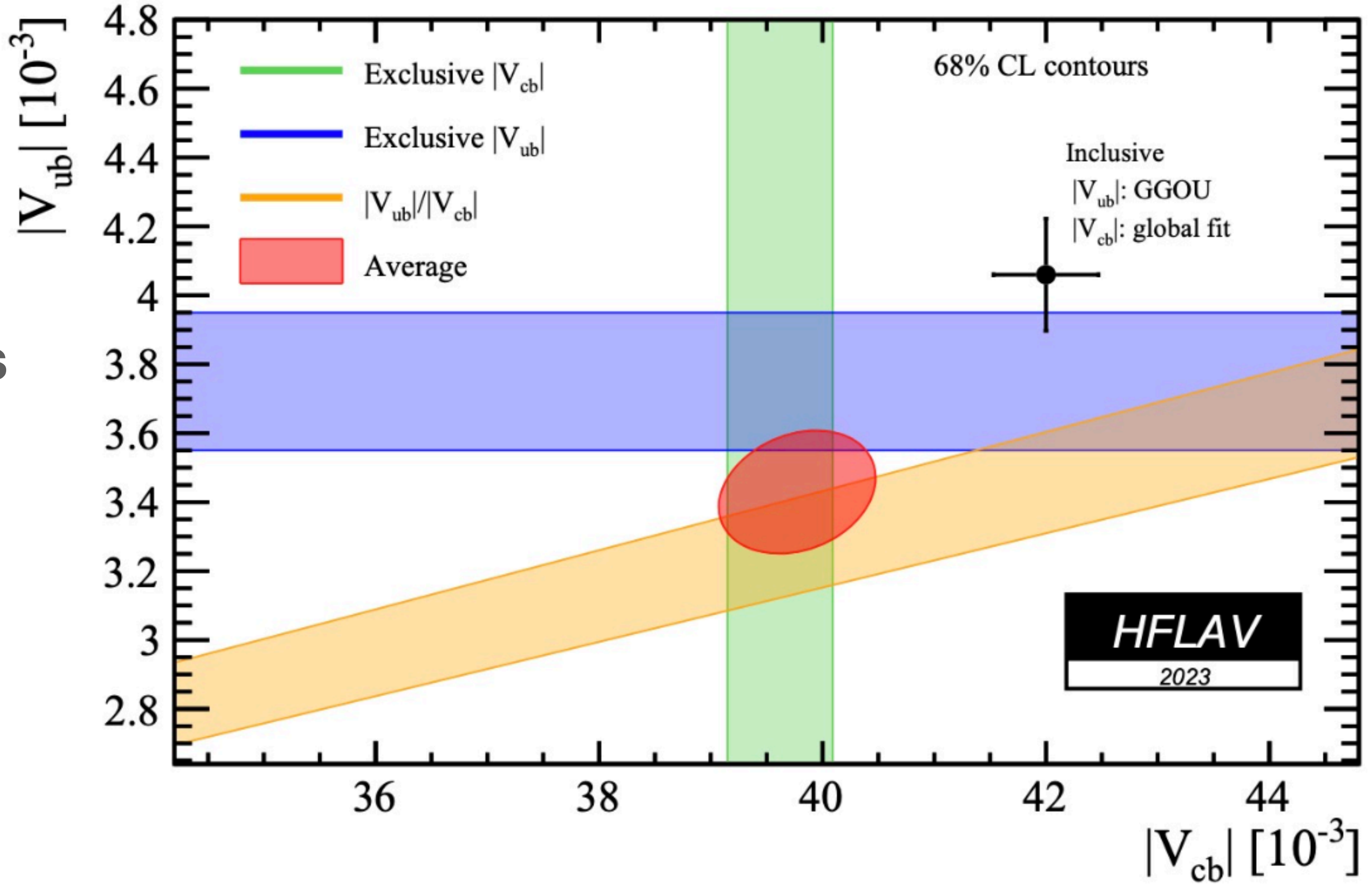
- **Compared to beauty decays: More sensitive to power corrections**

What and Why?

Flavor Puzzle V_{cb} and V_{ub}: Inclusive vs Exclusive

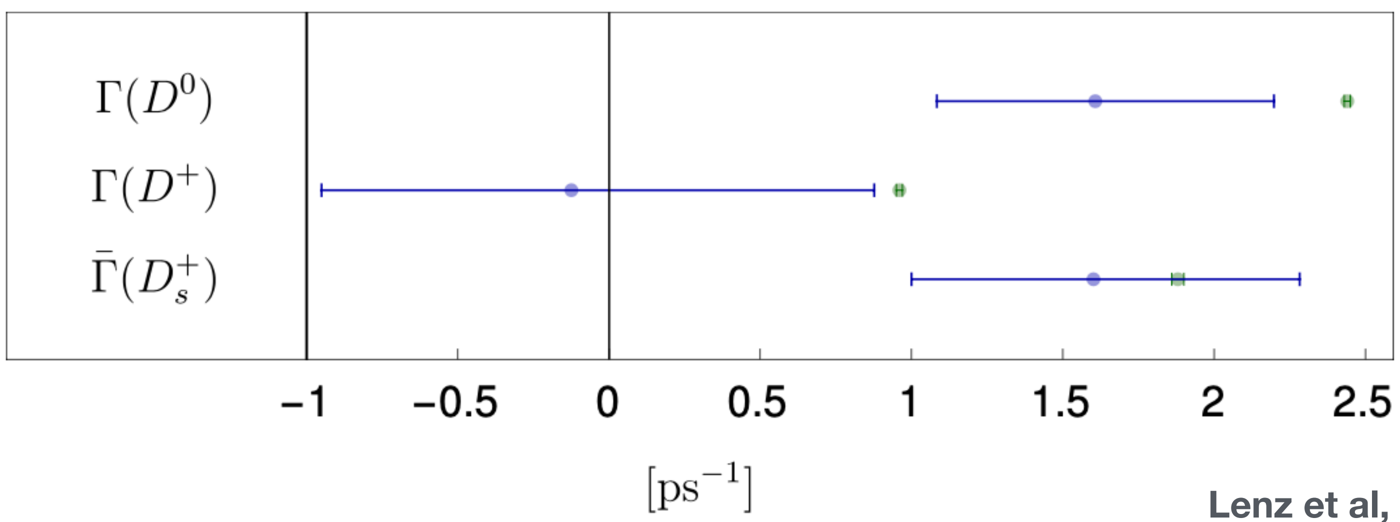
Key issue:
Systematic uncertainties from theoretical incl. and excl. frameworks

Hint:
Test Vcd and Vcs : incl. VS excl.
Await the first inclusive values



What and Why?

Charmed meson lifetimes:
theory vs experiment



Lenz et al, '22]

Fig 1

Meson

Experiment： High precision

Theory： significant uncertainty

2. Theoretical Framework

Theoretical Framework and Development

- **Optical theorem**

$$\sum \langle D|H|X\rangle\langle X|H|D\rangle \propto \mathrm{Im}\int \mathrm{d^4x\langle D|T\{H(x)H(0)\}|D\rangle}$$

- **Operator product expansion (OPE)**

  - ☆ Short distance : $x \sim 1/m_c$
  - ☆ Fluctuation in D meson $\sim \Lambda_{\mathrm{QCD}}$

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0) \to 1 + \frac{\Lambda_{\mathrm{QCD}}}{m_c} + \frac{\Lambda^2_{\mathrm{QCD}}}{m_c^2} + \dots$$

Systematic OPE in HQET.

- **Heavy Quark Effective Theory**

$$h_v(x) \equiv e^{-im_c v\cdot x}\frac{1+\gamma\cdot v}{2}c(x) \qquad v = (1,0,0,0)$$

$$L \ni \bar{h}_v iv\cdot Dh_v - \bar{h}_v\frac{D_\perp^2}{2m_c}h_v - a(\mu)g\bar{h}_v\frac{\sigma\cdot G}{4m_c}h_v + \dots$$

Subtract the big intrinsic momentum,

Leave only ~Λ_{QCD} degrees of freedom.

Theoretical Framework and Development

- **OPE**

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0)$$

$C_n(x)$

- ⭐ LO: $\alpha_s^0(m_c)$
- ⭐ NLO: $\alpha_s(m_c)$
- ⭐ **…**

$O_n(0)$

- ⭐ Dim-3: $\bar{h}_v h_v\ (\bar{c}\gamma^\mu c) \rightarrow$ **partonic decay rate.**
- ⭐ Dim-5: $\bar{h}_v D_\perp^2 h_v,\ g\bar{h}_v \sigma \cdot G h_v\,.$
- ⭐ Dim-6: $\bar{h}_v D_\mu (v \cdot D) D^\mu h_v,\ (\bar{h}_v \Gamma_1 q)(\bar{q}\Gamma_2 h_v),\ldots$
- ⭐ **…**

- **Contribute to inclusive decay rate and lifetime**

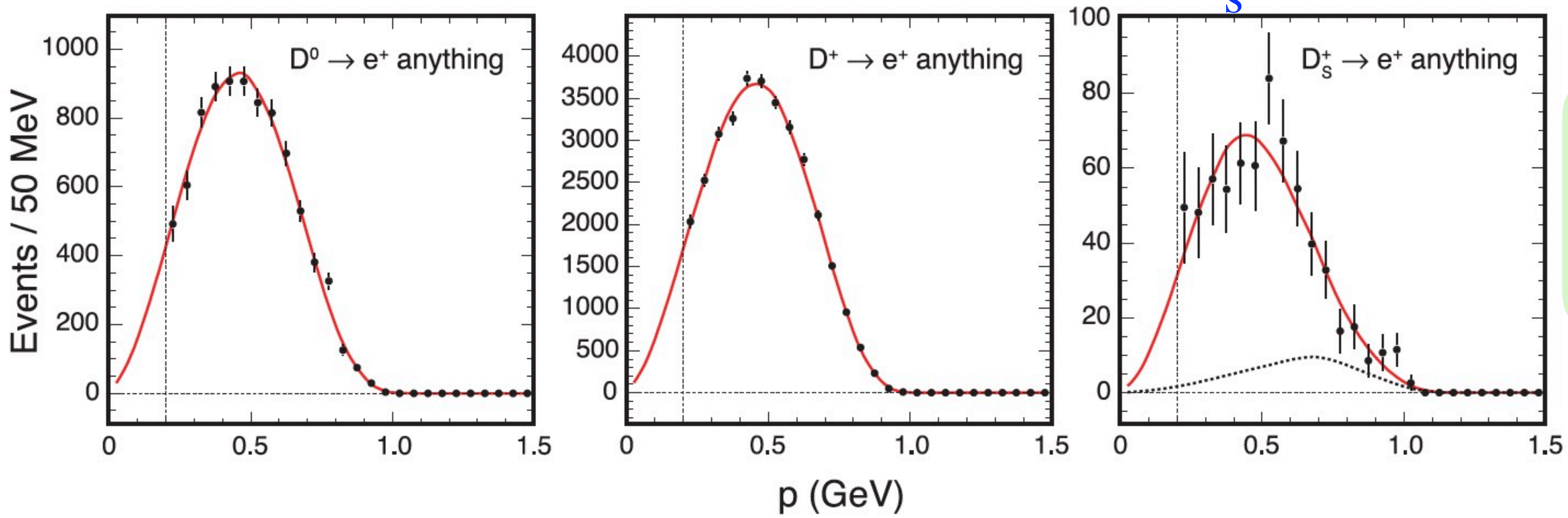
1. Matrix elements of the **same operators** (SL& NL)
2. Only different short-distance coefficients

3. Phenomenology

Experimental status

CLEO measurements

$D^0 \to e^+X$ $\quad D^+ \to e^+X$ $\quad D_s^+ \to e^+X$

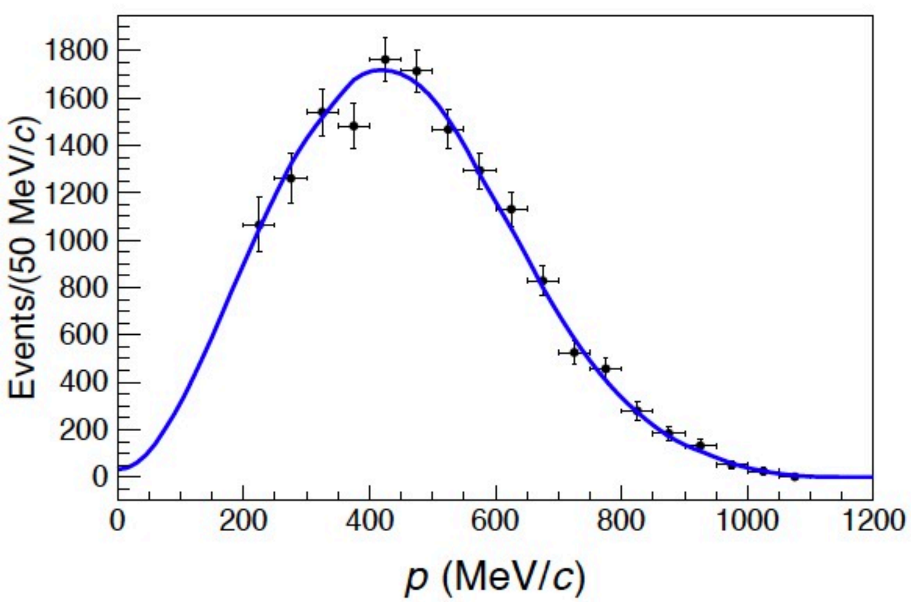


$$B\left(D^0 \to Xe^+\nu_e\right) = (6.46 \pm 0.09 \pm 0.11)\,\%$$
$$B\left(D^+ \to Xe^+\nu_e\right) = (16.13 \pm 0.10 \pm 0.29)\,\%$$
$$B\left(D_s^+ \to Xe^+\nu_e\right) = (6.52 \pm 0.39 \pm 0.15)\,\%$$

[CLEO, '09]

BESIII measurements

$D_s^+ \to e^+X$



[BESIII, '21]

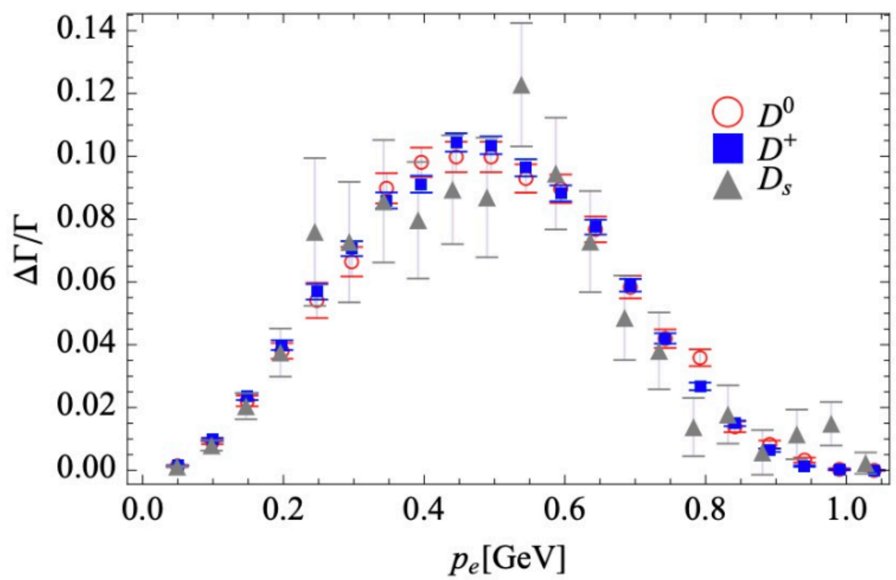
$$B\left(D_s^+ \to Xe^+\nu_e\right) = (6.30 \pm 0.13 \pm 0.10)\,\%$$

[BESIII, '21]

2% precision!

Phenomenological Progress

- Four quark operators were estimated



$$B_{\rm WA}^{(0),s}(D^0) = -0.001(3)\text{GeV}^3\,, \quad B_{\rm WA}^{(0),s}(D^+) = -0.001(3)\text{GeV}^3\,,$$
$$\bar{B}_{\rm WA}^{(1),s}(D^0) = -0.0001(6)\text{GeV}^3\,, \quad \bar{B}_{\rm WA}^{(1),s}(D^+) = -0.0001(6)\text{GeV}^3\,,$$
$$\bar{B}_{\rm WA}^{(2),s}(D^0) = -0.0001(10)\text{GeV}^3\,, \quad \bar{B}_{\rm WA}^{(2),s}(D^+) = -0.0002(10)\text{GeV}^3\,,$$
$$\bar{B}_{\rm WA}^{(\sigma),s}(D^0) = -0.0000(7)\text{GeV}^3\,, \quad \bar{B}_{\rm WA}^{(\sigma),s}(D^+) = -0.0000(7)\text{GeV}^3\,,$$

[Gambino, et al '10]

- Strong interaction running coupling constant have been estimated

$$\alpha_s = 0.377 \pm 0.008 \pm 0.114$$

[Wu, et al '24]

- Assuming non-perturbative parameters are **identical to those of B meson**
- Global Fitting under **kinetic mass scheme**

?

Starting from dimension-5!

3. Pheno-1: Determine the HQET parameters

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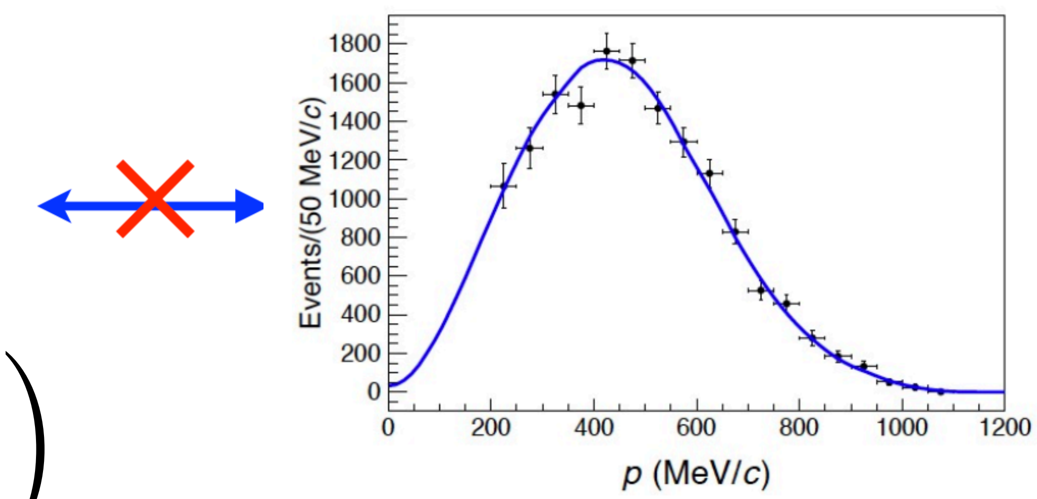
Global Fitting Strategy

Electronic Energy spectrum $\left(y \equiv 2E_e/m_c\right)$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = 12(1-y)y^2\theta(1-y)$$
$$+\frac{2\mu_\pi^2}{m_c^2}\left[-10y^3\theta(1-y) + 2\delta(1-y)\right.$$
$$-\frac{2\mu_G^2}{3m_c^2}\left[6y^2(6-5y)\theta(1-y)\right] + O\left(\alpha_s, \frac{\Lambda^3}{m_c^3}\right)$$



[BESIII, '21]

- Up to finite power, the obtained differential decay rate is **NOT** the experimental spectrum [Neubert, 1995] [Mannel et al,'94]
- Observables require integration over final states

$$\Gamma = \int \frac{d\Gamma}{dy}dy, \left\langle E_\ell^n \right\rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell^n dy, n = 1,2,3,4$$

[Gambino,Kamenik, '10]

Theoretical Results

Our main efforts on theory

NLO analytical integration

NNLO numerical results provided by Long Chen

$$\Gamma_{D_i} = \sum_{q=d,s} \hat{\Gamma}_0 |V_{cq}|^2 m_c^5 \Big\{ 1 + \frac{\alpha_s}{\pi} \frac{2}{3} \left(\frac{25}{4} - \pi^2 \right) + \frac{\alpha_s^2}{\pi^2} \left[\frac{\beta_0}{4} \frac{2}{3} \left(\frac{25}{4} - \pi^2 \right) \log\left(\frac{\mu^2}{m_c^2} \right) + 2.14690 n_l - 29.88311 \right]$$

[Chen, Chen, Guan, Ma, '23]

$$- 8\rho\delta_{sq} - \frac{1}{2} \frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{3}{2} \frac{\mu_G^2(D_i)}{m_c^2} + (6 + 8\log(\frac{\mu^2}{m_c^2})) \frac{\rho_D^3(D_i)}{m_c^3} + \frac{\tau_0(D_i)}{m_c^3} + \ldots \Big\},$$

Dim-5, $\Lambda_{\rm QCD}^2/{\rm m}_c^2$ Dim-6, $\Lambda_{\rm QCD}^3/{\rm m}_c^3$

$D \to e^+ X$ ($D \to e^+ \nu_e X$, only e^+ is detected)

Power correction： up to **dim-6** operators contributions

Perturbative correction： up to **NNLO** contributions

Phenomenological Analysis

The width of semi-leptonic inclusive decays in D mesons is **highly sensitive** to the charm quark mass.

- 1S mass scheme: **well perturbative behaviors** $1/2\ J/\psi$ mass

$$\Gamma/\Gamma_{\rm LO} \approx 1 - 13.1\% - 4.8\% + 1.8\,\%$$

[Hoang,Ligeti,Manohar, '98; Hoang,Teubner, '99]

- $\overline{\rm MS}$ mass scheme: **slow** convergence

$$\Gamma/\Gamma_{\rm LO} = 1 + 1.35\alpha_s + 3.02\alpha_s^2 + 7.69\alpha_s^3 \approx 1 + 52\% + 46\% + 44\,\%$$

[Melnikov,van Ritbergen, '99]

- Pole mass scheme**: no** convergence

Naive parameters of HQET

$$\Gamma/\Gamma_{\rm LO} = 1 - 0.77\alpha_s - 2.38\alpha_s^2 - 10.73\alpha_s^3 \approx 1 - 30\% - 36\% - 62\,\%$$

- Kinetic mass scheme: $\left(\alpha_s/\pi\right)\mu^n/m_c^n$ cut-off scale **somewhat subtle**

[Fael,Schönwald, Steinhauser, '20]

Theoretical Results

Our main efforts on theory

$$\langle E_e \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 m_c^6 \left[\frac{3}{10} + \frac{\alpha_s}{\pi} a_1^{(1)} + \frac{\alpha_s^2}{\pi^2} a_1^{(2)} - 3\rho\delta_{sq} - \frac{1}{2}\frac{\mu_G^2(D_i)}{m_c^2} + (\frac{139}{30} + 4\log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3(D_i)}{m_c^3} + \frac{3}{10}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \frac{\tau_0}{2m_c^3} + \dots \right],$$

$$\langle E_e^2 \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 m_c^7 \left[\frac{1}{10} + \frac{\alpha_s}{\pi} a_2^{(1)} + \frac{\alpha_s^2}{\pi^2} a_2^{(2)} - \frac{6}{5}\rho\delta_{sq} + \frac{1}{12}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{11}{60}\frac{\mu_G^2(D_i)}{m_c^2} + (\frac{17}{6} + 2\log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3(D_i)}{m_c^3} + \frac{7}{30}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \frac{\tau_0}{4m_c^3} + \dots \right],$$

$$\langle E_e^3 \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 m_c^8 \left[\frac{1}{28} + \frac{\alpha_s}{\pi} a_3^{(1)} + \frac{\alpha_s^2}{\pi^2} a_3^{(2)} - \frac{1}{2}\rho\delta_{sq} + \frac{1}{14}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{1}{14}\frac{\mu_G^2(D_i)}{m_c^2} + (\frac{223}{140} + \log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3(D_i)}{m_c^3} + \frac{1}{7}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \frac{\tau_0}{8m_c^3} + \dots \right],$$

$$\langle E_e^4 \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 m_c^9 \left[\frac{3}{224} + \frac{\alpha_s}{\pi} a_4^{(1)} + \frac{\alpha_s^2}{\pi^2} a_4^{(2)} - \frac{3}{14}\rho\delta_{sq} + \frac{3}{64}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{13}{448}\frac{\mu_G^2(D_i)}{m_c^2} + (\frac{481}{560} + \frac{1}{2}\log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3(D_i)}{m_c^3} + \frac{9}{112}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \frac{\tau_0}{16m_c^3} + \dots \right],$$

$$\left\langle E_e^2 \right\rangle_{\text{center}} \equiv \left\langle (E_e - \langle E_e \rangle)^2 \right\rangle$$

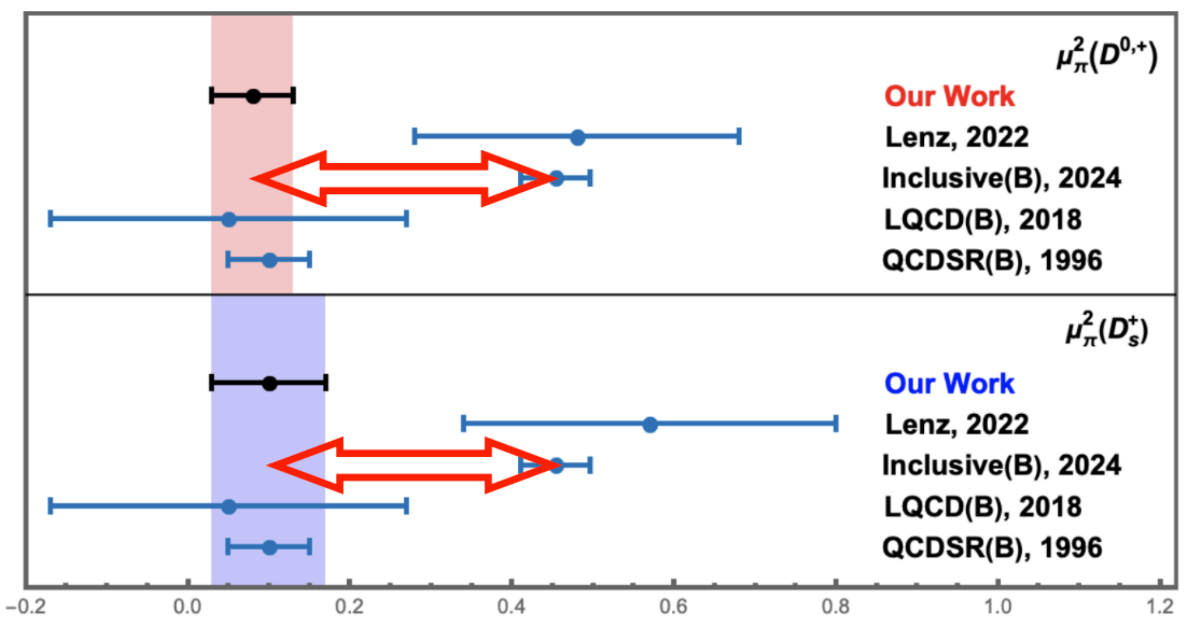
Fitting Results

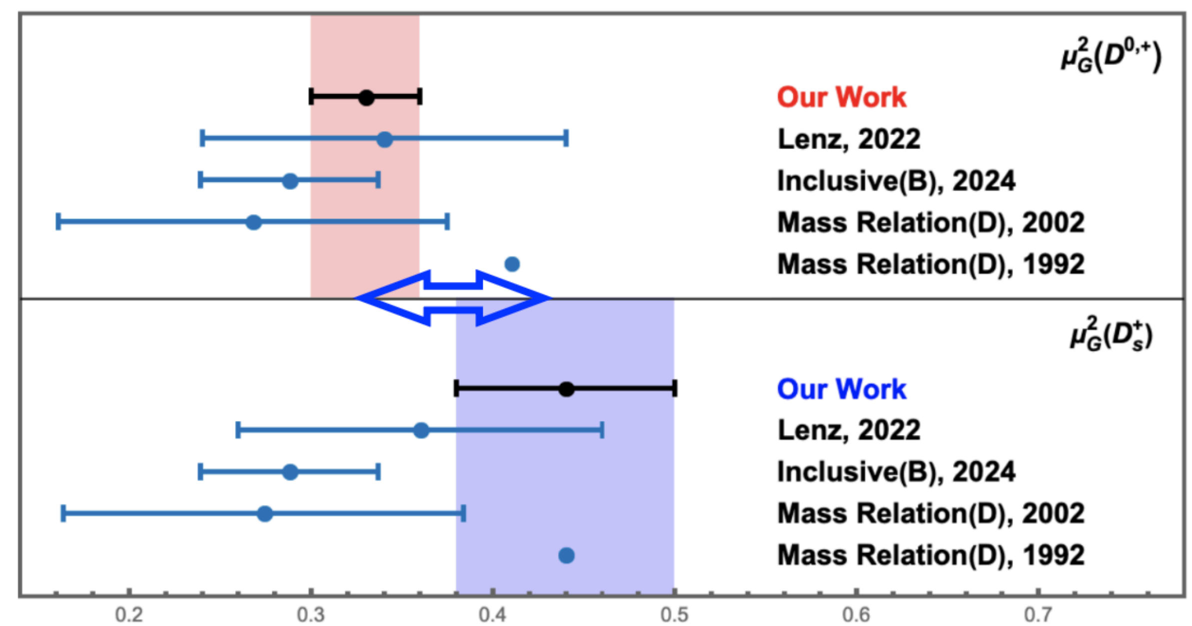
❖ Extracted nonperturbative HQE parameters

$$\mu_\pi^2(D^{0,+}) = (0.08 \pm 0.05)\mathrm{GeV}^2, \qquad \mu_\pi^2(D_s^+) = (0.10 \pm 0.07)\mathrm{GeV}^2,$$
$$\mu_G^2(D^{0,+}) = (0.33 \pm 0.03)\mathrm{GeV}^2, \qquad \mu_G^2(D_s^+) = (0.44 \pm 0.06)\mathrm{GeV}^2,$$
$$\rho_D^3(D^{0,+}) = (-0.003 \pm 0.002)\mathrm{GeV}^3, \qquad \rho_D^3(D_s^+) = (-0.004 \pm 0.002)\mathrm{GeV}^3,$$
$$\rho_{LS}^3(D^{0,+}) = (0.004 \pm 0.002)\mathrm{GeV}^3, \qquad \rho_{LS}^3(D_s^+) = (0.005 \pm 0.002)\mathrm{GeV}^3,$$

Sizable breaking effects of flavor SU(3) symmetry and heavy quark symmetry.

μ_π^2



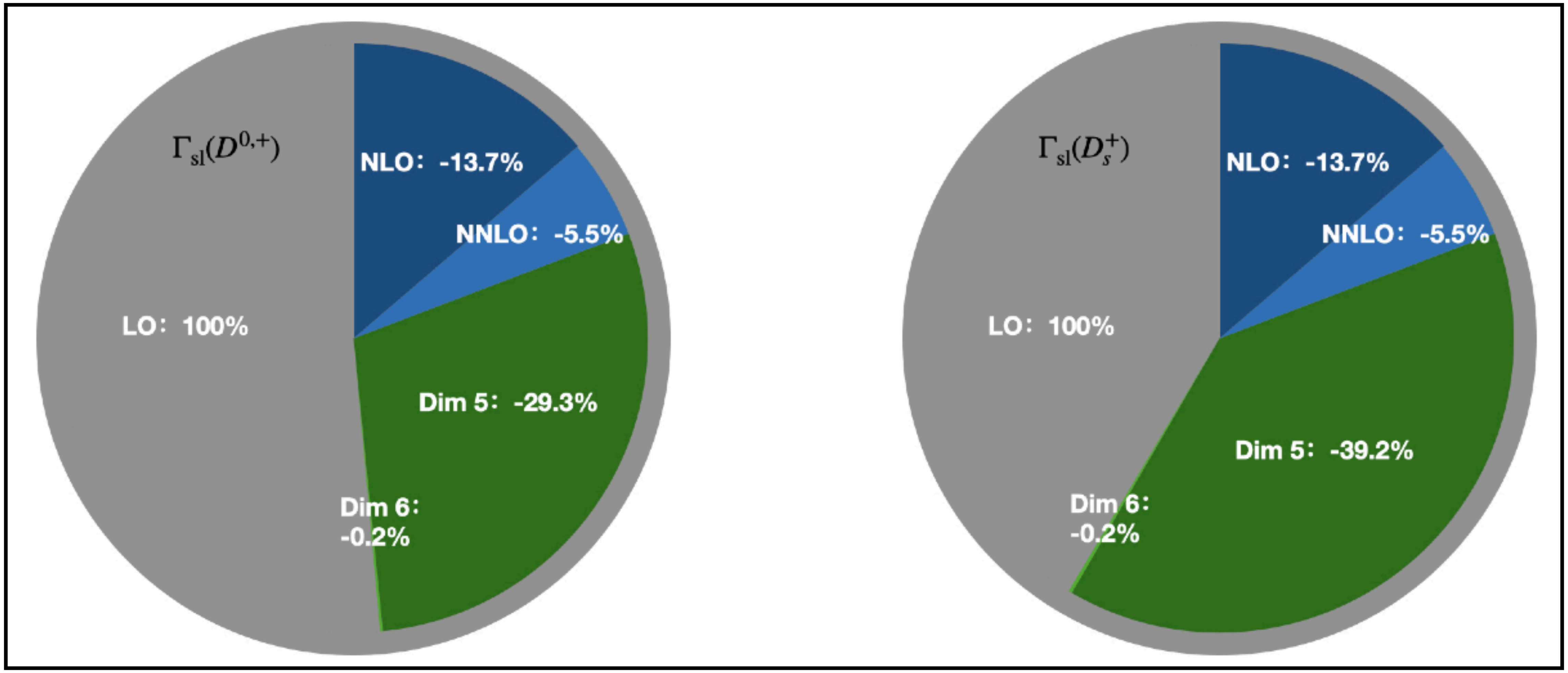


μ_G^2

Inappropriate to use beauty parameters for charm!

Convergence

The non-perturbative series exhibits **good** convergence behavior in D-meson semi-leptonic inclusive decays under the **1S mass** scheme



We propose the 1S mass scheme as the optimal choice for calculating charmed meson lifetimes

Weak Annihilation Uncertainty

❖ Redo the fits, adopting the HQET SR calculation for the weak annihilation contributions

$$\tau_0(D_d \to X_d) = \tau_0(D_s \to X_s) = \tau_{\rm val} = (-0.18 \pm 0.65)\ \text{GeV}^3$$ [King, Lenz, Piscopo, Rauh,'19]

$$\tau_0(D_u \to X_{d,s}) = \tau_0(D_d \to X_s) = \tau_0(D_s \to X_d) = \tau_{\rm nonval} = (0.45 \pm 2.10)\ \text{GeV}^3$$

$$\chi^2(\boldsymbol{\theta}) = \sum_{l=u,d,s} \sum_{i=1}^{5} \sum_{j=1}^{5} (y_{i,l} - \eta_{i,l})\, V_{ij,l}^{-1}\, (y_{j,l} - \eta_{j,l}) + \left(\frac{\tau_{\rm nonval} - (-0.18)}{0.65}\right)^2 + \left(\frac{\tau_{\rm val} - 0.45}{2.10}\right)^2$$

$$\mu_\pi^2(D^{0,+}) = 0.08\text{GeV}^2\ ,\ \mu_G^2(D^{0,+}) = 0.33\text{GeV}^2\ ,\ \rho_D^3(D^{0,+}) = -0.003\text{GeV}^3\ ,\ \rho_{LS}^3(D^{0,+}) = 0.004\text{GeV}^3$$
$$\mu_\pi^2(D_s) = 0.15\text{GeV}^2\ ,\ \mu_G^2(D_s) = 0.38\text{GeV}^2\ ,\ \rho_D^3(D_s) = -0.005\text{GeV}^3\ ,\ \rho_{LS}^3(D_s) = 0.006\text{GeV}^3\ ,$$
$$\tau_{\rm val} = -0.11\text{GeV}^3,\ \tau_{\rm nonval} = 0.002\text{GeV}^3\ .$$

The best-fit values only slightly change.

3. Pheno-2: Determine the CKM matrix elements

https://arxiv.org/abs/2509.11404

Kang Kang Shao, *Hai-Long Feng, Xue-Yin Han, Qin Qin, Fu-sheng Yu, Liang Sun*

Phenomenological Analysis

1. Treat Vcs and Vcd as free parameters rather than fixed inputs.
2. Difficult for incl. measurements to Xs and Xd → **Sum of excl. channels**

D^+ decays		D^0 decays		D_s decays	
Mode	BR(%)	Mode	BR(%)	Mode	BR(%)
$D^+ \to \bar{K}^0 e^+ \nu_e$	8.72 ± 0.09	$D^0 \to K^- e^+ \nu_e$	3.549 ± 0.026	$D_s \to \phi e^+ \nu_e$	2.34 ± 0.12
$(K^-\pi^+)_{\bar{K}^*(892)^0} e^+ \nu_e$	3.54 ± 0.09 [23]	$(\bar{K}^0\pi^-)_{\text{S-wave}} e^+ \nu_e$	0.079 ± 0.017	$D_s \to \eta e^+ \nu_e$	2.27 ± 0.06
$(K^-\pi^+)_{\text{S-wave}} e^+ \nu_e$	0.228 ± 0.011	$D^0 \to K^*(892)^- e^+ \nu_e$	2.04 ± 0.047 [24]	$D_s \to \eta' e^+ \nu_e$	0.81 ± 0.04
$D^+ \to \bar{K}_1(1270)^0 e^+ \nu_e$	0.230 ± 0.026	$D^0 \to \bar{K}_1(1270)^- e^+ \nu_e$	0.101 ± 0.018	$D_s \to f_0(980) e' \nu_e$	0.164 ± 0.013
$D^+ \to \eta e^+ \nu_e$	0.111 ± 0.007	$D^0 \to \pi^- e^+ \nu_e$	0.291 ± 0.004	$D_s \to K^0 e^+ \nu_e$	0.288 ± 0.026
$D^+ \to \pi^0 e^+ \nu_e$	0.372 ± 0.017	$D^0 \to \rho(770)^- e^+ \nu_e$	0.145 ± 0.007	$D_s \to K^*(892)^0 e^+ \nu_e$	0.205 ± 0.020
$D^+ \to \pi^+\pi^- e^+ \nu_e$	0.245 ± 0.008	$D^0 \to a(980)^- e^+ \nu_e$	$0.0133^{+0.0034}_{-0.0030}$		
$D^+ \to \pi^0\pi^0 e^+ \nu_e$	0.0315 ± 0.0027 [25]				
$D^+ \to \omega e^+ \nu_e$	0.169 ± 0.011				
$D^+ \to \eta' e^+ \nu_e$	0.020 ± 0.004				
$D^+ \to a(980)^0 e^+ \nu_e$	0.017 ± 0.008				

TABLE III. Branching ratios of selected exclusive decays of the D^+, D^0, and D_s mesons in units of percentage. All the values without references are taken from the PDG [21].

	D^+	D^0	D_s
$X_s e^+ \nu_e$	$14.60 \pm 0.16\%$	$5.81 \pm 0.06\%$	$5.58 \pm 0.14\%$
$X_d e^+ \nu_e$	$0.96 \pm 0.03\%$	$0.45 \pm 0.01\%$	$0.49 \pm 0.03\%$

TABLE I. The branching ratios of semi-leptonic inclusive D decays, derived from the sum of the experimentally well-measured exclusive decay modes.

$$B(D^+ \to X_{d+s} e^+ \nu_e) = 0.1602(32),$$
$$B(D^0 \to X_{d+s} e^+ \nu_e) = 0.0636(15),$$
$$B(D_s \to X_{d+s} e^+ \nu_e) = 0.0631(14),$$

In agreement with 1～2σ

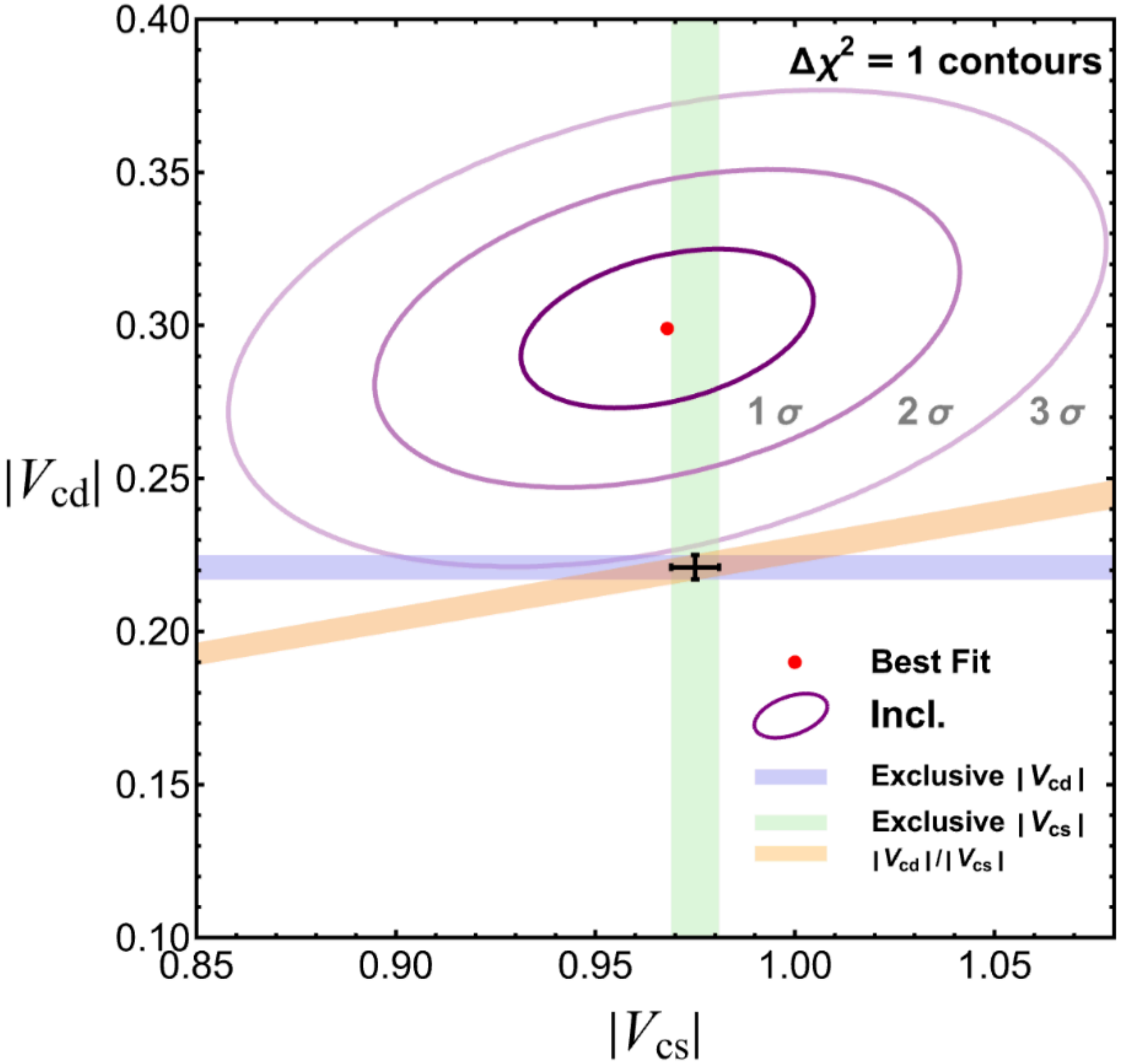
Phenomenological Analysis

- Data choice strategy: all inclusive X_{s+d} data are used
  - **S1**: Sum-of-exclusive X_s data, plus sum-of-exclusive X_d data,
  - **S2**: Sum-of-exclusive X_s data
  - **S3**: Sum-of-exclusive X_d data
- For each strategy, two scenarios for weak-annihilation contributions
  - **Scenario 1**: VIA, $\tau_0 = 0$
  - **Scenario 2**: HQET SR, input the previous best-fit values $\tau_{\rm val} = -0.11 \mathrm{GeV}^3, \ \tau_{\rm nonval} = 0.002 \mathrm{GeV}^3$.

Phenomenological Analysis

S1 $(X_{s,d})$	χ^2/d.o.f.	$\lvert V_{cs}\rvert$	$\lvert V_{cd}\rvert$
Scenario 1	3.24	$0.977 \pm 0.022 \pm 0.026$	$0.252 \pm 0.006 \pm 0.006$
Scenario 2	1.12	$0.969 \pm 0.022 \pm 0.026$	$0.253 \pm 0.006 \pm 0.006$
S2 (X_s)	χ^2/d.o.f.	$\lvert V_{cs}\rvert$	$\lvert V_{cd}\rvert$
Scenario 1	0.43	$0.974 \pm 0.022 \pm 0.026$	$0.279 \pm 0.031 \pm 0.007$
Scenario 2	0.29	$0.960 \pm 0.021 \pm 0.026$	$0.278 \pm 0.031 \pm 0.006$
S3 (X_d)	χ^2/d.o.f.	$\lvert V_{cs}\rvert$	$\lvert V_{cd}\rvert$
Scenario 1	4.08	$0.982 \pm 0.023 \pm 0.027$	$0.253 \pm 0.006 \pm 0.005$
Scenario 2	1.24	$0.974 \pm 0.022 \pm 0.026$	$0.254 \pm 0.006 \pm 0.006$

$$|V_{cs}| = 0.968 \pm 0.022 \pm 0.026 \pm 0.014$$
$$|V_{cd}| = 0.299 \pm 0.025 \pm 0.007 \pm 0.002$$



Inclusive vs exclusive: 3σ tension!

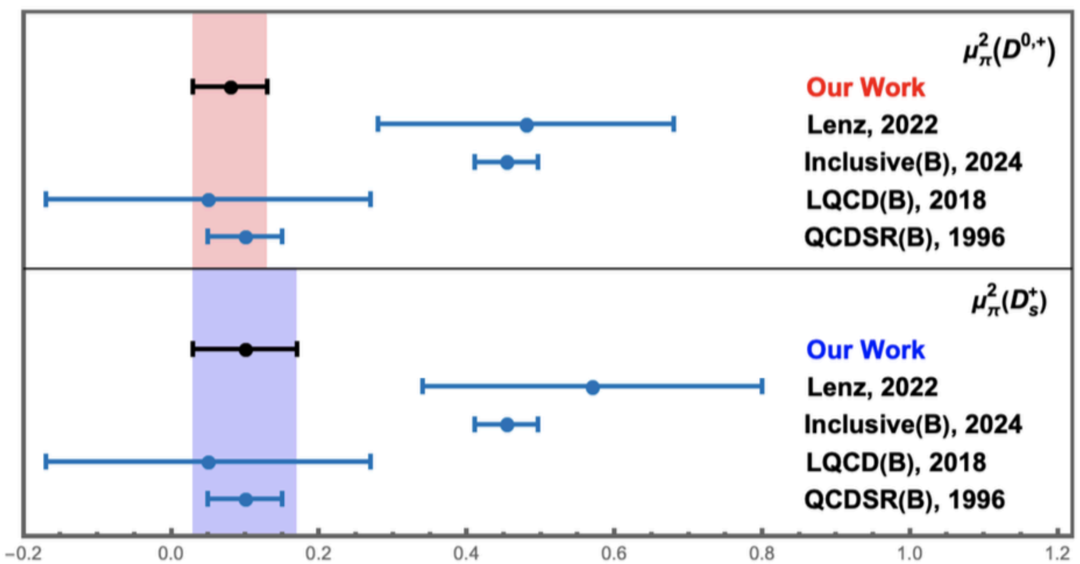
Summary

Phenomenology of inclusive D decays

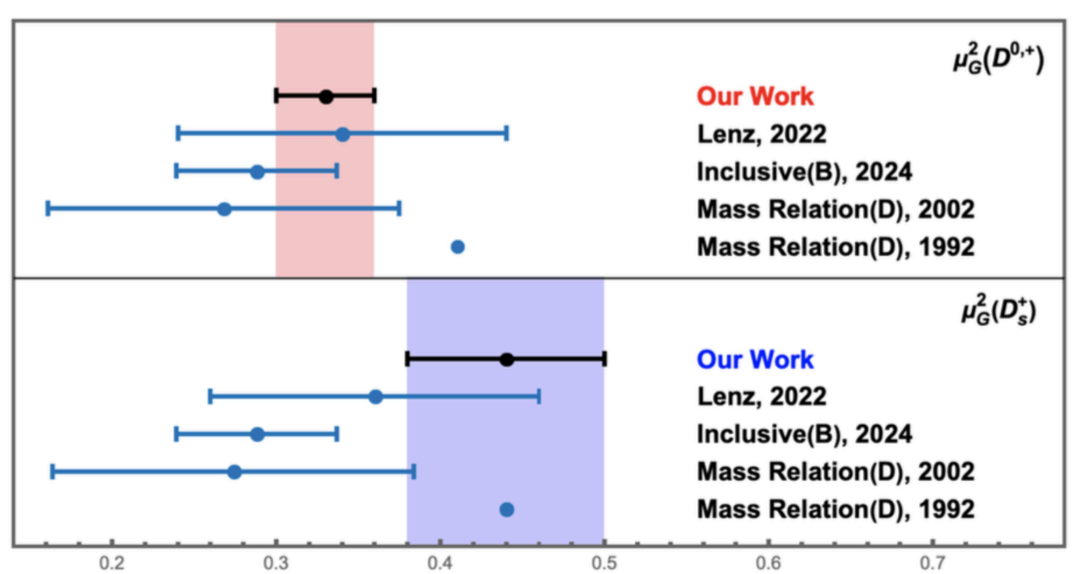
❖ By fitting the HQET formulas for inclusive D decays to data, we determine for the first time

➡ The **HQE parameters** for D mesons

μ_π^2



μ_G^2



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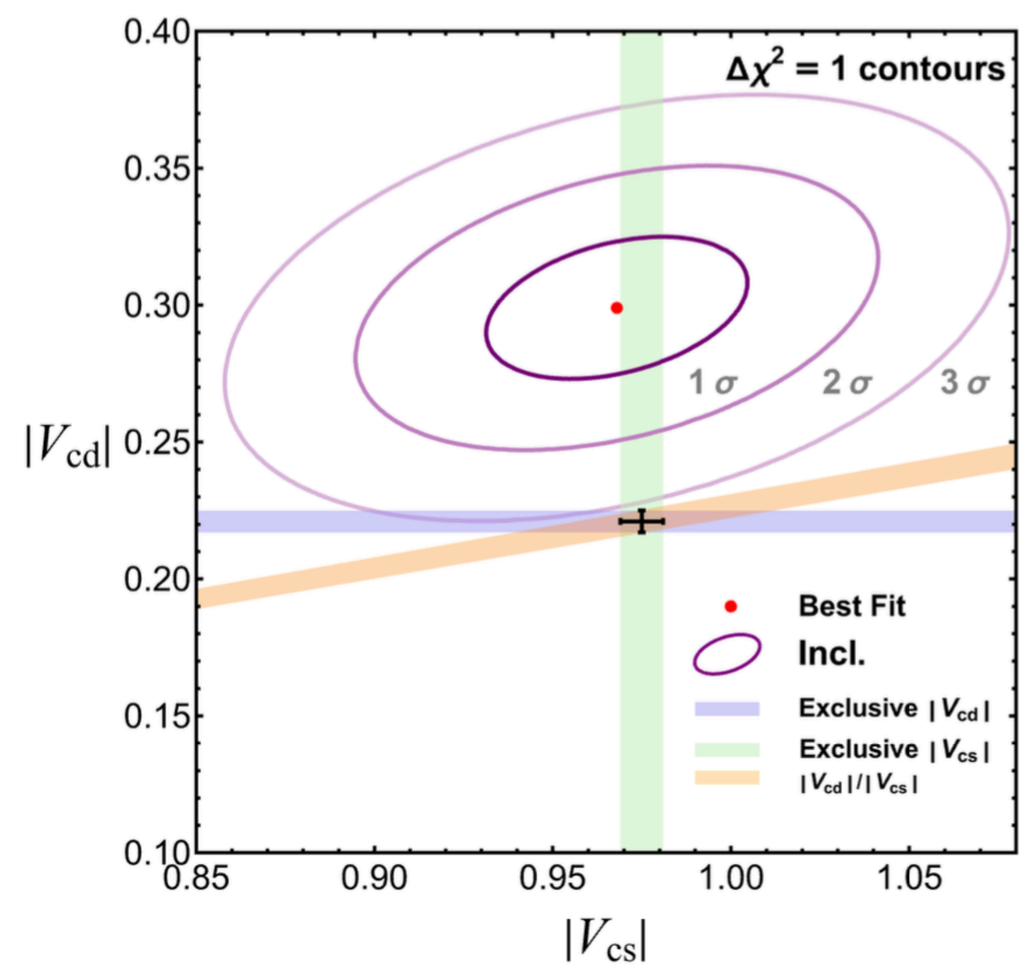
➡ The **CKM matrix elements** $V_{cs,cd}$ (inclusive)



[Shao,Huang,**QQ**,2502.05901]

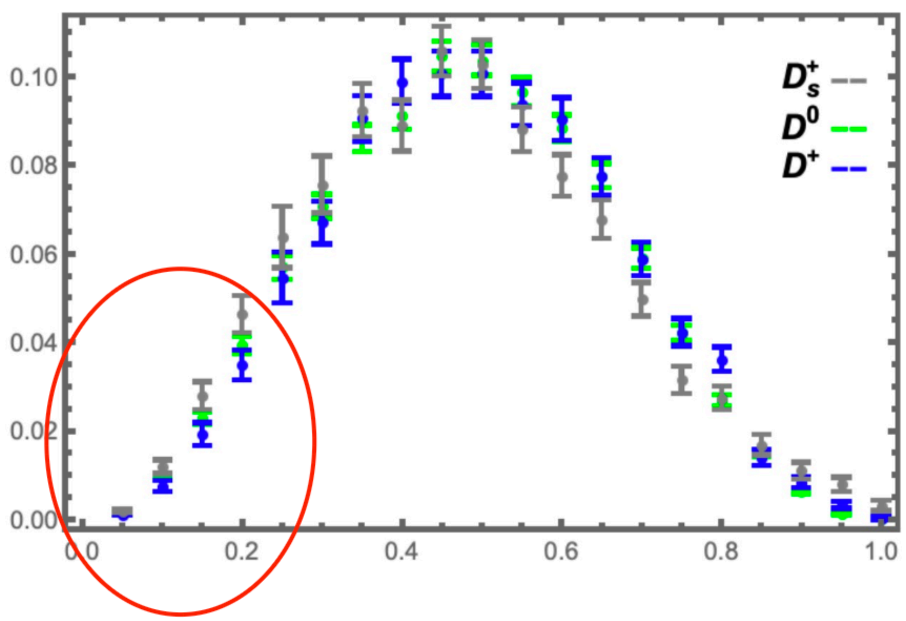
[Shao,Feng,Liu,**QQ**,Sun,Yu,2509.11404]

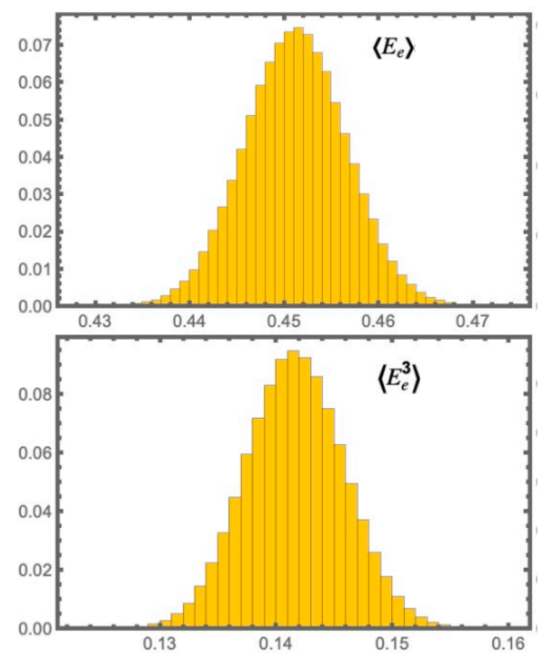
Appendix

Data Aspect

$$\frac{d\Gamma}{dy} = ay^2(1+by)(1-y)$$

Original data → **Extrapolation** [Gambino,Kamenik, '10] → **All-bin distribution** → **Lorentz boost** → **Rest frame** → **MC Simulation** → **Electron energy moments**





$$\langle E_e \rangle^{D_s}_{exp} = 0.437(6)\text{GeV}, \quad \langle E_e^2 \rangle^{D_s}_{exp} = 0.220(5)\text{GeV}^2$$
$$\langle E_e \rangle^{D^0}_{exp} = 0.462(5)\text{GeV}, \quad \langle E_e^2 \rangle^{D^0}_{exp} = 0.242(5)\text{GeV}^2$$
$$\langle E_e \rangle^{D^+}_{exp} = 0.455(4)\text{GeV}, \quad \langle E_e^2 \rangle^{D^+}_{exp} = 0.236(4)\text{GeV}^2$$

$$\langle E_e^3 \rangle^{D_s}_{exp} = 0.121(4)\text{GeV}^3, \quad \langle E_e^4 \rangle^{D_s}_{exp} = 0.072(3)\text{GeV}^4$$
$$\langle E_e^3 \rangle^{D^0}_{exp} = 0.138(4)\text{GeV}^3, \quad \langle E_e^4 \rangle^{D^0}_{exp} = 0.084(3)\text{GeV}^4$$
$$\langle E_e^3 \rangle^{D^+}_{exp} = 0.134(3)\text{GeV}^3, \quad \langle E_e^4 \rangle^{D^+}_{exp} = 0.081(3)\text{GeV}^4$$

Statistical uncertainties uncorrelated,
Systematic uncertainties fully correlated.

Fitting Results

❖ Global fit in two mass schemes, each with **Scenario 1** ($\Lambda_{\rm QCD}^2/m_c^2$) and **Scenario 2** ($\Lambda_{\rm QCD}^3/m_c^3$)

4-q operator contributions vanish under VIA, $\tau_0 = 0$.

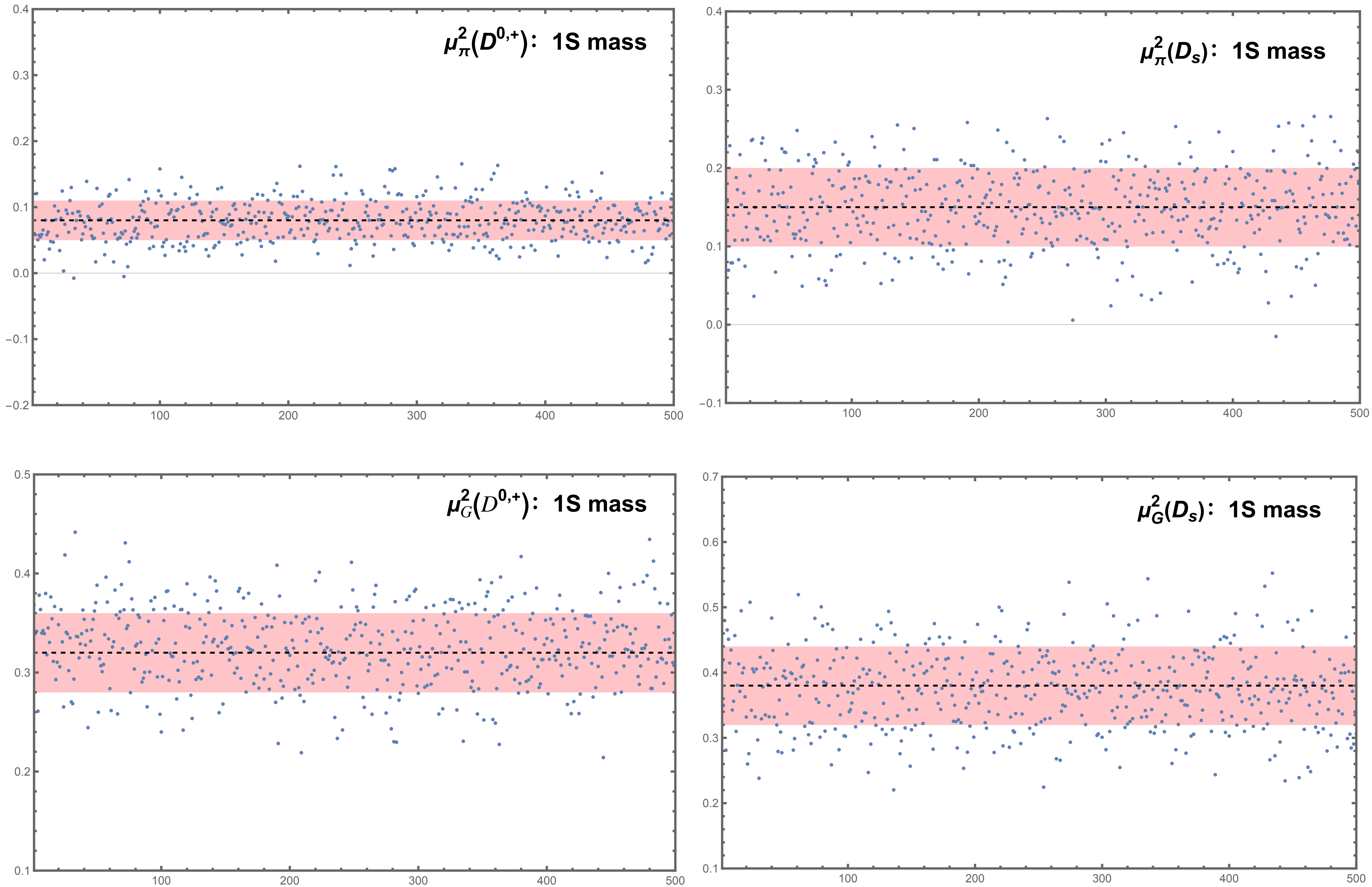
$\overline{\rm MS}$ scheme	χ^2/d.o.f.	D_i	$\mu_\pi^2/{\rm GeV}^2$	$\mu_G^2/{\rm GeV}^2$	$\rho_{\rm D}^3/{\rm GeV}^3$	$\rho_{LS}^3/{\rm GeV}^3$
Scenario 1	4.5	$D^{0,+}$	0.09 ± 0.01	0.27 ± 0.14	-	-
		D_s	0.09 ± 0.02	0.39 ± 0.12	-	-
Scenario 2	2.1	$D^{0,+}$	0.11 ± 0.02	0.26 ± 0.14	-0.002 ± 0.002	0.003 ± 0.002
		D_s	0.12 ± 0.02	0.38 ± 0.13	-0.003 ± 0.002	0.005 ± 0.002

1S scheme	χ^2/d.o.f.	D_i	$\mu_\pi^2/{\rm GeV}^2$	$\mu_G^2/{\rm GeV}^2$	$\rho_{\rm D}^3/{\rm GeV}^3$	$\rho_{LS}^3/{\rm GeV}^3$
Scenario 1	4.9	$D^{0,+}$	0.04 ± 0.01	0.33 ± 0.02	-	-
		D_s	0.06 ± 0.02	0.44 ± 0.02	-	-
Scenario 2	0.33	$D^{0,+}$	0.09 ± 0.02	0.32 ± 0.02	-0.003 ± 0.002	0.004 ± 0.002
		D_s	0.11 ± 0.02	0.43 ± 0.02	-0.004 ± 0.002	0.005 ± 0.002

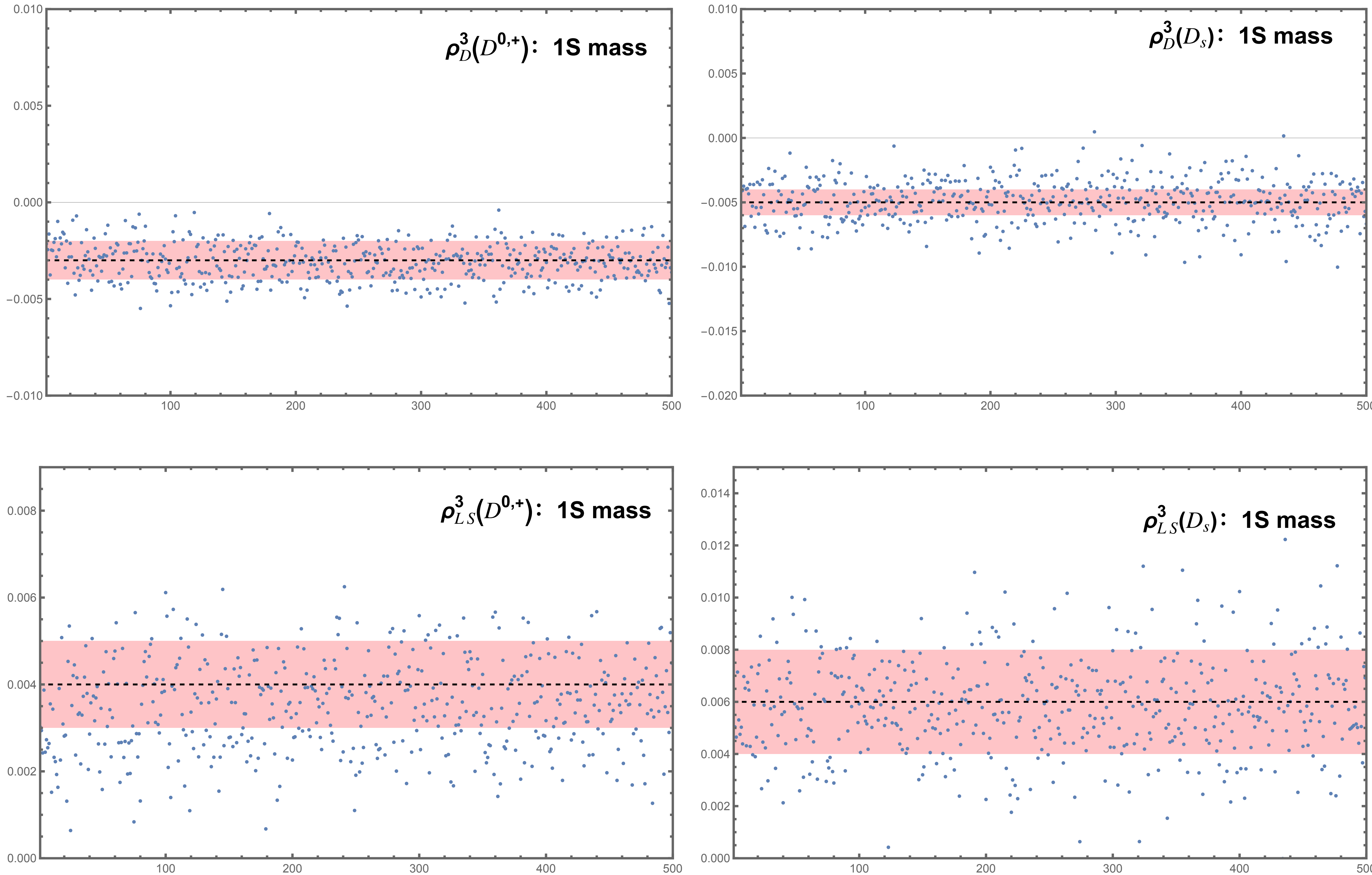
Reliable perturbative calculation ensures a good fit!

Differences between Scenarios 1 and 2 as systematic uncertainties.

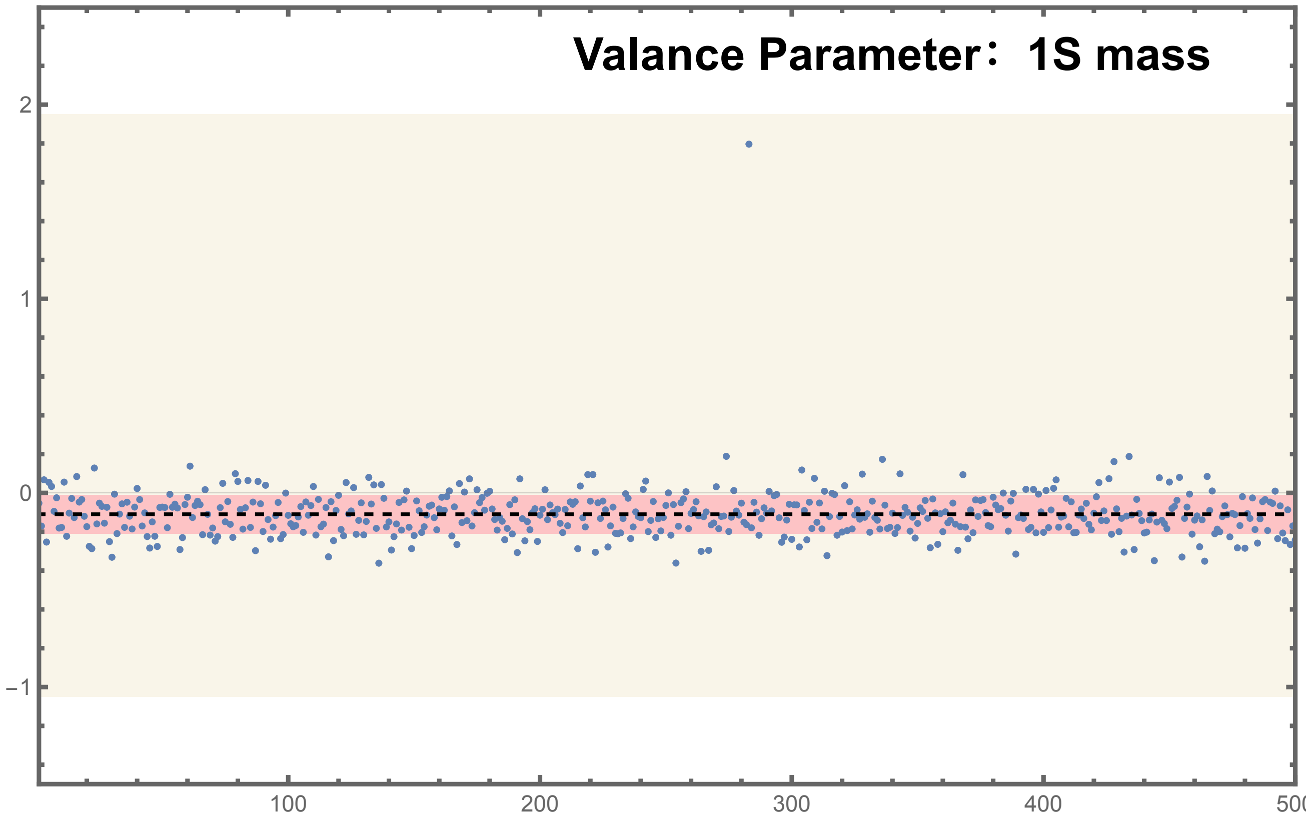
1S mass scheme robust Testing： scenario 3

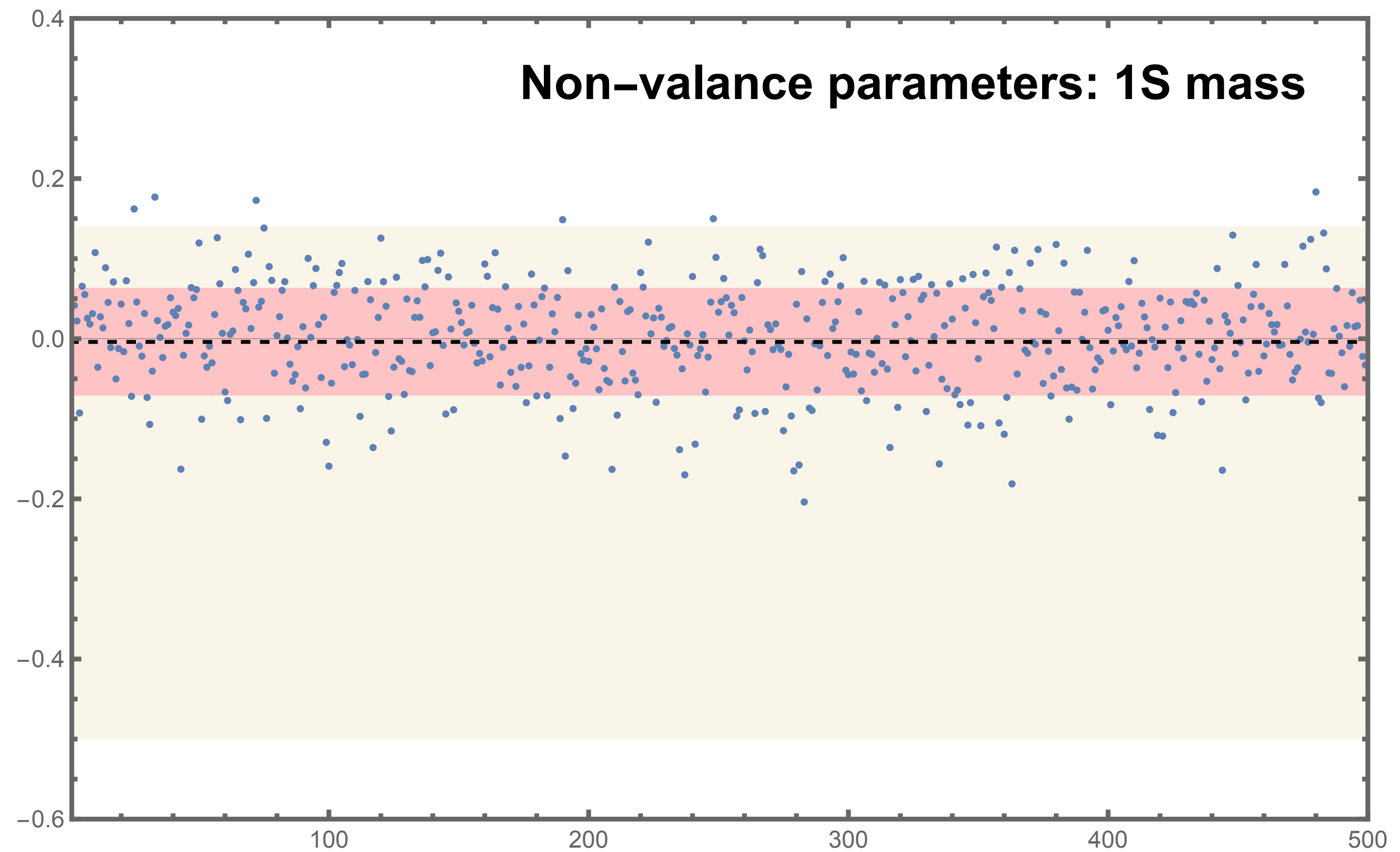


1S mass scheme robust Testing： scenario 3



1S mass scheme robust Testing： scenario 3





形状函数

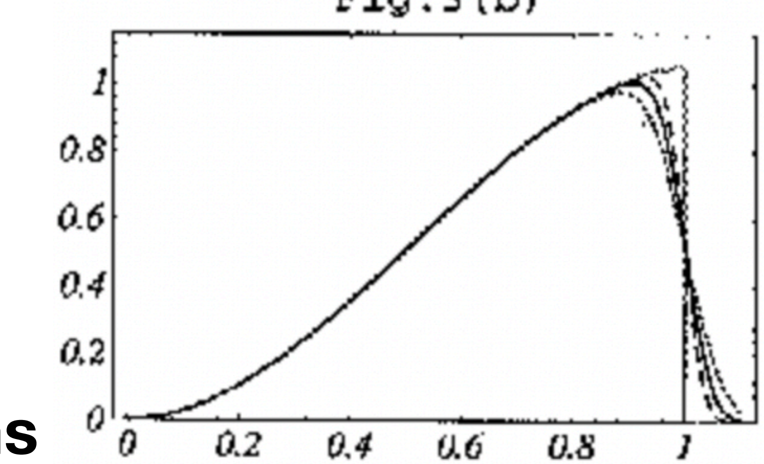
b to u 跃迁电子能谱

$$\frac{1}{2\Gamma_b}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = yF(y)\Theta(1-y) - \frac{\lambda_1+33\lambda_2}{6m_b^2}\delta(1-y) - \frac{\lambda_1}{6m_b^2}\delta'(1-y)$$

$$\frac{1}{2\Gamma_b}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = yF(y)\frac{1}{N}\sum_{i=1}^{N}\Theta\left(1-y+\varepsilon_i\right)$$

$$\delta y = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i = -\frac{\lambda_1+33\lambda_2}{6m_b^2},\quad \sigma_y^2 = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i^2 = -\frac{\lambda_1}{3m_b^2}$$

$$\frac{1}{N}\sum_{i=1}^{N}\Theta\left(1-y+\varepsilon_i\right) \xrightarrow{N\to\infty} \vartheta(y) = \Theta(1-y) + S(y)F(1)/F(y)$$



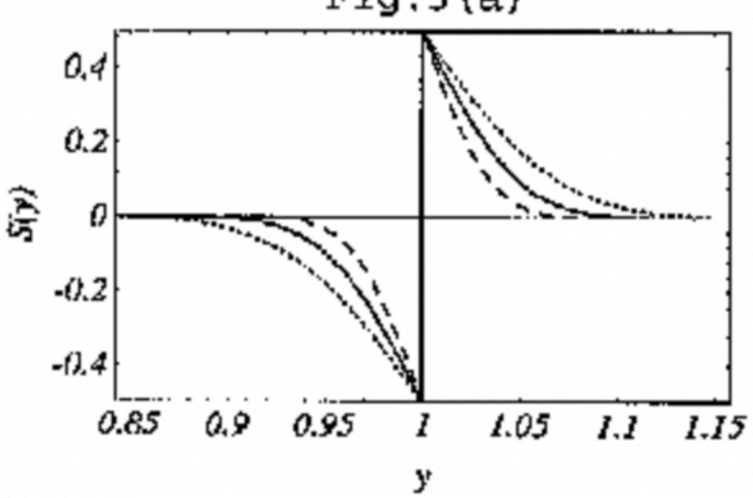
$$S(y) = \left\langle \Theta\left[1-y+\frac{2}{m_b}(v-\hat{p})\cdot iD\right] - \Theta(1-y)\right\rangle + \textbf{ less singular terms}$$

P: 轻子动量
D：b夸克动量

部分子模型
束缚态效应 $\phi\left(\left|\vec{p}_b\right|\right)$
忽略胶子场　A.C.M. 模型

$$\phi\left(\left|\vec{p}_b\right|\right) = \frac{4}{\sqrt{\pi}p_F^3}\exp\left(-\frac{\left|\vec{p}_b\right|^2}{p_F^2}\right)$$

$$S_{\mathrm{ACM}}(y) = \int\cdots\phi\left(\left|\vec{p}_b\right|\right)\cdots\mathrm{d}\vec{p}_b = \left[\frac{1}{2}-\Theta(1-y)\right]\Phi\left(\frac{m_b}{p_F}|y-1|\right)$$



粲介子寿命与半轻单举衰变宽度

- 同位旋对称性: D^0, D^+

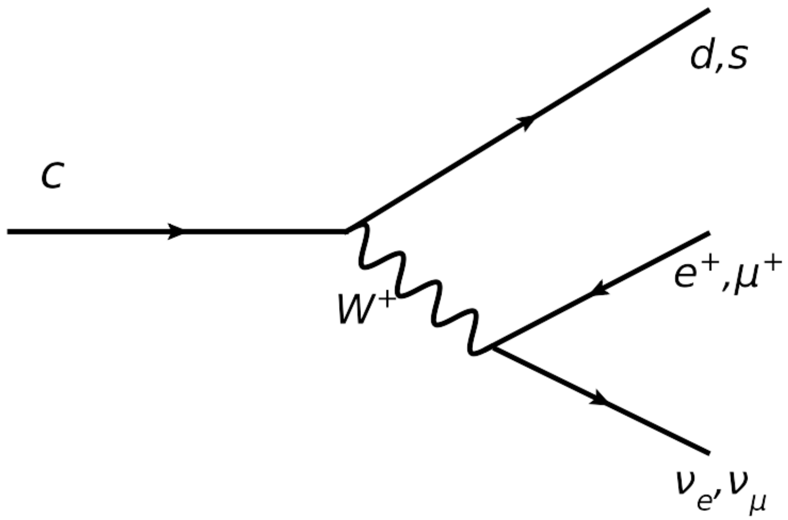
强子化过程中

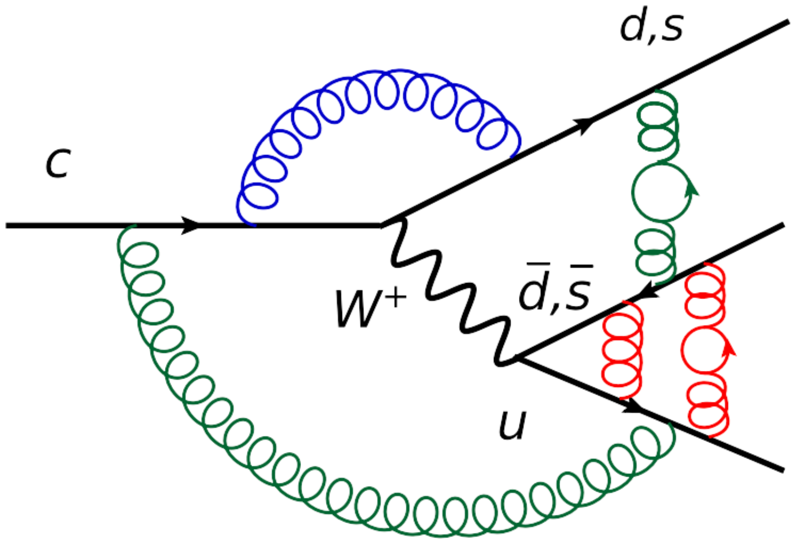
Quantity	D^0	D^+	D_s^+
$\tau\,[ps]$	0.4101 ± 0.0015	1.040 ± 0.007	0.504 ± 0.004
$\Gamma\,[ps^{-1}]$	2.438 ± 0.009	0.962 ± 0.006	1.984 ± 0.0016
$BR(D_i \to Xe\nu)\,[\%]$	6.49 ± 0.16	16.07 ± 0.30	6.30 ± 0.16
$\Gamma(D_i \to Xe\nu)\,[ps^{-1}]$	0.158 ± 0.004	0.155 ± 0.003	0.125 ± 0.003

旁观者效应贡献很小

HQETSR, VIA

- 粲介子寿命差异: 旁观者夸克效应：弱湮灭、弱交换和泡利干涉项





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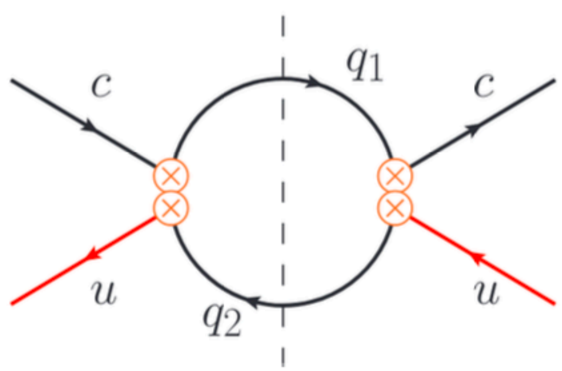
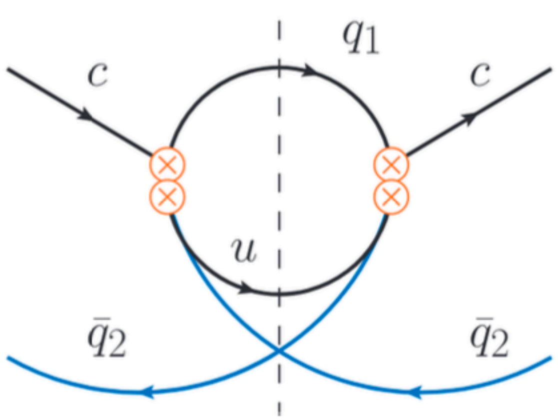
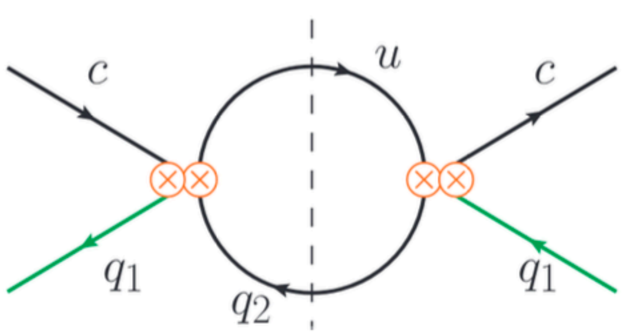


弱交换



泡利干涉



弱湮灭

半轻衰变过程仅包含弱湮灭过程

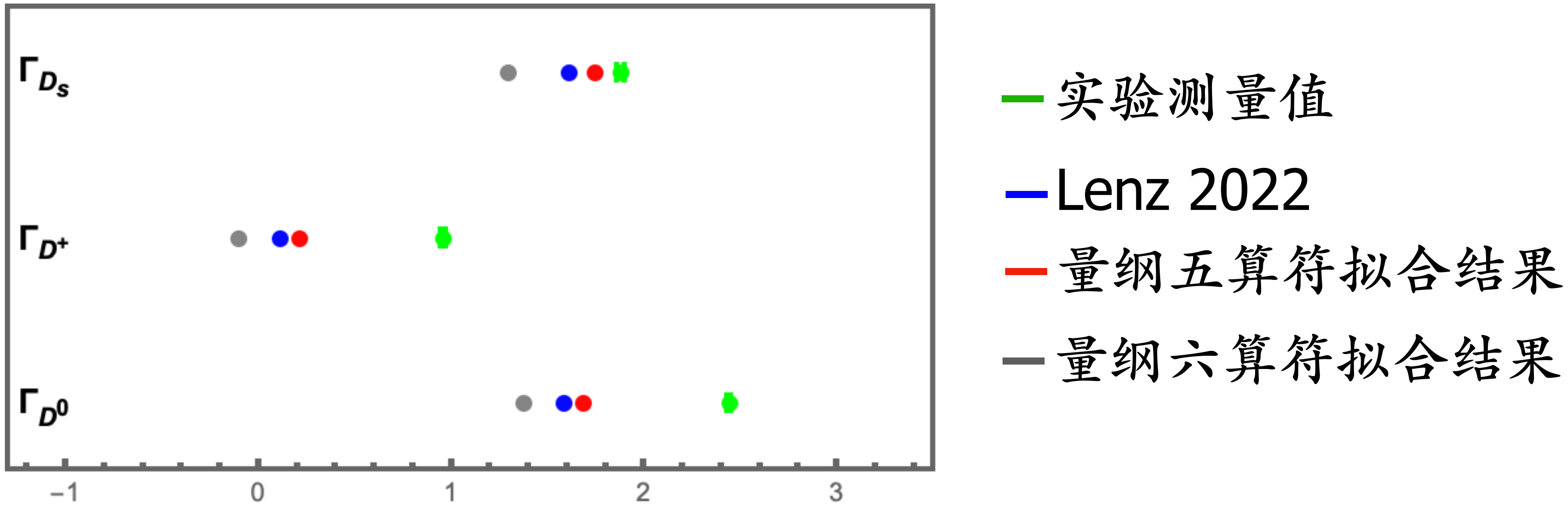
粲介子寿命与半轻单举衰变宽度

- 动力学质量方案下粲介子衰变宽度对非微扰矩阵元依赖形式为（$\mu = 0.5\text{GeV}$）：

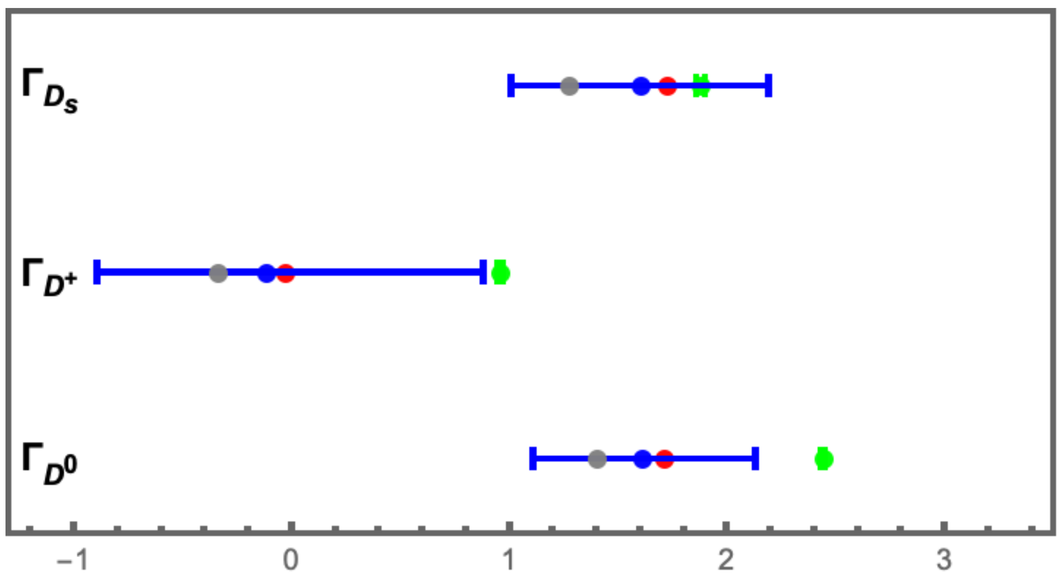
$$
\begin{aligned}
\Gamma\left(\mathrm{D}^0\right) = \Gamma_0[&\underbrace{6.15}_{c_3^{\mathrm{Lo}}} + \underbrace{2.95}_{\Delta c_3^{\mathrm{NLO}}} - 1.66\frac{\mu_\pi^2(D)}{\mathrm{GeV}^2} + 0.13\frac{\mu_G^2(D)}{\mathrm{GeV}^2} + 23.6\frac{\rho_D^3(D)}{\mathrm{GeV}^3} \\
&- 1.60\tilde{B}_1^q + 1.53\tilde{B}_2^q - 21.0\tilde{\epsilon}_1^q + 19.2\tilde{\epsilon}_2^q + \underbrace{0.00}_{\dim-7,\mathrm{VIA}} \\
&- 10.7\tilde{\delta}_1^{qq} + 1.53\tilde{\delta}_2^{qq} + 54.6\tilde{\delta}_3^{qq} + 0.13\tilde{\delta}_4^{qq} - 29.2\tilde{\delta}_1^{sq} + 28.8\tilde{\delta}_2^{sq} + 0.56\tilde{\delta}_3^{sq} + 2.36\tilde{\delta}_4^{sq}] \\
= 6.15\Gamma_0\Bigg[&1 + 0.48 - 0.13\frac{\mu_\pi^2(D)}{0.48\mathrm{GeV}^2} + 0.01\frac{\mu_G^2(D)}{0.34\mathrm{GeV}^2} + 0.31\frac{\rho_D^3(D)}{0.082\mathrm{GeV}^3} \\
&- \underbrace{0.01}_{\text{dim-6,VIA}} - 0.005\frac{\delta\tilde{B}_1^q}{0.02} + 0.005\frac{\delta\tilde{B}_2^q}{0.02} + 0.137\frac{\tilde{\epsilon}_1^q}{-0.04} - 0.125\frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{0.00}_{\dim-7,\mathrm{VIA}} \\
&- 0.0045r_1^{qq} - 0.0004r_2^{qq} - 0.0035r_3^{qq} + 0.0000r_4^{qq} \\
&- 0.0109r_1^{sq} - 0.0079r_2^{sq} - 0.0000r_3^{sq} + 0.0001r_4^{sq}\Bigg].
\end{aligned}
$$

粲介子寿命的初步定性讨论

动力学质量方案下基于VIA对粲介子衰变宽度的理论预言



动力学质量方案下基于HQETSR对粲介子衰变宽度的理论预言



- 实现了量纲五算符非微扰矩阵元模型无关的高精度全局拟合估计
- 该估计值使得衰变宽度更靠近实验测量值
- 该现象没有模型依赖