KalmanFilter-based alignment in ACTS

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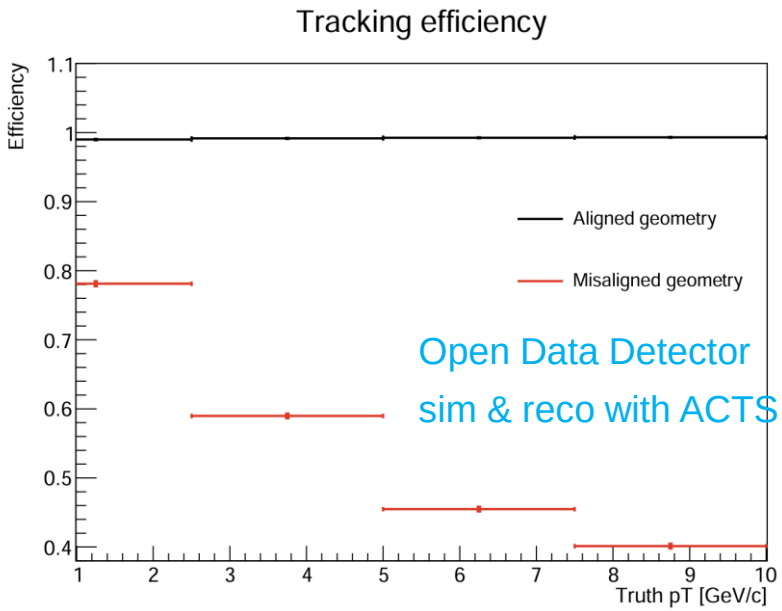
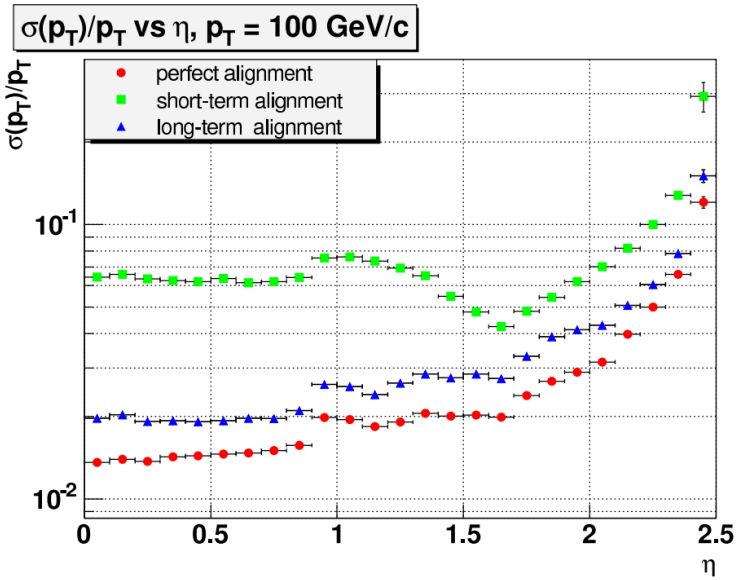
Why alignment matters?

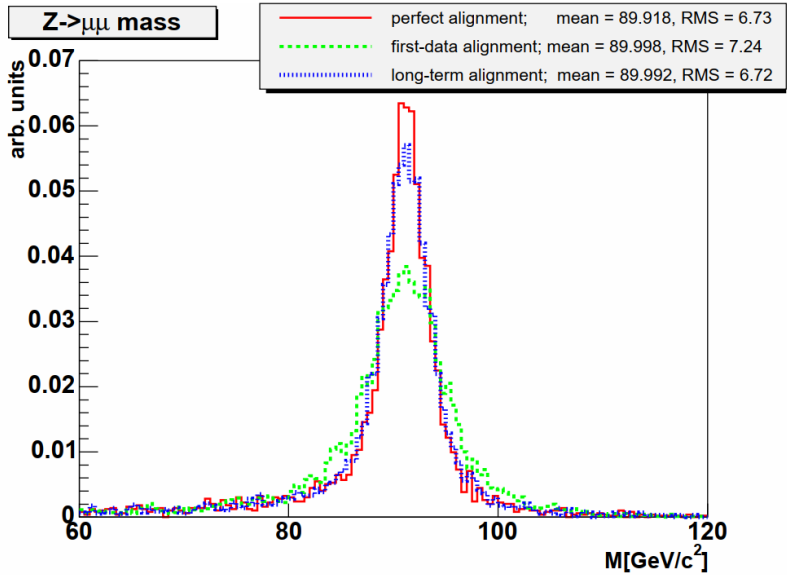
See more in Pawel Bruckman's slides

- Limited detector placement precision upon installation
- Misalignment is the dominant source of measurement resolution degradation

➡ Degradation of tracking precision and efficiency, and eventually **physics precision**!



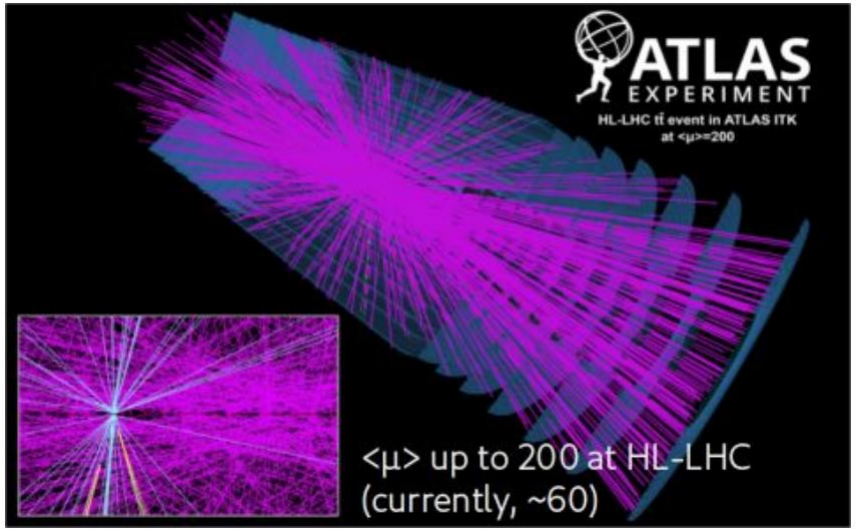




From G. Steinbruck

Why Kalman Filter based alignment?

- HL-LHC: recorded data (7-10x), pile-up <μ> = 200
  - Highly-performant tracking software
- Track-based detector alignment primarily relies on global χ^2 fitting methods:
  - High computational complexity
  - Difficulty in handling material effects and non-Gaussian noise



- Kalman Filter is widely used for track fitting in the high energy physics
  - Transition to a Kalman-Filter based alignment algorithm
  - Expected to enhance robustness and efficiency in detector alignment

ACTS recap

ACTS project

- A modern **open-source detector-independent** tracking toolkit for current&future HEP experiments based on LHC (and beyond) tracking experience
  - Data production by **ATLAS, FASER, sPHENIX**
  - Detector R&D by **CEPC, STCF, EIC, ePIC, LDMX**… *(see talk of Hao Li)*
- A R&D platform for ML-based tracking, heterogeneous computing and 4D tracking

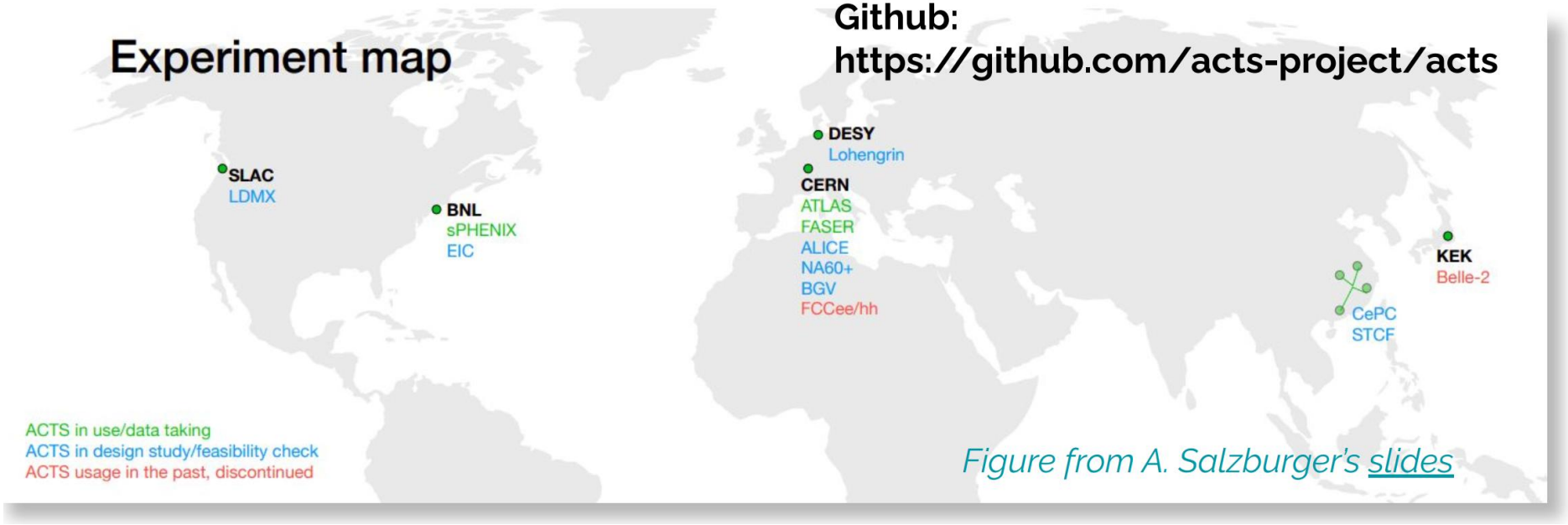
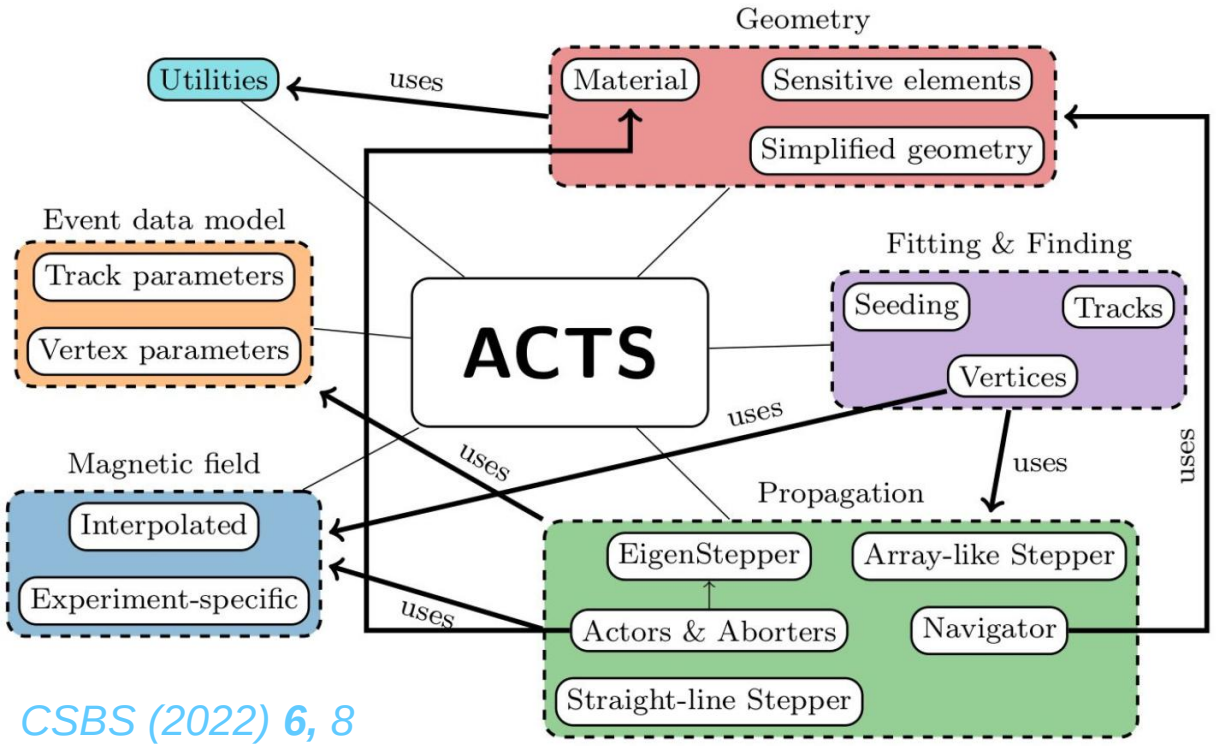


Figure from A. Salzburger's slides

ACTS design and functionalities

- Fully C++20 compliant, template design, strict thread-safety, contextual condition data …
  - More in *A. Salzburger's slides*

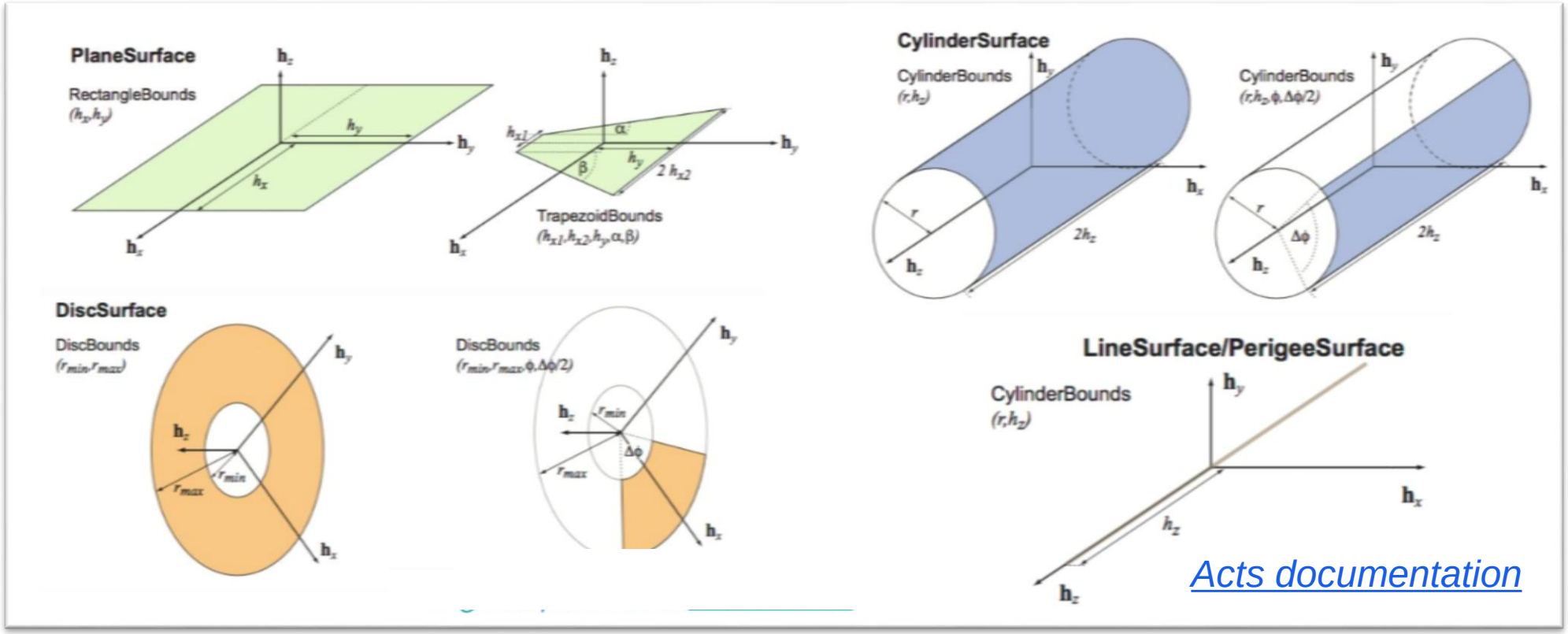


*CSBS (2022) **6**, 8*

- **Track fitting**
  - (Extended) Kalman Filter (KF), Gaussian Sum Filter, Non-linear KF
  - Global chisq fitter
- **Track finding**
  - Seeding, Combinatorial Kalman Filter (CKF), Graph Neural Networks, Hough Transform
- **Vertex finding&fitting**
  - Primary vertex: AMVF, IVF
- **KF-based Alignment** (prototype developed in 2022 and being slowly validated and optimized)

ACTS tracking geometry

- Tracking geometry is simplified from detailed full simulation geometry for fast navigation, but with material effects well taken into account
- Different concrete surfaces types for various tracking detectors
  - A surface has shape, bounds, **rotation+translation**, local coordinates and unique identifier…

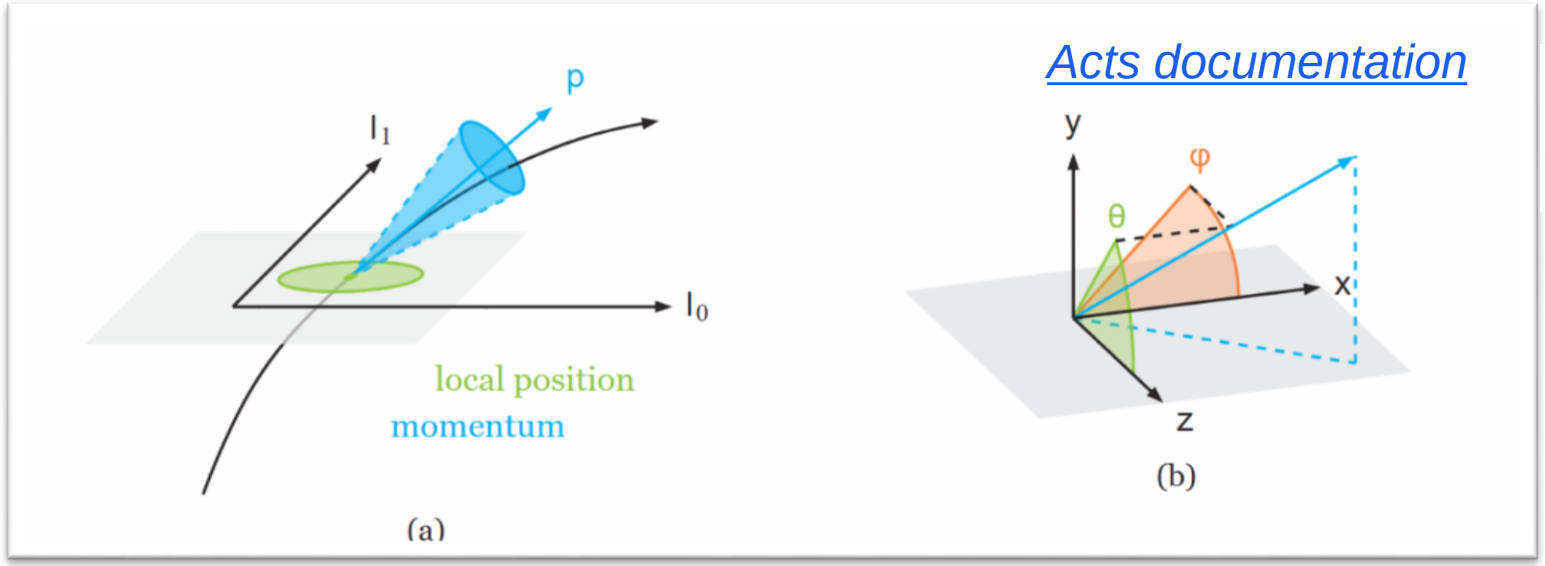


ACTS track parameterization

- 6-dimensional bound/local track parameters (integration of particle flight time in track propagation)

$$\vec{x} = (l_0, l_1, \phi, \theta, q/p, t)^T$$

- Measurement (1, 2, or 3 dimension) is a subset of the 6 parameters: $\vec{m} = H \cdot \vec{x}$



Acts documentation

e.g. $H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

Alignment in ACTS

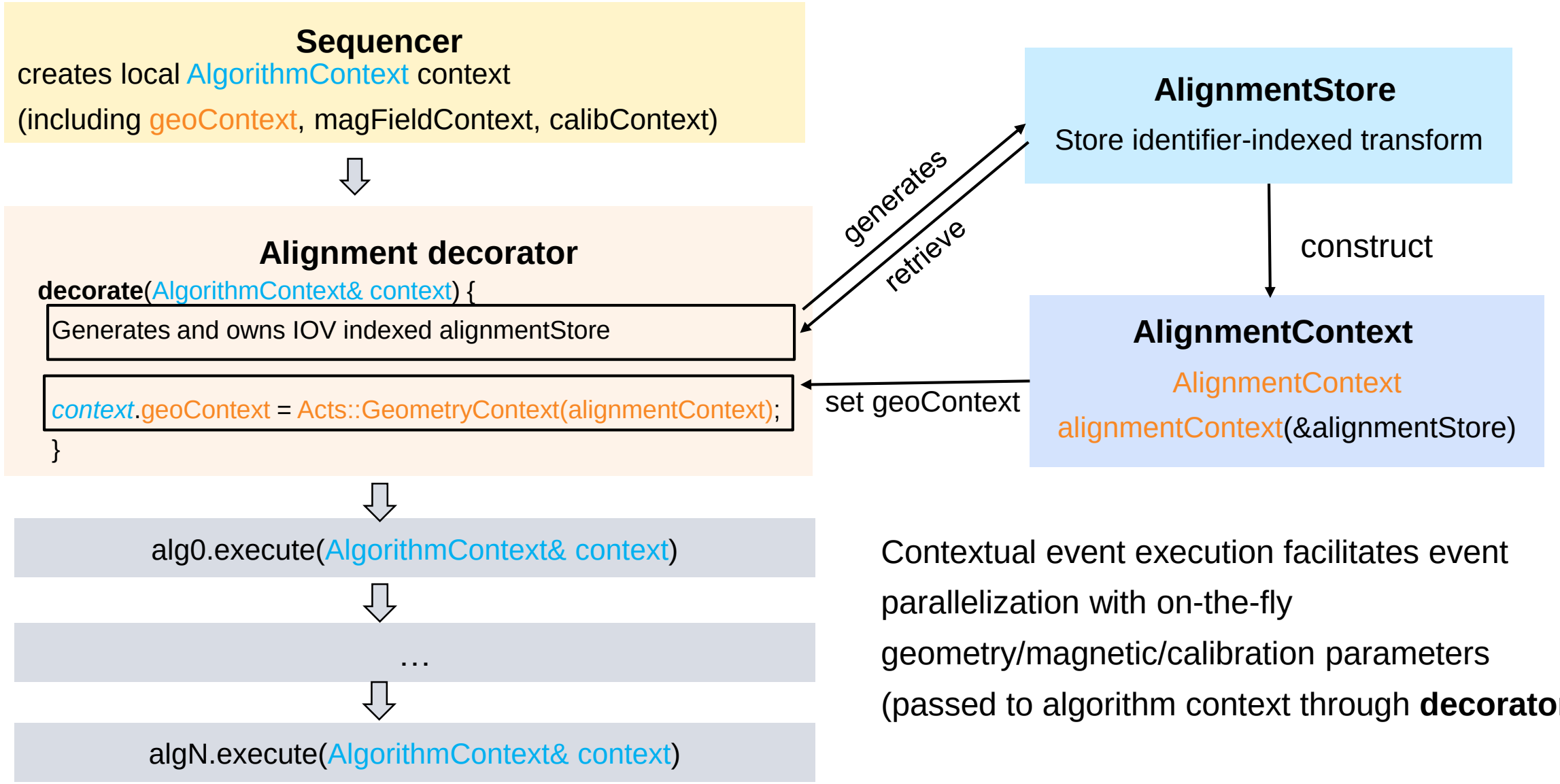
Alignment parameters

- Detector element placement description:
  - Translation (3 parameters) + Rotation (3x3 rotation matrix)
- Alignment parameters (6 parameters for now):
  - 3 parameters for translation along original local axes ($\vec{x}_L$, $\vec{y}_L$, $\vec{z}_L$)
  - 3 parameters for rotation about original local axes ($\vec{x}_L$, $\vec{y}_L$, $\vec{z}_L$) using Euler angles
    - Suppose rotation in the order: 1) around $\vec{x}_L$ about α → 2) around $\vec{y}_L$ about β → 3) around $\vec{z}_L$ about γ , then the new local axes become

$$(\vec{x}_L''', \vec{y}_L''', \vec{z}_L''') = (\vec{x}_L, \vec{y}_L, \vec{z}_L) \begin{pmatrix} cos\beta cos\gamma & sin\alpha sin\beta cos\gamma - cos\alpha sin\gamma & cos\alpha sin\beta cos\gamma + sin\alpha sin\gamma \\ cos\beta sin\gamma & sin\alpha sin\beta sin\gamma + cos\alpha cos\gamma & cos\alpha sin\beta sin\gamma - sin\alpha cos\gamma \\ -sin\beta & sin\alpha cos\beta & cos\alpha cos\beta \end{pmatrix}$$

Caveat: a rotation can be expressed in 24 equivalent sequence of Euler angles

Geometry Context



Contextual event execution facilitates event parallelization with on-the-fly geometry/magnetic/calibration parameters (passed to algorithm context through **decorators**)

Ideas of track-based alignment

- Tracks share the same detector geometry (a.k.a. **global track parameters** $\vec{\alpha}$) while they have their own track parameters (a.k.a. **local track parameters** $\vec{x_i}$)
- The global track parameters can be estimated by minimizing the χ^2 sum of a set of chosen good quality tracks:

$$\chi^2 = \sum_i {\chi_i}^2 = \sum_i [\vec{m}_i - \vec{h}_i(\vec{x}_i(\vec{\alpha}), \vec{\alpha})]^T V^{-1} [\vec{m}_i - \vec{h}_i(\vec{x}_i(\vec{\alpha}), \vec{\alpha})]$$

- This involves solving the non-linear equation iteratively, i.e. $\vec{\alpha}$ is updated iteratively to approach its optimal value:

$$\frac{d^2\chi^2}{d^2\vec{\alpha}} \Big|_{\vec{\alpha_0}} \Delta\vec{\alpha} = -\frac{d\chi^2}{d\vec{\alpha}} \Big|_{\vec{\alpha_0}}$$

The Kalman Filter fitter is used to fit each track in each iteration

Alignment ingredients

$r = m - h(x, \alpha)$ The track residual

V The measurement covariance

$H = \left.\frac{\partial h(x)}{\partial x}\right|_{x_0}$ The projection matrix from (bound) track parameters to measurement

C The covariance of track parameters at different measurements:
Straightforward with global chi2 fitter. Can provided by Kalman Filter as well

$A_{k\ell} \equiv \frac{\partial r_k}{\partial \alpha_\ell}$ The derivative of residual w.r.t. alignment parameters

The first and second derivatives for a single track and then summed over tracks

$$\frac{\mathrm{d}\chi^2}{\mathrm{d}\alpha} = 2A^T V^{-1}\left(V - HCH^T\right)V^{-1}r,$$

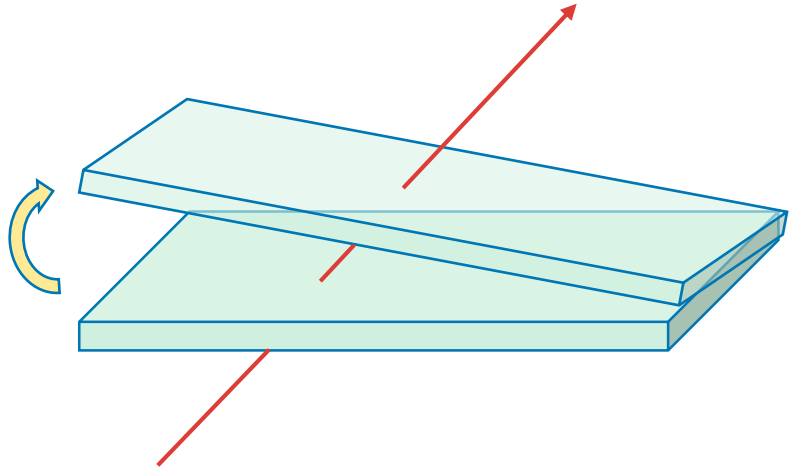
$$\frac{\mathrm{d}^2\chi^2}{\mathrm{d}\alpha^2} = 2A^T V^{-1}\left(V - HCH^T\right)V^{-1}A.$$

$$\left.\frac{\mathrm{d}^2\chi^2}{\mathrm{d}\alpha^2}\right|_{\alpha_0}\Delta\alpha = -\left.\frac{\mathrm{d}\chi^2}{\mathrm{d}\alpha}\right|_{\alpha_0}$$

Solved with Eigen LU decomposition
(claimed stable and well tested with large matrices)

Residual derivative

- Suppose the rotation (α, β, γ) around the original local axes is small, then



$$\frac{\partial \vec{x}_L'''}{\partial(\alpha,\beta,\gamma)} = \begin{pmatrix} \vec{x}_L & \vec{y}_L & \vec{z}_L \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = R \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\frac{\partial \vec{y}_L'''}{\partial(\alpha,\beta,\gamma)} = \begin{pmatrix} \vec{x}_L & \vec{y}_L & \vec{z}_L \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = R \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\frac{\partial \vec{z}_L'''}{\partial(\alpha,\beta,\gamma)} = \begin{pmatrix} \vec{x}_L & \vec{y}_L & \vec{z}_L \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = R \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Residual $\vec{r} = (r_x, r_y)$:

$r_x = (\vec{x}_{track} - \vec{o}_{module}) \cdot \vec{x}_L - x_{hit}$

$r_y = (\vec{x}_{track} - \vec{o}_{module}) \cdot \vec{y}_L - y_{hit}$

- $\vec{x}_{track}$ is the intersection of track with detector module
  - Changed with $\vec{o}_{module}$ and $\vec{x}_L$, $\vec{y}_L$, $\vec{z}_L$
- $\vec{o}_{module}$ is the center of the detector module
- x_{hit} and y_{hit} are hit local position on module

An alignment prototype

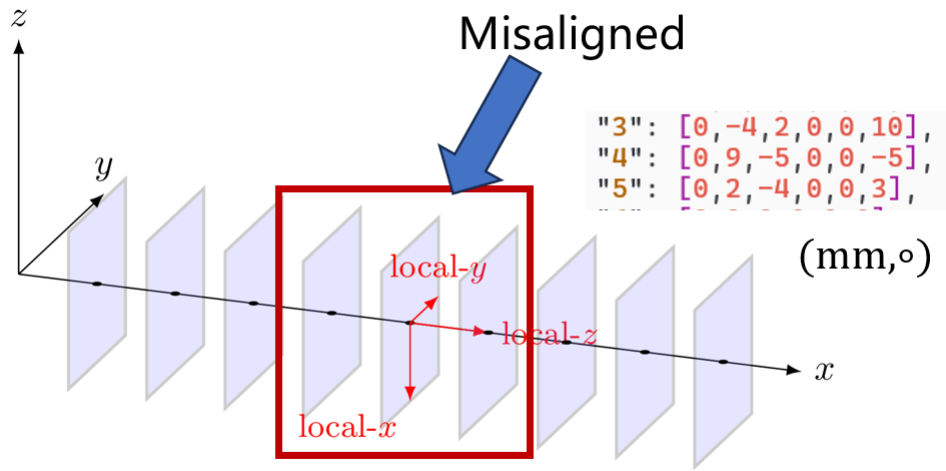
- It takes a set of tracks and sort out the sets of detector elements which can be and requested to be aligned => the initial value of $\vec{\alpha}$
- Perform the fit for each track using $\vec{\alpha}$, and estimate the χ^2 derivatives w.r.t. $\vec{\alpha}$
- Solve the equation to obtain $\Delta\vec{\alpha}$
- Update $\vec{\alpha}$ to become $\vec{\alpha}'$ using provided alignment parameter updater
- Stop iteration of the above three steps when provided converging criteria is met

```
template <typename trajectory_container_t,
          typename start_parameters_container_t, typename fit_options_t>
Acts::Result<AlignmentResult> align(
    const trajectory_container_t& trajectoryCollection,
    const start_parameters_container_t& startParametersCollection,
    const AlignmentOptions<fit_options_t>& alignOptions) const;
```

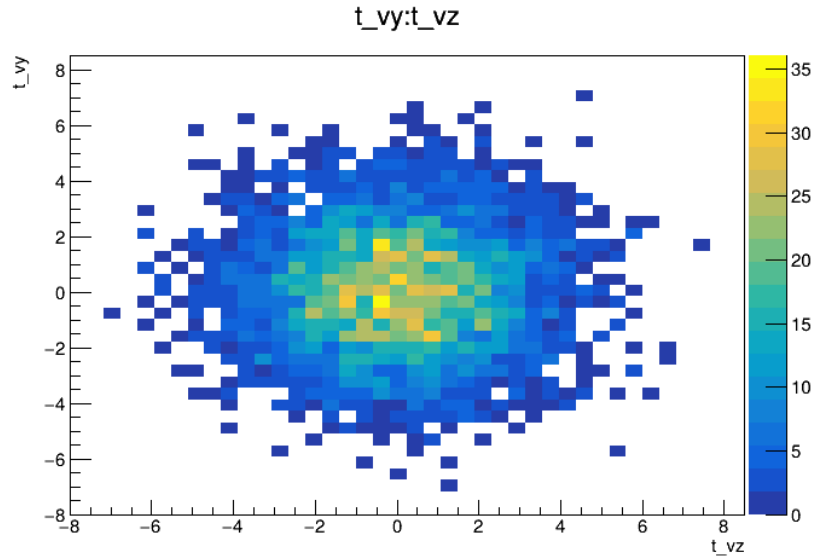
ACTS Alignment example

Sanity check with a telescope-like detector

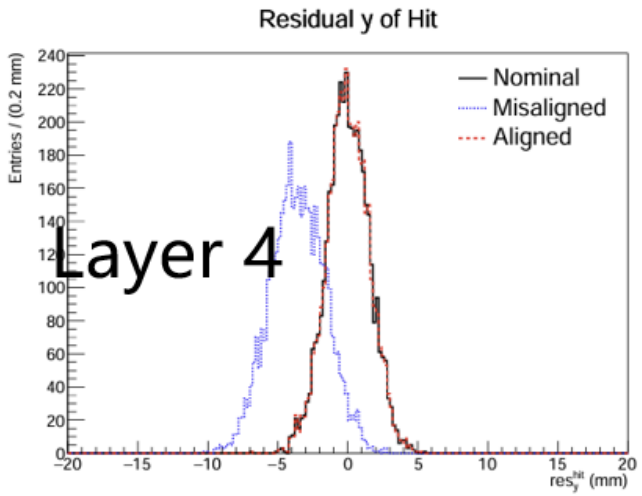
- Particle Gun:
  - 10k muon per event, $p = 10\ \mathrm{GeV}$, direction of momentum along global x-axis
  - Vertex: x, t=0, y and z~2D Gaussian with $\sigma_y = \sigma_z = 2\ \mathrm{mm}$
- Alignment degree of freedom: Translation-in-x/y+Rotation-around-z
- Only align the misaligned layers

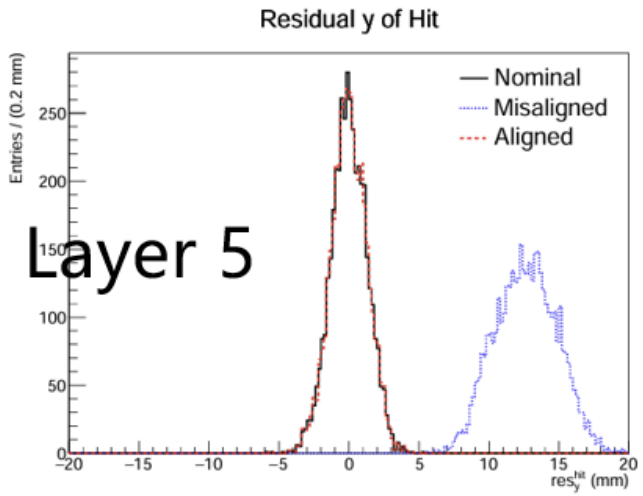


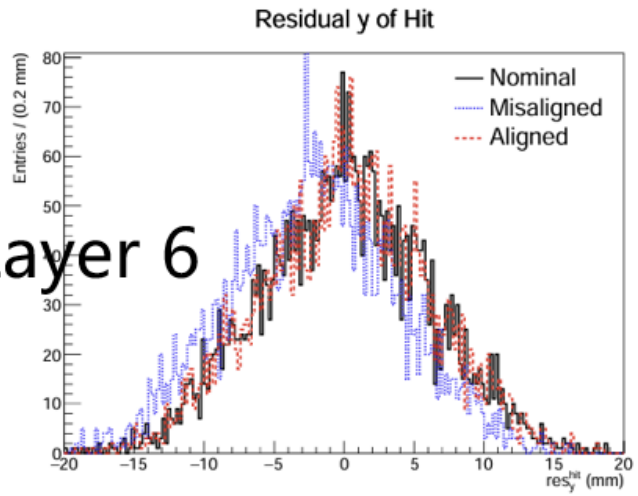
25 x 25 um2 resolution

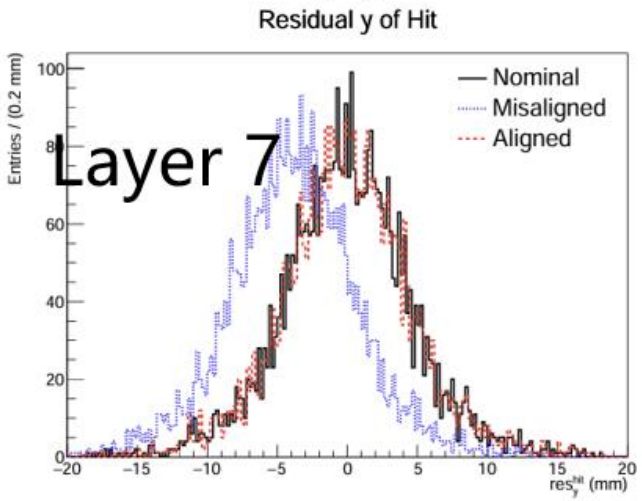


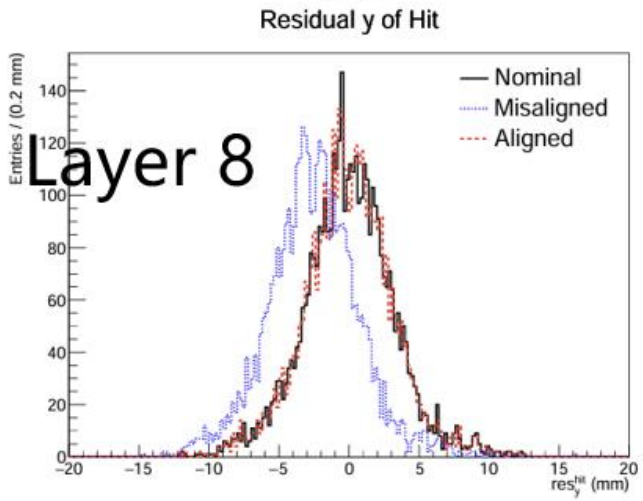
Residual at telescope layer

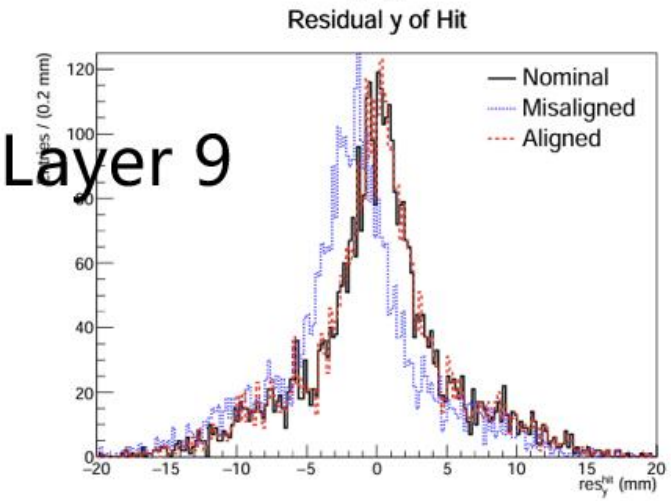








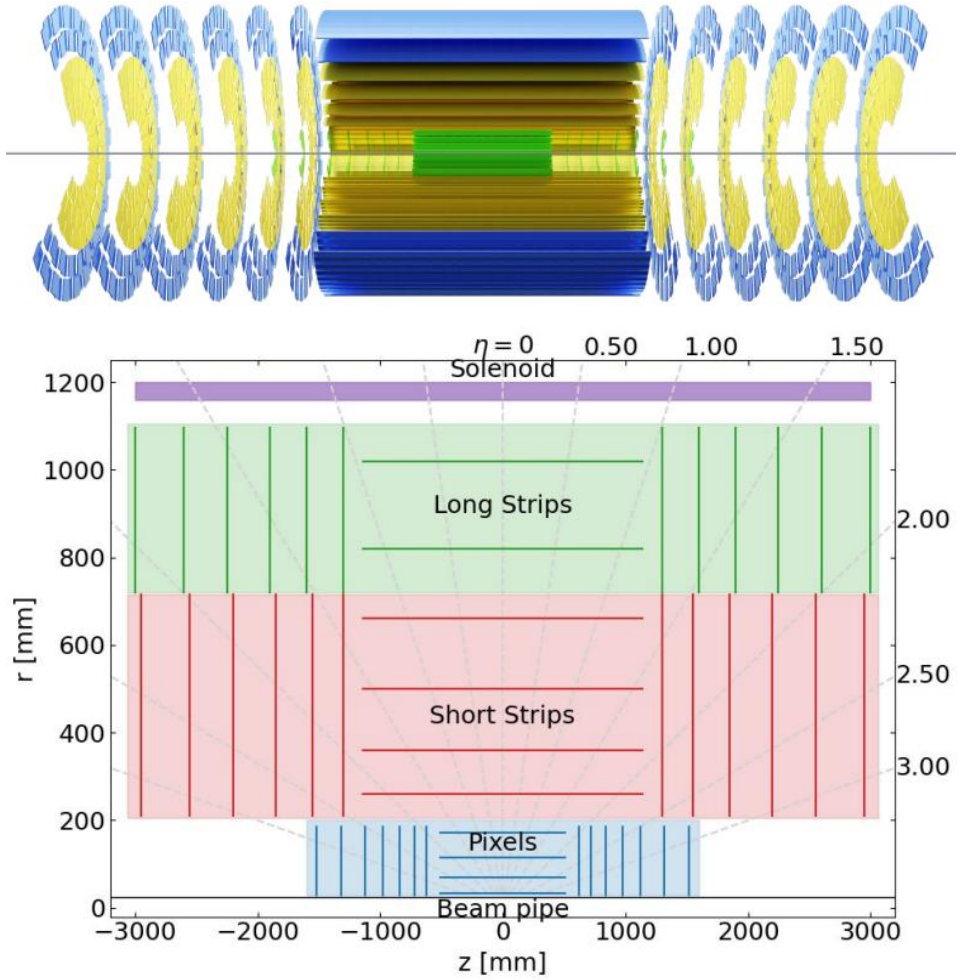




Validation with a realistic example detector

- Pixel
  - 2D Resolution: 15 μm
  - 4 barrel layers + 7 endcap disks
- Short strips
  - 2D Resolution: 43 μm / 1.2 mm
  - 4 barrel layers + 6 endcap disks
- Long strips
  - 1D, two sided with stereo angle (Resolution: 72 μm)
  - 2 barrel layers + 6 endcap disks

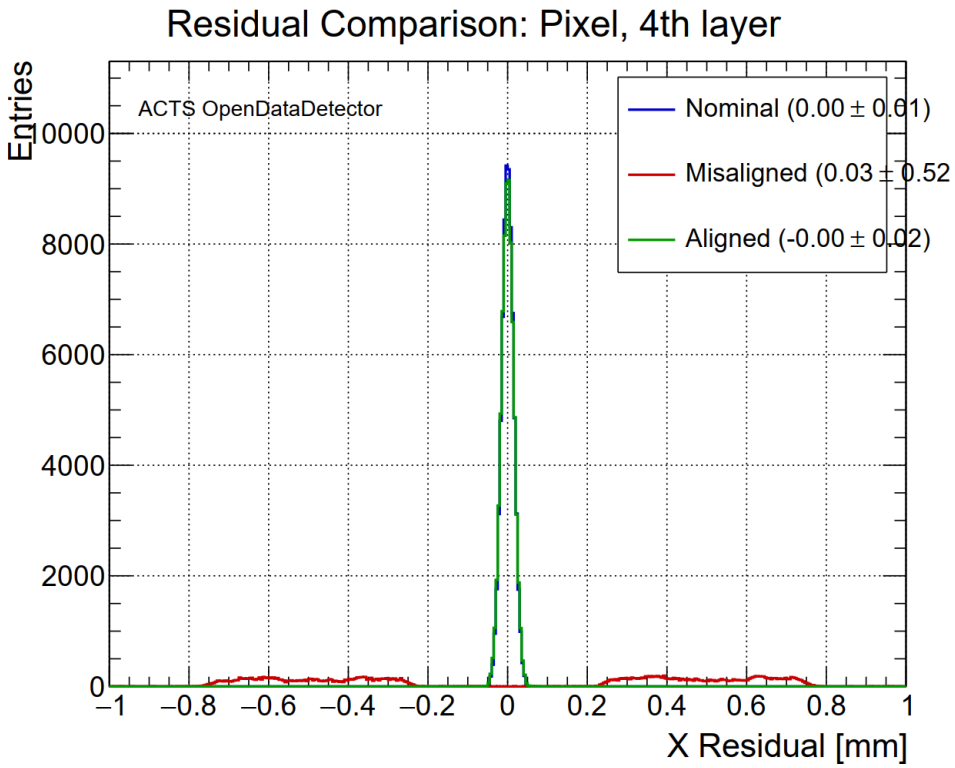
Misalignment decoration (testing two layers for now:
Pixel-4th layer and Short Strip-2nd layer):
→ Translation-in-x/y: +/- [0.25,0.75] mm
→ Rotation-about-z: +/- [0.01,0.03] rad

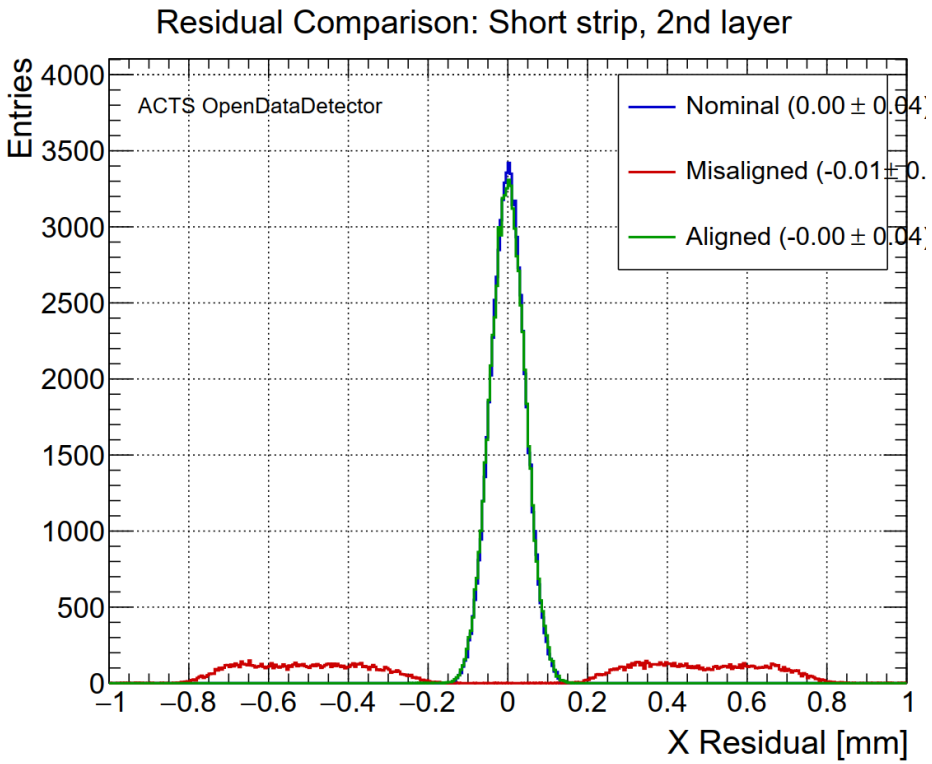


GitLab: https://gitlab.cern.ch/acts/OpenDataDetector
ACAT 2021: https://indico.cern.ch/event/855454/contributions/4596738
CHEP 2023: https://indico.jlab.org/event/459/contributions/11546

Residual

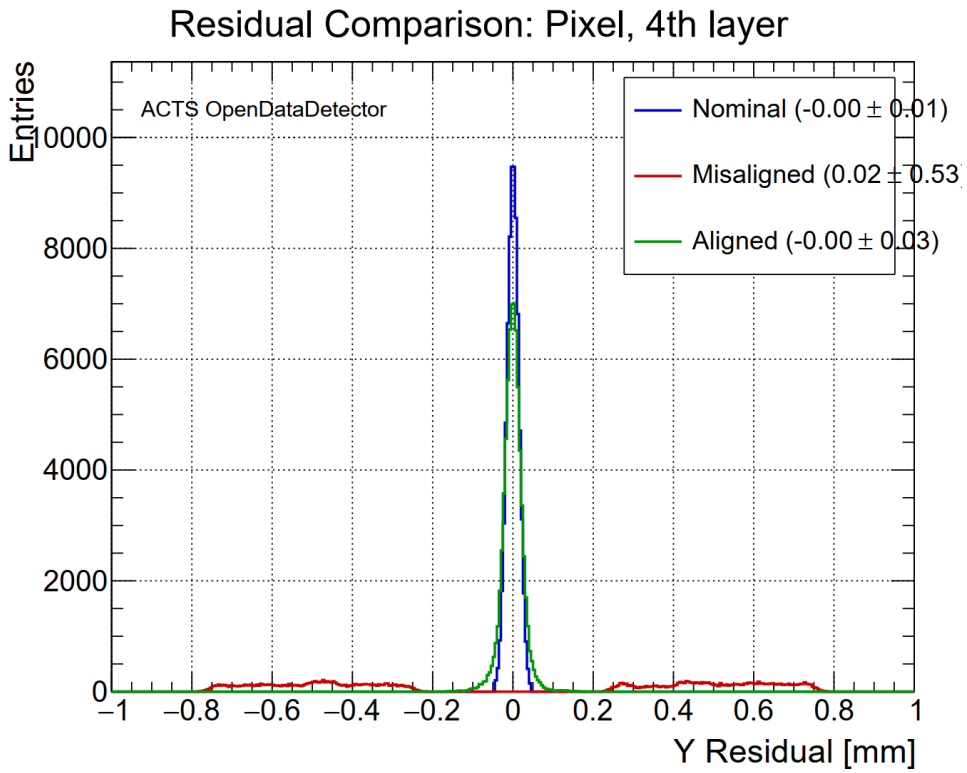
- The residual in local x direction looks good (consistent with results without ideal detector geometry)

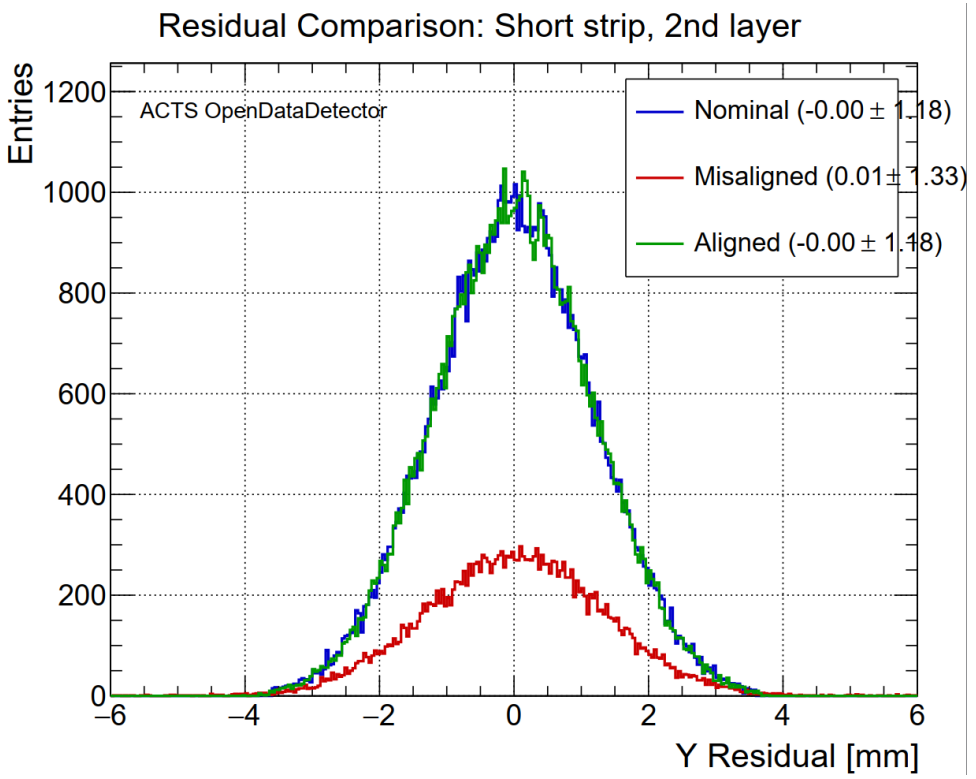




Residual

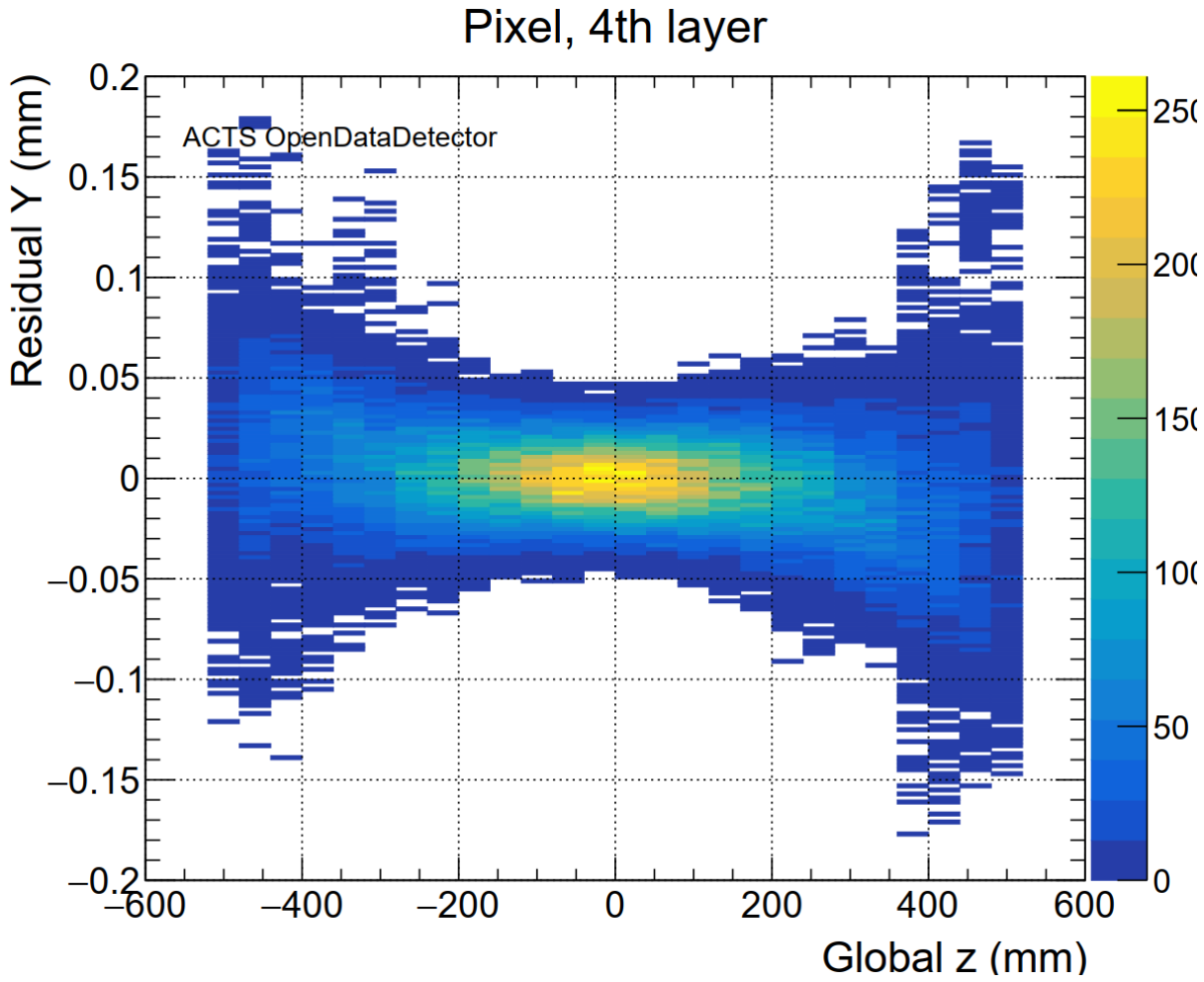
- The residual in local y direction at Pixel is sub-optimal (under investigation)





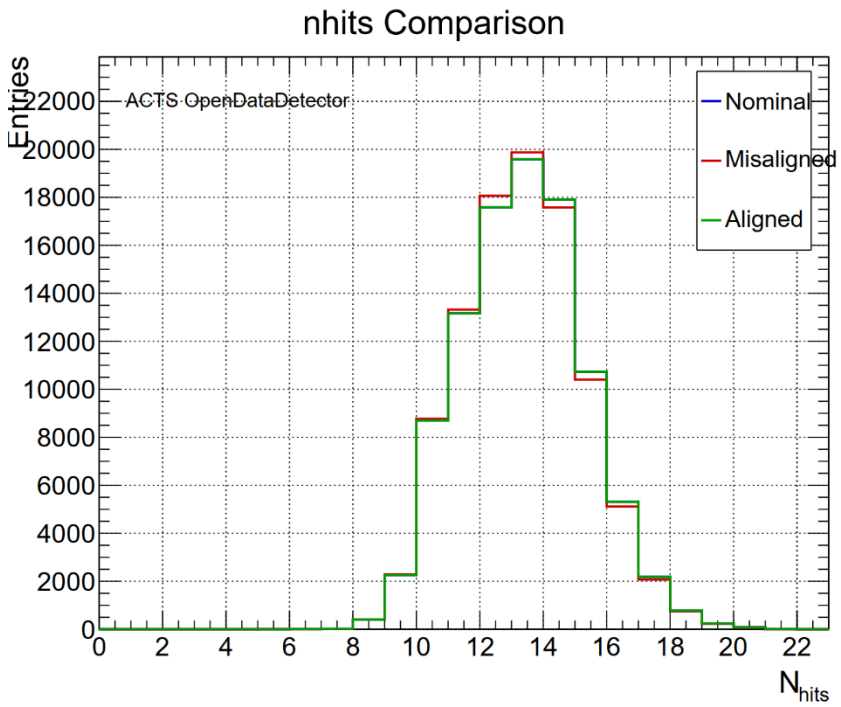
Residual

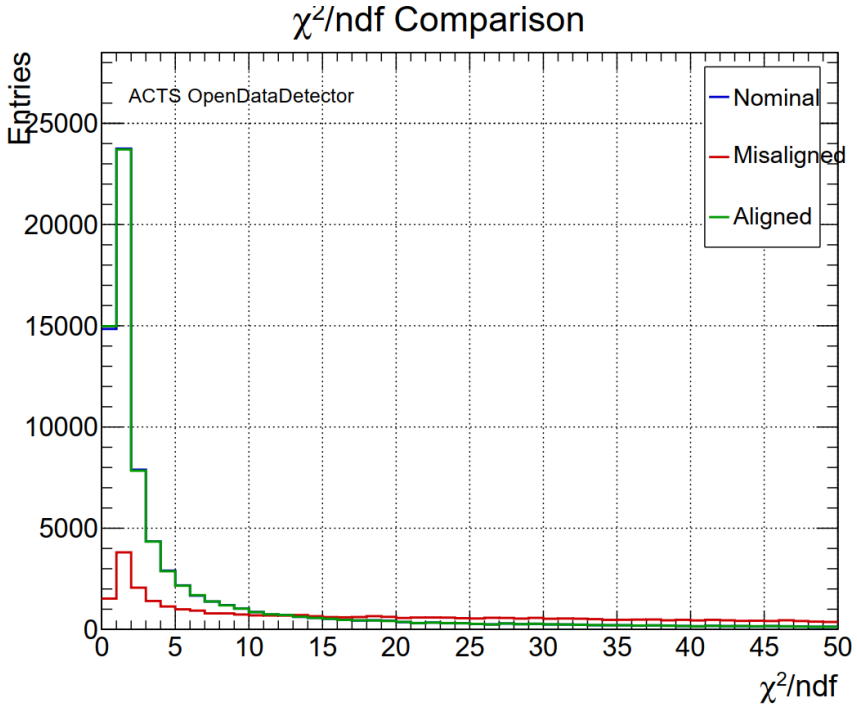
- The residual in local y direction at Pixel gets larger at larger global |z|



Track quality

- Improved nHits and χ^2/ndf for tracks with aligned geometry

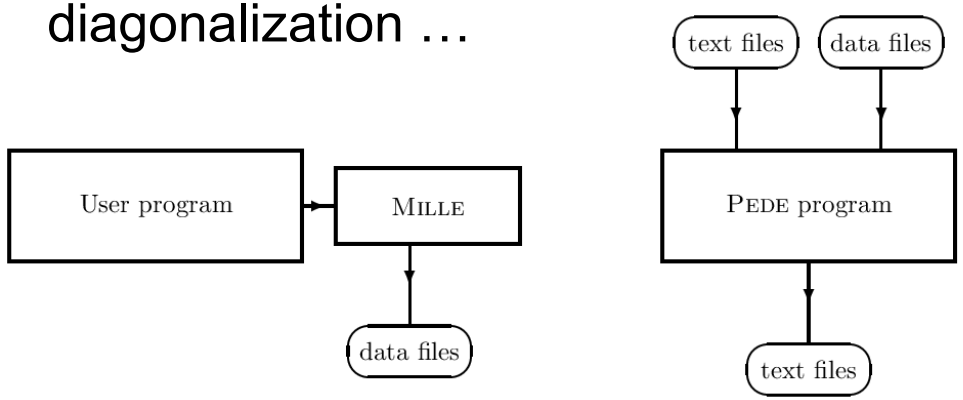




Minimization with MillePede ?

MillePede is a traditional alignment tool used and validated by BESIII, CMS …

- MillePede = Mille + Pede
  - **Mille**: write residual, derivatives of residual w.r.t. alignment parameters and track parameters into binary file
  - **Pede**: solve equation with dimension n to get solution for delta of track parameters :
    - Solver: Inversion, Cholesky decomposition, diagonalization …



Called for each dimension of a residual of each track

◆ mille()

```
void Mille::mille ( int          NLC,
                    const float * derLc,
                    int          NGL,
                    const float * derGl,
                    const int *  label,
                    float        rMeas,
                    float        sigma
                  )
```

Add measurement to buffer.

Parameters

- [in] **NLC** number of local derivatives
- [in] **derLc** local derivatives
- [in] **NGL** number of global derivatives
- [in] **derGl** global derivatives
- [in] **label** global labels
- [in] **rMeas** measurement (residuum)
- [in] **sigma** error

Summary

- ACTS is a tracking software, but also provides ingredients for alignment
  - Validated with a toy telescope-like detector and a realistic Open Data Detector
  - Only surface/module-wise for now. Superstructure alignment is in progress
- Minimization needs to be handled externally by the users, e.g. Mille-Pede.
  - A toy minimization based on Eigen solver is in place more for proof of principle
- Join us if you are interested:
  - https://mattermost.web.cern.ch/acts/channels/acts-alignment