Quantum entanglement of τ-pair and τ reconstruction at STCF

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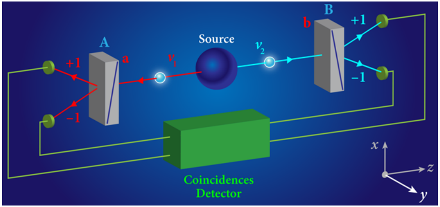
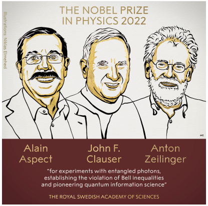
Entanglement & Quantum Tomography





Entanglement

- Quantum Entanglement (QE) — inseparability of past-interacted physical systems
- Violation of Bell Inequality — nonlocality beyond entanglement.
- QE can be observed in correlations of spin or flovor space.

Quantum Tomography

- Helicity and polarization amplitudes are very sensitive probes into the details of the underlying physics.
- Aim: reveal the full quantum state of the final state in a scattering process, which means to construct its density matrix.

Density Matrix

Density Matrix ρ

- Density matrix describe the statistical mixture of states $\{p_i, |\psi_i\rangle\}$

$$\rho = \sum_i p_i \, |\psi_i\rangle\langle\psi_i| \,, \quad p_i \geq 0, \quad \sum_i p_i = 1.$$

- For a biparticle qubits system,

$$\rho = \frac{1}{4}\Big[\mathbb{1}\otimes\mathbb{1} + \sum_i B_i^+(\sigma_i\otimes\mathbb{1}) + \sum_j B_j^-(\mathbb{1}\otimes\sigma_j) + \sum_{i,j} C_{ij}(\sigma_i\otimes\sigma_j)\Big].$$

σ_i are Pauli matrices, $B_i^{\pm}$ represent the polarization and C_{ij} represent the correlation of spins.

Entanglement & Bell Inequality [1] biparticle qubits system

- Entanglement can be discribed by concurrence $\mathcal{C}$

$$\mathcal{C}[\rho] = \max\{0, r_1 - r_2 - r_3 - r_4\} \; (? > 0), \quad R \equiv \rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y).$$

$r_1 \geq r_2 \geq r_3 \geq r_4$ are square roots of the eigenvalues of R.

- Bell Inequality is simplified as the Horodecki condition

$$\mathfrak{m}_{12}[\mathbf{C}] = m_1 + m_2 \; (? > 1), \quad M = \mathbf{C}\mathbf{C}^{\mathrm{T}}$$

$m_1 \geq m_2 \geq m_3$ are eigenvalues of M.

[1] A. J. Barr et al., "Quantum entanglement and Bell inequality violation at colliders", Prog. Part. Nucl. Phys. **139**, 104134 (2024)

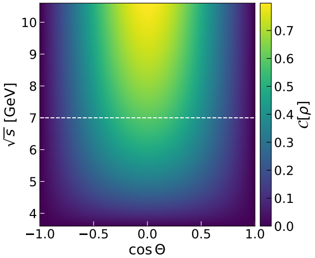
Theoretical Prediction

Analytical expression of $\mathcal{C}$ and $\mathfrak{m}_{12}$ [2]

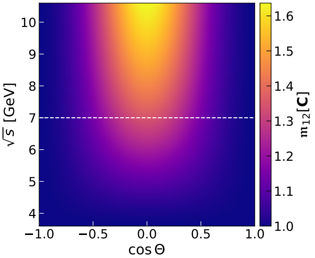
- Specifically, for tree-level QED process $e^+e^- \to \tau^+\tau^-$,

$$\mathcal{C} = \frac{\left(s - 4m_\tau^2\right)\sin^2\Theta}{4m_\tau^2\sin^2\Theta + s(\cos^2\Theta + 1)}, \quad \mathfrak{m}_{12} = 1 + \left[\frac{\left(s - 4m_\tau^2\right)\sin^2\Theta}{4m_\tau^2\sin^2\Theta + s(\cos^2\Theta + 1)}\right]^2.$$

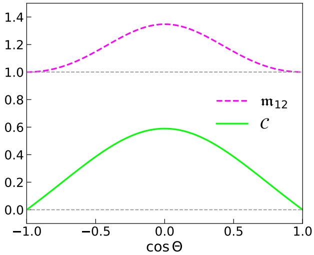
- Only represent the entanglement at production of the τ-pair.



- $\mathcal{C}(\sqrt{s}, \cos\Theta)$



- $\mathfrak{m}_{12}(\sqrt{s}, \cos\Theta)$



- $\mathcal{C}$ and $\mathfrak{m}_{12}$ at $\sqrt{s} = 7$ GeV

[2] K. Ehatäht et al., "Probing entanglement and testing Bell inequality violation with e+e-$\to\tau+\tau$- at Belle II", Phys. Rev. D **109**, 032005 (2024)

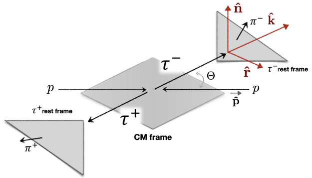
Reconstruction of Density Matrix

Helicity basis $\{\hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}\}$ defined in the c.m.s frame of τ-pair as

$$\hat{\mathbf{p}} \cdot \hat{\mathbf{k}} = \cos\Theta$$

$$\hat{\mathbf{n}} = (\hat{\mathbf{p}} \times \hat{\mathbf{k}}) / \sin\Theta$$

$$\hat{\mathbf{r}} = (\hat{\mathbf{p}} - \hat{\mathbf{k}}\cos\Theta) / \sin\Theta.$$



Reconstruction from $\mathbf{B}^\pm = \{B_i^\pm\}, \mathbf{C} = \{C_{ij}\}$ [2, 3]

- For a biparticle qubits system $(\hat{\mathbf{e}}_i = \hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}, \quad h_i^\pm = \mathbf{h}^\pm \cdot \hat{\mathbf{e}}_i)$

$$B_i^\pm = \frac{3}{\kappa_\pm} \cdot \frac{1}{\sigma} \int \mathrm{d}\Omega^\pm \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^\pm} (\mathbf{h}^\pm \cdot \hat{\mathbf{e}}_i) = \frac{3}{\kappa_\pm} \langle h_i^\pm \rangle,$$

$$C_{ij} = \frac{9}{\kappa_-\kappa_+} \cdot \frac{1}{\sigma} \iint \mathrm{d}\Omega^-\mathrm{d}\Omega^+ \frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega^-\mathrm{d}\Omega^+} (\mathbf{h}^- \cdot \hat{\mathbf{e}}_i)(\mathbf{h}^+ \cdot \hat{\mathbf{e}}_j) = \frac{9}{\kappa_-\kappa_+} \langle h_i^- h_j^+ \rangle .$$

- $\mathbf{h}^\pm$ is polarimeter vectors of $\tau^\pm$ in its c.m.s frame, $\kappa_\pm$ is analyzing power.
- Both vary with decay channels and depend on kinematic infomations of decay products.
- For $\tau \to \pi\nu$, $\kappa_\pm = \pm 1$ and $\mathbf{h}^\pm = -\hat{\mathbf{n}}_{\pi^\pm}$. $\hat{\mathbf{n}}_{\pi^\pm}$ is direction vector in $\tau^\pm$ rest frame.

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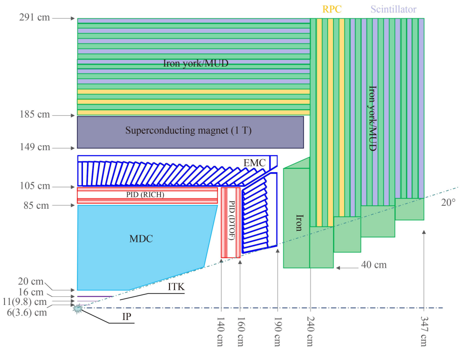
[2] K. Ehatäht et al., "Probing entanglement and testing Bell inequality violation with e+e-→τ+τ- at Belle II", Phys. Rev. D **109**, 032005 (2024)

[3] M. Fabbrichesi and L. Marzola, "Quantum tomography with τ leptons at the FCC-ee: Entanglement, Bell inequality violation, sinθW, and anomalous couplings", Phys. Rev. D **110**, 076004 (2024)

Detector Geometry [4]



Subdetector from inside out

- ITK: Inner Tracker
  - ITKW: cylindrical MPGD
  - ITKM: CMOS M-MAPS
- MDC: Main Drift Chamber
- PIDE: PID in the Endcap
  - DTOF: DIRC-like TOF
- PIDB: PID in the Barrel
  - RICH: CsI+MPGD
  - BTOF: Barrel-TOF
- ECAL (EMC): Electromagnetic Calorimeter (CsI+APD)
- MUD: Muon Detector RPC/scintillator+iron york

[4] M. Achasov et al., "STCF conceptual design report (Volume 1): Physics & detector", Front. Phys. (Beijing) **19**, 14701 (2024)

Data Set

Signal Monte Carlo

- OSCAR 2.6.2 fullsim + digi + reco
- $\sqrt{s} = 7$ GeV
- 30(100) million τ-pair events including main decay channels of τ with ISR used in $\pi\pi(\rho\rho)$ channel analysis.

Channel	Abs. BR	Norm. BR
$\mu^-\bar{\nu}_\mu\nu_\tau$	17.39%	19.30%
$e^-\bar{\nu}_e\nu_\tau$	17.82%	19.78%
$\pi^-\nu_\tau$	10.82%	12.01%
$\pi^-\pi^0\nu_\tau$	25.49%	28.29%
$\pi^-\pi^0\pi^0\nu_\tau$	9.26%	10.28%
$\pi^-\pi^+\pi^-\nu_\tau$	9.31%	10.33%
Sum	90.09%	100.00%

- Generator: aMC@NLO MadGraph

Advantage

- $\pi\pi$ channel

$$e^-e^+ \to \begin{cases} \tau^- \to \pi^-\nu_\tau \\ \tau^+ \to \pi^+\bar{\nu}_\tau \end{cases}$$

  - newly reported
  - best analyzing power $|\kappa| = 1$

- $\rho\rho$ channel

$$e^-e^+ \to \begin{cases} \tau^- \to \rho^-\nu_\tau, & \rho^- \to \pi^-\pi^0 \\ \tau^+ \to \rho^+\bar{\nu}_\tau, & \rho^+ \to \pi^+\pi^0 \end{cases}$$

  - previously reported
  - largest branching fraction

Background

- Other main decay channels of τ.
- Rare decay channels of τ. 30 million
- Hadronic process. 20 million

Initial Selection

Charged Track Selection

- $D_0 < 1$ cm, $Z_0 < 10$ cm
- $|\cos\theta| < 0.95$
- $N_{\text{charge}} = 2$, $Q_{\text{total}} = 0$

Shower Selection (no cut)

- Barrel $|\cos\theta| \in (0.00, 0.83)$
  $E_\gamma > 25$ MeV
- End cap $|\cos\theta| \in (0.83, 0.94)$
  $E_\gamma > 50$ MeV
- $t \in (-10, 10)$ ns
- $\theta_{\text{iso}} > 10^\circ$

Particle Identification

- GlobalPIDSvc based on data-driven machine learning methods, includes pre-trained BDT (based on XGBoost).
- For π
  - $\mathcal{P}_\pi(\pi) > \mathcal{P}_\pi(e), \mathcal{P}_\pi(K)$
- $N_\pi = 2$

$$e^- e^+ \to \begin{cases} \tau^- \to \pi^- \nu_\tau \\ \tau^+ \to \pi^+ \bar{\nu}_\tau \end{cases}$$

τ Reconstruction

Two-fold ambiguity [5] $\mathbf{k} = \mathbf{p}_{\tau^+}$

- In c.m.s frame of $\tau\tau$, assume no ISR.
- Invariant mass constraints

$$(p_\pi + p_\nu)^2 = m_\tau^2, \quad p_\nu^2 = 0.$$

- Initial four-momentum constraints

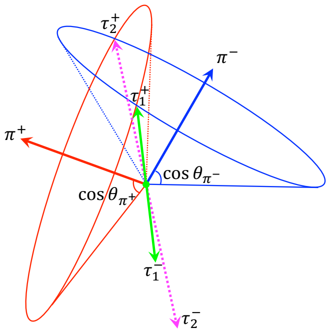
$$\sum_i p_i = p_{\rm ini} \equiv (E_{\rm cms}, 0, 0, 0).$$

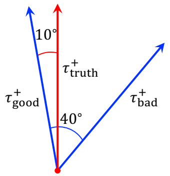
- Two-fold ambiguity of $\hat{\mathbf{k}}$
  - $p_{\rm ini}$ fixes $|\mathbf{k}|$

$$\cos\theta_i = \frac{2E_iE_\tau - m_i^2 - m_\tau^2}{2|\mathbf{k}||\mathbf{p}_i|} \;(i = \pi^-, \pi^+).$$

Selection

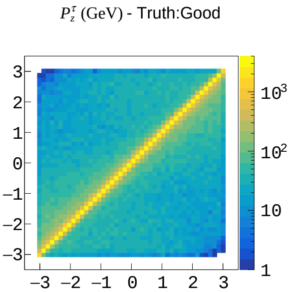
- $N_{\rm sol} > 0$.



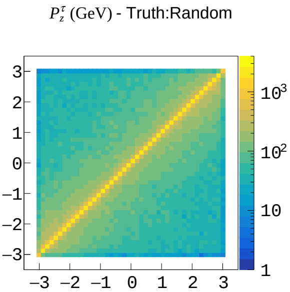


[5] K. Inami et al. (Belle), "An improved search for the electric dipole moment of the τ lepton", JHEP **04**, 110 (2022)

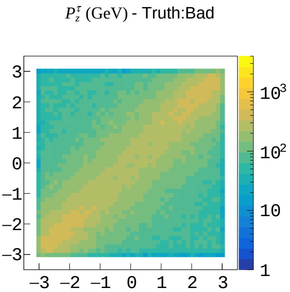
τ Reconstruction — deviation



- Truth : Good

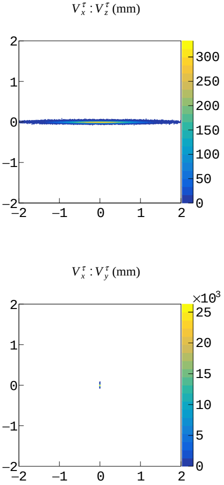


- Truth : Random

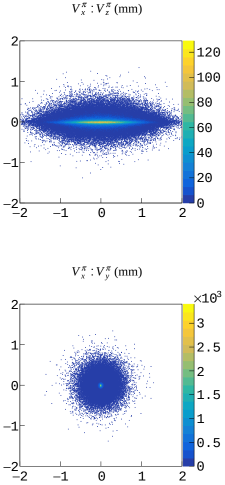


- Truth : Bad

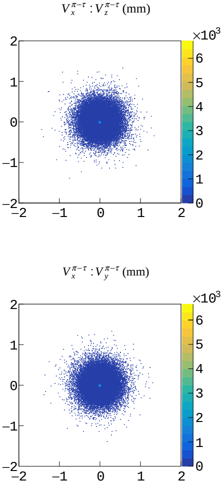
Difficulty of vertex resolution — truth vertex & reco POCA



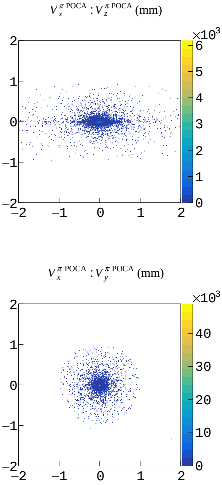
Production vertex of τ



Production vertex of π

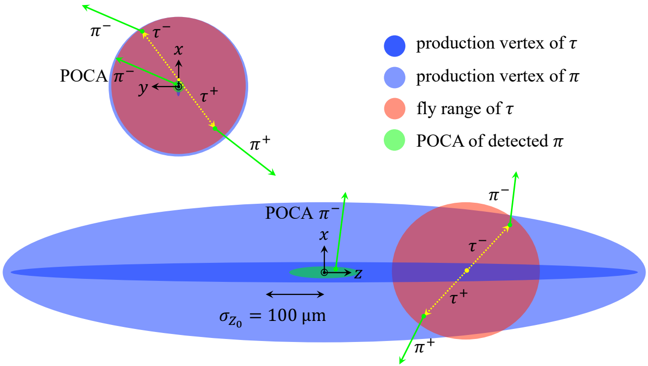


Fly vector of π



POCA of π

Difficulty of vertex resolution — distribution of vertex



Size

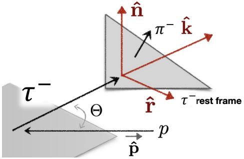
- Beam spot size at IP $\sigma_x \sim 10\ \mu m$
  $\sigma_y \sim 0.1\ \mu m$
- Bunch length (too large) $\sigma_z \sim 10$ mm
- Decay length of τ $L_\tau = 140\ \mu m$

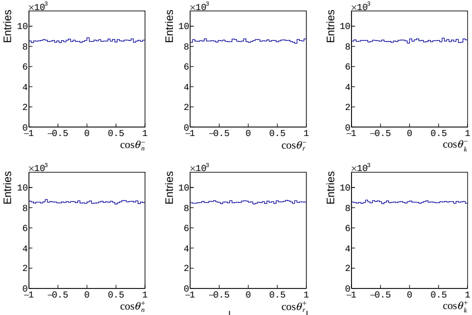
Resolution

- ITKM+MDC $\sigma_{D_0} \sim 20\ \mu m$
  (not good enough) $\sigma_{Z_0} \sim 100\ \mu m$
- Current resolution is insufficient to construct τ decay vertex.
- Two-fold ambiguity can't be eliminated.

Full PHSP — $\mathbf{B}^{\pm}, \mathbf{C}$

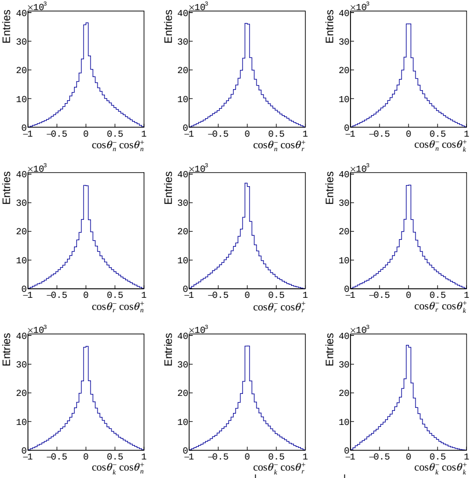
- Full PHSP: truth info, no detector effect.
- To check algorithm.





- $\cos\theta_i^{\pm} = -h_i^{\pm}$

- Experimentally, $h_i^{\pm} = \mathbf{h}^{\pm} \cdot \hat{\mathbf{e}}_i = \cos\theta_i^{\pm}$
$B_i^{\pm} = \pm 3 \langle h_i^{\pm} \rangle\,, \quad C_{ij} = -9 \langle h_i^- h_j^+ \rangle\,.$



- $\cos\theta_i^- \cos\theta_j^+ = h_i^- h_j^+$

Full PHSP — $\mathcal{C}, \mathfrak{m}_{12}$

- Values of $B_i^{\pm}, C_{ij}$ with statistical errors

i	n	r	k
B_i^-	0.0038 ± 0.0026	0.0012 ± 0.0026	0.0024 ± 0.0026
B_i^+	-0.0002 ± 0.0026	-0.0038 ± 0.0026	-0.0004 ± 0.0026
C_{ni}	-0.3147 ± 0.0046	0.0007 ± 0.0046	0.0022 ± 0.0046
C_{ri}	-0.0049 ± 0.0046	0.5659 ± 0.0045	0.0098 ± 0.0046
C_{ki}	-0.0006 ± 0.0046	-0.0010 ± 0.0046	0.7513 ± 0.0044

For simulation with ISR,

$$\mathcal{C} = 0.3136 \pm 0.0040, \quad \mathfrak{m}_{12} = 0.8850 \pm 0.0086.$$

For tree-level QED calculation [2] fixed in $\sqrt{s} = 7$ GeV,

$$\mathbf{B} = \mathbf{0}, \quad \mathbf{C} = \begin{pmatrix} -0.3287 & 0 & 0 \\ 0 & 0.5571 & 0 \\ 0 & 0 & 0.7717 \end{pmatrix}.$$

$$\mathcal{C} = 0.3287, \quad \mathfrak{m}_{12} = 0.9058.$$

The results are close and differences may come from ISR.

Statistical error

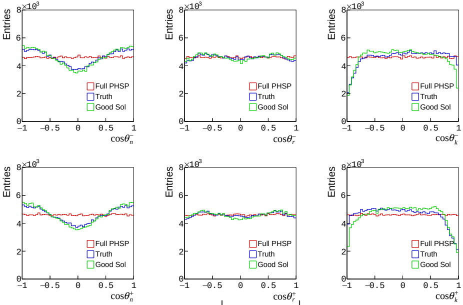
- Assume Gaussian distributed $B_i^{\pm}$, C_{ij}.
- Generate 10k sets of toy $B_i^{\pm}$, C_{ij} to get $\mathcal{C}$, $\mathfrak{m}_{12}$.

[2] K. Ehatäht et al., "Probing entanglement and testing Bell inequality violation with e+e-→τ+τ- at Belle II", Phys. Rev. D **109**, 032005 (2024)

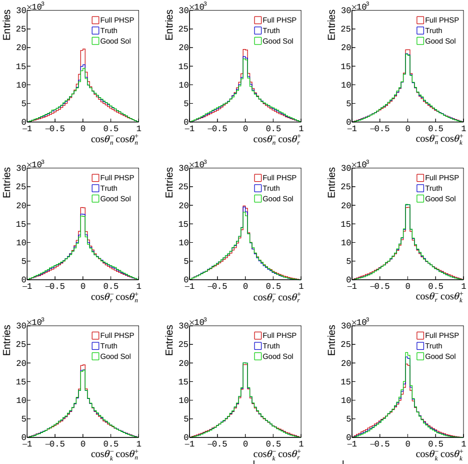
Truth & Good solution — $\mathbf{B}^{\pm}, \mathbf{C}$

- Good solution: detected info
- Truth: corresponding to Good solution

- Experimentally, $h_i^{\pm} = \mathbf{h}^{\pm} \cdot \hat{\mathbf{e}}_i = \cos\theta_i^{\pm}$
  $B_i^{\pm} = \pm 3 \langle h_i^{\pm} \rangle , \quad C_{ij} = -9 \langle h_i^- h_j^+ \rangle .$



- $\cos\theta_i^{\pm} = -h_i^{\pm}$



- $\cos\theta_i^- \cos\theta_j^+ = h_i^- h_j^+$

Truth & Good solution — $\mathcal{C}$

i		n	r	k
B_i^-	Full PHSP	0.0038 ± 0.0026	0.0012 ± 0.0026	0.0024 ± 0.0026
	Truth	-0.0001 ± 0.0038	-0.0027 ± 0.0036	0.1200 ± 0.0035
	Good solution	0.0005 ± 0.0038	-0.0021 ± 0.0036	0.0187 ± 0.0034
B_i^+	Full PHSP	-0.0002 ± 0.0026	-0.0038 ± 0.0026	-0.0004 ± 0.0026
	Truth	0.0002 ± 0.0038	-0.0054 ± 0.0036	0.1332 ± 0.0034
	Good solution	0.0013 ± 0.0038	-0.0074 ± 0.0036	0.0316 ± 0.0034
C_{ii}	Full PHSP	-0.3147 ± 0.0046	0.5659 ± 0.0045	0.7513 ± 0.0044
	Truth	-0.2962 ± 0.0067	0.7572 ± 0.0059	0.7081 ± 0.0056
	Good solution	-0.2795 ± 0.0068	0.6976 ± 0.0060	0.6934 ± 0.0053

- Result

	$\mathcal{C}$
Full PHSP	0.3136 ± 0.0040
Truth	0.2961 ± 0.0067
Good solution	0.2794 ± 0.0068

Events Selection

Step

- Charged Track: $N_{\pi^-} = 1, N_{\pi^+} = 1$
- Neutral Track: $N_\gamma = 4$
  - Use BDTG to select γ from π^0.
- Reconstruct π^0: γ pairing
- Reconstruct ρ: $\pi^0, \pi^\pm$ pairing
- Event-level Selection: BDTG

Efficiency

- Signal efficiency 5.00%
- Signal purity 87.12%
  - γ_{π^0}-γ_{ISR} misidentify 4.55%
- Background level 12.88%

$$e^-e^+ \to \begin{cases} \tau^- \to \rho^-\nu_\tau,\ \rho^- \to \pi^-\pi^0 \\ \tau^+ \to \rho^+\bar{\nu}_\tau,\ \rho^+ \to \pi^+\pi^0 \end{cases}$$

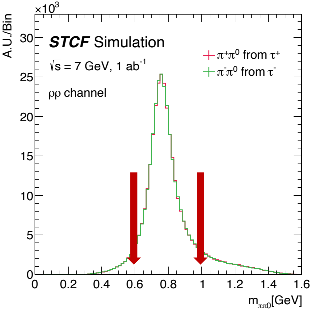
Events Selection

Reconstruct π^0, ρ

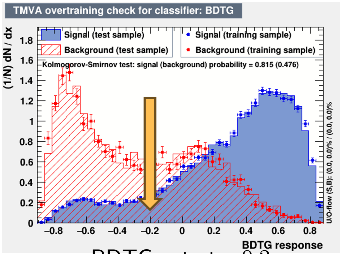
- Kinematic fit for $\chi^2 < 20$
- π^0 mass region $m_{\gamma\gamma} \in (115, 150)$ MeV
- ρ mass region $m_{\pi\pi^0} \in (0.6, 1.0)$ GeV
- Apply these cuts at final step to optimize efficiency and reconstruction quality

Event-level BDTG

- Input train variables
  - momentum: $\mathbf{p}_\nu, \mathbf{p}_\pi, \mathbf{p}_\gamma$
  - kinematic fit: χ^2
  - PID: $\mathcal{P}_\pi(i), i = \pi, \mu, K$
  - N_γ before γ BDTG cut

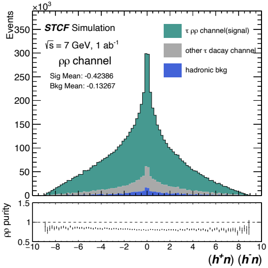


- $m_{\pi\pi^0}$ (GeV)

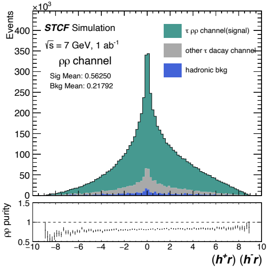


- BDTG cut at -0.2

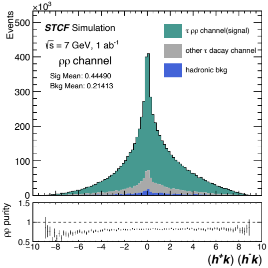
Random solution — $\mathcal{C}$



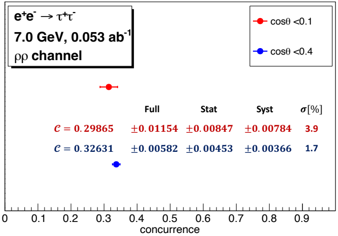
- $h_n^- h_n^+$



- $h_r^- h_r^+$



- $h_k^- h_k^+$



- $\mathcal{C}$

Result

- Red and blue dots indicate two choices of $\cos\theta$ cut.
- Promising for a witness ($> 5\sigma$) of the entanglement ($\mathcal{C} > 0$).

- Study of τ reconstruction and vertex resolution.
- Quantum entanglement of $\tau\tau$ via $\pi\pi$ or $\rho\rho$ channel.

$\pi\pi$ Channel — intermediate stage

- Consistency check between theory calculation and simulation.
- With good solution, $\mathcal{C} = 0.2794 \pm 0.0068$.
- Signal efficiency is 53.76%.
- Next steps
  - Random solution
  - Backgound

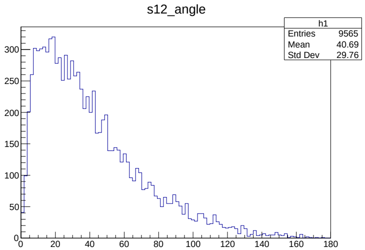
$\rho\rho$ Channel — final stage

- Event selection and background study.
- With random solution, $\mathcal{C} = 0.3263 \pm 0.0058$ $(> 5\sigma)$.
- Next steps
  - Systematic uncertainty

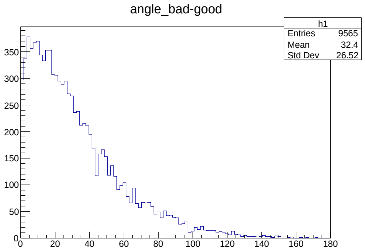
- $\tau \to 3\pi\nu$ channel is also under consideration.

Thanks for your attention !

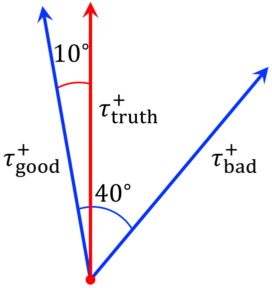
τ Reconstruction — two-fold ambiguity



- $\theta_{s_1 s_2}$ $(^\circ)$, $\mu = 41$

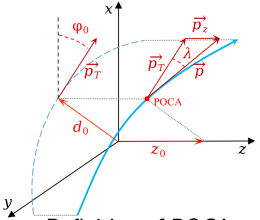


- Truth-Reco: $\theta_{\text{good}} - \theta_{\text{bad}}$ $(^\circ)$, $\mu = 32$

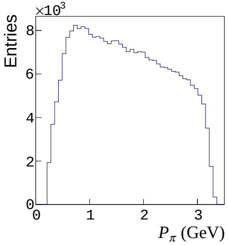


- Two solutions are kinematically equivalent.
- Need vertex information to distinguish them.

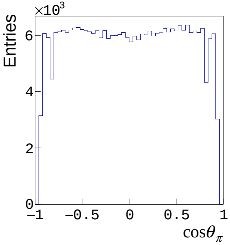
Issue of abnormally large ΔP_π



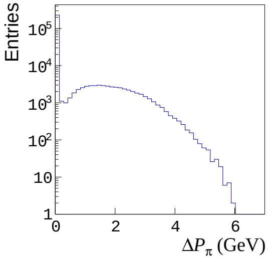
- Definition of POCA (Point Of Closest Approach)



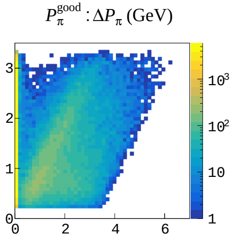
- P_π (GeV)



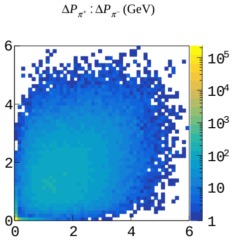
- $\cos\theta_\pi$



- $P_\pi^{\text{good}} - P_\pi^{\text{truth}}$ (GeV)



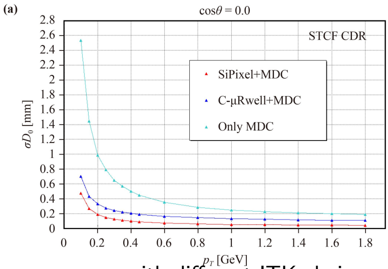
- ΔP_π-P_π (GeV)



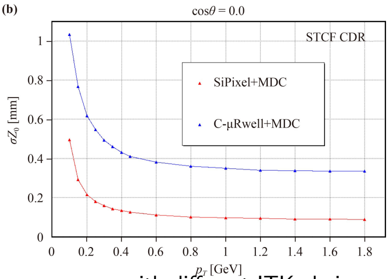
- ΔP_{π^-}-ΔP_{π^+} (GeV)

Cut on $\Delta P_{\pi^\pm} < 0.5$ GeV/c

Difficulty of vertex resolution — limitation of spatial resolution [5]

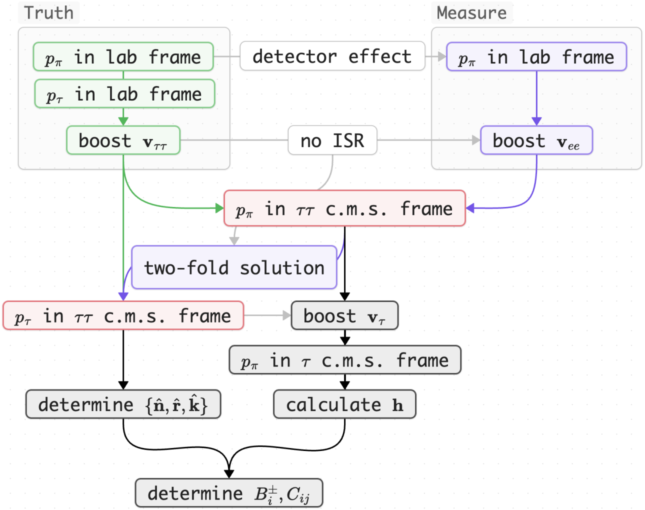


- σ_{D_0}-p_{T} with different ITK choice

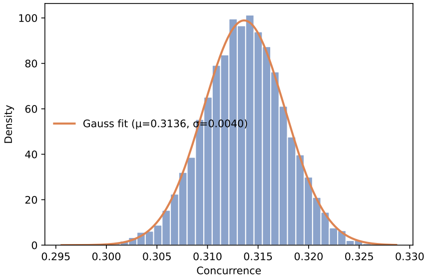


- σ_{Z_0}-p_{T} with different ITK choice

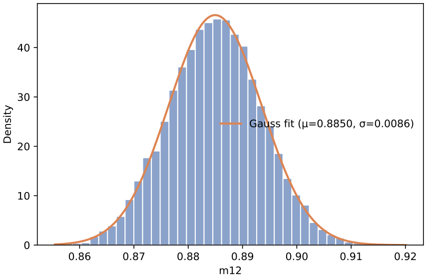
$\mathbf{B}, \mathbf{C}$



Full PHSP — $\mathcal{C}, \mathfrak{m}_{12}$ TOY



- $\mathcal{C}$



- $\mathfrak{m}_{12}$

Full PHSP — $\mathcal{C}, \mathfrak{m}_{12}$ TOY

Issue

- $\mathfrak{m}_{12}$ is sensitive and its systematic uncertainty is hard to determined.
- $\mathfrak{m}_{12}$ does not decrease as $\mathcal{C}$ decreases.

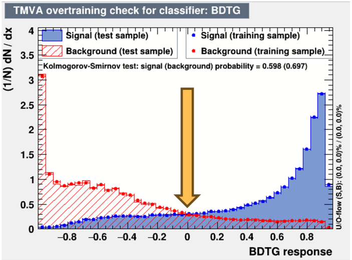
	$\mathcal{C}$	$\mathfrak{m}_{12}$
Full PHSP	0.3136 ± 0.0040	0.8850 ± 0.0086
Truth	0.2961 ± 0.0067	1.0751 ± 0.0122
Good solution	0.2794 ± 0.0068	0.9679 ± 0.0114

Events Selection — select γ from π^0

γ-level BDTG

- Use BDTG to distinguish γ_{π^0} from γ_{ISR}.
- ISR feature: low energy, relatively forward.
- Input train variables
  - N_{hit}
  - E_{seed}
  - E_{tot}
  - $\cos\theta_\gamma$
  - lateral moment $\frac{\sum_{i=3}^{n} E_i r_i^2}{E_1 r_1^2 + E_2 r_2^2 + \sum_{i=3}^{n} E_i r_i^2}$
  - secondary moment $\frac{\sum E_i r_i}{\sum E_i}$
  - A_{42} moment $\sum_i \frac{E_i}{E_{\mathrm{tot}}} f_{4,2}(r_i/R_0) e^{im\phi}$
  - variable2 $= 1 - \frac{E_{\mathrm{seed}}}{E_{3\times 3}}$
  - variable3

$$\begin{cases} 0 & , N_{\mathrm{hit}} = 1 \\ (E_{\mathrm{tot}}/E_{\mathrm{seed}} - 1)/(N_{\mathrm{hit}} - 1) & , N_{\mathrm{hit}} > 1 \end{cases}$$



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[1] A. J. Barr et al., "Quantum entanglement and Bell inequality violation at colliders", Prog. Part. Nucl. Phys. **139**, 104134 (2024).

[2] K. Ehatäht et al., "Probing entanglement and testing Bell inequality violation with e+e-$\longrightarrow\tau+\tau$- at Belle II", Phys. Rev. D **109**, 032005 (2024).

[3] M. Fabbrichesi and L. Marzola, "Quantum tomography with τ leptons at the FCC-ee: Entanglement, Bell inequality violation, sinθW, and anomalous couplings", Phys. Rev. D **110**, 076004 (2024).

[4] M. Achasov et al., "STCF conceptual design report (Volume 1): Physics & detector", Front. Phys. (Beijing) **19**, 14701 (2024).

[5] K. Inami et al. (Belle), "An improved search for the electric dipole moment of the τ lepton", JHEP **04**, 110 (2022).